Question 1:

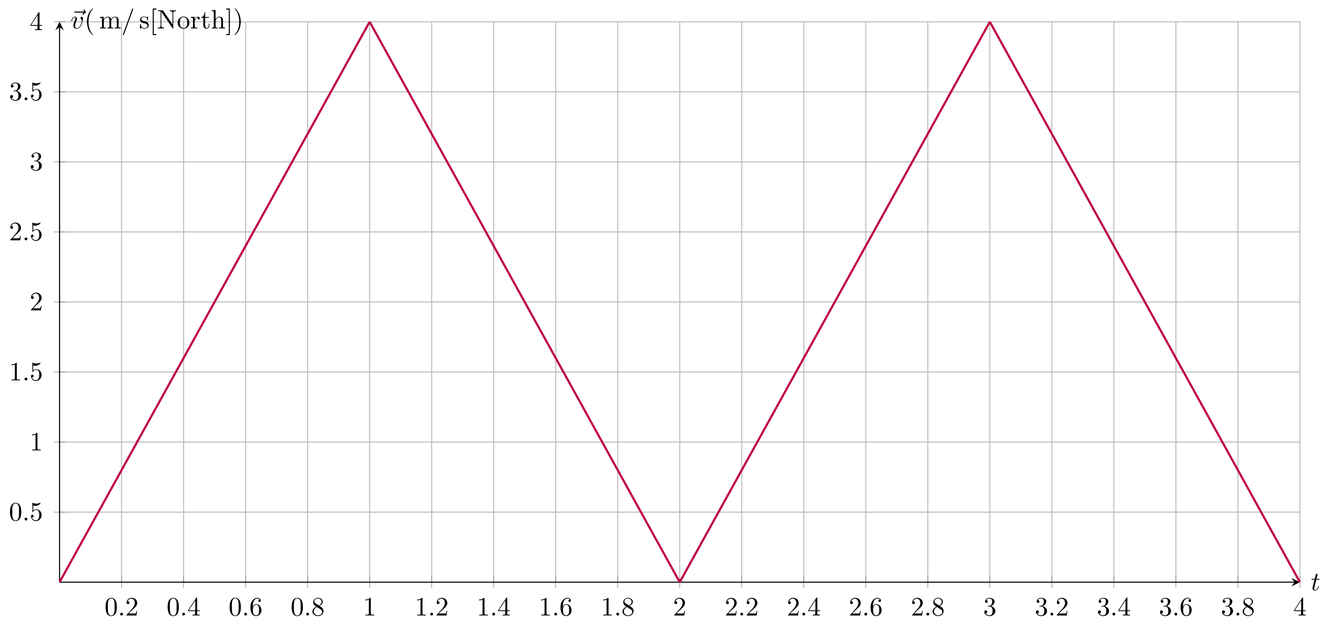
Answer the following True/False questions (**Assume [East] is positive**)

1. Consider an object under uniform motion in the negative direction.
   (a) The object has a non-zero average acceleration in the negative direction. (T / F) : *F*
   (b) At the end of the trip, the object <u>may</u> remain [East] relative to the reference point. (T / F): *T*
2. De-acceleration is just acceleration in the same direction of motion (T / F) : *F*
3. Suppose that a bullet accelerates at $\vec{a}_{av} = +1.068\,\mathrm{km/s^2}$ from rest to a final velocity of $\vec{v}_f = +356\,\mathrm{m/s}$. Then,
   (a) The time elapsed was $\Delta t = 3\,\mathrm{s}$ (T / F): *F*
   (b) If I double the acceleration of the bullet, then Δt doubles as well. (T / F): *F*
4. Suppose a Velocity V. Time plot is represented by $y = 2x + 4$,
   (a) The average acceleration is uniform (T / F): *T*
   (b) The initial velocity of the body at $t = 0$ was $\vec{v}_i = +4\,\mathrm{m/s}$ (T / F): *T*
   (c) The displacement over the time interval $[0, 2]$ was $\Delta\vec{d} = +12\,\mathrm{m}$ (T / F): *F*
   (d) The average acceleration is $\vec{a}_{av} = +2\,\mathrm{m/s^2}$ (T / F): *T*
5. A secant line on a Velocity V. Time graph over the interval $[t_1, t_2]$ gives me the instantaneous acceleration over the time interval $[t_1, t_2]$. (T / F): *F*
6. Suppose a Position V. Time plot is represented by $y = x^2 + 4$. Then,
   (a) The object is slowing down in the positive direction. (T / F): *F*
   (b) The object is experiencing uniform motion. (T / F): *F*
   (c) The object <u>may</u> be experiencing uniform acceleration (T / F): *F*
   (d) The initial position vector of the object at $t = 0$ is $\vec{d}_i = +2\,\mathrm{m}$ (T / F): *F*
7. Suppose that the tangent line to a Position V. Time plot at $t = 4$ was represented by the equation $y = -3x + 7$. Then,
   (a) The instantaneous velocity of the object at $t = 4$ was $\vec{v} = +3\,\mathrm{m/s}$ (T / F) : *F*
   (b) Suppose that the Position V. Time plot happened to be linear, then the average velocity of the object must have been $\vec{v}_{av} = -3\,\mathrm{m/s}$. (T / F) : *F*
8. Suppose a Velocity V. Time plot is represented by $y = -x + 3$, then the displacement over the time interval $[0, 6]$ is $\Delta\vec{d} = +0\,\mathrm{m}$. (T / F): *T*
9. Suppose that the average acceleration of an object in motion differs at two distinct points in time, then the Velocity V. Time plot must have been non-uniform. (T / F): *T*

