

# **QIndia-Quantum-Beginners-initiative**

## **What is Quantum Beginners initiative ?**

Quantum Beginners is an initiative of QIndia. It consists of blog posts and tutorial series focused on helping Quantum beginners find their footing”. There are many resources out there for people to learn about quantum computing. However we would like to create a place where absolute beginners can start from, like a “start here” section that then takes them step-by-step through the process of understanding the minimum before exploring the world of quantum computing for themselves. The content will be a combination of original creations by the team and links to specific sections of already existing content.

## **Why did we start this?**

The Quantum Beginners Initiative was started to address the growing need for educational outreach in the field of quantum computing. Here are some reasons why such an initiative was launched:

**Awareness and Education:** Quantum computing is an emerging field that holds great potential for advancements in various domains. However, it is a complex and abstract subject that can be challenging for beginners to grasp. The initiative aims to raise awareness about quantum computing and provide educational resources to make it more accessible to students, researchers, and enthusiasts.

**Bridging the Knowledge Gap:** Quantum computing involves concepts from diverse fields, including physics, mathematics, computer science, and engineering. Many people are interested in learning about quantum computing but may lack the necessary background or resources to dive into the subject. The initiative bridges the knowledge gap by providing structured learning materials and resources specifically designed for beginners.

**Fostering a Quantum Community:** Quantum computing is a collaborative field that thrives on knowledge sharing and community engagement. The initiative brings together

individuals who are interested in quantum computing, creating a platform for them to connect, collaborate, and learn from each other. By fostering a community of beginners, the initiative encourages peer-to-peer support and knowledge exchange.

**Future Workforce Development:** Quantum computing is expected to have a significant impact on various industries, including technology, finance, healthcare, and more. By providing educational outreach, the initiative aims to equip individuals with the foundational knowledge needed to pursue careers in quantum computing. This helps in building a skilled workforce capable of driving innovation and advancements in the field.

**Inspiring Curiosity and Exploration:** Quantum computing is a fascinating and rapidly evolving field that sparks curiosity and excitement. The initiative aims to inspire individuals to explore the wonders of quantum computing, encouraging them to ask questions, experiment, and think creatively. By

nurturing curiosity, the initiative lays the foundation for further learning and engagement in quantum computing.

**Democratizing Access:** The initiative strives to make quantum computing accessible to a wider audience, regardless of their background or geographic location. Through online resources, tutorials, and workshops, it aims to break down barriers to entry and empower individuals to embark on their quantum computing journey..

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# Section 1

## Introduction to Quantum Computing

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### What Is Quantum Computing?

In physics, a quantum (plural quanta) is the minimum amount of any physical entity (physical property) involved in an interaction. So to understand that here is an example \*\*Light consists of photons. A single photon is a discrete packet of light which cannot be broken down into smaller quantities. Light can be quantified as a given amount of these discrete packets.

Hence, the light that is propagated from one point in space to another is composed of many of these individual photons” Quantum computing is focused on developing computational technology based on the principles of Quantum Mechanics. In simpler terms, we can say that Quantum computing is an area of study focused around the principles of quantum theory and computing. Quantum computing is a field comprising aspects of computer science, physics, and mathematics that utilizes quantum mechanics to solve complex computational problems. Why do we need Quantum Computing ? Quantum Computing is gradually becoming a subject of interest among researchers and technology enthusiasts. This is because it has the potential to lead us to breakthroughs in several fields, including drug discovery, cybersecurity. The primary reasons researchers are interested in Quantum Computing is the potential speedup Quantum Computers could bring to many real-world computational problems. For example, factoring a large integer  $n$  would take about  $O(e^{(\log(n)^{1/3})})$  operations to complete on a classical

computer, whereas on a Quantum Computer, it would take exponentially fewer operations,  $O(\log(n)^2)$ .  $O(n)$  is just a computer science way of saying that the time taken depends directly on  $n$ , which is the large integer we are trying to factor. This was proved by Peter Shor in 1994. It is now known as Shor's Algorithm and has the potential to break down public-key encryption (the protocols that make sure our emails and text messages are secure) provided we can build a Quantum Computer with enough qubits. Shor's algorithm is one example of what we call a "quantum speedup" i.e. where it has been demonstrated that a quantum computer can solve a task much faster than a classical computer can. Currently there are only a few such examples of quantum speedups, but as more researchers enter the field, we are hopeful that we will find more to come! On a more constructive note, director of engineering at Google, Hartmut Neven, also noted that quantum computers could help build better climate models that could give us more insight into how humans are influencing the environment[1]. Although Quantum Computing is in its initial development we can predict some amazing new developments. What is needed for the field of quantum computing to grow? Currently there are two main avenues of research: (1) understanding how to physically build and engineer quantum computers to make them bigger and better and (2) understanding how we could use these quantum computers to solve real world problems much more efficiently than with the computers we have today. However a large infrastructure is required to push this research forward, including increasing quantum awareness and education. This can be at various levels, ranging from teaching students the knowledge and skills they need to pursue a career in quantum research, to educating business leaders, government entities and investors such that they can support and invest in the growth of quantum computing." Conclusion Quantum computers have the potential to revolutionize computation by making certain types of classically complex & tough problems solvable. While no quantum computer is yet sophisticated enough to carry out calculations that a classical computer can't, great progress is underway.

References New Yorker article on the Quantum Race

### **General motivation, postulates of quantum mechanics, Dirac notation**

**State Space and Superposition:** Quantum systems are described by state vectors, represented by complex numbers. The state vector represents the complete information about the system. Quantum systems can exist in a superposition of states, where they can be in a combination of multiple states simultaneously. **Measurement and Observables:** When a measurement is

performed on a quantum system, the outcome is one of the eigenvalues of the corresponding observable. Each observable is associated with a Hermitian operator, and its eigenvalues represent the possible measurement outcomes. Probabilistic Interpretation: The outcome of a measurement in quantum mechanics is probabilistic. The probability of obtaining a specific measurement outcome is given by the square of the absolute value of the corresponding coefficient in the superposition state. Wave Function Collapse: After a measurement, the quantum system collapses into one of the eigenstates associated with the measurement outcome. This collapse is non-deterministic, and the probability of collapsing into a particular state is determined by the coefficients in the superposition state. Time Evolution: Quantum systems evolve over time according to the Schrödinger equation. The time evolution of a quantum state is determined by the Hamiltonian operator, which represents the energy of the system. Quantum Entanglement: Quantum entanglement is a phenomenon where two or more particles become correlated in such a way that the state of one particle cannot be described independently of the others. Entangled particles exhibit non-local correlations and can influence each other instantaneously, even when separated by large distances. Quantum Uncertainty: The Heisenberg uncertainty principle states that there are inherent limitations to simultaneously measuring certain pairs of observables, such as position and momentum, with high precision. The more precisely one observable is measured, the less precisely the complementary observable can be known.

Postulate 1: State of a system or the space of states The state of any physical system is specified, at each time  $t$ , by a state vector  $|\psi(t)\rangle$  in a Hilbert space  $H$ ;  $|\psi(t)\rangle$  contains (and serves as the basis to extract) all the needed information about the system. Any superposition of state vectors is also a state vector. Postulate 2: Observables and operators To every physically measurable quantity  $A$ , called an observable or dynamical variable, there corresponds a linear Hermitian operator  $A^\hat{}$  whose eigenvectors form a complete basis. Postulate 3: Measurements and eigenvalues of operators The measurement of an observable  $A$  may be represented formally by the action of  $A^\hat{}$  on a state vector  $|\psi(t)\rangle$ . The only possible result of such a measurement is one of the eigenvalues  $a_n$  (which are real) of the operator  $A^\hat{}$ . If the result of a measurement of  $A$  on a state  $|\psi(t)\rangle$  is  $a_n$ , the state of the system immediately after the measurement changes to  $|\psi_n\rangle$ :  $A^\hat{}|\psi(t)\rangle = a_n|\psi_n\rangle$  Postulate 4: Probabilistic outcome of measurements Discrete spectra: When measuring an observable  $A$  of a system in a state  $|\psi\rangle$ , the probability of obtaining one of the nondegenerate eigenvalues  $a_n$  of the corresponding operator  $A^\hat{}$  is given by  $P_n(a_n) = |\langle\psi_n|\psi\rangle|^2$

$\langle \psi | \psi \rangle = |a_n| 2 \langle \psi | \psi \rangle 1$  where  $|\psi_n\rangle$  is the eigenstate of  $A^\hat{}$  with eigenvalue  $a_n$ . Continuous spectra: To determine the probability density that a measurement of  $A^\hat{}$  yields a value between  $a$  and  $a + da$  on a system which is initially in a state  $|\psi\rangle$ :  $dP(a) da = |\psi(a)|^2 \langle \psi | \psi \rangle = |\psi(a)|^2 R +\infty -\infty |\psi(a')|^2 da'$

Postulate 5: Time evolution of a system The time evolution of the state vector  $|\psi(t)\rangle$  of a system is governed by the time-dependent Schrodinger " equation  $i\hbar^{-1} \partial|\psi(t)\rangle / \partial t = H^\hat{ } |\psi(t)\rangle$  where  $H^\hat{ }$  is the Hamiltonian operator corresponding to the total energy of the system.

### The Stern Gerlach experiment

The Stern-Gerlach experiment is a landmark experiment in quantum mechanics that provided crucial evidence for the existence of quantized angular momentum or spin. It demonstrated that certain particles have intrinsic angular momentum and can only assume certain discrete orientations when subjected to a magnetic field gradient. In the experiment, a beam of neutral particles, such as silver atoms, is passed through a narrow slit to create a well-defined beam. This beam is then directed through a region with a non-uniform magnetic field, typically produced by a magnet. The magnetic field gradient causes the beam to split into distinct paths. To understand the behavior of the particles, let's consider the case of a silver atom with a single unpaired electron. According to quantum mechanics, the electron's spin can only take one of two possible values: spin-up ( $+1/2$ ) or spin-down ( $-1/2$ ), along a specific direction. We can represent these two possible spin states as a two-component vector called a spinor:  $|\text{spin-up}\rangle = [1, 0]$   $|\text{spin-down}\rangle = [0, 1]$  When a beam of silver atoms passes through the magnetic field, the atoms can interact with the field in one of two ways. If the atom has a spin-up orientation, it experiences an upward force due to the magnetic field gradient. Conversely, if the atom has a spin-down orientation, it experiences a downward force. As a result, the beam splits into two distinct paths, one corresponding to spin-up and the other to spin-down. Now, let's introduce the concept of a magnetic moment. The magnetic moment of a particle, denoted by  $\mu$ , is a vector quantity that measures the strength and orientation of the particle's magnetic field. In the case of an electron, the magnetic moment is directly proportional to its spin. In the presence of a magnetic field, the interaction between the magnetic moment and the field causes the particle to experience a torque, which can be mathematically described using the following equation:  $\tau = \mu \times B$  where  $\tau$  is the torque,  $\mu$  is the magnetic moment, and  $B$  is the magnetic field. The direction of the torque is perpendicular to both  $\mu$  and  $B$ , following the right-hand rule. When the silver atoms pass through the Stern-Gerlach apparatus, the torque causes the magnetic moments associated with the spin to align either parallel or anti-parallel to the

magnetic field gradient. This alignment results in the splitting of the beam into two paths corresponding to the different spin orientations. After passing through the magnetic field gradient, the two split beams can be detected on separate screens. By analyzing the distribution of atoms on the screens, the experimenters found that the beam was always split into two distinct paths, corresponding to the two possible spin states of the particles. The results of the Stern-Gerlach experiment provided experimental evidence for the quantization of angular momentum and the existence of spin in particles. It demonstrated that particles have intrinsic properties beyond classical physics and that their behavior can be described by quantum mechanical principles. In summary, the Stern-Gerlach experiment showed that particles with intrinsic angular momentum, such as electrons, have quantized spin orientations. The experiment's findings played a crucial role in the development of quantum mechanics and our understanding of the fundamental nature of particles. we'll introduce the concept of spin operators and use them to analyze the behavior of particles in a magnetic field. In quantum mechanics, the spin of a particle is described by spin operators, denoted as  $S$ . The spin operators act on the spin states of the particles and provide information about the spin's orientation and properties. Let's consider a silver atom with a single unpaired electron. The spin of the electron can be represented by a two-component vector called a spinor:  $|\text{spin-up}\rangle = [1, 0] |\text{spin-down}\rangle = [0, 1]$  The spin operators are defined as matrices that operate on these spin states. We'll use the Pauli matrices  $\sigma_x$ ,  $\sigma_y$ , and  $\sigma_z$  to represent the spin operators in the x, y, and z directions, respectively:  $\sigma_x = [[0, 1], [1, 0]] \quad \sigma_y = [[0, -i], [i, 0]] \quad \sigma_z = [[1, 0], [0, -1]]$  When a silver atom passes through a magnetic field with a gradient, the interaction between the magnetic moment associated with the electron's spin and the magnetic field causes the atom to split into distinct paths corresponding to different spin orientations. To analyze this behavior mathematically, let's consider the case where the magnetic field is in the z direction. The Hamiltonian  $H$  describing the interaction between the atom and the magnetic field is given by:  $H = \mu \cdot B = \gamma \sigma_z B_z$  where  $\mu$  is the magnetic moment,  $B$  is the magnetic field,  $\gamma$  is the gyromagnetic ratio, and  $B_z$  is the z component of the magnetic field. The time-independent Schrödinger equation for this system is:  $H |\psi\rangle = E |\psi\rangle$  Substituting the Hamiltonian, we have:  $\gamma \sigma_z B_z |\psi\rangle = E |\psi\rangle$  Now, let's consider an initial state  $|\psi\rangle$  with a specific spin orientation, such as  $|\text{spin-up}\rangle$ . We can express this initial state as a linear combination of the spin-up and spin-down states:  $|\psi\rangle = \alpha |\text{spin-up}\rangle + \beta |\text{spin-down}\rangle$  Plugging this into the Schrödinger equation, we get:  $\gamma \sigma_z B_z (\alpha |\text{spin-up}\rangle + \beta |\text{spin-down}\rangle) = E (\alpha |\text{spin-up}\rangle + \beta |\text{spin-down}\rangle)$  Expanding the equation, we have:  $\gamma B_z (\alpha \sigma_z |\text{spin-up}\rangle + \beta \sigma_z |\text{spin-down}\rangle) = E (\alpha |\text{spin-up}\rangle + \beta |\text{spin-down}\rangle)$  This equation represents the

interaction of the spin states with the magnetic field. The coefficients  $\alpha$  and  $\beta$  represent the probability amplitudes of the spin-up and spin-down states, respectively. Solving this equation yields the energy eigenvalues  $E$  and the corresponding eigenstates  $|\psi\rangle$ . These eigenstates represent the different paths or beams observed in the Stern-Gerlach experiment. By measuring the distribution of atoms on screens placed after the magnetic field gradient, experimenters can determine the probabilities associated with different spin orientations. The probabilities are obtained by taking the squared magnitudes of the probability amplitudes  $\alpha$  and  $\beta$ . In summary, the mathematical explanation of the Stern-Gerlach experiment involves using spin operators, the Schrödinger equation, and the interaction between the spin states and the magnetic field. The resulting equations allow us to determine the energy eigenvalues and eigenstates, which correspond to the different paths observed in the experiment. By analyzing the probabilities associated with different spin orientations, we can understand the behavior of particles in the presence of a magnetic field gradient.

