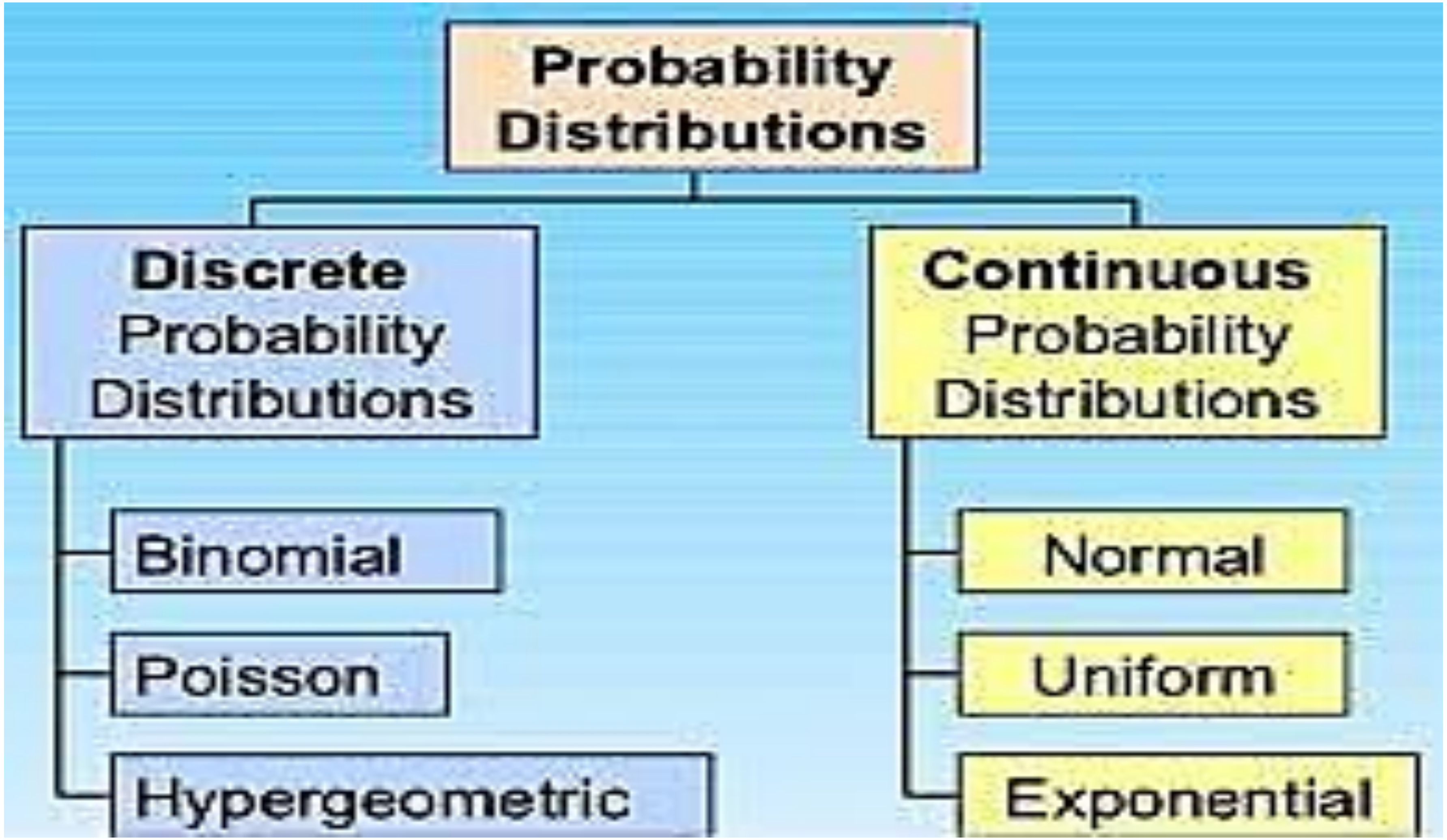

Probability Distributions
Discrete Probability Distributions
Continuous Probability Distributions
Binomial
Poisson
Hypergeometric
Normal
Uniform
Exponential

Binomial Distribution

Definition

Suppose a random experiment has the following characteristics.

1. *There are n identical and independent trials of a common procedure.*
2. *There are exactly two possible outcomes for each trial, one termed "success" and the other "failure."*
3. *The probability of success on any one trial is the same number p.*

Then the discrete random variable X that counts the number of successes in the n trials is the binomial random variable with parameters *n* and *p*.
We also say that X has a binomial distribution with parameters *n* and *p*.

Binomial Distribution

1.
Success or Failure
Probability of success =p
Probability of Failure =q
p=1 - q

2.
$$p(x=r) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

3.
Binomial Distribution=(q+p)^n

4.
Mean =np
variance =npq
Std. Deviation=$\sqrt{\text{variance}}$

- **P** is the probability of success on any trail.
- **q = 1- P** – the probability of failure
- **n** – the number of trails/experiments
- **r** – the number of successes, it can take the values 0, 1, 2, 3, . . . **r** .

Example:01

Determine the binomial distribution whose mean is 9 and whose standard deviation is 3/2.

Binomial Distribution=(q+p)^n

np=9 --------- eq (i)

$\sqrt{npq} = 3/2$

Squaring both sides

npq=9/4 ---------- eq(ii)

eq(ii) divided by eq(i)

$$\frac{\cancel{np}q}{\cancel{np}} = \frac{9/4}{9/1}$$

$q=\cancel{9}/4 \times 1/\cancel{9}$

q=1/4

p=1-q

p=1-(1/4)

p=3/4

From eq(i)

np=9

nx(3/4)=9

$n=(\cancel{9}^{3}x4)/\cancel{3}$

n=12

Binomial Distribution=(q+p)^n

=(1/4+3/4)^12

Example:02

The probability of a man hitting a target is 1/4. He fires 7 times. What is the probability of his hitting atleast twice the target.

P=1/4 -------- Success

q=1-P

q=1-(1/4)

q=3/4 --------- Failure

n=7

$$P(x \geq 2)$$

$$P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)+P(x=7)$$

OR

$$1-[P(x=0)+P(x=1)]$$

$$1-[P(x=0)+P(x=1)]$$

$${}^{n}C_{r}\,P^{r}\,q^{n-r}$$

$$=1-\left[{}^{7}C_{0}\,P^{0}\,q^{7}+{}^{7}C_{1}\,P^{1}\,q^{6}\right]$$

=1 - [1x1xq^7 + 7P^1 q^6

=1 - [q^7 + 7Pq^6]

=1 - q^6 [q + 7P]

=1- (3/4)^6 [(3/4) + 7(1/4)]

= 1 -(3/4)^6 [10/4]

= 1-(729/4096)[10/4]

= 1 - (7290/16384)

= (16384 - 7290)/16384

= 9094/16384

= 4547/8192 Ans

$${}^{3}C_{0}=1$$

$${}^{4}C_{0}=1$$

$${}^{7}C_{1}=7$$

Example:03 (H.W)

A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of

a. **no success**
b. **6 successes**
c. **At least 6 successes**
d. **At most 6 successes**

n=7

S={(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)}

P=6/36

P=1/6

q=1-P

q=1- (1/6)

q=5/6

(a)

P(x=0)

n=7 ; r=0

P=1/6 ; q=5/6

$^{n}C_{r}\ P^{r}\ q^{n-r}$

$^{7}C_{0}\ P^{0}\ q^{7}$

=1x1x(5/6)^7

= (5/6)^7

=78125/279936

=0.279

(b)

P(x=6)

r=6 ; n=7

$$ {}^{7}C_{6}\, P^{6}\, q^{7-6} $$

$$ {}^{7}C_{6} = \frac{7!}{6!\,1!} = \frac{7\times\cancel{6!}}{\cancel{6!}\,1!} = 7 $$

7 x (1/6)^6 x (5/6)^1

35/(6)^7

=35/279936

=0.000125

(c) $P(x \geq 6)$

P(x=6) + P(x=7)

$$ {}^{7}C_{6}\, P^{6}\, q^{7-6} + {}^{7}C_{7}\, P^{7}\, q^{7-7} $$

7(P)^6(q)^1 + 1(P)^7(q)^0

7P^6q + 1xP^7x1

P^6 (7q+ P)

(1/6)^6 (7(5/6) + 1/6)

(1/6)^6 x 36/6

(1/6)^6 x 6

(1/46656) x 6

1/7776

(d)

$$P(x \leq 6)$$

$$P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6)$$

OR

1 - P(x=7)

$1 - {}^{7}C_{7}\, P^{7}\, q^{7-7}$

1 - 1xP^7xq^0

1 - P^7

1 - (1/6)^7

1 - (1/279936)

(279936 - 1)/279936

279935/279936

Example:04

Find the mean of binomial distribution B(4,1/3).

mean=?
B(n,p)
mean=(4x1/3)
mean=4/3

Example:05 (H.W)

The mean and variance of binomial distribution are 4 and 4/3 respectively. Find $P(x \geq 1)$.

mean=4
variance=4/3
np=4
npq=4/3

$$\frac{\cancel{npq}}{\cancel{np}} = \frac{4/3}{4/1}$$

$q=(\cancel{4}/3) \times (1/\cancel{4})$
$q=1/3$
$p=1-q$
$p=1-(1/3)$
$p=2/3$

$np=4$
$n(2/3)=4$
$n=(4\times\cancel{3})/\cancel{2}$
$n=6$

$P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)$

OR

$1 - P(x=0)$

$$1 - \left[{}^{6}C_{0}\, p^{0}\, q^{6-0} \right]$$

$1 - (1 \times 1 \times q\text{^}6)$
$1-q\text{^}6$
$1-(1/3)\text{^}6$
$1-(1/729)$
$(729 - 1)/729$
$728/729$

Example:06 (H.W)

The probability that a pen manufactured by a company will be defective is 1/10.If 12 such pens are manufactured. Find the probability that none will be defected.

