**TOPIC 1**

# BASIC IDEAS ON LOGIC AND FUZZY SETS

FUZZY

1. Introduction to the idea of SD
2. The multi-valued logic
3. Basic definitions of theory of SD
4. Representation Theorem
5. Principle Extension

**INTRODUCTION**

Fuzzy sets were introduced by Zadeh in 1965

for processing / handling information and data affected

uncertainty / probabilistic imprecision.

- LAZadeh, Fuzzy Sets, Information and Control, 8 (1965)

  338-353.

They were designed for represented mathematically

uncertainty and vagueness and provide tools

formalized to work with the inherent imprecision

many problems.

However the story begins much Fuzzy Logic

before...

You really have to go back to Aristotle who introduced

called laws of thought as the basis for

DEVELOP A concise theory of logic and

subsequently mathematics.

**THE law of excluded**

This "Basic law of thought" states that any proposition alone It can be true or false and that no other intermediate truth value is allowed.

Aristotle himself and even Parminedes (300 BC), who proposed the first version of this law, and found serious and immediate objections (contingent propositions)

Heraclito proposed things that could be simultaneously true and false. Generally formulated many cases where this law was false

Plato who would put the "cornerstone" of Fuzzy Logic stating that "there is a third region between the true and the false where opposites are presented together "

THE Lukasiewicz valued logic

One of the first systematic formulations of

bivalued alternative to the logic of Aristotle was formulated by

J. Lukasiewicz between 1917 and 1920.

This author introduced a third truth value, "possible", and

consequently he made a trivaluada logic.

Lukasiewicz also assigned a numerical value between 0 and 1 to

possible term and build relevant mathematics

that logic. Lukasiewicz proposed a full notation and

axiomatic system to derive what he called "mathematical

modern ".

**Valued logic of Lukasiewcz**

Future contingent propositions (propositions future

things not know if they will be V or F):

"I win the lottery tomorrow"

w (p) • { v, f, n}

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | ¬A | | TO<br>^<br>B | VFN A | | | | v<br>B | VFN A | | | | •<br>B | VFN | | |
| V | F | | vvfnvvvvvvfn | | | | | | | | | | | | | |
| F | V | | F | fff | | | | fvfn | | | | | fvvv | | | |
| nnnnfnnvnnnvn V ** | | | | | | | | | | | | | | | | |

Symbolic truth values can be given numerically

in different ways.

It should be mentioned that DE has also proposed Knutz

1968-1973 three logic similar to that of Lukasiewicz values.

Knuth argued that its logic allowed a development of

sleeker than bivalued math logic.

**OTHER LOGICAS multivalued**

Kleene logic

undecidable mathematical propositions

w (p) • { v, f, i}

| TO | ¬A | | TO ^ B | vfi | A | | | v B | vf | | i | | TO • B | vf | | i |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| v | F | | vvfi | | | | | vvvvvvf | | | | | | | | i |
| F | v | | F | fff | | | | fvf | | | i | | fvvv | | | |
| i | i | | i | ifi | | | | ivi | | | i | | ivi | | | i |

epistemological application • i • it is not known.

The only difference with valued logic of Lukasiewicz is that

here

If A and B are undecidable A • B is indecidible

While there

If A and B are future contingent A • B is true

I win the lottery tomorrow • I'll buy a car,

It is true "

<u>Bochvar logic</u>

Paradoxical semantic (propositions carrying a denial

whose implicit statement implies its falsity).

w (q) = {v, f, p}

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| TO | ¬A | | | TO ^ B | VFP A | | | v B | VFP A | | | | • B | VFP | | |
| v | F | | | vvfpvvvpvvfp | | | | | | | | | | | | |
| F | v | | | F | FFP | | | fvfp | | | | | fvvp | | | |
| ppppppvpppvpp | | | | | | | | | | | | | | | | |

Belnap logic

Representative of the 4 logical truth values. oriented

the deduction of truth from a knowledge base

B: knowledgebase

Q: proposition

W (p) = v sii {B | p, B | / - ¬p}

W (p) = f sii {B | / p, B | - ¬p}

W (p) = c sii {B | p, B | - ¬p}

W (p) = {B i iff | / p, B | / - ¬p}

Can pass three states as v, f, c and f i gets in,

Hypothesis using the closed world.

Lukasiewicz later explored the possibility of handling Logical four, five, .... truth values, reaching the conclusion that there was a formal impediment to derivation of an infinite-valued logic.

