2⇒a

(i) (2,3) & (8,9)

Direction vector $\vec{D} = (8-2, 9-3)$

$= (6, 6)$

∴ Parametric form of line $= (2,3) + (6,6) \times t$

$= (2+6t, 3+6t)$

(ii) (-5,6) & (7,8)

direction vector, $\vec{D} = (\cancel{-7}, 8) - (-5, 6)$

$= (12, 2)$

∴ Parametric form $= (-5, 6) + (12, 2) \times t$

$= (-5+12t, 6+2t)$

(iii) (1,1) & (10,1)

direction vector $\vec{D} = (10, 1) - (1, 1)$

$= (9, 0)$

∴ Parametric form $= (1,1) + (9,0) \times t$

$= (1+9t, 1)$

(iv) $(1,2)$ & $(10,2)$

Direction vector $= (10,2) - (1,2)$

$= (9,0)$

Parametric form $= (1,2) + (9,0) \times t$

$= (1+9t\ ,\ 2)$

This are a the parametric form of those line.

Now,

The parametric form of line (i) is,

$AB = (2+6t\ ,\ 3+6t)$

The direction vector $\vec{d} = (6,6)$

So, The direction vector $\vec{d} = (-6,6)$

To find the foot of the perpendicular from (7,

to the line.

(i) Let the point on the line be (x, y)

$x = 2+6t$

$y = 3+6t$.

So the dot product of the direction vector $(6,6)$

& connecting vector $(x-7\ ,\ y-2)$ must be zero

$6(x-7)+6(y-2)=0$

$6(2+6t-7)+6(3+6t-2)=0$

$6(-5+6t)+6(1+6t)=0$

$\Rightarrow 72t-24=0$

$t=\frac{24}{72}=\frac{1}{3}$

$\therefore$ the point, $x=2+6t=2+6\times\frac{1}{3}=4$

$y=3+6t=3+6\times\frac{1}{3}=5$

$(x,y)=(4,5)$ Ans

for line (ii)

Parametric form $=(-5+12t, 6+2t)$

direction vector $=(12,2)$

A line perpendicular to it will have a directi[on] vector that satifies the dot product:

$(12)\times a+(2\times b)=0$

$b=-6a$

if $a=1$ then $b=-6$

So the $\overline{d^{\perp}}=(1,-6)$

line (iii) $= (1+9t, 1)$

(iv) $= (1+9t, 2)$

~~th~~ here,

the (iii) & (iv) line are parallel because of their constant y values.

So, they do not intersect.

(b)

(i) We have to find r where R & r are given.

fig-1

From the fig-1

ΔOC_1T_1 & ΔOC_2T_2

$\angle C_1T_1O = \angle C_2T_1O$

$C_1T_1 = C_2T_2$

$T_1O = T_1O$

So, $\Delta OC_1T_1 \cong \Delta OC_2T_2$

hence,

$$2n\theta = 360^\circ$$

$$\theta = \frac{360^\circ}{2n} = \frac{180^\circ}{n} = \frac{\pi}{n}$$

And We know,

$$\sin\theta = \frac{C_1T_1}{OC_1}$$

$$\sin\theta = \frac{r}{R-r}$$

$$r = R\sin\theta - r\sin\theta$$

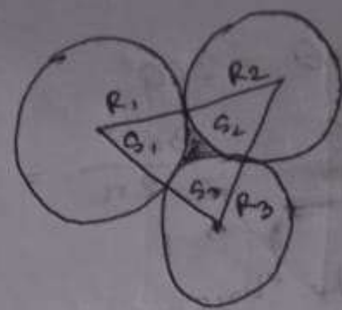
$$r + r\sin\theta = R\sin\theta$$

$$r(1+\sin\theta) = R\sin\theta$$

$$r = \frac{R\sin\theta}{1+\sin\theta}$$

This is a generalize formula to compute 'r

(ii)
Calculate the shaded area.