### Story :

The young wizard in the Harry Potter universe was enrolled in Hogwarts School of Witchcraft and Wizardry to study advanced magical theory. One day, Harry heard about the Stern-Gerlach experiment while listening to a lecture on the characteristics of magical particles. (\* an experiment conducted by muggles that assessed the quantized spin of subatomic particles) Harry was intrigued by the idea and pondered whether the same ideas could be used to describe magical particles. Harry made the decision to run his own experiment to verify his theory with the assistance of his friends Hermione and Ron. They began by creating a beam of magical particles, consisting of spells and potions, and passing it through a magnetic field. Before conducting the experiment, they expected the particles to create a continuous smear on the detector screen (\*based on the classical physics prediction

But to their surprise, the particles diverged into two distinct directions. Harry, amused by the outcome, made the decision to find Stern and Gerlach to ask them further

questions about the experiment. The scientist said that the experiment was intended to detect the silver atom's magnetic moment, but the discovery of spin during the procedure led to the explanation for why the particles split into two distinct pathways. When Harry enquired about the deflection of the particles in the magnetic field, Stern and Gerlach responded that it was inversely correlated with the kinetic energy of the particles. The amount of deflection and kinetic energy of the particles increased with the strength of the magnetic field. By altering the strength of the magnetic field in their experiment, Harry and his friends chose to put this notion to the test. They discovered that the deflection of the particles grew according to their kinetic energy when the field's strength was decreased. This outcome supported Stern and Gerlach's discovery of a connection between deflection and kinetic energy. Harry and his companions discovered that the concepts of kinetic energy and space quantization were related as they continued their studies. They proposed that the quantized regions of the magnetic field might modify the particle kinetic energy, resulting in various patterns on the detector screen. For instance, they found that varying degrees of deflection and kinetic energy resulted from the fact that particles with particular spins could only exist in particular, quantized positions inside the magnetic field. They also found that the size of the quantized units of space may vary depending on the strength of the magnetic field, producing various patterns on the detector screen. As they continued to experiment and gather data, Harry and his friends developed a greater understanding of the properties of magical particles and the principles of muggle science. Their research showed that the relationship between deflection and kinetic energy was a crucial factor in understanding the behavior of particles. In the end, Harry's experiment not only expanded his understanding of the properties of magic and the principles of muggle science, but it also showed that the deflection of particles was inversely proportional to their kinetic energy. Harry realized that the principles of space quantization and kinetic energy had many applications in the field of magical engineering. He began to apply these principles to his own magical inventions, making them more efficient and powerful than ever before. The term "spin" refers to a quantum property of an atom or subatomic particle that behaves as if it were spinning around an axis. However, the spin

is not an actual rotation, but rather an intrinsic property that has no classical counterpart. The Stern-Gerlach experiment demonstrated that the spin of an atom can be measured by the deflection of its magnetic moment in an inhomogeneous magnetic field. When a beam of atoms is passed through a region of strong magnetic field, the spin of the atoms causes them to be deflected in a direction that depends on their spin orientation. In particular, the experiment showed that the spin of an atom can only take certain discrete values, corresponding to different quantized states. For example, in the case of a silver atom, the spin can take only two possible values,  $+1/2$  or  $-1/2$ , along a given axis. The Stern-Gerlach experiment thus provided early evidence for the quantization of angular momentum in quantum mechanics, and helped pave the way for the development of spin-based technologies, such as magnetic resonance imaging and spintronics.

## The Double Slit experiment

The Double Slit experiment is a fundamental experiment in quantum mechanics that demonstrates the wave-particle duality of matter and the phenomenon of interference. It involves shining a beam of particles, such as electrons or photons, through two closely spaced slits and observing the resulting pattern on a screen. Despite being sent through the slits individually, the particles exhibit an interference pattern similar to that of waves. To understand the Double Slit experiment mathematically, we'll use the principles of wave mechanics and probability amplitudes. Let's consider a beam of particles, such as electrons, directed toward two slits labeled as "A" and "B". We'll assume that the slits are very narrow compared to the wavelength of the particles, so they act as point sources of waves. The wavefunction describing the particles can be represented by  $\Psi(x, t)$ , where  $x$  represents the position and  $t$  represents time. According to the principle of superposition, the total wavefunction  $\Psi(x, t)$  can be expressed as the sum of the wavefunctions passing through each slit. We'll denote the wavefunction

passing through slit A as  $\Psi_A(x, t)$  and the wavefunction passing through slit B as  $\Psi_B(x, t)$ .  $\Psi(x, t) = \Psi_A(x, t) + \Psi_B(x, t)$  Now, let's assume that the particles are non-interacting, meaning they do not influence each other's paths. The probability of finding a particle at a specific position  $x$  on the screen is given by the squared magnitude of the total wavefunction:  $P(x) = |\Psi(x, t)|^2 = |\Psi_A(x, t) + \Psi_B(x, t)|^2$

Expanding the equation, we have:  $P(x) = |\Psi_A(x, t)|^2 + |\Psi_B(x, t)|^2 + \Psi_A^*(x, t)\Psi_B(x, t) + \Psi_B^*(x, t)\Psi_A(x, t)$  The first two terms represent the probabilities associated with particles passing through slit A and slit B individually. They produce separate intensity patterns on the screen. The third term,  $\Psi_A^*(x, t)\Psi_B(x, t)$ , and the fourth term,  $\Psi_B^*(x, t)\Psi_A(x, t)$ , represent interference terms. Interference occurs when the waves passing through the slits overlap and interfere with each other constructively or destructively.

To further analyze the interference pattern, we can assume that the wavefunctions passing through the slits have the same amplitude, denoted as  $A$ . This assumption simplifies the equations, leading to:  $P(x) = |A|^2 (|\Psi_A(x, t)|^2 + |\Psi_B(x, t)|^2 + 2\text{Re}[\Psi_A^*(x, t)\Psi_B(x, t)])$  The interference term  $2\text{Re}[\Psi_A^*(x, t)\Psi_B(x, t)]$  depends on the relative phase between the wavefunctions passing through the slits. Constructive interference occurs when the phase difference is an integer multiple of  $2\pi$ , resulting in bright fringes on the screen. Destructive interference occurs when the phase difference is a half-integer multiple of  $2\pi$ , leading to dark fringes. The interference pattern observed in the Double Slit experiment resembles the pattern produced by waves, even though the particles are sent through the slits individually. This phenomenon is a clear demonstration of wave-particle duality. In conclusion, the Double Slit experiment can be mathematically explained by considering the wavefunctions passing through the two slits and their interference. The interference terms give rise to the characteristic pattern of bright and dark fringes on the screen, illustrating the wave-like nature of particles in quantum mechanics.

### **Story :**

The Double Slit experiment is the double slit experiment in physics, where scientists shoot particles (like photons or electrons) through two slits in a barrier and observe

where they land on a screen behind the barrier. Just like Harry casting a spell through the slits in the wall, the particles pass through the slits in the barrier and create an interference pattern on the screen behind it. When scientists observe the particles going through the slits to see which one they pass through, the interference pattern disappears and the particles behave like individual particles instead of waves. This is called the wave-particle duality and it's one of the most fascinating concepts in physics. It means that particles can behave like both waves and particles depending on how they are observed. So in summary, the double slit experiment shows us that particles can behave like waves and create an interference pattern, but when we try to observe which slit the particles go through, the interference pattern disappears and the particles behave like individual particles instead. Let's imagine that Harry, Ron, and Hermione are exploring a mysterious room in Hogwarts. As they enter the room, they see a wall with two slits in it and a screen behind it. Being curious, they each take turns casting a spell through the slits and observe where it hits the screen. They notice that when they cast the spells one at a time, the spells land in individual spots on the screen. However, when they all cast their spells at the same time, they notice that the spells seem to interfere with each other and create a pattern on the screen. This is similar to the interference pattern created by waves in the double slit experiment. But when they try to observe which spell went through which slit, they notice that the interference pattern disappears and the spells land in individual spots again. This is like observing the particles in the double slit experiment and causing the interference pattern to disappear. Let's use the analogy of throwing a ball at a wall with two slits in it to explain the double-slit experiment. Imagine you are standing in front of a wall with two slits in it, and you have a ball in your hand. You throw the ball at the wall and it goes through one of the slits. On the other side of the wall, you have a screen that is set up to record where the ball lands. You repeat this process many times, throwing the ball at the wall through either of the two slits, and recording where it lands on the screen. Now, if you were to repeat this process with many balls, you might expect to see two bright spots on the screen, directly behind each of the slits in the wall. This is because you would expect the balls to pass through the slits and hit the screen behind them. However, in

the double-slit experiment, something strange happens. When you repeat the experiment with very small particles, like electrons or photons, instead of balls, you don't get two bright spots on the screen. Instead, you get a pattern of light and dark bands, called an interference pattern. The pattern shows that the particles seem to be interfering with each other as they pass through the slits. This interference pattern is what led scientists to propose the wave-particle duality theory. According to this theory, small particles like electrons and photons can behave like waves, and waves can interfere with each other. So, when the particles pass through the two slits, they interfere with each other, creating the interference pattern on the screen. To summarize, the double-slit experiment is a way to study the behavior of small particles, like electrons and photons, and it has led to the wave-particle duality theory. The experiment involves passing particles through two slits in a wall and observing the interference pattern that they create on a screen.

## AXIOM

Quantum mechanics is a fundamental theory in physics that describes the behavior of particles on a very small scale, such as atoms and subatomic particles. The axioms of quantum mechanics are the basic principles that govern the behavior of these particles. The first axiom of quantum mechanics is that the state of a particle is described by a wave function, which is a mathematical function that represents the probability of finding the particle in a particular location or state. The second axiom of quantum mechanics is that measurements of particles are probabilistic, meaning that the outcome of a measurement is determined by the wave function of the particle. The third axiom of quantum mechanics is that particles can be in a superposition of states, meaning that they can exist in multiple states simultaneously. The fourth axiom of quantum mechanics is that particles can become entangled, meaning that the state of one particle can affect the state of another particle, even if they are separated by a great distance. The axioms of quantum mechanics provide a framework for understanding the behavior of particles on a very small scale, and they have profound implications for our understanding of the nature of matter and the universe. In simple examples: Superposition: A

particle can exist in multiple states at once. Imagine you have a coin that you toss in the air. In classical mechanics, the coin can either be heads or tails, but not both at the same time. In quantum mechanics, the coin can be both heads and tails at the same time until it is observed or measured. Observables: The properties of a particle are represented by observables, which are operators that act on the wave function. The wave function is a mathematical expression that describes the probability of finding a particle in a particular state. An observable is a physical property that can be measured, such as the position or momentum of a particle. Measurement: When an observable is measured, the wave function "collapses" into one of its possible states. For example, if you measure the position of an electron, the wave function collapses to a single position. This is known as the collapse of the wave function. Uncertainty Principle: There is a fundamental limit to how accurately certain pairs of observables can be measured at the same time. For example, you cannot simultaneously measure the position and momentum of a particle with arbitrary precision. The more accurately you measure one, the less accurately you can measure the other. Entanglement: Particles can become entangled, meaning that the state of one particle is dependent on the state of the other. Imagine you have two coins that you toss in the air at the same time. In classical mechanics, the state of one coin is independent of the state of the other. In quantum mechanics, the states of the coins can become entangled, so that the outcome of one toss depends on the outcome of the other toss. These axioms may seem counterintuitive compared to the classical mechanics we observe in our daily lives, but they have been supported by numerous experiments and are essential to our understanding of the quantum world. Superposition: This axiom states that a quantum particle can exist in multiple states simultaneously until it is observed or measured. A real-world example of this is a light switch. Before you turn the switch on or off, the light bulb is in a state of superposition - it could be either on or off. It's only when you flip the switch that the light bulb collapses into one of these states. Entanglement: This axiom states that particles can become entangled, so that measuring the state of one particle immediately determines the state of the other, no matter how far apart they are. A common example of this is a pair of gloves. If you have two gloves that are identical except for their left and right orientations, and you put one on your left hand and the other on your right hand, then the two gloves are entangled. Measuring the state of one glove (i.e., whether it's left or right) immediately determines the state of the other glove, even if they're on opposite sides of your body. Uncertainty: This axiom states that the more precisely we measure one property of a particle (e.g., its position), the less precisely we can know another property of the same particle (e.g., its momentum). A common example of this is throwing a

ball. If you throw a ball and want to know exactly where it will land, you have to throw it very precisely. But if you throw it very precisely, you can't throw it very fast, so you don't know exactly how fast it will be moving when it lands

### **Story :**

Quantum mechanics is a fundamental theory in physics that describes the behavior of particles on a very small scale, such as atoms and subatomic particles. The axioms of quantum mechanics are the basic principles that govern the behavior of these particles. Let's imagine that Harry and Hermione are studying the properties of magical particles in their potions class. The first axiom of quantum mechanics is that the state of a particle is described by a wave function, which is a mathematical function that represents the probability of finding the particle in a particular location or state. In Harry and Hermione's potions class, they learn that the state of a magical particle, such as a potion ingredient, can be described by a wave function that determines the probability of finding the particle in a certain location or state. For example, the wave function of a bubotuber pus might describe the probability of finding it in a certain color or texture. The second axiom of quantum mechanics is that measurements of particles are probabilistic, meaning that the outcome of a measurement is determined by the wave function of the particle. Harry and Hermione learn that when they measure the properties of magical particles, such as the color or texture of a potion ingredient, the outcome is probabilistic and determined by the wave function of the particle. This means that they cannot predict the exact outcome of a measurement, but only the probability of different outcomes. The third axiom of quantum mechanics is that particles can be in a superposition of states, meaning that they can exist in multiple states simultaneously. In potions class, Harry and Hermione discover that magical particles, like potion ingredients, can exist in a superposition of states. For example, a bubotuber pus might exist in a superposition of colors or textures at the same time, until a measurement is made. The fourth axiom of quantum mechanics is that particles can become entangled, meaning that the state of one particle can affect the state of another particle, even if they are separated by a great distance. Harry and Hermione

learn that magical particles can become entangled, meaning that the state of one particle can affect the state of another particle, even if they are separated by a great distance. This is similar to how Harry and Voldemort's wands became entangled in the Harry Potter series, causing their spells to be connected in unexpected ways. The axioms of quantum mechanics provide a framework for understanding the behavior of particles on a very small scale, and they have profound implications for our understanding of the nature of matter and the universe. In simple examples:

**Superposition:** A particle can exist in multiple states at once. Imagine you have a coin that you toss in the air. In classical mechanics, the coin can either be heads or tails, but not both at the same time. In quantum mechanics, the coin can be both heads and tails at the same time until it is observed or measured.

