Intro to Calculus

What is calculus?

Calculus is the study of change

Calculus revolves around derivatives and integrals

Calculus is used throughout data science

Used to train machine learning models:

- Gradient Descent
- Maximize accuracy
- Minimize error

Linear Functions and Slope

Learning Objectives

- Describe the form of a **linear function.**
- Recognize what the **slope** and **intercept** of a linear function are and how to calculate them.

What is a function?

$$\mathbf{y} = \boldsymbol{f}(\mathbf{x})$$

x: Input → **y**: Output

What is a linear function?

$$\mathbf{y = mx + b}$$

Given multiple <**x**, **y**> combinations, solve for **m** and **b**.

Linear Functions in Two Dimensions

Let <$\mathbf{x_1}$, $\mathbf{y_1}$> = <1, 3>, <$\mathbf{x_2}$, $\mathbf{y_2}$> = <2, 5>:

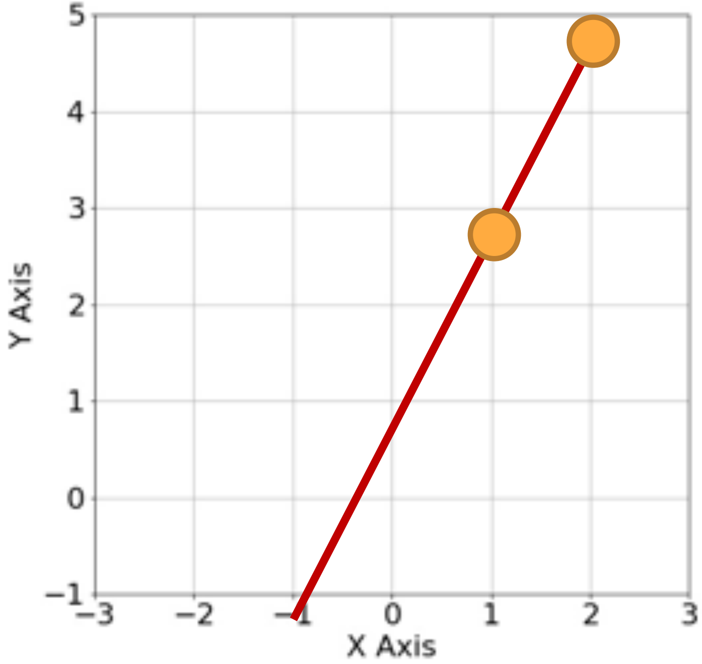
$\mathbf{y_1} = \mathbf{mx_1} + \mathbf{b}$, $\mathbf{y_2} = \mathbf{mx_2} + \mathbf{b}$

