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Leadership and Ideas for Tomorrow

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$\boxed{\text{chapter : General Vector Spaces}}$

(1)

let $P_1 = ax^n + bx^{n-1} + c$
$P_2 = -ax^n + dx^{n-1} + ex^{n-2}$

Now $P_1 + P_2 = (b+d)x^{n-1} + ex^{n-2} + c \notin P^n$
as degree is x^{n-1}

(2) $B = \{V_1, V_2 \dots V_n\}$
B is basis for V and
$S = \{V_1, V_2 \dots V_m\}$ is linearly independent
then $m \leq n$.

Since first condition of basis is linearly independent, so $V_1 \dots V_n$ are all linearly independent adding a vector may violate the bases conditions of linearly independent or spanning so m is either equal or less than n
i.e $m \leq n$.

(3) $A_1 = \begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 0 & b \end{pmatrix}$

$A + B = \begin{pmatrix} 1+1 & 0 \\ 0 & a+b \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & a+b \end{pmatrix}$

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first entry not 1
$\Rightarrow$ Not a vector space

(4) Not closed under ~~s~~ scaler multiplication
let $k = -1$ $\quad k\begin{pmatrix} x \\ y \end{pmatrix} = -1\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$

here $-x \neq 0$

so, not a vector space.

(5) $V_1 = t^2 - 1 \quad V_2 = t^2 + 3t - 5 \quad V_3 = t$

$X = 7t^2 - 15$

$X = C_1V_1 + C_2V_2 + C_3V_3$?

$C_1V_1 + C_2V_2 + C_3V_3 = C_1(t^2 - 1) + C_2(t^2 + 3t - 5) + C_3 t$
$= (C_1 + C_2)t^2 + (3C_2 + C_3)t + (-C_1 - 5C_2)$
$= 7t^2 - 15$

equating coeficients $\quad C_1 + 2 = 7 \Rightarrow \boxed{C_1 = 5}$

$C_1 + C_2 = 7 \Rightarrow C_1 = 7 - C_2$

$3(2) = -C_3 \quad\quad 3C_2 + C_3 = 0$

$\boxed{C_3 = -6}$ $\quad\quad -C_1 - 5C_2 = -15$ or $C_1 + 5C_2 = 15$

$7 - C_2 + 5C_2 = 15$

$4C_2 = 8$

$\boxed{C_2 = 2}$

$$\boxed{X = 5V_1 + 2V_2 - 6V_3}$$

(6) $M_{2\times2}$ require 4 linearly independent matrices to span $M_{2\times2}$

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(7) To find if $D = k_1 A + k_2 B + k_3 C$

$$= k_1\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} + k_2\begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix} + k_3\begin{pmatrix} 0 & 1 \\ 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ -4 & -2 \end{pmatrix} = \begin{pmatrix} k_1 & -k_1 \\ k_1 & 2k_1 \end{pmatrix} + \begin{pmatrix} 0 & 2k_2 \\ 0 & -k_2 \end{pmatrix} + \begin{pmatrix} 0 & k_3 \\ 5k_3 & 2k_3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ -4 & -2 \end{pmatrix} = \begin{pmatrix} k_1 & -k_1 + 2k_2 + k_3 \\ k_1 + 5k_3 & 2k_1 - k_2 + 2k_3 \end{pmatrix}$$

$\boxed{k_1 = 1}$

$k_1 + 5k_3 = -4$

$k_3 = \frac{-4-1}{5}$

$\boxed{k_3 = -1}$

$2k_1 - k_2 + 2k_3 = -2$

$2(1) - k_2 - 2 = -2$

$\cancel{2} - \cancel{2} + 2 = k_2$

$\boxed{k_2 = 2}$

$\boxed{D = A + 2B - C}$

(8) $V_1 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ $V_2 = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$ $V_3 = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$

$2V_1 = V_2$ scaler multiples

so are linearly dependent

$V = \{V_1, V_2, V_3\}$ is linearly dependent set.

$\Rightarrow V_1, V_2, V_3$ are L.I in $\mathbb{R}^3$ if atleast 2 vectors are scaler multiples or any vector in $\{V_1, V_2, V_3\}$ are linearly commination of other 2.

