COURSE CODE: ASC201

COUSE TITLE: PROBABILITY & STATISTICS

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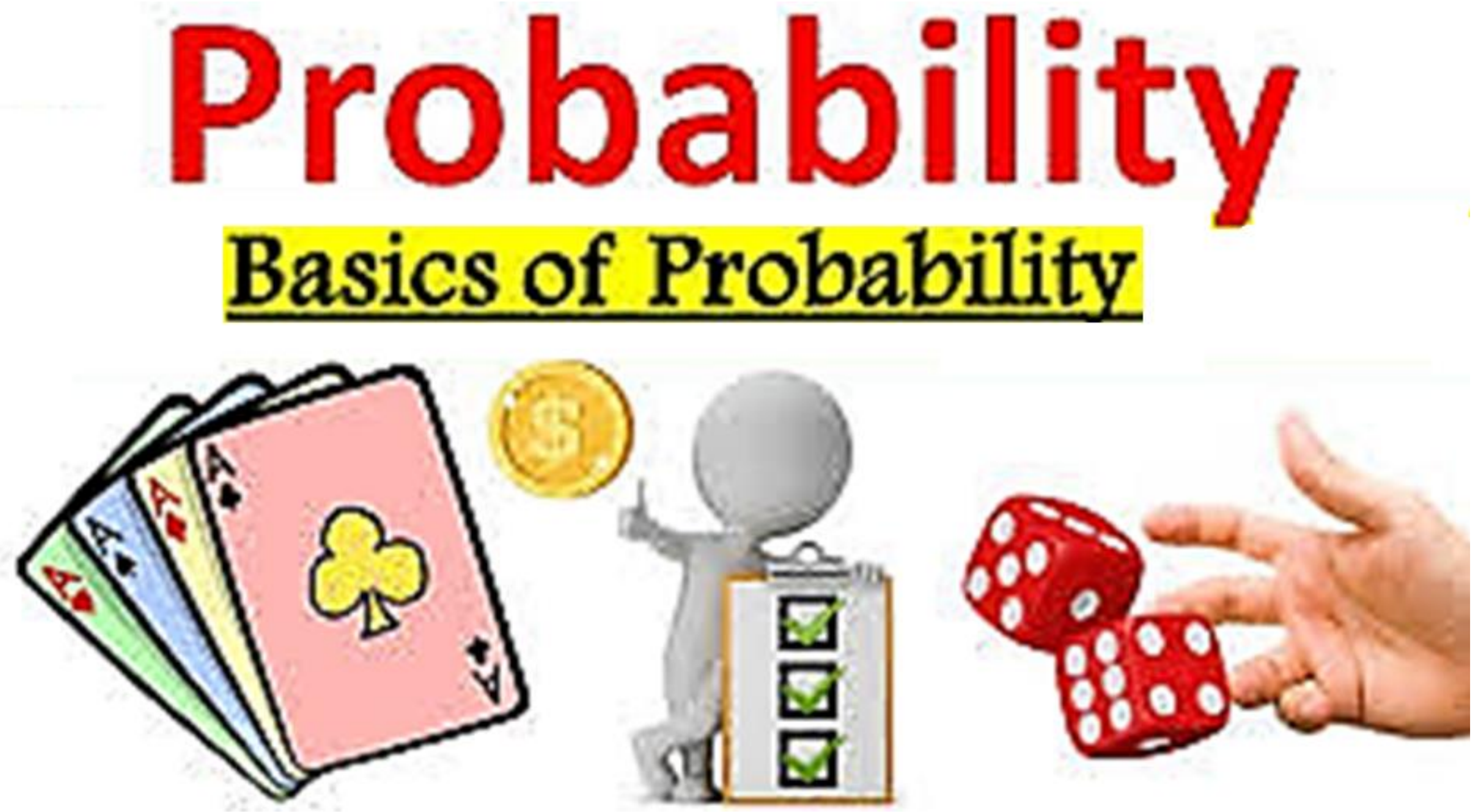
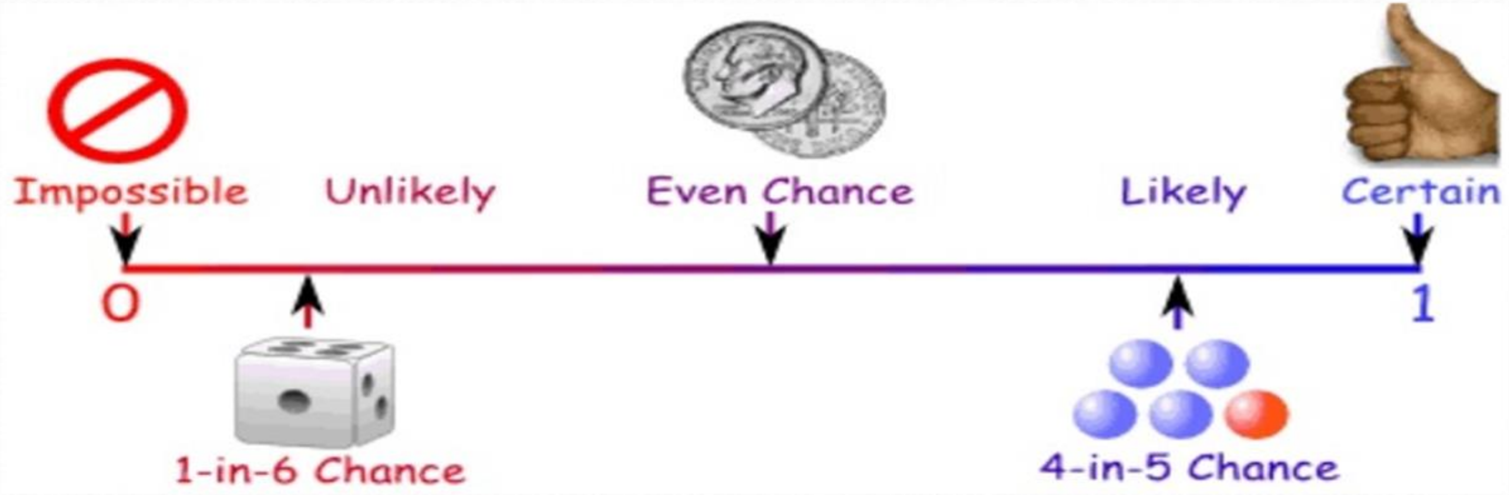

Probability
Basics of Probability

What is Probability?

Probability is the chance that something will happen - how likely it is that some event will happen.

Sometimes you can measure a probability with a number like "10% chance of rain", or you can use words such as impossible, unlikely, possible, even chance, likely and certain.

Example: "It is unlikely to rain tomorrow".



Definitions

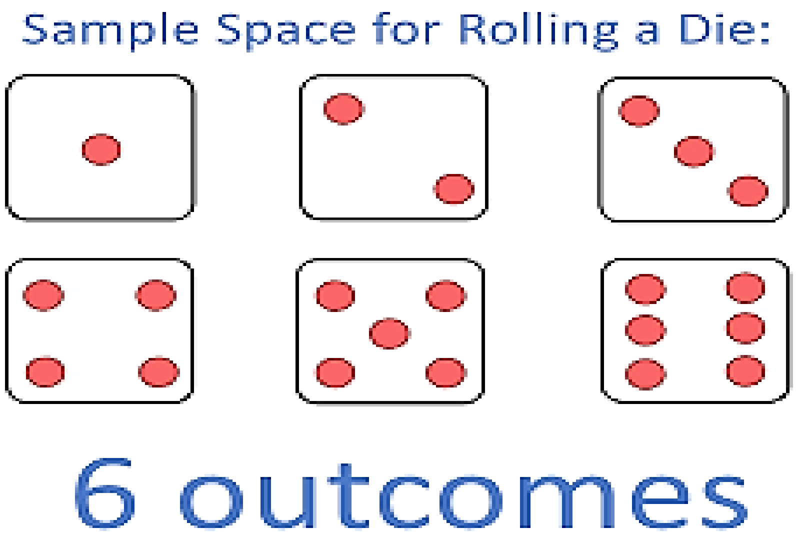
- **Experiment:** When a trial is conducted to obtain some statistical information.
  - Ex: Tossing of a fair coin
  - Ex: Rolling of a die

- **Event:** The possible outcomes of a trial, denoted by capital letters A,B,C etc.
  - Ex: In tossing of a coin, the outcomes – head or tail are events.

- **Equally Likely Events:** The events are said to be equally likely if the chance of happening of each event is equal or same.
  - Ex: In tossing of a coin, the events H or T are equally likely.

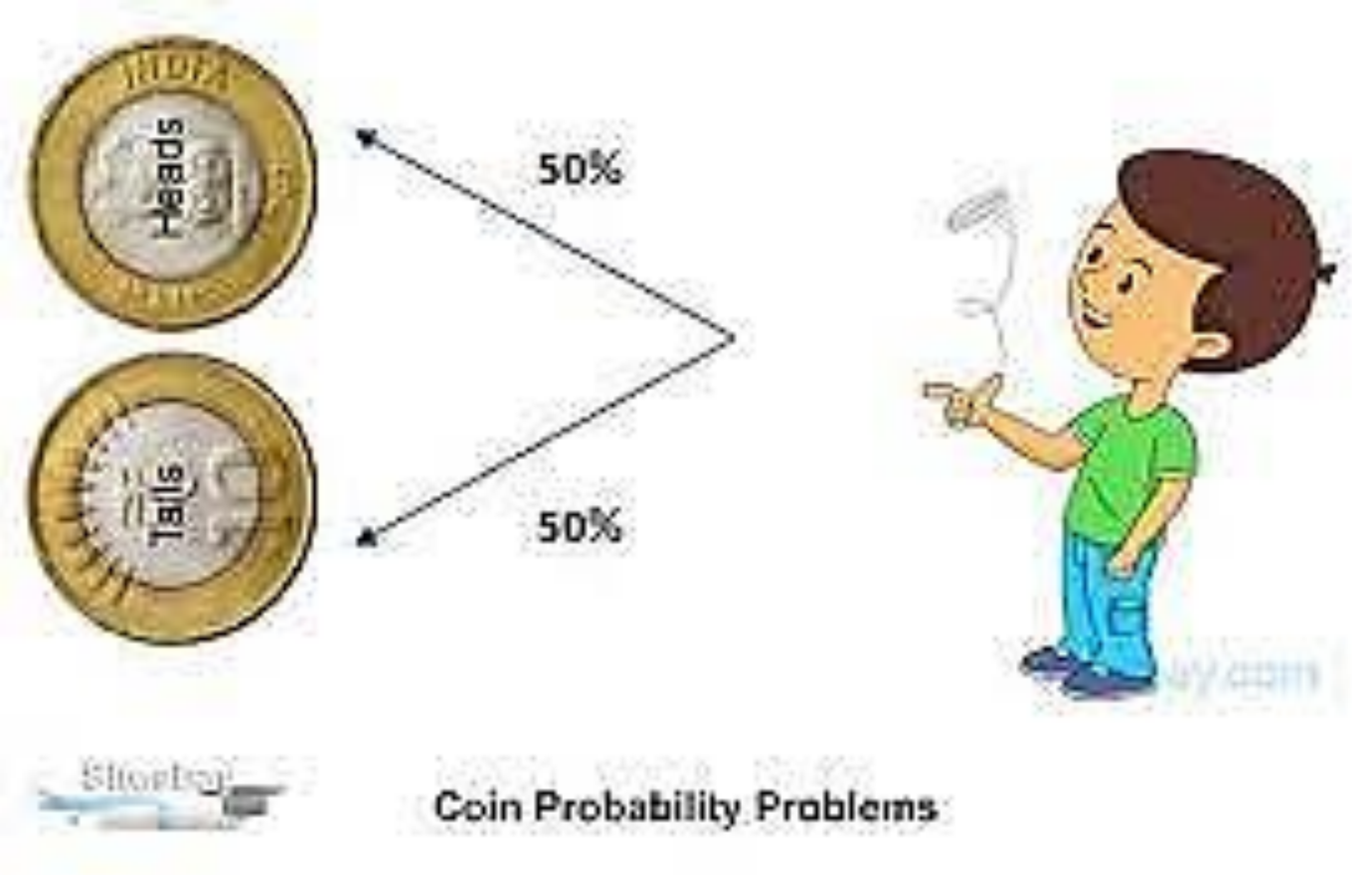
Definitions

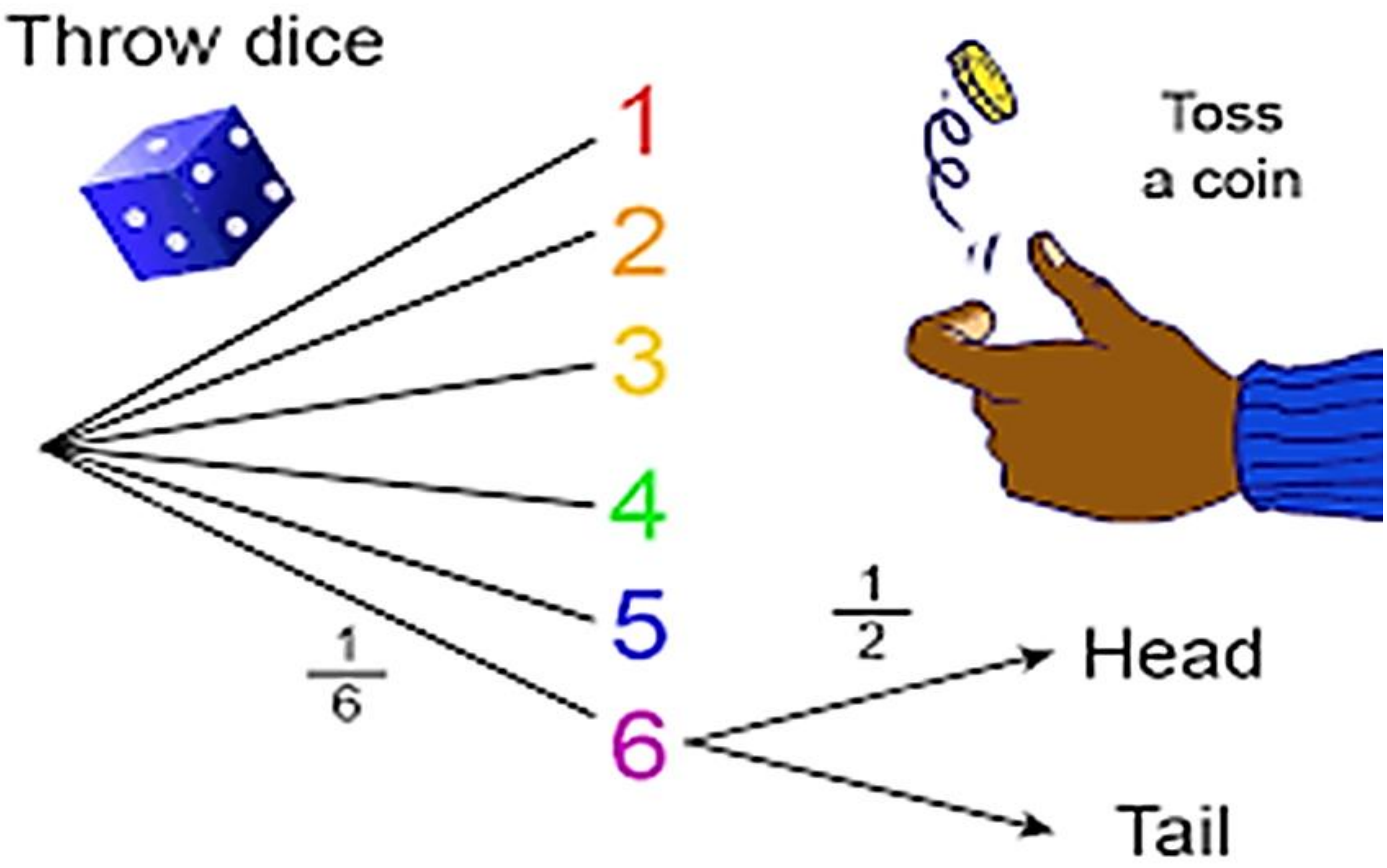
- **Sample Space:** The total number of possible outcomes taken together. These are also called exhaustive events.
  - Ex: The set of H,T in tossing a coin
  - Ex: The set of 1,2,3,4,5,6 in rolling a die

