Computational Geometry

Date: 07/05/23

Lec-1

What is geometry

Shape of geometric object (Properties)

Surface (types of surface)
- flat surface
- Curve surface

~~Eve~~ Based on surface, geometric objects

Geometry
- Euclidean (Flat Surface) or ~~plane~~ Surface Plane (Plane Geometry)
- Non-Euclidean (Curve Surface)

→ The elements

Axiom → It is a proposition which is always considered true.
Postulate — Has no proof ~~oit~~ of its being ~~true~~ true.

Axiom:

1. $a = b$, $b = c$ or, $a = c$

2. $x = 2z$, $y = 2z$ or $x = y$

3. $x = \frac{1}{2}z$; $y = \frac{1}{2}z$ $\therefore x = y$

4. $a = b$ —— (i)

$x = y$ —— (ii)

$\therefore a + x = b + y$

Axiom is general statement. It is true for all branch.

5. $a = b$ —— (i)

$x = y$ —— (ii)

$\therefore a - x = b - y$

6. The total is greater than the part.

$5 + 7 = 12$

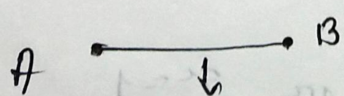
Postulate: It is also an axiom which is field specified.

Example: The sum of three angles is 180: it is a proposition which is only true for geometry.

Dimension → It is also a variable.

Point is zero dimensional.

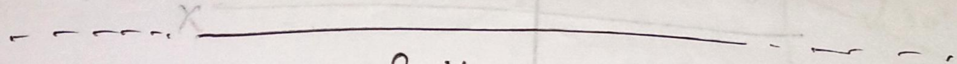
1. A line segment can be drawn connecting two different points.



Line segment

Properties of line segment
→ fixed length

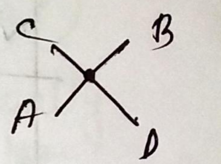
2. Any line segment can be extended in both direction to form a line.



A line is infinite. It has no fixed length.

3. A circle can be drawn by any radius and center.

4. Line realize করতে ২৫ line segment এর perspective এ.

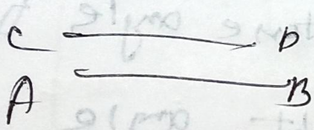


Intersecting lines

Intersecting line: If two lines meet in a common point.

If two line segment have one common point, then they are ~~ina~~ intersecting line.

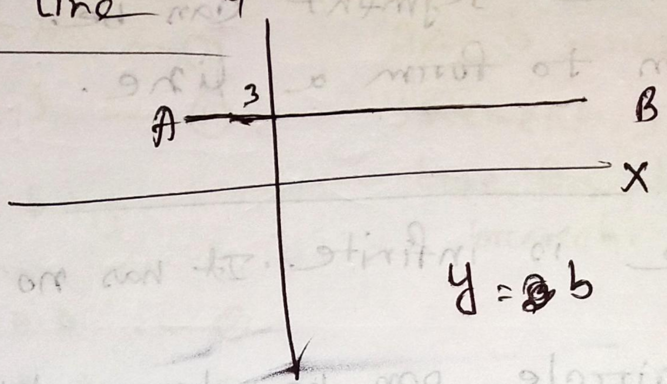
Non intersecting line: Two lines must be parallel to each other



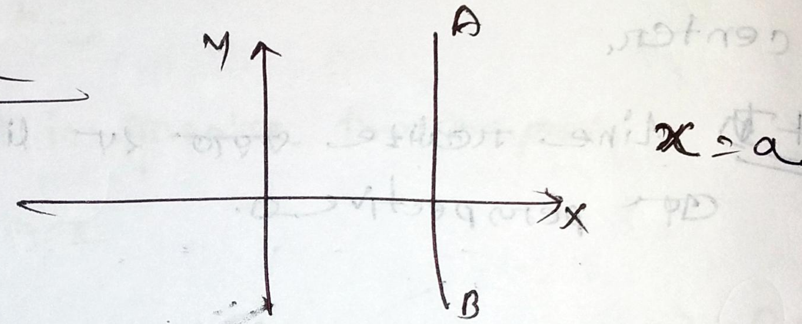
$AB \parallel CD$

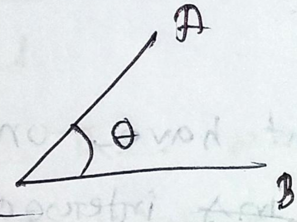
Equidistant lines:

Horizontal line



Vertical line





$\angle ACB$ or $\angle C$

1) $\theta < 90^\circ \rightarrow$ acute angle

2) $\theta > 90^\circ \rightarrow$ obstuse angle

3) $\theta = 90^\circ \rightarrow$ Right angle

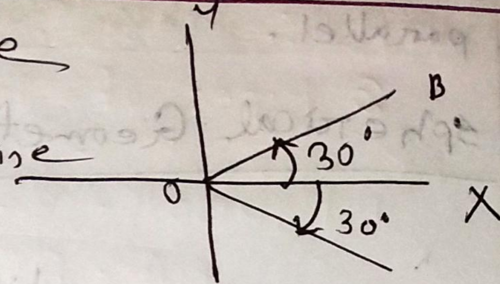
4) $\theta = |180^\circ| \rightarrow$ Straight angle

5) $\theta > 180^\circ$ and $\theta < 360^\circ \rightarrow$ reflex angle

6) $\theta = 360^\circ \rightarrow$ complete angle

$\theta = +30° \rightarrow$ anticlockwise

$\theta = -30 \rightarrow$ ~~anti~~clockwise



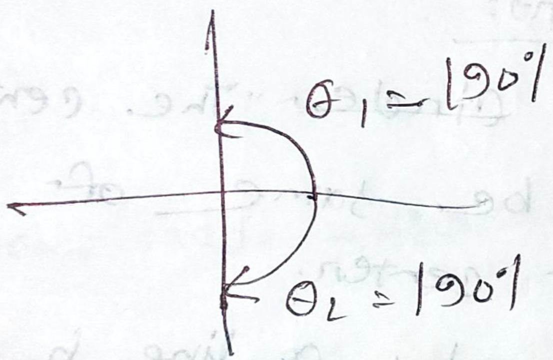
4. If $\theta_1 = 90°$, $\theta_2 = 90°$, $\theta_3 = 90°$

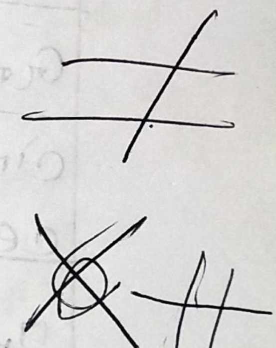
$\boxed{\theta_1 = \theta_2 = \theta_3}$

All 90° angles are equal. (equality / equity)

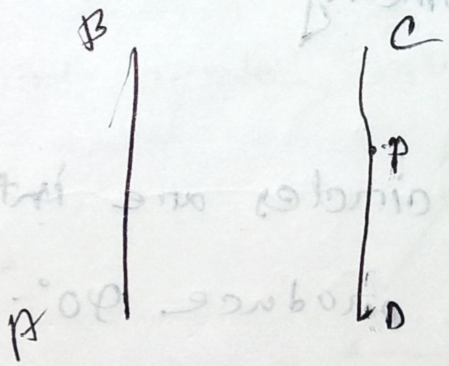
↑ Eventually equality

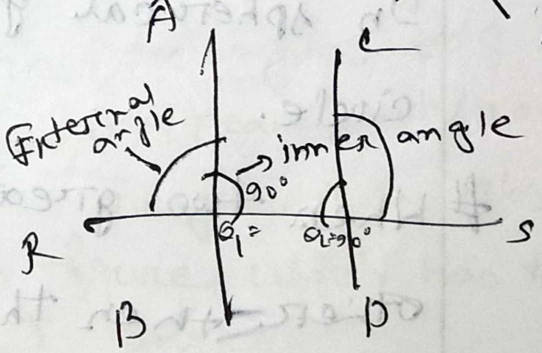
Congruent: Being equal for all perspective.





5.





$\theta_1 + \theta_2 = 180$

if, $\theta_1 + \theta_2 = 180°$ then two lines are parallel.

$\theta_1 + \theta_2 \neq 180°$, then they will be intersecting line not parallel.

Spherical Geometry → (Surface based)

Non euclidieam Geometry

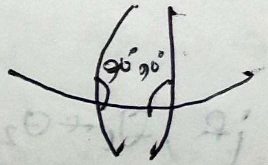
- Spherical elliptic (Positive curvature)
- Hyperbolic (Negative curvature.

Spherical Geometry:

Great circle : ~~circle~~ The center of circle should be same of the ~~vertical~~ center spheres center.

In spherical geometry, a line is a great circle.

When two great circles are intersected each other then they produce 90°.



Euclid 5th postulate is not valid for spherical geometry.

The sum of 3 angles of = 180° is not valid for spherical geometry.

<u>Computational Geometry:</u>

Lec - 2

Date: 18/05/23

~~Eucl~~

Euclidean Geometry

→ 2D

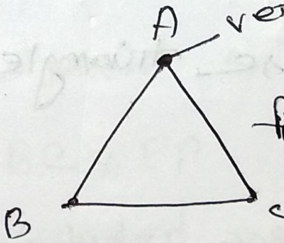
→ 3D / Solid Geometry

Flat / local area → ~~flat~~ Euclidean Geometry

<u>Geometric object:</u> Any geometric shape. like point, circle, square, triangle etc.

vertex / peak point / node

<u>Triangle:</u>



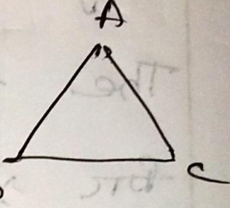
Triangle is a closed figure which has three angles.

Pair of intersecting line, AB, BC
AC, BC
AB, AC

Types of triangle:

3 types of triangle according to side.

i) Equilateral triangle



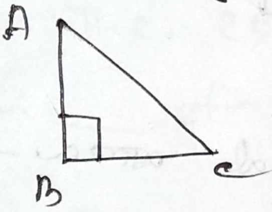
AB = AC = BC

ii) If two sides are equal only,
Isosceles triangle

iii) If three sides are not equal,
Scalene triangle.

3 types of triangle according to angle

i) Right triangle

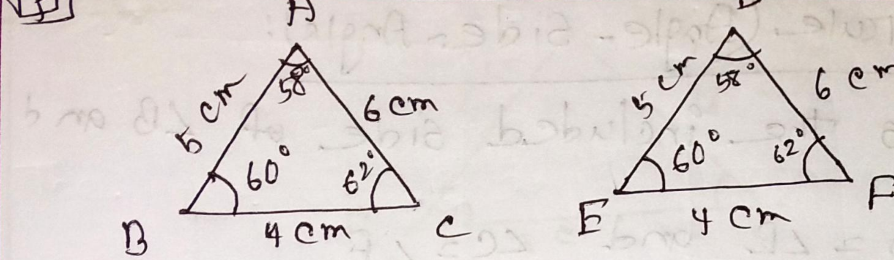


$\angle B = 90^\circ$

ii) Acute triangle, every angles are less than 90°

iii) Obtuse triangle.

[I]



These two triangles have same shape and size. They will fit other perfectly. These triangles are called congruent triangle.

$\triangle ABC \cong \triangle DEF$ [$\triangle ABC \cong EDF$ This is wrong] Because E is not corresponding point of A.

$\angle BAC = \angle EDF$ / $\angle A = \angle D$

$\therefore BC = EF$ [corresponding side]

$\angle B = \angle E$ [corresponding angle]

[II] There are some rules to determining congruent triangle:

i) SAS rule (side - Angle - Side)

AB and BC are adjacent sides of $\angle B$.

Included angle, adjacent side

If $AB = DE$ and $BC = EF$

and ~~$\angle B = \angle E$~~ included angle $\angle B = \angle E$

$\therefore \triangle ABC \cong \triangle DEF$

(ii) ASA rule (Angle-Side-Angle):

BC is the included side of $\angle B$ and $\angle C$

If $\angle B = \angle E$ and $\angle C = \angle F$

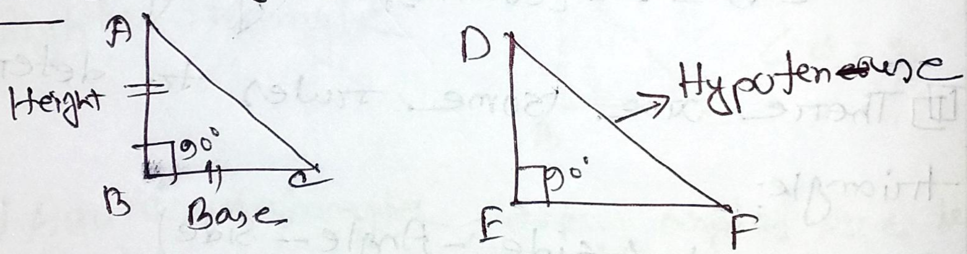
and included side $BC = EF$

$\therefore \triangle ABC \cong \triangle DEF$

(iii) SSS rule (Side-Side-Side):

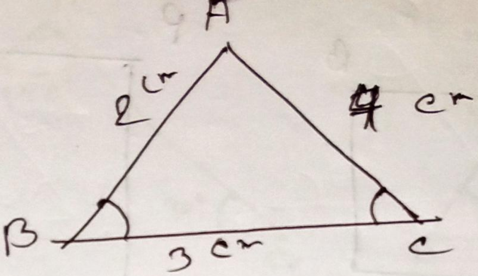
If all the three sides are equal they are congruent.

** (iv) RHS: (Right Hypotenuse Side)



$AC = DF$ and $AB = DE$

Theorem:



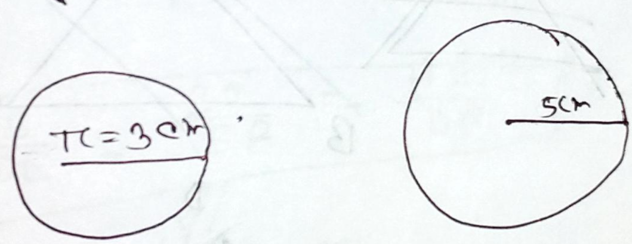
(1) The sum of any two side is greater than 3rd side.

(11) The side opposite to greater angle is always greater

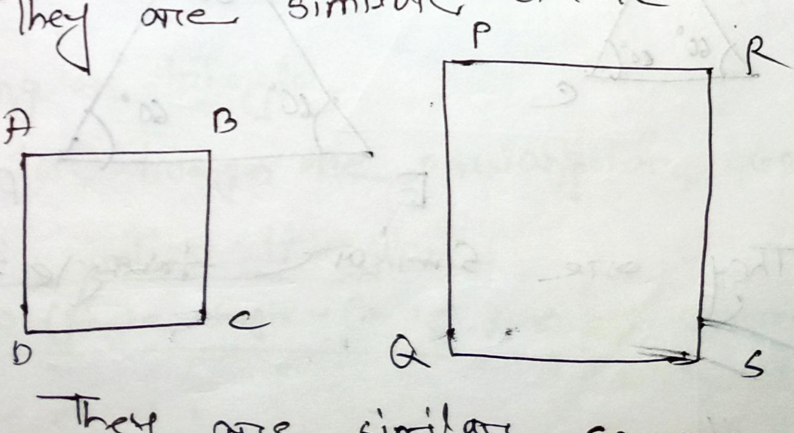
$\angle B > \angle C$

$AC > AB$

Similarity of geometric objects: They look similar but they are not same ~~shape or~~ size.

