Question 1:

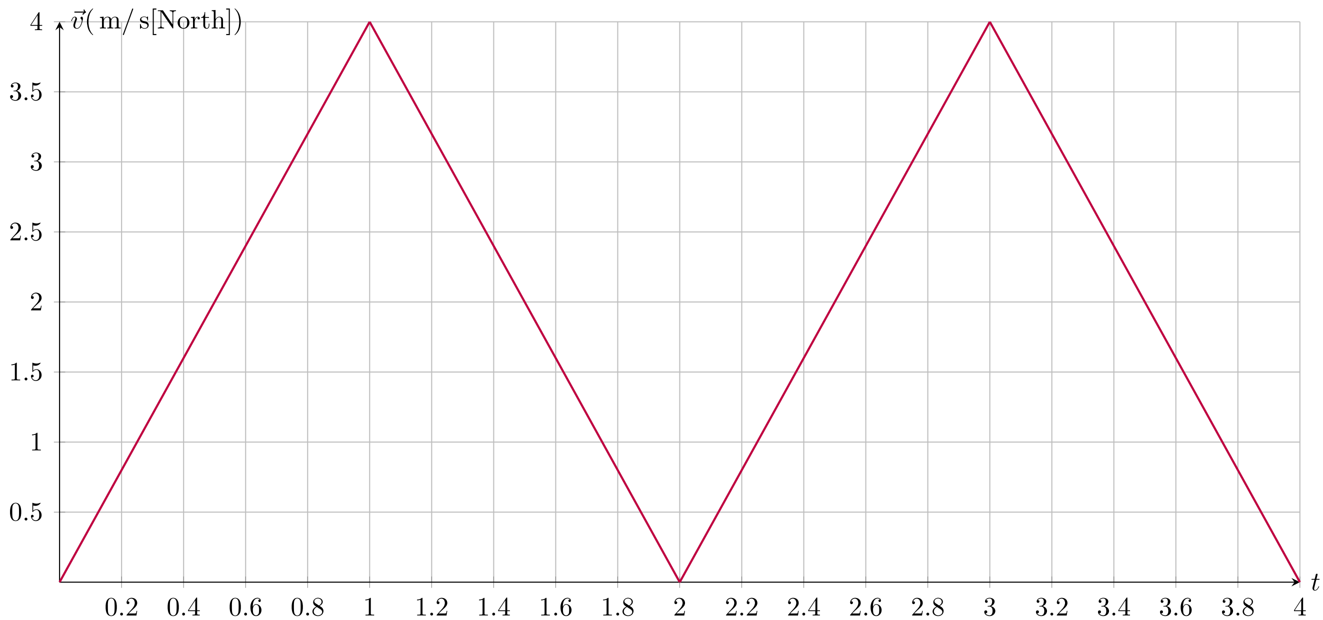
Answer the following True/False questions (**Assume [East] is positive**)

1. Consider an object under uniform motion in the negative direction.
   (a) The object has a non-zero average acceleration in the negative direction. (T / F) : *F*
   (b) At the end of the trip, the object may remain [East] relative to the reference point. (T / F): *T*
2. De-acceleration is just acceleration in the same direction of motion (T / F) : *F*
3. Suppose that a bullet accelerates at $\vec{a}_{av} = +1.068\,\mathrm{km/s^2}$ from rest to a final velocity of $\vec{v}_f = +356\,\mathrm{m/s}$. Then,
   (a) The time elapsed was $\Delta t = 3\,\mathrm{s}$ (T / F): *T*
   (b) If I double the acceleration of the bullet, then Δt doubles as well. (T / F): *F*
4. Suppose a Velocity V. Time plot is represented by $y = 2x + 4$,
   (a) The average acceleration is uniform (T / F): *T*
   (b) The initial velocity of the body at $t = 0$ was $\vec{v}_i = +4\,\mathrm{m/s}$ (T / F): *T*
   (c) The displacement over the time interval $[0, 2]$ was $\Delta\vec{d} = +12\,\mathrm{m}$ (T / F): *F*
   (d) The average acceleration is $\vec{a}_{av} = +2\,\mathrm{m/s^2}$ (T / F): *T*
5. A secant line on a Velocity V. Time graph over the interval $[t_1, t_2]$ gives me the instantaneous acceleration over the time interval $[t_1, t_2]$. (T / F): *F*
6. Suppose a Position V. Time plot is represented by $y = x^2 + 4$. Then,
   (a) The object is slowing down in the positive direction. (T / F): *F*
   (b) The object is experiencing uniform motion. (T / F): *F*
   (c) The object may be experiencing uniform acceleration (T / F): *F*
   (d) The initial position vector of the object at $t = 0$ is $\vec{d}_i = +2\,\mathrm{m}$ (T / F): *F*
7. Suppose that the tangent line to a Position V. Time plot at $t = 4$ was represented by the equation $y = -3x + 7$. Then,
   (a) The instantaneous velocity of the object at $t = 4$ was $\vec{v} = +3\,\mathrm{m/s}$ (T / F) : *F*
   (b) Suppose that the Position V. Time plot happened to be linear, then the average velocity of the object must have been $\vec{v}_{av} = -3\,\mathrm{m/s}$. (T / F) : *F*
8. Suppose a Velocity V. Time plot is represented by $y = -x + 3$, then the displacement over the time interval $[0, 6]$ is $\Delta\vec{d} = +0\,\mathrm{m}$. (T / F): *T*
9. Suppose that the average acceleration of an object in motion differs at two distinct points in time, then the Velocity V. Time plot must have been non-uniform. (T / F): *T*

