Question 1:

True/False, are the following quantities vectors?

(a) $500\,\mathrm{m}$ (T / F) : *F*

(b) $500\,\mathrm{kg}$ (T / F) : *F*

(c) $600\,\mathrm{m}$[East] (T / F) : *T*

(d) $600\,\mathrm{m}$[East] $-$ $500\,\mathrm{m}$[North] (T / F) : *T*

(e) 603×50 (T / F) : *F*

(f) $-52°C$ (T / F) : *F*

Question 2:

Compute the **displacement** (or net displacement) given the position vectors. Assume that the reference point is $(0, 0)$ for all vectors.

(a) $\vec{d_1} = 500\,\text{m}[\text{East}]$, $\vec{d_2} = 500\,\text{m}[\text{East}]$

Solution.

We start by assigning [East] as the $+$ direction, this means that $\vec{d_1} = +500$ and $\vec{d_2} = +500$. Then we compute the displacement using the displacement equation.

$$\begin{aligned}\Delta\vec{d} &= \vec{d_2} - \vec{d_1}\\ &= +500\,\text{m} - (+500\,\text{m})\\ &= +0\,\text{km}\end{aligned}$$

Hence $\Delta\vec{d} = 0[\text{East}]$ (Remember that 0 belongs to the non-negative integers so -0 doesn't make sense).

(b) $\vec{d_1} = +500\,\text{km}$, $\vec{d_2} = -801\,\text{km}$, $\vec{d_3} = -120\,\text{km}$, $\vec{d_4} = +61\,\text{km}$, $\vec{d_5} = +400\,\text{km}$, $\vec{d_6} = -742\,\text{km}$.

Solution.

Solution 1: We choose the $x-$dimension as our coordinate system and assign $(x \rightarrow)$ as our positive direction. By Corollary 1.2.0.1, when we have more than two position vectors we are only concerned with the final and initial position vectors. Hence,

$$\Delta\vec{d_T} = \vec{d_F} - \vec{d_I} = \vec{d_6} - \vec{d_1} = -742\,\text{km} - (+500\,\text{km}) = -1242\,\text{km}$$

Hence $\Delta\vec{d_T} = -1242\,\text{km}$

(c) $\vec{d_i} = 601\,\text{m}[\text{Left}]$, $\vec{d_f} = 234\,\text{m}[\text{Right}]$

Solution.

We start by assigning [Right] to be the positive direction of motion, this means that $\vec{d_i} = -601, \vec{d_f} = +234$. Next we simply use the displacement equation to compute the displacement.

$$\begin{aligned}\Delta\vec{d} &= \vec{d_f} - \vec{d_i}\\ &= +234\,\text{m} - (-601\,\text{m})\\ &= +832\,\text{m}\end{aligned}$$

Hence $\Delta\vec{d} = 832\,\text{Rightm}$ (Or $\Delta\vec{d} = +832\,\text{m}$, both are acceptable).

Question 3:

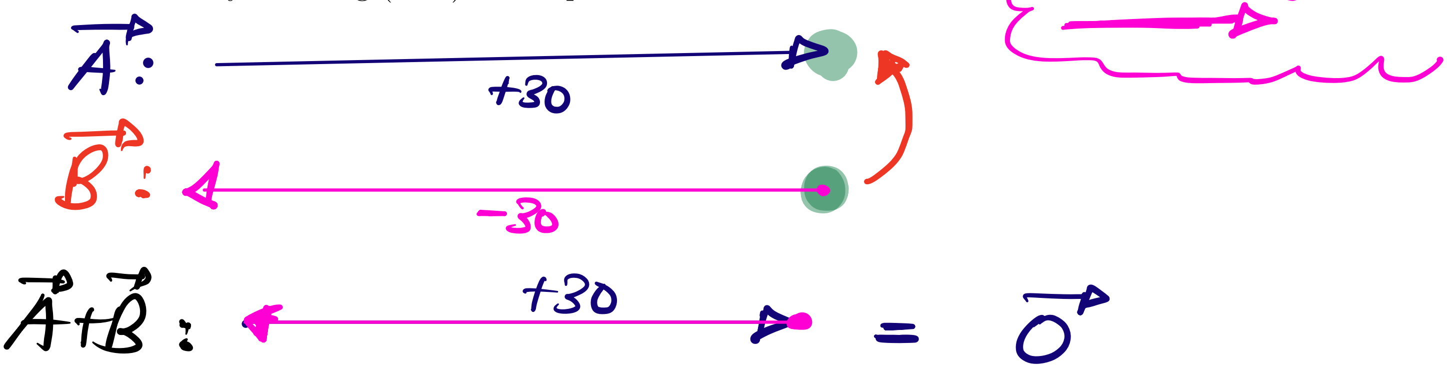
Determine the sum/difference of the following vectors **geometrically**. Use the x−dimensional co-ordinate system.

