Lec # 14
Pg # 6

Example 2

$$f(x) = \begin{cases} \frac{2}{5}(2x+3y) & 0 \le x \le 1 \\ & 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Verify condition 2
i.e $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

& Find
$P[(x,y) \in A]$ where $A = \{(x,y) \mid 0 \le x \le 1/2 ; 1/4 \le y \le 1/2\}$

Solution:-

$$\int_0^1 \left(\int_0^1 f(x)\,dx \right) dy$$

So first, integrating inside.

$$\Rightarrow \int_0^1 \frac{2}{5}(2x+3y)\,dx$$

$$\Rightarrow \frac{2}{5}\int_0^1 (2x+3y)\,dx$$

$$\Rightarrow \frac{2}{5}\left(\cancel{2}\frac{x^2}{\cancel{2}} + 3xy\right)\Big|_0^1$$

$$\Rightarrow \frac{2}{5}\left((1)^2 + 3(1)y - 0\right)$$

$\Rightarrow \frac{2}{5}(1+3y)$

integration of outside.

$\Rightarrow \frac{2}{5}\int_0^1 (1+3y)\,dy.$

$\Rightarrow \frac{2}{5}\left[y+\frac{3}{2}y^2\right]_0^1$

$\Rightarrow \frac{2}{5}\left[1+\frac{3}{2}(1)^2-0\right]$

$\Rightarrow \frac{2}{5}\left[1+\frac{3}{2}\right.$

$\Rightarrow \frac{\cancel{2}}{\cancel{5}}\left[\frac{\cancel{5}}{\cancel{2}}\right] \Rightarrow 1$

So 1st condition is satisfied.

Secondly; $P[(x,y)\in A]$ where $A=\{(x,y) \mid 0 \le x \le 1/2,\ 1/4 \le y \le 1/2$

Solution:-

$$\int_{1/4}^{1/2}\left(\int_0^{1/2} f(x)\,dx\right)dy$$

$$\int_{1/4}^{1/2}\int_{0}^{1/2} \frac{2}{5}(2x+3y)\,dx\,dy$$

$$\frac{2}{5}\int_{1/4}^{1/2}\int_{0}^{1/2}(2x+3y)\,dx\,dy.$$

$$\frac{2}{5}\int_{1/4}^{1/2}\left[x^2+3xy\right]_0^{1/2}dy.$$

$$\frac{2}{5}\int_{1/4}^{1/2}\left[\frac{1}{4}+\frac{3}{2}y\right]dy$$

$$\frac{1}{5}\int_{1/4}^{1/2}\left(\frac{1}{2}+3y\right)dy$$

$$\frac{1}{5}\left[\frac{y}{2}+\frac{3y^2}{2}\right]_{1/4}^{1/2}$$

$$\frac{1}{10}\left[y+3y^2\right]_{1/4}^{1/3}$$

$$\Rightarrow \frac{1}{10}\left[\frac{1}{2}+3\left(\frac{1}{2}\right)^2-\left(\frac{1}{4}+3\left(\frac{1}{4}\right)^2\right)\right]$$

$$\Rightarrow \frac{1}{10}\left[\frac{1}{2}+\frac{3}{4}-\left(\frac{1}{4}+\frac{3}{16}\right)\right]$$

$$\Rightarrow \frac{1}{10}\left[\frac{5}{4}-\frac{7}{16}\right]$$

$$\Rightarrow \frac{1}{10}\left[\frac{13}{16}\right]$$

$$\Rightarrow \frac{13}{160}$$

Example # 2.

Lecture 16.

Solution:-

$$f(x,y) = \frac{x(1+3y^2)}{4}$$

$$g(x) = \int_0^1 \frac{x(1+3y^2)}{4}\,dy$$

$$\Rightarrow \frac{1}{4}\int_0^1 (x+3xy^2)\,dy.$$

$$\Rightarrow \frac{1}{4}\left[xy + \frac{\cancel{3}xy^3}{\cancel{3}}\right]_0^1$$

$$\Rightarrow \frac{x}{4}\left[y+y^3\right]_0^1$$

$$\Rightarrow \frac{x}{4}\left[1+1-0\right]$$

$$\Rightarrow \frac{x}{2}$$

$$h(y) = \int_0^2 \frac{x(1+3y^2)}{4}\,dx$$

$$\Rightarrow \frac{1}{4}\int_0^2 (x+3xy^2)\,dx$$

$$\Rightarrow \frac{1}{4}\left[\frac{x^2}{2} + \frac{3x^2y^2}{2}\right]$$

$$\Rightarrow \frac{x^2}{8}\left(1+3y^2\right)\Big|_0^2$$

$$\Rightarrow \frac{(2)^2}{8}\left(1+3y^2\right)$$

$$\Rightarrow \frac{1}{2}\left(1+3y^2\right)$$

Combining $h(y)$ & $g(x)$

$$g(x)\cdot h(y) = \frac{x}{2} \times \frac{1}{2}\left(1+3y^2\right)$$

$$= \frac{x\left(1+3y^2\right)}{4}$$

$$= f(x, y)$$

So it is statistically independent

Example # 3.