$q=1/10$

$p=1-q$

$p=1-(1/10)$

$p=9/10$

$n=12$

$$P(x=12) = {}^{12}C_{12}\ p^{12}\ q^{12-12}$$

p(x=12)=1 x p^12 x 1

p(x=12)=p^12

p(x=12)=(9/10)^12

p(x=12)=(2.824 x 10^11)/(1x10^12)

p(x=12)=353/1250

p(x=12)=0.2824

Binomial Frequency distribution

The binomial probability distribution is multiplied by N (the number of sets or experiment), it is called binomial frequency distribution.

$$N.P(X = x) = N.\binom{n}{x} p^x q^{n-x}$$

Binomial Frequency Distribution

Example:01

Six dice are thrown 729 times. How many time do you expect at least three Dice to show five or a six?

$P(x \geq 3)$

n=6
N=729
S={1,2,3,4,5,6}
P=2/6=1/3
q=1-P
q=1- (1/3)
q=2/3

$$=1 -[P(x=0) + P(x=1) + P(x=2)]$$

$$729\left[1 - \left({}^{6}C_{0}\, p^{0} q^{6-0} + {}^{6}C_{1}\, p^{1} q^{6-1} + {}^{6}C_{2}\, p^{2} q^{6-2}\right)\right]$$

=729[1 - (1x1xq^6 + 6Pq^5 + 15 P^2q^4)]
=729[1- q^4(q^2 + 6Pq + 15P^2)]
=729[1 -(2/3)^4((2/3)^2 + 6(1/3)(2/3) +15(1/3)^2)]
=729[1-(16/81)(31/9)]
=729[1-(496/729)]
=~~729~~[(729 - 796)/ ~~729~~]
=233

POISSION DISTRIBUTION

Hint:

1. Poisson Distribution (Word)
2. Value of e (Given)

Question (Hint)

1. No. of phone calls record
2. No. of Accidents
3. No. of print mistakes in a book
4. No. of death
5. No. of defective product in factory

POISSION DISTRIBUTION

(1)

n ↑ Maximum trial

p ↓ Low Probability Of Occurance
(For Exp. Defective bulb)

(2)

Mean=λ=np

(3)

P(x=1) + P(x=1)

$$P(x=r)=\frac{e^{-\lambda}\lambda^{r}}{r!}$$

$$\mathrm{P}(x=r)=\frac{e^{-\lambda}\lambda^{r}}{r!}$$

Example:01

Suppose x has Poisson distribution. If P(x=2)=2/3 P(x=1), then find P(x=0)

Solution

$$P(x=2)=\frac{2}{3}P(x=1) \qquad P(x=0)\ ?$$

$$P(x=r)=\frac{e^{-\lambda}\lambda^r}{r!}$$

$$\frac{\cancel{e^{-\lambda}}\lambda^2}{2!}=\frac{2}{3}\,\frac{\cancel{e^{-\lambda}}\lambda^1}{1!}$$

$$\frac{\lambda^2}{2\mathrm{x}1}=\frac{2}{3}\,\frac{\lambda^1}{1\mathrm{x}1}$$

$$\lambda^2=\frac{4}{3}\ \lambda$$

$$3\lambda^2=4\lambda$$

$$3\lambda^2-4\lambda\ =0$$

$$\lambda(3\lambda-4)=0$$

$$\boxed{\lambda=0} \qquad 3\lambda-4=0$$

$$\boxed{\lambda=4/3}$$

$$P(x=r)=\frac{e^{-\lambda}\lambda^r}{r!}$$

$$P(x=0)=\frac{e^{-\lambda}\lambda^0}{0!}$$

$$P(x=0)=\frac{e^{-4/3}(4/3)^0}{1}$$

$$P(x=0)=e^{-4/3}$$

$$P(x=0)=0.2636$$

Example:02

The number of emergency admission each day to hospital is found to have Poisson distribution with mean 4.Find the probability that on a particular day there will be no emergency admission.

$$\lambda = 4$$

$$P(x = 0)$$

$$P(x = 0) = \frac{e^{-\lambda}\lambda^{0}}{0!}$$

$$P(x = 0) = \frac{e^{-4}(4)^{0}}{0!}$$

$$P(x = 0) = \frac{e^{-4}\, X\, 1}{1}$$

$$P(x = 0) = e^{-4}$$

$$P(x = 0) = \boxed{0.0183}$$

Example:03

Five coins are tossed 3200 times. What is the probability of getting 5 heads Two times?

p(x=2)

5	5	5	5	- - - -

3200 Times

n NO. of Trial (3200 Times)

p

$$\lambda = np$$

$$p = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$p = \left(\frac{1}{2}\right)^5$$

$$\lambda = np$$

$$\lambda = (3200)\left(\frac{1}{2}\right)^5$$

$$\lambda = (\overset{100}{\cancel{3200}})/\cancel{32}$$

$$\lambda = 100$$

$$P(x = 2) = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$P(x = 2) = \frac{e^{-100}(100)^2}{2x1}$$

$$P(x = 2) = \frac{e^{-100}\ X\ \overset{5000}{\cancel{10000}}}{\cancel{2}}$$

$$P(x = 2) = e^{-100}\ X\ 5000$$

$$P(x = 2) = 5000.e^{-100}$$

Example:04

Assume that the probability that a bomb dropped from an aeroplane will strike certain target is 1/5.If 6 bombs are dropped, find the probability that

a) Exactly 2 will strike the target.

b) At least 2 will strike the target.

Solution:

(a)

Solution:

(a)

$$p = \frac{1}{5}$$

$$n = 6$$

$$\lambda = nP$$

$$\lambda = 6X(\frac{1}{5})$$

$$\lambda = 1.2$$

$$\mathrm{P}(x = r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

$$\mathrm{P}(x = 2) = \frac{e^{-\lambda}\lambda^{2}}{2!}$$

$$\mathrm{P}(x = 2) = \frac{e^{-1.2}(1.2)^{2}}{2\text{x}1}$$

$$\mathrm{P}(x = 2) = \frac{0.3011\ X\ 1.44}{2}$$

$$\mathrm{P}(x = 2) = 0.2167$$

(b)

$$P(x \geq 2)$$

2,3,4,5,6

$$P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6)$$

OR

$$1 - [P(x = 0) + P(x = 1)]$$

$$1 - \left[\frac{e^{-1.2}(1.2)^0}{0!} + \frac{e^{-1.2}(1.2)^1}{1!}\right]$$

$$1 - \left[\frac{e^{-1.2} X\, 1}{1} + \frac{e^{-1.2}(1.2)}{1}\right]$$

$$1 - e^{-1.2}[1+1.2]$$

$$P(x \geq 2 = 1 - (0.6626))$$

$$P(x \geq 2) = 0.3373$$

Example:05

A car firm has two cars which it hires out day to day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5.Calculate the proportion of days,

1. **On which there is no demand**
2. **On which demand is refused**

X= Demand

(1)

$$\lambda = 1 \cdot 5$$

$$P(x = 0)$$

$$\mathrm{P}(x = r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

$$\mathrm{P}(x = 0) = \frac{e^{-1.5}(1.5)^{0}}{0!}$$

$$\mathrm{P}(x = 0) = \frac{e^{-1.5}X1}{1}$$

$$\mathrm{P}(x = 0) = e^{-1.5}$$

$$\mathrm{P}(x = 0) = 0.2231$$

(2) $P(x > 2)$

$$1 - [P(x = 0) + P(x = 1) + P(x = 2)]$$

$$= 1 - \left[\frac{e^{-1.5}(1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$$