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(9) For 'n' vectors $V_1, \ldots V_n$ in vector space V to be basis,

i) $V_1 \ldots V_n$ should span V

ii) $V_1, \ldots V_n$ should be linearly independent

(10) Zero vector is linearly dependent

$3\bar{0} = \bar{0}$ Non-zero solution exist
Non-trivial solution exist

zero vector $\bar{0}$ is linearly dependent

(11) A $n \times n$ matrix

$AX = b$ $b \in R^n$ exactly one solution

Facts about A

(1) Columns of A are linearly independent

(2) A is invertible $\det A \neq 0$ and its inverse exists.

(3) Only trivial solution exist to $AX = 0$.

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chapter : Determinants

(12) Det (D) = 8

(13) Det (A) = -24

(14) A square matrix $A \in R^{n \times n}$ is called upper triangular if $a_{jk} = 0$ whenever $j > k$. It is called lower triangular if $a_{jk} = 0$ whenever $j < k$.

e.g

upper triangular $= U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 5 & -2 \\ 0 & 0 & 3 \end{bmatrix}$

lower triangular $= L = \begin{bmatrix} 1 & 0 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

(15) det A = 3 det B = 0 det C = 7

yes invertible
i) $\det(AC) = \det A \times \det C = 3 \times 7 = 21$
ii) $\det(AB) = \det A \times \det B = 3 \times 0 = 0$
Not invertible

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(16) Square matrix $A \in R^{n \times n}$
suppose $m < n$ is a zero row or a zero column, the expanding by that row or column will give
$\det(A) = 0$

(17) An elementary matrix is a matrix obtained by a single row operation on the identity matrix

$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ 2x2 example multiplying row 2 of I_2 by 2

$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(18) A square matrix is invertible if $\det(A) \neq 0$. i.e., Its inverse exists.

(19) $\det(M) = 6$

(20) $\det(A) = 3$ $\det(B) = -4$

a) $\det(-2AB) = (-2)^3 \times 3 \times -4 = 96$

b) $\det(A^5 B^6) = (\det A)^5 \times (\det B)^6 = 3^5 \times (-4)^6$
$= 995328$

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Q1) i) $P = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 6 \\ 1 & 4 & 3 & 7 \\ 1 & 6 & 1 & 9 \end{pmatrix}$

$R_2 - R_1$
$R_3 - R_1$
$R_4 - R_1$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 0 & 3 \\ 0 & 4 & -2 & 5 \end{pmatrix}$$

$R_3 - 2R_1$
$R_4 - 4R_1$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -4 & -1 \\ 0 & 0 & -10 & -3 \end{pmatrix}$$

$-\frac{1}{4}R_3$

$$(-4)\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{1}{4} \\ 0 & 0 & -10 & -3 \end{pmatrix}$$

$R_4 + 10R_3$

$$-4\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{1}{4} \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \quad -3 + \frac{10}{4}$$

$$\det P = (-4)\left(-\frac{1}{2}\right) = 2$$

ii) $\det R = (1)(2)(3) = 6$

already triangular form.

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Chapter : linear transformation

(22) $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x-y \\ x-z \end{pmatrix}$

i) $T\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1-2 \\ 1-3 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

ii) $T\begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} -1+2 \\ -1+3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(23) $T(x) = AX \quad A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ linear?

a) $T(U+V) = T(U) + T(V)$? $\quad U = \begin{bmatrix} x \\ y \end{bmatrix} \; V = \begin{bmatrix} s \\ t \end{bmatrix}$

LHS $\quad T(U+V) = T\begin{pmatrix} x+s \\ y+t \end{pmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x+s \\ y+t \end{bmatrix} = \begin{bmatrix} x+s \\ -y-t \end{bmatrix}$

RHS

$T(U) + T(V) = T\begin{bmatrix} x \\ y \end{bmatrix} + T\begin{bmatrix} s \\ t \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} s \\ t \end{bmatrix}$

$= \begin{bmatrix} x \\ -y \end{bmatrix} + \begin{bmatrix} s \\ -t \end{bmatrix} = \begin{bmatrix} x+s \\ -y-t \end{bmatrix}$ = LHS

