2 Matrix Multiplication and Systems of Linear Equations

2.1 Matrix Multiplication

Definition 1: Matrix Multiplication

Let $A \in M_{m\times p}(\mathbb{F})$, $B \in M_{p\times n}$. We define the *matrix product* to be $C = AB \in M_{m\times n}$. The entries c_{ij} of the matrix C are determined by,

$$c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

Where $1 \leq i \leq m$, $1 \leq j \leq n$.

Definition 2: Indentity Matrix

We define $I_n \in M_{n\times n}$ to be,

$$I_n = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

Properties 1: Matrix Multiplication

1. For $A \in M_{m\times n}(\mathbb{F})$, $B \in M_{n\times p}(\mathbb{F})$, $C \in M_{p\times r}(\mathbb{F})$,
$$(AB)C = A(BC) \qquad \text{(Associative)}$$
2. For $A \in M_{m\times n}(\mathbb{F})$, $B, C \in M_{n\times p}(\mathbb{F})$,
$$A(B + C) = AB + AC \qquad \text{(Left distributivity)}$$
3. FOr $A, B \in M_{m\times n}(\mathbb{F})$, $C \in M_{n\times p}$
$$(A + B)C = AC + BC \qquad \text{(Right distributivity)}$$