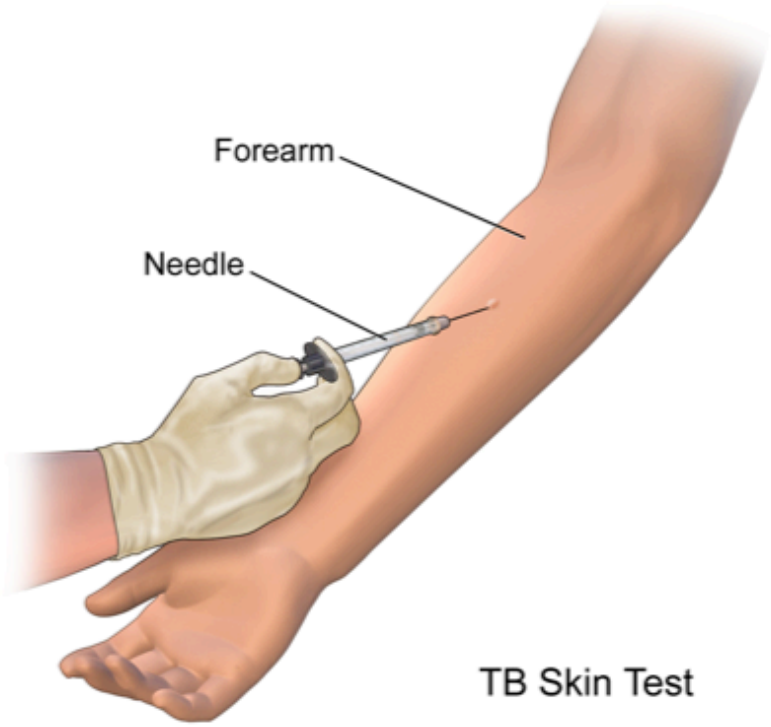
Bayes' Theorem

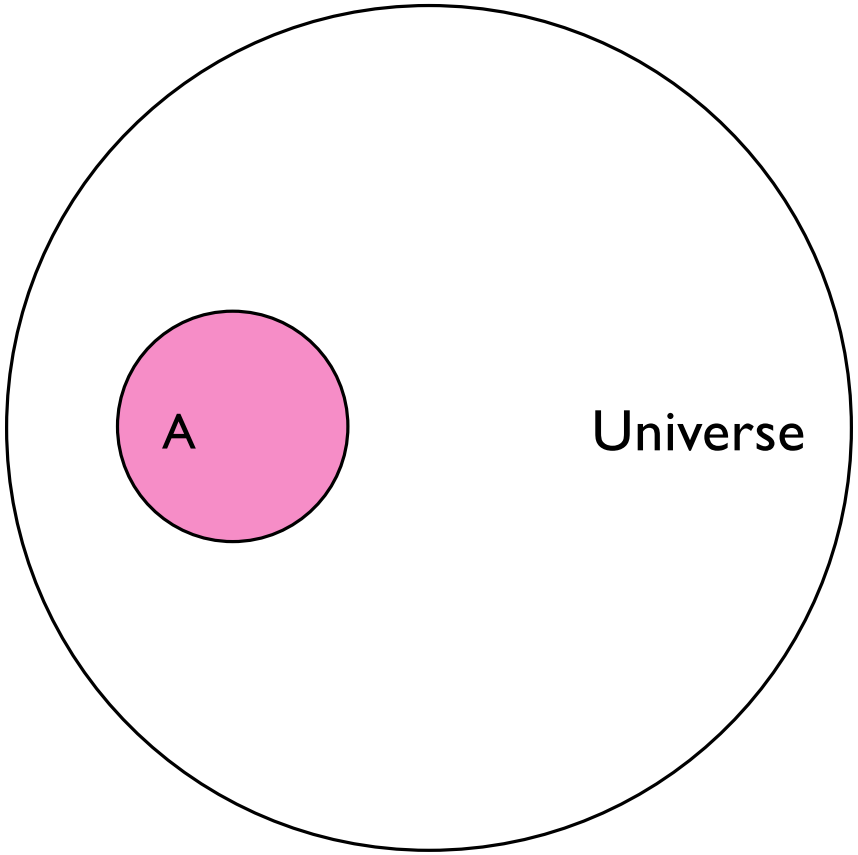
Thomas Bayes, ca. 1701-1761



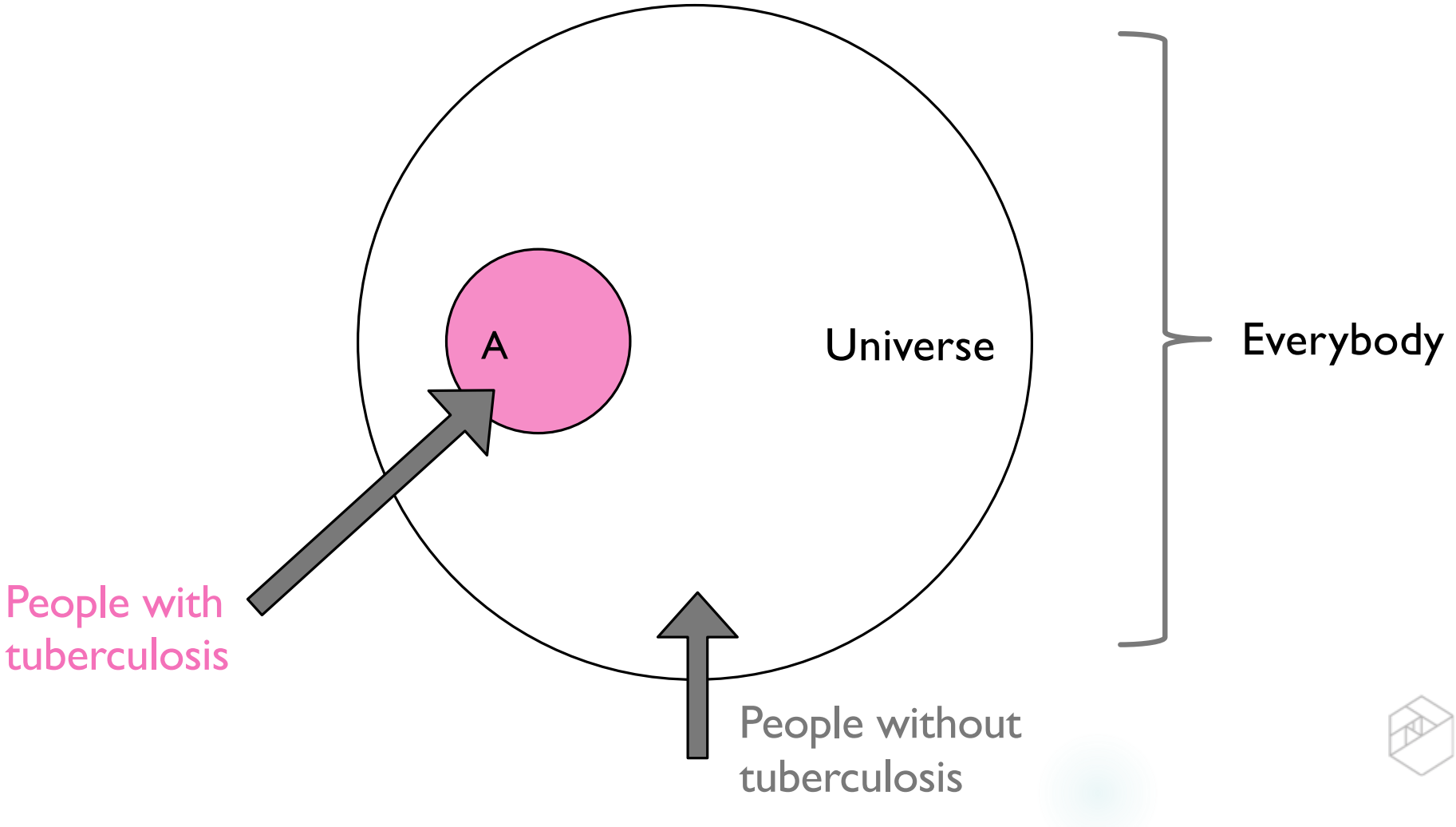
TB Skin Test

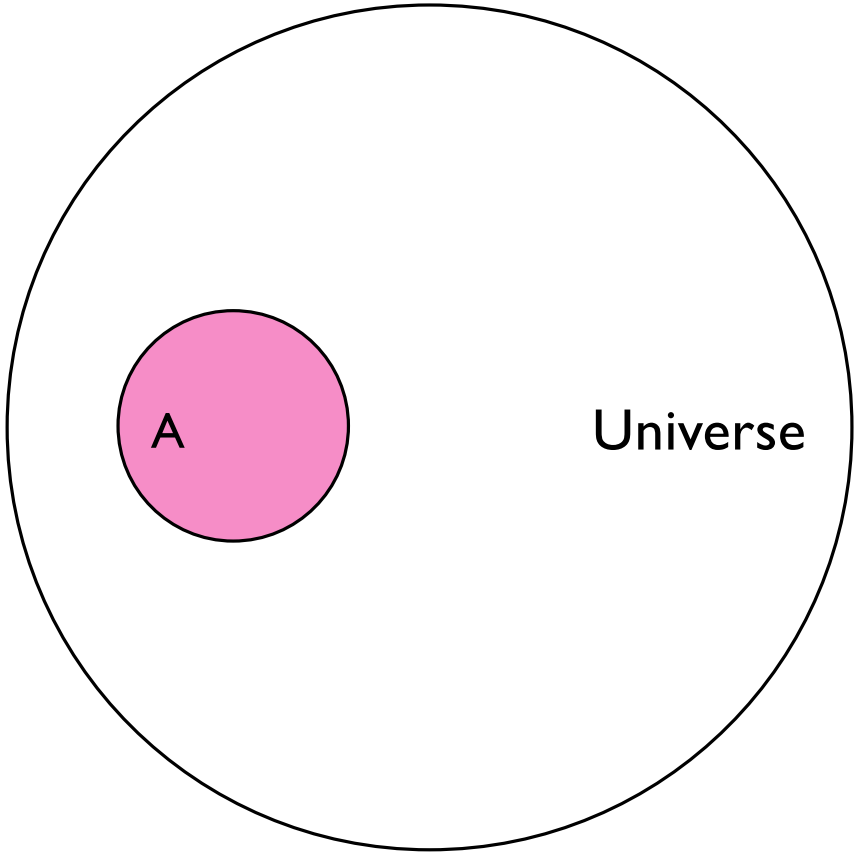
If a person **tests positive** for