BLG202E Numerical Methods in Comp. Eng.

Spring 2023 - Homework II

Due: April 19, 2023

By turning in this assignment, I agree by the ITU honor code and declare that all of this is my own work.

Important Notes

- You are required to submit a PDF document and Python source codes to Ninova before the deadline.
- Solve the questions by hand with necessary explanations of your steps. You may write your answers to a paper by hand, scan the papers and transform the scans to a PDF document. In that case, please make sure that the scans are readable.
- For questions 1 and 4, write necessary `Python` programs and add the screenshots of the execution results to the document. Make sure that the output of the programs are appropriately structured. Submit the `Python` codes for questions 1 and 4 as well.
- Please make sure that you write your full name and student identification number to every file you submit.
- If you have any questions, please contact Res. Asst. Sümeyye Öztürk via ozturks20@itu.edu.tr.

Question 1

Let the equation $f(x) = 4ln(x) - x$. Assume initial values $x_0 = 1$ for the *Newton* method and $x_0 = 1$, $x_1 = 2$ for the *Secant* method.

a) Perform the first 6 iterations for both methods. For each n, write $x_n, f(x_n), f'(x_n)$, and $|x_n - x_{n-1}|$ in a table format.

b) Perform error estimation for the result obtained.

c) Write the equation that relates the error between the *n.* and *(n + 1).* steps for the error analysis of the *Newton* method.

d) Additionally, write a `Python` program to implement the *Newton* method and the *Secant* method for solving the equation $f(x) = 4ln(x) - x$, where $f'(x) = 4/x - 1$. The program should take in the initial values and the maximum number of iterations as input and output a list of the iterates for each method. Implement the error estimation and convergence rate calculation functions as well.

Question 2

Let us consider the matrices.

$$A_1 = \begin{bmatrix} -1 & 3 & 0 \\ -4 & -1 & 3 \\ 0 & -4 & -1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix} \tag{1}$$

You are expected to perform all of the computations by hand.

a) Compute the factorizations of the form $PA = LU$ for each one of A_1 and A_2 by applying the LU *factorization* algorithm with partial pivoting.

b) Use the factorization computed in part (a) to solve the linear system $A_2 x = [3 \; -4 \; -2]^T$.

Question 3

Find the inverse of the matrix A using LU decomposition.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & -2 \end{bmatrix} \tag{2}$$

Question 4

i	1	2	3	4	5	6	7	8	9	10
x_i	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
y_i	0.72	1.63	1.88	3.39	4.02	3.89	4.25	3.99	4.68	5.03
i	11	12	13	14	15	16	17	18	19	20
x_i	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
y_i	5.27	4.82	5.67	5.95	5.72	6.01	5.5	6.41	5.83	6.33

a) Find the coefficients a and b of the linear function $l(x) = a + bx$ that best represents the given dataset in terms of least squares.

b) Find the coefficients c, d, and e of the second-degree polynomial $p(x) = cx^2 + dx + e$ that best represents the given dataset in terms of least squares.

c) Find the coefficients k and m of the logarithmic function $f(x) = k + mln(x)$ that best represents the given dataset in terms of least squares.

d) Plot the given data points and the estimated polynomials in your Python program.