Lesson 2:
Probability Rules

Introduction

METIS

Lecture Overview:

Goals of the lecture:

1. Understand some basic probability rules

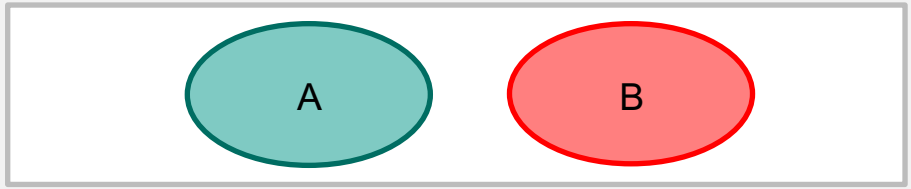
Probability Rules

METIS

Disjoint Events

Definition:

If A and B are disjoint events
$\mathbb{P}(A \text{ or } B) = \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$



Example 1:

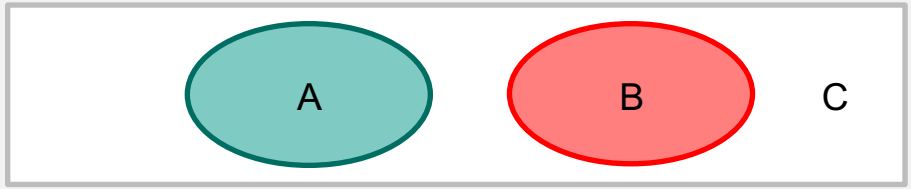
1. Probability of throwing a die and getting a 2 or 3

$\mathbb{P}(2) + \mathbb{P}(3) = 1/6 + 1/6 = 2/6 = 1/3$

Normalization of Total Probability

Definition:

If A, B and C are disjoint events
$\mathbb{P}(\Omega) = \mathbb{P}(A \cup B \cup C) =$
$\mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) = \sum_i \mathbb{P}(x_i) = 1$



Example 2:

1. Probability of throwing a die and getting a 1, 2, 3, 4, 5 or 6

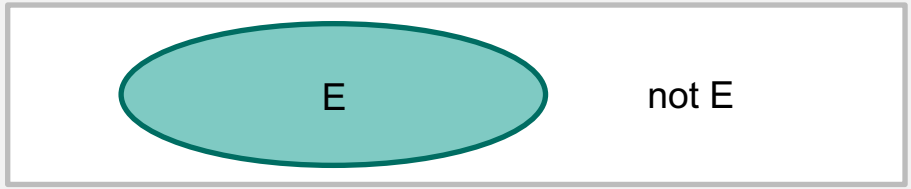
$$\mathbb{P}(1) + \mathbb{P}(2) + \mathbb{P}(3) + \mathbb{P}(4) + \mathbb{P}(5) + \mathbb{P}(6) = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 1$$

Complementary

Definition:

For any event E

$$\mathbb{P}(\text{not } E) = 1 - \mathbb{P}(E)$$



Example 3:

1. Probability of throwing a die and not getting a 2

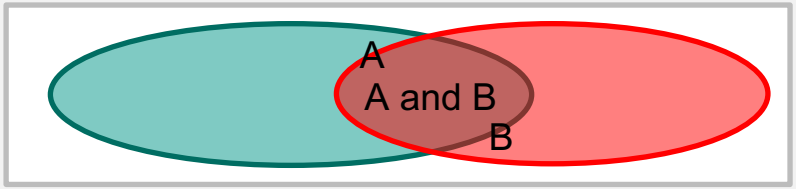
$$\mathbb{P}(\text{not } 2) = 1 - \mathbb{P}(2) = 1 - 1/6 = 5/6$$

Joint Events

Definition:

If A and B are joint events

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$



Example 4:

1. Probability of throwing a die and getting <4 OR even (e.g. {1,2,3,4,6})

$$\mathbb{P}(<4) + \mathbb{P}(\text{even}) - \mathbb{P}(<4 \cap \text{even}) = 3/6 + 3/6 - 1/6 = 5/6$$

Joint Events

Example 5:

Out of the students in this class,

- 30% live in Chicago,
- 50% have never programmed before, and
- 20% live in Chicago AND have never programmed before.

What is the probability that a randomly selected student does not live in Chicago and has programmed before?