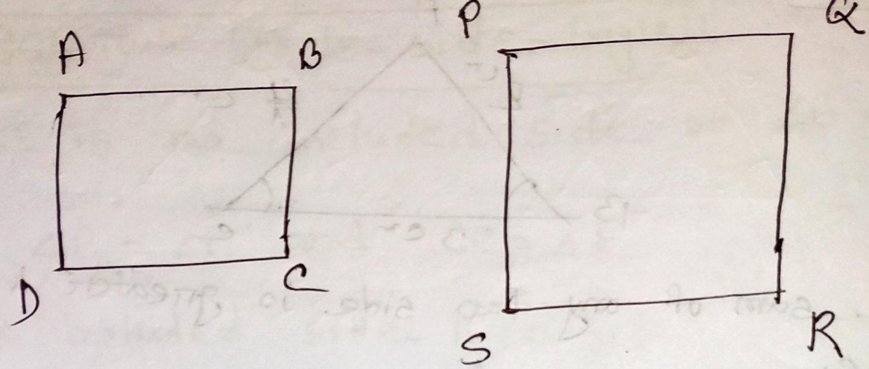


They are similar circle



They are similar square

$ABCD \sim PRSQ$



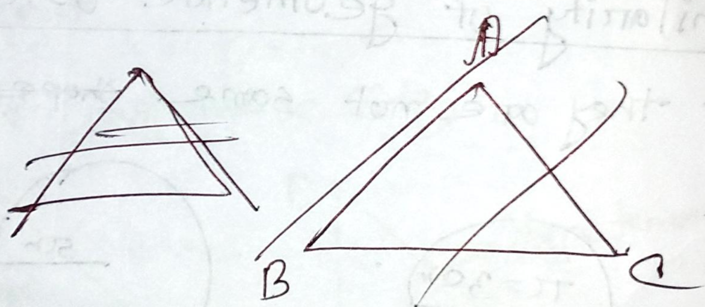
Shape same but size is not same

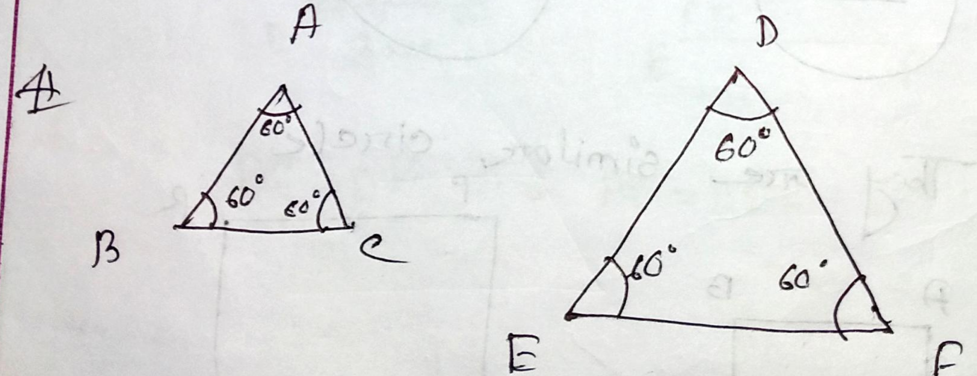
Similar → similar object

Similar/same geometric object but not congruent

→ size and shape same

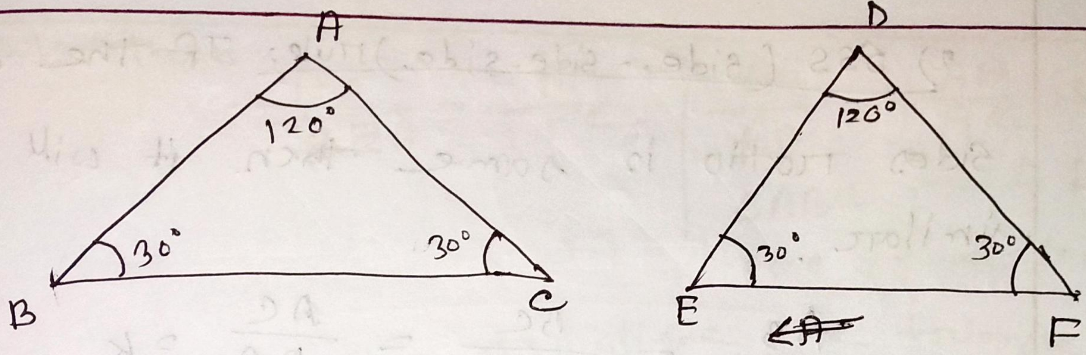
#





They are similar triangle.

#



They are similar triangle.

Similar triangle অর্থাৎ হল যখন একটা triangle কে অন্য একটা triangle এ convert করা যায়।

Greek Mathematician

Thales: If two triangles will be similar if the corresponding angles are same.

$\angle A = \angle D \qquad \angle B = \angle E \qquad \angle C = \angle F$

$\frac{BC}{EF} \qquad \frac{AC}{DF} \qquad \frac{AB}{DE}$

$$\boxed{\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}}$$

Basic proportionality Theorem

Equiangular triangle

All similar triangles are equiangular triangle.

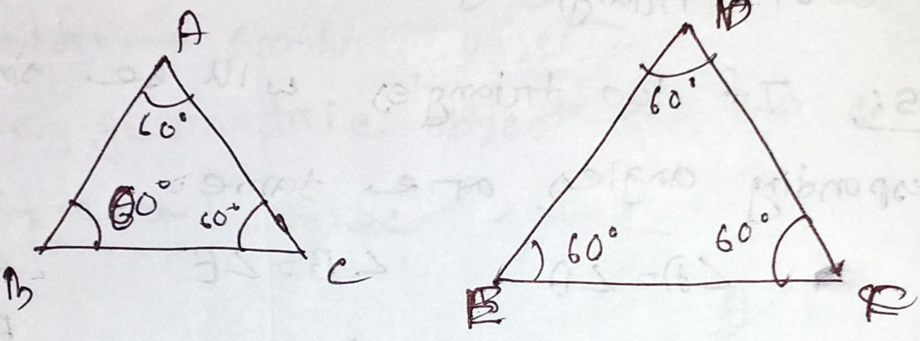
<u>Rules of similar triangle:</u>

1) <u>AAA (Angle-Angle-Angle) Rule:</u> All corresponding angles must be equal.

2) **SSS (side - side - side) rule:** If the corresponding sides ratio is same then it will be similar.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = K$$

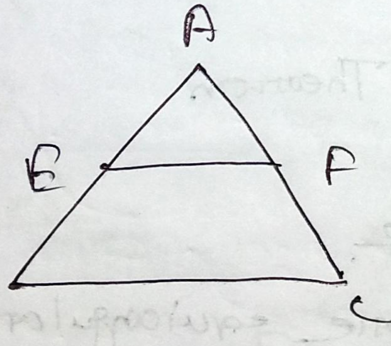
3) **SAS (Side - Angle - side) rule:**



if $\angle B = \angle E$

and $\frac{AB}{DE} = \frac{BC}{EF}$

#



$EF \parallel BC$

$\frac{AE}{EB} = \frac{AF}{FC}$

E, F is any point of AB and AC.

If a line is drawn similar to one side that divide the triangle divide into two distinct points then $\frac{AE}{EB} = \frac{AF}{FC}$ | $\frac{AE}{AB} = \frac{AF}{AC}$, since ΔABC and ΔDEF are similar triangle.

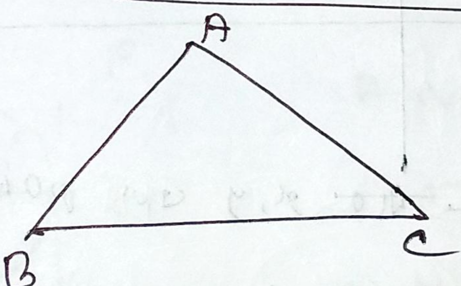
If we draw a line to divide the triangle into distinct point the $EF \parallel BC$

Given

$\Delta ABC \sim \Delta DEF$

$$\frac{\Delta ABC}{\Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AC}{DF}\right)^2 = \left(\frac{BC}{EF}\right)^2$$

Sine Law of Triangle:

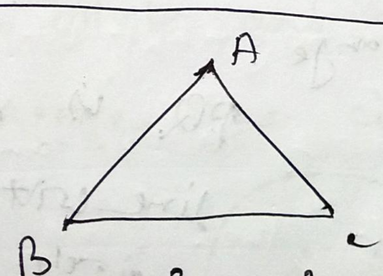


Given $\begin{cases} AB \\ BC \\ AC \end{cases}$

$\angle A, \angle B, \angle C$

$$\frac{AB}{\sin\angle C} = \frac{AC}{\sin\angle B} = \frac{BC}{\sin\angle A}$$

cosine law of Triangle:



Given, AB, AC, BC

$$\cos\angle B = \frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC}$$

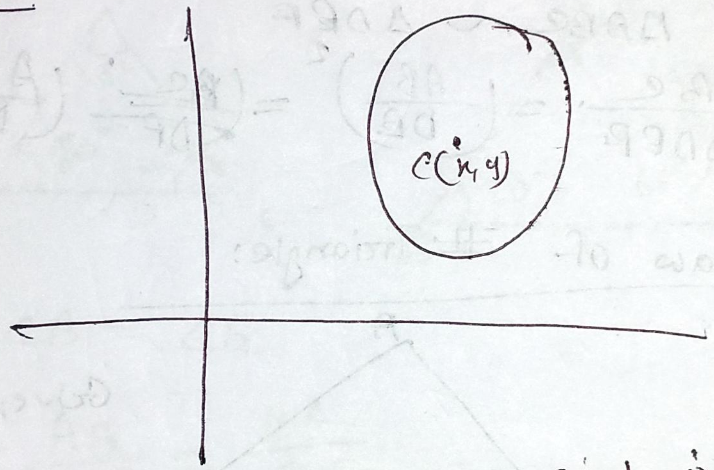
$$\Rightarrow \angle B = \cos^{-1}\left(\frac{AB^2+BC^2-AC^2}{2 \times AB \times BC}\right)$$

$$\cos\angle A = \frac{AB^2+AC^2-BC^2}{2 \times AB \times AC}$$

$$\therefore \angle A = \cos^{-1}\left(\frac{AB^2+AC^2-BC^2}{2 \times AB \times AC}\right)$$

AL kashi's Theorem

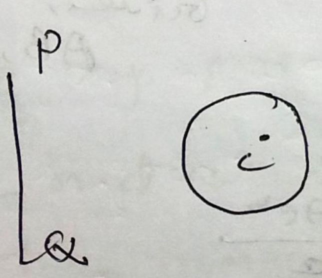
Circle:



Lattise point ~~At~~ x, y are point integer.

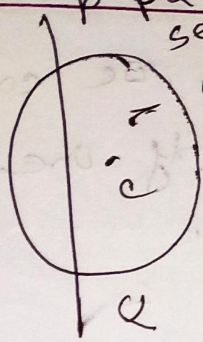
The numbers of integer value on the curve line can't be infinite.

Given a circle, and a line and how many type will be rearrange.



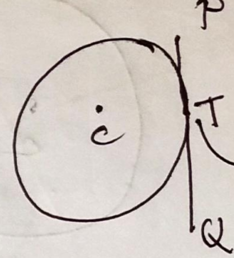
PQ is non-intersecting line with respect to circle.

Figure A



PQ is a secant of the circle.

Fig B



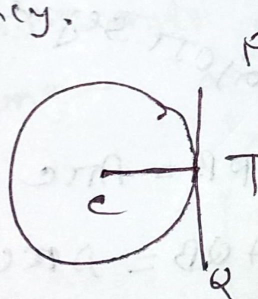
PQ has touched the circle.

touching point/ point of contact/ point of tangency

Fig C

Through a tangency, only one tangent can be drawn.

If we draw any radius through the point of tangency is perpendicular to the tangency.



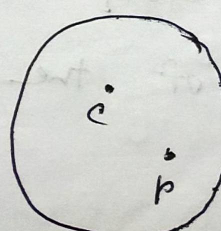
$QT \perp CT$

Prove this theorem

$CT \perp PQ$

1) Mathematical induction
2) Contradiction
3) construction

Given a circle and a point, how many ~~radius~~ to rearrange the circle.

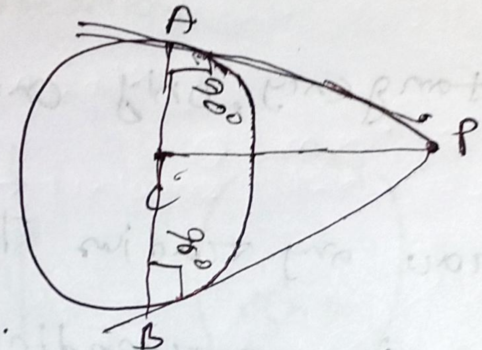


~~[illegible]~~ tangent draw ~~try~~ possible m.

Fig D

Here we can draw exactly one tangent.

fig - B



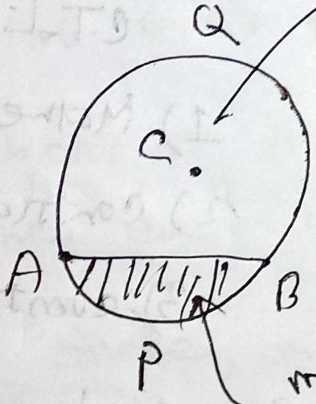
Here we can draw two tangents. The tangents are equal length.

$\boxed{PA = PB}$

$AC \perp AP$

$CB \perp PB$

Major segment



APB = Arc (Minor Arc)

AQB = Arc (Major Arc)

AB = Chord

minor segment of the circle

The region between chord and arc is called segment of the circle.

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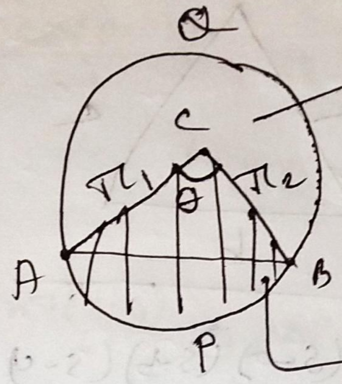
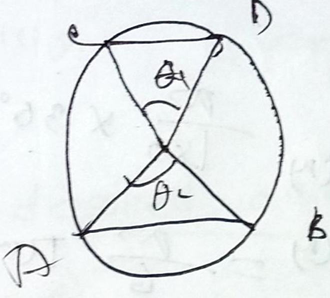


θ = Angle of the minor sector

Subtend

Minor arc APB has subtended an angle θ at the center of the circle.