$\mathbf{y_1} - \mathbf{y_2} = \mathbf{m}(\mathbf{x_1} - \mathbf{x_2})$

$\mathbf{m} = (5 - 3) / (2 - 1) = 2$

$\mathbf{b} = \mathbf{y_1} - \mathbf{mx_1} = \mathbf{y_2} - \mathbf{mx_2} = 1$

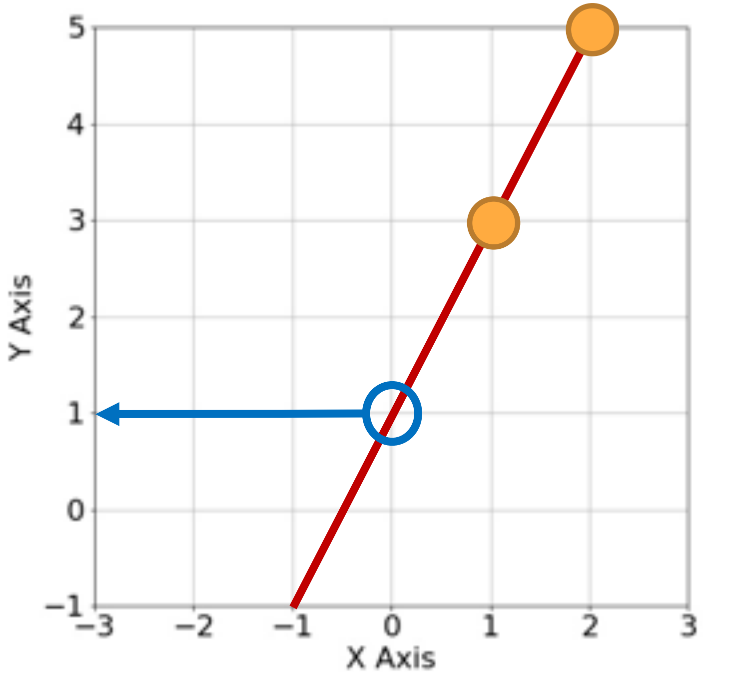
<m, b> = <2, 1>



What is **b**?

The **y-intercept**:
the point where the line crosses the y axis

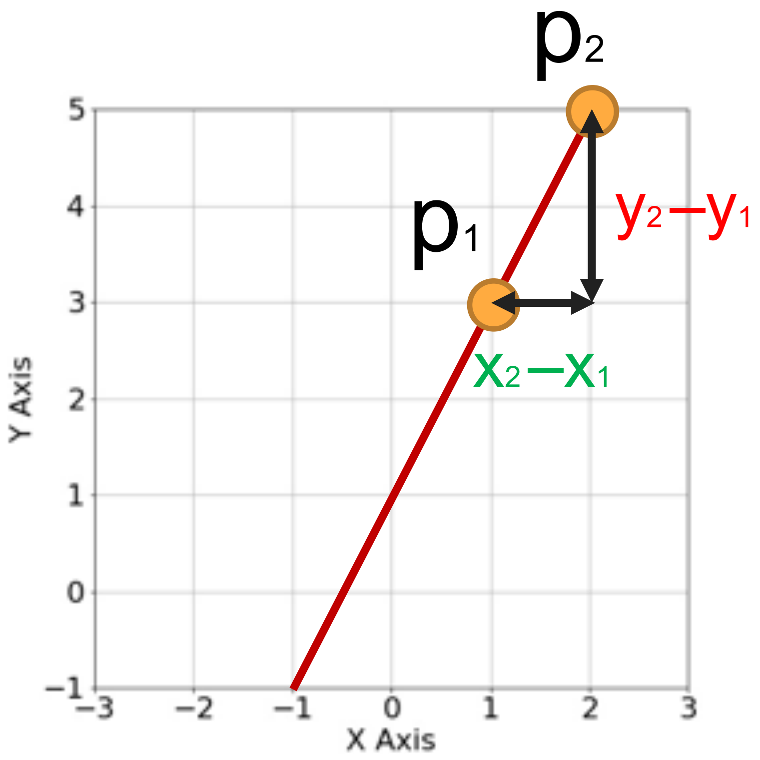
$$y = mx + b$$
$$y = 2x + 1$$



What is **m**?

The **slope,** or **derivative**:
How much **y** changes as **x** changes

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$



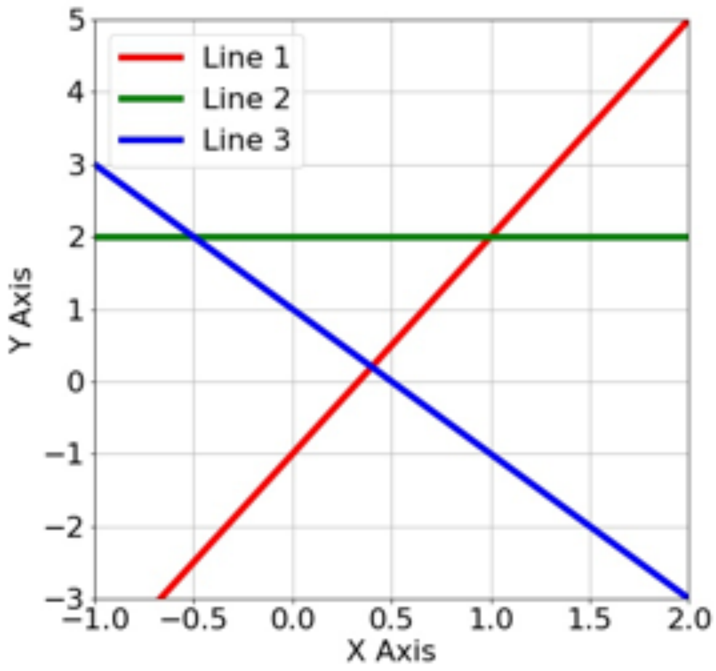
Exercise - Line Equations

Problem 1:

Calculate the line equation for the following lines. Helper equations:

$$y = mx + b$$

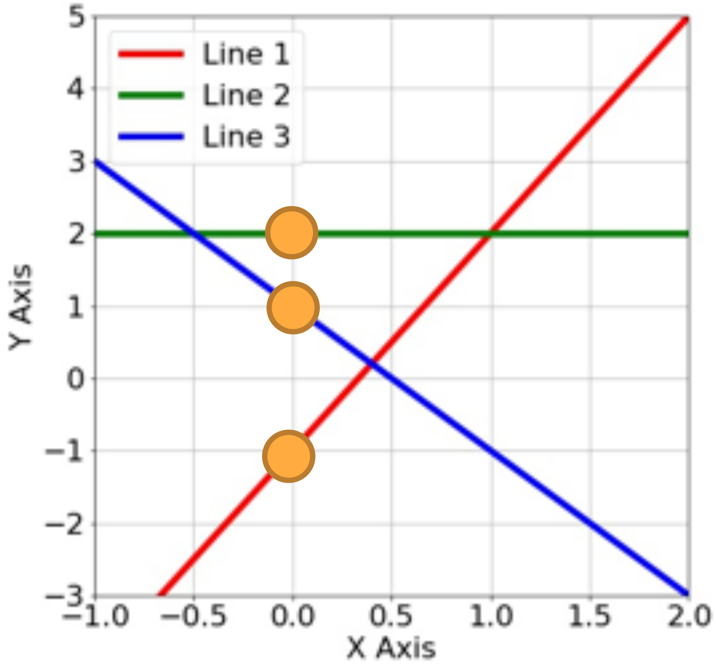
$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$



Exercise - Line Equations

Let's first extract the intercept:

$$y = mx + b = mx - 1$$
$$y = mx + b = mx + 2$$
$$y = mx + b = mx + 1$$



Exercise - Line Equations

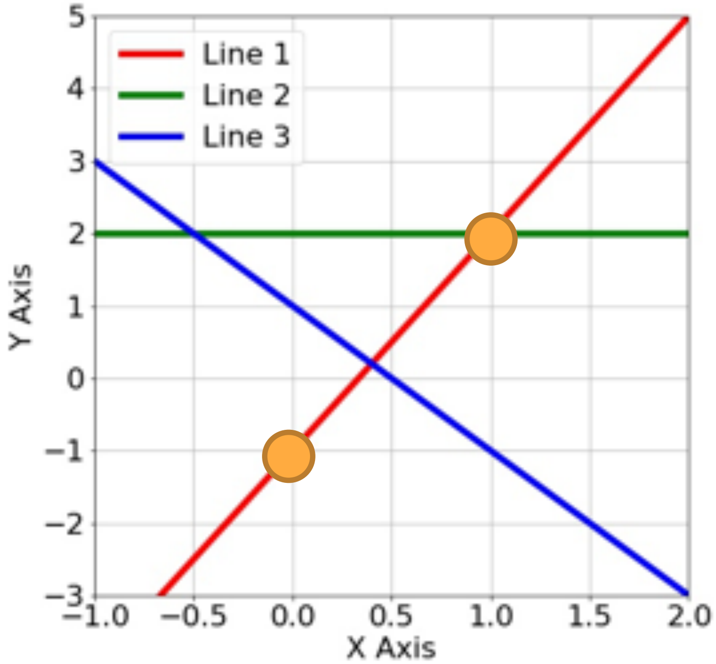
Let’s extract the slope:

