

## 2 Matrix Multiplication and Systems of Linear Equations

### 2.1 Matrix Multiplication

#### Definition 1: Matrix Multiplication

Let  $A \in M_{m \times p}(\mathbb{F})$ ,  $B \in M_{p \times n}$ . We define the *matrix product* to be  $C = AB \in M_{m \times n}$ . The entries  $c_{ij}$  of the matrix  $C$  are determined by,

$$c_{ij} = \sum_{k=1}^n a_{ik}b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

Where  $1 \leq i \leq m$ ,  $1 \leq j \leq n$ .

#### Definition 2: Identity Matrix

We define  $I_n \in M_{n \times n}$  to be,

$$I_n = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

### Properties 1: Matrix Multiplication

1. For  $A \in M_{m \times n}(\mathbb{F})$ ,  $B \in M_{n \times p}(\mathbb{F})$ ,  $C \in M_{p \times r}(\mathbb{F})$ ,

$$(AB)C = A(BC) \quad (\text{Associative})$$

2. For  $A \in M_{m \times n}(\mathbb{F})$ ,  $B, C \in M_{n \times p}(\mathbb{F})$ ,

$$A(B + C) = AB + AC \quad (\text{Left distributivity})$$

3. For  $A, B \in M_{m \times n}(\mathbb{F})$ ,  $C \in M_{n \times p}$

$$(A + B)C = AC + BC \quad (\text{Right distributivity})$$

4. For  $A \in M_{m \times n}(\mathbb{F})$ ,

$$I_m A = A$$

$$A I_n = A$$

For  $Z_m = 0 \in M_{m \times m}$ ,  $Z_n = 0 \in M_{n \times n}$ ,

$$Z_m A = Z_m$$

$$A Z_n = Z_n$$

5. For  $A \in M_{m \times n}(\mathbb{F})$ ,  $B \in M_{n \times p}(\mathbb{F})$ , and  $c \in \mathbb{F}$ ,

$$c(AB) = (cA)B = A(cB)$$

## 2.2 Solving Systems of Linear Equations