Intro to Calculus

Finding Extrema

Learning Objectives

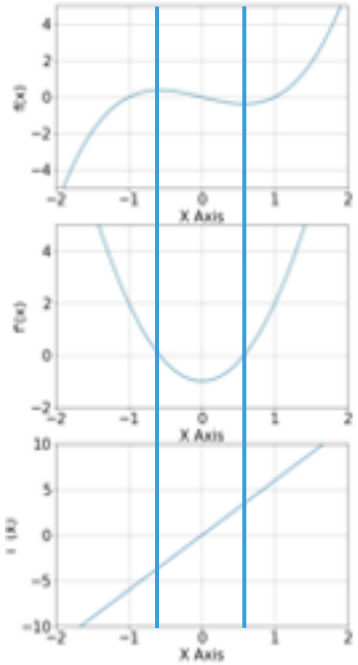
- Compute the second derivative of a function and describe how it relates to the first derivative.
- Find function **maxima** and **minima** using the first and second derivatives.

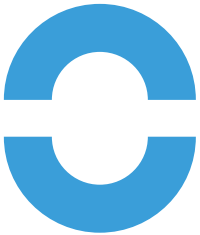
Finding Minimum and Maximum

$$f(x) = x^3 - x$$

$$f'(x) = 3x^2 - 1$$

$$f''(x) = 6x$$





Concave shapes f ’’(x) < 0

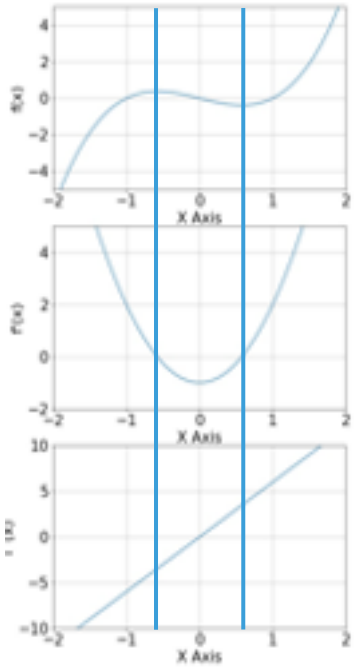
Convex shapes f ’’(x) > 0

Finding Minimum and Maximum

$$f(x) = x^3 - x$$

$$f'(x) = 3x^2 - 1$$

$$f''(x) = 6x$$



Definition:

Maximum: f ’(x) = 0 and f ’’(x) < 0

Minimum: f ’(x) = 0 and f ’’(x) > 0

Inflection Point: f’’(x) = 0

Exercise

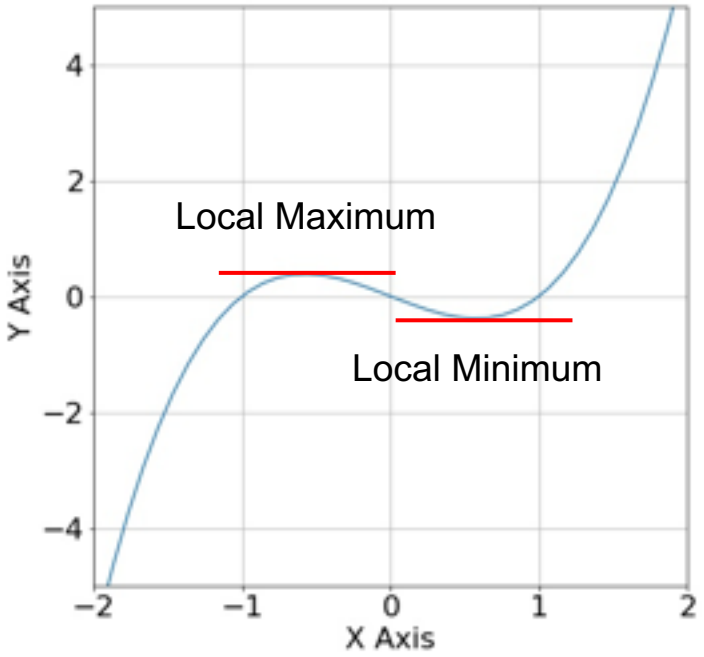
$$f(x) = x^3 - x$$

For which values of x do we have a maximum and/or minimum?

$$f'(x) = 3x^2 - 1 = 0$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3} \qquad x = \pm\sqrt{\frac{1}{3}} = \pm 0.54$$



Partial Derivatives and Gradients

Learning Objectives

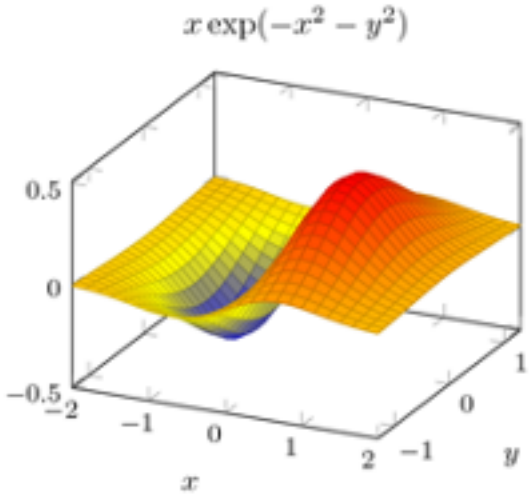
- Be able to compute **partial derivatives** and **gradients** for multidimensional functions.

Partial Derivatives

$$f(x,y) = x^2 - xy$$

$$\frac{\partial}{\partial x} f(x,y) = 2x - y$$

$$\frac{\partial}{\partial y} f(x,y) = -x$$

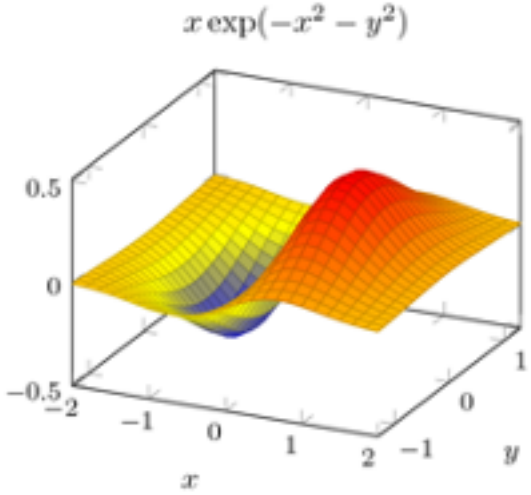


Gradient

$$f(x,y) = x^2 - xy$$

$$\nabla f(x,y) = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix}$$

$$\nabla f(x,y) = \begin{bmatrix} 2x - y \\ -x \end{bmatrix}$$



Summary of Derivatives

Operator	Symbol	Example
Derivative	$\frac{d}{dx}$	$\frac{d}{dx}x^3 = 3x^2$
Partial Derivative	$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial x}x^3y = 3x^2y$
Gradient	∇	$\nabla x^3y = \begin{bmatrix} 3x^2y \\ x^3 \end{bmatrix}$