$V = P_2[x]$

$\mathcal{E} = \{x^2, x, 1\}$ $\quad B = \{\underset{v_1}{1+x^2}, \underset{v_2}{1+x}, \underset{v_3}{1}\}$

$v = p(x) = x^2 + 2x + 1$

$[p(x)]_{\mathcal{E}} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Solve:

$x_1 v_1 + x_2 v_2 + x_3 v_3 = v$

$x_1(1+x^2) + x_2(1+x) + x_3(1) = x^2 + 2x1$

$[p(x)]_B = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

$(x_1 + 0 + 0)x^2 + (0 + x_2 + 0)x + (x_1 + x_2 + x_3) = x^2 + 2x + 1$

$x_1 = 1$

$x_2 = 2$

$x_3 = -2$

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$C = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

Exampe $\mathbb{R}^2$

$B = \left\{ \overset{v_1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}, \overset{v_2}{\begin{bmatrix} -1 \\ 1 \end{bmatrix}} \right\}$

$v = 2e_1 + 3e_2$

$\mathcal{E} = \left\{ \overset{e_1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}, \overset{e_2}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} \right\}$

$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$[v]_B = \begin{bmatrix} 5/2 \\ 1/2 \end{bmatrix}$

$[v]_{\mathcal{E}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Solve:

$x_1 v_1 + x_2 v_2 = v$

$\Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 1 & 1 & 3 \end{array}\right]$

$\left\{ \begin{array}{l} x_1 = 5/2 \\ x_2 = 1/2 \end{array} \right.$

Example $\mathbb{R}^2$ $B = \left\{ \overset{v_1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}, \overset{v_2}{\begin{bmatrix} -1 \\ 1 \end{bmatrix}} \right\}$

$V = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $\mathcal{E} = \left\{ \overset{e_1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}, \overset{e_2}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} \right\}$

$[V]_B$

Solve:

$x_1 v_1 + x_2 v_2 = V$

$\Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 1 & 1 & 3 \end{array}\right]$

Def (Coordinate vector)

Let $B = \{v_1, \dots, v_n\}$ be a basis of V and $v \in V$.

(Since we know that

$$v = \underline{\underline{\alpha_1}} v_1 + \underline{\underline{\alpha_2}} v_2 \cdots \underline{\underline{\alpha_n}} v_n$$

in a unique way)

$$[v]_B = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} \in \mathbb{R}^n.$$

$\parallel$

Coordinate vector of v w.r.t B

Theorem. Let $B = \{v_1, \dots, v_n\}$ be a basis of V, then any $v \in V$ could be written as a unique linear combination of $v_1, v_2, \dots, v_n$ i.e.

There exists unique $\alpha_1, \alpha_2, \dots, \alpha_n$ such that

$$V = \underline{\alpha_1} v_1 + \underline{\alpha_2} v_2 + \dots \underline{\alpha_n} v_1$$

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