Sampling Distribution

Sampling With Replacement

Sampling is said to be with replacement when from a population a sampling unit is drawn, observed and then returned to the population before another unit is drawn. The population in this case remains the same and a sampling unit might be selected more than once. In, with-replacement case, The possible samples are N^n

- If population consist of 6 units and you have to draw random samples of size n=2. then total sample size would $N^n = 6^2 = 36$

Sampling Without Replacement

- On the other hand, a sampling unit is chosen and not returned to the population after it has been observed, the sampling is said to be without replacement. Here the sampling units cannot be selected again for the sample as the unit drawn are not replaced.
- In without replacement case: The possible samples are ${}^{N}C_{n}$

$$\frac{N!}{(N-n)! \times n!}$$

Sampling Distribution

The Probability distribution or Relative frequency distribution of the values of Statistic (i.e mean , standard deviation, Proportion etc.) that can be drawn either **with or without replacement** is called sampling distribution of that Statistic.

Sampling Distribution of Sample Mean

The probability distribution of the values of sample mean computed from all possible samples that can be drawn either with or without replacement from the given population is called Sampling distribution of sample Mean .

Sampling Distribution

$$\mu_{\bar{X}} = \frac{\sum f\bar{X}}{\sum f}$$

$$\sigma_{\bar{X}}^2 = \frac{\sum f\bar{X}^2}{\sum f} - \left(\frac{\sum f\bar{X}}{\sum f}\right)^2$$

$$\sigma_{\bar{X}} = \sqrt{\frac{\sum f\bar{X}^2}{\sum f} - \left(\frac{\sum f\bar{X}}{\sum f}\right)^2}$$

Question No 1 : - A population consists of values 3, 6, 9, 12, 15.

(a) Take all possible samples of size 2 with replacement.

(b) Construct the sampling distribution of $\bar{X}$

(c) Compute the mean and variance of the sampling distribution of $\bar{X}$

Answer: -

Population = 3, 6, 9, 12, 15

N = 5 ; n = 2 (with replacement)

Total samples = $(N)^n = (5)^2 = 25$

Sample	Mean	Sample	Mean	Sample	Mean	Sample	Mean	Sample	Mean
3, 3	3	3, 6	4.5	3, 9	6	3, 12	7.5	3, 15	9
6, 3	4.5	6, 6	6	6, 9	7.5	6, 12	9	6, 15	10.5
9, 3	6	9, 6	7.5	9, 9	9	9, 12	10.5	9, 15	12
12, 3	7.5	12, 6	9	12, 9	10.5	12, 12	12	12, 15	13.5
15, 3	9	15, 6	10.5	15, 9	12	15, 12	13.5	15, 15	15

Sampling distribution of sample means

$\bar{X}$	f	$f\bar{X}$	$\bar{X}^2$	$f\bar{X}^2$
3	1	3	9	9
4.5	2	9	20.25	40.5
6	3	18	36	108
7.5	4	30	56.25	225
9	5	45	81	405
10.5	4	42	110.25	441
12	3	36	144	432
13.5	2	27	182.25	364.5
15	1	15	225	225
	25	**225**		**2250**

$$\mu_{\bar{X}} = \frac{\sum f\bar{X}}{\sum f} = \frac{225}{25} = 9$$

$$\sigma_{\bar{X}}^2 = \frac{\sum f\bar{X}^2}{\sum f} - \left(\frac{\sum f\bar{X}}{\sum f}\right)^2 = \frac{2250}{25} - \left(\frac{225}{25}\right)^2 = 90 - 81 = 9$$

$$\sigma_{\bar{X}} = \sqrt{\frac{\sum f\bar{X}^2}{\sum f} - \left(\frac{\sum f\bar{X}}{\sum f}\right)^2} = \sqrt{\frac{2250}{25} - \left(\frac{225}{25}\right)^2} = \sqrt{90 - 81} = 3$$

$$\mu_{\bar{X}} = \frac{\sum f\bar{X}}{\sum f} = \frac{162}{27} = 6$$

$$\sigma_{\bar{X}}^2 = \frac{\sum f\bar{X}^2}{\sum f} - \left(\frac{\sum f\bar{X}}{\sum f}\right)^2 = \frac{1026}{27} - \left(\frac{162}{27}\right)^2 = 38 - 36 = 2$$

$$\sigma_{\bar{X}} = \sqrt{\frac{\sum f\bar{X}^2}{\sum f} - \left(\frac{\sum f\bar{X}}{\sum f}\right)^2} = \sqrt{\frac{1026}{27} - \left(\frac{162}{27}\right)^2} = \sqrt{38 - 36} = \sqrt{2} = 1.41$$

Question No 2 : - A population consists of values 2, 4, 6, 8, 10, 12. Take all possible samples of size 2 without replacement

(a) **Construct the sampling distribution of $\bar{X}$**
(b) **Compute the mean and variance of the sampling distribution of $\bar{X}$**

Answer: -
Population = 2, 4, 6, 8, 10, 12.
N = 6 ; n = 2 (without replacement)
Total samples = $^{N}C_n = {}^{6}C_2 = 15$

Sample	Mean	Sample	Mean	Sample	Mean	Sample	Mean	Sample	Mean
2, 4	3	2, 10	6	4, 8	6	6, 8	7	8, 10	9
2, 6	4	2, 12	7	4, 10	7	6, 10	8	8, 12	10
2, 8	5	4, 6	5	4, 12	8	6, 12	9	10, 12	11

Sampling distribution of sample means

$\bar{X}$	f	$f\bar{X}$	$\bar{X}^2$	$f\bar{X}^2$
3	1	3	9	9
4	1	4	16	16
5	2	10	25	50
6	2	12	36	72
7	3	21	49	147
8	2	16	64	128
9	2	18	81	162
10	1	10	100	100
11	1	11	121	121
	15	**105**		**805**

$$\mu_{\bar{X}} = \frac{\sum f\bar{X}}{\sum f} = \frac{105}{15} = 7$$

$$\sigma_{\bar{X}}^2 = \frac{\sum f\bar{X}^2}{\sum f} - \left(\frac{\sum f\bar{X}}{\sum f}\right)^2 = \frac{805}{15} - \left(\frac{105}{15}\right)^2 = 53.67 - 49 = 4.67$$

$$\sigma_{\bar{X}} = \sqrt{\frac{\sum f\bar{X}^2}{\sum f} - \left(\frac{\sum f\bar{X}}{\sum f}\right)^2} = \sqrt{\frac{805}{15} - \left(\frac{105}{15}\right)^2} = \sqrt{53.67 - 49} = \sqrt{4.67} = 2.16$$

