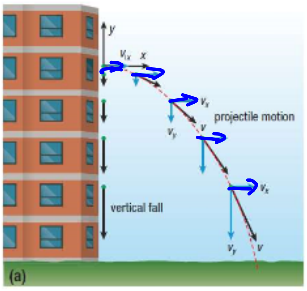
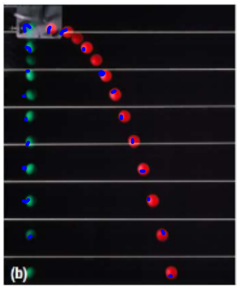
SPH3U: 2.3 Projectile Motion

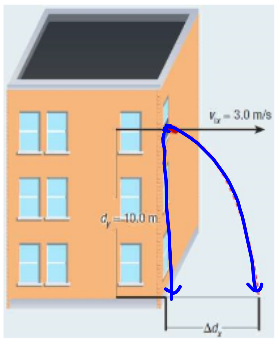
1. Projectile motion

Projectile:	an object that moves on a curved path due to gravity (falling).
projectile motion	horizontal motion (x) and vertical (y) are separate, or "independent".
projectile motion vs. river crossing	river crossing: both velocities are constant. projectile: vertical has acceleration.
range	$\vec{\Delta d_x}$, how far it goes horizontally.
convention	+ for up and right, − for down and left.





A beanbag is thrown from a window 10.0 m above the ground with an initial horizontal velocity of 3.0 m/s.



a. How long will it take the beanbag to reach the ground (what is its time of flight)?

y: $v_i = 0\ m/s$, $\Delta d = -10.0\ m$, $a = -9.8\ m/s^2$

$\Delta d = \cancel{v_i \Delta t}^{\,0} + \frac{1}{2} a \Delta t^2$

$\Delta t = \sqrt{\frac{2\Delta d}{a}} = \sqrt{\frac{-20}{-9.8}} = 1.4\ s.$

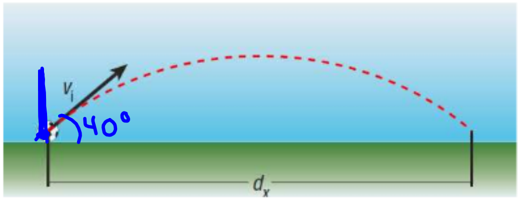
b. How far will the beanbag travel horizontally (what is its range)?

x: $\Delta t = 1.4\ s$, $v = 3.0\ m/s$.

$\Delta d = v\Delta t = 3.0(1.4) = 4.2\ m$.

2. Launching a projectile at an angle

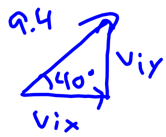
A soccer player running on a level playing field kicks a soccer ball with a velocity of 9.4 m/s at an angle of 40° above the horizontal. Determine the soccer ball's:



a. time of flight

Y $a = -9.8\ m/s^2$, $\Delta d = 0\ m$,

$v_{iy} = 9.4 \sin 40° = 6.042\ m/s$.



$\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$

$0 = 6.042 \Delta t + \frac{1}{2}(-9.8)\Delta t^2$

$0 = \Delta t (6.042 - 4.9 \Delta t) \longrightarrow 6.042 - 4.9\Delta t = 0$.

$\Delta t = 0$ or $\Delta t = \frac{6.042}{4.9} = 1.233 s$.

∴ the time of flight is 1.2 s.

b. range

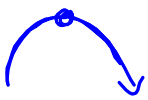
X $\Delta t = 1.233 s$, $v_x = 9.4 \cos 40°$

$\Delta d = v \Delta t = (9.4 \cos 40)(1.233)$

$= 8.879\ m$

∴ the range is 8.9 m.

c. maximum height



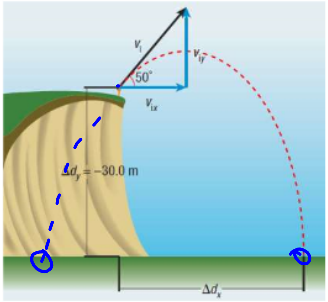
Y $v_i = 6.042\ m/s$, $a = -9.8\ m/s^2$, $v_f = 0\ m/s$.

$v_f^2 = v_i^2 + 2a\Delta d$

$\Delta d = \frac{v_f^2 - v_i^2}{2a} = \frac{0^2 - 6.042^2}{2(-9.8)} = 1.9\ m$.

∴ the max height is 1.9 m.

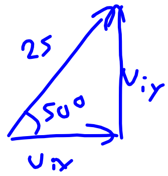
A golfer is trying to improve the range of her shot. To do so she drives a golf ball from the top of a steep cliff, 30.0 m above the ground where the ball will land. If the ball has an initial velocity of 25 m/s and is launched at an angle of 50° above the horizontal, determine the ball's:



a. time of flight

Y $\Delta d = -30\,m$, $a = -9.8\,m/s^2$,

$v_{iy} = 25\sin 50° = 19.151\,m/s.$



$\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$

$-30 = 19.151\Delta t + \frac{1}{2}(-9.8)\Delta t^2 \rightarrow 0 = -4.9\Delta t^2 + 19.151\Delta t + 30.$

$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-19.151 \pm \sqrt{19.151^2 - 4(-4.9)(30)}}{2(-4.9)} = -1.197,\ \underline{5.115\,s}$

$\therefore \Delta t = 5.1\,s.$

b. range

X $\Delta d_x = v_x \Delta t$ $\qquad v_x = 25\cos 50° = 16.070\,m/s.$

$= (16.070)(5.115) = 82.19\,m$

$\therefore \Delta d_x = 82\,m.$

c. final velocity (just before it hits the ground)

$v_x = 16.070\,m/s.$

Y $v_{iy} = 19.151\,m/s$, $a_y = -9.8\,m/s^2$, $\Delta d_y = -30\,m.$

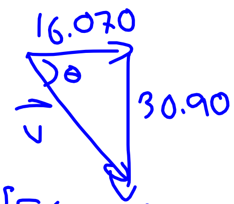
$v_f^2 = v_i^2 + 2a\Delta d$

$v_f = \pm\sqrt{v_i^2 + 2a\Delta d} = \sqrt{19.151^2 + 2(-9.8)(-30)}$

$= 30.90\,m/s.$

Both $v = \sqrt{16.070^2 + 30.90^2} = 35\,m/s.$

$\theta = \tan^{-1}\left(\frac{30.90}{16.070}\right) = 63°.$



$\therefore \vec{v}_f = 35\,m/s\ [F\,63°\,D].$

Homework: page 81: #1-2, 4, 6