

Numerical Analysis

Project 1

Final Report

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6771

Gui functions:

Create figures for building apps using `uifigure`.

Create a table user interface component using `uitable`.

`msgbox`: Generate a message box.

Create a dialogue box called `inputdlg` to collect user input.

`uiwait`: Stop running programmes and wait for them to start again.

Convert integers to a character array using `num2str`.

Convert strings to double precision values using `str2double`.

Functions used:

`Fzero`: The root of a nonlinear function. `Feval`: Evaluates a function using its name or its handle and the input arguments $x_1 \dots x_M$.

Data Structures:

1-Toc:

`Toc` reads the amount of time that has passed since the `tic` function's call to start the stopwatch timer. When the `toc` function is executed, MATLAB® reads the internal time and shows the amount of time that has passed since the most recent call to the `tic` function without producing anything. The amount of time elapsed is shown in seconds. The time since the call to the `tic` function corresponding to `timerVal` is shown by the function `toc(timerVal)`.

2-Tic:

`Tic` uses the `toc` function to calculate the passage of time. The `toc` function takes the recorded value from the `tic` function to compute the amount of time that has passed since the current time was recorded. In order to explicitly send the current time to the `toc` method, `timerVal = tic` puts the time in `timerVal`. When there are many calls to `tic` to time various sections of the same code, passing this value is helpful. An integer called `timerVal` only has significance for the `toc` function.

3-HPF decimal class:

I frequently observe folks requesting a tool with an accuracy range of greater than 16 digits. Numerical analysis guiding principles are significantly more valuable than any high precision instrument. However, there are situations when having a little additional accuracy is useful. Some of you will just want to play in the sandbox of enormous numbers. While some of you might make use of

Ben Barrowes' tools, HPF is entirely built in MATLAB, so compilers are not necessary. Whatever your motivations, I offer HPF, a high precision floating point tool, to all of you.

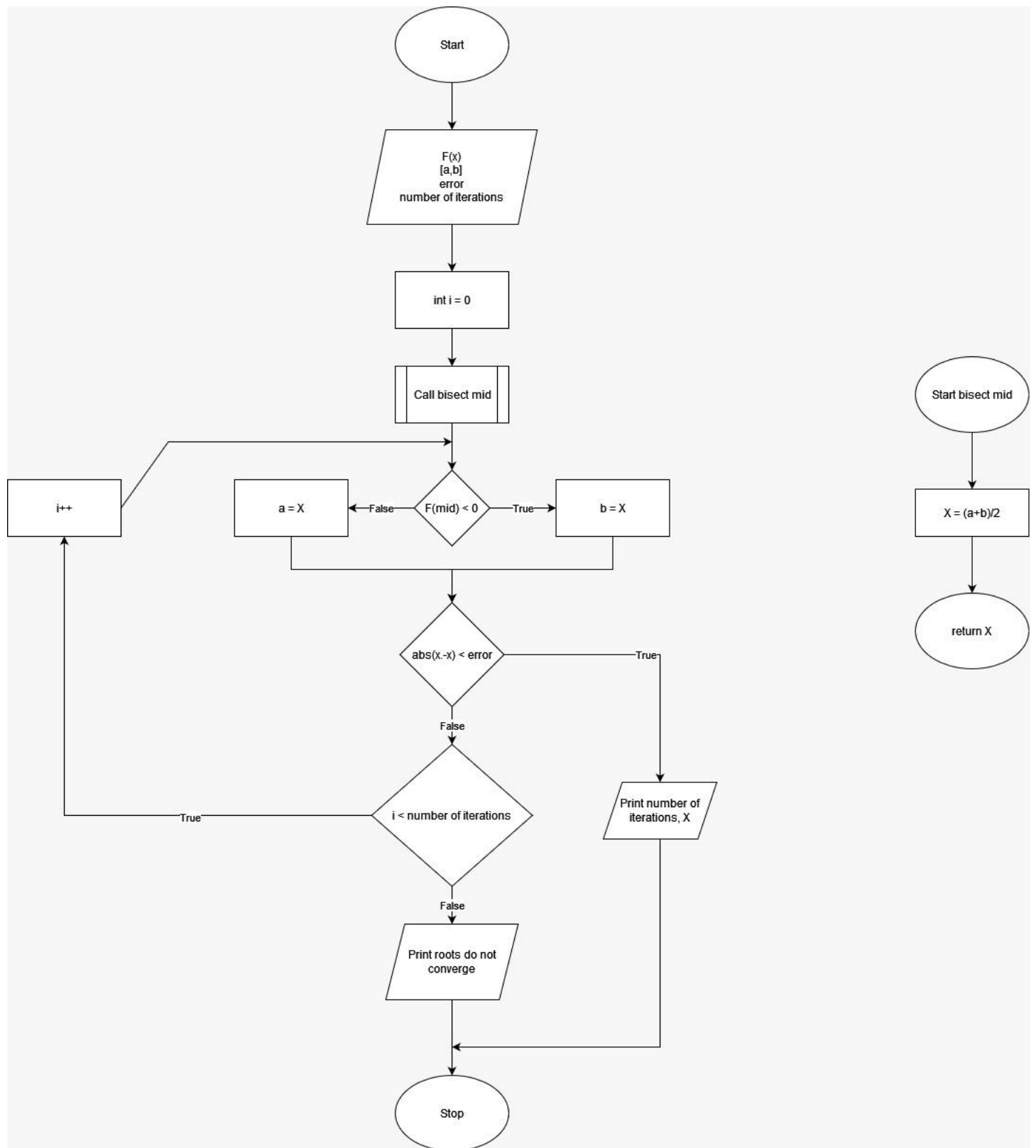
4- `B=ArrayFun(func,A)`:

One by one, apply the function `func` to the components of `A`. The outputs from `func` are then concatenated by `arrayfun` into the output array `B`, making `B(i)` equal to `func(A(i))` for the *i*th element of `A`. A function handle to a function that accepts one input parameter and returns a scalar is provided as the input argument `func`. As long as objects of that type can be concatenated, any data type can be used for `func`'s output. The dimensions of arrays `A` and `B` are the same.

