Exam online 3 hours

There will be eight questions

40% each question is of 5 points

## lecture 26

Trapezoidal Rule / Composite form of them
Simpson's Rule

Newton's open / closed Cotes formulas

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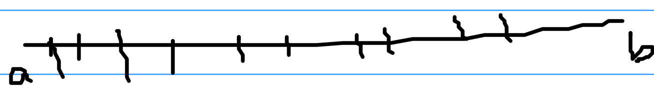
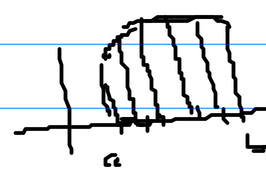


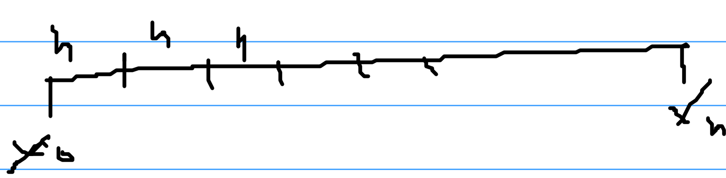

## Gaussian Quadrature

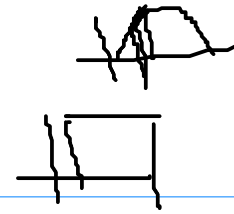



$$\int_a^b f(x)\,dx \approx \sum_{i=0}^{n} c_i f(x_i)$$

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3} f(x_0) + \frac{4h}{3} f(x_1) + \dots$$

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] + \frac{h^2}{2} f'(\xi)$$



$$\int_a^b f(x)\,dx \simeq \sum_{i=1}^{n} c_i \, f(x_i)$$

Total number of $c_i$ is $n$

Total number of $x_i$ is also $n$

$2n$ , $(2n-1)$

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] + \frac{h^2}{2} f''(\xi).$$

We are going to find out $c_i$ & $x_i$ for $n=2$ in the interval $[-1, 1]$

$(0, 1)$

$f(x) = ax + b$

$$\int_{-1}^{1} f(x)\,dx \simeq \sum_{n=1}^{2} c_i \, f(x_i)$$

$$= c_1 f(x_1) + c_2 f(x_2)$$

$n=2$ Total number of unknown is 4.

This is going give us accurate result for polynomial of degree less than or equal to 3

$1, \quad x, \quad x^2, \quad x^3$ $\qquad ax^3 + bx^2 + cx + d$

$f(x) = 1$

$$\int_{-1}^{1} 1\,dx = c_1(1) + c_2(1)$$

$$\left| x \right|_{-1}^{1} = c_1 + c_2, \qquad 1 - (-1) = c_1 + c_2$$

$$\boxed{2 = c_1 + c_2} \quad (1)$$

$f(x) = x$

$$\int_{-1}^{1} x\,dx = c_1 x_1 + c_2 x_2, \qquad \left| \frac{x^2}{2} \right|_{-1}^{1} = c_1 x_1 + c_2 x_2$$

$$\frac{1}{2}\{(1)^2 - (-1)^2\} = c_1 x_1 + c_2 x_2, \qquad \boxed{c_1 x_1 + c_2 x_2 = 0} \quad (2)$$

$f(x) = x^2$

$$\int_{-1}^{1} x^2\,dx = c_1 x_1^2 + c_2 x_2^2, \qquad \left| \frac{x^3}{3} \right|_{-1}^{1} = c_1 x_1^2 + c_2 x_2^2$$

$$\frac{(1)^3}{3} - (-1)^3/3 = c_1 x_1^2 + c_2 x_2^2$$

$$\boxed{2/3 = c_1 x_1^2 + c_2 x_2^2} \quad (3)$$

$f(x) = x^3$

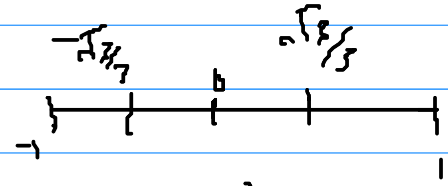
$$\int_{-1}^{1} x^3\,dx = c_1 x_1^3 + c_2 x_2^3, \qquad \left| \frac{x^4}{4} \right|_{-1}^{1} = c_1 x_1^3 + c_2 x_2^3$$

$$\boxed{0 = c_1 x_1^3 + c_2 x_2^3} \quad (4)$$

After solving equations (1) – (4) we get

$$c_1 = 1, \qquad c_2 = 1$$

$$x_1 = -\sqrt{3}/3, \qquad x_2 = \sqrt{3}/3$$



$n=2$

$$\int_a^b f(x)\,dx \simeq (1)\, f\left(-\sqrt{3}/3\right) + 1\, f\left(\sqrt{3}/3\right)$$