Sampling Distribution of Sample Proportion ($\hat{p}$)

MEASURE	Population Value (PARAMETER)	Sample Statistic (POINT ESTIMATE)
Mean	μ	$\bar{x}$
Standard deviation	σ	s
Proportion	p	$\hat{p}$

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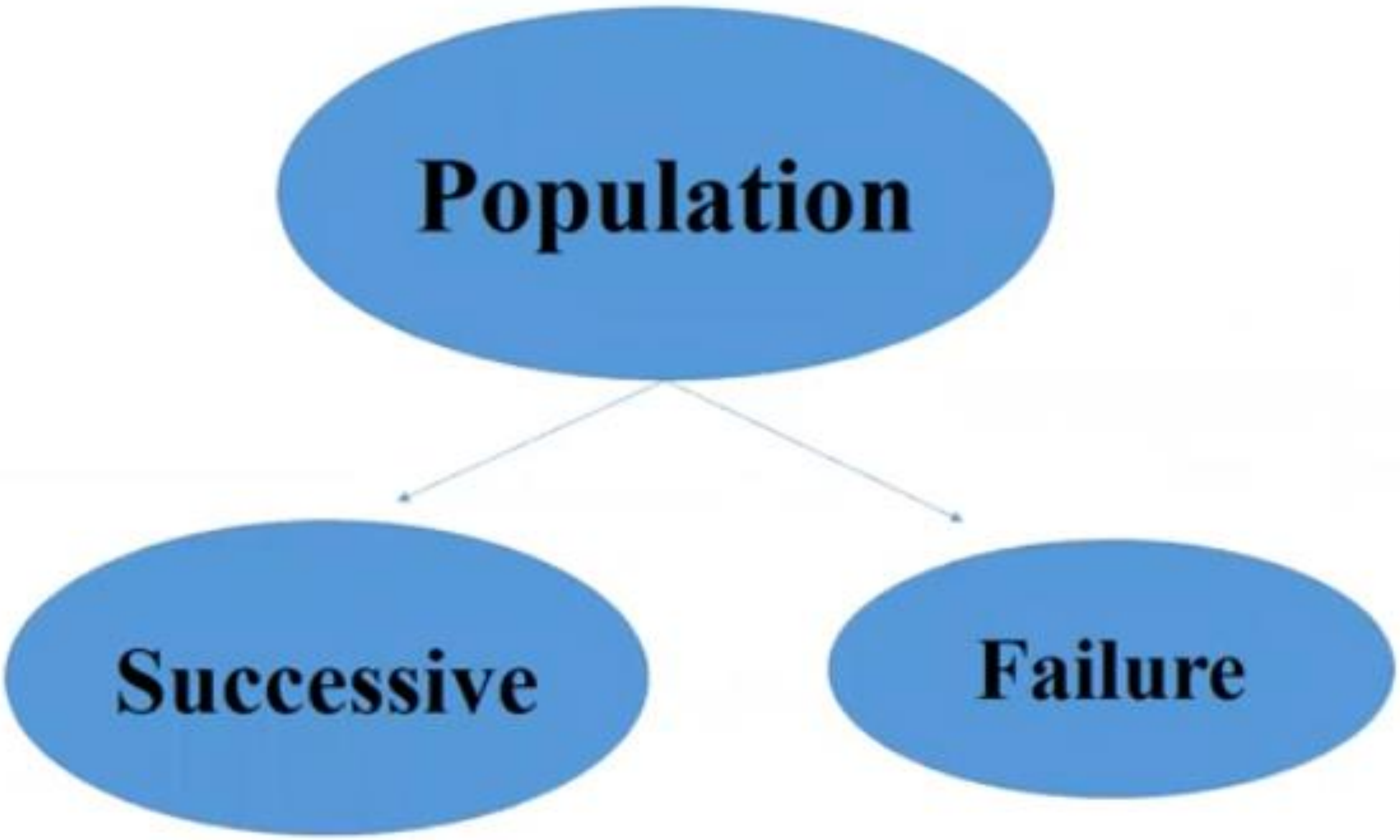
Sampling Distribution of Sample Proportion (p)

- The **Sampling Distribution of proportion** measures the proportion of success, i.e. a chance of occurrence of certain events, by dividing the number of successes i.e. chances by the sample size ’n’.
- Thus, the sample proportion is defined as **p = x/n**.


Population
Successive
Failure

Example

A random sample of 200 students found that 128 drove to school alone. Find $\hat{p}$ and $\hat{q}$ where $\hat{p}$ is the proportion of students who drove to school alone.

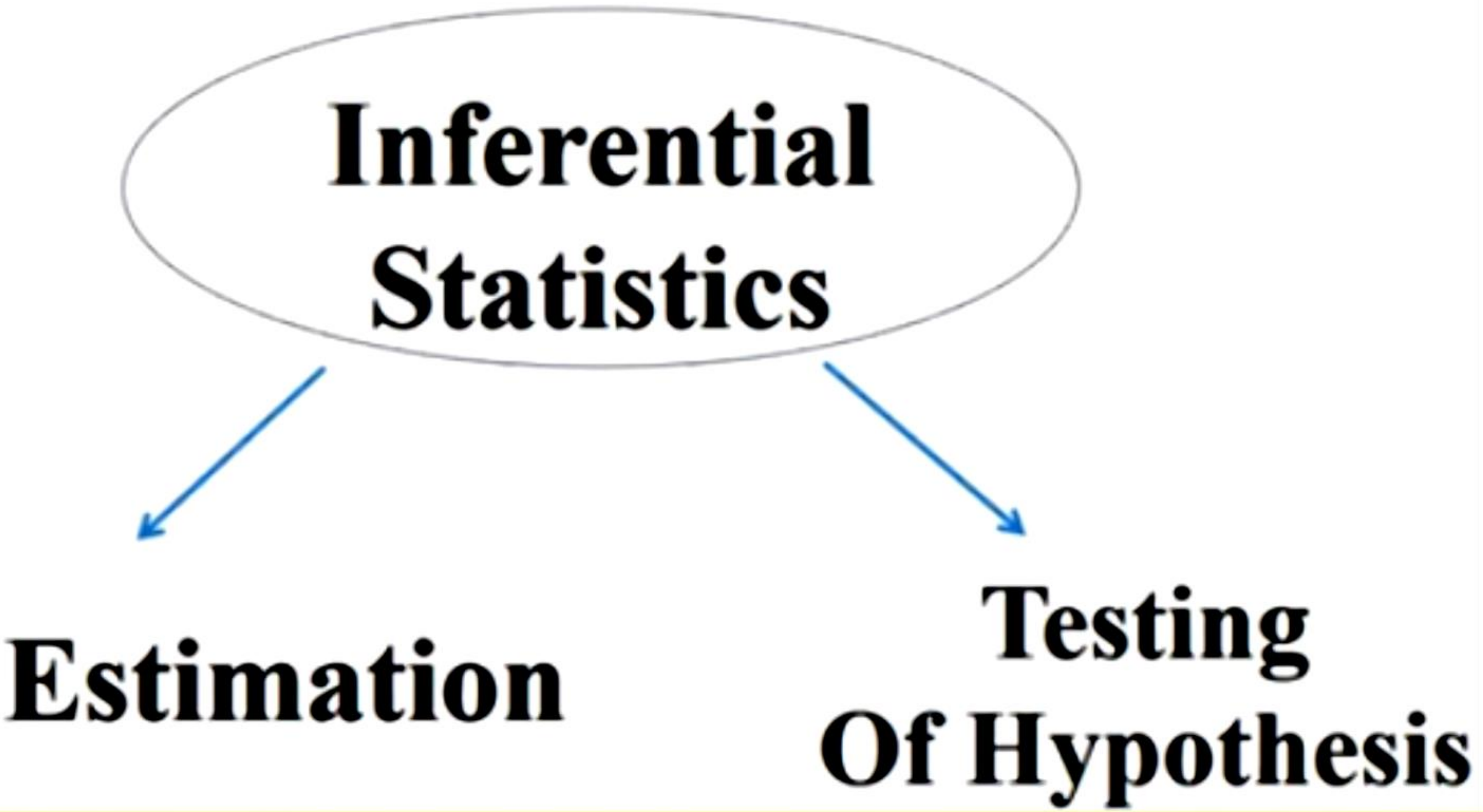
$$\hat{p} = \frac{X}{n} = \frac{128}{200} = 0.64 = 64\%$$

