METIS

Lesson 3: Derivatives

Introduction

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Lecture Overview:

Goals of the lecture:

1. Understanding what are derivatives

Derivatives

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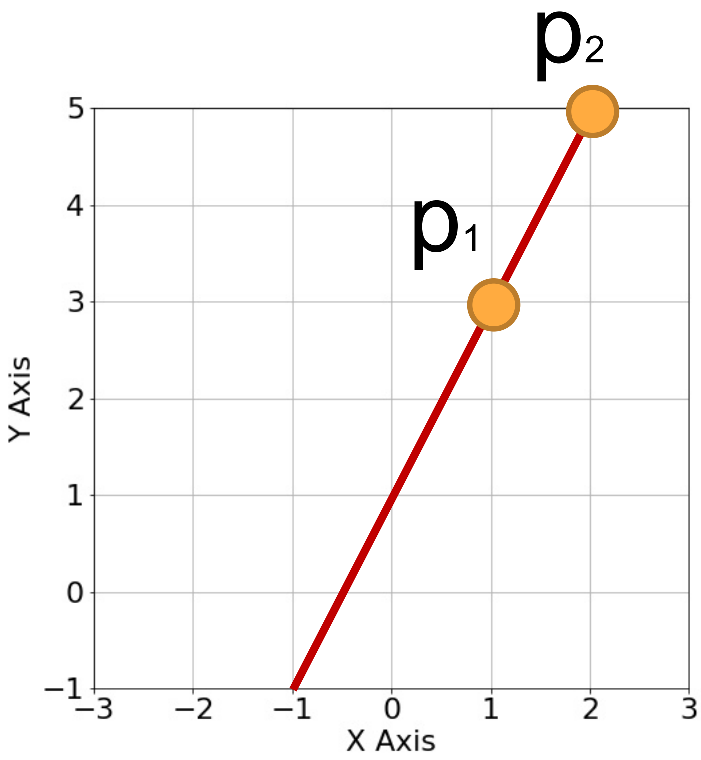
What is “m”?

“m” a.k.a. slope:

Indicates how steep the line is

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(5-3)}{(2-1)} = \frac{2}{1} = 2$$

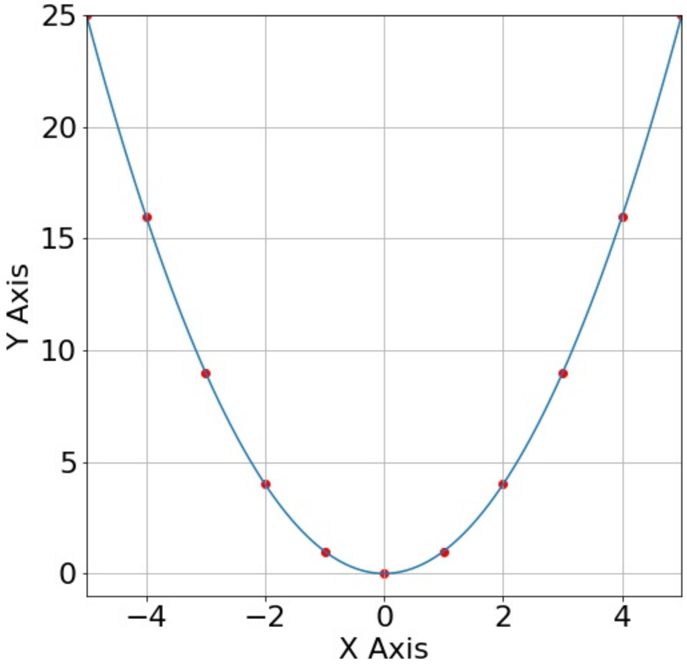


Derivative Notation

$$f'(x) = \frac{d}{dx} f(x)$$

Derivative of a Function

$$f(x) = x^2$$



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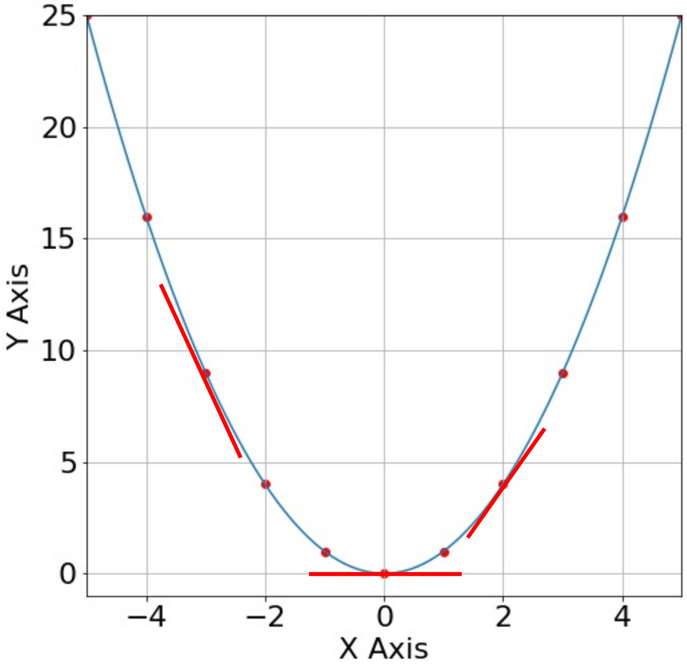

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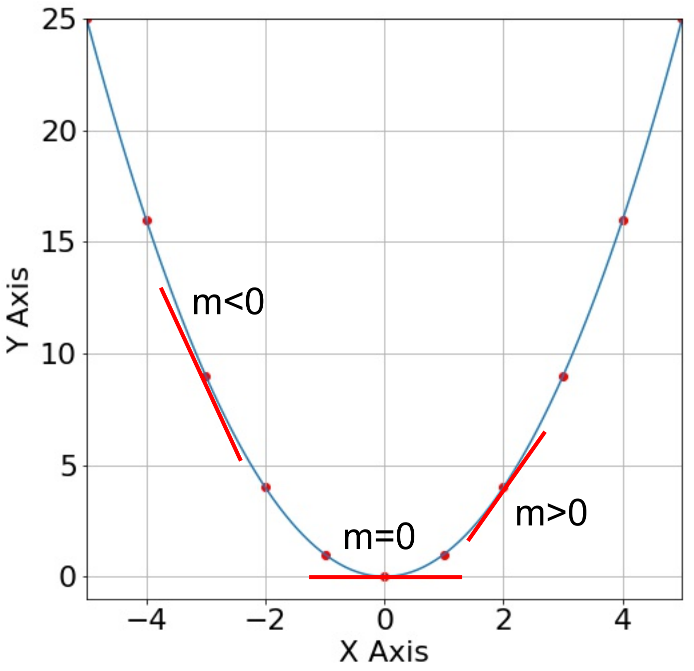

