Tugas_Aljabar Linear I Abdullah_Azzam _UTS_20.01.013.075.

1.] $A = \begin{bmatrix} 4a & 8 & 4 \\ 6 & -1 & -3b \\ 5 & 3c & 9 \end{bmatrix}$ dan $B = \begin{bmatrix} 12 & 8 & 4 \\ 6 & -1 & -3a \\ :5 & b & 9 \end{bmatrix}$

Jika $A = B$, $a + b + c = \dots$?

Jawaban :

$4a = 12$ $-3b = -3a$ $3c = b$

$a = 12/4$ $-3b = -9$ $3c = 3$

$a = 3$,, $b = -9/-3$ $c = 3/3$

             $b = 3$ $c = 1.$

$a + b + c = \dots$?

$3 + 3 + 1 = 7$,,

2. $A = \begin{bmatrix} a & 2 \\ 1 & b \end{bmatrix}$ $B = \begin{bmatrix} 4 & 1 \\ 2 & b+1 \end{bmatrix}$,

$C = \begin{bmatrix} -2 & b \\ -a & b^2 \end{bmatrix}$. $\boxed{A \times B^t - C = \begin{bmatrix} 6 & 2 \\ 5 & 4 \end{bmatrix}}$

a dan $b \dots ??$

Jawaban:

$\begin{bmatrix} a & 2 \\ 1 & b \end{bmatrix} \times \begin{bmatrix} 4 & 2 \\ 1 & b+1 \end{bmatrix} - \begin{bmatrix} -2 & b \\ -a & b^2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 5 & 4 \end{bmatrix}$

$= \begin{bmatrix} 4a+2 & 2a+2b+2 \\ b+4 & b^2+b+2 \end{bmatrix} - \begin{bmatrix} -2 & b \\ a & b^2 \end{bmatrix}$

$= \begin{bmatrix} 4a+4 & 2a+b+2 \\ a+b+4 & b+2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 5 & 2 \end{bmatrix}$

Maka, $b + 2 = 4$ $\left\{ \begin{array}{l} a + 2 + 4 = 5 \end{array} \right.$

$b = 4 - 2$ $\qquad a = 5 - 6$

$b = \boxed{2}$, $\qquad = \boxed{-1}$ //

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3. $P = \begin{bmatrix} 12 & 4 \\ 0 & -11 \end{bmatrix}$, $Q = \begin{bmatrix} x & 2y \\ -3 & 4 \end{bmatrix}$,

$R = \begin{bmatrix} 96 & -20 \\ 66 & -44 \end{bmatrix}$ $\boxed{P.Q^T = R, \quad 2x + y = \dots ??}$

Jawaban :

$$\begin{bmatrix} 12x + 8y & -20 \\ -22y & -44 \end{bmatrix} = \begin{bmatrix} 96 & -20 \\ 66 & -44 \end{bmatrix}$$

$\rightarrow 8(-3)$

$\Rightarrow \quad -22y = 66$ $\left\{ \begin{array}{l} 12x + (-3) = 96 \\ 12x = 96 + 24 \\ x = 120/12 \end{array} \right.$

$y = -3$

$\qquad x = 120/12 = 10$

$2x + y = 2(10) + (-3) = 10$

$\qquad = \boxed{17}$ "

4. $A = \begin{bmatrix} x+y & x \\ y & x-y \end{bmatrix}$, $B = \begin{bmatrix} 1 & -\frac{1}{2}x \\ -2y & 3 \end{bmatrix}$

$A^T = B$, $x + 2y = \ldots$??

Jawaban :

$$\begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}x \\ -2y & 3 \end{bmatrix}$$

$$= \quad \begin{aligned} x+y &= 1 \\ \underline{x-y} &= 3 \\ 2x &= 4 \\ x &= 2 \end{aligned} \Biggm\} + \begin{aligned} 2 + y &= 1 \\ y &= 1-2 \\ y &= -1 \end{aligned}$$

$$x + 2y = 2 + 2(-1)$$
$$= \boxed{0} ,$$

5. $A^T = \begin{bmatrix} 6/x & -1 \\ -10/x & 2 \end{bmatrix}$

$B^{-1} = \dfrac{1}{3x-10} \begin{bmatrix} 3 & -2 \\ -5 & x \end{bmatrix}$

$$= \begin{bmatrix} \dfrac{3}{3x-10} & \dfrac{-2}{3x-10} \\ \dfrac{-5}{3x-10} & \dfrac{x}{3x-10} \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{6}{x} & -1 \\ \dfrac{-10}{x} & 2 \end{bmatrix} = \begin{bmatrix} \dfrac{3}{3x-10} & \dfrac{-2}{3x-10} \\ \dfrac{-5}{3x-10} & \dfrac{x}{3x-10} \end{bmatrix}$$

