Physics 11 Test 1 - **SOLUTION**
Time : 1.5 hrs

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1 Preamble

This is a test covering what we have learnt so far in lecture, that being Sections 1,2,3. The time expected to complete this test is just under an hour and half granted that the student is adequately prepared. Throughout the test, the student must show all work to receive full marks.

2 Allowed Aids

The following aids are allowed on the Test

- Pencil, Pen, Eraser, Ruler, Protractor, Spare sheets of **blank** paper.
- Reference sheet **(FROM WEBSITE ONLY)**
- Basic scientific calculator

3 Restrictions:

The student is not allowed to communicate with any outside source, nor allowed to access any external aid on the internet,etc. The student may ask the instructor (ME) any questions or confusions but is advised to do so only when all else has failed due to the time constraints.

4 Name and Date:

Print your name and todays date below;

Name	Date

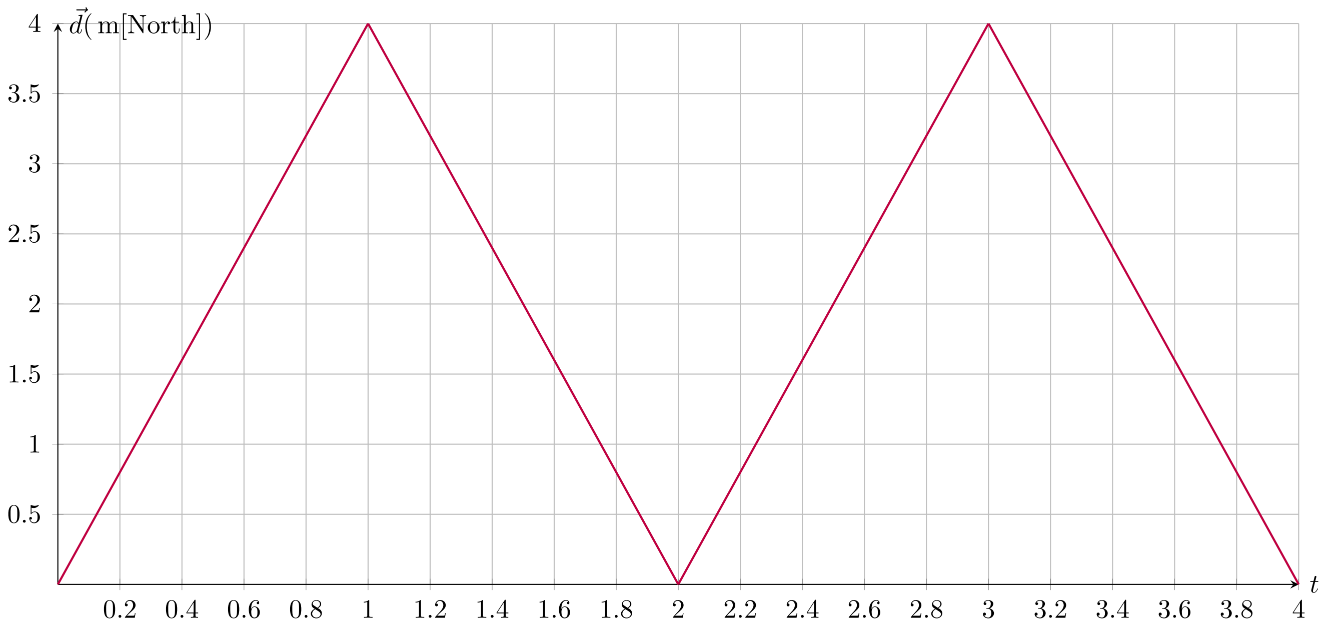
Question 1:

Answer the following True / False questions **(Assume [North],[East] is positive)**

