1) a) All my work was done by hand then copied to a digital format to ease reading when possible

Newton method:

Xn	F(Xn)	F'(Xn)	|Xn-Xn-1|
1	-1	3	N/A
1.333333333	-0.182605044	2	0.333333333
1.424635855	-0.008970892	1.807735033	0.091302522
1.429598359	-2.42112E-05	1.797988662	0.004962504
1.429611825	-2E-10	1.797962308	1.34657E-05
1.429611825	0	1.797962308	1E-10
1.429611825	0	1.797962308	0

Secant method:

Xn	F(Xn)	F'(Xn)	|Xn-Xn-1|
1	-1	N/A	N/A
2	0.772588722	1.772588722	1
1.564146656	0.225214974	1.255866808	0.435853344
1.384816354	-0.082546216	1.716169476	0.179330302
1.432915459	0.005929147	1.839438849	0.048099105
1.429692114	0.000144351	1.794655983	0.003223345
1.42961168	-2.596E-07	1.797883883	8.04336E-05
1.429611825	0	1.797962449	1.444E-07

Here are the equations that I used for my B and C parts

b) I used $E_n=X_n - r$ to calculate the error estimate. since I don't actually know the correct exact root I used the last iteration calculated instead of r in my code.

For the newton method:

Xn	Error estimate
1	0.429612
1.333333333	0.096278
1.424635855	0.004976
1.429598359	1.35E-05
1.429611825	9.87E-11
1.429611825	0
1.429611825	Value used as root, thus 0

For the secant method:

Xn	Error estimate
1	0.429612
2	0.570388
1.564146656	0.134535
1.384816354	0.044795
1.432915459	0.003304
1.429692114	8.03E-05
1.42961168	1.44E-07
1.429611825	Value used as root, thus 0

c) we know the error estimate is $E_n=X_n - r$ from part b, thus by using the newton formula

$X_{n+1} = X_n - f(X_n)/f'(x_n)$

$E_{n+1} + r = E_n + r - f(X_n)/f'(x_n)$

$E_{n+1} = E_n - f(X_n)/f'(x_n)$

If what is meant by the question is to find the relative error however, then the answer would be: $|X_{n+1} - X_n| / |X_{n+1}|$

|d)

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please enter x0: 1
please enter x1: 2
please enter max number of iterations: 6

Newton method iterates:

|      Xn       |     F(Xn)     |    F'(Xn)     |   |Xn-Xn-1|    |
| 1.0000000000  | -1.0000000000 | 3.0000000000  |      N/A      |
| 1.3333333333  | -0.1826050435 | 2.0000000000  | 0.3333333333  |
| 1.4246358551  | -0.0089708920 | 1.8077350333  | 0.0913025218  |
| 1.4295983589  | -0.0000242112 | 1.7979886624  | 0.0049625038  |
| 1.4296118246  | -0.0000000002 | 1.7979623077  | 0.0000134657  |
| 1.4296118247  | 0.0000000000  | 1.7979623075  | 0.0000000001  |
| 1.4296118247  | 0.0000000000  | 1.7979623075  | 0.0000000000  |
Newton method errors: [0.4296118247255556, 0.09627849139222233, 0.004975969629117394, 1.3465829812986385e-05, 9.869105532800404e-11, 0.0]
Newton method convergence rates: [1.980844580838516, 1.9956021413565381, 1.999869969985465, nan, nan]


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Secant method iterates:

|      Xn       |     F(Xn)     |    F'(Xn)     |   |Xn-Xn-1|    |
| 1.0000000000  | -1.0000000000 |      N/A      |      N/A      |
| 2.0000000000  | 0.7725887222  | 1.7725887222  | 1.0000000000  |
| 1.5641466559  | 0.2252149741  | 1.2558668084  | 0.4358533441  |
| 1.3848163539  | -0.0825462164 | 1.7161694759  | 0.1793303021  |
| 1.4329154592  | 0.0059291466  | 1.8394388489  | 0.0480991053  |
| 1.4296921139  | 0.0001443506  | 1.7946559830  | 0.0032233453  |
| 1.4296116803  | -0.0000002596 | 1.7978838829  | 0.0000804336  |
| 1.4296118247  | 0.0000000000  | 1.7979624493  | 0.0000001444  |
Secant method errors: [0.4296118247318659, 0.5703881752681341, 0.13453483120332188, 0.04479547084896951, 0.0033036344845749355, 8.028918980440558e-05, 1.444095529823386e-07]
Secant method convergence rates: [-5.096384163786625, 0.7613157914036462, 2.370688057022437, 1.4257859638231163, 1.7004285330264914, nan]
PS C:\Users\abdul\Desktop\numericHW2>
```

for my convergence calculation, since it was not explained in class I did some research and discussed the question with some other students, I decided to use this equation:

$$\alpha \approx \frac{\log |e_{n+1}/e_n|}{\log |e_n/e_{n-1}|} = \frac{\log |(x_{n+1}-r)/(x_n-r)|}{\log |(x_n-r)/(x_{n-1}-r)|}.$$

2)

2) a)

$$A_1 = \begin{bmatrix} -1 & 3 & 0 \\ -4 & -1 & 3 \\ 0 & -4 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -4 & 1 & 3 \\ 0 & \frac{13}{4} & -\frac{3}{4} \\ 0 & -4 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -4 & -1 & 3 \\ 0 & -4 & -1 \\ 0 & 0 & -\frac{25}{16} \end{bmatrix} = U$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{4} & -\frac{13}{16} & 1 \end{bmatrix} = L \qquad P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = P_{12}P_{23}$$

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} -1 & 3 & 0 \\ -4 & -1 & 3 \\ 0 & -4 & -1 \end{bmatrix}}_{A_1} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{4} & -\frac{13}{16} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} -4 & -1 & 3 \\ 0 & -4 & -1 \\ 0 & 0 & -\frac{25}{16} \end{bmatrix}}_{U}$$

$$A_2 = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 4 & 0 \\ 0 & 0 & 2 \\ 0 & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 4 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{bmatrix} = U$$

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 1 & 1 & 2 \\ 4 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix}}_{A_2} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 4 & 4 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{bmatrix}}_{U}$$

b) $Ax = b \rightarrow PAx = Pb,\ Ux = c$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{c} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix}}_{b} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 3 \end{bmatrix}$$

$c_1 = -4,\ c_2 = 0,\ c_3 = 4$

$Ux = c$

$$\begin{bmatrix} 4 & 4 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 4 \end{bmatrix} \qquad \begin{aligned} x_1 &= -2 \\ x_2 &= 1 \\ x_3 &= 2 \end{aligned}$$

3)

3)

$$A=\begin{bmatrix}2&-1&0\\-1&2&-1\\0&-1&-2\end{bmatrix}\rightarrow\begin{bmatrix}2&-1&0\\0&3/2&-1\\0&-1&-2\end{bmatrix}\rightarrow\begin{bmatrix}2&-1&0\\0&3/2&-1\\0&0&-\frac{8}{3}\end{bmatrix}=U$$

