1 Speed and Velocity

1.1 Average Speed

Definition 1.1.1
The **average speed** of an object is the ratio of the total distance traveled to the time elapsed.

$$v_{av} = \frac{d}{\Delta t}$$

Here $\Delta t = t_f - t_i$, or in other words the elapsed time.

We can conclude of course that v_{av} is of course a *scalar*, it has no associated direction, you could also argue that it is the ratio of two scalar quantities, time and distance, and hence it must be a scalar. The average speed is a common quantity we encounter everyday. Over a given distance traversed, our speed varies quite often, the average speed tells us the most common speed we were traveling at throughout the journey.

Remark: The units of v_{av} are the units of d divided by the units of Δt. (m /s for example).

Example 1.1.1
I am currently at the Library, 500 m[East] relative to my home. I decide to walk 350 m[West] to the Store. Compute my average speed if the trip took an hour. (In m/s)

Solution
$\Longrightarrow$

1.2 Average Velocity

Definition 1.2.1
The **average velocity** of an object is the ratio of the displacement to the time elapsed

$$\vec{v}_{av} = \frac{\overrightarrow{\Delta d}}{\Delta t}$$

We can conclude that $\vec{v}_{av}$ is a *vector quantity*, this is true because we can think of$(1/\Delta t)$ as a scalar, and a scalar multiplied by a vector ($\overrightarrow{\Delta d}$) is always a vector. The average velocity cares about our final position vector as well as our initial position vector.

Example 1.2.1
I am currently at the Library, 500 m[East] relative to my home. I decide to walk 350 m[West] to the Store. Compute my average *velocity* if the trip took an hour.

Solution
$\Longrightarrow$

Definition 1.2.2
A **position-time graph** is a graph of position versus time. We plot a set of position vectors on the vertical axes with their corresponding time on the horizontal axes.

For example let us consider a simple position versus time graph for a Ball rolling across a road starting at coordinates (0,0).

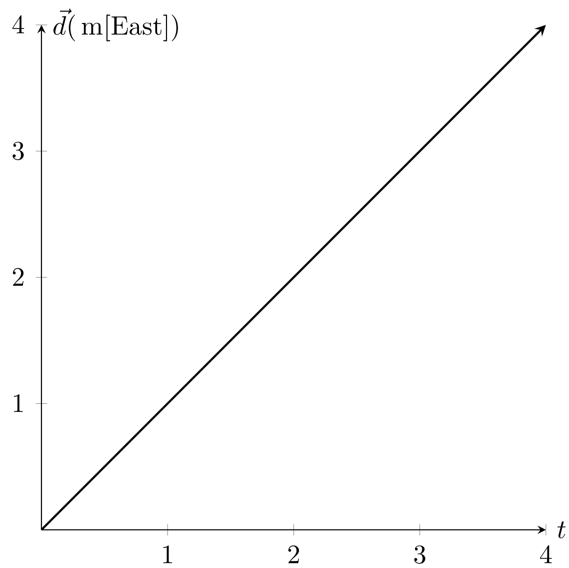


Figure 1: Position V. Time

At various time intervals, we can immediately extract the position of the Ball relative to the reference point (0,0). For example, at $t = 3$, the balls position vector was $\vec{d} = 3\,\text{m[East]}$.

Remark: Note that this graph assigns [East] as the positive direction of motion, this is because all y-points on the vertical axes are positive.

We can also consider the case of the Ball travelling [West] starting fom (0,0). We give the graph below,

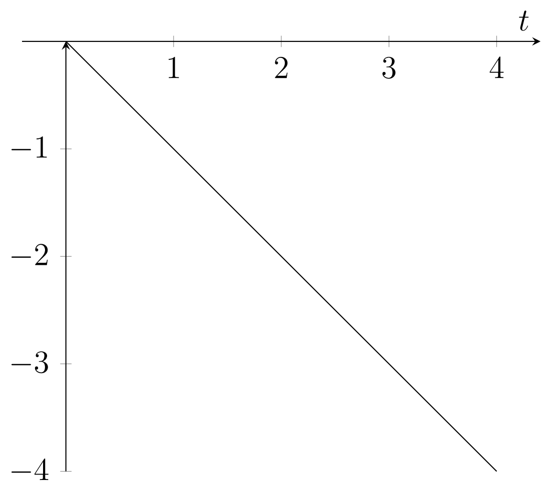


Figure 2 : Position V. Time (West case)

Example 1.2.2
I start at the Shopping center, 500 m[Right] of my house. I then travel 240 m[Left] to the Library. Preform the following,

(a) Draw the Graph of the trip.

