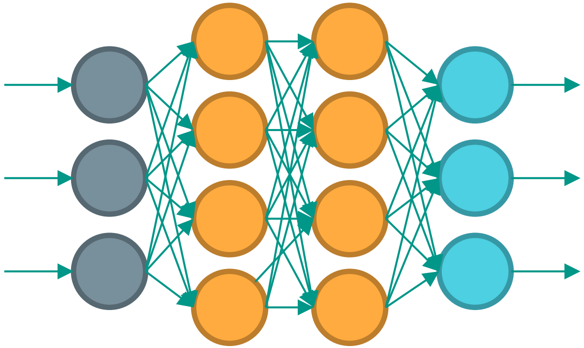
Introduction to Neural Nets

METIS

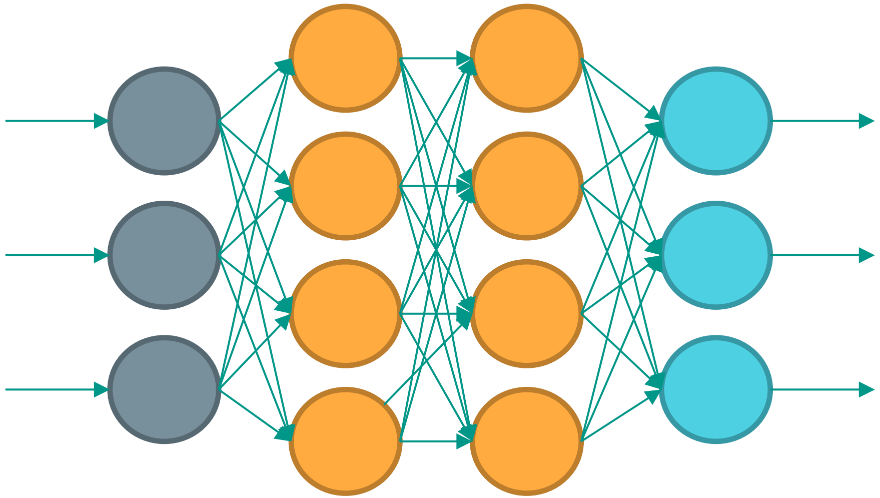
Motivation for Neural Nets

- Use biology as inspiration for mathematical model
- Get signals from previous neurons
- Generate signals (or not) according to inputs
- Pass signals on to next neurons
- By layering many neurons, can create complex model



Neural Net Structure

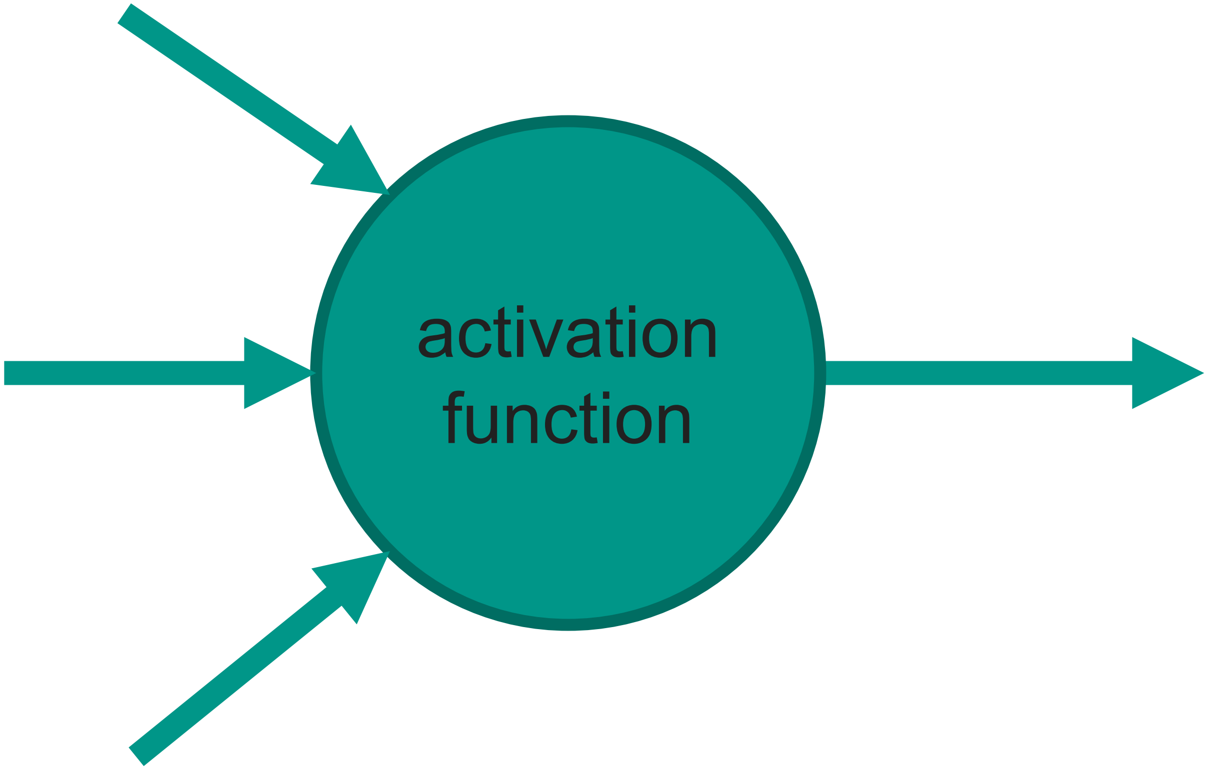
Input
(Feature Vector)



Output
(Label)

- Can think of it as a complicated computation engine
- We will "train it" using our training data
- Then (hopefully) it will give good answers on new data

Basic Neuron Visualization



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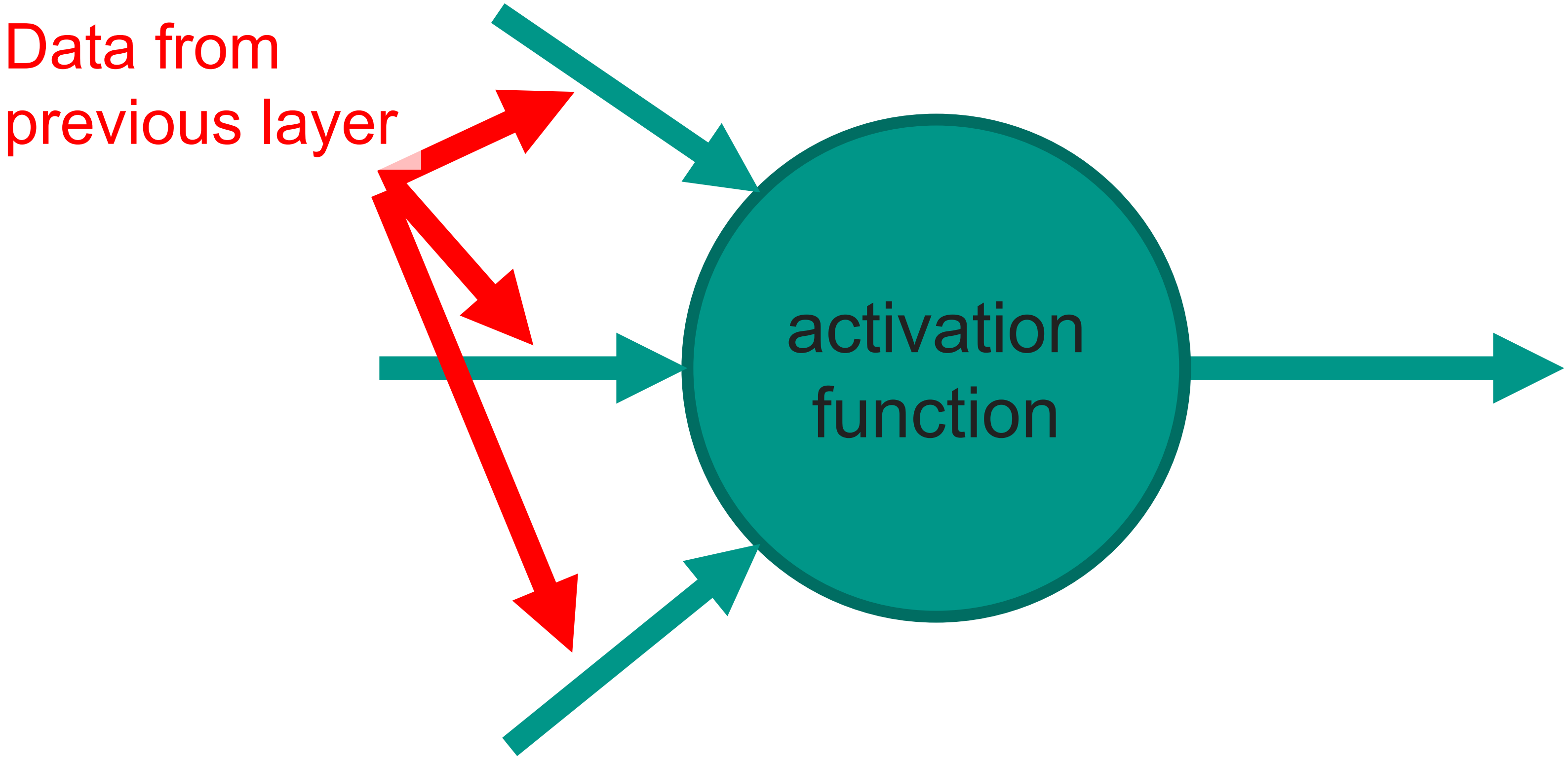