$$\hat{q} = \frac{n - X}{200} = \frac{200 - 128}{200} = \frac{72}{200} = 0.36 = 36\%$$

64% of the students in the survey drive to school alone, and 36% drive with others.

Estimation of Parameters

Areas of Inferential Statistics



Estimation

- The process of finding numerical value for the unknown population parameter is called an Estimation.

OR

- An **estimation** is the tool that is used to make inferences about populations from data.

Estimator and Estimate

A formula used to find the unknown value of population parameter is called an Estimator.

While

the numerical value obtained for the unknown population parameter by putting the sample data in the formula is called an Estimate.

$$\underbrace{\overline{X}}_{\text{Estimator}} = \underbrace{75.5}_{\text{Estimate}}$$

Types of Estimation

Estimation

- Point Estimation
- Interval Estimation

Point Estimate

When an estimate for the unknown population parameter is expressed by a single value, then it is called Point Estimate.

Example:

- Suppose it is desired to estimate the average height of 500 students in a University. If we draw sample of 70 students and find their average height as 5.4 feet. Then 5.4 is called the point Estimate for μ.
- And in this case the $\bar{x}$ is the Estimator.

Interval Estimate

When an estimate for the unknown value of a population parameter is expressed by the range of values within which the true value of population parameter is believed to lie, then it is called an Interval Estimate.

Example:

If we state that the true average height(Population Mean) of the university students is between 64" and 66" then the range 64" to 66" is called an Interval Estimate.

Example # 1

- A random sample of n = 3 has the elements 1, 3, and 5.
- Compute a point estimates of

➢ **Population mean**

➢ **Population standard deviation**

Solution:

- Population Mean (μ): Since sample data is given and we are required to find population mean which is not possible, so therefore we use estimator of "μ" which is "$\bar{x}$".

- Population Standard deviation(δ): Since sample data is given and we are required to find population standard deviation which is not possible , so therefore we use estimator of "δ" which is "s".

SOLUTION:

(Point Estimator of μ)

a) $\bar{X} = \frac{\sum x}{n} = \frac{1+3+5}{3} = \frac{9}{3} = 3$

(Point Estimate of μ)

b) Standard Deviation

X	X^2
1	1
3	9
5	25
9	**35**

$$S = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

$$= \sqrt{\frac{35}{3} - \left(\frac{9}{3}\right)^2}$$

$$= \sqrt{11.67 - 9} = \sqrt{2.67}$$

(Point Estimator of δ) $S = 1.634$ (Point Estimate of δ)

Practice Question # 1

A random sample of n = 6 has the elements 6, 10, 13, 14, 18 and 20.

Compute a point estimates of;

a. Population mean
b. Population standard deviation

Confidence Interval

Confidence Interval

- A confidence interval, in statistics, refers to the probability that a population parameter will fall between a set of values for a certain proportion of times.
- Confidence intervals measure the degree of uncertainty or certainty in a sampling method.
- For example, It tells you how confident you can be that the results from a poll or survey reflect what you would expect to find if it were possible to **survey the entire population.**
- They can take any number of probability limits, with the most common being a 95% or 99% confidence level.

Confidence Interval

- Provides Range of Values
  - Based on Observations from 1 Sample
- Gives Information about Closeness to Unknown Population Parameter
- Stated in terms of Probability

Never 100% Sure

Example: If you wanted to find out the Average Age of cigarette smokers. And based on sample survey, 90% confidence interval of average age smokers is between 18years and 25 years.

$$P(L<\theta<U) = 1- \alpha$$

Then confidence interval for the population Average "μ" is written as

$$P\,(18< \mu <25) = 90\%$$

Where

18 = lower value of the estimator "μ"

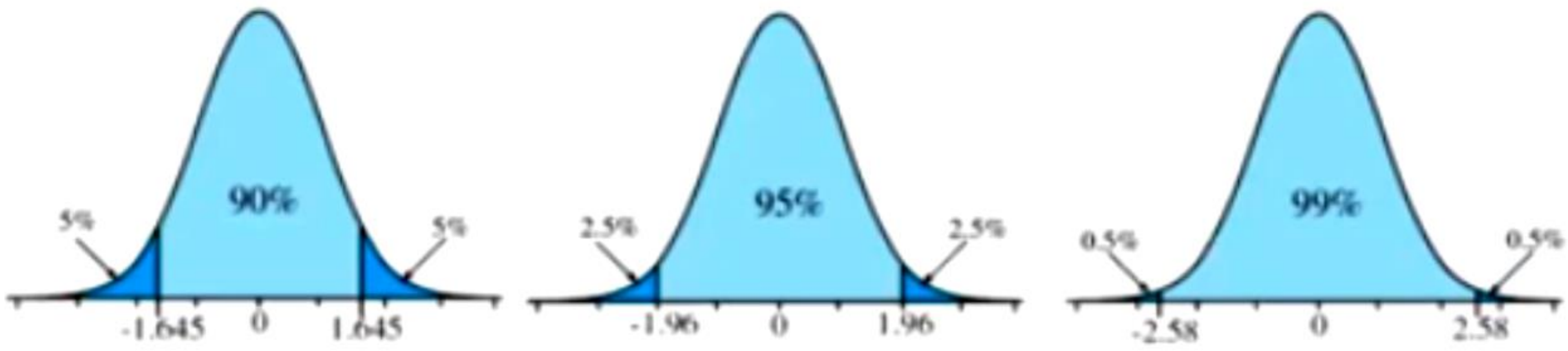
25 = Upper value of the estimator "μ"

