Intro to Probability

Experiments and Events

Learning Objectives

- Be able to recognize what a **sample space** is
- Be able to recognize that an **event** is a subset of the sample space

Data scientists run experiments

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Example Experiment: Dice Rolling

Suppose a data scientist wants to record the results of rolling a six-sided die.

The **sample space** is {1, 2, 3, 4, 5, 6}.

- The **event** for rolling an even number is {2, 4, 6}
- The **event** for rolling an odd number is {1, 3, 5}

Sets and Venn Diagrams

Learning Objectives

- Be able to distinguish if sets are **joint** or **disjoint** via Venn diagrams
- Be able to identify the **union** and **intersection** of two sets via Venn diagrams

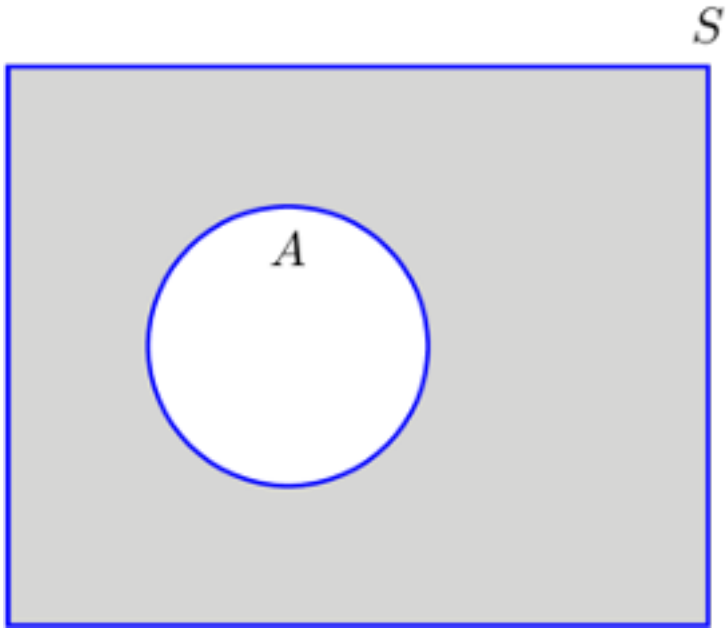
Venn diagrams are used to visualize sets

Denote the **sample space** as *S*.

- *A* is an **event** in the sample space
  - *A* is a subset of *S*

The sample space not contained within A, i.e. *S* - *A*, is the **complement** of *A*.

Example: Let *S* represent a coin flip. If *A* represents heads, then the complement of A represents not heads, i.e. tails.



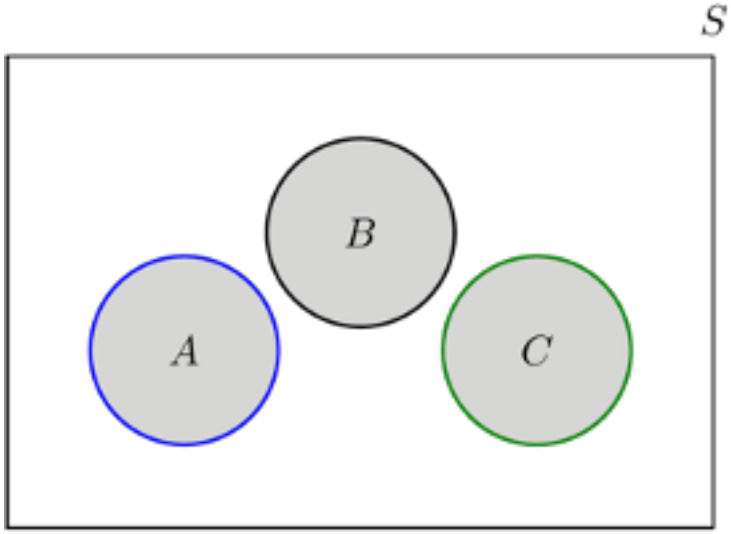
Disjoint sets do not have any shared elements

Let *S* be the result of a six-sided die roll

- Let *A* represent the dice rolls {1, 2}
- Let *B* represent the dice rolls {3, 4}
- Let *C* represent the dice rolls {5, 6}

A, B, and C are **disjoint**

- Sets that have shared elements (overlap) are called **joint** sets.