Example

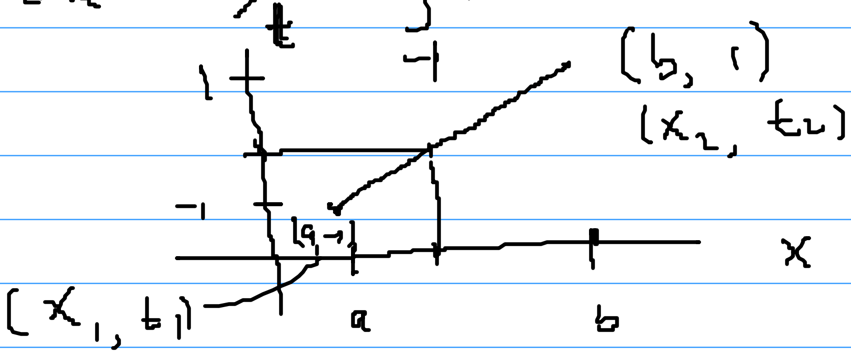
$$\int_{-1}^{1} e^{x^2}\,dx \simeq (1)\, e^{\left(\sqrt{3}/3\right)^2} + (1)\, e^{\left(\sqrt{3}/3\right)^2}$$

$n=5$

$$\int_{-1}^{1} e^{x^2}\,dx = \sum_{i=1}^{5} c_i f(x_i) = c_1 e^{x_1^2} + c_2 e^{x_2^2} + c_3 e^{x_3^2} + c_4 e^{x_4^2}$$

For any general interval $[a, b]$

$$\int_a^b f(x)\,dx \longrightarrow \int_{-1}^{1} f(\;)$$



$$\frac{t_2 - t_1}{x_2 - x_1} = \frac{t - t_1}{x - x_1}$$

$$\frac{1 - (-1)}{b - a} = \frac{t - (-1)}{x - a}$$

$$\frac{2}{b-a} = \frac{t+1}{x-a}$$

$x = a$

$t = 1$

$x = b$

$t = -1$

$$\frac{2(x-a)}{b-a} = t + 1$$

$$\frac{2(x-a)}{(b-a)} - 1 = t$$

$x \to t$

$$\int_a^b f(x)\,dx = \int_{-1}^{1} f(t)\,d(t)$$

$$x - a = \frac{(t+1)}{2}(b-a), \quad x = \frac{(t+1)}{2}(b-a) + a$$

$$= \int_{-1}^{1} f\left(\frac{(t+1)(b-a)}{2} + a\right) d\left(\frac{(t+1)}{2}(b-a) + a\right)$$

$$\boxed{\int_a^b f(x) = \int_{-1}^{1} f\left[\frac{(t+1)(b-a)}{2} + a\right] \left(\frac{b-a}{2}\right) dt}$$

Example $\int_1^2 e^{x^2}\,dx$ $\quad a=1$, $b=2$

$$\int_1^2 e^{x^2}\,dx = \int_{-1}^{1} f\left(\frac{t+1}{2}+1\right)\frac{1}{2}\,dt$$

$$= \frac{1}{2}\int_{-1}^{1} f\left(\frac{t+1}{2}+1\right)dt = \frac{1}{2}\int_{-1}^{1} e^{\left(\frac{t+1}{2}+1\right)^2}dt$$

$n=2$

$$= \frac{1}{2}\left[C_1 f(t_1) + C_2 f(t_2)\right]$$

$$= \frac{1}{2}\left[(1)\, e^{\left[\frac{-\sqrt{3}/3+1}{2}+1\right]^2} + (1)\, e^{\left[\frac{\sqrt{3}/3+1}{2}+1\right]^2}\right]$$

$C_1 = 1 \quad t_1 = -\sqrt{3}/3$

$C_2 = 1 \quad t_2 = \sqrt{3}/3$

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Next Topics are about

Numerical solution of differential Equation