$$y = mx + b = 3x - 1$$

$$y = mx + b = mx + 2$$

$$y = mx + b = mx + 1$$

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(2 - (-1))}{(1 - 0)} = 3$$



Exercise - Line Equations

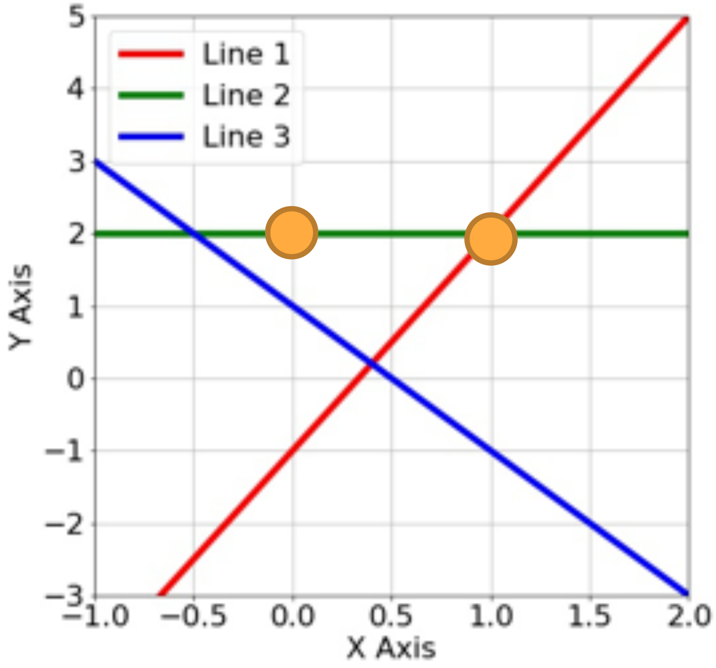
Let's extract the slope:

$$y = mx + b = 3x - 1$$

$$y = mx + b = 2$$

$$y = mx + b = mx + 1$$

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(2 - 2)}{(1 - 0)} = 0$$



Exercise - Line Equations

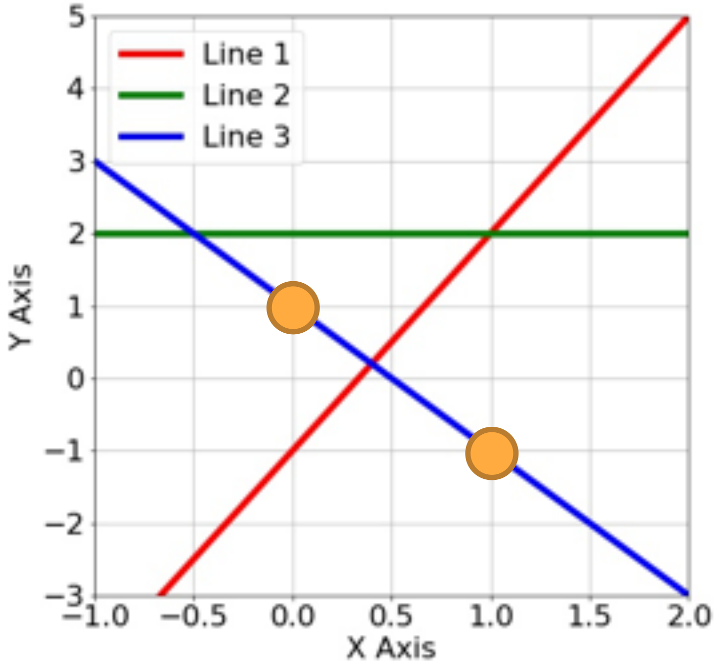
Let's extract the slope:

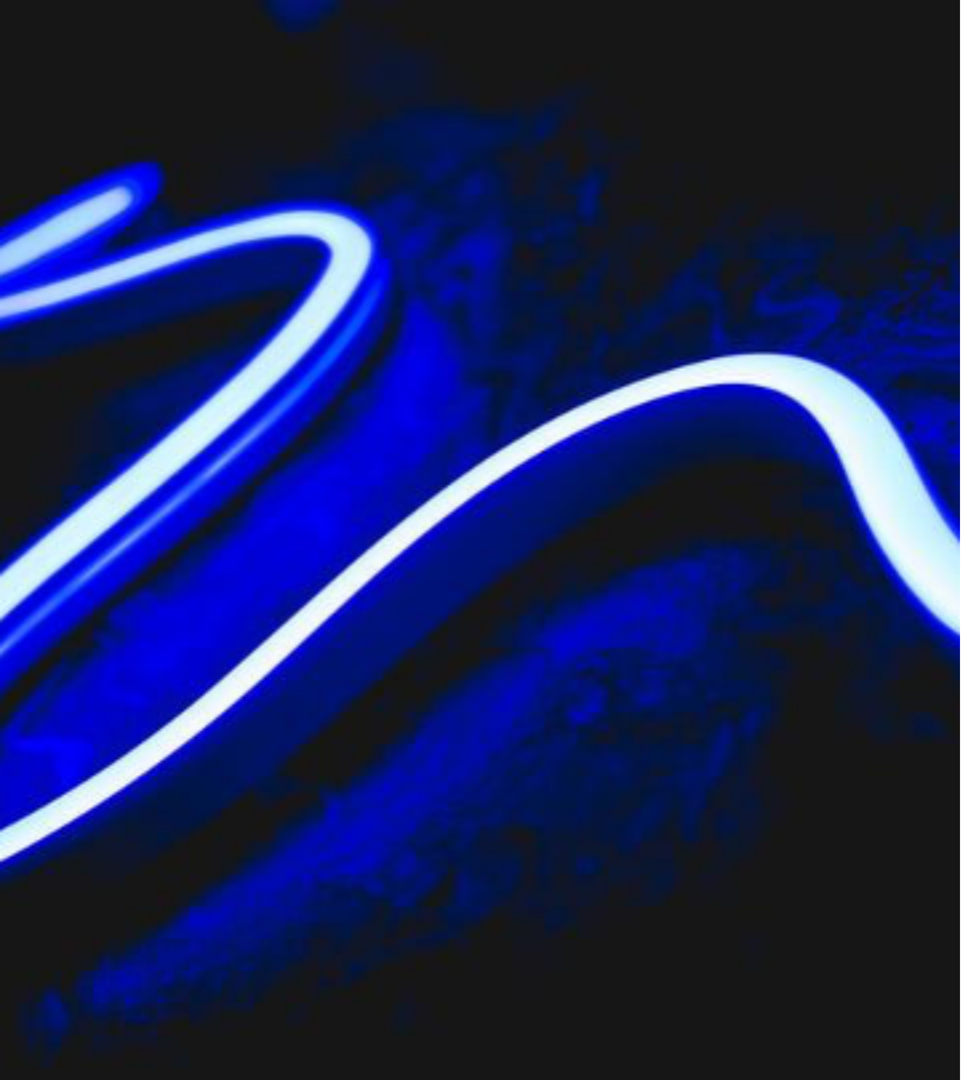
$$y = mx + b = 3x - 1$$

$$y = mx + b = 2$$

$$y = mx + b = -2x + 1$$

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(-1 - 1)}{(1 - 0)} = -2$$





Derivatives

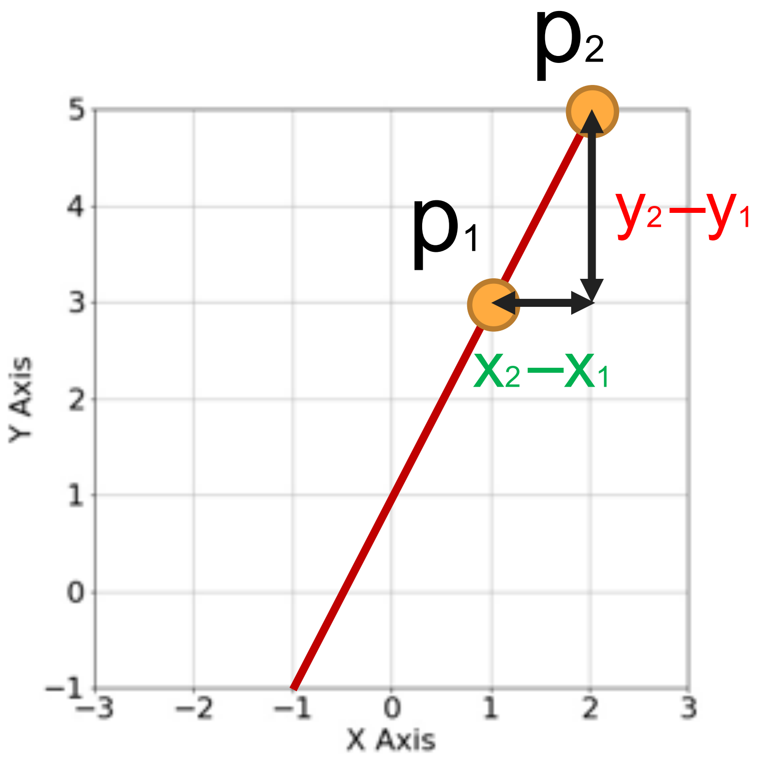
Learning Objectives

- Recognize that the **derivative** is a non-constant slope.
- Calculate the first derivative of a function.

