Math 138 (Online) Assignment 3 Due: Friday, Oct. 1, 5:00 pm EST

W1 and W2 are each worth 5 marks, and each MC question is worth 2 marks. W3-W6 are not to be handed in and just meant for extra practice. I will give the discord role of 'solver of an integral' to anyone who posts a (correct) solution to any of W3-W6 on Piazza, before the due date of A3, provided you claim to have solved it on your own.

Written Question(s).

W1 Calculate $\displaystyle\int \frac{x+1}{\sqrt{x^2+2x+3}}\,dx$

W2 Calculate $\displaystyle\int_0^2 xe^{\sqrt{x}}\,dx$

W3 Calculate $\displaystyle\int_0^{\frac{\pi}{2}} x\cos x\,dx$

W4 Calculate $\displaystyle\int_0^1 x\sqrt{1+x}\,dx$

W5 Calculate $\displaystyle\int_0^1 x^3\sqrt{1-x^2}\,dx$

W6 For this question, pretend that you have never heard of logarithms before. For $x \geq 1$, we define $L(x) = \displaystyle\int_1^x \frac{1}{t}\,dt$. (If you were to take Math 147, this is how they would define $\ln x$.)

(a) Let $0 < x < 1$. Prove that $L(x) = -L(\frac{1}{x})$.

(b) Prove that $L(xy) = L(x) + L(y)$ for all $x, y \geq 1$.

Multiple Choice Questions

For each question, select all correct options. Please note that I went out of my way to say 'select all correct options' and not 'select the correct option'.

MC1 Assume that $f(x)$ and $f'(x)$ are continuous on $[0,1]$ and differentiable on $(0,1)$.

(a) $\displaystyle\int_0^1 f(x)f'(x)\,dx = f(x)f'(x)\Big|_0^1 - \int_0^1 f(x)f''(x)\,dx$

(b) $\displaystyle\int_0^1 f(x)f'(x)\,dx = f''(x)f(x)\Big|_0^1 - \int_0^1 f'(x)^2\,dx$

(c) $\displaystyle\int_0^1 f(x)f'(x)\,dx = xf(x)f'(x)\Big|_0^1 - \int_0^1 x(f'(x)^2 + f(x)f''(x))\,dx$

(d) $\displaystyle\int_0^1 f(x)f'(x)\,dx = \frac{1}{2}[f(1)^2 - f(0)^2]$

(e) None of the above

MC2 Decide if each given substitution transforms the given integral into the new integral.

(a) $u = x^2$ changes $\displaystyle\int \frac{x^4}{x^2+1}\,dx$ into $\displaystyle\int \frac{u^2}{u+1}\,du$

(b) $u = x^3$ changes $\displaystyle\int x^2 e^{x^3}\,dx$ into $\displaystyle\int e^u\,du$.

(c) $u = \cos x$ changes $\displaystyle\int (\sin x) \times e^{\cos x}\,dx$ into $\displaystyle\int e^u\,du$

(d) $u = x^5 + 1$ changes $\displaystyle\int \frac{x^9}{x^5+1}\,dx$ into $\displaystyle\frac{1}{5}\int \frac{u+1}{u}\,du$

(e) None of the above do as claimed.

MC3 Which equalities are valid?

(a) $\displaystyle\int \frac{x}{x^4+1}\,dx = \int \frac{y}{y^4+1}\,dy$

(b) $\displaystyle\int_0^1 \frac{x}{x^4+1}\,dx = \int_0^1 \frac{y}{y^4+1}\,dy$

(c) $\displaystyle\int_3^x t^2\,dt = \int_3^x (pony)^2\,dpony$

(d) $\displaystyle\int_0^\pi \sin t\,dt = \int_0^{\sqrt[3]{6}} t^2\,dt$

(e) None of the above.

MC4 Select all true statements

(a) $\displaystyle\frac{d}{dt}\int_t^{t^2} x\,dx = 2t^3 - t$

(b) There exists some $C \in \mathbb{R}$ such that, for all $x \in \mathbb{R}$ we have that

$$\sin x \; - \int_{-\pi-\sqrt{3}-e^5}^{x} \cos t\,dt = C$$

(c) If $\displaystyle\int_\pi^{10} f(x)\,dx = \int_\pi^{10} g(x)\,dx$ then $f(x) = g(x)$ for all $x \in [\pi, 10]$.

(d) $\displaystyle\frac{d}{dx}\int_{cosx}^{x^3} \sin(y^2)\,dy = 3x^2\sin(x^6) + \sin(\cos^2 x) \times \sin x$

(e) Regardless of $f(x)$ we have that $\displaystyle\int f'(x)\,dx = f(x)$.