Quantum teleportation

Quantum teleportation is an intriguing phenomenon that allows the transfer of quantum states from one location to another, even without a direct quantum communication channel. It was proposed by Charles Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres, and William Wootters in 1993. To understand this concept, let's dive into a high-level explanation while incorporating a storyline.

Meet Alice and Bob, two friends who have known each other for a long time. However, circumstances have separated them, and they now live far apart. They were once fascinated by quantum physics and conducted an experiment together, generating a special pair of entangled quantum particles called an EPR pair. When they parted ways, each of them took possession of one qubit from this EPR pair.

Years later, Bob finds himself in a precarious situation, needing a specific quantum state, denoted as |ψ. Unfortunately, Alice is the only person who has access to |ψ, but she faces multiple challenges in transferring it to Bob. Firstly, she does not know the exact state of |ψ, and secondly, she can only send classical information to Bob—ordinary information that follows the laws of classical physics.

At first glance, Alice's situation seems rather dire. Quantum mechanics prevents her from determining the state of |ψ when she only has a single copy in her possession. Moreover, even if she did somehow know |ψ, precisely describing it would require an infinite amount of classical information since |ψ exists within a continuous space. It appears that time and information constraints would make it impossible for Alice to assist Bob. However, there is hope.

Quantum teleportation comes to the rescue. It is a remarkable technique that utilizes the entangled EPR pair that Alice and Bob created years ago. Here's how it works:

1. Alice begins the teleportation process by interacting the qubit |ψ she wishes to send with her own qubit from the EPR pair.

2. Alice then performs a joint measurement on both qubits in her possession, obtaining one of four possible classical outcomes: 00, 01, 10, or 11.

3. Alice communicates this measurement result, which represents classical information, to Bob through a classical channel such as a phone call or email.

4. Bob, upon receiving Alice's message, knows which of the four possible measurement results Alice obtained. Based on this information, Bob performs a specific operation on his half of the EPR pair.

5. Remarkably, by applying the appropriate operation corresponding to Alice's measurement outcome, Bob can effectively recover the original quantum state |ψ!

In summary, despite the limitations imposed by quantum mechanics and classical communication, quantum teleportation offers a solution for Alice to transmit the unknown quantum state |ψ to Bob. By leveraging the entanglement of the EPR pair and a small overhead of classical communication, Bob can successfully reconstruct |ψ on his end, enabling Alice to fulfill her mission of delivering the qubit |ψ to him.

< infographics of quantum teleportation >

Quantum Teleportation protocol:

Step 1 → A third party called Telamon creates a Bell pair, consisting of two entangled qubits. One qubit is given to Alice, and the other is given to Bob. Alice possesses a qubit containing the quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, which she aims to teleport to Bob. The entangled qubits can be represented in Dirac notation as:

$$|e\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

The three qubit quantum system can be represented as follows :

$$\begin{aligned}|\psi\rangle \otimes |e\rangle &= \frac{1}{\sqrt{2}}(\alpha|0\rangle \otimes (|00\rangle + |11\rangle) + \beta|1\rangle \otimes (|00\rangle + |11\rangle)) \\ &= \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle)\end{aligned}$$

Step 2 → As part of the protocol, Alice applies a CNOT gate between her two qubits, with the qubit containing the required state as the control and the entangled qubit as the target. This is followed by a Hadamard gate applied to her first qubit. These operations take Alice's qubits to the Bell basis.

$$
\begin{aligned}
&(H \otimes I \otimes I)(CNOT \otimes I)(|\psi\rangle \otimes |e\rangle) \\
&= (H \otimes I \otimes I)(CNOT \otimes I)\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle) \\
&= (H \otimes I \otimes I)\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle) \\
&= \frac{1}{2}(\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle) + \beta(|010\rangle + |001\rangle - |110\rangle - |101\rangle))
\end{aligned}
$$

By regrouping terms, this state can be represented more simply as:

$$
\begin{aligned}
= \frac{1}{2}(\quad & |00\rangle(\alpha|0\rangle + \beta|1\rangle) \\
& + |01\rangle(\alpha|1\rangle + \beta|0\rangle) \\
& + |10\rangle(\alpha|0\rangle - \beta|1\rangle) \\
& + |11\rangle(\alpha|1\rangle - \beta|0\rangle) \quad)
\end{aligned}
$$

As we can observe, Alice has successfully teleported the state of her qubit to Bob's qubit. However, the state is not precisely accurate.

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Infographics

$$\frac{1}{\sqrt{2}}(\alpha|0\rangle \otimes (|00\rangle + |11\rangle) + \beta|1\rangle \otimes (|00\rangle + |11\rangle))$$

→

$$
\begin{aligned}
\frac{1}{2}(\quad & |00\rangle(\alpha|0\rangle + \beta|1\rangle) \\
&+ |01\rangle(\alpha|1\rangle + \beta|0\rangle) \\
&+ |10\rangle(\alpha|0\rangle - \beta|1\rangle) \\
&+ |11\rangle(\alpha|1\rangle - \beta|0\rangle) \quad)
\end{aligned}
$$

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Step 3 → Alice proceeds to measure her first two qubits, which she owns, and shares the measurement result with Bob. Based on the outcome of her measurement, the state of Bob's qubit can be determined.

$$
\begin{aligned}
|00\rangle &\rightarrow (\alpha|0\rangle + \beta|1\rangle) \\
|01\rangle &\rightarrow (\alpha|1\rangle + \beta|0\rangle) \\
|10\rangle &\rightarrow (\alpha|0\rangle - \beta|1\rangle) \\
|11\rangle &\rightarrow (\alpha|1\rangle - \beta|0\rangle)
\end{aligned}
$$

Step 4 → On receiving the measurement result from Alice, Bob has to to make some transformations in order to obtain the state .
Upon receiving the measurement result from Alice, Bob performs a series of transformations in order to obtain the state Iψ〉that Alice initially possessed. The transformations Bob needs to apply are as follows:

Bob's State	Bits Received	Gate Applied
$(\alpha\|0\rangle + \beta\|1\rangle)$	00	I
$(\alpha\|1\rangle + \beta\|0\rangle)$	01	X
$(\alpha\|0\rangle - \beta\|1\rangle)$	10	Z
$(\alpha\|1\rangle - \beta\|0\rangle)$	11	ZX

After implementing these transformations, Bob successfully obtains the state |ψ⟩ that Alice had, while Alice loses the state |ψ⟩ from her qubit.

If storyline is not needed:

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Quantum teleportation is a remarkable phenomenon where Alice, who possesses a quantum state called |ψ⟩, wants to transfer this state to Bob, even though they are far apart and can only communicate through classical means. Telamon, a third party, assists in the process. By utilizing an entangled pair of qubits, Alice entangles her qubit with her half of the pair and performs measurements. She shares the measurement results with Bob. Based on Alice's measurements, Bob applies specific operations to his qubit from the entangled pair. Through this remarkable protocol, Bob successfully obtains the exact quantum state |ψ⟩ that Alice initially possessed, while Alice loses the state in her qubit. This process is accomplished without physically transmitting the quantum state itself, showcasing the counterintuitive nature of quantum mechanics and the potential for information transfer beyond traditional means.