And

90% = Level of confidence or Confidence coefficient ($1- \alpha$)

Common Levels of Confidence

Confidence level $1-\alpha$	Alpha level α	Z value $z_{1-(\alpha/2)}$
.90	.10	1.645
.95	.05	1.960
.99	.01	2.576



Confidence Interval of Population Mean(μ) When (n≥30)

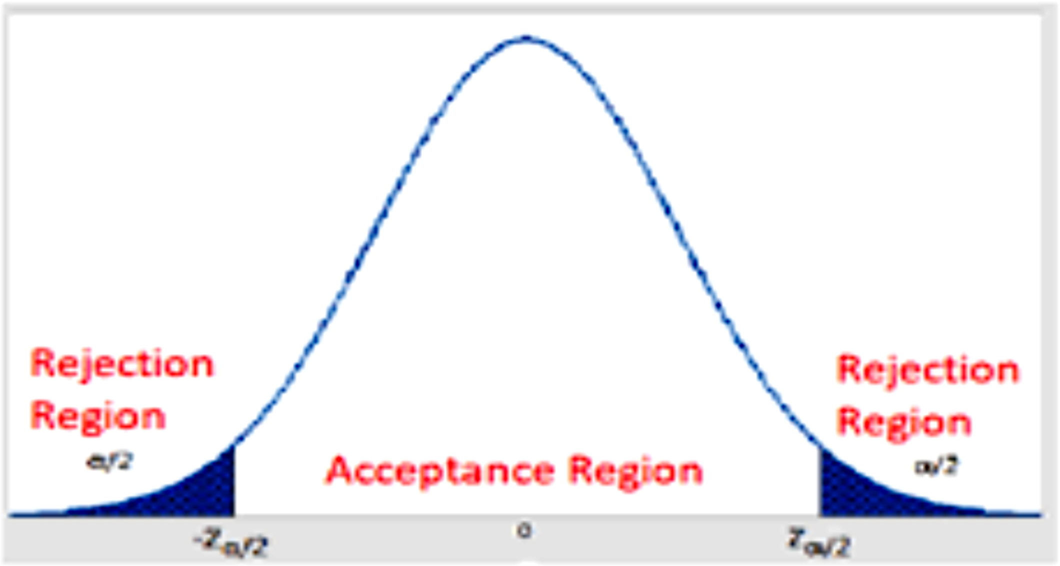
$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Population Standard Deviation

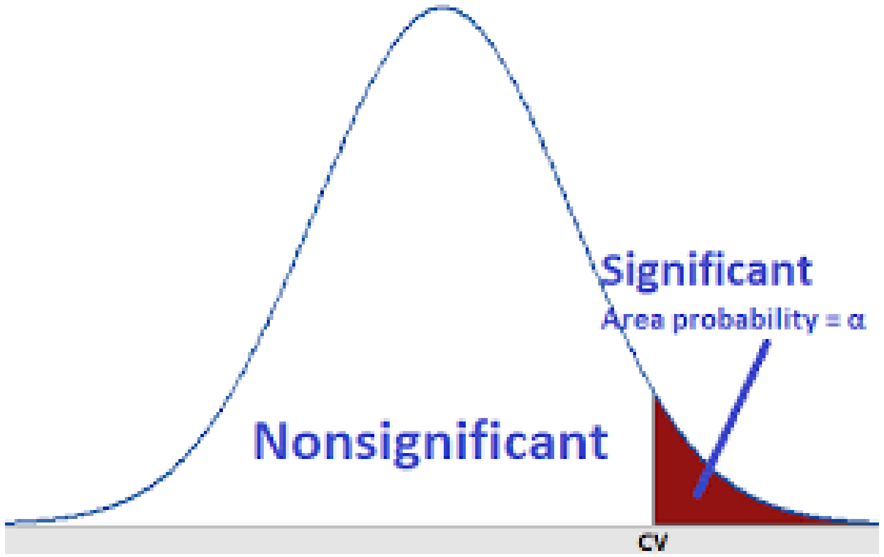
Sample mean

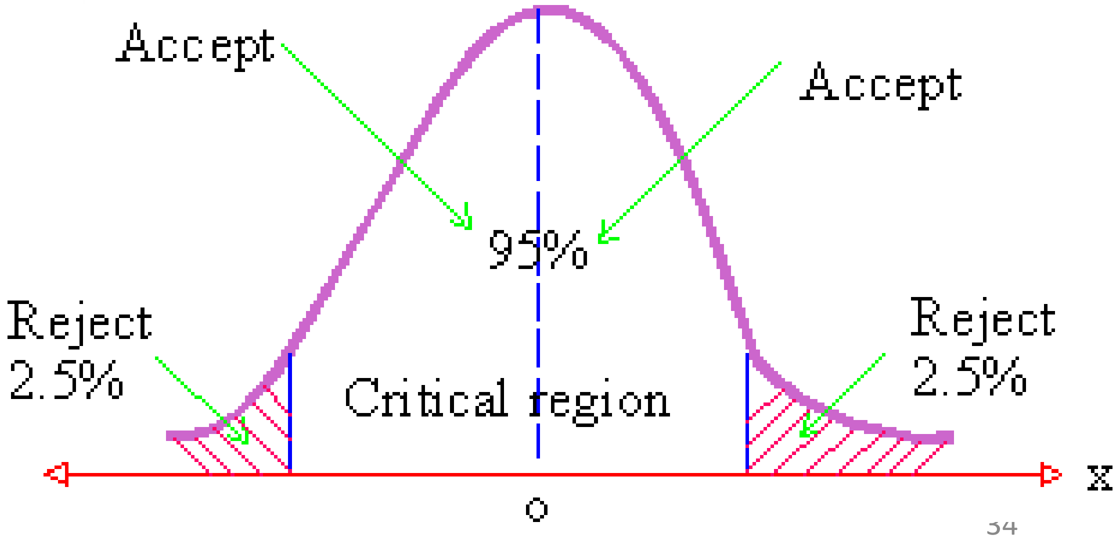
Critical Value
Or
Table Value

Sample Size



Critical Value





Practice Problems:

- Find:

1. $z_{\alpha/2}$ for 96% confidence interval 2.06
2. $z_{\alpha/2}$ for 99% confidence interval 2.58
3. $z_{\alpha/2}$ for 95% confidence interval 1.96
4. $z_{\alpha/2}$ for 94% confidence interval 1.88

Example#01

- The heights of a random sample of **50** college students show a mean of **174.5** cm and a standard deviation of **6.9** cm.