1. The maximum height I can jump on a trampoline is $d = 5000\,\text{m}$. I jump 3 times on the trampoline and the time elapsed was $\Delta t = 20\,\text{s}$. (**Assume** that a single jump means I reached my maximum height **and** landed back on the trampoline)
   (a) My average velocity relative to the trampoline was $\vec{v}_{av} = +1700\,\text{m/s}$. (T / F) : *F*
   (b) My average speed was $v_{av} = 1.5\,\text{km/s}$. (T / F): *T*
2. Suppose that relative to the center of a field, a batsmen stands at $\vec{d_i} = 50\,\text{m}$[East]. The batsmen bats a baseball at an average velocity of $\vec{v}_{av} = 350\,\text{m/s}$[West]. The time elapsed was $\Delta t = 15\,\text{s}$.
   (a) $\vec{d_f} = 5200\,\text{m}$[East] is the final position vector. (T / F): *F*
   (b) The magnitude of the average velocity is equal to the average speed. (T / F): *T*
3. Consider the Moon orbiting the Earth
   (a) The average velocity of the Moon is always non-zero after $t = 0$. (T / F): *F*
   (b) The average speed of the Moon is always non-zero after $t = 0$. (T / F): *T*
4. Consider the equation of motion of Car A : $x = -\frac{5}{2}t + 12$ and Car B : $x = \frac{7}{2}t - 6$
   (a) Car A has a greater average speed than Car B. (T / F): *F*
   (b) Car B is initially [East] relative to the reference point. (T / F): *F*
   (c) Car A is initially [West] relative to the reference point. (T / F) : *F*
   (d) Both drivers experienced uniform motion. (T / F): *T*
   (e) Car A and Car B will meet at $t = 3\,\text{s}$. (T / F): *T*
5. I kick a soccer ball at an average speed v_{av} and it takes Δt seconds to reach a distance of d meters.
   (a) Kicking the soccer ball at $2v_{av}$ will allow it to travel $\frac{d}{2}$ meters in Δt seconds. (T / F): *F*
   (b) Kicking the soccer ball at $\frac{v_{av}}{2}$ implies that it would take $2\Delta t$ seconds to travel d meters. (T / F): *T*

Question 2:

Answer the following multiple choice questions. Refer to the plot below for all Q1,Q2.



1. Which of the following scenarios best describe the motion depicted in the plot,
   (a) A ball rolling [North] across a flat road
   (b) A sprinter running on a circular track.
   (c) A man jumping on a trampoline.

   Solution: c)

2. Which of the following statements are correct about the plot?
   (a) The body experienced uniform motion within the time interval $[1, 2]$.
   (b) The body experienced uniform motion within the time interval $[0, 4]$
   (c) Within the time interval $[0, 2]$, the average velocity was $\vec{v}_{av} = +0\,\mathrm{m/\,s}$.
   (d) Within the time interval $[2, 3]$, the average velocity was $\vec{v}_{av} = +4\,\mathrm{m}/\ s$.
   (e) The average speed within the time interval $[0, 4]$ was $v_{av} = 4\,\mathrm{m/\,s}$.

   Solution: a),c),d),e)

3. I label three points on a straight line, F, G, H. Which of the following statements are true?
   (a) $\vec{d}_{FG} = \vec{d}_{GF} + \vec{d}_{GH}$
   (b) $\vec{d}_{HF} = (-\vec{d}_{FG}) + (-\vec{d}_{HG})$
   (c) $\vec{d}_{FH} = (-\vec{d}_{GF}) + (-\vec{d}_{HG})$
   (d) $-\vec{d}_{FG} = \vec{d}_{GH} + \vec{d}_{HF}$

   Solution: c),d)

Question 3:

Covert the following units to km/ h.

(a) 44200 m/ s

Solution.

$$44200\frac{\cancel{m}}{\cancel{s}}\left(\frac{1\,\text{km}}{1000\,\cancel{m}}\right)\left(\frac{3600\,\cancel{s}}{1\,\text{h}}\right) = 159120\frac{\text{km}}{\text{h}} = 1.59\times 10^5\frac{\text{km}}{\text{h}}$$

(b) 5512×10^4 in/ min **(1 inch = 2.54cm, 1 m = 100 cm)**

Solution.

$$5512\times 10^4\frac{\cancel{\text{in}}}{\cancel{\text{min}}}\left(\frac{60\,\cancel{\text{min}}}{1\,\text{h}}\right)\left(\frac{2.54\,\cancel{\text{cm}}}{1\,\cancel{\text{in}}}\right)\left(\frac{1\,\cancel{m}}{100\,\cancel{\text{cm}}}\right)\left(\frac{1\,\text{km}}{1000\,\cancel{m}}\right) = 84002.88\frac{\text{km}}{\text{h}}$$

(c) $336\frac{\text{km}}{\text{week}}$

Solution.

$$66\frac{\cancel{\text{km}}}{\cancel{\text{week}}}\left(\frac{1\cancel{\text{week}}}{7\cancel{\text{week}}}\right)\left(\frac{1\cancel{\text{day}}}{24\,\text{h}}\right) = 2\frac{\text{km}}{\text{h}}$$

Question 4:

Compute the **displacement** (or net displacement) given the position vectors. Assume that the reference point is $(0, 0)$ for all vectors.

