Example. 1

⇒ 8 computers.
⇒ 3 defective.

⇒ School purchased random "2" comp.

To Find: Probability distribution for the no of defectives.

Solution:
for defective (3 are defective) sample space will be from 2 random purchases.
i.e

{NN , ND , DD}

$\{0, 1, 2\} \longrightarrow$ Assigning random variable.

⇒ keep in mind that only 2 computer are purchased
⇒ probability that both are non-defective.

$$f(0) = P(X=0) = \frac{\binom{3}{0}\binom{5}{2}}{\binom{8}{2}}$$

$\binom{5}{2}$ means that we are taking 2 non-defective pieces out of 5

$\binom{3}{0}$ means that we are taking 0 defective pieces out of 3 defective

$\binom{8}{2}$ means that we are total taking 2 computer out of 8 computer.

⇒ we are taking combination because order does not matters i.e which computer to purchase.

⇒ moving towards solution.

$$f(0) = P(X=0) = \frac{\binom{3}{0}\binom{5}{2}}{\binom{8}{2}} = \frac{10}{28}$$

$$f(1) = P(X=1) = \frac{\binom{3}{1}\binom{5}{1}}{\binom{8}{2}} = \frac{15}{28}$$

$$f(2) = P(X=2) = \frac{\binom{3}{2}\binom{5}{0}}{\binom{8}{2}} = \frac{3}{28}$$

Checking that it is "PMF"

$$\sum f(x) = 1$$

$$\Rightarrow \frac{10}{28} + \frac{15}{28} + \frac{3}{28} = 1$$

$$\Rightarrow \frac{28}{28} = 1$$

$$\Rightarrow 1 = 1$$

So it is PMF.

Example 2

Cars to Sold = 4.
50% of inventory is equipped with air bags.
To find: Probability distribution of no of cars with 4 air bags among next 4 cars.

Solution:

Sample Space will be.

{N,N,N,N , NNNB , NNBB, NBBB , BBBB}
{ 0 1 2 3 4} assigning random variables.

n = total cars = 4
$P = 0.5$ becas 50% of probability of owning bag "
$x = (0, 1, 2, 3, 4)$

formula $= P(X=x) = \binom{n}{x}(P)^x(1-P)^{4-x}$

$= \binom{4}{x}(0.5)^x(1-0.5)^{4-x}$

$= \binom{4}{x}(0.5)^x(0.5)^{4-x}$ bases are same Power will be adde

$= \binom{4}{x}(0.5)^4$

$= \frac{1}{16}\binom{4}{x}$

$= \frac{\binom{4}{x}}{16}.$

alternatively,

$$\overset{f(0)}{\binom{4}{0}}, \overset{f(1)}{\binom{4}{1}}, \binom{4}{2}, \binom{4}{3}, \binom{4}{4}$$

$\Sigma f(x)$

$= 1 + 4 + 6 + 4 + 1$

$= 16$

so

$$f(x) = \frac{\binom{4}{x}}{16}$$

wher $x = \{0, 1, 2, 3, 4\}$

"Commulative Distribution"

$$P(X \leq x) = \sum_{t \leq x} f(t)$$

i.e to add previous terms.

CPDF

Calculating Cumulative Distribution function of example 2.

$f(0) = \frac{1}{16}$, $f(1) = \frac{1}{4}$, $f(2) = \frac{3}{8}$, $f(3) = \frac{1}{4}$

$f(4) = \frac{1}{16}$

x	PMF		CPDF
0	1/16		1/16
1	1/4	$\Rightarrow$ 1/4 + 1/16	5/16
2	3/8	$\Rightarrow$ 5/16 + 3/8	11/16
3	1/4	$\Rightarrow$ 11/16 + 1/4	15/16
4	1/16	$\Rightarrow$ 11/16 + 1/16	16/16 = 1.

note that in CPDF last answel will be each and every tim 1
as $\Sigma f(x) = 1 \Rightarrow$ in Probability distribution function.

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Example 1

$$f(x) = \begin{cases} \frac{x^2}{3} & \text{for } -1 < x < 2 \\ 0 & \text{"otherwise"} \end{cases}$$

To find:- i) verify $f(x) = $ pdf.

ii) find $P(0 < x < 1)$.

Solution:-

$f(x)$ is pdf if $\int_{-\infty}^{\infty} f(x)dx = 1$.

& $f(x) \geq 0$

So verifying.

