

1. Let $k \geq 1$ be an integer. Let T denote the set of multisets of size n with t types such that for all $i \in \{1, \dots, t\}$, the number of elements of the i -th type is a multiple of k , or in other words $k \mid i$ for each i . What is $|T|$? (Assume $k \mid n$)
2. Let $k \geq 0$ be an integer. Let H denote the set of multisets of size n with t types such that for each $i \in \{1, \dots, t\}$, the number of elements of the i -th type is at least k . What is $|U|$? (Assume $n \geq kt$)

Solution:

1. Let the set H be the set of all multisets of size n/k and t types. We claim a bijection between T and H . Note that each element in T may be written as (kc_1, \dots, kc_t) , where each $c_i \in \mathbb{Z}^+$. Let $f: T \rightarrow H$ be defined by mapping each $\alpha = (kc_1, \dots, kc_t) \in T$ to $f(\alpha) = (c_1, \dots, c_t)$. By the construction of T , $c_i \in \mathbb{Z}^+$ and $kc_1 + \dots + kc_t = n$ which implies that $c_1 + \dots + c_t = n/k$ hence $f(\alpha)$ is a multiset of size n/k and t types, hence $f(\alpha) \in H$. We define the inverse function $g: H \rightarrow T$ by simply assigning each $f(\alpha) = (c_1, \dots, c_t)$, as constructed previously, to $g(f(\alpha)) = (kc_1, \dots, kc_t)$, which simply multiplies each part of the multiset by k .