tuberculosis, what is the probability of him/her actually having TB?

https://www.dovemed.com/common-procedures/procedures-laboratory/tuberculin-skin-test-for-tuberculosis/



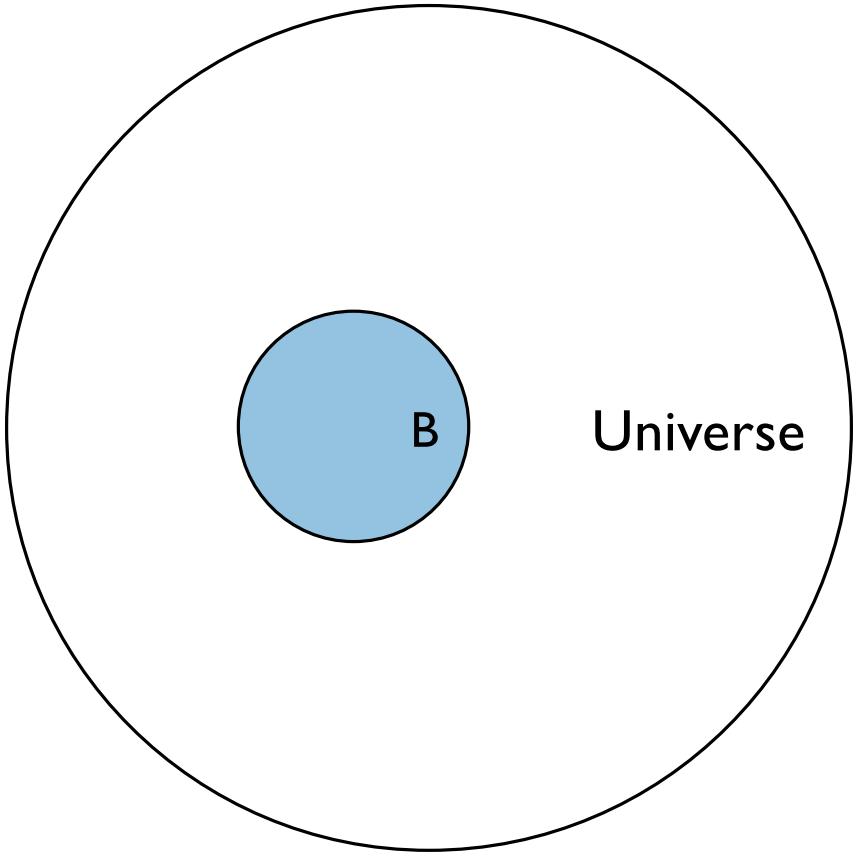
http://oscarbonilla.com/2009/05/visualizing-bayes-theorem/

