Solutions - Lecture 3 - Homework

Question 1.

Solution. Yes, $f(x) = x$ is precisely the identity function on $\mathbb{R}$ since it simply returns whatever is input. For a more thorough justification, we can show that,

$$\mathrm{id}_{\mathbb{R}}(x) = x = f(x).$$

Question 2.

Solution. Solution:

(a) The function g fails to be surjective since $2 \in \mathcal{B}$ is not mapped to. It also fails to be surjective since $g(-2) = g(-1) = 1$ (Two distinct inputs are mapped to the same output). Hence g is neither surjective nor injective. (And of course not invertible)

(b) Each element in $\mathcal{T}$ is mapped to and hence we can conclude that f is surjective. Also notice that no two elements from $\mathcal{H}$ are mapped to the same element in $\mathcal{T}$, hence we conclude that f is injective. Since f is both injective and surjective, we conclude that f is indeed invertible.

(c) Each element in $\mathcal{W}$ is mapped to and hence we can conclude that T is surjective. However note that $T(0) = T(1) = 2$ and hence T fails to be injective. We conclude that T is only surjective.

(d) Since no two elements from $\mathcal{Q}$ map to the same element in $\mathcal{P}$ we conclude that S is injective. However since $1 \in \mathcal{P}$ is not mapped to, we conclude that S fails to be surjective. And hence S is at most injective.

(e) Each element in $\mathcal{C}$ is mapped to and hence we can conclude that P is surjective. Also notice that no two elements from $\mathcal{D}$ are mapped to the same element in $\mathcal{C}$, hence we conclude that P is injective. Since P is both injective and surjective, we conclude that P is indeed invertible.

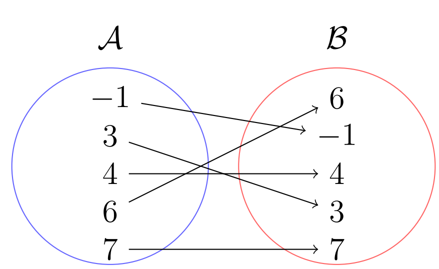
Question 3.

Solution. Since $\mathcal{R}(2) = \mathcal{R}(4) = 0$, we conclude that the function fails to be injective. Since the function fails to be injective, we conclude that it is not invertible.

Question 4. We can prove that the identity function on $\mathcal{A}$ will work here, that is,

$$\begin{gathered} \mathrm{id}_{\mathcal{A}} \colon \mathcal{A} \to \mathcal{B} \\ \mathrm{id}_{\mathcal{A}}(a) = a. \end{gathered}$$

Lets see how the mapping diagram will look like,



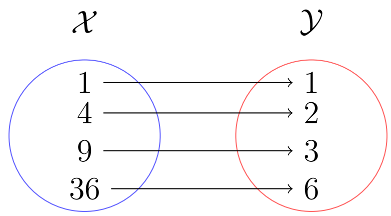
From the diagram we can see that no two elements from $\mathcal{A}$ are mapped to the same element in $\mathcal{B}$, hence $\mathrm{id}_{\mathcal{A}}$ is injective. Also since each element in $\mathcal{B}$ is mapped to, we conclude that $\mathrm{id}_{\mathcal{A}}$ is surjective. Hence $\mathrm{id}_{\mathcal{A}}$ is an invertible function.

Question 5.

Solution. It is indeed true that $\mathcal{R}_T = \mathcal{B}$. This is because if T is invertible, then T is surjective, if T is surjective, then every element in $\mathcal{B}$ is mapped to, meaning that the set of all outputs for T is $\mathcal{B}$, in other words, $\mathcal{R}_T = \mathcal{B}$.

Question 6.

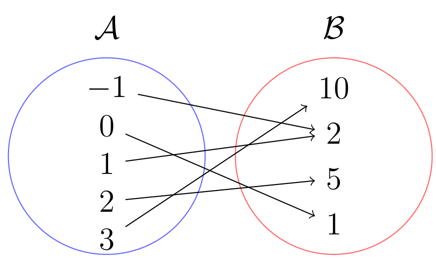
Solution. The following is the mapping diagram for $\mathcal{L}$,



From the diagram we can see that no two elements from $\mathcal{X}$ are mapped to the same element in $\mathcal{Y}$, hence $\mathcal{L}$ is injective. Also since each element in $\mathcal{Y}$ is mapped to, we conclude that $\mathcal{L}$ is surjective. Hence $\mathcal{L}$ is an invertible function.

Question 7.

Solution. The following is the mapping diagram for $\mathcal{P}$,



Notice that since $\mathcal{P}(-1) = \mathcal{P}(1) = 2$, we conclude that $\mathcal{P}$ fails to be injective. Since $\mathcal{P}$ fails to be injective, it fails to be invertible.