Computational Geometry
Chapter-0
(Orientation)

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What is Geometry?

- **Geometry is a branch of mathematics that studies the sizes, shapes, positions, angles and dimensions of objects.**

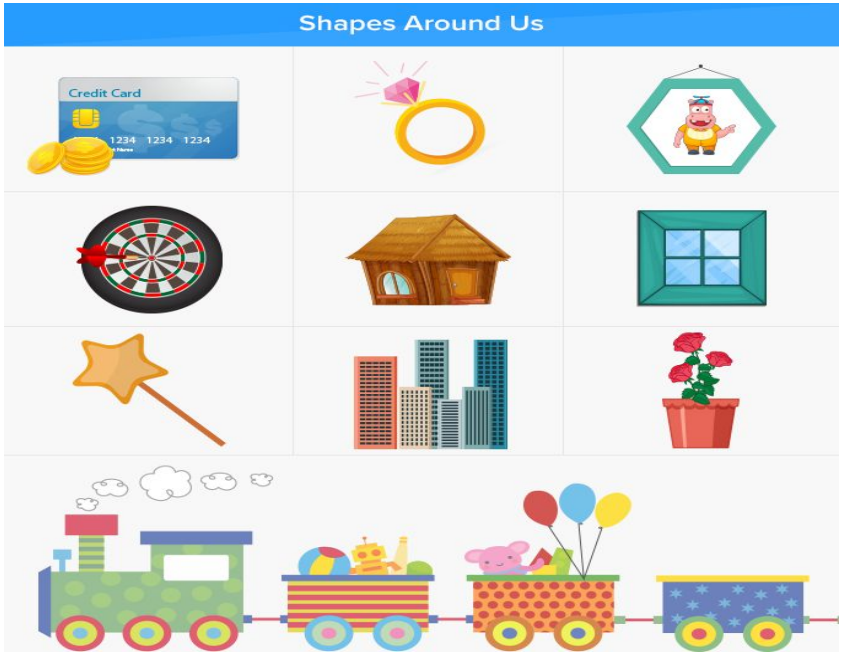
  Putting simply, Geometry is all about the shapes and their properties.

- **Everything we see in the world has a shape. Some shape has only length and width. They are known as 2D (two dimensional) shapes.**

- **There are some shapes which have an extra dimension height besides length and width. They are known as 3D dimensional shapes.**

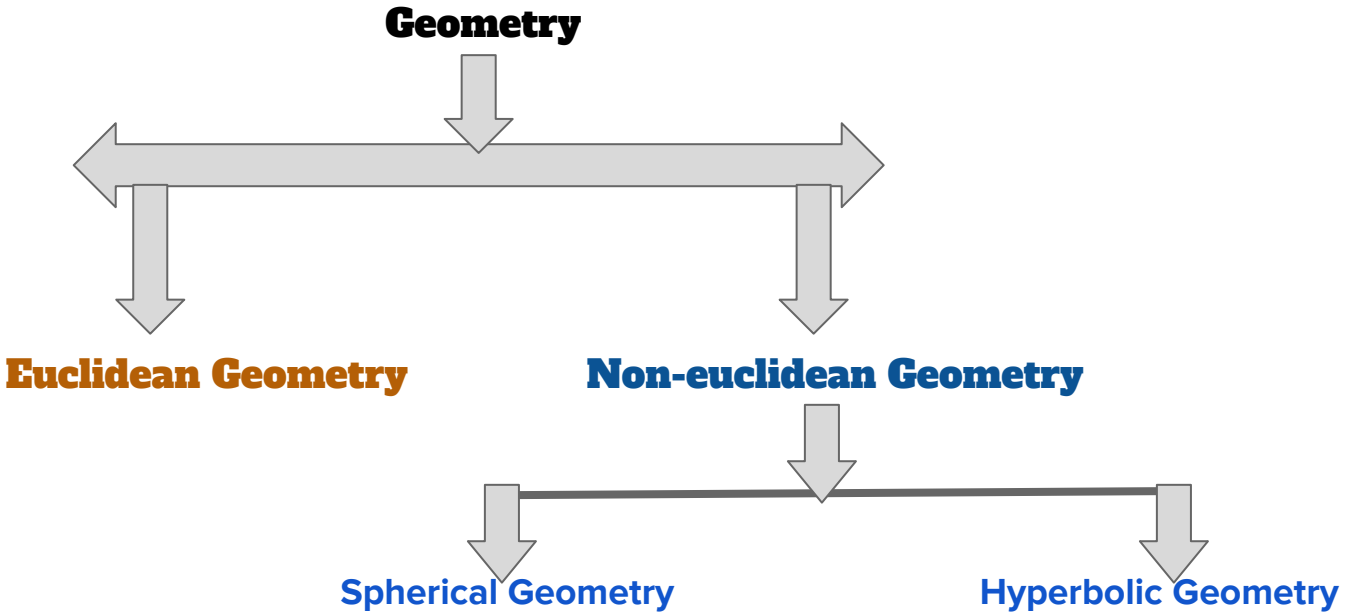
What is Geometry?

