BLG202E Numerical Methods in Comp. Eng.

Spring 2023 - Homework III

Due: May 22, 2023

By turning in this assignment, I agree by the ITU honor code and declare that all of this is my own work.

Important Notes

- You are required to submit a PDF document and Python source codes to Ninova before the deadline.
- Solve questions 1, 2, 3, and 4 by hand with necessary explanations of your steps. You may write your answers to a paper by hand, scan the papers and add them to the PDF document. In that case, please make sure that the scans are readable.
- For part (b) of question 2, and part (b) of question 3 write the necessary Python programs and add the screenshots of the execution results to the document. Make sure that the output of the program is appropriately structured. Submit the Python code for part (b) of question 2, and part (b) of question 3.
- Please make sure that you write your full name and student identification number on every file you submit.
- If you have any questions, please contact Nursena Bölük via boluk21@itu.edu.tr.

Question 1

Given the A matrix

$$A = \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix}$$

The power method has been applied to the matrix A. The result is a long list of vectors that seem to settle down to a vector of the form $\begin{pmatrix} h & 1 \end{pmatrix}^T$, where $|h| < 1$. Approximately, what is the largest eigenvalue in terms of that number h?

Question 2

Given the data points

x	0	1	2	4	6
$f(x)$	1	9	23	93	259

a) Construct divided difference table.

b) Construct a program implementing the divided difference for part a with Python

c) Determine f at 4.2, using Newton's interpolation polynomial (Hint: using the table in part a, find interpolating polynomial via divided difference method).

Question 3

Given the data points

x	-1.2	0.3	1.1
y	-5.76	-5.61	-3.69

a) Find a quadratic polynomial ($ax^2 + bx + c$) interpolating these point via:

i) setting a linear system

ii) Lagrange Interpolation method

b) Construct a program implementing the Lagrange Interpolation method for part a.ii with Python

c) Determine y at $x = 0$.

Question 4

For a function f, the forward-divided difference (i.e., divided difference) is given by

$$
\begin{array}{lll}
x_0 = 0.0 & f[x_0] = ? & \\
 & f[x_0, x_1] = ? & \\
x_1 = 0.4 & f[x_1] = ? & f[x_0, x_1, x_2] = \frac{50}{7} \\
 & f[x_1, x_2] = 10 & \\
x_2 = 0.7 & f[x_2] = 6 &
\end{array}
$$

a) Find the missing entries.

b) Construct the polynomial when the data is given in the order of x_0, x_1, x_2.

c) Construct the polynomial when the data is given in the order of x_2, x_1, x_0.

d) Are the polynomials that you found in part (a) and part (b) the same? Justify your answers.