Superdense Coding

In quantum information theory, **superdense coding** is a quantum communication protocol to communicate a number of classical bits of information by only transmitting a smaller number of qubits, under the assumption of sender and receiver pre-sharing an entangled resource.

In its simplest form, the protocol involves two parties, often referred to as Alice and Bob in this context, which share a pair of maximally entangled qubits, and allows Alice to transmit two bits (*i.e.*, one of 00, 01, 10 or 11) to Bob by sending only one qubit. This protocol was first proposed by Charles H. Bennett and Stephen Wiesner in 1970 (though not published by them until 1992) and experimentally actualized in 1996 by Klaus Mattle, Harald Weinfurter, Paul G. Kwiat and Anton Zeilinger using entangled photon pairs.

Superdense coding can be thought of as the opposite of quantum teleportation, in which one transfers one qubit from Alice to Bob by communicating two classical bits, as long as Alice and Bob have a pre-shared Bell pair.

Quantum teleportation and superdense coding are closely related, to avoid confusion we need to clarify the difference.

Quantum teleportation is a process by which the state of qubit can be transmitted from one location to another, using two bits of classical communication and a Bell pair. In other words, we can say it is a protocol that destroys the quantum state of a qubit in one location and recreates it on a qubit at a distant location, with the help of shared entanglement. Superdense coding is a procedure that allows someone to send two classical bits to another party using just a single qubit of communication.

Steps for protocol

Step 1

The process starts with a third party, who we'll call Charlie. Two qubits are prepared by Charlie in an entangled state. He initially starts the 2 qubits in the basis state |0> . He applies Hadamard gate to the first qubit to create superposition. He then applies CNOT gate using the first qubit as a control and the second as the target. This is the entangled state (Bell pair) we mentioned earlier.

We start in the state |00>

$$|00\rangle = |0\rangle_A \otimes |0\rangle_B$$

where the qubit to be sent to Alice is labeled with A and the qubit to be sent to Bob is labeled B. Charlie first applies a Hadamard gate to the first qubit, which creates superposition and we get the state:

$$|+0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

Then Charlie applies the CNOT gate. The CNOT gate entangles both qubits, i.e. it flips the target if the control is . Note that the control qubit is our leftmost qubit.

$$\text{CNOT}\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Step 2

Charlie sends the first qubit to Alice and the second qubit to Bob. The goal of the protocol is for Alice to send 2 classical bits of information to Bob using her qubit. But before she does, she needs to apply a set of quantum gates to her qubit depending on the 2 bits of information she wants to send:

Encoding Rules for Superdense Coding (Alice protocol):

- **00 apply I**
- **01 apply X**
- **10 apply Z**
- **11 apply ZX**

Thus if she wants to send a 00, she does nothing to her qubit (apply the identity gate). If she wants to send a 01, then she applies the X gate. Depending on what she wants to send, she applies the appropriate gate, then sends her qubit to Bob for the final step in the process.

Step 3

Bob receives Alice's qubit (leftmost qubit) and uses his qubit to decode Alice's message. Notice that he does not need to have knowledge of the state in order to decode it — he simply uses the restoration operation.

Bob applies a CNOT gate using the leftmost qubit as control and the rightmost as target. Then he applies a Hadamard gate and finally performs a measurement on both qubits to extract Alice's message.