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b) $T(kU) = kT(U)$? $U = \begin{bmatrix} x \\ y \end{bmatrix}$

LHS

$$T(kU) = T\begin{bmatrix} kx \\ ky \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} kx \\ ky \end{bmatrix} = \begin{bmatrix} kx \\ -ky \end{bmatrix}$$

RHS

$$kT(U) = kT\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= k\begin{bmatrix} x \\ -y \end{bmatrix} = \begin{bmatrix} kx \\ -ky \end{bmatrix} = L.H.S$$

Given transformation is linear.

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(24) $T: V \rightarrow W$ linear tranformation
$\{V_1 \dots V_n\}$ bases for V n-dim
prove if $U \in V$, then we can write
$T(U)$ as linear combination of $\{T(V_1) \dots T(V_n)\}$

$U \in V$

$U = C_1 V_1 + C_2 V_2 + \dots C_n V_n$

applying transformation both sides

$T(U) = T(C_1 V_1 + \dots C_n V_n)$

since T is linear

$T(U) = T(C_1 V_1) + T(C_2 V_2) + \dots T(C_n V_n)$

since T is linear

$T(U) = C_1 T(V_1) + C_2 T(V_2) + \dots C_n T(V_n)$

$T(U)$ is linear combination of
$\{T(V_1), \dots T(V_n)\}$.

(25) $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \\ y \end{pmatrix}$

$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

i) $T(u+v) = T(u) + T(v)$

LHS $T(u+v) = T\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix}\right)$

$= \begin{bmatrix} u_1+v_1+u_2+v_2 \\ u_1+v_1-u_2-v_2 \\ u_2+v_2 \end{bmatrix}$

RHS $T(u)+T(v) = T\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) + T\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right)$

$= \begin{bmatrix} u_1+u_2 \\ u_1-u_2 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1+v_2 \\ v_1-v_2 \\ v_2 \end{bmatrix}$

$= \begin{bmatrix} u_1+u_2+v_1+v_2 \\ u_1+v_1-u_2-v_2 \\ u_2+v_2 \end{bmatrix}$ = LHS

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ii) $T(CU) = CT(U)$

LHS

$$T(CU) = T\left(\begin{bmatrix} CU_1 \\ CU_2 \end{bmatrix}\right) = \begin{bmatrix} CU_1 + CU_2 \\ CU_1 - CU_2 \\ CU_2 \end{bmatrix}$$

RHS $CT(U) = CT\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$

$$= C\begin{bmatrix} U_1 + U_2 \\ U_1 - U_2 \\ U_2 \end{bmatrix} = \begin{bmatrix} CU_1 + CU_2 \\ CU_1 - CU_2 \\ CU_2 \end{bmatrix} = LHS$$

given T is linear transformation.

(26) $T(U+V) = \pm\sqrt{U+V} \neq \pm\sqrt{U} \pm \sqrt{V}$ for $U = 2$, $V = 3$

(27) P_2 vector space of degree 2 or less

a) $T: P_2 \longrightarrow P_2$ $\quad T(C_2x^2 + C_1x + C_0) = C_0x^2 + C_1x + C_2$

$P_a = C_2x^2 + C_1x + C_0$, $\quad P_b = a_2x^2 + a_1x + a_0$

i) $T(P_a + P_b) = T(P_a) + T(P_b)$

LHS $\quad T(P_a + P_b) = T((C_2 + a_2)x^2 + (C_1 + a_1)x + (a_0 + C_0))$

$= (a_0 + C_0)x^2 + (C_1 + a_1)x + (C_2 + a_2)$

RHS $\quad T(P_a) + T(P_b) = T(C_2x^2 + C_1x + C_0) + T(a_2x^2 + a_1x + a_0)$

$= C_0x^2 + C_1x + C_2 + a_0x^2 + a_1x + a_2$

$= (C_0 + a_0)^2 + (C_1 + a_1)x + (a_2 + C_2) =$

$$X = d_2 x^2 + d_1 x + d_0$$

ii) $T(cX) = cT(X)$

LHS

$T(cX) = T(cd_2 x^2 + cd_1 x + dc_0) =$
$cd_0 x^2 + cd_1 x + cd_2$

RHS

$cT(X) = cT(d_2 x^2 + d_1 x + d_0)$
$= c\,(d_0 x^2 + d_1 x + d_2)$
$= cd_0 x^2 + cd_1 x + cd_2 = LHS$

Hence T is linear.

