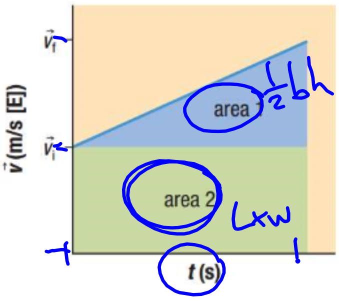
SPH3U: 1.5 Five Key Equations for Motion with Uniform Acceleration

1. A displacement equation

Area under a V-T graph:	displacement.

Find the area under the graph to the right. This is Equation 1!



$$\vec{\Delta d} = Lw + \frac{1}{2}bh = \Delta t\,\vec{v}_i + \frac{1}{2}\Delta t(\vec{v}_f - \vec{v}_i)$$
$$= \Delta t(\vec{v}_i + \frac{1}{2}\vec{v}_f - \frac{1}{2}\vec{v}_i)$$
$$= \frac{1}{2}\Delta t(\vec{v}_f + \vec{v}_i)$$

$$\boxed{① \quad \vec{\Delta d} = \frac{v_f + v_i}{2}\Delta t .}$$

Solve the average acceleration equation for v_f. This is Equation 2!

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\boxed{② \quad \vec{v}_f = \vec{a}\Delta t + \vec{v}_i}$$

Substitute v_f into the first equation. This is Equation 3!

$$\vec{\Delta d} = \frac{\vec{a}\Delta t + \vec{v}_i + \vec{v}_i}{2}\Delta t$$

$$\boxed{③ \quad \vec{\Delta d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2}$$

2. The five key equations of accelerated motion

	Equation	Variables in the equation	Variables not in the equation
Equation 1	$\Delta\vec{d} = \left(\frac{\vec{v_f} + \vec{v_i}}{2}\right)\Delta t$	$\vec{\Delta d}, \vec{v_f}, \vec{v_i}, \Delta t$	$\vec{a}$
Equation 2	$\vec{v_f} = \vec{v_i} + \vec{a}\Delta t$	$\vec{v_f}, \vec{v_i}, \vec{a}, \Delta t$	$\vec{\Delta d}$
Equation 3	$\Delta\vec{d} = \vec{v_i}\Delta t + \frac{1}{2}\vec{a}\Delta t^2$	. . .	$\vec{v_f}$
Equation 4	$\Delta\vec{d} = \vec{v_f}\Delta t - \frac{1}{2}\vec{a}\Delta t^2$	. . .	$\vec{v_i}$
Equation 5	$\vec{v_f}^2 = \vec{v_i}^2 + 2\vec{a}\Delta d$	. . .	Δt

A sports car approaches a highway on-ramp at a velocity of 20.0 m/s [E]. If the car accelerates at a rate of 3.2 m/s² [E] for 5.0 s, what is the displacement of the car?

G	iven
R	equired
E	quations
S	olution
S	tatement

G: $\vec{v}_i = 20.0$ m/s [E], $\vec{a} = 3.2$ m/s² [E], $\Delta t = 5.0$ s.

R: $\vec{\Delta d}$

E: $\vec{\Delta d} = v_i \Delta t + \frac{1}{2} a \Delta t^2$

S: $\vec{\Delta d} = (20)(5) + \frac{1}{2}(3.2)(5.0)^2$

$= 140$ m [E].

S: ∴ the displacement is 140 m [E].

$\vec{a}$, $\cancel{\vec{v}_f}$, $\vec{v}_i$, $\Delta\vec{d}$, Δt

A sailboat accelerates uniformly from 6.0 m/s [N] to 8.0 m/s [N] at a rate of 0.50 m/s² [N]. What distance does the boat travel?

G: $\vec{v}_i = 6.0$ m/s [N], $\vec{v}_f = 8.0$ m/s [N], $\vec{a} = 0.50$ m/s² [N]

R: Δd

E: $v_f^2 = v_i^2 + 2a\Delta d$

S: $\Delta d = \frac{v_f^2 - v_i^2}{2a} = \frac{8^2 - 6^2}{2(0.5)} = \frac{64 - 36}{1} = 28$ m.

S: ∴ the displacement is 28 m.

$\vec{a}$, $\vec{v}_f$, $\vec{v}_i$, $\Delta\vec{d}$, $\cancel{\Delta t}$

A dart is thrown at a target that is supported by a wooden backstop. It strikes the backstop with an initial velocity of 350 m/s [E]. The dart comes to rest in 0.0050 s.

a. What is the acceleration of the dart?

G: $\vec{v}_i = 350$ m/s [E], $\Delta t = 0.0050$ s, $\vec{v}_f = 0$ m/s

R: $\vec{a}$

E: $v_f = v_i + a\Delta t$

S: $\vec{a} = \frac{v_f - v_i}{\Delta t} = \frac{0 - 350}{0.005} = -70\,000$ m/s² [E]

$= 70\,000$ m/s² [W].

S: ∴ the acceleration is 70 000 m/s² [W].

$\vec{a}$, $\vec{v}_f$, $\vec{v}_i$, $\cancel{\Delta\vec{d}}$, Δt

b. How far does the dart penetrate into the backstop?

G: $\vec{v}_i = 350$ m/s [E], $\Delta t = 0.0050$ s, $\vec{v}_f = 0$ m/s.

R: $\vec{\Delta d}$

E: $\vec{\Delta d} = \left(\frac{v_f + v_i}{2}\right)\Delta t$

S: $\vec{\Delta d} = \left(\frac{0 + 350}{2}\right)(0.0050) = 0.875$ m

$= 0.88$ m.

$\cancel{\vec{a}}$, $\vec{v}_f$, $\vec{v}_i$, $\Delta\vec{d}$, Δt

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S: ∴ the displacement is 0.88 m.