1 Functions

1.1 Introduction

Definition 1.1.1
Let X, Y be sets. A function f is a rule which assigns to each element $x \in X$ an *unique* element $y \in Y$. Formally, we say that $f\colon X \to Y$, and $f(x) = y$.

A function f is an abstract mapping operation, it takes elements from set a A for which the function is well defined, called the **domain**, and maps it to elements in a set B, called the **co-domain** (I will formalize these definitions below). This mapping operation is more concisely written as $f\colon A \to B$, where if $x \in A$, then $f(x) = y \in B$, where we call y the **image point** and x the **preimage**. You may refer to the example below to help illustrate the concept, here we let $f(x) = x^2$, $A = \{-2, -1, 1, 2\}$ and $B = \{1, 2, 4, 6\}$

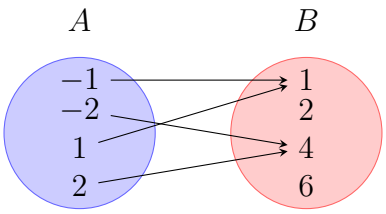

Figure 1: Mapping diagram of f

Notice that we require the rule $f(a) = b$ to map each element a to *unique* elements b. This would of course imply that if a rule $f(a) = b$ assigns some a more than one element in the co-domain, then we say that this rule $f(a) = b$ is **not** a function.

Definition 1.1.2
Let f be a function,

1. The **domain** D of f is the set of all x such that $f(x)$ is well defined.
2. The **range** (or image) R of f is defined as

$$R = \{f(x)\colon x \in D\}$$

Example 1.1.1
State the domain and range of f in Figure 1.1

Solution
Clearly the domain $D = A$, the range can be computed by determining all image points of f. We list them, $f(-1) = (-1)^2 = 1, f(-2) = (-2)^2 = 4, f(1) = 1^2 = 1, f(2) = 2^2 = 4$. Hence, $R = \{1, 4\}$

Example 1.1.2
State the domain for $f(x) = \sqrt{-3x - 9}$

Solution
We determine the set D of all possible values of x such that $f(x)$ is well defined. This occurs precisely when $-3x-9$ is a non-negative value, hence we require $-3x-9 \geq 0$ which would imply that $x \leq -3$. Hence the domain is any real number x such that $x \leq -3$, or to put more concisely using set-builder notation $D = \{x \in R \colon x \leq -3\}$ (Try to determine what the universe of discourse was here and the **statement** used).

Example 1.1.3
State the domain and range for $f(x) = \frac{-3}{4}(x + 3)^2 - 5$.

Solution
Clearly f is well defined for any $x \in \mathbb{R}$, hence $D = \{x \in \mathbb{R}\}$. Determining the range of f can be tricky, I find the best way to determine the range is to quickly sketch the function, this is a parabola that has $y-$intercept $= -5$ and points *downwards*, hence $R = \{y \in \mathbb{R} \colon y \leq -5\}$.