Assignment 2017–2018

Advanced Econometrics

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Notes and instructions:

1. This assignment is mandatory.
2. The assignment is to be made in groups of three or four students. No more, no less.
3. The assignment must be delivered no latter than Tuesday, the 17th of October, at 10:30. There will be no tolerance period for late deliveries. Deliveries after the assigned deadline imply that you have a final grade of zero for the assignment.
4. The assignment must be uploaded in pdf format to Canvas before the deadline mentioned above.
5. You must also upload all the MATLAB files used to obtain the answers. The code should be clear and well commented. **Note:** code in other languages like GAUSS or OX is also acceptable. Please inform me in advance if you will not be using MATLAB.
6. The delivered document must state the names and student numbers of each member of the group.
7. This is not homework. You must deliver a detailed econometric report. The answers should be as complete as possible and accompanied by appropriate justifications and insightful comments and remarks. Please consider all the theory that you know in analyzing the empirical results.
8. To be fair, I will not answer any questions concerning the assignment.

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PART I: Real Exchange rates

In economics, the *law of one price* and the *purchase power parity (PPP) hypothesis* both predict that, under certain conditions, *real exchange rates* should remain fixed over time. In particular, these predictions should hold true if there were no transaction costs of any kind.

Given the presence of transport costs and barriers to trade, there is wide consensus among economists that, in practice, real exchange rates should be 'free to fluctuate', but only when they are near equilibrium. In other words, while non-stationary dynamics seem acceptable close to equilibrium, overall, the real exchange rates should be overall 'stationary', 'stable' or 'mean reverting', and hence return to their equilibrium values in the long-run.

This description of the behavior of real exchange rates suggests the existence of nonlinear dynamics. As an econometrician, you surely want to test the validity of this claim! *Does the data support these theories? Are exchange rates 'free to fluctuate' close to equilibrium but overall stationary and mean-reverting?*

The file `AE2016_assign_p1.mat` contains monthly real effective exchange rates from five different groups of OECD countries, spanning from 1974 to 2016.

1. Plot each of the time-series that you have at your disposal.
2. For each time-series, estimate a Gaussian AR(1), AR(2), AR(3) and Exponential SESTAR models by maximum likelihood. Report the parameter estimates for all models. (Tip: selecting initial values for the ML estimation of the SESTAR parameters is a tricky business. You may consider using the estimated parameters for the AR(1) as guide)
3. Report the likelihoods achieved by each of the above models and provide a likelihood-based model comparison. Which model do you think is best? Do you find evidence of SESTAR dynamics in the data? Justify your answer.
4. Use the last 50 observations as a validation sample to compare the models in terms of their one-step-ahead forecast accuracy. Use the FRMSE and FMAE as your selection criterion.
5. Report the Diebold-Mariano test statistic for each model and each criterion. Does the evidence support the law of one price and the purchase power parity (PPP) hypothesis? Explain your findings.

Figure 1 plots the monthly *real exchange rate* of a composite currency from 15 *European Union* (EU 15) countries against the *Danish Kroner*, from 1992 to 2013.

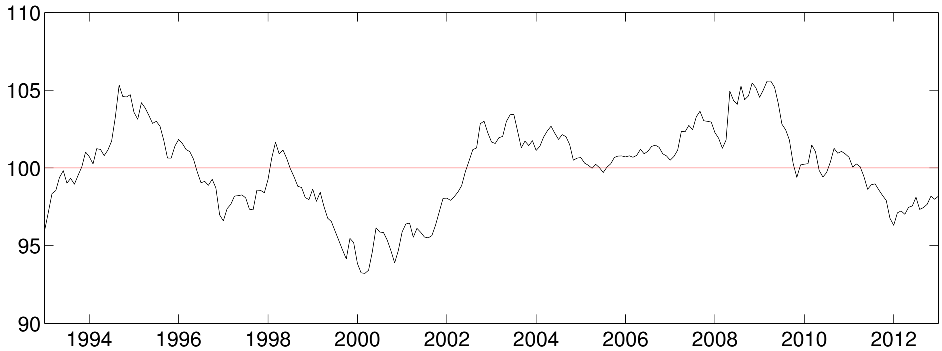

Figure 1: Real exchange rate of EU 15 vs Danish Kroner.

