Lesson 7:

Partial Derivatives and Gradients

METIS

Introduction

METIS

Lecture Overview:

Goals of the lecture:

1. Understand partial derivatives and gradients

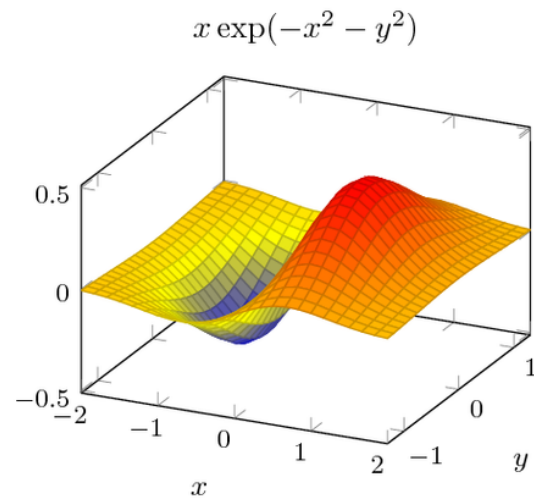# Partial Derivatives and Gradients

METIS

Partial Derivatives

$$f(x, y) = x^2 - xy$$

$$\frac{\partial}{\partial x} f(x, y) = 2x - y$$

$$\frac{\partial}{\partial y} f(x, y) = -x$$
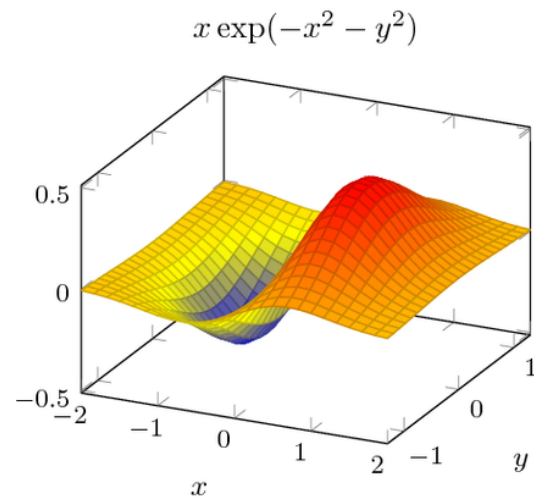

$x \exp(-x^2 - y^2)$

Gradient

$$f(x, y) = x^2 - xy$$

$$\nabla f(x, y) = \begin{bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{bmatrix}$$

$$\nabla f(x, y) = \begin{bmatrix} 2x - y \\ -x \end{bmatrix}$$

$x \exp(-x^2 - y^2)$

Summary

Operator	Symbol	Example
Derivative	$\dfrac{d}{dx}$	$\dfrac{d}{dx}x^3 = 3x^2$
Partial Derivative	$\dfrac{\partial}{\partial x}$	$\dfrac{\partial}{\partial x}x^3 y = 3x^2 y$
Gradient	∇	$\nabla x^3 y = \begin{bmatrix} 3x^2 y \\ x^3 \end{bmatrix}$

Problem 1:

$$f(x_1, x_2) = x_1 \ln(x_2) + \sin(x_1)$$

Problem 1:

$$f(x_1, x_2) = x_1 \ln(x_2) + \sin(x_1)$$

$$\frac{\partial f}{\partial x_1} = 1 \cdot \ln(x_2) + \cos(x_1) \qquad \frac{\partial f}{\partial x_2} = \frac{x_1}{x_2}$$

$$\nabla f = \begin{bmatrix} \ln(x_2) + \cos(x_1) \\ \\ x_1 / x_2 \end{bmatrix}$$

QUESTIONS?