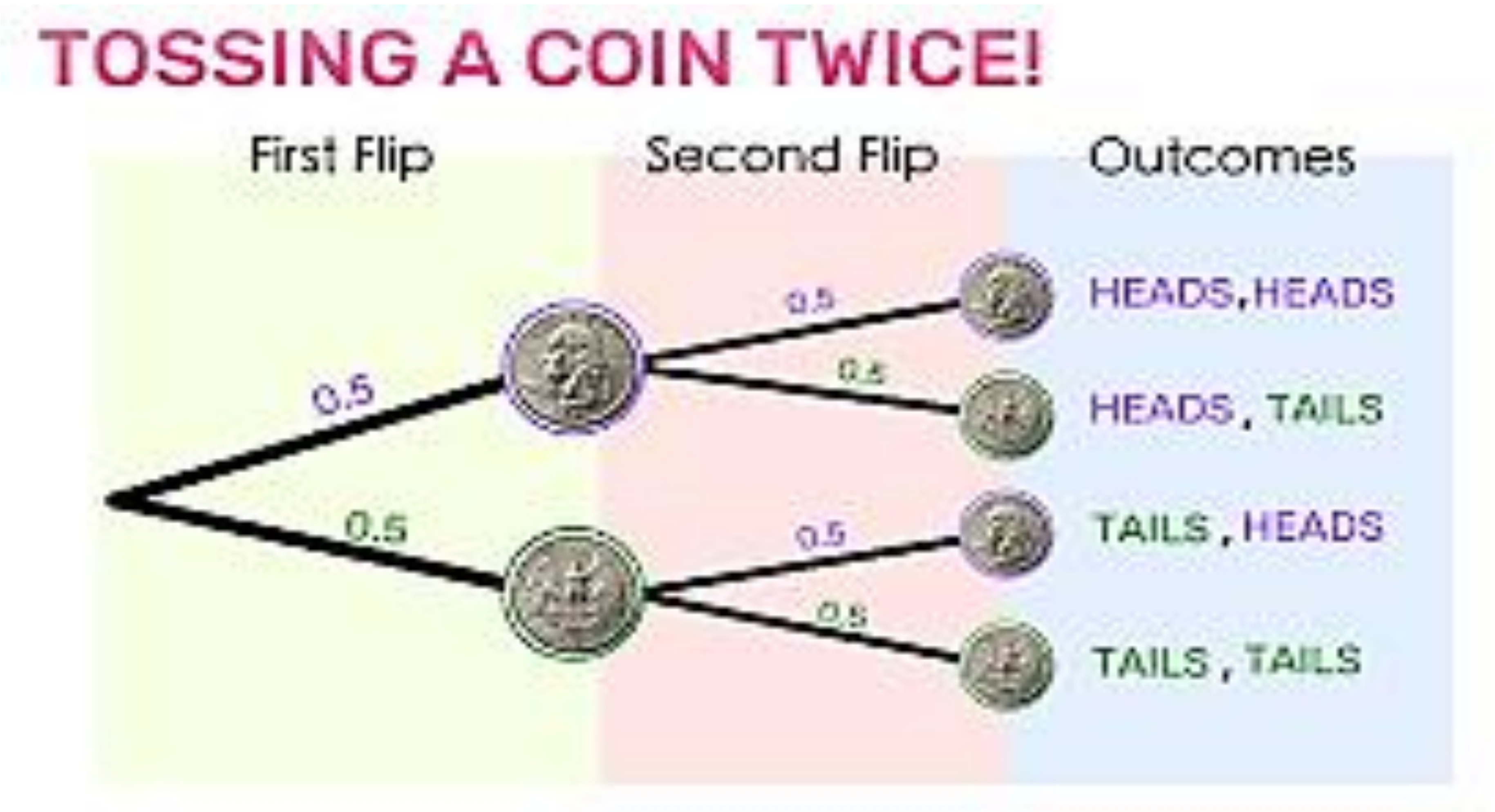


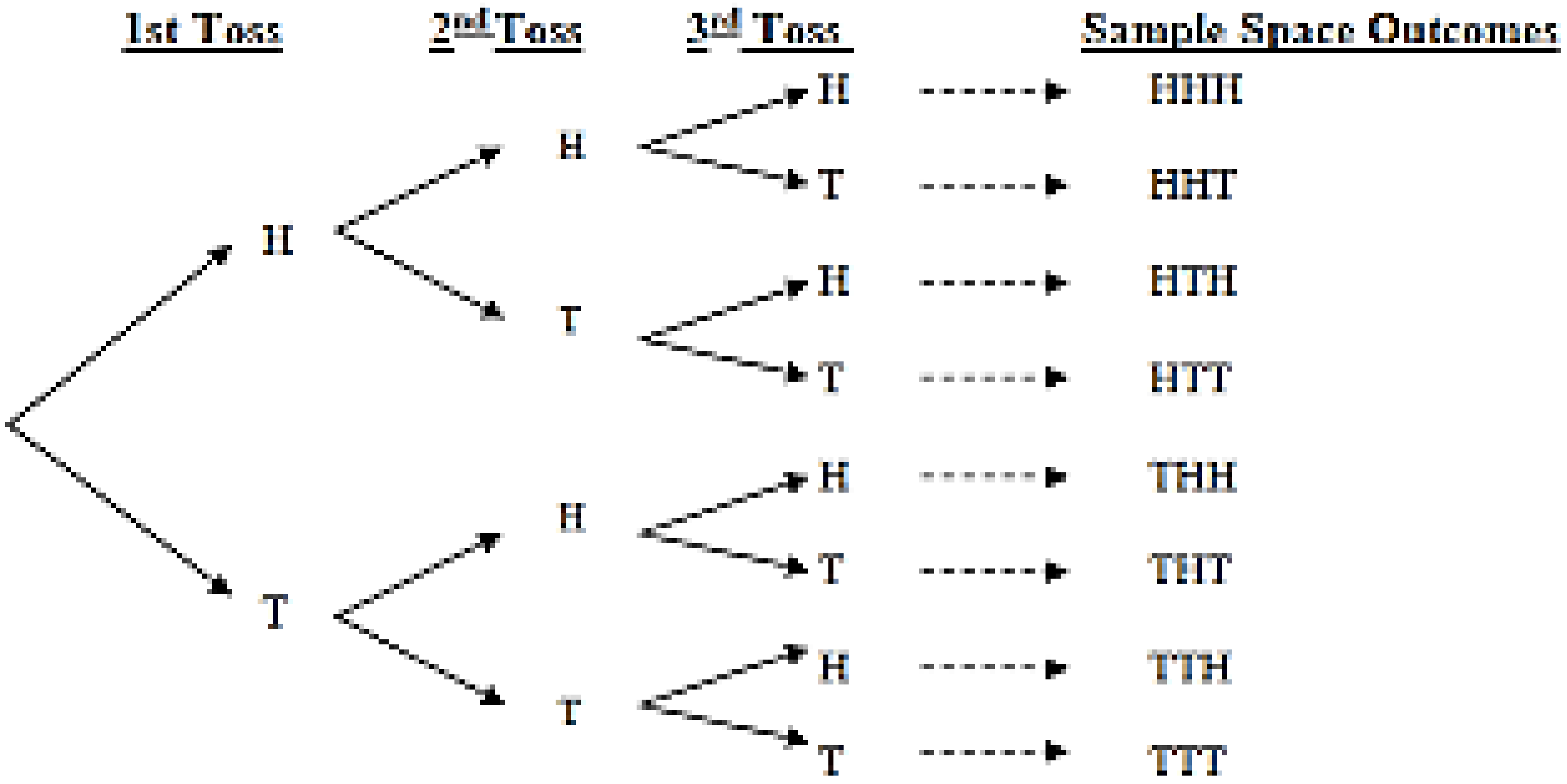
- **Favorable Events:** The event which we are considering for study.
  - Ex: Getting a head in case of tossing a coin
  - Ex: Getting a 6 in case of rolling a die

- An event is a subset of a sample space


Heads
Tails
50%
50%
Coin Probability Problems


Throw dice
1
2
3
4
5
6
1/6
Toss
a coin
1/2
Head
Tail


TOSSING A COIN TWICE!
First Flip
Second Flip
Outcomes
0.5
0.5
0.5
0.5
0.5
0.5
HEADS, HEADS
HEADS, TAILS
TAILS, HEADS
TAILS, TAILS


1st Toss
2nd Toss
3rd Toss
Sample Space Outcomes
H
T
H
T
H
T
H
T
H
T
H
T
H
T
HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

Roll a six-sided die

Outcomes: 1, 2, 3, 4, 5, 6

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Sum	Possible Rolls
2	(1,1)
3	(1,2) (2,1)
4	(1,3) (2,2) (3,1)
5	(1,4) (2,3) (3,2) (4,1)
6	(1,5) (2,4) (3,3) (4,2) (5,1)
7	(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)
8	(2,6) (3,5) (4,4) (5,3) (6,2)
9	(3,6) (4,5) (5,4) (6,3)
10	(4,6) (5,5) (6,4)
11	(5,6) (6,5)
12	(6,6)

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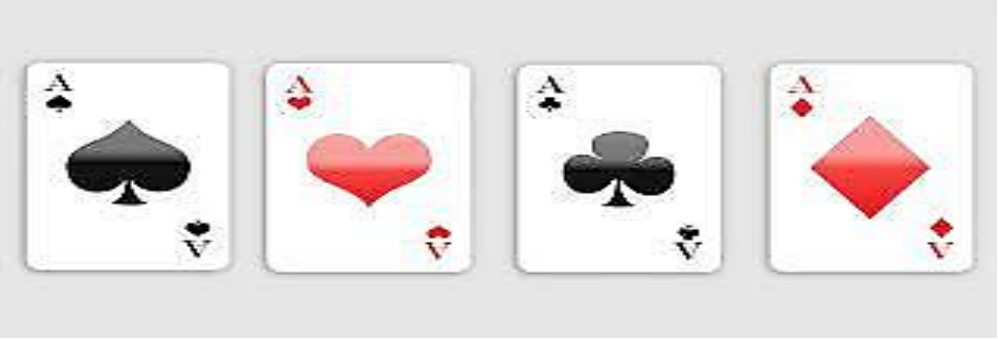
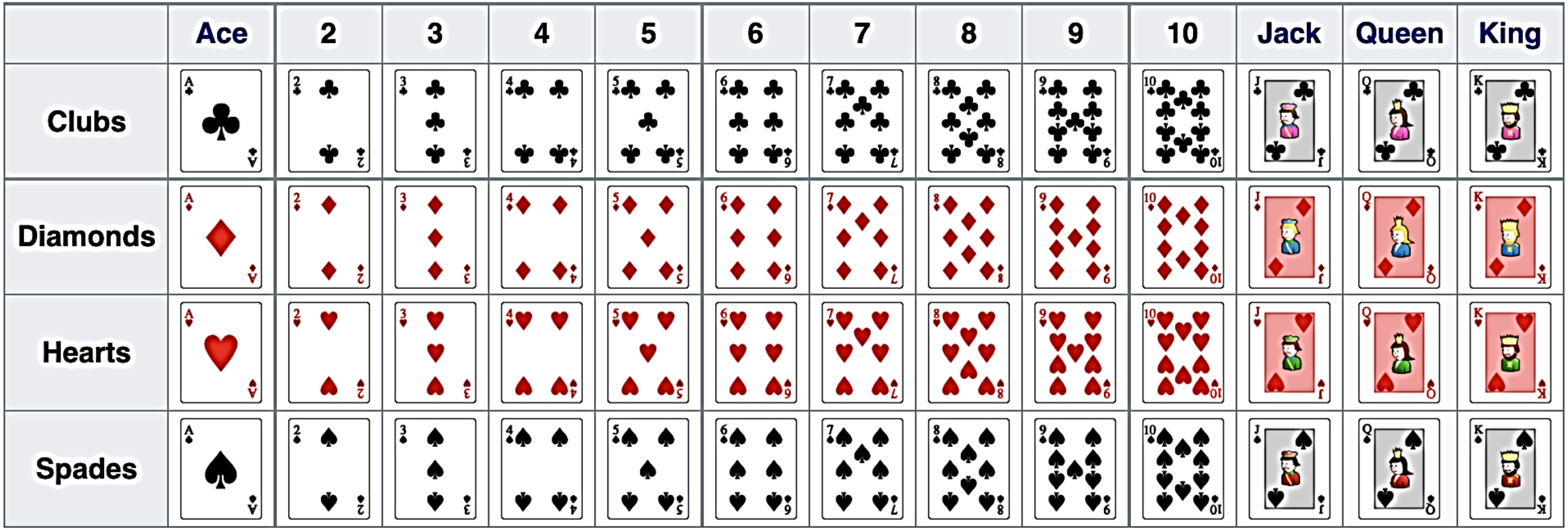
Cards (52)

Spade	Club	Diamond	Heart
1 King	1 King	1 King	1 King
1 Queen	1 Queen	1 Queen	1 Queen
1 Jack	1 Jack	1 Jack	1 Jack
1 Ace	1 Ace	1 Ace	1 Ace
2-10 Cards	2-10 Cards	2-10 Cards	2-10 Cards
Total = 13	Total = 13	Total = 13	Total = 13



Example set of 52 playing cards; 13 of each suit: clubs, diamonds, hearts, and spades




How Many Face Cards Are
In A Deck?

EPP

Experiment | Possibilities | Probability (H)

2 (H,T) Toss

6 (1,2,3,4,5,6) Dice

52 Cards

Probability = Favorable outcomes / Total outcomes

Example=01

What is probability of getting an even number in the throw of unbiased die.

Probability = Favorable outcomes / Total outcomes

$$P(\text{even no.}) = 3/6 = 1/2$$

Example=02

A bag contains 5 black and 10 white balls. What is the probability of drawing.

1) **Black Ball**
2) **White Ball**

Probability = Favorable outcomes / Total outcomes

P(Black Ball)=5/15=1/3

P(White Ball)=10/15=2/3

Example=03

In a lottery 10 prices and 90 blanks. If a person holds one ticket, what are the chances/Probability of .

1) **getting price**
2) **Not getting price**

Probability = Favorable outcomes / Total outcomes

P(getting price)=10/100=1/10=0.1

P(Not getting price)=90/100=9/10=0.9

OR

P(Not getting price)=1-P(getting price)

P(Not getting price)=1-0.1

P(Not getting price)=0.9

Example=04

What is the probability of getting a king in a draw from a pack of cards?

Probability = Favorable outcomes / Total outcomes

$$P(King)=4/52=1/13$$

Example=05

Find the probability of drawing a face card in a single random draw from a well shuffled pack of 52 cards.

Probability = Favorable outcomes / Total outcomes

$$P(\text{Face}) = 12/52 = 3/13$$

Permutations

A permutation is an arrangement of objects in a definite order. The members or elements of sets are arranged here in a sequence or linear order. For example, the permutation of set A={1,6} is 2, such as {1,6}, {6,1}.there are no other ways to arrange the elements of set A.

Number of permutations (order matters) of n things taken r at a time:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Number of different permutations of n objects where there are n_1 repeated items, n_2 repeated items, ... n_k repeated items

$$\frac{n!}{n_1!\,n_2!\ldots n_k!}$$

Permutations

A permutation is an arrangement of all or part of a set of objects.

Permutation can be classified in two different categories:

•**Permutation** of n **different** objects (when repetition is not allowed) $$P(n,r)=\frac{n!}{(n-r)!}$$

•**Permutation** when the objects are not distinct (when repetition is allowed) $$\frac{n!}{n_1!n_2!\ldots n_k!}$$

A B C D

Using 2 out of 4 boxes in a set:

Possible arrangements: 12

AB BA CA DA

AC BC CB DB

AD BD CD DC

PERMUTATIONS

Possible selections of distinct items: 6

AB = BA

BC = CB

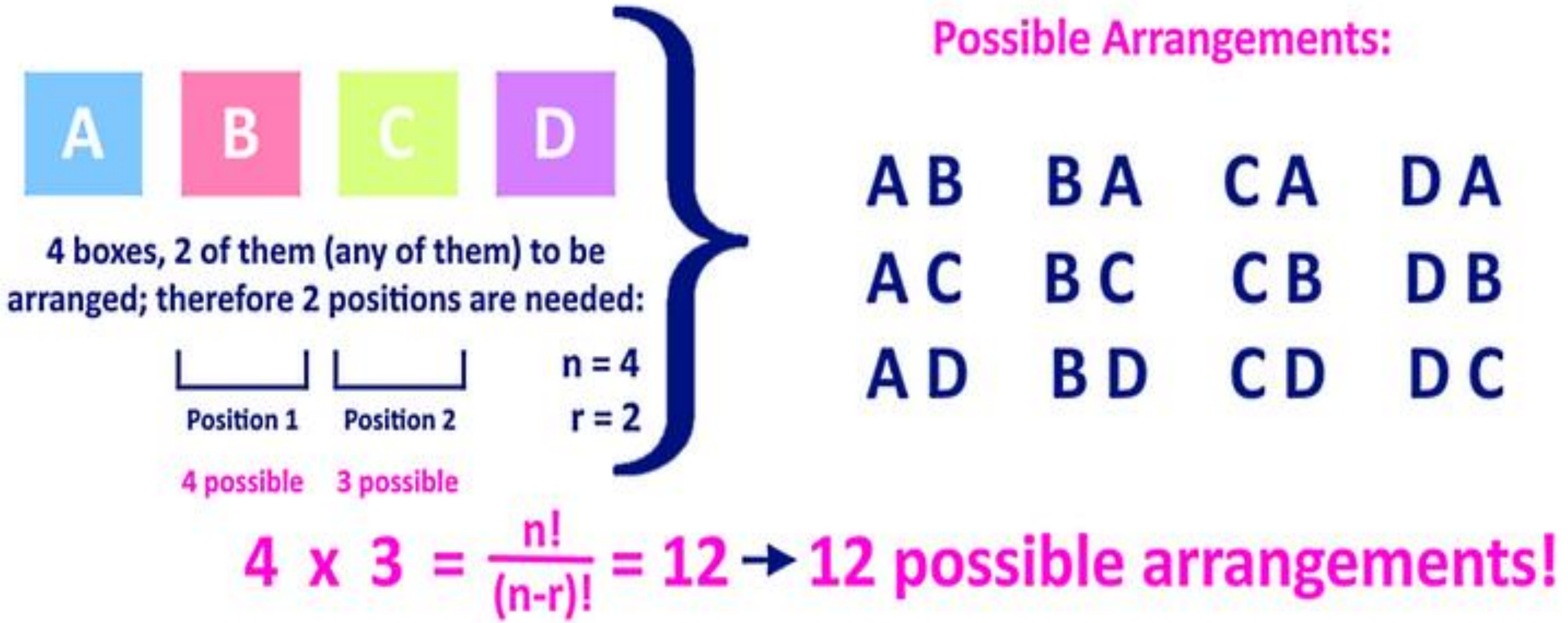
AC = CA

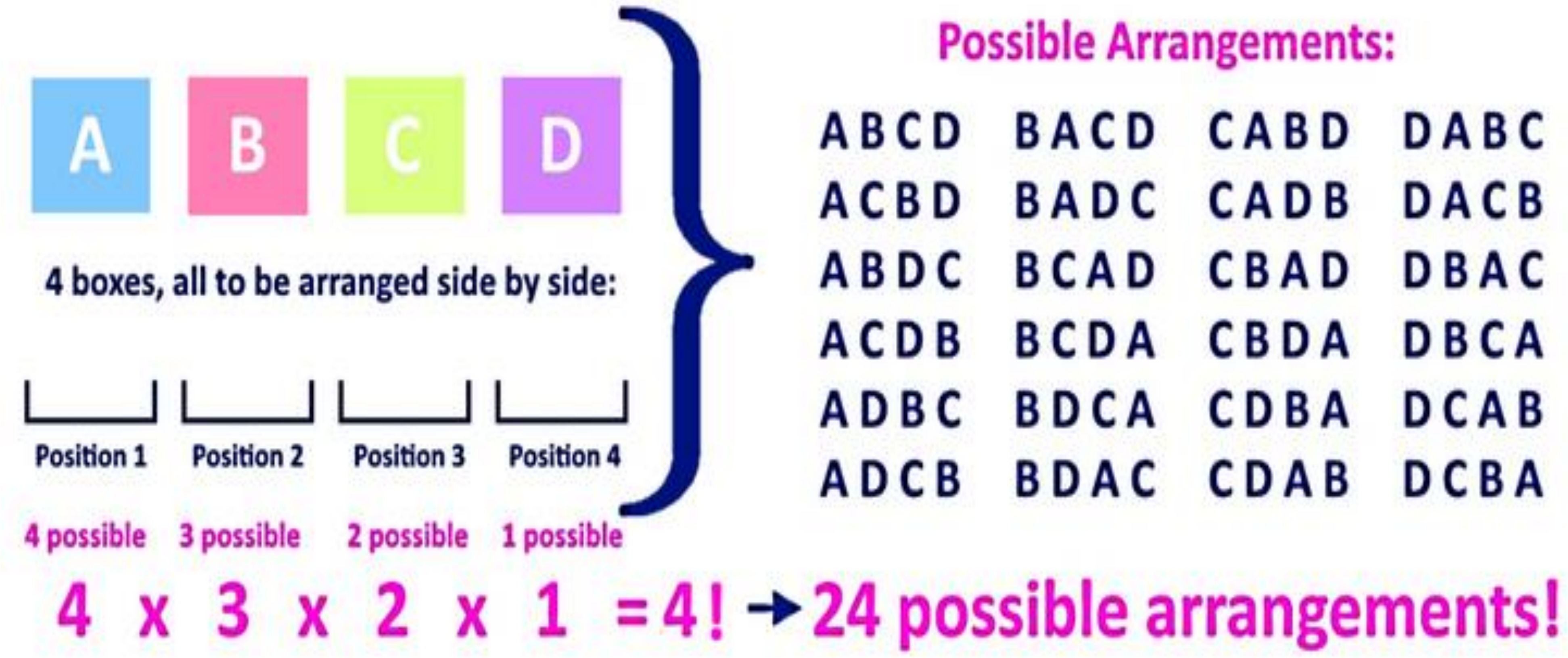
BD = DB

AD = DA

CD = DC

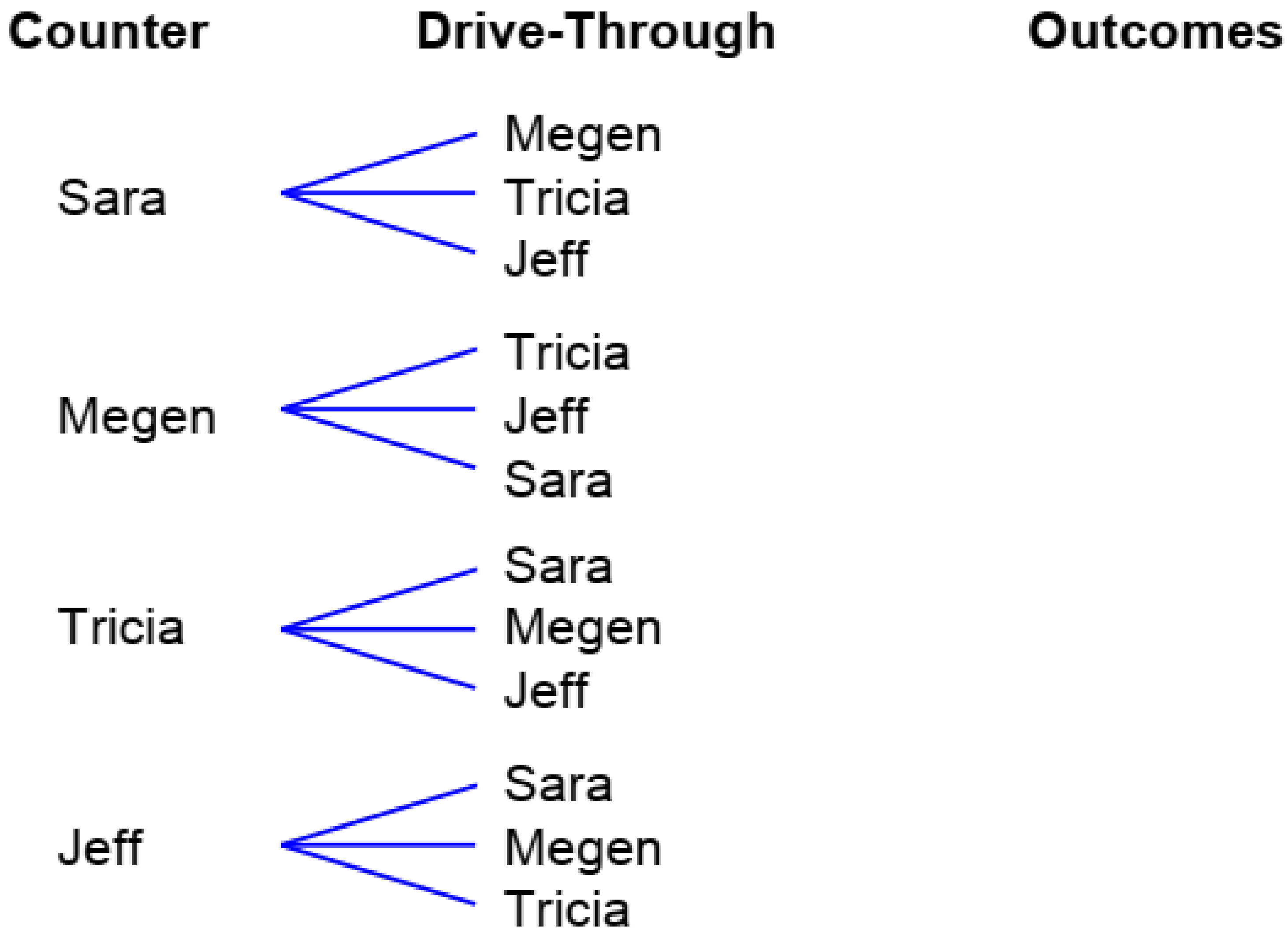
COMBINATIONS

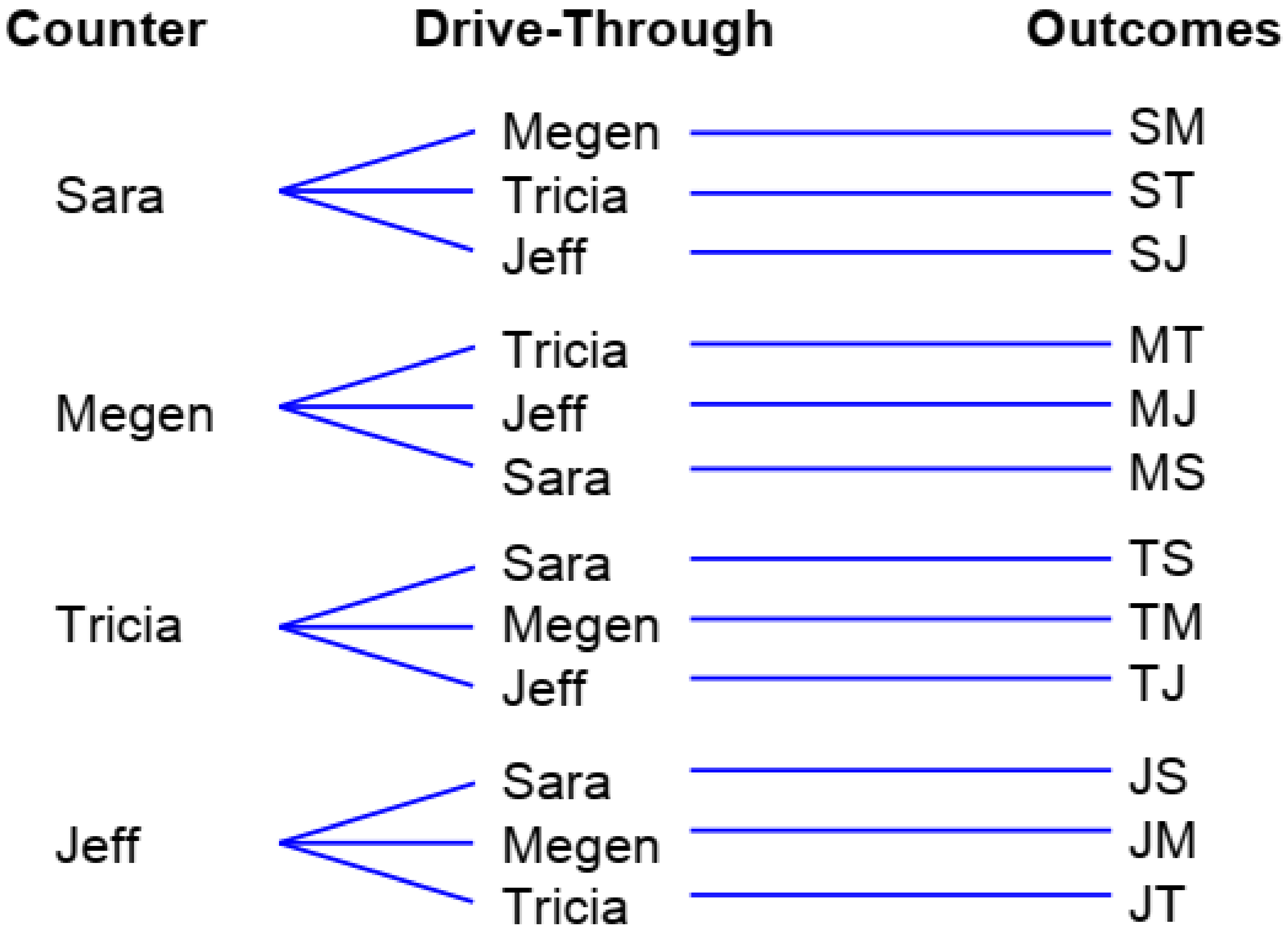

A
B
C
D
4 boxes, 2 of them (any of them) to be arranged; therefore 2 positions are needed:
Position 1
Position 2
n = 4
r = 2
4 possible
3 possible
Possible Arrangements:
A B   B A   C A   D A
A C   B C   C B   D B
A D   B D   C D   D C
4 x 3 = n!/(n-r)! = 12 → 12 possible arrangements!

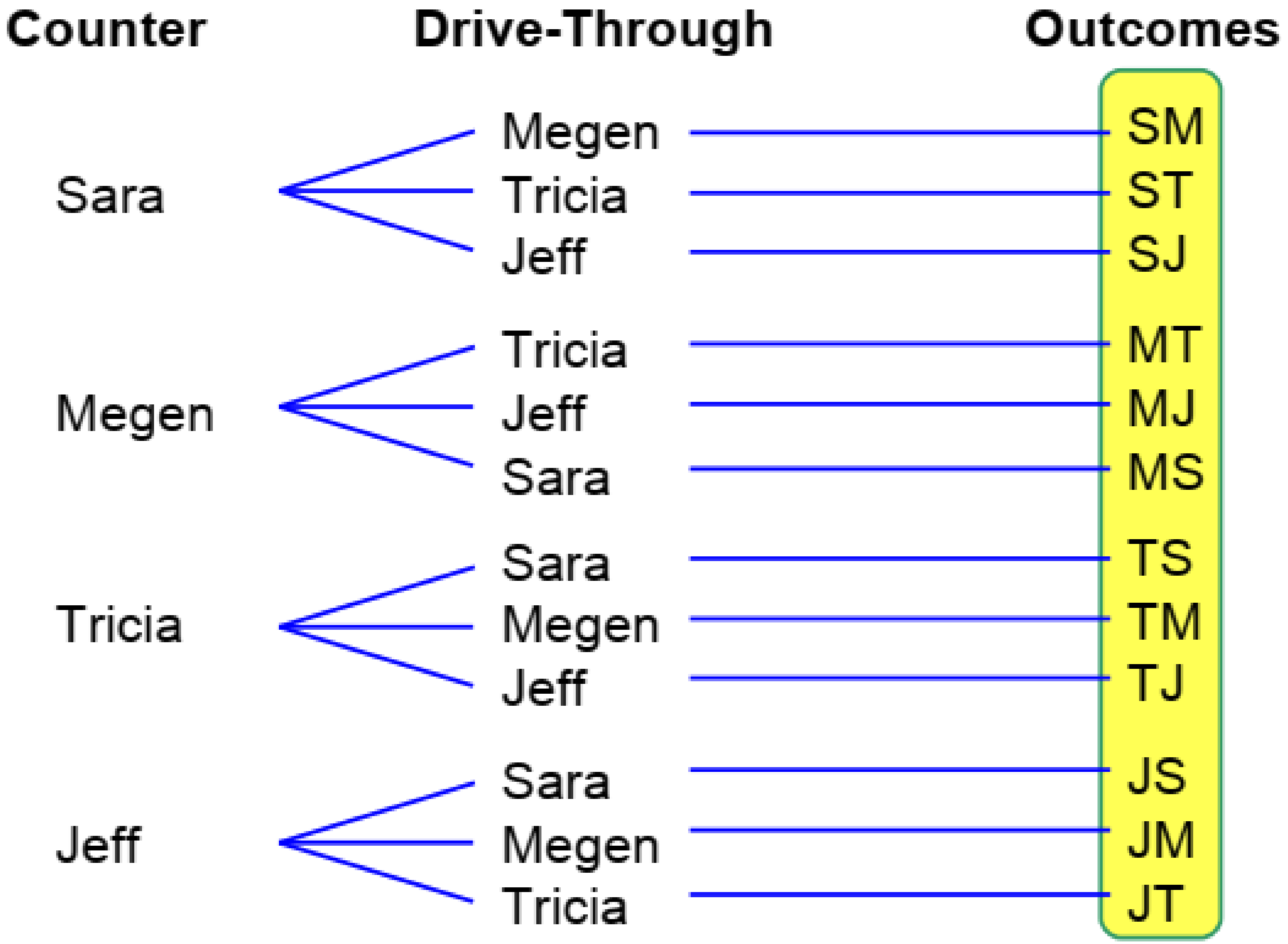

A
B
C
D
4 boxes, all to be arranged side by side:
Position 1
Position 2
Position 3
Position 4
4 possible
3 possible
2 possible
1 possible
Possible Arrangements:
ABCD BACD CABD DABC
ACBD BADC CADB DACB
ABDC BCAD CBAD DBAC
ACDB BCDA CBDA DBCA
ADBC BDCA CDBA DCAB
ADCB BDAC CDAB DCBA
4 x 3 x 2 x 1 = 4! → 24 possible arrangements!

The manager of a coffee shop needs to hire two employees, one to work at the counter and one to work at the drive-through window. Sara, Megen, Tricia and Jeff all applied for a job. How many possible ways are there for the manager to place the applicants?

Counter	Drive-Through	Outcomes
Sara		
Megen		
Tricia		
Jeff		


Counter
Drive-Through
Outcomes
Sara
Megen
Tricia
Jeff
Megen
Tricia
Jeff
Sara
Tricia
Sara
Megen
Jeff
Jeff
Sara
Megen
Tricia


Counter
Drive-Through
Outcomes
Sara
Megen
Tricia
Jeff
SM
ST
SJ
Megen
Tricia
Jeff
Sara
MT
MJ
MS
Tricia
Sara
Megen
Jeff
TS
TM
TJ
Jeff
Sara
Megen
Tricia
JS
JM
JT



There are 12 different ways for the 4 applicants to hold the 2 positions

In the previous example, the positions are in specific order, so each arrangement is unique.