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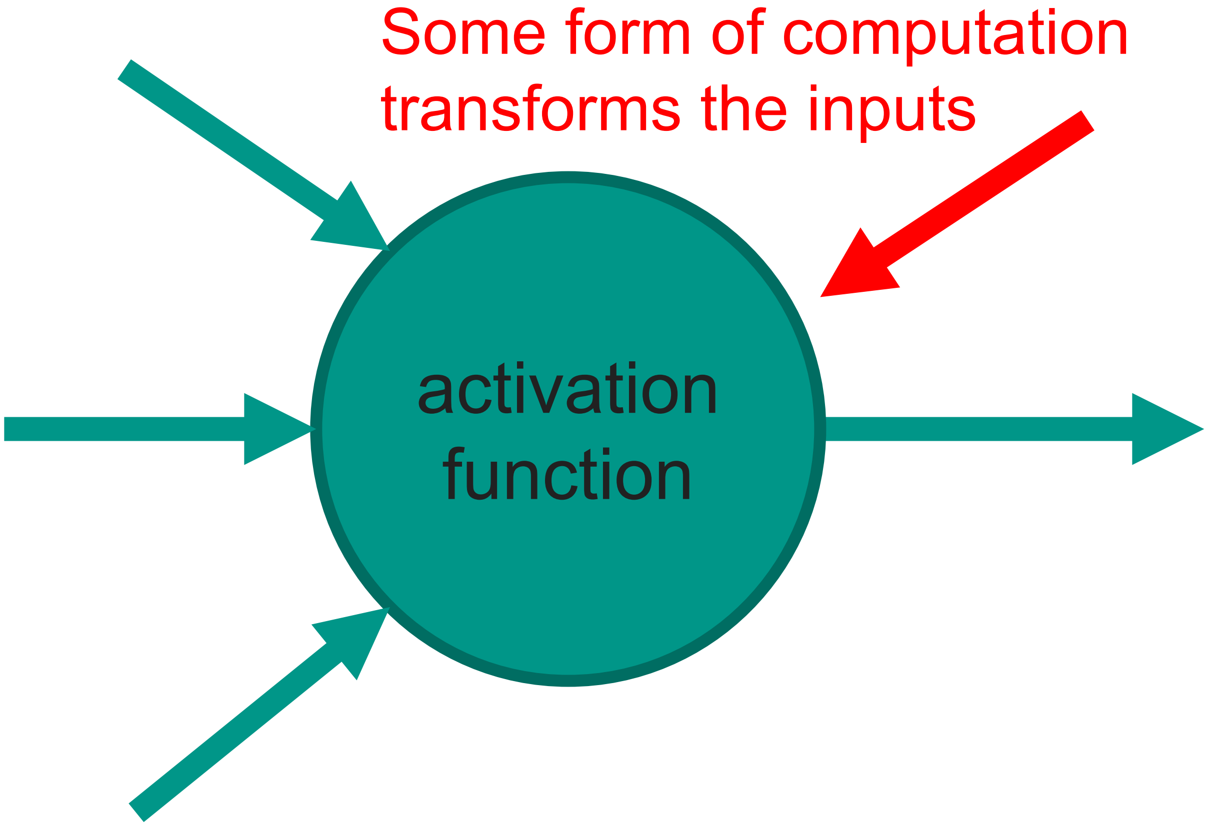
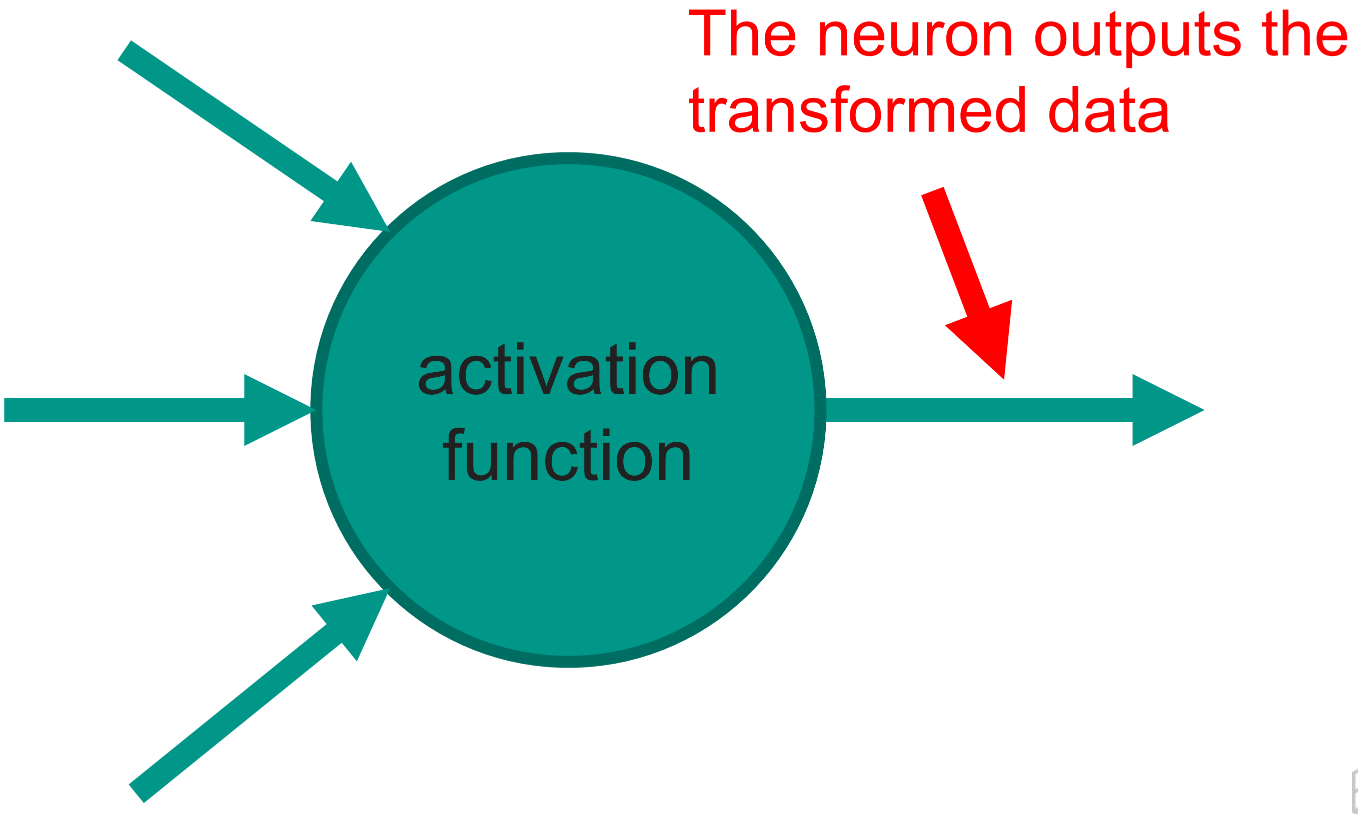
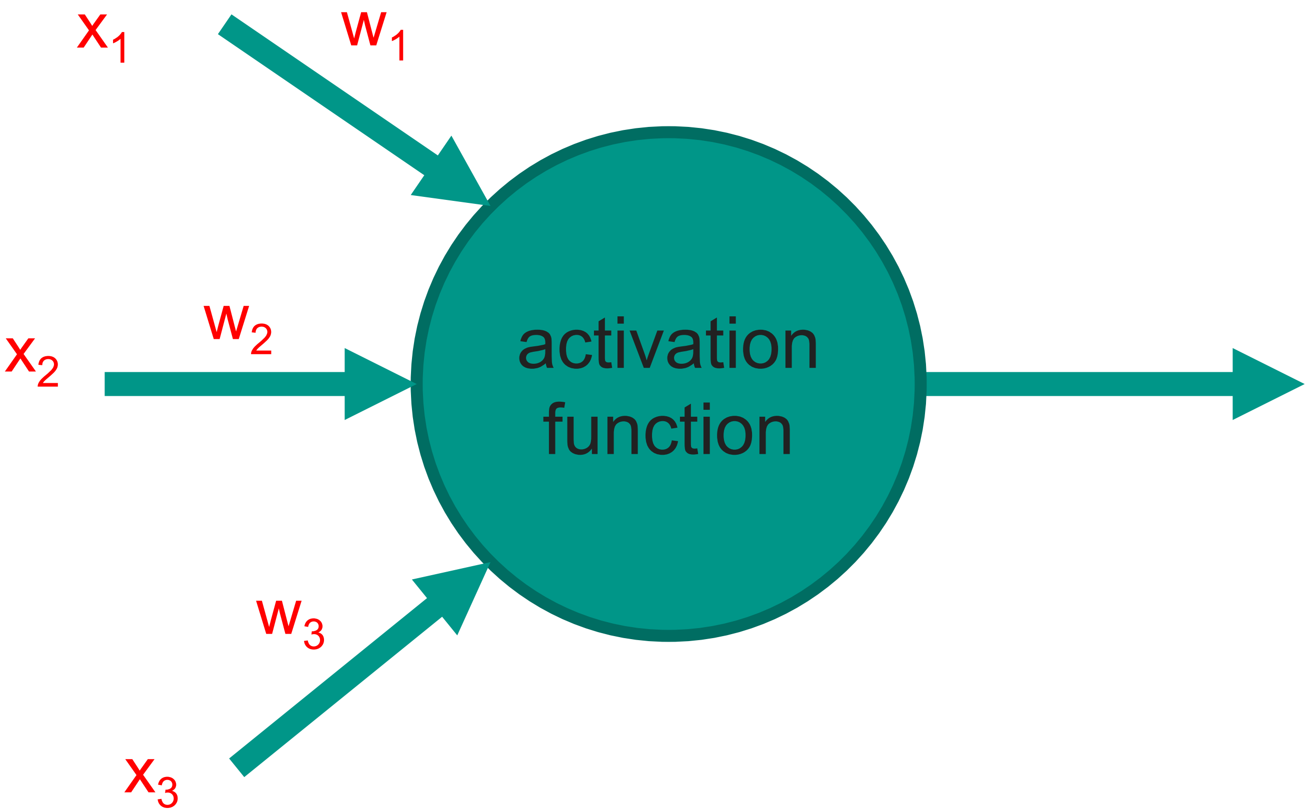


