Class -1

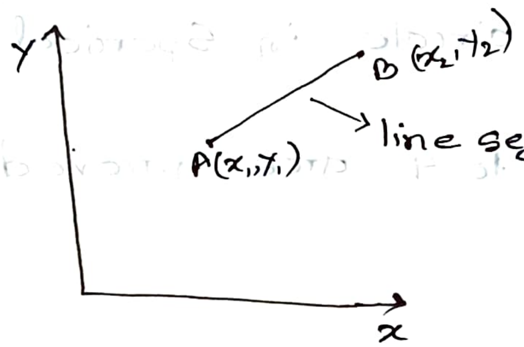
Geometry :- Branch of mathematics in which we discuss or study about the shape of different object

Space/Surface → Flat → Euclidian/playing geometry
Space/Surface → Curve → Non-Euclidian geometry.

Axiom :- a proposition which considered to be proved

Postuletes:-

(i)



(ii) If the line segment is extended in infinite it produce a line. Line has no end point.

(iii) With any centre & radius, anyone can draw a circle.

(iv) All rightangles are equal to each other.

(v) If you give a line segment and a point (not intersect), there should draw only one equidistant line.

Non-Euclidian → Sperical Geometry (earth → positive curvature.
→ Hiperbolic " (negative Curvature

L - 2

Sperical Geometry — Great Circle.

Each line is a great circle in Sperical geometry

*Euclidian postuletes 1 to 4 are proved in sperical geometry.

L-3

Euglidian geometry — 2D or 3D.

We study Computational Geometry, So that we can solve different computer Science related geometric algorithm.

Ex - Gaming, Graphics, programing, VSLI Design

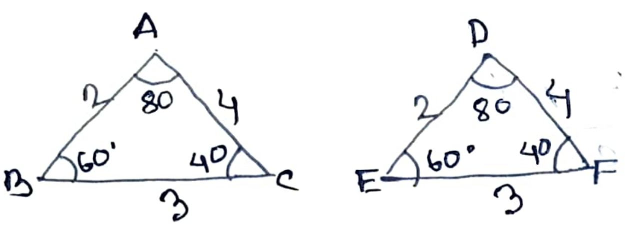
□ Complex property :-

□ Similar property :-

□ A triangle has 3 sides, 3 angles & 3 peak point.

□ A triangle is a three pair of intersecting line.

Congruence of triangle : $\Delta ABC \cong \Delta DEF$



$\Delta ABC \cong \Delta DEF$

$$\left.\begin{aligned} AB &= DE \\ BC &= EF \\ CA &= FD \end{aligned}\right\} \text{corresponding sides.}$$

□ When two triangle are congruence:

(I) Side - Side - Side.

(II) S - A - S.

(III) A - S - A.

(IV) RHS → side.

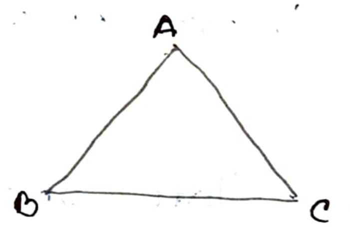
↓ ↓→ Hypoteneous

Right Triangle

Low of Triangles

(I) Sine law of Triangle

(II) Cosine law of Triangle



$$\frac{AB}{\sin\angle C} = \frac{AC}{\sin\angle B} = \frac{BC}{\sin\angle A} = k.$$

"The ratio of side & its opposite angle value with sine is always same" → law of sine

Law of cosine

$$\cos\angle B = \frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC}$$

$$\angle B = \cos^{-1}\left(\frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC}\right)$$

[-1 to +1) acos(x) in C++ $\theta = [0, \pi]$

[-1 to +1) asin(x) in C++ $\theta = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

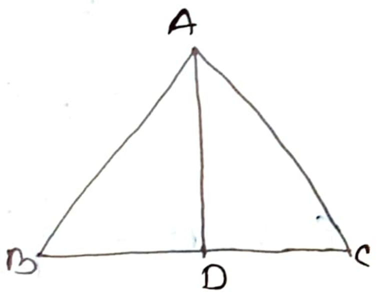
L-4

$\cos\frac{\pi}{2} = 0$

$\frac{\pi}{2} = \cos^{-1}(0)$

$\pi = 2\cos^{-1}(0)$

$= 2\,acos(0.0)$ → Double.



$$S = \frac{AB+BC+CA}{2}$$

$$\Delta ABC = \sqrt{S(S-AB)(S-BC)(S-AC)}$$

Algorithm for calculate area of Triangle :

```
get_Tri_area (double AB, double BC, double AC)
{
      S = (AB+BC+CA) / 2.0

   return sqrt(S*(S-AB)(S-BC)(S-AC));

}
```

$AD \perp BC$

$\Delta ABC = \frac{1}{2} \times BC \times AD$.