If two chords of a circle subtends two same angle at the centre then the angles are same.



area of the sector = πr^2

if $\theta = 360°$

$360°$ —— πr^2

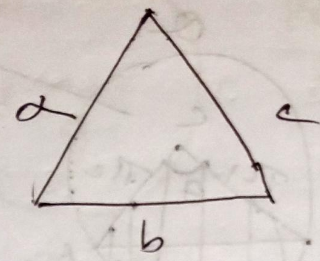
$1°$ —— $\frac{\pi r^2}{360°}$

θ —— $\frac{\pi r^2 \theta}{360°}$

Area of a sector

$= \frac{\theta}{360} \times \pi r^2$

Heron's Formula



$$S = \frac{a+b+c}{2}$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$$

Math.h or cmath

θ must be in radian

$\sin(\theta)$ → returns the sin of radian angle θ

$\cos(\theta)$

$\tan(\theta)$

$180° = \pi^c$

$\Rightarrow 1° = \left(\frac{\pi}{180}\right)^c$

$\theta = 30°$

$\frac{\pi}{180} \times 30°$

$\theta = \sin^{-1}(x) \rightarrow a\sin(x)$

$\theta = \cos^{-1}(x) \rightarrow a\cos(x)$ $= \frac{\pi}{6}$ radian

$\theta = \tan^{-1}(x) \rightarrow a\tan(x)$

$1^c = \left(\frac{180}{\pi}\right)°$

$\cos\frac{\pi}{2} = 0$

$\Rightarrow \frac{\pi}{2} = \cos^{-1}0$

$\Rightarrow \pi = 2\cos^{-1}0$

$\Rightarrow \pi = 2 * a\cos(0)$

$\Rightarrow \boxed{\pi = 2\,acos(0.0)}$

$\cos\pi = -1$

$\Rightarrow \pi = \cos^{-1}(-1)$

$\Rightarrow \boxed{\pi = acos(-1)}$

$\sin\frac{\pi}{2} = 1$

$\Rightarrow \pi = 2\sin^{-1}(1.0)$

obtained answer

Desired answer

\# $a = 3.13$ $b = 3.14$

a and b are equal ~~after~~ with ~~1 digit decimal~~ 1 digit precision. error = 0.01

$a = 3.132$ $b = 3.13\cancel{4}$ error = 0.001

a and b are equal with 2 digit precision.

$a = 3.132\cancel{3}$, $b = 3.132\cancel{4}$ a and b are equal with 3 digit precision. error = 0.0001

error = desired value - obtained value

$\boxed{\text{if } (b-a) <= 0.01) \quad \text{then } a = b}$

$0.01 \rightarrow 1\times10^{-2}$ or 1e-2

$0.001 \rightarrow 1\times10^{-3}$ or 1e-3

0.01

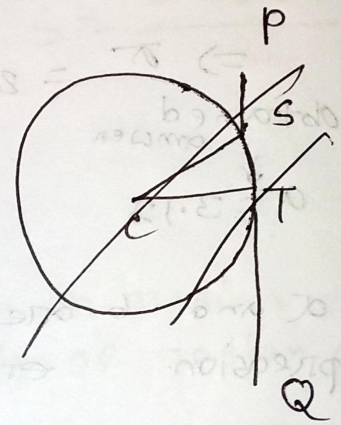
0.01 er immediate smallest $\rightarrow$ 0.009...

continuous number

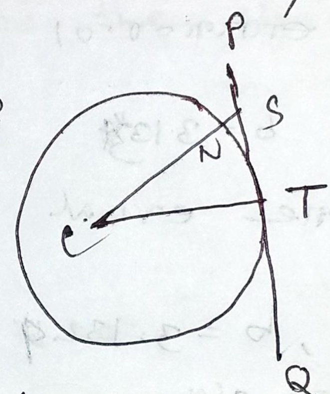
[illegible]

[II] Prove that the tangent at any point of a circle is perpendicular to the radius ~~the~~ through the points of contact. ($CT \perp PQ$)

⇒ Construction :



Taking a point S, other than T, on the tangent PQ. Joining CS meets the circle at N.



$CT = CN$ [Radius of circle]

$CS = CN + NS$

Therefore, $CS > CN$

$\Rightarrow CS > CT$

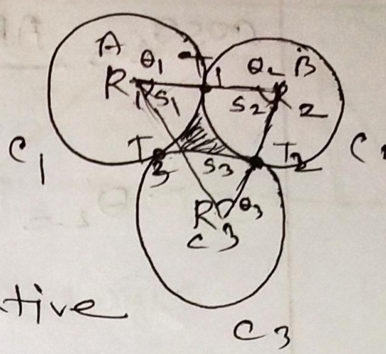
$\Rightarrow CT < CS$

S is an arbitary point on PQ, then CT is shorter than any other line segment, Thus $CT \perp PS$.

Lec-4

Date: 22/05/23

Problem - 04 (Slide) Exam



Error

Absolute Error

$= \left| \text{Expected result} - \text{Obtained result} \right|$

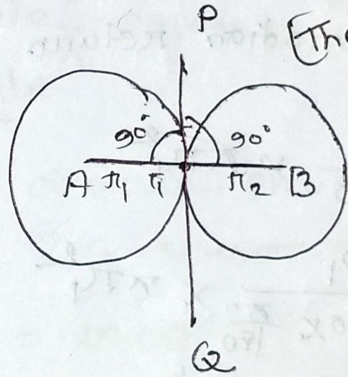
Relative Error

$= \dfrac{\left| \text{Expected result} - \text{Obtained result} \right|}{\text{Expected Result}}$

Problemএ ~~Er~~ কোন Errorএ mention থাকে না থাকলে absolute ~~error~~ consider করতে হবে।

Soln:

[Theorem]



$AT_1 \perp \cancel{PQ}\ PT_1$

$BT_1 \perp PT_1$

$\angle AT_1B = 180°$

A, T_1, B are lie on the same line.

$AB = r_1 + r_2$

Soln:

$AB = R_1 + R_2$

$AC = R_1 + R_3$

$BC = R_2 + R_3$

$S = \dfrac{AB + AC + BC}{2}$

Area of $\Delta ABC = \sqrt{S(S-AB)(S-AC)(S-BC)}$

$$\cos\theta_1 = \frac{AB^2 + AC^2 - BC^2}{2 \times AB \times AC}$$

$$\therefore \theta_1 = \cos^{-1}\left(\frac{AB^2 + AC^2 - BC^2}{2 \times AB \times AC}\right)$$

$$\cos\theta_2 = \frac{AB^2+BC^2-AC^2}{2\times AB\times BC}$$

$$\therefore \theta_2 = \cos^{-1}\left(\frac{AB^2+BC^2-AC^2}{2\times AB\times BC}\right)$$

$$\cos\theta_3 = \frac{AC^2+BC^2-AB^2}{2\times AC\times BC}$$

$$\therefore \theta_3 = \cos^{-1}\left(\frac{AC^2+BC^2-AB^2}{2\times AC\times BC}\right)$$

→ radian return করে।

$\frac{\pi}{180}\times 360$

$$S_1 = \frac{\theta_1}{360}\times \pi r_1^2$$

$$= \frac{\theta_1}{360\times \frac{\pi}{180}}\times \pi r_1^2$$

$$= \frac{\theta_1 \cancel{\pi} r_1^2}{2\cancel{\pi}}$$

$$= \frac{\theta_1 r_1^2}{2} = \frac{\theta_1}{2} r_1^2$$

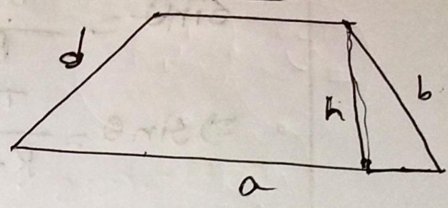
$$S_2 = \frac{\theta_2}{2} r_2^2$$

$$S_3 = \frac{\theta_3}{2} r_3^2$$

Area of the shaded region

$= \text{Area of } \Delta ABC \cancel{(S_1+S_2+S_3)} - (S_1+S_2+S_3)$

Problem-03

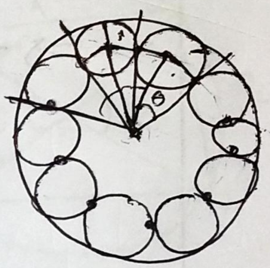


Area of trapezium

$= \frac{1}{2}$ (sum of parallel side) $\times$ height

$= \frac{1}{2}(a+c) \times h$

Problem-08



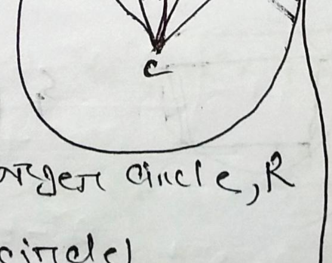
soln: n টি ~~tri~~ circle produce
2n triangle.

n টি ~~tria~~ circle produce
তারা 2n সমান angle.

total ~~a~~ $= 2n \times \theta$

$2n \times \theta = 360°$

$\Rightarrow \boxed{\theta = \frac{360°}{2n}}$ → degree



SAS rule

ΔCAT_2 and ΔCBT_2

$\angle CT_2A = \angle CT_2B$

$AT_2 = BT_2$

$CT_2 = CT_2$

$\therefore \Delta CAT_2 \cong \Delta CBT_2$

CP = radius of larger circle, R

BP = r (small circle)

BC = CP − BP

= R − r

$$\sin\theta = \frac{BT_2}{BC}$$

$$\Rightarrow \sin\theta = \frac{r}{R-r}$$

$$\Rightarrow r = \sin\theta(R-r)$$

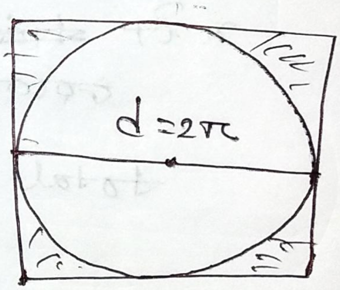
$$\Rightarrow r = R\sin\theta - r\sin\theta$$

$$\Rightarrow r + r\sin\theta = R\sin\theta$$

$$\Rightarrow r(1+\sin\theta) = R\sin\theta$$

$$\therefore r = \frac{R\sin\theta}{1+\sin\theta} \rightarrow \text{radian}$$

Problem:5

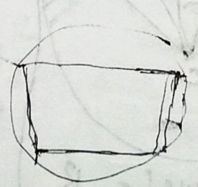


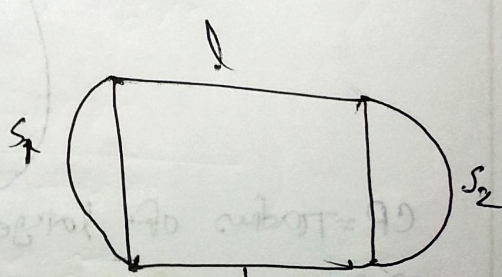
diameter, $d = 2r$

diameter = side of the square

Area of square $= d^2$

Problem:6





perimeter given

perimeter $= 2 \times$ (Length + width)

$P = l + S_2 + l + S_1$

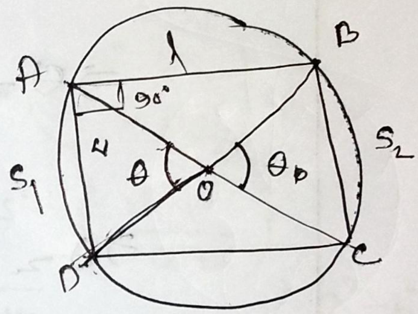
$\cancel{= 2l + 2S_2}$ $2l + S_1 + S_2$

$= \cancel{2(l+S_2)}$

$P = 2l + S_1 + S_2$ $\qquad S = S_1 = S_2$

$= 2l + 2S$

$= 2(l+S)$



prove O is the center (must)

$\angle AOD = \angle BOC$ [reciprocal angle]

OA = radius

Arc = perimeter of circle

$\theta = 360°$ হলে arc $\rightarrow 2\pi r$

$1°$ " " $\rightarrow \frac{2\pi r}{360}$

$\theta°$ " " $\frac{\theta}{360°} \times 2\pi r$

$= \frac{\theta}{180°}\pi r$

$l : w = a : b$

$\Rightarrow \frac{l}{w} = \frac{a}{b}$

$\Rightarrow l = ax$

$w = bx$

$BD = \sqrt{l^2 + w^2}$

$= \sqrt{a^2x^2 + b^2x^2}$

$\cancel{OB}$ $OD = \frac{BD}{2}$

$= \frac{\sqrt{a^2x^2+b^2x^2}}{2} = \frac{x\sqrt{a^2+b^2}}{2}$

$OA = OD = \frac{x\sqrt{a^2+b^2}}{2}$

$$\cos\theta = \frac{OA^2 + OD^2 - AD^2}{2 \times OA \times OD}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{OA^2 + OD^2 - AD^3}{2 \times OA \times OD}\right)$$

$$\cancel{S_T} \Rightarrow \theta = \cos^{-1}\left(\frac{2OA^2 - AD^2}{2OA^2}\right)$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{\frac{2 \times x^2(a^2+b^2)}{4} - b^2x^2}{2 \times \frac{x^2(a^2+b^2)}{4}}\right)$$

$$\Rightarrow \theta = \cos^{-1}\left\{\frac{\frac{x^2(a^2+b^2)}{2} - b^2x^2}{\frac{x^2(a^2+b^2)}{2}}\right\}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{\frac{x^2(a^2+b^2) - 2b^2x^2}{2}}{\frac{x^2(a^2+b^2)}{2}}\right)$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{x^2a^2 + b^2x^2 - 2b^2x^2}{a^2x^2 + b^2x^2}\right)$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{x^2a^2 - b^2x^2}{x^2a^2 + b^2x^2}\right)$$

$$\therefore \theta = \cos^{-1}\left(\frac{a^2 - b^2}{a^2 + b^2}\right) \text{ [Radian]}$$

Arc, $S_1 = \frac{\theta}{360^\circ} \times 2\pi r$

$$= \frac{\theta}{360 \times \frac{\pi}{180}} \times 2\pi r$$

$$= \frac{\theta \times 2\pi r}{2\pi}$$

$$= \theta r$$

$$= r \times \cos^{-1}\left(\frac{a^2-b^2}{a^2+b^2}\right)$$

$$= OA \times \cos^{-1}\left(\frac{a^2-b^2}{a^2+b^2}\right)$$

$$= \frac{x\sqrt{a^2+b^2}}{2} \times \cos^{-1}\left(\frac{a^2-b^2}{a^2+b^2}\right)$$

$P = 2l + 2s$

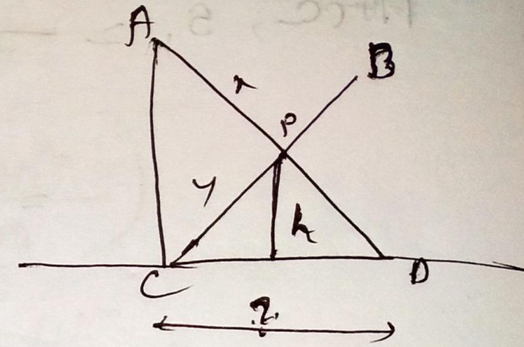
$$\Rightarrow P = 2 \times ax + \cancel{2} \times \frac{x\sqrt{a^2+b^2}}{\cancel{2}} \times \cos^{-1}\left(\frac{a^2-b^2}{a^2+b^2}\right)$$

$$\Rightarrow P = 2ax + x\sqrt{a^2+b^2}\cdot\theta$$

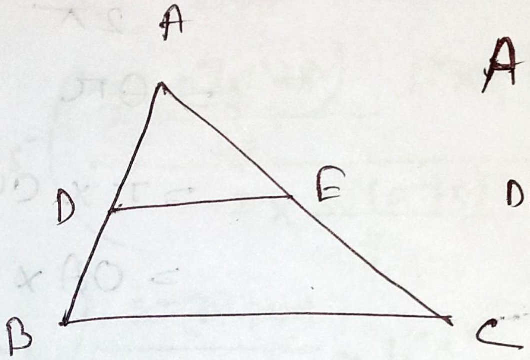
$$\Rightarrow P = x\left(2a + \sqrt{a^2+b^2}\cdot\theta\right)$$

$$\Rightarrow x = \frac{P}{2a + \theta\sqrt{a^2+b^2}}$$

Problem-05



Problem-7



AB, AC, BC

DE || BC

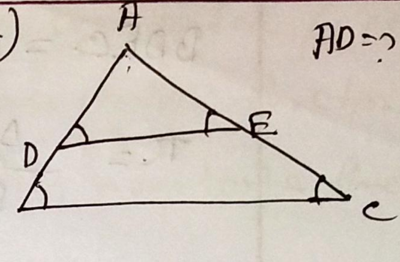
AD=?

i) Note All Solution

ii) Binary Search (Dictionary ও word (খোঁজা)

Lec-5 Date: 28/05/23

Problem 7: (Triangle partitioning problem)



AD = ?