$$= 1 - e^{-1.5}\left[1 + 1.5 + \frac{(1.5)^2}{2}\right]$$

$$= 1 - e^{-1.5}[3.625]$$

$$P(x > 2) = 0.1911$$

NORMAL DISTRIBUTION

$p(0 \leq x \leq 2)$ **Continuous**

1. Formula
2. Table
3. Curve

Probability Density Formula for Normal Distribution

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$-\infty < x < +\infty$$

$$-\infty < \mu < +\infty$$

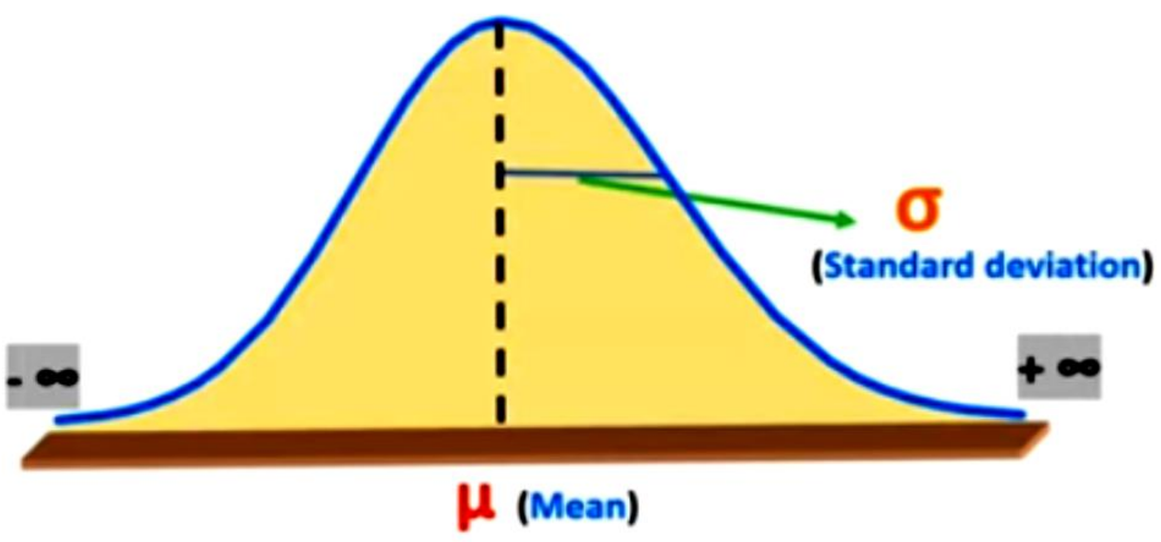
$$\sigma > 0$$

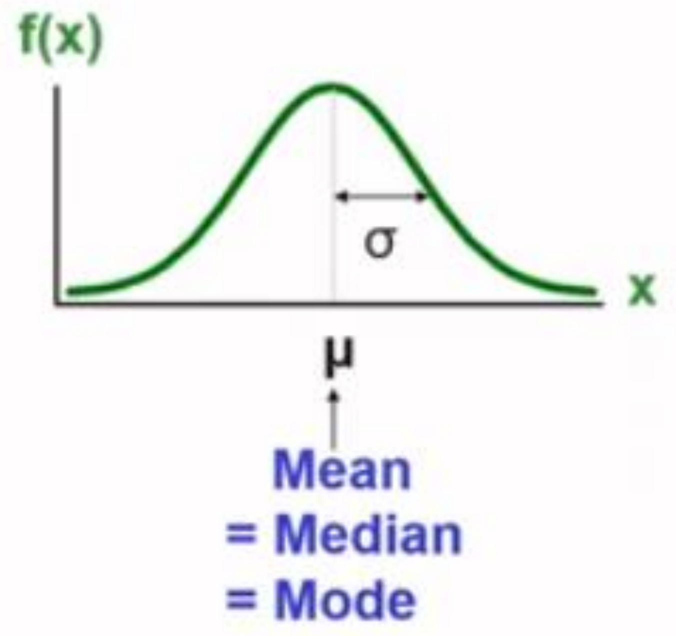
σ = Standard Deviation

μ = Mean

Introduction to Normal Distribution

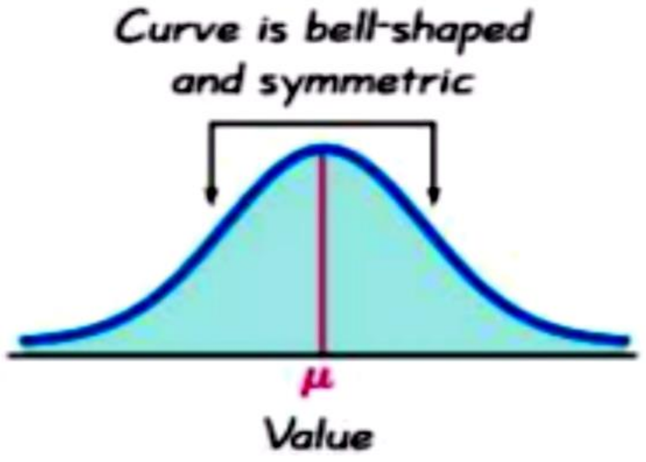
- Normal Distribution is a Symmetric, Bell-shaped distribution.
- Symmetrical about the mean (μ). Centerline represent the mean of distribution.
- Extends from - ∞ to +∞ .
- Property of normal distribution is that area under the curve.
- Total area under the curve is 1.

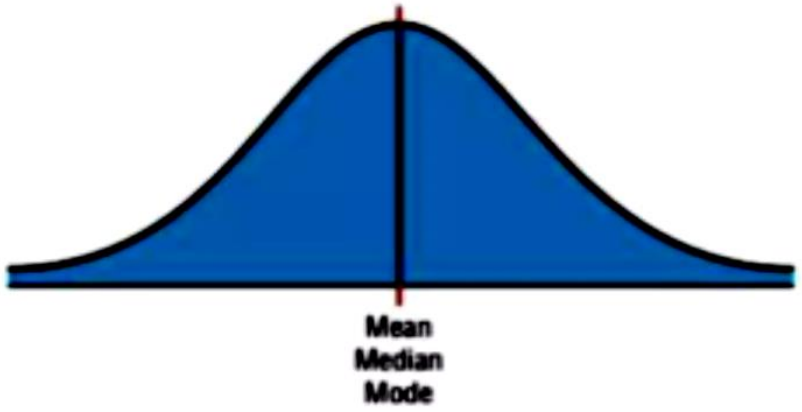




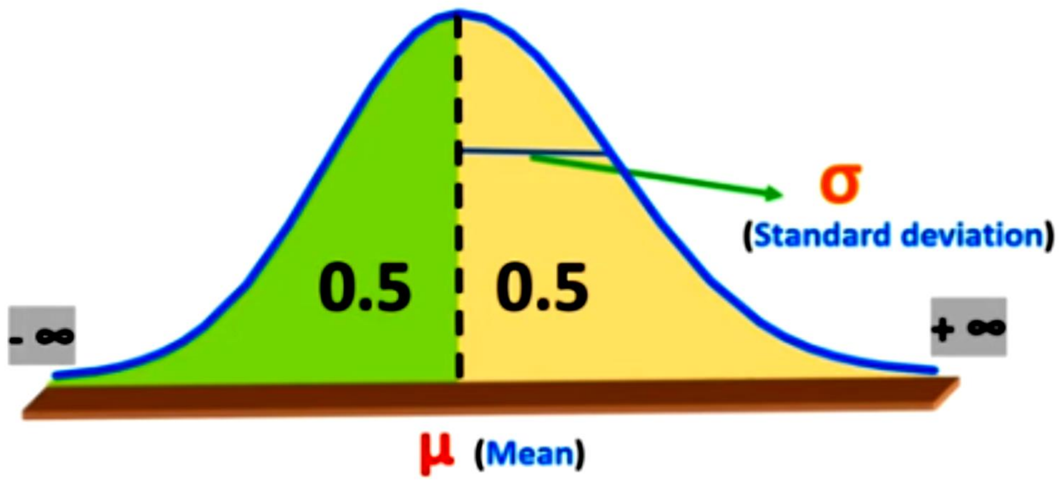
Properties of the Normal Distribution

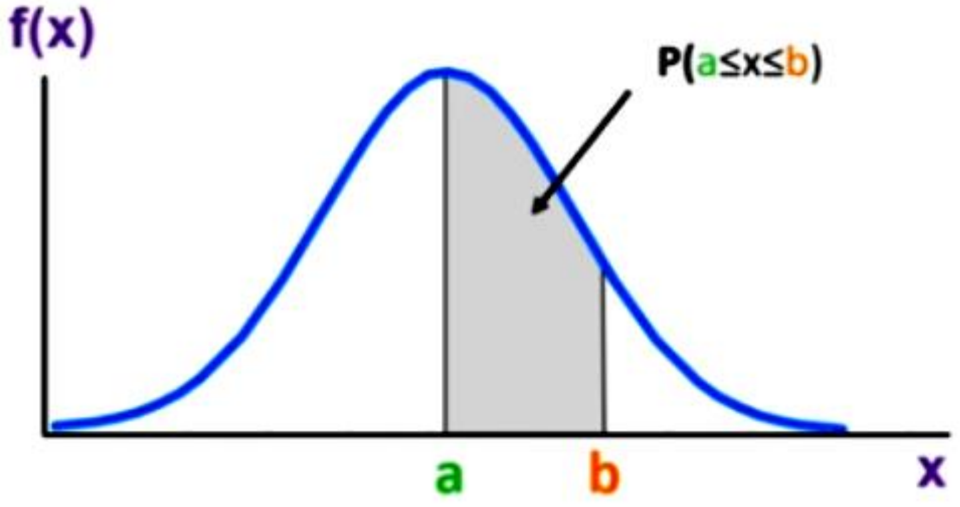
1. The distribution curve is **bell-shaped**.
2. The curve is symmetric about its center, the mean.
3. The mean, the median, and the mode coincide at the center.
4. The width of the curve is determined by the standard deviation of the distribution.



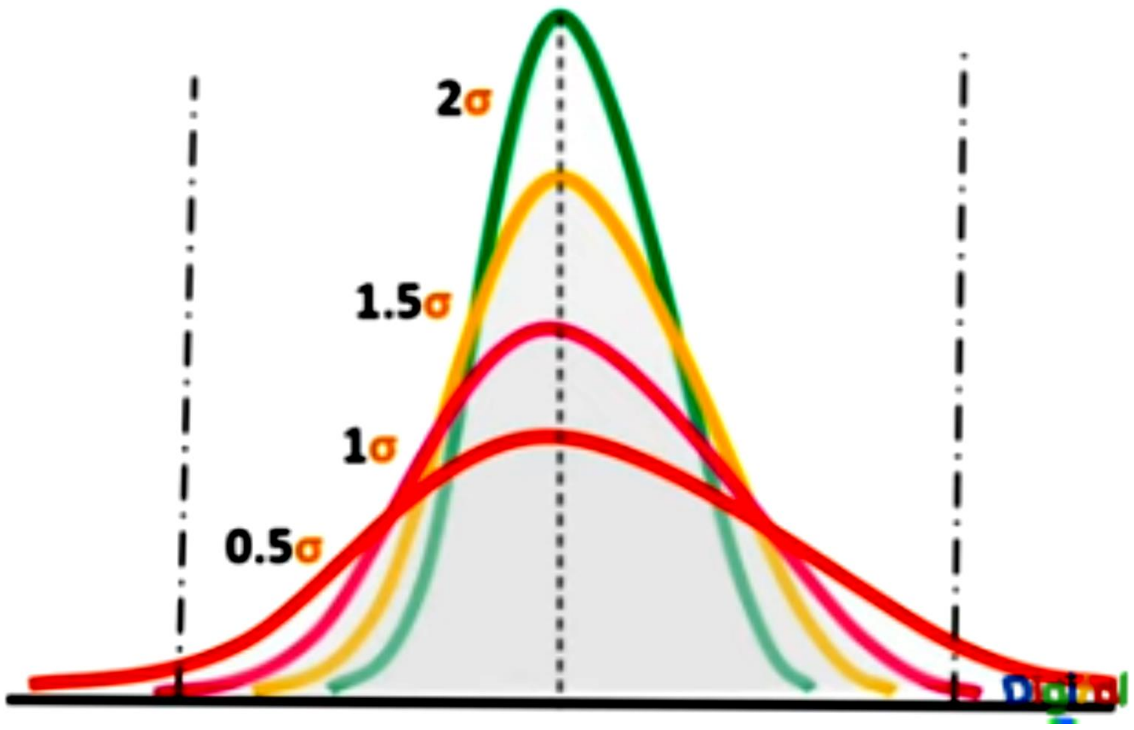


- Total area under the curve is 1. Half of this curve is 0.5
- Shaded area under the curve is the probability that X is between a and b

