Naive Bayes

Updating the state of knowledge

step by step

with new information

What time is it? Is it 9 AM?

What time is it? Is it 9 AM?

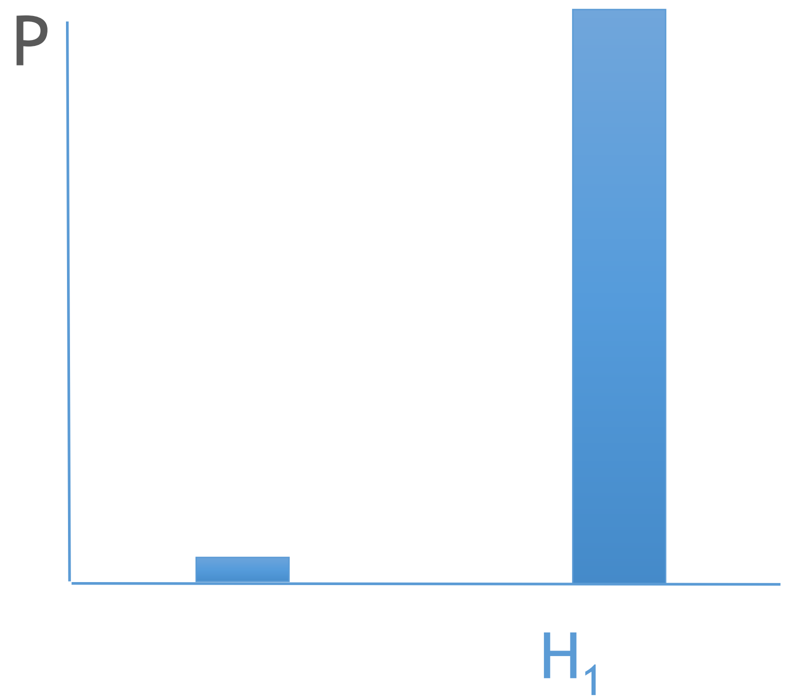
Decide between the two hypotheses (9AM/Not 9AM)
Using all the information we have

4.16% Right Time

What time is it? Is it 9 AM?

H_1 : It is 9 AM (9 to 10)

H_2 : It is not 9 AM



H_2

prior

dist.

$P(H_1) = P($ 9AM $) = 4.16\%$

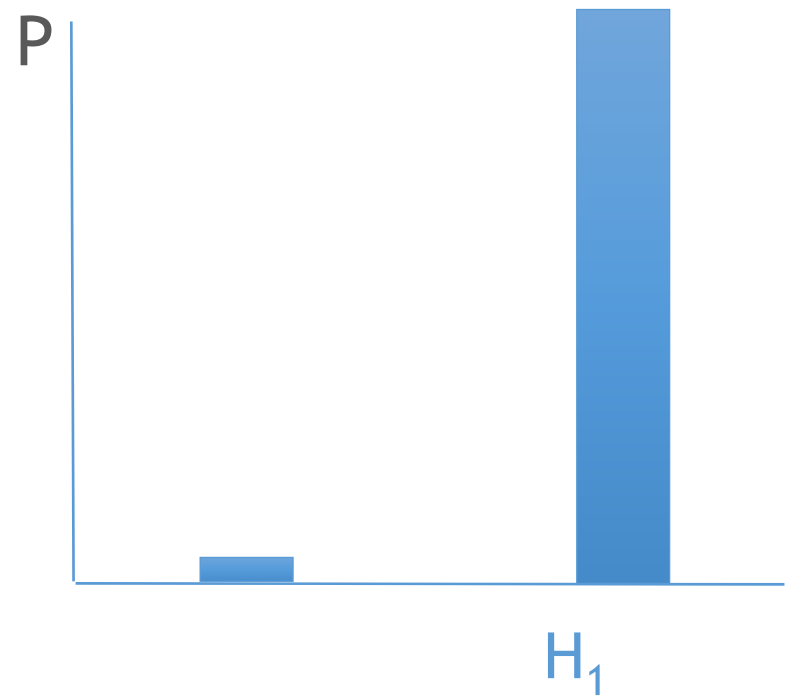
$P(H_2) = P(\text{not } 9AM) = 95.84\%$

4.16%
Right
Time

What time is it? Is it 9 AM?

H_1 : It is 9 AM (9 to 10)

H_2 : It is **not** 9 AM



New information:
5 train is full

H_2

prior

dist.

$P(H_1) = P(\text{9AM}) = 4.16\%$

$P(H_2) = P(\text{not 9AM}) = 95.84\%$

4.16% Right Time

What time is it? Is it 9 AM?

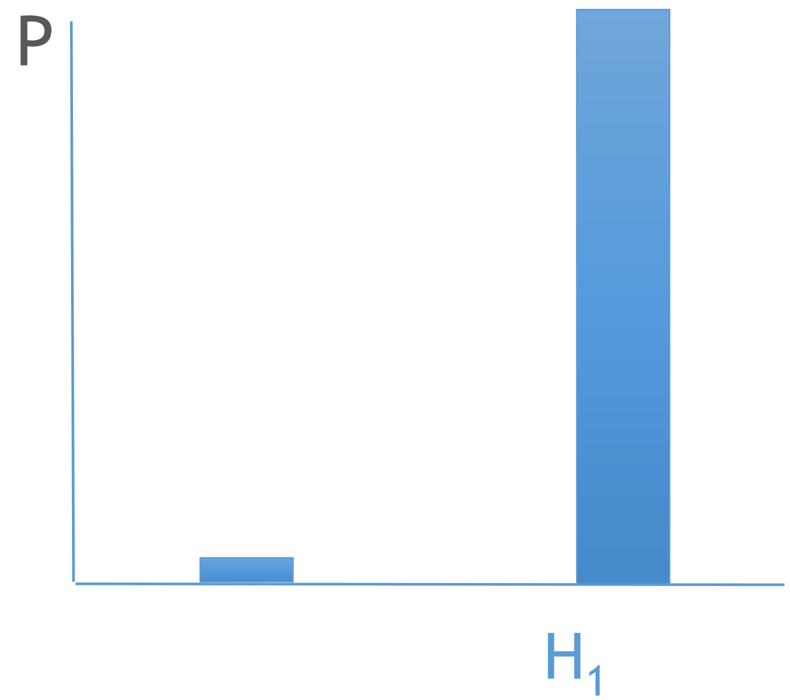
H_1 : It is 9 AM (9 to 10)

H_2 : It is not 9 AM

New information:

5 train is full

$$P(9AM|5tf) = \frac{P(5tf|9AM)\,P(9AM)}{P(5tf)}$$



$P(H_1) = P(\quad 9AM \quad) = 4.16\%$

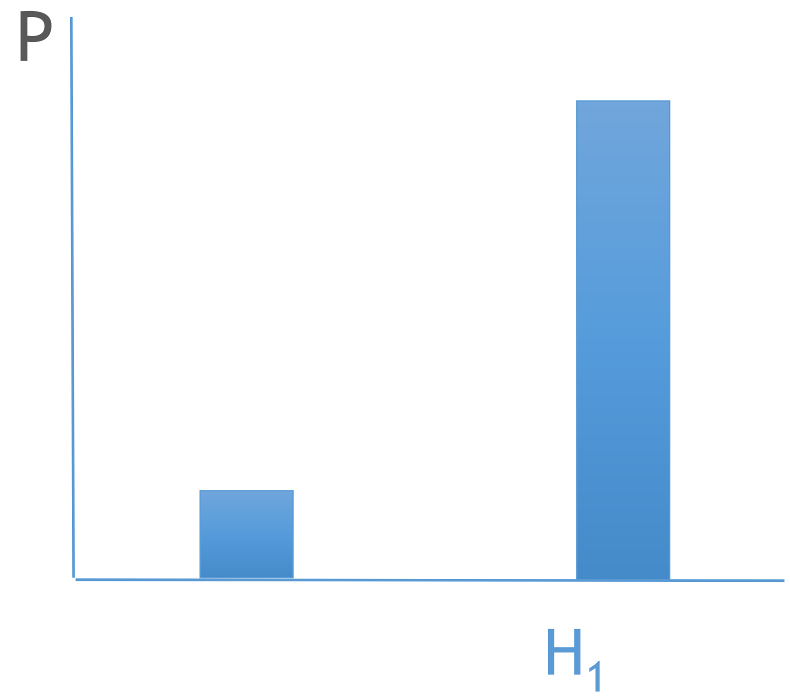
$P(H_2) = P(\text{not } 9AM) = 95.84\%$

13.19%
Right
Time

What time is it? Is it 9 AM?

H_1 : It is 9 AM (9 to 10)

H_2 : It is **not** 9 AM



H_2

posterior

dist.

