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1)

$$A = \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} \quad \text{using} \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} = x_0$$

$$x_1 = \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 24 \\ 57 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 24 \\ 57 \end{bmatrix} = \begin{bmatrix} 234 \\ 195 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 234 \\ 195 \end{bmatrix} = \begin{bmatrix} 2160 \\ 4401 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2160 \\ 4401 \end{bmatrix} = \begin{bmatrix} 19602 \\ 39447 \end{bmatrix}$$

I can continue but this enough for an approximation

$$\lambda = \frac{x_4^T x_5}{x_4^T x_4} \qquad \begin{bmatrix} 2160 & 4401 \end{bmatrix} \begin{bmatrix} 19602 \\ 39447 \end{bmatrix} = 215946567$$

$$\lambda = \frac{215946567}{24034401} \qquad \begin{bmatrix} 2160 & 4401 \end{bmatrix} \begin{bmatrix} 2160 \\ 4401 \end{bmatrix} = 24034401$$

$$\lambda = 8.984894901\ldots$$

$\lambda_1 \simeq 9$ if I continued with further iterations

its eigen vector would be $e_1 = \begin{pmatrix} 19602/39447 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.4969199179\ldots \\ 1 \end{pmatrix}$

$$e_1 \simeq \begin{pmatrix} 0.5 \\ 1 \end{pmatrix}$$

can also be solved in the following method:

$$\begin{bmatrix}5 & 2\\4 & 7\end{bmatrix}\underbrace{\begin{bmatrix}0\\1\end{bmatrix}}_{x_0} = \begin{bmatrix}2\\7\end{bmatrix} = 7\begin{bmatrix}0.285714\ldots\\1\end{bmatrix} = x_1$$

$$x_2 = \begin{bmatrix}5 & 2\\4 & 7\end{bmatrix}\begin{bmatrix}2/7\\1\end{bmatrix} = \begin{bmatrix}24/7\\57/7\end{bmatrix} = 8.14\ldots\begin{bmatrix}9/19\\1\end{bmatrix} = x_2$$

$$x_3 = \begin{bmatrix}5 & 2\\4 & 7\end{bmatrix}\begin{bmatrix}9/19\\1\end{bmatrix} = \begin{bmatrix}83/19\\169/19\end{bmatrix} = 8.89\ldots\begin{bmatrix}83/169\\1\end{bmatrix} = x_3$$

$$x_4 = \begin{bmatrix}5 & 2\\4 & 7\end{bmatrix}\begin{bmatrix}83/169\\1\end{bmatrix} = \begin{bmatrix}753/169\\1515/169\end{bmatrix} = 8.96\ldots\begin{bmatrix}251/505\\1\end{bmatrix} = x_4$$

$$x_5 = \begin{bmatrix}5 & 2\\4 & 7\end{bmatrix}\begin{bmatrix}251/505\\1\end{bmatrix} = \begin{bmatrix}453/101\\4539/505\end{bmatrix} = 8.988\begin{bmatrix}0.499\ldots\\1\end{bmatrix} = x_5$$

again $\lambda_1 \simeq 9$ and $e_1 \simeq \begin{pmatrix}0.5\\1\end{pmatrix}$

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2)

	x_1	x_2	x_3	x_4	x_5
x	0	1	2	4	6
$f(x)$	1	9	23	93	259
	f_1	f_2	f_3	f_4	f_5

a)

x	$f(x)$	1st	2nd	3rd	4th
0	1	$\frac{9-1}{1-0}=8$	$\frac{14-8}{2-0}=3$	$\frac{7-3}{4-0}=1$	0
1	9	$\frac{23-9}{2-1}=14$	$\frac{35-14}{4-1}=7$	$\frac{12-7}{6-1}=1$	
2	23	$\frac{93-23}{4-2}=35$	$\frac{83-35}{6-2}=12$		
4	93	$\frac{259-93}{6-4}=83$			
6	259				

4 stages, thus; $f_4(x) = b_1 + b_2(x-x_1) + b_3(x-x_1)(x-x_2) + b_4(x-x_1)(x-x_2)(x-x_3)$
$+ b_5(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

$$\boxed{\begin{aligned}b_1&=1\\b_2&=8\\b_3&=3\\b_4&=1\\b_5&=0\end{aligned}}$$

thus the equation is: $f_4(x) = 1 + 8(x-0) + 3(x)(x-1) + (x)(x-1)(x-2)$

$f_4(x) = 1 + 8x + 3x^2 - 3x + x^3 - 2x^2 - x^2 + 2x$

$f_4(x) = x^3 + 7x + 1$ at 4.2

$(4.2)^3 + 7(4.2) + 1 = 104.488$

to check using easy interpolation

$$\frac{6-4}{4.2-4} = \frac{259-93}{x-93}$$

$x = 109.6$ kinda close

3)

	x_1	x_2	x_3
x	-1.2	0.3	1.1
y	-5.76	-5.61	-3.69
	y_1	y_2	y_3

a) i)

