1 Acceleration

Definition 1.0.1
Average Acceleration, $\vec{a}_{av}$, refers to the rate of change of velocity, or in other words the ratio of the change of velocity to the time elapsed. (**Units:** m/s^2)

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

First we note that acceleration is a vector quantity because $\Delta \vec{v}$ is a vector quantity. Acceleration is experienced any time an object is increasing or decreasing its velocity, *any* change in velocity results in acceleration. For example, you must initially accelerate your vehicle in order for it to reach the desired velocity, similarly you must first *accelerate* your vehicle in order to come to a stop and change your velocity to ($+0\,\mathrm{m/s}$). In this course we will consider only uniform acceleration of a moving body and avoid situations where the acceleration a given body is non-uniform.

Remark : It is common to hear the term *de-accelerate*, however this term is rather redundant because the term acceleration refers to any change in velocity, regardless of weather you would like to increase your velocity or bring yourself to a holt ($\vec{v} = +0\,\mathrm{m/s}$).

Remark : If you are wondering why we are no longer working with $\vec{v}_{av}$, it is because when we were working with average velocity, we were not concerned with the precise velocity of the moving body at a given point in time but rather the "most common" velocity over a time interval. Average acceleration is concerned with changes in *exact* velocities, we will discuss these differences in a latter subsection.

Example 1.0.1
A vehicle on the highway changes his velocity from $\vec{v}_i = 500\,\mathrm{m/s}$[East] to $\vec{v}_f = 612\,\mathrm{m/s}$ [West] in $\Delta t = 2\,\mathrm{min}$. Compute his average acceleration,

Solution
$\Longrightarrow$

Definition 1.0.2
A **velocity-time graph** is a plot describing the motion of an object, with velocity on the vertical axis and time on the horizontal axes.

Similar to the analogy of how a Pos V. Time plot helps us understand velocity better, a Velocity v. Time plot will help us understand acceleration better. Again we mention some basic properties, again we take the reference point to be $(0, 0)$. Also we take the positive direction of motion to be above the vertical axes. We now mention a proposition similar to a one we have seen earlier.

Proposition 1.0.2
Given a **Linear** Velocity v. Time plot of a moving body, the slope m of the plot represents the average acceleration, $\vec{a}_{av}$, of the body.

Proof
We prove the result similar to the method we used in the previous section. Let the slope of the Velocity v. Time graph be m, let us compute this slope by using the slope formula, namely,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Since the $y-$coordinates on a Velocity v. Time graph are velocity vectors $\vec{v}$, and the $x-$coordinates are time points, t, we can translate this slope formula to the equivalent,

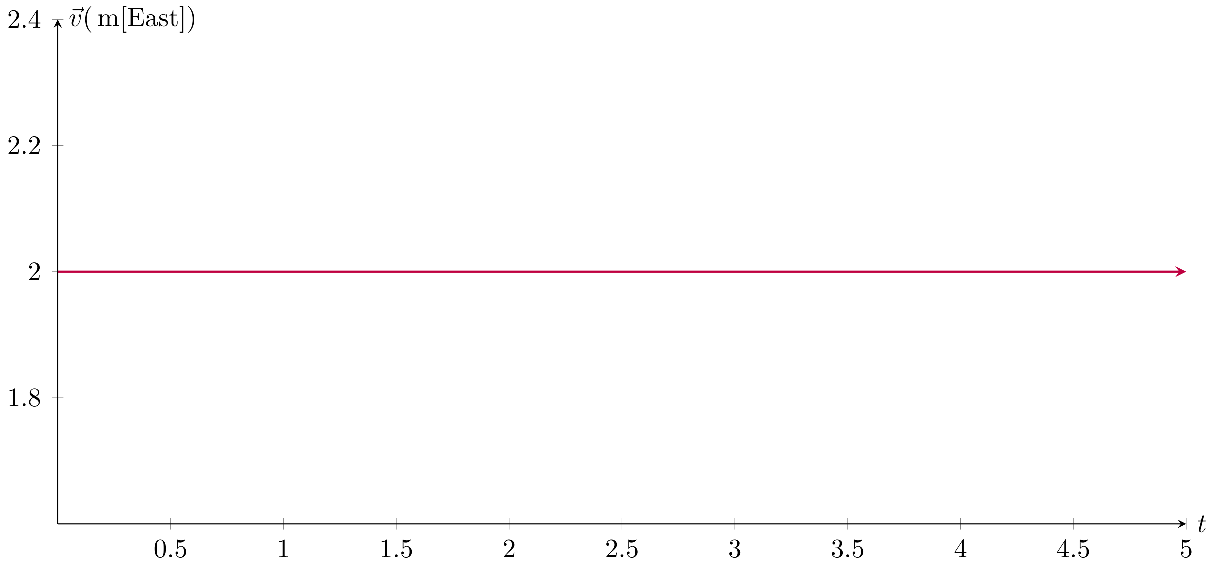
$$m = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