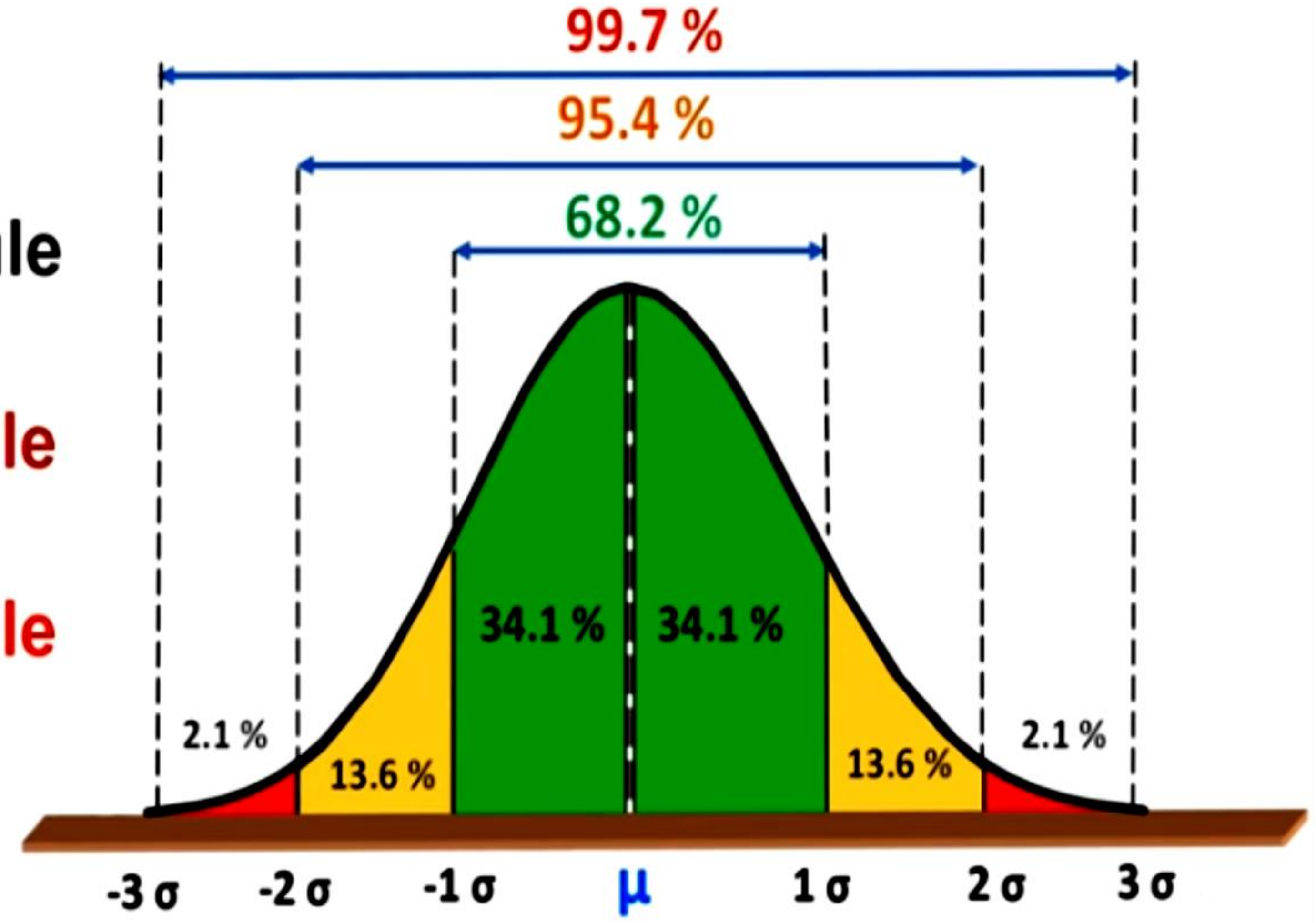




- The Shape of the curve depends upon value of Standard deviation σ.



3 Sigma σ Rule
OR
Empirical Rule
OR
68-95-99.7 rule



What is Standard Normal Distribution ?

The Standard Normal Distribution

The STANDARD NORMAL DISTRIBUTION of a random variable is a normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$.

The letter Z is used to denote the standard normal random variable. The specific value z of the r.v. Z is called the **z-score**.

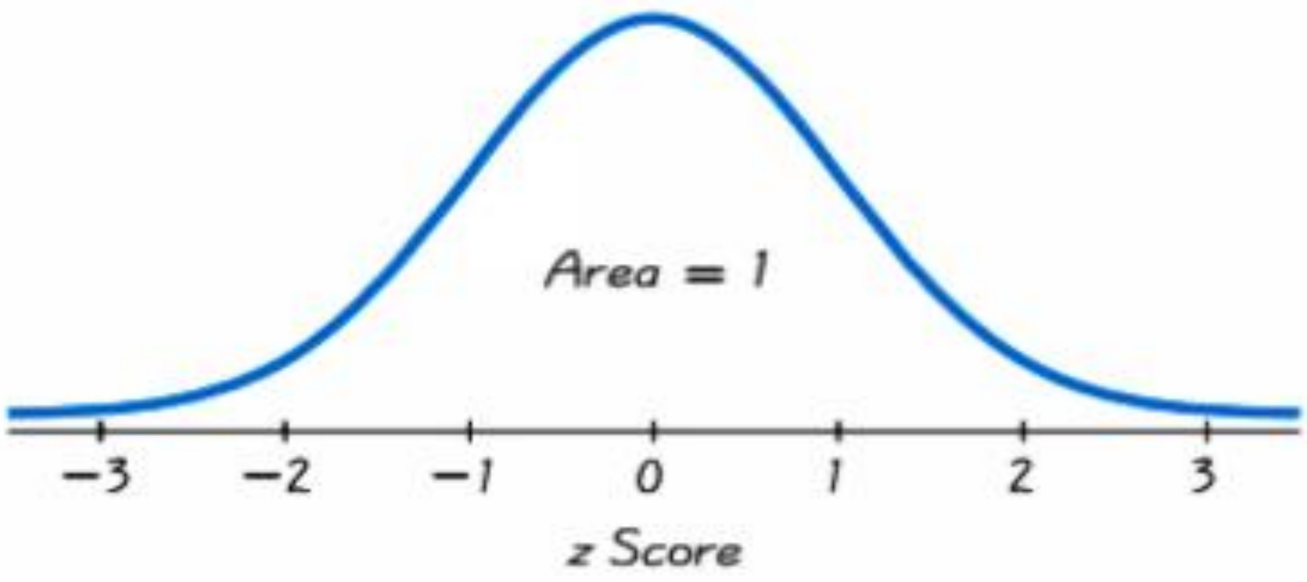
The Standard Normal Distribution

The probability function of a random variable Z with a standard normal distribution by is given by

$$y = p(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

The Standard Normal Distribution

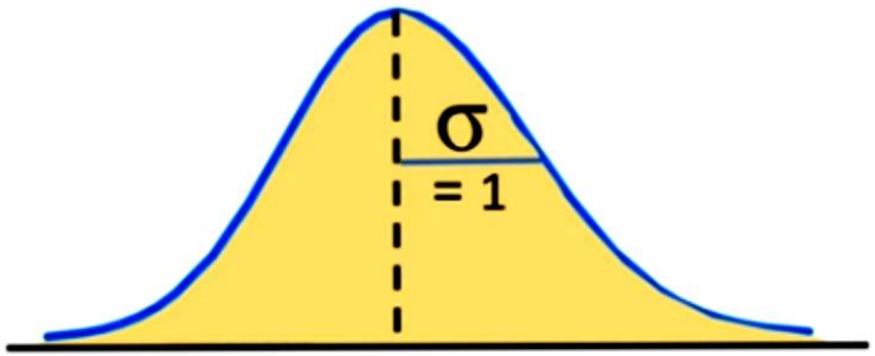
The graph of the standard normal distribution