(b) $T: P_2 \longrightarrow P_2 \quad T(c_2 x^2 + c_1 x + c_0) = c_0^2 x^2 + c_1^2 x + c_2^2$

$P_a = c_2 x^2 + c_1 x + c_0 \quad , \quad P_b = d_2 x^2 + d_1 x + d_0$

$T(P_a + P_b) = T((c_2 + d_2)x^2 + (c_1 + d_1)x + (c_0 + d_0))$
$= (c_0 + d_0)^2 x_2 + (c_1 + d_1)^2 x + (c_2 + d_2)^2$

$T(P_a) + T(P_b) = T(c_2 x^2 + c_1 x + c_0) + T(d_2 x^2 + d_1 x + d_0)$
$= c_0^2 x^2 + c_1^2 x + c_2^2 + d_0^2 x^2 + d_1^2 x + d_2^2$
$= (c_0^2 + d_0^2)x^2 + (c_1^2 + d_1^2)x + (c_2^2 + d_2^2)$

$\neq$

$(c_0 + d_0)^2$

Hence T is not linear.

(28) $p \to q \cong \sim q \to \sim p$

$T(0) \neq 0 \to$ T is not linear transform. $\cong$ T is linear transf. $\to T(0) = 0$

let T is linear

$x \neq 0$

$x - x = 0$

$T(0) = T(x-x) = T(x) + T(-x) = T(x) - T(x)$
$= 0$

Hence we proved the contradiction.

$\Rightarrow$ If $T(0) \neq 0$, then T is not linear.

(31) zero linear transformation

$T: V \longrightarrow W \quad T(V) = 0 \quad \forall v \in V$

prove $\ker(T) = V$

Since every vector in V will give us 0, so $\ker(T) = V$.

(32) $T(V) = 0$ range (T) image of T

$T: V \to W$

Every vector image is zero vector

So, image of T is zero vector in W.

(33) $T: V \to W$ $\to$ L.T $u \in \ker(T)$, $v \in \ker(T)$

$\cancel{k}T(u+v) = T(u+v) = \cancel{k}T(u) + \cancel{k}T(v)$
$= k(0) + k(0)$
$= 0 \quad \Rightarrow ku + kv \in \ker(T)$

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Transpose of any square matrix is linear transformation

$T: V \longrightarrow V$

$T(A) \longrightarrow A^T$ claim T is linear

where A is square matrix.

i) $T(A+B) = T(A) + T(B)$

$T(A+B) = (A+B)^T$
$= A^T + B^T$
$T(A+B) = T(A) + T(B)$

ii) $T(CA) = C\,T(A)$

$T(CA) = (CA)^T$
$= CA^T$
$T(CA) = C\,T(A)$

$\Rightarrow$ T is linear transformation

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$T: R^3 \to R^2 \qquad T(X) = AX$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

i) $\ker T$

i.e $T(X) = AX = 0 \qquad X = ?$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

Reduced echelon (Do complete steps yourself.

Let $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

then $x - z = 0$ homogeneous syste

$y + 2z = 0 \Rightarrow x = z$

$y = -2z \qquad AX = 0$

$z = z$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} z$$

$\ker T = \{ zu \mid u = (1\ -2\ 1)^T$ z is any real number $\}$

ii) Null space T $= \ker T = \{ ru \mid u = (1\ -2\ 1)^T$ r is any real number $\}$

Solution Practice Worksheet #02

chapter	Coordinate system

(34) $B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$

Since both vectors of B are linearly independent (Not multiple of each other). they are basis as they span $\mathbb{R}^2$ too.