(a) $\vec{d_1} = 514\,\text{m}[\text{West}]$, $\vec{d_2} = 332\,\text{m}[\text{West}]$

Solution.

Lets take [East] to be the positive direction of motion,

$$\Delta\vec{d} = \vec{d_f} - \vec{d_i} = -332\,\text{m} - (-514\,\text{m}) = +182\,\text{m} = 182\text{m [East]}$$

(b) $\vec{d_1} = 51\,\text{m}[\text{S}]$, $\vec{d_2} = 33\,\text{m}[\text{S}]$, $\vec{d_3} = 27\,\text{m}[\text{N}]$, $\vec{d_4} = 93\,\text{m}[\text{N}]$,$\vec{d_5} = 298\,\text{m}[\text{S}]$, $\vec{d_6} = 432\,\text{m}[\text{N}]$

Solution.

Lets take [North] to be the positive direction of motion,

$$\Delta\vec{d} = \vec{d_f} - \vec{d_i} = +432\,\text{m} - (-51\,\text{m}) = +483\,\text{m} = 483\text{m [N]}$$

(c) $\vec{d_1} = 4\,\text{m}[\text{East}]$, $\vec{d_2} = 4\,\text{m}[\text{West}]$, $\vec{d_3} = 4\,\text{m}[\text{North}]$, $\vec{d_4} = 4\,\text{m}[\text{East}]$

Solution.

Lets take [North] and [East] to be positive direction of motion,

$$\Delta\vec{d} = \vec{v}_f - \vec{v}_i = +4 - (+4) = +0 = 0\text{m [East]}$$

Question 5:

Determine the sum/difference of the following vectors **geometrically**. Use the $x-$dimensional coordinate system.

(a) $\vec{A} = +2$, $\vec{B} = -8$

$$\vec{A} + \vec{B}$$

(b) $\vec{A} = +4$, $\vec{B} = -3$, $\vec{C} = +10$, $\vec{D} = -12$, $\vec{E} = -13$, $\vec{F} = +20$

$$(\vec{A} + \vec{B}) - (\vec{C} - \vec{D}) + (\vec{E} - \vec{F})$$

Question 6:

Suppose a train took the following route the other day to the following cities; Oshawa, Pickering, Markham, London (Starting at Oshawa). Given below are all of his position vectors along the trip (All relative to **Toronto**). Compute his average velocity as well as his average speed if the trip took 4 h.

- $\vec{d}_{OSH} = 224\,\text{km}[\text{East}]$
- $\vec{d}_{PKR} = 154\,\text{km}[\text{East}]$
- $\vec{d}_{MRK} = 72\,\text{km}[\text{West}]$
- $\vec{d}_{LND} = 556\,\text{km}[\text{East}]$

Solution.

Let us take [East] as the positive direction of motion. We start by computing the average velocity, this strictly relies on the initial and final position vectors relative to Toronto. The initial position vector relative to Toronto is $\vec{d}_{OSH} = +224\,\text{km}$. The final position vector is $\vec{d}_{LND} = +556$, since it is of course our final location and the vector is already given relative to Toronto. Hence,

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} = \frac{+556\,\text{km} - (+224\,\text{km})}{4\,\text{h}} = +83\,\text{km/h} = 82\,\text{km/h}[\text{East}]$$

To compute the average speed we must compute all displacements. We analyze the number of displacements the train took throughout the journey. We have the following route ($OSH \to PKR \to MRK \to LND$), hence you have three displacements which we compute separately.