At this point we are free to choose any two coordinate pairs $(\vec{v}_1, t_1)$, $(\vec{v}_2, t_2)$, let us choose $(\vec{v}_f, t_f)$, $(\vec{v}_i, t_i)$, the final and initial coordinate pairs of the moving body. This gives,

$$m = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \vec{a}_{av}$$

1.1 Types of motion from Velocity v. Time Plots

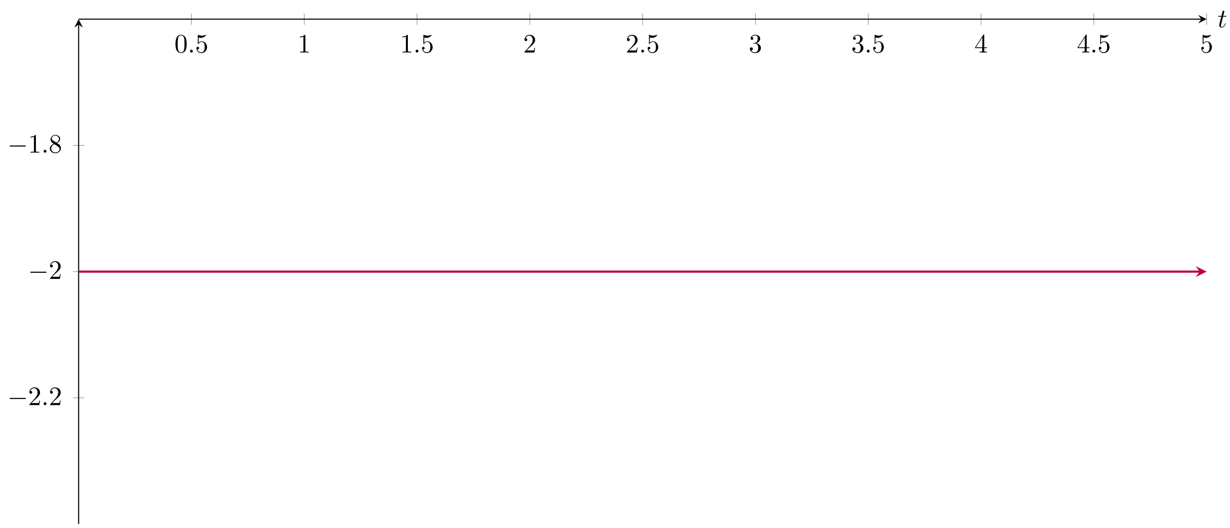
Similar to before, we will encounter common types of motion and hence it would be useful to make mention of their plots and what they look like.



(a)

Properties of type (a):

