Rain-Flood Model of Pakistan

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CS166: Modeling and Analysis of Complex Systems

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Purpose

The purpose of this simulation is to model the water flow during the monsoon season in Pakistan and find whether the country is susceptible to floods or not. In case there is susceptibility to floods, the simulation will aim to find a solution that can reduce the occurrence of floods in the country. The reason this simulation is important is that this year, Pakistan has experienced the worst flash floods and landslides due to water flow in the past 30 years with almost 10 million children in need of life-support due to diseases, homelessness, etc (UNICEF, 2022). With the impacts of climate change becoming more prominent with each passing year, this issue needs to be addressed as soon as possible (Economist, 2022).

For this simulation, I take information from Salma et al.'s paper from 2012 which studied rainfall patterns in Pakistan. They divided Pakistan into five different zones based on annual rainfall in each of the cities so that they could be grouped. Moreover, I studied the general flood path that is followed in Pakistan taken from (UN OCHA, n.d.). Looking at the path, I took five cities from each of the zones, trying to make them as equidistant as possible while also choosing cities for which data was available in the paper or could be found through Google searches.

In the simulation, we will measure the water levels in each of the cities as well as the state of flooding which we will keep track of to see what path the flood takes in the country as well as try to counteract it. Finally, we will explore the probability of natural drainage i.e. the probability with which water is displaced within the soil naturally has an impact on the percolation threshold i.e. transition of flood from the North to the South.

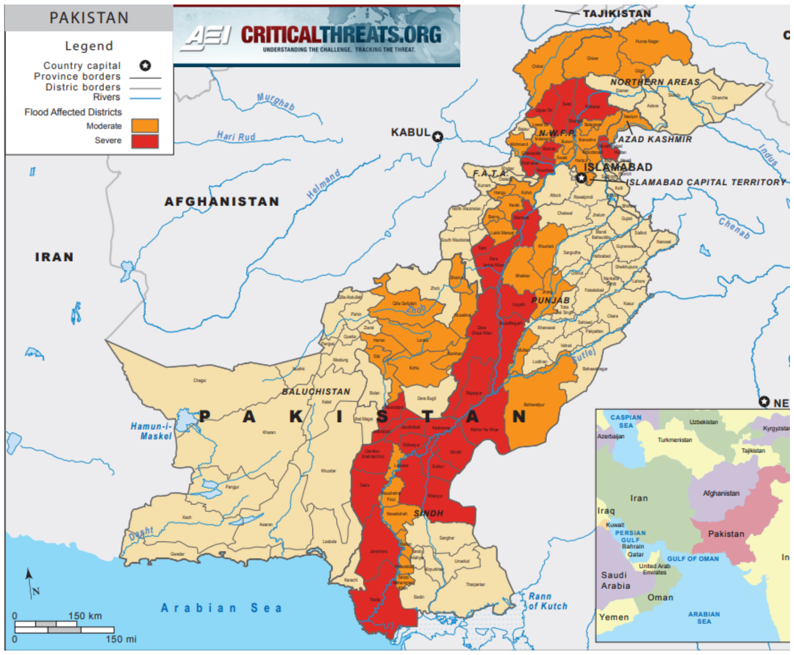

Figure 1: Flood route map of Pakistan from UN OCHA. The blue lines represent the flow of the flood from the mountains and hills of the north all the way to the Arabian sea in the south.

The Data

	Names	Altitude in meters	Annual Precipitation in meters	Rainy days as %age	Average Rainfall in meters	Soil	Flooded	Water Body present?	Barrage
0	Swat	980	0.67	50	0.022333	0	False	0	0
1	Skardu	2228	0.67	50	0.022333	0	False	0	0
2	Kohistan	1650	0.67	45	0.024815	1	False	0	0
3	Gilgit	1500	0.67	50	0.022333	0	False	0	0
4	Muzaffarabad	737	0.67	45	0.024815	0	False	0	0
5	Swabi	340	0.65	30	0.036111	0	False	1	0
6	Mianwali	210	0.65	30	0.036111	1	False	1	0
7	Gujrat	1110	0.65	30	0.036111	1	False	1	0
8	Nowshera	552	0.65	30	0.036111	1	False	1	0
9	Jhelum	234	0.65	30	0.036111	1	False	1	0
10	TTK	149	0.33	20	0.027500	0	False	1	0
11	Chiniot	179	0.33	20	0.027500	1	False	1	0
12	Lahore	217	0.33	20	0.027500	0	False	1	0
13	Bhakkar	159	0.33	20	0.027500	0	False	1	0
14	Jhang	158	0.33	20	0.027500	0	False	1	0
15	Lodhran	106	0.22	10	0.036667	0	False	1	0
16	Muzaffargarh	123	0.22	10	0.036667	1	False	1	0
17	Multan	122	0.22	10	0.036667	0	False	1	0
18	DGK	390	0.22	10	0.036667	0	False	1	0
19	Bahawalpur	118	0.22	10	0.036667	0	False	1	0
20	Karachi	10	0.29	15	0.032222	1	False	1	0
21	Sukkur	67	0.29	15	0.032222	1	False	1	0
22	Larkana	147	0.29	15	0.032222	0	False	1	0
23	Nawabshah	34	0.29	15	0.032222	0	False	0	0
24	Kashmore	66	0.29	15	0.032222	0	False	0	0

Rules of the simulation

We have set up the system using two-dimensional Cellular Automata. The rules of the system are relatively simple:

1. If the water level of the soil is below ground level, water drains away internally with a certain probability (drainage probability).
2. If water is at ground level, the soil becomes water-logged.
3. If the water level rises above ground level i.e. flooding occurs, the excess water is displaced to the nearest neighbor with the steepest descent.
4. If there is a water body e.g. river near the neighbor, only a certain percentage of water flows into the city soil, the rest drains away into the river.
5. The updating rules follow a Von-Neumann neighborhood model with water flow possible towards the North, South, East, or West but not diagonally. This rule is important as Pakistan's natural water flow system is North to South with canals being made horizontally to supply water to further away areas.

Modeling Assumptions

The model makes quite a few assumptions about certain things. The first assumption comes with modeling the system as Cellular Automata as this means that we assume the cells (cities) are equidistant from each other. This assumption might not hold in practice but I have tried my best to choose cities that are equidistant from each other. The second assumption is the choice of neighbors. The model uses a Von-Neumann neighborhood which is the best fit that I could find to the actual terrain of Pakistan which is in practice quite complex forged over years of natural water flow.

The model also assumes that the excess water will only flow to the steepest descent neighbor. Here I took inspiration from sampling methods such as MCMC samplers. Again, this might not be true as water flow to neighbors follows particular patterns based on topography but for the sake of the model, we shall assume that it flows to the neighbor with the steepest descent.

The rainfall in each city is assumed to follow a Gamma distribution centered at the mean amount of rainfall during the Monsoon season which I took from Salma et. al. The reason behind choosing a Gamma distribution is that rain as a variable is theoretically continuous and constrained to be positive even if that might not be the case in practice where most rainfall is under a certain range. The Gamma distribution has the same range as the theoretical value of rain and has a light tail meaning that big variations are less-likely though still possible. This also brings in the assumption that the rain each day is independent of the previous days. Again, this assumption is used for modeling purposes even though rain on a certain day might increase the chances of rain on the next day in practice.

Next, we consider the assumptions for the amount of water that flows from one cell to another. If a water body is present, only 30% of the water is assumed to reach the neighbor. This value is chosen by me based on my knowledge of how water bodies can help to reduce floods. Furthermore, barrages are assumed to completely block water flow which is a valid assumption in my opinion as only big floods rather than those caused by rain can overcome barrages.

With regard to the soil, it is assumed that once a city's soil is water-logged, it will remain in that state as it takes quite a few years for the soil to displace that water with air. Moreover, we assume that the natural drainage probability mentioned above is 60%, while 40% of the time, it drains a certain amount of water but not all of it. Also, if the soil is water-logged, the natural drainage probability is reduced by a factor of six, a number chosen by me to model differences between water-logged and non-water-logged soils. Finally, I assume that non-water-logged soils can absorb up to 2 meters of water while water-logged soils can only absorb 0.6 meters.

Finally, we assume that percolation only occurs vertically as compared to diagonally. As Pakistan's natural water cycle flows from the North to the South and our model only models a certain part of it, this assumption appears to be valid as the diagonal flow would take the water into neighboring countries which are beyond our concern for the sake of this simulation because these cases are not reported in real life.

Parameter(s)

Since the simulation is for a specific region, the model doesn't have a lot of parameters that can be altered. I have made the natural drainage probability a parameter for the sake of this simulation which can be changed by an expert using this later on with more knowledge than me[1].

Outputs

I keep track of various attributes for each of the cells including their flooding status which is done using a Boolean variable, and their water levels at each point to keep track of how much water the soil has. These two values used together can help us find a target threshold for percolation as that is the thing that we are trying to stop. If we find a threshold for the average water level after which Percolation always occurs, we can implement measures to keep the average for all cities below this level and ensure that the flood can be contained.

Python Implementation

For the code, I have used Object-Oriented Programming to keep track of my simulation. The two classes in the code are Grid and Cell. The Cell class keeps track of each city including its attributes and its own independent rain. Meanwhile, the Grid class has methods that allow it to find the neighbors, keep track of water flow, return values to us at any point for a certain

[1] My parameter was based on reference from (University of California, 2009)

attribute, make plots for a certain attribute, and analyze whether Percolation has occurred or not at any stage. It does this by making a grid of instances of the Cell class with each instance/object representing one city. For more details, please refer to the Appendix where all of the code is present.

For theoretical proof of how the simulation works, please refer to Appendix A.

Strategies

Since Pakistan is a developing country, the strategy that I have used is Barrages which are relatively cheap to build so they aren't a burden on the economy as well as provide extensive cover from floods. The strategy is used already in Pakistan and I suggest which cities we should enclose so as to stop Percolation.

Choosing which cities need to be protected can be done by just plotting the cases where Percolation occurs and look which route is most impacted. Let us first look at the plot for the initial conditions when flooding occurs.

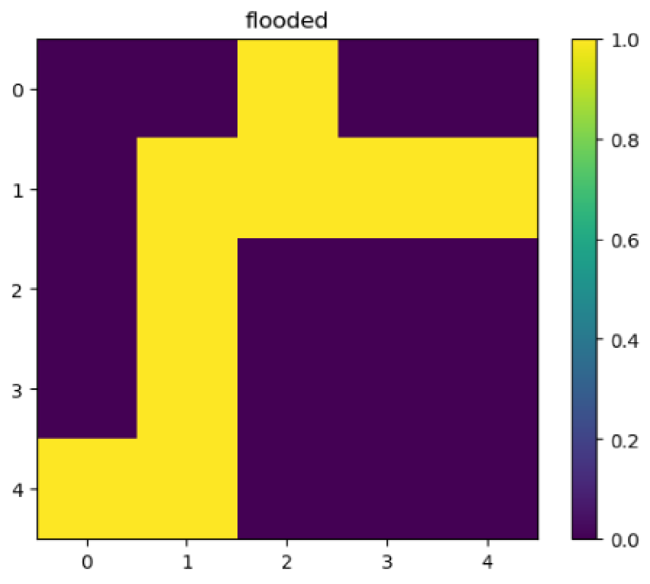

Figure 2: Color plot showing the status of flooding in cells after two months of simulation. While this specific run does not have Percolation, we can see that the second and third columns are most susceptible to it, especially the second column.