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{-1} f(x)dx + \int_{-1}^{2} \frac{x^2}{3}dx + \int_{2}^{\infty} f(x)dx$$

$-\infty$ to 1 && 2 to ∞ = 0

So

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-1}^{2} \frac{x^2}{3}dx.$$

$$= \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{3}\left[\frac{8}{3} - \left(\frac{-1}{3}\right)\right]$$

$$= \frac{1}{3}\left(\frac{9}{3}\right)$$

$$= 1$$

so it is pdf.

finding $P(0 < x < 1)$

$$\Rightarrow \int_0^1 f(x)\,dx$$

$$\Rightarrow \int_0^1 \frac{x^2}{3}\,dx$$

$$\Rightarrow \frac{1}{3}\int_0^1 x^2\,dx \qquad \text{By powe rule}$$

$$\Rightarrow \frac{1}{3}\left[\frac{x^3}{3}\right]_0^1$$

$$\Rightarrow \frac{1}{3}\left[\frac{1}{3} - 0\right]$$

$$\Rightarrow \frac{1}{9}$$

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Example 2

$$f(x) = \begin{cases} 0.75(1-x^2) & -1 \le x \le 1 \\ 0 & \text{"otherwise"} \end{cases}$$

To find:- i) x such that $P(X \le x) = 0.95$
ii) $P(-0.5 \le x \le 0.5)$
iii) $P(0.25 \le x \le 2)$

Solution.

i) $P(-0.5 \le x \le 0.5)$

$$\Rightarrow \int_{-0.5}^{0.5} f(x)\,dx$$

$$\Rightarrow \int_{-1/2}^{1/2} 0.75(1-x^2)\,dx$$

$$\Rightarrow 0.75 \int_{-1/2}^{1/2} (1-x^2)\,dx$$

$$\Rightarrow 0.75 \left[x - \frac{x^3}{3} \right]_{-1/2}^{1/2}$$

$$\Rightarrow 0.75 \left[\frac{1}{2} - \frac{\frac{1}{8}}{3} - \left\{ \frac{-1}{2} - \frac{\left(\frac{-1}{8}\right)}{3} \right\} \right]$$

$$\Rightarrow 0.75 \left[\frac{1}{2} - \frac{1}{24} - \left\{ -\frac{1}{2} + \frac{1}{24} \right\} \right]$$

$$\Rightarrow 0.75 \left[\frac{11}{24} - \left\{ \frac{-11}{24} \right\} \right]$$

$$\Rightarrow 0.75 \left[\frac{22}{24} \right] \Rightarrow 0.6875$$

ii $\Rightarrow$ $P(0.25 \le x \le 2)$

$$\Rightarrow \int_{0.25}^{1} f(x)dx + \int_{1}^{2} f(x)dx$$

$$\Rightarrow \int_{1/4}^{1} 0.75(1-x^2)dx$$

$$\Rightarrow 0.75\left[x - \frac{x^3}{3}\right]_{1/4}^{1}$$

$$\Rightarrow 0.75\left[1-\frac{1}{3} - \left\{\frac{1}{4} - \frac{\frac{1}{4}}{3}\right\}\right]$$

$$\Rightarrow 0.75\left[\frac{2}{3} - \left\{\frac{1}{4} - \frac{1}{12}\right\}\right]$$

$$\Rightarrow 0.75\left[\frac{2}{3} - \frac{2}{12}\right]$$

$$\Rightarrow 0.75\left[\frac{6}{12}\right]$$

$$\Rightarrow 0.375.$$

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Example 3

$$f(x) = \begin{cases} \frac{x^2}{3} & \text{for } (-1 < x < 2) \\ 0 & \text{otherwise.} \end{cases}$$

find CDF

$\therefore$ divide $-\infty$ to ∞ in 3 parts.

Lets divide the interval $-\infty$ to $+\infty$ into three parts

for i $\Rightarrow (-\infty < x < -1)$

for ii $\Rightarrow (-1 < x < 2)$

for iii $\Rightarrow (2 < x < \infty)$

let $F(x)$ = CDF & $f(x)$ = PDF then $\rightarrow$ A

$$\text{CDF} = \int_{-\infty}^{x} (\text{PDF})\, dx.$$

for i $\Rightarrow$ CDF $= 0$

for ii $\Rightarrow$ $F(x) = \int_{-1}^{2} f(x)\, dx$ (Checking whether is PDF

$$= \int_{-1}^{2} \frac{x^2}{3} dx \Rightarrow \frac{1}{3} \int_{-1}^{2} x^2 dx \Rightarrow \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{3}\left[\frac{8}{3} + \frac{1}{3}\right] \Rightarrow 1$$

it is PDF using A then.

$$F_x = \int_{-1}^{x} f(t)\, dt \Rightarrow \int_{-1}^{x} \frac{t^2}{3} \Rightarrow \left[\frac{t^3}{9}\right]_{-1}^{x}$$

$$= \frac{x^3}{9} + \frac{1}{9} \Rightarrow \frac{x^3 + 1}{9}$$

So CDF

$$\begin{cases} 0 & \infty < x < -1 \\ \frac{x^3+1}{9} & -1 < x < 2 \\ 1 & 2 < x < \infty \end{cases}$$

Example 4 Lec # 13 pg #

$$f(y) = \begin{cases} \frac{5}{8b} & \text{for } \left(\frac{+2b}{5}\right) < y < 2b \\ 0 & \text{otherwise.} \end{cases}$$

find $F(y)$ "commulative distribution function"

Solution.