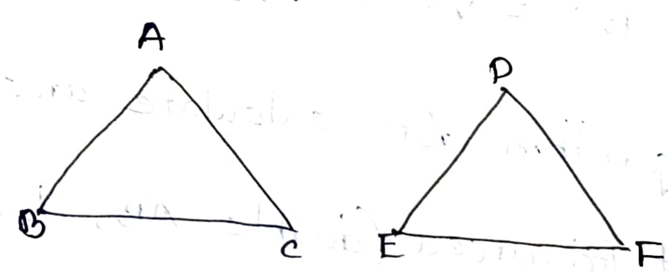
$= \frac{1}{2} \times BC \times AB \sin \angle B$.

$\sin \angle B = \frac{2 \Delta ABC}{BC \times AB}$

$\angle B = \sin^{-1} \left(\frac{2 \Delta ABC}{BC \times AB} \right)$.

Similarity of Triangles.

If $\Delta ABC \cong DEF$



$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$.

ΔABC & ΔDEF are equiangular Triangle

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = k \rightarrow$ Scaling factor
or
Representative Fraction

If $\triangle ABC \sim \triangle DEF$

$$\frac{\triangle ABC}{\triangle DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AC}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2$$

$$\longrightarrow \frac{AD}{DB} = \frac{AE}{BC}$$

L-5

Integral point / lattice point :- It is a point whether its value of all axes is integer.

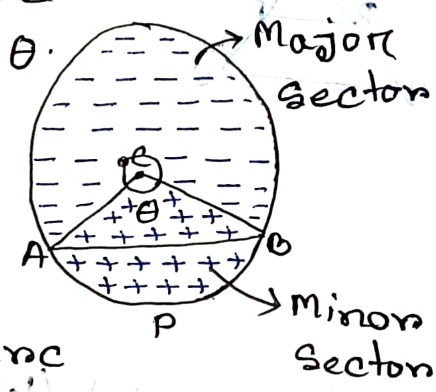
Circle :- Whether the all points of a curveture which are quidistance of a fixed point is called ~~is~~ circle.

Arc :- An Arc is a portion of a circle.

Cord :- A cord is a line segment obtained by containing two point in circle.

A segment of a circle is inner region of that circle with the cord & its corres--ponding Arc (magor & minor segment).

Cord AB/It's corresponding arc APB have subtainded an angle θ.



Minor sector is the region between the two radius & their corresponding minor arc

If the whole circle considered as a sector then,

$$360^\circ \rightarrow \pi r^2$$

$$1^\circ \rightarrow \frac{\pi r^2}{360}$$

$$\theta^\circ \rightarrow \frac{\pi r^2}{360^\circ}\theta$$

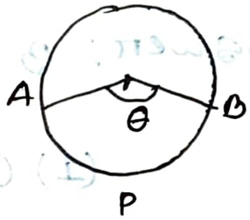
$$\rightarrow \frac{\pi r^2 \times \theta^c}{360 \times \frac{\pi}{180}}$$

$$\rightarrow \frac{\theta^c}{2} r^2$$ → The area of minor sector of angle θ^c

Area of minor segment = Area of minor sector - Area of $\triangle ABC$.

~~Cur~~

Circumference of a circle with radius r is $= 2\pi r$.



Here,

$\theta \rightarrow$ is the angle subtended by arc APB

When $\theta = 360°$, the length of the arc is $= 2\pi r$.

" $\theta = 1°$ " " " " " " $= \dfrac{2\pi r}{360°}$

" $\theta = \theta°$ " " " " " $= \dfrac{2\pi r}{360°} \times \theta$.

$$\therefore s = \frac{2\pi r}{360°} \times \theta$$

$$= r\theta^c$$

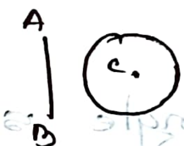
Q

& a line AB.

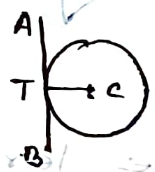
(i) How many ways can we arrange the line & the circle with each other

Answer: 3 ways.