(a) $\vec{A} = +30$, $\vec{B} = -30$

$$\vec{A} + \vec{B}$$

Solution.

We start by choosing $(x \rightarrow)$ as the positive direction.

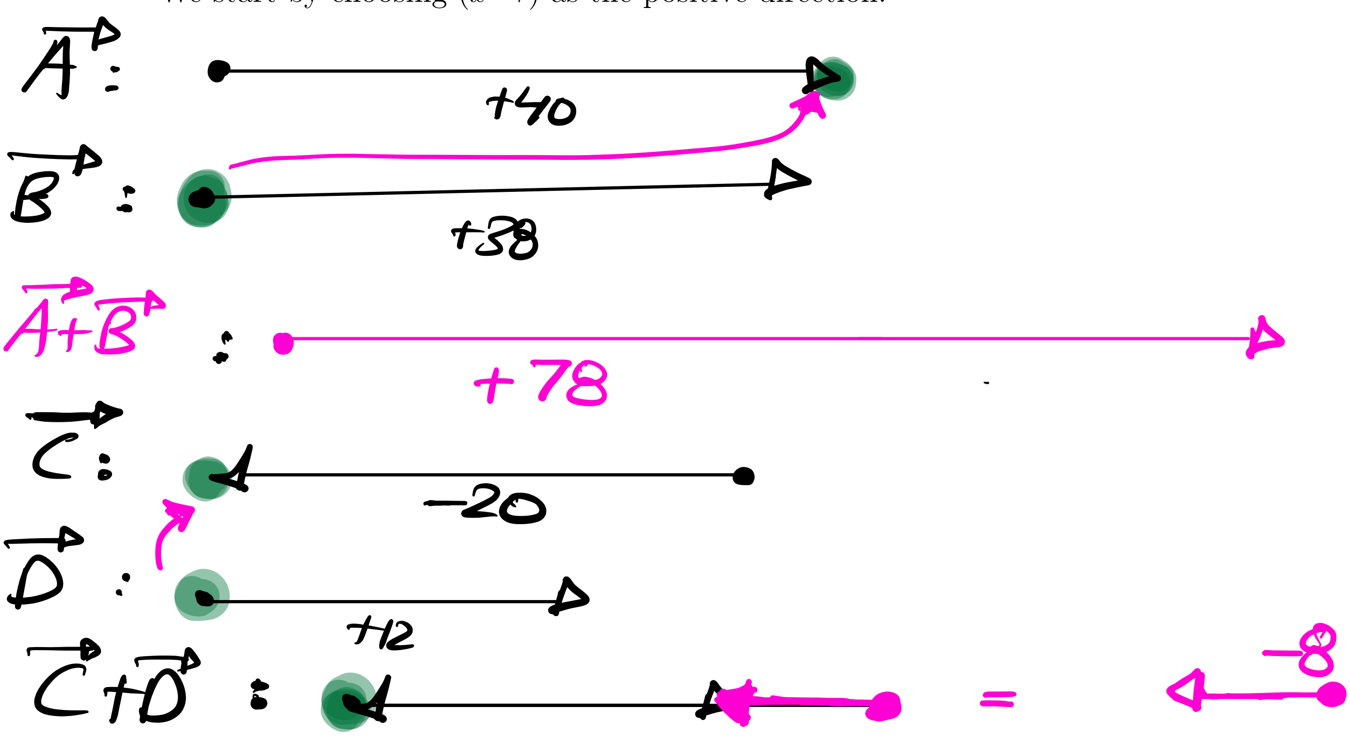


(b) $\vec{A} = +40$, $\vec{B} = +38$, $\vec{C} = -20$, $\vec{D} = +12$

$$\vec{A} + \vec{B} - (\vec{C} + \vec{D})$$

Solution.

We start by choosing $(x \rightarrow)$ as the positive direction.



$\vec{A}+\vec{B}$: $\bullet\xrightarrow{\quad +78 \quad}$

$\vec{C}+\vec{D}$: $\xleftarrow{-8}\bullet$

- Let $\vec{A}+\vec{B} = \vec{Y}$ $\vec{C}+\vec{D} = \vec{X}$ then

$$\vec{A}+\vec{B}-(\vec{C}+\vec{D}) = \vec{Y}-\vec{X} = \vec{Y}+(-\vec{X})$$