- **We can find different basic geometric shapes appearing in objects such as credit cards, bills and coins, finger rings, photo frames, dart boards, huts, windows, magician's wands, tall buildings, flower pots, toy trains, and balloons.**

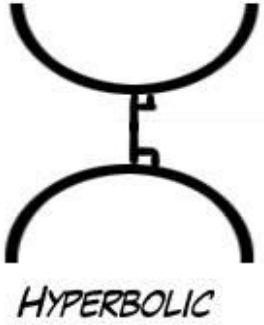


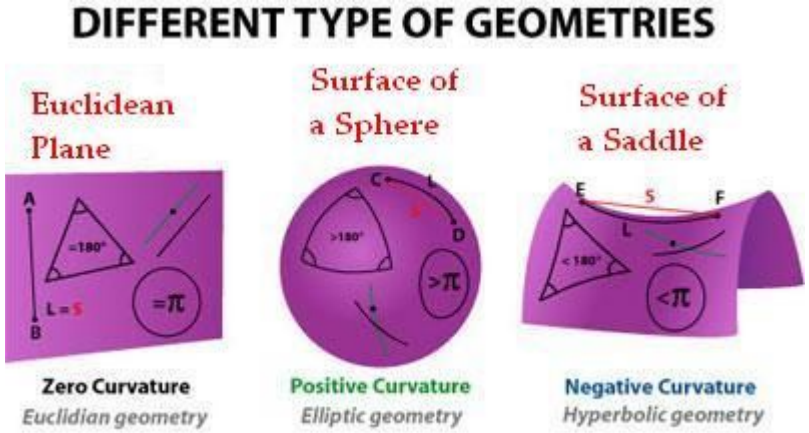
Category of Geometry

- **(Based on dimension) Geometry can be divided into two types:**



Category of Geometry





Euclidean Geometry

- **The type of geometry we typically learn in school is known as Euclidean Geometry.**

- **Euclidean geometry is named after the ancient Greek mathematician Euclid who wrote axioms and postulates in order to explain the properties of geometric objects in space of two and three dimensions in his book called The Elements over 2,000 years ago.**

- **In Euclidean geometry, geometric objects are thought to be in the plane (flat) surface. Hence, it is also known as Plane Geometry.**

- **What is Geometric Object? (The geometrics shapes we see around us)**
  - ★ **Points, Lines**
  - ★ **Triangles, rectangles, circles, polygons, solids (cube, cuboid, cone, cylinder) and so on.**

Euclidean Geometry

- **Axiom:**
  - ★ **An axiom is a statement based on assumption, which is universally considered to be true but has no clearly defined proof (only from the intuition, we can perceive it).**
  - ★ **Everybody agrees with it, but nobody can prove that it is correct or disprove that it is incorrect.**
  - ★ **In a more formal note, an axiom is given as a proposition which is self-evidently true.**
  - ★ **Example: Euclid's 5th axiom "The whole is greater than the part" is evident to anybody as a true statement.**

Euclidean Geometry

- **Postulate:**
  - ★ **A postulate is the same as an axiom, a proposition which is self-evidently true.**
  - ★ **Example: Euclid's 1st postulate "A straight line segment can be drawn connecting two distinct points given".**

- **Difference between axiom and postulate:**
  The difference between the terms axiom and postulates is not in its definition but in the perception and interpretation. An axiom is a statement, which is common and general, and has a lower significance and weight. A postulate is a statement with higher significance and relates to a specific field. Since an axiom has more generality, it is often used across many scientific and related fields.

Axiom is an archaic (much) older term while postulate is a new term in mathematics.