(1) Outer →



(2) Tangent →



(T is touching point)

Here, $AB \perp TC$.

(3) Intersect / secant →

⊞ Can be,

Proved by Mathematical induction technique

Construction " technique

Contradiction technique

HW Prove that $CT \perp AB$.

Q) Given,  & a point P

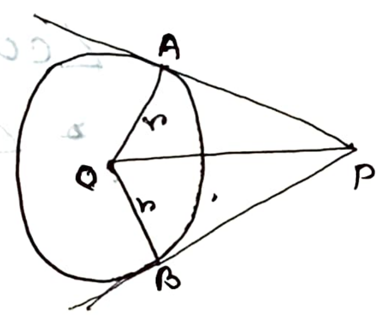
It can be arranged in 3 ways.

(i)  → outside (can draw 2 tangents)

(ii)  → on the circle (draw 1 tangent)

(iii)  → inside (no tangent can drawn)

HW If a point is outside circle than prove we can draw two tangent & the tangent are equal length.



Here,

$AP \perp AO$

$BP \perp BO$

So, $\angle OAP = 90° = \angle OBP = 90°$

$\triangle APO$ & $\triangle BPO$

$AO = BO = r$

$OP = OP$ (common side)

$\angle OAP = \angle OBP = 90°$

$\therefore$ ~~ADA~~ $\triangle APO \cong \triangle BPO$

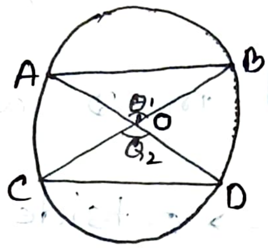
So, $AP = BP$ (proved).

$\boxed{\text{H.W}}$ Given,

$AB = CD$

then prove that

$\theta_1 = \theta_2$



Answer :-

Given,

$AB = CD$

$\angle BAD = \angle ADC$ & $\angle BCD = \angle CBA$)

$\therefore \angle A = \angle D$ & $\angle C = \angle B$

So, $\triangle AOB \cong \triangle COD$

and so,

$\angle COD = \angle AOB$

$\angle \theta_1 = \angle \theta_2$

(proved)

L-6

error = Expected result − obtained result.

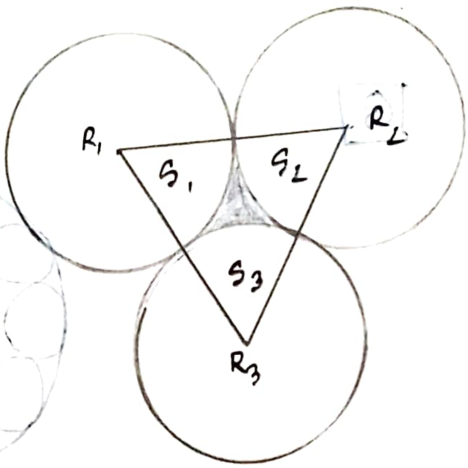
Absolute error = |error| {scale measurment same}

Relative error = $\frac{|error|}{\text{Expected result}}$

↙ scale of measurment same না হলে

$= \frac{\text{absolute error}}{\text{Expected result}}$

$\boxed{Q}$ Caculate the area (shaded).



(i) $R_1R_2 = R_1 + R_2$

$R_1R_3 = R_1 + R_3$

$R_2R_3 = R_2 + R_3$

(ii) $S = (R_1R_2 + R_1R_3 + R_2R_3)/2.0$

(iii) $\Delta R_1R_2R_3$ = get_Tri_area (R_1R_2, R_2R_3, R_1R_3)

get_angle function ← (iv) $\left[\theta = \cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right) \right.$

return acos$\left.\left(((a*a)+(b*b)-(c*c))/(2*a*b)\right) \right]$

(v) θ_1 = get_angle (R_1R_2, R_2R_3, R_1R_3)

(vi) θ_2 = get_angle (R_1R_2, R_1R_3, R_2R_3)

(vii) θ_3 = get_angle (R_1R_3, R_2R_3, R_1R_2)

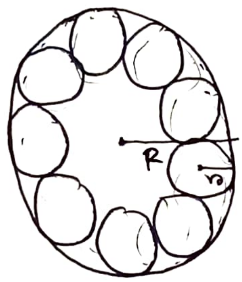
(VIII) $S_1 = R_1^2 \dfrac{\theta_1}{2}$