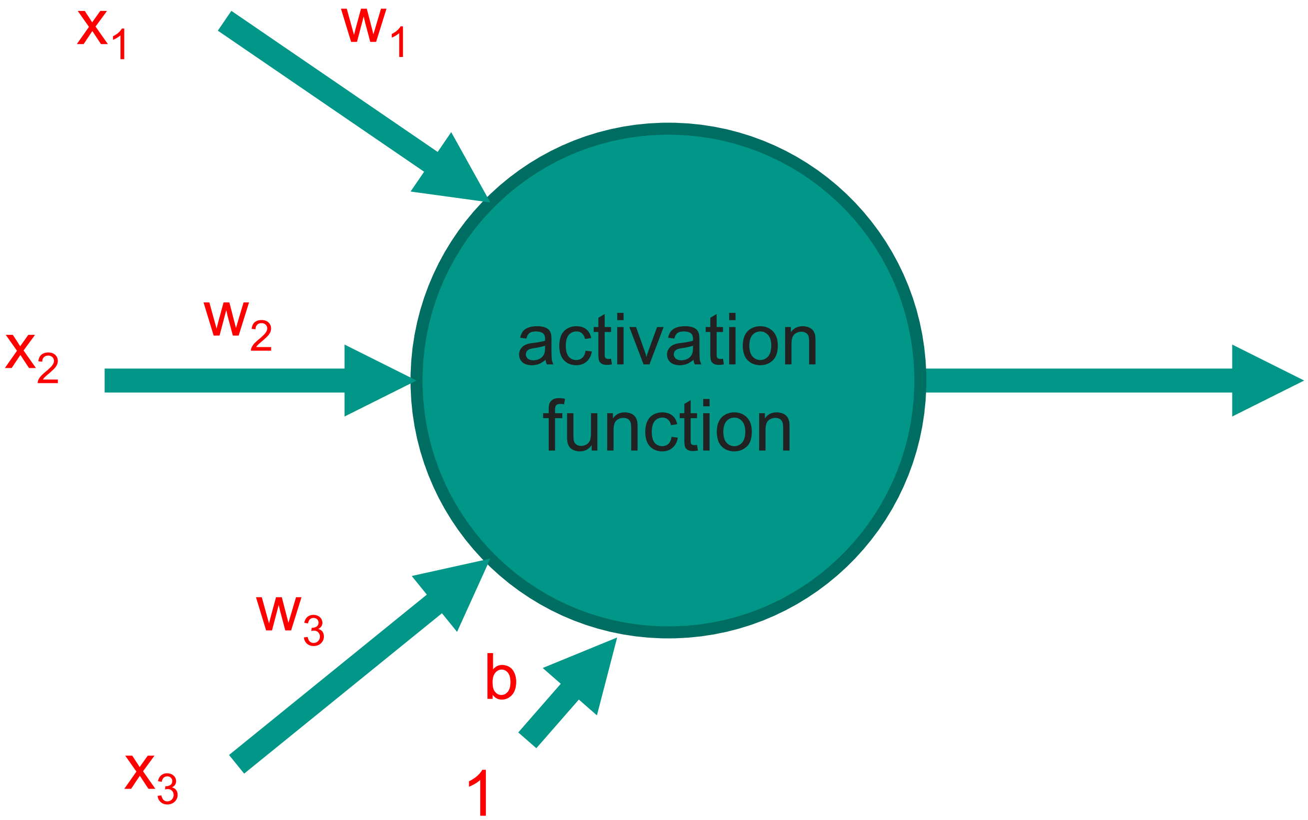
Basic Neuron Visualization



Basic Neuron Visualization



Basic Neuron Visualization



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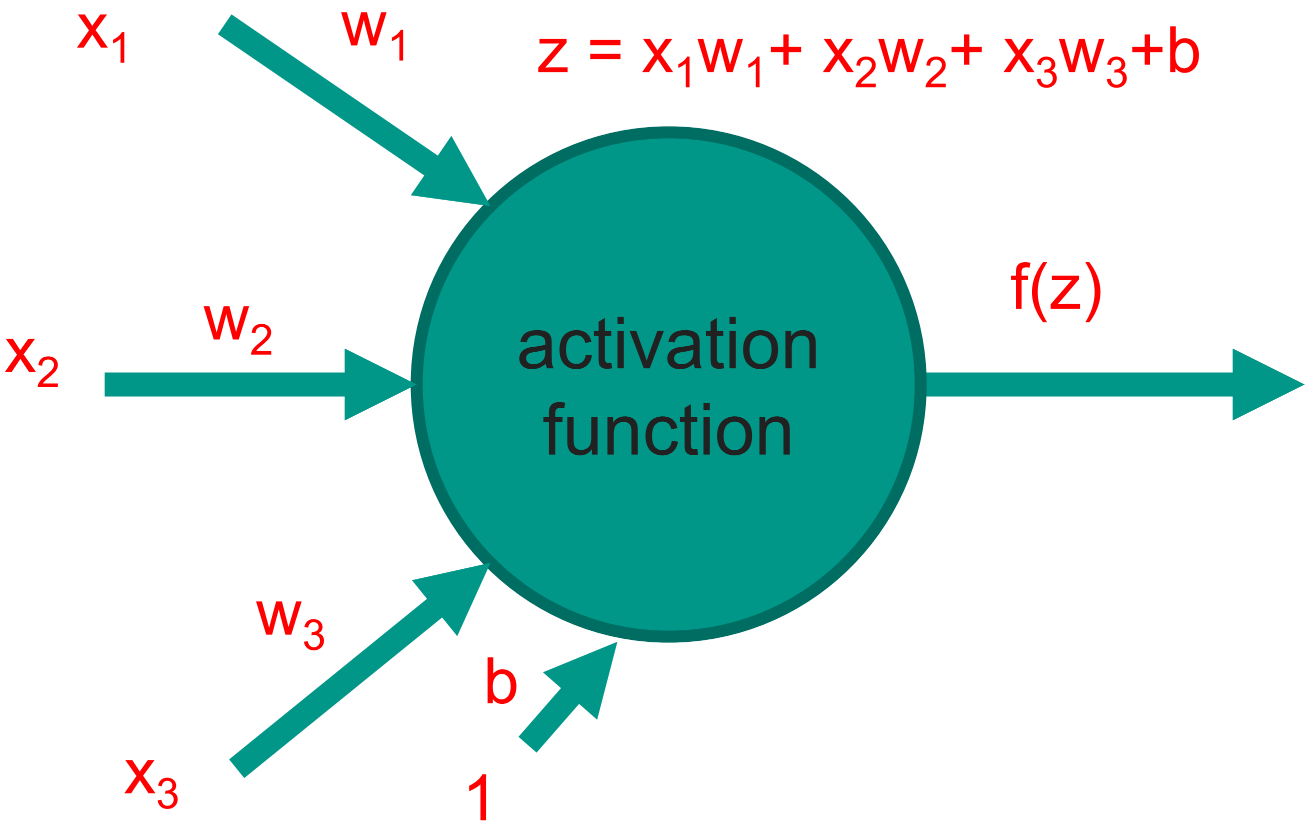


In Vector Notation

z = “net input”

b = “bias term”

f = activation function

a = output to next layer

$$z = b + \sum_{i=1}^{m} x_i w_i$$

$$z = b + x^T w$$

$$a = f(z)$$

Relation to Logistic Regression

When we choose: $f(z) = \frac{1}{1+e^{-z}}$

$$z = b + \sum_{i=1}^{m} x_i w_i = x_1 w_1 + x_2 w_2 + \cdots + x_m w_m + b$$

Then a neuron is simply a "unit" of logistic regression!

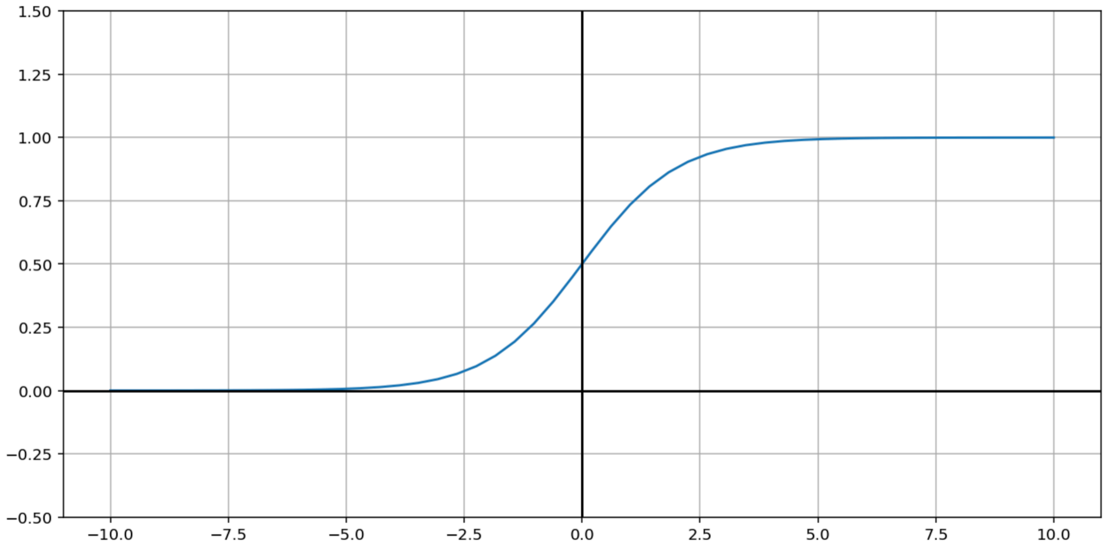
weights ⇔ coefficients

inputs ⇔ variables

bias term ⇔ constant term

Relation to Logistic Regression

This is called the "sigmoid" function: $\sigma(z) = \frac{1}{1+e^{-z}}$



Nice Property of Sigmoid Function

Quotient rule

$$\frac{d}{dx} \cdot \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma'(z) = \frac{0-(-e^{-z})}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})^2}$$

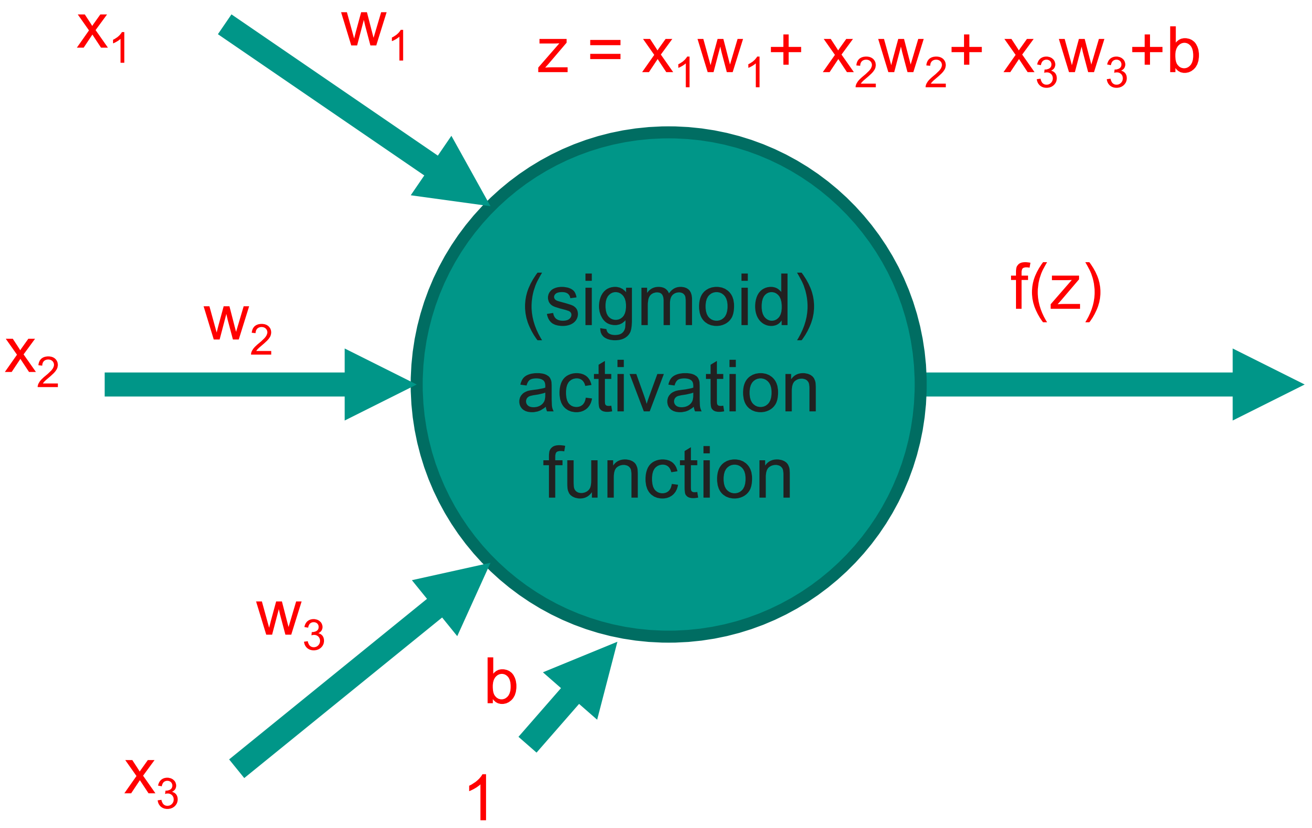
$$= \frac{1+e^{-z}-1}{(1+e^{-z})^2} = \cancel{\frac{1+e^{-z}}{(1+e^{-z})^2}} - \frac{1}{(1+e^{-z})^2}$$

$$= \frac{1}{1+e^{-z}} - \frac{1}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$$