- Recall that $-\vec{X}$ is the vector that points in opposite direct of $\vec{X}$

$\vec{X} = \vec{C}+\vec{D}$: $\xleftarrow{-8}\bullet$

$-\vec{X}$: $\bullet\xrightarrow{+8}$

$\vec{Y}$: $\bullet\xrightarrow{\quad +78 \quad}$

$-\vec{X}$: $\bullet\xrightarrow{+8}$

$\vec{Y}+(-\vec{X}) =$ $\bullet\xrightarrow{\quad +78 \quad}\bullet\xrightarrow{+8}$

$= \bullet\xrightarrow{\quad +86 \quad}$

(c) $\vec{A} = +2$, $\vec{B} = +18$, $\vec{C} = -12$, $\vec{D} = -8$, $\vec{E} = +7$

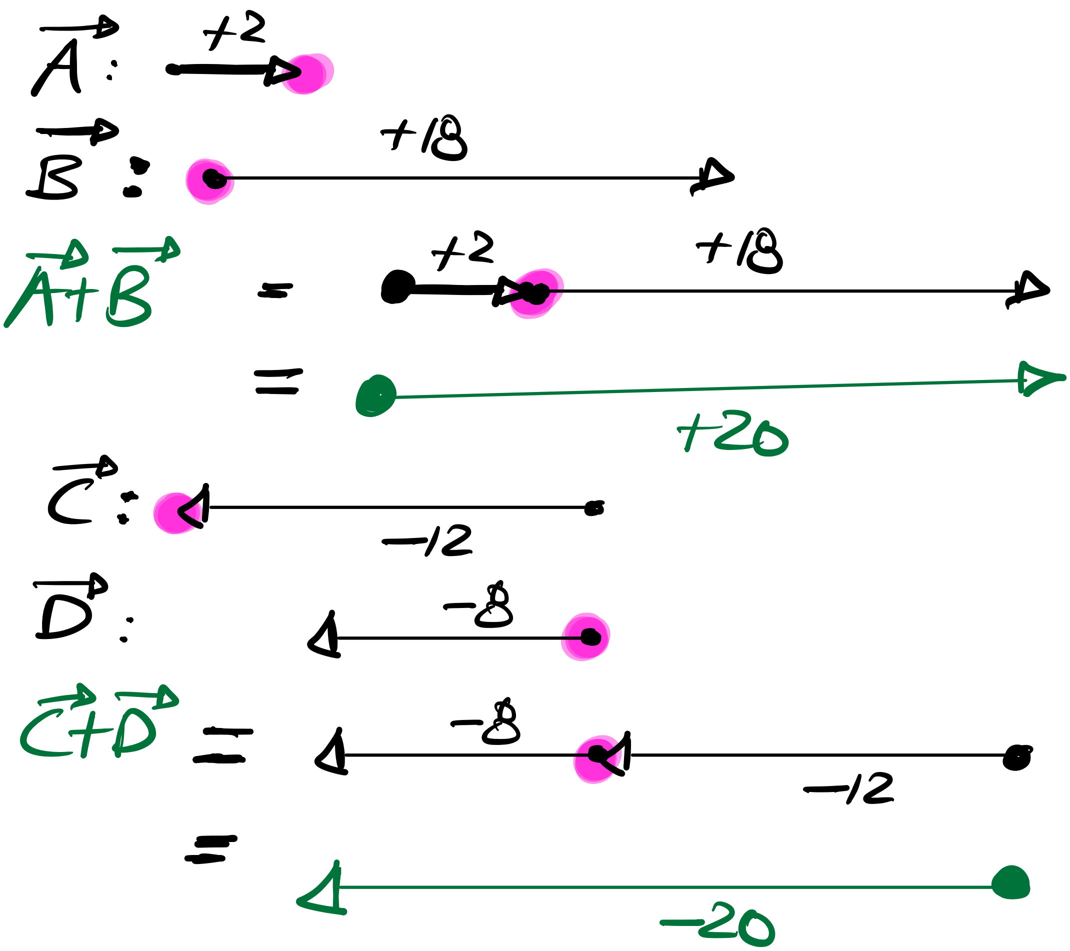
$$\vec{A} + \vec{B} + \vec{C} + \vec{D} - \vec{E}$$

