

CS 341 Final

Name:

1. Prove by mathematical induction that

$$\sum_{i=1}^n i^3 = \left(\sum_{i=1}^n i\right)^2.$$

Hint $\sum_{i=1}^n i = \frac{n(n+1)}{2}$.

2. Prove by mathematical induction that

$$T(n) = n^2 + n$$

is the solution to the recurrence relation

$$T(0) = 0 \text{ and } T(n) = 2n + T(n - 1).$$

3. A binary search tree is to be built, using the algorithm given in class. The following numbers are input in the order given: 30, 50, 15, 60, 40, 10, 20, 70, 55, 45, 47, 46, 56, 59 and 57. Draw the resulting tree.

4. Next the numbers 50, 30, 47 and 55 are deleted, using the algorithm given in class, from the tree resulting from the previous problem and in the given order. Draw the tree after each deletion.

5. An AVL tree is to be built, using the algorithm given in class. The following numbers are input in the order given: 30, 50, 15, 60, 40, 10, 20, 70, 55, 45, 47, 46, 56, 59 and 57. Draw the resulting tree. Be sure to redraw the tree for each rotation and indicate if it is a single or a double rotation.

6. What is the error in the following program segment. Explain in detail what will happen if it is not fixed?

```
#include <iostream>
using namespace std;
void main() {
    struct link{ int info; link *next;};
    link *top, *curr, *prev, *ptr;
    int ival;

    /*
       A missing segment of code builds a linked list
       with top as a pointer to the top.
    */

    cin >> ival;
    prev = 0; curr = top;

    while(curr && curr->info != ival){
        curr = curr->next;
    }
    if(curr){
        if(!prev)
            top = top->next;
        else
            prev->next = curr->next;
        delete curr;
    }
}
```

7. The list

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$$

is stored in a linked list. The linked list is stored in two arrays as discussed in class. Fill in the missing information.

top

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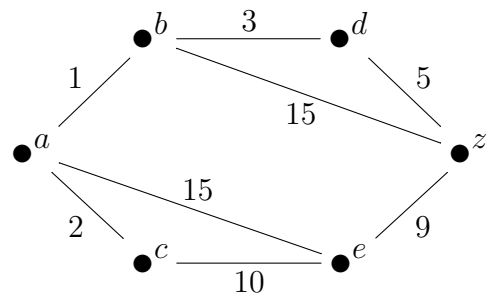
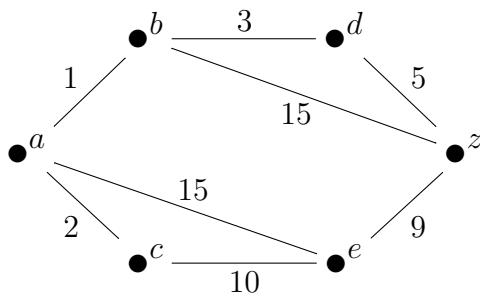
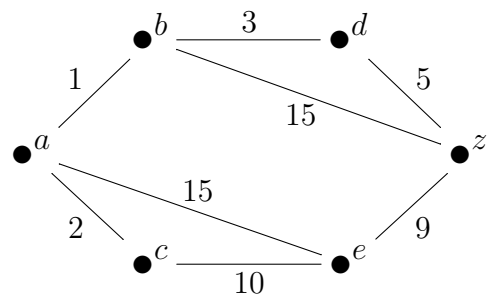
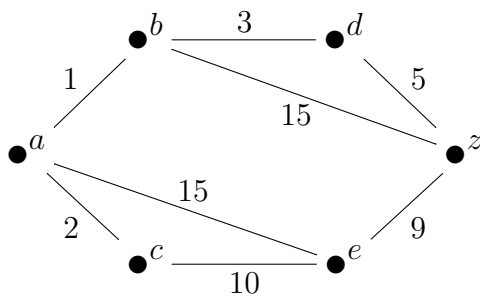
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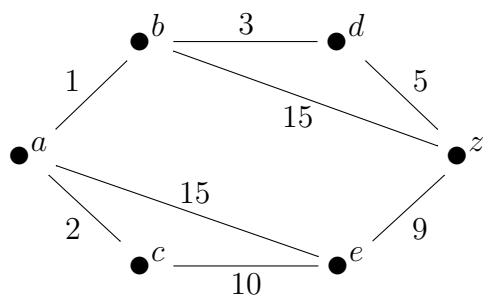
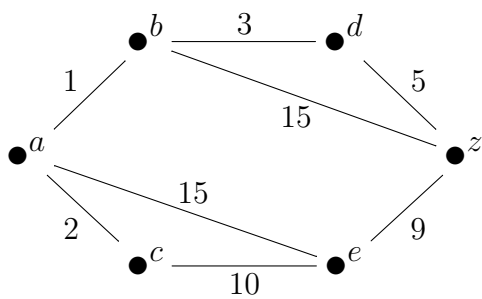
1	2	3	4	5	6	7	8
x_4	x_2	x_8	x_6	x_7	x_1	x_3	x_5

Next

1	2	3	4	5	6	7	8

8. For the weighted graph given below, use the algorithm discussed in class to find the shortest path from a to z . Be sure to show all of your steps and to redraw the graph along with the indices and candidate shortest paths each time that the nodes are re-indexed. For your convenience you are provided with several copies of the graph so that you need not waste time redrawing the graph. You may or may not need all of the copies given.





9. The quicksort algorithm given in class is used to sort the array with entries 30, 20, 25, 28, 50, 15, 49, 14, 6 Draw the array each time elements are swapped. Be sure to indicate which elements are being swapped.

10. The array 1, 9, 30, 25, 10, 50 is to be sorted by heap sort using the algorithm given in class, redraw the array each time that an entry changes. You may redraw only once for each swap. Be sure to indicate which elements are being swapped.

