

## Assignment due November 25th before class

CSCE 2202: Analysis and Design of Algorithms

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*Implementation is optional, unless otherwise stated. Design with clear comments and pseudo-code is enough. Do not forget to derive your complexities. It is good practice for you if you decide to implement.*

### Meals Calculator (8 points)

Your friend who decided lately to stick to a healthy lifestyle went to a nutrition specialist who confined him to eat certain meals and gave him the freedom to organize these meals as he likes, as long as he eats **exactly** a certain amount of calories per day (not less and not more).

Your friend is a little confused on what to do and is complaining about having a limited amount of options. You want to motivate him by showing him **how many options** he actually has every day. So you will build a calculator that will take the calories of the allowed meals and the calorie allowance on a day and tell your friend how many options he has.

Examples:

Calories of allowed meals = {300, 400, 600}

Calorie allowance = 1300 calories

# varieties = 2 (either 1 of each meal or 3 meals of 300 and 1 of 400)

Calories of allowed meals = {200, 500, 300, 600}

Calorie allowance = 1000 calories

# varieties = 5 ( 5 meals of 200, or 2 meals of 200 and 2 meals of 300 or 2 meals of 200 and 1 meal of 600, 1 meal of 200, 1 meal of 300 and 1 meal of 500, or 2 meals of 500)

**Assuming that your friend has infinite access to all meal types, implement the calculator.**

### Cyberfight (7 points)

In a cyberattack event, ethical hackers are brought in to be digital detectives that are required to track malicious hackers down. You are given a list of different zones, each either dominated by an ethical hacker (E) or a malicious hacker (M). Assume you know which type dominates each zone. You also know that any ethical hacker can only detect malicious hackers that are **at most k** zones away from them.

The event starts and the requirement is that **each ethical hacker catches at most one malicious hacker**. The malicious hackers will “hide” by constantly changing their hacking techniques while the ethical hackers will try to track them down by also trying out different cybersecurity defense strategies. To optimize, you decide to give hints to the ethical hackers by telling them which malicious hacker they should focus on. So you start matching ethical hackers to zones that you know are occupied by malicious hackers. Design an efficient algorithm that determines the maximum number of matches you will be able to make.

## Shortest Path Revisited (5 points)

A weighted graph  $G(V,E)$  with  $V = (A \dots F)$  and  $E = (AB, AD, AF, BC, BD, BE, BF, CD, DE, EF)$  has edge weights  $(1,12,16,1,10,11,14,9,1,1)$ , respectively.

- Trace Dijkstra's algorithm to determine the shortest paths from node (A) to all other nodes.
- Modify the algorithm to produce a **count of the number of edges** for each of the shortest paths and implement it for any general graph.

## Water System for the New Capital (5 points)

You're helping engineers in a construction project in Egypt's new capital city install water pipes and fittings in homes. Water pipes come in various lengths and shapes and you can virtually connect any two pipes with no difference in function but at varying cost. The cost of connecting two pipes is equal to the sum of their respective lengths. You would like to connect all pipes into one pipe to secure the pipework at minimum cost. Design an algorithm that takes the lengths of  $n$  pipes and returns the minimum cost of joining the pipes into one pipe system. What is the worst-case complexity of your algorithm?

## Covid Peaks (8 points)

One wave of COVID-19 is characterized by the following pattern: a daily increase in positive cases, where each day brings additional cases until the wave reaches its maximum. Sometimes the maximum stays for a couple of days before the decrease starts happening. Once a decline happens, the number of cases keeps decreasing. If the number increases again, this marks the start of a new wave.

**Implement** a function that receives a time series of daily number of cases in New York (multiple waves could exist), then returns the total length of the peak periods (= total number of days the cases remained at the maximum of their respective waves). Derive the complexity of your design.

## Parallel Building of MST (3 points)

Consider a complete weighted graph with 512 vertices. You want to build the minimum spanning tree using parallel programming. You will divide the adjacency matrix into equal submatrices and assign each to one of 256 processors to compute the MST for the corresponding subgraph in parallel with the other processors. After that, you combine the sub-trees into one MST. Is your approach better than building the MST using Kruskal's algorithm? Explain your answer by using complexity analysis as reasoning.

## Genetic Inheritance (6 points)

Suppose that a given biological species acquires a certain genetic feature with probability  $x$ . For subsequent generations, the probability becomes 70% of the average of probabilities for past generations. For example, the 0th generation will have  $x$  probability, then the 1st generation will have  $0.7x$  probability, then the 2nd generation will have  $0.7*((1 + 0.7)/2)x$ , and so on.

Write a recursive brute force algorithm which receives  $x$  to compute the probability for a given generation  $g$  and find the number of arithmetic operations for it. (Pseudo-code is enough)

Using a DP approach, **implement** an algorithm that improves the efficiency of this computation and give its complexity.

## Midterm After-Party (8 points)

After your second midterm, we want to hang out together at the gardens again but multiple classes will be doing so at the same time. AUC set a policy that students from the same class can sit close to each other but there needs to be enough social distancing between groups of different classes. They also will not allow more than 10 classes in total to occupy the space.

So they plan a 2D grid of the garden and let everyone report where they plan to sit. Because all students miss the gardens, they try as much as possible to plan together to optimize the space, such that every student is sitting next to at least one other student from the same class and a whole class will leave at least one empty space between itself and other classes. Assume the edges for the grid are reserved for security.

The students mark an X at their reserved spots then AUC asks for your help to take the grid and count the classes on the grid to report whether that is within their threshold.

Example

Grid[ ][ ] =

{0, 0, 0, 0, 0, 0, 0, X},

{0, X, X, X, X, 0, 0, X}, // The X's in red mark the classes.

{0, X, 0, X, 0, 0, 0, X}, // There are two classes but one of them has only one student.

{0, X, X, X, X, 0, X, 0}, // The X's in blue mark security locations that do not count

{0, 0, 0, 0, 0, 0, 0, X}

Design an algorithm to take the grid and do the counting for you.