$$f(x, y) = 10xy^2$$

$$g(x) = \frac{10x(1-x^3)}{3}$$

$$h(y) = 5y^4$$

Chk are these statistically independent.

$$g(x) \cdot h(y) = \frac{10x(1-x^3)}{3} \cdot 5y^4$$

$$\Rightarrow \frac{50xy^4(1-x^3)}{3}$$

$$\neq f(x, y)$$

not

so X & Y are statistically independent.

Example # 4. lec # 16.

after seeing Q.
Chk Statistical independence of 3/14.

Solution:-

$f(0,1) = \frac{3}{14}.$

$g(0) = ?$ & $h(1) = ?$

treat 0, & 1 as x then things will be easy to understands

$$g(0) = \sum_{y=0}^{2} f(0,y) \Rightarrow f(0,0) + f(0,1) + f(0,2)$$

$$\Rightarrow \frac{3}{28} + \frac{3}{14} + \frac{1}{28}$$

$$\Rightarrow \frac{10}{28} \Rightarrow \frac{5}{14}.$$

$$h(1) = \sum_{x=0}^{2} f(x,1) \Rightarrow f(0,1) + f(1,1) + f(2,1)$$

$$\Rightarrow \frac{3}{14} + \frac{3}{14} + 0$$

$$\Rightarrow \frac{6}{14} \Rightarrow \frac{3}{7}$$

$$g(0) \cdot h(1) \Rightarrow \frac{5}{14} \cdot \frac{3}{7} \Rightarrow \frac{15}{98} \neq f(0,1)$$

So, it is not statistically independent.

Example # 5

⇒ Total components = 7
⇒ Good components = 4
⇒ Defective components = 3

⇒ Sample taken = 3

To find: no of good components from 3.

Solution:

Sample space will be as:

{GGG, GGD, GDD, DDD}

{0, 1, 2, 3} ⇒ assigning random

$$f(0) \Rightarrow P(X=0) \Rightarrow \frac{({}^4C_3)({}^3C_0)}{({}^7C_3)} \Rightarrow \frac{4 \cdot 1}{35} \Rightarrow \frac{4}{35}$$

$$f(1) \Rightarrow P(X=1) \Rightarrow \frac{({}^4C_2)({}^3C_1)}{({}^7C_3)} \Rightarrow \frac{6 \cdot 3}{35} \Rightarrow \frac{18}{35}$$

$$f(2) \Rightarrow P(X=2) \Rightarrow \frac{({}^4C_1)({}^3C_2)}{({}^7C_3)} \Rightarrow \frac{4 \cdot 3}{35} \Rightarrow \frac{12}{35}$$

$$f(3) \Rightarrow P(X=3) \Rightarrow \frac{({}^4C_0)({}^3C_3)}{({}^7C_3)} \Rightarrow \frac{1 \cdot 1}{35} \Rightarrow \frac{1}{35}$$

from defination of mean.

$$\mu = E(x) = \sum_{x=0}^{3} x f(x)$$

$$\Rightarrow x_0 f(0) + x_1 f(1) + x_2 (f(2)) + x_3 f(3)$$

$$\Rightarrow \frac{4}{35} + \frac{18}{35} + \frac{12}{35} + \frac{1}{35}$$

$$\Rightarrow \frac{\cancel{60}}{\cancel{35}} \Rightarrow \frac{12}{7}$$

Example 6 lec # 16.

$$f(x) = \frac{20000}{x^3} ; \quad x > 100$$
Otherwise 0.

Expected life = ?

Solution.

$$E(x) \Rightarrow \int_{-\infty}^{100} x f(x) + \int_{100}^{\infty} x f(x)$$

$$E(x) \Rightarrow 0 + \int_{100}^{\infty} \cancel{x} \cdot \frac{20,000}{x^{\cancel{3}2}}$$

$$E(x) = 20000 \int_{100}^{\infty} x^{-2}\, dx$$

$$E(x) = 20000 \left[\frac{x^{-1}}{-1} \right]_{100}^{\infty}$$

$$E(x) = 20000 \left[\frac{1}{-x} \right]_{100}^{\infty}$$

$$E(x) = 20000 \left[\frac{1}{-\infty} + \left(\frac{1}{+100} \right) \right]$$

$$E(x) = 20{,}000 \left[\frac{1}{100} \right.$$

$$E(x) = 200.$$