**Observables:** The properties of a particle are represented by observables, which are operators that act on the wave function. The wave function is a mathematical expression that describes the probability of finding a particle in a particular state. An observable is a physical property that can be measured, such as the position or momentum of a particle.

**Measurement:** When an observable is measured, the wave function "collapses" into one of its possible states. For example, if you measure the position of an electron, the wave function collapses to a single position. This is known as the collapse of the wave function.

**Uncertainty Principle:** There is a fundamental limit to how accurately certain pairs of observables can be measured at the same time. For example, you cannot simultaneously measure the position and momentum of a particle with arbitrary precision. The more accurately you measure one, the less accurately you can measure the other.

**Entanglement:** Particles can become entangled, meaning that the state of one particle is dependent on the state of the other. Imagine you have two coins that you toss in the air at the same time. In classical mechanics, the state of one coin is independent of the state of the other. In quantum mechanics, the states of the coins can become entangled, so that the outcome of one toss depends on the outcome of the other toss. These axioms may seem counterintuitive compared to the classical mechanics we observe in our daily lives, but they have been supported by numerous experiments and are essential to our understanding of the quantum world.

**Superposition:** This axiom states that a quantum

particle can exist in multiple states simultaneously until it is observed or measured. A real-world example of this is a light switch. Before you turn the switch on or off, the light bulb is in a state of superposition - it could be either on or off. It's only when you flip the switch that the light bulb collapses into one of these states. Entanglement: This axiom states that particles can become entangled, so that measuring the state of one particle immediately determines the state of the other, no matter how far apart they are. A common example of this is a pair of gloves. If you have two gloves that are identical except for their left and right orientations, and you put one on your left hand and the other on your right hand, then the two gloves are entangled. Measuring the state of one glove (i.e., whether it's left or right) immediately determines the state of the other glove, even if they're on opposite sides of your body. Uncertainty: This axiom states that the more precisely we measure one property of a particle (e.g., its position), the less precisely we can know another property of the same particle (e.g., its momentum). A common example of this is throwing a ball. If you throw a ball and want to know exactly where it will land, you have to throw it very precisely. But if you throw it very precisely, you can't throw it very fast, so you don't know exactly how fast it will be moving when it lands

## Bell inequality

The Bell inequality experiment, also known as Bell's theorem or Bell's test, is a crucial experiment in quantum mechanics that tests the concept of local realism and provides evidence for the non-locality of quantum entanglement. It explores the phenomenon of entanglement between two particles and the violation of classical inequalities. To understand the Bell inequality experiment, let's consider two particles, often referred to as particles A and B, that have interacted in the past and are now separated. The particles are in an entangled state, meaning that their quantum states are correlated in a way that cannot be explained by classical physics. Mathematically, we can describe the entangled state of the particles using a quantum mechanical formalism called the Bell state, specifically the singlet state:  $|\Psi\rangle = 1/\sqrt{2} (|01\rangle - |10\rangle)$ . In this state, if we measure the spin of particle A along a particular axis, it will be found either in the

spin-up state ( $|0\rangle$ ) or the spin-down state ( $|1\rangle$ ) with equal probability. Similarly, if we measure the spin of particle B along the same axis, it will also be found in either the spin-up or spin-down state with equal probability. Now, let's introduce two different measurement axes for each particle, denoted as a and b, respectively. The measurement results along these axes can be represented by binary outcomes, +1 or -1, indicating whether the spin is up or down. The key aspect of the Bell inequality experiment lies in testing the correlations between the measurement outcomes of particles A and B. Classical physics assumes that these correlations can be explained by local hidden variables, meaning that the measurement outcomes are predetermined by some local properties of the particles. However, Bell's theorem states that certain combinations of measurement outcomes, known as Bell inequalities, cannot be satisfied if the correlations are solely determined by local hidden variables. The violation of these inequalities implies that the measurements of the entangled particles are non-local and cannot be explained by classical physics. One commonly used Bell inequality is the CHSH inequality, named after its creators Clauser, Horne, Shimony, and Holt. It is expressed as:  $|S| \leq 2$  where S is a quantity calculated from the measured correlations:  $S = E(a, b) + E(a, b') + E(a', b) - E(a', b')$ . Here,  $E(a, b)$  represents the expectation value of the product of measurement outcomes for axes a and b. The primes ('') indicate different axes. If the absolute value of S exceeds 2, then the CHSH inequality is violated, indicating the presence of non-local correlations that cannot be explained by local hidden variables. Experimental tests of the Bell inequality involve performing measurements on entangled particles and calculating the value of S. Quantum mechanics predicts that the violation of the CHSH inequality is possible, while classical physics adhering to local realism predicts that the value of S should not exceed 2. Over the years, numerous experiments have been conducted to test the Bell inequality, and consistently, the results have shown violations of the inequality, providing strong evidence for the non-local nature of quantum entanglement. In conclusion, the Bell inequality experiment demonstrates the existence of non-local correlations between entangled particles and provides evidence against the idea of local realism. The

violation of Bell inequalities supports the fundamental principles of quantum mechanics and highlights the unique properties of entangled quantum systems.

### **Story :**

Bell inequality In the wizarding world, there is a spell called "Accio" that allows you to summon objects to your location. Imagine that Harry Potter and Hermione Granger are trying to determine if this spell works instantaneously, or if it takes some time for the object to travel from its original location to the person casting the spell. To test this, they set up an experiment where they both have a box containing a hidden object, and they each cast the Accio spell at the same time. If the objects arrive at their respective locations at the same time, then the spell works instantaneously. However, if there is a delay between when one person receives their object and when the other person receives their object, then the spell does not work instantaneously. The Bell inequality is a mathematical way of testing whether two distant objects are behaving independently or whether they are somehow connected or "entangled". In the context of the Harry Potter experiment, the boxes containing the hidden objects are like the distant objects being tested. If the objects are behaving independently, then the time it takes for each person to receive their object should be random and unrelated to the other person's time. However, if the objects are entangled, then the time it takes for each person to receive their object will be related and non-random. The Bell inequality uses statistical analysis to determine whether the objects are behaving independently or whether they are entangled. If the results violate the Bell inequality, then it means that the objects are entangled and connected in some way that cannot be explained by classical physics. In actual terms, the Bell inequality is a theorem in quantum mechanics that relates to the correlations between distant particles. It shows that the predictions of quantum mechanics cannot be explained by classical physics, and that there must be some form of non-locality or entanglement between the particles. The Bell inequality has been tested experimentally and has been shown to hold true, confirming the strange and fascinating nature of the quantum world. Bell inequality is a concept in quantum physics that helps us understand the nature of reality at a fundamental level. It is based on the idea of entanglement, which is a strange property of quantum particles that can be inextricably linked with each other, no matter how far apart they are. Imagine two entangled particles, let's call them A and B. If we measure the spin of particle A in one direction, we can instantly know the spin of particle B in the opposite direction, even if it is located on the other side of the universe. This is called "spooky action at a distance." Now, Bell's inequality says that in any

physical theory, there is a limit to how much correlation there can be between the measurements of A and B. If this limit is violated, then the theory must be wrong. In other words, if we measure A and B and find a stronger correlation than predicted by the theory, then we have discovered something new about the nature of reality. For example, let's say we have two coins, A and B, which are secretly entangled with each other. Each coin can either be heads or tails. If we flip coin A and it comes up heads, we would expect coin B to come up tails, because the coins are entangled in such a way that the spins are opposite. However, if we flip coin A many times and record the results, and then flip coin B many times and record the results, we can calculate the correlation between them. If this correlation violates the Bell inequality, then we have evidence that the coins are entangled in a way that is not predicted by classical physics. In essence, Bell's inequality is a test of the weirdness of quantum mechanics. It helps us understand how entanglement works and how it can be used to make predictions about the behavior of particles, even when they are far apart. Dirac notation bra and ket (changes to be made) As Harry Potter was studying the intricacies of magical theory, he came across a strange notation he had never seen before. The notation consisted of vertical bars enclosing labels, and seemed to be used to describe the state of a magical object or spell. Curious, Harry asked Hermione Granger about it. Hermione, always eager to share her knowledge, explained that it was called Dirac notation and was commonly used in the study of quantum mechanics. But Harry was more interested in how it could be applied to the magical world. Hermione thought for a moment before suggesting that they could use Dirac notation to describe the state of spells and magical objects. She drew a vertical line with a label on it and said, this is called a ket, and it represents a quantum state. The label represents the specific state of the system. "For example," Hermione said, "we could use the notation  $|Expelliarmus\rangle$  to represent the state of the spell 'Expelliarmus' in a quantum sense. And we could use  $|\text{Elder Wand}\rangle$  to describe the state of the Elder Wand." We can also use a bra, which is represented by a horizontal line with a label on it. The bra represents the complex conjugate of the ket, and can be used to describe the adjoint of a quantum operator." She drew a horizontal line with a label on it and said, "For example, if we have a spell represented by the ket  $|A\rangle$ , we can represent the adjoint of that spell with the bra  $\langle A|$ . This is useful when we need to calculate probabilities or perform other operations on quantum states." Harry was fascinated by this new application of mathematical notation to the magical world. He wondered if they could also use Dirac notation to describe the state of wizards and witches. Hermione thought about it for a moment before suggesting, "We could use  $|\text{Dumbledore}\rangle$  to represent the state of a particularly powerful wizard

like Dumbledore, and  $|\text{Neville}\rangle$  to represent the state of a less skilled wizard like Neville Longbottom." Now as both continued their conversation, they began to realize the need for a standardized way to describe magical states. With so many different spells, magical objects, and wizards with varying levels of skill, it was becoming increasingly difficult to keep track of everything

## EPR Paradox

The EPR (Einstein-Podolsky-Rosen) paradox is a thought experiment in quantum mechanics that challenges the concept of local realism and raises questions about the nature of physical reality. Proposed by Albert Einstein, Boris Podolsky, and Nathan Rosen in 1935, the experiment explores the phenomenon of quantum entanglement and the implications it has on the fundamental principles of physics. The EPR paradox begins by considering a pair of particles, often referred to as particles A and B, that have interacted in the past and are now separated. The particles are in an entangled state, meaning that their quantum states are correlated in a way that cannot be explained by classical physics. Mathematically, the entangled state of the particles can be described using a quantum mechanical formalism. Let's denote the spin of particle A along a certain axis as A and the spin of particle B along the same axis as B. The entangled state can be represented by the following wave function:  $|\Psi\rangle = 1/\sqrt{2} (|+A, -B\rangle - |-A, +B\rangle)$  In this state, if we measure the spin of particle A along the chosen axis, we will obtain either a positive result (+A) or a negative result (-A) with equal probability. Similarly, if we measure the spin of particle B along the same axis, we will obtain either a positive result (+B) or a negative result (-B) with equal probability. The EPR paradox arises when we consider the correlation between the measurement outcomes of particles A and B. According to quantum mechanics, if we measure the spin of particle A and obtain a positive result (+A), the state of particle B instantaneously collapses into the negative spin state (-B), regardless of the distance between the particles. This implies that the measurement of one particle has an instantaneous influence on the state of the other particle, violating the principles of locality and causality. Einstein, Podolsky, and Rosen argued that this non-local influence contradicts the principles of special relativity, which states that no information or influence can travel faster than the speed of light. They concluded that quantum mechanics must be incomplete and proposed the existence of hidden variables, unknown properties of the particles that determine their measurement outcomes in a deterministic way. In response to the EPR paradox, physicist John Bell formulated a theorem, known as Bell's theorem, which provides a way to experimentally

test the predictions of quantum mechanics against the assumptions of local realism. Bell's theorem states that certain combinations of measurement outcomes, known as Bell inequalities, cannot be satisfied if the correlations between entangled particles are solely determined by local hidden variables. Experimental tests of the EPR paradox and Bell's theorem have been conducted using various setups, such as measuring the polarization of entangled photons. The results consistently support the predictions of quantum mechanics, showing violations of Bell inequalities and providing evidence against the existence of local hidden variables. The implications of the EPR paradox and Bell's theorem are profound. They suggest that quantum entanglement leads to non-local correlations between particles, challenging our understanding of space, time, and causality. The experiments reinforce the probabilistic nature of quantum mechanics and the need to accept the fundamental concept of entanglement, where the state of one particle cannot be described independently of the other. In conclusion, the EPR paradox highlights the counterintuitive nature of quantum mechanics and raises profound questions about the nature of reality. The non-local correlations observed in entangled systems challenge classical notions of locality and causality, paving the way for further investigations into the fundamental principles of quantum physics.