The symbol ${}_4P_2$ denotes the number of permutations when arranging 4 applicants in two positions.

You can also use the Fundamental Counting Principle to determine the number of permutations.

$$\underbrace{\text{ways to choose first employee}}\ \times\ \underbrace{\text{ways to choose second employee}}$$

$${}_4P_2 = \quad 4 \quad \times \quad 3$$

Permutation

From 2 letter word from a pool of 3 alphabets (A,B,C)

AB AC BC BA CA CB

6 Ways

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$^{3}P_{2} = 3!/(3-2)!$$

From 3 letter word from a pool of 3 alphabets (A,B,C)

ABC ACB BCA BAC CAB CBA

6 Ways

$$^{3}P_{3} = 3!/(3-3)!$$

Examples:

E1: In how many ways can 5 men be seated in a row having 3 seats?

Ans:

Here $n = 5$ & $r = 3$

Therefore:

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{5!}{(5-3)!} = \frac{120}{2} = 60 \text{ ways} $$

E2: In how many ways can 5 men form a line?

Ans:

Here $n = 5$ & $r = 5$

Therefore:

$$ {}^{n}P_{r} = {}^{5}P_{5} = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5! = 120 \text{ ways} $$

Examples:

E1: In how many ways can the word "STATISTICS" be arranged?

Ans: There are in all 10 letters consisting of:

3S's 3T's, 2I's, 1A's & 1C's

Therefore, the number of permutations are:

$$P = \frac{10!}{3! \cdot 3! \cdot 2! \cdot 1! \cdot 1!} = 50{,}400$$

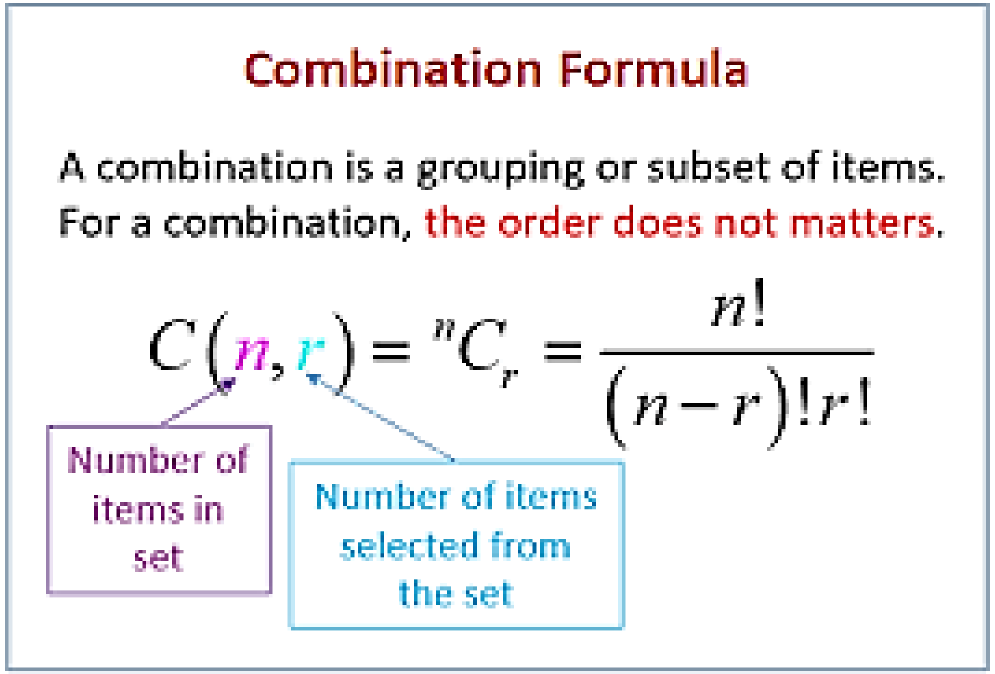
E2: In how many ways can 2 red, 3 blue and 4 green chips be arranged in a row

Ans:

$$P = \frac{9!}{2! \cdot 3! \cdot 4!} = 1260$$

Use Of Combinations

A combination is a mathematical technique that determines the number of possible arrangements in a collection of items where the order of the selection does not matter. In combinations, you can select the items in any order.



Combinations

Selecting 4 fruits out of 10 fruits

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$\begin{aligned} ^{10}C_{4} = C(n, r) = C(10, 4) &= \frac{10!}{(4!(10-4)!)} \\ &= \frac{10!}{4! \times 6!} \\ &= 210 \text{ ways} \end{aligned}$$

Example=01

A bag contains 5 green and 7 red balls, two balls are drawn. What is the probability that one is green and other is red.

Probability = Favorable outcomes / Total outcomes

$$P = \frac{{}^{5}C_1 \times {}^{7}C_1}{{}^{12}C_2}$$

$${}^{5}C_1 = 5!/1!\ 4! = 5\times4!/1!\ 4! = 5$$

$${}^{7}C_1 = 7!/1!\ 6! = 7\times6!/1!\ 6! = 7$$

$${}^{12}C_2 = 12!/2!\ 10! = 12\times11\times10!/2!\ 10! = 12\times11/2\times1 = 66$$

$$P = (5\times7)/66 = 35/66$$

Example=02

From a pack of 52 cards, two cards are drawn at random. Find a probability that one a king and the Other is queen.

Probability = Favorable outcomes / Total outcomes

$$P = \frac{{}^{4}C_{1} \times {}^{4}C_{1}}{{}^{52}C_{2}}$$

$$P=(4x4)/1326=16/1326=0.012$$

Example=03

A bag contain 9 red balls, 7 white balls and 4 green balls. Three(3) balls are drawn randomly Without replacement. Find the probability of getting one(1) ball of each color.

Probability = Favorable outcomes / Total outcomes

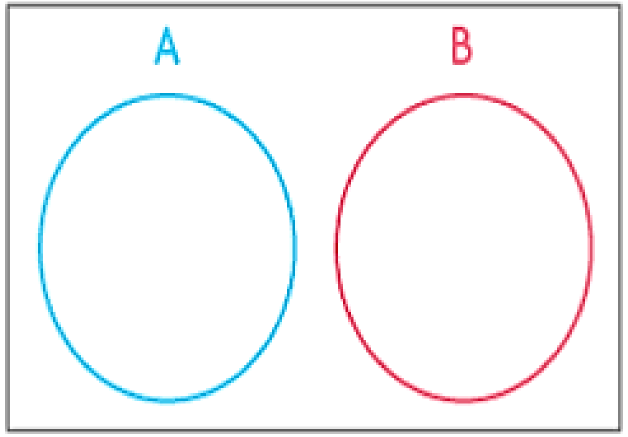
$$P = \frac{{}^{9}C_1 \times {}^{7}C_1 \times {}^{4}C_1}{{}^{20}C_3}$$

$$P=(9x7x4)/1140=0.221$$

ADDITION THEOREM

Mutually Exclusive

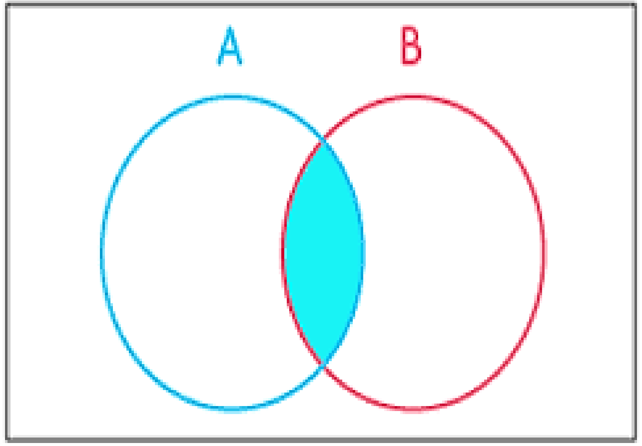
Mutually Exclusive



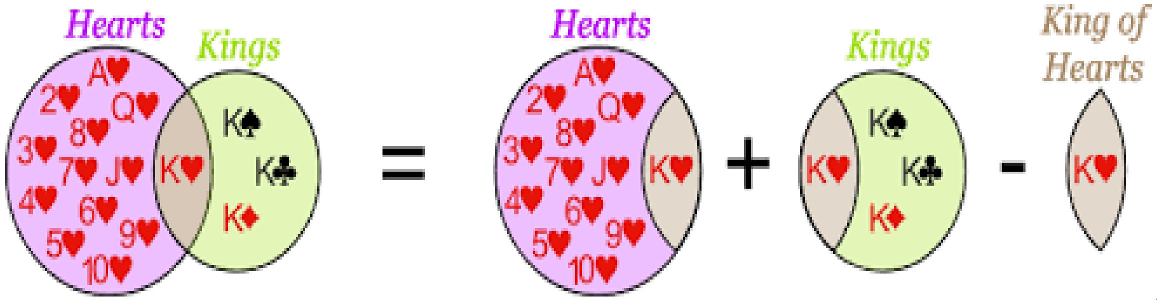
$$P(A+B)=P(A) + P(B)$$

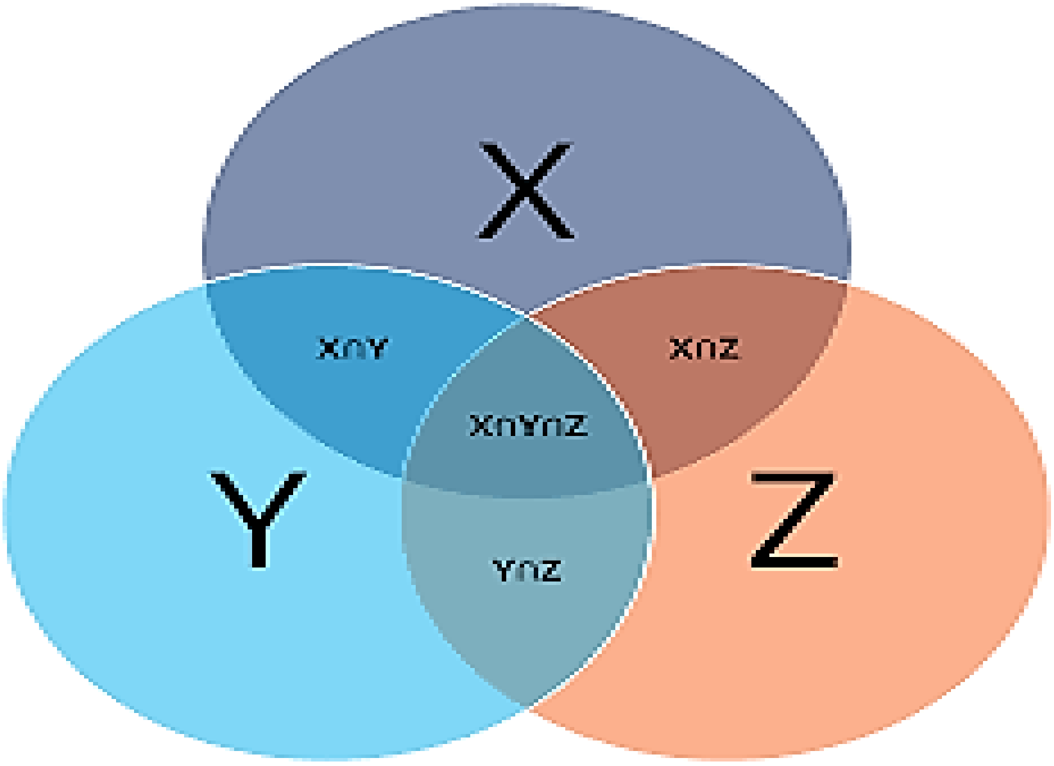
Non- Mutually Exclusive

Non Mutually Exclusive



$$P(A+B)=P(A) + P(B) - P(AB)$$





Mutually Exclusive

P(A+B+C)=P(A) + P(B) + P(C)

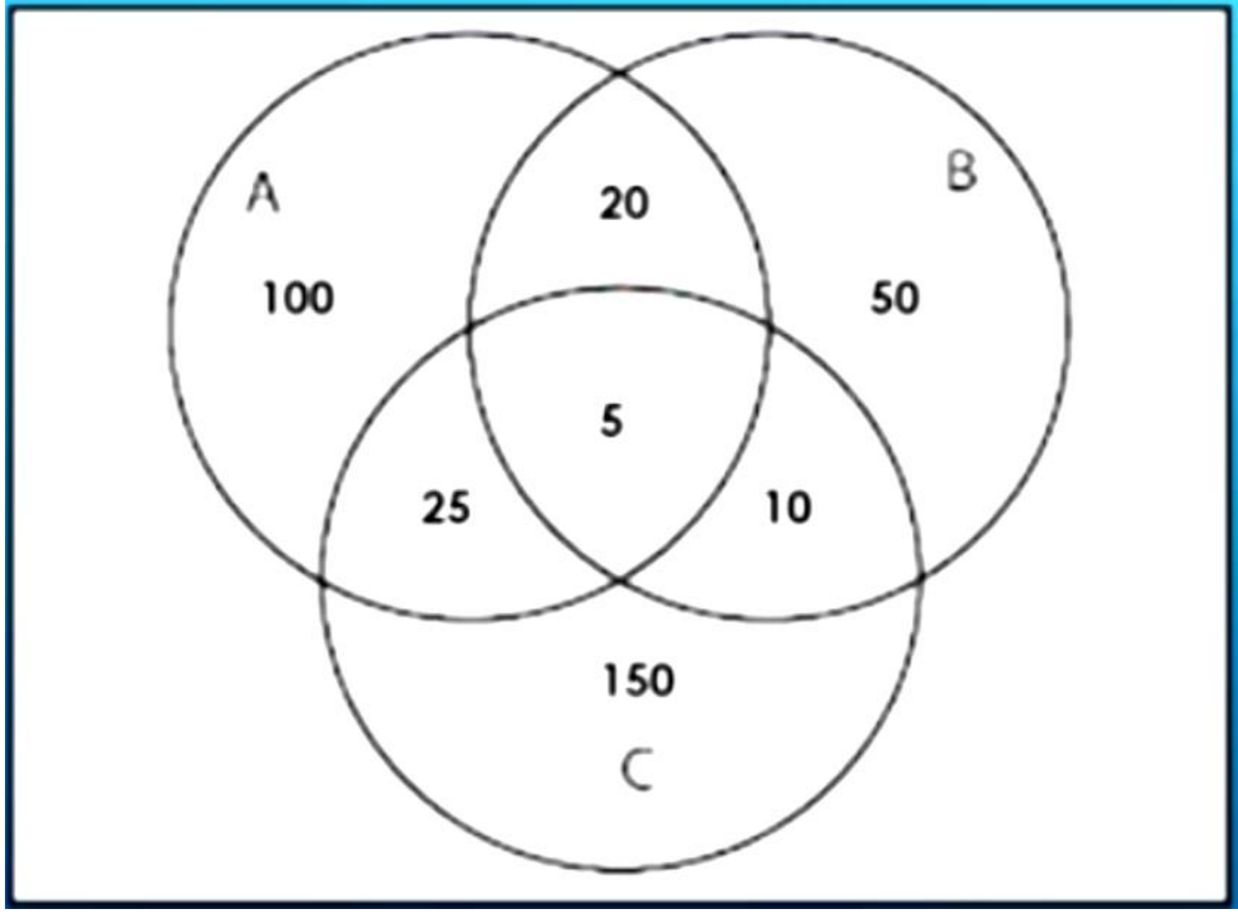
Non- Mutually Exclusive

P(A+B+C)=P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)

