2 Properties of Vector Spaces, Intro to Subspaces

Theorem 1

In any vector space, the zero vector is unique. That is if $0, z \in V$ have property where $v + 0 = v$ and $v + z = v$, then $z = 0$.

Theorem 2

In any vector space, the additive inverse of a vector is unique. That is, $\forall v \in V$, if there exists vectors $y, x \in V$ such that $v + y = 0$ and $v + x = 0$, then $y = x$.

Definition 1: Subtraction of vectors

Let V be a vector space, suppose $v, w \in V$. The *difference* of these two vectors, denoted $v - w$, is simply that sum $v + (-w)$, that is the sum of v and the additive inverse of w.

Lemma 1

Let V be a vector space. The following properties hold for V:

1. $\forall \mathbf{v} \in V$, $0 \in \mathbb{F}$, $0 \cdot \mathbf{v} = \mathbf{0}$.
2. $\forall c \in \mathbb{F}$, $c \cdot \mathbf{0} = \mathbf{0}$.
3. Given $c \in \mathbb{F}$, $\mathbf{v} \in V$, if $c \cdot \mathbf{v} = \mathbf{0}$, then $c = 0$ or $\mathbf{v} = \mathbf{0}$.

Lemma 2

Let V be a vector space. The following properties hold for V:

1. $\forall v \in V$, $(-1) \cdot v = -v$. (The additive inverse of v)
2. $\forall c \in \mathbb{F}$, $v \in V$, $c \cdot (-v) = (-c) \cdot v = -(c \cdot v)$.
3. $\forall c \in \mathbb{F}$, $v, w \in V$, $c(v - w) = cv - cw$.