Question 2:

Answer the following multiple choice questions.

1. Which of the following statements are correct about the plot below? (Assume that the motion lasted for 4 seconds)



   (a) The body experienced uniform acceleration throughout the entire trip.

   (b) Within the time interval $[0, 2]$ the average acceleration was $\vec{a}_{av} = +0\,\mathrm{m/\,s^2}$

   (c) Within the time interval $[3, 4]$ the average acceleration was $\vec{a}_{av} = -4\,\mathrm{m/\,s^2}$

   (d) Within the time interval $[1, 4]$ the average acceleration was $\vec{a}_{av} = -1.333\,\mathrm{m/\,s^2}$

   (e) At $t = 2\,\mathrm{s}$, the instantaneous acceleration was $\vec{a}_{av} = +4\,\mathrm{m/\,s^2}$

   (f) At $t = 3.4\,\mathrm{s}$, the instantaneous acceleration was $\vec{a}_{av} = -4\,\mathrm{m/\,s^2}$

   (g) The average acceleration is <u>not</u> the same as the instantaneous acceleration for each point in time.

   Solution: b), c), d), e), f)

2. The Velocity V. Time plot for a body in motion is similar to $y = 4x + 7$.

   (a) The displacement over the first $t = 4\,\mathrm{s}$ was $\Delta\vec{d} = +23\,\mathrm{m}$.

   (b) The object experienced uniform motion.

   (c) The object experienced uniform acceleration.

   (d) The object was speeding up in the positive direction

   Solution: c), d)

Question 3:

Answer the following inquires about the plot below,

(a) The displacement over the time interval $[1, 3]$.

(b) The displacement over the time interval $[3, 8]$.

(c) The displacement by the end of the trip ($\Delta t = 12\,\mathrm{s}$)

Solution.

(a)

$$\Delta\vec{d} = \text{Area}[1,3] = 2 \times 3 = +6\,\mathrm{m}$$

(b)

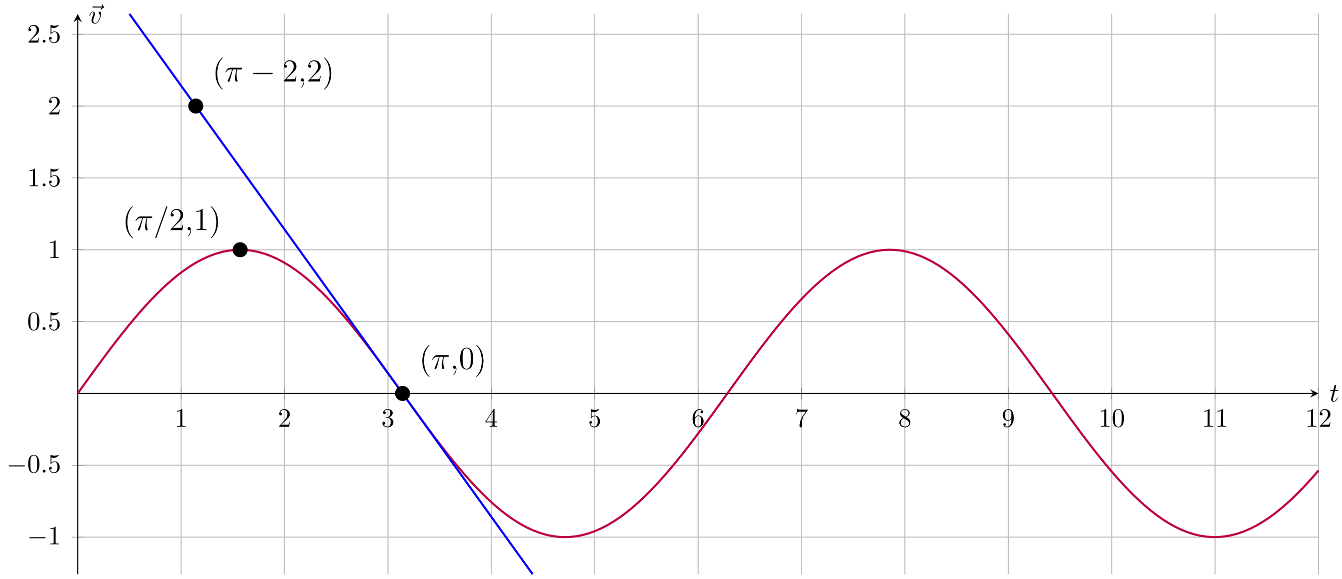
$$\begin{aligned}\Delta\vec{d} &= \text{Area}[3,8] \\ &= \text{Area}[3,4] + \text{Area}[4,6] + \text{Area}[6,8] \\ &= (+3.5)\ +\ (+4)\ +\ (-4) \\ &= +3.5\,\mathrm{m}\end{aligned}$$