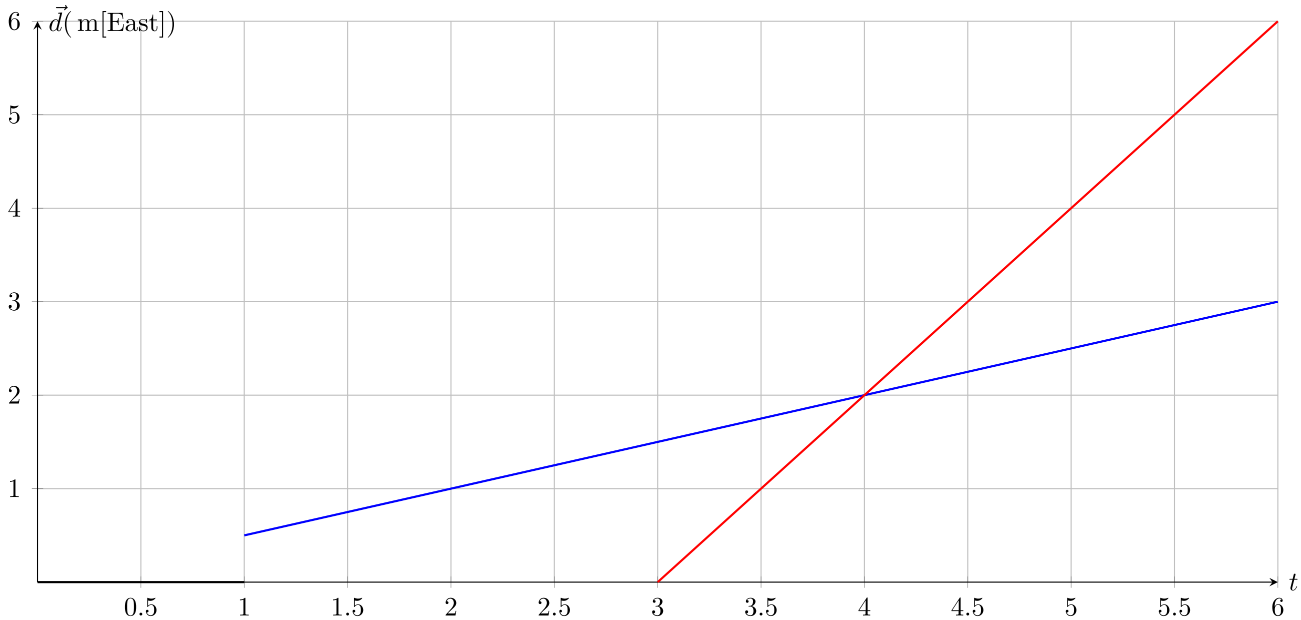
$$\begin{aligned}
\Delta\vec{d}_1 &= \vec{d}_{PKR} - \vec{d}_{OSH} = +154\,\text{km} - (+224\,\text{km}) = -70\,\text{km} \\
\Delta\vec{d}_2 &= \vec{d}_{MRK} - \vec{d}_{PKR} = -72\,\text{km} - (+154\,\text{km}) = -226\,\text{km} \\
\Delta\vec{d}_3 &= \vec{d}_{LND} - \vec{d}_{MRK} = +556\,\text{km} - (-72\,\text{km}) = +628\,\text{km} \\
d &= \sum_i |\overrightarrow{\Delta d_i}| \\
&= |\overrightarrow{\Delta d_1}| + |\overrightarrow{\Delta d_2}| + |\overrightarrow{\Delta d_3}| \\
&= |-70\,\text{km}| + |-226\,\text{km}| + |+628\,\text{km}| \\
&= 924\,\text{km}
\end{aligned}$$

We can now compute the average speed,

$$v_{av} = \frac{d}{\Delta t} = \frac{924\,\text{km}}{4\,\text{h}} = 231\,\text{km/h}$$

Question 8:

Two tourists, Tourist A, Tourist B, decide to tour a city, below we depict their Position V. Time plots, however, we were only able to record information of Tourist A after $t = 1$, and information about Tourist B after $t = 3$. Your task is to determine the equations of motion for both Tourists using the plots given.



Solution.