The last three rows in the grid represent the provinces which are mainly plains and easy to build on. Keeping this in mind, the strategy that I have implemented is introducing barrages in the last three rows of the second and third columns of the grid to look at how that impacts percolation.

Results

Below, I will compare two kinds of histograms for both the current conditions and the new strategy.

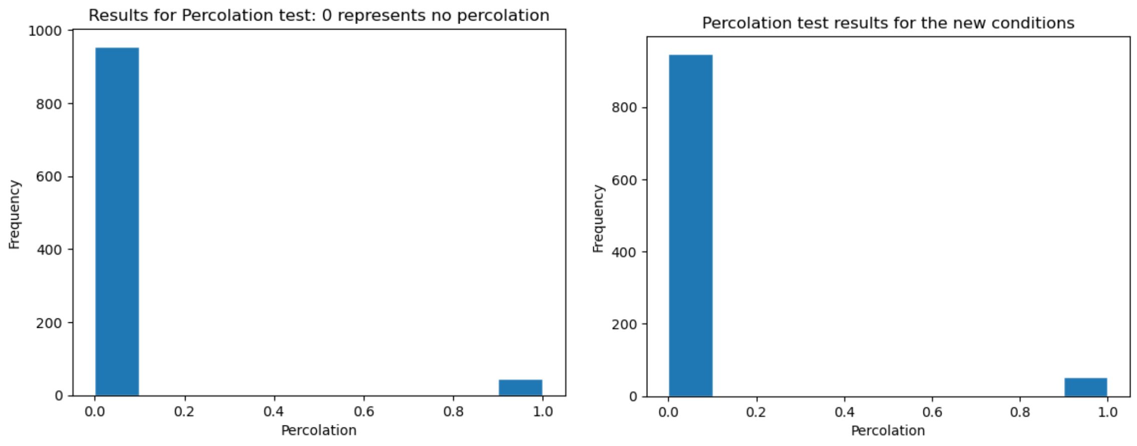

Figure 3: Histograms representing percolation in both conditions, current conditions on the left, after barrages are built on the right. There is almost no noticeable difference between the two conditions.

Next, let us compare the average water levels for the soil at the end of the simulation.

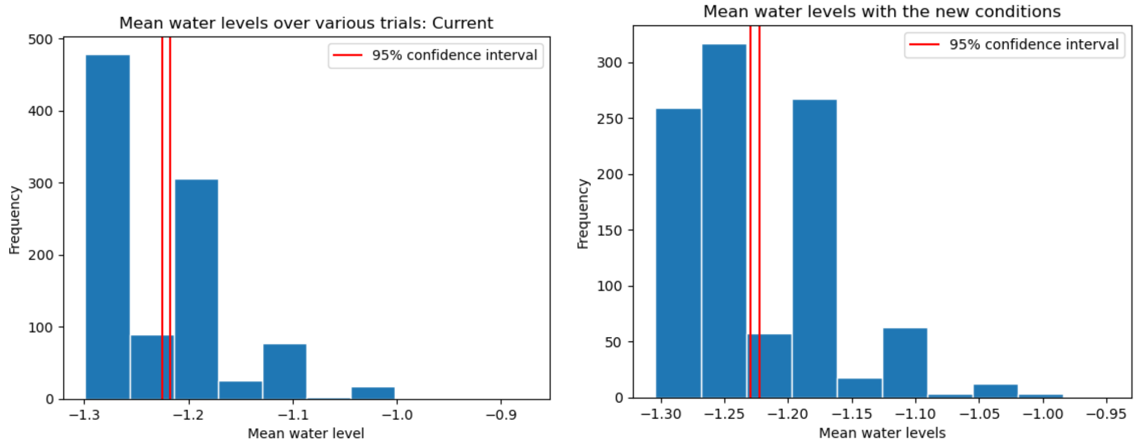

Figure 4: Histograms showing average water levels in the soil at the end of the simulation. We can see that the strategy has managed to contain the water within cities as shown by more values being closer to 0 as compared to the current conditions.

Looking at both the plots above, it is hard to arrive at a conclusion as to whether the new strategy has helped improve the situation or not. Finally, we will compare the two metrics with respect to one another to see if there is a relationship between them.

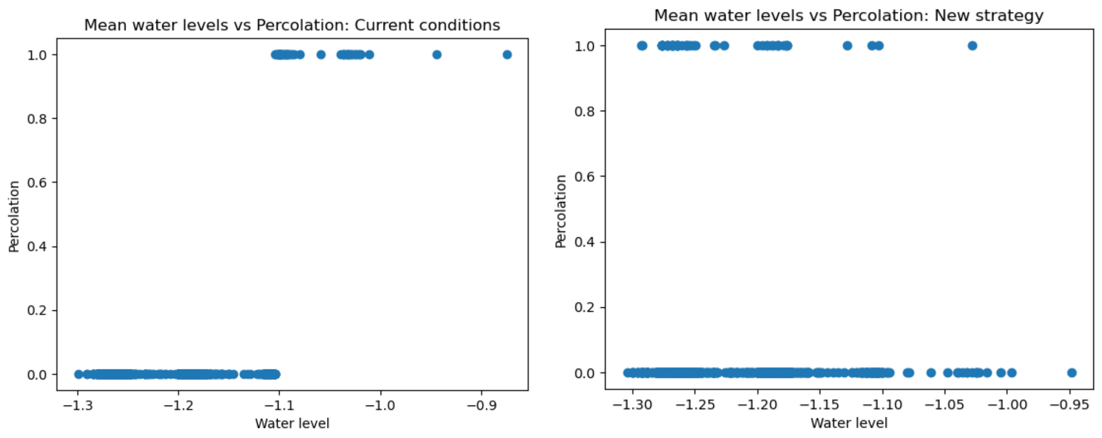

Figure 5: Scatter plots showing trends of how Percolation changes with average water levels in both cases.

As we can see in the figure above, currently we have a threshold appearing clearly for an average water level after which Percolation occurs. However, for the new strategy, there is no evident threshold indicating that the Percolation occurs only because of random occurrences rather than certain conditions being met. This is because the water is contained and Percolation is only occurring in the rare cases that each city on its own is flooded by the amount of rainfall it is receiving which is something outside of our control.

Based on the results above, my advice is to build barrages along the following cities: Chiniot, Lahore, Muzaffargarh, Multan, Sukkur, Larkana. These barrages should help to stop floods from flowing all across the country and actually enable us to contain them which could lead to more concentrated efforts in these particular cities.

Uncertainty

If we look at the 95% confidence interval for the average water levels, the intervals represent how certain we are of the results. The closer the intervals are, the lesser the uncertainty.Here, the intervals for the old strategy are: [-1.2249, -1.2175]. Meanwhile, the intervals for the new strategy are: [-1.2291, -1.2220]. The small size of the intervals show that our simulation is quite stable and we can be quite confident in our results since the intervals are quite narrow. Running for more steps might bring these intervals down even more and we could do this given the availability of computational resources.

References

Economist, T. (2022, August 31). *Why are Pakistan's floods so bad this year?* The Economist. Retrieved December 14, 2022, from https://www.economist.com/graphic-detail/2022/08/31/why-are-pakistans-floods-so-bad-this-year?gclid=CjwKCAiAheacBhB8EiwAItVO240kB_grv5Aer1XRaPQqnyamPhKKgharzjOgCK4eOEW9WVYQViGLCRoCThEQAvD_BwE&gclsrc=aw.ds

Salma, S., Rehman, S., & Shah, M. A. (2012, July). *Rainfall trends in different climate zones of Pakistan - PMD*. Rainfall Trends in Different Climate Zones of Pakistan. Retrieved December 14, 2022, from https://www.pmd.gov.pk/rnd/rnd_files/vol8_issue17/4.pdf

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UNICEF, O. (2022). *Devastating floods in Pakistan*. UNICEF. Retrieved December 14, 2022, from https://www.unicef.org/emergencies/devastating-floods-pakistan-2022

University of California, D. of A. and N. R. (2009, August). *Soil Water Holding characteristics*. Center for Landscape & Urban Horticulture. Retrieved December 14, 2022, from https://ucanr.edu/sites/UrbanHort/Water_Use_of_Turfgrass_and_Landscape_Plant_Materials/Soil_Water_Holding_Characteristics/

Appendix A

Let probability of having a lower altitude neighbor be p

If we have only one cell, the probability of flow across that cell is 1

If we scale this to a $2x2$ grid, we have 4 possible situations. We can have any number ranging from 1 to 4 for lower altitude neighbors in the grid.

If we have 1 lower altitude neighbor, we can't percolate. Hence, we can ignore these.

If we have 2 lower altitude neighbors, we have 2 cases of percolation, shown below.

If we have 3 lower altitude neighbors, we have 4 cases of percolation, shown below.

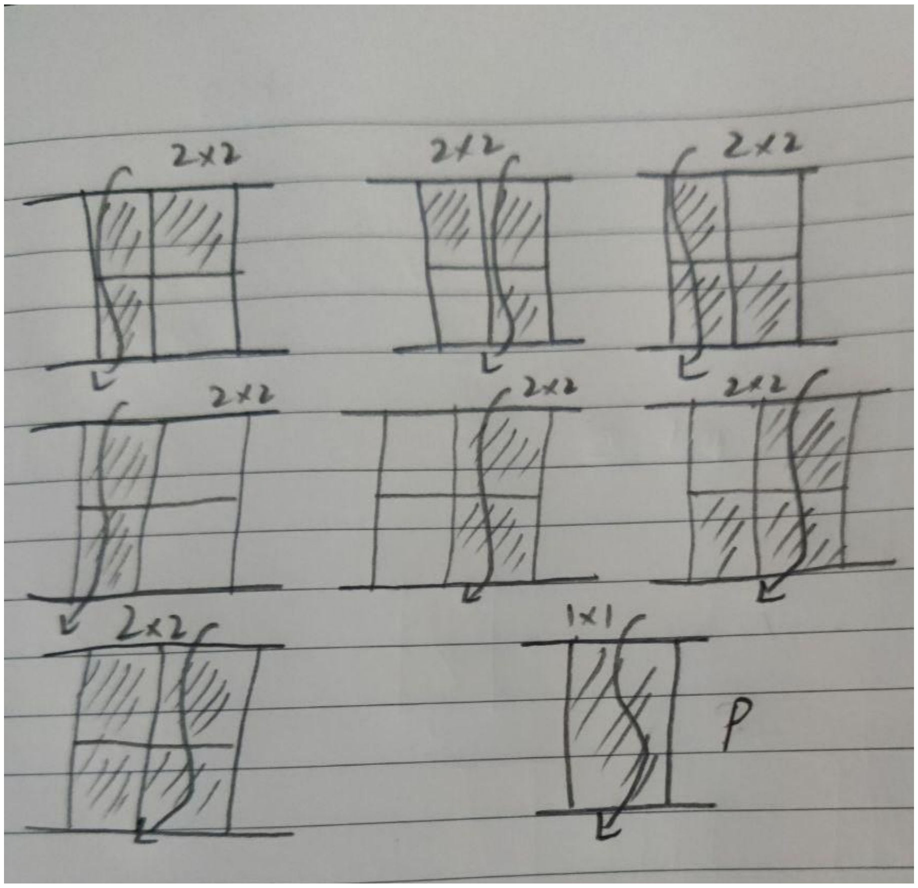
If we have 3 lower altitude neighbors, we have 1 cases of percolation, shown below.