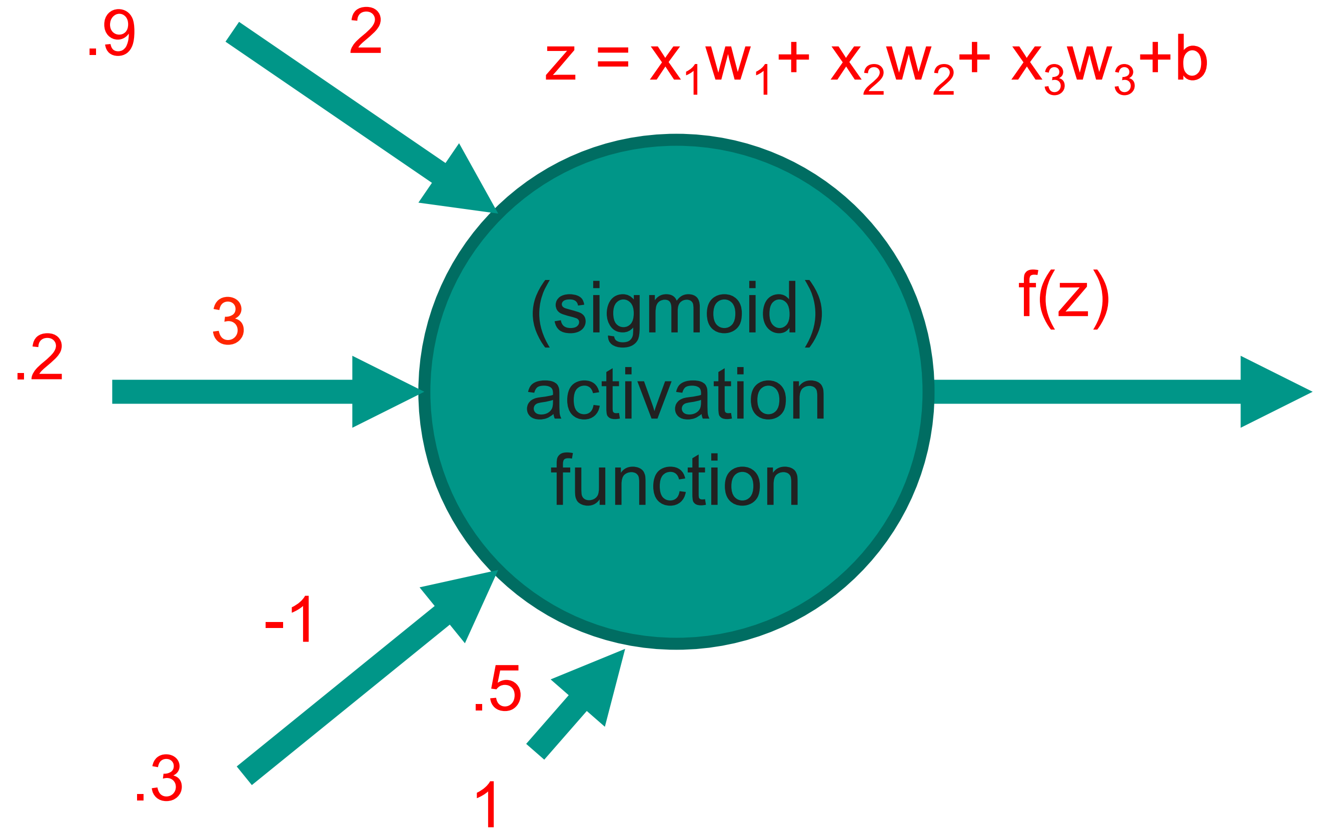
$$\sigma'(z) = \sigma(z)(1-\sigma(z))$$

This will be helpful!