This would be completely logical formalized by 1930.

Lukasiewicz believed that logic and infinite trivalorada valued were the most interesting from the point of view their properties, although the tetravalorada was the most easily Aristoteleans adaptable to classic principles.

Infinitely valued Lukasiewicz logic

w (q) • [ 0.1]

a degree of truth is given to proposals:

w (q): 0: completely false

w (q) = 1: absolutely true

w (¬q) = 1 w (q)

w (q • r) = min {w (q), w (r)}

w (q • r) = max {w (q), w (r)}

w (q • r) = min {1, 1-w (q) + w (r)}

min • { t-rules}

max • { t-conorms}

min (1, 1 -r $_{\text{one}}$ + r $_{2}$) • { } function implication

You can get other logical infinitely valued

using different operators of these families.

- Fuzzy set theory
- fuzzy logic (fuzzy).

**Some applications valued logics**

It is not a mathematical or logical game but have

Actual applications:

Linguistics Treatment of assumptions. For example, to say

"The current prime minister is the Galician"

you are assuming that Spain has a president

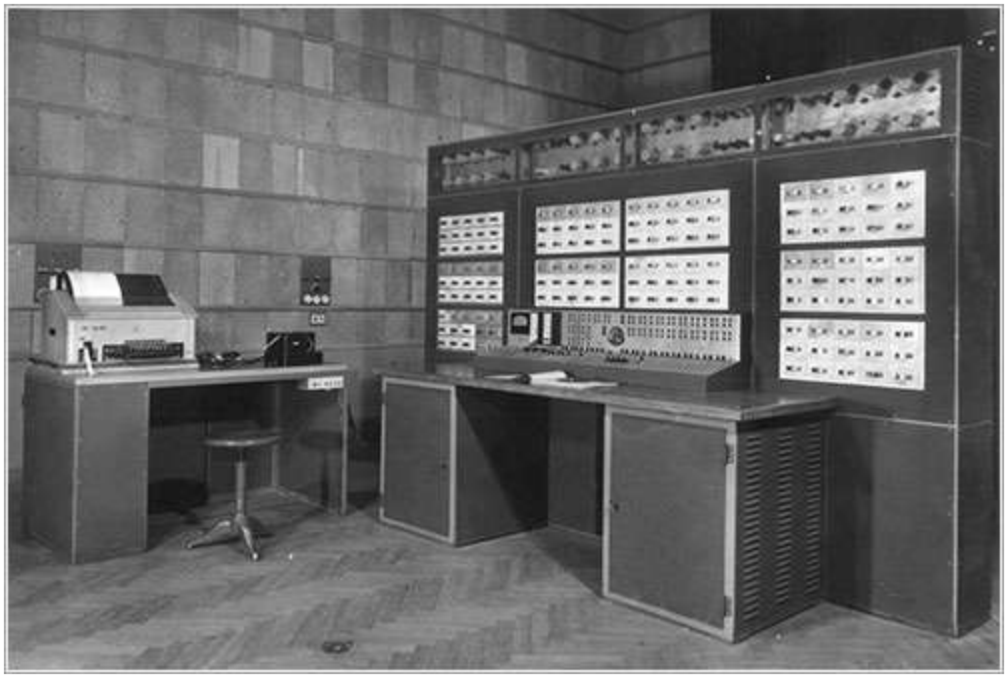
government.

Hardware design: N-valued logic to design and verify

circuits with n states.

Maths: Inaccurate precise handling entities.

**Curiosity: The computer Setun**



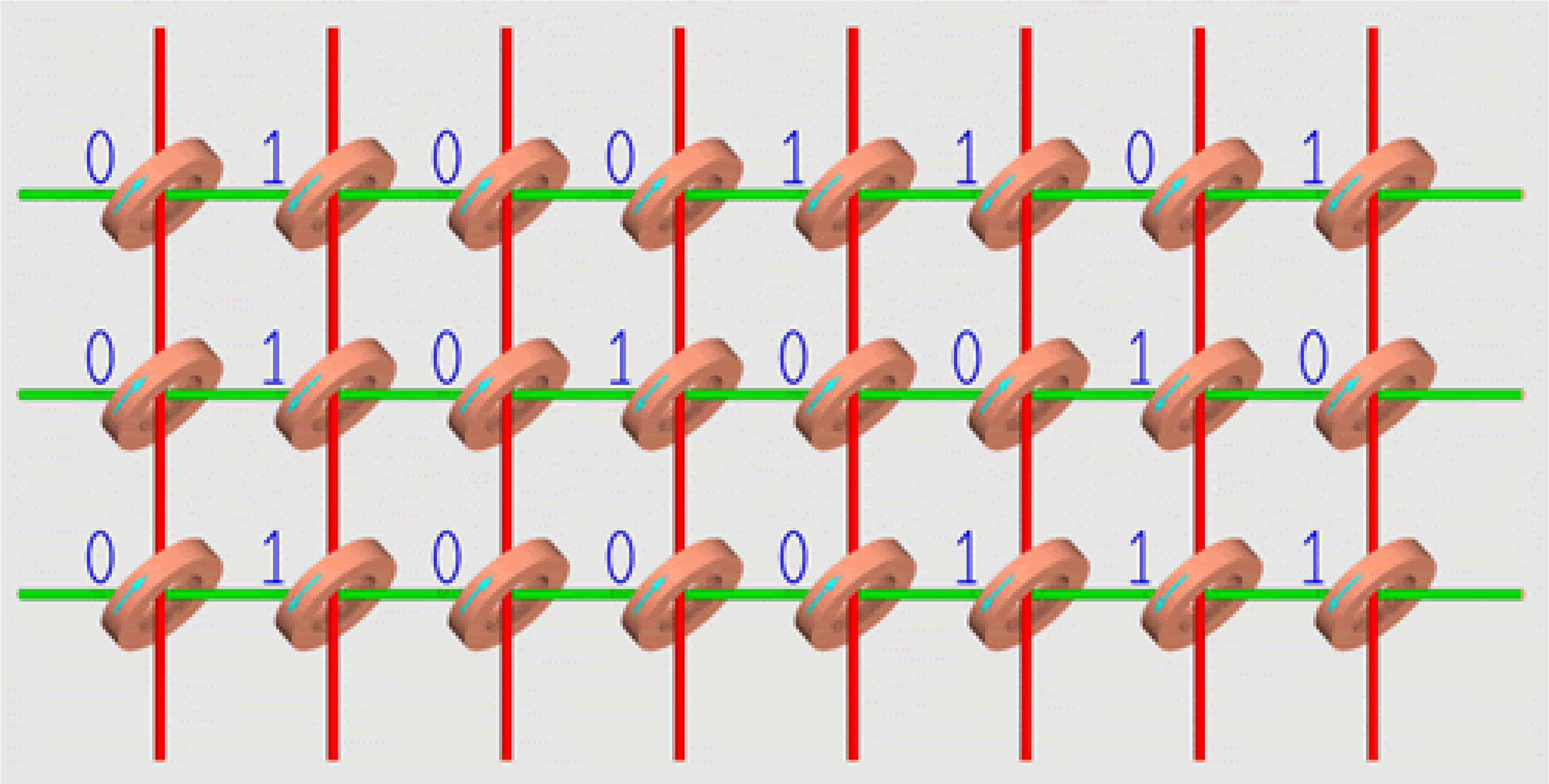
Proposed by Nikolay Brusentsov in State University