Euclidean Geometry

- **Some of the Euclid's Axioms:**

★ **Things which are equal to the same thing are equal to one another.**
★ **If equals are added to equals, the wholes are equal.**

★ **If equals are subtracted from equals, the remainders are equal.**
★ **Things which coincide with one another are equal to one another.**

★ **The whole is greater than the part.**

★ **Things which are double of the same things are equal to one another.**
★ **Things which are halves of the same things are equal to one another**

Euclidean Geometry

- **Euclid's Five Postulates:**
  - ★ **Postulate-1: A line segment can be drawn connecting two distinct points given.**
  - ★ **Postulate-2: A line segment can be extended in either direction to form a line.**
  - ★ **Postulate-3: A circle can be drawn with any center and radius.**
  - ★ **Postulate-4: All right angles are equal to one another (congruent).**
  - ★ **Postulate-5: If two lines are intersected by a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines will intersect each other on that side if produced indefinitely.**

Euclidean vs. Non-euclidean Geometry

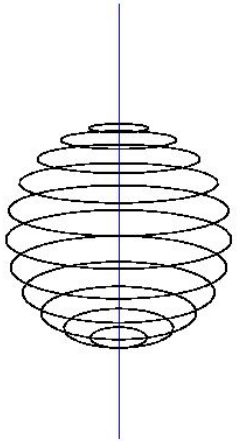
- **Euclid's Fifth Postulate (simpler version)**
  - ★ **In euclid's geometry, lines are straight and go on forever. Given a line and a point not on the line, you can draw only one parallel line.**
  - ★ **The shortest distance between two points is a straight line.**
  - ★ **The sum of the angles of a triangle is 180 degrees.**

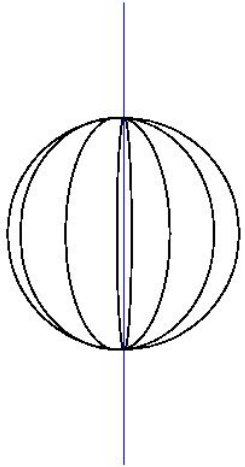
Everything is placed or built on a flat or plane surface.

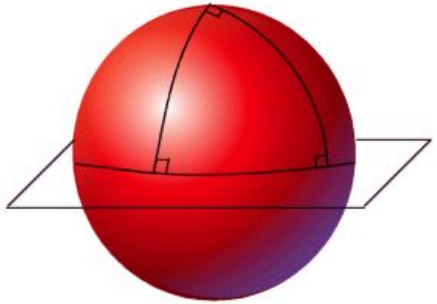
But the world is not flat. So euclidean geometry always does not work for things in the world which brings us the introduction to non-euclidean geometry (spherical geometry and hyperbolic geometry)

Non-euclidean Geometry

- **Spherical Geometry**
  - ★ Everything is built on the surface of a sphere. In spherical geometry, lines are circles (great circles) on the sphere that have the same center as the sphere.
  - ★ Given a line and a point not on the line, you can draw no parallel lines.
  - ★ The shortest distance between two points is a great circle.
  - ★ The sum of the angles of a triangle is always greater than 180 degrees.