This gives us the equation:

$$p_2 = p_1^4 + 4p_1^3(1 - p_1) + 2p_1^2(1 - p_1)^2$$

Generalizing this to bigger scales, we get

$$p_{s+1} = p_s^4 + 4p_s^3(1 - p_s) + 2p_s^2(1 - p_s)^2$$



We can make a cobweb plot, in which the intersection points will indicate the values of **p** where the threshold is possible.

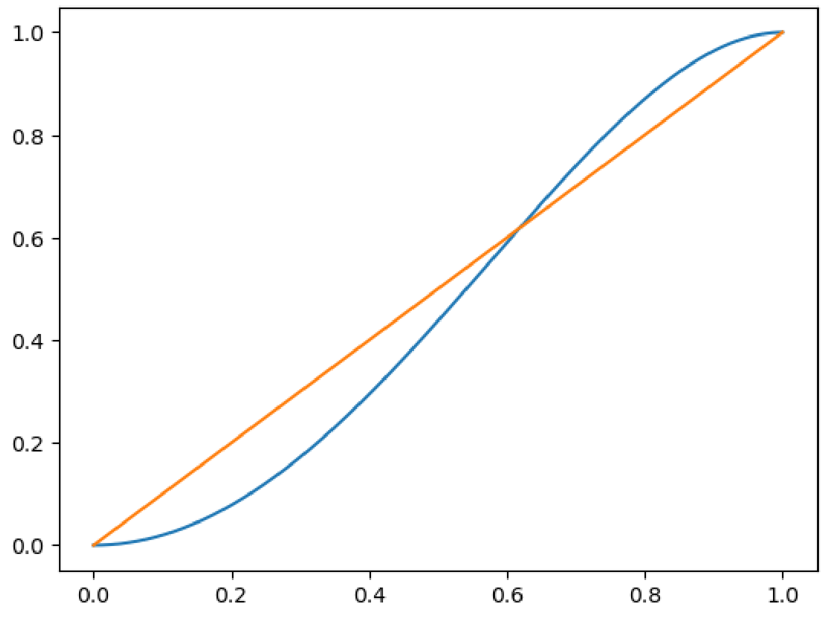


Figure 6: Cobweb plot showing potential percolation thresholds. We see that the intersection point is at 0.60.

If, for the entire grid, our probability of having a lower altitude neighbor is higher than 0.6, we should see that percolation is possible. For our simulation, this is 0.88 which means that percolation is possible.

However, our simulation has a lot of randomness which comes from various sources such as probability of rain, probability of water body being present, probability of barrage being present etc. This leads to a variation in our results. However, because percolation is possible, we see that there are still instances where we see flood water flowing across an entire column. This would not have been the case if our probability had been lower. Unfortunately, this is hard to prove because designing a grid with that exact probability is not feasible.

HC & LO Appendix

#PythonImplementation: I implemented a working simulation making sure that my model follows certain rules and then displayed the code as well. I displayed all the relevant plots along the way and made sure to use appropriate data structures as per my requirement. Moreover, I also implemented Object-Oriented programming in a very replicable way with a dataset.

#CodeReadability: I included docstrings and in-line comments for all of my code and made sure to label what is happening so that a person can read and understand it. Moreover, I also included parameters and attributes for all the methods and classes respectively so someone can further use my code. Finally, I also followed a consisten coding style so that a person doesn't get confused when reading my code.

#EmpiricalAnalysis: I ran the simulation 1000 times for each setting with each trial having 60 updates of its own to show the updates per day over the course of two months of Monsoon that Pakistan experiences. I made plots of the measurements and showed how they vary over time. Moreover, I also reported confidence intervals along with comments on what they mean, how wide they are, and how their width can be reduced.

#Professionalism: I used all the APA guidelines, and presented my report in a professional manner with suitable figure captions. I prepared my Jupyter notebook in a properly formatted manner and then merged it with my PDF so that it is easy flowing and easily referred to. Before submitting, I restarted my Kernel and ran all my cells to make sure that my code is running without errors.

#Modeling: I designed a model based on the given rules and conditions along with incorporating the region of interest by collecting data and then including that in my model. I outlined the assumptions and rules of my model clearly along with assessing the validity of these assumptions.

#TheoreticalAnalysis: I included a Renormalization Group Analysis from which I derived a percolation threshold. I showed the mathematical derivation along with a small explanation of what Renormalization Group Analysis gives us. Moreover, I showed a visual representation of how we get to our equation. Finally, I made a cobweb plot to find the threshold probability and then compared this to the value from our simulation to prove that my simulation is working correctly.

HCs

#Professionalism: I followed nuanced conventions of APA formatting including but not limited to double spacing, Times New Roman formatting in a 12 Font size, proper indentations, page numbers included in headers, and Figure captions being a font size smaller than 12.

#ConfidenceIntervals: I calculated the confidence intervals for the average water level across the grid at the end of the season and interpreted this. I also commented on the width of the interval along with how it can be reduced. Finally, I made an automated function that calculates the confidence intervals for any given list.

#Audience: While writing the paper, I kept in mind my audience and tried to tailor my explanations to someone who was reading the paper as a student of the class but at the same time making sure that someone with not as much knowledge would be able to understand it as well. This allowed me to make sure that my paper flows smoothly and conveys the exact meaning that I want it to a broader audience.

#Modeling: I clearly identified the assumptions and rules of my model and implemented a working version of it as well. I also evaluated the validity of the model using various sources and made sure to justify any constant that I was using either from a paper or clearly mentioning that this came from my knowledge and should be changed by a person more knowledgeable in the field than me.

Code Appendix

rainfall_flood

December 14, 2022

0.0.1 Importing necessary packages

```
[1]: import numpy as np
     import random
     import matplotlib.pyplot as plt
     import scipy.stats as sts
     import pandas as pd
```

0.0.2 Data preparation

A rainfall study divided Pakistan into 5 zones (Salma et.al., 2012) based on rainfall trends. The main essence of this division is latitude. I have taken 5 cities from each of the zones to design the CA model with the 5 cities forming one row of the final grid.

```
[2]: # names of all the cities
     names = ['Swat', 'Skardu', 'Kohistan', 'Gilgit', 'Muzaffarabad', 'Swabi',␣
      ↪'Mianwali', 'Gujrat', 'Nowshera',
              'Jhelum', 'TTK', 'Chiniot', 'Lahore', 'Bhakkar', 'Jhang', 'Lodhran',␣
      ↪'Muzaffargarh', 'Multan', 'DGK',
              'Bahawalpur', 'Karachi', 'Sukkur', 'Larkana', 'Nawabshah', 'Kashmore']
     # respective altitudes of the cities
     altitudes = [980, 2228, 1650, 1500, 737, 340, 210, 1110, 552, 234, 149, 179,␣
      ↪217, 159, 158, 106, 123, 122, 390, 118,
                  10, 67, 147, 34, 66]
     # average rain fall per annum
     precipitation = [0.67]*5 + [0.65]*5 + [0.33]*5 + [0.22]*5 + [0.29]*5
     # percentage of days it rains on average in respective cities
     percentage_days = [50]*2 + [45, 50, 45] + [30]*5 + [20]*5 + [10]*5 + [15]*5
     # average rainfall per rain
     height_per_rain = [precipitation[i]/(60*(percentage_days[i]/100)) for i in␣
      ↪range(len(precipitation))]
     # whether soil is waterlogged or not
     soil = [0,0,1] + [0]*3 + [1]*4 + [0,1,0,0,0]*2 + [1]*2 + [0]*3
     # all cities start off as not flooded
     flooded = [False]*25
     # does the city have a water body nearby to act as natural drainage
     water_body = [0]*5 + [1]*18 + [0,0]
     barrage = [0]*25
```

```
data = pd.DataFrame({
    'Names': names,
    'Altitude in meters': altitudes,
    'Annual Precipitation in meters': precipitation,
    'Rainy days as %age': percentage_days,
    'Average Rainfall in meters': height_per_rain,
    'Soil': soil,
    'Flooded': flooded,
    'Water Body present?': water_body,
    'Barrage': barrage
})
data
```