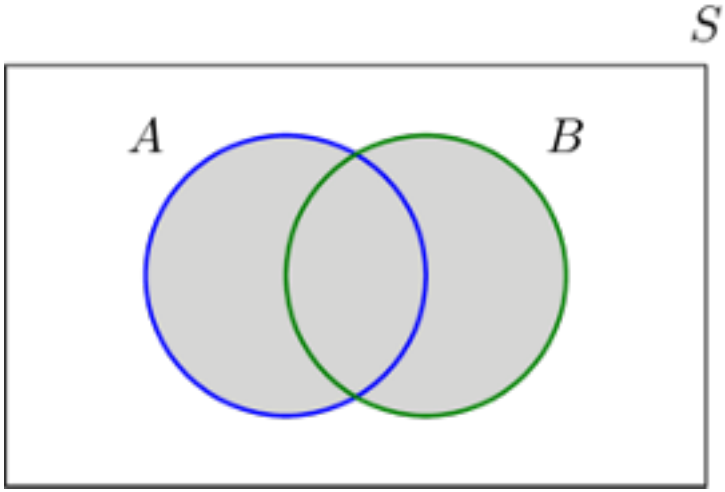


The union of joint sets contains all elements in either set

Let *S* be the result of a six-sided die roll

- Let *A* represent an even number roll, i.e. {2, 4, 6}
- Let *B* represent a die roll less than 4, i.e. {1, 2, 3}

Then the *A* U *B* is {1, 2, 3, 4, 6} and the complement of A U B is {5}

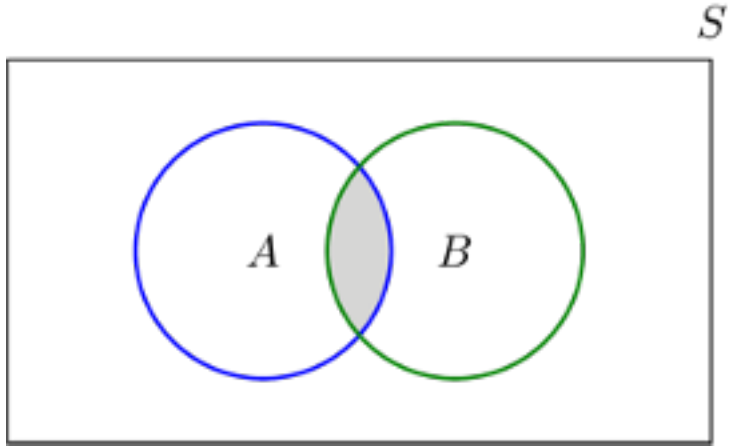


The intersection of joint sets contains all elements in both sets

Let *S* be the result of a six-sided die roll

- Let *A* represent an even number roll, i.e. {2, 4, 6}
- Let *B* represent a die roll less than 4, i.e. {1, 2, 3}

Then the intersection of *A* and *B* is {2}

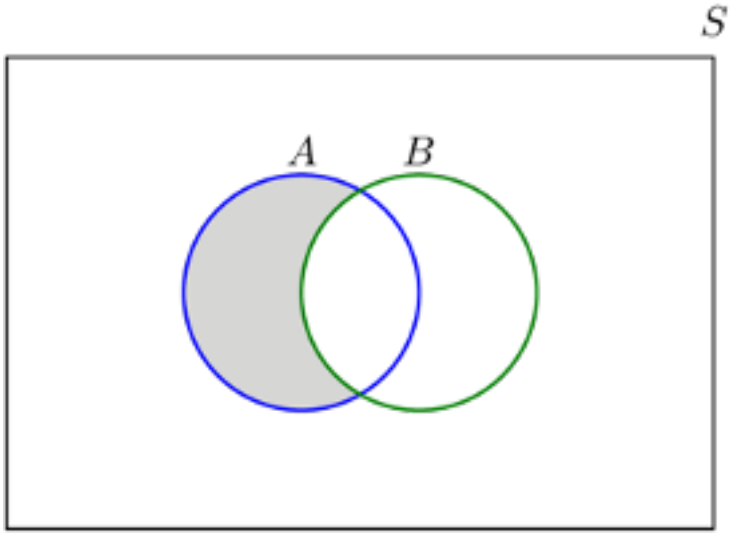


The difference of joint sets, *A* - *B*, contains the elements of *A* that are not in *B*

Let *S* be the result of a six-sided die roll

- Let *A* represent an even number roll, i.e. {2, 4, 6}
- Let *B* represent a die roll less than 4, i.e. {1, 2, 3}

Then *A* - *B* is {4, 6}



Probability and Unions of Events

Learning Objectives

- Recognize that probabilities only consists of numbers between 0 and 1
- Be able to calculate probabilities for unions of joint and disjoint events

What is probability?

The probability of an event A happening is

P(A) = Number of ways event A can happen / Total number of possible outcomes

Probabilities fall between 0 and 1.

Probability for Unions of Events

Given two events **A** and **B**, the probability of either happening, i.e. their union is

$$P(\mathbf{A} \text{ or } \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \cup \mathbf{B})$$

Probability Exercise: Rolling Odd or Even

Given a fair six-sided die:

- Let *A* represent rolling an odd number, i.e. {1, 3, 5}
  - Then P(A) = 3/6 = ½
- Let *B* represent rolling an even number, i.e. {2, 4, 6}
  - Then P(B) = 3/6 = ½
- Note that A and B are disjoint of each other
  - Then P(A and B) = 0

Then the probability of rolling an odd or even number is simply

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{1}{2} + \frac{1}{2} + 0 = 1$$