ΔADE and ΔABC are similar triangle.

$\therefore \Delta ADE \sim \Delta ABC$

$\angle ADE = \angle ABC$

$\angle AED = \angle ACB$

$$\frac{\Delta ADE}{BDEC} = k \qquad AD = ?$$

Soln: ΔABC, ΔADE

$BDEC = \Delta ABC - \Delta ADE$

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$\Rightarrow AE \times AB = AD \times AC$

$$\Rightarrow AE = \frac{AD \times AC}{AB} \quad \text{--- (1)}$$

$$\frac{AD}{AB} = \frac{DE}{BC}$$

$\Rightarrow AB \times DE = AD \times BC$

$$\Rightarrow DE = \frac{AD \times BC}{AB} \quad \text{--- (11)}$$

For some value of AD, we can calculate,

ΔADE,
$$S_{\Delta ADE} = \frac{AD + AE + DE}{2}$$

$$\Delta ADE = \sqrt{S_{\Delta ADE}\left(S_{\Delta ADE} - AD\right)\left(S_{\Delta ADE} - DE\right)\left(S_{\Delta ADE} - AE\right)}$$

1sec এ 10^8 loop execute করে

$BDEC = \Delta ABC - \Delta ADE$

$$r = \frac{\Delta ADE}{BDEC}$$

AD এর lowest value 0 and max value

$AD = \cancel{BC}\ AB$ $\qquad$ $AD = [0, AB]$

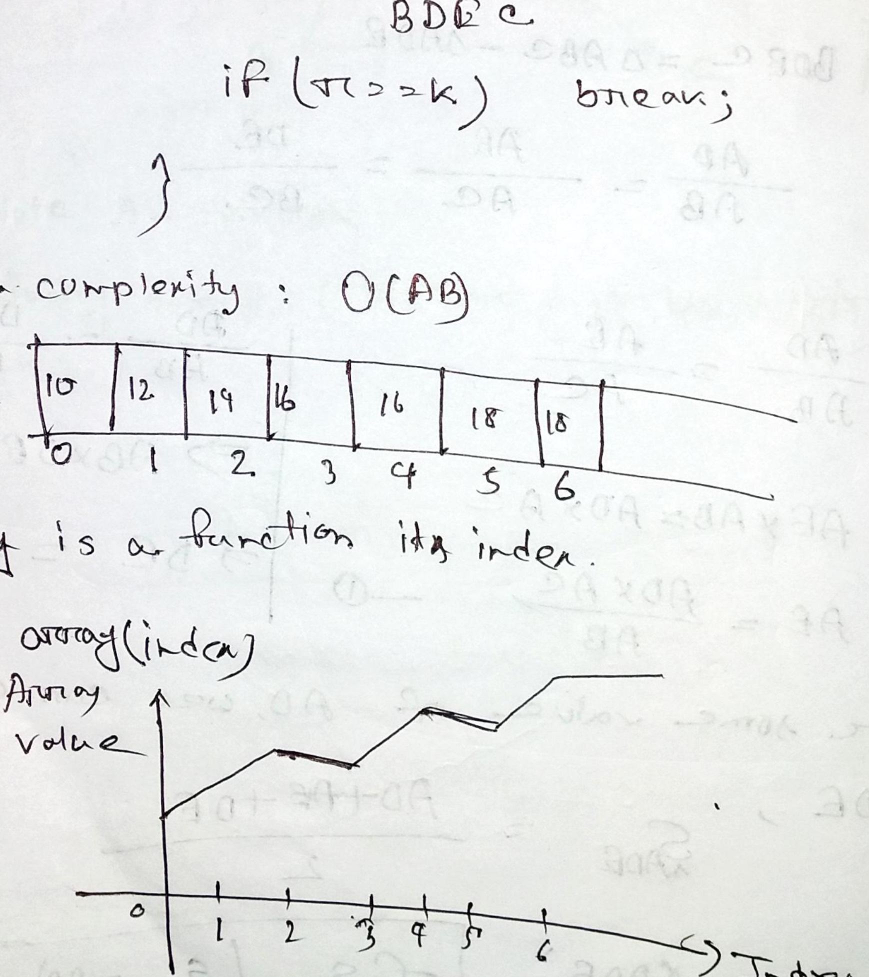
```
for (AD = 0 ; AD <= AB ; AD++)
{
    r = ΔADE / BDEC
    if (r == k)  break;
}
```

Time complexity : $O(AB)$

Arr =

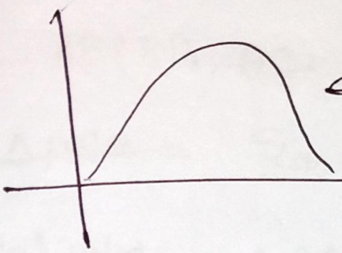
10	12	14	16	16	18	18
0	1	2	3	4	5	6

array is a function its index.



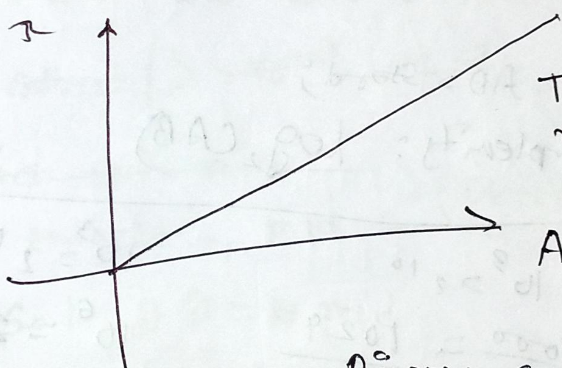
All the time function will be decreasing order / increasing order then it is called monotonic function.

=> Binary search can be applied only monotoric function.



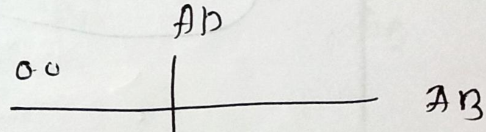
← The function is not a monotoric ~~function~~ function.

=> Ternary search can be applied non monotoric function



This is a monotoric function

Binary search.



Start = 0.0
end = AB
while (~~Start <= end~~) abs (Start - end) > 10^{-7})
{
mid = (Start + end) / 2.0;
AD = mid;
x = $\frac{\triangle AD E}{BDEC}$;

$\boxed{10^{-6} \to \text{give wrong answer}}$

~~if(T(==k) break;~~

```
if (T(>k){
    start = a0;
    end = mid;
}
else{
    start = mid;
}
}
```

AD = start;

complexity: $\log_2(AB)$

$10^3 \approx 2^{10}$

$\frac{1000}{1k} \approx \frac{1024}{1k}$

$10^3 = 2^{10}$

$10^6 = 2^{20}$

$10^8 = 10^6 \times 10^2 = 2^{20} \times 2^7$

$= 2^{27}$

$\log_2 2^{27} = 27$

error = |end − start|

error $< 10^{-6}$

Algorithm:

1. What are the inputs of the algorithm
2. What are the steps of the algorithm
3. What are the outputs of the algorithm

get_AD(AB, AC, BC, K)

1) ~~$\Delta ABC =$~~ $S_{\Delta ABC} = (AB + AC + BC)/2.0$

2) Calculate, $\Delta ABC = \sqrt{S_{ABC}(S_{ABC} - AB)(S_{\Delta ABC} - AC)(S_{\Delta ABC} - BC)}$

3) set Start = 0.0 and end = AB

4) while (|end - start| >= ~~10^{-7}~~ 1e-7)

4.a) ~~mid set mid~~ calculate, mid = (Start + end)/2.0;

4.b) Set AD = mid.

4.c) ~~Calculate, $S_{\Delta ADE}$ =~~ $AE = \dfrac{AD \times AC}{AB}$

4.d) $DE = \dfrac{AD \times BC}{AB}$

4.e) $S_{\Delta ADB} = (AD + AE + DE)/2.0$

4.f) $\Delta ADE = \sqrt{S_{\Delta ADB}(S_{\Delta ADE} - AD)(S_{\Delta ADE} - AE)(S_{\Delta ADE} - DE)}$

4.g) BDEC = $\Delta ABC - \Delta ADE$

4.h) $r = \Delta ADE / BDEC$

4.i) if $r > k$, set end = mid

~~4.j)~~ else, start = mid.

5) AD = start.

6) return AD.

Soln (2)

$$\frac{\Delta ADE}{\Delta ABC} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{DE}{BC}\right)^2 = \left(\frac{AE}{AC}\right)^2$$

~~ADE~~

$$\frac{\Delta ADE}{BDEC} = k$$

$$\Rightarrow \frac{BDEC}{\Delta ADE} = \frac{1}{k}$$

$$\Rightarrow \frac{BDEC + \Delta ADE}{\Delta ADE} = \frac{1+k}{k}$$

$$\Rightarrow \frac{\Delta ABC}{\Delta ADE} = \frac{1+k}{k}$$

$$\Rightarrow \frac{\Delta ADE}{\Delta ABC} = \frac{k}{1+k}$$

$$\frac{\Delta ADE}{\Delta ABC} = \left(\frac{AD}{AB}\right)^2$$

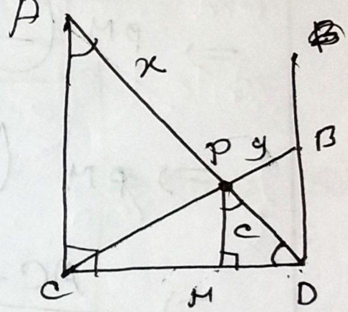
$$\Rightarrow \frac{k}{1+k} = \left(\frac{AD}{AB}\right)^2$$

$$\Rightarrow \frac{AD}{AB} = \sqrt{\frac{k}{1+k}}$$

$$\therefore \boxed{AD = AB \times \sqrt{\frac{k}{1+k}}}$$

Lec-6

Date: 30/05/23



Problem:5 Given

$AD = x$

$BC = y$

~~AD and~~ CD = ?

$PM \perp CD$

$PM = c$

ΔACD

ΔBCD

ΔACD and ΔPMD

$\Delta ACD \sim \Delta PMD$

$$\frac{PM}{AC} = \frac{MD}{CD} = \frac{PD}{AD}$$

ΔBCD and ΔPCM

$\Delta BCD \sim \Delta PCM$

$$\frac{PM}{BD} = \frac{CM}{CD} = \frac{PC}{BC}$$

$CD \downarrow PM \uparrow$

$CD \uparrow PM \downarrow$

$$\frac{PM}{AC} = \frac{MD}{CD} \quad \text{--- (I)}$$

$$\frac{PM}{BD} = \frac{CM}{CD} \quad \text{--- (II)}$$

(I) + (II)

$$\frac{PM}{AC} + \frac{PM}{BD} = \frac{MD}{CD} + \frac{CM}{CD}$$

$$\Rightarrow PM\left(\frac{1}{AC} + \frac{1}{BD}\right) = \frac{MD+CM}{CD}$$

$$\Rightarrow PM\left(\frac{BD+AC}{AC \cdot BD}\right) = \frac{CD}{CD}$$

$$\Rightarrow PM\left(\frac{AC+BD}{AC \cdot BD}\right) = 1$$

$$\Rightarrow \frac{AC+BD}{AC \cdot BD} = \frac{1}{PM}$$

$$\Rightarrow \frac{AC \cdot BD}{AC+BD} = PM$$

$$\Rightarrow \frac{\left(\sqrt{AD^2 - CD^2}\right)\left(\sqrt{BC^2 - CD^2}\right)}{\left(\sqrt{AD^2 - CD^2}\right) + \left(\sqrt{BC^2 - CD^2}\right)} = C$$

$$\Rightarrow \frac{\left(\sqrt{x^2 - CD^2}\right)\left(\sqrt{y^2 - CD^2}\right)}{\left(\sqrt{x^2 - CD^2}\right) + \left(\sqrt{y^2 - CD^2}\right)} = C$$

minimum of $CD_{min} = 0$

if CD = 0,

$$\frac{\sqrt{x^2}\ \sqrt{y^2}}{\sqrt{x^2}+\sqrt{y^2}} = c$$

$$\Rightarrow c = \frac{xy}{x+y}$$

$$PM = c = \frac{xy}{x+y}$$

if, $y > x$

$CD_{max} = x$

~~$x \neq y$~~

$x = y$

$CD = min(x, y)$

let $x > y$,

~~CDmax~~ $CD_{max} = y$

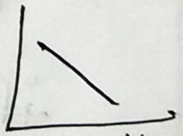
$CD_{max} = min(x, y)$

```
for ( CD = 0; CD <= min(x,y) ; CD++ )
{
    Calculate → π.
    if (π == c)          Ans = CD

}
```

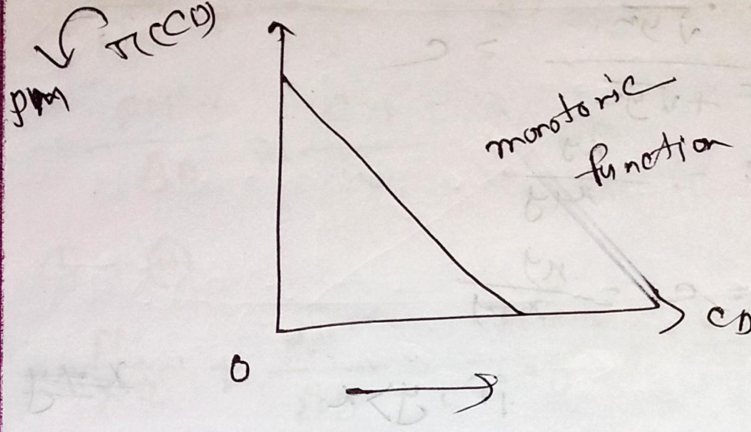
→ Wrong Solution

Monotonic functions:



Strictly decreasing monotonic function

1) Strictly increasing monotonic function
2) " decreasing
3) Increasing monotonic function
4) Decreasing "




```
Start = 0;
end = min(x, y);
while (~~start <= end~~ (abs|end - start| >= 10^-7))
{
    mid = (start + end) / 2.0;

    CD = mid;

    r = getRation(CD);

    ~~if (r == c) break;~~

    if (r > c)   start = mid;
    if (r < c)   end = mid;
}
```

get_ration (CD, x, y)

1. $\cancel{CD}\ r = \dfrac{\cancel{CD} \ast \text{sqrt}(x^2 - CD^2) \times \text{sqrt}(y^2 - CD^2)}{\text{sqrt}(x^2 - CD^2) + \text{sqrt}(y^2 - CD^2)}$

2. return ~~CD~~ r

$\boxed{\text{Vector}}$

Date: 04/06/23

Vector

→ Vector has magnitude and angle.

Tail $\qquad\qquad$ Head.