- Find a **80%** and **95%** confidence interval for the mean height of all college students.

Solution:

Here

n = 50 $\quad \bar{X} = 174.5 \quad S = 6.9 \quad 1-\alpha = 80\% \quad$ **Z-value =** 1.29

The 80% confidence interval for μ

$$\bar{X} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$174.5 \pm 1.29 \frac{6.9}{\sqrt{50}}$$

For Lower Value of μ

174.5 - 1.25 = 173.25

For Upper Value of μ

174.5 + 1.25 = 175.75

So therefor 80% confidence interval for average height of all college students is (173.25, 175.75)

Solution:

Here

n = 50 $\quad \bar{X} = 174.5 \quad$ **S = 6.9** $\quad 1-\alpha = 95\% \quad$ **Z-value = 1.96**

The 95% confidence interval for μ

$$\bar{X} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$174.5 \pm 1.96 \frac{6.9}{\sqrt{50}}$$

For Lower Value of μ

174.5 - 1.91 = 172.59

For Upper Value of μ

174.5 + 1.91 = 176.41

So therefor 95% confidence interval for average height of all college students is (172.59, 176.41)

Solution:

So therefor 80% confidence interval for average height of all college students is $(173.25 \leq \mu \leq 175.75)$

So therefor 95% confidence interval for average height of all college students is $(172.59 \leq \mu \leq 176.41)$

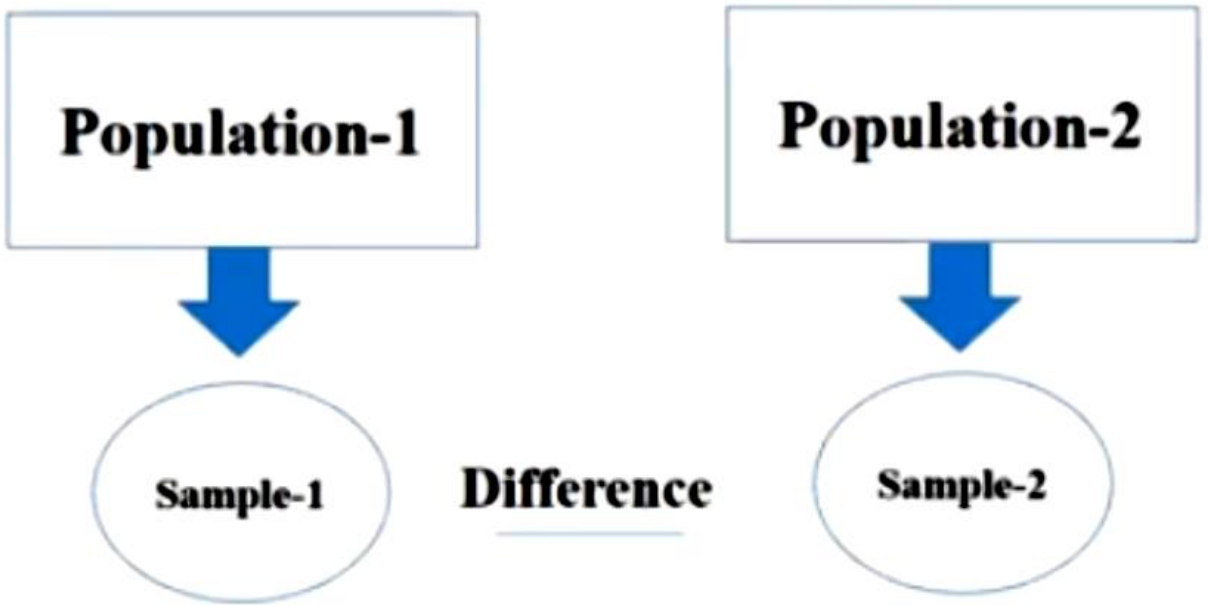
Practice Question # 1

- A random sample of **100** automobile owners shows that an automobile is driven on the average **23500** miles per year , in the state of America, with a standard deviation of **3900** miles.

- Construct a **90% and 99%** confidence interval for the average number of miles an automobile is driven annually in America.

For the difference between the means of two Populations ($\mu_1-\mu_2$)

Confidence Interval for the difference between the means of two Populations ($\mu_1 - \mu_2$)



Introduction

- There are many situations where it is of interest to compare two groups with respect to their mean scores.
- For example we are interested to compare weight of smokers and non-smokers, or marks of male and female etc. Both of these situations involve comparisons between two independent samples, meaning that there are different people in the groups being compared.
- We could begin by computing the sample sizes (n_1 and n_2), means ($\bar{X}1$ & $\bar{X}$ 2) and standard deviations (s_1 and s_2) in each sample.
- The point estimate for the difference in population means is the difference in sample mean $\bar{x}_1 - \bar{x}_2$

Confidence Interval for the difference between the means of two Populations $(\mu_1 - \mu_2)$

When n_1 and $n_2 \geq 30$

$$(\bar{x}_1 - \bar{x}_2) \pm Z\alpha/2\sqrt{\frac{\sigma_1{}^2}{n_1} + \frac{\sigma_2{}^2}{n_2}}$$

Note: **When σ_1 and σ_2 is unknown then replace σ_1 by s_1 and σ_2 by s_2.**

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},$$

Confidence Level	Z value
80%	1.28
90%	1.645
95%	1.96
98%	2.33
99%	2.58
99.8%	3.08
99.9%	3.27

Example

- A standardized chemistry test was given to sample of **75** Boys and **50** girls. The girls made an average grade of **76** with a standard deviation of **6**, while the boys made an average grade of **82** with a standard deviation of **8**.
- Find a **95%** and **99%** confidence interval for the difference($\mu_1 - \mu_2$).