To provide a rigorous mathematical explanation of the EPR (Einstein-Podolsky-Rosen) paradox, we will delve into the formalism of quantum mechanics. In quantum mechanics, the state of a system is described by a wave function. In the case of two entangled particles, we can represent their joint state using a tensor product. Let's consider two spin-1/2 particles, denoted as A and B, and let  $\sigma$  be the Pauli spin operator. The initial entangled state can be written as:  $|\Psi\rangle = (|+\rangle_A \otimes |-\rangle_B - |-\rangle_A \otimes |+\rangle_B) / \sqrt{2}$ , where  $|+\rangle$  and  $|-\rangle$  represent the spin-up and spin-down states along a chosen axis. This state signifies that the spins of the particles are correlated in such a way that if one is measured in the spin-up state, the other will be in the spin-down state, and vice versa. Now, let's consider measuring the spin of particle A along a specific direction, denoted by a unit vector  $n_A$ . The possible outcomes are +1 and -1, corresponding to spin-up and spin-down, respectively. We can represent these measurement outcomes using projection operators:  $P_{+A} = (|+\rangle_A \langle +|)$ ,  $P_{-A} = (|-\rangle_A \langle -|)$ . Similarly, for particle B along a direction specified by  $n_B$ , we have:  $P_{+B} = (|+\rangle_B \langle +|)$ ,  $P_{-B} = (|-\rangle_B \langle -|)$ . The probability of obtaining a particular measurement outcome is given by the Born rule:  $P(+1, -1) = |\langle \Psi | P_{+A} \otimes P_{-B} | \Psi \rangle|^2$ ,  $P(-1, +1) = |\langle \Psi | P_{-A} \otimes P_{+B} | \Psi \rangle|^2$ . To analyze the paradox, let's consider measuring the spins of particles A and B along the same direction  $n = n_A = n_B$ . We obtain:  $P(+1, -1) = |\langle \Psi | P_{+A} \otimes P_{-B} | \Psi \rangle|^2 = 1/2$ ,  $P(-1, +1) = |\langle \Psi | P_{-A} \otimes P_{+B} | \Psi \rangle|^2 = 1/2$ . These probabilities are independent of the separation between the particles. However,

according to quantum mechanics, the measurement outcomes of particle A and particle B are instantaneously correlated. If we measure +1 for particle A, we are certain to obtain -1 for particle B, and vice versa. This non-local correlation raises the question of how information seems to be transmitted instantaneously between the particles, seemingly violating the principles of causality and locality. It is important to note that the EPR paradox does not provide a means for faster-than-light communication. No useful information can be transmitted through this entanglement correlation, as the measurement outcomes are probabilistic and do not allow for deterministic communication. The paradox challenges the classical notion of local realism, which posits that physical properties are predetermined and that distant objects should not have instantaneous correlations. The violation of local realism observed in experiments based on Bell's theorem suggests that quantum mechanics provides a more accurate description of reality, where entangled systems exhibit non-local correlations. In summary, the EPR paradox demonstrates the existence of non-local correlations between entangled particles, highlighting the unique nature of quantum mechanics. These correlations, despite their seemingly paradoxical properties, are a fundamental aspect of quantum theory and have been experimentally confirmed. The EPR paradox has profound implications for our understanding of the nature of reality, causality, and the limits of classical physics.

### **Properties of Basis (ket/Bra vectors):orthogonality, inner product and hilbert spaces, outer product, matrix**

In quantum mechanics, a ket vector (denoted as  $|\psi\rangle$ ) represents the state of a quantum system, while a bra vector (denoted as  $\langle\phi|$ ) represents the dual state. These vectors belong to a vector space called a Hilbert space, denoted as H.

**Orthogonality:** Two vectors  $|\psi\rangle$  and  $|\phi\rangle$  in a Hilbert space H are said to be orthogonal if their inner product  $\langle\phi|\psi\rangle$  is zero. The inner product is a mathematical operation that combines a ket vector with a bra vector, resulting in a scalar. If  $\langle\phi|\psi\rangle = 0$ , it implies that the vectors are perpendicular or independent of each other.

**Inner Product:** The inner product between two vectors  $|\psi\rangle$  and  $|\phi\rangle$  is denoted as  $\langle\phi|\psi\rangle$ . It is a complex-valued scalar obtained by taking the complex conjugate of the bra vector  $\langle\phi|$  and then performing a dot product with the ket vector  $|\psi\rangle$ . The inner product has several properties:

**Linearity:**  $\langle\phi|(a|\psi\rangle + b|\chi\rangle) = a\langle\phi|\psi\rangle + b\langle\phi|\chi\rangle$ , where a and b are complex numbers.

**Conjugate Symmetry:**  $\langle\phi|\psi\rangle = \langle\psi|\phi\rangle^*$ , where \* denotes complex conjugation.

**Positive Definiteness:**  $\langle\psi|\psi\rangle \geq 0$ , and  $\langle\psi|\psi\rangle = 0$  if and only if  $|\psi\rangle = 0$  (the zero vector).

**Hilbert Spaces:** A Hilbert space, denoted as H, is a complete inner product space. It is a vector space equipped with an inner product that

satisfies certain properties. Hilbert spaces provide a mathematical framework for describing quantum states and operations. Hilbert spaces have the following properties: Linearity: For any two vectors  $|\psi\rangle$  and  $|\phi\rangle$  in H, the sum  $(|\psi\rangle + |\phi\rangle)$  and scalar multiples  $(a|\psi\rangle)$  are also in H.

Completeness: Every Cauchy sequence in H converges to a limit that is also in H. Inner Product: The inner product satisfies the properties mentioned earlier. Outer Product: The outer product is an operation that combines a ket vector  $|\psi\rangle$  with a bra vector  $\langle\phi|$ . It results in a matrix, known as an outer product matrix, denoted as  $|\psi\rangle\langle\phi|$ . The elements of the outer product matrix are given by  $(|\psi\rangle\langle\phi|)_{ij} = \psi_i * \phi_j$ , where  $\psi_i$  and  $\phi_j$  are the components of the ket and bra vectors, respectively. The outer product matrix has the following properties: Hermitian: If  $|\psi\rangle$  and  $|\phi\rangle$  are complex vectors, the outer product matrix  $|\psi\rangle\langle\phi|$  is Hermitian, meaning it is equal to its own conjugate transpose:  $(|\psi\rangle\langle\phi|)^\dagger = |\psi\rangle\langle\phi|$ . Rank-1: The outer product matrix  $|\psi\rangle\langle\phi|$  has rank 1, which means it can be written as the outer product of two vectors:  $|\psi\rangle\langle\phi| = |\psi\rangle\otimes|\phi\rangle$ . Matrices: In quantum mechanics, matrices are used to represent quantum operators and transformations. A matrix can be expressed as a collection of numbers arranged in rows and columns. The elements of a matrix can be complex numbers. Matrices have the following properties: Addition and Scalar Multiplication: Matrices can be added together element-wise, and they can be multiplied by scalars. Matrix Product: Matrices can be multiplied using the matrix product operation, where the elements of the resulting matrix are obtained by taking the dot product of the corresponding row of the first matrix with the corresponding column of the second matrix.

Hermitian Matrices: A matrix is Hermitian if it is equal to its own conjugate transpose:  $A = A^\dagger$ .

Unitary Matrices: A matrix is unitary if its conjugate transpose is equal to its inverse:  $U^\dagger U = U U^\dagger = I$ , where I is the identity matrix. Eigenvalues and Eigenvectors: Matrices have eigenvalues and eigenvectors, which represent the values and vectors that satisfy the equation  $A|v\rangle = \lambda|v\rangle$ , where A is the matrix,  $|v\rangle$  is the eigenvector, and  $\lambda$  is the eigenvalue. These are the essential mathematical concepts related to basis vectors, orthogonality, inner product, Hilbert spaces, outer product, and matrices in the context of quantum mechanics. They provide the foundation for understanding quantum states, measurements, and transformations within the formalism of quantum theory.

Basis Vectors: In quantum mechanics, basis vectors form the foundation of the vector space that describes the state of a quantum system. A basis is a set of linearly independent vectors that span the entire vector space. In quantum mechanics, the basis vectors are often represented as kets, denoted as  $|\psi\rangle$ . The ket  $|\psi\rangle$  represents the state of the quantum system.

Orthogonality: In a Hilbert space, two vectors  $|\psi\rangle$  and  $|\phi\rangle$  are said to be orthogonal if their inner product is zero. Mathematically, orthogonality can be expressed as  $\langle\phi|\psi\rangle = 0$ . This

means that the vectors are perpendicular or independent of each other. Orthogonal vectors are crucial in quantum mechanics for representing mutually exclusive states. Inner Product: The inner product is a mathematical operation that combines a bra vector  $\langle \phi |$  with a ket vector  $|\psi\rangle$ , resulting in a scalar quantity. The inner product between  $|\phi\rangle$  and  $|\psi\rangle$  is denoted as  $\langle \phi | \psi \rangle$ . It is defined as the complex conjugate of the bra vector  $\langle \phi |$  multiplied by the ket vector  $|\psi\rangle$ . The inner product plays a fundamental role in quantum mechanics, as it provides a way to compute probabilities and measure the similarity between quantum states. Hilbert Spaces: A Hilbert space is a complete inner product space that serves as the mathematical framework for quantum mechanics. It is a vector space equipped with an inner product that satisfies certain properties. Hilbert spaces provide a rigorous foundation for describing quantum states, operators, and measurements. Hilbert spaces possess the following properties:

- Linearity:** For any two vectors  $|\psi\rangle$  and  $|\phi\rangle$  in the Hilbert space, the sum  $(|\psi\rangle + |\phi\rangle)$  and scalar multiples  $(a|\psi\rangle)$  are also elements of the Hilbert space.
- Completeness:** Every Cauchy sequence in the Hilbert space converges to a limit that is also in the Hilbert space.
- Inner Product:** The inner product defined in the Hilbert space satisfies the properties of linearity, conjugate symmetry, and positive definiteness.
- Outer Product:** The outer product is an operation that combines a ket vector  $|\psi\rangle$  with a bra vector  $\langle \phi |$ , resulting in an outer product matrix denoted as  $|\psi\rangle\langle \phi |$ . The elements of the outer product matrix are given by  $(|\psi\rangle\langle \phi |)_{ij} = \psi_i * \phi_j$ , where  $\psi_i$  and  $\phi_j$  are the components of the ket and bra vectors, respectively.
- Properties of the outer product matrix include:**

  - Hermitian:** If  $|\psi\rangle$  and  $|\phi\rangle$  are complex vectors, the outer product matrix  $|\psi\rangle\langle \phi |$  is Hermitian, meaning it is equal to its own conjugate transpose:  $(|\psi\rangle\langle \phi |)^\dagger = |\psi\rangle\langle \phi |$ .
  - Rank-1:** The outer product matrix  $|\psi\rangle\langle \phi |$  has rank 1, which means it can be expressed as the outer product of two vectors:  $|\psi\rangle\langle \phi | = |\psi\rangle\otimes|\phi\rangle$ .

- Matrices:** In quantum mechanics, matrices are used to represent quantum operators, transformations, and observables. A matrix is a rectangular array of complex numbers organized in rows and columns. Matrices exhibit the following properties:

  - Addition and Scalar Multiplication:** Matrices can be added together element-wise and can be multiplied by scalars.
  - Matrix Product:** Matrices can be multiplied using the matrix product operation, where the elements of the resulting matrix are obtained by taking the dot product of the corresponding row of the first matrix with the corresponding column of the second matrix.
  - Hermitian Matrices:** A matrix is Hermitian if it is equal to its own conjugate transpose:  $A = A^\dagger$ .
  - Unitary Matrices:** A matrix is unitary if its conjugate transpose is equal to its inverse:  $U^\dagger U = U U^\dagger = I$ , where  $I$  is the identity matrix.
  - Eigenvalues and Eigenvectors:** Matrices have eigenvalues and eigenvectors, which represent the values and vectors that satisfy the equation  $A|v\rangle = \lambda|v\rangle$ , where  $A$  is the matrix,  $|v\rangle$  is the

eigenvector, and  $\lambda$  is the eigenvalue. These properties are essential in the mathematical framework of quantum mechanics, providing the tools to describe quantum states, transformations, and measurements accurately.

## Story :

Properties of Basis (ket/Bra vectors): orthogonality, inner product and hilbert spaces, outer product, matrix Orthogonality: In the wizarding world, different spells can have distinct effects. Similarly, in the realm of Basis vectors, orthogonality refers to the concept that two different spells (ket vectors) are unrelated and have no overlap. It's like casting a "Lumos" spell, which produces light, and a "Silencio" spell, which cancels any sound. These spells have no effect on each other and are orthogonal. Orthogonality: In the wizarding world, different spells have distinct effects and do not interfere with each other. Similarly, Basis vectors are orthogonal when they have no overlap or correlation. In quantum mechanics, orthogonality implies that the inner product of two orthogonal Basis vectors is zero. This property allows us to represent different states or magical effects as separate vectors that do not interact. For example, casting the "Lumos" spell to produce light and the "Silencio" spell to cancel sound are orthogonal spells. The "Lumos" vector and the "Silencio" vector in our Basis are independent of each other, and their inner product is zero. Inner Product and Hilbert Spaces: The inner product is like combining two spells to create a new magical effect. In Harry Potter terms, it's like casting "Wingardium Leviosa" and "Alohomora" together to unlock levitating objects. The inner product allows us to calculate the degree of correlation or similarity between two Basis vectors. A Hilbert space is like a magical realm where these vectors reside, representing all possible states and allowing for various magical combinations. Inner Product and Hilbert Spaces: The inner product is a mathematical operation that quantifies the correlation or similarity between two Basis vectors. It measures the overlap between these vectors and provides a measure of their compatibility. In the magical realm of Harry Potter, let's consider the "Wingardium Leviosa" spell for levitating objects and the "Alohomora" spell for unlocking doors. These spells can be seen as Basis vectors. The inner product between these two Basis vectors captures how well they work together. A larger inner product suggests a stronger correlation, indicating that casting these spells in combination produces a more powerful effect. Hilbert spaces are the magical realms where these Basis vectors reside. They represent the complete set of possible states or magical combinations. Just like the wizarding world contains endless magical possibilities, Hilbert spaces encompass a vast array of states and transformations that can be represented by

Basis vectors. Outer Product: The outer product of Basis vectors is like combining spells to create more complex magical effects. It's like merging "Expelliarmus" with "Protego" to create a powerful shield that deflects any incoming spells. The outer product allows us to create composite spells by multiplying two Basis vectors together. Outer Product: The outer product of Basis vectors allows us to combine them to create more complex magical effects. It represents a composition of spells, where the resulting vector captures the joint effect of the individual Basis vectors. In the context of Harry Potter, consider the "Expelliarmus" spell for disarming opponents and the "Protego" spell for creating a protective shield. Combining these spells through the outer product results in a new vector that represents a powerful shield capable of deflecting any incoming spells. Matrix: In the wizarding world, a matrix is like a magical recipe book that contains instructions for combining different spells in specific ways. Each entry in the matrix represents a combination of Basis vectors that produces a unique magical outcome. For example, a matrix might describe how to combine "Aguamenti" with "Incendio" to create a stream of water that turns into a wall of fire. Matrix: In quantum mechanics, a matrix is a mathematical object that describes the transformations between Basis vectors. It serves as a "recipe book" for combining different Basis vectors in specific ways. In the magical world, a matrix can be seen as a collection of instructions for combining spells. Each entry in the matrix represents a particular combination of Basis vectors, producing a unique magical outcome. For instance, a matrix might describe the combination of the "Aguamenti" spell, which creates a stream of water, with the "Incendio" spell, which ignites fire. Following the matrix's instructions, these Basis vectors can be multiplied together to produce a new magical effect, such as water turning into a wall of fire. By understanding and manipulating the properties of orthogonality, inner product and Hilbert spaces, outer product, and matrices, we gain the ability to explore and harness the magical possibilities within the realm of Basis vectors. These concepts not only provide insights into quantum mechanics but also offer an enchanting way to connect with the fascinating world of Harry Potter. Just like Harry Potter's world is filled with spells and magical possibilities, the realm of Basis vectors encompasses a rich set of states and transformations. By understanding the properties of orthogonality, inner product and Hilbert spaces, outer product, and matrices, we can unlock the secrets of these magical vectors and harness their power for various applications in quantum mechanics and mathematics.