Solution.

We start by choosing $(x \rightarrow)$ as the positive direction.

① First, rewrite sum,

$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + (-\vec{E})$$



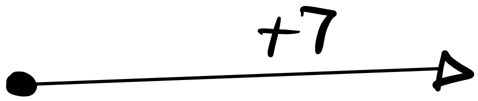
$\vec{A}+\vec{B}$:

$\vec{C}+\vec{D}$:

$(\vec{A}+\vec{B})+(\vec{C}+\vec{D}) =$

$= \vec{0}$

E:



$-\vec{E}$:

$$(\vec{A}+\vec{B})+(\vec{C}+\vec{D})+(-\vec{E}) = \vec{0}+(-\vec{E})$$

$$= -\vec{E}$$

$=$

Question 4:

An amazon driver had to make a round of package deliveries to the following cities; Oshawa, Pickering, Waterloo, London (Starting form AMZ headquarters). Given below are all of his position vectors along the trip (All relative to his AMZ headquarters). Compute his net **displacement** relative to AMZ headquarters as well as his total **distance** traveled (Assume that the driver strictly drives from location to location).

- $\vec{d}_{OSH} = 400\,\text{km}[\text{East}]$
- $\vec{d}_{PKR} = 350\,\text{km}[\text{West}]$
- $\vec{d}_{WTL} = 84000\,\text{m}\ [\text{East}]$
- $\vec{d}_{LND} = 712\,\text{km}[\text{West}]$

Solution.

We start by choosing [East] as the positive(+) direction motion. Since the driver started at AMZ headquarters his initial position vector is $\vec{d}_I = +0\,\text{km}$ (Or $\vec{d}_{AMZ} = +0\,\text{km}$). Since the final position vector is simply $\vec{d}_F = \vec{d}_{LND} = -712\,\text{km}$ we can simply compute the net displacement.

$$\Delta\vec{d}_T = \vec{d}_F - \vec{d}_I = -712\,\text{km} - 0\,\text{km} = -712\,\text{km}$$

Hence $\Delta\vec{d}_T = 712\,\text{km}[\text{West}]$. To compute the total distance traveled we must compute each displacement from the following pairs of cities ; $(AMZ \to OSH)$, $(OSH \to PKR)$, $(PKR \to WTL)$, $(WTL \to LND)$. We can label these pairs $\Delta\vec{d}_0, \Delta\vec{d}_1, \Delta\vec{d}_2, \Delta\vec{d}_3$.

