Linear Bèzier curves:

$$B(t) = P_0 + t(P_1 - P_0) \quad ; \quad 0 \leq t \leq 1$$

displacement vector from P_0 to P_1. $\Rightarrow B(t) = (1-t)P_0 + tP_1$.

Quadratic Bèzier curves:

$$B(t) = (1-t)[(1-t)P_0 + tP_1] + t[(1-t)P_1 + tP_2] \quad ; \quad 0 \leq t \leq 1$$

$$B'(t) = 2(1-t)(P_1 - P_0) + 2t(P_2 - P_1)$$

tangents to the curves at P_0 and P_2 intersect at P_1.

$$B(t) = (1-t)^2 P_0 + 2(1-t)tP_1 + t^2 P_2$$

cubic Bèzier Curves:

$$B(t) = \underbrace{(1-t)B_{P_0, P_1, P_2}(t) + tB_{P_1, P_2, P_3}(t)}_{\text{Quadratic Bèzier curves}}$$

$$B(t) = (1-t)^3 P_0 + 3(1-t)^2 tP_1 + 3(1-t)t^2 P_2 + t^3 P_3$$

Recursive Definitions:

$B_{P_0 P_1 P_2 \cdots P_k} \rightarrow$ Bèzier curves denoted by points $P_0 P_1 P_2 \cdots P_k$

$$B_{P_0}(t) = P_0$$

$$B_{P_0 P_1 \cdots P_n}(t) = (1-t) B_{P_0 P_1 \cdots P_{n-1}}(t) + tB_{P_1 P_2 \cdots P_n}(t)$$

Explicit Definition:

$$B(t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} \cdot t^i P_i$$

$$B(t) = (1-t)^n P_0 + \binom{n}{1}(1-t)^{n-1} tP_1 + \cdots + \binom{n}{n-1}(1-t)t^{n-1} P_{n-1} + t^n P_n$$