Functions Quiz 2

January, 2022

1 Name and Date:

Print your name and todays date below;

Name	Date

Question 1: (8 points)

Answer the following True/False questions,

1. Let $\mathrm{id}_{\mathbb{R}} \colon \mathbb{R} \to \mathbb{R}$ be the identity function on $\mathbb{R}$, then

$$\mathrm{id}_{\mathbb{R}}(\mathrm{id}_{\mathbb{R}}(\mathrm{id}_{\mathbb{R}}(\mathrm{id}_{\mathbb{R}}(\mathrm{id}_{\mathbb{R}}(\mathrm{id}_{\mathbb{R}}(2)))))) = 2.$$

   Circle the correct answer: **True** **False**

2. Let $\phi \colon \mathcal{A} \to \mathcal{B}$ be an *injective* function, then every element in $\mathcal{B}$ is mapped to.
   Circle the correct answer: **True** **False**

3. Let $\mathcal{L} \colon \mathcal{A} \to \mathcal{B}$ be a *surjective* function, then $|\mathcal{R}_{\mathcal{L}}| = |\mathcal{B}|$, where $\mathcal{R}_{\mathcal{L}}$ is the range of $\mathcal{L}$.
   Circle the correct answer: **True** **False**

4. Let $\lambda \colon \mathcal{A} \to \mathcal{B}$ be a function, suppose λ is one of surjective or injective, then λ is invertible.
   Circle the correct answer: **True** **False**

5. Let $\mathcal{X} = \{-1, 0, 1\}$ and $\mathcal{Y} = \{-1, 1, 2\}$ be sets, lets define the following function,

   - $\eta \colon \mathcal{X} \to \mathcal{Y}$.
   - $\eta(x) = 2x^2 - 1$.

   Then η is an invertible function.
   Circle the correct answer: **True** **False**

6. Let $\mathcal{V} = \{-2, 0\}$ and $\mathcal{W} = \{0, 4\}$ be sets, define the following function,

   - $\mathcal{T} \colon \mathcal{V} \to \mathcal{W}$.
   - $\mathcal{T}(v) = v^2$.

   Then the function,

   - $\mathcal{T}^{-1} \colon \mathcal{W} \to \mathcal{V}$.
   - $\mathcal{T}^{-1}(w) = \sqrt{w}$.

   is the inverse function for $\mathcal{T}$.
   Circle the correct answer: **True** **False**

7. Let $\mathbf{A}$ and $\mathbf{B}$ be binary strings, then $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.
   Circle the correct answer: **True** **False**

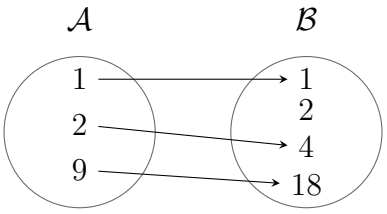
8. Let $G \colon \mathcal{H} \to \mathcal{T}$ be a function. If $|\mathcal{H}| = |\mathcal{T}|$, then G is invertible.
   Circle the correct answer: **True** **False**

Question 2: (8 points)

For each of the following, you are given a function and its definition. For each question,

- Prove that the function is invertible **or** prove that the function is not invertible.
- Determine the range of the function.

(a) $g\colon \mathcal{A} \to \mathcal{B}$,



(b) Let $\mathcal{X} = \{-3-2, 0, 1\}$, $\mathcal{Y} = \{0, 1, 2, 3\}$ be sets and define,

- $H\colon \mathcal{X} \to \mathcal{Y}$.
- $H(x) = |x|$.

Question 3: (7 marks)

Let $\mathcal{X} = \{3, 7, 4\}$ and $\mathcal{Y} = \{2, 0, 1\}$ be sets, define the following function,

- $A\colon \mathcal{X} \to \mathcal{Y}$.
- $A(x) = \sqrt{x-3}$.

Prove that the function,

- $A^{-1}\colon \mathcal{Y} \to \mathcal{X}$.
- $A^{-1}(y) = y^2 + 3$.

is the inverse function for $\mathcal{L}$.
Hint: Use mapping tables.

Question 4: (3 marks)

Let $\mathcal{S} = \{101, 010, 111\}$ and $\mathcal{R} = \{11, 01, 10\}$ be sets of binary strings, define the following function,

- $Q \colon \mathcal{S} \to \mathcal{R}$.
- $Q(\mathbf{S}) = \mathbf{s}_1\mathbf{s}_2$.

Prove that the function,

- $Q^{-1} \colon \mathcal{R} \to \mathcal{S}$.
- $Q^{-1}(\mathbf{R}) = \mathbf{R} + 1$.

is **NOT** the inverse function for Q.
Hint: Which of the two conditions in Definition 4.1 does it fail to preserve?