$A \longrightarrow B$

→ Vector is a directed line segment. That has both direction and magnitude. Magnitude means the length of line.

Tail $\qquad\qquad$ Head

$A \longrightarrow B$

$(x_1, y_1) \qquad\qquad (x_2, y_2)$

$$\|AB\| = \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$$

↳ It represents the length of the line ~~AB~~ segment AB.

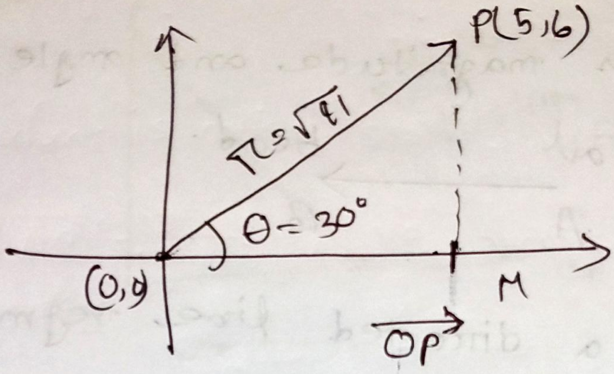
→ r = ~~mg~~ magnitude, θ = angle.

→ Vector = $(r, \angle\theta)$ [polar representation of vector]

→ Integer point also known as lattice point.

→ Component form of vector;

Component form of vectors:



OM = x component

PM = y component

$OM = r\cos\theta$

$\boxed{x = r\cos\theta}$

$PM = r\sin\theta$

$\boxed{y = r\sin\theta}$

Vector has two component.

1) x-component
2) y-component.

Vector = (x, y)

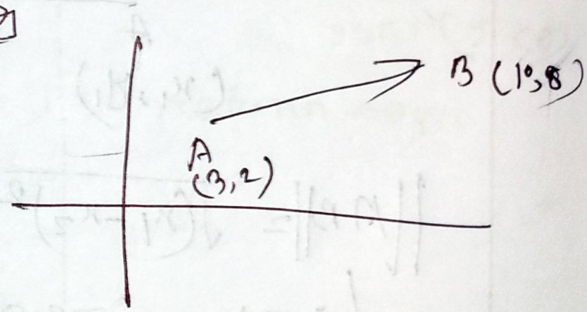
$\overrightarrow{OP} = P - O$

$= (5,6) - (0,0)$

$= 5-0,\ 6-0$

$= (5,6)$

2)



$\overrightarrow{AB} = B - A$

$= (10,8) - (3,2)$

$= (10-3,\ 8-2)$

$= (7,6)$

Vector Operation:

1. Addition
2. Subtraction
3. ~~Multiplication~~ Product.

Addition: $A = (x_1, y_1)$ $B = (x_2, y_2)$ They are vector ~~Ita~~ according to origin.

$\overrightarrow{OA} = (x_1, y_1)$

$\overrightarrow{OA} + \overrightarrow{OB} = (x_1 + x_2, y_1 + y_2)$

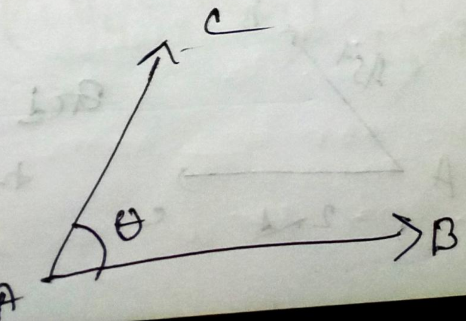
$$\overrightarrow{OA} + \overrightarrow{OB} \; \cancel{\overrightarrow{OA}} = \underset{OA}{\begin{vmatrix} x_1 \\ y_1 \end{vmatrix}} + \underset{\overrightarrow{OB}}{\begin{vmatrix} x_2 \\ y_2 \end{vmatrix}} = \begin{vmatrix} x_1 + x_2 \\ y_1 + y_2 \end{vmatrix}$$

$$\overrightarrow{OA} + \overrightarrow{OB} = |x_1 \;\; y_1| + (x_2 \;\; y_2)$$

$$= (x_1 + x_2, \; y_1 + y_2)$$

Product → Dot product

Product → Cross product



1st vector
2nd vector

$\vec{AB} \cdot \vec{AC} = |AB| \times |AC| \times \cos\theta$.

smallest angle

Dot product is called scalar product.

$\vec{AB} \times \vec{AC} = |\vec{AB}| \times |AC| \sin\theta \; \hat{\eta}$

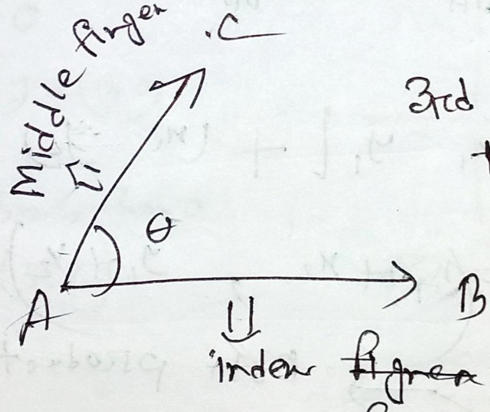
first vector

second vector

vector product বোঝায়।

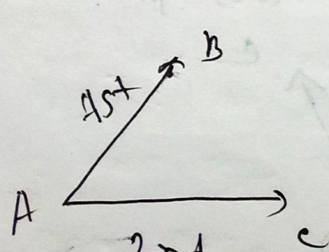
cross product can't take in 2D.

" " always take in 3D.



3rd vector outside the board.

Above the surface → vector direction positive



3rd vector inside the board.

below the surface → vector direction negative.

· Cross product → Magnitude, direction
↓
Right hand rule

Unit vector: $\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$ $\hat{n}$ → unit vector

according to x axis unit vector → $\hat{i}$

" " y " " " → $\hat{j}$

" " z " " " → $\hat{k}$

$\overrightarrow{AB} = (x_1, y_1)$

$\overrightarrow{AC} = (x_2, y_2)$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = (x_1 x_2 + y_1 y_2) = \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}$

$= (x_1 y_2 - x_2 y_1) \cdot \hat{n}$

↳ the direction of the new vector.

When can we take the product of two vectors?

⇒ They are originated from same point. or they should be made from same point.

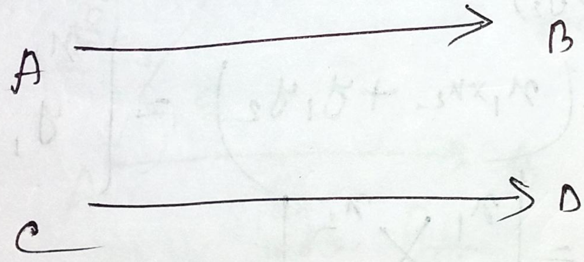
সব vectors কে shift করে same point এ আনা possible.

Categorization of vectors:

1) **Unit vector:** $\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

2) **Zero vector:** It is just a concept. This vector does not exist.

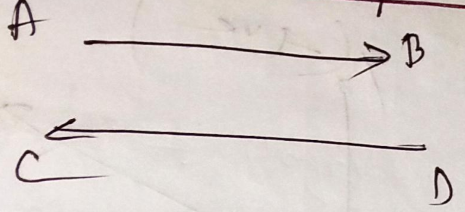
3) **Parallel vector:** Angle between two parallel vector is 0°.



If two vectors are always parallel, then they are parallel vector. Two vectors working in the same direction.

Two vectors can be same vector when their magnitude and ~~te~~ direction are same.

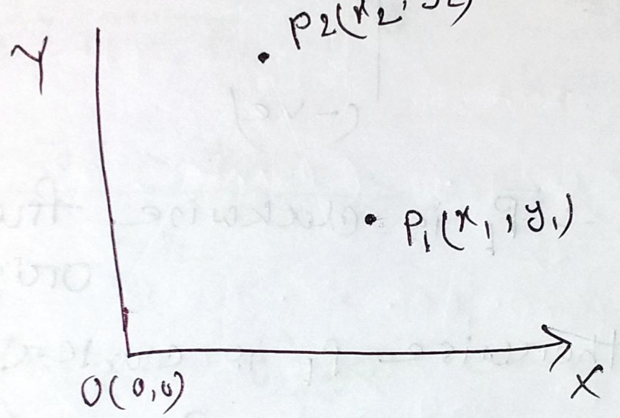
4) **Anti parallel vector / Opposite vector:**



They are anti parallel vectors

Angle between two anti parallel vector is $180°$

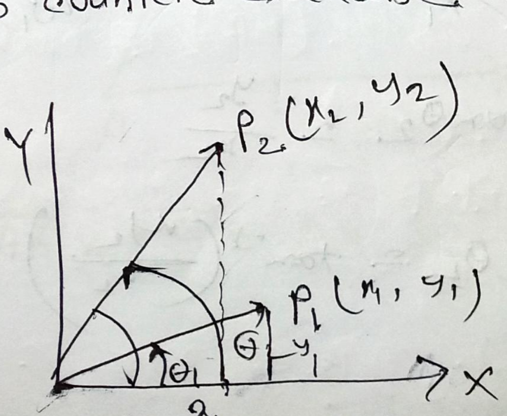
Q



Determine ~~the~~ if P_1 is clockwise or counter clockwise from P_2 w.r.t. origin.

Soln; P_1 is clockwise w.r.t. P_2

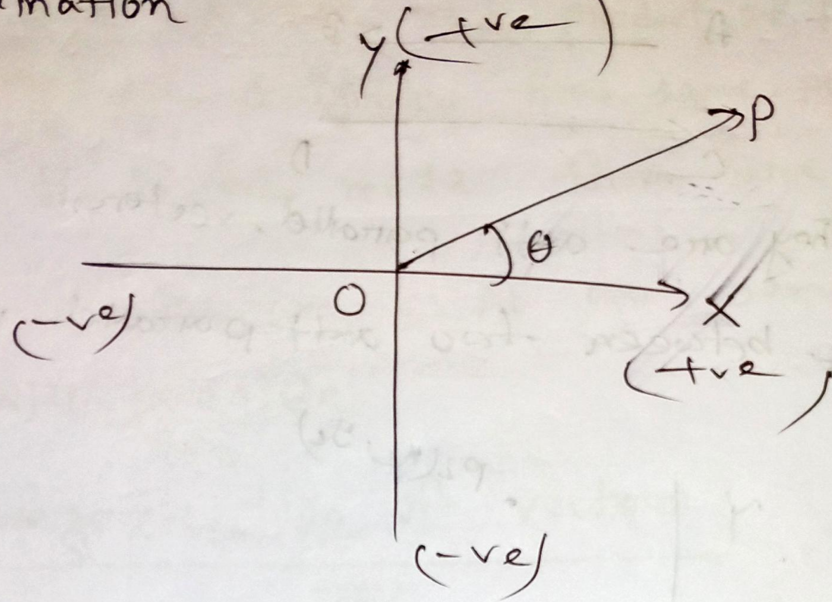
P_2 is counter clockwise w.r.t. P_1



θ_1 = the angle made by the first point and origin

θ_2 = the angle made by the 2nd point and origin

Inclination



$\theta_2 > \theta_1$ ∴ P_1 is clockwise from P_2 w.r.t. origin.

Otherwise P_1 is counterclockwise from P_2 w.r.t. origin

Slope: Inclination or tan

$m_1 = \tan\theta_1 = \cancel{\frac{x_1}{y_1}} \; \frac{y_1}{x_1}$

$\Rightarrow \cancel{m_1 = \frac{y_1}{x_1}} \quad \theta_1 = \tan^{-1}\left(\cancel{\frac{x_1}{y_1}}\right)\left(y_1/x_1\right)$

$m_2 = \tan\theta_2 = \frac{y_2}{x_2}$

$\therefore \theta_2 = \tan^{-1}\left(\frac{y_2}{x_2}\right)$

Another solution

$\theta_1 \cancel{>} < \theta_2$

$\Rightarrow \tan\theta_1 \cancel{>} < \tan\theta_2$

$\Rightarrow m_1 \cancel{>} < m_2$

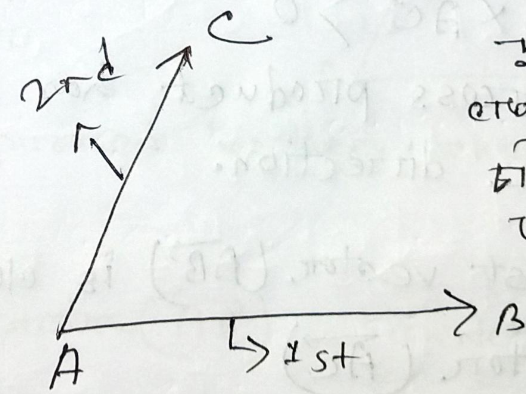
$\Rightarrow \frac{y_1}{x_1} < \cancel{>} \frac{y_2}{x_2}$

$\Rightarrow \frac{y_1}{x_1} - \frac{y_2}{x_2} < \cancel{>} 0$

$\Rightarrow \frac{y_1 x_2 - y_2 x_1}{x_1 x_2} \cancel{>} < 0$

$\Rightarrow (x_2 y_1 - x_1 y_2) \cancel{> 0}$ can we say P_1 is clockwise P_2 w.r.t origin

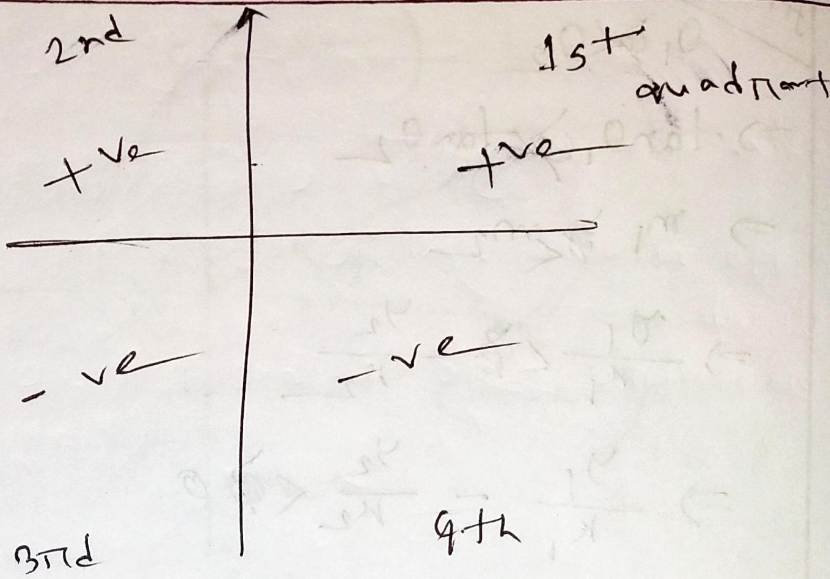
11



দুইটা vector এর cross product নিতে চাইলে ১ম vector থেকে ২য় vector এর দিকে smaller angle এ move করা উচিত।

$\vec{AB} \times \vec{AC} = AB \cdot AC \cdot \sin\theta$

Sine



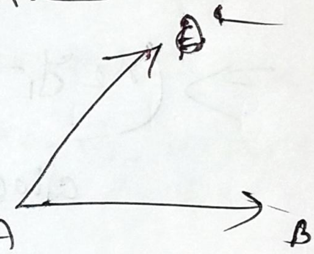
$0 \le \theta \le 180$ (counter clockwise direction)

⇓

$\sin\theta$ can be positive

$\sin(\theta) = \sin\theta$

$\sin(-\theta) = -\sin\theta$



→ $\vec{AB} \times \vec{AC} > 0$

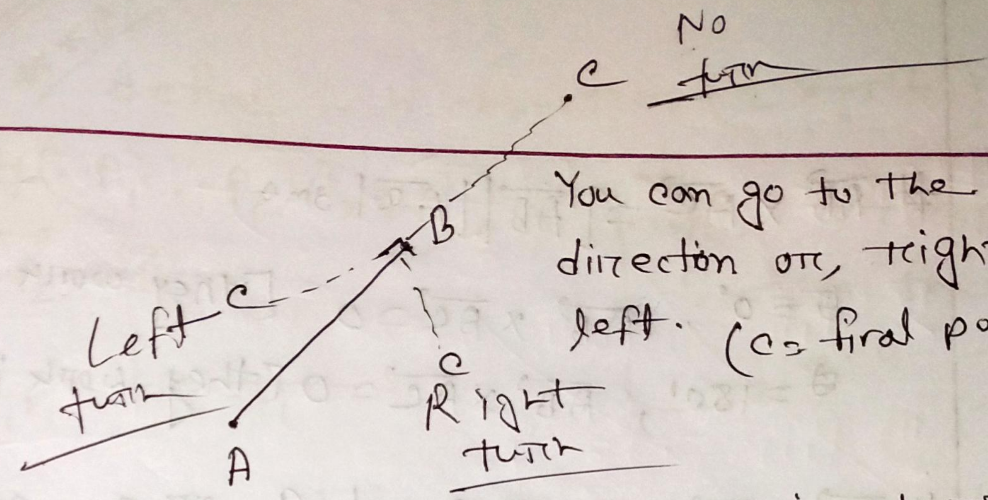
i) Cross product has been taken counter clockwise direction.

ii) 1st vector ($\vec{AB}$) is clockwise from the 2nd vector ($\vec{AC}$)

iii) point B is in clockwise orientation w.r.to point C.

iv) Point A, B, C are in counter clockwise orientation.