Let the sample of real exchange rates $\{x_t\}_{t=1}^T$ at your disposal be a subset of the realized path of a strictly stationary and ergodic time-series $\{x_t\}_{t\in\mathbb{Z}}$ with bounded moments of fourth order $\mathbb{E}|x_t|^4 < \infty$. Consider a *Gaussian Exponential SESTAR model* for the real exchange rate

$$x_t = \alpha + g(x_{t-1};\boldsymbol{\theta})(x_{t-1}-\mu) + \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\epsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,\sigma_\epsilon^2)$$

$$\text{and} \quad g(x_{t-1};\boldsymbol{\theta}) := \delta + \frac{\gamma}{1+\exp(\beta(x_{t-1}-\mu)^2)} \quad \text{for every} \quad t \in \mathbb{Z}.$$

Suppose that the parameters $\boldsymbol{\theta} = (\alpha,\delta,\gamma,\beta,\sigma_\varepsilon^2)$ of the model are estimated by maximum likelihood (ML) on a compact parameter space Θ with $\beta \geq 0$ and $\sigma_\varepsilon^2 > 0$.

6. Give the expression for the log likelihood function.

7. Show that the log likelihood function converges uniformly to a limit deterministic function. (Tip: note that $|g(x;\boldsymbol{\theta})| \leq |\delta| + |\gamma|/2$ for all x and $\sigma_\varepsilon^2 \geq a$ for some $a > 0$.)

8. Suppose that there exists a $\boldsymbol{\theta}_0 \in \Theta$ that is the unique maximizer of the limit function. Show that the ML estimator $\hat{\boldsymbol{\theta}}_T$ is consistent for $\boldsymbol{\theta}_0$. Is it asymptotically normal? What is the approximate distribution of the ML estimator?

9. Suppose that the parameters of the model were estimated by *maximum likelihood*, using the data set spanning from November 1992 to April 2009. The estimation yielded the following results:

Log likelihood	-216.15
Observations	196

Parameter	Estimate	Std Error
α	98.59	0.5231
δ	0.631	0.2101
γ	0.719	0.0751
β	0.061	0.0180
μ	98.12	2.4971
σ^2_ε	0.998	0.1881

Do you reject the hypothesis that $\gamma = 0$? What does that tell you about the dynamic behavior of this *real exchange rate*?

10. Suppose that the model is well specified, and suppose for a moment that the 'true' parameter $\boldsymbol{\theta}_0$ is exactly equal to the point estimates reported in the table above. Would you say that the *real exchange rate* time-series $\{x_t\}_{t\in\mathbb{Z}}$ is stationary? Does the evidence support the *law of one price* and the *purchase power parity (PPP) hypothesis*? Is it reasonable to assume that the data has four bounded moments $\mathbb{E}|x_t|^4 < \infty$?

11. Suppose that you also obtained parameters estimates for the linear Gaussian AR(1) model

$$x_t = \omega + \phi x_{t-1} + v_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{v_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0, \sigma^2_v).$$

and obtained the following results

Log likelihood	-238.12
Observations	196

Parameter	Estimate	Std Error
ω	1.296	0.4131
ϕ	0.991	0.9193
σ^2_v	1.021	0.1779

Which model do you think is best? The linear AR(1) or the SESTAR?

12. If these models are mis-specified, then to which parameter is the ML estimator converging to? What is the meaning of the limit parameter $\boldsymbol{\theta}_0$ in each model?

13. Suppose that you find yourself on the month of April, 2009. The real exchange rate has just peaked at its highest value of 105,33. Use both the AR(1) and exponential SESTAR models above to forecast the real exchange rate for the following 12 months. In particular, use simulations to calculate the conditional expectation of X_{T+n} for $n = 1, 2, ..., 12$.