Example Neuron Computation

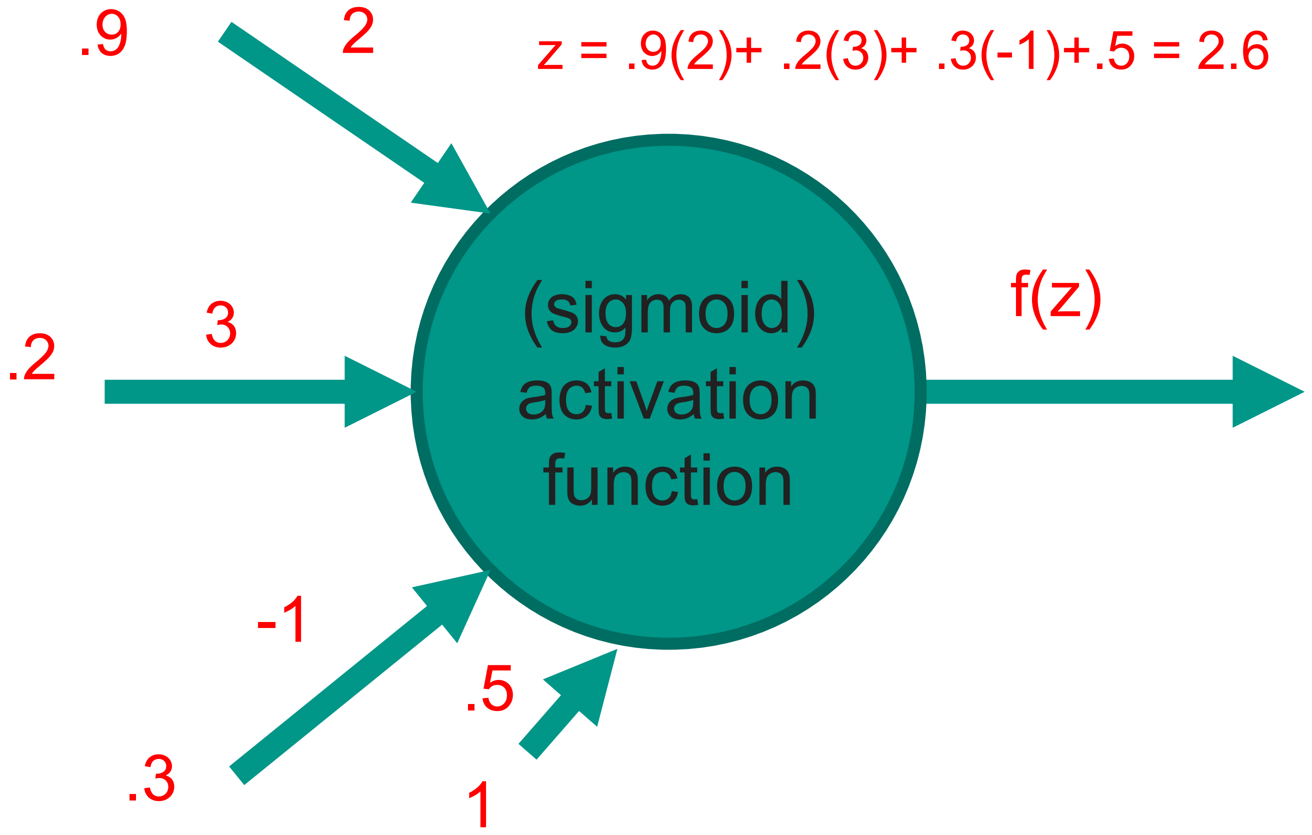


Example Neuron Computation

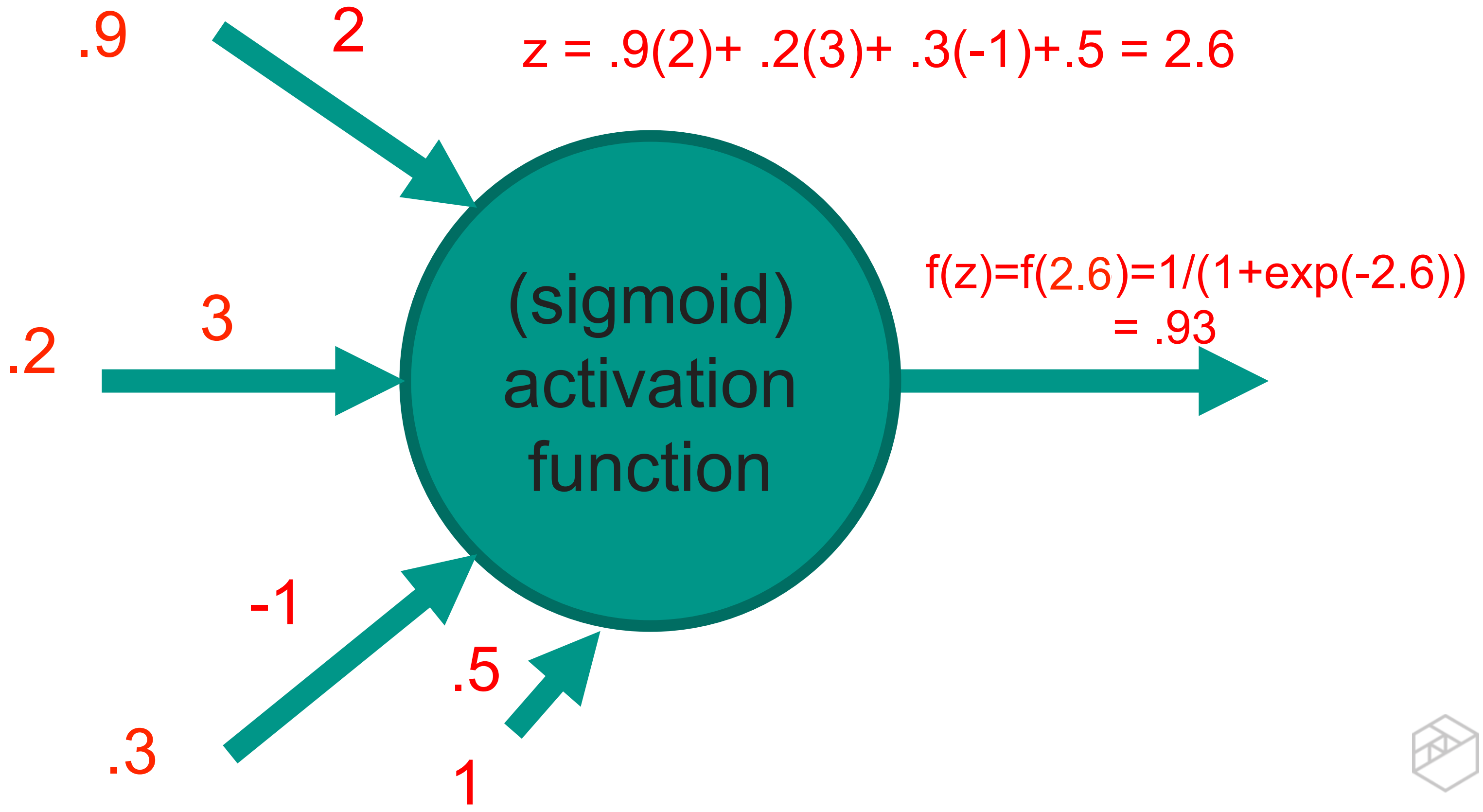


Example Neuron Computation

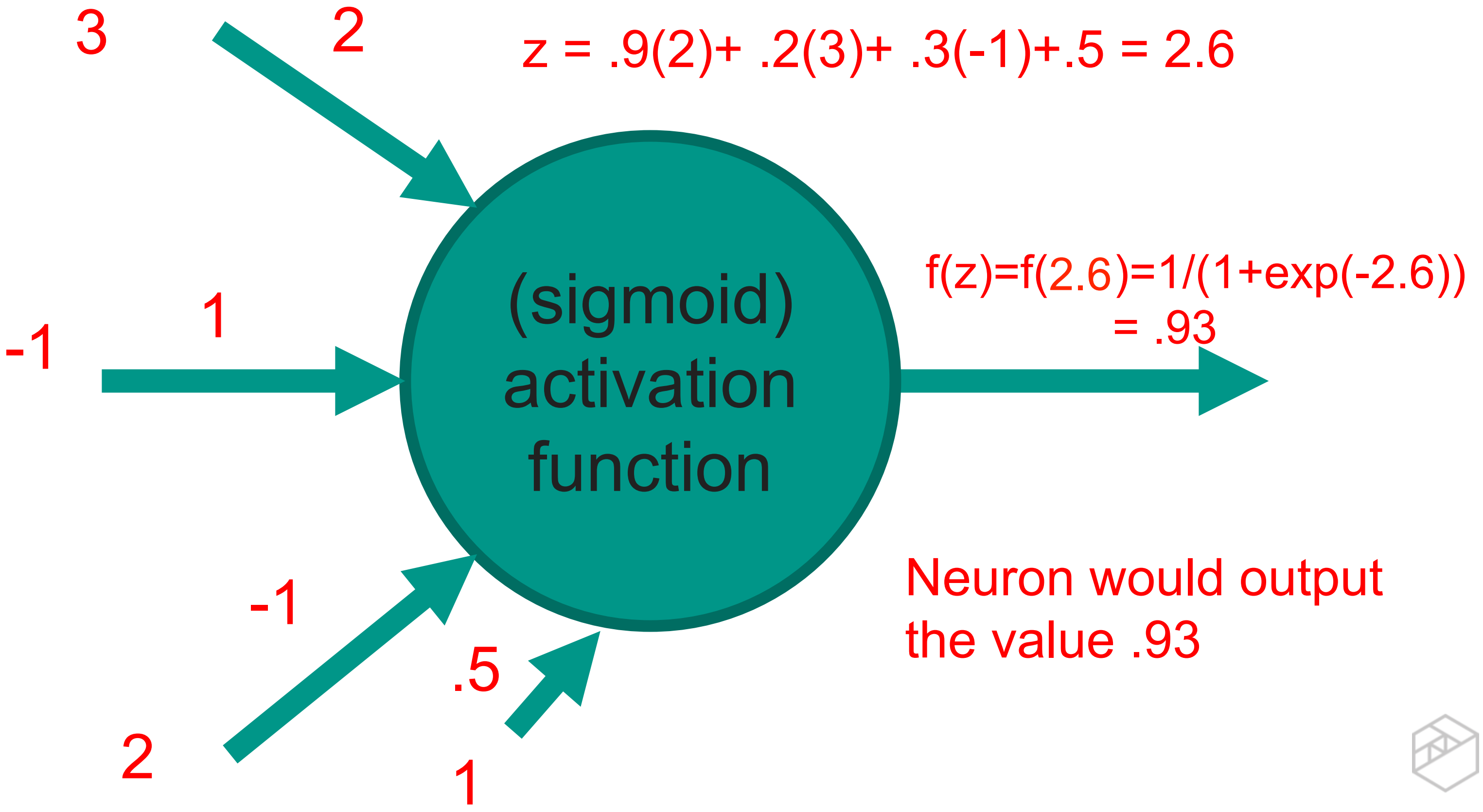
$z = .9(2) + .2(3) + .3(-1) + .5 = 2.6$



Example Neuron Computation

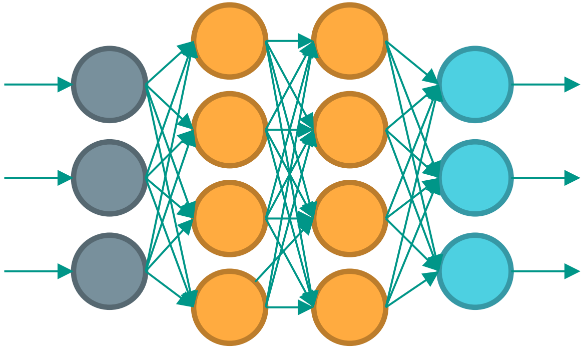


Example Neuron Computation



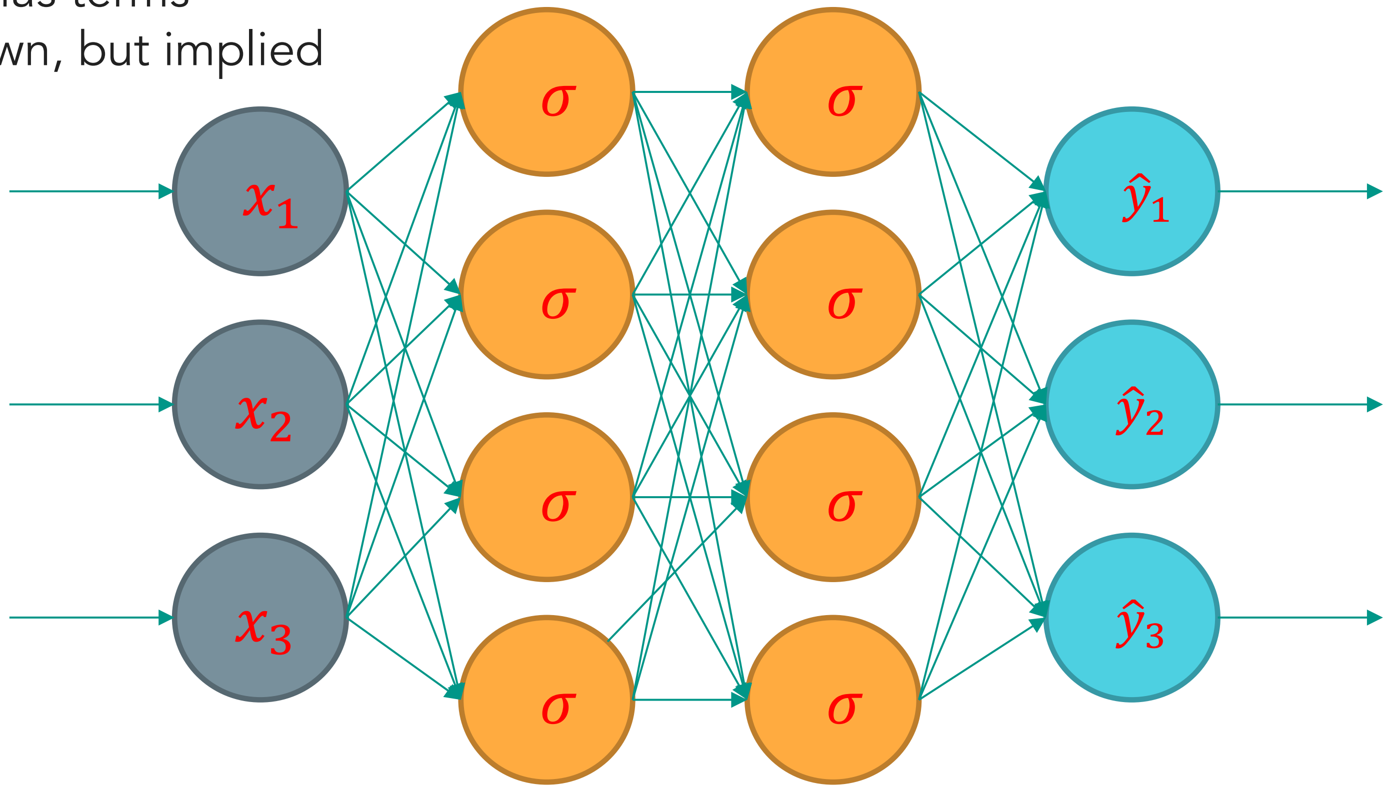
Why Neural Nets?

- Why not just use a single neuron? Why do we need a larger network?
- A single neuron (like logistic regression) only permits a linear decision boundary.
- Most real-world problems are considerably more complicated!