[2]:

```
           Names  Altitude in meters  Annual Precipitation in meters  \
0           Swat                 980                            0.67
1         Skardu                2228                            0.67
2       Kohistan                1650                            0.67
3         Gilgit                1500                            0.67
4   Muzaffarabad                 737                            0.67
5          Swabi                 340                            0.65
6       Mianwali                 210                            0.65
7         Gujrat                1110                            0.65
8       Nowshera                 552                            0.65
9         Jhelum                 234                            0.65
10           TTK                 149                            0.33
11       Chiniot                 179                            0.33
12        Lahore                 217                            0.33
13       Bhakkar                 159                            0.33
14         Jhang                 158                            0.33
15       Lodhran                 106                            0.22
16  Muzaffargarh                 123                            0.22
17        Multan                 122                            0.22
18           DGK                 390                            0.22
19    Bahawalpur                 118                            0.22
20       Karachi                  10                            0.29
21        Sukkur                  67                            0.29
22       Larkana                 147                            0.29
23     Nawabshah                  34                            0.29
24      Kashmore                  66                            0.29

    Rainy days as %age  Average Rainfall in meters  Soil  Flooded  \
0                   50                    0.022333     0    False
1                   50                    0.022333     0    False
2                   45                    0.024815     1    False
3                   50                    0.022333     0    False
4                   45                    0.024815     0    False
```

```
5                   30                    0.036111     0    False
6                   30                    0.036111     1    False
7                   30                    0.036111     1    False
8                   30                    0.036111     1    False
9                   30                    0.036111     1    False
10                  20                    0.027500     0    False
11                  20                    0.027500     1    False
12                  20                    0.027500     0    False
13                  20                    0.027500     0    False
14                  20                    0.027500     0    False
15                  10                    0.036667     0    False
16                  10                    0.036667     1    False
17                  10                    0.036667     0    False
18                  10                    0.036667     0    False
19                  10                    0.036667     0    False
20                  15                    0.032222     1    False
21                  15                    0.032222     1    False
22                  15                    0.032222     0    False
23                  15                    0.032222     0    False
24                  15                    0.032222     0    False

    Water Body present?  Barrage
0                     0        0
1                     0        0
2                     0        0
3                     0        0
4                     0        0
5                     1        0
6                     1        0
7                     1        0
8                     1        0
9                     1        0
10                    1        0
11                    1        0
12                    1        0
13                    1        0
14                    1        0
15                    1        0
16                    1        0
17                    1        0
18                    1        0
19                    1        0
20                    1        0
21                    1        0
22                    1        0
23                    0        0
24                    0        0
```

```
[3]: class Cell:
         '''
         Class to form a cell in the grid

         Attributes
         ----------
             name: str
                 Name of the city
             altitude: int
                 Altitude above sea level of the city
             precipitation: int
                 total rainfall in the city per annum
             rainy_days: int
                 Percentage of days in the year the city experiences rainfall
             height_per_rain: float
                 millimeters of rain per rainfall on average
             soil: int
                 1 if soil is waterlogged, 0 otherwise
             flooded: boolean
                 whether the city is flooded or not
             water_body: int
                 1 if city has a water body nearby, 0 otherwise
             net_height: float
                 Total height of the city depending on the water level of the soil␣
      ↪as well
             overflow: float
                 Excess water that is not absorbed by the soil and needs to be␣
      ↪drained
             net_rain: float
                 Total amount of rain over a period of time
             barrage: int
                 1 if the city is protected by a barrage, 0 otherwise

         '''
         def __init__(self, name, altitude, precipitation, percentage_days,␣
      ↪height_per_rain, soil, flooded, water_body, barrage):
             '''
             Method to make the instance of the class.

             Parameters
             ----------
                 name: str
                 altitude: int
                 precipitation: int
                 percentage_days: int
                 height_per_rain: float
                 soil: int
```

```
            flooded: boolean
            water_body: int
            barrage: int

        Features
        --------
        If the soil is not water-logged, it still has capacity to absorb water␣
↪so we its capacity is to absorb
        1.5 m of water. If the soil is water-logged, it only has a capacity of␣
↪0.5 m of water.
        '''
        self.name = name
        self.water_body = water_body
        self.altitude = altitude
        self.precipitation = precipitation
        self.rainy_days = percentage_days
        self.height_per_rain = height_per_rain
        self.soil = soil
        self.flooded = flooded
        if self.soil == 0:
            self.water = -2
        else:
            self.water = -0.6
        self.net_height = self.altitude + self.water
        self.overflow = 0
        self.net_rain = 0
        self.barrage = barrage

    def update(self, prob):
        '''
        Method to update the water condition within the cell and simulate the␣
↪rain
        '''
        # check whether rain happens given the probability of a rainy day
        if random.random() < self.rainy_days/100:
            # if it is gonna rain, simulate amount of rain through a gamma␣
↪distribution
            # a gamma distribution was chosen because it is defined between 0␣
↪and +inf which
            # is the range of amount of rain
            rain = sts.gamma(self.height_per_rain, 0.3).rvs()
        else:
            rain = 0
        # add rain amount to total rain
        self.net_rain += rain

        # add rain to water levels in the city
        self.water += rain
        # if water level has reached ground level, soil is water logged
        if self.water == 0:
            self.soil = 1
        # if water level is above ground level
        elif self.water > 0:
            # how much water is overflowing
            self.overflow += self.water
            # set water level to ground as water overflows
            self.water = 0
            # city is flooded
            self.flooded = True
            # soil is water-logged
            self.soil = 1
            # net height of the city is now altitude + water level +␣
↪overflowing water
            self.net_height = self.altitude + self.water + self.overflow

        else:
            # if soil is not water logged
            if self.soil == 0:
                # water is absorbed into the ground with a 60% probability,␣
↪restoring original levels
                if random.random() < prob:
                    self.water = -2
                # else: water level goes down by 1 mm per day
                elif self.water-0.2 >= -2:
                    self.water -= 0.2
            else:
                # if soil is water logged, 10% chance of being restored to␣
↪original levels
                if random.random() < prob/6:
                    self.water = -0.6
                else:
                    # water level does not change
                    self.water = self.water

    def update_flood(self):
        '''
        Method to update the flood status of the cell
        '''
        # if water level is below ground level, not flooded
        if self.water + self.overflow <= 0:
            self.flooded = False
        # if above ground level, it is flooded
        else:
            self.flooded = True
```

```
[4]: class Grid:
         '''
         Class for the grid and the simulation

         Attributes
         ----------
             n: int
                 Number of rows and columns
             rows: int
                 Number of rows in the grid
             columns: int
                 Number of columns in the grid
             cells: list
                 the grid which contains all the instances of the cells
             prob: float
                 probability with which water level restores to its original level
         '''
         def __init__(self, n = 5, drainage_prob = 0.6):
             '''
             Method to initiate the class when it is called

             Parameters
             ----------
                 n: int
                     default value = 5. Number of rows and columns
                 drainage_prob: float
                     default value = 0.6. Probability with which old water levels␣
   ↪are restored.
             '''
             self.n = n
             self.prob = drainage_prob
             self.rows = self.n
             self.columns = self.n
             self.cells = []
             self.flow = []

             # make the grid
             for i in range(n):
                 # make an empty row
                 row = []
                 for j in range(n):
                     # make instances for the cells
                     cell = self.getCell(names[(i*5)+j], altitudes[(i*5)+j],␣
   ↪precipitation[(i*5)+j], percentage_days[(i*5)+j],
                            height_per_rain[(i*5)+j], soil[(i*5)+j],␣
   ↪flooded[(i*5)+j], water_body[(i*5)+j], barrage[(i*5)+j])
                     # append the cell to the row
```

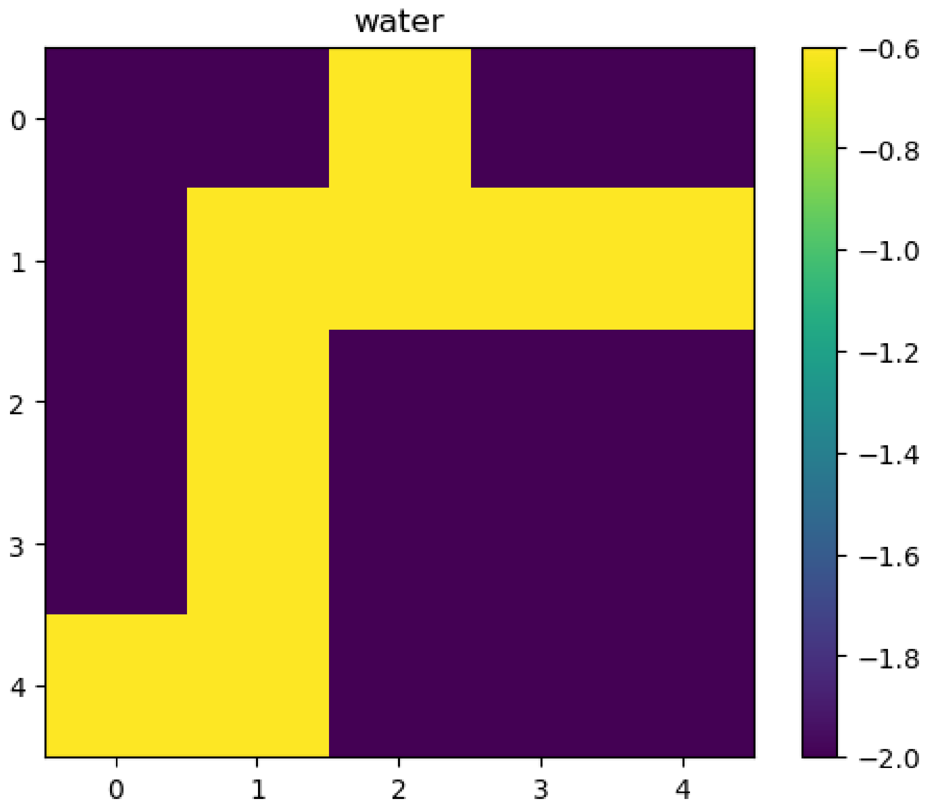
```
                row.append(cell)
            # append the row to the grid
            self.cells.append(row)

    def getCell(self, name, altitude, precipitation, percentage_days,␣
↪height_per_rain, soil, flooded, water_body, barrage):
        '''
        Method to make an instance of a cell

        Parameters
        ----------
            name: str
            altitude: int
            precipitation: int
            percentage_days: int
            height_per_rain: float
            soil: int (0 or 1)
            flooded: boolean
            water_body: int (0 or 1)
            barrage: int (0 or 1)
        '''
        # makes an instance of the Cell class and returns it
        return Cell(name, altitude, precipitation, percentage_days,␣
↪height_per_rain, soil, flooded, water_body, barrage)

    def getCells(self):
        '''
        Method to return all the cells in the grid
        '''
        return self.cells

    def observe(self, attribute):
        '''
        Method to plot the grid

        Parameters
        ----------
            attribute: str
                the attribute that you want to observe. Can be any one from the␣
↪attributes of the Cell class.
        '''
        # get all the cells from the grid
        a = self.getCells()
        # make an empty list to flatten the cells
        all_cells = []
        # loop over the grid and append all the cells into a flat array
        for i in range(len(a)):
            for j in range(len(a)):
                all_cells.append(a[i][j])
        # make an array by retrieving the attribute under consideration for all␣
↪the cells and make a grid again
        attributes = np.array([getattr(all_cells[i], attribute) for i in␣
↪range(len(all_cells))]).reshape(5,5)
        # make the plot
        plt.figure()
        c = plt.imshow(attributes)
        plt.title(str(attribute))
        plt.colorbar(c)
        plt.show()

    def retrieve(self, attribute):
        '''
        Method to retrieve a grid of the specific attribute that we are looking␣
↪for
        '''
        # get all the cells
        a = self.getCells()
        # flatten the grid
        flattened = np.array(a).flatten()
        # retrieve attribute and reshape to form the grid
        attribute_grid = np.array([getattr(flattened[i], attribute) for i in␣
↪range(len(flattened))]).reshape(self.n, self.n)
        return attribute_grid

    def percolation_test(self):
        '''
        Method to check for percolation
        '''

        # set default percolation to be False
        percolation = False
        # loop over each of the columns
        for y in range(self.n):
            # set flow to be True by default
            flow = True
            # retrieve cells for the columns
            a = [g.getCells()[x][y] for x in range(self.n)]
            # get water levels for all the cells in the column
            water_levels = [x.water for x in a]
            # loop over the cells and check if any is not flooded
            for level in water_levels:
                if level < 0:
                    # if any of the cells is not flooded, percolation doesn't␣
↪occur and water is not flowing
```