2//

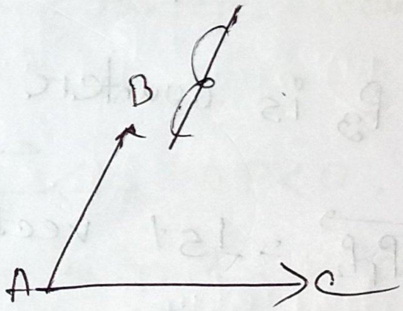


You can go to the same direction or, right or left. (C = final position)

C is at left of AB or C is at right of BC or C is at along the AB.

⇒ When 3 points are belonged in the same line then they are known as co-lineare points.

1) Turn
2) Orientation
3) Direction



$\vec{AB} \times \vec{AC} < 0$

1) Cross product has been taken clockwise direction.

2) 1st vector ($\vec{AB}$) in counter clockwise from the 2nd vector (AC)

3) Point B is in counter clockwise w.r.to point C.

4) Point A, B and C are in clockwise orientation

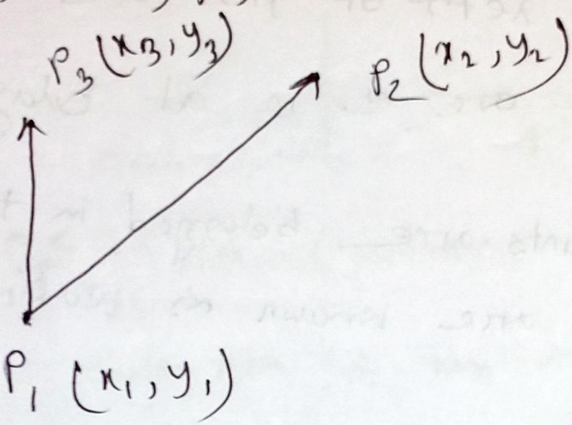
$\vec{AB} \times \vec{AC} = |\vec{AB}||\vec{AC}| \sin\theta$

$\theta = 0°$, $\vec{AB} \times \vec{AC} = 0$ [they work in same directi

$\theta = 180°$, $\vec{AB} \times \vec{AC} = 0$ [they work in opposite direction]

$\vec{AB} \times \vec{AC} = 0$, A, B and C are colinear points

Q



P_3 is counter clockwise w.r.t. to P_2

$\vec{P_1P_2}$ = 1st vector

$\vec{P_1P_3}$ = 2nd vector

originate from same point

$\vec{P_1P_2} \times \vec{P_1P_3} > 0 \rightarrow P_3$ is counter clockwise P_1 and P_2

$\vec{P_1P_2} \times \vec{P_1P_3} < 0 \rightarrow P_3$ is clockwise P_1 and P_2

$\vec{P_1P_2} \times \vec{P_1P_3} = 0 \rightarrow P_1, P_2$ and P_3 are colinear points

CCW = counter clockwise orientation

~~From~~ *** define ccw function (2 marks)

ccw(P_1, P_2, P_3)

[first crossproduct function then ccw]

i) ~~$\overrightarrow{P_1P_2} = P_2 - P_1$~~

ii) ~~$\overrightarrow{P_1P_3} = P_3 - P_1$~~

iii) ~~cross pro~~ i) $\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$ = cross product (~~$\overrightarrow{P_1P_2}$, $\overrightarrow{P_1P_3}$~~) (P_1, P_2, P_3)

CP

CP>0

iv) ~~If~~ if ~~$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} > 0$~~, P_3 is counter clockwise w.r.to P_1 and P_2.

P_3 ~~is lef~~ has made left turn at point P_2 w.r.to P_1 ~~and P_2~~.

P_1, P_2, P_3 ~~is~~ are in counter clockwise orientation.

else if, ~~$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} < 0$~~, CP<0,

P_3 is clockwise w.r.t. P_1 and P_2

P_3 has made right turn at point P_2 w.r.t. Point P_1

P_1, P_2 and P_3 are in clockwise orientation.

CP = 0

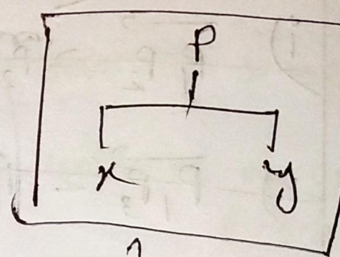
else, ~~$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = 0$~~, P_1, P_2 and P_3 are colinear points.

P_1, P_2 are either in same direction or opposite direction

Cross Product ($\cancel{\overrightarrow{P_1P_2}, \overrightarrow{P_1P_3}}$) (P_1, P_2, P_3)

i) $\cancel{\overrightarrow{P_1P_2} = P_2 - P_1}$

ii) $\cancel{\overrightarrow{P_1P_3} = P_3 - P_1}$



$p\{x, y\}$

$p.x,\ p.y$

i) $\overrightarrow{P_1P_2} = P_2 - P_1$

$= (P_2.x - P_1.x,\ P_2.y - P_1.y)$

ii) $\overrightarrow{P_1P_3} = P_3 - P_1$

$= (P_3.x - P_1.x,\ P_3.y - P_1.y)$

iii) ~~cross~~ $\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$

$= \{(P_2.x - P_1.x) \times (P_3.y - P_1.y)\} - \{(P_2.y - P_1.y) \times (P_3.x - P_1.x)\}$

iv) return, $\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$

dotProduct (P_1, P_2, P_3)

i) $\overrightarrow{P_1P_2} = P_2 - P_1$

$= (P_2.x - P_1.x,\ P_2.y - P_1.y)$

ii) $\overrightarrow{P_1P_3} = P_3 - P_1$

$= (P_3.x - P_1.x,\ P_3.y - P_1.y)$

iii) $\overrightarrow{P_1P_2} \cdot \overrightarrow{P_1P_3} = \{(P_2.x - P_1.x) \times (P_3.x - P_1.x)\} +$

$\{(P_2.y - P_1.y) \times (P_3.y - P_1.y)\}$

iv) ~~Return~~ return ~~$\overrightarrow{P_1P_2} \cdot \overrightarrow{P_1P_3}$~~ $\overrightarrow{P_1P_2} \cdot \overrightarrow{P_1P_3}$

~~#~~ তিন point এর orientation find করার জন্য CCW function হলো।

Computational Geometry
Lec-8

Date: 05/06/23

~~Stenrating~~

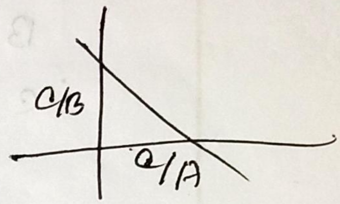
Straddle

$Ax + By = c$ → particular point of line

$\frac{x}{c/A} + \frac{y}{c/B} = 1$

$P_1 (x_1, y_1)$ —— $q_1 (x_2, y_2)$

$P_2 (x_3, y_3)$ —— $q_2 (x_4, y_4)$



P_1 $(P_1 . x, P_1 . y)$ —— q_1 $(q_1 . x, q_1 . y)$ $\longrightarrow A_1 x + B_1 y = c_1$ — (i)

P_2 $(P_2 . x, P_2 . y)$ —— q_2 $(q_2 . x, q_2 . y)$ $\longrightarrow A_2 x + B_2 y = c_2$ — (ii)

$q(x, y)$

$P_1 (x_1, y_1)$ —— $P_2 (x_2, y_2)$

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$$

$\Rightarrow x (y_1 - y_2) - x_1 y_1 + x_1 y_2 = (x_1 - x_2) y - x_1 y_1 + x_2 y_1$

$\Rightarrow (y_1 - y_2) x - x_1 y_1 + x_1 y_2 - (x_1 - x_2) y + x_1 y_1 - x_2 y_1 = 0$

$\Rightarrow (y_1 - y_2) x - (x_1 - x_2) y = x_2 y_1 - x_1 y_2$

$$\Rightarrow -(y_2-y_1)x-(x_1-x_2)y=-(x_1y_2-x_2y_1)$$

$$\Rightarrow (y_2-y_1)x+(x_1-x_2)y = x_1y_2-x_2y_1$$

$A = y_2-y_1$

$B = x_1-x_2$

$C = x_1y_2-x_2y_1$

getLine(P_1, P_2)

i) $A = P_2.y - P_1.y$

ii) $B = P_1.x - P_2.x$

iii) $C = P_1.x\, P_2.y - P_2.x\, P_1.y$

iv) return ~~A,B,C~~ A, B, C

if, $\frac{A_1}{A_2} = \frac{B_1}{B_2}$ these two lines are parallel

$$\Delta = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} = A_1B_2 - A_2B_1$$

$$x = \frac{\begin{vmatrix} C_1 & B_1 \\ C_2 & B_2 \end{vmatrix}}{\Delta} = \frac{B_2C_1 - B_1C_2}{\Delta}$$

$$y = \frac{\begin{vmatrix} A_1 & C_1 \\ B_1 & C_2 \end{vmatrix}}{\Delta} = \frac{A_1 C_2 - C_1 B_1}{\Delta}$$

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}$$

$\Rightarrow A_1 B_2 = A_2 B_1$

$\Rightarrow A_1 B_2 - A_2 B_1 = 0$

do_intersect(P_1, q_1, P_2, q_2)

i) A_1, B_1, C_1 = get_line(P_1, q_1)

ii) A_2, B_2, C_2 = get_line(P_2, q_2)

iii) $\Delta = A_1 B_2 - A_2 \cancel{B_2} B_1$

iv) If $\Delta = 0$, lines are parallel

else { $x = (B_2 C_1 - B_1 C_2)/\Delta$

$y = (A_1 C_2 - A_2 C_1)/\Delta$

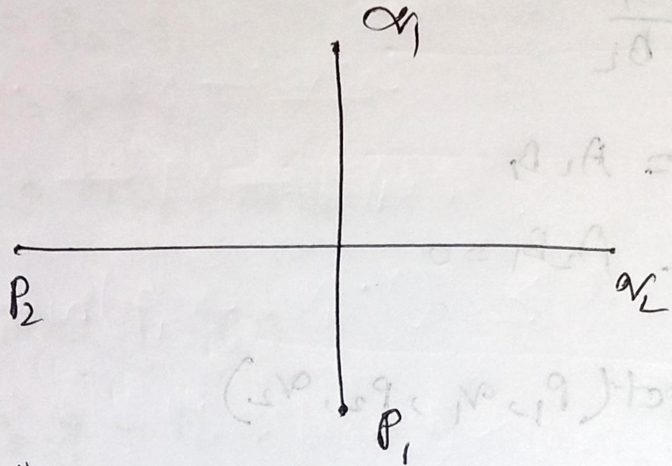
return x, y }

Q Given, line segment (line segment intersect করে কিনা)

p_1, q_1

p_2, q_2

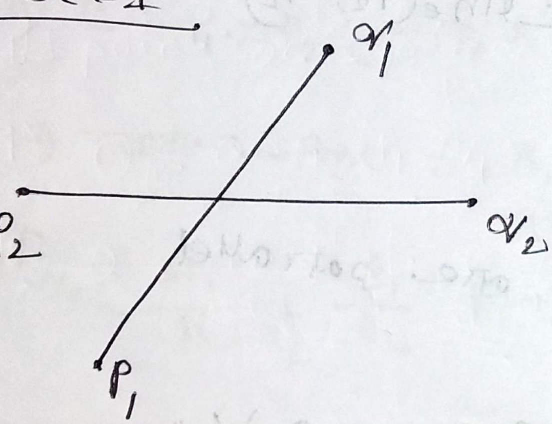
Soln



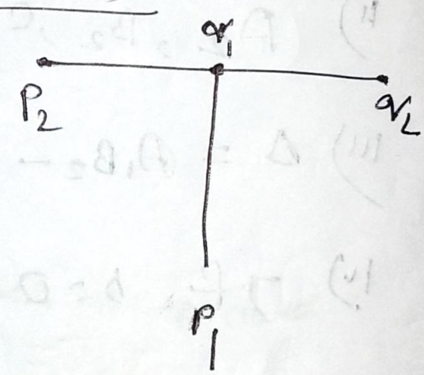
1st line কে reference ধরে second line যদি 1st line কে straddle করে এবং ওরা intersect করে।

Intersecting case:

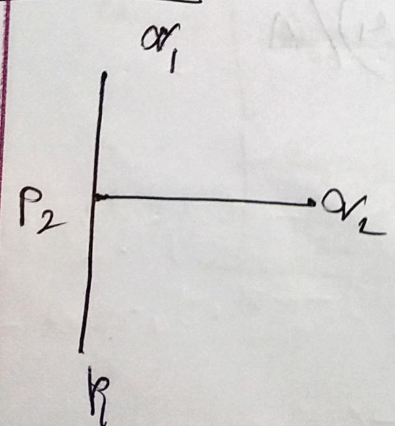
Case-1



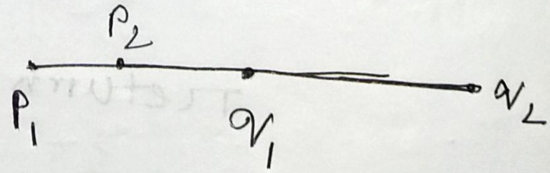
Case-2



Case-3



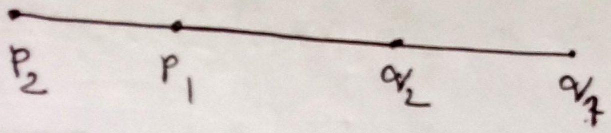
Case-4



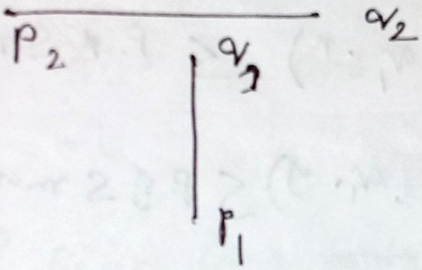
$p_1 = 0$
$q_1 = 0$

$p_2 = 0$
$q_2 = 0$

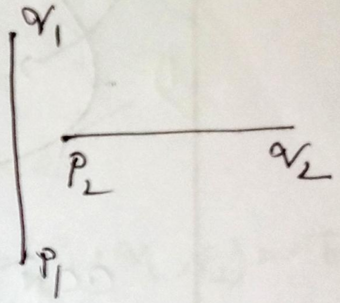
Case-5



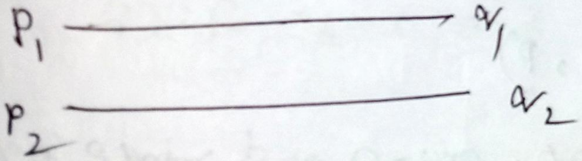
Case 6:



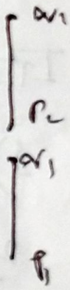
Case-7



Case 8:



Case 9:



$p_1 = 0$
$q_1 = 0$
$p_2 = 0$
$q_2 = 0$

$\boxed{T}$ ~~$ccw(p_1, q_1, p_2)$~~

$O_1 = ccw(p_1, q_1, p_2)$
$O_2 = ccw(p_1, q_1, q_2)$
ref = 1st line segment

$O_3 = ccw(p_2, q_2, p_1)$
$O_4 = ccw(p_2, q_2, \cancel{q_4}\, q_1)$
ref = 2nd line segment

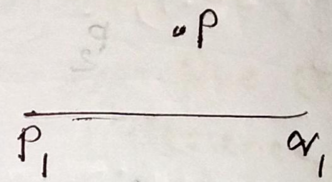
if ~~($O_1 \neq O_2$ and $O_3 \neq O_4$~~

P.T.O

if $O_1 \,!= O_2$ and $O_3 \,!= O_4$

Line segment have intersected.

