Introduction to Reinforcement Learning

github.com/kengz/openai_lab

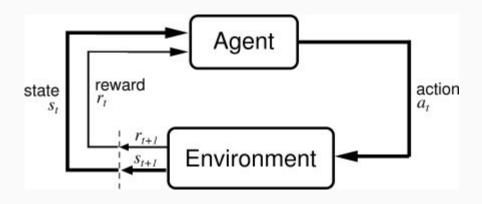
Wah Loon Keng Laura Graesser

RL system

- Agent (OpenAl Lab)
- Environment (OpenAl gym)
- SARS (state, action, reward, state_next) time transition

Characteristics

- Goal oriented, reward signals
- On-line, interactive
- Stateful environment (MDP)
- No full access to the function to optimize (no labeled data)



RL as MDP

Can be specified with:

S: state space (discrete/cont.)

A: action space (discrete/cont.)

R: reward distribution, $(\cdot|s,a)$

P: transition probability, $(\cdot|s,a)$

MDP:

- 1. Init, t = 0, env samples state $s_0 \sim p(s_0)$
- 2. For t = 0 until done:
 - 2.1. Agent selects action, a
 - 2.2. Env samples reward, $r_t \sim R(\cdot|s_t, a_t)$
 - 2.3. Env samples next state, $s_{t+1} \sim P(\cdot|s_t, a_t)$
 - 2.4. Agent receives r_t, s_{t+1}

CartPole-v0

State space (continuous, 4 dims):

- position
- velocity
- angle
- angular velocity
- e.g.: [0.023, 0.004, -0.061, 0.028]

Action space (discrete, 2 actions):

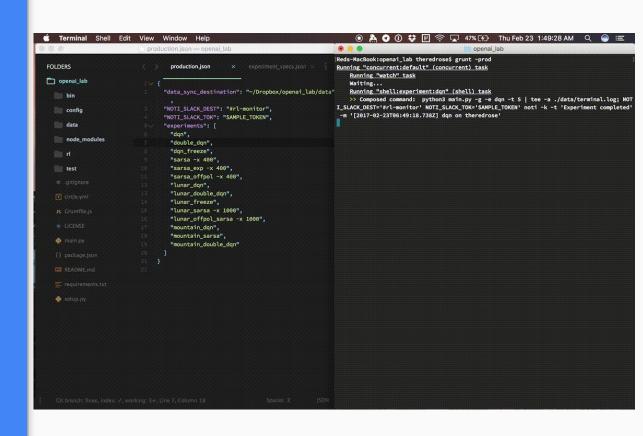
- push left: 0
- push right: 1
- e.g.: 0

Reward: +1 per timestep for staying up

Solution/epi: total rewards = 200

Solution: mean rewards > 195.0 over 100 consecutive episodes

github.com/kengz/openai lab



What to learn in an RL problem?

- 1. What action to take? → **Policy** based algorithms
- 2. How good actions are given states? → **Value** based algorithms
- 3. The dynamics of the system? \rightarrow **Model** based algorithms

Some or all of the above?

Deep-RL hint: use deep neural networks to approximate these functions.

Policy, Objective, Value

A policy is a function $\pi:S\to A$

Find a policy π^* that max. the cum. discounted reward $\sum_{t\geq 0} \gamma^t r_t$

$$\pi^* = arg \max_{\pi} Eig[\sum_{t \geq 0} \gamma^t r_t | \piig]$$

with
$$s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(s_t, a_t)$$

Value function: how good is a state?

$$V^{\pi}(s) = Eig[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, \piig]$$

Q-value function: how good is a state-action pair?

$$Q^{\pi}(s,a) = Eig[\sum_{t>0} \gamma^t r_t | s_0 = s, a_0 = a, \piig]$$

Q-learning

Q-Learning

Use a neural net to approximate the Q-function

$$Q^{\pi}(s,a) = Eig[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \piig]$$

Neural net approx. with param $heta:Q(s,a; heta)pprox Q^*(s,a)$

With a perfect Q function we know how to act optimally - policy for free

$$\pi^*(s) = \max_a Q(s,a)$$

How to learn it? Bootstrapping

Bellman equation for optimal policy:

$$egin{aligned} Q^*(s,a) &= r(s,a) + \gamma Eig[V(s'|s,a)ig] \ Q^*(s,a) &pprox r(s,a) + \gamma \, \max_{a'} Q^*(s',a')|s,a \end{aligned}$$

DQN Algorithm

Algorithm DQN

For i = 1 N:

Gather data (s_i, a_i, r_i, s'_i) by acting in the environment using some policy for j = 1 ... K:

1. Calculate target values for each example

$$y_i = r_i + \gamma \; \max_{a'} Q(s_i', a'; heta_{i-1}) | s_i, a_i$$

2. Update network parameters, using MSE loss

$$L_j(heta) = rac{1}{2} \sum_i \left| \left| \left(y_i - Q(s_i, a_i; heta_i)
ight)
ight|^2$$

Properties

Pros & Cons

- Value-based
- Off-policy
- Model-free
- Online/Offline

Pros:

- Low variance Q function estimates the expected cumulative discounted rewards from (s^t, a^t)
- Fairly sample efficient

Cons:

- High bias (critic imperfect)
- Q function has no convergence guarantee
- Exploration problem

Improving Q-learning

- Replay memory: somewhat decorrelates data, more sample efficient
- Target networks: make the moving target move less
- Different networks to select next state action and to evaluate it