Question 2:

Answer the following multiple choice questions.

1. Which of the following statements are correct about the plot below? (Assume that the motion lasted for 4 seconds)



   (a) The body experienced uniform acceleration throughout the entire trip.

   (b) Within the time interval $[0, 2]$ the average acceleration was $\vec{a}_{av} = +0\,\mathrm{m/s^2}$

   (c) Within the time interval $[3, 4]$ the average acceleration was $\vec{a}_{av} = -4\,\mathrm{m/s^2}$

   (d) Within the time interval $[1, 4]$ the average acceleration was $\vec{a}_{av} = -1.333\,\mathrm{m/s^2}$

   (e) At $t = 2\,\mathrm{s}$, the instantaneous acceleration was $\vec{a}_{av} = +4\,\mathrm{m/s^2}$

   (f) At $t = 3.4\,\mathrm{s}$, the instantaneous acceleration was $\vec{a}_{av} = -4\,\mathrm{m/s^2}$

   (g) The average acceleration is <u>not</u> the same as the instantaneous acceleration for each point in time.

   Solution: b), c), d), e), f)

2. The Velocity V. Time plot for a body in motion is similar to $y = 4x + 7$.

   (a) The displacement over the first $t = 4\,\mathrm{s}$ was $\Delta\vec{d} = +23\,\mathrm{m}$.

   (b) The object experienced uniform motion.

   (c) The object experienced uniform acceleration.

   (d) The object was speeding up in the positive direction

   Solution: c), d)

Question 3:

Answer the following inquires about the plot below,

(a) The displacement over the time interval $[1, 3]$.

(b) The displacement over the time interval $[3, 8]$.

(c) The displacement by the end of the trip ($\Delta t = 12\,\mathrm{s}$)

Solution.

(a)

$$\Delta\vec{d} = \text{Area[1,3]} = 2 \times 3 = +6\,\mathrm{m}$$

(b)

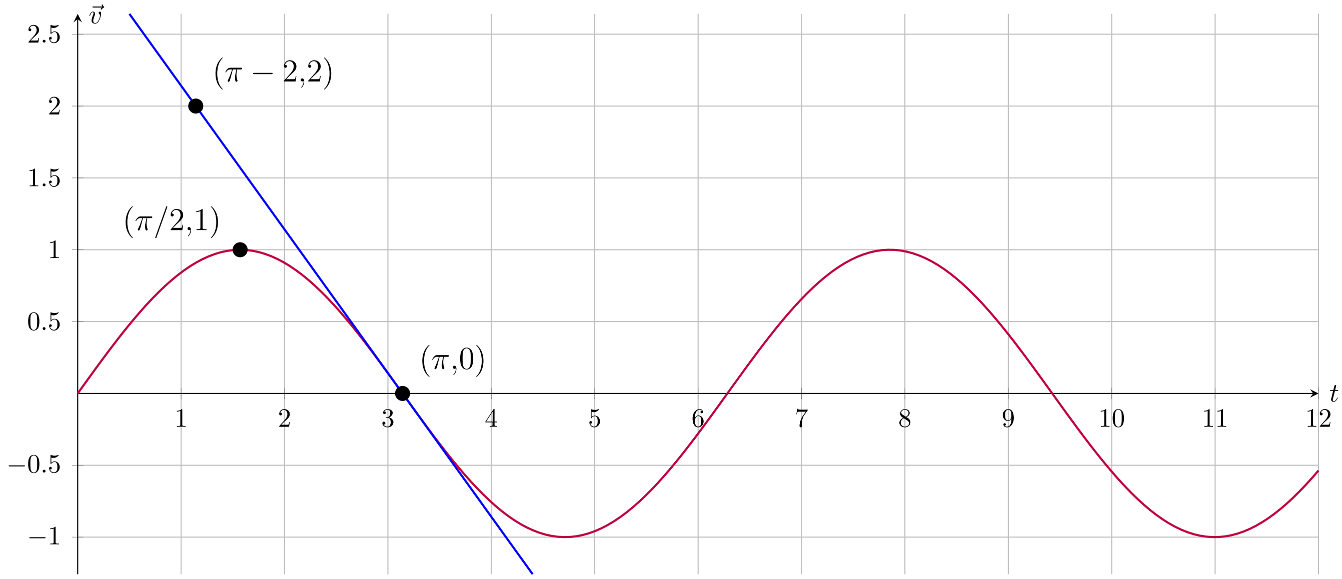
$$\begin{aligned}\Delta\vec{d} &= \text{Area[3,8]}\\ &= \text{Area[3,4]} + \text{Area[4,6]} + \text{Area[6,8]}\\ &= (+3.5)\ +\ (+4)\ +\ (-4)\\ &= +3.5\,\mathrm{m}\end{aligned}$$

