METIS

Lesson 1:

Probability Introduction

Introduction

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Lecture Overview:

Goals of the lecture:

1. Develop your probabilistic intuition to solve real-world problems

Probabilistic Intuition

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Probability

Definition:

Probability is the study of theoretical possibilities and their likelihood of occurring

Testing your Probabilistic Intuition

Before we begin, let us test your intuition by discussing your perceived outcomes for a series of phenomena:

Example 1:

1. What is the probability of flipping heads with a fair coin?
2. What is the probability of getting a 1 or 2 with a fair die?
3. What is the probability of getting a Heart card from a 52 deck of cards?

Testing your Probabilistic Intuition

Before we begin, let us test your intuition by discussing your perceived outcomes for a series of phenomena:

Example 1:

1. What is the probability of flipping heads with a fair coin?
   0.5
2. What is the probability of getting a 1 or 2 with a fair die?
   2/6 = 1/3 = 0.33
3. What is the probability of getting a Heart card from a 52 deck of cards?
   13/52 = 1/4 = 0.25

Definitions

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Random Experiment

Definition:

Random Experiment is the process of observing a phenomenon that has an uncertain outcome. Random Experiments are unpredictable in the short term and predictable in the long term

Example 2:

1. **Flipping a coin** (the outcome is uncertain – we may get heads or tails). In the short term, the experiment is unpredictable (i.e. probability of heads is unknown), but in the long term, the experiment becomes predictable (i.e. probability of heads becomes roughly 0.5 if fair coin)
2. **Rolling a 6-sided die** (the outcome is uncertain – we may get 1,2,3,4,5,6). In the short term, we can't predict if we get a 1, but in the long term, it converges to 1/6.

Sample Space

Definition:

Sample Space describes a list of all the possible outcomes of an experiment.

The Sample Space is typically denoted by Omega or S, enclosed in curly brackets and separated by commas: Ω = {Outcome_1, Outcome_2, Outcome_3, …, Outcome_n}

Example 3:

1. A fair coin has a sample space of {*H, T*}
2. A 6-sided die has a sample space of {1, 2, 3, 4, 5, 6}
3. Sentiment Analysis has a sample space of {“Positive”, “Negative”, “Neutral”}
4. A pregnancy test has a sample space of {“Pregnant”, “Not Pregnant”}

Random Experiment Outcomes

Definition:

The **Outcome** of a Random Experiment is simply the result that is obtained after running the experiment

Example 4:

1. We can flip a coin and observe the outcome as **“Heads”**
2. We can roll a die and observe the outcome as **“5”**
3. We can choose a card from a 52-card deck and observe the outcome as **“Ace of Spades”**
4. We can interpret the sentiment of a corpus of documents and observe the outcome as **“Positive”**
5. We can run a pregnancy test and observe the outcome as **“Not Pregnant”**

Events of a Random Experiment

Definition:

The **Events** of a Random Experiment are subsets of the outcomes in the Sample Space that we are interested in observing.

Example 5:

1. While a Sample Space of rolling a 6-sided die may contain all the numbers between 1 – 6, we may actually only be interested in the even numbers. Therefore, S = {1,2,3,4,5,6}, but E = {2,4,6}
2. While a Sample Space of a 52-card deck may contain all the combinations of 2-10,J,Q,K,A with the 4 suits, we may actually only be interested in the 4 aces. Therefore E = {A_spades, A_hearts, A_clubs, A_diamonds}

Problem 1

Problem 1:

If a Random Experiment consists of flipping a coin twice:

1. What is the Sample Space of this experiment?

2. What are the possible events where the first toss results in a Heads?

*Assume that the coin consist of two sides – “Heads” and “Tails”

Problem 1

Solution 1:

1. The Sample Space of the Random Experiment described above is:

   S = {TT, TH, HT, HH}

2. The events of the experiment where the first toss results in a Heads is:

   E = {HT, HH}

Probability Measure

Definition (1):

Probability Measure describes the chances that a specific event will occur and is denoted by $\mathbb{P}(E)$.

- If we assumed that each event had the same chance of occurring, then we can easily calculate the probability measure for any event in the experiment as:

$$\mathbb{P}(E) = \frac{\mathcal{N}(E)}{\mathcal{N}(S)} = \frac{\#\, of\ favorable\ outcomes}{\#\, total\ possible\ outcomes}$$

- OR, we can repeat the experiment for a large number of times and observe the frequency in which the event occurred:

$$\mathbb{P}(E) = \lim_{n \to \infty} \frac{f_n(E)}{n}$$

Probability Measure

Definition (2):

- The Probability Measure assigns a real number to each Event
- The probability of an event will have a value between 0 and 1
- The sum of the probabilities of the sample space is equal to 1

Example 6:

1. In the Sample Space associated with the Random Experiment of flipping a coin, the Probability Measure assigns a real number to each event: $\mathbb{P}(Heads) = 0.5$ and $\mathbb{P}(Tails) = 0.5$. Here, I am assigning numbers based on a probabilistic assumption of a fair coin
2. In the Sample Space associated with the Random Experiment of rolling a 6-sided die, the Probability Measure assigns a real number to each event: $\mathbb{P}(1) = \mathbb{P}(2) = \mathbb{P}(3) = \mathbb{P}(4) = \mathbb{P}(5) = \mathbb{P}(6) = \frac{1}{6}$.

Problem 2

Problem 2:

A roulette wheel has 37 numbers. 18 numbers are red, 18 numbers are black and 1 number is green.

1. What is the probability of getting a red number?
2. What is the probability of getting a black number?
3. What is the probability of getting a green number?

Problem 2

Solution 2:

$$\mathbb{P}(E) = \frac{\mathcal{N}(E)}{\mathcal{N}(S)} = \frac{\#\, of\ favorable\ outcomes}{\#\, total\ possible\ outcomes}$$

1. $\mathbb{P}(Red) = 18/37 = 0.486$
2. $\mathbb{P}(Black) = 18/37 = 0.486$
3. $\mathbb{P}(Green) = 1/37 = 0.027$

QUESTIONS?