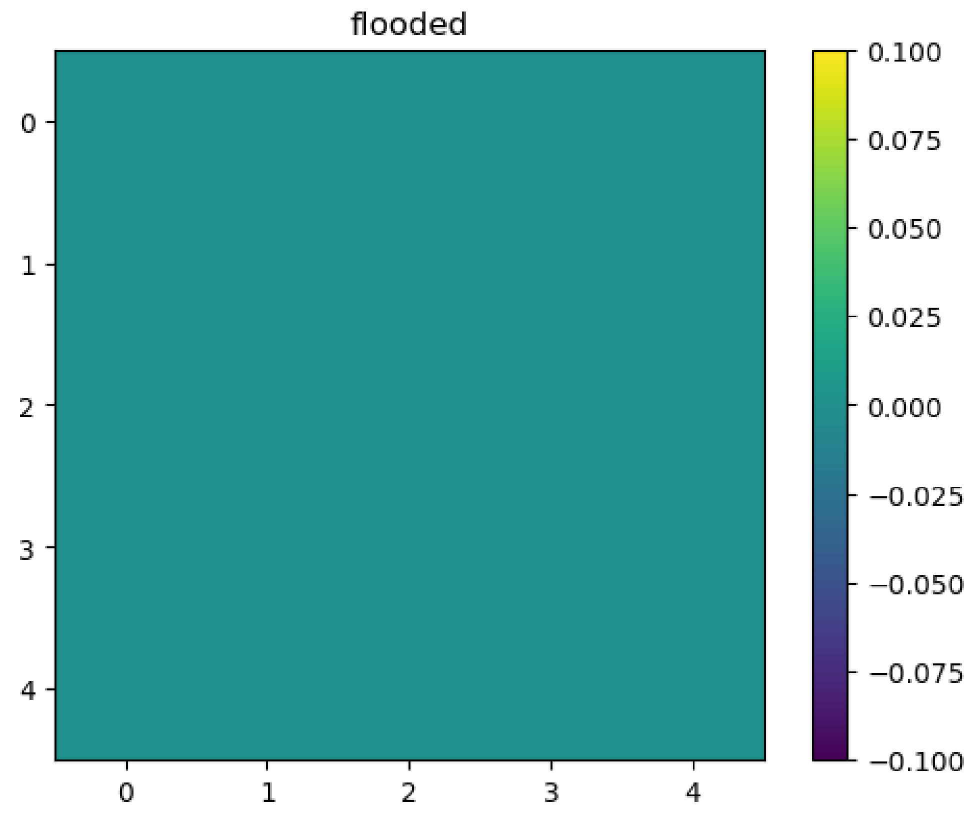
```
                    flow = False
            if flow == True:
                # if flow is true i.e. all cells are flooded, set percolation␣
↪to be True
                percolation = True
        # return the percolation variable
        return percolation


    def update(self):
        '''
        Method to update the entire system
        '''
        flow = 0

        #update all cells on their own first i.e. check to see if rain happened
        for x in range(self.n):
            for y in range(self.n):
                self.cells[x][y].update(self.prob)

        # update top row
        for y in range(self.n):
            x = 0
            current = self.cells[x][y]
            # make an empty array for neighbors
            neighbors = []
            # using a Von-Neuman neighborhood but not periodic conditions.␣
↪Boundaries on the vertical axes are cut off.
            for dx, dy in [(1, 0), (0, -1), (0, 1)]:
                # append all the neighbors into the array
                neighbors.append(self.cells[dx][(y+dy)%self.n])
            # get height of the cell under consideration
            current_height = current.net_height
            # set steepest neighbor to be None by default in case our cell is␣
↪in a valley
            steepest_neighbor = None
            # loop through the neighbors
            for n in neighbors:
                # if the neighbor is lower than current cell
                if n.net_height < current_height:
                    # if the steepest neighbor is None, set it to current␣
↪neighbor
                    if steepest_neighbor == None:
                        steepest_neighbor = n
                        flow += 1
                    # if there is already a neighbor, compare the heights and␣
↪choose the one with steepest decline
```

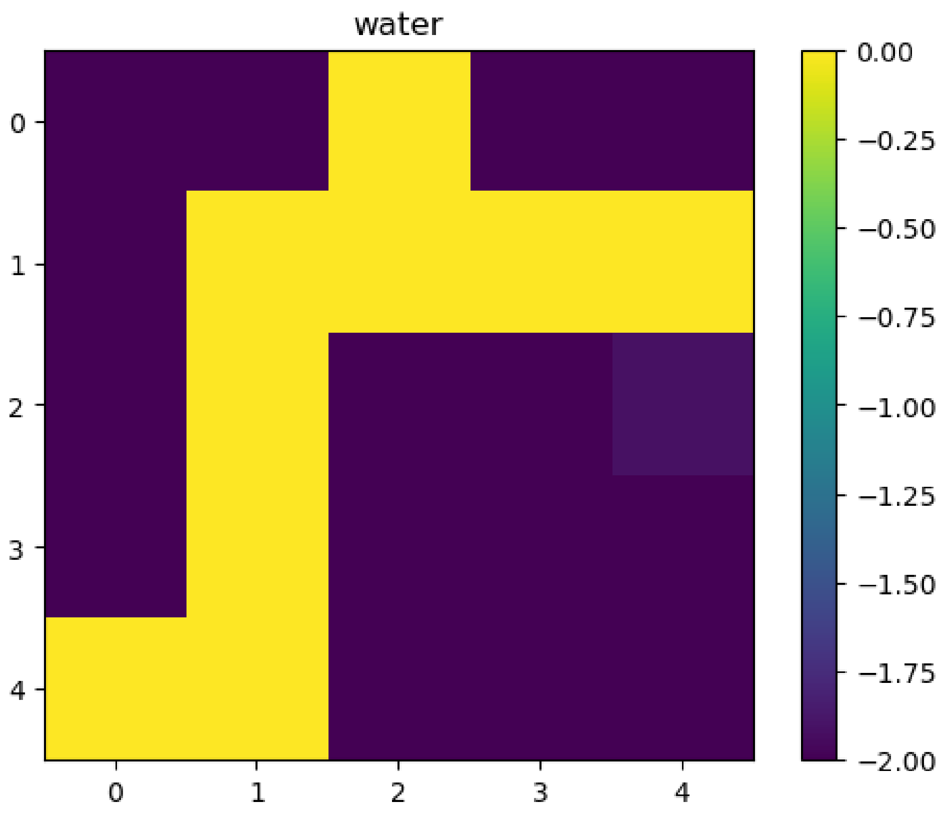
```
                    else:
                        if n.net_height < steepest_neighbor.net_height:
                            steepest_neighbor = n

            #water flows to steepest neighbor
            if current.overflow > 0 and steepest_neighbor != None:
                # if the neighbor has a barrage, the water will be stopped by␣
↪it and absorbed into the surrounding soil
                if steepest_neighbor.barrage == 1:
                    steepest_neighbor.water += 0
                else:
                    # if the steepest neighbor has a water body near it, most␣
↪of the water drains into the water body
                    # rather than to the city. Only 30% of the overflow makes␣
↪it to the neighboring cell
                    if steepest_neighbor.water_body == 1:
                        steepest_neighbor.water += current.overflow * 0.3
                    else:
                        steepest_neighbor.water += current.overflow
                # once water has flown to the neighbor, current overflow␣
↪becomes 0
                current.overflow = 0
                # update the neighbor based on water flow
                # neighbor becomes water logged if overflow caused its water␣
↪level to come level with the ground
                if steepest_neighbor.water == 0:
                    steepest_neighbor.soil = 1
                # if water level is above the ground level
                elif steepest_neighbor.water > 0:
                    # set the overflow value
                    steepest_neighbor.overflow += steepest_neighbor.water
                    steepest_neighbor.water = 0
                    # set neighbor to be flooded
                    steepest_neighbor.flooded = True
                    # eighbor's soil becomes water logged
                    steepest_neighbor.soil = 1
                    # update neighbor's height
                    steepest_neighbor.net_height = steepest_neighbor.altitude +␣
↪steepest_neighbor.water + steepest_neighbor.overflow

        # update all except boundaries
        for x in range(1, self.n-1):
            for y in range(self.n):
                current = self.cells[x][y]
                neighbors = []
                for dx, dy in [(-1, 0), (1, 0), (0, -1), (0, 1)]:
                    neighbors.append(self.cells[(x+dx)%self.n][(y+dy)%self.n])
                # find steepest neighbor
                current_height = current.net_height
                steepest_neighbor = None
                for n in neighbors:
                    if n.net_height < current_height:
                        # steep comparison
                        if steepest_neighbor == None:
                            steepest_neighbor = n
                            flow += 1
                        else:
                            if n.net_height < steepest_neighbor.net_height:
                                steepest_neighbor = n

                #water flows to steepest neighbor
                if current.overflow > 0 and steepest_neighbor != None:
                    # if the neighbor has a barrage, the water will be stopped␣
↪by it and absorbed into the surrounding soil
                    if steepest_neighbor.barrage == 1:
                        steepest_neighbor.water += 0
                    else:
                        # if the steepest neighbor has a water body near it,␣
↪most of the water drains into the water body
                        # rather than to the city. Only 30% of the overflow␣
↪makes it to the neighboring cell
                        if steepest_neighbor.water_body == 1:
                            steepest_neighbor.water += current.overflow * 0.3
                        else:
                            steepest_neighbor.water += current.overflow
                    current.overflow = 0
                    if steepest_neighbor.water == 0:
                        steepest_neighbor.soil = 1
                    elif steepest_neighbor.water > 0:
                        steepest_neighbor.overflow += steepest_neighbor.water
                        steepest_neighbor.water = 0
                        steepest_neighbor.flooded = True
                        steepest_neighbor.soil = 1
                        steepest_neighbor.net_height = steepest_neighbor.
↪altitude + steepest_neighbor.water + steepest_neighbor.overflow


        # update bottom row
        for y in range(self.n):
            x = 4
            neighbors = []
            current = self.cells[x][y]
            for dx, dy in [(-1, 0), (0, -1), (0, 1)]:
                neighbors.append(self.cells[(x+dx)%self.n][(y+dy)%self.n])
            # find steepest neighbor
            current_height = current.net_height
            steepest_neighbor = None
            for n in neighbors:
                if n.net_height < current_height:
                    # steep comparison
                    if steepest_neighbor == None:
                        steepest_neighbor = n
                        flow += 1
                    else:
                        if n.net_height < steepest_neighbor.net_height:
                            steepest_neighbor = n

            #water flows to steepest neighbor
            if current.overflow > 0 and steepest_neighbor != None:
                # if the neighbor has a barrage, the water will be stopped by␣
↪it and absorbed into the surrounding soil
                if steepest_neighbor.barrage == 1:
                    steepest_neighbor.water += 0
                else:
                    # if the steepest neighbor has a water body near it, most␣
↪of the water drains into the water body
                    # rather than to the city. Only 30% of the overflow makes␣
↪it to the neighboring cell
                    if steepest_neighbor.water_body == 1:
                        steepest_neighbor.water += current.overflow * 0.3
                    else:
                        steepest_neighbor.water += current.overflow
                current.overflow = 0
                if steepest_neighbor.water == 0:
                    steepest_neighbor.soil = 1
                elif steepest_neighbor.water > 0:
                    steepest_neighbor.overflow += steepest_neighbor.water
                    steepest_neighbor.water = 0
                    steepest_neighbor.flooded = True
                    steepest_neighbor.soil = 1
                    steepest_neighbor.net_height = steepest_neighbor.altitude +␣
↪steepest_neighbor.water + steepest_neighbor.overflow

        self.flow.append(flow/(self.n * self.n))
```