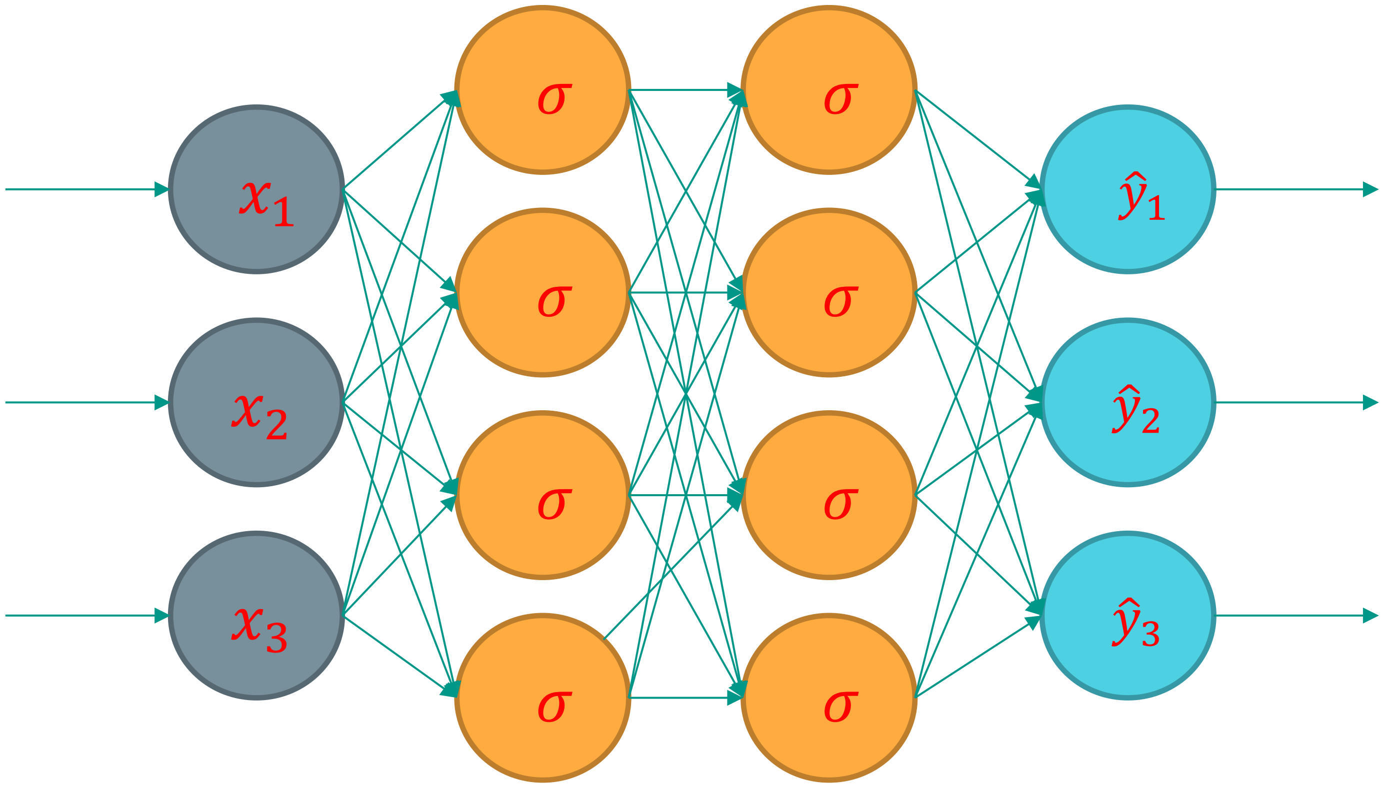


Feedforward Neural Network

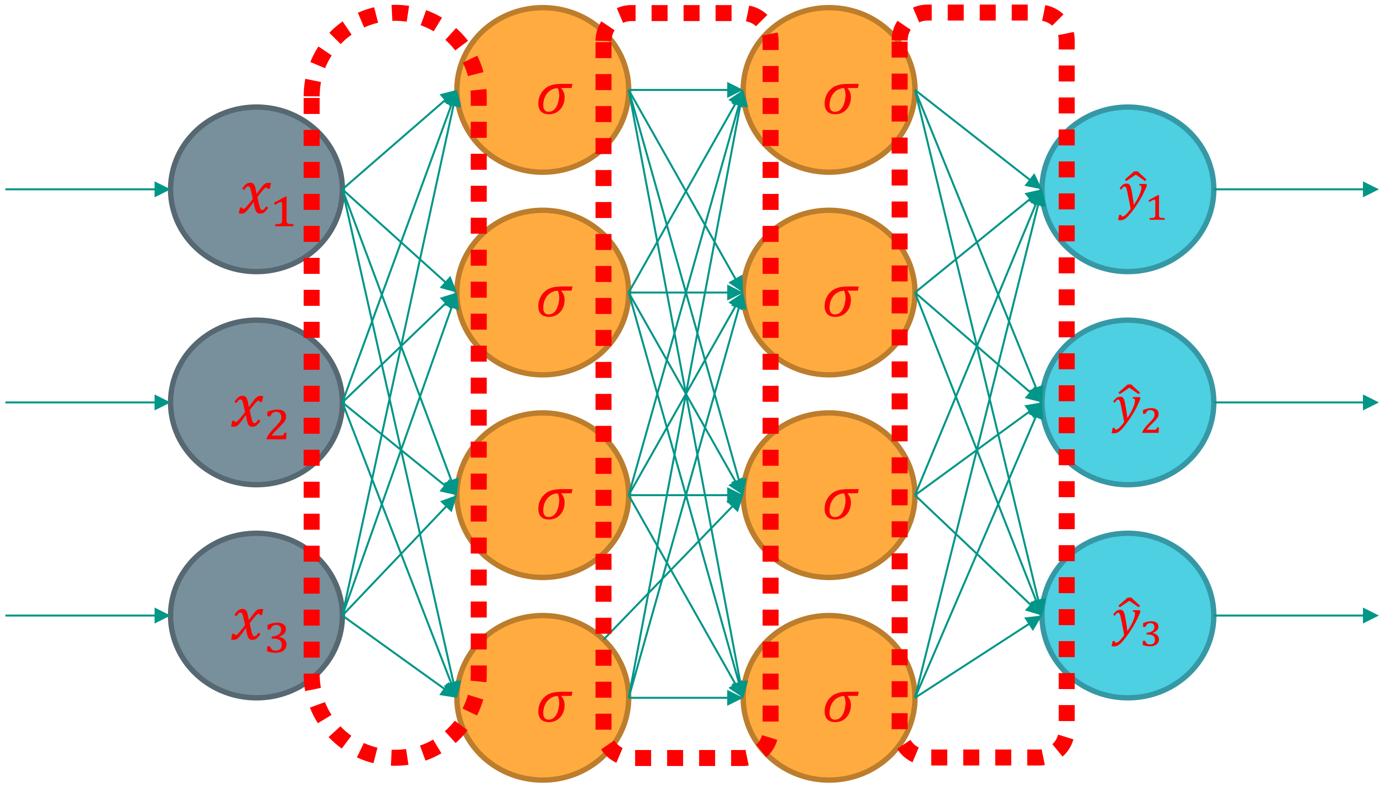
Note: bias terms
not drawn, but implied



Feedforward Neural Network

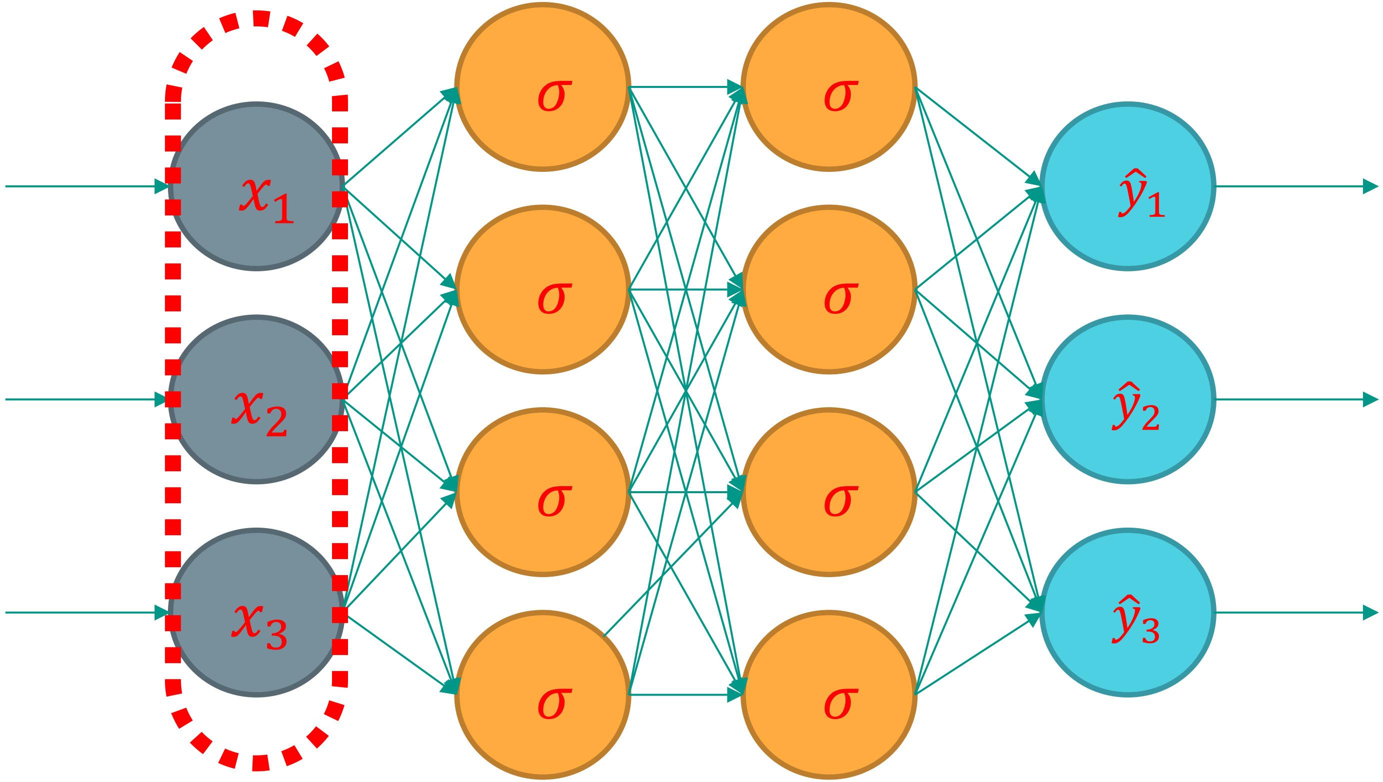


Weights

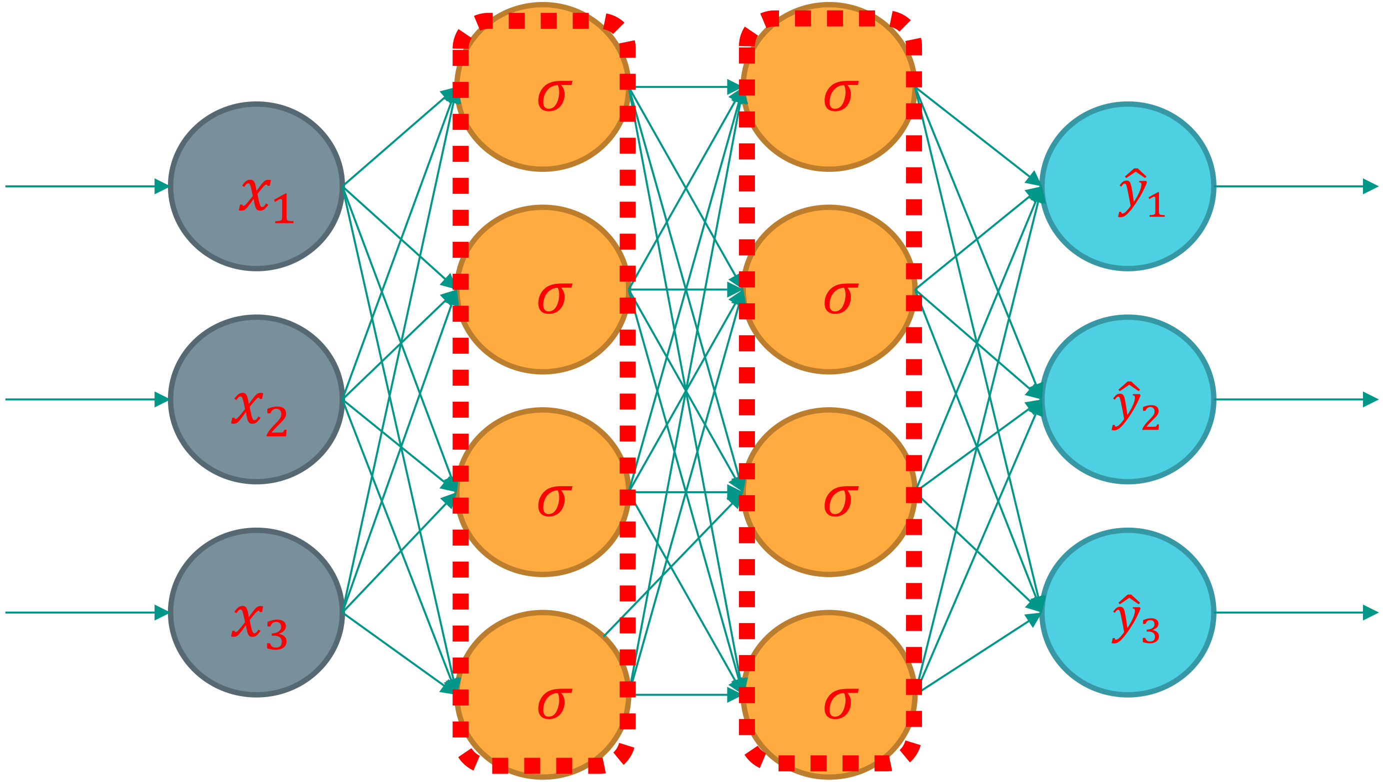


x_1
x_2
x_3
σ
σ
σ
σ
σ
σ
σ
σ
$\hat{y}_1$
$\hat{y}_2$
$\hat{y}_3$

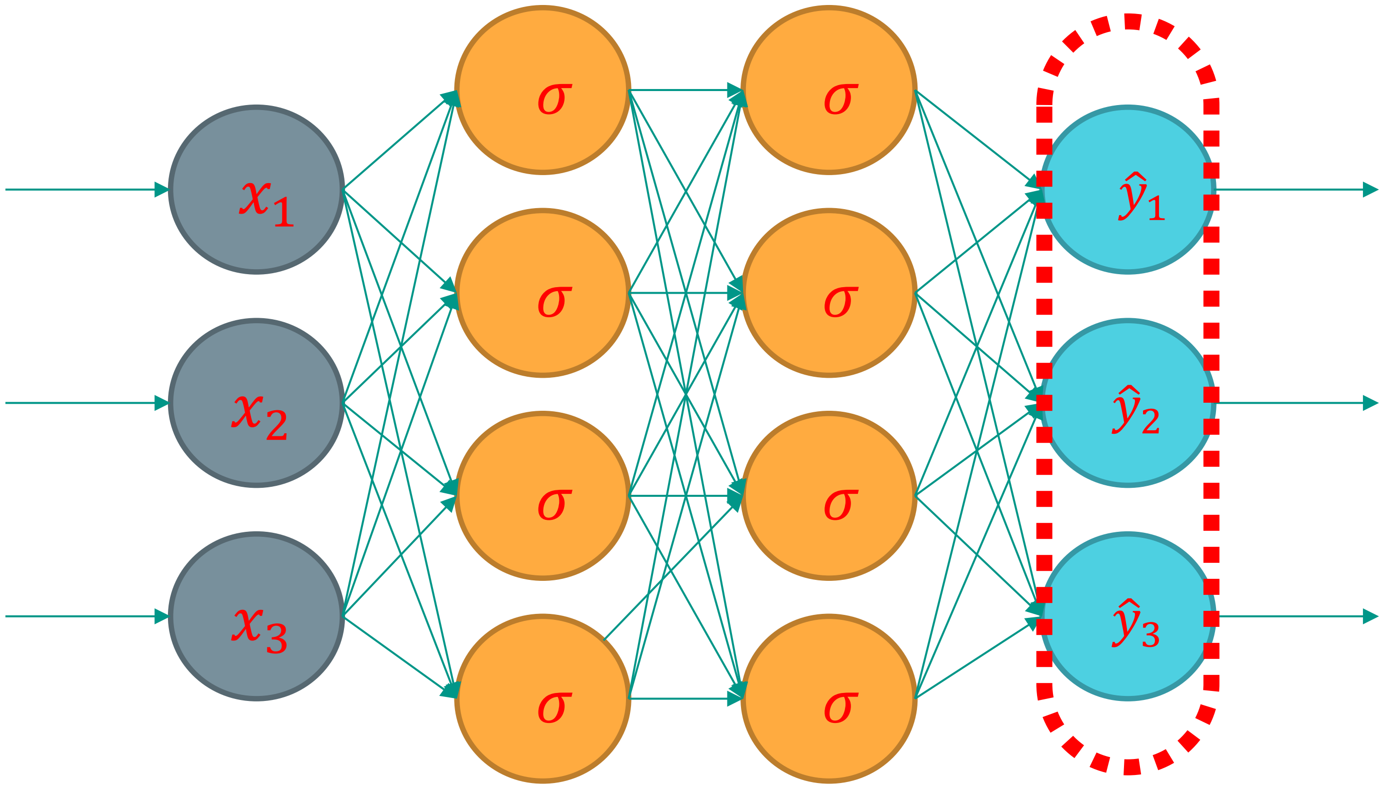
Input Layer


x_1
x_2
x_3
σ
σ
σ
σ
σ
σ
σ
σ
$\hat{y}_1$
$\hat{y}_2$
$\hat{y}_3$

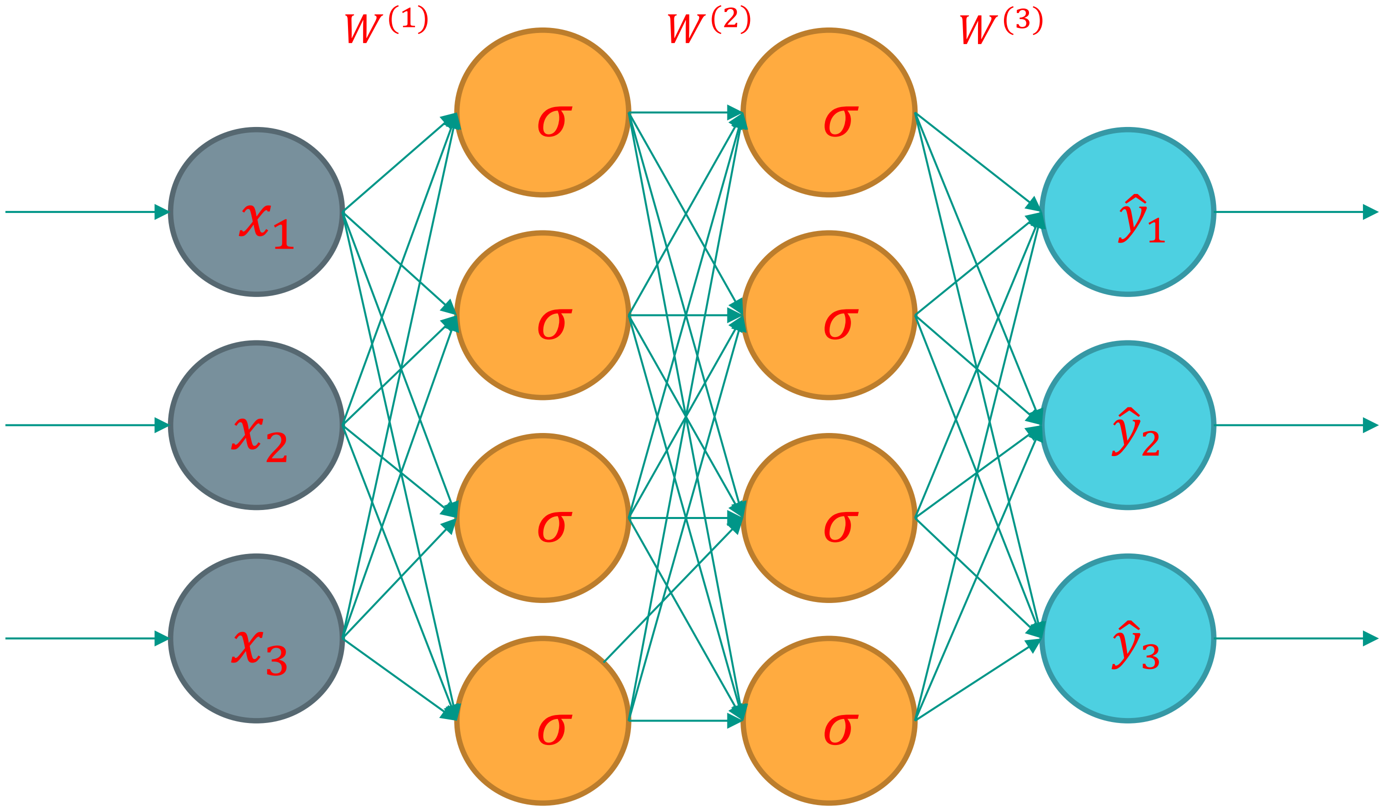
Hidden Layers



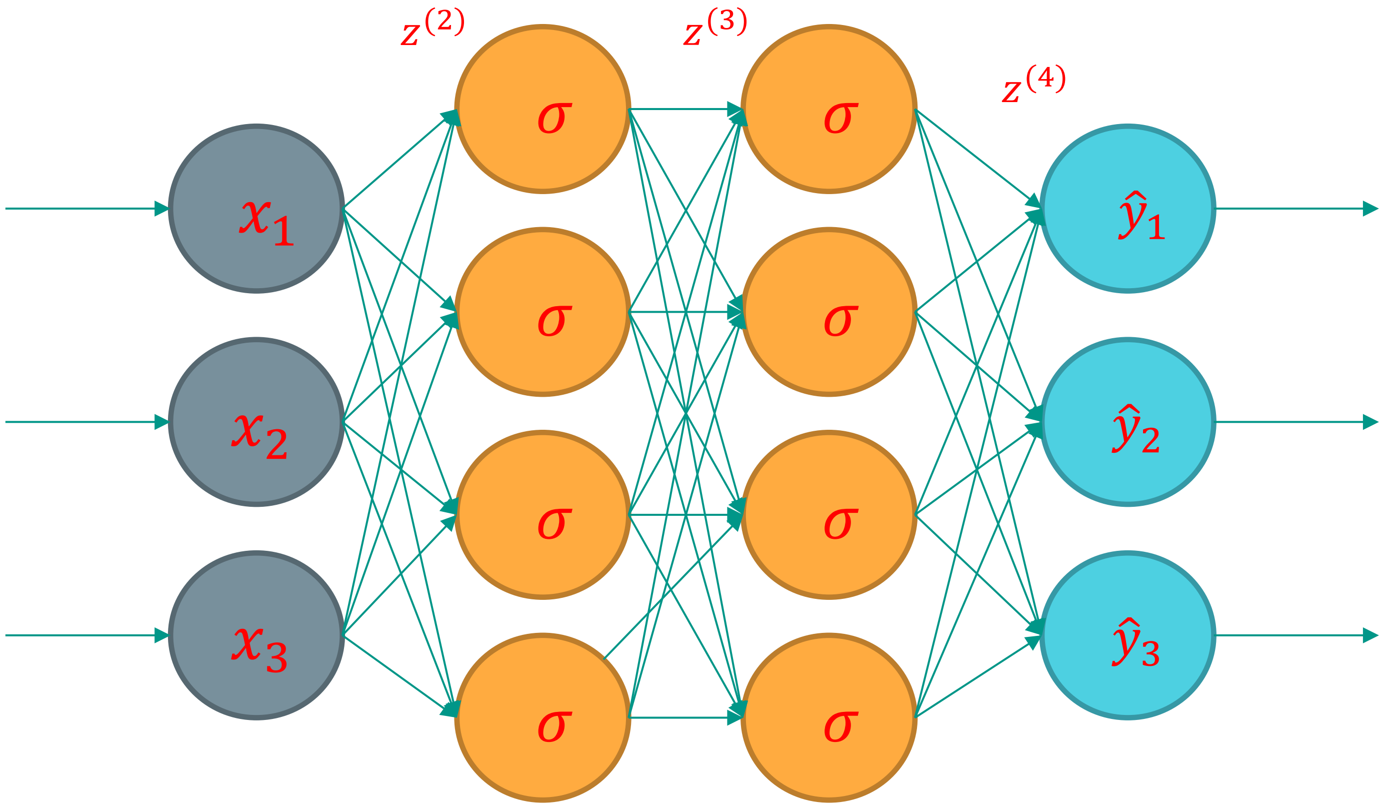
Output Layer



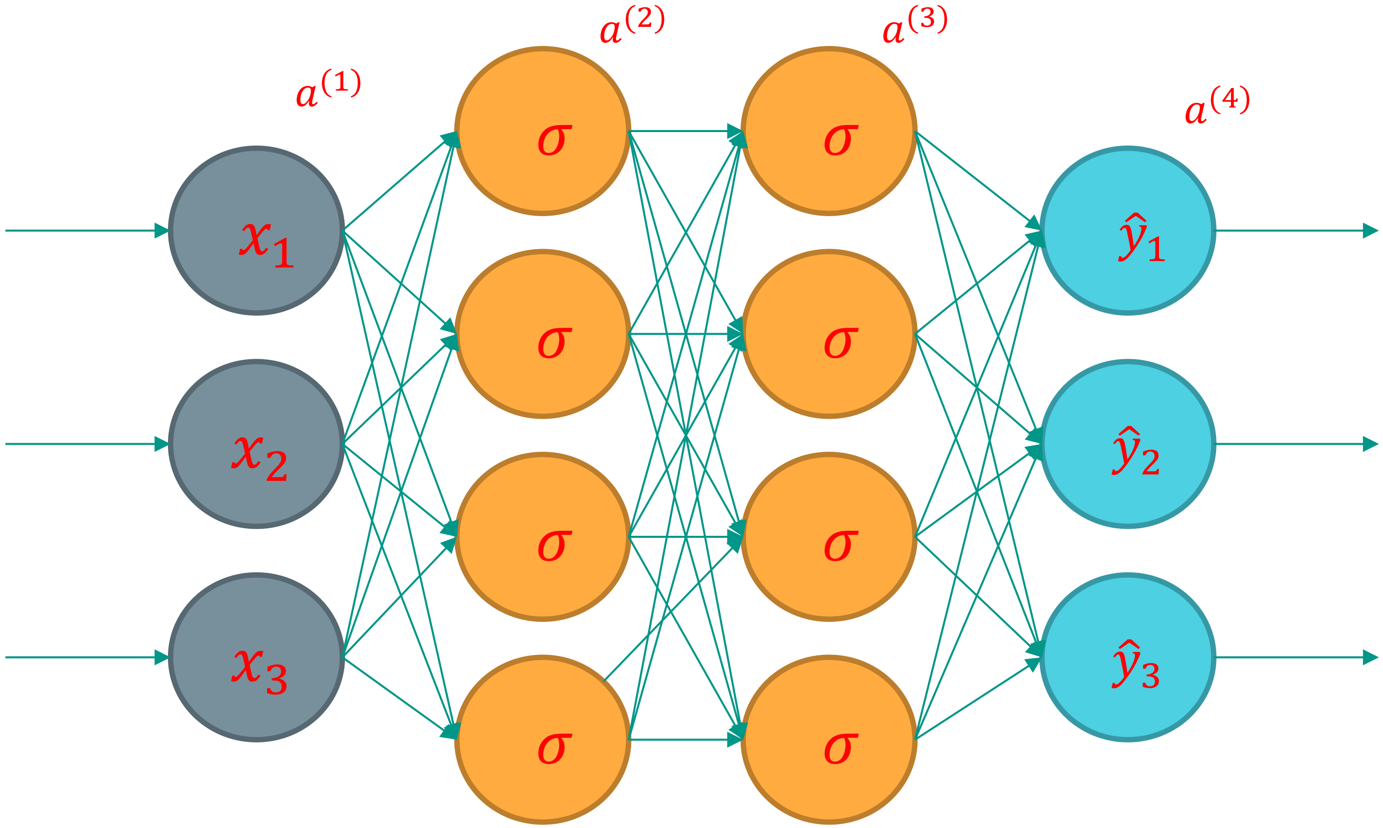
Weights (represented by matrices)



Net Input (sum of weighted inputs, before activation function)