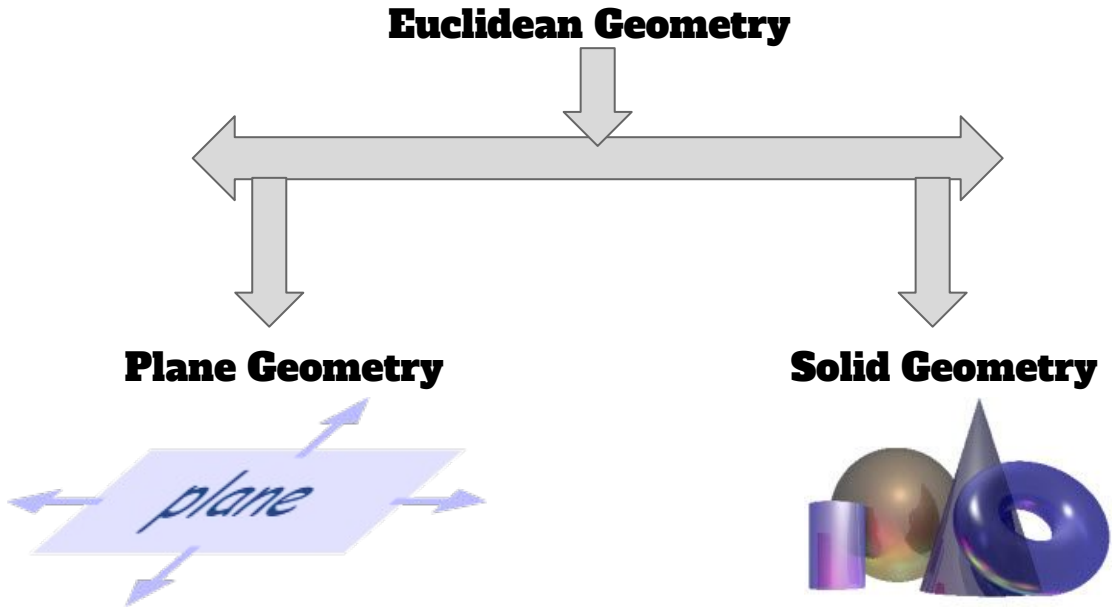




Non-euclidean Geometry

Category of Geometry (Euclidean)

- **(Based on dimension) Euclidean geometry can be divided into two types:**



Category of Euclidean Geometry

- **Plane Shape Geometry:**
  it is all about flat shapes such as points, lines, circles, triangles, polygons etc. A point has no dimension or 0-dimensional, line is 1-dimensional and triangle, circles, polygons are 2-dimensional.

- **Solid Geometry:**
  It is about the study of three dimensional objects such as prisms, cube, cuboid, pyramid, cylinder, and so on.

So, What is Computational Geometry!!!

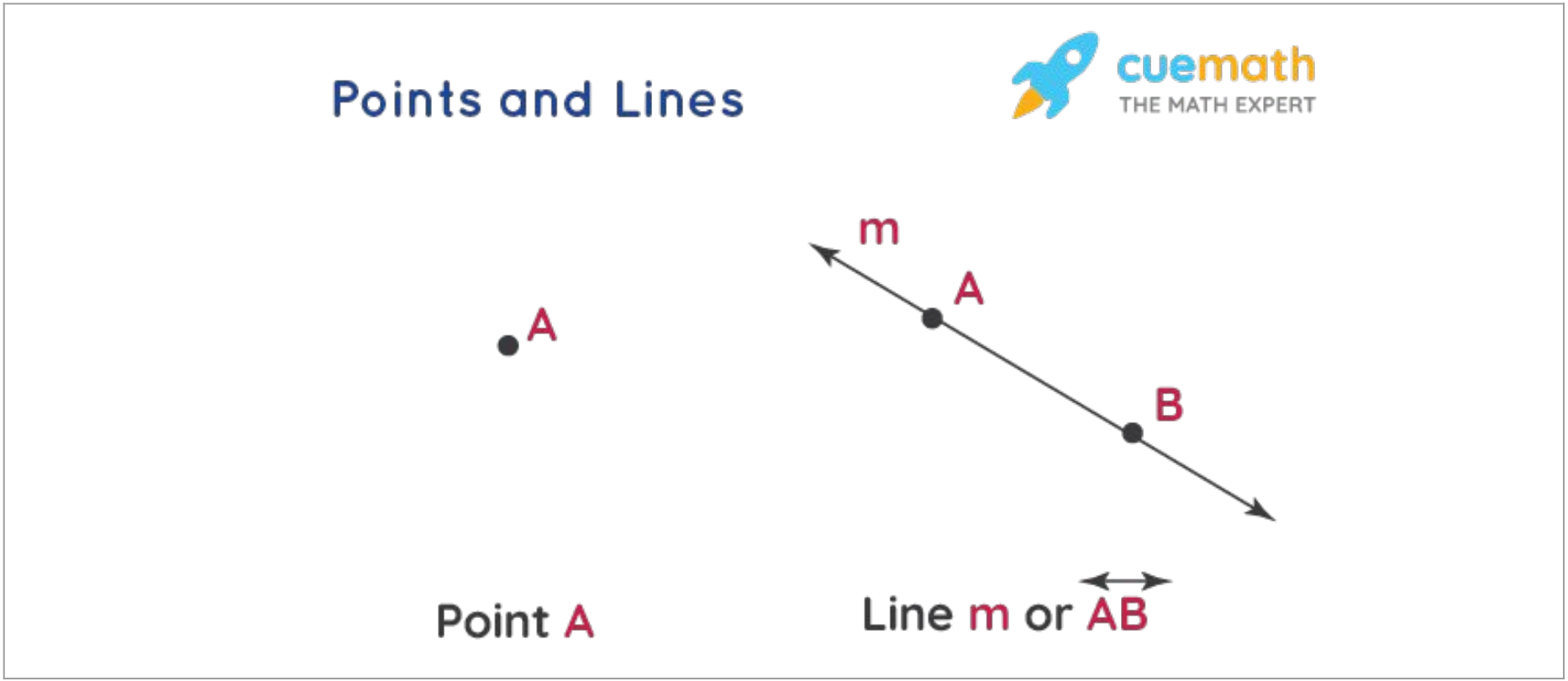
Point, Line, Angle and Plane Surface (Plane Geometry)