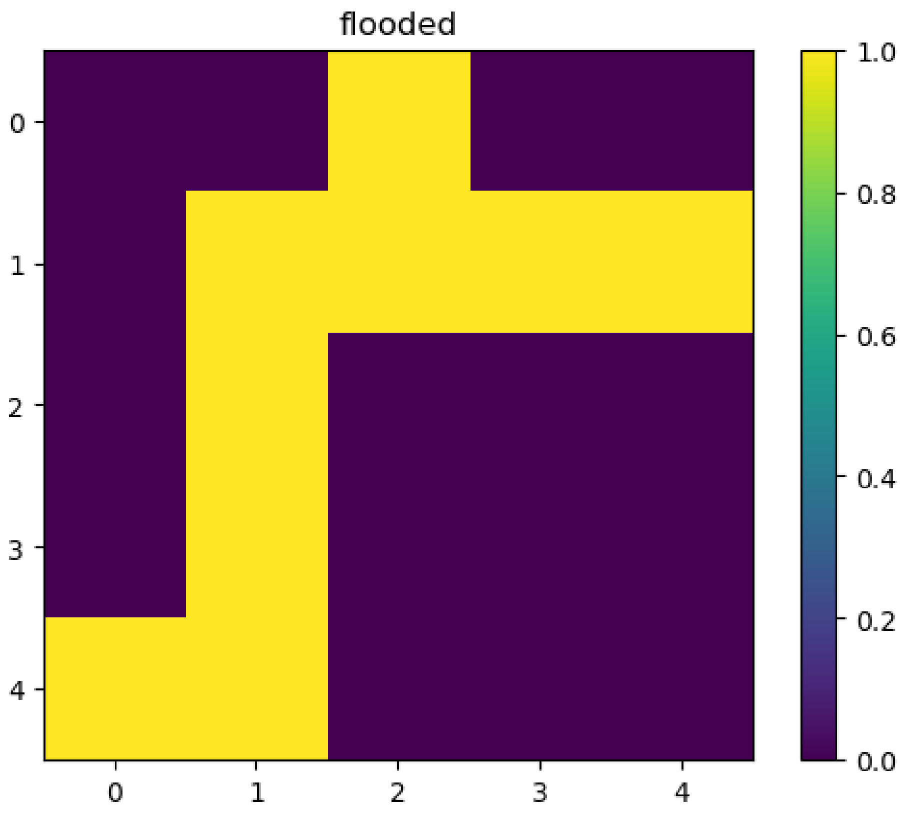
[5]:
```
# make an instance of the Grid
g = Grid()
# observe initial conditions of water levels and flooding
g.observe('water')
g.observe('flooded')
```

```
# update for 60 days i.e. length of the moonsoon season
for i in range(60):
    g.update()
# observe final conditions
g.observe('water')
g.observe('flooded')
# check whether percolation occurs or not
print(g.percolation_test())
```




flooded
0
1
2
3
4
0
1
2
3
4
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100


water
0
1
2
3
4
0
1
2
3
4
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
−1.75
−2.00

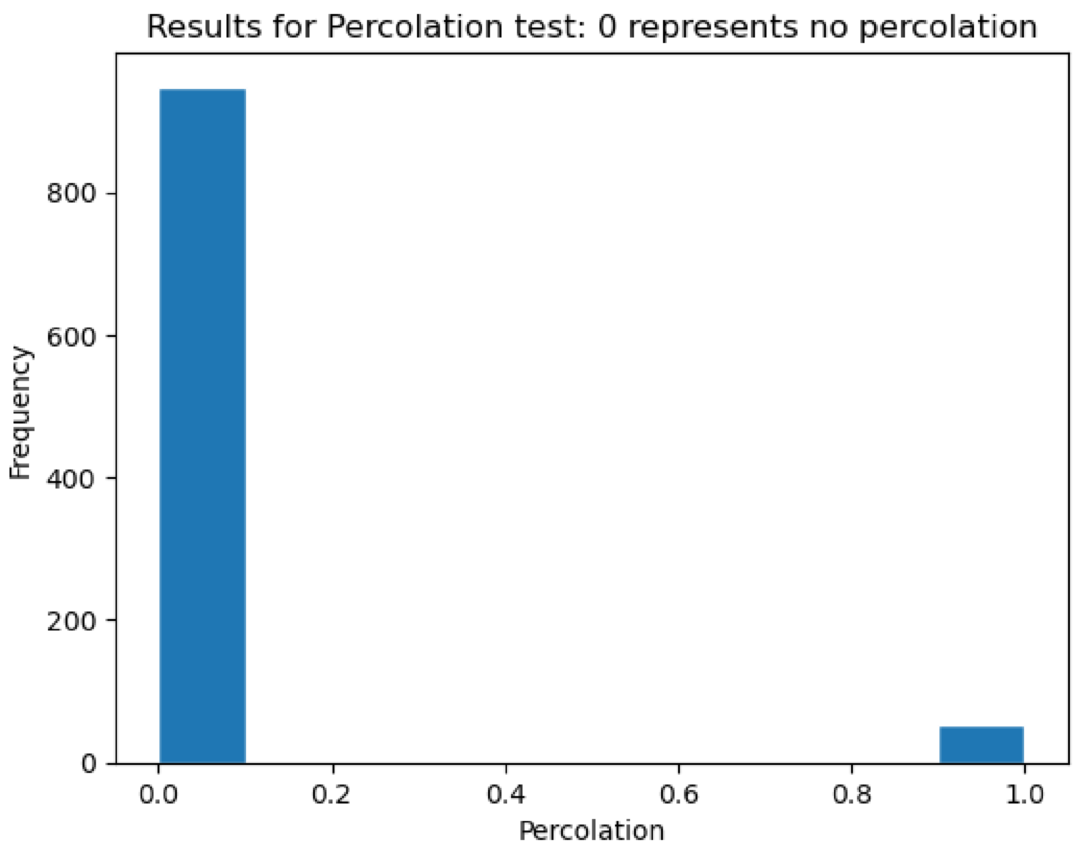


```
False
```

[6]:
```
for i in range(100):
    g = Grid()
    for j in range(365):
        g.update()
    if g.percolation_test == True:
        g.observe('flooded')
```

[7]:
```
#empirical runs
results_old = []
for i in range(1000):
    g = Grid()
    for j in range(60):
        g.update()
    if g.percolation_test() == False:
        results_old.append(0)
    else:
        results_old.append(1)
```

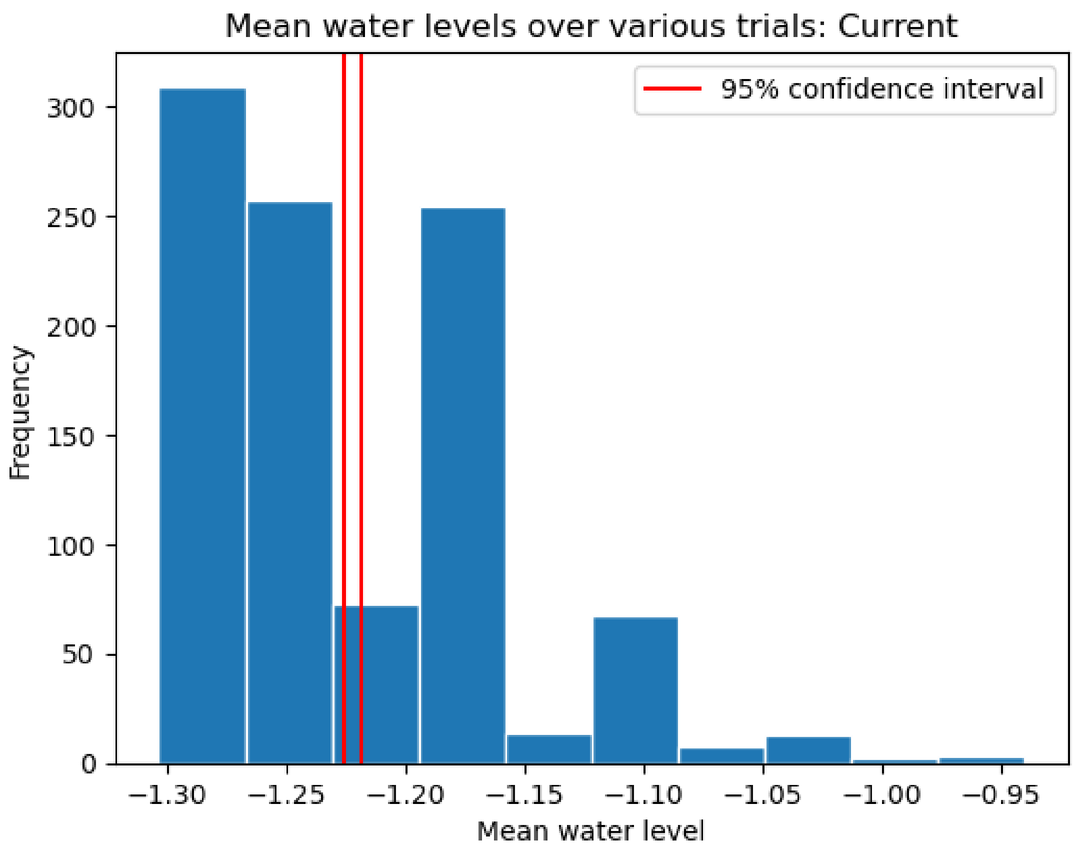
[8]:
```
plt.figure()
plt.title('Results for Percolation test: 0 represents no percolation')
plt.hist(results_old, edgecolor = 'white')
plt.xlabel('Percolation')
plt.ylabel('Frequency')
plt.show()
```