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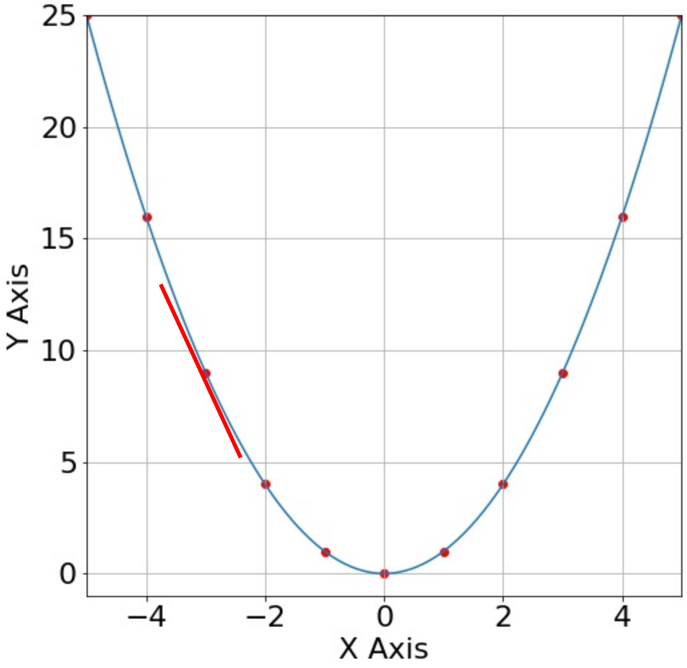

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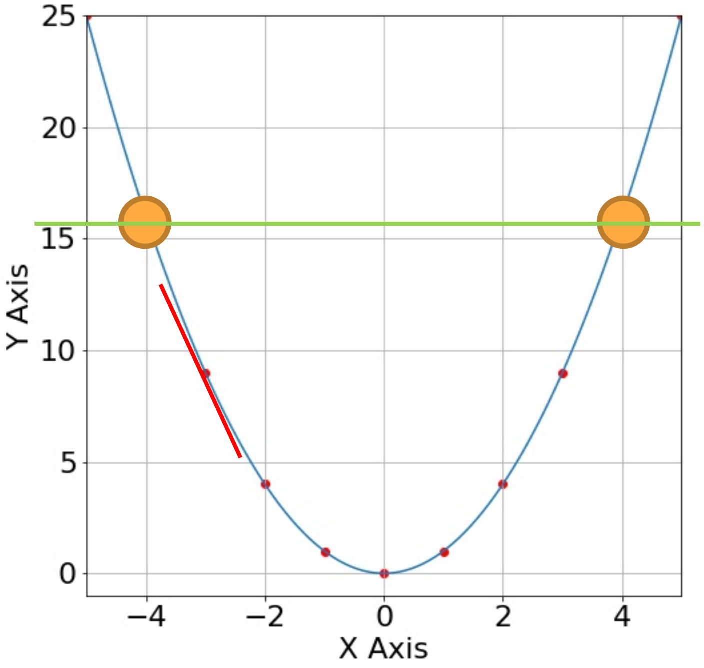

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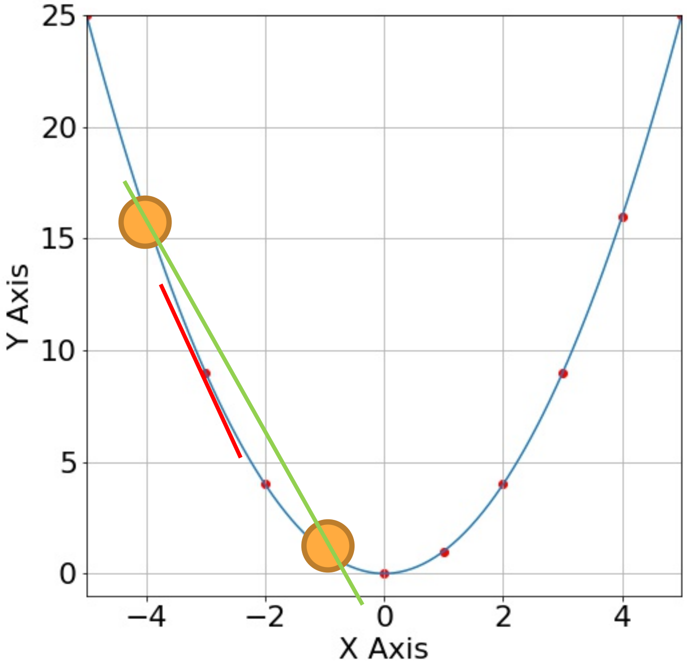

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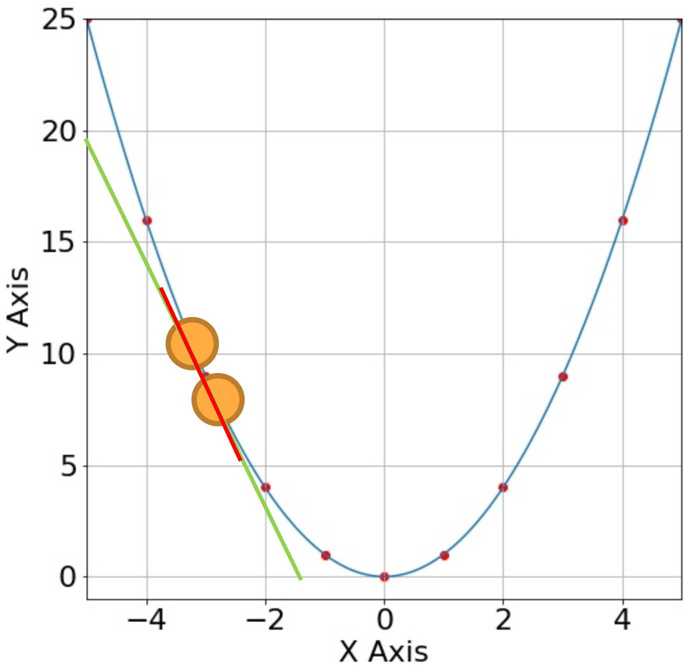