(b) Compute my average speed

(c) Compute my average velocity

Solution
$\Longrightarrow$

1.3 Motion with Uniform and Non-uniform velocity

Definition 1.3.1
Uniform motion (Or constant velocity) is motion where the velocity is fixed.

Uniform motion has some unique properties which we will uncover later on and is hence important to make mention of. We usually encounter constant velocity motion on the road, is it true that you (or the driver) is always in a state of acceleration (changing velocity)?

Definition 1.3.2
Non-uniform motion (Or accelerated motion) is motion where the velocity is *not* fixed.

Proposition 1.3.1
Given a **Linear** Position v. Time plot of a moving body, the slope m represents the average velocity (or speed) of the body.

Proof
Let the slope of the Position V. Time graph be m, let us compute this slope by using the slope formula, namely,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Since the $y-$coordinates on a Position V. Time graph are position vectors $\vec{d}$, and the

$x-$coordinates are time points, t, we can translate this slope formula to the equivalent,

$$m = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$$

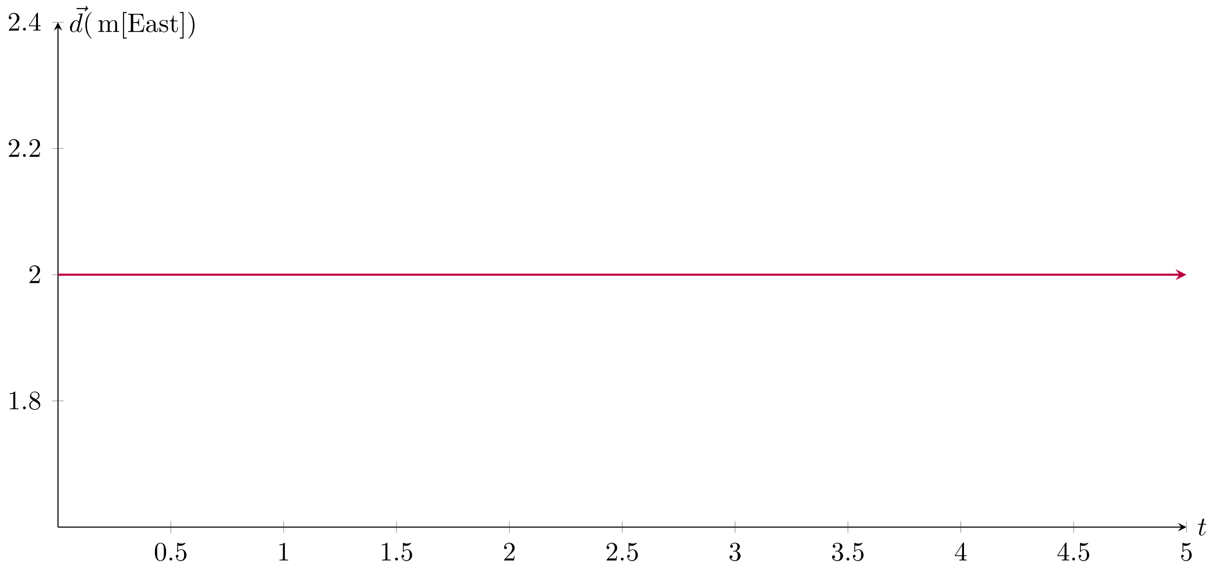
At this point we are free to choose any two coordinate pairs $(\vec{d}_1, t_1)$, $(\vec{d}_2, t_2)$, let us choose $(\vec{d}_f, t_f)$, $(\vec{d}_i, t_i)$, the final and inital coordinate pairs of the moving body. This gives,

$$m = \frac{\vec{d}_f - \vec{d}_i}{t_f - t_i} = \vec{v}_{av}$$

Since we consider motion in a single direction, the average speed and the average velocity only differ in direction. ■

