How to determine the inverse of a function

Given a function $f(x)$ we would like to determine the inverse function $f^{-1}(x)$. Follow the procedure given below to do so,

Step 1. Replace $f(x)$ with the variable y.

Step 2. Isolate for x.

Step 3. Replace x with $f^{-1}(x)$, and replace y with x. And your done!

These steps seem simple in theory but can be tricky in practice, lets find the inverse function for the following examples **(In class)**,

1. $f(x) = 3x + 7$
2. $g(x) = -2\sqrt{4x + 16} + 1$
3. $h(x) = -\frac{4}{2x-1} - 3$

Practice Problems:

Question 1. Determine the inverse of the following functions,

(a) $T(x) = \frac{1}{2x-1} + 4$

(b) $\mathcal{L}(x) = \sqrt{4x - 1} + 7$

(c) $\mathcal{H}(x) = \frac{3}{4}x - 1$

(d) $\mathcal{F}(x) = -\frac{3}{x+1} + 6$

(e) $\mathcal{P}(x) = -4\sqrt{2x + 8}$

Question 2. A linear function passes through the points $(1, 3)$ and $(2, 5)$. Determine its inverse function.

Solutions to Practice Problems:

Question 1. Solution:

(a) $T^{-1}(x) = \frac{1}{2(y-4)} + \frac{1}{2}$.

(b) $\mathcal{L}^{-1}(x) = \frac{1}{4}(x-7)^2 + \frac{1}{4}$.

(c) $\mathcal{H}^{-1}(x) = \frac{4}{3}x + \frac{4}{2}$.

(d) $\mathcal{F}^{-1}(x) = -\frac{3}{x-6} - 1$.

(e) $\mathcal{P}^{-1}(x) = \frac{x^2}{8} - 4$.

Question 2. We can use the points given to find the slope of the equation of the line using the slope formula. How you lable the points is up to you, ill label them as $P_1(1,3)$ and $P_2(2,5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{2-1} = 2.$$

So far we know that,

$$f(x) = 2x + b.$$

At this point we need to determine the b value, to do so we can plug either P_1 or P_2 into the function to solve, ill use P_1.

$$\begin{aligned} f(1) &= 2(1) + b \\ 3 &= 2 + b \\ b &= 1. \end{aligned}$$

Hence we have our final form,

$$f(x) = 2x + 1.$$

Now we move on to determine the inverse function,

$$\begin{aligned} y &= 2x + 1 \\ y - 1 &= 2x \\ x &= \frac{y}{2} - \frac{1}{2} \\ f^{-1}(x) &= \frac{x}{2} - \frac{1}{2}. \end{aligned}$$