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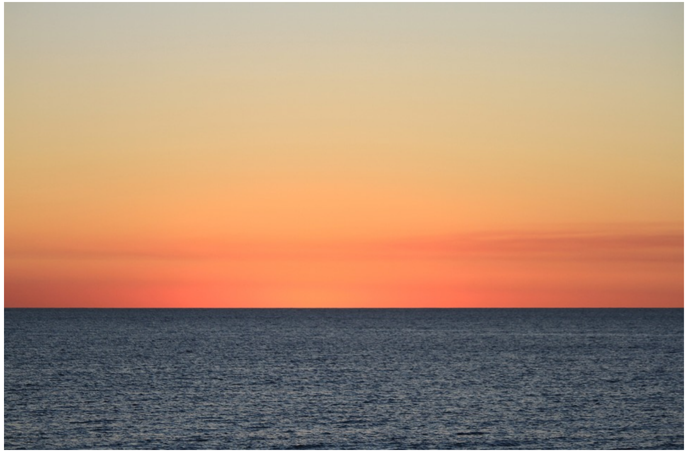
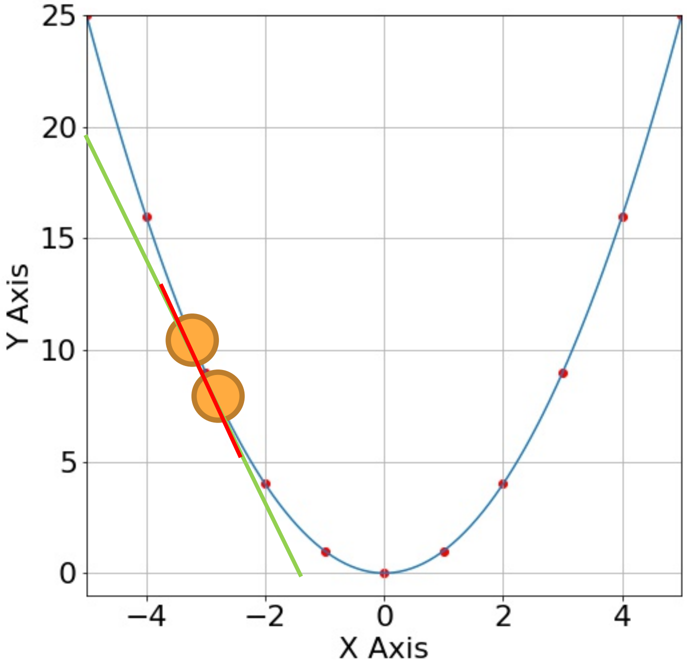
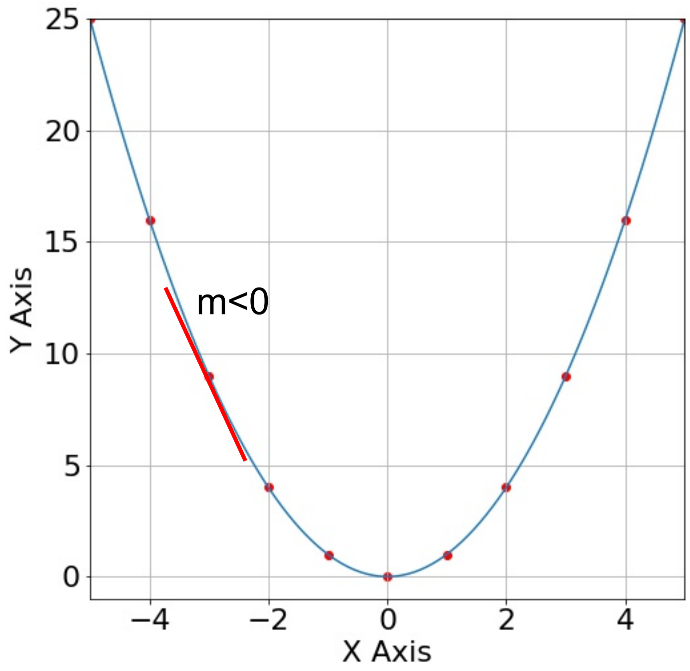




Derivative of a Function

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$



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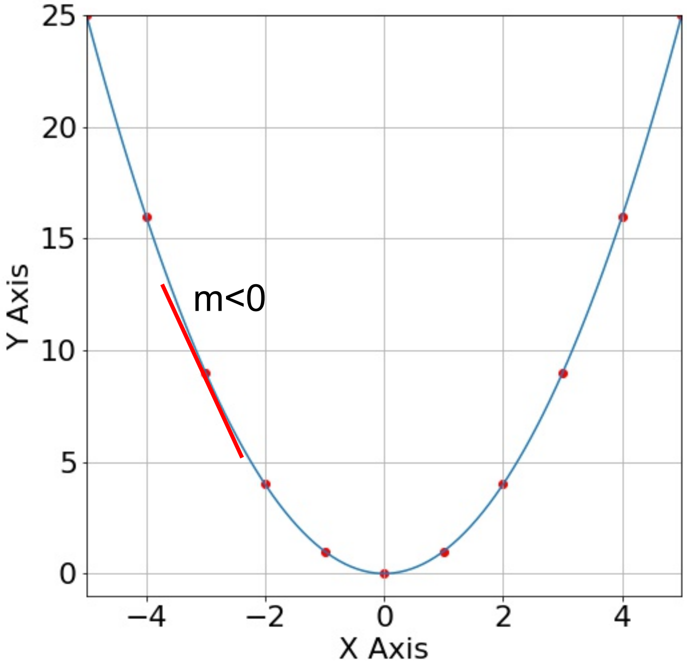

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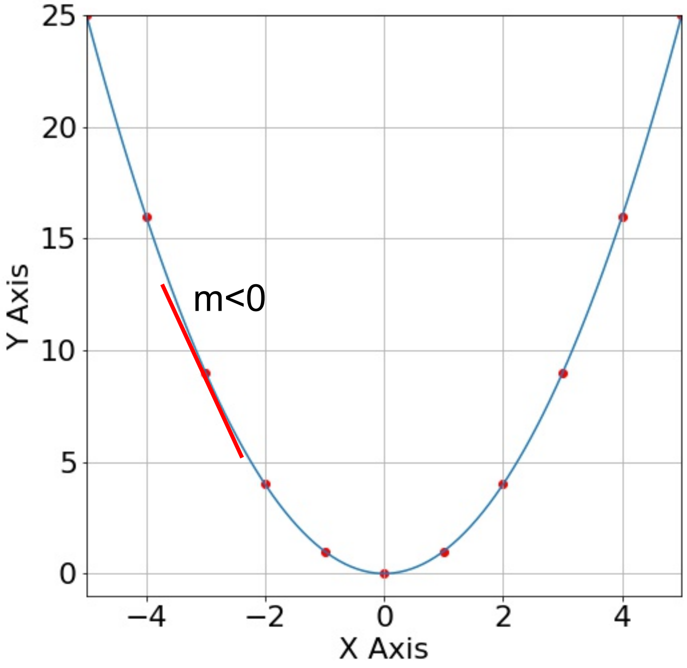
$$m = \frac{\left(f(x_2) - f(x_1)\right)}{(x_2 - x_1)}$$

$$x_2 = x_1 + h$$



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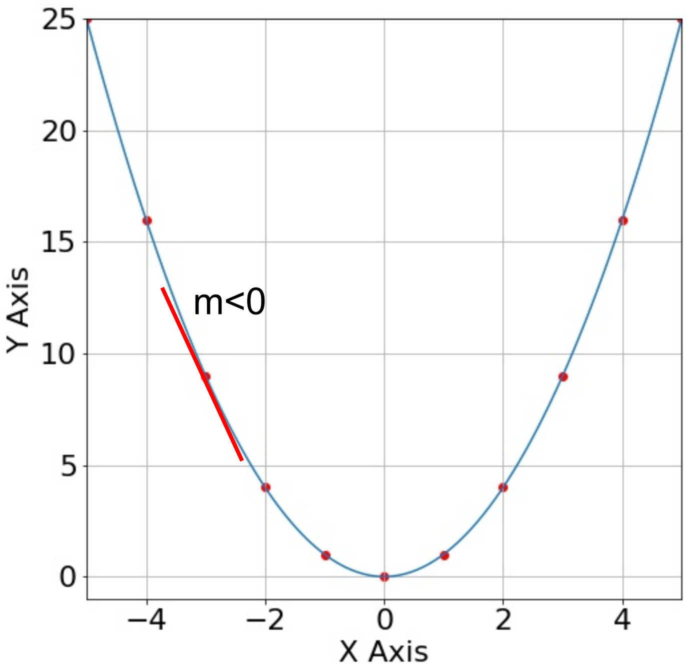