1.4 Types of motion from Position V. Time Plots

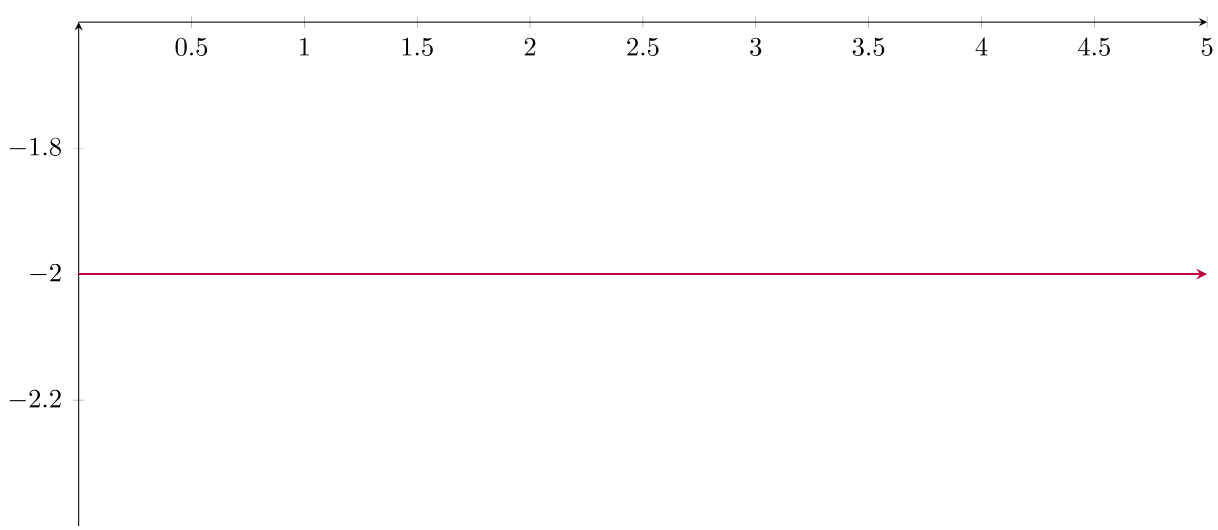
In this course we will encounter common types of motion and hence it would be useful to make mention of their plots and what they look like, as this should in turn enhance the students comprehension of the particular motion they are dealing with.



(a)

Properties of type (a):

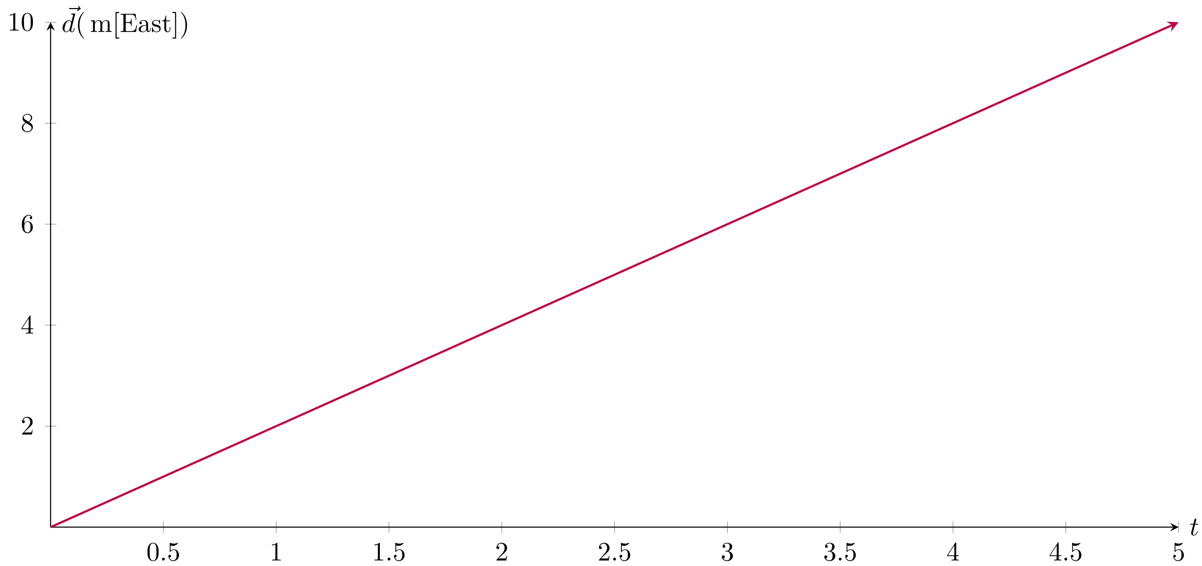
- The slope of the graph is zero, hence $\vec{v}_{av} = +0\,\mathrm{m/\,s}$.
- The item is at **rest**.
- The object is [East] relative to the reference point (0,0).



