## Parsing: Part-II

## PREDICTIVE PARSING

- The goal is to construct a top-down parser that never backtracks
- Always leftmost derivations
- We must transform a grammar in two ways:
  - eliminate left recursion
  - perform left factoring
- These rules eliminate most common causes for backtracking although they do not guarantee a completely backtrack-free parsing

# LEFT RECURSION: INFINITE LOOPING PROBLEM

- ☐ A grammar is left-recursive if it has a non-terminal A, such that there is a derivation :
  - $A \stackrel{+}{\Rightarrow} A\alpha$ , for some string  $\alpha$ .
- □ Top-Down parsing can't reconcile this type of grammar, since it could consistently make choice which wouldn't allow termination.

$$A \Rightarrow A\alpha \Rightarrow A\alpha\alpha \Rightarrow A\alpha\alpha\alpha \dots \text{ etc. } A \rightarrow A\alpha \mid \beta$$

- ☐ So we have to convert our left-recursive grammar into an equivalent grammar which is not left-recursive.
- The left-recursion may appear in a single step of the derivation (immediate left-recursion), or may appear in more than one step of the derivation.

## IMMEDIATE LEFT RECURSION

- ightharpoonup A ightharpoonup A ightharpoonup A ightharpoonup Where ho does not start with A
  - eliminate immediate left recursion
- $A \rightarrow \beta A'$  where A' is a new nonterminal
- $A' \rightarrow \alpha A' \mid \epsilon$  an equivalent grammar

More General (but still immediate):

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid A\alpha_3 \mid \dots \mid \beta_1 \mid \beta_2 \mid \beta_3 \mid \dots$$

Transform into:

$$\begin{array}{l} A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \beta_3 A' \mid ... \\ A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \alpha_3 A' \mid ... \mid \epsilon \end{array}$$

## IMMEDIATE LEFT RECURSION ELIMINATION: EXAMPLE

### Our Example:

$$E \rightarrow E + T \mid T \longrightarrow \begin{cases} E \rightarrow TE' \\ E' \rightarrow + TE' \mid \in \end{cases}$$

$$T \rightarrow T * F \mid F \longrightarrow FT' \longrightarrow F$$

## LEFT RECURSION IN MORE THAN ONE STEP

- A grammar cannot be immediately left-recursive, but it still can be left-recursive.
- By just eliminating the immediate left-recursion, we may not get a grammar which is not left-recursive.

## Example:

```
S \to A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}
A \to A\underline{\mathbf{c}} \mid S\underline{\mathbf{d}} \mid \underline{\mathbf{e}}
```

Is A left recursive? Yes.

Is S left recursive? Yes, but not immediate left recursion.  $S \Rightarrow Af \Rightarrow Sdf$ 

### Approach:

Look at the rules for S only (ignoring other rules)... No left recursion.

Look at the rules for A...

Do any of A's rules start with S? Yes.

$$A \rightarrow S\underline{d}$$

Get rid of the S. Substitute in the righthand sides of S.

$$A \rightarrow Afd \mid bd$$

The modified grammar:

$$S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}$$
  
 $A \rightarrow A\underline{\mathbf{c}} \mid A\underline{\mathbf{fd}} \mid \underline{\mathbf{bd}} \mid \underline{\mathbf{e}}$ 

Now eliminate immediate left recursion involving A.

$$S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}$$
  
 $A \rightarrow \underline{\mathbf{bd}}A' \mid \underline{\mathbf{e}}A'$   
 $A' \rightarrow \mathbf{c}A' \mid \mathbf{fd}A' \mid \underline{\mathbf{e}}$ 

## The Original Grammar:

```
S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}

A \rightarrow A\underline{\mathbf{c}} \mid S\underline{\mathbf{d}} \mid B\underline{\mathbf{e}}

B \rightarrow A\underline{\mathbf{g}} \mid S\underline{\mathbf{h}} \mid \underline{\mathbf{k}}
```

### So Far:

```
S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}
A \rightarrow \underline{\mathbf{b}}\underline{\mathbf{d}}A' \mid \underline{\mathbf{B}}\underline{\mathbf{e}}A'
A' \rightarrow \underline{\mathbf{c}}A' \mid \underline{\mathbf{f}}\underline{\mathbf{d}}A' \mid \epsilon
```

## The Original Grammar:

```
S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}

A \rightarrow A\underline{\mathbf{c}} \mid S\underline{\mathbf{d}} \mid B\underline{\mathbf{e}}

B \rightarrow A\underline{\mathbf{g}} \mid S\underline{\mathbf{h}} \mid \underline{\mathbf{k}}
```

### So Far:

```
S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}
A \rightarrow \underline{\mathbf{bd}}A' \mid B\underline{\mathbf{e}}A'
A' \rightarrow \underline{\mathbf{c}}A' \mid \underline{\mathbf{fd}}A' \mid \varepsilon
B \rightarrow A\underline{\mathbf{g}} \mid \underline{\mathbf{Sh}} \mid \underline{\mathbf{k}} \rightarrow
```

Look at the B rules next; Does any righthand side start with "S"?