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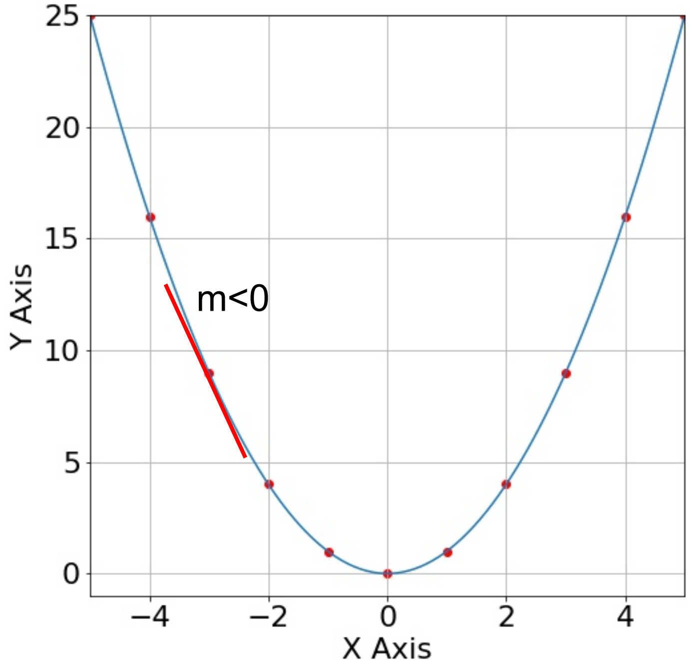

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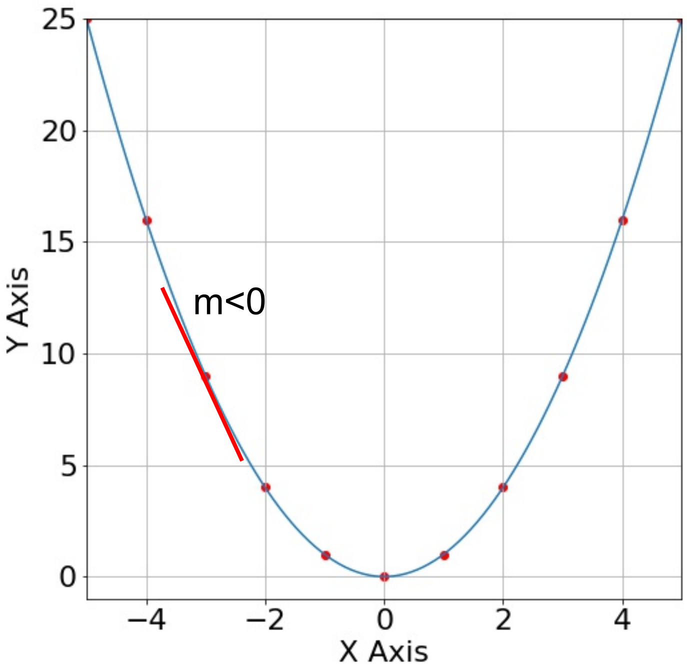
$$m = \frac{\left(f(x_1 + h) - f(x_1)\right)}{h}$$



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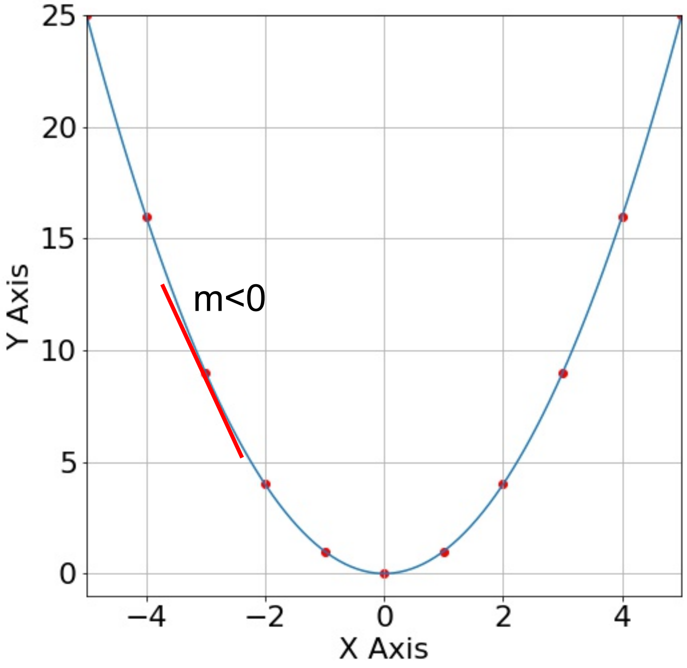

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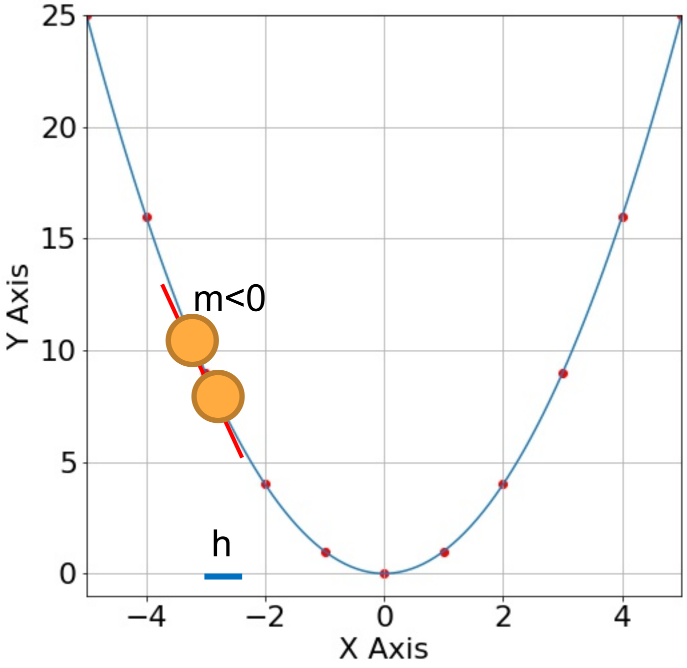
$$m = \frac{\left(f(x_1 + h) - f(x_1)\right)}{h} \qquad f(x) = x^2$$