(c)

$$\begin{aligned}\Delta\vec{d} &= \text{Area[0,12]}\\ &= \text{Area[0,1]} + \text{Area[1,3]} + \text{Area[3,8]} + \text{Area[8,10]} + \text{Area[10,12]}\\ &= (+1.5)\ +\ (+6)\ +\ (+3.5)\ +\ (-8)\ +\ (-4)\\ &= -1\,\text{m}\end{aligned}$$

Question 4:

Answer the following inquires about the Velocity V. Time plot below,



(a) Determine the average acceleration within the time interval $[\pi/2, \pi]$.

(b) Determine the instantaneous acceleration at time $t = \pi$.
(**Hint:** The line in blue is a tangent line to the plot at $t = \pi$)

(c) Prove that $\vec{a}_{av} = +0\,\mathrm{m/s^2}$ over the interval $[0, \pi]$.

Solution.

(a) We proceed by using proposition 1.0.2, which relates the slope of a secant line on a Velocity V. Time to the average acceleration. To determine the slope of the secant line, we choose any two points on the secant line, I will consider $P1\colon (\pi/2, 1)$, $P2\colon (\pi, 0)$

$$\vec{a}_{av} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{\pi - (\pi/2)} = \frac{-1}{\pi/2} = -\frac{2}{\pi}\,\mathrm{m/s^2}$$

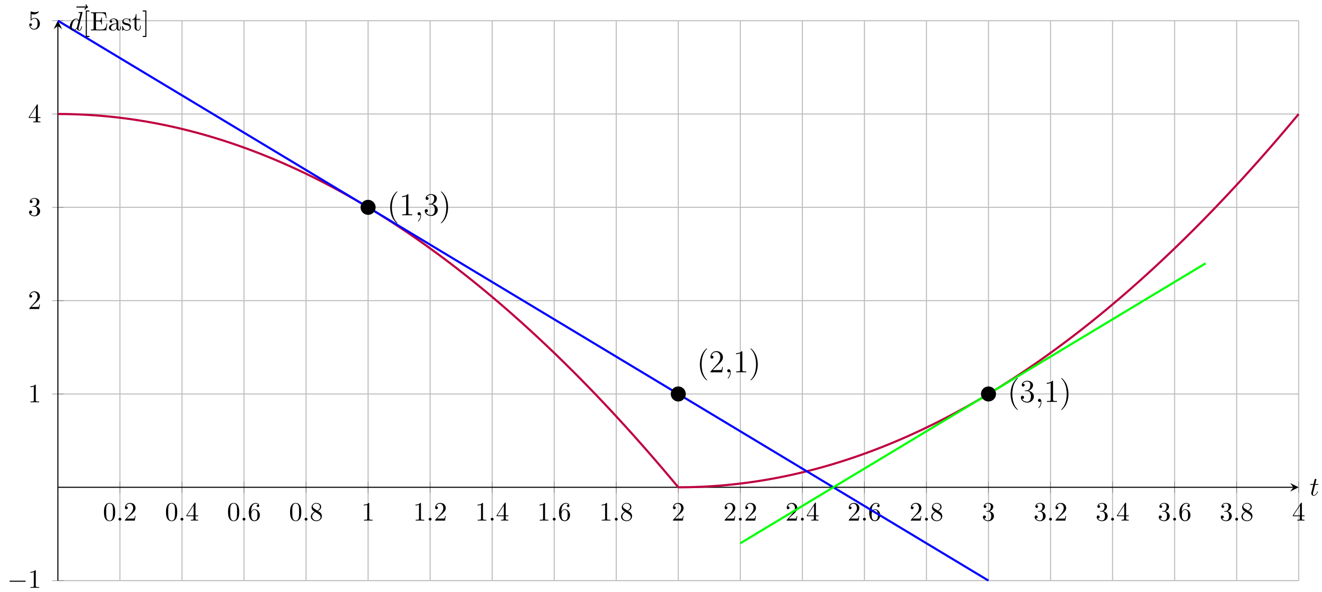
(b) To determine the instantaneous acceleration we turn to theorem 1.3.2 which states that the instantaneous acceleration at some time t is the slope of the tangent line to the plot at time t. The question already gives us the tangent line at time $t = \pi$. All that is left is to compute the slope of the tangent line, we can do so by choosing any two points, I will choose $P1\colon (\pi, 0)$, $P2\colon (\pi - 2, 2)$. (We denote the acceleration at $t = \pi$ as $\vec{a}(\pi)$)