(IX) $S_2 = \dfrac{\theta_2}{2} \times R_2^2$

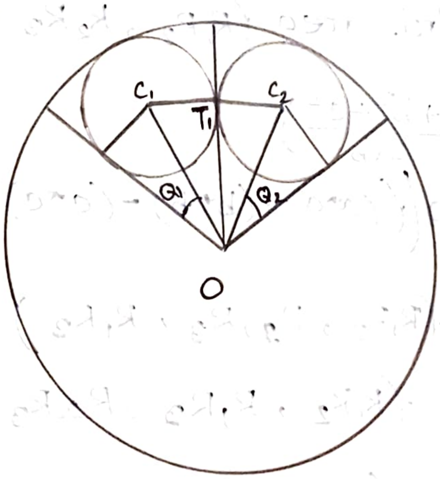
(X) $S_3 = \dfrac{\theta_3}{2} \times R_3^2$

(XI) Shaded area $= \Delta R_1R_2R_3 - (S_1 + S_2 + S_3)$

$\boxed{Q}$



find the r where given R (radius) & n (number of small circle



ΔOC_1T_1 & ΔOC_2T_2

$\angle C_1T_1O_1 = \angle C_2T_1O$

$C_1T_1 = C_2T_1 = r$

$T_1O = T_1O$ = common line/side.

$\therefore \angle\theta_1 = \angle\theta_2$

$2n\theta = 360^\circ$

$\theta = \dfrac{360^\circ}{2n} = \dfrac{180^\circ}{n} = \dfrac{\pi}{n}$

We know,

$\sin\theta = \dfrac{C_1T_1}{OC_1}$

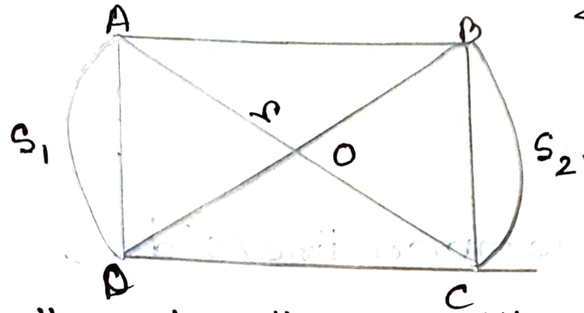
$\sin\theta = \dfrac{r}{R-r}$

$r = R\sin\theta - r\sin\theta$.

$r(1+\sin\theta) = R\sin\theta$.

$r = \dfrac{R\sin\theta}{1+\sin\theta}$

$\boxed{Q}$



given

$\frac{L}{W} = \frac{4}{3}$, P.

Calculate the length & width.

Ans:-

$S_1 + S_2 + 2 * \text{length} = P$ (পরিসীমা)

$2S_1 + 2L = P$ ——— (1)

$\triangle ADC$, $\angle ADC = 90°$

$\frac{L}{W} = \frac{4x}{3x} = \frac{ax}{bx}$

$\therefore L = ax$, $W = bx$.

$$AC = \sqrt{AD^2 + DC^2}$$

$$= \sqrt{(ax)^2 + (bx)^2}$$

$$= \sqrt{a^2+b^2} \cdot x.$$

$$r = \frac{AC}{2} = \frac{\sqrt{a^2+b^2} \cdot x}{2}$$

$\triangle OAD$

$$\theta = \cos^{-1}\left(\frac{OA^2 + OD^2 - AD^2}{2OA \cdot OD}\right)$$

$$= \cos^{-1}\left(\frac{2r^2 - AD^2}{2r^2}\right)$$

$$= \cos^{-1}\left(\frac{2 \times \frac{a^2+b^2}{4} \cdot x^2 - 2b^2x^2}{2\,\frac{a^2+b^2}{4} \cdot x^2}\right)$$

$$= \cos^{-1}\left[\frac{a^2x^2 - b^2x^2}{(a^2+b^2)\,x^2}\right]$$

$$\theta = \cos^{-1}\left[\frac{a^2-b^2}{a^2+b^2}\right]$$

$$s_1 = r\theta$$

$$= r\,\frac{\sqrt{a^2+b^2}\;x}{2} \times \theta \,.$$

from ①

$$2 \times \frac{\sqrt{a^2+b^2}\cdot x}{2} \times \theta + 2ax = P \,.$$

$$x\left\{\left(\sqrt{a^2+b^2}\right)\theta + 2a\right\} = P \,.$$

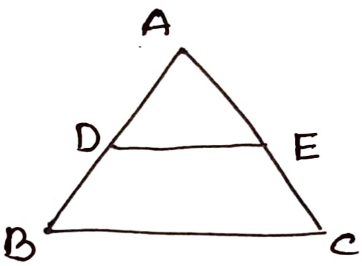
$$\therefore x = \frac{P}{\sqrt{a^2+b^2}\cdot\theta + 2a} \,.$$

$\boxed{\text{H.W}}$

(i) Binary search

(ii) Ternary search.