- **Point:**
  A point is defined as a location in any space and is represented by a dot (.). It does not have any length, height, shape, or size.

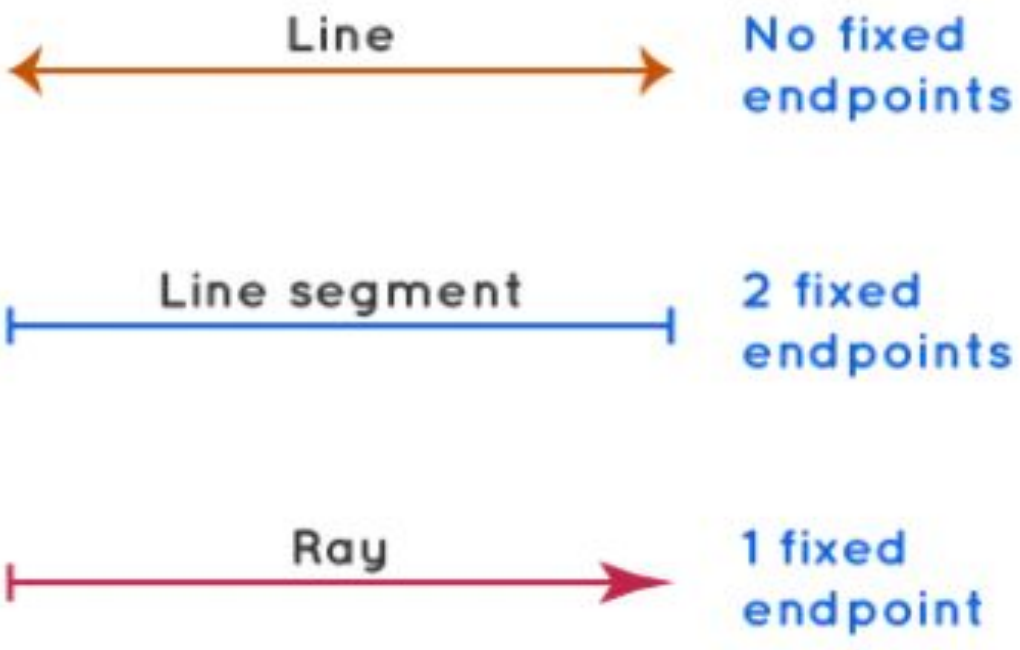
- **Line:**
  - ★ **A series of points connected by a straight path is defined as a line. A line is usually defined by two points.**
  - ★ **A line has no thickness and can extended or produced indefinitely in both directions.**
  - ★ **The length of a line is undefined and it can have infinite numbers of points.**
- **Line Segment:**
  A line segment is a part of a line that has two endpoints and a fixed length.

So, What is Computational Geometry!!!

Point, Line and Plane Surface (Plane Geometry)



Point, Line and Plane Surface (Plane Geometry)



Category of Points and Lines (Plane Geometry)

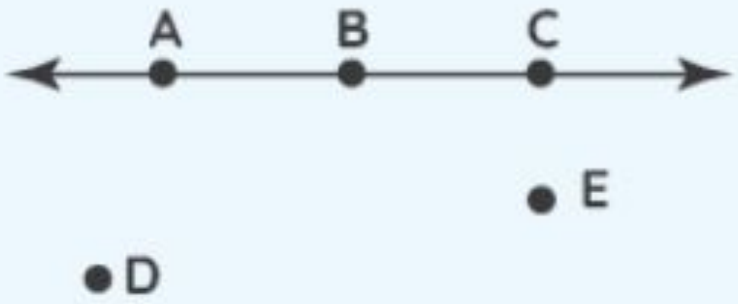
- **Category of Points:**
  Points can be classified into several types: such as collinear and non-collinear points as well as coplanar and non-coplanar points.