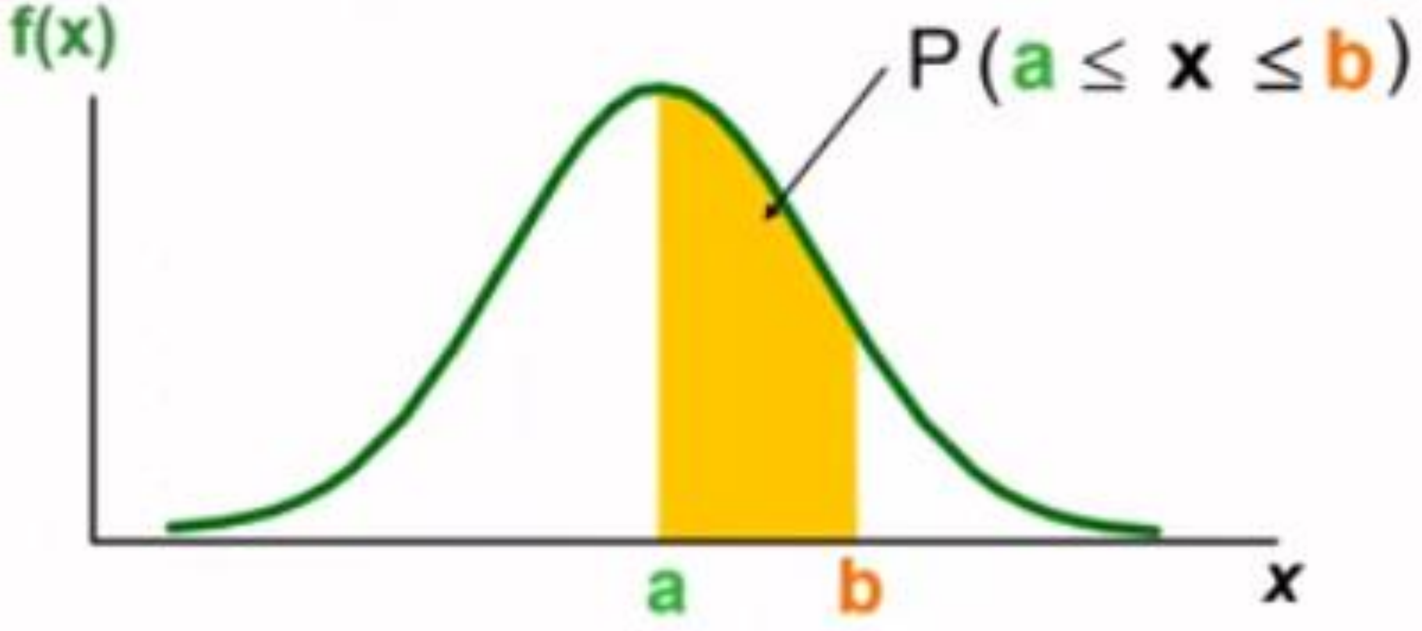
- **Normal Distribution curve can be transformed to Standard normal distribution.**

$$Z = \frac{X - \mu}{\sigma}$$

X = random variable
σ = Standard deviation
μ = Mean
Z = Number of standard deviations from x to the mean



Probability is measured by the area under the curve



Example:01

The random variable x is normally distributed with mean 9 and standard Deviation 3.Find the probability $x \geq 15$, $x \leq 15$, $0 \leq x \leq 9$

Solution

$x \geq 15$

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{15-9}{3}$$

$$z = \frac{6}{3}$$

$z = 2$

$P(x \geq 15)$

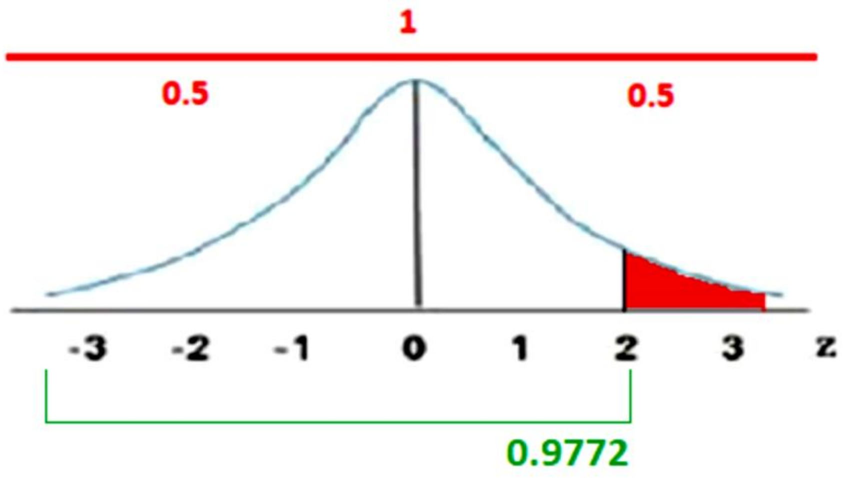
$P(z \geq 2)$

From Table=0.9772

$P(z \geq 2) = 1 - P(z \leq 2)$

$P(z \geq 2) = 1 - 0.9772$

$P(z \geq 2) = 0.0228$



$P(x \leq 15)$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{15 - 9}{3}$$

$$z = \frac{6}{3}$$

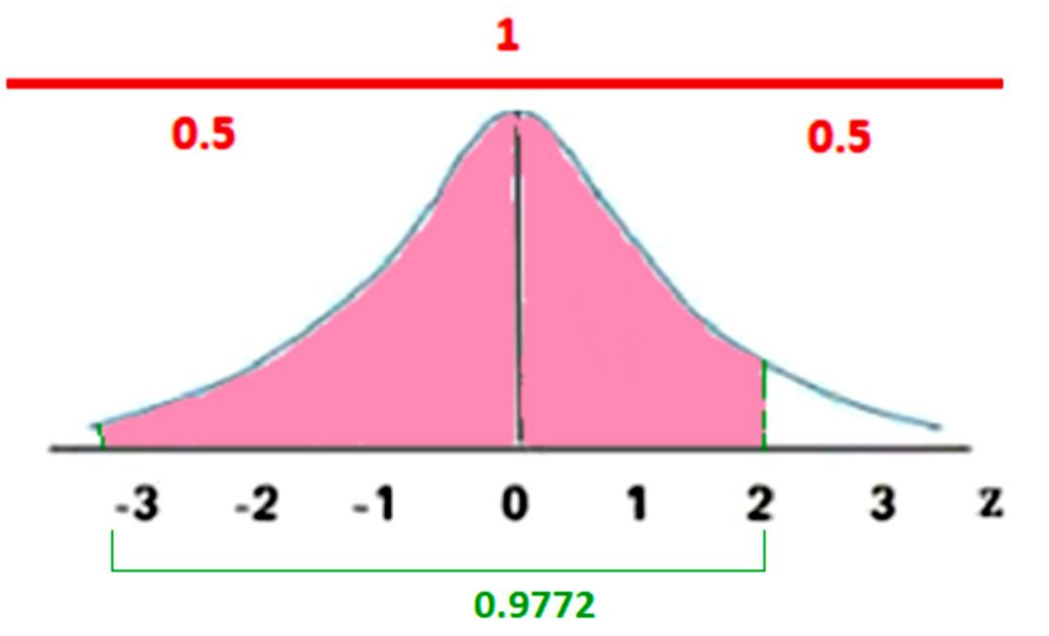
$z = 2$

$P(x \leq 15)$

$P(z \leq 2)$

$P(z \leq 2)$=0.9772

From Table=0.9772



$0 \leq x \leq 9$

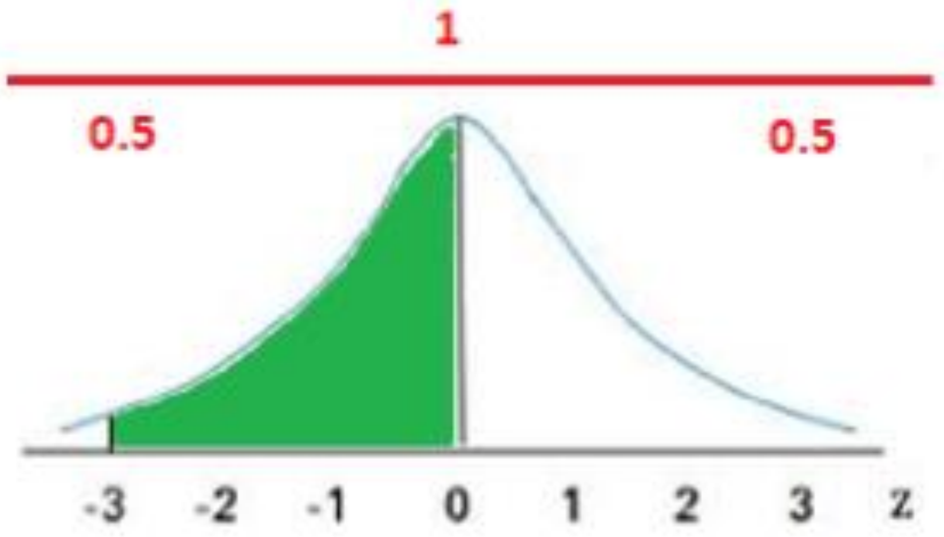
From Table= P(-3)=0.0013 & P(0)=0.5000

$$z = \frac{x - \mu}{\sigma} \qquad z = \frac{x - \mu}{\sigma}$$

$$z = \frac{0 - 9}{3} \qquad z = \frac{9 - 9}{3}$$

z =-3 $\qquad$ z =0

P($-3 \leq z \leq 0$)



P($-3 \leq z \leq 0$) $= 0.5000 - 0.0013 = 0.4987$

Example:02

Let x denote the number of scores in a test. If X is normally distributed with mean 100 and standard deviation 15.Find the probability that X does not exceed 130.