Assume h = 0.1

$$m = \frac{f(-3 + 0.1) - f(-3)}{0.1} = \frac{8.41 - 9}{0.1} = -5.1$$



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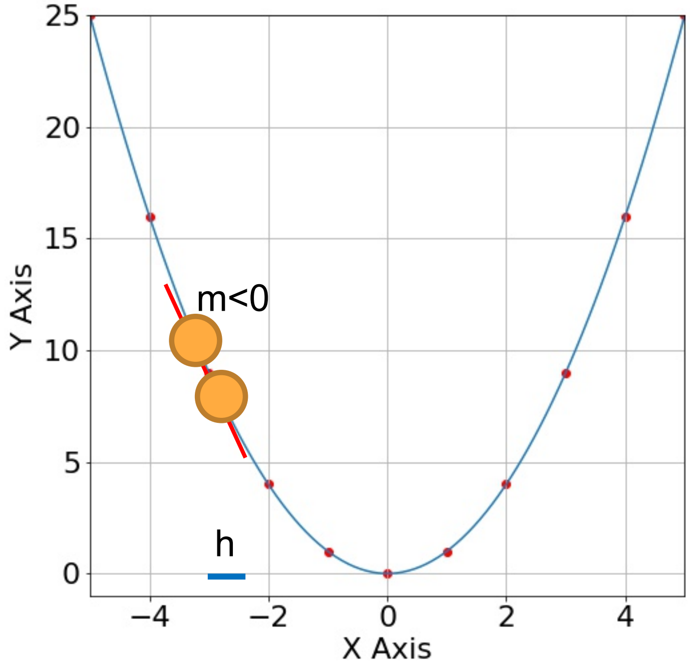
Assume h = 0.01

$$m = \frac{f(-3 + 0.01) - f(-3)}{0.01} = \frac{8.94 - 9}{0.01} = -5.99$$



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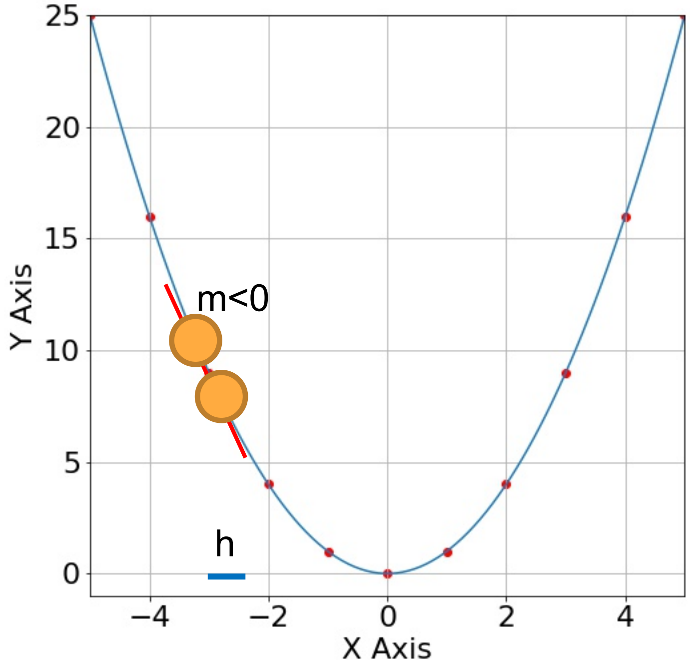

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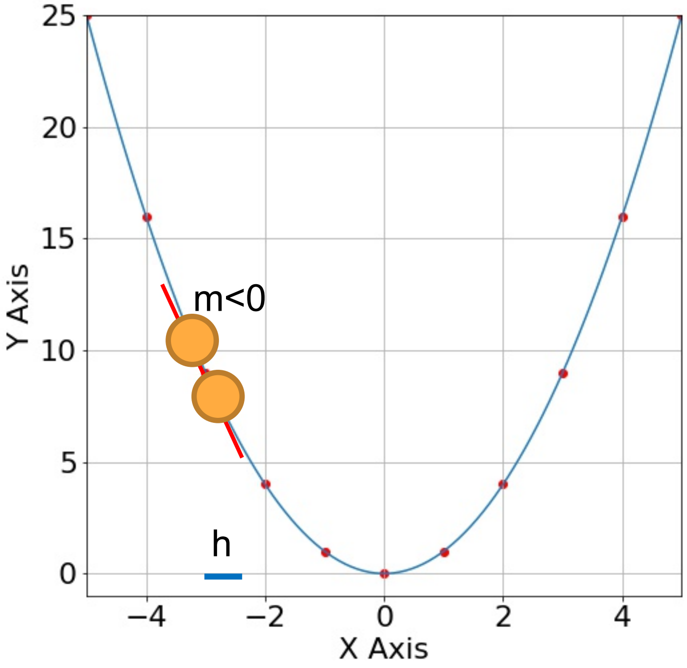

Derivative of a Function

$$m = \frac{\left(f(x_1 + h) - f(x_1)\right)}{h}$$

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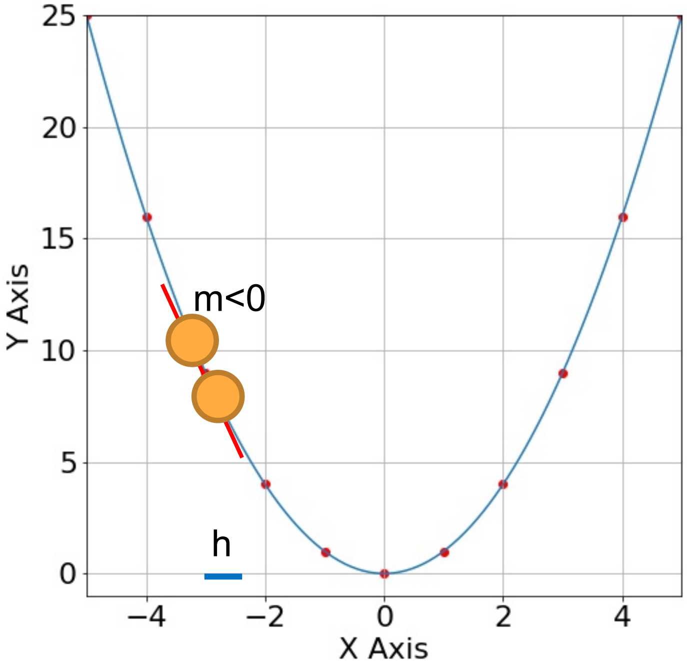

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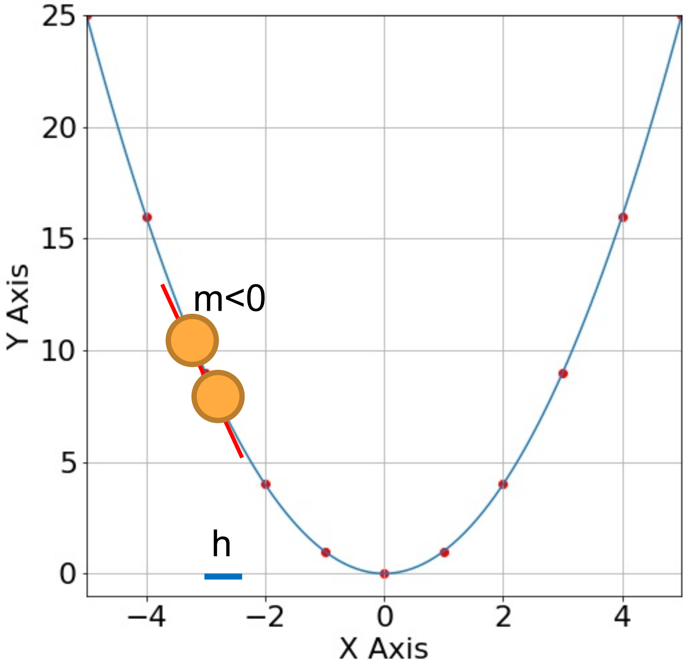