$\boxed{iv}$



$p_1.x \rightarrow p.x \rightarrow q_1.x$

$p_1.y \leftarrow p.y \rightarrow q_1.y$

$\min(p_1.x,\ q_1.x) \le p.x \le \max(p_1.x,\ q_1.x)$

$\min(p_1.y,\ q_1.y) \le p.y \le \max(p_1.y,\ q_1.y)$

$ccw(p_1, q_1, p) = 0$

$\boxed{v}$ point_on_segment(p_1, q_1, p)

1) $O_1 =$ ccw(p_1, q_1, p)

2) ~~O = ccw~~ if ~~O = 0~~ $O_1 = 0$ and $\min(p_1.x, q_1.x) \le p.x \le \max(p_1.x, q_1.x)$ and $\min(p_1.y, q_1.y) \le \max(p_1.y, q_1.y)$

~~3) return~~ return true;

4) return false.

LS = Line Segment

Algorithm:

do_LS_intersect(P_1, q_1, P_2, q_2)

1) $O_1 = CCW(P_1, q_1, P_2)$

2) $O_2 = CCW(P_1, q_1, q_2)$

3) $O_3 = CCW(P_2, q_2, P_1)$

4) $O_4 = CCW(P_2, q_2, q_1)$

5) if $O_1 != O_2$ and ~~O3~~ $O_3 != O_4$

   return true.

6) if $O_1 = 0$ and point_on_segment(P_1, q_1, P_2) == True

   return true.

7) if $O_2 = 0$ and point_on_segment(P_1, q_1, q_2) == True

   return true.

8) return False.

Numerical a line segment দিয়ে দিতে হবে condition calculate করে দেখাতে হবে। (Simulation করে দেখাতে হবে)

দুটো given point দেওয়া ~~আছে~~ আছে তার distance

⊞ get_distance (p, q)

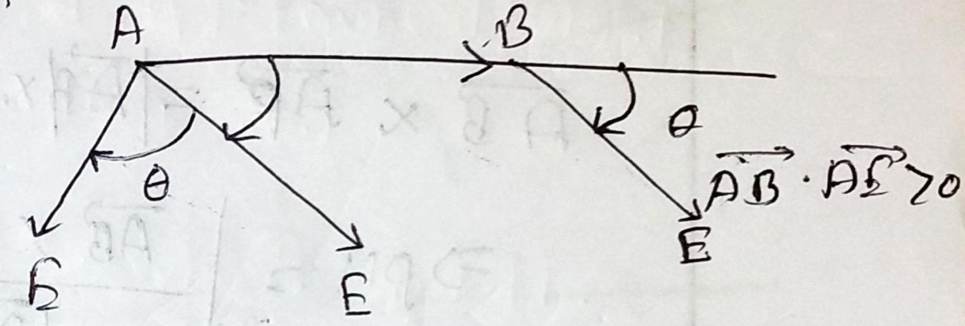
1. return sqrt((p.x - q.x) * (p.x - q.x) +
(p.y - q.y) * (p.y - q.y));

Lec-9

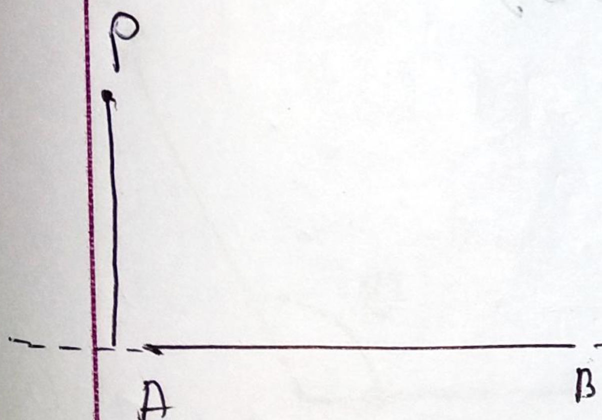
Date: 11/06/23

Given a point, p and a line, AB.

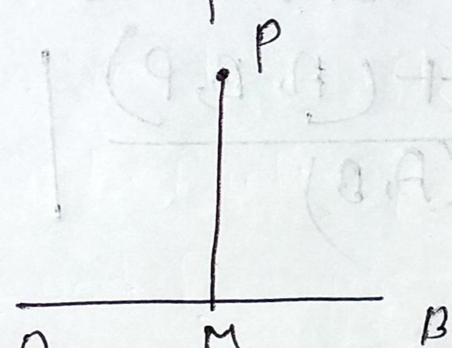
dist (P, AB)



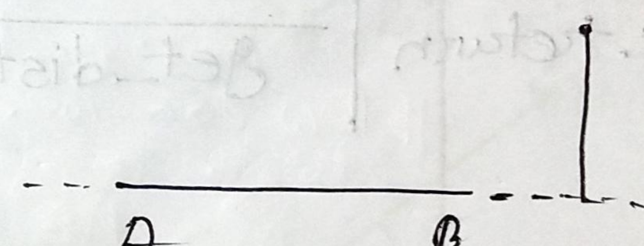
$AB.AE < 0$



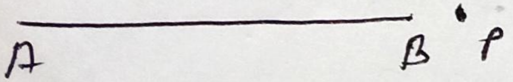
Case-I



case-2



case-3



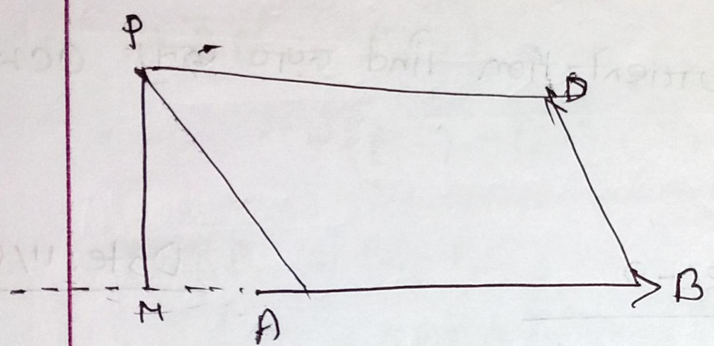
case-4

Line equation:

$y = mx + c \rightarrow$ It cannot represent perpendicular line.

$Ax + By = C$

$y - y_1 = m(x - x_1)$



$\vec{AB} \times \vec{AP} = |\vec{AB}| \times PM$

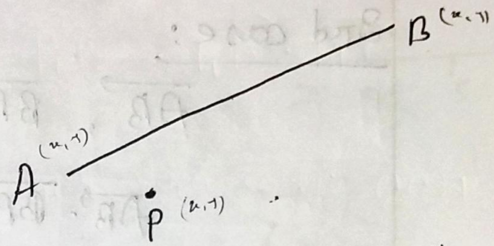
$\Rightarrow PM = \left| \dfrac{\vec{AB} \times \vec{AP}}{|\vec{AB}|} \right|$

dist_point_to_Line (P, A, B)

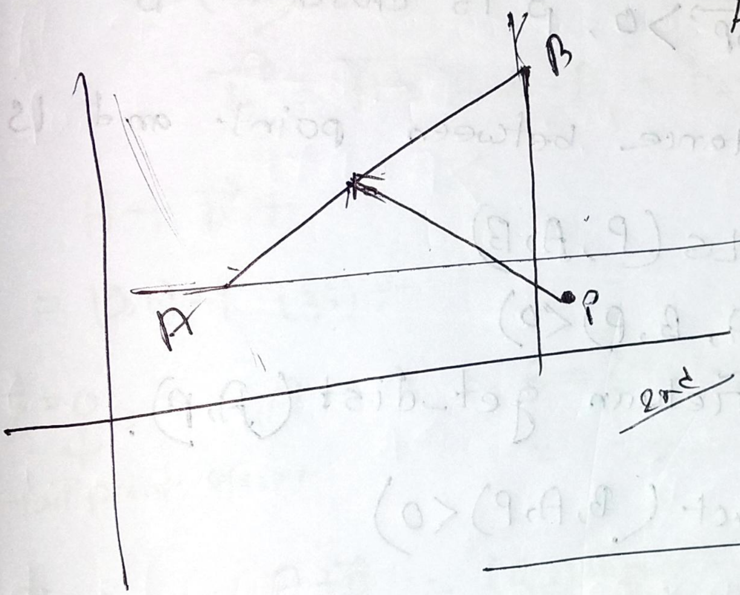
1. return $\left| \dfrac{\text{crossProduct}(A, B, P)}{\text{get\_dist}(A, B)} \right|$

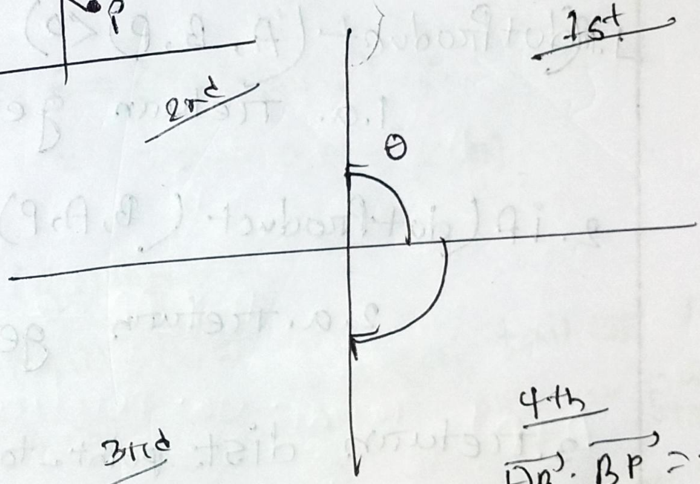
② Given a point, P and a line segment (LS), (AB) calculate the distance line segment from point P. (Here we can not extend line segment)

__Ans__



Angle calculation

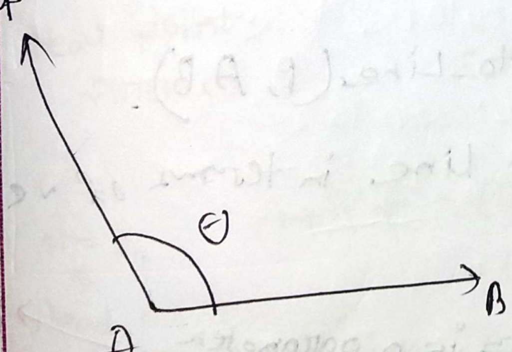


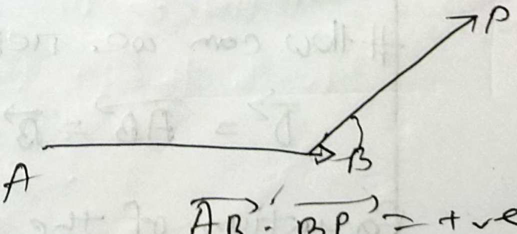


3rd

4th

$\vec{AB} \cdot \vec{BP} = +ve$





$\vec{AB} \cdot \vec{AP} = -ve < 0$

A is close to P

$\vec{AB} \cdot \vec{BP} = +ve > 0$

B is close to P.

1st case:

$\vec{AB}, \vec{AP}$

$\vec{AB} \cdot \vec{AP} < 0$, P is closest to A.

3rd case:

$\vec{AB}, \vec{BP}$

$\vec{AB} \cdot \vec{BP} > 0$, p is closer to B.

Algorithm: Distance between point and LS.

distance_b_P_LS (P, A, B)

1. if(dotProduct (A, B, P) < 0)

   1.a. return get_dist (A, P).

2. if (dotProduct (B, A, P) < 0)

   2.a. return get_dist (B, P).

3. return dist_point_to_Line (P, A, B).

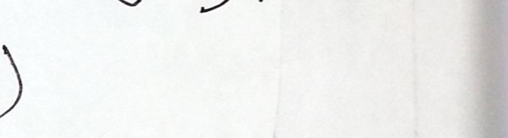
How can we represent a line in terms of vectors

$\vec{D} = \vec{AB} = \vec{B} - \vec{A}$

Equation of the line is: t is a parameter $(-\infty, +\infty)$

$\underbrace{A + \vec{D}t}$ or $[A + \vec{AB} \cdot t]$

Parametric form of line.

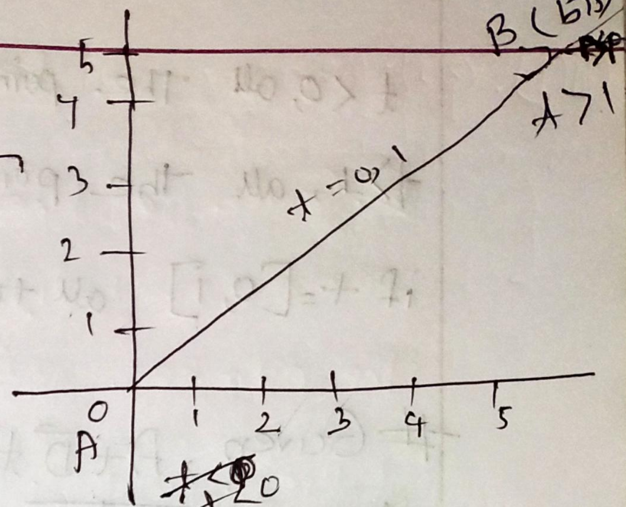


$D = \overrightarrow{BA}$

$B + \vec{D} t$

↓

tail এর point



$A = (0,0)$

$B = (5,5)$

$\vec{D} = \overrightarrow{AB} = B - A = (5-0, 5-0) = (5,5)$

$A + \vec{D} t$

$= (0,0) + (5,5) t$

$t = 0, \quad A + 0 = A = (0,0) + (5,5) \cdot 0$

↓

tail point আসবে

$= (0,0) + (0,0) = (0,0)$

$t = 1, \quad A + \vec{B} t = (0,0) + (5,5) \times 1$

↓

Head point আসবে.