Draw a diagram representing the Sample Space, and Events and label each part of the diagram.

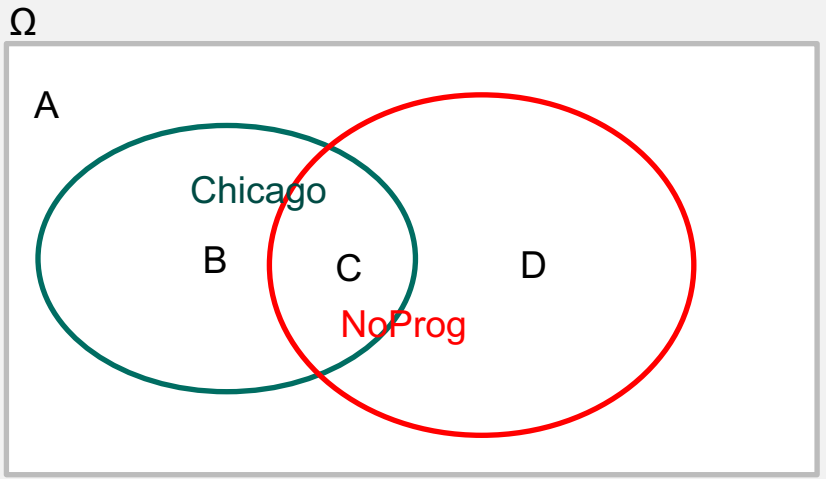
Joint Events

Example 5 (Solution – Part 1):

$\mathbb{P}(Chicago) = \mathbb{P}(B \cup C) = 0.3$
$\mathbb{P}(NoProg) = \mathbb{P}(C \cup D) = 0.5$
$\mathbb{P}(Chicago \cap NoProg) = \mathbb{P}(C) = 0.2$

A = NoChicago AND YesProg
B = Chicago AND YesProg
C = Chicago AND NoProg
D = NoProg AND NoChicago

Ω = A + B + C + D

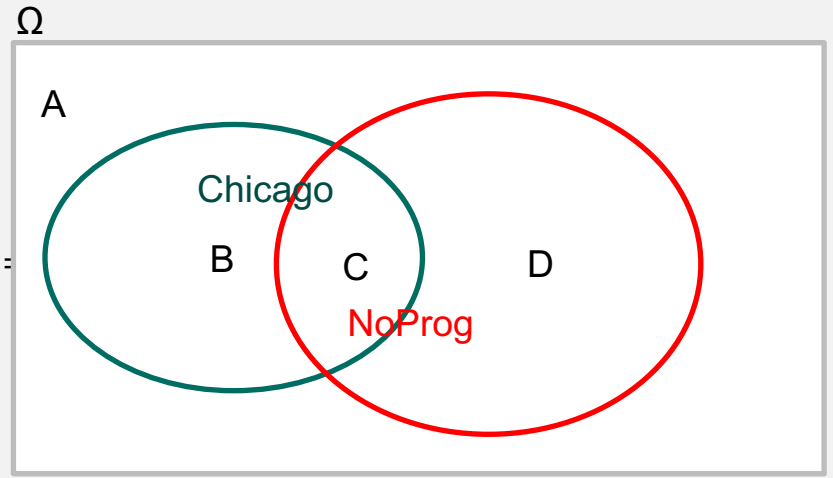


Joint Events

Example 5 (Solution – Part 1):

$\mathbb{P}(Chicago) = \mathbb{P}(B \cup C) = 0.3$
$\mathbb{P}(NoProg) = \mathbb{P}(C \cup D) = 0.5$
$\mathbb{P}(Chicago \cap NoProg) = \mathbb{P}(C) = 0.2$

$\mathbb{P}(A) = 1 - \big(\mathbb{P}(Chicago) + \mathbb{P}(NoProg) - \mathbb{P}(C)\big) =$
1-0.3-0.5+0.2 = 0.4



Problem 1

Problem 1:

1. What is the probability of throwing two dies and getting the same number on both (e.g. (1,1), (2,2), etc)?

Problem 1

Solution 1:

6 outcomes out of 36 meet the criteria:

$$\mathbb{P}(same) = \frac{6}{36} = \frac{1}{6}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	4,5	5,5	5,6
6,1	6,2	6,3	4,6	6,5	6,6

Problem 2

Problem 2:

1. What is the probability of throwing two dies and the sum is an even number AND is greater than 7?

Problem 2

Solution 2:

9 outcomes out of 36 meet the criteria even AND >7:

$$\mathbb{P}(same) = \frac{9}{36} = \frac{1}{4}$$

1,1=2	1,2=3	1,3=4	1,4=5	1,5=6	1,6=7
2,1=3	2,2=4	2,3=5	2,4=6	2,5=7	2,6=8
3,1=4	3,2=5	3,3=6	3,4=7	3,5=8	3,6=9
4,1=5	4,2=6	4,3=7	4,4=8	4,5=9	4,6=10
5,1=6	5,2=7	5,3=8	4,5=9	5,5=10	5,6=11
6,1=7	6,2=8	6,3=9	4,6=10	6,5=11	6,6=12

Problem 3

Problem 3:

1. What is the probability of throwing two dies and the sum is an even number OR is greater than 7?

Problem 3

Solution 3:

$$\mathbb{P}(even) = \frac{18}{36} = \frac{1}{2}$$

1,1=2	1,2=3	1,3=4	1,4=5	1,5=6	1,6=7
2,1=3	2,2=4	2,3=5	2,4=6	2,5=7	2,6=8
3,1=4	3,2=5	3,3=6	3,4=7	3,5=8	3,6=9
4,1=5	4,2=6	4,3=7	4,4=8	4,5=9	4,6=10
5,1=6	5,2=7	5,3=8	4,5=9	5,5=10	5,6=11
6,1=7	6,2=8	6,3=9	4,6=10	6,5=11	6,6=12

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

Problem 3

Solution 3:

$$\mathbb{P}(even) = \frac{18}{36} = \frac{1}{2}$$

$$\mathbb{P}(> 7) = \frac{15}{36} = \frac{5}{12}$$

1,1=2	1,2=3	1,3=4	1,4=5	1,5=6	1,6=7
2,1=3	2,2=4	2,3=5	2,4=6	2,5=7	2,6=8
3,1=4	3,2=5	3,3=6	3,4=7	3,5=8	3,6=9
4,1=5	4,2=6	4,3=7	4,4=8	4,5=9	4,6=10
5,1=6	5,2=7	5,3=8	4,5=9	5,5=10	5,6=11
6,1=7	6,2=8	6,3=9	4,6=10	6,5=11	6,6=12

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

Problem 3

Solution 3:

$$\mathbb{P}(even) = \frac{18}{36} = \frac{1}{2}$$

$$\mathbb{P}(> 7) = \frac{15}{36} = \frac{5}{12}$$

$$\mathbb{P}(> 7 AND\ even) = \frac{9}{36} = \frac{1}{4}$$

1,1=2	1,2=3	1,3=4	1,4=5	1,5=6	1,6=7
2,1=3	2,2=4	2,3=5	2,4=6	2,5=7	2,6=8
3,1=4	3,2=5	3,3=6	3,4=7	3,5=8	3,6=9
4,1=5	4,2=6	4,3=7	4,4=8	4,5=9	4,6=10
5,1=6	5,2=7	5,3=8	4,5=9	5,5=10	5,6=11
6,1=7	6,2=8	6,3=9	4,6=10	6,5=11	6,6=12

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

Problem 3

Solution 3:

$$\mathbb{P}(even) = \frac{18}{36} = \frac{1}{2}$$

$$\mathbb{P}(> 7) = \frac{15}{36} = \frac{5}{12}$$

$$\mathbb{P}(> 7 AND\ even) = \frac{9}{36} = \frac{1}{4}$$

1,1=2	1,2=3	1,3=4	1,4=5	1,5=6	1,6=7
2,1=3	2,2=4	2,3=5	2,4=6	2,5=7	2,6=8
3,1=4	3,2=5	3,3=6	3,4=7	3,5=8	3,6=9
4,1=5	4,2=6	4,3=7	4,4=8	4,5=9	4,6=10
5,1=6	5,2=7	5,3=8	4,5=9	5,5=10	5,6=11
6,1=7	6,2=8	6,3=9	4,6=10	6,5=11	6,6=12

$\mathbb{P}(\text{even U } >7) = 1/2 + 5/12 - 1/4 = 8/12 = 2/3$

$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

QUESTIONS?