  ★ **Collinear and Non-collinear Points:**
  If three or more points lie on a single straight line then the points are called collinear points. If the group of points do not lie on the same line then those points are called non-collinear points.

  ★ **Coplanar and Non-coplanar Points:**
  If a group of points lie on the same plane then they are said to be coplanar points. A set of points that do not lie on the same plane are non-coplanar points.

Category of Points and Lines (Plane Geometry)

Collinear Points and Non-Collinear Points



Here,
A, B, C are Collinear Points
D, E are Non-Collinear Points

Coplanar Points and Non-Coplanar Points

Here,
A, B, C are Coplanar Points
A, B, C and D are Non-Coplanar Points

Category of Points and Lines (Plane Geometry)

- **Types of Lines:**
  There are several types of lines that can distinguished easily based on their unique properties. Such as:

  ★ **Horizontal Line – A line that is mapped from left-to-right or right-to-left and is parallel to the x-axis in a plane is called a horizontal line.**

  ★ **Vertical Line – A line that is mapped from up to down or down to up and is parallel to the y-axis in a plane is called a vertical line.**

  ★ **Intersecting Lines – When two lines cross each other and meet at a point, they are known as intersecting lines. The point at which they meet is known as the point of intersection.**

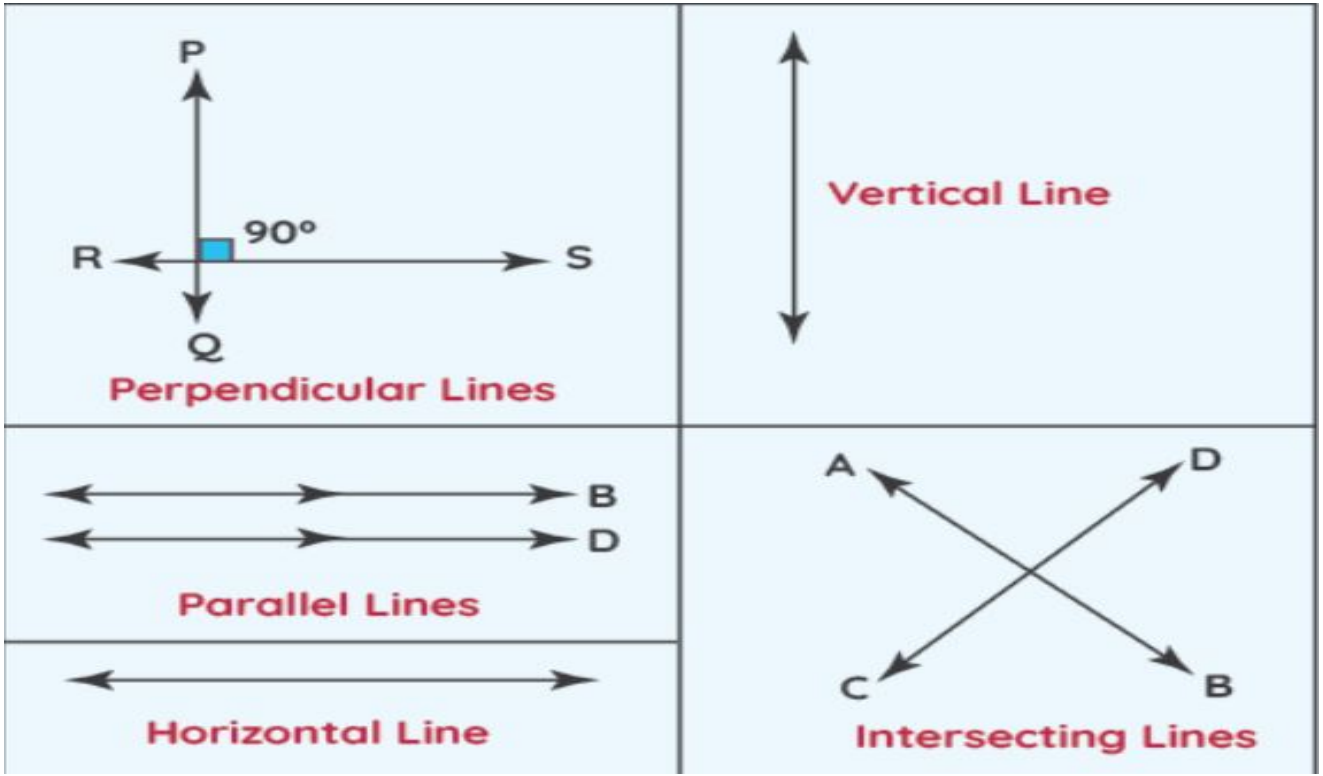
Category of Points and Lines (Plane Geometry)

