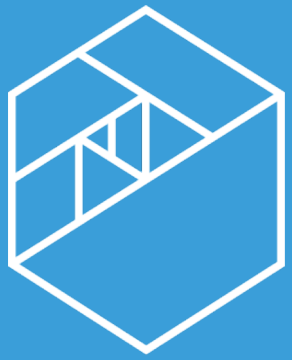

Lesson 3: Independence

Introduction

METIS

Lecture Overview:

Goals of the lecture:

1. Understand the concepts of independence and conditional independence

Independence

METIS

Independence

Definition:

When events are independent (e.g. the occurrence of one event does not influence the occurrence of another event), the probabilities can be multiplied.

$$\mathbb{P}(A,B) = \mathbb{P}(A) \times \mathbb{P}(B)$$

Example 1:

1. Probability of throwing a coin three times and getting exactly HTH

$$\mathbb{P}(HTH) = \mathbb{P}(H) \times \mathbb{P}(T) \times \mathbb{P}(H) = 1/2 \times 1/2 \times 1/2 = 1/8$$

Problem 1

Problem 1:

What is the probability of throwing a die 6 times and getting 1, 2, 3, 4, 5, and 6 (in that order)?

Problem 1

Solution 1:

$$\mathbb{P}(1,2,3,4,5,6) = \mathbb{P}(1) \times \mathbb{P}(2) \times \mathbb{P}(3) \times \mathbb{P}(4) \times \mathbb{P}(5) \times \mathbb{P}(6) = \frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6}\frac{1}{6} = \frac{1}{6^6} = \frac{1}{46656}$$

Problem 2

Problem 2:

The probability of someone blinking on a photograph is 0.1. A picture of five people is being taken.

1. What is the probability that no one is blinking?

2. What is the probability that at least one person is blinking?

3. What is the probability that all are blinking?

Problem 2

Solution 2:

For one individual:

$\mathbb{P}(blink) = 0.1$

$\mathbb{P}(no\_blink) = 1 - 0.1 = 0.9$

For group:

1) $\mathbb{P}(no\_blink) = 0.9^5 = 0.59$
2) $\mathbb{P}(1\ or\ more\ blink) = 1 - 0.59 = 0.41$
3) $\mathbb{P}(all\_blink) = 0.1^5 = 0.00001$

Conditional Probability

Definition:

Two events are said to be dependent when the probability of one event occurring influences the likelihood of the other event occurring.

$$\mathbb{P}(A|B) = \mathbb{P}(A \cap B) / \mathbb{P}(B)$$

Example 2:

1. If you draw two cards without replacement,

What is the probability that the second card is an Ace if the first card is an Ace?

$$\mathbb{P}(A_2|A_1) = 3/51$$

What is the probability that the second card is an Ace if the first card is not an Ace?

$$\mathbb{P}(A_2|\text{not } A_1) = 4/51$$

Problem 3

Problem 3:

A bag contains 5 red balls, 3 blue balls and 4 green balls. If we chose two balls without replacement, what is the probability that the first ball will be green and the second blue?

Problem 3

Solution 3:

$R = 5, B = 3, G = 4$

$\mathbb{P}(B \cap G) = \mathbb{P}(B|G)\,\mathbb{P}(G) = \frac{3}{11}\frac{4}{12} = \frac{1}{11}$

Problem 4

Problem 4:

I like dogs, they make me happy. Sometimes dogs greet me. What is the probability that I am happy?

$$\mathbb{P}(greet) = \mathbb{P}(G) = \frac{1}{4}$$

$$\mathbb{P}(happy|G) = \mathbb{P}(H|G) = \frac{9}{10}$$

$$\mathbb{P}(happy|NG) = \mathbb{P}(H|NG) = \frac{2}{10}$$

Problem 4

Solution 4:

$$\mathbb{P}(greet) = \mathbb{P}(G) = \frac{1}{4} \qquad \mathbb{P}(no\ greet) = \mathbb{P}(NG) = \frac{3}{4}$$

$$\mathbb{P}(happy|G) = \mathbb{P}(H|G) = \frac{9}{10} \qquad \mathbb{P}(not\ happy|G) = \mathbb{P}(NH|G) = \frac{1}{10}$$

$$\mathbb{P}(happy|NG) = \mathbb{P}(H|NG) = \frac{2}{10} \qquad \mathbb{P}(not\ happy|NG) = \mathbb{P}(NH|NG) = \frac{8}{10}$$

Problem 4

Solution 4:

$$\mathbb{P}(greet) = \mathbb{P}(G) = \frac{1}{4} \qquad \mathbb{P}(no\ greet) = \mathbb{P}(NG) = \frac{3}{4}$$

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Problem 4

Solution 4:

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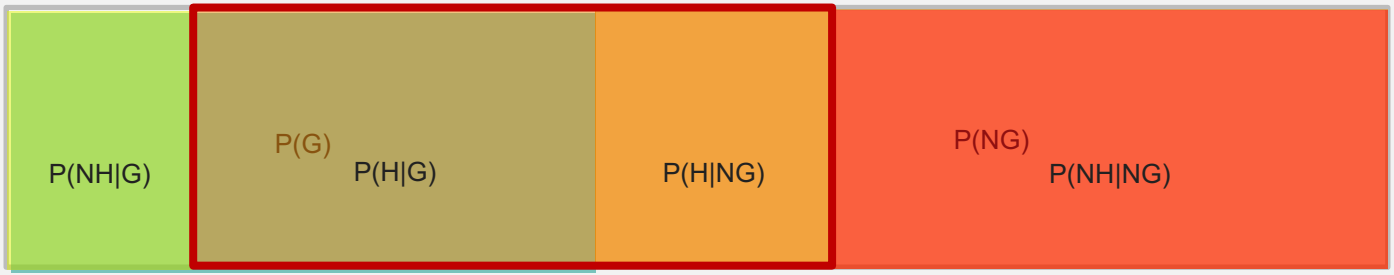
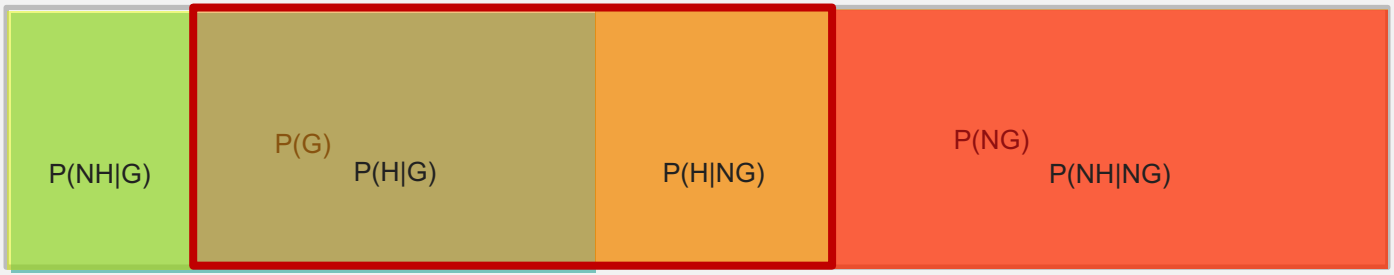


Problem 4

Solution 4:

$$\mathbb{P}(H) = \mathbb{P}(H|G)\mathbb{P}(G) + \mathbb{P}(H|NG)\mathbb{P}(NG) = \frac{9}{10}\frac{1}{4} + \frac{2}{10}\frac{3}{4} = \frac{15}{40} = \frac{3}{8} = 0.375$$



QUESTIONS?