Algorithm

(a) $R_1R_2 = R_1+R_2$

$R_1R_3 = R_1+R_3$

$R_2R_3 = R_2+R_3$

(b) $S = \cancel{R_1+R_2}\left(R_1R_2+R_1R_3+R_2R_3\right)/2.0$

(c) $\Delta R_1R_2R_3$ = get_Tri_area (R_1R_2, R_1R_3, R_2R_3)

~~(d)~~ $\cancel{\theta = \cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right)}$

(d) θ_1 = get_angle (R_1R_2, R_2R_3, R_1R_3)

(e) θ_2 = get_angle (R_1R_2, R_1R_3, R_2R_3)

(f) θ_3 = get_angle (R_1R_3, R_2R_3, R_1R_2)

(g) $S_1 = R_1^2 \times \frac{\theta_1}{2}$

(h) $S_2 = \frac{\theta_2}{2} \times R_2^2$

(i) $S_3 = \frac{\theta_3}{2} \times R_3^2$

(j) Shaded area $= \Delta R_1R_2R_3 - (S_1+S_2+S_3)$

(d)

If $(CCW(P_1, P_2, P_3)) = 0$

then P_1, P_2 & P_3 points are collinear.

If P_3 is on the segment joined by P_1 & P_2

then

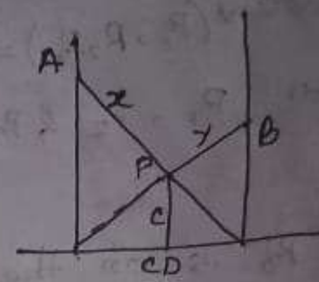
(i) $CCW(P_1, P_2, P_3) = 0$

(ii) $\min(x_1, x_2) \le x_3 \le \max(x_1, x_2)$

$\min(y_1, y_2) \le y_3 \le \max(y_1, y_2)$.

(3) a

AD = 10, BC = 10
x   y
c = 3



Algorithm

get_CD (x, y, c)

(i) set low = 0.0

(ii) set high = min(x, y)

(iii) while (high − low ≥ 10^{-6})

(a) compute mid = (low + high)/2.0

(b) set CD = mid

(c) compute r = get-ratio (x, y, CD)

(d) if (r > c) low = mid
else high = mid

(iv) return high

get_ration (x, y, CD)

(i) compute $r = \dfrac{sqrt(x*x - CD*CD) * sqrt(y*y - CD*CD)}{sqrt(x*x - CD*CD) + sqrt(y*y - CD*CD)}$

(ii) return r.

Execution trace

~~iteration 1~~

(i) $low = 0.0$

(ii) $high = (min(10, 10)) = 10.0$

(iii) $high - low \geq 10^{-6}$

$10 - 0.0 \geq 10^{-6}$ } iteration 1

$\therefore$ (a) $mid = \frac{10+0}{2} = 5$

(b) $CD = 5$

(c) $r = \frac{\sqrt{(100-25)} \times \sqrt{(100-25)}}{\sqrt{(100-25)} + \sqrt{(100-25)}} = \frac{75}{2\sqrt{75}} = \frac{\cancel{75}^{25}}{2 \times \cancel{5}\sqrt{3}}$

$= \frac{25}{2\sqrt{3}} = 12.99$

(d) $12.99 > 3$

$\therefore low = 5$

iteration 2

(i) $10 - 5 \geq 10^{-6}$

(a) $mid = \frac{10+5}{2} = 7.5$

(b) $CD = 7.5$

(c) $r = \frac{\sqrt{100-(7.5)^2} \times \sqrt{100-(7.5)^2}}{\sqrt{100-(7.5)^2} + \sqrt{100-(7.5)^2}}$

$= 3.3072$

(d) $3.3072 > 3$

Iteration 3

1 $10 - 3.3072 \geq 10^{-6}$

(a) mid $= 6.6536$.

(b) CD $= 6.6536$

(c) $r = \dfrac{\sqrt{(100-(6.6536)^2} \times \sqrt{100-(6.6536)^2}}{\sqrt{100-(6.6536)^2} + \sqrt{100-(6.6536)^2}}$

$= 3.7326$.

Iteration 3

$10 - 7.5 \geq 10^{-6}$

(a) mid $= 8.75$.

(b) CD $= 8.75$

(c) $r = 2.42062$

(d) $2.42062 < 3$

so high = mid

high $= 8.75$.

iterration 4

(a) $mid = \frac{8.75+7.5}{2} = 8.125$

(b) $CD = 8.125$

(c) $r = \frac{33.984375}{11.6592238} = 2.9148$

(d) $2.9148 < 3$

$high = 8.125$.

iterration 5

(a) $mid = \frac{8.125+7.5}{2} = 7.8125$.

(b) $CD = 7.8125$

(c) $r = 3.1210913$

(d) $3.1210913 > 3$

$low = mid$

$low = 7.8125$.

(d)

(i) $P_1 = (10, 0)$ $Q_1 = (0, 10)$

$P_2 = (0, 0)$ $Q_2 = (10, 10)$

Evaluation (if they intersect or not)

(a) $Q_1 = ccw(P_1, Q_1, P_2)$

$= \cancel{ccw(\overrightarrow{P_1 Q} \times}$

$\overrightarrow{P_1 Q_1} = (-10, 10)$

$\overrightarrow{P_1 P_2} = (-10, 0)$

$|\overline{P_1 Q_1} \times \overline{P_1 P_2}| = (P_1Q_1.x * P_1P_2.y - P_1Q_1.y * P_1P_2.x)$

$= (-10 \times 0) - (-10 * 10)$

$= -0 + 100 = 100$

$Q_1 = 0$

(b) $Q_2 = ccw(P_1, Q_1, Q_2)$

$= |\overline{P_1 Q_1} \times \overline{P_1 Q_2}|$

$= (-10 \times 10 - 0) = -100$

(c) $Q_3 = ccw(P_2, Q_2, P_1)$

$= (10, 10) \times (10, 0)$

$= (0 * 100) = -100$

(d) $Q_4 = ccw(P_2, Q_2, Q_1)$
$= (10, 10) \times (0, 10)$
$= (100 - 0) = 100$.

(e) $Q_1 = \cancel{0}$ ~~&&~~
point-on-line $(P_1, Q_1, P_2) == true$
(e) $Q_1 != Q_2$ && $Q_3 != Q_4$,
so, line (P_1, Q_1) intersect line (P_2, Q_2)

(ii) $P_1 = (-5, -5)$, $Q_1 = (0, 0)$
$P_2 = (1, 1)$ $Q_2 = (10, 10)$

(a) $O_1 = ccw(P_1, Q_1, P_2)$
$= \overline{P_1Q_1} \times \overline{P_1P_2}$
$= (5, 5) \times (6, 6)$
$= 30 - 30 = 0$.

(b) $O_2 = ccw(P_1, Q_1, Q_2)$
$= \overline{P_1Q_1} \times \overline{P_1Q_2} = (5, 5) \times (15, 15)$
$= 0$.

(c) $O_3 = ccw(P_2, Q_2, P_1)$

$= \overline{P_2Q_2} \times \overline{P_2P_1}$

$= (9,9) \times (-6,-6)$

$= 0$

(d) $O_4 = ccw(P_2, Q_2, Q_1)$

$= \overline{P_2Q_2} \times (\overline{P_2Q_1})$

$= (9,9) \times (-1,-1)$

$= 0$.

(e) $O_1 = 0$ && point_on_line $(P_1 Q_1, P_2) ==$ false

(f) $O_2 = 0$ && point_on_line $(P_1, Q_1, Q_2) ==$ false

(g) $O_3 = 0$ && " " " $(P_2, Q_2, P_1) ==$ false

(h) $O_4 = 0$ && " " " $(P_2, Q_2, Q_1) ==$ false.

(i) The lines do not intersect each other.

1
(a)

Algorithm

Area_of_simple_polygon (P, n)

(i) Set area = 0.0 , O(0, 0)

(ii) (i = 0; i ≤ n-1 ; i++)

(a) $\vec{a}$ = P[i] − O

(b) $\vec{b}$ = P[(i+1) % n] − O

(c) Area = Area + get_cross_product ($\vec{a}$, $\vec{b}$)

(iii) return $\left|\left(\frac{1}{2.0} * Area\right)\right|$

Simulation for,

$P_1 = (5, 5)$ $P_2 = (-3, 4)$, $P_3(-4, -4)$, $P_4(1, -5)$

$P_5(1, 0)$

(i) Area = 0.0

iteration i = 1

(a) $\vec{a} = (5, 5) - (0, 0) = (5, 5)$

(b) $\vec{b} = (-3, 4)$

(c) Area $= 0.0 + \begin{vmatrix} 5 & 5 \\ -3 & 4 \end{vmatrix} = 0.0 + (20 + 15)$

$= 35$.

when
$i = 2$,

(a) $\vec{a} = (-3, 4)$

(b) $\vec{b} = (-4, 4)$

(c) Area $= 35 + \begin{vmatrix} -3 & 4 \\ -4 & -4 \end{vmatrix}$

$= 35 + (12 + 16)$

$= 63$

when $i = 3$.

(a) $\vec{a} = (-4, -4)$

(b) $\vec{b} = (1, -5)$

(c) Area $= 63 + \begin{vmatrix} -4 & -4 \\ 1 & -5 \end{vmatrix}$

$= 63 + (20 + 4)$

$= 87$

when $i = 4$

(a) $\vec{a} = (1, -5)$

(b) $\vec{b} = (9, 0)$

(c) Area $= 87 + \begin{vmatrix} 1 & -5 \\ 9 & 0 \end{vmatrix}$

$= 87 + 45$

$= 132$

$2\overline{)177}(88$
16
17
16

when $i=5$,

(a) $\vec{a} = (9,0)$

(b) $\vec{b} = (5,5)$

(c) $Area = 132 + \left(\begin{vmatrix} 9 & 0 \\ 5 & 5 \end{vmatrix} \right)$

$= 132 + (45)$

$= 177$.

$Area = \frac{1}{2} \times Area$

$= \frac{1}{2} \times 177 = 88.5$.

Ans.

(c)

Algorithm

Point_in_convex_polygon ($P[\,]$, n, Q)

(i) P_0, P_0_index = get_reference_point ($P[\,]$, n)

(ii) rotate (P.begin(), P.begin() + P_0_index, P.end())

(iii) if(ccw (P_0, P_{n-1}, Q) > 0 || ccw.(P_0, P_1, Q) < 0)

return false;

(iv) start = 1, end = n−2;

(v) while (start <= end)

$m = \frac{start + end}{2}$;

$i = m$.

if (ccw.(P_0, P_i, Q) > 0)

start = m+1;

else,

end = m−1;

(vi) if (ccw.(P[i], P[i+1], Q) >= 0)

return true.

(vii) return false.

```
get-reference-point (P[ ], n)
1. min_x = point[0].x
2. max_p = 0;
   for (i=1 ; i<=n-1 ; i++)
   { if ((P[i].x < min_x) || (P[i].x == min_x &&
            P[i].y > p[max_p].y))
        { min_x = P[i].x
          max_p = i ;
        }
3. return (min_x , max_p).

int main( )
{ int n, P[n+1];
  for (i=0 ; i≤n-1 ; i++)
      cin>>P[i].x >>P[i].y ;
  P0, P0_index = get_reference_point (P[ ], n)
  Point_in_convex_polygon (P[ ], n, Q).
}
```

d)

We have to inspect an algorithm to test the convexity of a given polygon.

Algorithm

Is_convex (P[ ], n)

(i) set left_turn = 0 , right_turn = 0 .

(ii) for (i=0 ; i<n-1 ; i++)

(a) if (ccw (P[i], P[(i+1)%n], P[(i+2)%n]) ≥ 0

left_turn ++ ;

(b) if (ccw (P[i], P[(i+1)%n], P[(i+2)%n]) ≤ 0

right_turn ++ ;

(iii) return (left_turn == n || right_turn == n)

Pick's theorem,

$$A = I + \frac{B}{2} - 1$$

here,

A = Area of the polygon

I = Number of interior lattice points

B = " " boundary " "

(B)(i) graham's scan is better for surrounding the trees by fence because the time complexity is $O(n \log n)$.

(ii)

Simulation of Graham's Scan Algorithm.

Step: 1

b_m_p = I

Step: 2

Swap (P[0], b_m_p)

A	B	C	D	E	F	G	H	I

after swaping

I	B	C	D	E	F	G	H	A

Step - 3

Sort (P[0], P+n, compare)

~~I~~	~~A~~	~~E~~	~~G~~	~~D~~	~~B~~	~~C~~	~~H~~	~~F~~

I	A	E	D	B	G	C	H	F

Step - 4

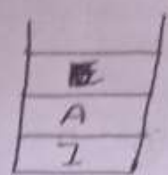
Remove co-linear (P[0], P, n)

I	A	E	B	C	H	F
0	1	2	3	4	5	6

Step-5

for $i = 2$ to $n-1$

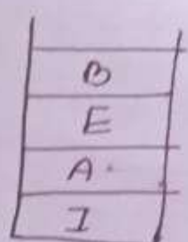
iteration - 1



$ccw(I, A, P[2]) > 0$

$CH \in P[2]$

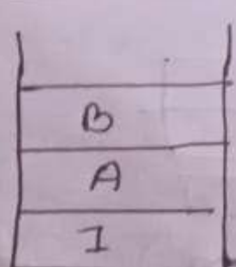
iteration - 2



$ccw(A, E, P[3]) < 0$

remove E.

iteration - 3



$ccw(I, A, B) > 0$

$CH \in B$.

iterration-4

C
B
A
I

$ccw(A, B, C) > 0$

$CH \in C$

iterration-5

H
C
B
A
I

$ccw(B, C, H) > 0$

$CH \in H$

iterration - 6

F
H
C
B
A
I

$ccw(C, H, F) < 0$

rremove H

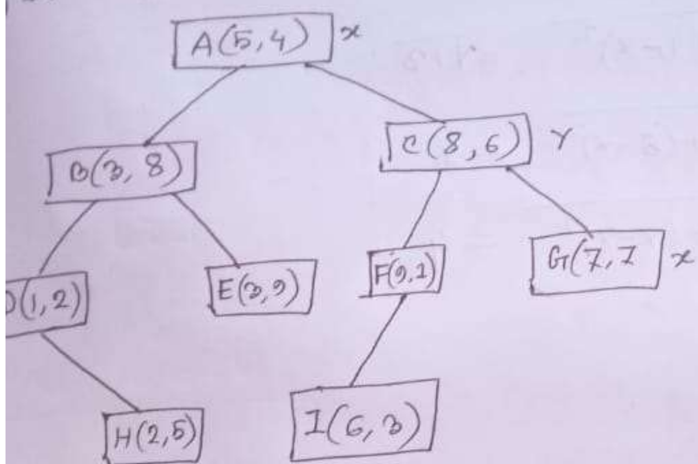
Iterration - 7

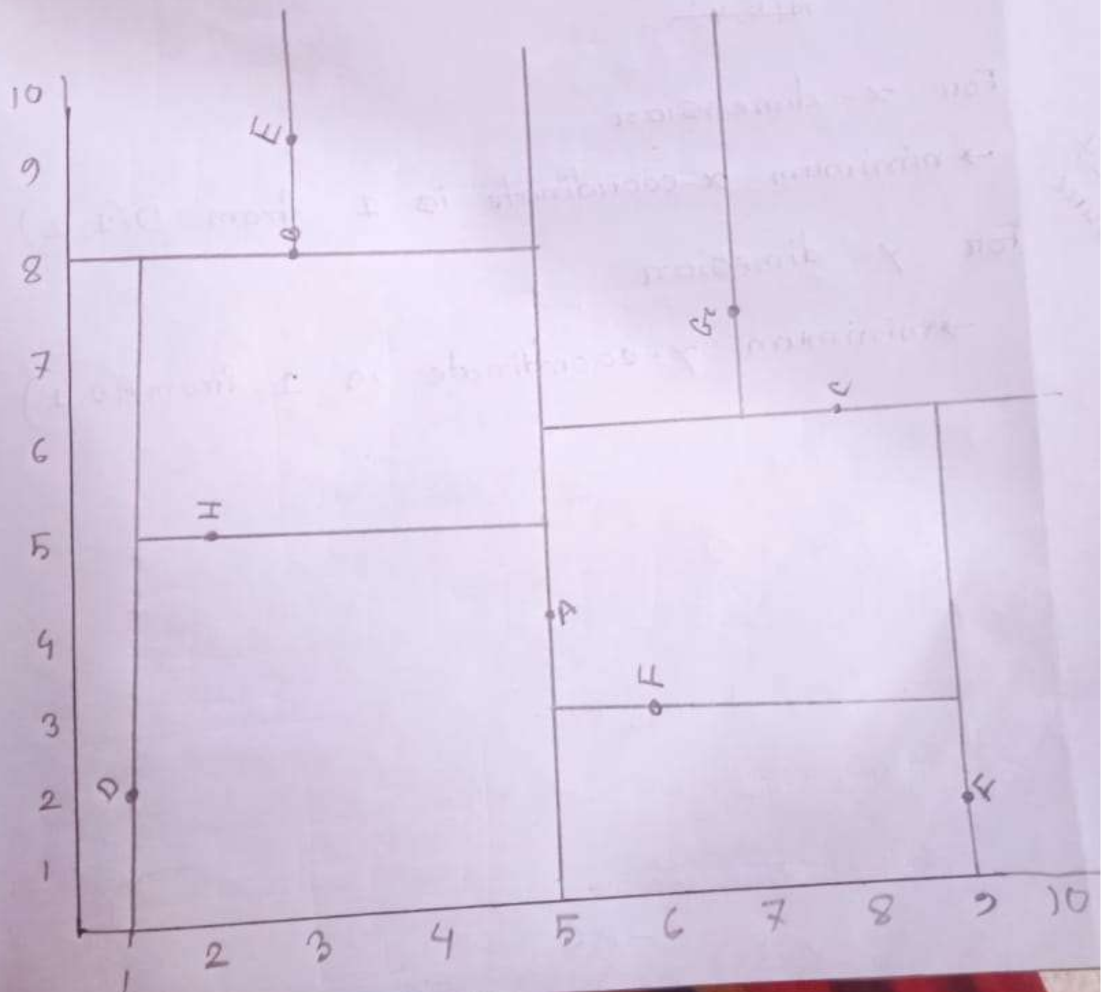
F
C
B
A
I

$ccw(B, C, F) < 0$

rremove C

If insert I(6,3)





(ii)

$D_1 = \sqrt{(5-3)^2 + (4-7)^2} = \sqrt{13}$

$D_2 = \sqrt{(3-3)^2 + (8-7)^2} = 1$

$D_3 = \sqrt{(7-3)^2 + (7-7)^2} = 4$

(iii)

From Points on K-D tree,

~~A(5,4)~~,

For x- dimension

Not sure.

→ minimum x-coordinate is 1 from D(1,

For y dimesion

→ minimum y-coordinate is 1 from F(?,

2

a) ΔOC_1T_1 & ΔOC_2T_1

i)

$\angle C_1T_1O = \angle C_2T_1O$

& $C_1T_1 = C_2T_1$

$T_1O = T_1O$

$\therefore \Delta OC_1T_1 \cong \Delta OC_2T_1$

$\angle\theta_1 = \angle\theta_2 = \theta$

& $2n\theta = 360°$

$\theta = \frac{360°}{2n} = \frac{\pi}{n}$

we know,

$$\sin\theta = \frac{C_1T_1}{OC_1}$$

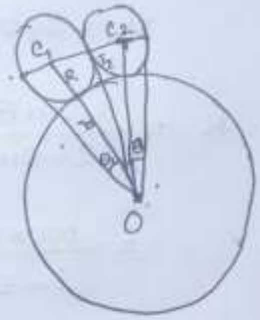
$$\Rightarrow \sin\theta = \frac{R}{r+R}$$

$$\Rightarrow r\sin\theta + R\sin\theta = R$$

$$\Rightarrow R - R\sin\theta = r\sin\theta$$

$$\Rightarrow R = \frac{r\sin\theta}{1-\sin\theta}$$

Ans



(ii) If $n = 100$

$r = 100$

$\therefore R = \dfrac{r \sin\theta}{1 - \sin\theta}$

$= \dfrac{100 \times \sin \cancel{180}^{\circ} \frac{\pi}{100}}{1 - \sin \cancel{180}^{\circ} \frac{\pi}{100}}$

$= 3.24$

$\theta = \dfrac{\pi}{\cancel{18}n}$

$= \dfrac{\cancel{180^{\circ}}}{\cancel{100}} = \cancel{1.80^{\circ}}$

$= \dfrac{3.14}{100} = 0.0314$

(b)

(i) If $AC + BC \leq AB$ then

AC, BC can not be determined.

Not sure.

(ii) Algorithm:

(a) Set $low = 0$

(b) Set $high = S$.

(c) while $(high - low \geq 10^{-6})$

(c) <u>Algorithm</u>

minimum_line(P[ ], n, Q)

(i) for (i=0, i<n, i++)

(a) $\Delta x = P[i].x - Q.x$

(b) $\Delta y = P[i].y - Q.y$

(c) calculate $T = \left|\frac{\Delta y}{\Delta x}\right|$

(d) Push T to the set S[ ].

(ii) calculate number_of_line = ~~length of S[ ]~~ . S[

(iii) return number_of_line;

(d) same as (2→a in 28)

3

(a) not in syllabus.

(b) same algorithm as (4 → d in 28)

(c) same " as (4 → c in 28)

Section - IV

(a) $c_1(-10, 8)$, $r_1 = 30$ cm

$c_2(14, -24)$, $r_2 = 10$ cm.

(i) ~~If~~ ~~leet~~

Let d = distance between c_1 & c_2

(a) if $d > r_1 + r_2$ (does not intersect)

(b) if $d == r_1 + r_2$ (touches externally)

(c) if $d == |r_1 - r_2|$ (" internally)

(d) if $d < |r_1 - r_2|$ (c_2 is on the c_1 c

(e) else c_1 & c_2 intersect each other

(ii) $d = \sqrt{(-10-14)^2 + (8+24)^2}$

$= 40$

$d = 40 = 30 + 10 = r_1 + r_2$

So, the circle touches each other.

The point will divide the line $(-10, 8)$ & $(14, -24)$ into $30:10$.

$$P(x, y) = \left(\frac{-10\times 10 + 14\times 30}{40}, \frac{8\times 10 + (-24\times 30)}{40}\right)$$

$$= (8, -16)$$

$\therefore$ The intersection/ ~~poin~~ touch point is $(8, -16)$

Ans

(b)

(i) Area $= \frac{1}{2} \begin{vmatrix} 7 & 13 & 14 & 10 & 6 & 5 & 4 & 4 & 7 \\ 1 & 2 & 7 & 9 & 10 & 9 & 6 & 3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\left[\{14 + 91 + 126 + 100 + 54 + 30 + 12 + 4\} - \{13 + 28 + 70 + 54 + 50 + 36 + 24 + 21\}\right]$$

$$= 67.5$$

(ii) $B = n = 8$

(iii) for $(i = 0;\ i < n-1;\ i{+}{+})$

$B = B + \text{get\_n\_lattice\_points}\,(P[i], P[(i+1)\%n])$

~~= 8 +~~

<u>1st iteration,</u> P_0, P_1

$$B = 8 + \gcd(\Delta x, \Delta y) - 1$$
$$= 8 + \gcd(6, 1) - 1$$
$$= 8 + 0 = 8$$

<u>2nd iteration</u> P_1, P_2

$$B = 8 + \gcd(\Delta x, \Delta y) - 1$$
$$= 8 + \gcd(1, 5) - 1$$
$$= 8$$

3rd iterration (P_2, P_3)

$$B = 8 + \gcd(4, 2) - 1$$
$$= 8 + 2 - 1$$
$$= 9$$

4th iterration (P_3, P_4)

$$B_0 = 9 + \gcd(4, 1) - 1$$
$$= 9$$

5th iterration (P_4, P_5)

$$B = 9 + \gcd(1, 1) - 1 = 9$$

6th iterration (P_5, P_6)

$$B = 9 + \gcd(2, 3) - 1 = 9$$

7th iterration (P_6, P_7)

$$B = 9 + \gcd(0, 3) - 1 = 9$$

8th iterration

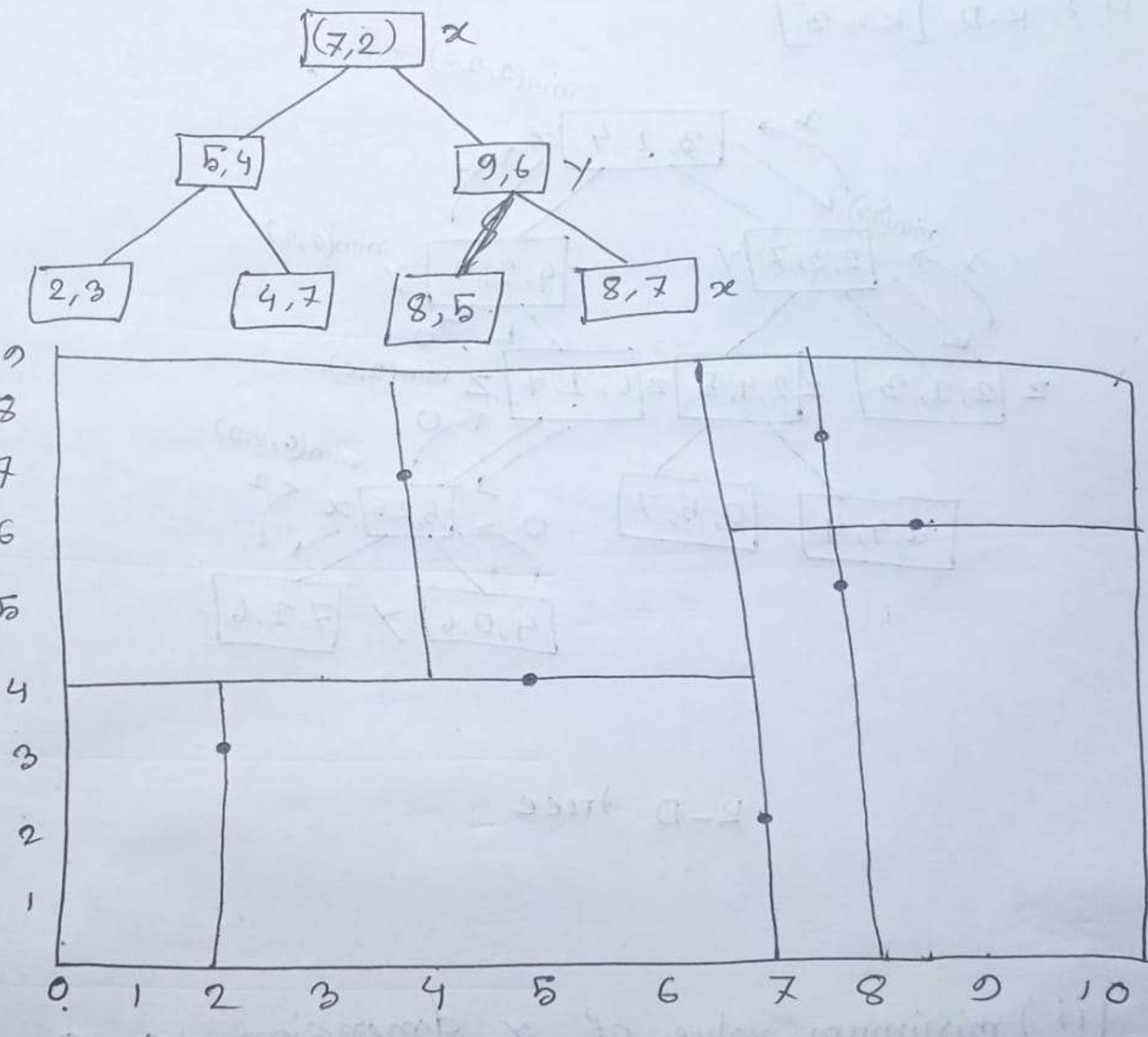
$$B = 9 + \gcd(3, 2) - 1 = 9$$

Now

$$I = A - \frac{B}{2} + 1 = 67.5 - \frac{9}{2} + 1$$
$$= 67.5 - 4.5 + 1$$
$$= 64$$

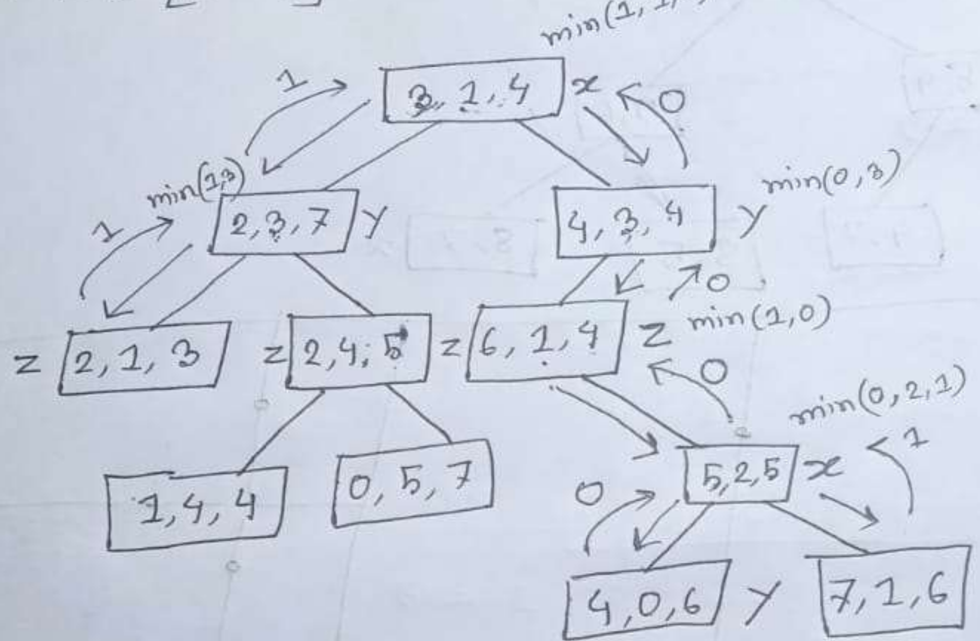
Ans

d insert (8,5)



Spliting plane

(i) K-D [K = 3]



K-D tree

(ii) minimum value of Y dimension is 0

<u>not sure</u>