Activations (output of neurons to next layer)



Matrix representation of computation

For a single data point (instance)

$$x = [x_1, x_2, x_3]$$

$$(x = a^{(1)})$$

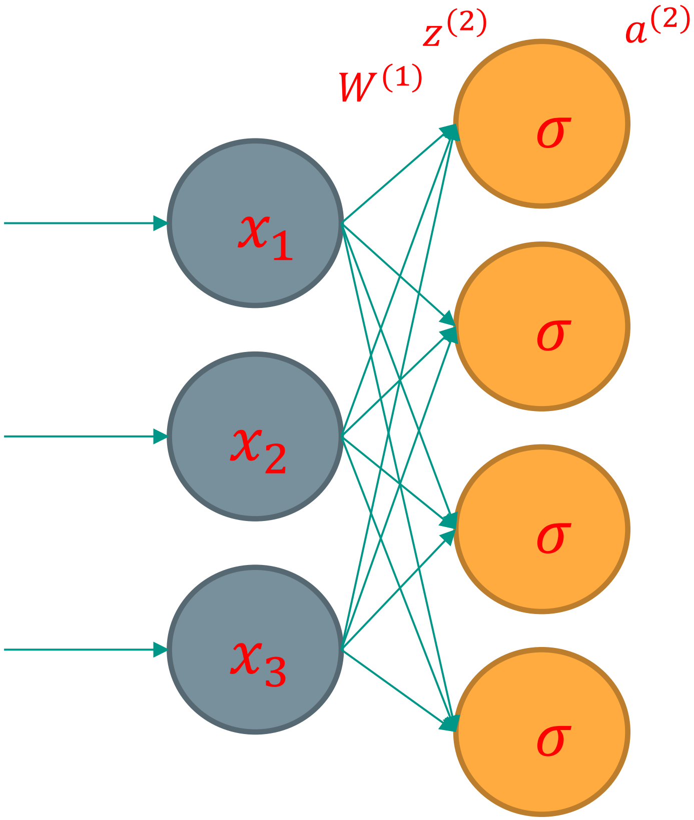
$$z^{(2)} = xW^{(1)}$$

$$a^{(2)} = \sigma(z^{(2)})$$

$W^{(1)}$ is a 3x4 matrix

$z^{(2)}$ is a 4-vector

$a^{(2)}$ is a 4-vector



Continuing the Computation

For a single training instance (data point)

Input: vector x (a row vector of length 3)

Output: vector $\hat{y}$ (a row vector of length 3)

$$z^{(2)} = xW^{(1)} \qquad a^{(2)} = \sigma(z^{(2)})$$

$$z^{(3)} = a^{(2)}W^{(2)} \qquad a^{(3)} = \sigma(z^{(3)})$$

$$z^{(4)} = a^{(3)}W^{(3)} \qquad \hat{y} = softmax(z^{(4)})$$

Multiple data points

In practice, we do these computation for many data points at the same time, by "stacking" the rows into a matrix.
But the equations look the same!

Input: matrix x (an nx3 matrix) (each row a single instance)
Output: vector $\hat{y}$ (an nx3 matrix) (each row a single prediction)

$$z^{(2)} = xW^{(1)} \qquad a^{(2)} = \sigma(z^{(2)})$$

$$z^{(3)} = a^{(2)}W^{(2)} \qquad a^{(3)} = \sigma(z^{(3)})$$

$$z^{(4)} = a^{(3)}W^{(3)} \qquad \hat{y} = softmax(z^{(4)})$$

Now we know how feedforward NNs do Computations.

Next, we will learn how to adjust the weights to learn from data.

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