Solution for 95% confidence interval between the means $(\mu_1 - \mu_2)$

Here

Boys	**Girls**
$n_1 = 75$	$n_2 = 50$
$\bar{X}_1 = 82$	$\bar{X}_2 = 76$
$s_1 = 8$	$s_2 = 6$
$s_1^2 = 64$	$s_2^2 = 36$

$1-\alpha = 95\%$ Z-value = 1.96

Solution for 95% confidence interval between the means ($\mu_1 - \mu_2$)

Therefore 95% confidence interval for $\mu_1 - \mu_2$

$$(\bar{x}_1 - \bar{x}_2) \pm Z\alpha/2 \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(82 - 76) \pm 1.96 \sqrt{\frac{64}{75} + \frac{36}{50}}$$

$$6 \pm 1.96\sqrt{0.853 + 0.72} \Rightarrow 6 \pm 1.96\sqrt{1.573}$$

$$6 \pm 1.96(1.254) \Rightarrow 6 \pm 2.458$$

For Lower Value of $\mu_1 - \mu_2$

6 – 2.458 = 3.542

For Upper Value of $\mu_1 - \mu_2$

6 + 2.458 = 8.458

So therefore 95% confidence interval for $\mu_1 - \mu_2$ is (3.542, 8.458)

Solution for 99% confidence interval between the means ($\mu_1 - \mu_2$)

Therefore 99% confidence interval for $\mu_1 - \mu_2$

$$(\bar{x}_1 - \bar{x}_2) \pm Z\alpha/2 \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(82 - 76) \pm 2.58 \sqrt{\frac{64}{75} + \frac{36}{50}}$$

$$6 \pm 2.58\sqrt{0.853 + 0.72} \Rightarrow 6 \pm 2.58\sqrt{1.573}$$

$$6 \pm 2.58(1.254) \qquad 6 \pm 3.235$$

For Lower Value of $\mu_1 - \mu_2$

6 – 3.235 = 2.765

For Upper Value of $\mu_1 - \mu_2$

6 + 3.235 = 9.235

So therefor 99% confidence interval for $\mu_1 - \mu_2$ is (2.765, 9.235)

Practice Question # 1

- For a random sample of 61 accounting students in a class using group learning techniques, the mean examination score was 322.12, and the sample standard deviation was 54.53.
- For an independent random sample of 61 students in the same course but in a class not using group learning techniques, the sample mean and standard deviation of the scores were 304.61 and 62.61, respectively.
- **Find a 90% and 98% confidence interval for the difference between the two population mean scores.**

Practice Question # 1

With Learning Techniques	Without Learning Techniques
$n_1 = 61$	$n_2 = 61$
$\overline{X}_1 = 322.12$	$\overline{X}_2 = 304.61$
$S_1 = 54.53$	$S_2 = 62.61$

$1-\alpha = 90\%$ and 98%

Confidence Interval for Population Proportion “P”.

Introduction

- When a characteristic being measured is categorical — for example, opinion on an issue (support, oppose, or are neutral), gender, political party, or type of behavior (do/don't wear a seatbelt while driving) — most people want to estimate the proportion (or percentage) of people in the population that fall into a certain category of interest.
- For example, Investors in the stock market are interested in the true proportion of stocks that go up and down each week. Businesses that sell personal computers are interested in the proportion of households in the Pakistan that own personal computers.
- In each of these cases, the object is to estimate a population proportion, p, using a sample proportion, ρ, plus or minus a margin of error.
- **The result is called a *confidence interval for the population proportion, p.***

Formula for CI for a population proportion

Confidence Interval for Population Proportion "P".

$$\hat{p} \pm Z\alpha / 2 \sqrt{\frac{\hat{p}\left(1 - \hat{p}\right)}{n}}$$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Commonly used confidence levels are 90%, 95%, and 99%

Confidence Level	Z value
80%	1.28
90%	1.645
95%	1.96
98%	2.33
99%	2.58
99.8%	3.08
99.9%	3.27

Example

- In a random sample of **500** people eating lunch at a hospital cafeteria on various Fridays, it was found that **160** preferred seafood.

- Find a **95%** confidence interval for the actual proportion of people who eat seafood on Friday at this cafeteria.

Solution....

Here $\hat{p}$ = X/n = 160 / 500 = 0.32

Therefore 95% confidence interval for P

$$\hat{p} \pm Z\alpha/2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.32 \pm 1.96\sqrt{\frac{0.32(1-0.32)}{500}}$$

$$0.32 \pm 1.96\sqrt{\frac{0.32(0.68)}{500}}$$

Solution....

$$0.32 \pm 1.96 \sqrt{\frac{0.2176}{500}}$$

$$0.32 \pm 1.96 \sqrt{0.0004352}$$

$$0.32 \pm 1.96 (0.020)$$

$$0.32 \pm 0.0392$$

$$0.32 \pm 0.0392$$

For Lower Value of "P"

$0.32 - 0.0392 = 0.2808$

For Upper Value of

$0.32 + 0.0392 = 0.3592$

So therefor 95% confidence interval for "P" is (0.2808, 0.3592) **28% , 35.9%**

Example

A survey conducted by Fatima and Manahil of 1404 respondents found that 323 students paid for their education by student loans. Find the 90% confidence of the true proportion of students who paid for their education by student loans.

Determine $\hat{p}$ and $\hat{q}$

$$\hat{p} = \frac{X}{n} = \frac{323}{1404} = 0.23$$

$$\hat{q} = 1 - \hat{p} = 1.00 - 0.23 = 0.77$$

Determine the critical value

$$\alpha = 1 - 0.90 = 0.10$$

$$\frac{\alpha}{2} = \frac{0.10}{2} = 0.05$$

$$z_{\alpha/2} = 1.65$$

Since $\alpha = 1 - 0.90 = 0.10$, $z_{\alpha/2} = 1.65$.

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\hat{p} = \frac{323}{1404} = 0.23 \qquad \hat{q} = 1 - \hat{p} = 1 - 0.23 = 0.77$$

$$0.23 - 1.65\sqrt{\frac{(0.23)(0.77)}{1404}} < p < 0.23 + 1.65\sqrt{\frac{(0.23)(0.77)}{1404}}$$

$$0.23 - 0.019 < p < 0.23 + 0.019$$

$$0.211 < p < 0.249$$

$$21.1\% < p < 24.9\%$$

Practice Question

- A random sample of 2,000 current BBA students reveals, 80% are satisfied with the level of education in Pakistan.
- Develop a **90** percent confidence interval for the Population Proportion.
- For **90% Z=1.645**

These values are taken from the **Student *t* distribution**, most often called the ***t* distribution**.