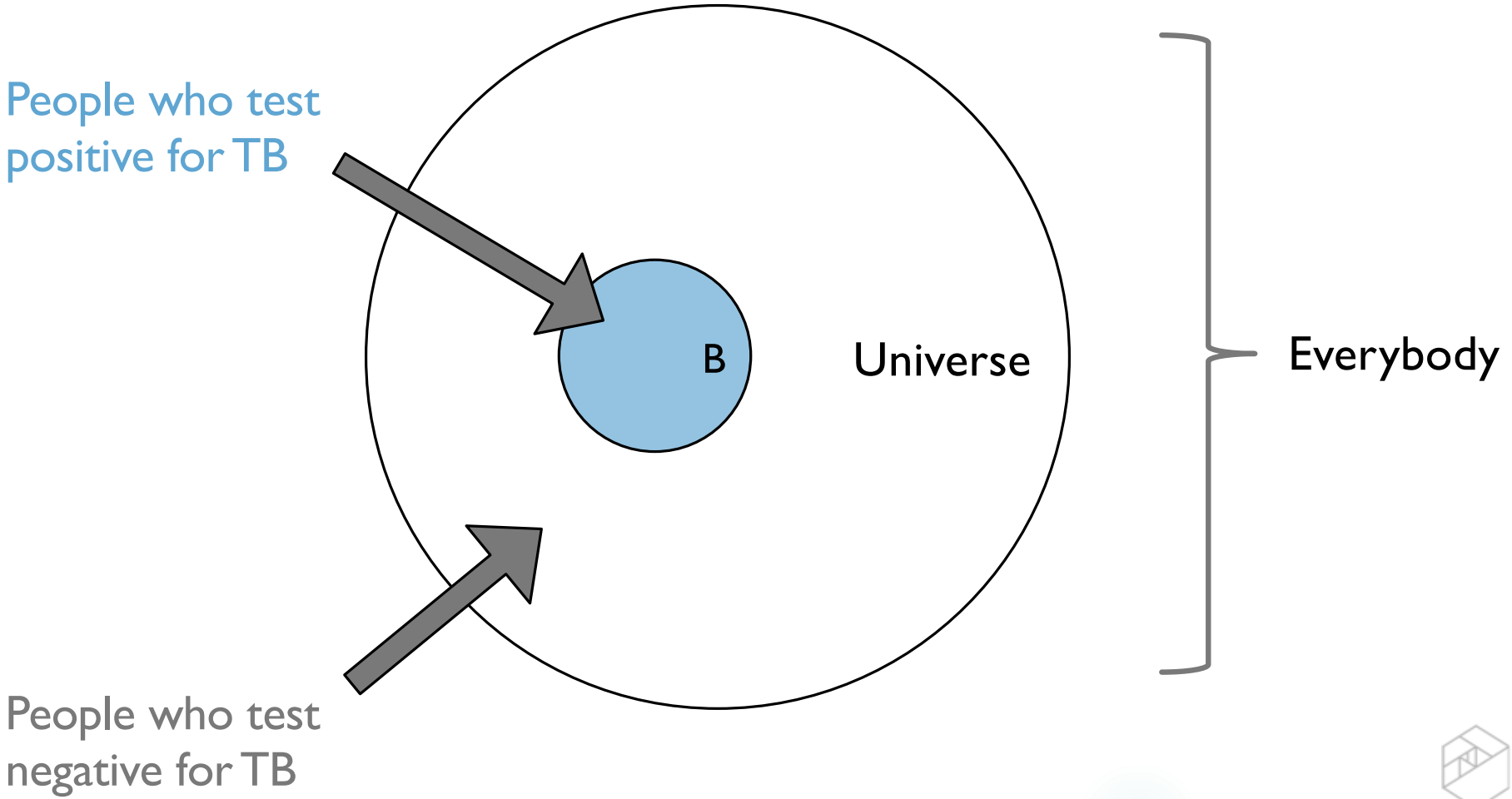

A
Universe
Everybody
People with tuberculosis
People without tuberculosis

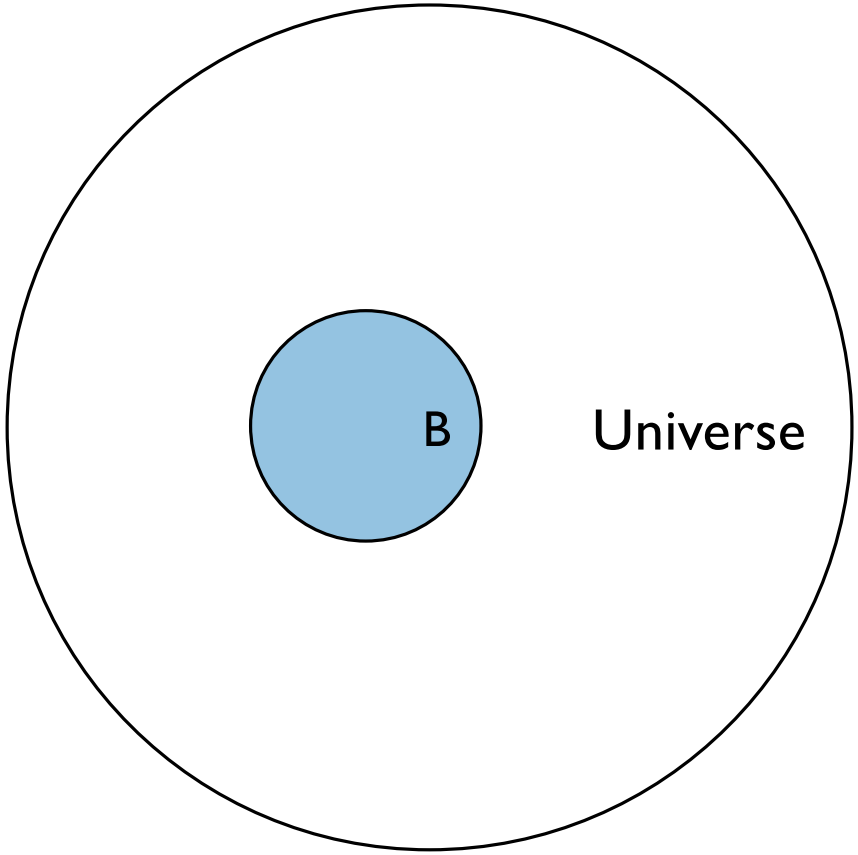


If I pick a random person in the universe, what's the probability of him/her **having** tuberculous?

$$P(A) = \frac{|A|}{|U|}$$

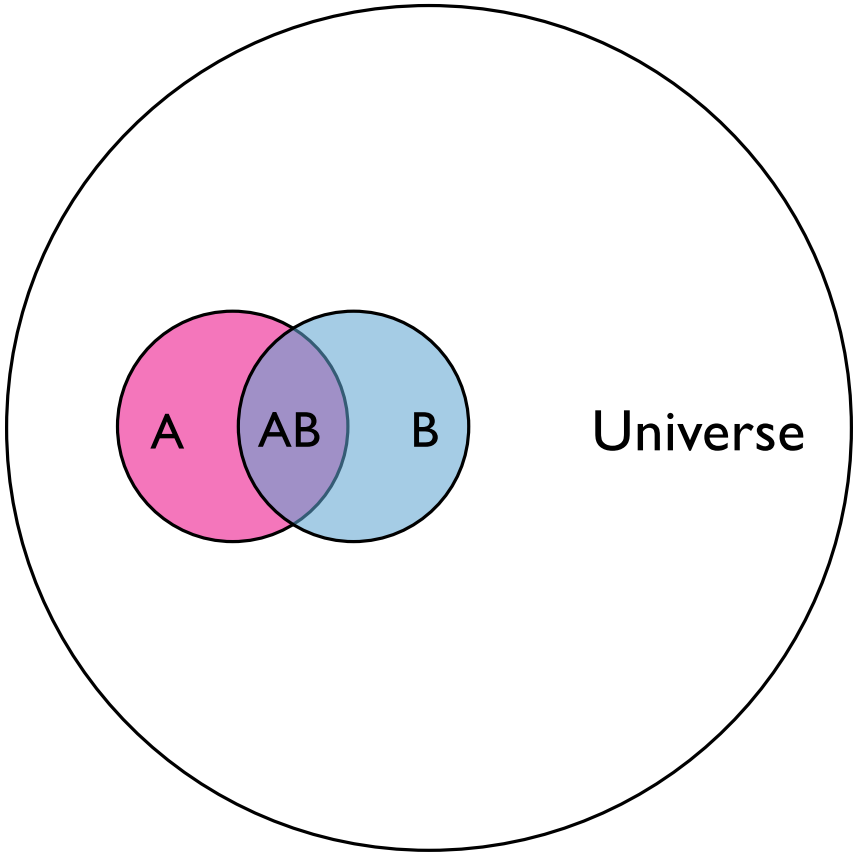

B
Universe


People who test positive for TB
B
Universe
Everybody
People who test negative for TB