$$L=\begin{bmatrix}1&0&0\\-1/2&1&0\\0&-2/3&1\end{bmatrix}$$

$$U^{-1}=\left[\begin{array}{ccc|ccc}2&-1&0&1&0&0\\0&3/2&-1&0&1&0\\0&0&-8/3&0&0&1\end{array}\right]\rightarrow\left[\begin{array}{ccc|ccc}2&-1&0&1&0&0\\0&3/2&-1&0&1&0\\0&0&1&0&0&-\frac{3}{8}\end{array}\right]$$

$$\rightarrow\left[\begin{array}{ccc|ccc}2&-1&0&1&0&0\\0&1&0&0&2/3&-1/4\\0&0&1&0&0&-3/8\end{array}\right]\rightarrow\left[\begin{array}{ccc|ccc}1&0&0&1/2&1/3&-1/8\\0&1&0&0&2/3&-1/4\\0&0&1&0&0&-3/8\end{array}\right]$$

$$\underbrace{\begin{bmatrix}1/2&1/3&-1/8\\0&2/3&-1/4\\0&0&-3/8\end{bmatrix}}_{U^{-1}}$$

$$L^{-1}=\left(\begin{array}{ccc|ccc}1&0&0&1&0&0\\-1/2&1&0&0&1&0\\0&-2/3&1&0&0&1\end{array}\right)\rightarrow\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&0&1/2&1&0\\0&-\frac{2}{3}&1&0&0&1\end{array}\right]$$

$$L^{-1}=\begin{bmatrix}1&0&0\\1/2&1&0\\1/3&2/3&1\end{bmatrix}$$

$$A^{-1}=U^{-1}L^{-1}=\begin{bmatrix}\frac{1}{2}&1/3&-1/8\\0&2/3&-1/4\\0&0&-3/8\end{bmatrix}\begin{bmatrix}1&0&0\\1/2&1&0\\1/3&2/3&1\end{bmatrix}=\begin{bmatrix}5/8&1/4&-1/8\\1/4&1/2&-1/4\\-1/8&-1/4&-3/8\end{bmatrix}$$

4)

4) $f(x) = a + bx$

a)

$$\boxed{\begin{aligned} &\sum x_i = 105 \\ &\sum x_i^2 = 717.5 \\ &\sum y_i = 90.99 \\ &\sum x_i y_i = 560.87 \\ &n = 20 \end{aligned}}$$

$$\frac{\partial}{\partial a} \sum (y - a - bx)^2$$

$$= \boxed{\sum (y_i - a - bx_i) = 0}$$

$$\frac{\partial}{\partial b} \sum (y - a - bx)^2$$

$$\boxed{\sum (y_i x_i - a x_i - b x_i^2) = 0}$$

Thus after filling my values and rearranging the equations

$$\begin{bmatrix} 20 & 105 \\ 105 & 717.5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 90.99 \\ 560.87 \end{bmatrix}$$

$\boxed{a \approx 1.923 \quad b \approx 0.5}$ $\boxed{f(x) = 1.923 + 0.5x}$

b) $P(x) = cx^2 + dx + e$

$$\begin{aligned} &\sum x_i^2 = 717.5 \\ &\sum x_i^3 = 5512.5 \\ &\sum x_i^4 = 45166.625 \\ &\sum y_i x_i^2 = 4062.34 \end{aligned}$$

$$\frac{\partial}{\partial c} \sum (y - cx^2 - dx - e)^2$$

$$\boxed{\sum (y_i x_i^2 - c x_i^4 - d x_i^3 - e x_i^2) = 0} \quad ①$$

$$\frac{\partial}{\partial e} \sum (y - cx^2 - dx - e)^2 \rightarrow \boxed{\sum (y_i - c x_i^2 - d x_i - e) = 0} \quad ③$$

$$\frac{\partial}{\partial d} \sum (y - cx^2 - dx - e)^2 \rightarrow \boxed{\sum (y_i x_i - c x_i^3 - d x_i^2 - e x_i) = 0} \quad ②$$

$$\begin{bmatrix} 45166.625 & 5512.5 & 717.5 \\ 5512.5 & 717.5 & 105 \\ 717.5 & 105 & 20 \end{bmatrix} \begin{bmatrix} c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 4062.335 \\ 560.87 \\ 90.99 \end{bmatrix}$$

$$\boxed{\begin{aligned} &c \approx -0.0686 \quad d \approx 1.22 \quad e \approx 0.603 \\ &P(x) = -0.0686x^2 + 1.22x + 0.603 \end{aligned}}$$

c) $f(x) = k + m \ln(x)$

$$\frac{\partial}{\partial k} \sum \left[y - k - m\ln(x)\right]^2$$

$$\boxed{\sum \left[y_i - k - m\ln(x_i)\right] = 0}$$

$$\frac{\partial}{\partial m} \sum \left[y - k - m\ln(x)\right]^2 \rightarrow \boxed{\sum \left[y\ln(x) - k\ln(x) - m\ln^2(x)\right] = 0}$$

$\sum \ln(x_i) = 28.4727$

$\sum \ln^2(x_i) = 53.0842$

$\sum y_i \ln(x_i) = 153.95$

$$\begin{bmatrix} 20 & 28.4727 \\ 28.4727 & 53.0842 \end{bmatrix} \begin{bmatrix} k \\ m \end{bmatrix} = \begin{bmatrix} 90.99 \\ 153.95 \end{bmatrix}$$

$k \simeq 1.78 \quad m \simeq 1.945 \quad f(x) = 1.78 + 1.945\ln(x)$

4)d)