Solution:

$P(x < 130)$ $\sigma = 15$ $\mu = 100$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{130 - 100}{15}$$

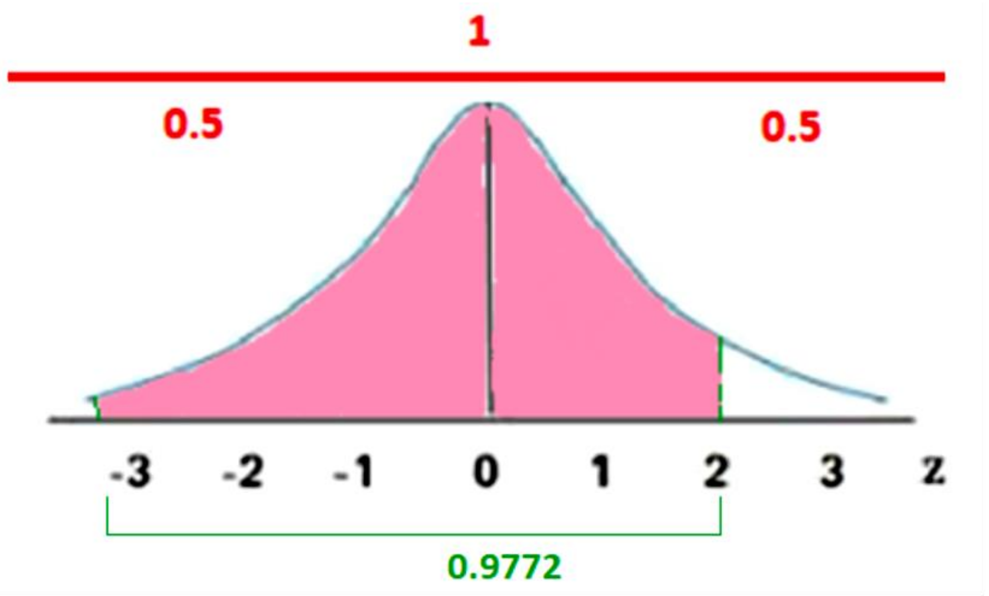
$$z = \frac{30}{15}$$

$z = 2$

$P(z \leq 2)$

From Table=0.9772

$P(z \leq 2)$=0.9772



Example:03

The average daily sale of 500 branch offices was 150 thousand and the standard deviation 15 thousand. Assuming the distribution to be normal

Indicate how many branches have sales between

1) **120 thousand and 145 thousand**
2) **140 thousand and 165 thousand**

P(120$\leq x \leq 145$)

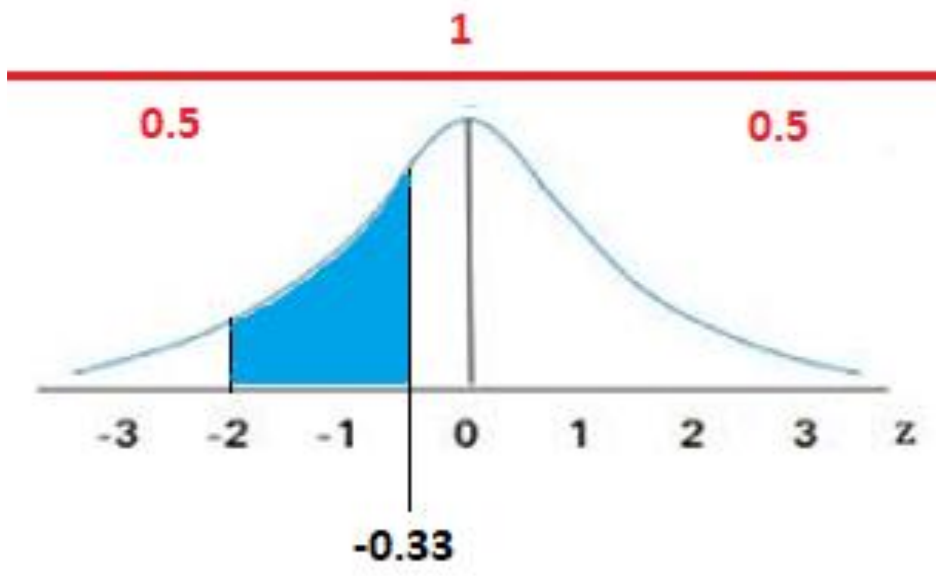
$\mu = 150 \qquad \sigma = 15$

$$z = \frac{x-\mu}{\sigma} \qquad z = \frac{x-\mu}{\sigma}$$

$$z = \frac{120-150}{15} \qquad z = \frac{145-150}{15}$$

$$z = \frac{-30}{15} \qquad z = \frac{-5}{15}$$

$$z = -2 \qquad z = -0.33$$



P(-2$\leq z \leq -0.33$)= P(-0.33) – P(-2)

P(-2$\leq z \leq -0.33) = 0.3707 - 0.0228$ From Table

$= 0.3479$

P($140 \le x \le 165$)

$\sigma = 15$ $\mu = 150$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{140 - 150}{15}$$

$$z = \frac{-10}{15}$$

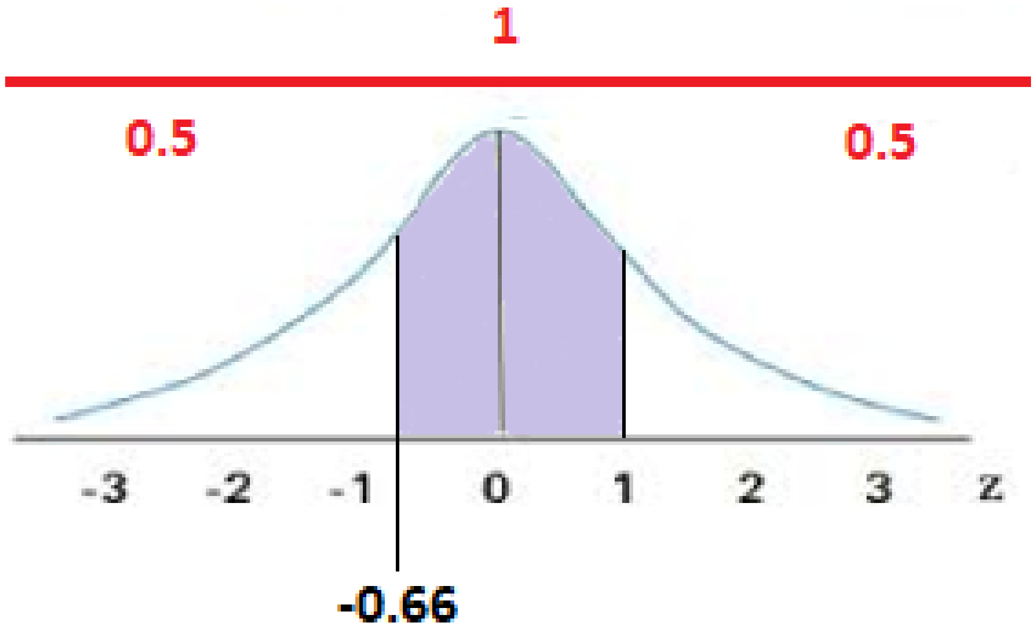
z =-0.66

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{165 - 150}{15}$$

$$z = \frac{15}{15}$$

$$z = 1$$



P(-0.66$\le z \le$ 1)= P(1) – P(-0.66)

$= 0.8413$ – 0.2546 From Table

=0.5867

Example:04

Scores on an exam are normally distributed with a mean of 65 and a standard deviation of 9.Find the percent of the scores.

1) **Less than 54**
2) **At least 80**
3) **Between 70 and 86**

(1)

$$P(x < 54)$$

$\mu = 65 \qquad \sigma = 9$

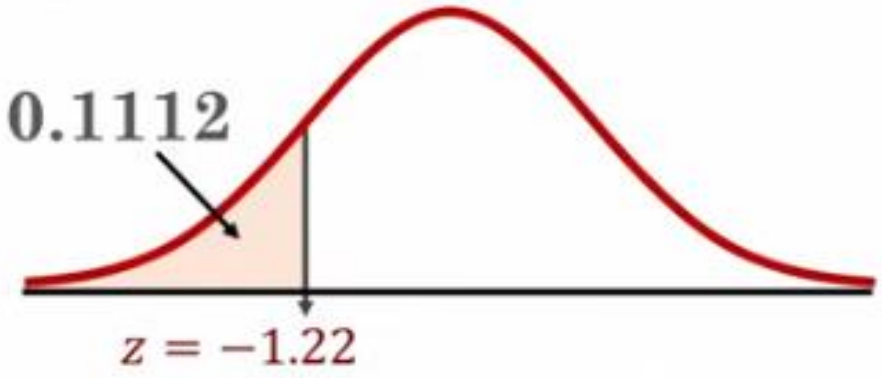
$$x = 54$$

$$z = \frac{x - \mu}{\sigma} = \frac{54 - 65}{9}$$

$$z = -1.22$$

$$P(x < 54) = P(z < -1.22) = 0.1112$$

11.12%



(2)

$$P(x \geq 80) = P(x > 80)$$

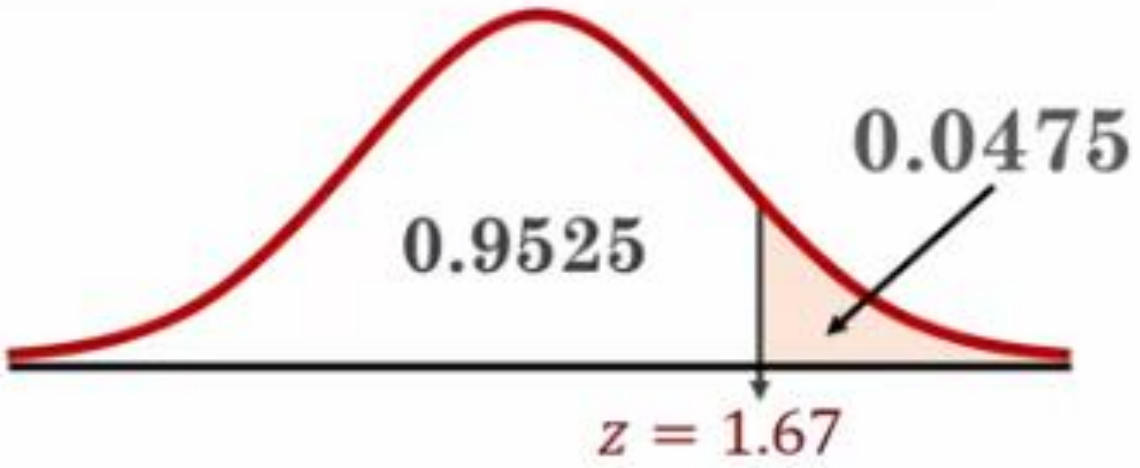
$$x = 80$$

$$z = \frac{x - \mu}{\sigma} = \frac{80 - 65}{9}$$

$$z = 1.67$$

$$P(x > 80) = P(z > 1.67)$$
$$= 1 - P(z < 1.67)$$
$$= 1 - 0.9525$$
$$= 0.0475$$

4.75%



(3)