Now let $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, by definition

$$V = C_1 V_1 + C_2 V_2 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

coordinate vector

$$V = T_B [V]_B$$

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

Augmented Matrix

$$R_1 - R_2 \quad \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 1 & 1 & 1 \end{array}\right]$$

$R_2 + R_1$

$$-\tfrac{1}{2} R_2 \quad \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & -2 & 2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 1 & -1 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$[V]_B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ coordinates of V relative to B.

(35) $V_1 = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$, $V_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $X = \begin{bmatrix} 3 \\ 12 \\ 7 \end{bmatrix}$ $B = \{V_1, V_2\}$

V_1 and V_2 not multiples of each other so, $B = \{V_1, V_2\}$ is linearly independent and so basis for $W = \text{Span}\{V_1, V_2\}$

X in W? i.e

$$X = C_1 V_1 + C_2 V_2 \quad ?$$

$3 - \frac{2}{2}$

Augmented Matrix

$$\begin{array}{l} R_1 - \frac{1}{2}R_2 \\ R_1 - \frac{3}{2}R_3 \end{array} \left[\begin{array}{cc|c} 3 & -1 & 3 \\ 6 & 0 & 12 \\ 2 & 1 & 7 \end{array}\right] \overset{-R_2}{\sim} \left[\begin{array}{cc|c} 3 & -1 & 3 \\ 0 & -1 & -3 \\ 0 & -\frac{5}{2} & -\frac{15}{2} \end{array}\right]$$

$$\begin{array}{l} R_2 + R_1 \\ R_2 + \frac{2}{5}R_3 \end{array} \left[\begin{array}{cc|c} 3 & -1 & 3 \\ 0 & 1 & 3 \\ 0 & -\frac{5}{2} & -\frac{15}{2} \end{array}\right]$$

$$\frac{1}{3}R_1 \left[\begin{array}{cc|c} 3 & 0 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

coordinates of X relative to B.

$[\vec{X}]_B = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Yes X is in W.

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

(36) $V = \begin{bmatrix} 3 \\ 11 \\ -7 \end{bmatrix}$ coordinates of V in $E = \{e_1, e_2, e_3\}$

$$V = 3\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 11\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 7\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[V]_E = \begin{bmatrix} 3 \\ 11 \\ -7 \end{bmatrix}$$

(37) coordinates of $V(t) = 3 - t^2 - 7t^3$ relative to $B = \{1, t, t^2, t^3\}$

$$V(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 = 3 - t^2 - 7t^3$$

$$[V(t)]_B = \begin{bmatrix} 3 \\ 0 \\ -1 \\ -7 \end{bmatrix}$$

(38) let $B = \{V_1, \dots, V_n\}$ be a basis for V and let $X \in V$. The coordinates of X relative to the basis B are the unique scalars $C_1, C_2 \dots C_n$ such that

$$X = C_1 V_1 + C_2 V_2 + \dots C_n V_n$$

In vector notation, the B-coordinates of X will be denoted by $[X]_B = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}$ and we will call $[X]_B$ the coordinate vector of X relative to B.

(39) $P = \begin{bmatrix} 1 & 3 & 3 \\ -1 & -4 & -2 \\ 0 & 0 & -1 \end{bmatrix}$ column's form basis for $\mathbb{R}^3$

$[x]_B = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

(a) $X = ?$

$X = P[X]_B$

$= \begin{bmatrix} 1 & 3 & 3 \\ -1 & -4 & -2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

(b) $[V]_B = ?$ $V = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

compute Yourself

$[V]_B = P^{-1} V$ $P^{-1} = \begin{bmatrix} 4 & 3 & 6 \\ -1 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 3 & 6 \\ -1 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$[V]_B = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}$

(40) let $B = \{V_1, V_2 \dots V_n\}$ be a basis of $\mathbb{R}^n$ and let $P = [V_1 \dots V_n] \in M_{n \times n}$. If $X \in \mathbb{R}^n$ and $[X]_B$ are B-coordinates of X relative to B then $X = P[X]_B$. where P maps B-coordinate vectors to coordinate vectors relative to standard basis of $\mathbb{R}^n$. P is called change of coordinate ~~vecto~~ matrix from basis B to standard basis in $\mathbb{R}^n$.