Let the intersection of the two plots be $A(4, 2)$. We need to find the linear equations of motion; $x_A = m_A t + b_A$, $x_B = m_B t + b_B$. We first compute the slope the plot and then afterwards determine the y-intercept of each plot,

$$\begin{aligned}
m_A &= \frac{x_2 - x_1}{t_2 - t_1} && \text{(I will use points P1 : (6,3), P2 : (4,2))} \\
&= \frac{2-3}{4-6} \\
&= \frac{1}{2} \\
m_B &= \frac{x_2 - x_1}{t_2 - t_1} && \text{(I will use points P1 : (6,6), P2 : (4,2))} \\
&= \frac{2-6}{4-6} \\
&= 2
\end{aligned}$$

We now have the slope of each plot, we now simply determine the b value for each plot by using a point that exists on their plots. You are free to choose any point, I will choose point $A(4,2)$,

$$\begin{aligned}
x_A &= m_A t + b_A \\
x_A &= \frac{1}{2}t + b_A \\
2 &= \frac{1}{2}(4) + b_A && \text{(Inserting point (4,2))} \\
b_A &= 0 \\
x_B &= m_B t + b_B \\
x_B &= 2t + b_B \\
2 &= 2(4) + b_B && \text{(Inserting point (4,2))} \\
b_B &= -6
\end{aligned}$$

Hence we can construct the equations of motions;

$$x_A = \frac{1}{2}t \quad x_B = 2t - 6$$

Question 9:

A bunny takes a tour around his neighborhood starting at his shelter. He travels 600 m[East] to House A, then from House A he travels 754 m[West] to House B, then from House B he travels 550 m[West] to House C, and then finally from House C he travels 2 km[East] to House D. Compute his average velocity as well as his average speed if the elapsed time was $\Delta t = 2\,\text{min}$. (Assume that the shelter is the reference point)

Solution.

Let us assign the [East] direction as the positive direction of motion, for the average velocity we must first obtain the initial position vector relative to the shelter, this happens to simply be $\vec{d}_I = +0\,\text{m}$. Next we must obtain the final position vector, the final location the bunny ends up is House D, hence we need the position vector $\vec{d}_{DS}$, where the subscript S represents the shelter. For this we use relative vector math operations, observe that we have the following vectors $\vec{d}_{AS}, \vec{d}_{BA}, \vec{d}_{CB}, \vec{d}_{DC}$, hence we can apply the relative vector proposition over multiple steps to get to desired result.

$$\begin{aligned}
\vec{d}_{DB} &= \vec{d}_{DC} + \vec{d}_{CB} \\
&= +2000\,\text{m} + (-550\,\text{m}) \\
&= +1450\,\text{m} \\
\vec{d}_{DA} &= \vec{d}_{DB} + \vec{d}_{BA} \\
&= +1450\,\text{m} + (-754\,\text{m}) \\
&= +696\,\text{m} \\
\vec{d}_{DS} &= \vec{d}_{DA} + \vec{d}_{AS} \\
&= +696\,\text{m} + (+600\,\text{m}) \\
&= +1296\,\text{m}
\end{aligned}$$

Hence the final position vector is $\vec{d}_F = \vec{d}_{DS} = +1296$, this allows use to compute the average velocity,

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} = \frac{+1296\,\text{m}}{120\,\text{s}} = +10.8\,\text{m/s} = 10.8\,\text{m/s[East]}$$

The average speed is much simpiler, the question gives us the amount he displaces as he traverses his journey ($A \to B \to C \to D$). That is, we precisely have all displacement vectors $\Delta\vec{d}_1 = \vec{d}_{AS}, \Delta\vec{d}_2 = \vec{d}_{BA}, \Delta\vec{d}_3 = \vec{d}_{CB}, \Delta\vec{d}_4 = \vec{d}_{DC}$. Therefore we compute the distance traveled using the Corollary, afterwards we compute the average speed,

$$\begin{aligned}
d &= \sum_i |\overrightarrow{\Delta d_i}| \\
&= |\overrightarrow{\Delta d_1}| + |\overrightarrow{\Delta d_2}| + |\overrightarrow{\Delta d_3}| + |\overrightarrow{\Delta d_4}| \\
&= |+600\,\text{m}| + |-754\,\text{m}| + |-550| + |+2000\,\text{m}| \\
&= 600\,\text{m} + 754\,\text{m} + 550\,\text{m} + 2000\,\text{m} \\
&= 3904\,\text{m} \\
v_{av} &= \frac{d}{\Delta t} = \frac{3904\,\text{m}}{120s} = 32.533\,\text{m/s}
\end{aligned}$$