$$P(70 < x < 86)$$

$x = 70$

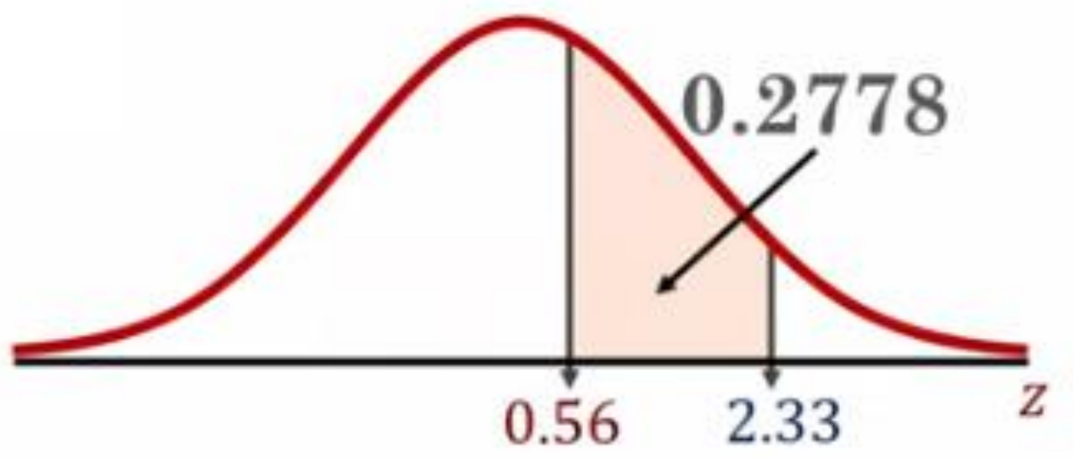
$$z = \frac{x-\mu}{\sigma} = \frac{70-65}{9} = 0.56$$

$x = 86$

$$z = \frac{x-\mu}{\sigma} = \frac{86-65}{9} = 2.33$$

$$\begin{aligned} &P(70 < x < 86) \\ &= P(0.56 < z < 2.33) \\ &= P(z < 2.33) - P(z < 0.56) \\ &= 0.9901 - 0.7123 \\ &= 0.2778 \end{aligned}$$

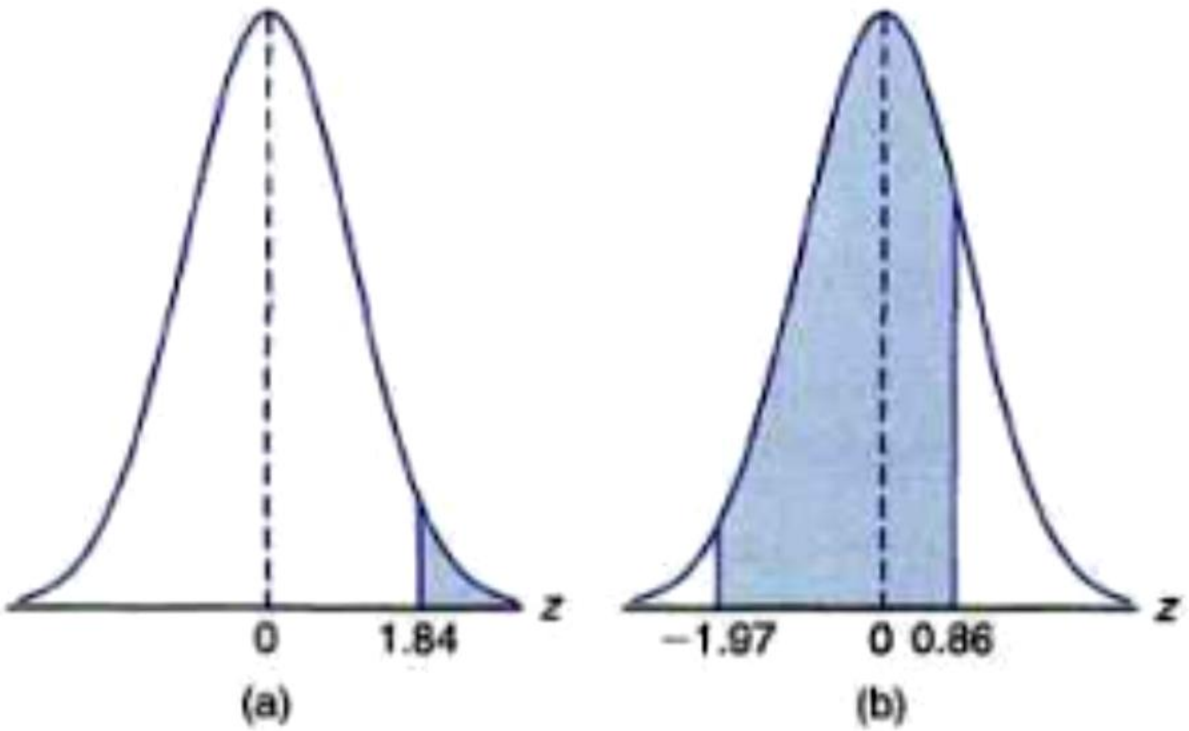
27.78%



Example :I Given a standard normal distribution, find the area under the curve that lies

(a) to the right of $z = 1.84$, and

(b) between $z = -1.97$ and $z = 0.86$.



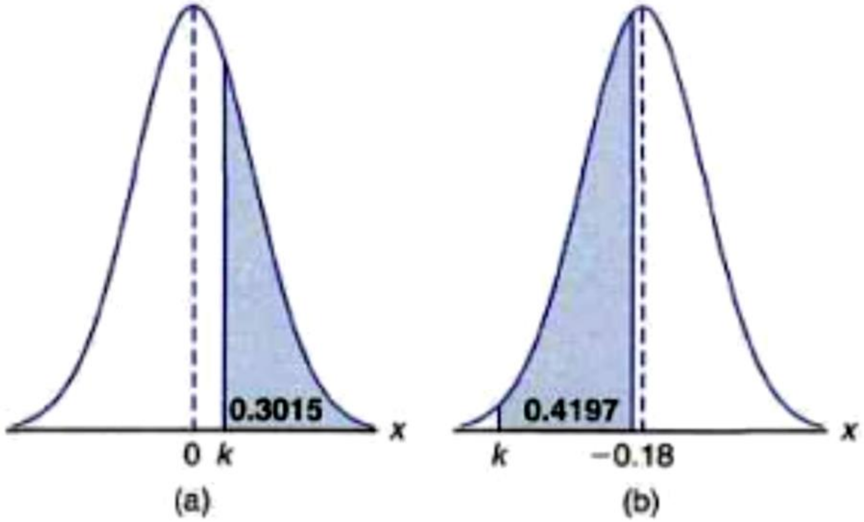
Solution: (a) The area in Figure 6.9(a) to the right of $z = 1.84$ is equal to 1 minus the area in Table A.3 to the left of $Z = 1.84$, namely, $1 - 0.9671 = 0.0329$.

(b) The area in Figure 6.9(b) between $z = -1.97$ and $z = 0.86$ is equal to the area to the left of $z = 0.86$ minus the area to the left of $z = -1.97$. Table A.3 we find the desired area to be $0.8051 - 0.0244 = 0.7807$.

Example :! Given a standard normal distribution, find the value of k such that

(a) $P(Z > k) = 0.3015$, and

(b) $P(k < Z < -0.18) = 0.4197$.



Solution: (a) In Figure 6.10(a) we see that the A: value leaving an area of 0.3015 to the right must then leave an area of 0.6985 to the left. From Table A.3 it follows that $k = 0.52$.

(b) From Table A.3 we note that the total area to the left of —0.18 is equal to 0.4286. In Figure 6.10(b) we see that the area between k and -0.18 is 0.4197 so that the area to the left of k must be $0.4286 - 0.4197 = 0.0089$. Hence, from Table A.3, we have $k = -2.37$.

Example : Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value between 45 and 62.

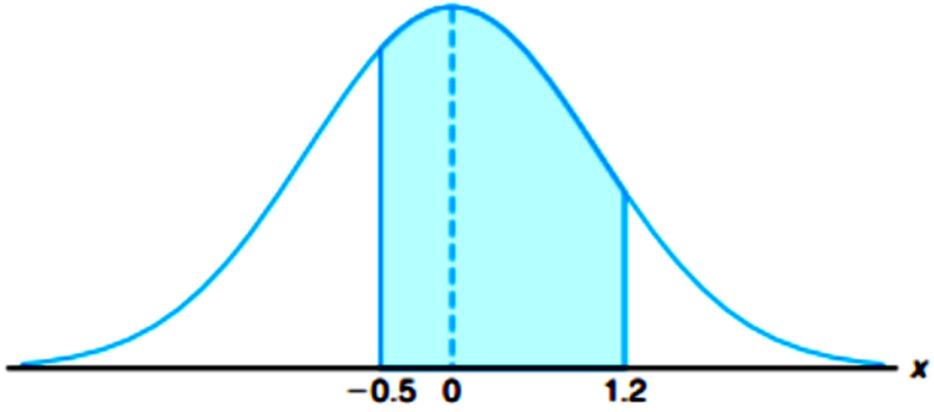


Figure 6.11: Area for Example 6.4.