**notation, unitary, operators and projectors, eigenvalues and eigenvectors, probabilities and measurements**

**Notation in Quantum Mechanics:** In quantum mechanics, a ket vector is denoted by  $|\psi\rangle$ , where  $\psi$  represents the state of a quantum system. The corresponding bra vector is denoted by  $\langle\psi|$ , which is the complex conjugate transpose of the ket vector. The inner product between two vectors is denoted as  $\langle\phi|\psi\rangle$ , representing the complex conjugate of the bra vector multiplied by the ket vector.

**Unitary Operators:** A unitary operator is a linear transformation that preserves the inner product between vectors. In quantum mechanics, unitary operators represent quantum gates that can manipulate the state of a quantum system. Mathematically, an operator  $U$  is unitary if it satisfies the condition  $U^\dagger U = UU^\dagger = I$ , where  $U^\dagger$  is the conjugate transpose of  $U$  and  $I$  is the identity operator. Unitary operators are reversible and preserve the norm and inner product of vectors.

**Operators and Projectors:** Operators in quantum mechanics represent observables or transformations. They act on ket vectors to produce new ket vectors. Hermitian operators are particularly important as they represent physical observables, and their eigenvalues correspond to possible measurement outcomes. A projector is a type of operator that projects a ket vector onto a specific subspace. A projector  $P$  is defined as  $P = |\phi\rangle\langle\phi|$ , where  $|\phi\rangle$  is a normalized ket vector. Applying a projector to a ket vector  $|\psi\rangle$  projects  $|\psi\rangle$  onto the subspace spanned by  $|\phi\rangle$ . Projectors are idempotent ( $P^2 = P$ ) and Hermitian ( $P = P^\dagger$ ).

**Eigenvalues and Eigenvectors:** Eigenvalues and eigenvectors play a fundamental role in quantum mechanics. For an operator  $A$ , an eigenvector  $|v\rangle$  and its corresponding eigenvalue  $\lambda$  satisfy the equation  $A|v\rangle = \lambda|v\rangle$ . Eigenvectors represent stable states of the system, and eigenvalues represent the possible values that can be obtained when measuring the corresponding observable.

**Probabilities and Measurements:** In quantum mechanics, probabilities are associated with measurement outcomes. Given a quantum system in state  $|\psi\rangle$ , the probability of obtaining a specific measurement outcome corresponding to an eigenvalue  $\lambda$  and eigenvector  $|v\rangle$  of an observable  $A$  is given by  $P(\lambda) = |\langle v|\psi\rangle|^2$ . The squared modulus of the inner product between the eigenvector and the state represents the probability of obtaining the measurement outcome associated with that eigenvector. Measurements in quantum mechanics involve the collapse of the state vector. When a measurement is performed, the state of the system changes instantaneously to the eigenvector corresponding to the measured outcome, and the probability of obtaining that outcome is given by the squared modulus of the inner product. The measurement process is non-deterministic, and different measurement outcomes occur with probabilities according to the state of the system. The rigorous mathematical framework of quantum mechanics provides a precise way to describe the behavior of quantum systems using notation, unitary operators, projectors, eigenvalues, eigenvectors, and probabilities. These concepts allow for the

calculation of measurement outcomes, prediction of system behavior, and understanding of quantum phenomena.

## Story :

notation, unitary, operators and projectors, eigenvalues and eigenvectors, probabilities and measurements Notation: In the wizarding world, spells and magical phenomena are often represented using special symbols and incantations. Similarly, in quantum mechanics, we use mathematical notation to represent states and operations. We can think of these notations as magical spells that capture the essence of quantum phenomena. For example, the symbol  $|\psi\rangle$  represents a quantum state called "psi," analogous to a specific magical state. This notation allows us to describe and manipulate quantum states just as wizards use spells to perform magical actions. Unitary Operators: In the Harry Potter universe, certain spells have the remarkable property of preserving the overall magical energy. Similarly, in quantum mechanics, unitary operators are like powerful spells that preserve the total probability or energy of a quantum system. For instance, the "Finite Incantatem" spell in Harry Potter can neutralize the effects of other spells without changing the underlying magic. Similarly, a unitary operator in quantum mechanics preserves the norm of a quantum state while performing transformations. It ensures that the total probability of finding the system in any state remains constant. Projectors: In the magical world, wizards often use spells to focus their magical energy on specific targets or objects. In quantum mechanics, projectors are operators that allow us to "project" a quantum state onto a specific subspace. For example, the "Accio" spell in Harry Potter is used to summon a particular object from a distance. We can think of the Accio spell as a projector that selects the desired object from the surrounding environment. Eigenvalues and Eigenvectors: In the realm of magic, wizards possess unique affinities for certain types of magic. Similarly, in quantum mechanics, eigenvalues and eigenvectors represent the inherent properties of quantum systems. In the Harry Potter world, a wizard may have a special affinity for the "Patronus Charm," and their particular affinity is analogous to an eigenvalue. The associated eigenvector represents the specific form or representation of the Patronus that the wizard can summon. Probabilities and Measurements: In the magical world, wizards often perform spells with the expectation of achieving specific outcomes. Similarly, in quantum mechanics, we deal with probabilities to determine the likelihood of obtaining certain results from measurements. For instance, when a wizard casts the "Petrificus Totalus" spell, they expect the target to be paralyzed. However, the success rate may vary depending on factors such as the

wizard's skill and the resilience of the target. Similarly, in quantum mechanics, we can calculate the probabilities of obtaining certain measurement outcomes based on the quantum state and the measurement process. In the context of Harry Potter, understanding probabilities and measurements allows us to anticipate the effects of magical spells and make predictions about the outcome of specific magical actions. By relating notation, unitary operators, projectors, eigenvalues and eigenvectors, and probabilities and measurements to the magical world of Harry Potter, we can grasp the fundamental concepts of quantum mechanics in an enchanting and relatable manner. Just as wizards navigate the intricacies of their magical realm, we explore the wonders of quantum mechanics through the lens of Harry Potter's magical universe.

## Linear Algebra

Linear algebra plays a fundamental role in the field of quantum computing, providing the mathematical framework for understanding and manipulating quantum systems. In this explanation, we will explore the key concepts of linear algebra used in quantum computing, including vectors, matrices, inner products, eigenvectors and eigenvalues, unitary transformations, and tensor products.

**Vectors:** In quantum computing, vectors are used to represent quantum states. A vector represents the state of a quantum system and is typically written in the form of a column vector. For example, a qubit state can be represented as a two-dimensional column vector with complex entries:  $|\psi\rangle = [\alpha] [\beta]$ . Here,  $\alpha$  and  $\beta$  are complex probability amplitudes that satisfy the normalization condition  $|\alpha|^2 + |\beta|^2 = 1$ .

**Matrices:** Matrices are used to represent quantum operations, such as quantum gates, which manipulate the quantum state. A matrix acts on a vector to produce a new vector. For example, a quantum gate acting on a qubit can be represented by a  $2 \times 2$  matrix:  $U = [a \ b] [c \ d]$ . To apply the gate to a qubit state  $|\psi\rangle$ , we multiply the matrix  $U$  with the vector  $|\psi\rangle$  to obtain the new state  $U|\psi\rangle$ .

**Inner Products:** The inner product is a fundamental operation in quantum mechanics. It is used to calculate the probability of obtaining a certain outcome when measuring a quantum state. The inner product of two vectors  $|\psi\rangle$  and  $|\phi\rangle$  is denoted as  $\langle\psi|\phi\rangle$  and is defined as the sum of the products of the complex conjugates of corresponding elements:  $\langle\psi|\phi\rangle = \sum \alpha^* \beta$ .

**Eigenvectors and Eigenvalues:** Eigenvectors and eigenvalues are important concepts in quantum mechanics. An eigenvector of a matrix  $A$  is a non-zero vector  $v$  such that  $Av = \lambda v$ , where  $\lambda$  is the corresponding eigenvalue. In quantum computing, eigenvectors and eigenvalues represent stable states and their associated energies or probabilities.

**Unitary Transformations:** Unitary transformations are fundamental in quantum computing as they preserve the normalization and reversibility of

quantum states. A unitary matrix  $U$  satisfies the condition  $U^\dagger U = I$ , where  $U^\dagger$  is the conjugate transpose of  $U$ . Unitary transformations are used to represent quantum gates, which are the building blocks of quantum circuits.

**Tensor Products:** Tensor products are used to represent composite quantum systems, such as multiple qubits. The tensor product of two vectors  $|\psi\rangle$  and  $|\phi\rangle$  is denoted as  $|\psi\rangle \otimes |\phi\rangle$  and represents the joint state of the two systems. For example, the tensor product of two qubits  $|0\rangle$  and  $|1\rangle$  is:  $|0\rangle \otimes |1\rangle = |01\rangle$

Tensor products are also used to represent multi qubit gates and entanglement.

**Tensor Product of Matrices:** The tensor product can also be applied to matrices. Given two matrices  $A$  and  $B$ , their tensor product  $A \otimes B$  is defined as a block matrix where each element of  $A$  is multiplied by the entire matrix  $B$ :  $A \otimes B = [a_{11}B \ a_{12}B \ \dots \ a_{1n}B] \ [a_{21}B \ a_{22}B \ \dots \ a_{2n}B] \ [\dots \ \dots \ \dots] \ [a_{m1}B \ a_{m2}B \ \dots \ a_{mn}B]$  The resulting matrix will have dimensions  $m \times n$  times the dimensions of  $B$ .

**Orthogonal and Orthonormal Vectors:** In quantum computing, orthogonal and orthonormal vectors play a crucial role. Two vectors  $|\psi\rangle$  and  $|\phi\rangle$  are orthogonal if their inner product  $\langle\psi|\phi\rangle$  is zero. Orthogonal vectors are linearly independent and span different directions in a vector space. If in addition, both vectors have unit length, they are called orthonormal vectors.

**Hermitian and Unitary Matrices:** A matrix  $A$  is Hermitian if it is equal to its conjugate transpose:  $A = A^\dagger$ . Hermitian matrices have real eigenvalues and orthogonal eigenvectors. They are commonly used to represent observables in quantum mechanics. A matrix  $U$  is unitary if its conjugate transpose is equal to its inverse:  $U^\dagger U = UU^\dagger = I$ . Unitary matrices preserve the inner product, norm, and angles between vectors.

Unitary matrices represent reversible operations in quantum computing and are often used to construct quantum gates.

**Projection Operators:** A projection operator  $P$  is a Hermitian matrix that squares to itself:  $P^2 = P$ . Projection operators project a vector onto a subspace by "projecting" it onto the space spanned by the eigenvectors associated with a specific eigenvalue. They are used in quantum computing to represent measurements and state preparations.

**Quantum Entanglement:** Quantum entanglement is a fundamental concept in quantum mechanics. It occurs when two or more qubits become correlated in such a way that the state of the combined system cannot be described independently. The entangled state is represented as a tensor product of two or more qubits, and the measurement of one qubit affects the measurement outcomes of the other qubits. Entanglement plays a crucial role in various quantum algorithms and quantum communication protocols. These additional concepts build upon the foundations of linear algebra and extend its applications to the realm of quantum computing. Understanding these concepts provides a deeper understanding of the mathematical underpinnings of quantum mechanics and enables the manipulation and analysis of quantum

systems. By utilizing these concepts from linear algebra, quantum computing enables the description, manipulation, and measurement of quantum states. The mathematical framework provided by linear algebra allows for precise calculations and predictions in quantum mechanics. Understanding these concepts is essential for studying and advancing the field of quantum computing.

### **Superposition ,entanglement teleportation**

**Superposition:** Superposition is a fundamental concept in quantum mechanics that describes the ability of a quantum system to exist in multiple states simultaneously. In classical physics, objects have definite properties, such as position and momentum. However, in quantum mechanics, a particle can be in a superposition of multiple states, each with an associated probability amplitude. Mathematically, the state of a quantum system is represented by a ket vector  $|\psi\rangle$ , which can be expressed as a linear combination of basis states. For example, in a two-level quantum system (qubit), the state can be written as  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ , where  $\alpha$  and  $\beta$  are complex probability amplitudes and  $|0\rangle$  and  $|1\rangle$  are the basis states (eigenstates of a specific observable, e.g., spin-up and spin-down).

**Entanglement:** Entanglement is a phenomenon in quantum mechanics where two or more particles become correlated in such a way that the state of one particle cannot be described independently of the others. This correlation persists even when the particles are physically separated by large distances. Mathematically, entanglement is represented by a composite system in a joint state that cannot be expressed as a product of individual states. For example, consider two qubits A and B. The entangled state can be written as  $|\psi\rangle = \alpha|00\rangle + \beta|11\rangle$ , where  $|00\rangle$  and  $|11\rangle$  are the joint basis states. In an entangled state, measurements on one qubit can instantly affect the state of the other qubit, regardless of the physical distance between them.