[9]:
```
# check for mean water levels in the grid at the end of the moonsoon season
mean_water_old = []
for i in range(1000):
    g = Grid()
    for j in range(60):
        g.update()
    water_levels = np.array(g.retrieve('water')).flatten()
    mean_water_old.append(np.mean(water_levels))
```

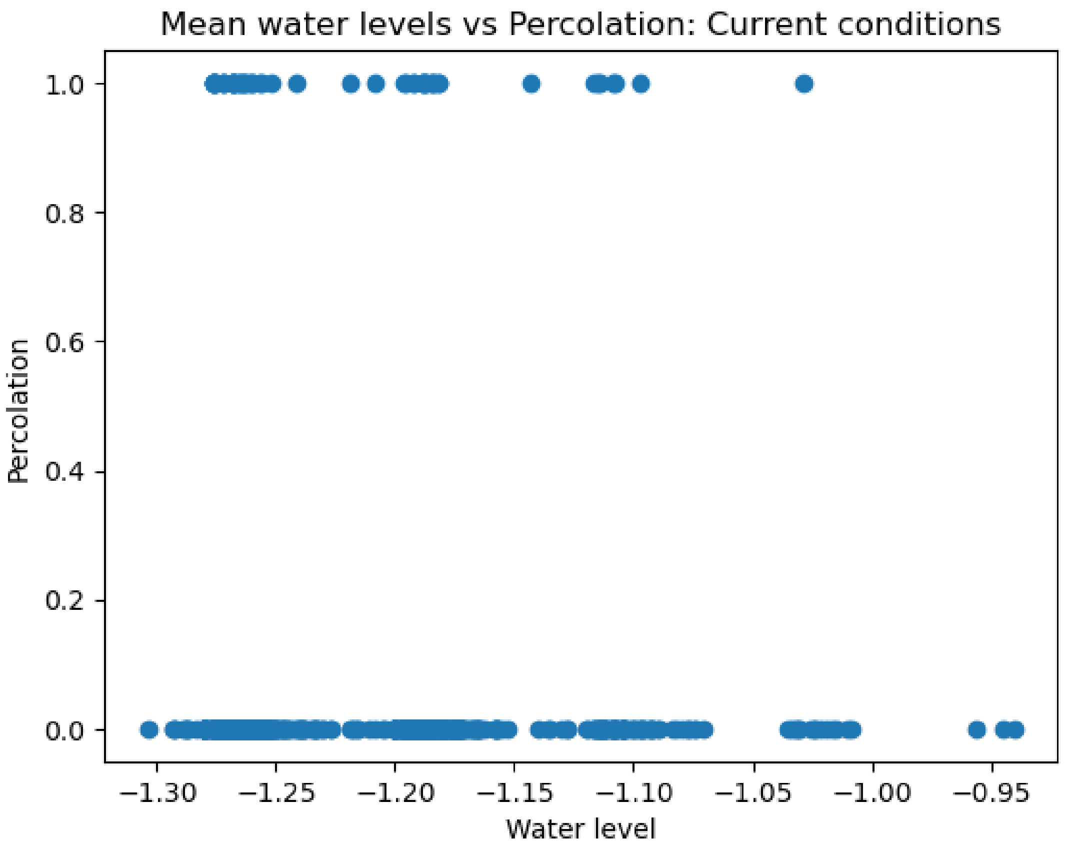
[10]:
```
def confint(lst):
    '''
    Function for calculating confidence intervals of a given list
    Inputs
    ------
        lst: list
            list of the data
    Outputs
    -------
        confint: list
            list of two values of confidence intervals.
    '''
    mean = np.mean(lst)
    standard_error = sts.sem(lst)
    deviation = standard_error * 1.96
    return [mean - deviation, mean + deviation]

confint_old = confint(mean_water_old)
```

[11]:
```
plt.figure()
plt.title('Mean water levels over various trials: Current')
plt.hist(mean_water_old, edgecolor = 'white', bins = 10)
plt.axvline(confint_old[0], label = '95% confidence interval', color = 'red')
plt.axvline(confint_old[1], color = 'red')
plt.legend()
plt.xlabel('Mean water level')
plt.ylabel('Frequency')
plt.show()
print('95% Confidence interval: ', confint_old)
```



```
95% Confidence interval:  [-1.2258314132398211, -1.218445097265094]
```

[12]:
```
plt.figure()
plt.scatter(mean_water_old, results_old)
plt.title('Mean water levels vs Percolation: Current conditions')
plt.xlabel('Water level')
plt.ylabel('Percolation')
plt.show()
```



0.1 New conditions

[13]:
```
# run a small number of trials to see where the problem is
for i in range(5):
    g = Grid()
    for j in range(60):
        g.update()
    if g.percolation_test() == True:
        g.observe('flooded')
```

[14]:
```
# add barrages
barrage = [0]*10 + [1,1,1,0,0]*3

data = pd.DataFrame({
    'Names': names,
    'Altitude in meters': altitudes,
    'Annual Precipitation in meters': precipitation,
    'Rainy days as %age': percentage_days,
    'Average Rainfall in meters': height_per_rain,
    'Soil': soil,
    'Flooded': flooded,
    'Water Body present?': water_body,
    'Barrage': barrage
})
data
```

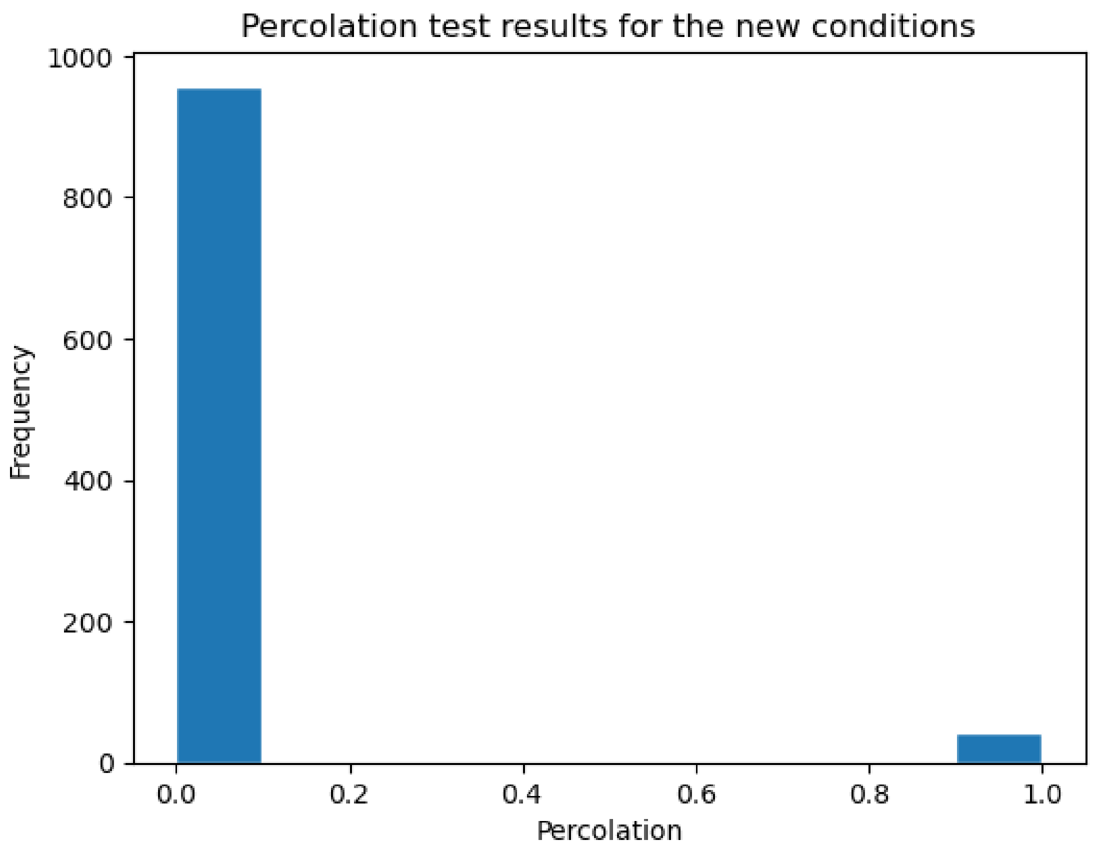
[14]:

```
           Names  Altitude in meters  Annual Precipitation in meters  \
0           Swat                 980                            0.67
1         Skardu                2228                            0.67
2       Kohistan                1650                            0.67
3         Gilgit                1500                            0.67
4    Muzaffarabad                737                            0.67
5          Swabi                 340                            0.65
6       Mianwali                 210                            0.65
7         Gujrat                1110                            0.65
8       Nowshera                 552                            0.65
9         Jhelum                 234                            0.65
10           TTK                 149                            0.33
11       Chiniot                 179                            0.33
12        Lahore                 217                            0.33
13       Bhakkar                 159                            0.33
14         Jhang                 158                            0.33
15       Lodhran                 106                            0.22
16   Muzaffargarh                123                            0.22
17        Multan                 122                            0.22
18           DGK                 390                            0.22
19    Bahawalpur                 118                            0.22
20       Karachi                  10                            0.29
21        Sukkur                  67                            0.29
22       Larkana                 147                            0.29
23     Nawabshah                  34                            0.29
24      Kashmore                  66                            0.29

    Rainy days as %age  Average Rainfall in meters  Soil  Flooded  \
0                   50                    0.022333     0    False
1                   50                    0.022333     0    False
2                   45                    0.024815     1    False
3                   50                    0.022333     0    False
4                   45                    0.024815     0    False
5                   30                    0.036111     0    False
6                   30                    0.036111     1    False
7                   30                    0.036111     1    False
8                   30                    0.036111     1    False
9                   30                    0.036111     1    False
10                  20                    0.027500     0    False
11                  20                    0.027500     1    False
12                  20                    0.027500     0    False
```

```
13                   20                     0.027500     0    False
14                   20                     0.027500     0    False
15                   10                     0.036667     0    False
16                   10                     0.036667     1    False
17                   10                     0.036667     0    False
18                   10                     0.036667     0    False
19                   10                     0.036667     0    False
20                   15                     0.032222     1    False
21                   15                     0.032222     1    False
22                   15                     0.032222     0    False
23                   15                     0.032222     0    False
24                   15                     0.032222     0    False

    Water Body present?  Barrage
0                     0        0
1                     0        0
2                     0        0
3                     0        0
4                     0        0
5                     1        0
6                     1        0
7                     1        0
8                     1        0
9                     1        0
10                    1        1
11                    1        1
12                    1        1
13                    1        0
14                    1        0
15                    1        1
16                    1        1
17                    1        1
18                    1        0
19                    1        0
20                    1        1
21                    1        1
22                    1        1
23                    0        0
24                    0        0
```