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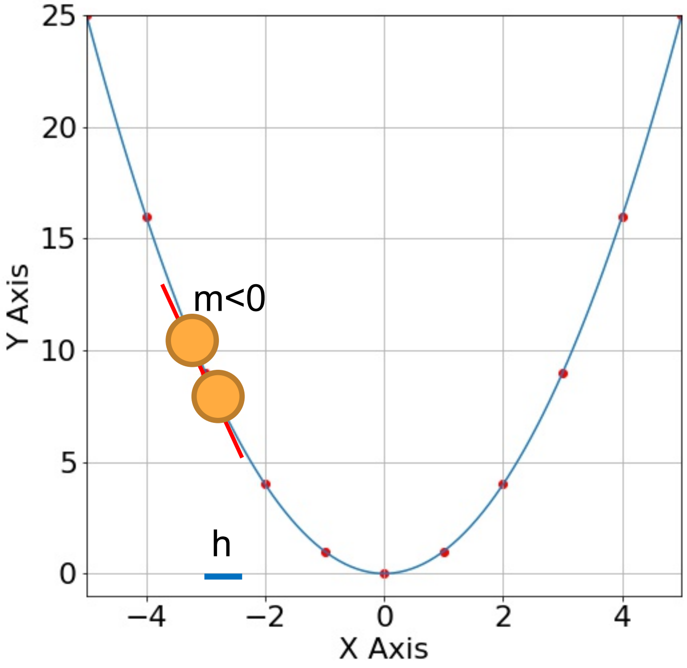

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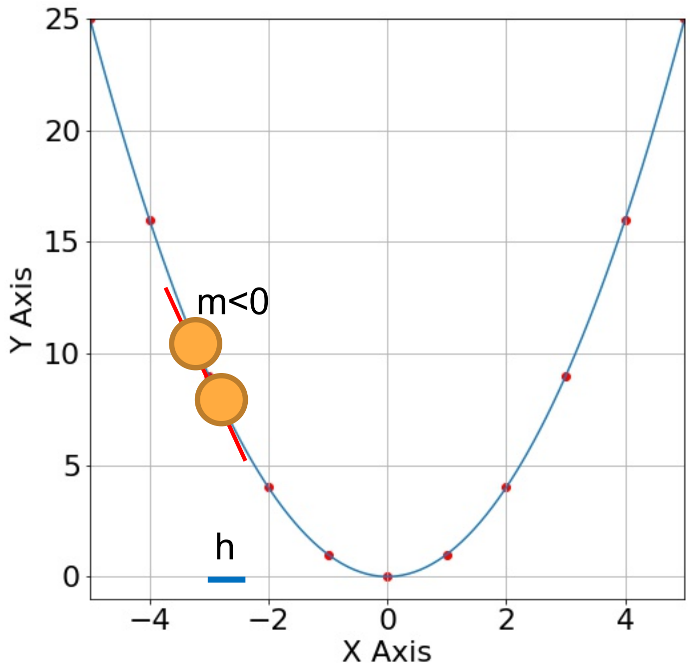
$$lim_{h\to 0}\frac{(-3+h)^2-9}{h} =$$



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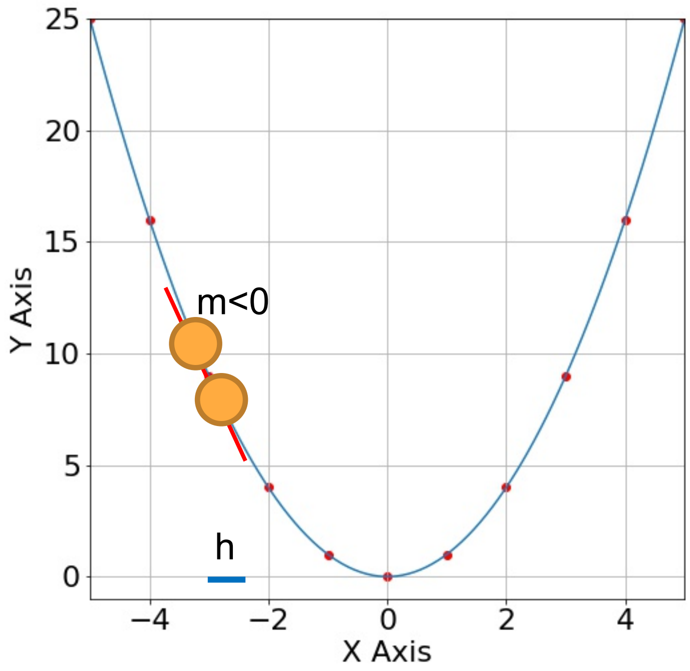
$$lim_{h \to 0} \frac{(-3 + h)^2 - 9}{h} =$$

$$lim_{h \to 0} \frac{9 - 6h + h^2 - 9}{h} =$$



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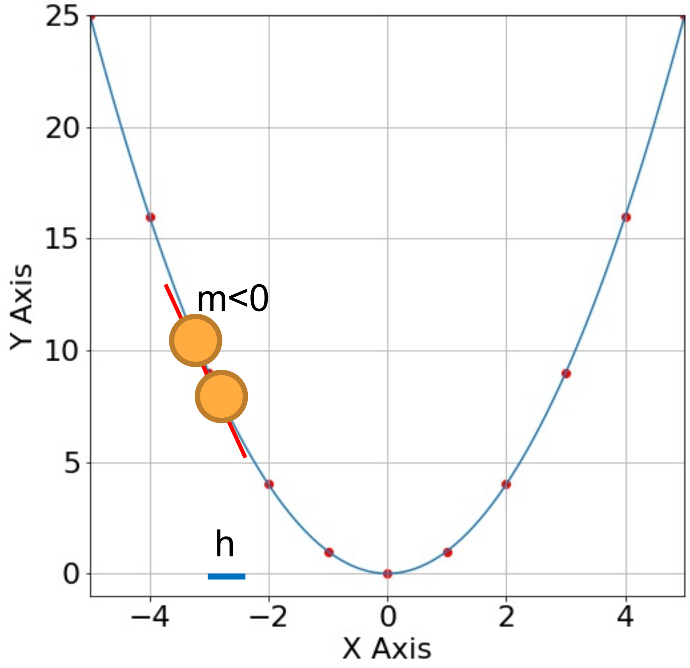