1)Bisection

The algorithm we used:

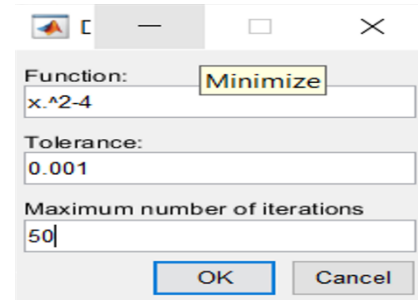
1. Start
2. Define function $f(x)$
3. Input a. Lower and Upper guesses x_0 and x_1 b. tolerable error e
4. If $f(x_0)*f(x_1) > 0$ print "Incorrect initial guesses" goto 3 End If
5. Do $x_2 = (x_0+x_1)/2$ If $f(x_0)*f(x_2) < 0$ $x_1 = x_2$
Else $x_0 = x_2$ End If while $\text{abs}(f(x_2)) > e$
6. Print root as x_2
7. Stop



A) $f(x) = x^2 - 4$

Tolerance = 0.001

Interval = [0,3]

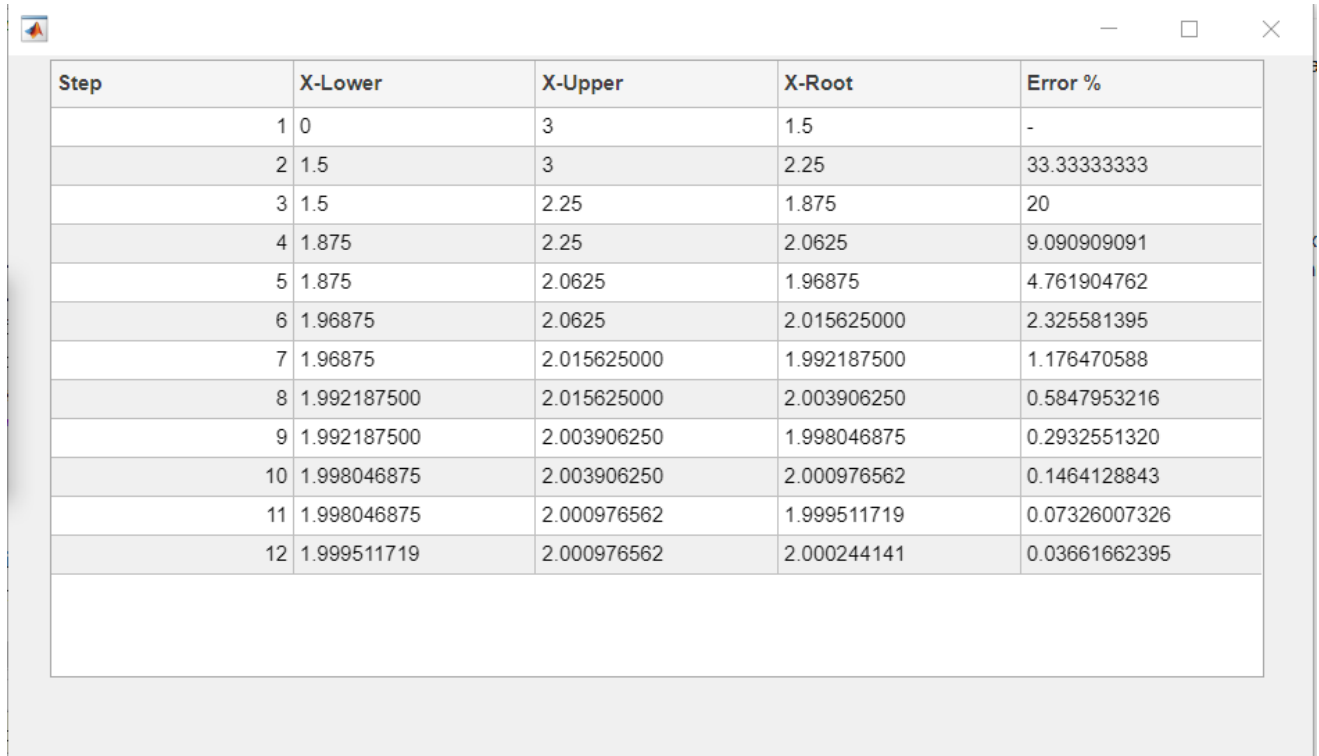


Function: Minimize

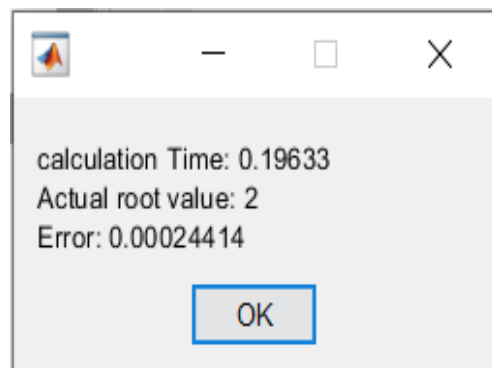
Tolerance:

Maximum number of iterations:

OK Cancel



Step	X-Lower	X-Upper	X-Root	Error %
1	0	3	1.5	-
2	1.5	3	2.25	33.33333333
3	1.5	2.25	1.875	20
4	1.875	2.25	2.0625	9.090909091
5	1.875	2.0625	1.96875	4.761904762
6	1.96875	2.0625	2.015625000	2.325581395
7	1.96875	2.015625000	1.992187500	1.176470588
8	1.992187500	2.015625000	2.003906250	0.5847953216
9	1.992187500	2.003906250	1.998046875	0.2932551320
10	1.998046875	2.003906250	2.000976562	0.1464128843
11	1.998046875	2.000976562	1.999511719	0.07326007326
12	1.999511719	2.000976562	2.000244141	0.03661662395



calculation Time: 0.19633
Actual root value: 2
Error: 0.00024414





OK

$$B) f(x) = 3x^4 + 6.1x^3 - 2x^2 + 3x + 2$$

Tolerance = 0.001

Interval = [-1,0]

Step	X-Lower	X-Upper	X-Root	Error %
1	-1	0	-0.5	-
2	-0.5	0	-0.25	100
3	-0.5	-0.25	-0.375	33.33333333
4	-0.5	-0.375	-0.4375	14.28571429
5	-0.4375	-0.375	-0.40625	7.692307692
6	-0.4375	-0.40625	-0.421875	3.703703704
7	-0.4375	-0.421875	-0.4296875	1.818181818
8	-0.4296875	-0.421875	-0.4257812500	0.9174311927
9	-0.4257812500	-0.421875	-0.4238281250	0.4608294931
10	-0.4257812500	-0.4238281250	-0.4248046875	0.2298850575
11	-0.4248046875	-0.4238281250	-0.4243164063	0.1150747986
12	-0.4243164063	-0.4238281250	-0.4240722656	0.05757052389
13	-0.4240722656	-0.4238281250	-0.4239501953	0.02879355024

calculation Time: 0.0623
 Actual root value: -0.42406
 Error: 0.00011112

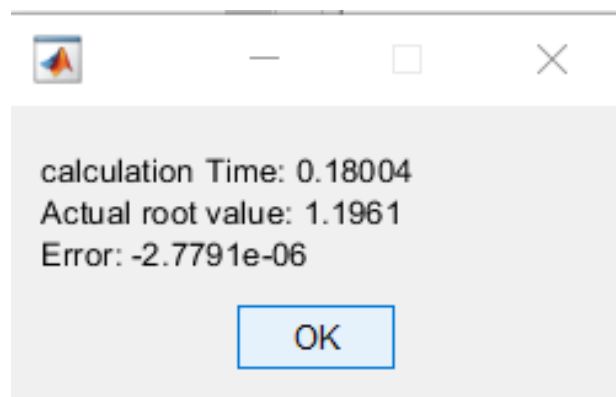
OK

C) $f(x) = x^2 - \sin(x) - 0.5$

Tolerance = 0.001

Interval = [0, 2]

