Intro to Probability

Intersections of Independent Events

Learning Objectives

- Recognize if two events are **independent**
- Be able to calculate probability for intersecting independent events

Independent events are not influenced by previous events

The result of a coin toss coming up heads is always ½.
The result of rolling a one using a six-sided fair die is always ⅙.

Probability for Intersections of Independent Events

If two events *A* and *B* are independent, than the probability of both happening simultaneously, i.e. their intersection, is

$$P(A \text{ and } B) = P(A) * P(B)$$

Probability Exercise: Two Dice Rolls

Given a fair six-sided die:

- Let *A* represent rolling an odd number, i.e. {1, 3, 5}
    - Then P(*A*) = 3/6 = ½
- Let *B* represent rolling an even number, i.e. {2, 4, 6}
    - Then P(*B*) = 3/6 = ½
- Note that *B* is independent of A

Then the probability of rolling an even number, then rolling an odd number is

$$P(A \text{ and } B) = P(A) * P(B) = 1/4$$

Dependent Events and Conditional Probability

Learning Objectives

- Recognize if an event is **dependent** on another
- Be able to calculate probability of dependent events using **conditional probability**

Dependent events are influenced by previous events

What's the probability of drawing an ace from a deck of 52 cards?
If we don't replace the drawn card, what's the probability of drawing a second ace?

Let A = draw an ace from the deck, and B = draw a second ace:

$$P(A) = 4/52 = \mathbf{1/13}$$

Use conditional probability for dependent events

What's the probability of drawing an ace from a deck of 52 cards? **1/13**
If we don't replace the drawn card, what's the probability of drawing a second ace?

Let **P(*B*|*A*)** be the **conditional probability** of B happening, given that event *A* has already occurred.

$$P(B|A) = 3/51 = 1/17$$

Intersection of Dependent Events

What's the probability of drawing two cards from a deck of 52 cards, and both cards are aces?

Let *A* = drawing an ace from the deck, and *B* = drawing a second ace:

$$P(A) = 4/52 = 1/13,\ P(B|A) = 3/51 = 1/17$$

Then **P(*A* and *B*) = P(*A*) * P(*B*|*A*)** = 1/221.

Bayes' Theorem

Learning Objectives

- Learn how **Bayes' Theorem** is derived from conditional probability
- Be able to use Bayes' Theorem to calculate probabilities of dependent events

Bayes' Theorem

Recall **P(*A* and *B*) = P(*A*) * P(*B*|*A*)**

By symmetry, P(*B* and *A*) = P(*B*) * P(*A*|*B*)

Since P(*A* and *B*) = P(*B* and *A*),

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$