William. Sealy Gosset Publicly research was not allowed

Sigma is unknown	sigma is known
n < 30	n > 30
t distribution	z distribution

Characteristics of the t Distribution

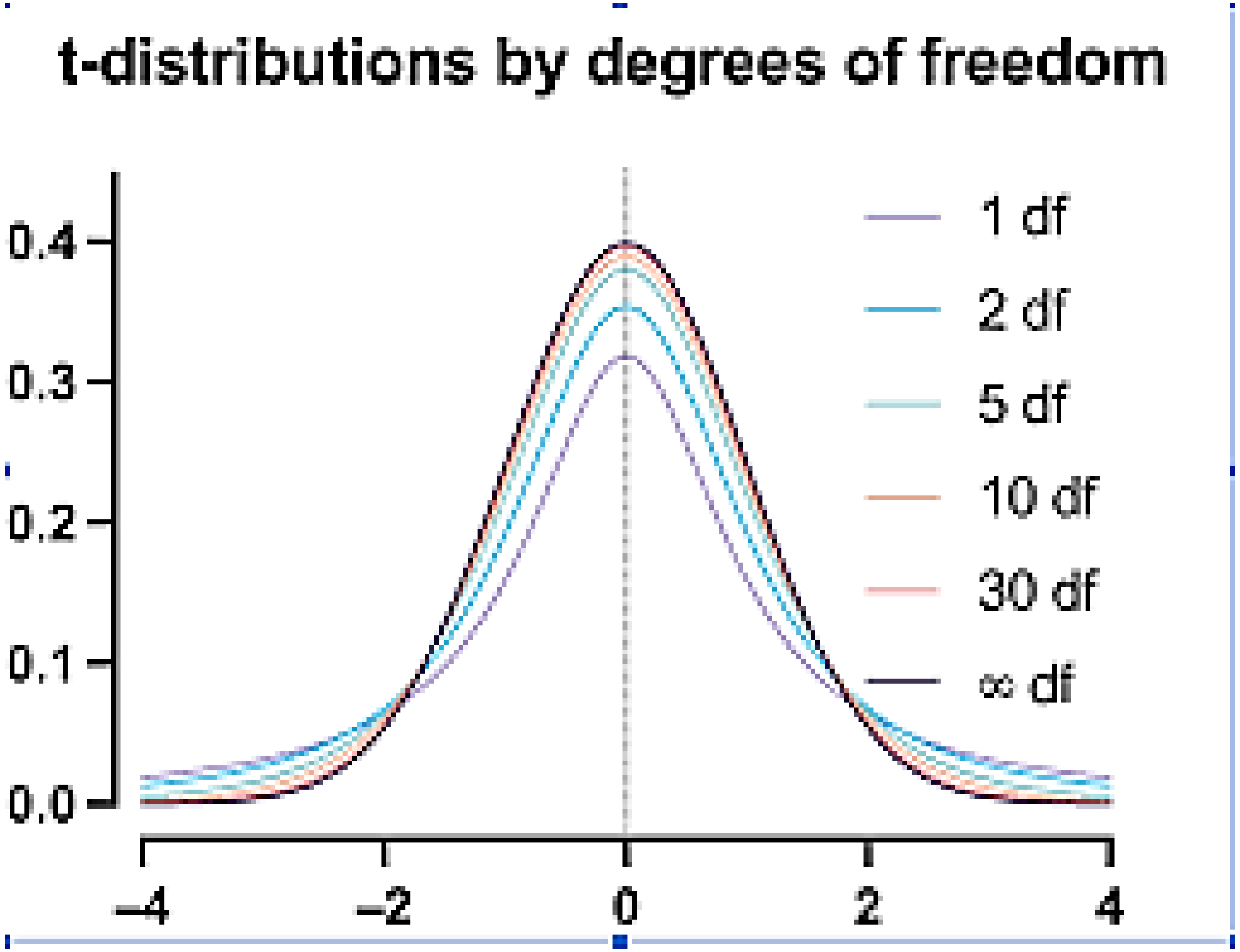
The *t* distribution is similar to the standard normal distribution in these ways:

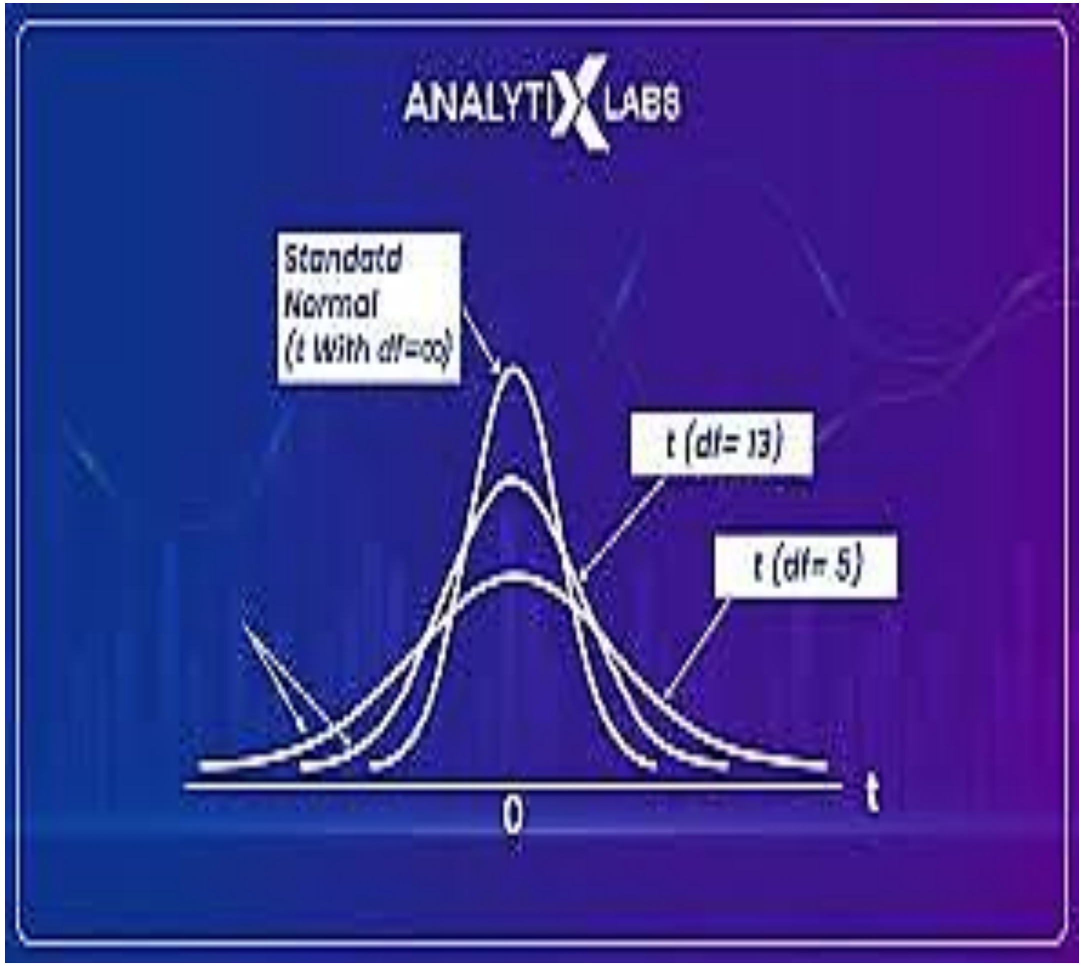
1. **It is bell-shaped.**
2. **It is symmetric about the mean.**
3. **The mean, median, and mode are equal to 0 and are located at the center of the distribution.**
4. **The curve never touches the *x* axis.**