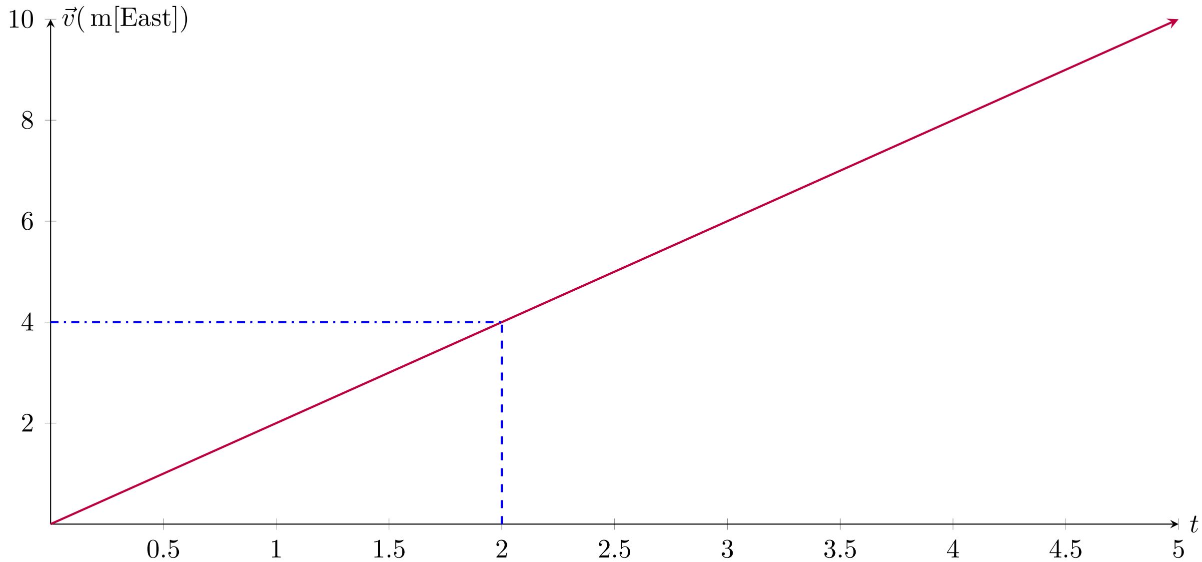
- The slope of the graph is zero, hence $\vec{a}_{av} = +0\,\mathrm{m/\,s^2}$.
- The object is experiencing **uniform motion**.
- The object is moving [East] relative to the reference point (0,0).



(b)

Properties of type (b):

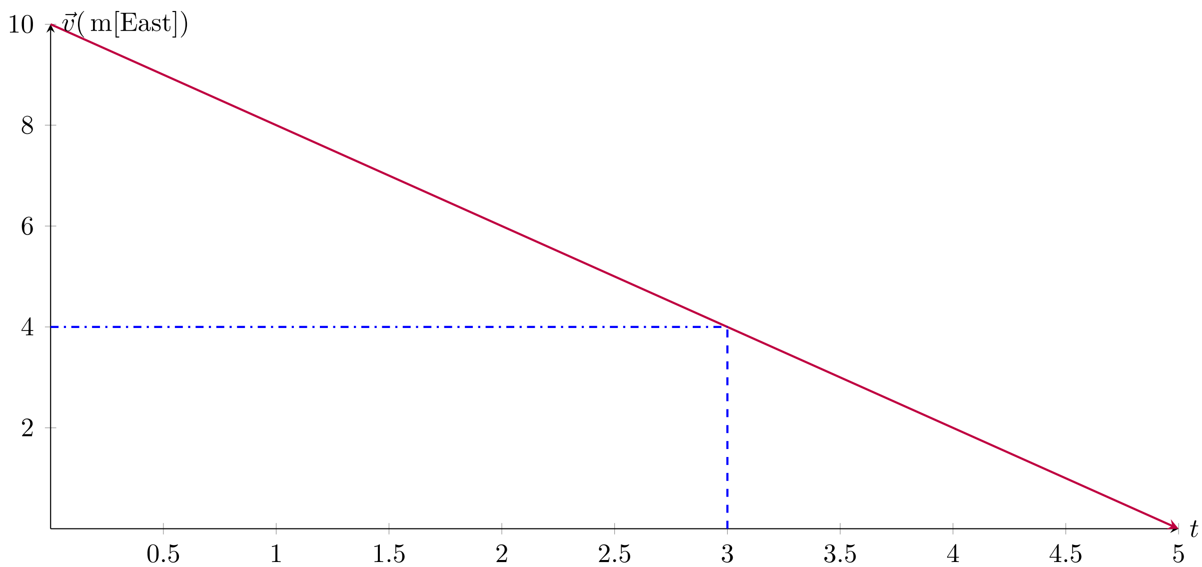
- The slope of the graph is zero, hence $\vec{a}_{av} = +0\,\mathrm{m/\,s^2}$.
- The object is experiencing **uniform motion**.
- The object is moving [West] relative to the reference point (0,0).



(c)

Properties of type (c):

- The slope of the graph is $m = +2$, hence $\vec{a}_{av} = +2\,\mathrm{m/\,s^2}$.
- The object experiencing **uniform acceleration**.
- The object is traveling in the [East] direction.



(d)

Properties of type (d):

- The slope of the graph is $m = -2$, hence $\vec{a}_{av} = -2\,\mathrm{m/\,s^2}$.
- The object experiencing **uniform acceleration**.
- The object is traveling in the [West] direction.