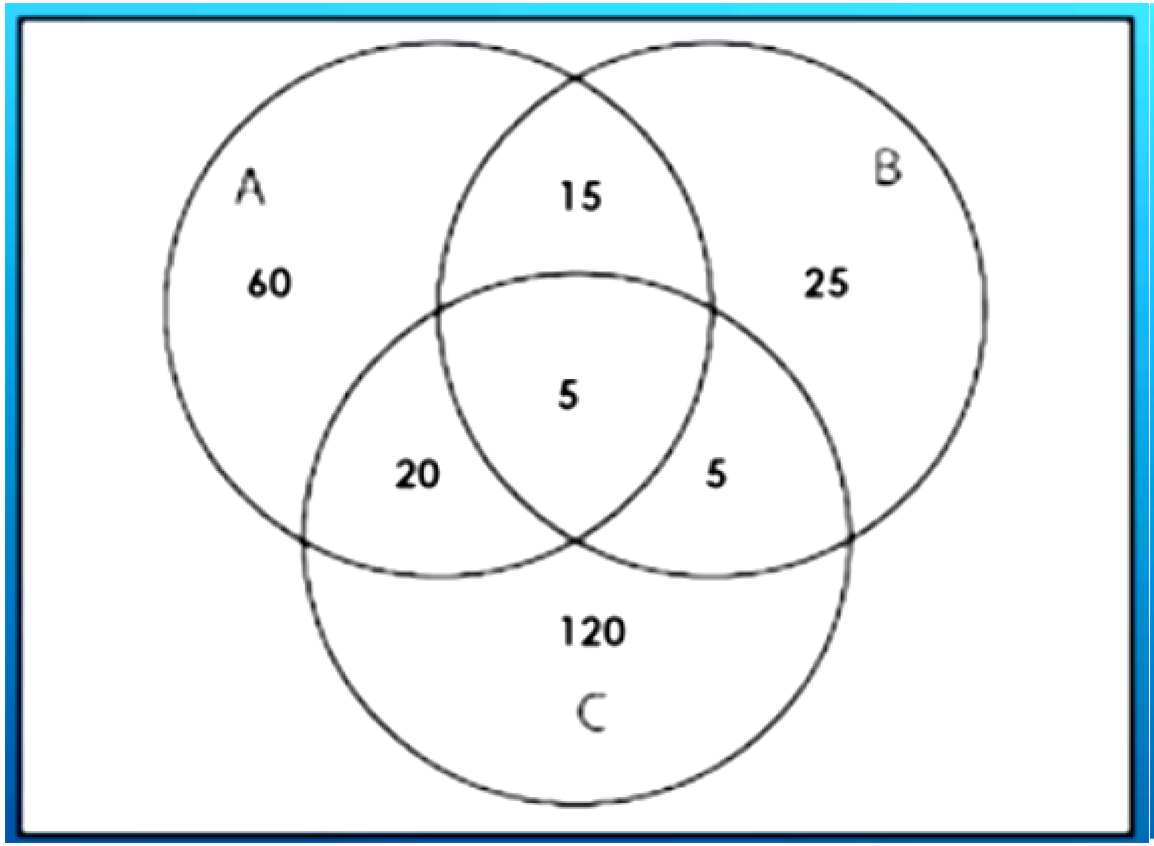
EXAMPLE OF UNION OF THREE SETS

- In a junior class, the number of students in electrical engineering (EE) is 100, in math (MA) 50, and in computer science (CS) 150. Among these 20 are taking both EE and MA, 25 are taking EE and CS, and 10 are taking MA and CS. Five of them are
  taking EE, CS and MA. Find the total number of students in the junior class. (Ref #)
- Solution: Let sets A = EE = 100, B = MA = 50, C = CS = 150.
- Students taking both EE and MA = $A \cap B = 20$
- Students taking both EE and CS = $A \cap C = 25$
- Students taking both MA and CS = $B \cap C = 10$
- Students taking EE, MA and CS = $A \cap B \cap C = 5$
- Total number of students = $A \cup B \cup C$

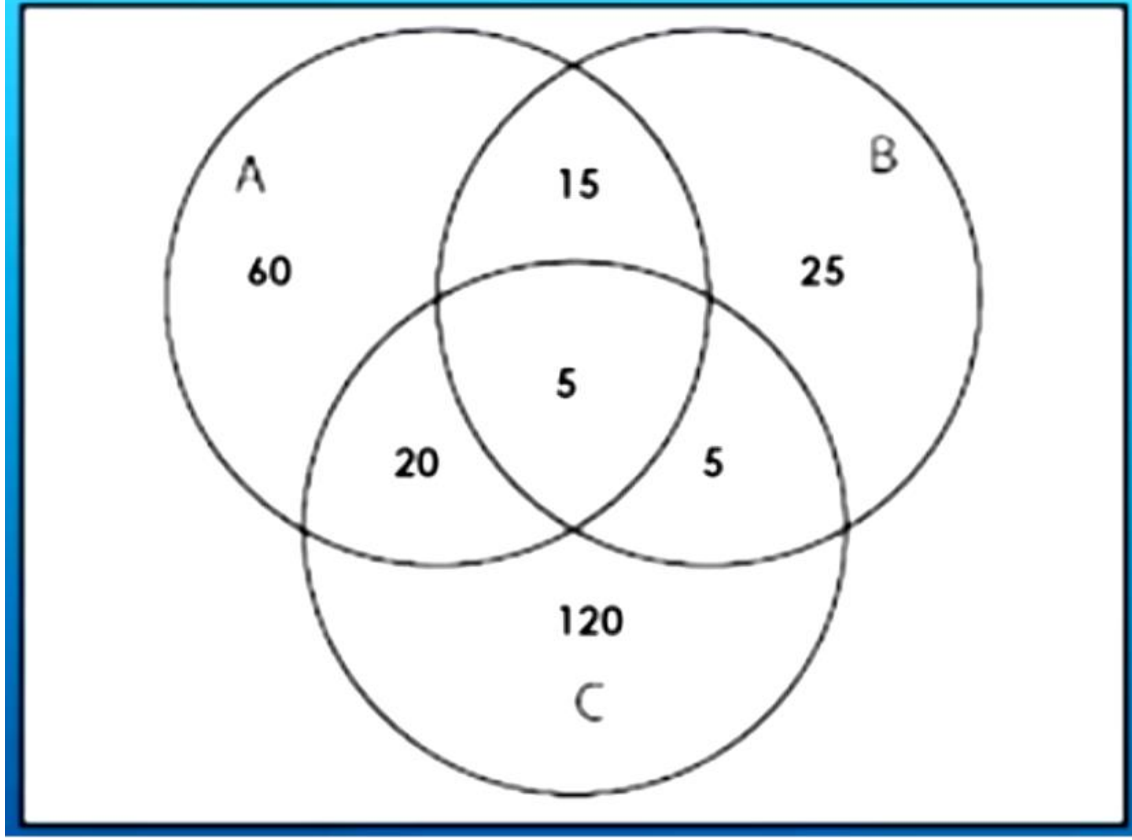
$$P(A+B+C)=P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$$



- Is this Venn diagram correct?
- **NO**
- Because for set A, number of students = 100 +20 + 5 + 25 = 150.
- However we know that total number of students in set A =100



- After adding all the values in set A it must be equal to 100.
- So start with the value common for A, B and C, i.e. 5.
- We know that students taking both EE and MA = A ∩ B = 20.
- But there are 5 students who are taking all three courses.
- Therefore 20 – 5 =15 students who are taking both EE and MA.
- We know that students taking both EE and CS = A ∩ C = 25.
- Similarly, 25 – 5 = 20
- A = 100 – 5 – 15 – 20 = 60



- Students taking EE only = 60
- Students taking MS only = 25
- Students taking CS only = 120
- Students taking EE and MS = 15
- Students taking EE and CS = 20
- Students taking MS and CS = 5
- Students taking EE, MS and CS = 5

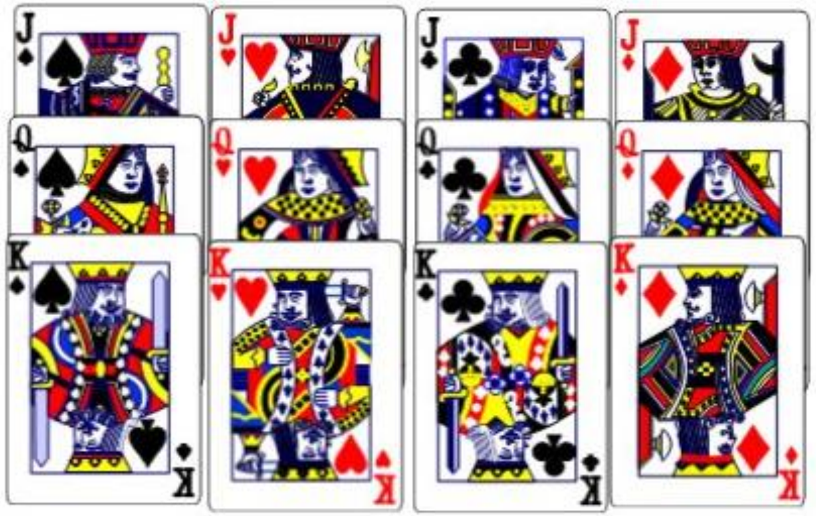
$$P(A+B+C)=P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$$

Example=01 (Mutually Exclusive)

A card is drawn from a pack of 52 cards. What is the probability of getting either a king or Queen?

$$P(A+B)=P(A) + P(B)$$
$$=4/52 + 4/52$$
$$P(A+B)= 8/52=4/26=2/13$$

Face Cards



Example=02 (Mutually Exclusive)

A Perfect die is throw. What is the probability of throwing 3 or 5?

$$P(A+B)=P(A)+P(B)$$
$$=1/6+1/6$$
$$P(A+B)=2/6=1/3$$

A=1,2,3,4,5,6

B=1,2,3,4,5,6

Example=03 (Non-Mutually Exclusive)

A bag contains balls numbered from 1 to 30.One ball is drawn at random find the probability that The number of ball is a multiple of 5 or 6.

P(A+B)=P(A) + P(B) -P(AB)

P(A+B)=6/30 + 5/30 - 1/30

P(A+B)=(6+5-1)/30=10/30=1/3

A=5,10,15,20,25,30

B=6,12,18,24,30

P(A)=6/30

P(B)=5/30

P(AB)=1/30

Example=04 (Non-Mutually Exclusive)

A Card is drawn out of pack of 52 cards. Find the probability that the card is an Ace, a king, a Queen Or a card of Clubs.

$$P(A+B)=P(A) + P(B) - P(AB)$$

$$=12/52 + 13/52 - 3/52$$

$$P(A+B)=(12+13-3)/52=22/52=22/52=11/46=0.239$$

Example=05 (Mutually Exclusive)

P(A)=0.3

P(AUB)=0.5

P(B)=?

For what value of P(B) are A and B mutually Exclusive?

P(B)=0.2

Example=06

A bag contains 30 balls numbered from 1 to 30 one ball is drawn at random. Find the probability That the number of the ball is multiple of 5 or 8.

(Mutually Exclusive OR Not Mutually Exclusive)

?

Mutually Exclusive

A=5,10,15,20,25,30

B=8,16,24

P(A)=6/30

P(B)=3/30

P(A+B)=P(A) + P(B)

=6/30 + 3/30

P(A+B)=9/30=3/10=0.3

Example=07

A card is drawn from a pack of 52 cards. What is the probability that.

a) It is either a king or Queen.

b) It is either a king or black card.

(Mutually Exclusive OR Not Mutually Exclusive)

?

a) It is either a king or Queen (Mutually Exclusive)

A=4

B=4

P(A)=4/52

P(B)=4/52

$$P(A+B)=P(A)+P(B)$$
$$=4/52+4/52$$
$$=8/52=2/13=0.153$$

b) It is either a king or black card (Non-Mutually Exclusive)

A=king(4)

B=black card(26)

$$P(A+B)=P(A)+P(B)-P(AB)$$
$$=4/52+26/52-2/52$$
$$P(A+B)=28/52=7/13=0.538$$

Example=08

What is the probability of drawing a heart or a king card from a pack of cards?

(Mutually Exclusive OR Not Mutually Exclusive)

?

Not Mutually Exclusive

A=heart (13)

B=king (4)

$P(A+B)=P(A) + P(B) - P(AB)$

$=13/52 + 4/52 - 1/52$

$P(A+B)=4/13$

Example=09 (Non-Mutually Exclusive)

A number was drawn at random from the number 1 to 50.What is the probability that it will be a multiple of 2 or 3 or 10.

A=(2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42,44,46,48,50)

B=(3,6,9,12,15,18,21,24,27,30,33,36,39,42,45,48)

C=(10,20,30,40,50)

P(A)=25/50 ; P(B)=16/50 ; P(C)=5/50

P(AB)=8/50 ; P(BC)=1/50 ; P(AC)=5/50

P(ABC)=1/50

$$P(A+B+C)=P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$$

$$=25/50 + 16/50 + 5/50 - 8/50 - 1/50 - 5/50 + 1/50$$

$$P(A+B+C)=(25+16+5-8-1-5+1)/50=33/50=0.66$$

Example=10

A bag contains 30 balls numbered 1 to 30.One ball is drawn at random. Find the probability that the number of drawn ball will be a multiple of 3 or 5.

(Mutually Exclusive) / (Non-Mutually Exclusive)

?

Multiplication Theorem in Probability

Multiplication Rule

Independent Events

$$P(A \cap B) = P(A) \,.\, P(B)$$

Dependent Events

$$P(A \cap B) = P(A) \,.\, P(B \mid A)$$

Multiplication Theorem

- **Independent (with replacement)**
- **Dependent (Without replacement)**

Example 1:

A die is rolled and a penny is tossed. What is the probability that a 3 is rolled on the die and a head is tossed on the penny?

Solution:

Sample space (die) = {1, 2, 3, 4, 5, 6}

Event A: Rolling a 3 on a die: $P(A) = 1/6$

Sample space (penny)= {H, T}

Event B: Tossing a head with the penny: $P(B) = 1/2$

Probability of BOTH: These are independent events. Event *A* does not have an effect on event *B*.

Answer: $P(A \text{ and } B) = P(A) \cdot P(B) = 1/6 \cdot 1/2 = \mathbf{1/12}$

Example=02 (Independent)

From a pack of 52 cards, two cards are drawn at random one after the another with replacement. What is the probability that both cards are kings?

$P(E1)=4/52$

$P(E2)=4/52$

$P(E)=P(E1) \times P(E2)$

$\quad =4/52 \times 4/52$

$P(E)=16/2704=1/169=0.00591$

Example=03 (Independent)

A bag contains 5 white and 3 black balls. Two balls are drawn at random one after another With replacement. Find the probability that both the balls drawn are black.

$P(E1)=3/8$

$P(E2)=3/8$

$$P(E)=P(E1) \times P(E2)$$
$$=3/8 \times 3/8$$

$P(E)=9/64$

Example=04 (Independent)

Probability of getting 3 tails in three tosses of a coin.

Example=05 (Independent)

.

Two (02) Vacancies

Probability of selection of Husband=4/5

Probability of selection of Wife =3/4

A. Both of them will be selected.

B. None of them will be selected.

C. Only Wife will be selected.

A. $P(E1) \times P(E2)$

$4/5 \times 3/4 = 3/5$

B. P(Not selected) x P(Not selected)

$(1-4/5) \times (1-3/4)$

$1/5 \times 1/4 = 1/20$

C. P(Wife selected) x P(Husband Not selected)

$3/4 \times 1/5 = 3/20$

Example=06 (Independent)

A problem in statistics is given to three students A,B,C whose probability of solving it are 1/2 ,1/3 and 1/4 .What is the probability that the problem will be solved?

$P(\text{Not solved})=P(\overline{A}) \times P(\overline{B}) \times P(\overline{C})$

$P(A)=1/2$; $P(\overline{A})=1-1/2$

$P(\overline{A})=1/2$

$P(B)=1/3$; $P(\overline{B})=1-1/3$

$P(\overline{B})=2/3$

$P(C)=1/4$; $P(\overline{C})=1-1/4$

$P(\overline{C})=3/4$

$P(\text{Not solved})=1/2 \times 2/3 \times 3/4$

$P(\text{Not solved})=1/4$

$P(\text{solved})=1-P(\text{Not solved})$

$=1-1/4$

$P(\text{solved})=3/4$

Example=07 (Independent)

A Student Mr. X is interviewed for 3 posts. For the first post, there are 3 Students, for the second post , there are 4 Students and for third there are 2 Students. What are the Chances of Mr. X being getting selected?

$P(\text{Not getting selected}) = P(\overline{A}) \times P(\overline{B}) \times P(\overline{C})$

$P(A)=1/3$; $P(\overline{A})=1-1/3$

; $P(\overline{A})=2/3$

$P(B)=1/4$;$P(\overline{B})=1-1/4$

;$P(\overline{B})=3/4$

$P(C)=1/2$;$P(\overline{C})=1-1/2$

;$P(\overline{C})=1/2$

$P(\text{Not getting selected})=2/3 \times 3/4 \times 1/2$

$P(\text{Not getting selected})=1/4$

$P(\text{getting selected})=1-P(\text{Not getting selected})$

$P(\text{getting selected})=1-1/4$

$P(\text{getting selected})=3/4$

CONDITIONAL PROBABILITY

Multiplication Theorem

Dependent
(Without replacement)

Example

Two cards are drawn from a standard deck of cards. What is the probability that the first card is a club and the second card is a heart?

The first card was not put back into the deck after being drawn.

Solution:

Sample space = {52 cards in the deck}

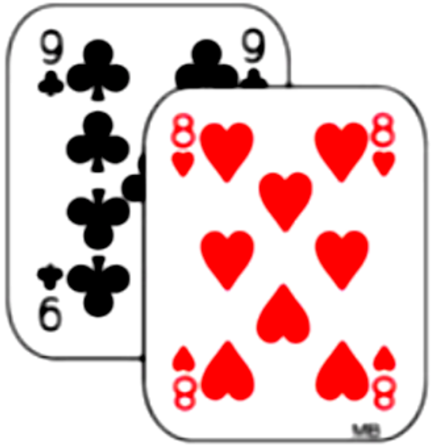
Event A: Drawing a club: {13 clubs} $P(A) = 13/52 = 1/4$

Sample space = {51 cards - one card in now missing}

Event B: Drawing a heart: {13 hearts} $P(B) = 13/51$

Probability of BOTH: These are dependent events. Event *B* is affected by event *A*. Without replacing the card, the sample space for the second draw is changed to only 51 available cards.

