(A-8-2 MATH 239, Winter 2021) For each $n \in \mathbb{N}$, deterine the number of distinct spanning trees of $K_{2,n}$.

Solution: We denote $K_{2,n}$ to denote the complete graph for some $n \in \mathbb{N}$, with bipartition (A, B), where $|V(A)| = 2$. Denote $\varkappa(G)$ to denote the number of distint spanning trees of $K_{2,n}$. We count the spanning trees as follows, for the vertex $v \in V(A)$, we connect it to some i vertices in $V(B)$, and for the reaminig $n - i$ vertices in $V(G)$, we connect vertex $w \in V(A)$ to them. To complete the connection to ensure the spanning tree T is connected, we must pair $w \in V(A)$ to one of the i verties $v \in V(A)$ is paired to. This gives us i options for each of the $\binom{n}{i}$ options of pairing v to i vertices, this gives the following sum which we simplify using results from T-1-2,

$$\varkappa(G) = \sum_{i=1}^{n} \binom{n}{i} i = n \cdot 2^{n-1}$$

$\blacksquare$