How to determine the inverse of a function

So far we have been dealing with invertible functions equipped with relatively small domains and co-domains. In such a scenario, it was easy to have intuition for the inverse function, or to perhaps guess it and confirm with mapping tables, However for domains like $\mathbb{R}, \mathbb{Z}, \mathbb{N}$, its not so obvious. Hence we would like to formulate an algorithm to assist in determine the inverse function. Given an **invertible** function $f(x)$, to determine the inverse function $f^{-1}(x)$, follow the procedure given below,

Step 1. Replace $f(x)$ with the variable y.

Step 2. Isolate for x.

Step 3. Replace x with $f^{-1}(x)$, and replace y with x. And your done!

Example. Determine the inverse function for the following function, **(In class)**

(a) $f(x) = 3x + 7$

(b) $g(x) = -2\sqrt{4x + 16} + 1$

(c) $h(x) = -\frac{4}{2x-1} - 3$

Horizontal Line Test

The **Horizontal Line Test** is a quick way to check weather or not a function is invertible (This is very similar to the vertical line test). Preform the following,

1. Graph or sketch the function then draw a Horizontal through **the ENTIRE** the plot.

2. **IF** the Horizontal line hits the graph **once**, then the function is invertible.

3. **ELSE**, the function is **not** invertible.

Example. Determine weather or not the following functions are invertible or not using the Horizontal line test, **(In class)**,

(a) $g(x) = \sqrt{x}$.

(b) $f(x) = 2x - 3$.

(c) $G(x) = x^2 - 3$.

Practice Problems:

Question 1. Determine the inverse of the following functions,

(a) $T(x) = \frac{1}{2x-1} + 4$

(b) $H(x) = -2x + 22$.

(c) $F(x) = \sqrt{4x+4}$.

(d) $\mathcal{L}(x) = \sqrt{4x-1} + 7$

(e) $\mathcal{H}(x) = \frac{3}{4}x - 1$

(f) $\mathcal{F}(x) = -\frac{3}{x+1} + 6$

(g) $\mathcal{P}(x) = -4\sqrt{2x+8}$

Question 2. A linear function passes through the points $(1,3)$ and $(2,5)$. Determine its inverse function.

Question 3. Determine weather or not the following functions are invertible or not using the Horizontal line test,

(a) $f(x) = -x + 3$.

(b) $g(x) = -x^2 + 6x - 8$.

(c) $h(x) = |x|$.

Question 4. Textbook, Pg 138 Q4.

Question 5. Textbook, Pg 138 Q12.

Question 6. Textbook, Pg 138 Q18. (**Important Question ***)

Question 7. Textbook, Pg 138 Q21.(**Important Question ***)

Solutions to Practice Problems:

Question 1.

(a) $T^{-1}(x) = \frac{1}{2(y-4)} + \frac{1}{2}$.

(b) $H^{-1}(x) = -\frac{1}{2}(x - 22)$

(c) $F^{-1}(x) = \frac{1}{4}(x^2 - 4)$

(d) $\mathcal{L}^{-1}(x) = \frac{1}{4}(x - 7)^2 + \frac{1}{4}$.

(e) $\mathcal{H}^{-1}(x) = \frac{4}{3}x + \frac{4}{2}$.

(f) $\mathcal{F}^{-1}(x) = -\frac{3}{x-6} - 1$.

(g) $\mathcal{P}^{-1}(x) = \frac{x^2}{8} - 4$.

Question 2. $f^{-1}(x) = \frac{1}{2}(x - 1)$.

Question 3.

(a) Invertible.

(b) Not Invertible.

(c) Not Invertible.

Question 4 - 7. (Refer to Textbook, ASK me if your confused by their answers)