(c)

$$\begin{aligned}
\Delta\vec{d} &= \text{Area}[0,12] \\
&= \text{Area}[0,1] + \text{Area}[1,3] + \text{Area}[3,8] + \text{Area}[8,10] + \text{Area}[10,12] \\
&= (+1.5)\ +\ (+6)\ +\ (+3.5)\ +\ (-8)\ +\ (-4) \\
&= -1\,\text{m}
\end{aligned}$$

Question 4:

Answer the following inquires about the Velocity V. Time plot below,



(a) Determine the average acceleration within the time interval $[\pi/2, \pi]$.

(b) Determine the instantaneous acceleration at time $t = \pi$.
(**Hint:** The line in blue is a tangent line to the plot at $t = \pi$)

(c) Prove that $\vec{a}_{av} = +0\,\mathrm{m/s^2}$ over the interval $[0, \pi]$.

Solution.

(a) We proceed by using proposition 1.0.2, which relates the slope of a secant line on a Velocity V. Time to the average acceleration. To determine the slope of the secant line, we choose any two points on the secant line, I will consider $P1\colon (\pi/2, 1)$, $P2\colon (\pi, 0)$

$$\vec{a}_{av} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{\pi - (\pi/2)} = \frac{-1}{\pi/2} = -\frac{2}{\pi}\,\mathrm{m/s^2}$$

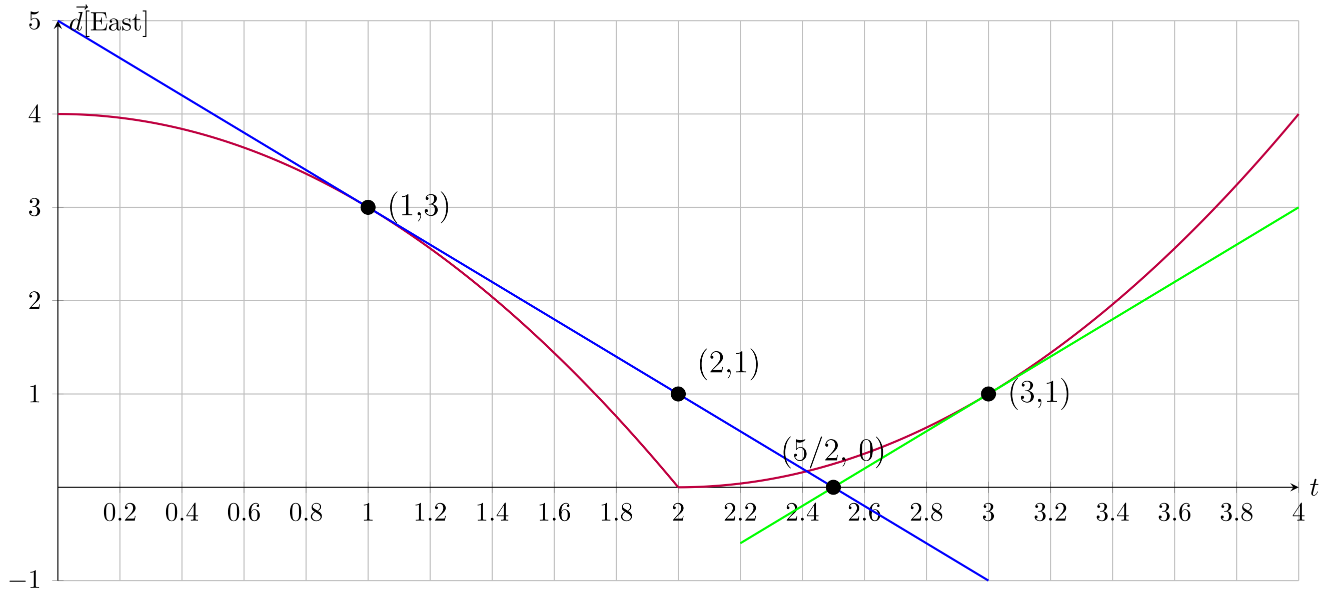
(b) To determine the instantaneous acceleration we turn to theorem 1.3.2 which states that the instantaneous acceleration at some time t is the slope of the tangent line to the plot at time t. The question already gives us the tangent line at time $t = \pi$. All that is left is to compute the slope of the tangent line, we can do so by choosing any two points, I will choose $P1\colon (\pi, 0)$, $P2\colon (\pi - 2, 2)$. (We denote the acceleration at $t = \pi$ as $\vec{a}(\pi)$)

$$\vec{a}(\pi) = m_T = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{\pi - 2 - \pi} = \frac{2}{-2} = -1\,\mathrm{m/s^2}$$

(c) *Proof.* Over the time interval $[0, \pi]$, the final and initial velocity vectors are equivalent, in other words, $\vec{v}_i = \vec{v}_f = +0\,\mathrm{m}$. This would imply that $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = +0\,\mathrm{m/s}$. This would imply that $\vec{a}_{av} = +0\,\mathrm{m/s^2}$. ■

Question 5:

Given the Position V. Time plot below, answer the following inquires.



(a) Determine the average velocity over the time interval $[0, 2]$.

(b) Describe the motion over the time interval $[0, 2]$

(c) Determine the instantaneous velocity at $t = 1$.
(**Hint:** The line in blue is a tangent line to the plot at $t = 2$)

(d) Describe the motion of the plot after $t = 2$ seconds.

(e) The slope of the tangent line in green is $m = +12$. Determine the equation of the line ($y = mx + b$).

Solution.

(a) The average velocity over the time interval $[0, 2]$ is simply the slope of the secant line over $[0, 2]$. To determine the slope we choose any two points on the secant line, I will choose $P1\colon (0, 5)$, $P2\colon (2, 0)$

$$\vec{v}_{av} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 5}{2 - 0} = -\frac{5}{2}\,\mathrm{m/s}$$

(b) The object is slowing down in the positive direction.

(c) To determine the instantaneous velocity at some time t we need to compute the slope of the tangent line to the graph at time t. The question already gives us the tangent line in blue at time $t = 1$. All that is left is to compute the slope of the tangent line, to do so we can choose any two points on the tangent line, I will choose $P1\colon (2, 1)$, $P2\colon (1, 3)$. (We denote the velocity at $t = 1$ as $\vec{v}(1)$)f

$$\vec{v}(1) = m_T = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{1 - 2} = -2\,\mathrm{m/s}$$

(d) d

Question 6:

A ball is kicked with an initial velocity of $\vec{v}_i = 80\,\mathrm{m/s}$[South]. It experiences a drag force and de-accelerates at $\vec{a}_{av} = 5\,\mathrm{m/s^2}$[North].

(a) Determine the final velocity of the ball after $\Delta t = 40\,\mathrm{s}$

(b) At what time t did ball start to travel in the Northward direction.

Question 7:

Patrick has decided to embark on a journey throughout the sea on a boat. The boat has a relative velocity of $\vec{v}_{PG} = 400\,\mathrm{m/s}$[East] relative to the ground (G). On the boat, Patrick is walking with a relative velocity of $\vec{v}_{PB} = +50\,\mathrm{m/s}$ relative to the boat. Determine the average acceleration of patrick relative to the ground. Determine,

(a) The velocity of patrick relative to the ground ($\vec{v}_{PG}$)
(**Hint:** Use the exact same technique from when we were working with position vectors, i.e $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$)

(b) The average acceleration of Patrick relative to the ground over a time period of $\Delta t = 40\,\mathrm{s}$ if everything was <u>initially</u> at rest.

Question 8:

A car is initially traveling at an initial velocity $\vec{v}_i = 412\,\mathrm{m/\,s}$[East]. The car then de-accelerates at an average acceleration of $\vec{a}_{av}$ to come to a rest at a red light over a duration of Δt. When the light turns green, the car accelerates at an average acceleration $-\vec{a}_{av}$ over a time period $2\Delta t$, to reach a final velocity of $\vec{v}_f = 240\,\mathrm{m/\,s}$ [East] . Determine the the average acceleration $\vec{a}_{av}$.
(**Hint :** Setup the correct equations to get rid of Δt)