$$\begin{aligned}
\Delta\vec{d}_0 &= \vec{d}_{OSH} - \vec{d}_{AMZ} && \text{(Recall } \vec{d}_{AMZ} = \vec{d}_I = +0\,\text{km)}\\
&= +400\,\text{km} - (0\,\text{km})\\
&= +400\,\text{km}\\
\Delta\vec{d}_1 &= \vec{d}_{PKR} - \vec{d}_{OSH}\\
&= -350\,\text{km} - (+400\,\text{km})\\
&= -750\,\text{km}\\
\Delta\vec{d}_2 &= \vec{d}_{WTL} - \vec{d}_{PKR}\\
&= +84\,\text{km} - (-350\,\text{km})\\
&= +434\,\text{km}\\
\Delta\vec{d}_3 &= \vec{d}_{LND} - \vec{d}_{WTL}\\
&= -712\,\text{km} - (+84\,\text{km})\\
&= -796\,\text{km}
\end{aligned}$$

Hence the total distance traveled, d, is just the sum of the absolute values of the displacement vectors,

$$
\begin{aligned}
d &= \sum_i |\overrightarrow{\Delta d_i}| \\
&= |\overrightarrow{\Delta d_0}| + |\overrightarrow{\Delta d_1}| + |\overrightarrow{\Delta d_2}| + |\overrightarrow{\Delta d_3}| \\
&= |+400\,\mathrm{km}| + |-750\,\mathrm{km}| + |434\,\mathrm{km}| + |-796| \\
&= 2380\,\mathrm{km}
\end{aligned}
$$

Question 5:

A bird traverses 600 km[N] to London starting from Waterloo. From London, he traverses 312 km[N] to Clinton. Finally, he traverses 98 km[S] to a nearby forest relative to Clinton. Compute his net displacement relative to Waterloo as well as his total **distance** traveled

Solution.

Let us choose [N] as the positive(+) direction of motion. The bird starts at Waterloo, so his initial position vector is $d_I = 0\,\text{km[N]}$. Afterwards he traverses to London, hence his position vector at London relative to Waterloo is $\vec{d}_{LW} = +600\,\text{km}$. If afterwards he traverses 312 km[N] to Clinton relative to London, his position vector relative to London is $\vec{d}_{CL} = +312\,\text{km}$. Hence, his position vector at Clinton relative to Waterloo is,

$$\vec{d}_{CW} = \vec{d}_{CL} + \vec{d}_{LW} = +312\,\text{km} + 600\,\text{km} = +912\,\text{km}$$

Afterwards if he continues 98 km[S] to a forest relative to Clinton, his position vector at the forest relative to Clinton is $\vec{d}_{FC} = -98\,\text{km}$. Hence his position vector at the forest relative to Waterloo is

$$\vec{d}_{FW} = \vec{d}_{FC} + \vec{d}_{CW} = -98\,\text{km} + (+912\,\text{km}) = +814\,\text{km}$$

Hence our final position vector is $\vec{d}_F = \vec{d}_{FW} = +814\,\text{km}$, since our initial position vector was $\vec{d}_I = +0\,\text{km}$, we can compute,

$$\Delta\vec{d}_T = \vec{d}_F - \vec{d}_I = +814\,\text{km} - 0\,\text{km} = +814\,\text{km} = 814\,\text{km[N]}$$

To compute the total distance, notice that each time he "traverses", he displaces. Total distance is concerned about the sum of the magnitudes of the displacement vectors and **ignores** the direction of travel. Hence we can simply add up all the displacements (or traversals) to get d.

$$\begin{aligned} d &= \sum_i |\overrightarrow{\Delta d}| \\ &= |\overrightarrow{\Delta d_{LW}}| + |\overrightarrow{\Delta d_{CL}}| + |\overrightarrow{\Delta d_{FC}}| \\ &= |600\,\text{km}| + |312\,\text{km}| + |-98\,\text{km}| \\ &= 1010\,\text{km} \end{aligned}$$

Question 6:

Student X was travelling around UW campus the other day to get to all of classes. He was curious what his net displacement always was at the end of his tour so he decided to record his position vectors along his tour, however all of his position vectors are recorded relative to the previous building. Assuming that X starts at his residence, help him compute his net displacement relative to his residence (Abbreviated as RES).

