# 1 Definition and Examples of Vector Spaces

**Definition 1: Vector space**

A vector space over $\mathbb{F}$ is a set $V$ equipped with two operations, *addition* and *scalar multiplication*. The elements of $V$ are usually called *vectors*

The addition operation takes two elements $v, w \in V$ and produces a new element $v + w \in v$, called the sum of the vectors. Addition is defined with of vectors obeys four key properties:

1. For all $v, w, x \in V$, $(v + w) + x = v + (w + x)$. (Associativity of addition)
2. There is a vector $0 \in V$, called a *zero vector*, such that $v + 0 = v$ for all $v \in V$. (Existence of a zero vector)
3. For all $v \in V$, there is a vector $-v \in V$ such that $v + (-v) = 0$. (Existence of an *additive inverse*).
4. For all $v, w \in V$, $v + w = w + v$. (Commutativity of addition)

The scalar multiplication takes an element $c \in \mathbb{F}$ (called the scalar) along with a vector $v \in V$, and produces $cv \in V$. Scalar multiplication of vectors obey four properties:

1. $\forall c_1, c_2 \in F$, $\forall v \in V$, $c_1(c_2 v) = (c_1 c_2)v$
2. $\forall v \in v$, $1 \cdot v = v$
3. $\forall c_1, c_2 \in F$, $\forall v \in v$, $(c_1 + c_2)v = c_1 v + c_2 v$
4. $\forall c \in \mathbb{F}$, $\forall v, w \in V$, $c(v + w) = cv + cw$

In general, a *field* is a set where you can add, subtract, multiply, and divide elements from the set in "familiar" ways, as you would see in vector spaces for example.