$ax^2+bx+c=f(x)$

$$\left.\begin{array}{l} ① - 1.44a+1.2b+c=-5.76 \\ ② - 0.09a+0.3b+c=-5.61 \\ ③ - 1.21a+1.1b+c=-3.69 \end{array}\right\} \begin{bmatrix} 1.44 & -1.2 & 1 \\ 0.09 & 0.3 & 1 \\ 1.21 & 1.1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -5.76 \\ -5.61 \\ -3.69 \end{bmatrix}$$

solving the matrix by augmentation we get:

$$\left[\begin{array}{ccc|c} 1.44 & -1.2 & 1 & -5.76 \\ 0.09 & 0.3 & 1 & -5.61 \\ 1.21 & 1.1 & 1 & -3.69 \end{array}\right] = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -6 \end{bmatrix}$$

so the polynomial is $\boxed{x^2+x-6=f(x)}$

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a) ii) $P(x)=\sum_{i=1}^{3} y_i \cdot L_i(x)$

$+ L_1(x) = \frac{x-0.3}{-1.2-0.3} \cdot \frac{x-1.1}{-1.2-1.1} = \frac{x^2-1.1x-0.3x+0.33}{3.45} = \frac{x^2-1.4x+0.33}{3.45}$

$+ L_2(x) = \frac{x+1.2}{0.3+1.2} \cdot \frac{x-1.1}{0.3-1.1} = \frac{x^2+0.1x-1.32}{-1.2}$

$+ L_3(x) = \frac{x+1.2}{1.1+1.2} \cdot \frac{x-0.3}{1.1-0.3} = \frac{x^2+0.9x-0.36}{1.84}$

$P(x)= + (-5.76)\frac{x^2-1.4x+0.33}{3.45} + (-5.61)\frac{x^2+0.1x-1.32}{-1.2} + (-3.69)\frac{x^2+0.9x-0.36}{1.84}$

$P(x)=-1.67x^2+\underline{2.34x}-0.55+4.675x^2+\underline{0.4675x}-\underline{6.0775}-2.01x^2-\underline{1.80x}+\underline{0.72}$

$P(x) \simeq x^2+x-6$ → if I use the full values without rounding this is the equation that I get

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c) by subbing in the equation, we get $P(x)=-6$

4)

x	$f(x)$	1st	2nd
$x_0 = 0$	?①		
		?③	
			$\frac{50}{7}$
$x_1 = 0.4$	?②		
		10	
$x_2 = 0.7$	6		

?① = O
?② = N
?③ = M

a) $\frac{50}{7} = \frac{10-M}{0.7-0} \rightarrow \boxed{M=5}$, $10 = \frac{6-N}{0.3} \rightarrow \boxed{N=3}$

$5 = \frac{3-O}{0.4} \rightarrow \boxed{O=1}$

b)

x	$f(x)$	1st	2nd
$x_0 = 0$	1		
		5	
			$\frac{50}{7}$
$x_1 = 0.4$	3		
		10	
$x_2 = 0.7$	6		

$b_1 = 1$, $b_2 = 5$, $b_3 = \frac{50}{7}$

2 stages: $f_2(x) = b_1 + b_2(x - x_0) + b_3(x-x_0)(x-x_1)$

$= 1 + 5x + \frac{50}{7}(x)(x - \frac{2}{5})$

$= \frac{50}{7}x^2 - \frac{20}{7}x + 5x + 1 = \boxed{\frac{50}{7}x^2 + \frac{15}{7}x + 1}$

c)

x	$f(x)$	1st	2nd
$X_0 = 0.7$	6		
		$\frac{3-6}{0.4-0.7} = 10$	
			$\frac{50}{7}$
$X_1 = 0.4$	3		
		5	
$X_2 = 0$	1		

$b_1 = 6$, $b_2 =$, $b_3 = \frac{50}{7}$

2 stages, same equation thus;