$-1 = \dfrac{-2}{3x-10}$

$-3x + 10 = -2$

$-3x = -12$

$x = 4$

Nilai $2x = 2(4) = 8$

6. $A = \begin{bmatrix} 3 & 5 \\ -1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 5 \\ -1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} -17 & 20 \\ 6 & -7 \end{bmatrix}$

$AB^{-1} = \begin{bmatrix} -17 & 20 \\ 6 & -7 \end{bmatrix}^{(-1)}$

$= \dfrac{1}{-17 \cdot (-7) - 20 \cdot 6} \begin{bmatrix} -7 & -20 \\ -6 & -17 \end{bmatrix}$

$= \begin{bmatrix} 7 & 20 \\ 6 & 17 \end{bmatrix}$

7. $P = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$

$$P^{-1} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}^{(-1)} = \frac{1}{2\cdot3 - 5\cdot1} \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}$$

$$Q^{-1} = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}^{(-1)} = \frac{1}{5\cdot1 - 4\cdot1} \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$$

$$\text{Det } P^{(-1)} = \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix} = 3\cdot2 - (-5)\cdot1 = 11 \text{,,}$$

$$\text{Det } Q^{(-1)} = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix} = 1\cdot5 - (-4)\cdot(-1) = 1 \text{,,}$$

Nilai Det $Q^{-1} \cdot P^{-1} = 11\cdot1 = 11$

8. $\begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, $x^2 + 2xy + y^2 = \dots$??

$= \begin{bmatrix} 2x + 6y \\ x - 3y \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$

$= \left. \begin{array}{l} 2x + 6y = 2 \\ x - 3y = -5 \end{array} \right| \begin{array}{l} \times \frac{1}{2} \\ \times 1 \end{array} \left| \begin{array}{l} x + 3y = 1 \\ x - 3y = -5 \end{array} \right. +$

$\hspace{5cm} 2x = -4$

$\hspace{5cm} x = -2 .\text{,,}$

$= x - 3y = (-2) - 3y = -5$

$\hspace{4cm} -3y = -3$

$\hspace{4cm} y = 1 \text{,,}$

$x^2 + 2xy + y^2 = (-2)^2 + 2.(-2).1 + 1^2$

$\hspace{3cm} = 4 - 4 + 1$

$\hspace{3cm} = 2 .$

9. $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} x & x1 \\ x+y & 2-2 \end{bmatrix}$

$= \begin{bmatrix} 5x+3y & 32-4 \\ 5x+4y & 42-7 \end{bmatrix} = \begin{bmatrix} 21 & 8 \\ 23 & 9 \end{bmatrix}$

$\Rightarrow 5x + 3y = 21$ $\qquad\qquad$ $5x + 4(2) = 23$

$\underline{5x + 4y = 23}$ $\qquad\qquad$ $5x = 23 - 8$

$\qquad -y = -2$ $\qquad\qquad\qquad x = \dfrac{15}{5}$

$\qquad\quad y = 2,,$

$\Rightarrow \quad 32 - 4 = 8$ $\qquad\qquad\qquad x = 3$

$\qquad\quad 32 = 12$

$\qquad\quad 2 = 4,,$

$x + y - 2 = 3 + 2 - 4$

$\qquad\qquad = \boxed{1}$

10. $\begin{bmatrix} 5 & -2 \\ 9 & -4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ x & x+y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -2x+10 & -2x-2y-5 \\ -4x+18 & -4x-4y-9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad -2x+10 = 1 \qquad\qquad \Big| \quad -2x-2y-5 = 0$

$\qquad\qquad -2x = 1-10 \qquad \Big| \quad -2\left(-9/2\right)-2y-5 = 0$

$\qquad\qquad\qquad x = -\dfrac{9}{2} \text{ ,, } \qquad \Big| \quad 9-2y-5 = 0$

$\qquad\qquad\qquad\qquad\qquad\qquad \Big| \qquad y = 2 \text{ ,, .}$

Nilai $x-y = -\dfrac{9}{2} - 2 = \boxed{\dfrac{-13}{2}}$