Policy Gradient

Policy Gradient Formalism

Define a class of parametrized policies, $\Pi = \{\pi_{\theta}, \theta \in \mathbb{R}^m\}$

For each $\pi_{ heta}$, define its value $J(heta) = Eig[\sum_{t \geq 0} \gamma^t r_t | \pi_{ heta}ig]$

Find the optimal policy $heta^* = arg \max_{ heta} J(heta)$

For trajectory $au = (s_0, a_0, r_0, s_1, \ldots)$ and scalar score function f(au)

$$J(heta) = E_{ au \sim p(au; heta)}ig[f(au)ig] = \int_{ au} f(au)p(au; heta)d au$$

Weights the probability for action a given s

Gradient update for convergence (with some magic):

$$egin{aligned}
abla_{ heta} J(heta) &= E_{ au \sim p(au; heta)} \left[f(au)
abla_{ heta} log p(au; heta)
ight] \ &pprox \sum_{t \geq 0} f(au)
abla_{ heta} log \pi_{ heta}(a_t|s_t) \end{aligned}$$

Policy Gradient Algorithm (REINFORCE, Williams 1992)

```
Algorithm REINFORCE:
Initialize weights \theta, learning rate \alpha
for each episode (trajectory) \tau = \{s_0, a_0, r_0, s_1, \cdots, r_T\} \sim \pi_{\theta}
   for t=0 to T do
       	heta \leftarrow 	heta + lpha \ f(	au)_t 
abla_{	heta} log \pi_{	heta}(a_t|s_t)
   end for
end for
```

Properties

Pros & Cons

- Policy-based
- On-policy
- Model-free
- Offline (samples whole episodes)

Pros:

- Unbiased, uses direct sampling

Cons:

- High variance since the sampled reward varies per episode, credit assignment
- Inefficient sampling

Improving REINFORCE by lowering the variance:

Given $\nabla_{\theta} J(\theta) \approx \sum_{t>0} f(\tau) \nabla_{\theta} log \pi_{\theta}(a_t|s_t)$, improve baseline with:

- 1. reward as weightage $f(au) = \sum\limits_{t' > t} r_{t'}$
- 2. add discount factor $f(au) = \sum\limits_{t' > t} \gamma^{t'-t} r_{t'}$
- 3. introduce baseline $f(au) = \sum\limits_{t' > t} \gamma^{t'-t} r_{t'} b(s_t)$
- 4. advantage function $f(\tau) = Q^{\pi}(s_t, a_t) V^{\pi}(s_t) = A^{\pi}(s_t, a_t)$

$$abla_{ heta}J(heta)pprox \sum_{t\geq 0}ig(Q^{\pi_{ heta}}(s_t,a_t)-V^{\pi_{ heta}}(s_t)ig)
abla_{ heta}log\pi_{ heta}(a_t|s_t)$$

Actor-Critic

What to fit for the critic?

A, Q, or V?

$$egin{aligned}
abla_{ heta} J(heta) &pprox \sum_{t \geq 0} ig(Q^{\pi_{ heta}}(s_t, a_t) - V^{\pi_{ heta}}(s_t)ig)
abla_{ heta} log \pi_{ heta}(a_t|s_t) \ A^{\pi}(s_t, a_t) &= Q^{\pi}(s_t, a_t) - V^{\pi}(s_t) \end{aligned}$$

Q can be approximated with the reward from the current state and action, and V

$$Q^\pi(s_t,a_t)=r(s_t,a_t)+E_{s_{t+1}\sim p(s_{t+1}|s_t,a_t)}ig[V^\pi(s_{t+1})ig]$$

$$Q^\pi(s_t,a_t)pprox r(s_t,a_t)+V^\pi(s_{t+1})$$

$$A^{\pi}(s_t, a_t) = r(s_t, a_t) + V^{\pi}(s_t') - V^{\pi}(s_t)$$

How to learn V? The bootstrap again!

$$V(s_t) = r(s_t, a_t) + V(s_t^\prime)$$

We can just learn V

Actor-Critic Algorithm

For
$$i = 1 N$$
:

- 1. Gather data (s_i, a_i, r_i, s'_i) by acting in the environment using your policy
- 2. Update V

for
$$j = 1 ... K$$
:

Calculate target values for each example

$$y_i = r_i + V(s_i^\prime)$$

Update network parameters, using MSE loss

$$L_j(heta) = rac{1}{2} \sum_i \left| \left| \left(y_i - V(s_i; heta_i)
ight)
ight|^2$$

- 3. Evaluate A, $A^\pi(s_i,a_i) = r(s_i,a_i) + V^\pi(s_i') V^\pi(s_i)$
- 4. Calculate gradient, $\nabla_{\phi}J(\phi) pprox \sum_{i} A_{t}^{\pi}(s_{i},a_{i}) \nabla_{\phi}log\pi_{\phi}(a_{i}|s_{i})$
- 5. Use gradient to update parameters ϕ

Properties

- Value and policy -based
- On-policy
- Model-free
- Online/offline

Pros & Cons

Pros:

- Low bias
- Low variance

Cons:

Sample inefficient (from the policy gradient)

OpenAl Lab Demo

Appendix

Experience Replay

Memory unit

At each time step store <S, A, R, S'>

- State, S
- Action taken in S
- Reward after taking action A
- Next state, S'

<u>Parameters</u>

Max memory length

<u>Advantages</u>

- Can learn from past experiences as well as present experience
- Can learn about optimal policy whilst following an exploratory policy
- Decorrelates data (somewhat)