14. Suppose that the normality of the innovations was not rejected by a Jarque-Bera test. Add 90% confidence intervals to your forecasts. Furthermore, calculate the probability of the real exchange rate rising above $105,33$ in the following month of May 2009. Compare the results obtained from the two models.

15. Compare the forecasts of the two models to the actual observed values given below.

May 09	103.26	Sep 09	101.19	Jan 10	101.02
Jun 09	101.73	Oct 09	101.24	Feb 10	100.08
Jul 09	101.14	Nov 09	100.26	Mar 10	99.526
Aug 09	100.79	Dec 09	100.72	Apr 10	98.956

16. Produce and compare 24-month impulse response functions (IRFs) using both the SESTAR and AR(1) models estimated above. Please focus on IRFs generated by a positive shock of size 10, with origin at 100. Please comment on the shape of the IRFs and do not forget to include 90% confidence bounds.

PART II: Financial Returns

In economics and finance, it is well known that the conditional volatility of financial returns evolves slowly over time. Financial returns are thus characterized by the presence of 'clusters of volatility'. Recent evidence suggests that robust volatility filters might be needed for correctly capturing the time-varying conditional volatility in stock returns. Furthermore, there are also reasons to believe that large negative returns produce more volatility than large positive returns. This characteristic, first noted by Black (1976), is known as the *leverage effect*. Some analysts claim that the *leverage effect* plays a very important role in financial markets. As an econometrician, you surely want to test these claims! *Do robust filters really perform better? Does the data support the existence of a 'leverage effect'?*

1. Consider the following *Robust-GARCH-with-Leverage-Effect* model for time-varying volatilities

$$x_t = \sigma_t \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{TID}(\lambda),$$

$$\sigma_t^2 = \omega + \alpha \frac{x_{t-1}^2 + \delta x_{t-1}}{1 + \lambda^{-1} x_{t-1}^2} + \beta \sigma_{t-1}^2.$$

   Analyze, compare and intepret the news-impact-curves of this model for different values of $\lambda = 2, 5, 10, 50$, and different values of $\delta = 0, 0.1, 0.3, 1$. What happens as $\lambda \to \infty$? Do you find a familiar model? What if you let both $\lambda \to \infty$ and $\delta \to 0$?

2. In the file `AE2016_assign_p2.mat` you can find a data set of daily stock returns. This data set includes returns on five stock market indices that reflect the market value of large companies in the US, Europe and Japan. Make use of a GARCH model as well as the Robust GARCH-with-Leverage-Effect model above to filter the time-varying conditional volatility present in these stock market indices. Report the estimated parameters.

3. Use the parameter estimates obtained in the previous question to plot and compare the estimated news-impact-curve and the filtered volatilities obtained from the two alternative models.

4. Calculate and report the 1% and 5% value-at-risk (VaR) implied by the filtered volatilities obtained in the previous question. Compare the two models in terms of their implied VaRs.

5. Report the log likelihood, AIC and modified AIC attained by each model. Which one do you think is best? Justify your answer.

6. Consider only the two stocks with highest average returns over the sample. These are stocks 3 and 4 on your data set. Build a portfolio using these stocks and re-optimize your portfolio every day by maximizing the Sharpe ratio. Since the returns are uncorrelated, you can use the unconditional mean of returns to calculate your Sharpe ratio. Report the optimal portfolios obtained using the GARCH and Robust GARCH-with-Leverage-Effect models estimated above to filter the conditional volatilities.

Figure 2 plots the time-series of weekly S&P500 returns from 2005 to 2013. The S&P500 index is a stock market index based on the 500 largest companies in the United States.