$= (0,0) + (5,5)$

$= (5,5)$

$t < 0$, tail এর left side এর point আসবে.

$t > 1$, Head এর right side এর point আসবে.

$t = 2 \quad A + \vec{D} t = (0,0) + (5,5) \times 2$

↓

Head point এর right side এর point আসবে.

$= (0,0) + (10,10)$

$= (10,10)$

$t < 0$, all the points to left of $\vec{A}$

$t > 1$, all the points to the right of B.

if $t = [0, 1]$, all the points b/w A and B.

Given, $A + \vec{D}t$, Is a point P on the line?

A B

or $\vec{D} = B - A$

$\Rightarrow B = D + A$

.P

$$A \xrightarrow[\vec{D}]{} B$$

$$\vec{D} \times \vec{AP} = 0$$

$A + \vec{D}t$ কোনো এক particulare t এর জন্য P পাওয়া তাহলে P line এর উপরে থাকে।

for that t,

$$P = A + \vec{D}t$$

$$(P.x, P.y) = (A.x, A.y) + (D.x, D.y)t$$

$$= \begin{bmatrix} A.x \\ A.y \end{bmatrix} + \begin{bmatrix} D.x \\ D.y \end{bmatrix} t$$

$$\begin{bmatrix} P.x \\ P.y \end{bmatrix} = \begin{bmatrix} A.x + D.x\,t \\ A.y + D.y\,t \end{bmatrix}$$

$P.x = A.x + D.x\,t$ and

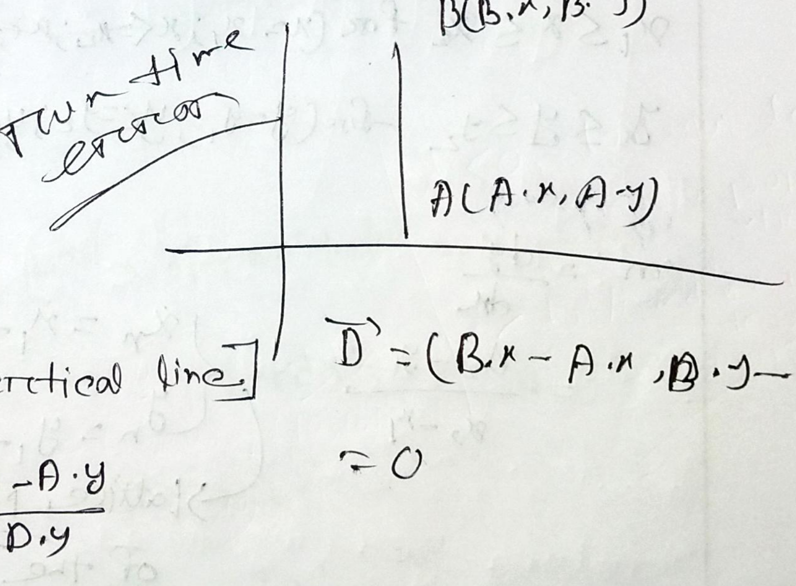
$P.y = A.y + D.y\,t$

$$t_1 = \frac{P.x - A.x}{D.x} \quad \text{and} \quad t_2 = \frac{P.y - A.y}{D.y}$$

2 টোর t এর value same হলে P given line এর উপর থাকবে,

if ($t_1 = t_2$), P is on $A + \vec{D}t$

otherwise, P is not on $A + \vec{D}t$



$\vec{D} = (B.x - A.x, B.y - A.y)$

$= 0$

$t_1 = t_2$ [vertical line]

$$\Rightarrow \frac{P.x - A.x}{D.x} = \frac{P.y - A.y}{D.y}$$

$\Rightarrow$ ~~$D.y(P.x) - D.y\,A.x = (D.x)(P.y)$~~

$$\Rightarrow \boxed{(P.x - A.x)\,D.y = (P.y - A.y)\,D.x}$$

Lec-10 Date: 15/06/23

Q. Given a line segment specified by two points $A(x_1, y_1)$ and $B(x_2, y_2)$ Find out the number of Lattice point on it.

Lattice point → with x, y of integer point.

~~Slanted line~~ Inclined line (Types of line)

Soln

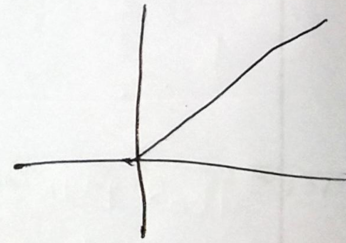
Horizontal line → y coordinate must be same

$abs(x_2 - x_1) + 1$ If Line segment is Horizontal if, $y_1 = y_2$

$abs(y_2 - y_1) + 1$ If Line segment is vertical. if, $x_1 = x_2$

$x_1 \leq x \leq x_2$ for (x = x1; x <= x2; x++)

$y_1 \leq y \leq y_2$ for (y = y1; y <= y2; y++)



$m = \frac{dy}{dx}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$\left.\begin{array}{l} x_n = x_1 + dx \\ y_n = y_1 + dy \end{array}\right\}$ Integer number

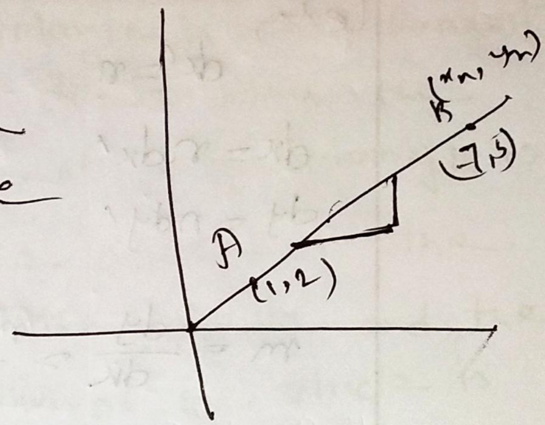
→ lattice point and end point of the line segment.

$dy = y_2 - y_1 = 5 - 2 = 3$

$dx = x_2 - x_1 = 7 - 1 = 6$

$$m = \frac{dy}{dx} = \frac{3}{6} = \frac{1}{2}$$

$\frac{3}{6}$ → original slope



A (1,2)

$B(1+6,\ 2+3) \equiv (7,5)$

or, $m = \frac{dy}{dx} = \frac{1}{2} = \frac{dy'}{dx'}$

$\frac{1}{2}$ → reduced slope

[given point এর সাথে reduced slope add করলে next lattice point পাবো]

$x_2 = x_1 + dx'$ → reduced difference

$y_2 = y_1 + dy'$

$x_3 = x_2 + \cancel{\frac{dy}{}}\,dx' = x_1 + dx' + dx' = x_1 + 2dx'$

$y_3 = \cancel{x}\cdot y_2 + dy' = y_1 + dy' + dy' = y_1 + 2dy'$

$x_4 = x_3 + dx' = x_1 + 3dx'$

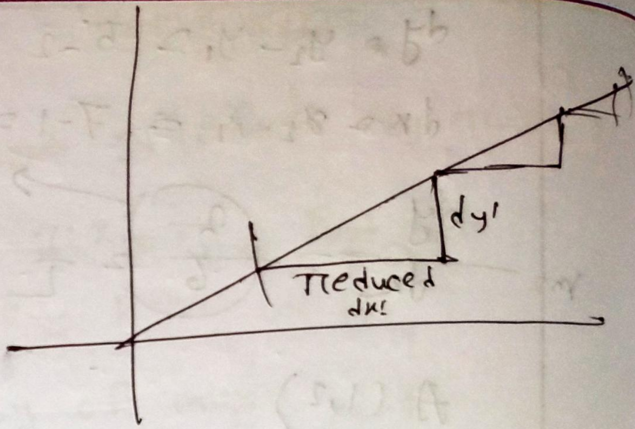
$y_5 = y_3 + dy' = y_1 + 3dy'$

let,

$dx' = n$

$dx = n dx'$

$dy = n dy'$



$m = \frac{dy}{dx} = \frac{n \times dy'}{n \times dx'}$

$n = gcd(dy, dx)$

Number of lattice point

$= gcd(|dy|, |dx|) + 1$ [A, B are both inclusive]

$gcd(|dy|, |dx|) \rightarrow$ [either A or B are excludes]

$gcd(|dy|, |dx|) - 1$ [A and B are both excluded]

Algorithm

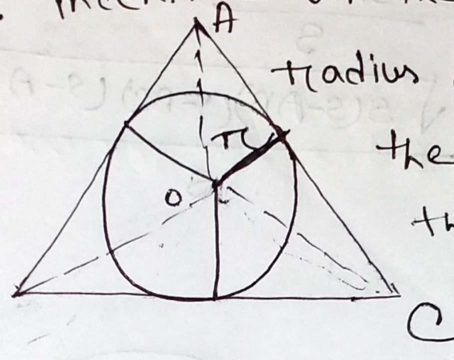
get_LatticePoint_onLS(A, B)

1) $dy = abs(y_2 - y_1)$

2) $dx = abs(x_2 - x_1)$

3) return $gcd(dy, dx) + 1$

An ~~intersect~~ incircle of a triangle is the largest possible circle that can be drawn within the ~~triangle~~ so that every side of the triangle is ~~tangent~~ largest to the circle. The center of the circle is the incentre of the triangle and the radius of the circle is the inradius of the triangle.



Given, the length of AB, AC, BC

Find out $r = ?$

ΔOAB
ΔOAC
ΔOBC

Soln

$\Delta OAB = \frac{1}{2} \times AB \times r$

~~$\Delta ABC = \frac{1}{2} \times BC \times$~~

$\Delta OAC = \frac{1}{2} \times AC \times r$

$\Delta OBC = \frac{1}{2} \times BC \times r$

$\Delta ABC = (\frac{1}{2} \times AB \times r) + (\frac{1}{2} AC \times r) + (\frac{1}{2} \times BC \times r)$

$\Rightarrow \Delta ABC = \frac{1}{2} r (AB + AC + BC)$

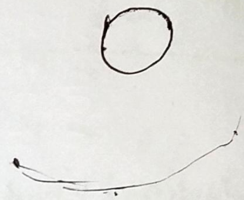
$\Rightarrow r \cdot \frac{1}{2} = \frac{\Delta ABC}{AB + AC + BC}$

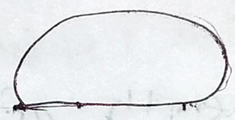
$$\Rightarrow r = \frac{2 \times \Delta ABC}{AB+AC+BC}$$

$$\Rightarrow r = \frac{\Delta ABC}{\frac{AB+AC+BC}{2}}$$

$$\Rightarrow r = \frac{\Delta ABC}{s}$$

$$\Rightarrow r = \frac{\sqrt{s(s-AB)(s-BC)(s-AC)}}{s}$$





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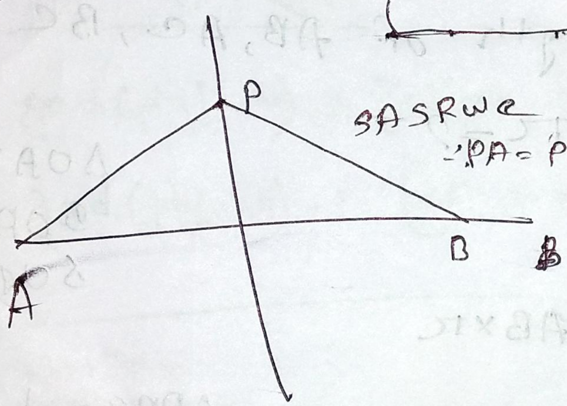
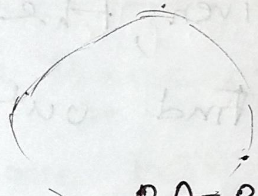
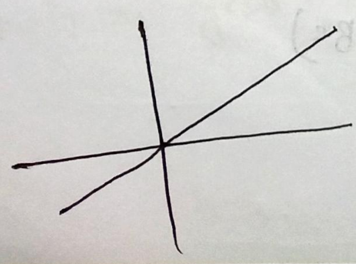


SASRule
$\therefore PA = PB$



$PA = PB$?

তিন non colinear point দিয়ে একটা circle draw করা possible. কারণ তিনটি point দিয়ে triangle draw করলে তার একটাই point এ intersect করে.



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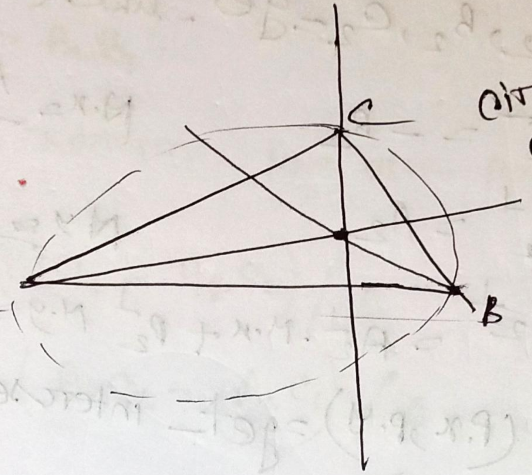

Q Given three non-coline are points. Find out the circle through it.

Sol^n: [Calculating intersecting point of bisectore]
Perpendiculare line draw করে intersecting point calculate করতে হবে।

Sol^n: ~~Ax + By = c~~ $A_1x + B_1y = c_1$

AB: A_1, B_1, c_1 calcalate করো।

$A_1, B_1, c_1 =$ get_Line(A, B)

$Ax + By = c$
↓
$-Bx + Ay = c$
↓
perpendicular line of eq.

$AB^{\perp}$:

$A_1^{\perp} = -B_1$

$B_1^{\perp} = A_1$

~~$c_1^{\perp} =$~~

$M.x = \frac{A.x + B.x}{2}$

$M.y = \frac{A.y + B.y}{2}$

$c_1^{\perp} = A_1^{\perp} M.x + B_1^{\perp} \times M.y$

$\underline{BC:}$

$A_2, B_2, C_2 = $ get_line (A_2, B_2)

$\underline{BC^{\perp}:}$ $A_2^{\perp} = -B_2$ $\qquad M.x = \dfrac{A_2.x + A_1.y}{2}$

$B_2^{\perp} = A_2$ $\qquad M.y = \dfrac{A_2.y + B_2.y}{2}$

$C_2^{\perp}$

$= A_2^{\perp}.M.x + B_2^{\perp}\ M.y$

$(P.x, P.y) = $ get_intersection_point $(A_1^{\perp}, B_1^{\perp}, C_1^{\perp}, A_2^{\perp}, B_2^{\perp}, C_2^{\perp})$

~~AC:~~

~~A_3, B_3, C_3~~

get_intersection_point$(A_1, B_1, C_1, A_2, B_2, C_2)$

1) $\cancel{A}\ \Delta = A_1 B_2 - A_2 B_1$

2) if $\Delta = 0$, return lines are parallel

3) else, $x = (B_2 C_1 - B_1 C_2) / \Delta$

~~4~~ $y = (A_1 C_2 - A_2 C_1) / \Delta$

4) return(x, y)

(After Bid tutorial
2nd class)