**Quantum Teleportation:** Quantum teleportation is a protocol that allows the transfer of the complete state of a quantum system from one location to another, without physically moving the system itself. It utilizes the principles of entanglement and superposition to transmit the quantum information. The quantum teleportation process involves three parties: the sender (Alice), the receiver (Bob), and an entangled pair of qubits shared between them. The steps of quantum teleportation are as follows:

- Initialization:** Alice and Bob share an entangled pair of qubits, which are in an entangled state such as  $|\psi\rangle = \alpha|00\rangle + \beta|11\rangle$ .
- Bell Measurement:** Alice performs a joint measurement, known as a Bell measurement, on the qubit she wants to teleport (let's call it qubit A) and her half of the entangled pair (qubit C). This measurement collapses the two qubits and obtains a classical result.
- Communication:** Alice sends the classical result obtained from the Bell measurement to Bob.
- Decomposition:** Bob performs a unitary operation on his qubit (qubit B) based on the classical result received from Alice. This operation, combined with the shared entangled pair, results in the reconstruction of the original state of qubit A.

communicates the classical result of her measurement to Bob using classical communication channels. Pauli Correction: Based on the classical result received from Alice, Bob applies a specific quantum gate or operation to his qubit (qubit B) to transform it into the desired state. Through this process, the state of qubit A is teleported to qubit B, and Alice's original qubit A is destroyed in the process. The teleportation is successful because of the entanglement between the initial state and the entangled pair, which allows for the transfer of information.

Mathematically, quantum teleportation relies on the principles of entanglement, superposition, and measurements. It involves the use of quantum gates, such as the Hadamard gate and controlled gates, to manipulate the qubits and perform the necessary transformations.,

superposition and entanglement are two fundamental concepts in quantum mechanics that defy classical intuition. Superposition allows for the existence of multiple states simultaneously, while entanglement creates correlations between particles that persist regardless of distance. Quantum teleportation utilizes these phenomena to transfer the complete state of a quantum system from one location to another. Through the mathematical formalism of quantum mechanics, these concepts and protocols can be rigorously described, providing a foundation for understanding and harnessing the power of quantum information. Superposition: In quantum mechanics, the state of a quantum system is represented by a ket vector, denoted as  $|\psi\rangle$ , which belongs to a Hilbert space. A ket vector can be expressed as a linear combination of basis states, where each basis state corresponds to an eigenstate of a specific observable. Mathematically, we can write:  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ , where  $\alpha$  and  $\beta$  are complex probability amplitudes, and  $|0\rangle$  and  $|1\rangle$  are the basis states (eigenstates). The coefficients  $\alpha$  and  $\beta$  determine the probabilities of measuring the system in the respective basis states. Entanglement: Entanglement arises when two or more quantum systems become correlated, resulting in a composite system that cannot be described independently. Consider a bipartite system with two qubits, A and B. The joint state of the system can be expressed as:  $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$ , where  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  are complex probability amplitudes. This entangled state cannot be factorized into individual states for qubit A and qubit B. Measurements on one qubit will instantaneously affect the state of the other, irrespective of their physical separation. Quantum Teleportation: Quantum teleportation allows for the transfer of the complete state of a quantum system from one location to another by exploiting entanglement. The process involves the following steps:

Initialization: Initially, we have three qubits: the sender's qubit (qubit A), the receiver's qubit (qubit B), and an entangled pair shared between them (qubits C and D). The joint state of the system is:  $|\psi\rangle = \alpha|0\rangle_A|00\rangle_{CD} + \beta|0\rangle_A|11\rangle_{CD} + \gamma|1\rangle_A|00\rangle_{CD} + \delta|1\rangle_A|11\rangle_{CD}$ . Bell Measurement: The sender, Alice,

performs a joint measurement, known as a Bell measurement, on qubit A and qubit C. This measurement collapses the two qubits into one of the four Bell states, which can be represented as:  $|\psi\rangle = \alpha|\Phi+\rangle AB|\Phi+\rangle CD + \beta|\Phi-\rangle AB|\Phi-\rangle CD + \gamma|\Psi+\rangle AB|\Psi+\rangle CD + \delta|\Psi-\rangle AB|\Psi-\rangle CD$ , where  $|\Phi+\rangle$ ,  $|\Phi-\rangle$ ,  $|\Psi+\rangle$ , and  $|\Psi-\rangle$  are the Bell states. Communication: Alice communicates the result of her measurement to Bob using classical channels. This requires two classical bits of information. Pauli Correction: Based on the classical information received, Bob applies a specific set of quantum operations (Pauli gates) on his qubit B. These operations transform qubit B into the desired state, which is the state originally possessed by qubit A. Through these steps, the state of qubit A is "teleported" to qubit B, while the entanglement between qubits C and D is destroyed in the process. In a mathematical sense, quantum teleportation relies on the principles of superposition, entanglement, and measurements. The mathematical formalism of quantum mechanics, including linear algebra and quantum gates, allows us to describe and manipulate these quantum phenomena accurately. By understanding and harnessing superposition, entanglement, and teleportation, we can explore the intriguing capabilities of quantum systems and pave the way for future advancements in quantum information processing and communication

## Story :

Superposition ,entanglement, teleportation Once upon a time, in the magical world of Hogwarts, Harry Potter and his friends embarked on a fascinating adventure that led them to discover the secrets of quantum mechanics. In their pursuit of knowledge, they encountered three remarkable phenomena: superposition, entanglement, and teleportation. Superposition, in the wizarding world, was akin to a magical spell that allowed objects to exist in multiple states simultaneously. Just like a wand could be both in Harry's hand and on the table at the same time, quantum particles could be in a combination of different states. Imagine a Sorting Hat that could simultaneously place a student in multiple Hogwarts houses. In quantum mechanics, this meant that a particle, such as an electron, could exist in a superposition of states, like being both spin-up and spin-down simultaneously. Entanglement, on the other hand, was like a powerful spell that linked particles together in such a way that the properties of one particle became intrinsically connected to the other, regardless of the distance between them. It was as if Harry and his best friend Ron Weasley shared an unbreakable bond, where whatever happened to one affected the other. In quantum entanglement, when two particles become entangled, their properties become intertwined, and any change to one particle

instantly affects the other, even if they were light-years apart. Teleportation was the most astonishing phenomenon of all. Just as Harry could magically transport himself from one place to another using Floo Powder or Apparition, quantum teleportation allowed for the instantaneous transfer of information from one particle to another, without any physical movement. It was like sending a message through an enchanted mirror, where the reflection on one mirror instantly appeared on the other. In quantum teleportation, the state of a particle, such as its spin or polarization, could be transferred to another distant particle, resulting in a perfect replica appearing at the destination. To understand these phenomena better, Harry, Hermione, and Ron turned to the brilliant Professor Snape, who revealed the magic behind quantum mechanics. He explained that in the quantum world, particles could exist in a superposition of states until they were measured or observed. It was like a magical potion that remained in an undetermined state until someone drank it, revealing its true nature. Entanglement, Professor Snape explained, was the result of a powerful enchantment that intertwined the fate of particles. When two particles were entangled, their states became entwined, and whatever happened to one would instantaneously affect the other, even if they were separated by vast distances. It was as if a pair of enchanted mirrors shared a mystical connection, enabling them to reflect each other's changes, regardless of the physical separation. Teleportation, the most mystical phenomenon, involved the delicate art of transferring the state of one particle to another. Just as a skilled wizard could use a Portkey to transport themselves to a different location, quantum teleportation involved manipulating the entangled particles to encode the state of the original particle onto the destination particle. The original particle's state would vanish, only to reappear instantaneously in the destination particle, as if it had magically teleported across space. In their quest to understand these phenomena, Harry, Hermione, and Ron engaged in numerous magical experiments. They used enchanted wands and potions to create superpositions of particles, observed the effects of entanglement on enchanted objects, and even attempted to teleport magical artifacts across the Hogwarts grounds. Through their adventures, they realized that these quantum phenomena held tremendous potential for the wizarding world. Superposition could lead to more powerful spells with a range of effects, entanglement could revolutionize long-distance communication between magical communities, and teleportation could enable instant travel across the vast magical realms. In the end, Harry and his friends discovered that the magic of quantum mechanics was great.

## Paradigm shift

The paradigm shift from quantum mechanics to quantum computing represents a fundamental transition in our understanding of physics and the potential applications of information processing. This bridge between the two fields involves harnessing the unique properties of quantum mechanics to design and develop powerful computing systems capable of solving problems beyond the reach of classical computers. In this concise explanation, I will outline the key concepts and principles underlying this bridge. Quantum mechanics, established in the early 20th century, revolutionized our understanding of the microscopic world. It introduced the notion of wave-particle duality, where particles like electrons and photons exhibit both particle-like and wave-like properties. Quantum mechanics also introduced the concept of superposition, where particles can exist in multiple states simultaneously, and entanglement, where particles become intricately correlated, even when separated by large distances.

Quantum computing builds upon these principles to revolutionize information processing. Instead of classical bits that exist in either a 0 or 1 state, quantum computing uses quantum bits, or qubits, which can exist in superposition states, representing both 0 and 1 simultaneously. This superposition enables quantum computers to perform computations in parallel, vastly increasing their processing power. One of the most critical algorithms in quantum computing is Shor's algorithm, which efficiently factors large numbers. Factoring large numbers is notoriously difficult for classical computers, but Shor's algorithm exploits the inherent parallelism and interference of quantum superposition to solve this problem exponentially faster. Another key concept in quantum computing is quantum entanglement. Entangled qubits exhibit strong correlations, allowing for faster communication and enhanced computational capabilities. Quantum teleportation, a process that transfers the state of one qubit to another through entanglement, is a prominent example of how entanglement can be harnessed for information transfer. To implement quantum computing in practice, various physical systems are explored as platforms for qubits. These include superconducting circuits, trapped ions, topological states of matter, and more. Each platform has its own advantages and challenges in terms of qubit stability, coherence time, and scalability. However, harnessing the power of quantum computing also poses significant challenges. Quantum systems are highly susceptible to noise and decoherence, which causes loss of information and errors in calculations. Quantum error correction techniques are being developed to mitigate these errors and protect quantum states from decoherence. The bridge between quantum mechanics and

quantum computing extends beyond the development of hardware and algorithms. It involves interdisciplinary research encompassing quantum information theory, quantum algorithms, quantum error correction, and quantum complexity theory. Theoretical advances in these areas further our understanding of the capabilities and limitations of quantum computers. The impact of quantum computing extends beyond traditional computing tasks. Quantum simulation allows researchers to model complex quantum systems, such as chemical reactions or materials, with unprecedented accuracy. Quantum machine learning explores how quantum computers can enhance pattern recognition and optimization tasks. In conclusion, the bridge between quantum mechanics and quantum computing represents a transformative shift in our approach to information processing. By leveraging the principles of quantum mechanics, such as superposition and entanglement, quantum computing offers the potential for exponential computational speedup and new solutions to previously intractable problems. As research and development in quantum computing continue to progress, the bridge between these fields will deepen, opening new frontiers in science, technology, and innovation.

## **Qubit Modalities and Type**

Qubits, or quantum bits, are the fundamental units of information in quantum computing. They represent the quantum mechanical analog of classical bits and are the building blocks of quantum algorithms and computations. In this explanation, I will discuss three types of qubits: superconducting qubits, NMR qubits, trapped ion qubits, and diamond-based qubits.

Superconducting qubits are a leading platform for implementing quantum computing systems. They are fabricated using superconducting materials, such as niobium, and operate at extremely low temperatures near absolute zero. Superconducting qubits are typically formed by creating a circuit with Josephson junctions, which are weak links between superconducting elements. One commonly used type of superconducting qubit is the transmon qubit. The transmon qubit is based on a superconducting loop interrupted by a Josephson junction. By controlling the voltage applied to the qubit, its energy levels can be manipulated, enabling state manipulation and readout. Transmon qubits benefit from long coherence times and scalability, making them attractive for large-scale quantum computers. Nuclear Magnetic Resonance (NMR) qubits utilize the principles of nuclear magnetic resonance to encode and manipulate quantum information. NMR qubits typically involve a collection of nuclear spins in a molecule or a solid-state material. These spins can be manipulated by applying radiofrequency pulses and magnetic fields. NMR qubits have been extensively used in quantum chemistry and quantum simulations.

due to their ability to accurately simulate the behavior of molecules. However, NMR qubits face challenges in scalability and measurement efficiency, limiting their applicability in large-scale quantum computing. Trapped ion qubits are based on the manipulation of individual ions using electromagnetic fields. In this approach, ions are trapped using electromagnetic traps, such as radiofrequency or Paul traps. Laser beams are then used to cool and manipulate the ions' internal quantum states. The internal energy levels of trapped ions serve as qubit states, and their manipulation is achieved through laser-induced transitions. The long coherence times and precise control offered by trapped ion qubits make them suitable for various quantum computing tasks, including high-fidelity gate operations and entanglement generation.

Diamond-based qubits are a relatively recent development in the field of quantum computing. They utilize defects in diamond crystals, such as nitrogen vacancy (NV) centers, as qubits. NV centers consist of a nitrogen atom adjacent to a vacant lattice site in the diamond lattice. The electronic spin of the NV center serves as the qubit, which can be initialized, manipulated, and measured using laser and microwave fields. Diamond-based qubits benefit from long coherence times, compatibility with room temperature operation, and the potential for integration with nanophotonic devices. In addition to these specific types of qubits, there are several other approaches being explored, such as topological qubits, photonics-based qubits, and more. Each type of qubit has its own strengths and challenges, and researchers are actively working to improve their coherence times, gate fidelity, and scalability. In summary, superconducting qubits, NMR qubits, trapped ion qubits, and diamond-based qubits represent different approaches to implementing qubits for quantum computing. Each type has its unique advantages and challenges, and researchers are continuously advancing these platforms to build more robust and scalable quantum computers. The choice of qubit platform depends on the specific requirements of the application and the ongoing technological developments in each field

## Section 2

### Introduction to circuit formalism

Quantum gates are unitary operators that act on qubits to perform specific transformations. These gates are represented by matrices, and their action on the qubit state can be described

using matrix-vector multiplication. Commonly used gates include the Pauli gates (X, Y, Z), the Hadamard gate (H), and the phase gate (S). For example, the Hadamard gate transforms the  $|0\rangle$  state to an equal superposition of  $|0\rangle$  and  $|1\rangle$ , and the  $|1\rangle$  state to an equal superposition with a phase shift:  $H|0\rangle = 1/\sqrt{2} (|0\rangle + |1\rangle)$   $H|1\rangle = 1/\sqrt{2} (|0\rangle - |1\rangle)$

**Quantum Circuit:** A quantum circuit consists of a sequence of quantum gates applied to qubits, followed by measurements. The circuit is represented as a directed acyclic graph, where the qubits flow from left to right. The state of the qubits evolves as the gates are applied, following the rules of quantum mechanics.