Answer: $P(A \text{ and } B) = P(A) \cdot P(B \mid A) = 13/52 \cdot 13/51$ **= 169/2652**



Dependent events since the first card was not replaced.

Example=01 (Dependent)

A bag contains 10 white and 5 black balls. Two (02) balls are drawn at random one after the other Without replacement. Find the probability that both balls drawn are black.

$$P(AB)=P(A) \times P(B/A)$$

$$P(AB)=5/15 \times 4/14$$

$$P(AB)=2/21$$

Example=02 (Dependent)

Find the probability of drawing a king, a queen, and a jack in that order from a pack of cards in three Consecutive draw the cards drawn not being replaced.

$$P(ABC)=P(A) \times P(B/A) \times P(C/AB)$$

$$P(ABC)=4/52 \times 4/51 \times 4/50$$

$$P(ABC)=0.000482$$

Example=03 (Dependent)

Four cards are drawn without replacement. What is the probability that they are all Kings?

$$P(ABCD) = P(A) \times P(B/A) \times P(C/AB) \times P(D/ABC)$$

$$P(ABCD) = 4/52 \times 3/51 \times 2/50 \times 1/49$$

$$P(ABCD) = 0.000003693$$

Example=04 (Independent + Dependent)

Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that they are both jacks if the first card is

i. **Replaced**
ii. **Not Replaced**

(i) $P(AB)=P(A) \times P(B)$

$=4/52 \times 4/52$

$P(AB)=1/169$

(ii) $P(AB)=P(A) \times P(B/A)$

$=4/52 \times 3/51$

$P(AB)=12/2652=1/221$

Example=05 (Independent + Dependent)

A bag contain 5 white and 8 red balls. Two(02) successive drawings of 3 balls are made such that

i. **The balls are replaced before the second trial**

ii. **The balls are not replaced before the second trial.**

Find the probability that the first drawing will give 3 white and the second 3 red balls in each case.

(i)

A-------------- 3 white balls in first draw

B--------------- 3 red balls in second draw

P(AB)=P(A) x P(B)

$$P(A)=\frac{{}^{5}C_{3}}{{}^{13}C_{3}}$$

${}^{5}C_{3}$ =5!/3!2!

=(5x4x3x2x1)/(3x2x1) (2x1)

=10

${}^{13}C_{3}$ =13!/3!10!=(13x12x11x10!)/(3x2x1)10!

=1716/6=286

P(A)=10/286=5/143=0.0349

$$P(B)=\frac{{}^{8}C_{3}}{{}^{13}C_{3}}$$

${}^{8}C_{3}$ =8!/3!5!=(8x7x6x5!)/(3x2x1)5!

=336/6=56

${}^{13}C_{3}$ =13!/3! 10!=(13x12x11x10!)/(3x2x1) 10!

=286

P(B)=56/286

P(AB)=10/286 x 56/286

P(AB)=140/20449=0.00684

(ii)

$P(AB)=P(A) \times P(B/A)$

$$P(A)=\frac{{}^{5}C_{3}}{{}^{13}C_{3}}$$

$=10/286$

$$P(B/A)=\frac{{}^{8}C_{3}}{{}^{10}C_{3}}$$

${}^{8}C_{3}$ $=8!/3!\ 5!=(8x7x6x5!)/(3x2x1)\ 5!$
$=336/6=56$

${}^{10}C_{3}$ $=10!/3!\ 7!=(10x9x8x7!)/(3x2x1)\ 7!$
$=120$

$P(B/A)=56/120$

$P(AB)=P(A) \times P(B/A)$

$P(AB)=10/286 \times 56/120 = 7/429$

Example= (H.W)

A bag contain 6 black and 10 red balls. Two(02) successive drawings of 4 balls are made such that

i. **The balls are replaced before the second trial**

ii. **The balls are not replaced before the second trial.**

Find the probability that the first drawing will give 4 black and the second 4 red balls in each case.

Example= (H.W)

A bag contain 7 black,8 white and 9 red balls. Three(03) successive drawings of 3 balls are made such that

i. **The balls are replaced before the second & third trial**

ii. **The balls are not replaced before the second & third trial**

Find the probability that the first drawing will give 3 black and second drawing 3 white and the third 3 red balls in each case.

Complementary Event

The complement of A, denoted by A', $\bar{A}$ or A^c, consists of all the outcomes in which the event A does not occur.

$$P(A) + P(A') = 1$$

$$P(A) = 1 - P(A')$$

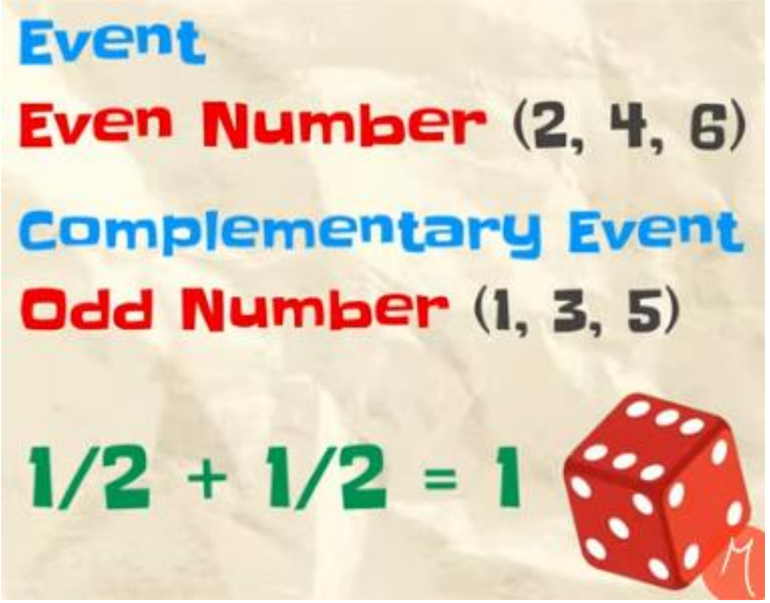
Depending on the problem, it may be easier to find $P(A')$ and then use the above equation to find $P(A)$.

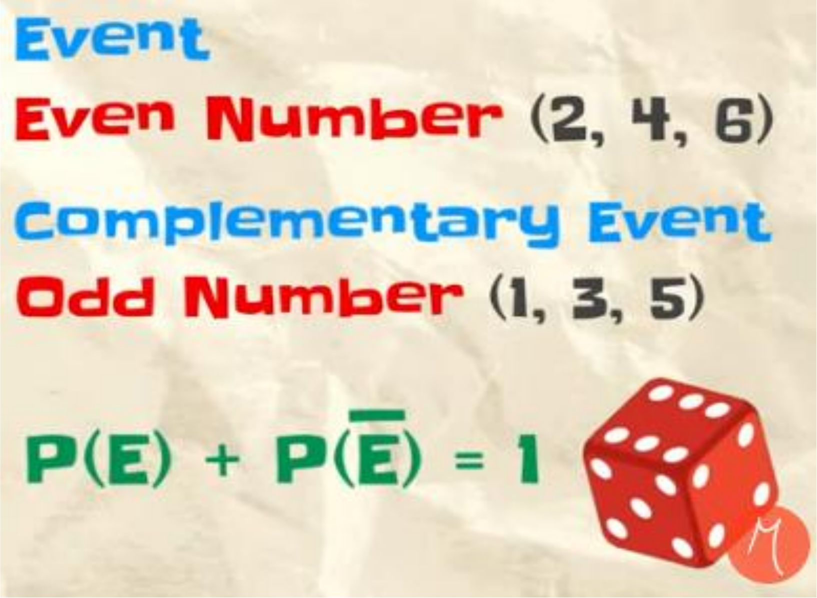

Complementary Events
Event
Even Number (2, 4, 6)
Complementary Event
(Not an Even Number)


Event
Even Number (2, 4, 6)
Complementary Event
Odd Number (1, 3, 5)
No common
Outcome


Event
Even Number (2, 4, 6)
Complementary Event
Odd Number (1, 3, 5)


Event
Even Number (2, 4, 6)
Complementary Event
Odd Number (1, 3, 5)
1, 2, 3, 4, 5, 6

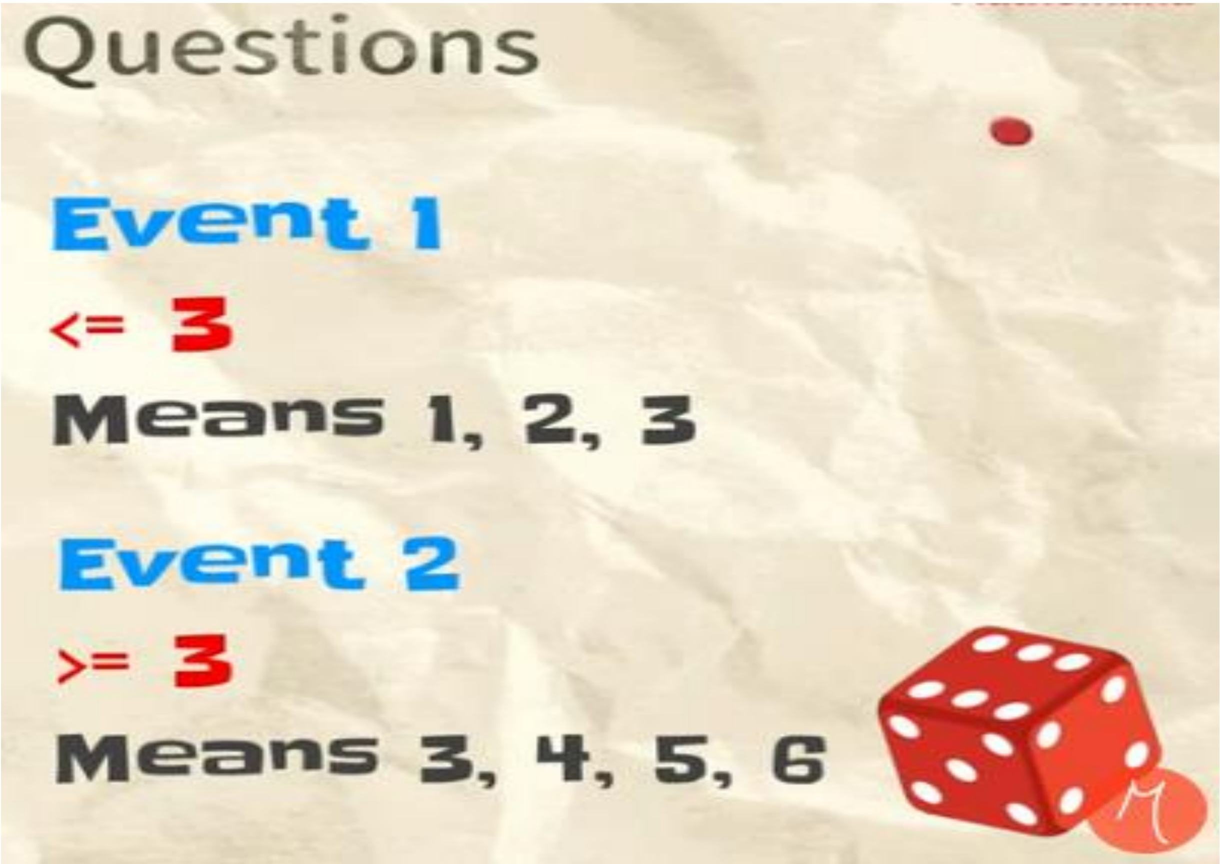

Event
Even Number (2, 4, 6)
Complementary Event
Odd Number (1, 3, 5)
1/2 + 1/2 = 1


Event
Even Number (2, 4, 6)
Complementary Event
Odd Number (1, 3, 5)
$P(E) + P(\bar{E}) = 1$

If a dice is rolled then the sample space S is given as S = {1 , 2 , 3 , 4 , 5 , 6 }. If event E_1 represents all the outcomes which is greater than 4, then E_1 = {5, 6} and E_1' = {1, 2, 3, 4}. Thus E_1' is the complement of the event E_1.


Questions
Event 1
Prime Number
2, 3, 5
Event 2
Even Number
2, 4, 6


Questions
1 ?
Event 1
Prime Number
2, 3, 5
Event 2
Even Number
2, 4, 6


Questions
Event 1
<= 3
Means 1, 2, 3
Event 2
>= 3
Means 3, 4, 5, 6

Questions

1,2,3,3,4,5,6

Event 1

<= 3

Means 1, 2, 3

Event 2

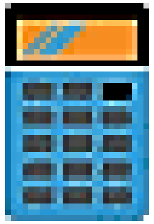
>= 3

Means 3, 4, 5, 6

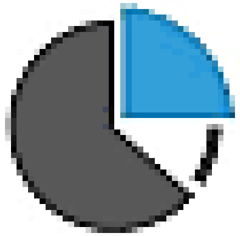

Questions
Event 1
<= 6
Event 2
> 6

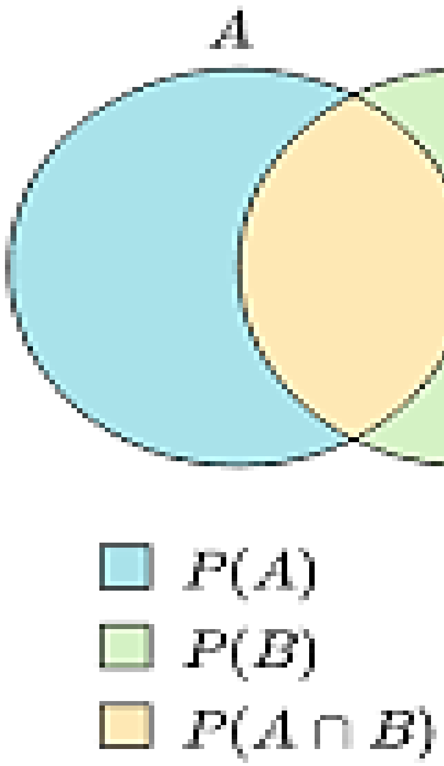

Questions
Event 1
<= 6
Means 1, 2, 3, 4, 5, 6
Event 2
> 6
Means

Conditional Probability Formula



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

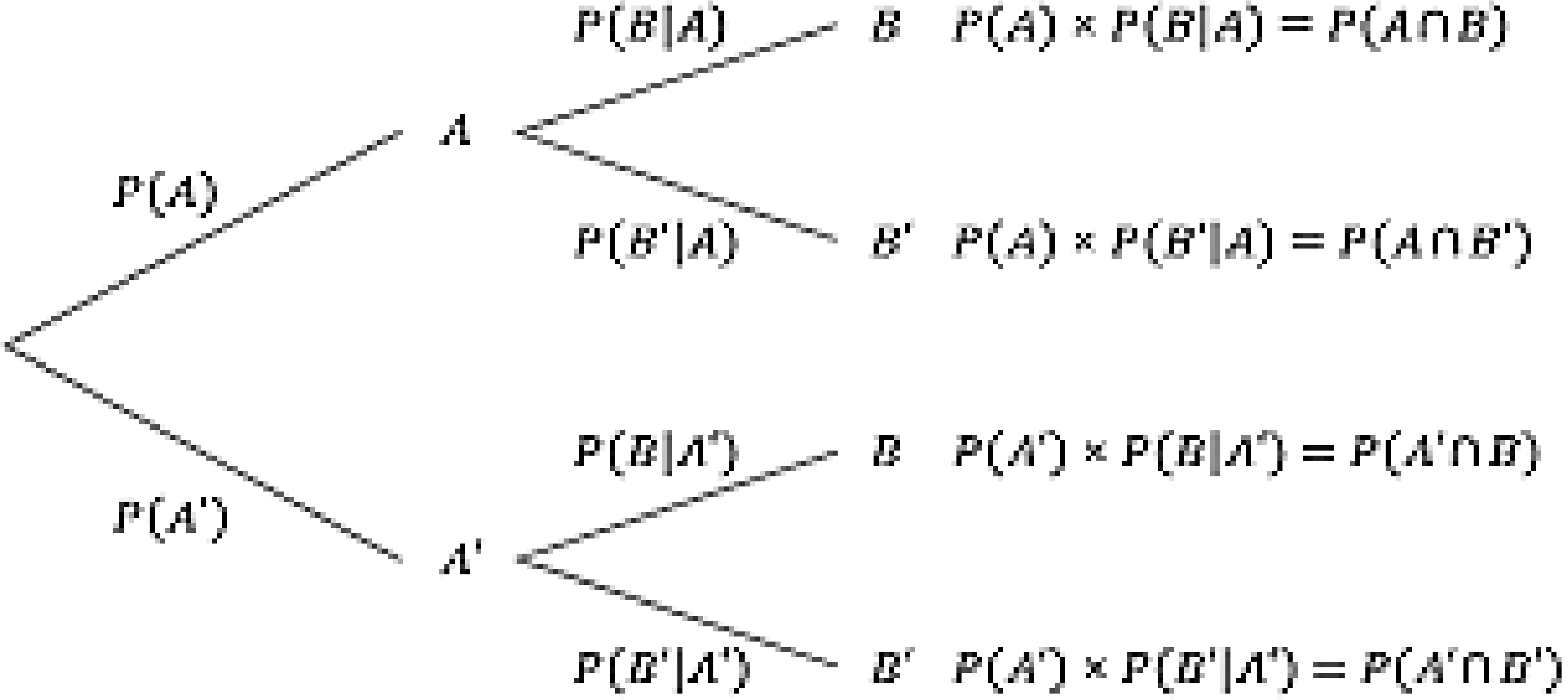




Conditional Probability Formula

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Probability that A occurs given that B has already occured


$P(A)$
$P(A')$
A
A'
$P(B|A)$
$P(B'|A)$
$P(B|A')$
$P(B'|A')$
B $P(A) \times P(B|A) = P(A \cap B)$
B' $P(A) \times P(B'|A) = P(A \cap B')$
B $P(A') \times P(B|A') = P(A' \cap B)$
B' $P(A') \times P(B'|A') = P(A' \cap B')$

Example 1:

Two Coins

Sample space{HH.HT,TH,TT}

Notation:

P(A/B)=Prob. Of A (B is already occur)

P(B/A)=Prob. Of B (A is already occur)

$P(A|B) = P(A\cap B)/P(B)$ **OR** $P(B|A) = P(A\cap B)/P(A)$

A--------Exact 2 Head A={HH}

B-------- At least 1 Head B={HH,HT,TH}

$P(A\cap B)=\{HH\}$

$P(A|B) = P(A\cap B)/P(B)=(1/4)/(3/4)=1/3$

$P(B|A) = P(A\cap B)/P(A)=(1/4)/(1/4)=1$

Example 2:

Two dies are thrown simultaneously, and the sum of the numbers obtained is found to be 7. What is the probability that the number 3 has appeared at least once?