$$\vec{a}(\pi) = m_T = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{\pi - 2 - \pi} = \frac{2}{-2} = -1\,\mathrm{m/s^2}$$

(c) *Proof.* Over the time interval $[0, \pi]$, the final and initial velocity vectors are equivalent, in other words, $\vec{v}_i = \vec{v}_f = +0\,\mathrm{m}$. This would imply that $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = +0\,\mathrm{m/s}$. This would imply that $\vec{a}_{av} = +0\,\mathrm{m/s^2}$. ■

Question 5:

Given the Position V. Time plot below, answer the following inquires.



(a) Determine the average velocity over the time interval $[0, 2]$.

(b) Describe the motion over the time interval $[0, 2]$

(c) Determine the instantaneous velocity at $t = 1$.
(**Hint:** The line in blue is a tangent line to the plot at $t = 2$)

(d) Describe the motion of the plot after $t = 2$ seconds.

(e) Determine the equation of the tangent line at $t = 3$ ($y = mx + b$).

Solution.

(a) The average velocity over the time interval $[0, 2]$ is simply the slope of the secant line over $[0, 2]$. To determine the slope we choose any two points on the secant line, I will choose $P1\colon (0, 4)$, $P2\colon (2, 0)$

$$\vec{v}_{av} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{2 - 0} = -\frac{4}{2}\,\mathrm{m/s} = -2\,\mathrm{m/s}$$

(b) The object is speeding up in the positive direction.

(c) To determine the instantaneous velocity at some time t we need to compute the slope of the tangent line to the graph at time t. The question already gives us the tangent line in blue at time $t = 1$. All that is left is to compute the slope of the tangent line, to do so we can choose any two points on the tangent line, I will choose $P1\colon (2, 1)$, $P2\colon (1, 3)$. (We denote the velocity at $t = 1$ as $\vec{v}(1)$)f

$$\vec{v}(1) = m_T = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{1 - 2} = -2\,\mathrm{m/s}$$

(d) Speeding up in the positive direction

(e) We first determine the slope of the tangent line, to do this we choose any two points on the tangent line. We observe that the first point is given at $P1\colon (3,1)$, the second point can be obtained by using the point of intersection between the blue tangent line and the green tangent line. To obtain this point, we observe that this is precisley the $x-$intercept of the blue tangent line, hence we can first construct the equation of the blue tangent line and then proceed to determine the $x-$intercept. From b) we know that the slope of the blue tangent line is $m_T = -2$, also the plot depicts the $y-$intercept of the tangent line which is $b = 5$, hence,

$$y = -2x + 5$$

The $x-$intercept must therefore be $x = 5/2 = 2.5$, hence the second point is $P2\colon (2.5, 0)$. We proceed by determining the slope of the green tangent line as well as the y-intercept,

$$\begin{aligned} m_T &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{2.5-3} = \frac{1}{.5} = +2 \\ y &= 2x + b \\ 1 &= 2(3) + b && \text{(Inserted point } (3,1)) \\ b &= -5 \end{aligned}$$

Hence the equation of the green tangent line is $y = 2x - 5$

Question 6:

A ball is kicked with an initial velocity of $\vec{v}_i = 80\,\mathrm{m/s}$[South]. It experiences a drag force and de-accelerates at $\vec{a}_{av} = 5\,\mathrm{m/s^2}$[North].