### The Original Grammar:

$$S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}$$

$$A \rightarrow A\underline{\mathbf{c}} \mid S\underline{\mathbf{d}} \mid B\underline{\mathbf{e}}$$

$$B \rightarrow A\underline{\mathbf{g}} \mid S\underline{\mathbf{h}} \mid \underline{\mathbf{k}}$$

### <u>So Far:</u>

```
S → Af | b

A → bdA' | BeA'

A' → cA' | fdA' | ε

B → Ag | Afh | bh | k

Substitute, using the rules for "S"

Af... | b...
```

### The Original Grammar:

```
S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}

A \rightarrow A\underline{\mathbf{c}} \mid S\underline{\mathbf{d}} \mid B\underline{\mathbf{e}}

B \rightarrow A\mathbf{g} \mid S\underline{\mathbf{h}} \mid \underline{\mathbf{k}}
```

### So Far:

```
S \rightarrow A\underline{f} \mid \underline{b}
A \rightarrow \underline{b}\underline{d}A' \mid B\underline{e}A'
A' \rightarrow \underline{c}A' \mid \underline{f}\underline{d}A' \mid \epsilon
B \rightarrow A\underline{g} \mid A\underline{f}\underline{h} \mid \underline{b}\underline{h} \mid \underline{k}
```

Does any righthand side start with "A"?

```
The Original Grammar:
       S \rightarrow Af \mid \underline{b}
       A \rightarrow Ac \mid Sd \mid Be
       B \rightarrow Ag \mid Sh \mid k
So Far:
       S \rightarrow A\mathbf{f} \mid \mathbf{b}
       A \rightarrow bdA' \mid BeA'
       A' \rightarrow \underline{c}A' \mid \underline{fd}A' \mid \varepsilon
       B \rightarrow \underline{bd}A'g \mid B\underline{e}A'g \mid A\underline{fh} \mid \underline{bh} \mid \underline{k}
                Substitute, using the rules for "A"
                              bdA'... | BeA'...
```

## The Original Grammar: $S \rightarrow Af \mid \underline{b}$ $A \rightarrow Ac \mid Sd \mid Be$ $B \rightarrow Ag \mid Sh \mid k$ So Far: $S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}$ $A \rightarrow bdA' \mid BeA'$ $A' \rightarrow \underline{c}A' \mid \underline{fd}A' \mid \epsilon$ $B \rightarrow \underline{bd}A'g \mid \underline{Be}A'g \mid \underline{bd}A'\underline{fh} \mid \underline{Be}A'\underline{fh} \mid \underline{bh} \mid \underline{k}$ Substitute, using the rules for "A" **bd**A'... | B**e**A'...

### The Original Grammar:

```
S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}
A \rightarrow A\underline{\mathbf{c}} \mid S\underline{\mathbf{d}} \mid B\underline{\mathbf{e}}
B \rightarrow A\underline{\mathbf{g}} \mid S\underline{\mathbf{h}} \mid \underline{\mathbf{k}}
```

#### So Far:

```
S \rightarrow A\underline{f} \mid \underline{b}

A \rightarrow \underline{bd}A' \mid B\underline{e}A'

A' \rightarrow \underline{c}A' \mid \underline{fd}A' \mid \varepsilon

B \rightarrow \underline{bd}A'\underline{\sigma} \mid \underline{Bc}A'\underline{\sigma} \mid \underline{bc}A
```

 $B \to \underline{bd}A'g \mid \underline{be}A'g \mid \underline{bd}A'\underline{fh} \mid \underline{be}A'\underline{fh} \mid \underline{bh} \mid \underline{k}$ 