- **Types of Lines:**
  There are several types of lines that can distinguished easily based on their unique properties. Such as:

  ★ **Perpendicular Lines– When two lines intersect exactly at 90°, they are known as perpendicular lines.**

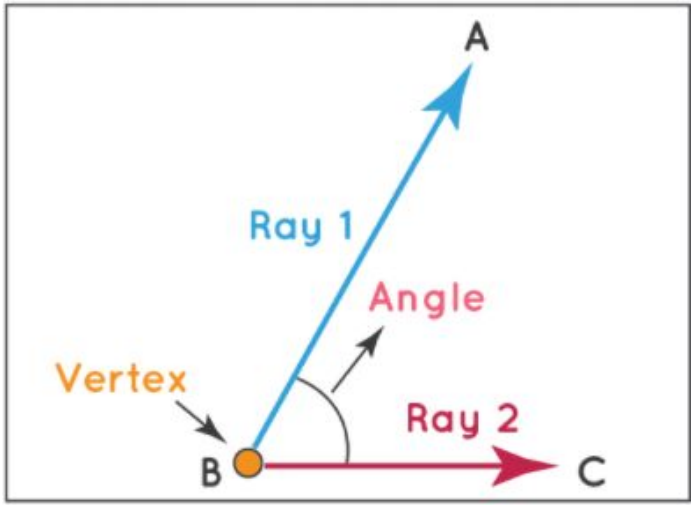
  ★ **Parallel Lines – Two lines are said to be parallel if they do not intersect at any point and are equidistant.**

Category of Points and Lines (Plane Geometry)



Angle and its Categories (Plane Geometry)

- **Angle:**
  When two rays intersect at a point, they form an angle. Angles are usually measured in degrees and denoted by ° (the degree symbol), which is a measure of rotation. An angle can have a value between 0° to 360° and it is denoted by the symbol ∠.Observe the following figure which shows ∠ ABC



Angle and its Categories (Plane Geometry)

- **Types of Angles:**

  Angles can be categorized into different types based on their measurements. Angles are generally of 6 types:

  ★ **Acute angle: If the measure of an angle is less than 90° then it is known as an acute angle.**

  ★ **Obtuse angle: If the measure of an angle is greater than 90° but less than 180°, then it is known as an obtuse angle.**

  ★ **Right angle: If the measure of an angle is exactly equal to 90° then it is known as a right angle.**

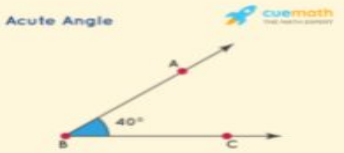
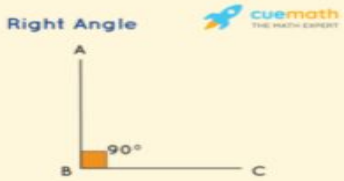
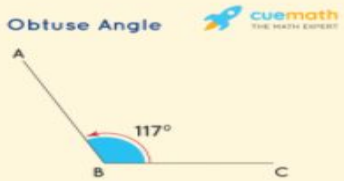
Angle and its Categories (Plane Geometry)

- **Types of Angles:**

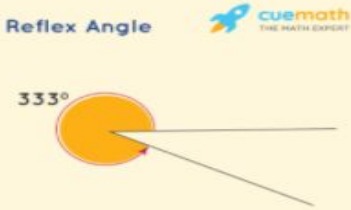
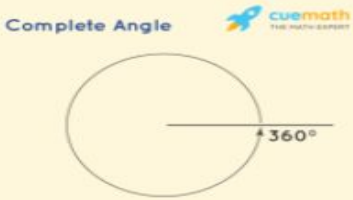
  Angles can be categorized into different types based on their measurements. Angles are generally of 6 types:

  - ★ **Straight angle - If the measure of an angle is 180° then it is known as a straight angle.**
  - ★ **Reflex angle - If the measure of an angle is greater than 180° but less than 360°, then it is known as a reflex angle.**
  - ★ **Complete angle - If the measure of an angle is 360° then it is known as a complete angle.**

Angle and its Categories (Plane Geometry)