$f_2(x) = 6 + 10(x - 0.7) + \frac{50}{7}(x-0.7)(x-0.4)$

$f_2(x) = 6 + 10x - 7 + \frac{50}{7}x^2 - \frac{20}{7}x - 5x + 2$

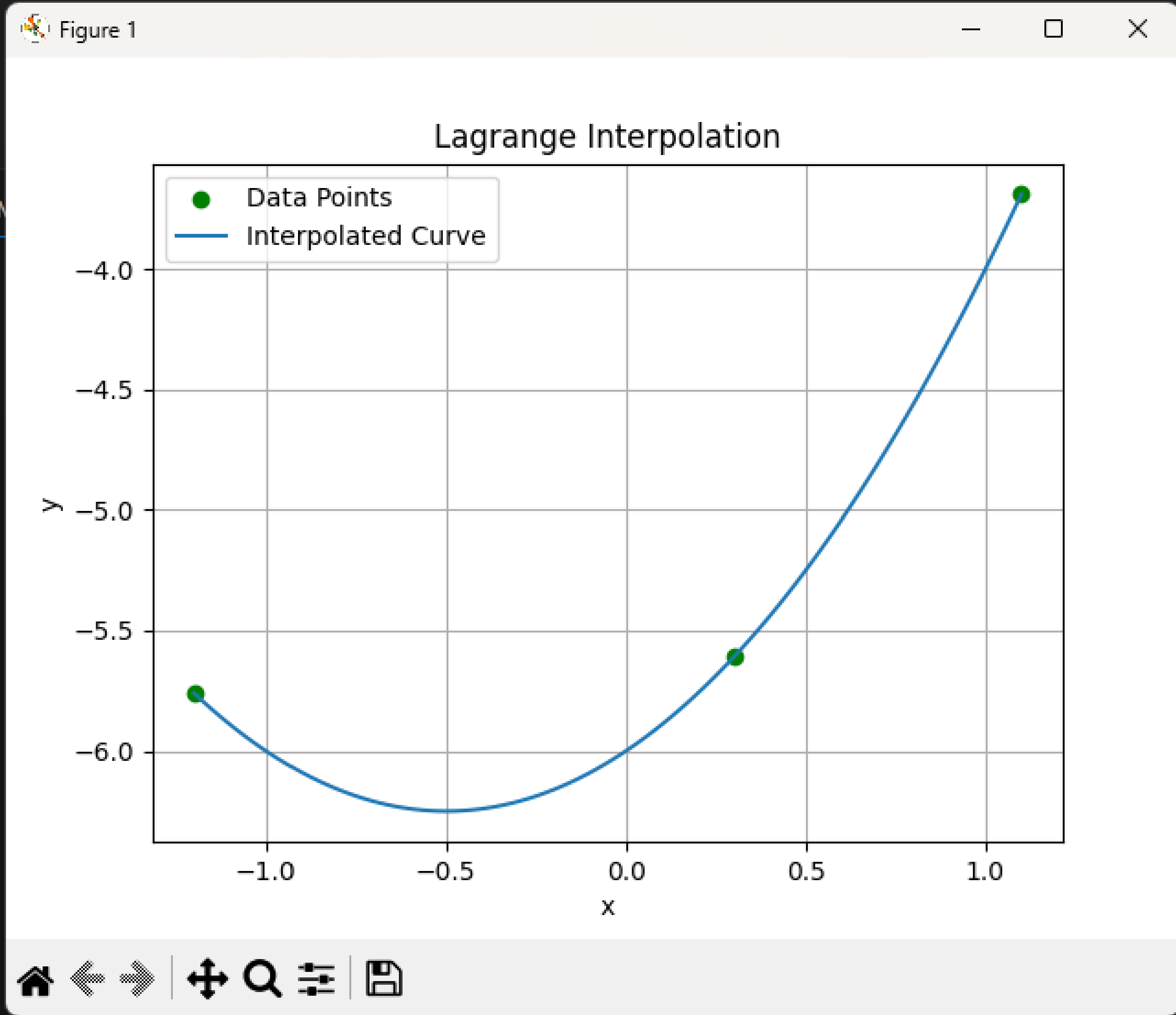
$f_2(x) = 1 + \frac{15}{7}x + \frac{50}{7}x^2$

d) The polynomials are the same, despite us changing the orders, the rating taken as we calculate as we progress up the table stay the same, since if x flips so does $f(x)$ so the ending value stays the same despite different arrangement.

```
        return sum
```

PROBLEMS   OUTPUT   DEBUG CONSOLE   TERM

```
PS C:\Users\abdul\Desktop\numtest> &
give the value to interpolate at: 0
value:  -6.0
```



```
[Running] python -u "c:\Users\abdul\Desktop\numerical_assignments\HW3\q2.py"
X   f(x)    1st    2nd    3rd    4th
0     1     8      3      1      0
1     9     14     7      1
2     23    35     12
4     93    83
6     259
##################################################################
part c: the value at 4.2 is 104.488
```