- $\vec{d}_{\text{MC rel RES}} = 400\,\text{m[East]}$
- $\vec{d}_{\text{RCH rel MC}} = 312\,\text{m[West]}$
- $\vec{d}_{\text{QNC rel RCH}} = 600\,\text{m[East]}$
- $\vec{d}_{\text{BIO rel QNC}} = 256\,\text{m[West]}$

Note : The notation $\vec{d}_{\text{A rel B}}$ represents the position vector at location B relative to A.

Solution.

We start by choosing [East] as our positive(+) direction of motion. Since X starts at his residence, his initial position vector is $\vec{d}_I = +0\,\text{m}$. We need to obtain our final position vector $\vec{d}_F$, which will be our final location relative to RES, or in other words, we need $\vec{d}_{\text{BIO rel RES}}$. Here we will make repeated use of Proposition 1.2.1 which states that,

$$\vec{d}_{\text{B rel A}} = \vec{d}_{\text{B rel C}} + \vec{d}_{\text{C rel A}}$$

We preform the following operations,

$$\begin{aligned}
\vec{d}_{\text{BIO rel RCH}} &= \vec{d}_{\text{BIO rel QNC}} + \vec{d}_{\text{QNC rel RCH}} \\
&= -256\,\text{m} + (+600\,\text{m}) \\
&= +344\,\text{m} \\
\vec{d}_{\text{BIO rel MC}} &= \vec{d}_{\text{BIO rel RCH}} + \vec{d}_{\text{RCH rel MC}} \\
&= +344\,\text{m} + (-312\,\text{m}) \\
&= +32\,\text{m} \\
\vec{d}_{\text{BIO rel RES}} &= \vec{d}_{\text{BIO rel MC}} + \vec{d}_{\text{MC rel RES}} \\
&= +32\,\text{m} + (+400\,\text{m}) \\
&= +432\,\text{m}
\end{aligned}$$

Hence our final position vector is $\vec{d}_F = \vec{d}_{\text{BIO rel RES}} = +432\,\text{m}$, we are now ready to compute the net displacement,

$$\Delta\vec{d}_T = \vec{d}_F - \vec{d}_I = +432\,\text{m} - 0\,\text{m} = +432\,\text{m} = 432\,\text{m[East]}$$

Question 7:

I throw a rock in the air and after a brief period of time it returns to my hand. Prove that the rocks vertical displacement relative to my hand was zero.

Solution.

Proof. We choose the y-dimension as our coordinate system and choose $(y \uparrow)$ as our positive direction. Since the initial position vector $(\vec{d_i} = +0\,\text{m})$ relative to my hand is equivalent to the final position vector relative to my hand $(\vec{d_i} = +0\,\text{m})$, the displacement must be zero. In other words, since $\vec{d_i} = \vec{d_f} = +0\,\text{m}$, $\Delta\vec{d} = \vec{d_f} - \vec{d_i} = +0 - 0 = +0\,\text{m}$. ∎

[redacted]

Question 8:

(BONUS) I throw a rock straight up into the air from a cliff 400 m[N] from the ground. After a brief period of time it lands on the ground. What was the balls vertical displacement relative to the cliff?

Solution.

We choose the y-dimension as our coordinate system and choose ($y \uparrow$) as our positive direction. Since the initial position vector of the rock relative to my hand is $\vec{d_i} = +0\,\text{m}$, and the final position vector of the rock relative to my hand was $\vec{d_f} = -400\,\text{m}$ we can compute the displacement vector,

$$\Delta\vec{d} = \vec{d_f} - \vec{d_i} = -400\,\text{m} - 0 = -400\,\text{m} = 400\,\text{m[S]}$$

Explanation using a Diagram: