EXPERIMENT 3

Aim-To study Rayleigh and Ricean Distribution

Software used-Octave

Theory-

In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. Every multipath component follows a Gaussian distribution on the receiver side. It is well known that the envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution. envelope as a function of time. The Rayleigh distribution has a probability density function (pdf) given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right) & (0 \le r \le \infty) \\ 0 & (r < 0) \end{cases}$$

where p(r) is the rms value of the received voltage signal before envelope detection, The probability that the envelope of the received signal does not exceed a specified value R is given by the corresponding cumulative distribution function (CDF).

$$P(R) = P_r(r \le R) = 1 - \exp\left(\frac{-R^2}{2\sigma^2}\right)$$

The mean and the variance of this distribution are given by:

Mean Value r_mean

$$r_{\text{mean}} = E[r] = \int_0^\infty rp(r)dr = \sigma\sqrt{\pi/2}$$

$$= 1.2533\sigma$$

Variance. $E[r^2]$ = second central moment-square of first central moment

$$E\left[r^2\right] = E\left[x^2\right] + E\left[y^2\right] = 2\sigma^2$$

$$= \sigma\sigma^2 E\left[r^2\right] - (E[r])^2 = 2\sigma^2 \cdot \sigma(\pi/2)$$

$$= 0.4292\sigma^2$$

The median value of r is found by solving

$$\frac{1}{2} = \int_0^{rmedian} p(r)dr$$

$$r_{median} = 1.177\sigma$$

Thus the mean and the median differ by only 0.55 dB in a Rayleigh fading signal. Note that the median is often used in practice, since fading data are usually measured in the field and a particular distribution cannot be assumed. By using median values instead of mean values it is easy to compare different fading distributions which may have widely varying means. Figure 18 illustrates the Rayleigh pdf.

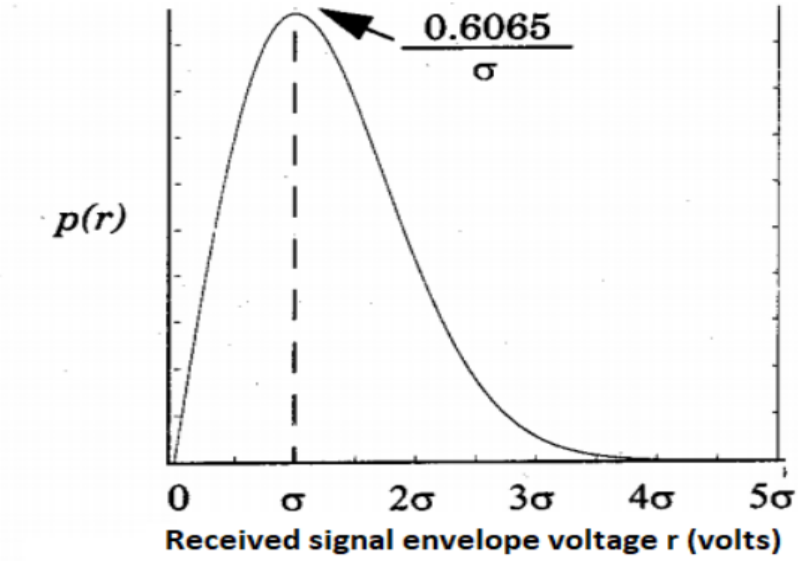

Figure 18: Rayleigh fading characteristics

Ricean Fading Distribution

When there is a dominant stationary (nonfading) signal component present, such as a line-of-sight propagation path, the small-scale fading envelope distribution is Ricean. In such a situation, random multipath components arriving at different

angles are superimposed on a stationary dominant signal. At the output of an envelope detector, this has the effect of adding a dc component to the random multipath. Just as for the case of detection of a sine wave in thermal noise , the effect of a dominant signal arriving with many weaker multipath signals gives rise to the Ricean distribution. As the dominant signal becomes weaker, the composite signal resembles a noise signal which has an envelope that is Rayleigh. Thus, the Ricean distribution degenerates to a Rayleigh distribution when the dominant component fades away.

The Ricean distribution is given by

$$p(r) = \begin{cases} \frac{r}{2} e^{-\frac{\left(r^2+A^2\right)}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) & \text{for } (A \geq 0, r \geq 0) \\ 0 & \text{for } (r < 0) \end{cases}$$

The parameter A denotes the peak amplitude of the dominant signal and Io is the modified Bessel function of the first kind and zero-order. The Ricean distribution is often described in terms of a parameter K which is defined as the ratio between the deterministic signal power and the variance of the multipath.

$$K(dB) = 10 \log \frac{A^2}{2\sigma^2} dB$$

The parameter K is known as the Ricean factor and completely specifies the Ricean distribution.

As $\mathrm{A}->0, \mathrm{K}->\infty \mathrm{dB}$, and as the dominant path decreases in amplitude, the Ricean distribution degenerates to a Rayleigh distribution.

Figure 19 shows the Ricean pdf. The Ricean CDF is compared with the Rayleigh CDF.

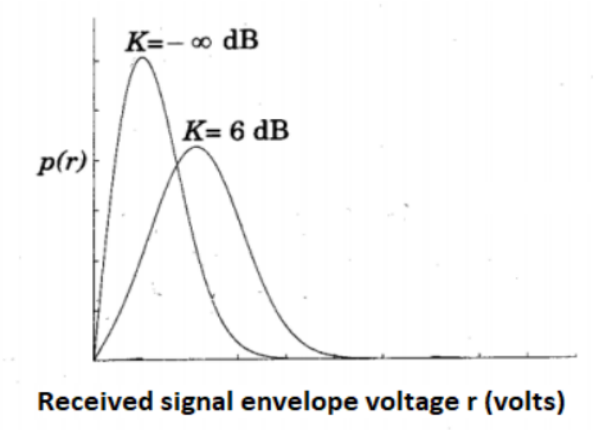

Figure 19: Rayleigh and Rician distributions

Conclusion