***Solution*:** The z values corresponding to $x_1 = 45$ and $x_2 = 62$ are

$$z_1 = \frac{45 - 50}{10} = -0.5 \text{ and } z_2 = \frac{62 - 50}{10} = 1.2.$$

Therefore,

$$P(45 < X < 62) = P(-0.5 < Z < 1.2).$$

$P(-0.5 < Z < 1.2)$ is shown by the area of the shaded region in Figure 6.11. This area may be found by subtracting the area to the left of the ordinate $z = -0.5$ from the entire area to the left of $z = 1.2$. Using Table A.3, we have

$$\begin{aligned} P(45 < X < 62) &= P(-0.5 < Z < 1.2) = P(Z < 1.2) - P(Z < -0.5) \\ &= 0.8849 - 0.3085 = 0.5764. \end{aligned}$$

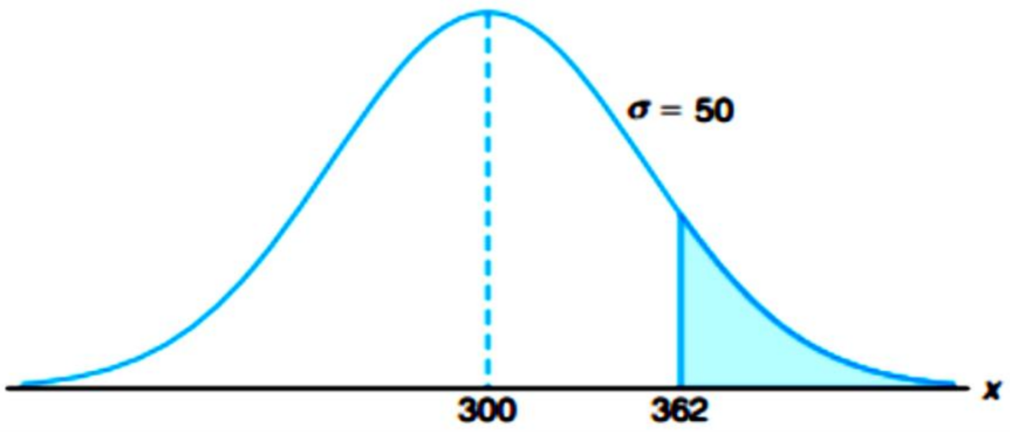
Example Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362.

Solution: The normal probability distribution with the desired area shaded is shown in Figure 6.12. To find $P(X > 362)$, we need to evaluate the area under the normal curve to the right of $x = 362$. This can be done by transforming $x = 362$ to the corresponding z value, obtaining the area to the left of z from Table A.3, and then subtracting this area from 1. We find that

$$z = \frac{362 - 300}{50} = 1.24.$$

Hence,

$$P(X > 362) = P(Z > 1.24) = 1 - P(Z < 1.24) = 1 - 0.8925 = 0.1075.$$



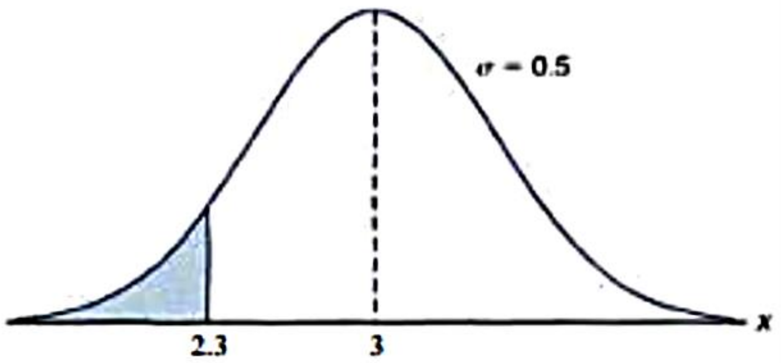
Example : A certain type of storage battery lasts, on average, 3.0 years with a standard deviation of 0.5 year. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 2.3 years.

Solution: First construct a diagram such as Figure 6.14, showing the given distribution of battery lives and the desired area. To find the $P(X < 2.3)$, we need to evaluate the area under the normal curve to the left of 2.3. This is accomplished by finding the area to the left of the corresponding z value. Hence we find that

$$z = \frac{2.3 - 3}{0.5} = -1.4,$$

and then using Table A.3 we have

$$P(X < 2.3) = P(Z < -1.4) = 0.0808.$$



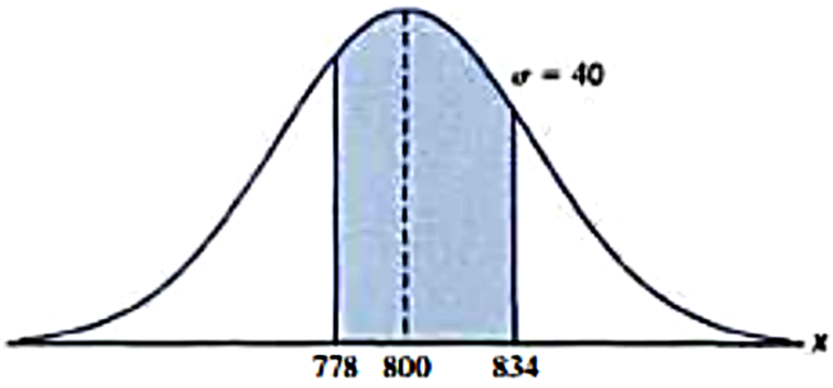
Example : An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

Solution: The distribution of light bulbs is illustrated by Figure 6.15. The z values corresponding to $x_1 = 778$ and $x_2 = 834$ are

$$z_1 = \frac{778 - 800}{40} = -0.55. \quad \text{and} \quad z_2 = \frac{834 - 800}{40} = 0.85.$$

Hence

$$P(778 < X < 834) = P(-0.55 < Z < 0.85) = P(Z < 0.85) - P(Z < -0.55)$$
$$= 0.8023 - 0.2912 = 0.5111.$$



Using the Normal Curve in Reverse

$$z = \frac{x - \mu}{\sigma} \quad \text{to give} \quad x = \sigma z + \mu.$$

Example : Given a normal distribution with $\mu = 40$ and $\sigma = 6$, find the value of x that has

(a) 45% of the area to the left and

(b) 14% of the area to the right.

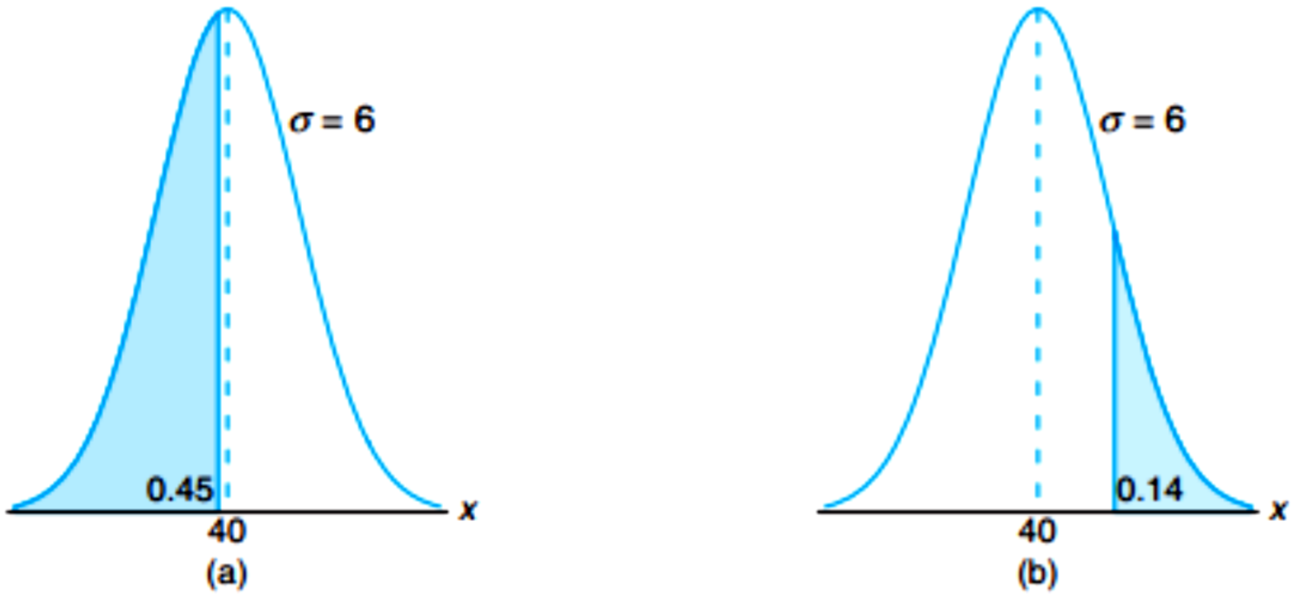


Figure 6.13: Areas for Example 6.6.

Solution: (a) An area of 0.45 to the left of the desired x value is shaded in Figure 6.13(a). We require a z value that leaves an area of 0.45 to the left. From Table A.3 we find $P(Z < -0.13) = 0.45$, so the desired z value is -0.13. Hence,

$$x = (6)(-0.13) + 40 = 39.22.$$

(b) In Figure 6.13(b), we shade an area equal to 0.14 to the right of the desired x value. This time we require a z value that leaves 0.14 of the area to the right and hence an area of 0.86 to the left. Again, from Table A.3, we find $P(Z < 1.08) = 0.86$, so the desired z value is 1.08 and

$$x = (6)(1.08) + 40 = 46.48.$$

Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.03.59	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0591	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5138	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7701	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9G78	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998