Question 10:

Suppose that I fire an arrow straight up into the air from a cliff at a position $\vec{d}_{CG} = 56\,\text{m}[\text{North}]$ relative to the ground. Suppose that a wooden box 14 m high is lying on the ground, and that the arrow lands directly on top of it. Compute the average velocity as well as the average speed of the arrow if the duration of the flight was $\Delta t = 45\,\text{s}$. (Hint : The reference point is your choice)

Solution.

Let A denote the subscript of the arrow. Let us choose [North] as the positive direction of motion, let us assign the reference point as the cliff (C). The initial position vector of the arrow (A) relative to the cliff is $d_I = +0\,\text{m}$. The final position vector $\vec{d}_{AC}$ of the arrow relative to the cliff may be computed by implementing the relative vector proposition as well as the proposition which states that $-\vec{d}_{AB} = \vec{d}_{BA}$. Note that because the arrow lands on a box that is 14 m from the ground, the position vector of the arrow relative to the ground is $\vec{d}_{AG} = +14\,\text{m}$,

$$\vec{d}_{AC} = \vec{d}_{AG} + \vec{d}_{GC} = \vec{d}_{AG} + (-\vec{d}_{CG}) = +14\,\text{m} + (-56\,\text{m}) = -42\,\text{m}$$

Hence we have the final position vector $\vec{d}_f = \vec{d}_{AC} = -42\,\text{m}$. We can now compute the average velocity,

$$\vec{v}_{av} = \frac{\Delta d}{\Delta t} = \frac{\vec{d}_F - \vec{d}_I}{45} = \frac{-42\,\text{m} - (+0\,\text{m})}{45\,\text{s}} = -0.933\,\text{m/s}$$

Question 11:

Runner A and Runner B run back and forth across a $50\,\mathrm{m}$ track initially starting at position $(0, 0)$ and facing [East] (Assume that [East] is the positive direction of motion). Runner A has an average speed $v_{av} = 15\,\mathrm{m/s}$ and Runner B has an average speed of $v_{av} = 20\,\mathrm{m/s}$. After an elapsed time of $\Delta t = 1\,\mathrm{min}$, what was the position vector of Runner A relative to Runner B (i.e $\vec{d}_{AB}$)

Solution.

Let us first determine the amount of time it will take each runner to reach the end of the track, let this time for Runner A be Δt_A and the time for Runner B to be Δt_B.

$$\Delta t_A = \frac{d}{v_{av}} = \frac{10}{3}\,\mathrm{s}$$
$$\Delta t_B = \frac{d}{v_{av}} = \frac{5}{2}\,\mathrm{s}$$

Key Observations:

1. If it takes Δt seconds to reach the end of the track, then any integer multiple of $\Delta t \cdot k$ would mean that you have covered the track k times. This means that after $\Delta t \cdot k$ seconds, the runner is either at the start or end of the track.

2. Let Δt be the time it takes for a runner to reach the end of the track. Then by the end of even multiples times of Δt (2Δ, $4\Delta t$, etc) correspond to a Runner facing in the [East] direction and by the end of odd multiple times of Δt correspond to a Runner facing in the [West] direction. This is because both runners start facing [East], by the end of Δt, they will begin facing [West], then after 2Δ they will face east again and so on.

Observation 1 tells us that if $\Delta t \cdot k = 60$ has an integer solution for k, then that Runner is either at the front of the room or at the back of the room. Observation 2 tells us that if k is even then that Runner is facing [East], else he is facing [West]. Lets compute these results,

$$\begin{aligned}
\Delta t_A \cdot k &= 60 \\
\frac{10}{3} \cdot k &= 60 \\
k &= 18 \\
\Delta t_B \cdot k &= 60 \\
\frac{5}{2} \cdot k &= 60 \\
k &= 24
\end{aligned}$$

Hence we have found solutions for k for both Runner A and Runner B, also both k values are even implying by the end of 60 seconds, both runners have returned to the start of the track and are at the same location. Hence $\vec{d}_{AB} = +0\,\mathrm{m}$.