It Moscow in 1956, was based on the idea that the ferrite cores

better suited to **ternary logic** that logic

binary, since the latter missed an one of three states

potential of these nuclei.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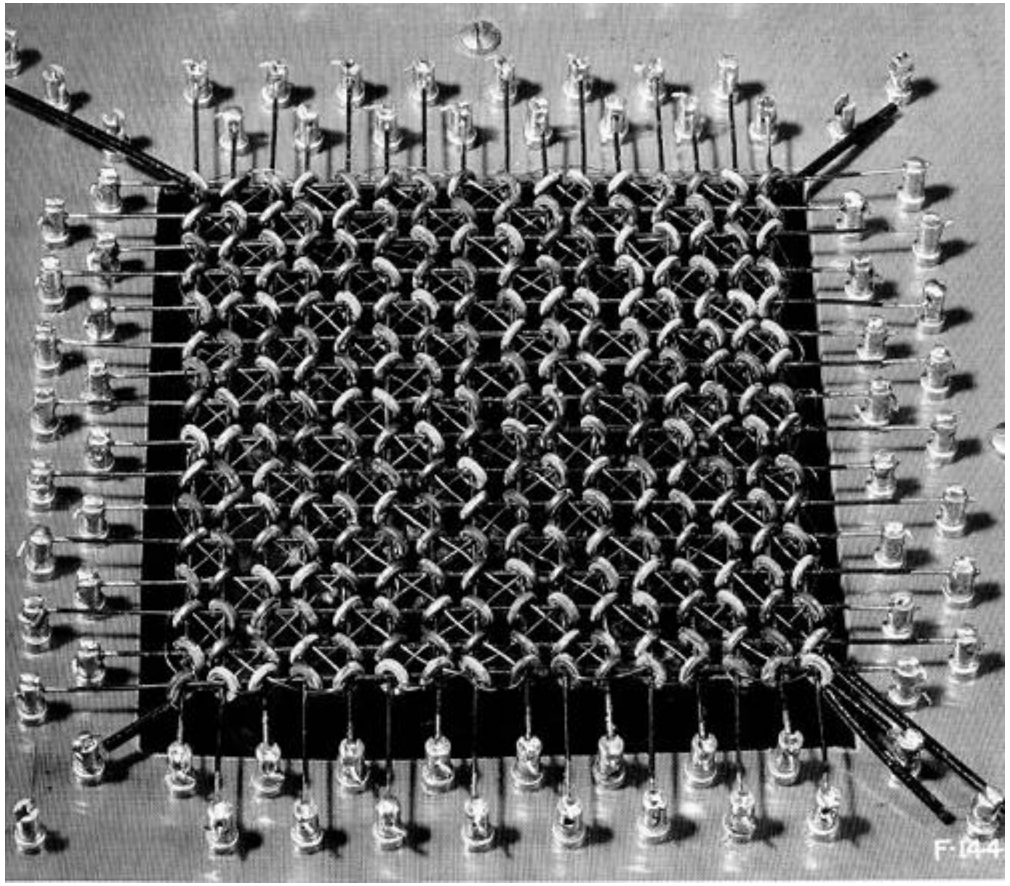
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After rapid development first became operational Setun

in 1958, although until 1960 did not go into production.

Thereafter he proved to be very reliable, and came to

produced about 50 units before authorities

They decided that enough was enough of these "bullshit

university "and canceled the production, even though

there were still many orders without serving.

Despite that came into production a new model, **Setun 70,**

but the obsolescence of technology ferrite cores

It marked the end of these projects.

**Fuzzy sets (Fuzzy Sets)**

The word fuzzy (photo) refers to "moved or blurry" in the sense of imagery with poorly defined contours. Hence the Diffuse or Blurry translation we use in Castilian

LA Zadeh in 1965 characterized the concept of SET And by extension FUZZY FUZZY LOGIC.

Is set theory associated infinite valued logic Lukasiewicz.

Zadeh's idea is to make the range of values an element belonging to a set can vary in [0,1] instead of just one of the torque values {0,1} (or what is the same False, True).

Then Zadeh extends the set operators classic (logical operators) to the new formulation, testing that the formulation thus obtained extends the logic (Theory Sets) Classic.

From Theory of Fuzzy Sets (fuzzy) Zadeh Fuzzy Logic introduced as an extension of the logic You multivalued.

What justifies the development of fuzzy logic is the need a conceptual framework for dealing with uncertainty not Probabilistic and lexical imprecision.

In the words of Zadeh (1992), most notable features Fuzzy logic are:

- Fuzzy Logic (LD) everything is a matter of degree
- The exact reasoning is a limiting case of Reasoning Approximate.
- LD knowledge is interpreted as a collection of elastic constraints (diffuse) on a set of variables.
- LD inference can be seen as the spread of a set of elastic constraints.
- Diffuse system (SD): result of "fuzzification" of a conventional system
- Fuzzy Systems operating with fuzzy sets instead of numbers.
- Essentially the representation of information Systems Diffuse mimics the mechanism Approximate Reasoning that makes the human mind.

- **When using fuzzy or diffuse technology?**

  or In complex processes, if there is no solution model

  simple.

  - In nonlinear processes.
  - When you have to enter experience a operator "expert" that is based on vague concepts obtained their experience.
  - When certain parts of a system to be controlled are unknown and can be measured reliably (with possible) errors.
  - When the setting of a variable can produce mismatch other.
  - In general, when they want to represent and operate concepts that have imprecision or uncertainty (such as decision-making processes in economics or finance).

**SOME BASIC deficiones**

X is a non-empty set of objects to consider

as a reference or universe of discourse.

Definition : A fuzzy set X is a set of pairs

values {(x, r), x • X, r • [ 0,1]}.

Each element x • X with its membership grade to A.