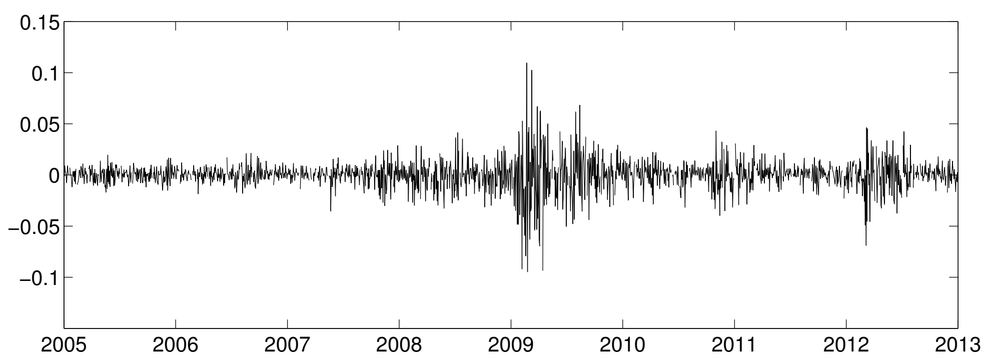

Figure 2: Weekly S&P 500 returns.

Let the sample of S&P500 returns $\{x_t\}_{t=1}^T$ at your disposal be a subset of the realized path of a strictly stationary and ergodic time-series $\{x_t\}_{t\in\mathbb{Z}}$ with bounded moments of sixth order $\mathbb{E}|x_t|^8 < \infty$. Consider the *Asymmetric GARCH* model for time-varying volatilities

$$x_t = \sigma_t \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,1),$$

$$\sigma_t^2 = \omega + \alpha(x_{t-1} - \delta)^2 + \beta\sigma_{t-1}^2.$$

Suppose that the parameters $\boldsymbol{\theta} = (\omega, \alpha, \delta, \beta)$ of the model are estimated by maximum likelihood (ML) on a compact parameter space Θ with α, δ and β satisfying

$$\omega > a, \quad \alpha > a, \quad \text{and} \quad a < \beta < 1 \quad \text{for some } a > 0.$$

7. Give the expression for the log likelihood function.

8. Show that the log likelihood function converges uniformly to a limit deterministic function. (Tip: note that $\omega > a$, $\alpha > a$ and $\beta > 0$ ensures $\sigma_t^2 > a$ for every t.)

9. Suppose that there exists a $\boldsymbol{\theta}_0 \in \Theta$ that is the unique maximizer of the limit log likelihood function. Show that the ML estimator $\hat{\boldsymbol{\theta}}_T$ is consistent for $\boldsymbol{\theta}_0$. Is it asymptotically normal? What is the approximate distribution of the estimator?

10. Suppose that the parameters of the model were estimated by *maximum likelihood* yielding the following results:

Log likelihood	-3305.81
AIC	6623.7

Parameter	Estimate	Std Error
ω	0.012	0.0035
α	0.096	0.0017
β	0.885	0.0154
δ	0.012	0.0015

Do you reject the hypothesis that $\delta = 0$? What does that tell you about the dynamic behavior of volatility? Do you think that there exists evidence for a *leverage effect* in financial returns?

11. Suppose that you also obtained parameters estimates for the GARCH model

$$x_t = \sigma_t \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{NID}(0,1).$$

$$\sigma_t^2 = \omega + \alpha x_{t-1}^2 + \beta \sigma_{t-1}^2.$$

and obtained the following results:

Log likelihood	-3197.1
AIC	6796.5

Parameter	Estimate	Std Error
ω	0.017	0.0027
α	0.095	0.0092
β	0.891	0.0115

A colleague of yours points out that the results reported in these tables for the GARCH model contain a mistake. Can you find it?

12. Suppose that the GARCH model is well specified, and suppose for a moment that the 'true' parameter $\boldsymbol{\theta}_0$ is exactly equal to the point estimates reported in the table above. Would you say that the true volatility is stationary? What about the S&P500 time-series $\{x_t\}_{t\in\mathbb{Z}}$? is it stationary?
(Tip: note that $\mathbb{E}|0.095\epsilon_t^2 + 0.891| < 1$ for $\epsilon_t \sim N(0,1)$)

13. Suppose that the Asymmetric GARCH model is correctly specified. What can you say about the specification of the GARCH model? If the Asymmetric GARCH is mis-specified, then what can you say about the GARCH specification?