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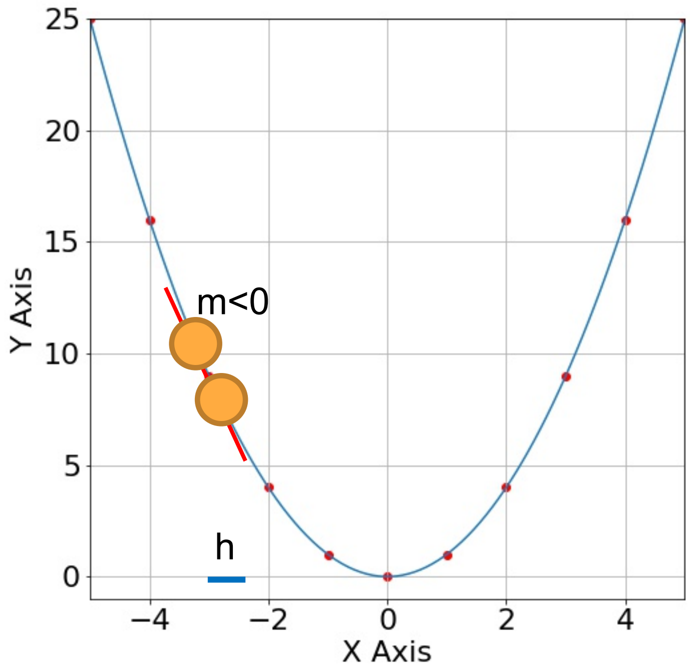
$$lim_{h\to 0}\frac{h^2 - 6h}{h} =$$

$$lim_{h\to 0}\frac{h-6}{1} = \frac{0-6}{1} = -6$$

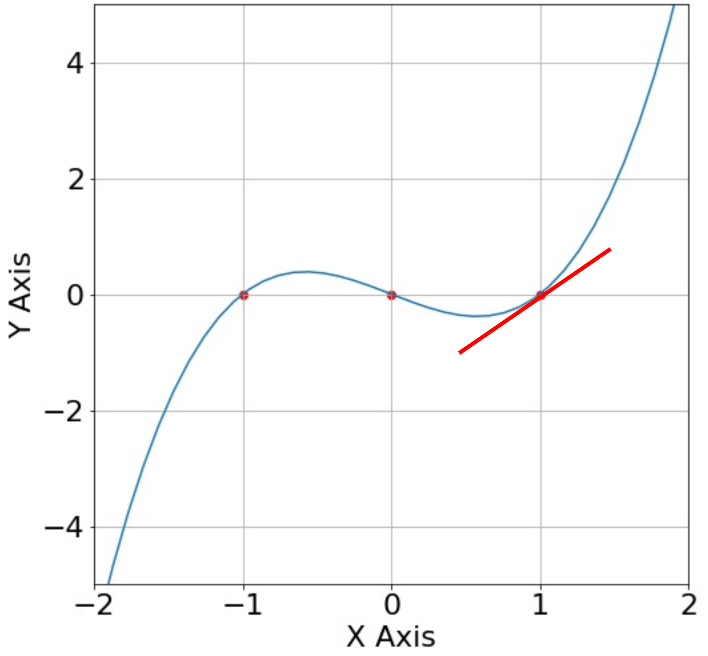


Problem 1:

Problem 1: Calculate f '(x) at x_1=1

$$f(x) = x^3 - x$$

$$f'(x_1) = lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$



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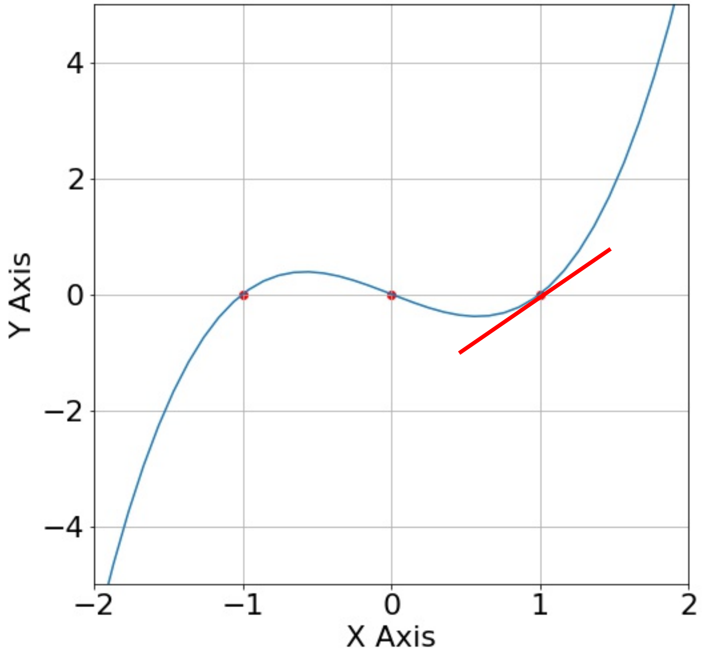

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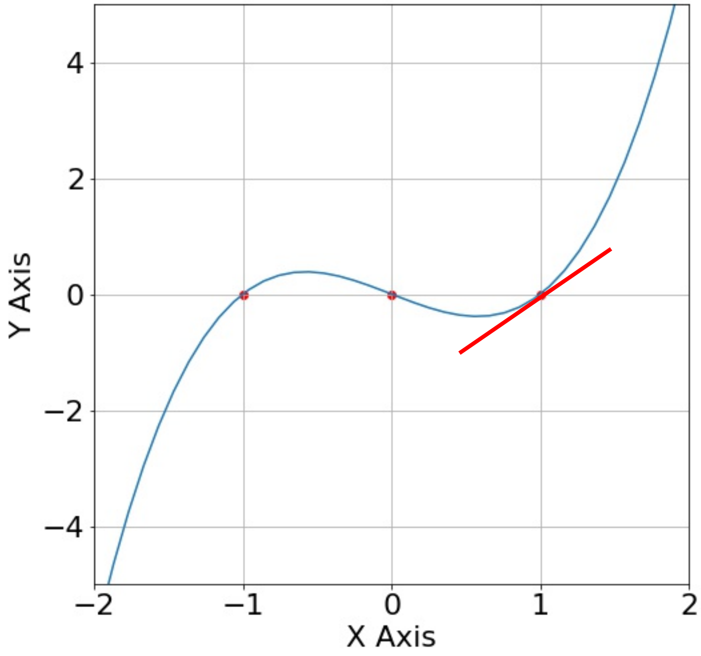
$$lim_{h \to 0} \frac{(1 + h)^3 - (1 + h) - (1 - 1)}{h} =$$