Step	X-Lower	X-Upper	X-Root	Error %
1	0	2	1	-
2	1	2	1.5	33.33333333
3	1	1.5	1.25	20
4	1	1.25	1.125	11.11111111
5	1.125	1.25	1.1875	5.263157895
6	1.1875	1.25	1.21875	2.564102564
7	1.1875	1.21875	1.203125000	1.298701299
8	1.1875	1.203125000	1.195312500	0.6535947712
9	1.195312500	1.203125000	1.199218750	0.3257328990
10	1.195312500	1.199218750	1.197265625	0.1631321370
11	1.195312500	1.197265625	1.196289063	0.08163265306
12	1.195312500	1.196289063	1.195800781	0.04083299306
13	1.195800781	1.196289063	1.196044922	0.02041232905
14	1.196044922	1.196289063	1.196166992	0.01020512297
15	1.196044922	1.196166992	1.196105957	0.005102821860
16	1.196044922	1.196105957	1.196075439	0.002551476029
17	1.196075439	1.196105957	1.196090698	0.001275721740
18	1.196075439	1.196090698	1.196083069	6.378649385e-4
19	1.196075439	1.196083069	1.196079254	3.189334865e-4



D) $f(x)=x^3-x-1$

Tolerance= 10^{-5}

Interval= [1,2]

from input file

1 $x.^3-x-1$

Step	X-Lower	X-Upper	X-Root	Error %
1	1	2	1.5	-
2	1	1.5	1.25	20
3	1.25	1.5	1.375	9.090909091
4	1.25	1.375	1.3125	4.761904762
5	1.3125	1.375	1.34375	2.325581395
6	1.3125	1.34375	1.328125000	1.176470588
7	1.3125	1.328125000	1.320312500	0.5917159763
8	1.320312500	1.328125000	1.324218750	0.2949852507
9	1.324218750	1.328125000	1.326171875	0.1472754050
10	1.324218750	1.326171875	1.325195313	0.07369196758
11	1.324218750	1.325195312	1.324707031	0.03685956506
12	1.324707031	1.325195312	1.324951172	0.01842638659
13	1.324707031	1.324951172	1.324829102	0.009214042200
14	1.324707031	1.324829102	1.324768066	0.004607233356
15	1.324707031	1.324768066	1.324737549	0.002303669746
16	1.324707031	1.324737549	1.324722290	0.001151848140
17	1.324707031	1.324722290	1.324714661	5.759273871e-4
18	1.324714661	1.324722290	1.324718475	2.879628643e-4

calculation Time: 0.039553
 Actual root value: 1.3247
 Error: 5.181e-07

OK

2)Regula Falsi:

The algorithm we used:

1. Start

2. Define function $f(x)$

3. Input a. Lower and Upper guesses x_0 and x_1 b. tolerable error e

4. If $f(x_0)*f(x_1) > 0$

print "Incorrect initial guesses" goto

End If

5. Do

$$x_2 = x_0 - ((x_0 - x_1) * f(x_0)) / (f(x_0) - f(x_1))$$

If $f(x_0)*f(x_2) < 0$

$$x_1 = x_2$$

Else $x_0 = x_2$

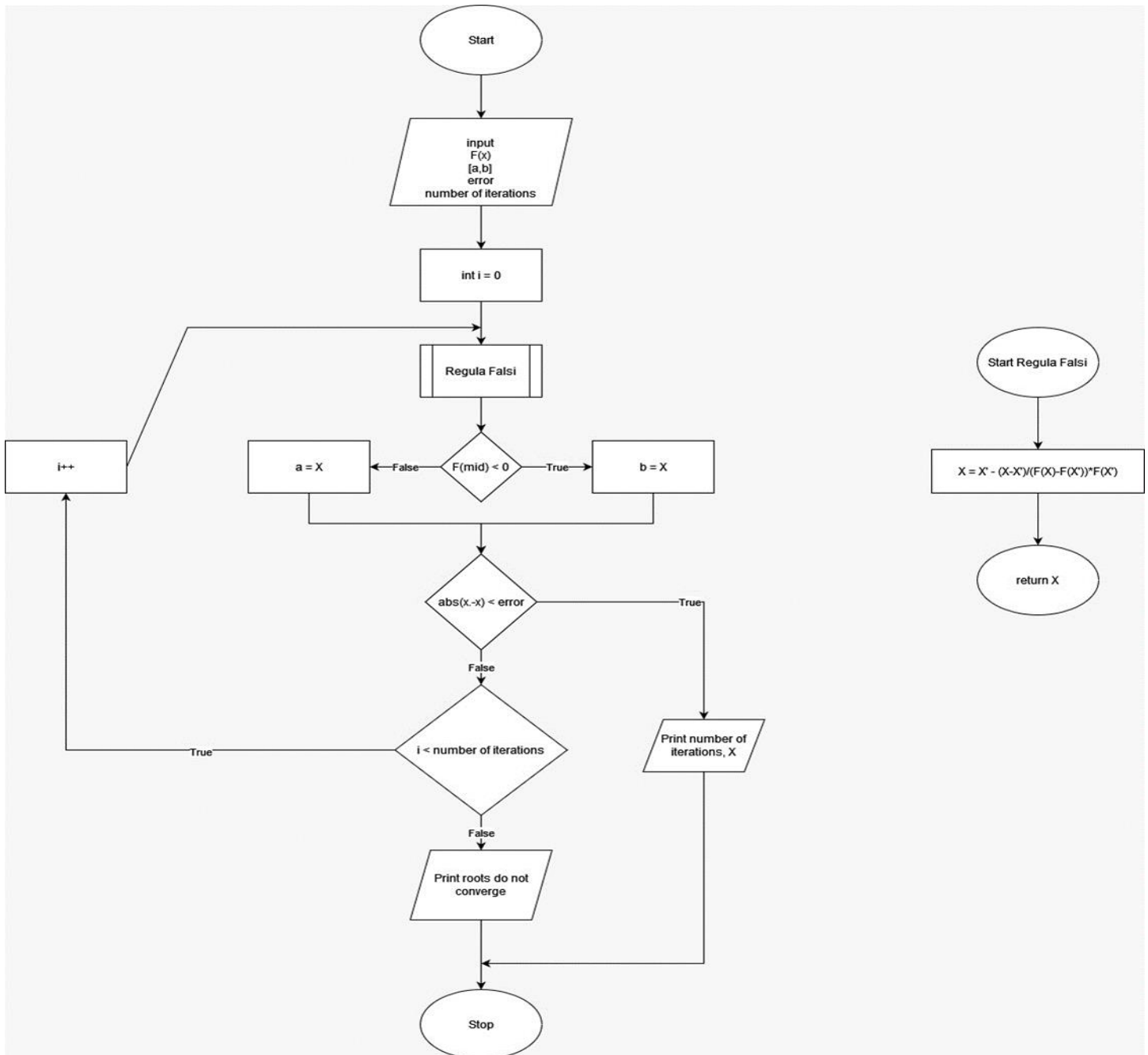
End If

While $\text{abs}(f(x_2)) > e$

6. Print root as x2

7. Stop

Flow Charts:



A) $f(x) = x^2 - 4$

Tolerance = 0.001

Interval = [0,3]

Step	X-Lower	X-Upper	X-Root	F(Xr)	Error %
1	0	3	1.333333333	-2.222222222	-
2	1.333333333	3	1.846153846	-0.5917159763	27.77777778
3	1.846153846	3	1.968253968	-0.1259763165	6.203473945
4	1.968253968	3	1.993610224	-0.02551827619	1.271876272
5	1.993610224	3	1.998720409	-0.005116724772	0.2556728696
6	1.998720409	3	1.999744016	-0.001023868941	0.05118689720
7	1.999744016	3	1.999948801	-2.047947572e-4	0.01023947575
8	1.999948801	3	1.999989760	-4.095979029e-5	0.002047979029

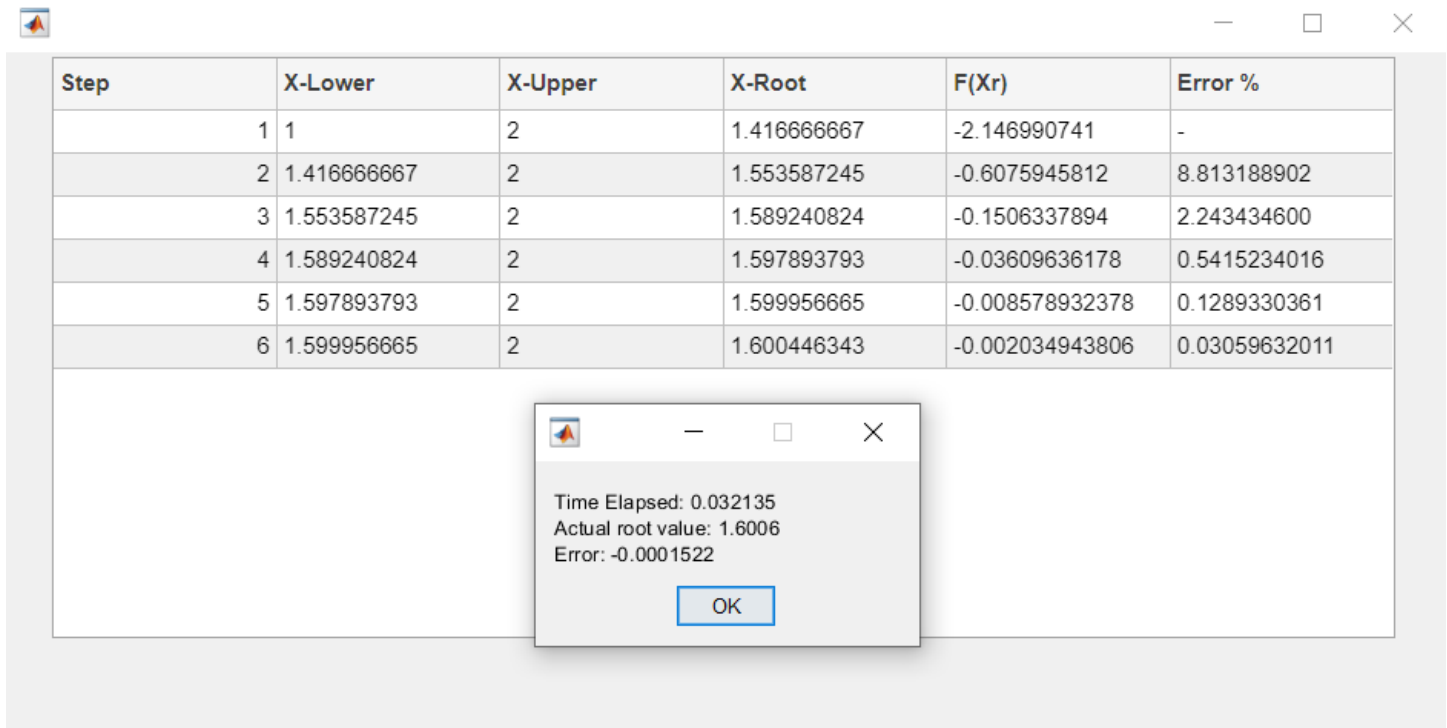
Time Elapsed: 0.025766
 Actual root value: 2
 Error: -1.024e-05

OK

B) $f(x) = 2x^3 - 2x - 5$

Tolerance = 0.001

Interval = [1,2]



Step	X-Lower	X-Upper	X-Root	F(Xr)	Error %
1	1	2	1.416666667	-2.146990741	-
2	1.416666667	2	1.553587245	-0.6075945812	8.813188902
3	1.553587245	2	1.589240824	-0.1506337894	2.243434600
4	1.589240824	2	1.597893793	-0.03609636178	0.5415234016
5	1.597893793	2	1.599956665	-0.008578932378	0.1289330361
6	1.599956665	2	1.600446343	-0.002034943806	0.03059632011

Time Elapsed: 0.032135
 Actual root value: 1.6006
 Error: -0.0001522

OK


c) $f(x) = e^x + 2x + 2 \cos(x) - 6$

Tolerance = 0.001

Interval= [1.8,1.9]



Step	X-Lower	X-Upper	X-Root	F(Xr)	Error %
1	2.5	2.6	2.543288644	-0.01772286218	-
2	2.543288644	2.6	2.545059151	-7.123835766e-4	0.06956644701
3	2.545059151	2.6	2.545130226	-2.857752421e-5	0.002792579465
4	2.545130226	2.6	2.545133077	-1.146305594e-6	1.120193865e-4

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Time Elapsed: 0.10399
Actual root value: 2.5451
Error: -1.1914e-07

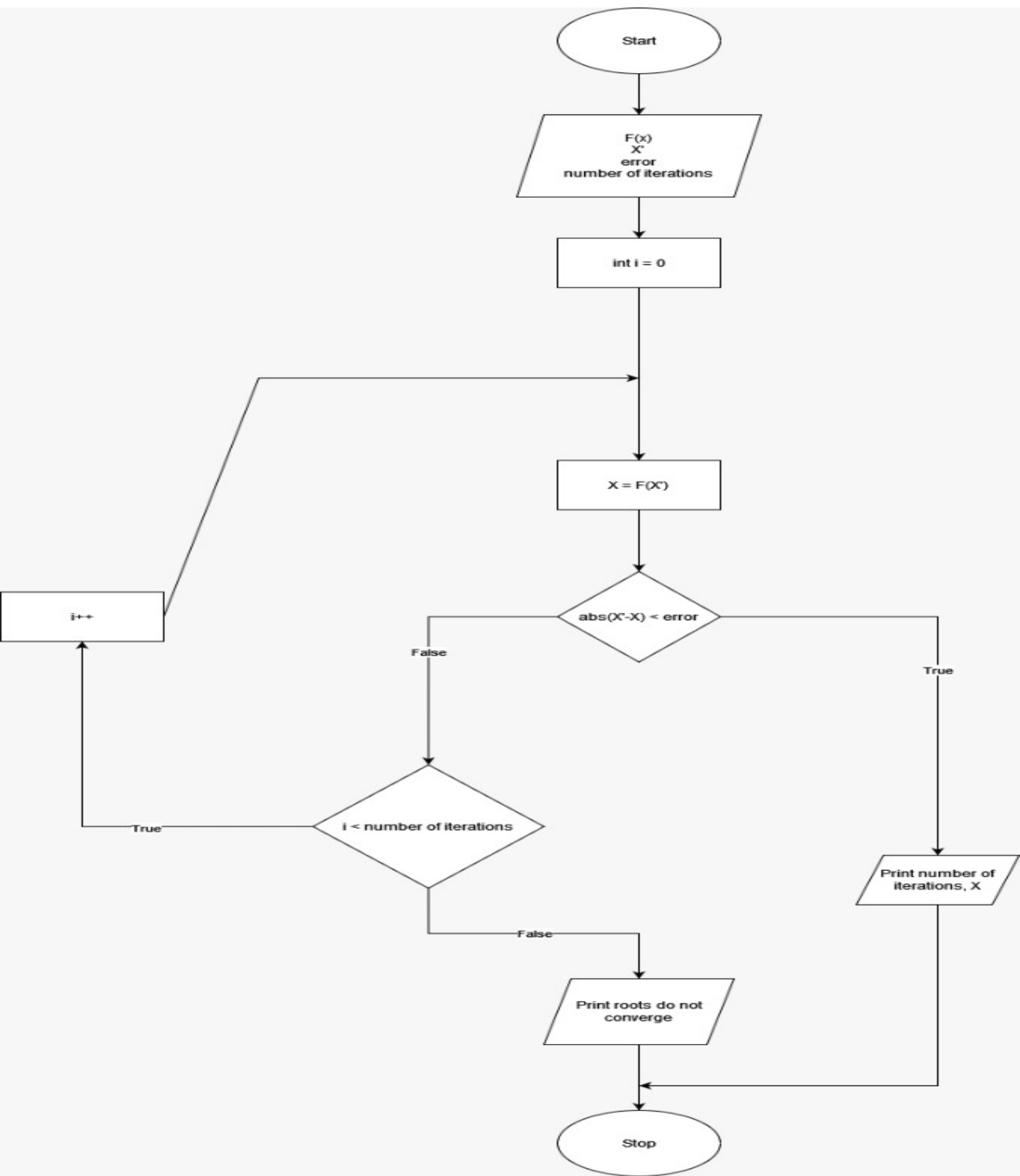
OK

False Position method has converged

3)Fixed point :

The algorithm we used:

1. Start
2. Define function as $f(x)$
3. Define convergent form $g(x)$
4. Input: a. Initial guess x_0 b. Tolerable Error e c. Maximum Iteration N
5. Initialize iteration counter: $\text{step} = 1$
6. Do
 - $x_1 = g(x_0)$ $\text{step} = \text{step} + 1$
 - If $\text{step} > N$
 - Print "Not Convergent" Stop
 - End If
 - $x_0 = x_1$
- While $\text{abs } f(x_1) > e$
7. Print root as x_1
8. Stop



a) $F(x)=x^2-x-1$

Interval[1,3]

Fixed Point Iteration

$$x^2 - x - 1 = 0$$

$$x_{n+1} = 1 + \frac{1}{x_n}$$

Pick $x_1 = 2$

$$x_2 = 1 + \frac{1}{2} = 1.5$$

$$x_3 = 1 + \frac{1}{1.5} = 1.666$$

$$x_4 = 1 + \frac{1}{1.666} = 1.6$$

Step	X(i)	g(x(i))	Error %
1	2	1.5	-
2	1.5	1.66666667	33.33333333
3	1.66666667	1.6	10
4	1.6	1.625	4.16666667
5	1.625	1.615384615	1.538461538
6	1.615384615	1.619047619	0.5952380952
7	1.619047619	1.617647059	0.226243439
8	1.617647059	1.618181818	0.08658008658
9	1.618181818	1.617977528	0.03304692664
10	1.617977528	1.618055556	0.01262626263
11	1.618055556	1.618025751	0.004822298307
12	1.618025751	1.618037135	0.001842027704
13	1.618037135	1.618032787	7.035812284e-4

b) $f(x)=e^{-x} - x$ interval =[0,1]

Find root of $f(x) = e^{-x} - x = 0$.
(Answer: $\alpha = 0.56714329$)
 Take $x_0 = 0$.

We put: $x_{i+1} = e^{-x_i}$

i	x_i	ϵ_a (%)	ϵ_t (%)
0	0	-	100.0
1	1.000000	100.0	76.3
2	0.367879	171.8	35.1
3	0.692201	46.9	22.1
4	0.500473	38.3	11.8
5	0.606244	17.4	6.89
6	0.545396	11.2	3.83
7	0.579612	5.90	2.20
8	0.560115	3.48	1.24
9	0.571143	1.93	0.705
10	0.564879	1.11	0.399

Approximate Percent Relative error

True Percent Relative error

—

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Time Elapsed: 0.020512

OK

Step	X(i)	g(x(i))	Error %
1	0	1	-
2	1	0.3678794412	100
3	0.3678794412	0.6922006276	171.8281828
4	0.6922006276	0.5004735006	46.85363946
5	0.5004735006	0.6062435351	38.30914659
6	0.6062435351	0.5453957860	17.44678968
7	0.5453957860	0.5796123355	11.15662253
8	0.5796123355	0.5601154614	5.903350814
9	0.5601154614	0.5711431151	3.480866980
10	0.5711431151	0.5648793474	1.930803931
11	0.5648793474	0.5684287250	1.108868242
12	0.5684287250	0.5664147331	0.6244191192
13	0.5664147331	0.5675566373	0.3555684138
14	0.5675566373	0.5669089119	0.2011965161
15	0.5669089119	0.5672762322	0.1142556402
16	0.5672762322	0.5670678984	0.06475156780
17	0.5670678984	0.5671860501	0.03673877244
18	0.5671860501	0.5671190401	0.02083120848
19	0.5671190401	0.5671570440	0.01181586888
20	0.5671570440	0.5671354902	0.006700779702
21	0.5671354902	0.5671477143	0.003800466624
22	0.5671477143	0.5671407815	0.002155356274
23	0.5671407815	0.5671447133	0.001222412855
24	0.5671447133	0.5671424834	6.932777789e-4

4) Newton Raphson:

The algorithm we used:

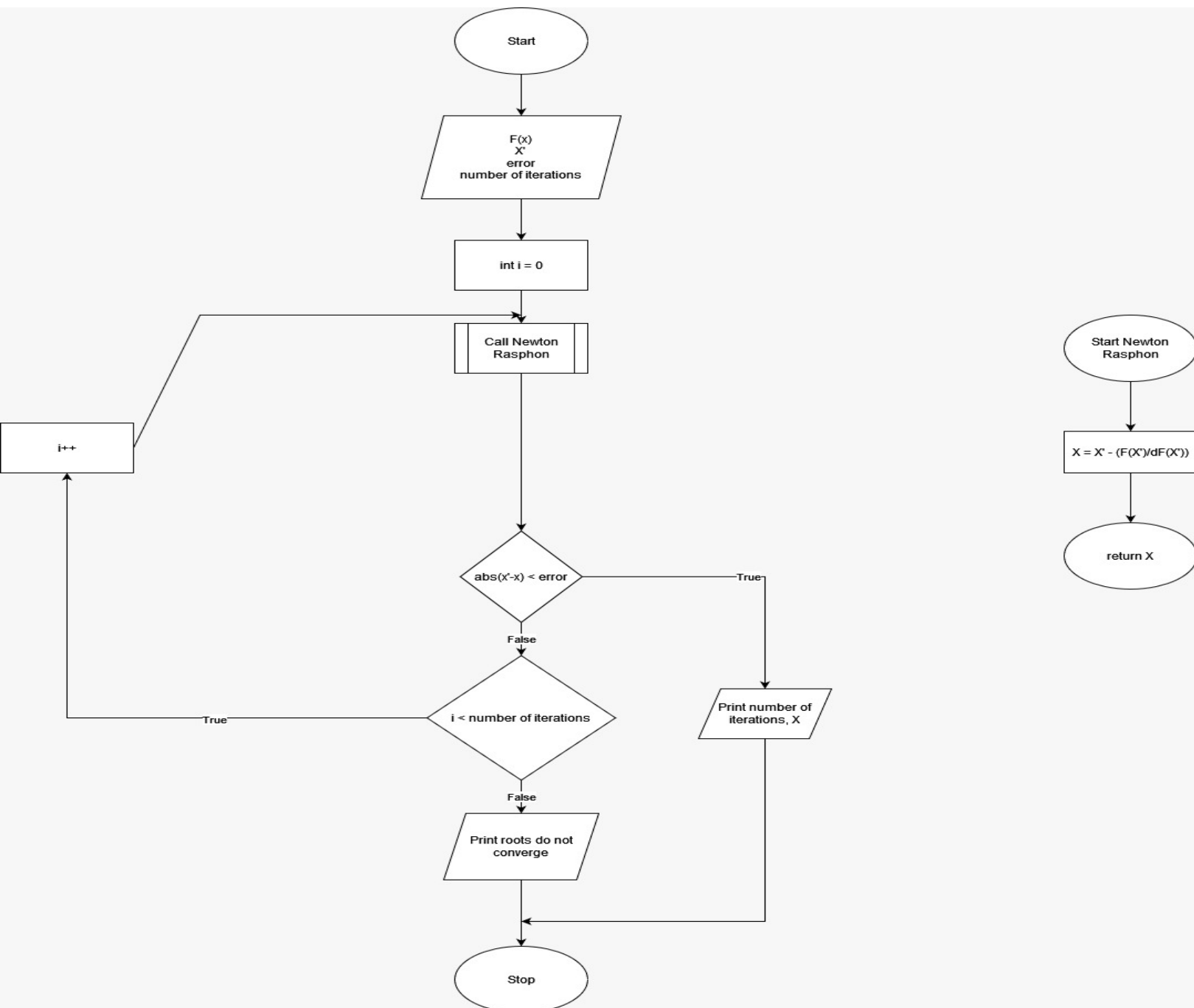
1. Start
2. Define function as $f(x)$
3. Define derivative of function as $g(x)$
4. Input: a. Initial guess x_0 b. Tolerable Error e c. Maximum Iteration N
5. Initialize iteration counter $step = 1$
6. Do
 - If $g(x_0) = 0$
 - Print "Mathematical Error" Stop
 - End If
 - $x_1 = x_0 - f(x_0) / g(x_0)$
 - $x_0 = x_1$ $step = step + 1$
 - If $step > N$
 - Print "Not Convergent" Stop

End If

While $\text{abs } f(x_1) > e$

7. Print root as x_1

8. Stop



A) $f(x) = x - 0.75 - 0.2 \sin(x)$

Tolerance = 10^{-1}

Interval = $[0, \pi/2]$

$X_0 = \pi/4$

Step	X(i)	F(Xi)	F'(Xi)	X(i+1)	Error%
1	0.785398	-0.1060233331	0.8585786207	0.9088850408	13.58665125
2	0.9088850408	0.001121251243	0.8770748735	0.9076066422	0.1408538112
3	0.9076066422	1.288740830e-7	0.8768732890	0.9076064952	1.619314302e-5

Time Elapsed: 10.3084
Actual root value: 0.90761
Error: 1.9984e-15

OK

$$B) f(x) = x^3 - 2x^2 - 4x + 8$$

Tolerance = 0.01

$$X_0 = 1.5$$

Step	X(i)	F(Xi)	F'(Xi)	X(i+1)	Error%
1	1.5	0.875	-3.25	1.769230769	15.21739130
2	1.769230769	0.2007282658	-1.686390533	1.888259109	6.303602058
3	1.888259109	0.04854890687	-0.8564690456	1.944944061	2.914477248
4	1.944944061	0.01195774231	-0.4313540392	1.972665472	1.405276805
5	1.972665472	0.002968282016	-0.2164346964	1.986379918	0.6904241303
6	1.986379918	7.394999129e-4	-0.1084041356	1.993201613	0.3422481004

Time Elapsed: 2.9318
Actual root value: -2
Error: 3.9932

OK

$$c) f(x) = x^2 - 4$$

$$X_0 = 4$$

Step	X(i)	F(Xi)	F'(Xi)	X(i+1)	Error%
1	4	12	8	2.5	60
2	2.5	2.25	5	2.05	21.95121951
3	2.05	0.2025	4.1	2.000609756	2.468759525
4	2.000609756	0.002439396193	4.001219512	2.000000093	0.03048315735
5	2.000000093	3.716891879e-7	4.000000186	2	4.646114626e-6

Time Elapsed: 7.8431
Actual root value: 2
Error: 2.2204e-15

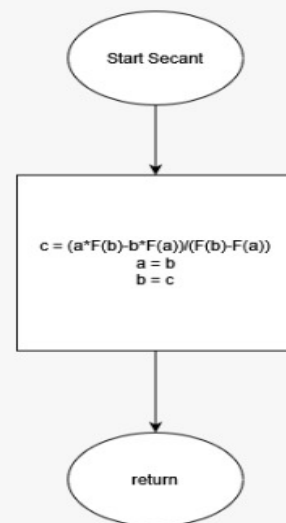
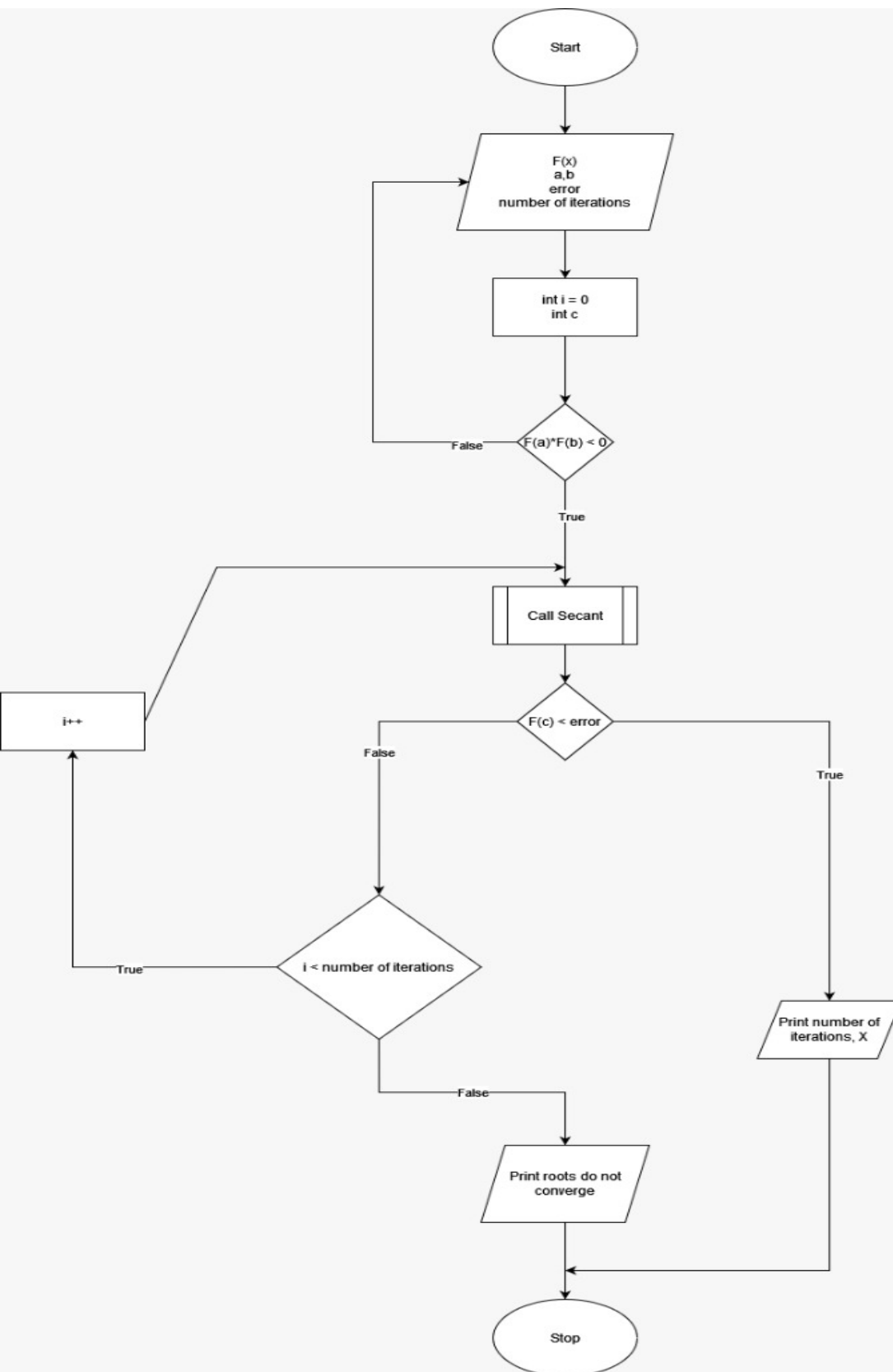
OK

5) Secant:

The algorithm we used:

1. Start
2. Define function as $f(x)$
3. Input: a. Initial guess x_0, x_1 b. Tolerable Error e c. Maximum Iteration N
4. Initialize iteration counter $step = 1$
5. Do If $f(x_0) = f(x_1)$
Print "Mathematical Error"
Stop
End If
$$x_2 = x_1 - (x_1 - x_0) * f(x_1) / (f(x_1) - f(x_0))$$
$$x_0 = x_1$$
$$x_1 = x_2$$
$$step = step + 1$$
 If $step > N$ Print "Not Convergent"
Stop
End If
While $abs f(x_2) > e$
6. Print root as x_2

7. Stop

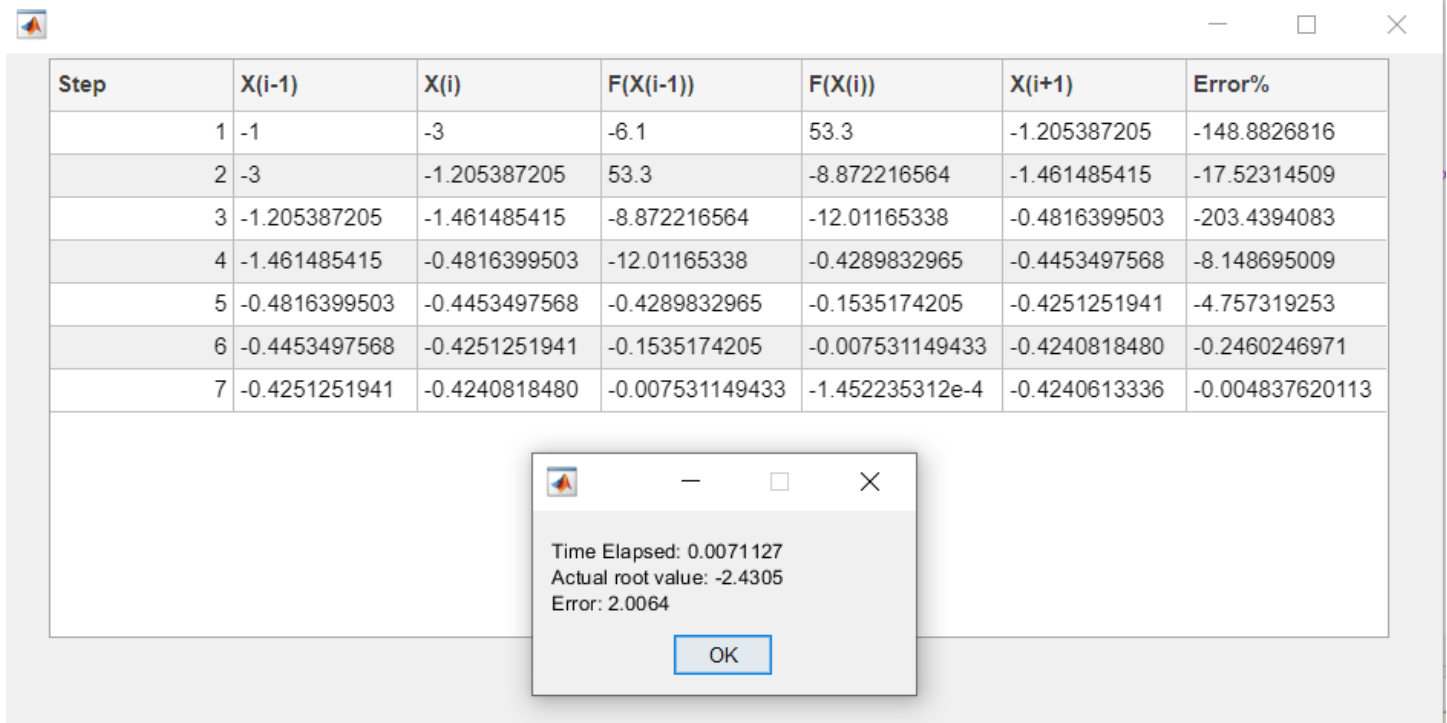


A)

$$f(x) = 3x^4 + 6.1x^3 - 2x^2 + 3x + 2$$

Interval = [-1, -3]