Characteristics of the t Distribution

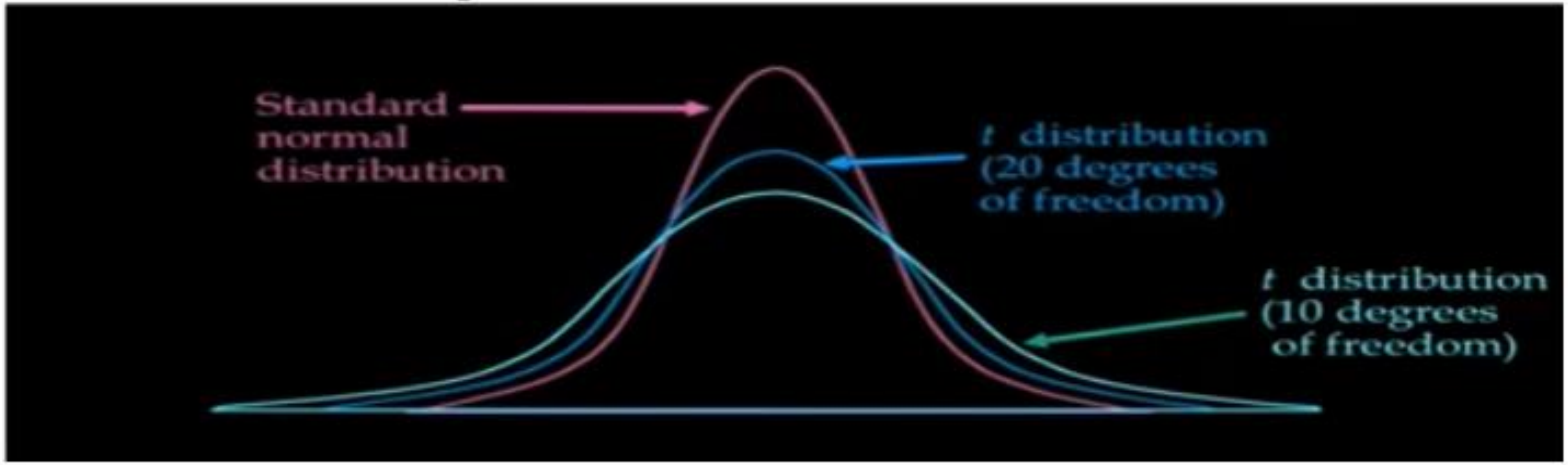
The t distribution differs from the standard normal distribution in the following ways:

1. **The variance is greater than 1.**
2. **The t distribution is actually a family of curves based on the concept of degrees of freedom, which is related to sample size.**
3. **As the sample size increases, the t distribution approaches the standard normal distribution.**


t-distributions by degrees of freedom
0.4
0.3
0.2
0.1
0.0
−4
−2
0
2
4
1 df
2 df
5 df
10 df
30 df
∞ df


ANALYTIX LABS
Standard
Normal
(t With df=∞)
t (df= 13)
t (df= 5)
0
t

The symbol d.f. will be used for degrees of freedom.
The degrees of freedom for a confidence interval for the mean are found by subtracting 1 from the sample size. That is, d.f. = $n - 1$.



Formula for a Specific Confidence Interval for the Mean When σ Is Unknown and $n < 30$

$$\bar{X} - t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

The degrees of freedom are $n - 1$.

Alpha = level of significance

= 1- level of confidence

=1-.95

=.05/2 .025

$$\bar{X} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

Table A.4 Critical Values of the t-Distribution

	α						
v	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Table A.4 (continued) Critical Values of the t-Distribution

	α						
v	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290

Example: A random sample of 10 children found that their average growth for the first year was 9.8 inches. Assume the variable is normally distributed and the sample standard deviation is 0.96 inch. Find the 95% confidence interval of the population mean for growth during the first year.

$$\overline{X} = 9.8 \qquad s = 0.96 \qquad n = 10$$

$$\overline{X} - t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) < \mu < \overline{X} + t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$9.8 - 2.262\left(\frac{0.96}{\sqrt{10}}\right) < \mu < 9.8 + 2.262\left(\frac{0.96}{\sqrt{10}}\right)$$

$$9.8 - 0.69 < \mu < 9.8 + 0.69$$

$$9.11 < \mu < 10.49$$

Therefore, one can be 95% confident that the population mean of the first-year growth is between 9.11 and 10.49 inches

Example: The data represent a sample of the number of home fires started by candles for the past several years. Find the 99% confidence interval for the mean number of home fires started by candles each year.

5460 5900 6090 6310 7160 8440 9930

Step 1: Find the mean and standard deviation. The mean is $\bar{X} = 7041.4$ **and standard deviation** $s = 1610.3$.

Step 2: Find $t_{\alpha/2}$ **in Table . The confidence level is 99%, and the degrees of freedom d.f. = 6**

$t_{.005} = 3.707$.

Step 3: Substitute in the formula.

$$\bar{X} - t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$7041.4 - 3.707\left(\frac{1610.3}{\sqrt{7}}\right) < \mu < 7041.4 + 3.707\left(\frac{1610.3}{\sqrt{7}}\right)$$

$$7041.4 - 2256.2 < \mu < 7041.4 + 2256.2$$

$$4785.2 < \mu < 9297.6$$

One can be 99% confident that the population mean number of home fires started by candles each year is between 4785.2 and 9297.6, based on a sample of home fires occurring over a period of 7 years.

Practice

- Estimate the mean population high temperature with 90% confidence, by using the random sample of the following temp.

60,88,73,86,103,79,67,72,89,88,76,87

Ans: 74.6-86.8