$P(H_1) = P(\text{ 9AM } | \text{5tf}) = 13.19\%$

$P(H_2) = P(\text{not 9AM} | \text{5tf}) = 86.81\%$

What is classification?

Deciding among hypotheses (labels),
using information we have (features)
for each example

3 Features: Votes on 3 Bills

2 Labels: Democrat / Republican

Prediction:

I know your votes, I'm trying to guess your party

3 Features: Votes on 3 Bills

2 Labels: Democrat / Republican

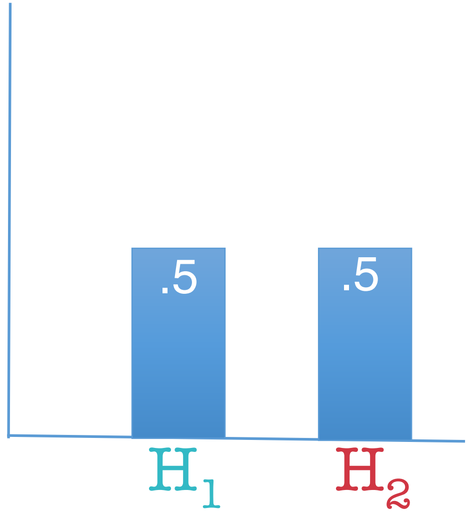
Prediction:

I know your votes, I'm trying to guess your party

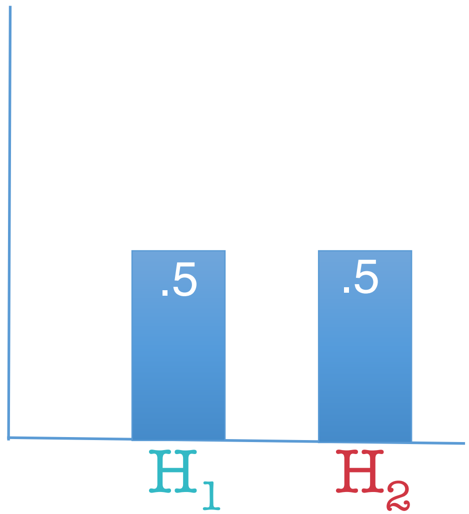
2 Labels

H_1: Democrat

H_2: Republican



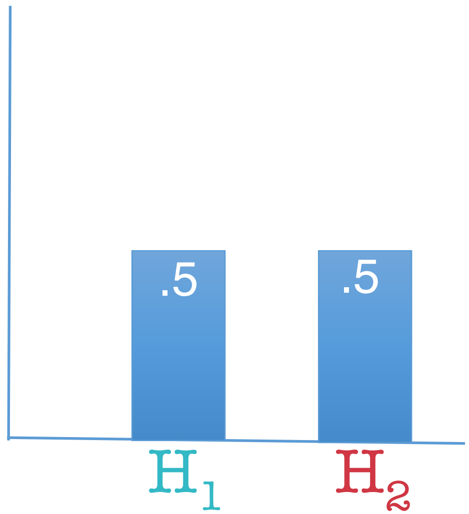
Prior: Initial belief



Prior: Initial belief

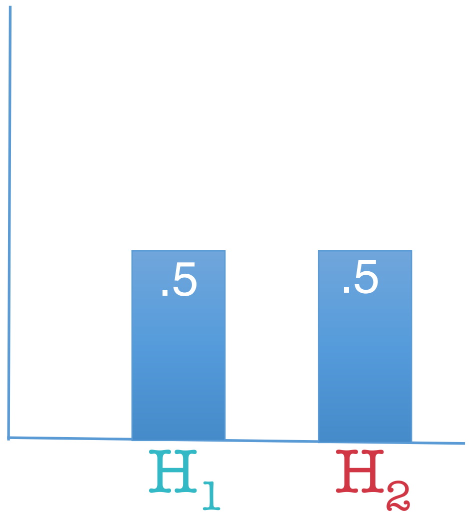
50 - 50 ?   P(Democrat) = 0.5 ?

(Uninformative prior)



Prior: Initial belief

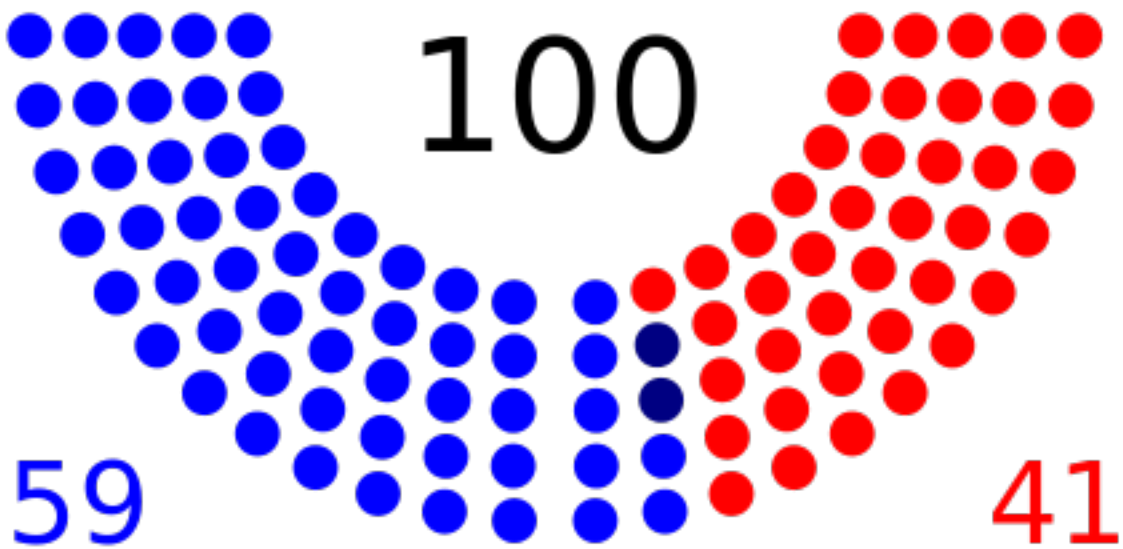
What's my best guess without any vote info?

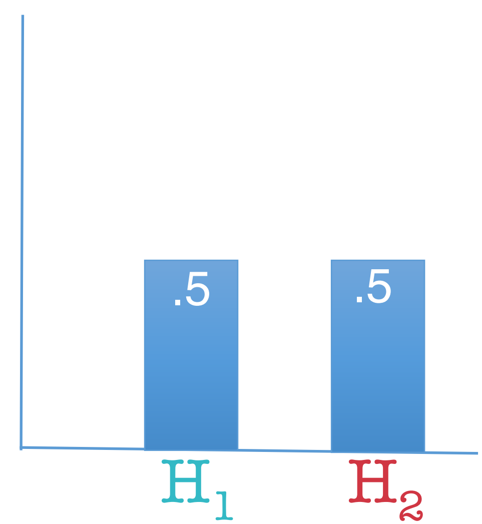


Prior: Initial belief

What's my best guess without any vote info?

I'd guess democrat since there are more of them.



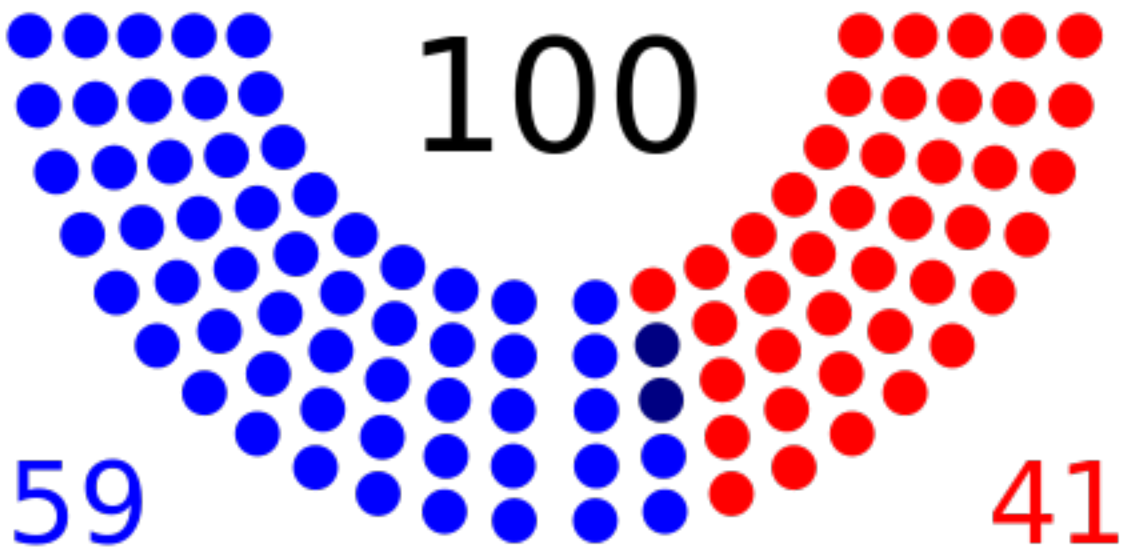


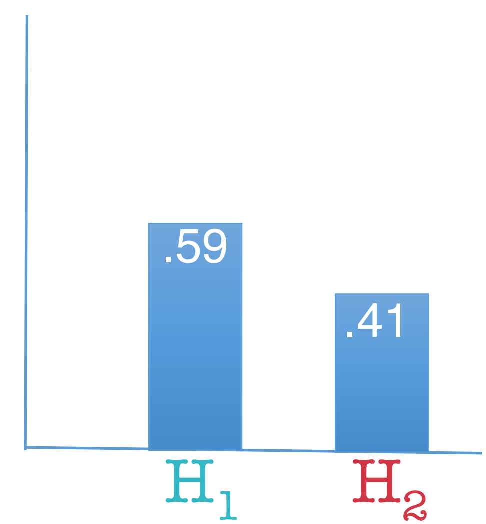
Prior: Initial belief

What's my best guess without any vote info?

I'd guess democrat since there are more of them.

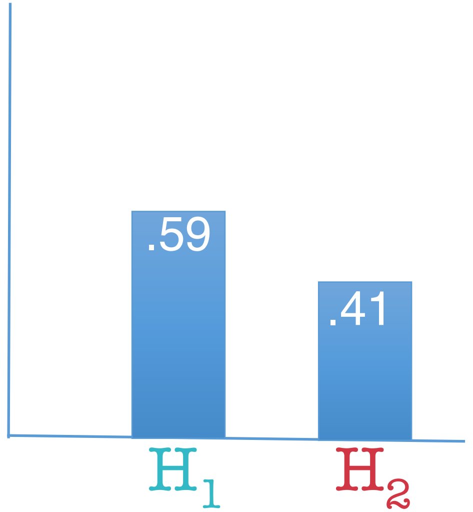
P(Democrat) = 0.59





Prior: Initial belief

P(Democrat) = 0.59

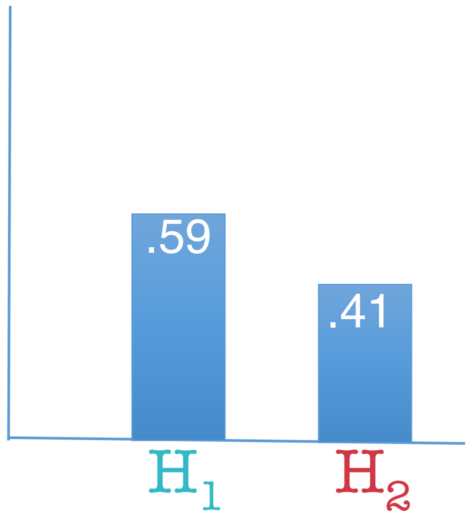


Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

Voted YES on Net Neutrality



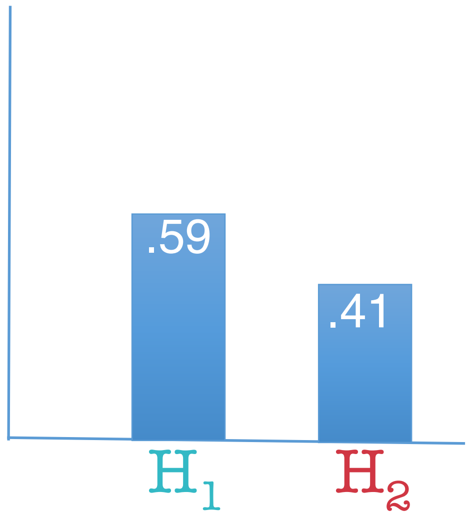
Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

Voted YES on Net Neutrality

$$P(Dem|Y_{NN}) = \frac{P(Y_{NN}|Dem)\ P(Dem)}{P(Y_{NN})}$$



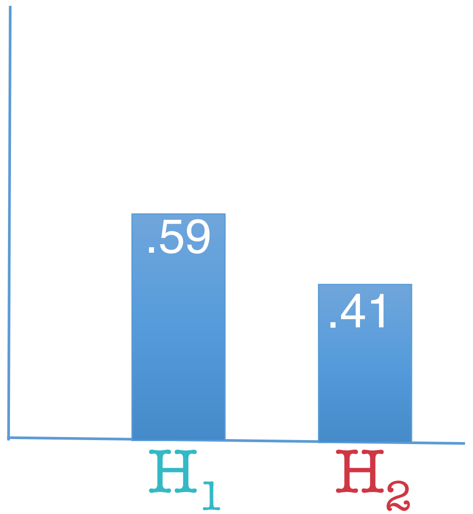
Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

Voted YES on Net Neutrality

$$\underbrace{P(Dem|Y_{NN})}_{\text{posterior}} = \frac{\overbrace{P(Y_{NN}|Dem)}^{\text{likelihood}}\ \overbrace{P(Dem)}^{\text{prior}}}{\underbrace{P(Y_{NN})}_{\text{evidence (normalization factor)}}}$$



Prior: Initial belief

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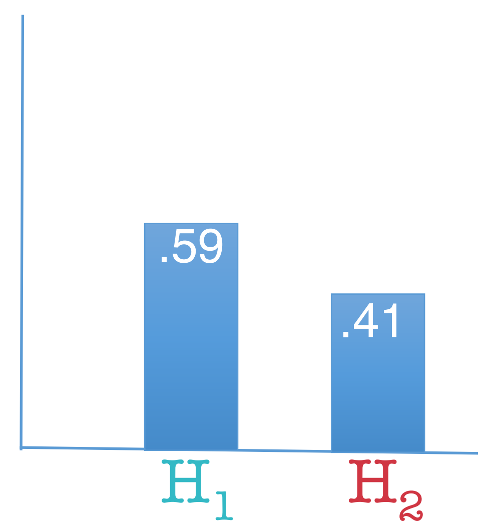
New information (feature 1):

Voted YES on Net Neutrality

$P(Y_{NN}|Dem)$

Prob. of voting yes on net neutrality if you're democrat

$$\underbrace{P(Dem|Y_{NN})}_{\text{posterior}} = \frac{\overbrace{P(Y_{NN}|Dem)}^{\text{likelihood}}\ \overbrace{P(Dem)}^{\text{prior}}}{\underbrace{P(Y_{NN})}_{\substack{\text{evidence} \\ \text{(normalization factor)}}}}$$



Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

Voted YES on Net Neutrality

$P(Y_{NN}|Dem)$

Prob. of voting yes on net neutrality if you're democrat

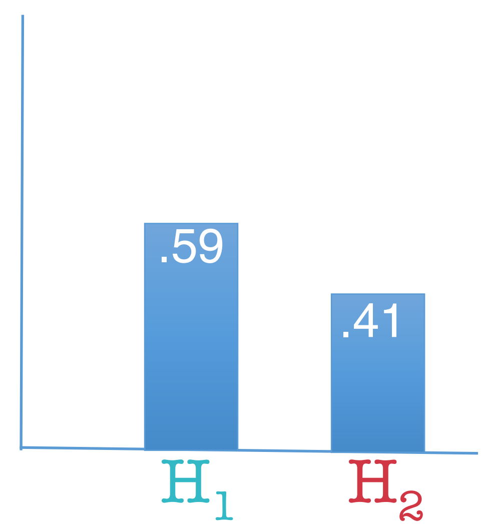
likelihood prior

posterior

$$P(Dem|Y_{NN}) = \frac{P(Y_{NN}|Dem)\,P(Dem)}{P(Y_{NN})}$$

evidence (normalization factor)

$$P(Rep|Y_{NN}) = \frac{P(Y_{NN}|Rep)\,P(Rep)}{P(Y_{NN})}$$



Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

Voted YES on Net Neutrality

$P(Y_{NN}|Dem)$

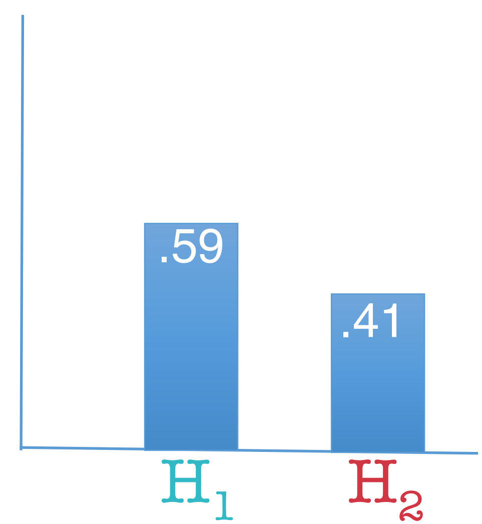
Prob. of voting yes on net neutrality if you're democrat

$$P(Dem|Y_{NN}) = \frac{P(Y_{NN}|Dem)\,P(Dem)}{P(Y_{NN})}$$

posterior, likelihood, prior, evidence (normalization factor)

$P(Y_{NN}|Rep)$

$$P(Rep|Y_{NN}) = \frac{P(Y_{NN}|Rep)\,P(Rep)}{P(Y_{NN})}$$

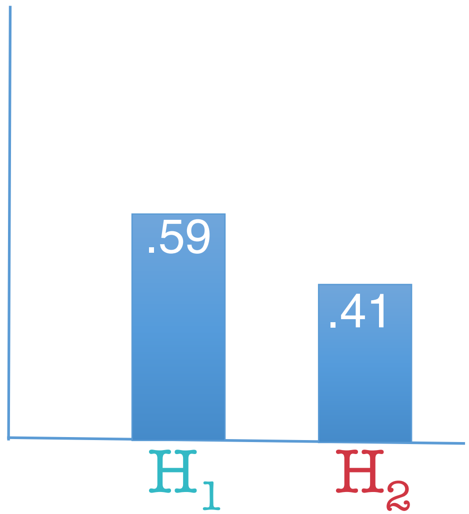


$P(Y_{NN}|Dem)$

Prob. of voting yes
on net neutrality
if you're democrat

$P(Y_{NN}|Rep)$

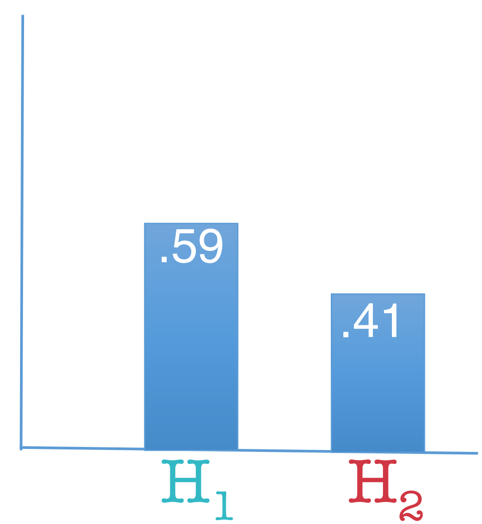
Prob. of voting
yes
on net neutrality
if you're
republican



Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$P(Y_{NN} | Dem)$
Prob. of voting yes on net neutrality if you're democrat

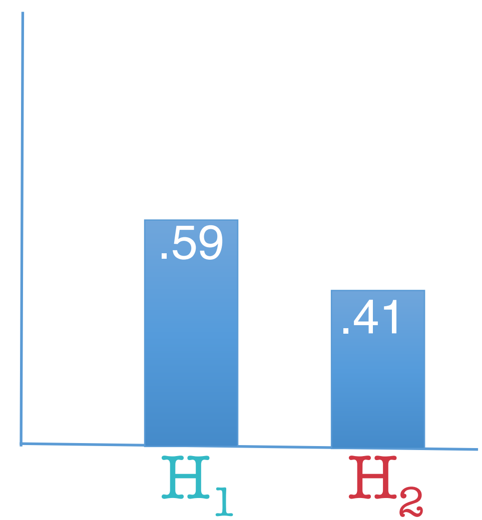
$P(Y_{NN} | Rep)$
Prob. of voting yes on net neutrality if you're republican



Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$$P(Y_{NN} \mid Dem) \approx \frac{\text{\# democrats that } Y_{NN}}{\text{\# all democrats}}$$

$$P(Y_{NN} \mid Rep) \approx \frac{\text{\# republicans that } Y_{NN}}{\text{\# all republicans}}$$

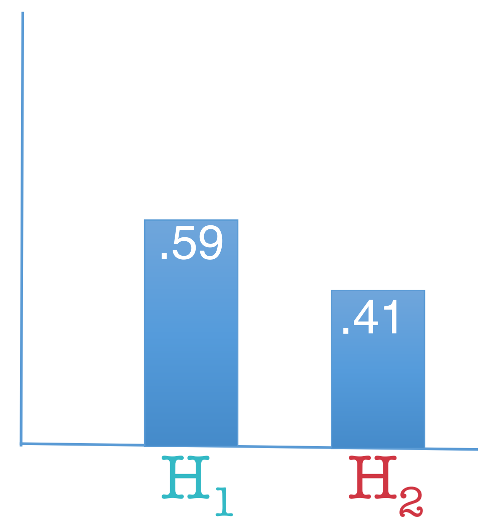


Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$$P(Y_{NN}|Dem) \approx \frac{\#\text{ democrats that } Y_{NN}}{\#\text{ all democrats}}$$

$$P(Y_{NN}|Rep) \approx \frac{\#\text{ republicans that } Y_{NN}}{\#\text{ all republicans}}$$

For likelihoods of discrete data,
training/fitting means counting!
(and estimating likelihoods by dividing counts)

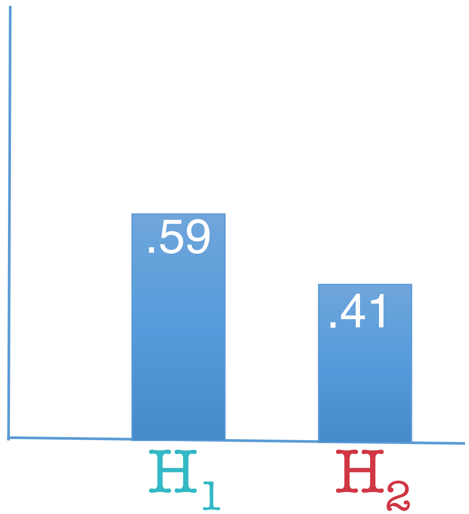


Training set has the answers!
We know Dem/Rep for each person, we know their votes!

$$P(Y_{NN} | Dem) \approx \frac{56}{59} = 0.949$$

$$P(Y_{NN} | Rep) \approx \frac{34}{41} = 0.829$$

For likelihoods of discrete data,
training/fitting means counting!
(and estimating likelihoods by dividing counts)



Prior: Initial belief

P(Democrat) = 0.59

New information (feature 1):

Voted YES on Net Neutrality

likelihood prior

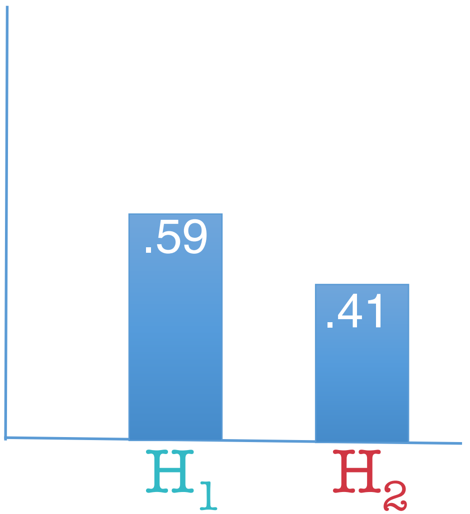
posterior

$$P(Dem|Y_{NN}) = \frac{P(Y_{NN}|Dem)\,P(Dem)}{P(Y_{NN})}$$

evidence

(normalization factor)

$$P(Rep|Y_{NN}) = \frac{P(Y_{NN}|Rep)\,P(Rep)}{P(Y_{NN})}$$



Prior: Initial belief

$P(\text{Democrat}) = 0.59$

New information (feature 1):

Voted YES on Net Neutrality

likelihood prior

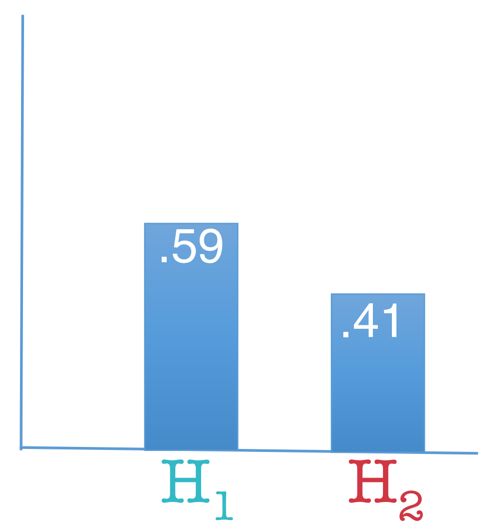
posterior

$$P(Dem|Y_{NN}) = \frac{0.949 * 0.59}{P(Y_{NN})}$$

evidence

(normalization factor)

$$P(Rep|Y_{NN}) = \frac{0.829 * 0.41}{P(Y_{NN})}$$



Current belief

$P(\text{Democrat} | Y_{NN}) = 0.62$

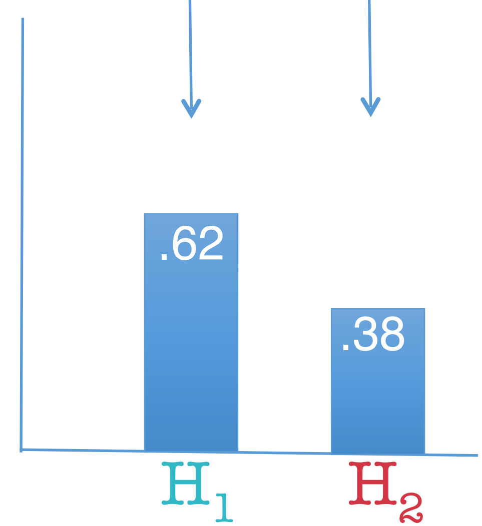
New information (feature 1):

Voted YES on Net Neutrality

$$\underbrace{P(\text{Dem}|Y_{NN})}_{\text{posterior}} = \frac{\overbrace{0.949}^{\text{likelihood}} * \overbrace{0.59}^{\text{prior}}}{\underbrace{P(Y_{NN})}_{\text{evidence (normalization factor)}}}$$

$$P(\text{Rep}|Y_{NN}) = \frac{0.829 * 0.41}{P(Y_{NN})}$$

New posterior distribution

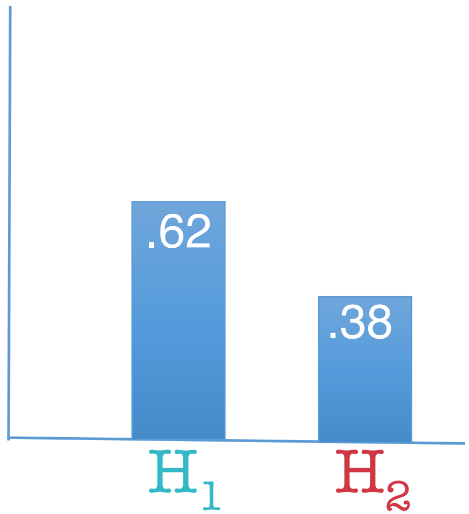


Current belief

$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts



Current belief

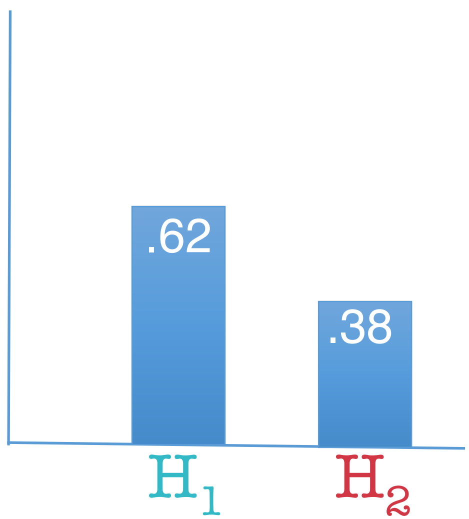
$P(\text{Democrat} | Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem}|Y_{NN}, Y_{TC}) = \frac{P(Y_{TC}|\text{Dem})\, P(\text{Dem}|Y_{NN})}{P(Y_{TC})}$$

$$P(\text{Rep}|Y_{NN}, Y_{TC}) = \frac{P(Y_{TC}|\text{Rep})\, P(\text{Rep}|Y_{NN})}{P(Y_{TC})}$$



Current belief

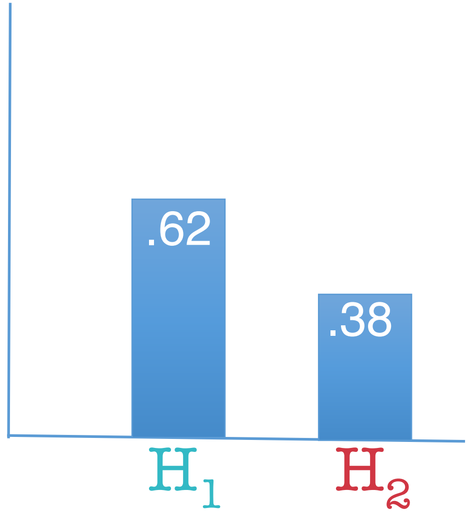
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

$P(Y_{TC} \mid \text{Dem})$

$$\approx \frac{10}{59} = 0.169$$

$P(Y_{TC} \mid \text{Rep})$

$$\approx \frac{35}{41} = 0.854$$



Current belief

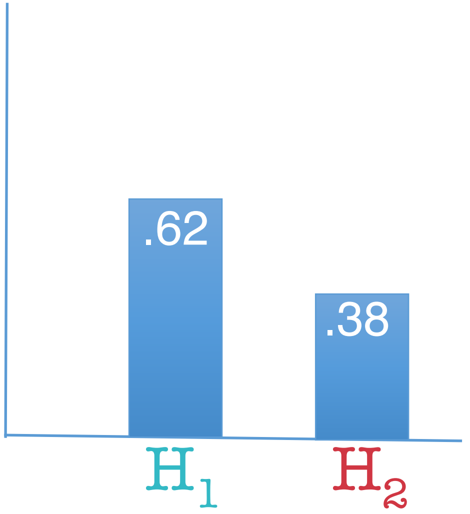
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{P(Y_{TC} \mid \text{Dem})\, P(\text{Dem} \mid Y_{NN})}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{P(Y_{TC} \mid \text{Rep})\, P(\text{Rep} \mid Y_{NN})}{P(Y_{TC})}$$



Current belief

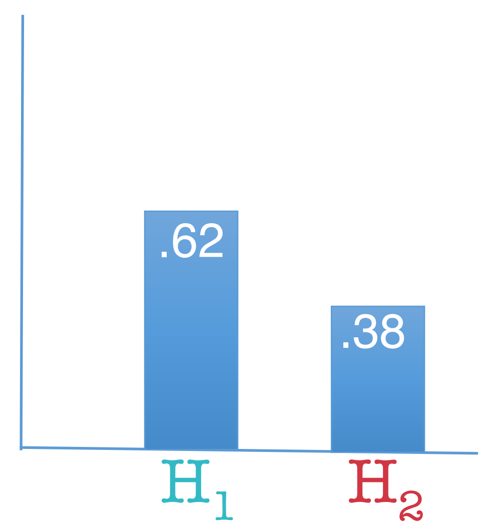
$P(\text{Democrat} \mid Y_{NN}) = 0.62$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{0.169 \quad * \quad 0.62}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{0.854 \quad * \quad 0.38}{P(Y_{TC})}$$



Current belief

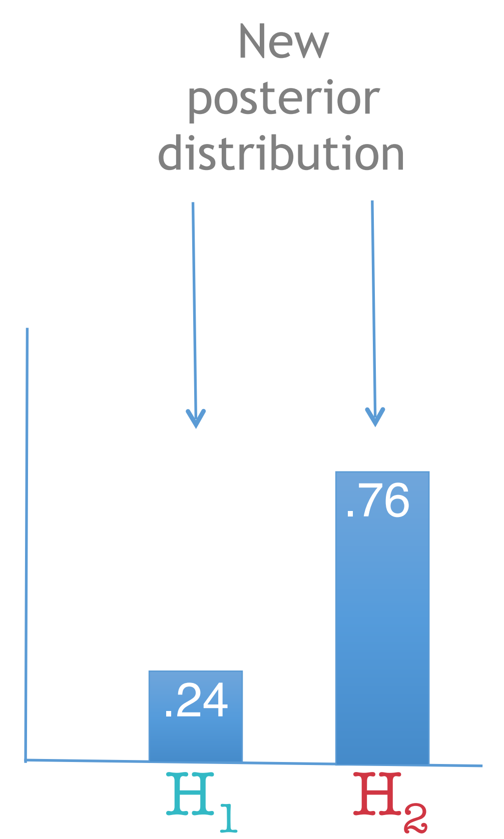
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 2):

Voted YES on Tax Cuts

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}) = \frac{0.169 \quad * \quad 0.62}{P(Y_{TC})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}) = \frac{0.854 \quad * \quad 0.38}{P(Y_{TC})}$$

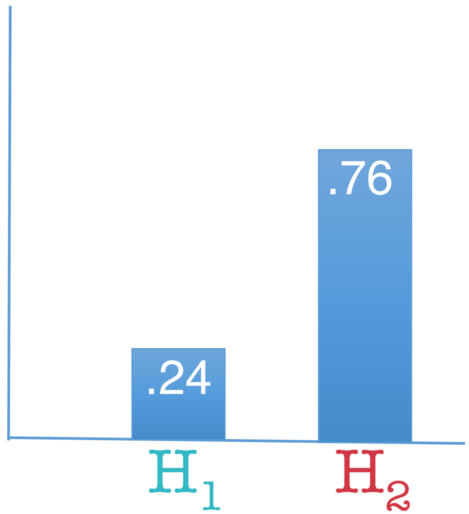


Current belief

P(Democrat | Y_{NN}, Y_{TC}) = 0.24

New information (feature 3):

Voted NO on License-free Guns



Current belief

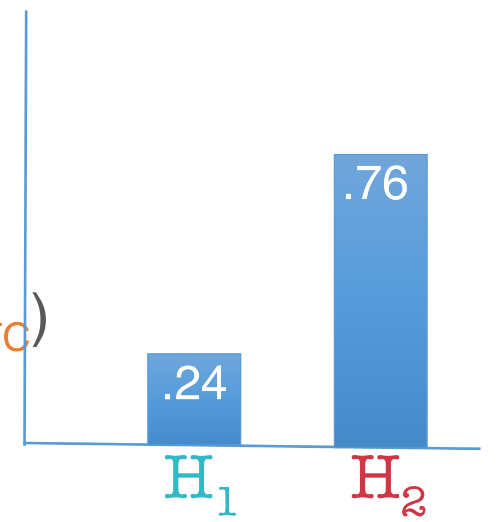
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Dem})\, P(\text{Dem} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Rep})\, P(\text{Rep} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$



Current belief

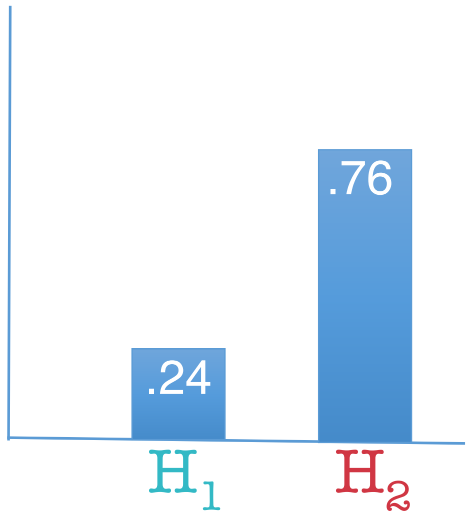
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

$P(N_{LG} \mid \text{Dem})$

$$\approx \frac{53}{59} = 0.898$$

$P(N_{LG} \mid \text{Rep})$

$$\approx \frac{23}{41} = 0.561$$



Current belief

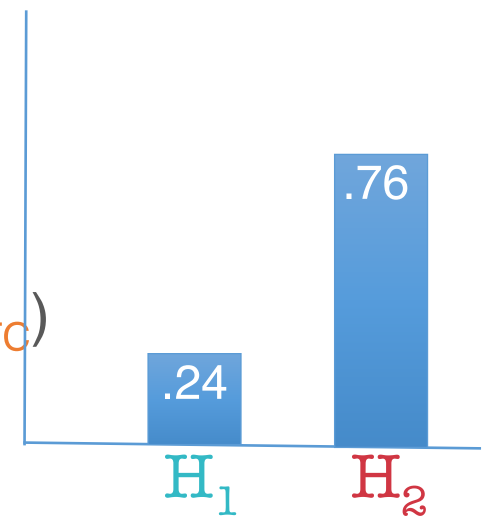
$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$

New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Dem})\, P(\text{Dem} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(N_{LG} \mid \text{Rep})\, P(\text{Rep} \mid Y_{NN}, Y_{TC})}{P(N_{LG})}$$



Current belief

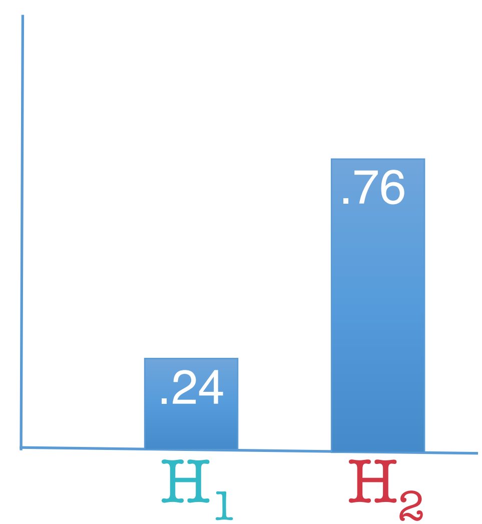
$$P(\text{Democrat} \mid Y_{NN}, Y_{TC}) = 0.24$$

New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.898 \quad * \quad 0.24}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.561 \quad * \quad 0.76}{P(N_{LG})}$$



Current belief

$P(\text{Democrat} \mid Y_{NN}, Y_{TC}, N_{LG}) = 0.34$

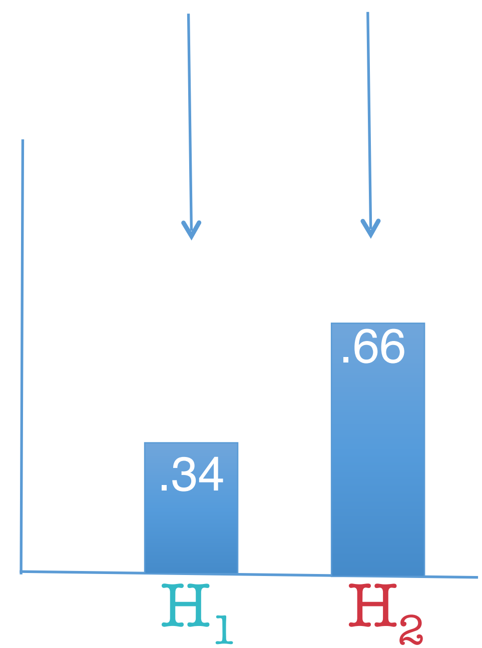
New information (feature 3):

Voted NO on License-free Guns

$$P(\text{Dem} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.898 \quad * \quad 0.24}{P(N_{LG})}$$

$$P(\text{Rep} \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{0.561 \quad * \quad 0.76}{P(N_{LG})}$$

New posterior distribution



Current belief

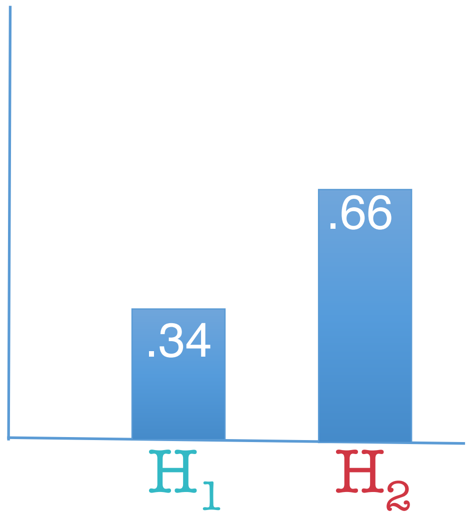
$P(\text{Democrat} | Y_{NN}, Y_{TC}, N_{LG}) = 0.34$

Classify this person that voted

Yes on Net Neutrality (Y_{NN}),

Yes on Tax Cuts (Y_{TC}),

No on License-free Guns (N_{LG})

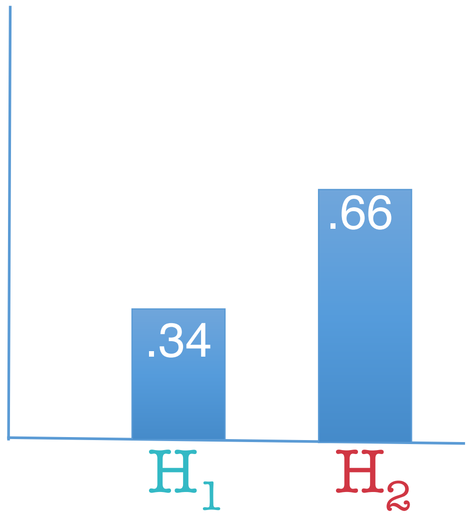


Current belief

$P(\text{Democrat} \mid Y_{NN}, Y_{TC}, N_{LG}) = 0.34$

Classify this person that voted
Yes on Net Neutrality (Y_{NN}),
Yes on Tax Cuts (Y_{TC}),
No on License-free Guns (N_{LG})

My strongest belief is in H_2,
I classify this person with the label Republican.



Naïve Bayes

Training:

Count and calculate the likelihood of each feature value for each class:

$P(Y_{NN}|Dem) \quad = 1 - P(N_{NN}|Dem)$

$P(Y_{NN}|Rep) \quad = 1 - P(N_{NN}|Rep)$

$P(Y_{TC}|Dem) \quad = 1 - P(N_{TC}|Dem)$

$P(Y_{TC}|Rep) \quad = 1 - P(N_{TC}|Rep)$

$P(Y_{LG}|Dem) \quad = 1 - P(N_{LG}|Dem)$

$P(Y_{LG}|Rep) \quad = 1 - P(N_{LG}|Rep)$

Prediction:

Use Bayes to update priors with the likelihoods,
Pick label with the highest posterior probability.

What was the naïve part?

We assumed each feature is independent

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Easier to see in a single update rather than sequential

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Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$P(Dem | Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(Y_{NN}, Y_{TC}, N_{LG} | Dem) \quad P(Dem)}{P(Y_{NN}, Y_{TC}, N_{LG})}$$

We assumed each feature is independent

Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
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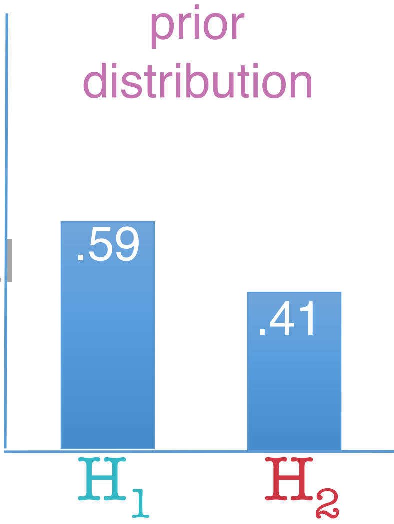
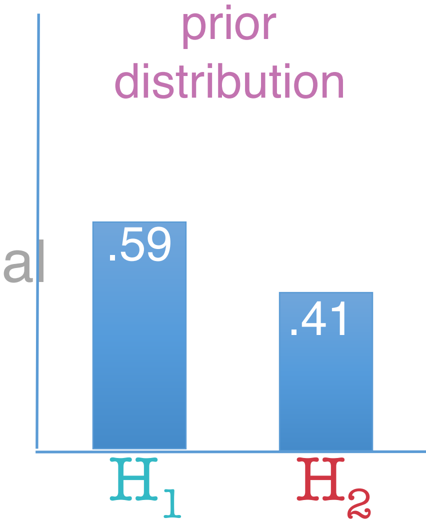


likelihood prior

posterior

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{P(Y_{NN},Y_{TC},N_{LG}|Dem)\ \ P(Dem)}{P(Y_{NN},Y_{TC},N_{LG})}$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

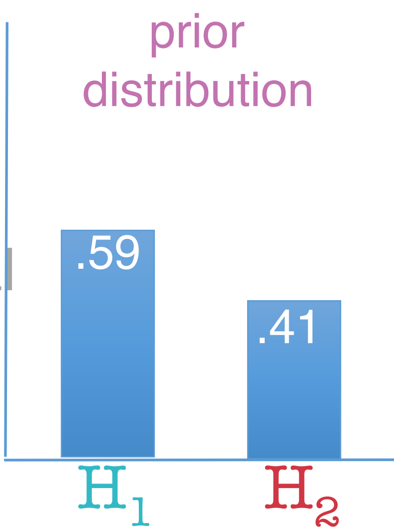
Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$\overset{\text{posterior}}{P(Dem \mid Y_{NN}, Y_{TC}, N_{LG})} = \frac{\overset{\text{likelihood}}{P(Y_{NN}, Y_{TC}, N_{LG} \mid Dem)} \quad \overset{\text{prior}}{P(Dem)}}{P(Y_{NN}, Y_{TC}, N_{LG})}$$

Independence Assumption:

$P(Y_{NN}, Y_{TC}, N_{LG} \mid Dem) = P(Y_{NN} \mid Dem)\, P(Y_{TC} \mid Dem)\, P(N_{LG} \mid Dem)$

We assumed each feature is independent



Easier to see in a single update rather than sequential

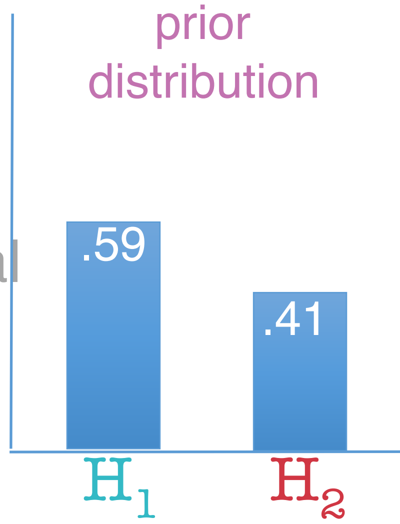
Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

$$\underbrace{P(Dem|Y_{NN},Y_{TC},N_{LG})}_{\text{posterior}} = \frac{\overbrace{P(Y_{NN},Y_{TC},N_{LG}|Dem)}^{\text{likelihood}}\ \overbrace{P(Dem)}^{\text{prior}}}{P(Y_{NN},Y_{TC},N_{LG})}$$

Independence Assumption:
$P(Y_{NN},Y_{TC},N_{LG}|Dem) = P(Y_{NN}|Dem)\ P(Y_{TC}|Dem)\ P(N_{LG}|Dem)$

Not even close in most cases!
Naïve Bayes still works well.

We assumed each feature is independent



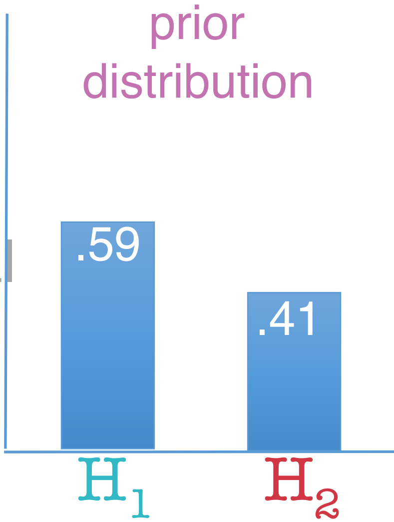
Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

posterior / likelihood / prior

$$P(Dem \mid Y_{NN}, Y_{TC}, N_{LG}) = \frac{P(Y_{NN} \mid Dem)\, P(Y_{TC} \mid Dem)\, P(N_{LG} \mid Dem)\, P(Dem)}{P(Y_{NN}, Y_{TC}, N_{LG})}$$

We assumed each feature is independent



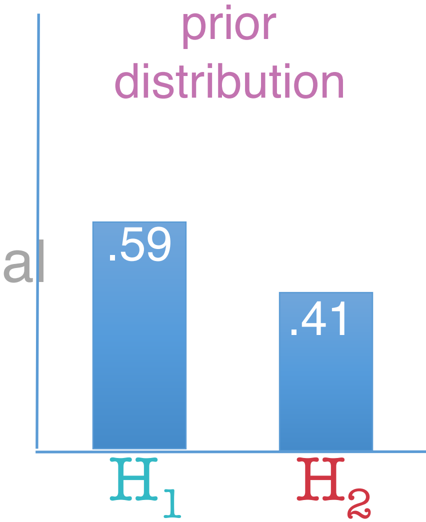
Easier to see in a single update rather than sequential

Prob. of this example having label Democrat, given the values Yes, Yes and No on the features NN, TC and LG

likelihood prior

posterior

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{0.949 * 0.169 * 0.898 \quad * \quad 0.59}{P(Y_{NN},Y_{TC},N_{LG})}$$



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Easier to see in a single update rather than sequential

Prob. of this example having label Democrat,
given the values Yes, Yes and No
on the features NN, TC and LG

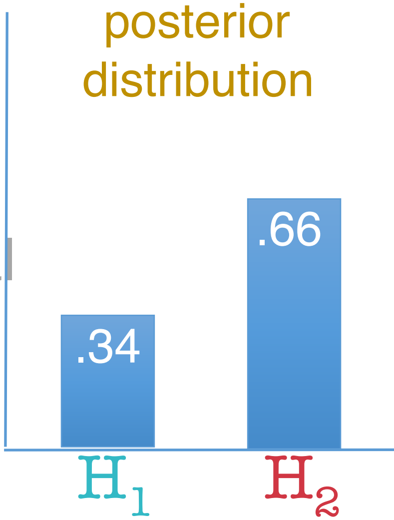
likelihood prior

posterior

$$P(Dem|Y_{NN},Y_{TC},N_{LG}) = \frac{0.949 * 0.169 * 0.898 \quad * \quad 0.59}{P(Y_{NN},Y_{TC},N_{LG})}$$

$$P(Rep|Y_{NN},Y_{TC},N_{LG}) = \frac{0.829 * 0.854 * 0.561 \quad * \quad 0.41}{P(Y_{NN},Y_{TC},N_{LG})}$$

We assumed each feature is independent



Easier to see in a single update rather than sequential

Prob. of this example having label Democrat, given the values Yes, Yes and No on the features NN, TC and LG

$P(Dem|Y_{NN},Y_{TC},N_{LG}) = 0.34$

$P(Rep|Y_{NN},Y_{TC},N_{LG}) = 0.66$ ← predict!

What about multiple classes?












Straightforward!

Update each hypothesis,
given the values Yes, Yes and No
on the features NN, TC and LG

$P(Dem|Y_{NN},Y_{TC},N_{LG})$
$P(Rep|Y_{NN},Y_{TC},N_{LG})$
$P(Indep|Y_{NN},Y_{TC},N_{LG})$

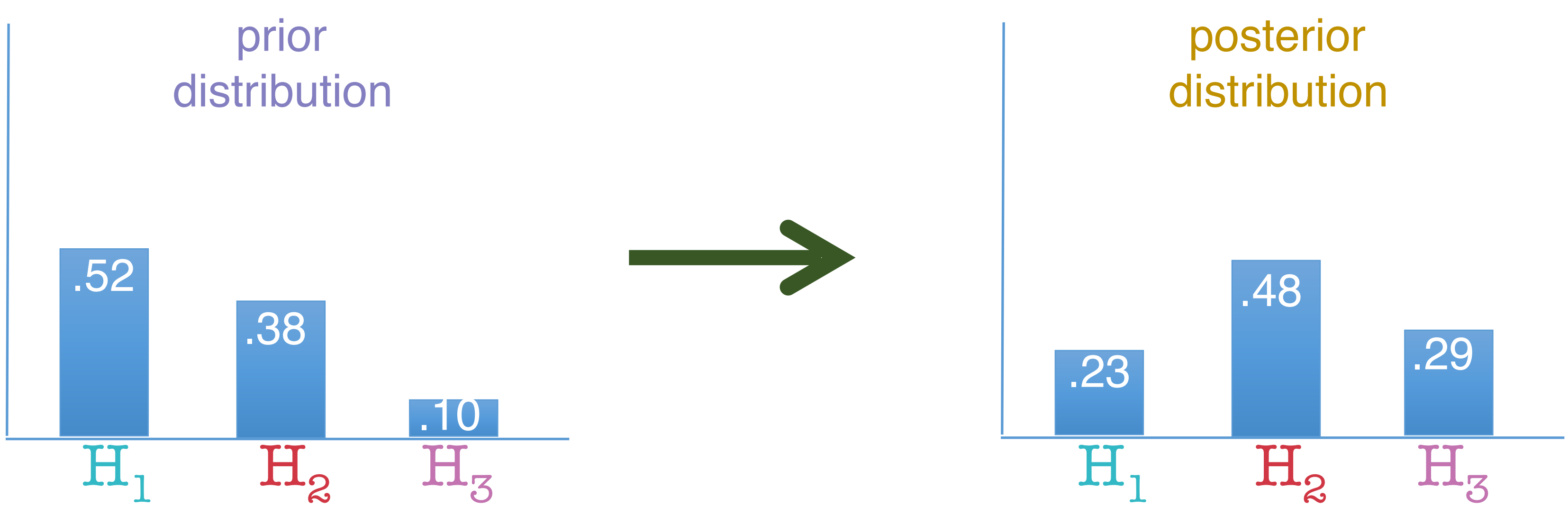
Straightforward!

Update each hypothesis,
given the values Yes, Yes and No
on the features NN, TC and LG

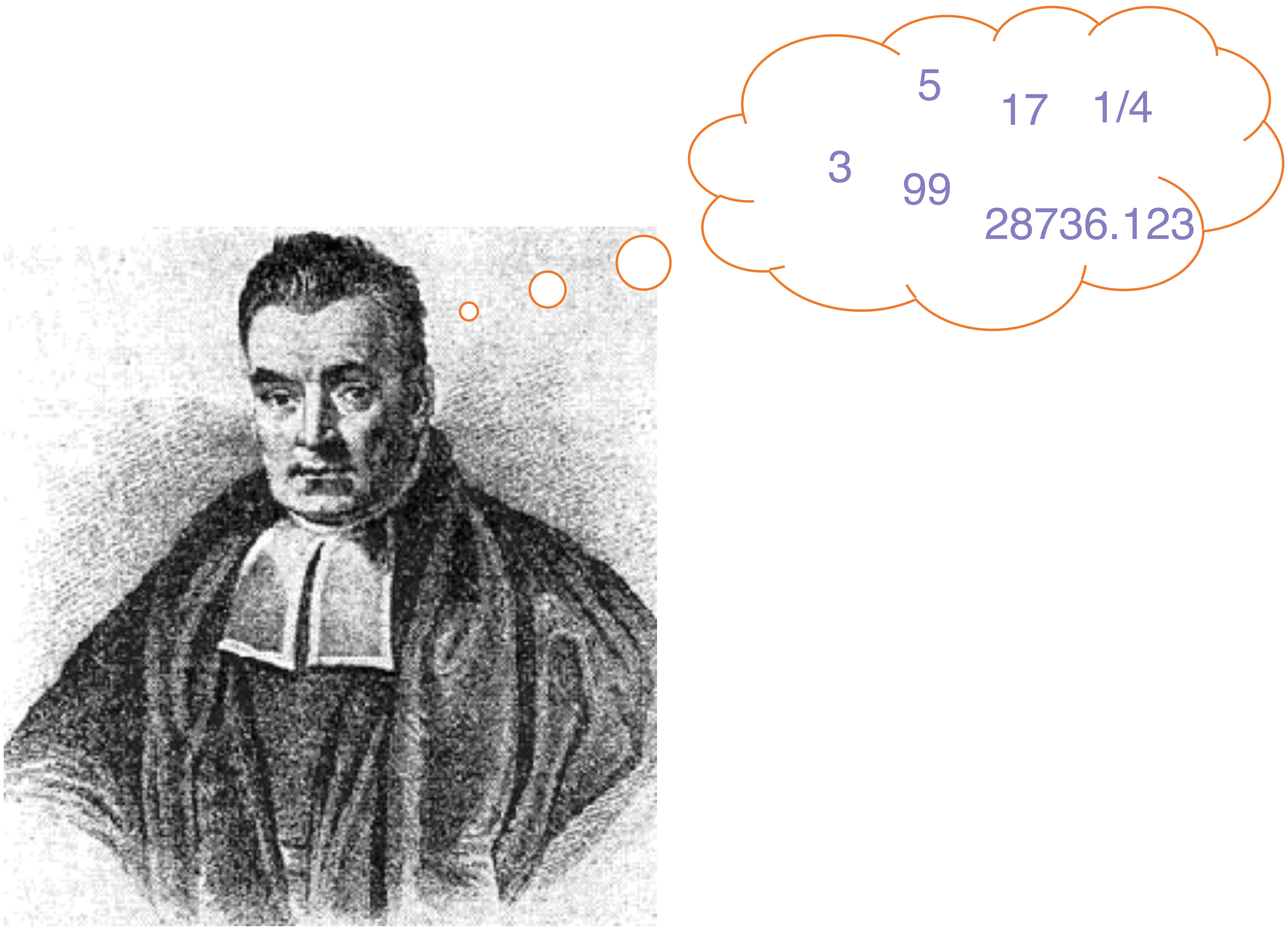
$P(Dem|Y_{NN},Y_{TC},N_{LG})$

$P(Rep|Y_{NN},Y_{TC},N_{LG})$

$P(Indep|Y_{NN},Y_{TC},N_{LG})$



How about numeric features?



Fit a likelihood function!

Instead of $P(Y_{NN}|Dem)$ or $P(N_{NN}|Dem)$,
you get a function $P(age{=}x \mid Dem)$

Fit a likelihood function!

Instead of $P(Y_{NN}|Dem)$ or $P(N_{NN}|Dem)$,
you get a function $P(age=x \mid Dem)$

Look at the training set, fit a Gaussian distribution to $P(age=x|Dem)$
Use this Gaussian likelihood when predicting.

Flavors of Bayes in `sklearn`:

Numeric Features:
Gaussian Naïve Bayes

Features that are 0 or 1 (and both matter):
Bernoulli Naïve Bayes

Features that are count-like (and only non-zero matters):
Multinomial Naïve Bayes

Flavors of Bayes in `sklearn`:

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