**Measurements** are performed at the end of the circuit to obtain classical information about the quantum system.

**Circuit Evolution:** The evolution of a quantum circuit can be described using the formalism of quantum gates and their matrix representations. Given an initial qubit state  $|\psi\rangle$  and a circuit C, the final state  $|\psi'\rangle$  after applying the circuit can be obtained by sequentially applying the gate operations:  $|\psi'\rangle = U_n U_{\{n-1\}} \dots U_1 |\psi\rangle$ . Here,  $U_i$  represents the unitary matrix representing the i-th gate in the circuit. The overall transformation of the circuit can be represented as the product of these matrices:  $U = U_n U_{\{n-1\}} \dots U_1$ . The final state can then be written as  $|\psi'\rangle = U|\psi\rangle$ .

**Measurement and Probabilities:** At the end of a quantum circuit, measurements are performed to extract classical information from the quantum system. Measurement collapses the quantum state into one of the basis states, with probabilities determined by the amplitudes of the corresponding basis states. Given a qubit state  $|\psi\rangle$ , the probability of measuring the qubit in the  $|0\rangle$  state is  $|\alpha|^2$ , and in the  $|1\rangle$  state is  $|\beta|^2$ .

**Tensor Product and Multi-Qubit States:** Quantum circuits can operate on multiple qubits, forming multi-qubit states. The state of a system with n qubits is described by a vector in a tensor product space of n individual qubit spaces. The tensor product of two qubits  $|a\rangle$  and  $|b\rangle$  is denoted as  $|ab\rangle$ . For example, the state of two qubits can be written as:  $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

The evolution of multi-qubit systems is described by applying gates to individual qubits or pairs of qubits, as specified by the circuit. In summary, the formalism of quantum circuits provides a mathematical framework for understanding and manipulating quantum systems. It involves representing qubit states as superpositions, applying unitary gates to transform the states, and performing measurements to extract classical information. The evolution of a quantum circuit can be described using matrix representations of the gates. By combining these elements, quantum circuit formalism enables the design and analysis of quantum algorithms and computations.

**-Story :**

In the enchanting world of Harry Potter, where magic reigns supreme, even the most extraordinary spells and potions can be understood through the lens of circuit formalism. Just as wizards and witches cast spells using specific wand movements and incantations, magic in the Harry Potter universe can be likened to a series of interconnected circuits that produce desired effects. In circuit formalism, a spell or potion is represented by a circuit diagram, where different components and symbols represent various magical elements. Just as a magical spell requires specific ingredients and precise steps to achieve the desired outcome, a circuit consists of components like gates, wires, and registers that manipulate and transform information.

Imagine Harry Potter brewing a Polyjuice Potion to transform into someone else. The potion-making process can be seen as a circuit with different stages. Each stage represents a specific step in the process, where ingredients are added, mixed, and transformed. Similarly, a circuit consists of individual gates and operations that act on input data to produce an output. In the magical world, a wand serves as a tool to channel and control magic. Similarly, in circuit formalism, we have quantum gates that serve as the building blocks for performing operations on quantum bits, also known as qubits. These gates include familiar magical concepts like the Polyjuice Potion-making cauldron or the Mirror of Erised, each designed to bring about a specific transformation. Just as Harry and his friends combine their magical abilities to tackle challenges, circuits in the magical world can be combined to create more complex and powerful effects. Connecting multiple circuits allows the flow of information and magical energy between them, resulting in a collective and synchronized outcome. Moreover, in the world of Harry Potter, there are enchantments and spells that exhibit unique properties, such as invisibility cloaks or the Patronus Charm. These magical phenomena can be compared to quantum superposition and entanglement in circuit formalism. Superposition, where an object can exist in multiple states simultaneously, is like an invisibility cloak rendering the wearer simultaneously visible and invisible. Entanglement, on the other hand, is akin to the deep connection between Harry and his Patronus, where their fates intertwine regardless of the physical distance. Just as Harry and his friends analyze and study spells and potions to understand their effects and limitations, circuit formalism provides a framework for analyzing and optimizing magical circuits. Through circuit analysis, the wizards and witches of Hogwarts can unravel the intricacies of complex spells, predict their outcomes, and refine their magical craft. In summary, circuit formalism in the context of Harry Potter helps us understand the magical world as a network of interconnected circuits. Just as wizards and witches cast spells by following precise steps and combining various components, circuits allow us to represent and

manipulate magical effects through gates and operations. By exploring this magical analog, we can gain a deeper understanding of the underlying principles that govern the enchanting world of Harry Potter.

## Types of Quantum Gates

Quantum gates are fundamental operations used in quantum computing to manipulate qubits and perform quantum computations. Each gate represents a specific transformation on the quantum state, and they can be combined to create complex quantum circuits. In this article, we will explore some common types of quantum gates and their mathematical representations.

**Pauli Gates:** Pauli gates are named after the physicist Wolfgang Pauli and play a crucial role in quantum computing. They include the Pauli-X gate ( $\sigma_x$ ), Pauli-Y gate ( $\sigma_y$ ), and Pauli-Z gate ( $\sigma_z$ ). These gates correspond to rotations around the X, Y, and Z axes of the Bloch sphere, respectively. The mathematical representations of Pauli gates are as follows: Pauli-X gate:  $\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$  Pauli-Y gate:  $\sigma_y = i|0\rangle\langle 1| - i|1\rangle\langle 0|$  Pauli-Z gate:  $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$  These gates perform different transformations on the qubit states and are frequently used in quantum algorithms.

**Hadamard Gate:** The Hadamard gate (H) is another fundamental gate in quantum computing. It creates superposition by transforming the  $|0\rangle$  state to an equal superposition of  $|0\rangle$  and  $|1\rangle$ , and the  $|1\rangle$  state to an equal superposition with a phase shift. The mathematical representation of the Hadamard gate is:  $H = 1/\sqrt{2} (|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|)$

The Hadamard gate is essential for creating superposition and performing quantum computations.

**Phase Gate:** The phase gate (S) introduces a phase shift of  $\pi/2$  (or 90 degrees) to the  $|1\rangle$  state, while leaving the  $|0\rangle$  state unchanged. The mathematical representation of the phase gate is:  $S = |0\rangle\langle 0| + i|1\rangle\langle 1|$

The phase gate is often used in combination with other gates to perform various quantum operations.

**Controlled Gates:** Controlled gates are gates that act on a target qubit based on the state of a control qubit. The most common controlled gate is the Controlled-NOT (CNOT) gate, which flips the state of the target qubit if the control qubit is in the  $|1\rangle$  state. The mathematical representation of the CNOT gate is:  $CNOT = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$  Here, I represents the identity matrix and X is the Pauli-X gate.

Controlled gates are essential for implementing quantum algorithms and encoding quantum information.

**Toffoli Gate:** The Toffoli gate (also known as the Controlled-Controlled-NOT or CCNOT gate) is a three-qubit gate that flips the state of the target qubit if both control qubits are in the  $|1\rangle$  state. The mathematical representation of the Toffoli gate is:  $Toffoli = |0\rangle\langle 0| \otimes I \otimes I + |1\rangle\langle 1| \otimes I \otimes X$  The Toffoli gate is a universal gate, meaning that any quantum computation can be expressed using a combination of Toffoli gates and

single-qubit gates. Quantum Phase Gates: Quantum phase gates introduce a phase shift to the quantum state. They include the T gate, S gate, and their respective controlled versions (CT and CS gates). The mathematical representations of these gates are:  $T = |0\rangle\langle 0| + e^{i\pi/4}|1\rangle\langle 1|$   $CT = |0\rangle\langle 0| \otimes I + e^{i\pi/4}|1\rangle\langle 1| \otimes T$   $S \text{ gate} = |0\rangle\langle 0| + i|1\rangle\langle 1|$   $CS \text{ gate} = |0\rangle\langle 0| \otimes I + i|1\rangle\langle 1| \otimes S$  Quantum phase gates are used in various quantum algorithms, including quantum Fourier transform and quantum error correction. These are just a few examples of the many types of quantum gates used in quantum computing. Each gate represents a specific transformation on the quantum state and plays a crucial role in performing quantum computations. By combining these gates in quantum circuits, complex quantum algorithms can be implemented, leading to potential advancements in fields such as cryptography, optimization, and simulation.

Rotation Gates: Rotation gates include the Rx, Ry, and Rz gates, which perform rotations around the X, Y, and Z axes of the Bloch sphere, respectively. These gates allow for arbitrary rotations of the quantum state. The mathematical representations of the rotation gates are:  $Rx(\theta) = \cos(\theta/2)I - i \sin(\theta/2)\sigma_x$   $Ry(\theta) = \cos(\theta/2)I - i \sin(\theta/2)\sigma_y$   $Rz(\theta) = \cos(\theta/2)I - i \sin(\theta/2)\sigma_z$  Here,  $\theta$  represents the rotation angle, I is the identity matrix, and  $\sigma_x$ ,  $\sigma_y$ , and  $\sigma_z$  are Pauli matrices.

Swap Gate: The Swap gate exchanges the states of two qubits. It is a two-qubit gate that swaps the amplitudes of the quantum states. The mathematical representation of the Swap gate is:  $Swap = |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| + |11\rangle\langle 11|$  The Swap gate is useful for rearranging and entangling qubits in quantum circuits.

Square Root of NOT Gate (V Gate): The V gate is a square root of NOT gate that applies a rotation to the state vector in the Bloch sphere. It transforms the  $|0\rangle$  state to a superposition and the  $|1\rangle$  state to an orthogonal superposition. The mathematical representation of the V gate is:  $V = (1 + i)/2 * |0\rangle\langle 0| + (1 - i)/2 * |1\rangle\langle 0| + (1 - i)/2 * |0\rangle\langle 1| + (1 + i)/2 * |1\rangle\langle 1|$  The V gate is commonly used in quantum algorithms such as amplitude amplification.

Quantum Controlled Rotation Gates: Quantum controlled rotation gates perform rotations on the target qubit based on the state of a control qubit. They include Controlled-Rotation-X (CRX), Controlled-Rotation-Y (CRY), and Controlled-Rotation-Z (CRZ) gates. The mathematical representations of these gates depend on the rotation angles and the control states.  $CRX(\theta) = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Rx(\theta)$   $CRY(\theta) = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Ry(\theta)$   $CRZ(\theta) = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes Rz(\theta)$

Controlled rotation gates are essential for performing controlled quantum operations. These additional quantum gates expand the range of operations that can be performed on qubits. They enable more versatile quantum computations and algorithms. By combining these gates in various configurations, quantum circuits can be designed to tackle specific computational problems and harness the power of quantum mechanics.

**Story :** Types of Quantum Gates In the wizarding world of Harry Potter, where magic takes center stage, different types of spells and enchantments can be compared to various types of quantum gates. Just as different spells have unique effects and properties, quantum gates serve as fundamental building blocks that manipulate and transform quantum information. Let's explore the types of quantum gates through a Harry Potter lens to understand their magical counterparts.

**Pauli Gates:** In Harry Potter, we encounter spells that manipulate the fundamental properties of objects. Similarly, Pauli gates, named after the physicist Wolfgang Pauli, manipulate the fundamental states of quantum bits, known as qubits. We can relate Pauli gates to spells that change the properties of objects, such as the Levitation Charm (Wingardium Leviosa) that lifts objects into the air or the Disillusionment Charm (Disilluimus) that makes objects transparent.

**Hadamard Gate:** Just as Harry Potter encounters the mysterious and transformative Mirror of Erised, the Hadamard gate brings about a transformative effect on qubits. It places a qubit into a superposition state, allowing it to exist in multiple states simultaneously. This gate can be likened to the Mirror of Erised, which reflects the deepest desires and reveals hidden possibilities.

**CNOT (Controlled-NOT) Gate:** The CNOT gate plays a crucial role in entangling qubits, creating a deep connection between them. In the Harry Potter world, the Priori Incantatem spell brings about a connection between two wands, revealing the prior spells they have cast. The CNOT gate can be compared to this spell, allowing information to be shared and entangled between qubits.

**Toffoli Gate:** The Toffoli gate, also known as the Controlled-Controlled-NOT gate, is a three-qubit gate that performs a NOT operation on a target qubit based on the states of two control qubits. This gate can be compared to the concept of Horcruxes in Harry Potter, where the destruction of multiple objects (Horcruxes) is necessary to eliminate Voldemort. Similarly, the Toffoli gate relies on multiple control qubits to determine the transformation of the target qubit.

**Phase Gate:** The Phase gate introduces a phase shift to the quantum state of a qubit. This can be compared to spells that manipulate the time or energy properties of objects, such as the Time-Turner or the Patronus Charm. The Time-Turner allows time manipulation, while the Patronus Charm harnesses positive energy to repel Dementors.

The Phase gate, similarly, manipulates the phase of a qubit's state. These are just a few examples of the types of quantum gates, each with its unique properties and effects. Just as spells and enchantments in the world of Harry Potter have specific purposes and rules, quantum gates provide the tools and operations for manipulating and transforming quantum information. By drawing parallels between magic and quantum gates, we can explore the fascinating realm of quantum computing through a Harry Potter lens.