By extension of the characteristic function is associated to each

A fuzzy membership function by a set that associates

each element X with a value of [0,1]:

Characterization . (Fuzzy set) A fuzzy set is characterized by

one uA function: X → [0, 1] such that uA (x) represents the degree of

A membership of each x • X.

Usually written is written A (x) instead of uA (x) or

is the same A = {A (x) / x, x • X}.

The membership values ranging from 0 (not belong in

all) and 1 (total membership).

Classical sets are a particular case of fuzzy set

with membership function (characteristic function) with values

{0,1}.

If X = {x1,. . . , Xn} is a finite set and A is a subset

Diffuse X, sometimes notation used

$$A = \mu 1 / x1 + \cdot \cdot \cdot + .mu.m / xn$$

**A FUZZY SET FEATURES**

- Diffuse height of a set (height): The greatest value of its membership function: sup {A (x) x • X}.

- Diffuse set Normalized (normal): one for which there an element belonging to the fuzzy set completely, is ie grade 1. In other Height (A) = 1 mode.

- Support for a Joint Diffuse (support): Elements X A belong to more degree to 0: Support (A) = {x • X | A (x)> 0}.

- Diffuse a core assembly (core): Elements of X belong to the set with grade 1: Core (A) = {x • X | A (x) = 1}. The core (A) is always included in the support (A).

- • -Court: X values with minimum degree of membership equal to
- : TO • = { x • X | • • A (x)}.

**Consistency restriction:** Yes • 1> • 2, then *TO*• one • *TO*• 2

- Diffuse convex or concave set: One whose function

membership meets

Convex: A ( • x1 + (1- •) x2) ≥min {A (x1), A (x2)}.

Concave: A ( • x1 + (1- •) x2) ≤ max {A (x1), A (x2)}

For any x1 and x2 of X and •• [ 0.1].

- Cardinality of a set Diffuse (very different definitions)

but in general it can be said that "this is not counting the

number of elements "that has but to determine" a measure

of its size. "

## Representation Theorem or Identity Principle

All conj. Diffuse can decompose and reconstructed from

in a family of conjs. not fuzzy.

1) A fuzzy set is represented by the set of

their •• cuts and • [0,1] α sup (X) : ; • • A α A α
• • [0,1] α

where to • ( x) = 1 or A • ( x) = 0, depending on whether x belongs or

not to •• A cut •.

2) Any family of indexed arrays and nested allows

define a fuzzy set of taking them as a family

alpha cuts {A •• •• [ 0,1]}.

.

**Conclusion:**

- Any problems formulated within the framework of the fuzzy sets can be resolved by transforming these fuzzy sets in his family •• cuts Nested, determining for each solution using no fuzzy techniques.

- Highlights the idea that fuzzy sets are a generalization.

**Principle Extension (Extension Principle)**

Transforms fuzzy sets of identical or different universes by a function.

Let X and Y be two universes f: X $\longrightarrow$ Y.

Be A sd on X.

Extension Principle states that B = f (A) is a Y sd with membership function

B (y) = sup {A (x) | x in X such that y = f (x)}

This principle can be generalized to the case in which the

Universe X is the Cartesian product of n Universos:

$$X = X1 \times X2 \times ... \times Xn$$

Let f: X • Y, y = f (x), with x = (x1, x2, ..., xn).

By extension principle n becomes Fuzzy Sets

A1, A2, ... An, of universes X1, X2, ... Xn, and respectively,

in a fuzzy set B = f (A1, A2, ..., An) in Y, with function

membership:

$$B (y) = \sup \{\min [A1 (x1), A2 (x2), ..., An (xn)] \mid x \bullet X, y = f (x)\}$$

**Examples**

Sean $X$ and $Y$, both the universe of natural numbers.

**E1.-** Function add 4: $y = f(x) = x + 4$:

$A =$ 0.1 / 2 + 0.4 / + 4.1 + 0.6 3/5;

$B = f(A) =$ 0.1 / 0.4 6 + / 7 + 1/8 + 0.6 / 9;

**E2.-** SUM function: $y = f(x$ one, $x$ 2) = $x$ 1 + $x$ 2 :

$TO$ 1 = 0.1 / 2 + 0.4 / 3 + 1/4 + 0.6 / 5;

$TO$ 2 = 0.4 / 5 + 1/6;

$B = f(A$ one, $TO$ 2) = 0.1 / 7 + 0.4 / 8 + 0.4 / 9 + 1/10 + 0.6 / 11;