The relation between PDF & CDF is ($f(x)$ $F(x)$)

$$F(x) = \int_{-\infty}^{x} f(t)\,dt \quad fo -\infty < x < \infty$$

Checking whether $f(y)$ is PDF or not.

$$f(y) = \int_{+2b/5}^{2b} \frac{5}{8b}\,dy$$

$$= \frac{5}{8b} \int_{+2b/5}^{2b} 1\,dy$$

$$\Rightarrow \frac{5}{8b}\left[y\right]_{+2b/5}^{2b}$$

$$\Rightarrow \frac{5}{8b}\left[2b-\frac{2b}{5}\right]$$

$$\Rightarrow \frac{\cancel{5}}{\cancel{8b}}\left[\frac{\cancel{8b}}{\cancel{5}}\right]$$

$\Rightarrow 1$ so it is PDF

Calculating CDF

$$F(y) = \int_{+2b/5}^{y} f(y)\,dt$$

$$F(y) = \int_{+2b}^{y} \frac{5}{8b}\,dt$$

$$= \frac{5}{8b}\left[t\right]_{+2b/5}^{y}$$

$$= \frac{5}{8b}\left(y \mp 2b/5\right)$$

so CDF is $\begin{cases} 0 & -\infty < y < -\frac{2b}{5} \\ \frac{5}{8b}(y+2b/5) & +\frac{2b}{5} < y < 2b \\ 1 & 2b < y < -\infty \end{cases}$

for $P(X \leq b)$

$$F(b) = \frac{5}{8b}\left(b - \frac{2b}{5}\right)$$

$$F(b) = \frac{5}{8b}\left(\frac{3b}{5}\right)$$

$$F(b) = \frac{3}{8}$$

The probability that the winning bid is less than the DOE's preliminary estimate b is 3/8.

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Example 5

Given: ⇒ Two ballpens are selected at random.
⇒ Total Ballpens = 8
out of ⇒ Green = 3 ; Red = 2 ; Blue 3

⇒ X = no of blue pens selected
⇒ Y = no of Red pens selected

To find joint probability $f(x,y)$ and $P[(x,y) \in A]$ wher A is the region $\{(x,y) \mid x+y \le 1\}$.

Solution

first make joint probability chart.

$f(x,y)$		Y 0	1	2	Red
X	0	3/28	6/28	1/28	10/28
	1	9/28	6/28	0	15/28
blue	2	3/28	0	0	3/28
		15/28	12/28	1/28	1

$$f(0,0) \Rightarrow \frac{{}^3C_2\,{}^2C_0\,{}^3C_0}{{}^8C_2} \Rightarrow 3/28$$

$$f(0,1) \Rightarrow \frac{{}^3C_1\,{}^2C_1\,{}^3C_0}{{}^8C_2} \Rightarrow 6/28$$

$$f(0,2) \Rightarrow {}^3C_0\,{}^2C_2\,{}^3C_0 \Rightarrow 1/28$$

$$f(1,0) \Rightarrow {}^3C_1\,{}^2C_0\,{}^3C_1 \Rightarrow 9/28$$

$$f(1,1) \Rightarrow {}^3C_0\,{}^2C_1\,{}^3C_1 \Rightarrow 6/28$$

$$f(2,0) \Rightarrow {}^3C_0\,{}^2C_0\,{}^3C_2 \Rightarrow 3/28$$

Now for condition

$\{(x,y) \mid x+y \leq 1$

$$p(x,y) = f(0,0), + f(0,1) + f(1,0)$$

$$= \frac{3}{28} + \frac{6}{28} + \frac{9}{28}$$

$$= \frac{18}{28} = \frac{9}{14}$$

Example #(6)1

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Simultaneous flip of two coins as Joint PDF.

Sampl Space = {HH, HT, TH, TT.}
Total possiblities are = 4.

let X represents ~~X~~ H comes
& Y represents T comes.
so Joint PDF x, y will be

$f(x,y)$	Y: 1 T	2 T	
X H 1	1/4	1/4	1/2
H 2	1/4	1/4	1/2
	1/2	1/2	1