L-7



Given,

AB, AC, BC

DE || BC

ADE : BDEC

AD = ?

Ans:-

From DE || BC

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\Rightarrow \frac{BD+AD}{AD} = \frac{AE+EC}{AE}$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$$

$\Delta ADE \sim \Delta ABC$

$\angle A = \angle A$

$$\frac{AD}{AB} = \frac{AE}{AC}$$

So, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$AE = \frac{AD}{AB} \times AC$$

$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$DE = \frac{AD}{AB} \times BC$$

For som AD,

AE & DE

$$S_{\Delta ADE} = \frac{AD + DE + AE}{2}$$

$$\Delta ADE = \sqrt{s(s-AE)(s-DE)(s-AD}$$

$$S_{\Delta ABC} = \frac{AB + AC + BC}{2}$$

$$\Delta ABC = \sqrt{s(s-AB)(s-AC)(s-BC}$$

$\therefore$ BDEC $= (\Delta ABC - \Delta ADE)$

$$R = \frac{\Delta ADE}{BDEC}$$

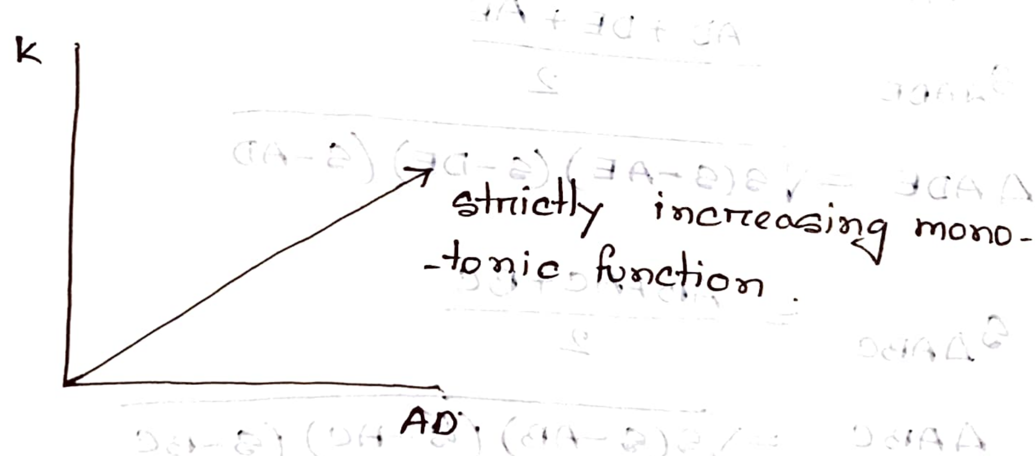
If $k == n$ for AD

Guess AD Technique.

(i) Random guess

(ii) Range of AD [0, AB]

[কোনো একটি function যদি either non-decreasing or increasing function হয় তখন এই function কে monotonic function বলে। Function monotonic হলেই binary search apply করতে পারব।



easy way,

$\triangle ADE \sim \triangle ABC$

$$\frac{\triangle ADE}{\triangle ABC} = \frac{AD^2}{AB^2} = \frac{AE^2}{AC^2} = \frac{DE^2}{BC^2}$$

$$AD = \sqrt{\frac{\Delta ADE}{\Delta ABC} \times AB^2}$$

Now,

$$\frac{\Delta ADE}{BDEC} = r$$

$$\frac{BDEC + \Delta ADE}{\Delta ADE} = \frac{r+1}{r}$$

$$\therefore \frac{\Delta ABC}{\Delta ADE} = \frac{r+1}{r}$$

$$\frac{\Delta ADE}{\Delta ABC} = \frac{r}{r+1}$$

$$\Delta ADE = \frac{r}{r+1} \times \Delta ABC.$$