Tolerance = 10^{-2}



The screenshot shows a software window with a table of iteration results and a summary dialog box. The table has 7 rows and 7 columns: Step, X(i-1), X(i), F(X(i-1)), F(X(i)), X(i+1), and Error%. The summary dialog box displays the following information:

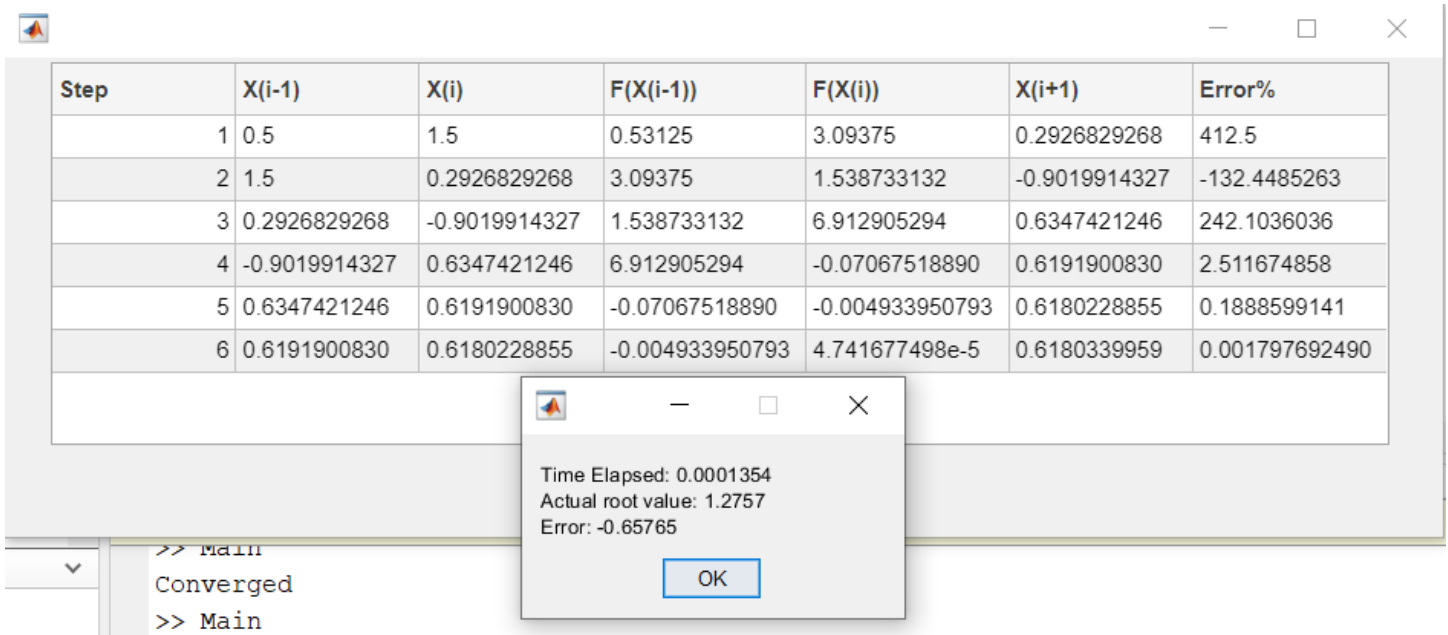
- Time Elapsed: 0.0071127
- Actual root value: -2.4305
- Error: 2.0064
- OK button

Step	X(i-1)	X(i)	F(X(i-1))	F(X(i))	X(i+1)	Error%
1	-1	-3	-6.1	53.3	-1.205387205	-148.8826816
2	-3	-1.205387205	53.3	-8.872216564	-1.461485415	-17.52314509
3	-1.205387205	-1.461485415	-8.872216564	-12.01165338	-0.4816399503	-203.4394083
4	-1.461485415	-0.4816399503	-12.01165338	-0.4289832965	-0.4453497568	-8.148695009
5	-0.4816399503	-0.4453497568	-0.4289832965	-0.1535174205	-0.4251251941	-4.757319253
6	-0.4453497568	-0.4251251941	-0.1535174205	-0.007531149433	-0.4240818480	-0.2460246971
7	-0.4251251941	-0.4240818480	-0.007531149433	-1.452235312e-4	-0.4240613336	-0.004837620113

$$B) f(x) = x^5 - 5x + 3$$

$$\text{Interval} = [0.5, 1.5]$$

$$\text{Tolerance} = 10^{-3}$$



The image shows a MATLAB Command Window with a table of iteration results for the bisection method. The table has 7 columns: Step, X(i-1), X(i), F(X(i-1)), F(X(i)), X(i+1), and Error%. The iterations are shown for steps 1 through 6. A dialog box is overlaid on the table, displaying the following information: Time Elapsed: 0.0001354, Actual root value: 1.2757, and Error: -0.65765. The dialog box has an OK button.

Step	X(i-1)	X(i)	F(X(i-1))	F(X(i))	X(i+1)	Error%
1	0.5	1.5	0.53125	3.09375	0.2926829268	412.5
2	1.5	0.2926829268	3.09375	1.538733132	-0.9019914327	-132.4485263
3	0.2926829268	-0.9019914327	1.538733132	6.912905294	0.6347421246	242.1036036
4	-0.9019914327	0.6347421246	6.912905294	-0.07067518890	0.6191900830	2.511674858
5	0.6347421246	0.6191900830	-0.07067518890	-0.004933950793	0.6180228855	0.1888599141
6	0.6191900830	0.6180228855	-0.004933950793	4.741677498e-5	0.6180339959	0.001797692490

Time Elapsed: 0.0001354
Actual root value: 1.2757
Error: -0.65765

OK

$$C) f(x) = x^3 - x - 1$$

$$\text{Interval} = [1, 2]$$

$$\text{Tolerance} = 10^{-4}$$



Step	$X(i-1)$	$X(i)$	$F(X(i-1))$	$F(X(i))$	$X(i+1)$	Error%
1	1	2	-1	5	1.166666667	71.42857143
2	2	1.166666667	5	-0.5787037037	1.253112033	6.898454746
3	1.166666667	1.253112033	-0.5787037037	-0.2853630296	1.337206446	6.288812988
4	1.253112033	1.337206446	-0.2853630296	0.05388058668	1.323850096	1.008901951
5	1.337206446	1.323850096	0.05388058668	-0.003698115440	1.324707937	0.06475692647
6	1.323850096	1.324707937	-0.003698115440	-4.273426281e-5	1.324717965	7.570533496e-4
7	1.324707937	1.324717965	-4.273426281e-5	3.458221465e-8	1.324717957	6.121414230e-7



Time Elapsed: 0.0005262
Actual root value: 1.3247
Error: -7.5717e-14

>> Main

From previous examples verifies This:

