Jahangirnagar University
Department of Computer Science and Engineering
3rd Year 1st Semester B.Sc. (Hons.) Tutorial Exam -2022

Course Title: Computational Geometry
Time: **80 minutes**

Course No: **CSE-305**
Full Marks: 30+30

[দেখি সব পারি কিনা]

Part-I

Q1. a) Upon the circumference of the inner circle with radius r are placed n outer circles each with radius R in such a way that two consecutive outer circles touch each other and the inner circle touches all the outer circles just as shown in the diagram below:

i. Formulate an equation to compute R given the value of r and n. [absolute error or relative error should not exceed 10^{-5}] [5]

ii. If $n = 100$, $r = 100$ then $R = ?$ [5]

b) Given three non-collinear points $A\ (x_1, y_1)$, $B\ (x_2, y_2)$ and $C\ (x_3, y_3)$ on a *2D* plane. Write an algorithm to find the minimum distance of the line segment specified by $P\ (x, y)$ and $Q\ (x, y)$ from the center, S of the circle passing through A, B and C. [6]

c) Write an algorithm to calculate the number of lattice points on a line segment with ends points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. [4]

d) Given N points as $P_i (x_i, y_i)$; $0 \leq i < N$ on a $2D$ space and a specific point $Q (x, y)$. Write an algorithm of at most **log linear** ($NlogN$) time complexity to find the minimum number of lines going through Q in order to cover all the points. [10]

Part-II

Q2.

a) Given two points as $A (10,20)$ and $B (30,10)$. Now answer the following:

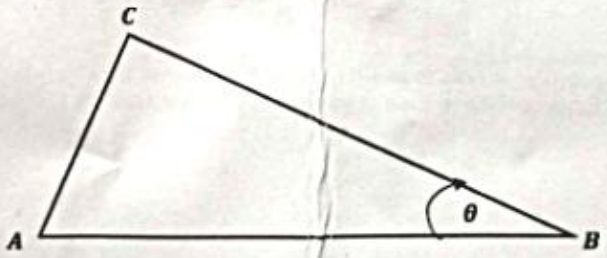
i. Represent the line AB in parametric form considering t as the parameter. [2]

ii. Analyze the characteristic of t. [2.5]

iii. Find a unit vector which will be perpendicular to $\overrightarrow{AB}$. [2.5]

b) Given two end points of a line segments as $M(0,0)$ and $N(2,0)$. Find the minimum distance of the line segment from the given point $P(4,0)$. [5]

c) A triangle has to be drawn in such a way that the length of the largest side is given as $AB = x$ and the sum of the other two sides is given as $AC + BC = S$ with a constraint that BC maintains an angle θ with AB as shown in the picture. [10]



i. Analyze the case when the values of AC and BC cannot be determined if any.

ii. Write an algorithm to find the value of AC and BC. [absolute error or relative error should not exceed 10^{-6}]

iii. Find the value of AC and BC given that $x = 12, S = 18, \theta = 90^0$. ***[only up to 3rd iteration if iterations are required]***

d) Two line-segments are defined as $P_1(1,2)$, $Q_1(1,7)$ and $P_2(5,5)$, $Q_2(-5,5)$. Demonstrate the execution trace of the algorithm to check if they intersect. [8]

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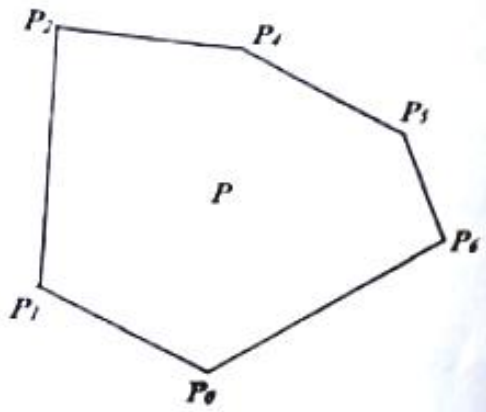
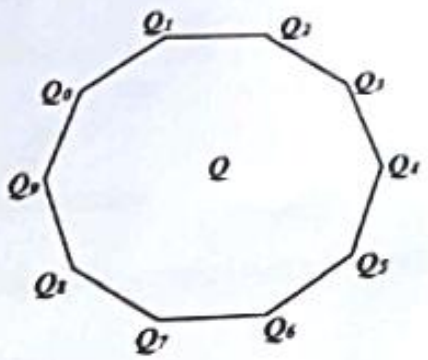
Q1. a) Let P and Q be two convex hulls such that $P_i.x < Q_j.x$; $\substack{0 \le i < n \\ 0 \le j < m}$. Show the execution trace of the algorithm ***merge_CH(P,n,Q,m)*** in order to merge P and Q so that the resultant polygon still becomes a convex hull of the point sets P and Q. (use below diagram to compute ***ccw(p,q,r)***) [8]





b) Let $P = (P_0, P_1, P_2 \ldots P_{n-1})$ be a sequence of n points where $(P.x, P.y) \in Z$ on a $2D$ plane. Someone claims that the following algorithm can identify whether or not P describes the boundary of a convex polygons in clockwise order. [6]

```
bool is_convex(p_0, ..., p_{n-1}) {
    for i = 0, ..., n - 1:
        if (p_i, p_{(i+1) mod n}, p_{(i+2) mod n}) form a rightturn:
            return false;
    return true;
}
```

Disprove the claim and describe a correct algorithm to solve the problem.

c) Let $P \subset Z^2$ be a convex polygon, given as an array **P[0], P[1],....P[n-1]** of its N ($\leq 10^6$) vertices and M ($\leq 10^6$) queries are provided. Each query contains a point $Q \in Z^2$. Construct an algorithm of not more than **log linear time complexity** to find if Q lies inside, outside or on the boundary of the polygon. [8]

d) Let $V \subset \mathbf{Z}^2$ be a **convex** polygon, defined with the following vertices.

V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7
(7, 1)	(13, 2)	(14,7)	(10, 9)	(6, 10)	(5, 9)	(4, 6)	(4, 3)

Determine the number of lattice points inside the convex polygon

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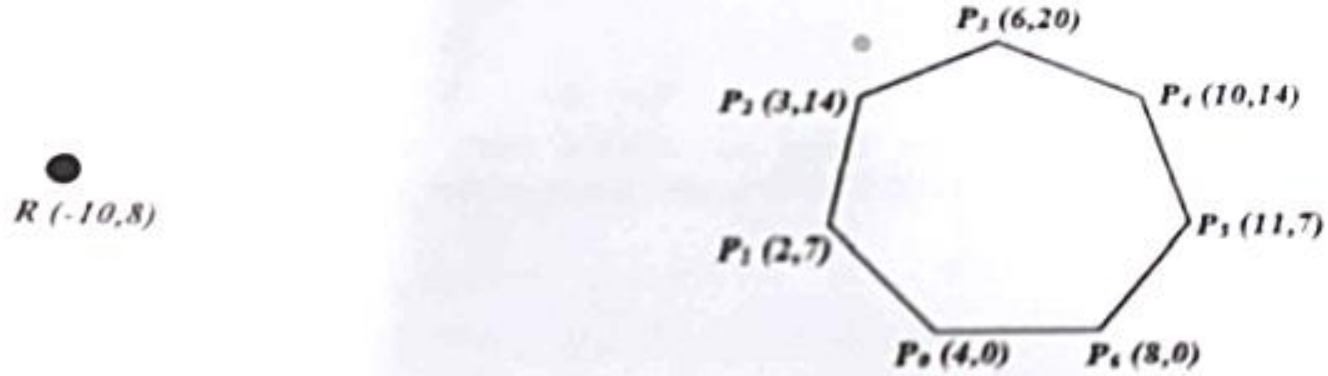
T-III

Q1. a) Determine the upper tangent from ***R*** to the convex polygon ***P*** in Z^2 with the condition that: $R.x < P_i.x; 0 \leq i < n$ using an algorithm of $log_2 n$ running time complexity. [10



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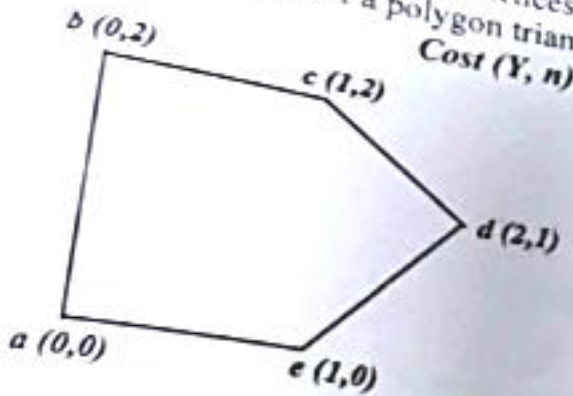
b) A convex polygon Y of n vertices can be triangulated into $n-2$ triangles with respect to any [10]
vertex. The cost of a polygon triangulation is defined as:



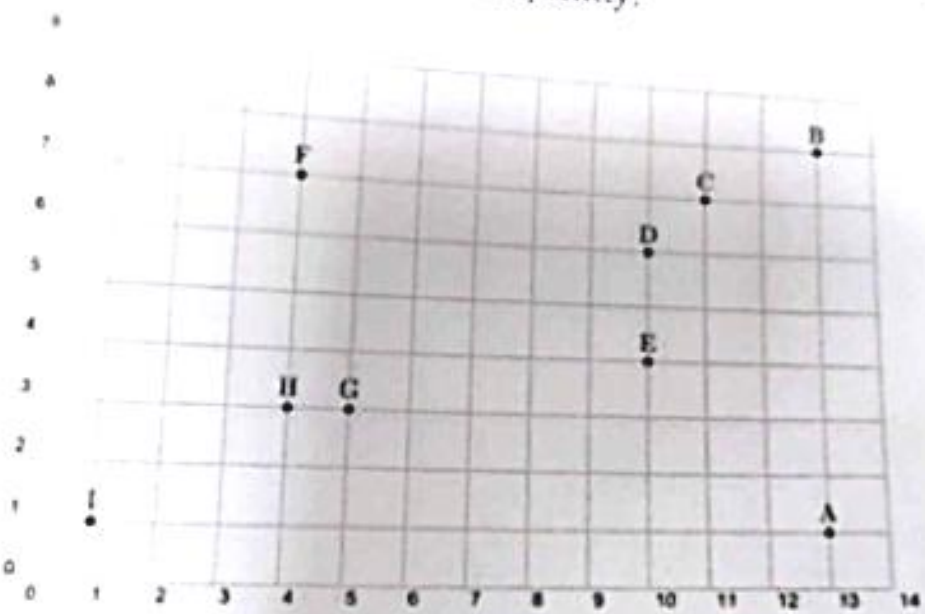
$$Cost(Y, n) = \Delta_1 + \Delta_2 + \ldots + \Delta_i + \cdots + \Delta_{n-2}$$

$$\Delta_i(a, b, c) = dist(a, b) + dist(b, c) + dist(a, c)$$

$$dist(p, q) = \sqrt{(p.x - q.x)^2 + (p.y - q.y)^2}$$

Determine the minimum cost of polygon triangulation considering the polygon to the left. Mention the time complexity of your algorithm.

c) Construct a convex hull of the following set of points given in the diagram using an algorithm of at most log linear ($NLog_2N$) running time complexity. [10]



If there are 10^6 points in the plane, could Jarvis March algorithm be used to solve this problem. Mention the case for which Jarvis March runs in $O(n)$ time complexity.

C. Geometry

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3) Write an algorithm of O(logn) time complexity in order to inspect whether a given point lies inside a convex polygon or not. [10]