Intro to Probability

Permutations with Repetition

Learning Objectives

- Understand that a **permutation** is an ordered sequence of events
- Learn the formula to count the number of possible permutations, where events can be repeated

Permutation with Repetition

How many ways can you fill this with numbers between 0 and 9?

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It’s just all numbers between 0 and 99, which is **100 ways**.

Permutation with Repetition

How many ways can you fill this with numbers between 0 and 9?

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10 ways * 10 ways * 10 ways * 10 ways = **10,000 ways**.

Permutation with Repetition

10 ways * 10 ways * 10 ways * 10 ways = **10,000 ways**

In this case, we had **N = 10** numbers to choose from,
and we chose **r = 4** of them.

Repetition is allowed and order matters.

Total Number of Permutations = N^r

Permutations without Repetition

Learning Objectives

- Learn what a **factorial** is and how to compute it
- Learn the formula to count the number of possible permutations, where events **cannot** be repeated

Permutation without Repetition

How many ways can you fill this with numbers between 0 and 9, without repetition?

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It’s just all numbers between 0 and 99, except 00, 11, 22, etc.
This ends up being 100 - 10 = **90 ways**.

Permutation without Repetition

How many ways can you fill this with numbers between 1 and 4, without repetition?

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4 ways * 3 ways * 2 ways * 1 ways = **24 ways**.

This expression is also known as a **factorial**:

$$N! = N * (N-1) * \ldots * 1$$

Permutation without Repetition

How many ways can you fill this with numbers between 0 and 9, without repetition?

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10 ways * 9 ways * 8 ways * 7 ways = **5,040 ways**.

Permutation without Repetition

10 ways * 9 ways * 8 ways * 7 ways = **5,040 ways**

In this case, we had **N = 10** numbers to choose from,
and we chose **r = 4** of them.

Repetition is **not** allowed and order matters.

Total Number of Permutations = $_NP_r = \frac{N!}{(N-r)!}$

$$\frac{10 \times 9 \times 8 \times 7 \times \cancel{6} \times \cancel{5} \ldots}{\cancel{6} \times \cancel{5} \times \cancel{4} \times \ldots}$$

Combinations

Learning Objectives

- Understand that a **combination** is a sequence in which order doesn’t matter (unlike permutations)
- Learn the formula to count the number of possible combinations of events

Combinations

How many ways can you fill this with numbers between 0 and 9, without repetition?
Order doesn't matter, so 1234, 2134, 4123, etc. are all the same.

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In this case, we need to uncount the various orderings.
How many ways can you order **r = 4** numbers? 4! = 24 ways or **r!** ways.

Combinations

How many ways can you fill this with numbers between 0 and 9, without repetition?
Order doesn't matter, so 1234, 2134, 4123, etc. are all the same.

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We had 5,040 numbers from before. Let's remove the 24 orderings.

5,040 / 24 = **210 ways**.

Combinations

5,040 / 24 = **210 ways**

In this case, we had **N = 10** numbers to choose from,
and we chose **r = 4** of them.

Repetition is **not** allowed and order **does not** matter.

Total Number of Combinations = $_{N}C_{r} = \frac{N!}{(N-r)!r!}$