(a) Determine the final velocity of the ball after $\Delta t = 40\,\mathrm{s}$

(b) At what time t did ball start to travel in the Northward direction.

Solution.

(a) We proceed by using the average acceleration equation, I will choose [South] as the positive direction of motion.

$$\begin{aligned}
\vec{a}_{av} &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \\
\vec{a}_{av}(\Delta t) &= \vec{v}_f - \vec{v}_i \\
\vec{v}_f &= \vec{v}_i + (\vec{a}_{av})\Delta t \\
&= +80 + (-5)(40) \\
&= -120\,\mathrm{m/s}
\end{aligned}$$

Hence the final velocity of the ball was $\vec{v}_f = 120\,\mathrm{m}/s$[North]

(b) We observe that since acceleration is in the opposite direction of motion, at some point in time the velocity must have been $\vec{v}_f = +0\,\mathrm{m/s}$. The time Δt when $\vec{v}_f = +0\,\mathrm{m/s}$ precisely corresponds to the time at which the ball switched its direction of motion and started traveling [North].

$$\begin{aligned}
\vec{a}_{av} &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \\
\Delta t &= \frac{\vec{v}_f - \vec{v}_i}{\vec{a}_{av}} \\
&= \frac{0 - 80}{-5} \\
&= 16\,\mathrm{s}
\end{aligned}$$

Therefore at $t = 16\,\mathrm{s}$, the ball changes its direction of motion.

Question 7:

Patrick has decided to embark on a journey throughout the sea on a boat. The boat has a relative velocity of $\vec{v}_{BG} = 400\,\mathrm{m/s}$[East] relative to the ground (G). On the boat, Patrick is walking with a relative velocity of $\vec{v}_{PB} = 50\,\mathrm{m/s}$[East] relative to the boat. Determine,

(a) The velocity of patrick relative to the ground ($\vec{v}_{PG}$)
(**Hint:** Use the exact same technique from when we were working with position vectors, i.e $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$)

(b) The average acceleration of Patrick relative to the ground over a time period of $\Delta t = 40\,\mathrm{s}$ if everything was <u>initially</u> at rest.

Solution.

(a) Let us assign [East] as the positive direction of motion. We use the statement from the hint, in other words for reference points A, B, C, $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$. Hence,

$$\vec{v}_{PG} = \vec{v}_{PB} + \vec{v}_{BG} = 50 + 400 = +450\,\mathrm{m/s}$$

Hence the velocity of patrick relative to the ground is $\vec{v}_{PG} = 450\,\mathrm{m/s}$[East].

(b) We proceed with the equation for average acceleration,

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{450 - 0}{40} = +11.25\,\mathrm{m/s^2}$$

Hence the average acceleration after $\Delta t = 40\,\mathrm{s}$, was $\vec{a}_{av} = 11.25\,\mathrm{m/s^2}$[East].

Question 8:

A car is initially traveling at an initial velocity $\vec{v}_i = 412\,\mathrm{m/s}$[East]. The car then de-accelerates at an average acceleration of $\vec{a}_{av}$ to come to a rest at a red light over a duration of Δt. When the light turns green, the car accelerates at an average acceleration $-\vec{a}_{av}$ over a time period $2\Delta t$, to reach a final velocity of $\vec{v}_f = 240\,\mathrm{m/s}$ [East] . Determine the the average acceleration $\vec{a}_{av}$.
(**Hint :** Setup the correct equations to get rid of Δt)

Solution.

Let us assign [East] as the positive direction of motion. We first setup the equation for the motion of the car until it reaches the red light by using the average acceleration equation,

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{0 - 412}{\Delta t} = \frac{-412}{\Delta t}$$

$$\vec{a}_{av} = \frac{-412}{\Delta t} \quad \Longrightarrow \quad \vec{a}_{av}\Delta t = -412 \tag{8.1}$$

We now setup the second equation of motion of the car from the moment the light turns green until he reaches his final speed of $\vec{v}_f = 240\,\mathrm{m/s}$[East].

$$-\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{2\Delta t} = \frac{240 - 0}{2\Delta t} = \frac{120}{\Delta t}$$

$$-\vec{a}_{av} = \frac{120}{\Delta t} \quad \Longrightarrow \quad -\vec{a}_{av}\Delta t = 120 \tag{8.2}$$