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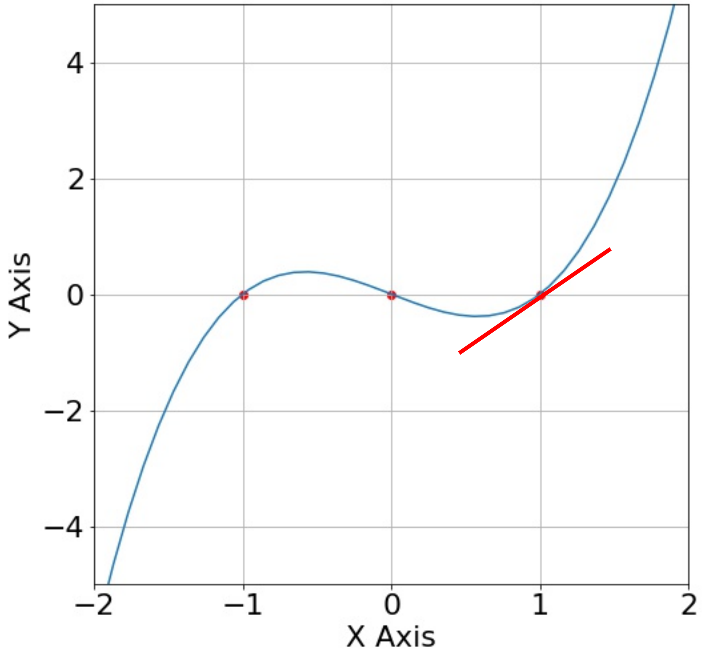
$$lim_{h\to 0}\frac{(1+h)^3-(1+h)-(1-1)}{h} =$$

$$lim_{h\to 0}\frac{h^3+3h^2+3h+1-1-h}{h} =$$



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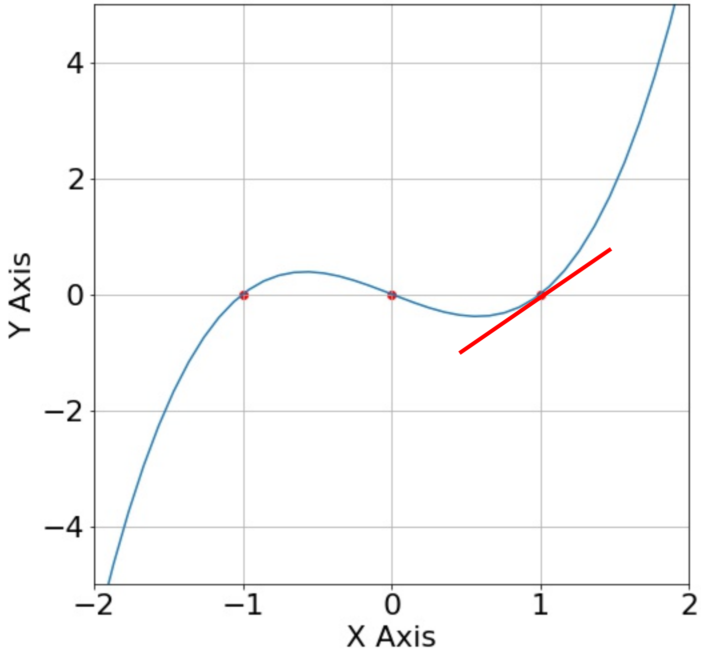

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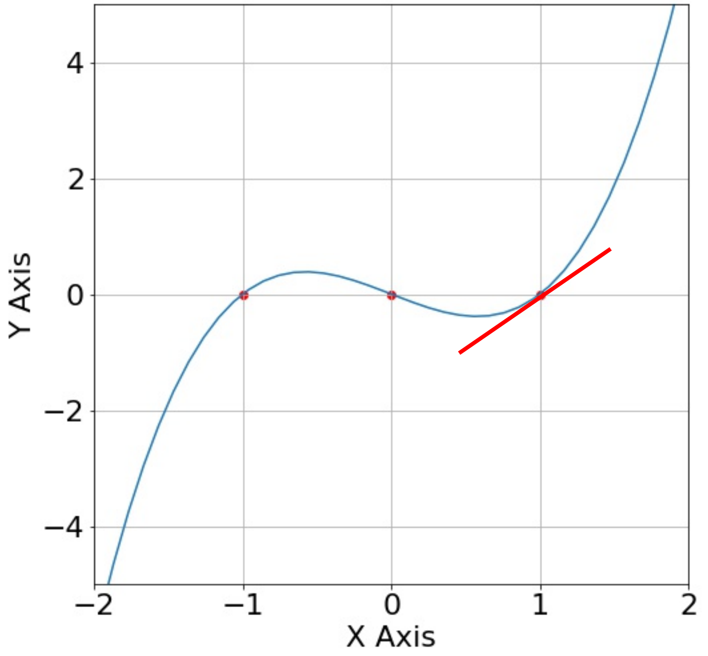

Continuous Functions

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A **continuous function** is a **function** for which sufficiently small changes in the input result in arbitrarily small changes in the output.

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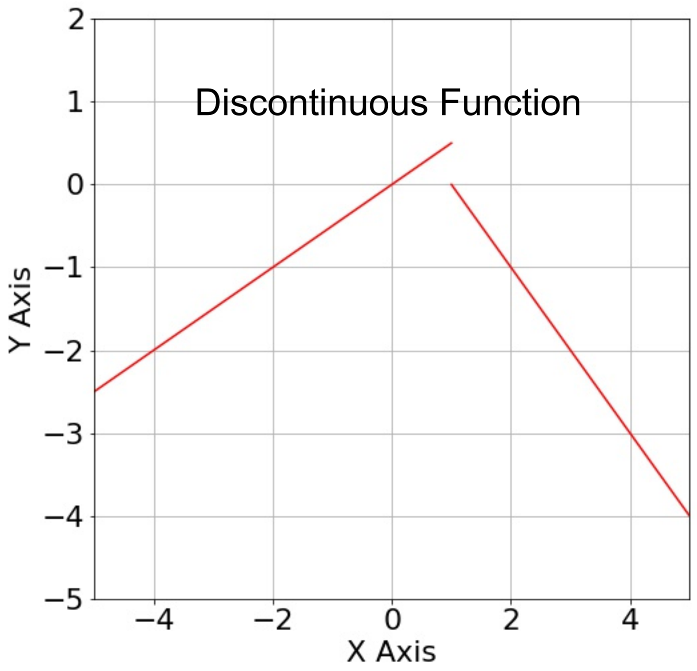

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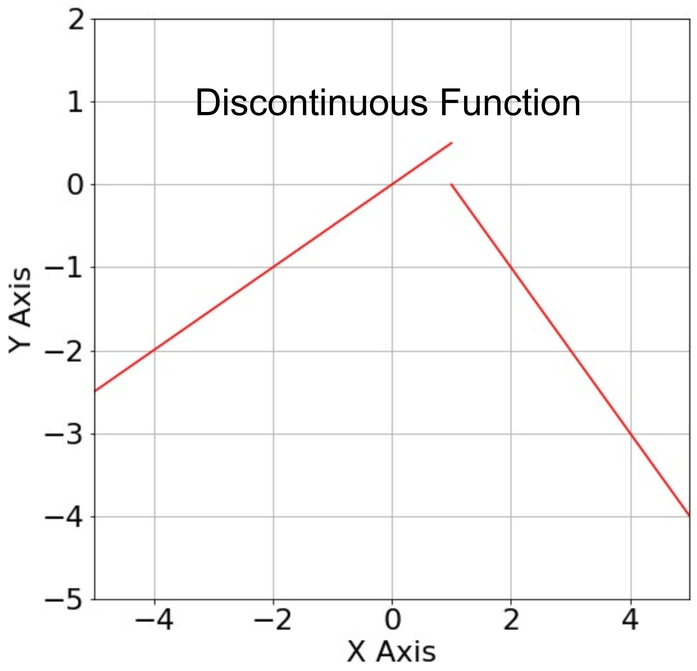
Derivatives exist only on continuous functions



QUESTIONS?