## Left recursion involving "B"

Finally, eliminate any immediate

#### Next Form

```
\begin{split} S &\to A\underline{f} \mid \underline{b} \\ A &\to \underline{b}\underline{d}A' \mid B\underline{e}A' \\ A' &\to \underline{c}A' \mid \underline{f}\underline{d}A' \mid \epsilon \\ B &\to \underline{b}\underline{d}A'\underline{g}B' \mid \underline{b}\underline{d}A'\underline{f}\underline{h}B' \mid \underline{b}\underline{h}B' \mid \underline{k}B' \\ B' &\to \underline{e}A'\underline{g}B' \mid \underline{e}A'\underline{f}\underline{h}B' \mid \epsilon \end{split}
```

## The Original Grammar:

```
S \rightarrow A\underline{\mathbf{f}} \mid \underline{\mathbf{b}}

A \rightarrow A\underline{\mathbf{c}} \mid S\underline{\mathbf{d}} \mid B\underline{\mathbf{e}} \mid C

B \rightarrow A\underline{\mathbf{g}} \mid S\underline{\mathbf{h}} \mid \underline{\mathbf{k}}

C \rightarrow B\mathbf{k}\mathbf{m}A \mid AS \mid \underline{\mathbf{j}} -
```

If there is another nonterminal, then do it next.

### So Far:

```
S \rightarrow A\underline{f} \mid \underline{b}
A \rightarrow \underline{b}\underline{d}A' \mid B\underline{e}A' \mid CA'
A' \rightarrow \underline{c}A' \mid \underline{f}\underline{d}A' \mid \epsilon
B \rightarrow \underline{b}\underline{d}A'\underline{g}B' \mid \underline{b}\underline{d}A'\underline{f}\underline{h}B' \mid \underline{b}\underline{h}B' \mid \underline{k}B' \mid CA'\underline{g}B' \mid CA'\underline{f}\underline{h}B'
B' \rightarrow \underline{e}A'\underline{g}B' \mid \underline{e}A'\underline{f}\underline{h}B' \mid \epsilon
```

# ALGORITHM FOR ELIMINATING LEFT RECURSION

```
Assume the nonterminals are ordered A_1, A_2, A_3,...
          (In the example: S, A, B)
\underline{\text{for}} \underline{\text{each}} nonterminal A_i (for i = 1 to N) \underline{\text{do}}
   for each nonterminal A_i (for j = 1 to i-1) do
      Let A_i \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \mid ... \mid \beta_N be all the rules for A_i
      if there is a rule of the form
         A_i \rightarrow A_i \alpha
      then replace it by
         A_i \rightarrow \beta_1 \alpha \mid \beta_2 \alpha \mid \beta_3 \alpha \mid \dots \mid \beta_N \alpha
      endIf
   endFor
   Eliminate immediate left recursion
            among the A_i rules
                                                                      Inner Loop
endFor
```

## Left Factoring: Common Prefix Problem

#### Problem: Uncertain which of 2 rules to choose:

```
stmt \rightarrow if expr then stmt else stmt
| if expr then stmt
```

### When do you know which one is valid?

### What's the general form of stmt?

 $A \rightarrow \alpha \beta_1 \mid \alpha \beta_2$   $\alpha : \text{if } expr \text{ then } stmt$   $\beta_1 : \text{else } stmt \quad \beta_2 : \in$ 

### Transform to:

 $A \rightarrow \alpha A'$ 

 $A' \rightarrow \beta_1 \mid \beta_2$ 

### **EXAMPLE:**

 $stmt \rightarrow if expr then stmt rest$ 

 $rest \rightarrow else\ stmt \mid \in$ 

## Left Factoring: Example

```
A \rightarrow \underline{abB} \mid \underline{aB} \mid \underline{cdg} \mid \underline{cdeB} \mid \underline{cdfB}
A \rightarrow aA' \mid cdg \mid cdeB \mid cdfB
A' \rightarrow bB \mid B
A \rightarrow aA' \mid cdA''
A' \rightarrow bB \mid B
A'' \rightarrow g \mid eB \mid fB
```

## Left Factoring: Example

 $A \rightarrow ad \mid a \mid ab \mid abc \mid b$ 



 $A \rightarrow aA' \mid b$ 

A'  $\rightarrow$  d |  $\epsilon$  | b | bc



 $A \rightarrow aA' \mid b$ 

 $A' \rightarrow d \mid \epsilon \mid bA''$ 

 $\textbf{A''} \rightarrow \epsilon \textbf{ | c}$ 

## Reading Materials

- Chapter -4 of your Text book:
  - o Compilers: Principles, Techniques, and Tools

## THE END