1.2 Instantaneous and Average Velocity

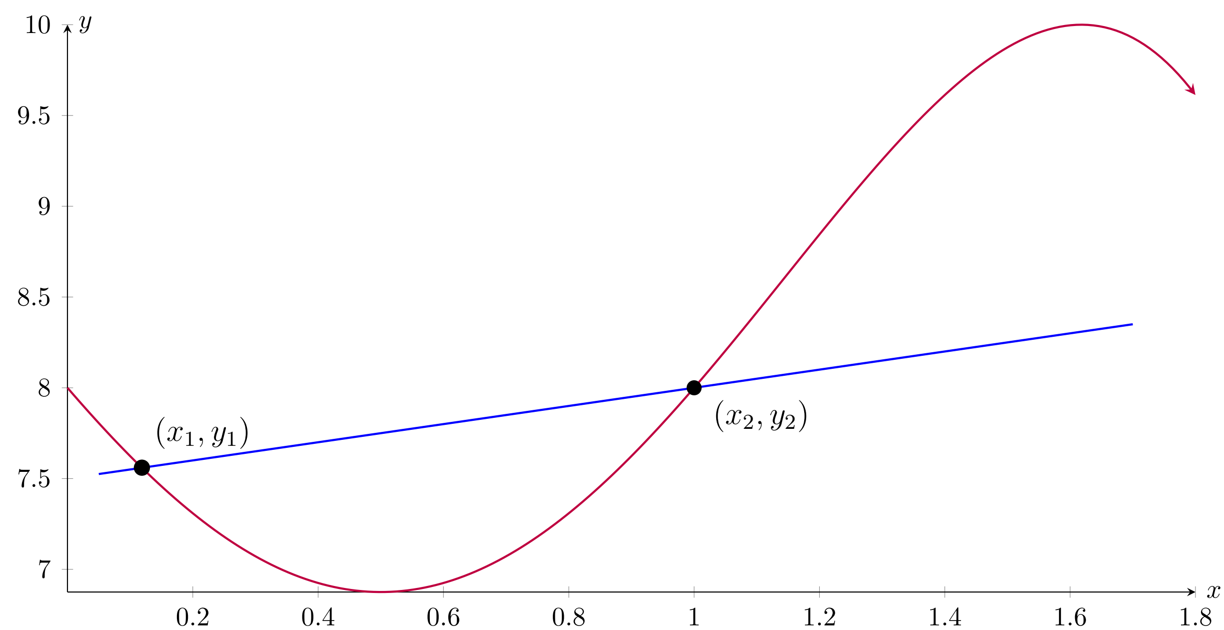
Definition 1.2.1
Uniform acceleration is motion where the acceleration of the body is fixed.

Definition 1.2.2
The **instantaneous velocity**, $\vec{v}$, of an object is the *exact* velocity of an object at a given time t

It remains to ask how we can compute the instantaneous velocity of an object at a given time t. To answer this question requires knowledge of basic Calculus, however we can still introduce the idea of secant and tangent lines.

Definition 1.2.3
A **secant line** is a line segment connecting two points, (x_1, y_1) and (x_2, y_2), on a graph.

It helps to understand the idea of a secant line using an illustration,



The slope of the secant line would be computed as,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If we replaced the $y-$coordinates with position vectors and the $x-$coordinates with time points, the slope of the secant line would represent the average velocity within the interval $[t_1, t_2]$.

Theorem 1.2.1
Given a Pos v. Time graph of a moving body, the slope of a secant line in the interval $[t_1, t_2]$ represents the average velocity between $[t_1, t_2]$.

Proof
We proceed with similar reasoning from above, the slope of the secant line would be given as,

$$m = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1} = \vec{v}_{av}$$

Definition 1.2.4
A **tangent line** is a secant line, with the property that x_2 and x_1 are infinitesimally close apart.

If x_2 and x_1 are infinitesimally close apart, then of course y_2 and y_1 must also be infinestesmly apart as well (How y changes with respect to x wouldn't matter at the infinitesimal level)). As a result of this restriction, the tangent line to a graph would look as if it merely touches the graph at a single point, but in fact it does not, if we were to zoom infinestesmly we would indeed observe that the tangent line touches the graph at two distinct points. Aggain