Solution:

The sample space S would consist of all the numbers possible by the combination of two dies. Therefore S consists of 6 × 6, i.e. 36 events.

Event A indicates the combination in which 3 has appeared at least once.

Event B indicates the combination of the numbers which sum up to 7.

$A = \{(3, 1), (3, 2), (3, 3)(3, 4)(3, 5)(3, 6)(1, 3)(2, 3)(4, 3)(5, 3)(6, 3)\}$

$B = \{(1, 6)(2, 5)(3, 4)(4, 3)(5, 2)(6, 1)\}$

$P(A) = 11/36$

$P(B) = 6/36$

$A \cap B = 2$

$P(A \cap B) = 2/36$

Applying the conditional probability formula we get,

$P(A|B) = P(A \cap B)/P(B) = (2/36)/(6/36) = 1/3$

Example 3:

A die is thrown twice and the sum of the numbers appearing is observed to be 6.What is the conditional probability that the number 4 has appeared at least once?

Solution:

The sample space S would consist of all the numbers possible by the combination of two dies. Therefore S consists of 6 × 6, i.e. 36 events.

Event A indicates the combination of the numbers which sum up to 6.

Event B indicates the combination in which 4 has appeared at least once.

A = {(5, 1), (1, 5), (3, 3)(4, 2)(2, 4)}

B = {(4, 1)(4, 2)(4, 3)(4, 4)(4, 5)(4, 6)(1,4)(2,4)(3,4)(5,4)(6,4)}

P(A) = 5/36

P(B) = 11/36

$A \cap B = 2$

$P(A \cap B) = 2/36$

Applying the conditional probability formula we get,

$P(B|A) = P(A \cap B)/P(A) = (2/36)/(5/36) = 2/5$

Question 4:

A bag contains 3 red and 7 black balls. Two balls are drawn at random without replacement. If the second ball is red, what is the probability that the first ball is also red?

Solution:

Let A: event of selecting a red ball in first draw

B: event of selecting a red ball in second draw

$P(A \cap B)$ = P(selecting both red balls) = $3/10 \times 2/9 = 1/15$

$P(B)$ = P(selecting a red ball in second draw) = P(red ball and red ball or black ball and red ball)

= P(red ball and red ball) + P(black ball and red ball)

$= 3/10 \times 2/9 + 7/10 \times 3/9 = 3/10$

$\therefore P(A|B) = P(A \cap B)/P(B) = 1/15 \div 3/10 = 2/9.$

Question 5:

The probability of a student passing in science is $\frac{4}{5}$ and the of the student passing in both science and math's is $\frac{1}{2}$. What is the probability of that student passing in math's knowing that he passed in science?

Solution:

Let A ≡ event of passing in science

B ≡ event of passing in math's

Given, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{1}{2}$

Then, probability of passing math's after passing in science = $P(B|A) = P(A \cap B)/P(A)$

$= \frac{1}{2} \div \frac{4}{5} = \frac{5}{8}$

∴ the probability of passing in math's is $\frac{5}{8}$.

Question 6:

In a class, 40% of the students like Mathematics and 25% of students like Physics and 15% like both the subjects. One student select at random, find the probability that he likes Physics if it is known that he likes Mathematics.

Solution:

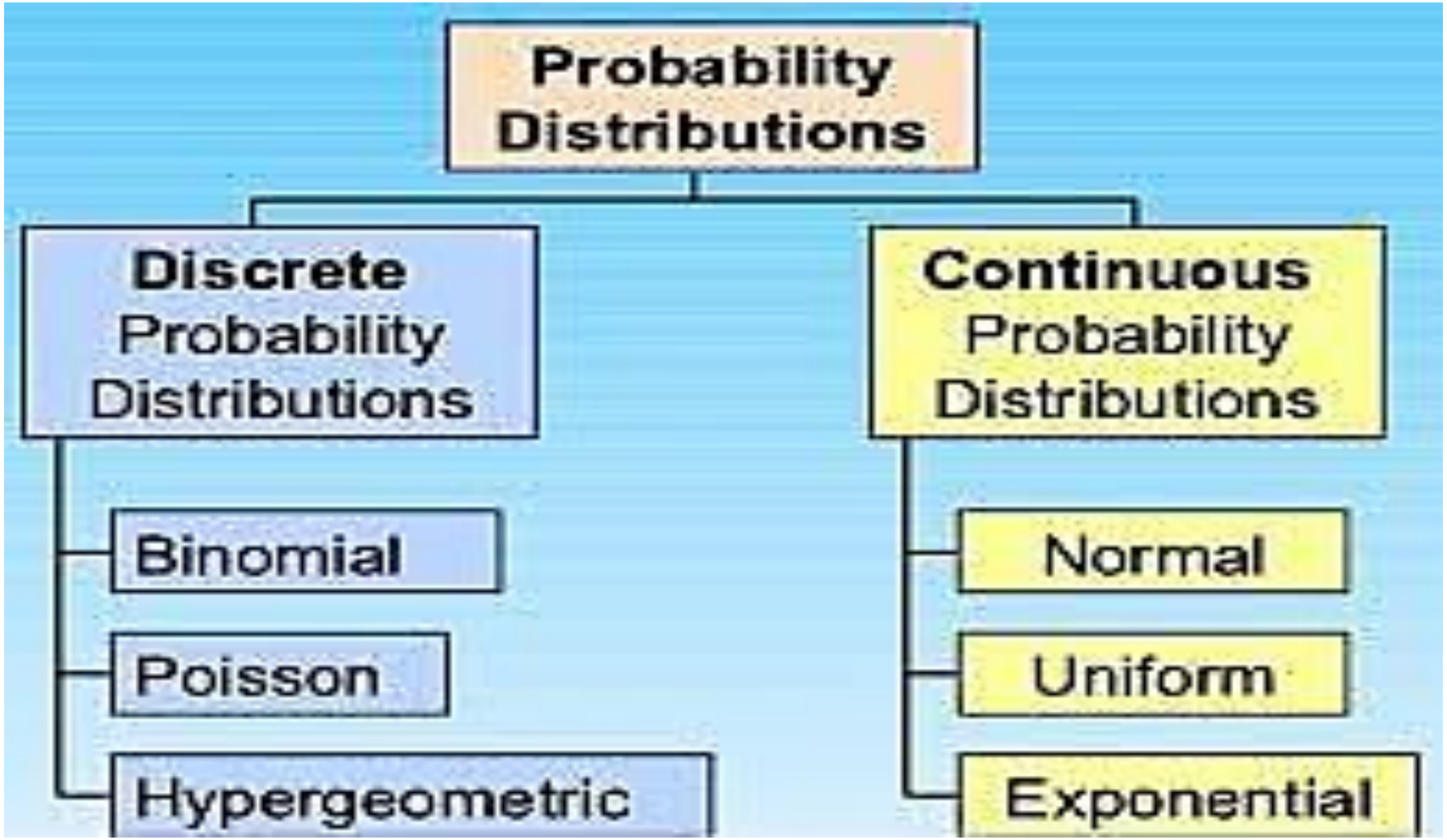
Number of students like Mathematics = n(A) = 40

Number of students like Physics = n(B) = 25

Number of students like both Mathematics and Physics = n(A ∩ B) = 15

Now, the probability that the student likes Physics if it is known that he likes Mathematics is given by –

$P(B|A) = n(A \cap B)/n(A) = 15/40 = 3/8$

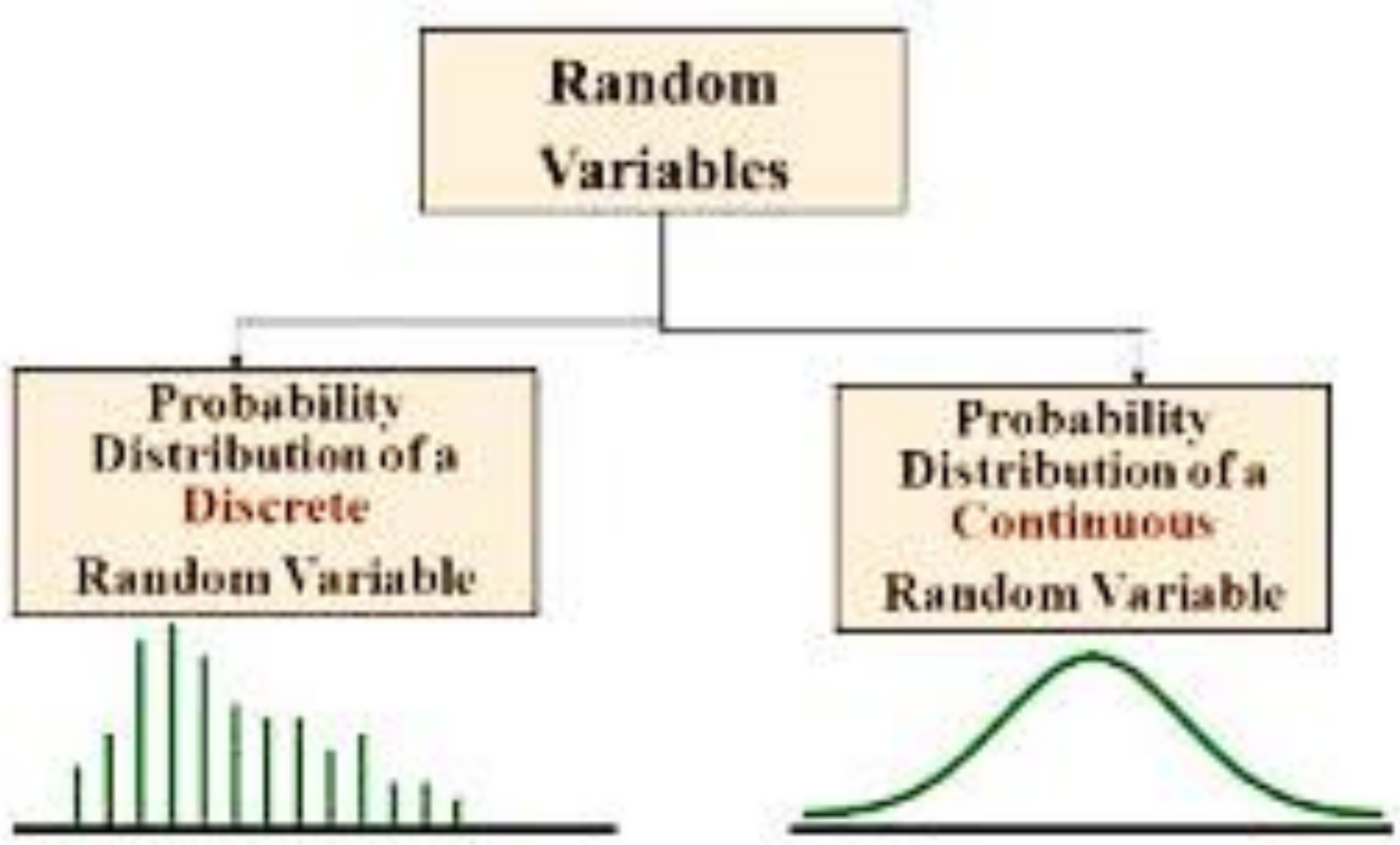

Probability Distributions
Discrete Probability Distributions
Continuous Probability Distributions
Binomial
Poisson
Hypergeometric
Normal
Uniform
Exponential

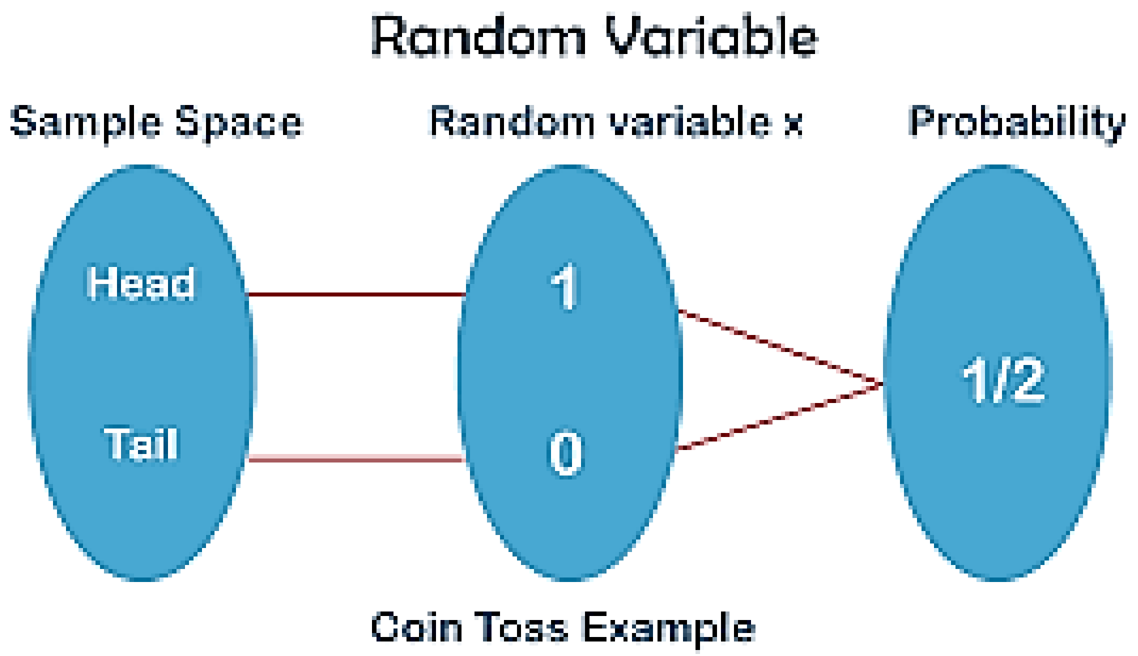
Definition

The **probability distribution** *of a discrete random variable X is a list of each possible value of X together with the probability that X takes that value in one trial of the experiment.*

The probabilities in the probability distribution of a random variable X must satisfy the following two conditions:

1. Each probability $P(x)$ must be between 0 and 1: $0 \le P(x) \le 1$.
2. The sum of all the probabilities is 1: $\Sigma P(x) = 1$.


Random
Variables
Probability
Distribution of a
Discrete
Random Variable
Probability
Distribution of a
Continuous
Random Variable


Random Variable
Sample Space
Random variable x
Probability
Head
Tail
1
0
1/2
Coin Toss Example

Find the probability distribution of

(i) number of heads in two tosses of a coin.

Let X: Number of heads

We toss coin twice

So, we can get 0 heads, 1 heads or 2 heads.

So, value of **X can be 0, 1, 2**

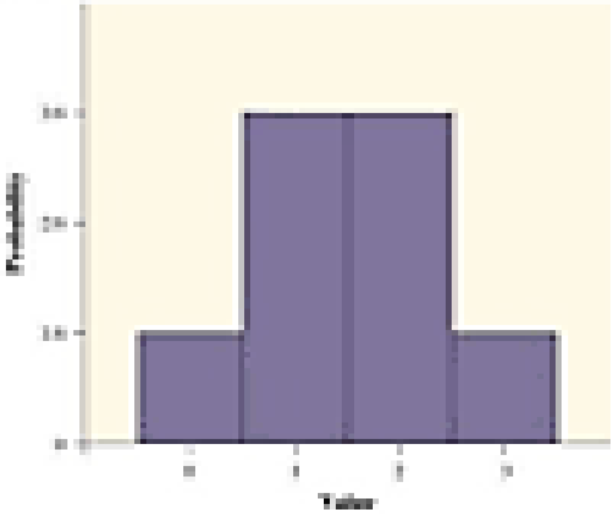
X	Outcomes	Number of outcomes	P(X)
0	{TT}	1	$\frac{1}{4}$
1	{TH, HT}	2	$\frac{2}{4} = \frac{1}{2}$
2	{HH}	1	$\frac{1}{4}$

The **probability distribution** of a discrete random variable is a list of probabilities associated with each of its possible values

Consider tossing a fair coin 3 times.
Define X = the number of heads obtained

X = 0: TTT
X = 1: HTT THT TTH
X = 2: HHT HTH THH
X = 3: HHH

Value	0	1	2	3
Probability	1/8	3/8	3/8	1/8



X	p(X)	Dice combinations
2	1/36	(1,1)
3	2/36	(1,2); (2,1)
4	3/36	(1,3); (3,1); (2,2)
5	4/36	(1,4); (4,1); (2,3); (3,2)
6	5/36	(1,5); (5,1); (2,4); (4,2); (3,3)
7	6/36	(1,6); (6,1); (2,5); (5,2); (3,4); (4,3)
8	5/36	(2,6); (6,2); (3,5); (5,3); (4,4)
9	4/36	(3,6); (6,3); (4,5); (5,4)
10	3/36	(4,6); (6,4); (5,5)
11	2/36	(5,6); (6,5)
12	1/36	(6,6)

Example:01

A fair die is tossed once. If the random variable X is the number of even numbers, find the probability Distribution of X.

X" No. of even number"

S={1,2,3,4,5,6}

P(even)=3/6 = 1/2

P(odd) =3/6 = 1/2

X	0	1
P	1/2	1/2

Example:02

Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of number of kings.

X"No. of kings"

P(X=0)=48/52 x 48/52=144/169

P(X=1)=(4/52x48/52) + (48/52x4/52)=24/169

P(X=2)=4/52x4/52=1/169

X	0	1	2
P	144/169	24/169	1/169

Example:03

The random variable X has the following probability distribution,
Determine K , P(X< 3) , P(0 < X < 3)

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	2K^2	7K^2+K

Sum of Probability=1

0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1

10k^2+9k-1=0 (Quadratic Equation)

10k^2+10k-k-1=0

10k(k+1)-1(k+1)=0

(k+1)(10k-1)=0

k+1=0 10k-1=0

k=-1 10k=1

k=1/10

P(X<3)

P(X=0)+P(X=1)+P(X=2)

0+k+2k

=3k

=3(1/10)

=3/10

P(0<X<3)

P(X=1)+P(X=2)

=k+2k

=3k

=3(1/10)

=3/10

For Any Discrete Probability Distribution: Formulas

Mean $\mu = \Sigma[x \cdot P(x)]$

Variance $\sigma^2 = [\Sigma x^2 \cdot P(x)] - \mu^2$

Std. Dev $\sigma = \sqrt{[\Sigma x^2 \cdot P(x)] - \mu^2}$

Example:04

Find the mean and variance for the following probability distribution.

X	0	1	2	3
P(X)	1/6	1/2	3/10	1/30

$$\frac{3}{5} \begin{matrix} \leftarrow \text{numerator} \\ \leftarrow \text{denominator} \end{matrix}$$

$$\mu = \Sigma[x \cdot P(x)]$$

Solution

X	P(X)*	P(X)	P(X).X	P(X).X^2
0	1/6 x 5/5	5/30	0	0
1	1/2 x 15/15	15/30	15/30	15/30
2	3/10 x 3/3	9/30	18/30	36/30
3	1/30	1/30	3/30	9/30
			$\sum P(x) \cdot x = \frac{36}{30} = 1 \cdot 2$	$\sum P(x) \cdot x^2 = \frac{60}{30} = 2$

$$\sigma^2 = \sum[P(x) \cdot x^2] - \mu^2$$

$=2 - (1.2)^2$

$=14/25 = 0.56$

Example:05

A fair coin is tossed twice. Let *X* be the number of heads that are observed.
1.Construct the probability distribution of *X*.
2.Find the probability that at least one head is observed.

Solution:

1. The possible values that X can take are 0, 1, and 2. Each of these numbers corresponds to an event in the sample space S={HH,HT,TH,TT} of equally likely outcomes for this experiment: X = 0 to {TT}, X = 1 to {HT,TH}, and X = 2 to {HH}. The probability of each of these events, hence of the corresponding value of X, can be found simply by counting, to give

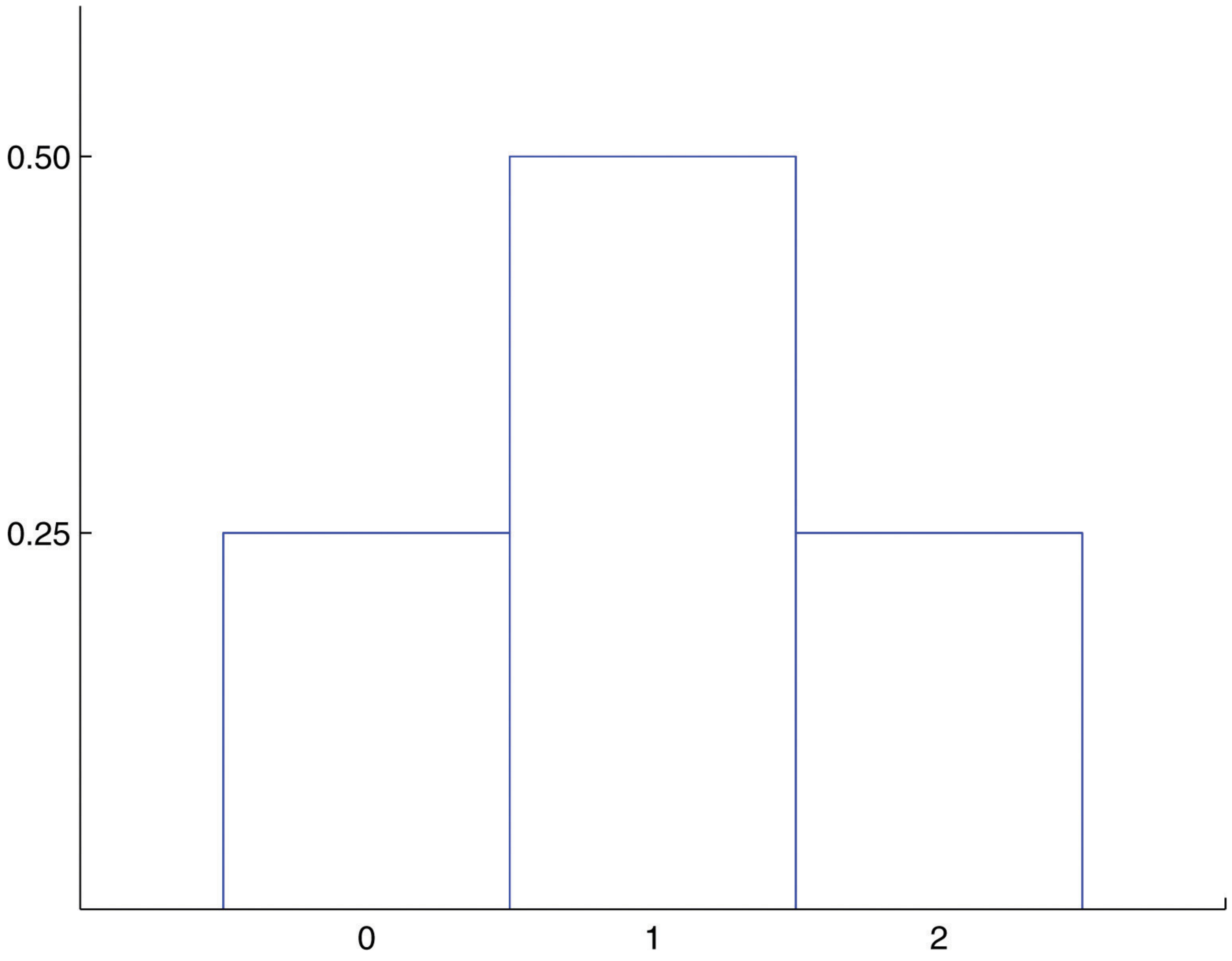
X	0	1	2
P(X)	0.25	0.50	0.25

This table is the probability distribution of X.

2. "At least one head" is the event $X \geq 1$, which is the union of the mutually exclusive events X = 1 and X = 2. Thus

$$P(X \geq 1) = P(1) + P(2) = 0.50 + 0.25 = 0.75$$

P(X)



Random Variable of X

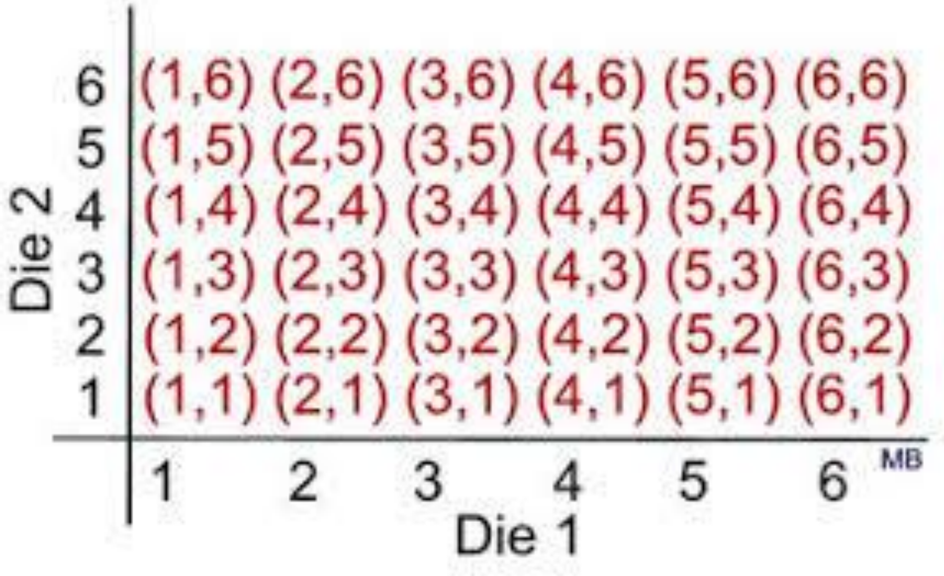
Example:06

A pair of fair dice is rolled. Let X denote the sum of the number.

1. Construct the probability distribution of X.
2. Find $P(X \geq 9)$.
3. Find the probability that X takes an even value.

Solution:

The sample space of equally likely outcomes is



X	p(X)	Dice combinations
2	1/36	(1,1)
3	2/36	(1,2); (2,1)
4	3/36	(1,3); (3,1); (2,2)
5	4/36	(1,4); (4,1); (2,3); (3,2)
6	5/36	(1,5); (5,1); (2,4); (4,2); (3,3)
7	6/36	(1,6); (6,1); (2,5); (5,2); (3,4); (4,3)
8	5/36	(2,6); (6,2); (3,5); (5,3); (4,4)
9	4/36	(3,6); (6,3); (4,5); (5,4)
10	3/36	(4,6); (6,4); (5,5)
11	2/36	(5,6); (6,5)
12	1/36	(6,6)

1. The possible values for X are the numbers 2 through 12. X = 2 is the event {11}, so P(2)=1⁄36.
X = 3 is the event {12,21}, so P(3)=2⁄36.
Continuing this way we obtain the table

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

This table is the probability distribution of X.

2. The event $X \geq 9$ is the union of the mutually exclusive events X = 9, X = 10, X = 11, and X = 12. Thus

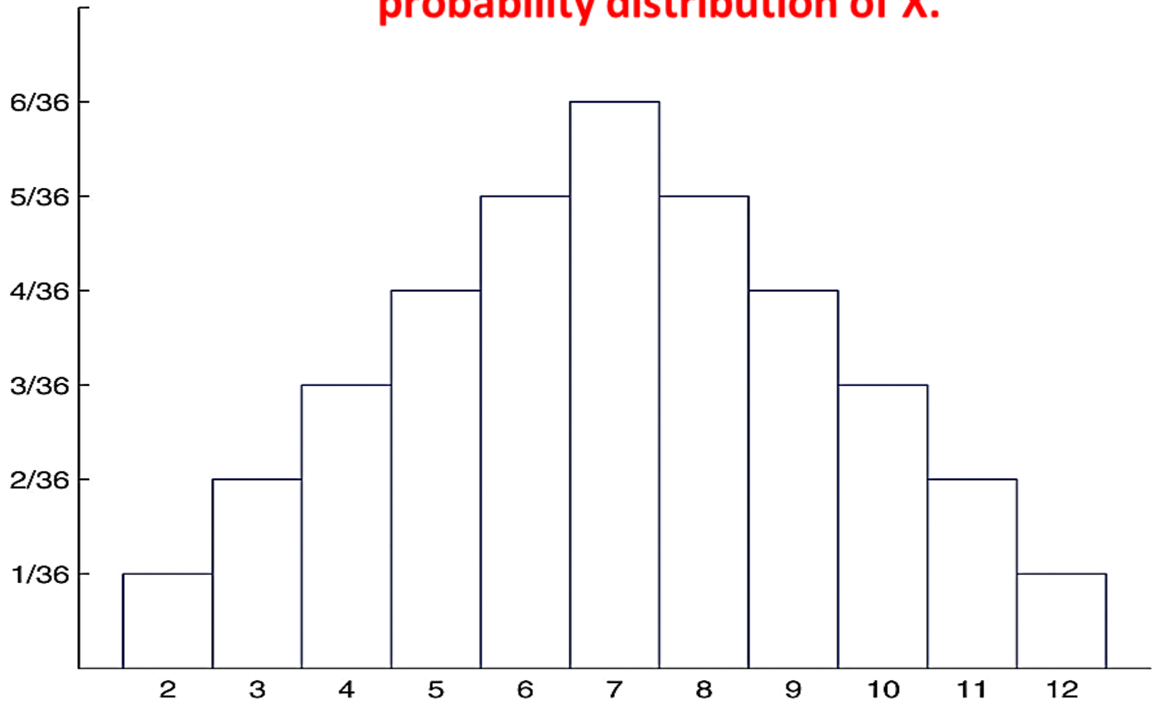
$P(X \geq 9) = P(9)+P(10)+P(11)+P(12)=4/36+3/36+2/36+1/36=10/36=0.27$

3. The probability that X takes an even value must be 0.5, note that X takes six different even values but only five different odd values. We compute

P(X is even)=P(2)+P(4)+P(6)+P(8)+P(10)+P(12)

1/36+3/36+5/36+5/36+3/36+1/36=18/36=0.5

probability distribution of X.



X	P(X)	P(X).X	P(X).X^2
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	180/36
7	6/36	42/36	294/36
8	5/36	40/36	320/36
9	4/36	36/36	324/36
10	3/36	30/36	300/36
11	2/36	22/36	242/36
12	1/36	12/36	144/36
		$\sum P(x) \cdot x = \frac{252}{36} = 7$	$\sum P(x) \cdot x^2 = \frac{1974}{36} = 54.83$

$$\mu = \sum [x \cdot P(x)]$$

$$\sigma^2 = \sum [P(x) \cdot x^2] - \mu^2$$

$$= 54.83 - (7)^2$$

$$= 54.83 - 49 = 5.83$$

$$\sigma = 2.41$$

Example:07

Find the mean and variance of the number of heads in the two tosses of a coin.

Let X: Number of heads

We toss coin twice

So, we can get 0 heads, 1 heads or 2 heads.

So, value of **X can be 0, 1, 2**

X	Outcomes	Number of outcomes	P(X)
0	{TT}	1	$\frac{1}{4}$
1	{TH, HT}	2	$\frac{2}{4} = \frac{1}{2}$
2	{HH}	1	$\frac{1}{4}$

$$\mu = \sum[x \cdot P(x)]$$

$=1$

$$\sigma^2 = \sum[P(x) \cdot x^2] - \mu^2$$

$=0.5$

Example:08

A discrete random variable *X* has the following probability distribution:

X	-1	0	1	4
P(X)	0.2	0.5	a	0.1

Find

1. a.
2. P(0).
3. P(X > 0).
4. P(X ≥ 0).
5. The mean μ of X.
6. The variance σ2 of X.
7. The standard deviation σ of X.

Solution:

1. Since all probabilities must add up to 1, $a=1-(0.2+0.5+0.1)=0.2$.
2. Directly from the table, $P(0)=0.5$.
3. From the table, $P(X>0)=P(1)+P(4)=0.2+0.1=0.3$.
4. From the table, $P(X\geq 0)=P(0)+P(1)+P(4)=0.5+0.2+0.1=0.8$
5. Mean=0.4
6. Variance=1.84
7. Std. Deviation=1.3565

Example:09

A discrete random variable X has the following probability distribution:

X	77	78	79	80	81
P(X)	0.15	0.15	0.20	0.40	0.10

Compute each of the following quantities.

a) $P(80)$.
b) $P(X > 80)$.
c) $P(X \leq 80)$.
d) The mean μ of X.
e) The variance $\sigma 2$ of X.
f) The standard deviation σ of X.

Example:10

A discrete random variable X has the following probability distribution:

X	13	18	20	24	27
P(X)	0.22	0.25	0.20	0.17	0.16

Compute each of the following quantities.

a) **P(18).**
b) **P(X > 18).**
c) **P(X ≤ 18).**
d) **The mean μ of X.**
e) **The variance σ2 of X.**
f) **The standard deviation σ of X.**