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Q1):-

$\sin x + x^2 - 1 = 0$. $[0, 1]$.

$x^2 = 1 - \sin x$ $\sin x = 1 - x^2$

$x = \pm\sqrt{1 - \sin x}$ $x = \sin^{-1}(1 - x^2)$

$g(0) = \sqrt{1 - \sin(0)} = 1$

$g(1) = 0.398$.

$0 \leq g(x) \leq 1$.

$$g'(x) = \frac{1}{2}(1 - \sin x)^{-1/2} \times -\cos x$$

$$\frac{-\cos x}{2\sqrt{1 - \sin x}}$$

$$g'(0) = \frac{-\cos(0)}{2\sqrt{1 - \sin(0)}} = \frac{-1}{2} \leq 1.$$

$g'(x) \leq k < 1$.

$P_0 = 0.5$.

$g(P_n) = \sqrt{1 - \sin(P_{n-1})}$

$g(P_1) = \sqrt{1 - \sin(0.5)} = 0.7215$

$g(P_2) = \sqrt{1 - \sin(0.7215)} = 0.58265$

$g(P_3) = \sqrt{1 - \sin(0.58265)} = 0.670642$

$g(P_4) = \sqrt{1 - \sin(0.670642)} = 0.61523$

root = 0.636732658.

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Q3):-

$f(x) = \sin \pi x$

x	0	$1/6$	$1/2$
$f(x)$	0	$1/2$	1.

$f(x, x_0, x_1, x_2, x_3) =$

$$\frac{f(x_0)(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + \frac{f(x_1)(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$\frac{f(x_2)(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$0 + \frac{1/2(x)(x-1/2)}{\left(\frac{1}{6}\right)\left(-\frac{1}{3}\right)} + \frac{(1/6)(x-0)(x-1/6)}{\left(\frac{1}{2}-0\right)\left(\frac{1}{2}-\frac{1}{6}\right)}$$

$$-9x\left(x-\frac{1}{2}\right) + x\left(x-\frac{1}{6}\right)$$

$$-9x^2 + \frac{9}{2}x + x^2 - \frac{1}{6}x$$

$$\frac{13}{3}x - 8x^2.$$

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Q4):- $f(x) = x^2$ $\quad N \leq x \leq N+1$.

$f(N) = N^2$

$f(N+1) = (N+1)^2$.

x	N	$N+1$
y	N^2	N^2+2N+1.

$$P_2(x) = \frac{f(x_0)(x-x_1)}{(x_0-x_1)} + \frac{f(x_1)(x-x_0)}{(x_1-x_0)}$$

$$\frac{N^2(x-N+1)}{N-N-1} + \frac{(N+1)^2(x-N)}{N+1-N}$$

$$E(x) = \frac{(x-N)(x-N-1)f''(\xi(x))}{(2!)}$$

$f'(x) = 2x$

$f''(x) = 2$.

$$\frac{(x-N)(x-N-1) \times 2(\xi(x))^0}{2}$$

replace $\xi(x)$ by $N+1$.

$E(x) = (x-N)(x-N-1)$.

$x^2 - xN - x - xN + N$.

$x^2 - 2xN - x + N$

$x^2 + x(-2N-1) + N$.

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$-1N^2(x-N+1) + (N+1)^2(x-N)$.

$-xN^2 - N^3 - N^2 + (N^2+2N+1)(x-N)$.

$-xN^2 - N^3 - N^2 + N^2x - N^3 + 2Nx - 2N^2 + x - N$

$P_2(x) = -2N^3 - 3N^2 + N(2x-1)$.

max of $(h(x))$.

$h(x) = (x-N)(x-N-1)$.

$= x^2 - Nx - x - Nx + N^2 + N$.

$x^2 - 2Nx + N - x$.

$x^2 + x(-2N-1) + N + N^2$.

$h'(x) = 2x - 2N - 1$.

$2x - 2N - 1 = 0$

$x = \frac{2N+1}{2}$.

$x = N + \frac{1}{2}$.

$h(N) = 0$

$h(N+1) = (N+1-N)(N+1-N-1)$

$(1)(0) = 0$.

$h(N+1/2) = \left|(1/2)(1/2 - 1)\right| = 1/4$ maxima.

$E(X) = \left(\frac{N+1}{2}\right)\left(\left(\frac{N+1}{2}\right) - N\right)\left(\frac{N+1}{2} - N - 1\right) = \left|\frac{1}{2}\left(\frac{-1}{2}\right)\right| = \frac{1}{4}$

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Q5):

$2x^6 - 5x^4 + 2 = 0$. $\varepsilon \leq 10^{-3}$.

$|c_n - c| \leq \varepsilon$ $[0, 1]$

$\frac{b-a}{2^n} \leq 10^{-3}$

$\frac{1-0}{2^n} \leq 10^{-3}$

$\frac{1}{10^{-3}} \leq 2^n$

$2^n \geq \frac{1}{10^{-3}}$

$2^n \geq 1000$

$\log 2^n \geq \log 1000$.

$n \geq \frac{\log(1000)}{\log(2)}$.

$n \geq 9.96$

$n = 10$.

Q6) from Lagrange:-

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	x_0	x_1	x_2	x_3
x	-2	-1	1	3
$f(x)$	-1	2	-1	19

$$f(x) = \frac{f(x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + \frac{f(x_1)(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$+ \frac{f(x_2)(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + \frac{f(x_3)(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$f(1.5) = \frac{(-1)(1.5-(-1))(1.5-1)(1.5-3)}{(-2+1)(-2-1)(-2-3)} + \frac{(2)(1.5+2)(1.5-1)(1.5-3)}{(-1+2)(-1-1)(-1-3)}$$

$$+ \frac{(-1)(1.5+2)(1.5+1)(1.5-3)}{(1+2)(1+1)(1-3)} + \frac{(19)(1.5+2)(1.5+1)(1.5-1)}{(3+2)(3+1)(3-1)}$$

$$\frac{13}{64} = 0.203125$$

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Q6) From Divided Difference Polynomial:-

x	-2	-1	1	3
f(x)	-1	2	-1	19

x	f(x)	I DD.	II DD	III DD
-2	-1	$f(x_0,x_1)$ $\frac{2+1}{-1+2} = \frac{3}{1}$	$f(x_0,x_1,x_2)$ $\frac{-3/2-3}{1+2} = -3/2$	$f(x_0,x_1,x_2,x_3)$ $\frac{23/8+3/2}{3+2} = \frac{7}{8}$
-1	2	$f(x_1,x_2)$ $\frac{-1-2}{1+1} = -\frac{3}{2}$	$\frac{10+3/2}{3+1} = 23/8$	
1	-1	$f(x_2,x_3)$ $\frac{19+1}{2} = 10$.	$f(x_1,x_2,x_3)$	
3	19			

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3).$$

$$f(x) = -1 + (x+2)(3) + (x+2)(x+1)(-3/2) + (x+2)(x+1)(x-1)\left(\frac{7}{8}\right).$$

$$f(x) = -1 + 3(x+2) - \frac{3}{2}(x+2)(x+1) + \frac{7}{8}(x+2)(x+1)(x-1).$$

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$$f(1.5) = -1 + 3(1.5+2) - \frac{3}{2}(1.5+2)(1.5+1) + \frac{7}{8}(1.5+2)(1.5+1)(1.5-1)$$

$$f(1.5) = \frac{13}{64} = 0.203125$$