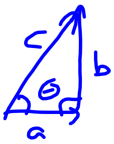
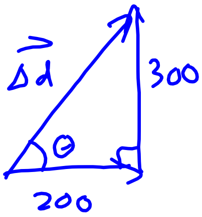
SPH3U: 2.2 Motion in Two Dimensions – Algebraic Approach

1. Adding displacements in two dimensions



Adding perpendicular vectors:	use basic trigonometry (Pythagorean Theorem and tan ratio).
magnitude	$a^2+b^2=c^2$
angle	$\tan\theta = \frac{b}{a} \rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

A jogger runs 200.0 m [E], turns at an intersection, and continues for an additional displacement of 300.0 m [N]. What is the jogger's total displacement?

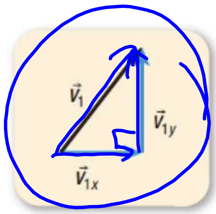


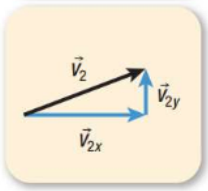
$$\Delta d = \sqrt{200^2+300^2} = 360 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{300}{200}\right) = 56^\circ$$

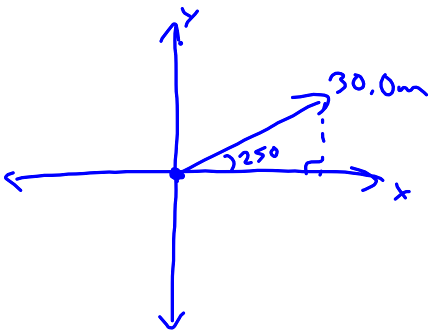
$$\therefore \vec{\Delta d} = 360 \text{ m } [E56^\circ N].$$

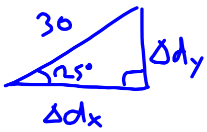
Component vectors:	perpendicular (x and y) vectors that add up to the original vector.





Break the displacement vector 30.0 m [E25°N] down into perpendicular component vectors.





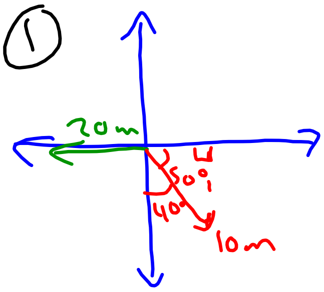
$$\cos 25^\circ = \frac{\Delta d_x}{30}$$

$$\rightarrow \vec{\Delta d_x} = 30\cos 25^\circ = \underline{27.19 \text{ m } [E]}$$

$$\sin 25^\circ = \frac{\Delta d_y}{30}$$

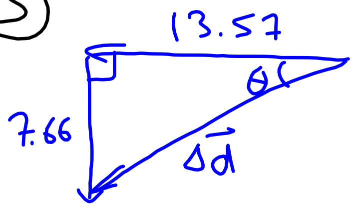
$$\rightarrow \vec{\Delta d_y} = 30\sin 25^\circ = \underline{12.68 \text{ m } [N]}$$

A cat walks 20.0 m [W] and then turns and walks a further 10.0 m [S40°E]. What is the cat's total displacement?

① 

②
$$\vec{\Delta d_x} = \vec{\Delta d_{1x}} + \vec{\Delta d_{2x}}$$
$$= 20\,m\ [W] + 10\cos 50°\ [E].$$
$$= -20\,m + 10\cos 50°$$
$$= -13.57\ m$$
$$\underline{= 13.57\,m\ [W].}$$

$$\vec{\Delta d_y} = \vec{\Delta d_{1y}} + \vec{\Delta d_{2y}}$$
$$= 0\,m - 10\sin 50°$$
$$= -7.66\ m$$
$$\underline{= 7.66\,m\ [S].}$$

