1 Sets

1.1 Introduction

Definition 1.1.1
Sets are defined to be a collection of *objects* composed inside a pair of braces .

To define what we mean by an *object* can be complicated, and hence I will refer to objects as anything that has been previously defined or "tangible" (although even this can get a little philosophical). For example if the object is an integer, then we can build a set with some integers, take $\{2, 3, 44, 5\}$ as an example or take the following tangible objects $\clubsuit, \heartsuit, \triangle$ and build a set with them $\{\clubsuit, \heartsuit, \triangle\}$. There are some key properties of sets to note. **Order does not matter**, meaning any rearrangement of the objects in a set yields the same set, for example we say that $\{1, 2, 3, 4, 5\} = \{2, 3, 4, 5, 1\} = \{1, 2, 3, 5, 4\}$, etc. Also, **duplicates are not allowed** so whenever we observe a duplicate object, we immediately remove it and yield an equivalent set, so $\{1, 2, 2, 3, 4\} = \{1, 2, 3, 4\}$. We say that the **cardinality** of a set $\mathcal{S}$ is the number of elements (or objects) in the set, and denote the quantity as $|\mathcal{S}|$, for example if $\mathcal{S} = \{1, 2, 3, 4, 5\}$ then $|\mathcal{S}| = 5$.

Definition 1.1.2
We denote $\emptyset$ as the set with no elements, and call it the **empty set**. This implies $|\emptyset| = 0$.

Notation: If some element x is contained within a set $\mathcal{S}$, then we say that x is an element of $\mathcal{S}$ and write $x \in S$. Consequently, if some element y is **not** an element of $\mathcal{S}$, then we say that y is not an element of $\mathcal{S}$ and write $y \notin \mathcal{S}$.

1.2 Common Sets

There are a few common recurring sets that are the building blocks for the objects we will manipulate throughout this book. We list them here, (note that the $\dots$ notation indicates a continuation following the logical pattern)

1. $\mathbb{Z}$ denotes the set of all integers $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$.
2. $\mathbb{N}$ denotes the set of all *positive* integers $\mathbb{N} = \{1, 2, 3, \dots\}$.
3. $\mathbb{R}$ denotes the set of all real numbers (rational or irrational).
4. $\mathbb{Q}$ denotes the set of all rational numbers.
5. $\mathbb{Z}^+$ denotes the set of all non-negative integers $\mathbb{Z}^+ = \{0, 1, 2, 3, \dots\}$
   Remark: Some texts will not allow 0 to be apart of $\mathbb{Z}^+$.
6. $\mathbb{R}^+$ denotes the set of all non-negative positive real numbers.
7. $\mathbb{Z}^-$ denotes the set of all negative integers $\mathbb{Z}^+ = \{-1, -2, -3, \dots\}$
8. $\mathbb{R}^-$ denotes the set of all negative real numbers.

Remark: In some very specific math subjects we like to say that N includes 0 as well, this can be particularly useful whenever there is some sort of correspondence to Computer Science.

The **universe of discourse** denoted $\mathcal{U}$, is the set of all objects we may be interested in a given scenario. In this book, we are mostly always working with the set $\mathbb{R}$, and hence the universe of discourse will almost always be $\mathcal{U} = \mathbb{R}$. (There may be a few special cases were we explicitly differentiate).

Definition 1.2.1
We say that the set $\{x \in \mathcal{U} \colon \textbf{statement}\}$ is the set of all elements x in $\mathcal{U}$ such that the **statement** is true for x. (The semicolon means "such that", some texts will use a $|$ instead)

Example 1.2.1
Write out all of the elements of the following sets,

(a) $S = \{n \in \mathbb{Z} \colon 0 \leq n \leq 4\}$.

(b) $F = \{y \in \mathbb{N} \colon y \geq 4\}$

(c) $H = \{x \in \mathbb{Z} \colon -x > 0\}$

Solution
$\implies$