METIS

Lesson 5:

Rules for Derivatives

Introduction

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Lecture Overview:

Goals of the lecture:

1. Understand some of the rules for derivatives

Rules for Derivatives

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Rules for Derivatives

Definition:

Addition: $\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$

Multiplication: $\left(f(x) \cdot g(x)\right)' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Composition: $\left(f(g(x))\right)' = f'\left(g(x)\right) \cdot g'(x)$

Example for Addition-Subtraction

$$\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$$

$$\frac{d}{dx}h(x) = \frac{d}{dx}(\sin(x) + x^2)$$

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$$= \frac{d}{dx}(\sin(x)) + \frac{d}{dx}(x^2)$$

Example for Addition-Subtraction

$$\big(f(x) \pm g(x)\big)' = f'(x) \pm g'(x)$$

$$\frac{d}{dx}h(x) = \frac{d}{dx}(\sin(x) + x^2)$$

$$= \frac{d}{dx}(\sin(x)) + \frac{d}{dx}(x^2) = \cos(x) + 2x$$

Example for Products

$$\left(f(x) \cdot g(x)\right)' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{d}{dx} h(x) = \frac{d}{dx} (x \cdot cos(x))$$

Example for Products

$$\left(f(x) \cdot g(x)\right)' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{d}{dx} h(x) = \frac{d}{dx}(x \cdot cos(x)) = \frac{d}{dx}(x) \cdot \cos(x) + x \cdot \frac{d}{dx}(\cos(x))$$

Example for Products

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$$\frac{d}{dx}h(x) = \frac{d}{dx}(x \cdot cos(x)) = \frac{d}{dx}(x) \cdot \cos(x) + x \cdot \frac{d}{dx}(\cos(x))$$

$$= 1 \cdot \cos(x) - x \cdot \sin(x)$$

Example for Composition (Chain Rule)

$$\left(f\left(g(x)\right)\right)' = f'\left(g(x)\right) \cdot g'(x)$$

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$$\frac{d}{dx} f(g(x)) = \frac{d}{dx}(\sin(x^3 - x^2))$$

Example for Composition (Chain Rule)

$$\big(f(g(x))\big)' = f'\big(g(x)\big) \cdot g'(x)$$

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx}(\sin(x^3 - x^2)) = \frac{d}{ds}(\sin(g(x))) \frac{d}{dx}(x^3 - x^2)$$

Example for Composition (Chain Rule)

$$\big(f(g(x))\big)' = f'\big(g(x)\big) \cdot g'(x)$$

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx}(\sin(x^3 - x^2)) = \frac{d}{ds}(\sin(g(x))) \frac{d}{dx}(x^3 - x^2)$$

$$= \cos(x^3 - x^2)\,(3x^2 - 2x)$$

Problem 1:

Problem 1: Calculate h '(x)

$$h(x) = \frac{x}{x^2 - 2}$$

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Problem 1: Calculate h ’(x)

$$h(x) = \frac{x}{x^2-2}$$

$$f(x) = x$$

$$f'(x) = 1$$

$$g(x) = \frac{1}{x^2-2} = (x^2-2)^{-1}$$

$$g'(x) = -(x^2-2)^{-2} \cdot 2x = -2x(x^2-2)^{-2}$$

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x) =$$

$$1 \cdot \frac{1}{x^2-2} + x \cdot \left(-2x(x^2-2)^{-2}\right) = \frac{1}{(x^2-2)} - \frac{2x^2}{(x^2-2)^2} = \frac{x^2-2-2x^2}{(x^2-2)^2} = \frac{-x^2-2}{(x^2-2)^2}$$

Problem 2:

Problem 2: Calculate m '(x)

$m(x) = x \cdot \ln(\cos x)$

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Problem 2: Calculate m '(x)

$$m(x) = \underbrace{X}_{f(x)} \cdot \underbrace{\ln(\cos X)}_{g(h(x))}$$

$h(x) = \cos X$

$h'(x) = -\sin X$

$f(x) = X$

$f'(x) = 1$

$g(h(x)) = \ln(\cos X)$

$g(h(x))' = g'(h(x))\,h'(x) = \frac{1}{\cos X}\cdot(-\sin X) = -\tan X$

$m'(x) = f'(x)\,g(h(x)) + f(x)\cdot g(h(x))' = 1\cdot\ln(\cos X) + X\cdot(-\tan X) =$

$\ln(\cos X) - X\cdot\tan X$

Problem 3:

Problem 3: Calculate m ’(x)

$m(x): \ln(e^x - x)$

Problem 3:

Problem 3: Calculate m '(x)

$$m(x): \ln(e^x - x)$$

$$f(g(x)) \qquad g(x): e^x - x$$

$$g'(x): e^x - 1$$

$$f(g(x))': f'(g(x)) \cdot g'(x): \frac{1}{e^x - x} \cdot (e^x - 1): \frac{e^x - 1}{e^x - x}$$

QUESTIONS?