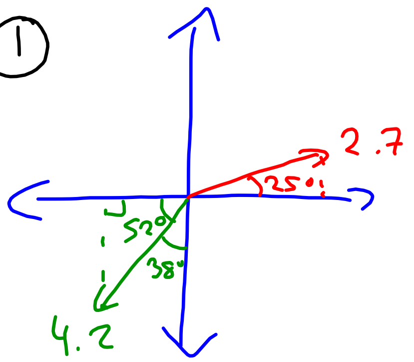
③ 

④
$$\Delta d = \sqrt{13.57^2 + 7.66^2}$$
$$= 15.58\,m$$
$$\theta = \tan^{-1}\left(\frac{7.66}{13.57}\right)$$
$$= 29°$$

⑤ ∴ the total displacement is $\vec{\Delta d} = 15.6\,m\ [W29°S]$.

A hockey puck travels a displacement of 4.2 m [S38°W]. It is then struck by a hockey player's stick and undergoes a displacement of 2.7 m [E25°N]. What is the puck's total displacement?

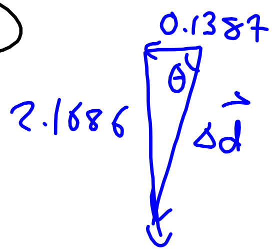
1



2

$$\overrightarrow{\Delta d_x} = 2.7\cos 25° - 4.2\cos 52°$$
$$= -0.1387 \text{ m}$$
$$= 0.1387 \text{ m [W]}.$$
$$\overrightarrow{\Delta d_y} = 2.7\sin 25° - 4.2\sin 52°$$
$$= -2.1686 \text{ m}$$
$$= 2.1686 \text{ m [S]}.$$

3



4

$$\Delta d = \sqrt{0.1387^2 + 2.1686^2}$$
$$= 2.1730 \text{ m}$$
$$\theta = \tan^{-1}\left(\frac{2.1686}{0.1387}\right)$$
$$= 86.34°$$

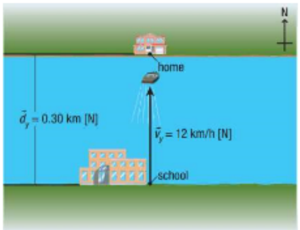
5

$$\therefore \overrightarrow{\Delta d} = 2.2 \text{ m [W86°S]}.$$

2. Adding velocities in two dimensions

River crossing problems:	2-dimensional problems with perpendicular vectors. the 2 dimensions are independent.

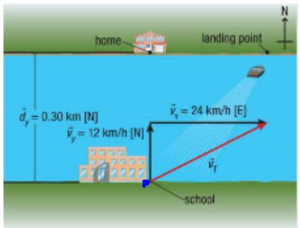
A physics student hops into her motorboat and steers straight across a river at a constant velocity of 12 km/h [N]. If the river is 0.30 km across and has no current, how long will it take her to cross the river?



$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ → $\Delta t = \frac{\Delta \vec{d}}{\vec{v}} = \frac{0.30\text{ km}}{12\text{ km/h}}$

$= 0.025\text{ h} \times \frac{3600\text{ s}}{\text{h}} = 90\text{ s}.$

Most rivers have a current moving in the direction of the river. The river now has a current of 24 km/h [E], as shown to the right. How long does it now take the boat to cross the river?



Still 90 s (x and y are independent).

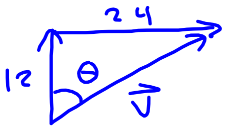
How far downstream does the boat land?

$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$ $\Delta \vec{d}_x = \vec{v}_x \Delta t$

$= (24\text{ km/h [E]})(0.025\text{ h})$

$= 0.6\text{ km [E]}.$

What is the boat's resultant velocity?



$v = \sqrt{12^2 + 24^2} = 27\text{ km/h}.$

$\theta = \tan^{-1}\left(\frac{24}{12}\right) = 63°$

$\therefore \vec{v} = 27\text{ km/h [N63°E]}.$

Homework: page 75: #1-3, 6b, 8-9