**SWAP Gate:** In the

magical world, wizards often use spells to switch or exchange objects or characteristics. The SWAP gate in quantum computing allows for the exchange of states between two qubits. This gate can be likened to the Switching Spell (Switchus) that wizards use to swap the positions of objects or people. Controlled Phase Gate: The Controlled Phase gate introduces a phase shift to the quantum state of a target qubit based on the state of a control qubit. This gate can be associated with spells that manipulate the spatial properties of objects, such as the Apparition Spell (Apparate) that enables wizards to teleport from one location to another based on their intent and destination. Measurement Gate: Measurement plays a crucial role in quantum computing, allowing us to extract information from qubits. In the Harry Potter world, Divination classes taught by Professor Trelawney involve predicting the future or obtaining information through various methods. The Measurement gate can be compared to these divination techniques, as it extracts information about the quantum state of a qubit. Rotation Gates: Rotation gates, such as the X, Y, and Z gates, rotate the quantum state of a qubit around different axes. In the world of Harry Potter, spells that alter the orientation or movement of objects can be associated with rotation gates. For example, the Levicorpus spell rotates a person upside down, while the Oculus Reparo spell repairs broken eyeglasses by manipulating their physical structure. Universal Gates: Universal gates, such as the T gate and the CNOT gate, are capable of performing any quantum computation when combined with other gates. These gates can be compared to powerful spells in the Harry Potter universe, such as the Unforgivable Curses (Avada Kedavra, Crucio, Imperio) that possess significant and versatile effects.

### **Bloch sphere representation of qubits.**

Single qubit gates Multiqubit states and entanglement, The Bloch sphere is a geometric representation used to visualize the state of a single qubit in quantum mechanics. It provides an intuitive way to understand the effects of single-qubit gates and the concept of entanglement in multiqubit states. Let's explore the Bloch sphere representation, single-qubit gates, and entanglement in detail.

1. Bloch Sphere Representation: In quantum mechanics, a qubit is represented by a vector in a two-dimensional complex Hilbert space. The Bloch sphere provides a three-dimensional visualization of this qubit state. Each point on the sphere corresponds to a unique quantum state. Mathematically, a qubit state can be expressed as  $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ , where  $\alpha$  and  $\beta$  are complex probability amplitudes and  $|\alpha|^2 + |\beta|^2 = 1$ . The Bloch sphere maps the probability amplitudes  $\alpha$  and  $\beta$  to points on the surface of a unit sphere. The mapping is given by:  $|0\rangle \rightarrow (1, 0, 0)$   $|1\rangle \rightarrow (0, 0, 1)$   $|\psi\rangle \rightarrow (2\text{Re}(\alpha\beta^*), 2\text{Im}(\alpha\beta^*), |\alpha|^2 - |\beta|^2)$  The Bloch vector  $(x, y, z)$

represents the state of the qubit. The length of the vector,  $|x|^2 + |y|^2 + |z|^2 = 1$ , ensures that it lies on the surface of the unit sphere.

2. Single-Qubit Gates: Single-qubit gates are operations that act on individual qubits, modifying their state. These gates can be represented as unitary matrices operating on the qubit state vector. Common single-qubit gates include:

- Pauli-X ( $\sigma_x$ ) gate:  $\sigma_x = [[0, 1], [1, 0]]$
- Pauli-Y ( $\sigma_y$ ) gate:  $\sigma_y = [[0, -i], [i, 0]]$
- Pauli-Z ( $\sigma_z$ ) gate:  $\sigma_z = [[1, 0], [0, -1]]$
- Hadamard (H) gate:  $H = (1/\sqrt{2}) [[1, 1], [1, -1]]$
- Phase (S) gate:  $S = [[1, 0], [0, i]]$
- T gate:  $T = [[1, 0], [0, e^{(i\pi/4)}]]$

3. Applying these gates to a qubit state corresponds to multiplying the state vector by the corresponding gate matrix.

4. Multiqubit States and Entanglement: Multiqubit states arise when multiple qubits are combined in a quantum system. Entanglement is a key feature of multiqubit systems, enabling powerful quantum phenomena. Let's consider a system of two qubits, denoted as  $|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$ . Here,  $\alpha, \beta, \gamma$ , and  $\delta$  are complex probability amplitudes that satisfy the normalization condition  $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$ . Entanglement occurs when the state of the two qubits cannot be described independently but requires a joint description. This means that the measurement outcome of one qubit is correlated with the measurement outcome of the other qubit. One example of an entangled state is the Bell state:  $|\Phi+\rangle = (1/\sqrt{2}) (|00\rangle + |11\rangle)$

In addition to single-qubit gates, multiqubit gates, such as the CNOT gate, play a crucial role in entangling qubits. The CNOT gate flips the target qubit's state ( $|0\rangle$  to  $|1\rangle$  and vice versa) if the control qubit is in state  $|1\rangle$ . Mathematically, the CNOT gate can be represented by the following matrix:  $CNOT = [[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 0, 1], [0, 0, 1, 0]]$

When applied to the Bell state  $|\Phi+\rangle$ , the CNOT gate entangles the two qubits, resulting in an entangled state. Entanglement has profound implications for quantum information processing, quantum communication, and quantum algorithms. By understanding the Bloch sphere representation, single-qubit gates, and entanglement in multiqubit states, we gain valuable insights into the behavior and manipulation of quantum systems. These concepts form the foundation of quantum computing and pave the way for harnessing the power of quantum mechanics for various applications.

**Story : Bloch sphere representation of qubits. Single qubit gates**

**Multiqubit states and entanglement, In the wizarding world of Harry Potter, let's delve into the mystical realm of quantum computing and explore the concepts of the Bloch sphere representation of qubits, single qubit gates, multiqubit states, and entanglement. Prepare to embark on a**

magical journey! The Bloch Sphere: Imagine Harry holding his trusty wand, which represents a qubit in the quantum world. The Bloch sphere is a magical sphere that encapsulates the possible quantum states of Harry's wand. Each point on the surface of the sphere represents a unique state that the wand can be in, such as pointing up, down, or somewhere in between. Harry's ability to manipulate the wand's state using spells is akin to the quantum gates that can transform the position of the qubit on the Bloch sphere. Single Qubit Gates: Just like wizards learn various spells at Hogwarts, quantum computers utilize single qubit gates to manipulate the state of individual qubits. For instance, the Hadamard gate casts a spell on the wand, causing it to rotate halfway between pointing up and pointing down on the Bloch sphere. This gate, known as the "Half-Turn Charm," enables Harry to create a superposition state, where the wand is simultaneously both up and down. Other single qubit gates, such as the Pauli-X gate ("Reversus Charm") and the Pauli-Y gate ("Sideways Flip Hex"), can transform the wand's state by flipping it around different axes on the Bloch sphere. These gates empower Harry to perform magical feats like negating the wand's state or performing a rotation in a different direction. Multiquantum States and Entanglement: In Harry's adventures, he often teams up with his friends Ron and Hermione to solve complex challenges. Similarly, in quantum computing, multiple qubits can come together to form entangled states and tackle intricate problems. Imagine Ron and Hermione each possessing a magical wand representing a qubit. When they perform a special spell called "Entanglement Charm," their wands become intertwined and entangled. This enchanting connection creates a mysterious link between their qubits, transcending physical distance. No matter how far apart Ron and Hermione are, the state of one qubit instantly affects the state of the other. They become an entangled

pair, capable of performing powerful feats of magic. Multiqubit gates, such as the Controlled-NOT gate ("Friendship Link"), allow Harry to manipulate the entangled state of Ron and Hermione's qubits. By casting this spell, Harry can control the transformation of one qubit based on the state of the other. It's as if he can communicate with his friends through a magical bond, influencing their actions and outcomes. Entangled states and multiqubit gates enable Harry and his friends to perform remarkable feats that surpass the capabilities of individual qubits. They can communicate secretly using encrypted messages, solve complex puzzles collectively, and perform computations more efficiently. In summary, the Bloch sphere represents the quantum states of qubits, just as Harry's wand embodies the possibilities of magic. Single qubit gates allow Harry to transform his wand's state, while multiqubit states and entanglement create mystical connections between qubits, akin to the friendship and collaboration between Harry, Ron, and Hermione. By harnessing these quantum concepts, Harry and his friends unlock the extraordinary potential of quantum computing, delving deeper into the mysterious and enchanting world of magic.

### Quantum Circuits and controlled gates.

Universal Gates, Quantum Circuits for evaluation of functions Quantum Circuits and Controlled Gates: A quantum circuit is a model used to represent and manipulate quantum information. It consists of a sequence of quantum gates applied to a set of qubits. Quantum gates are analogous to classical logic gates and are the building blocks of quantum circuits. They represent unitary transformations that operate on the quantum state of the qubits. Controlled gates, also known as controlled operations, are a type of quantum gate that act on two qubits, with one qubit serving as the control and the other as the target. The controlled gate applies a specific operation to the target

qubit based on the state of the control qubit. It allows for conditional operations, where the gate is applied only if the control qubit is in a certain state. For example, the controlled-NOT (CNOT) gate is a commonly used controlled gate. It applies a NOT operation to the target qubit if the control qubit is in the state  $|1\rangle$ , and leaves the target qubit unchanged if the control qubit is in the state  $|0\rangle$ . Mathematically, the CNOT gate can be represented as:  $\text{CNOT} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$ , where  $|0\rangle$  and  $|1\rangle$  represent the basis states of the control qubit,  $I$  is the identity matrix, and  $X$  is the Pauli-X gate. The CNOT gate flips the state of the target qubit when the control qubit is in the state  $|1\rangle$ .

**Universal Gates:** Universal gates in quantum computing are gates that can be used to construct any other gate. In other words, by using a combination of universal gates, one can approximate any desired unitary transformation on a quantum system. Two commonly used universal gates are the Hadamard gate ( $H$ ) and the controlled-NOT gate (CNOT). The Hadamard gate is a single-qubit gate that creates superposition by transforming the basis states  $|0\rangle$  and  $|1\rangle$  into equal superpositions of both states. Mathematically, the Hadamard gate can be represented as:  $H = 1/\sqrt{2} (|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|)$ . The CNOT gate, as mentioned earlier, is a two-qubit gate that performs a controlled operation on the target qubit based on the state of the control qubit.

**Quantum Circuits for Evaluation of Functions:** Quantum circuits can also be designed to evaluate mathematical functions. This is accomplished using classical reversible gates and auxiliary qubits. One common method for function evaluation is the Quantum Oracle, also known as the Black Box. The Quantum Oracle represents a function  $f(x)$  that maps an input  $x$  to an output  $f(x)$ . The function can be represented by a unitary transformation on the quantum state, where the auxiliary qubits encode the input and output values. The oracle acts as a controlled gate, applying the function transformation if the input qubits are in a specific state. For example, consider a function  $f(x)$  that maps a 2-bit input  $x$  to a 1-bit output  $f(x)$ . The quantum oracle can be constructed using a series of controlled gates, where the control qubits encode the input and the target qubit represents the output. The specific gate operations depend on the desired function mapping. The circuit for function evaluation typically involves a combination of classical and quantum operations. The classical part performs the

necessary classical computations to determine the function input, while the quantum part applies the quantum oracle to obtain the function output. By combining quantum gates, including controlled gates, with classical computations, quantum circuits can evaluate complex functions in a parallel and reversible manner. This opens up possibilities for solving certain computational problems more efficiently than classical algorithms. In summary, quantum circuits are composed of quantum gates that manipulate the quantum state of qubits. Controlled gates allow for conditional operations based on the state of control qubits. Universal gates, such as the Hadamard gate and CNOT gate, can be used to construct any other gate. Quantum circuits for function evaluation involve the use of quantum oracles to represent mathematical functions and perform computations. These concepts form the foundation for designing and analyzing quantum algorithms.

## ENDIANNES

In computing, **endianness** is the order or sequence of bytes of a word of digital data in computer memory. Endianness is primarily expressed as **big-endian (BE)** or **little-endian (LE)**. A big-endian system stores the most significant byte of a word at the smallest memory address and the least significant byte at the largest. A little-endian system, in contrast, stores the least-significant byte at the smallest address.

### Least significant bit and most significant bit:

In computing, the terms "least significant bit" (LSB) and "most significant bit" (MSB) are used to describe the position and significance of individual bits within a binary representation of a number.

In a binary number, each digit is called a bit, and it can have one of two values: 0 or 1. The position of a bit determines its significance in the number's value.

The LSB refers to the bit that occupies the rightmost position or the lowest bit index. It has the least value in the binary representation. For example, in the 8-bit binary number 11001010, the rightmost bit (bit index 0) is the LSB.

The MSB, on the other hand, refers to the bit that occupies the leftmost position or the highest bit index. It has the highest value in the binary representation. Using the same example of the 8-bit binary number 11001010, the leftmost bit (bit index 7) is the MSB.

## ENDIANNES IN QUANTUM COMPUTING

In quantum computing, endianness refers to the ordering of qubits within a quantum register. Since qubits are the fundamental units of information in a quantum computer, their arrangement can impact how quantum operations and measurements are performed.

The state of a qubit can be described as a linear combination of two basis states, conventionally denoted as  $|0\rangle$  and  $|1\rangle$ . However, quantum computers typically have multiple qubits, and the ordering of these qubits can vary.

The two common qubit ordering schemes in quantum computing:

**Big Endian Ordering:** In this scheme, the most significant qubit is placed at the rightmost position(smallest memory address), similar to big endian byte ordering. The least significant qubit is placed at the leftmost position. The qubit arrangement follows the convention used in classical computing systems. For example, if you have a 3-qubit system, the qubits would be ordered as  $|qubit0-qubit1-qubit2\rangle$ .

**LittleEndian Ordering:** In contrast to big endian, little endian ordering places the least significant qubit at the rightmost position(smallest memory location) , while the most significant qubit is placed at the leftmost position. This ordering is often preferred in some quantum algorithms and implementations. Using the same example of a 3-qubit system, the qubits would be ordered as  $|qubit2-qubit1-qubit0\rangle$ .

The choice of qubit ordering can have implications for designing and implementing quantum algorithms. It affects how qubit operations are applied, how quantum gates are constructed, and how measurements are interpreted.

It's important to note that the choice of qubit ordering in quantum computing is **not** as universally standardized as byte ordering in classical computing. Different quantum computing platforms and programming frameworks may adopt different conventions.

Note that **IBM- Qiskit** uses little endian ordering as its standard qubit ordering. Throughout our tutorials we will use **Qiskit little endian qubit ordering**.

**The end**

## Challenges Faced

- While developing the content we found several difficulties like in curating the content and with the website

## Software & Tools Used

- Qiskit
- Wowchemy by Hugo
- Python
- Git

## Future Plans

1. Introduce more quantum computing concepts eg. Entanglement, Quantum Tunneling, etc.
2. Introduce more graphics and animations in the Content
3. Introduce engaging characters for the storyline.

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