2) a)

$$A_1 = \begin{bmatrix} -1 & 3 & 0 \\ -4 & -1 & 3 \\ 0 & -4 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -4 & 1 & 3 \\ 0 & \frac{13}{4} & -\frac{3}{4} \\ 0 & -4 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -4 & -1 & 3 \\ 0 & -4 & -1 \\ 0 & 0 & -\frac{25}{16} \end{bmatrix} = U$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{4} & -\frac{13}{16} & 1 \end{bmatrix} = L \qquad P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = P_{12}P_{23}$$

$$\underset{P}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}} \underset{A_1}{\begin{bmatrix} -1 & 3 & 0 \\ -4 & -1 & 3 \\ 0 & -4 & -1 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{4} & -\frac{13}{16} & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} -4 & -1 & 3 \\ 0 & -4 & -1 \\ 0 & 0 & -\frac{25}{16} \end{bmatrix}}$$

$$A_2 = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 4 & 0 \\ 0 & 0 & 2 \\ 0 & -2 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 4 & 4 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{bmatrix} = U$$

$$\underset{P}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}} \underset{A_2}{\begin{bmatrix} 1 & 1 & 2 \\ 4 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 0 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 4 & 4 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{bmatrix}}$$

b) $Ax = b \rightarrow PAx = Pb, \; Ux = c$

$$\underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 0 & 1 \end{bmatrix}} \underset{c}{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}} = \underset{P}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}} \underset{b}{\begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix}} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 3 \end{bmatrix}$$

$c_1 = -4, \; c_2 = 0, \; c_3 = 4$

$Ux = c$

$$\begin{bmatrix} 4 & 4 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 4 \end{bmatrix}$$

$x_1 = -2$

$x_2 = 1$

$x_3 = 2$

3)

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & -2 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3/2 & -1 \\ 0 & -1 & -2 \end{bmatrix} \to \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3/2 & -1 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}$$

$$U^{-1} = \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 & 1 & 0 \\ 0 & 0 & -8/3 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{3}{8} \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2/3 & -1/4 \\ 0 & 0 & 1 & 0 & 0 & -3/8 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} 1/2 & 1/3 & -1/8 \\ 0 & 2/3 & -1/4 \\ 0 & 0 & -3/8 \end{matrix}}_{U^{-1}} \\ 0 & 0 & 1 & \end{array}\right]$$

$$L^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ -1/2 & 1 & 0 & 0 & 1 & 0 \\ 0 & -2/3 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & 1 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\longleftarrow$$

$$L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/3 & 2/3 & 1 \end{bmatrix}$$

$$A^{-1} = U^{-1}L^{-1} = \begin{bmatrix} \frac{1}{2} & 1/3 & -1/8 \\ 0 & 2/3 & -1/4 \\ 0 & 0 & -3/8 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/3 & 2/3 & 1 \end{bmatrix} = \begin{bmatrix} 5/8 & 1/4 & -1/8 \\ 1/4 & 1/2 & -1/4 \\ -1/8 & -1/4 & -3/8 \end{bmatrix}$$

4) $f(x) = a + bx$

a) $\boxed{\begin{aligned} &\sum x_i = 105 \\ &\sum x_i^2 = 717.5 \\ &\sum y_i = 90.99 \\ &\sum x_i y_i = 560.87 \\ &n = 20 \end{aligned}}$

$$\frac{\partial}{\partial b} \sum (y - a - bx)^2$$

$$\boxed{\sum (y_i x_i - a x_i - b x_i^2) = 0}$$

$$\frac{\partial}{\partial a} \sum (y - a - bx)^2$$

$$= \boxed{\sum (y_i - a - b x_i) = 0}$$

Thus after filling my values and rearranging the equations

$$\begin{bmatrix} 20 & 105 \\ 105 & 717.5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 90.99 \\ 560.87 \end{bmatrix}$$

$\boxed{a \approx 1.923 \quad b \approx 0.5}$ $\boxed{f(x) = 1.923 + 0.5x}$

b) $P(x) = cx^2 + dx + e$

$\sum x_i^2 = 717.5$
$\sum x_i^3 = 5512.5$
$\sum x_i^4 = 45166.625$
$\sum y_i x_i^2 = 4062.34$

$$\frac{\partial}{\partial c} \sum (y - cx^2 - dx - e)^2$$

$$\boxed{\sum (y_i x_i^2 - c x_i^4 - d x_i^3 - e x_i^2) = 0} \text{ — } 1$$

$$\frac{\partial}{\partial e} \sum (y - cx^2 - dx - e)^2 \rightarrow \boxed{\sum (y_i - c x_i^2 - d x_i - e) = 0} \text{ — } 3$$

$$\frac{\partial}{\partial d} \sum (y - cx^2 - dx - e)^2 \rightarrow \boxed{\sum (y_i x_i - c x_i^3 - d x_i^2 - e x_i) = 0} \text{ — } 2$$

$$\begin{bmatrix} 45166.625 & 5512.5 & 717.5 \\ 5512.5 & 717.5 & 105 \\ 717.5 & 105 & 20 \end{bmatrix} \begin{bmatrix} c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 4062.335 \\ 560.87 \\ 90.99 \end{bmatrix}$$

$c \approx -0.0686 \quad d \approx 1.22 \quad e \approx 0.603$

$P(x) = -0.0686x^2 + 1.22x + 0.603$

c) $f(x) = k + m\ln(x)$

$$\frac{\partial}{\partial k} \sum\left[y - k - m\ln(x)\right]^2$$

$$\sum\left[y_i - k - m\ln(x_i)\right] = 0$$

$$\frac{\partial}{\partial m} \sum\left[y - k - m\ln(x)\right]^2 \rightarrow \boxed{\sum\left[y\ln(x) - k\ln(x) - m\ln^2(x)\right] = 0}$$

$\sum \ln(x_i) = 28.4727$
$\sum \ln^2(x_i) = 53.0842$
$\sum y_i \ln(x_i) = 153.95$

$$\begin{bmatrix} 20 & 28.4727 \\ 28.4727 & 53.0842 \end{bmatrix} \begin{bmatrix} k \\ m \end{bmatrix} = \begin{matrix} 90.99 \\ 153.95 \end{matrix}$$

$k \simeq 1.78 \quad m \simeq 1.945 \quad f(x) = 1.78 + 1.945\ln(x)$