Type of Angles	Measurements	Illustration
Acute Angle	Here, (∠ABC = 40°) < 90°	
Right Angle	Here, ∠ABC = 90°	
Obtuse Angle	Here, 90° < (∠ABC = 117°) <180°	

Angle and its Categories (Plane Geometry)

Straight Angle	Here, ∠AOB = 180°	
Reflex Angle	180° < (Reflex Angle = 330°) < 360°	
Complete Angle	Angle = 360°	

Computational Geometry

- **Computational Geometry:**
  - ★ **It is a branch of computer science that studies algorithms for solving geometric problems.**
  - ★ **In modern engineering and mathematics, computational geometry has applications in diverse fields such as:**
    - ❖ **Computer Graphics**
    - ❖ **Robotics**
    - ❖ **VLSI Design**
    - ❖ **Computer Aided Design**
    - ❖ **Molecular Modeling**
    - ❖ **Textile Layout, Forestry and Statistics**

Last but not the least, for competitive programming!!

Input and Output to Computational Geometry Problems

- **Input:**
  A description of a set of geometric objects such as a set of points, a set of line segments or the vertices of a polygon in counterclockwise order.

- **Output:**
  A response to a query about the objects. For example,

  ★ if a point is in clockwise or counterclockwise order w.r.to two given points.
  ★ if two line segments intersect.
  ★ whether a point lies inside a given polygon or not.

Using Geometric algorithms, we can prepare responses to these query?? Ans: yes!

Contents

Chapter No.	Chapter Title	Description
CGCH-01	Basic Geometry	1. triangle and its category 2. Congruence of triangles 3. Similarity of triangles 4. Circles and their related theorems 5. Areas related to circles (areas of segment and sector of a circle) 6. Law of sine of triangle 7. Law of cosine of triangle 8. Circumcircle 9. Incircle 10. Application of binary search in geometry 11. Application of ternary search/differentiation in geometry

Contents

Chapter No.	Chapter Title	Description
CGCH-02	Points, Vectors, Lines and Line Segments	1. Point and line representation 2. Counterclockwise Function (CCW) 3. Vector representation 4. Dot product 5. Cross product 6. Lattice points 7. Closest pair of points (naive approach, divide & conquer) 8. Line segments intersection checking 9. Intersection point of lines 10. Finding intersection of two segments 11. Length of union of line segments 12. Minimum lines to cover all points 13. Area of union of triangles, vertical decomposition method

Contents

Contents

Chapter No.	Chapter Title	Description
CGCH-02	Polygon	1. Polygon and its classifications 2. Area of simple polygon with ordered vertices 3. Point in polygon checking 4. Art gallery theorem 5. Tangents between two convex polygons 6. Check if point belongs to convex polygon in O(logn) 7. Minkowski sum of convex polygon 8. Pick's theorem 9. Number of lattice points inside lattice polygon 10. Lattice points of non-lattice polygon 11. Finding the incircle in a convex polygon using ternary search in O(Nlog2C) 12. Catalan number 13. Non-intersecting chords in circle 14. Ways of polygon triangulation

Contents

Chapter No.	Chapter Title	Description
CGCH-02	Polygon	1. Minimum score polygon triangulation using matrix chain multiplication 2. Monotone chain, Monotone polygon, monotone mountain 3. Triangulating monotone polygon 4. Trapezoidalization of polygon 5. Centers of gravity of polygon and polyhedra

Contents

Chapter No.	Chapter Title	Description
CGCH-04	Convex Hull	1. Convex hull construction 1.1. Jarvis Algorithm or Wrapping 1.2. Monotone chain algorithm 1.3. Graham scan 1.4. Quick hull algorithm 1.5. Divide and conquer 2. Convex hull trick and Li Chao tree 3. Dynamic convex hull-adding points to an existing convex hull 4. Deleting points from convex hull

Contents

Chapter No.	Chapter Title	Description
CGCH-05	Sweep line	1. Search for a pair of intersecting segments 2. Point location in O(logn) 3. Closest pair of points

Contents

Chapter No.	Chapter Title	Description
CGCH-05	Geometric data structure	1. K-d trees 2. Orthogonal range searching 3. Priority search trees

Contents

Chapter No.	Chapter Title	Description
CGCH-06	Proximity	1. Delaunay triangulation 2. Voronoi diagram 3. Half plane intersection-S&I algorithm in O(nlon) 4. Geometric inversion transform

Computational Geometry

Basic Geometry

Computational Geometry