(b)

Properties of type (b):

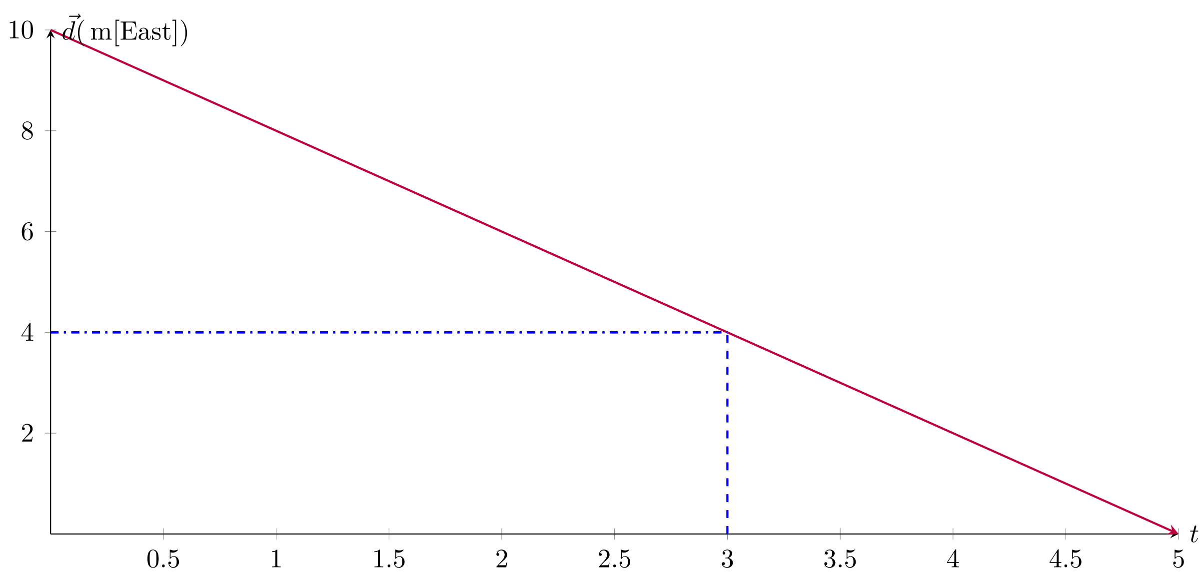
- The slope of the graph is zero, hence $\vec{v}_{av} = +0\,\mathrm{m/\,s}$.
- The item is at **rest**.
- The object is [West] relative to the reference point (0,0).



(c)

Properties of type (c):

- The slope of the graph is $m = +2$, hence $\vec{v}_{av} = +2\,\mathrm{m/\,s}$.
- The item experiencing **uniform motion** or **constant velocity**.
- The object is traveling in the [East] direction.



(d)

Properties of type (c):

- The slope of the graph is $m = -2$, hence $\vec{v}_{av} = -2\,\mathrm{m/\,s}$.
- The item experiencing **uniform motion** or **constant velocity**.
- The object is traveling in the [West] direction.