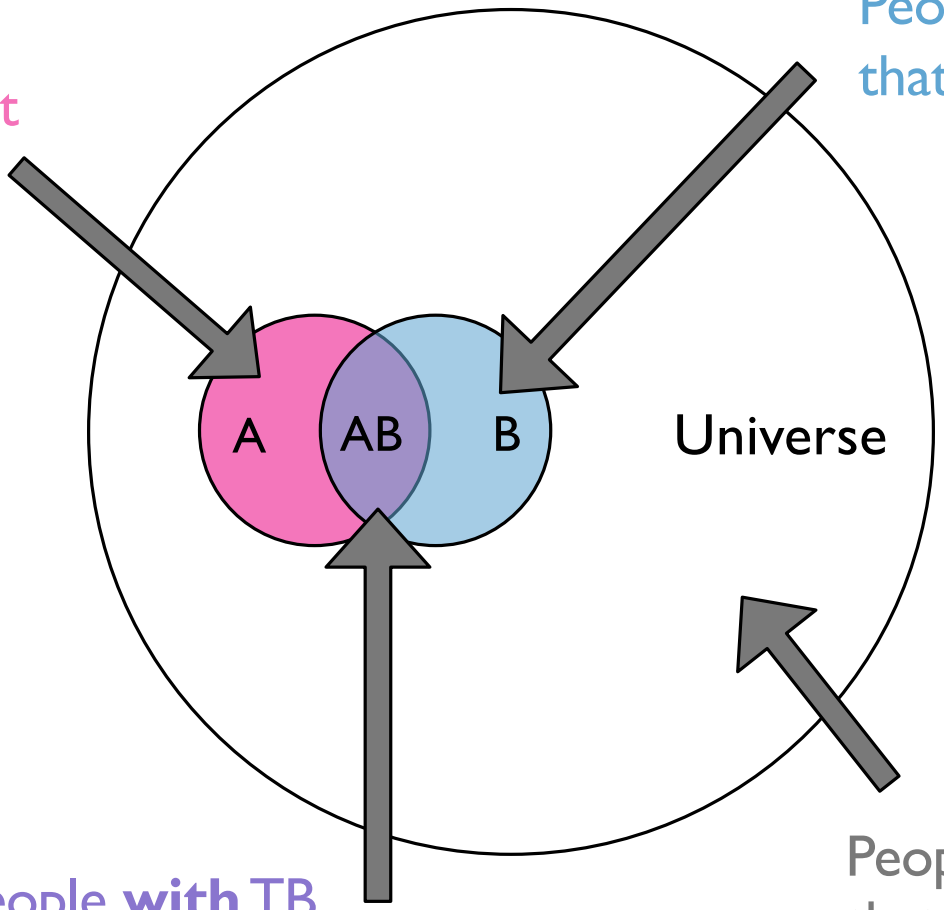
If I pick a random person in the universe, what's the probability of him/her **testing positive** for TB?

$$P(B) = \frac{|B|}{|U|}$$


A
AB
B
Universe

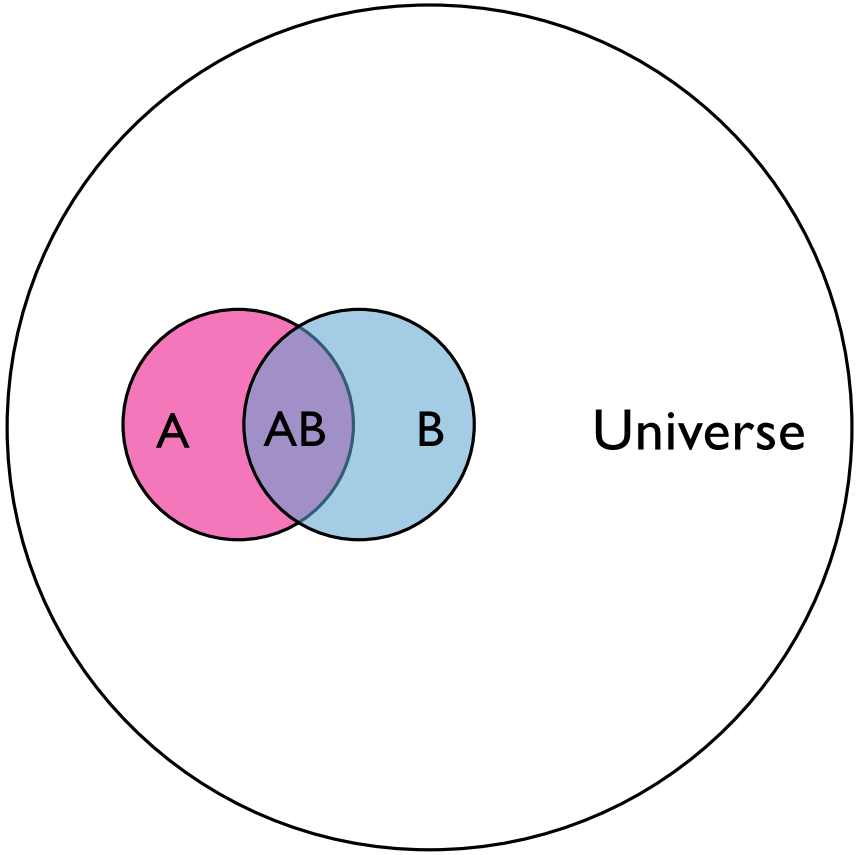
People **with** tuberculosis that test **negative**

People **without** TB that test **positive**



People **with** TB that test **positive**

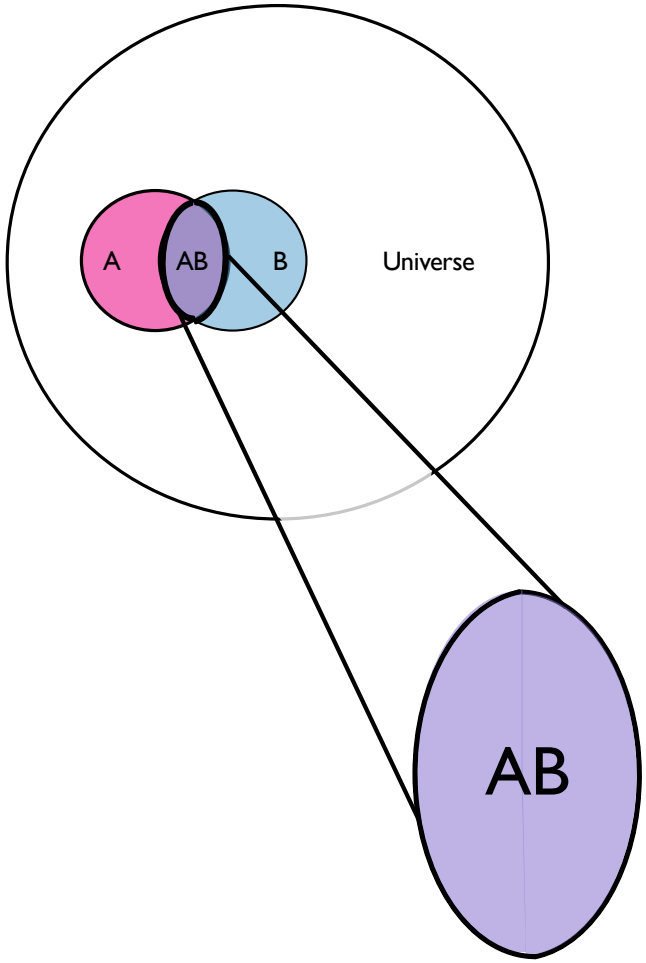
People **without** TB that test **negative**



If I pick a random person in the universe, what's the probability of him/her **having** tuberculous AND **testing positive** for it?