Slope of a Line

How much **y** changes as **x** changes

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

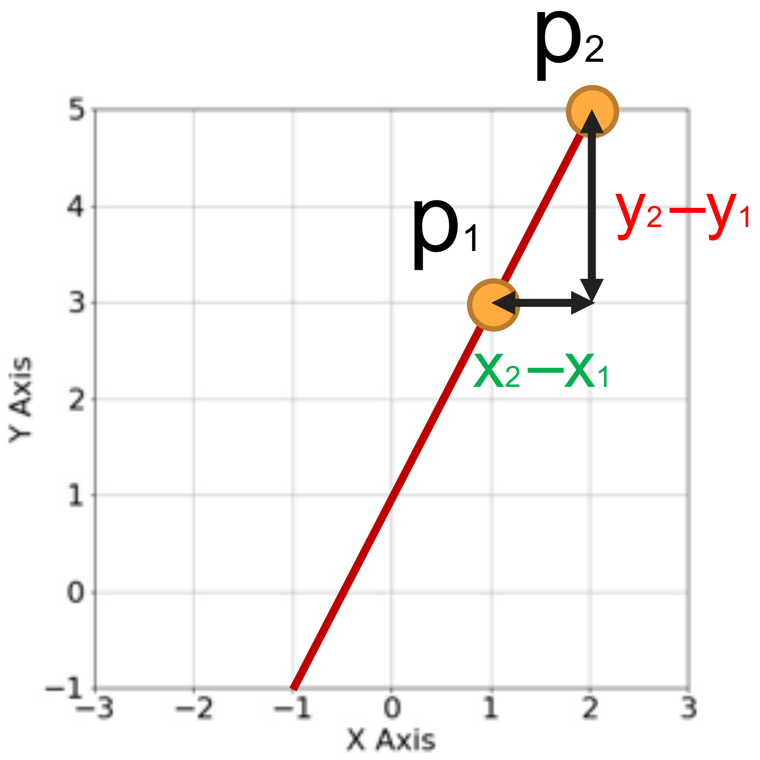


Derivative of a Function

How much **y** changes as **x** changes

$$f'(x) = \frac{d}{dx} f(x)$$

Slope = Derivative



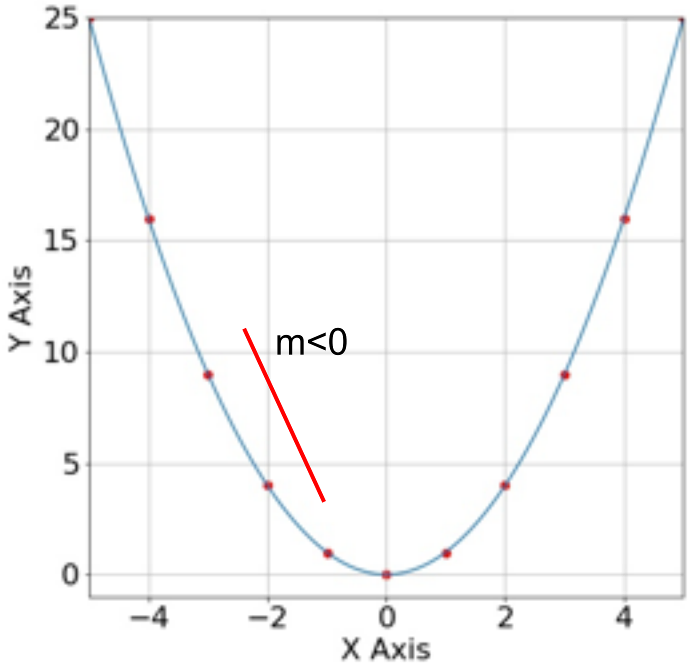
Derivative of x^2

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$m = \frac{(f(x_2) - f(x_1))}{(x_2 - x_1)}$$

$$x_2 = x_1 + h$$

$$m = \frac{(f(x_1 + h) - f(x_1))}{h}$$



Derivative of x^2

$$m = \frac{(f(x_1 + h) - f(x_1))}{h} \qquad f(x) = x^2$$

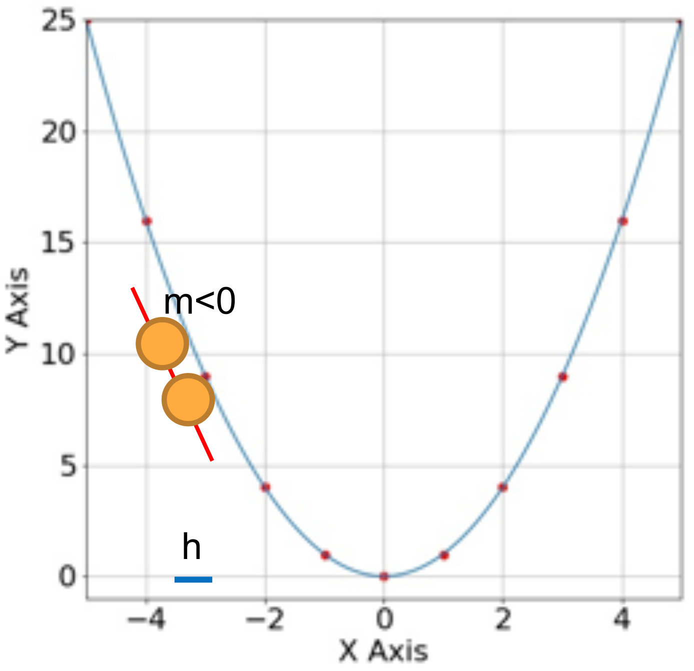
$m = ((x_1 + h)^2 - x^2) / h$

$m = (x_1^2 + 2hx_1 + h^2 - x_1^2) / h$

$m = (2hx_1 + h^2) / h = 2x_1 + h$

$$f'(x_1) = lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}$$

$f'(x_1) = 2x_1 + h \rightarrow \mathbf{2x_1}$



Common Derivatives

Polynomials

$$\frac{d}{dx}(ax^n) = a \cdot nx^{n-1}$$

Radicals

$$\frac{d}{dx}\sqrt[m]{x^n} = \frac{d}{dx}\left(x^{\frac{n}{m}}\right) = \frac{n}{m}x^{\frac{n}{m}-1}$$

Common Derivatives

Exponentials

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = \ln(a) \cdot a^x$$

Logarithms

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\log_b(x) = \frac{1}{\ln(b)x}$$

Rules for Derivatives

Definition:

Addition: $\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$

Multiplication: $\left(f(x) \cdot g(x)\right)' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Composition: $\left(f(g(x))\right)' = f'\left(g(x)\right) \cdot g'(x)$