[15]:
```
#empirical runs
results_new = []
for i in range(1000):
    g = Grid()
    for j in range(60):
        g.update()
    if g.percolation_test() == False:
        results_new.append(0)
    else:
        results_new.append(1)
```

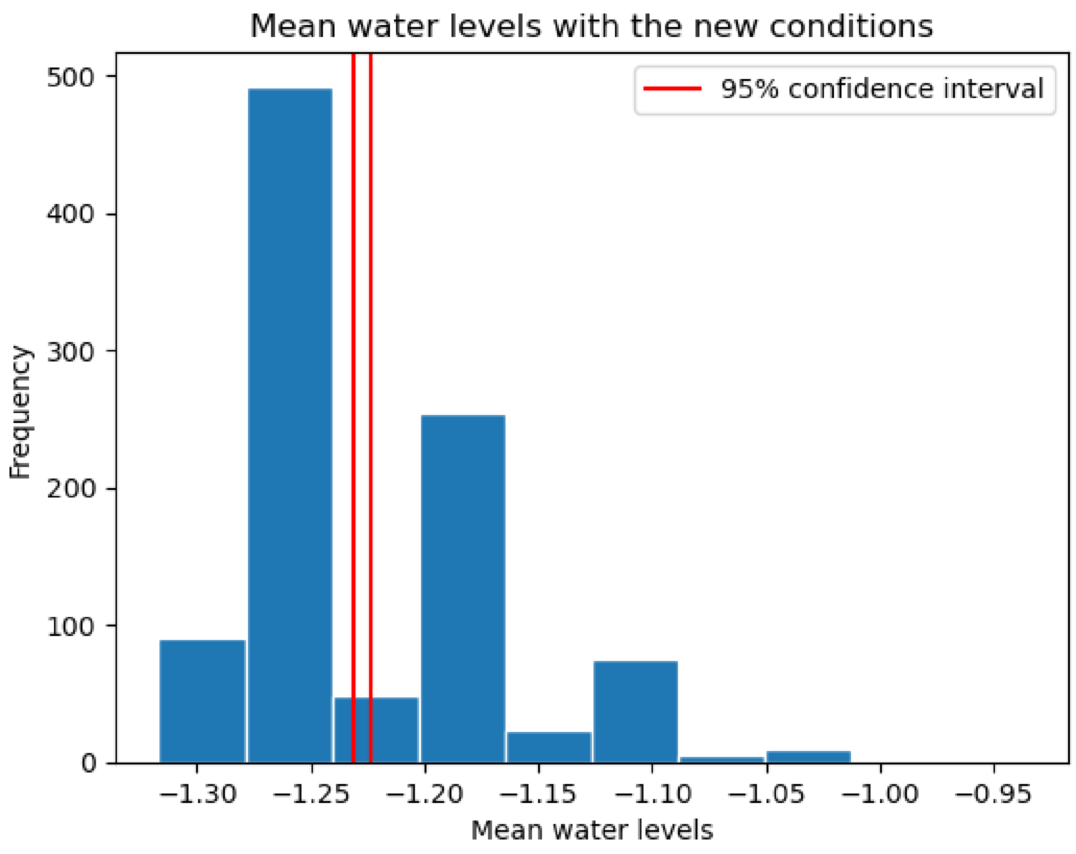
[16]:
```
plt.figure()
plt.title('Percolation test results for the new conditions')
plt.hist(results_new, edgecolor = 'white')
plt.xlabel('Percolation')
plt.ylabel('Frequency')
plt.show()
```



[17]:
```
mean_water_new = []
for i in range(1000):
    g = Grid()
    for j in range(60):
        g.update()
    water_levels = np.array(g.retrieve('water')).flatten()
    mean_water_new.append(np.mean(water_levels))
```

[18]:
```
confint_new = confint(mean_water_new)
```

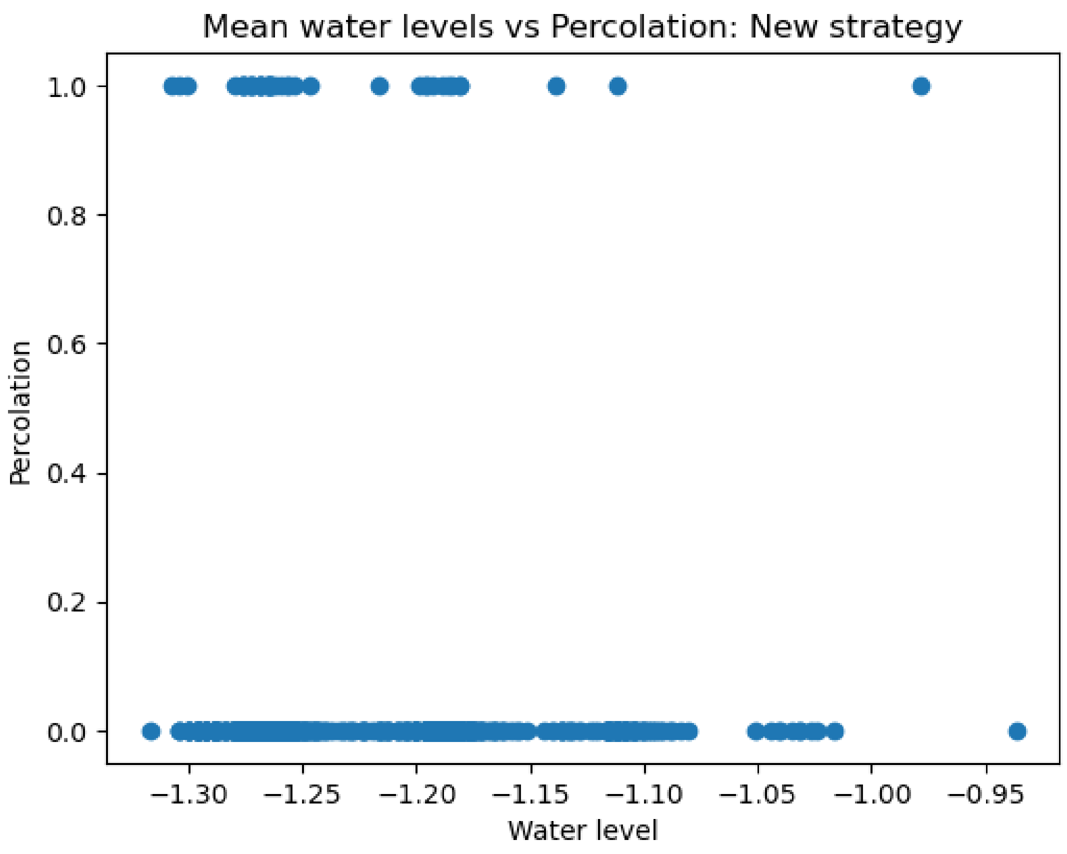
[19]:
```
plt.figure()
plt.title('Mean water levels with the new conditions')
plt.hist(mean_water_new, edgecolor = 'white', bins = 10)
plt.axvline(confint_new[0], label = '95% confidence interval', color = 'red')
plt.axvline(confint_new[1], color = 'red')
plt.legend()
plt.xlabel('Mean water levels')
plt.ylabel('Frequency')
plt.show()
print('95% confidence interval: ', confint_new)
```



```
95% confidence interval:  [-1.2307397064454368, -1.223509851716117]
```

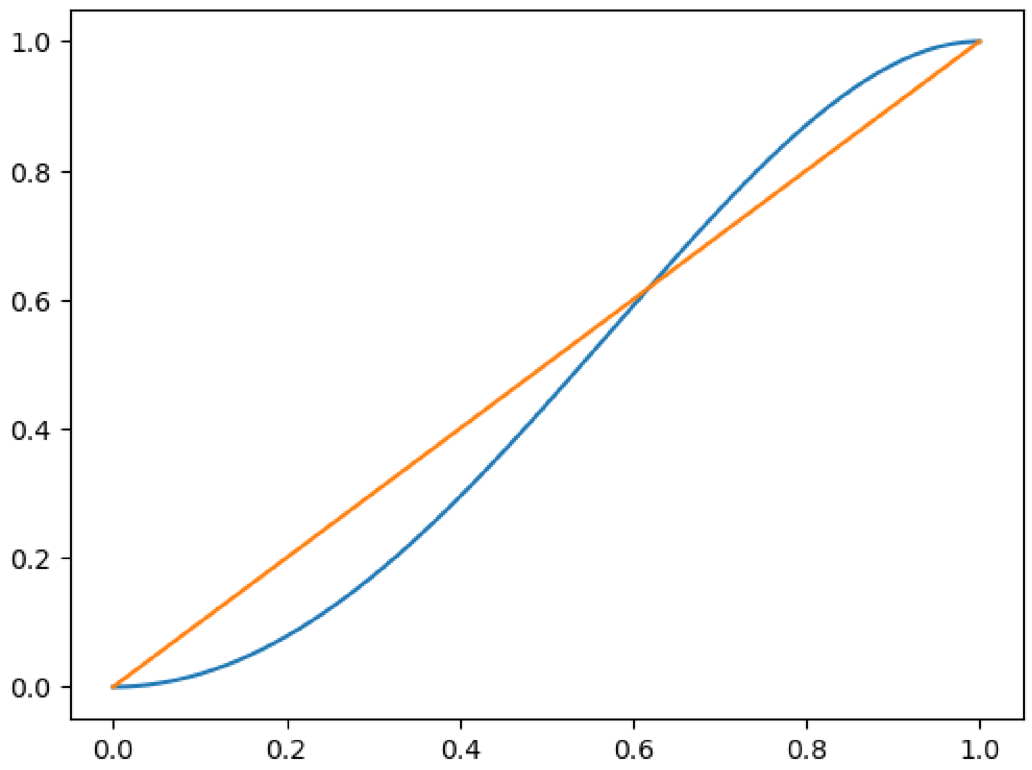
[20]:
```
for i in range(5):
    g = Grid()
    for j in range(60):
        g.update()
    if g.percolation_test() == True:
        g.observe('flooded')
```

[21]:
```
plt.figure()
plt.scatter(mean_water_new, results_new)
plt.title('Mean water levels vs Percolation: New strategy')
plt.xlabel('Water level')
plt.ylabel('Percolation')
plt.show()
```



0.1.1 Cobweb plot

[22]:
```
def f(x):
    return (x**4) + 4*(x**3)*(1-x) + 2*(x**2)*(1-x)**2

x = np.linspace(0,1,1000)
plot = [f(value) for value in x]
diagonal = [value for value in x]
plt.figure()
plt.plot(x, plot)
plt.plot(x, diagonal)
plt.show()
```



0.1.2 Average probability of having a lower altitude neighbor in our model

```
[23]: g = Grid()
      for i in range(60):
          g.update()
      np.mean(g.flow)
```

```
[23]: 0.8800000000000002
```