$$P(A, B) = \frac{|AB|}{|U|}$$

Joint Probability



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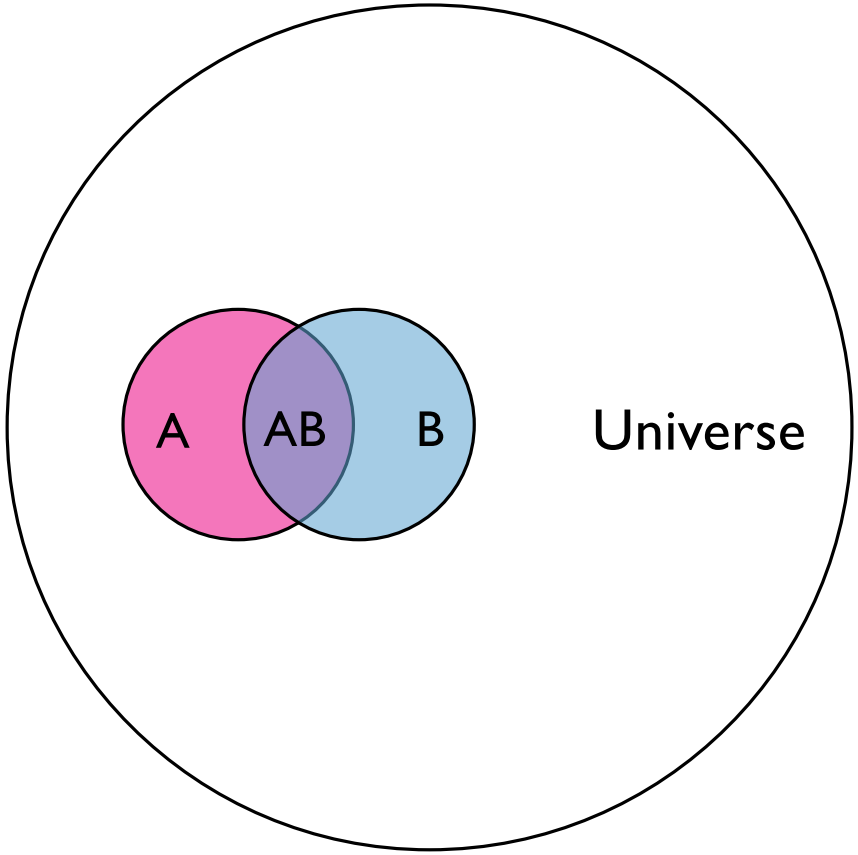
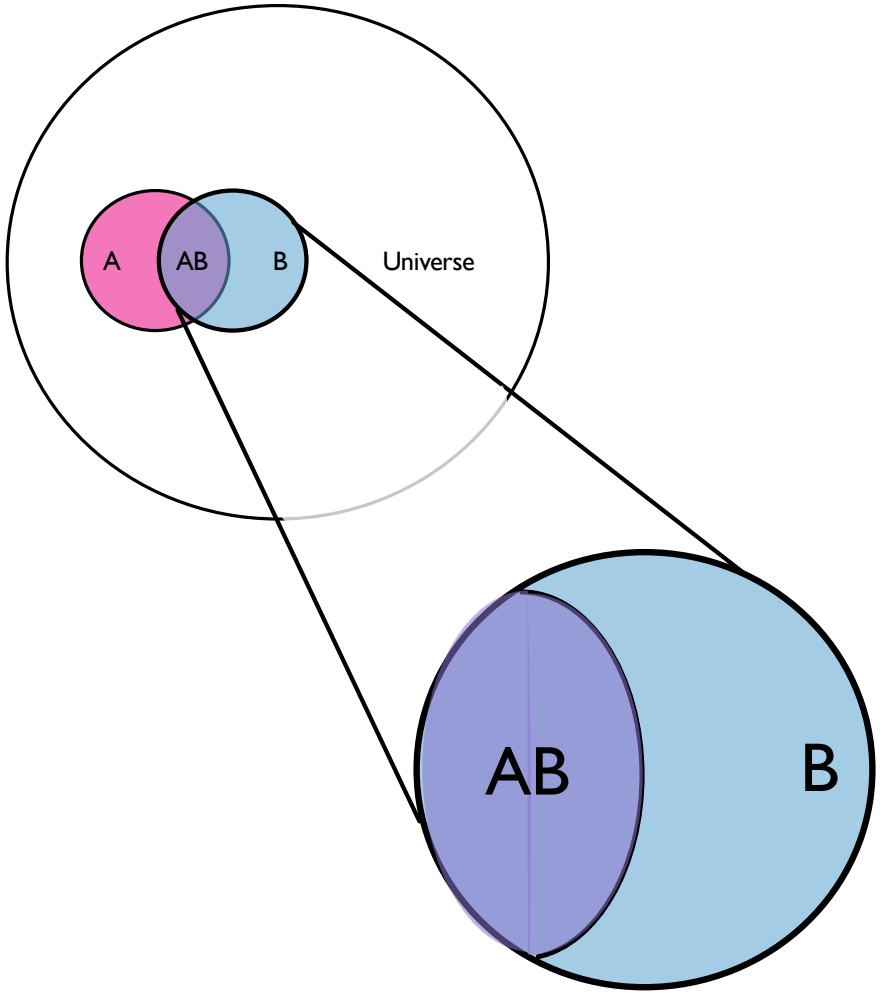


If I pick a random person **that tested positive**, what's the probability of him/her having tuberculous?

$$P(A|B) = \frac{|AB|}{|B|}$$

Conditional Probability



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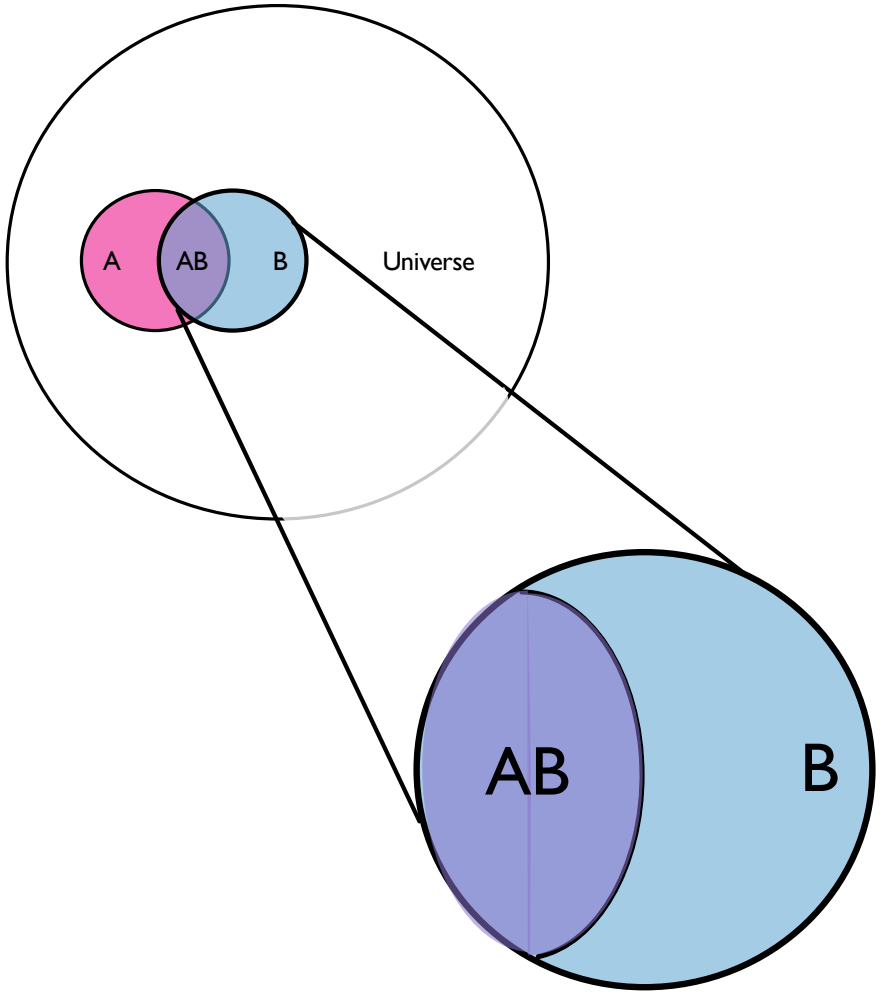
Conditional Probability

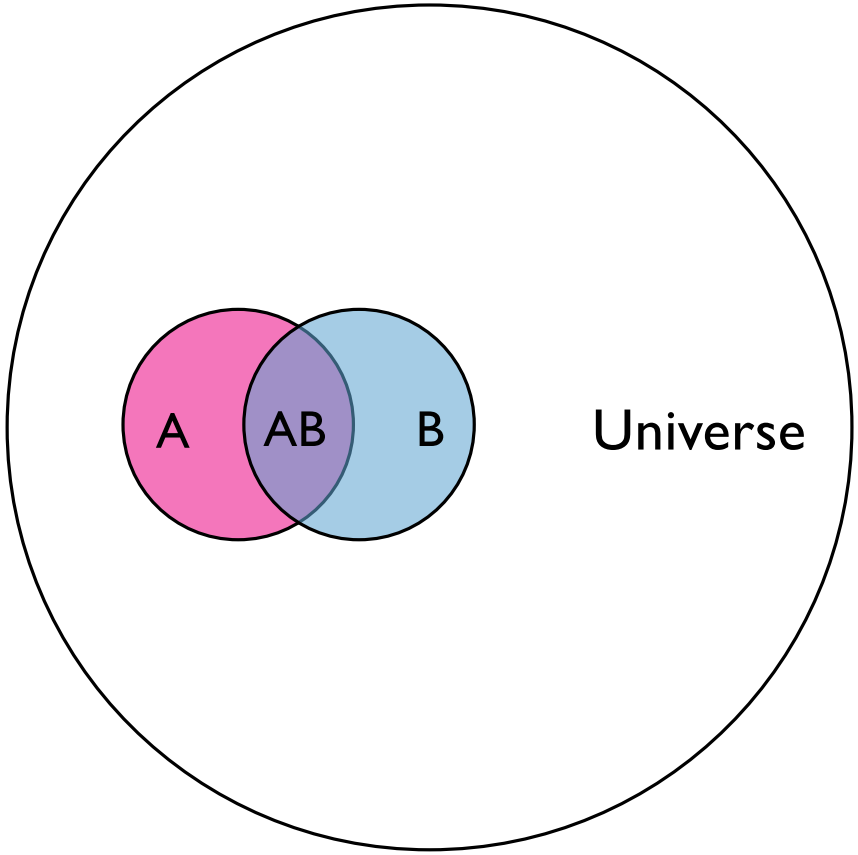


If I pick a random person **that tested positive**, what's the probability of him/her having tuberculous?

$$P(A|B) = \frac{|AB|}{|B|}$$

$$= \frac{|AB|/|U|}{|B|/|U|}$$

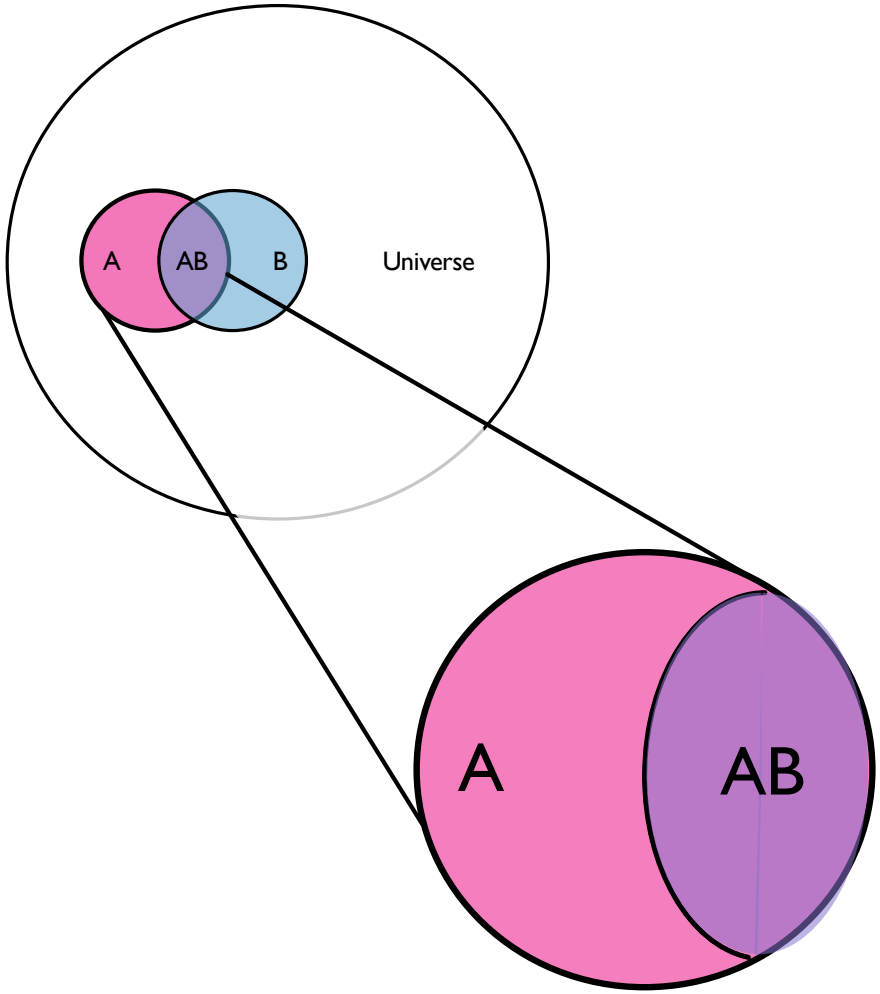
$$= \frac{P(A,B)}{P(B)}$$



If I pick a random person **that has tuberculous**, what's the probability of him/her testing positive?

$$P(B|A) = \frac{|AB|}{|A|}$$

Conditional Probability



If I pick a random person **that has tuberculous**, what's the probability of him/her testing positive?

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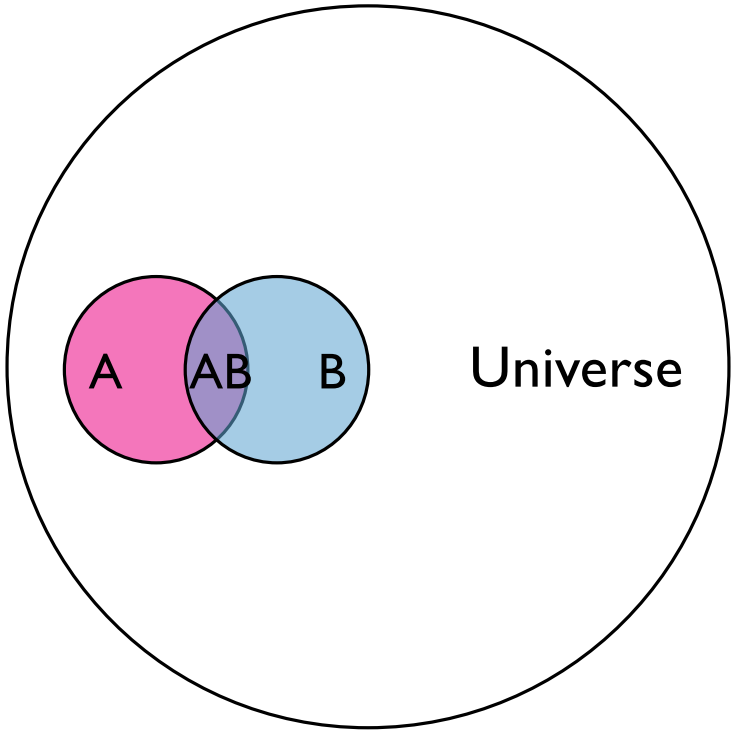
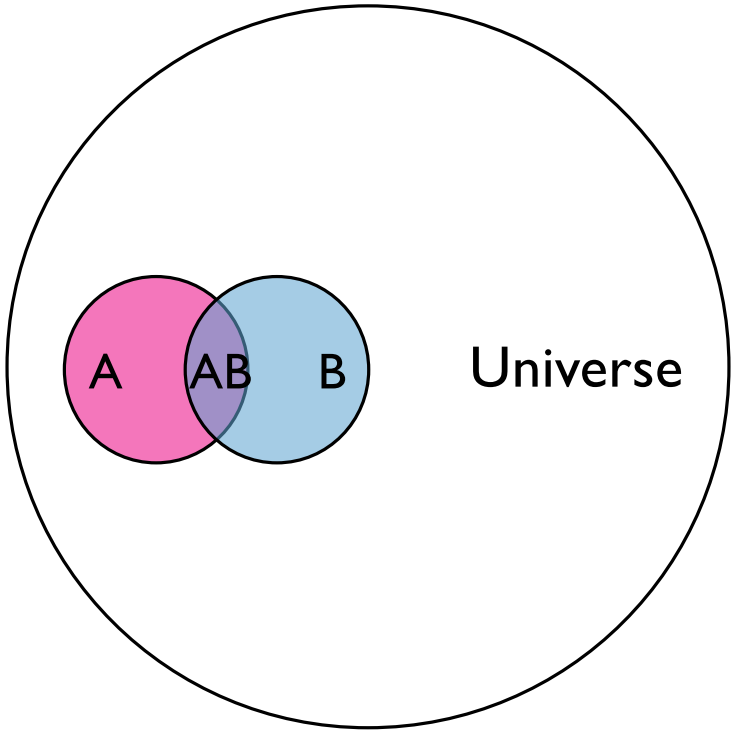


P(test positive | has TB)

$$P(B|A) = \frac{P(A,B)}{P(A)}$$

P(has TB | test positive)

$$P(A|B) = \frac{P(A,B)}{P(B)}$$

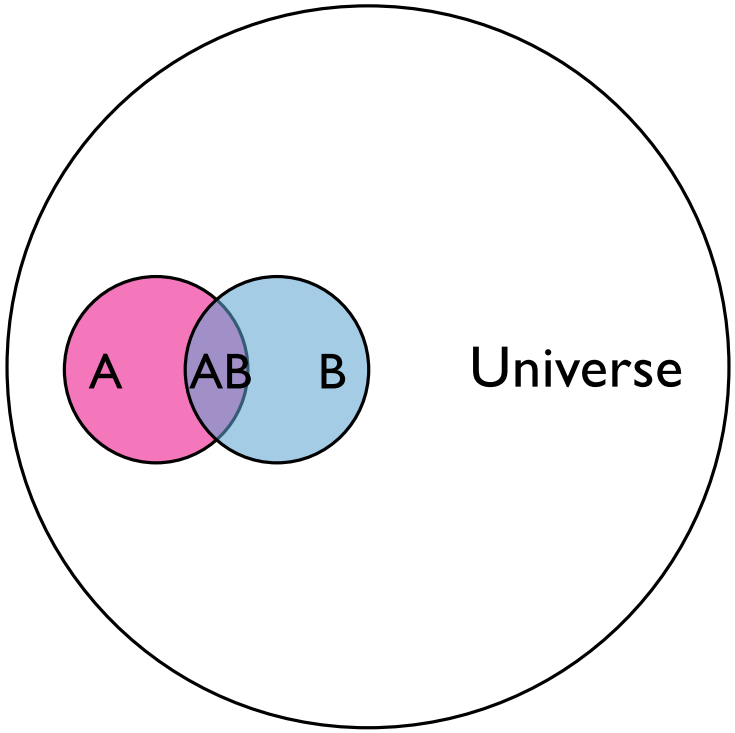


P(test positive | has TB)

$$P(B|A)P(A) = P(A,B)$$

P(has TB | test positive)

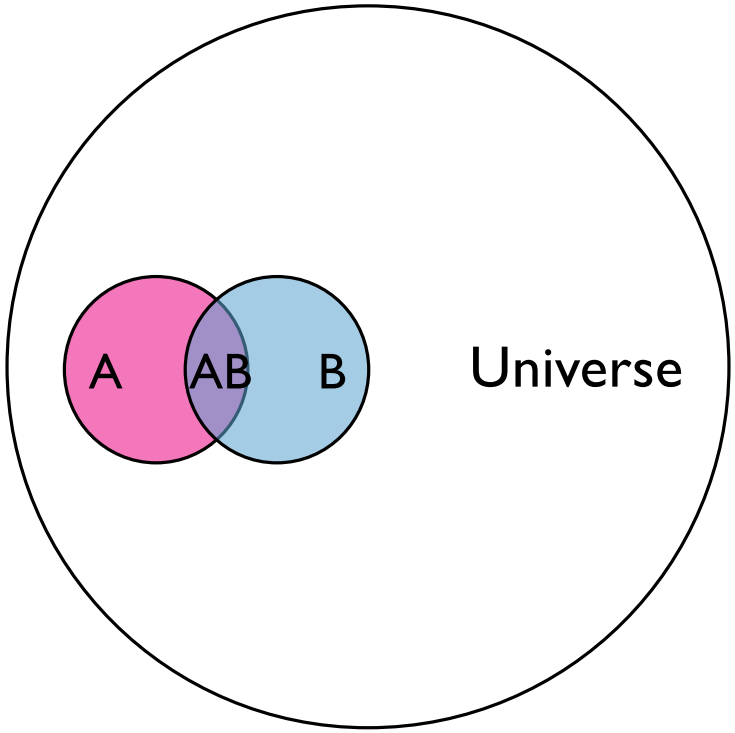
$$P(A,B) = P(A|B)P(B)$$



P(test positive | has TB)

$$P(B|A)P(A) = P(A,B) = P(A|B)P(B)$$

P(has TB | test positive)



P(test positive | has TB)

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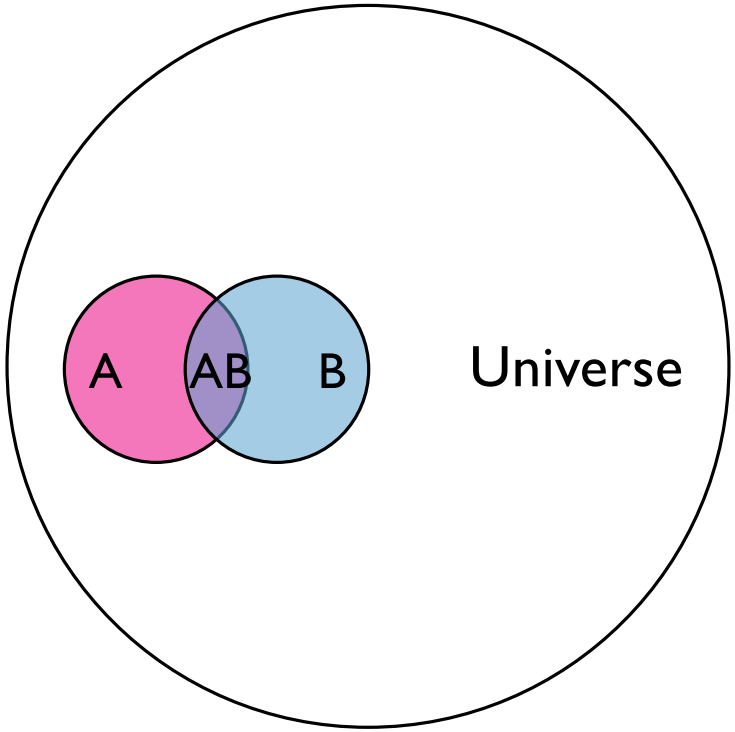
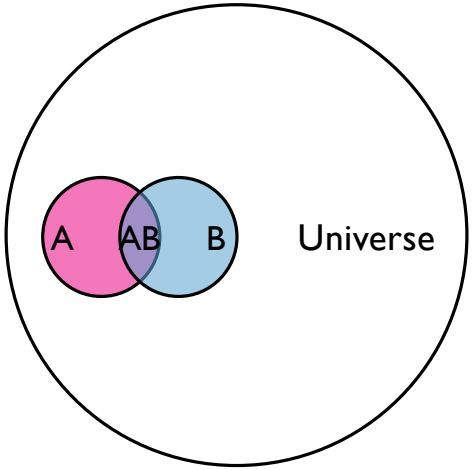


P(test positive | has TB) P(has TB | test positive)

$$P(B|A)P(A) = P(A|B)P(B)$$

P(has TB) P(test positive)

If a person **tests positive** for tuberculosis, what is the probability of him/her actually having TB?



$$P(\text{has TB} \mid \text{test positive}) = \frac{P(\text{test positive} \mid \text{has TB}) \times P(\text{has TB})}{P(\text{test positive})}$$

BAYES THEOREM

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

EXAMPLE

The United States sees about 3 new cases of TB per 100,000 people each year. An estimated 80% of people who have TB will yield a positive skin test. An additional 1% of people that do not have the disease will also test positive. If a person from the US first tests positive during a yearly TB screening, what is the probability that he/she actually has tuberculous?

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Recall Bayes' Theorem:

$$P(\text{has TB} \mid \text{test positive}) = \frac{P(\text{test positive} \mid \text{has TB}) \times P(\text{has TB})}{P(\text{test positive})}$$

The United States sees about 3 new cases of TB per 100,000 people each year. An estimated 80% of people who have TB will yield a positive skin test. An additional 1% of people that do not have the disease will also test positive. If a person from the US first tests positive during a yearly TB screening, what is the probability that he/she actually has tuberculous?

P(has TB) = 3 cases /100,000 people = 0.00003

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P(test positive | has TB) = 0.80

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$$P(\text{test positive}) = P(\text{test positive}, \text{has TB}) + P(\text{test positive}, \text{does not have TB})$$

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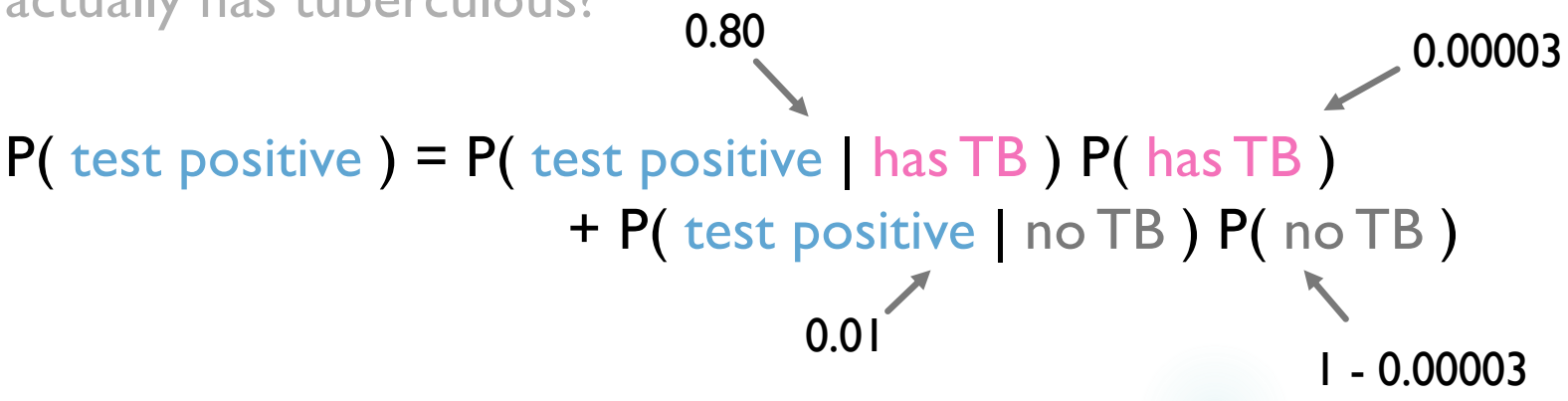

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$$
\begin{aligned}
P(\text{test positive}) &= P(\text{test positive} \mid \text{has TB})\, P(\text{has TB}) \\
&\quad + P(\text{test positive} \mid \text{no TB})\, P(\text{no TB}) \\
&= 0.80 \cdot 0.00003 + 0.01 \cdot (1 - 0.00003) \\
&\approx 0.01002
\end{aligned}
$$

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$$P(\text{has TB} \mid \text{test positive}) = \frac{P(\text{test positive} \mid \text{has TB}) \times P(\text{has TB})}{P(\text{test positive})}$$

$$\approx \frac{0.80 \cdot 0.00003}{0.01002} \approx 0.0024$$

ONLY 0.24%!

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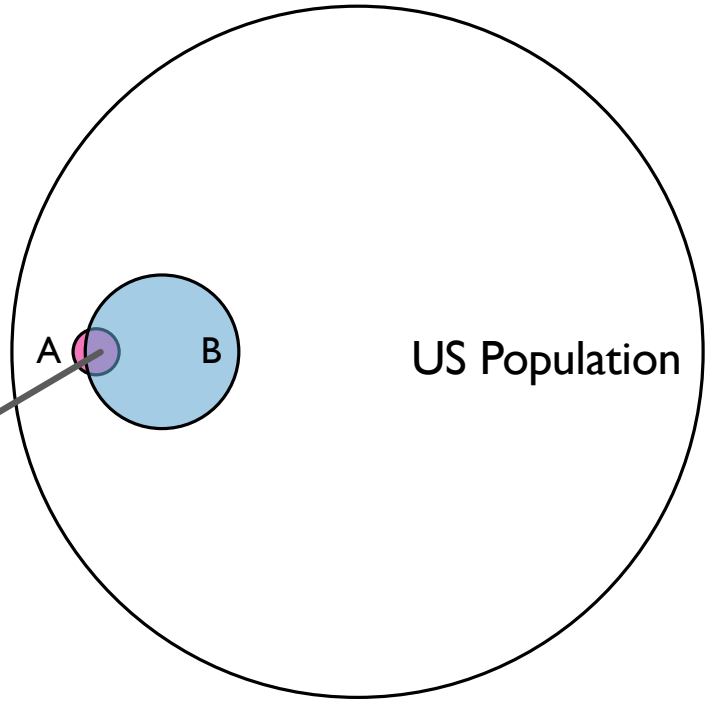
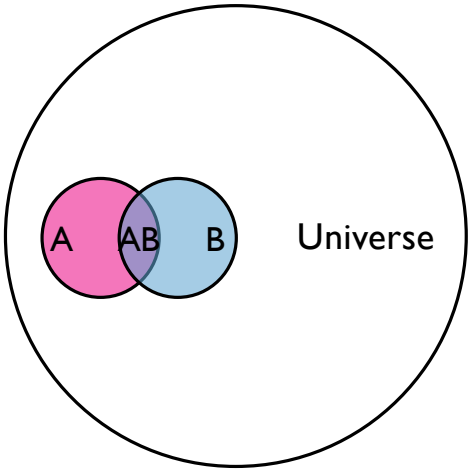


$$P(A|B) = \frac{|AB|}{|B|}$$

$$\approx \quad 0.24\%$$

RECAP



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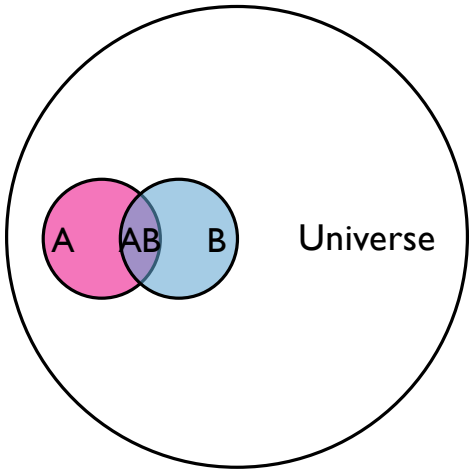

$$P(\text{has TB} \mid \text{test positive}) = \frac{P(\text{test positive} \mid \text{has TB}) \times P(\text{has TB})}{P(\text{test positive})}$$

Another perspective:

Updating Knowledge

$$\underbrace{P(A|B)}_{\text{posterior}} = \frac{\overbrace{P(B|A)}^{\text{likelihood}}}{\underbrace{P(B)}_{\text{evidence}}} \cdot \overbrace{P(A)}^{\text{prior}}$$