Solutions Test 1 - Review

Question 1. Solution: (Check with me)

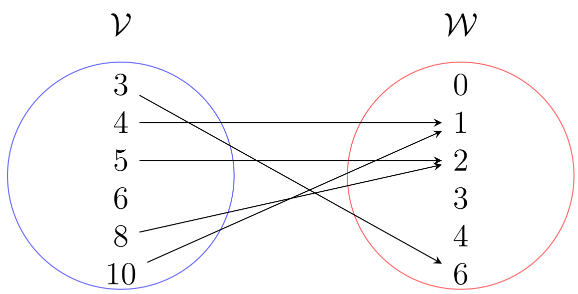
Question 2. Solution:

(a) $H = \{4, 5, 6, 7, 8, \cdots\}$.

(b) $R = \{-1, 0, 1, 2, 3, 4\}$.

(c) $A = \{2, -2\}$.

Question 3. Solution:



Question 4. Solution:

(a) $T(T(1)) = 175$.

(b) $H(H(-2)) = -4$.

(c) $T(H(0)) = -1$.

(d) $T(x) = x(3x + 4)$.

(e)

$$\begin{aligned} T(H(x)) &= 3(x-1)^2 + 4(x-1) \\ &= 3(x^2 - 2x + 1) + 4(x-1) \\ &= 3x^2 - 6x + 3 + 4x - 4 \\ &= 3x^2 - 2x - 1. \end{aligned}$$

The two integers you should get are $p = 1, q = -3$. (How you label it is up to you), afterwards we need.

- $t = \gcd(|a|, |p|) = \gcd(|3|, |1|) = \gcd(3, 1) = 1$.
- $k = \gcd(|q|, |c|) = \gcd(|-3|, |-1|) = \gcd(3, 1) = 1$.

Since $a \cdot q = 3 \cdot -3 = -9$, we conclude that $a \cdot q < 0$ and hence,

$$T(H(x)) = (tx - k)\left(\frac{a}{t}x + \frac{p}{t}\right) = (x - 1)(3x + 1).$$

Question 5. Solution:

(a) $\mathcal{D} = \{x \in \mathbb{R} \mid x \leq 2\}$, $\mathcal{R} = \{y \in \mathbb{R} \mid y \leq -7\}$.

(b) $\mathcal{D} = \mathbb{R}$, $\mathcal{R} = \{y \in \mathbb{R} \mid y \leq 6\}$.

(c) $\mathcal{D} = \mathbb{R}$, $\mathcal{R} = \mathbb{R}$.

(d) $\mathcal{D} = \mathbb{R}$, $\mathcal{R} = \{y \in \mathbb{R} \mid y \geq -5\}$.

(e) $\mathcal{D} = \{x \in \mathbb{R} \mid x \neq \frac{2}{5}\}$, $\mathcal{R} = \{y \in \mathbb{R} \mid y \geq 4\}$.

(f) $\mathcal{D} = \{x \in \mathbb{R} \mid -3 \leq x \leq 1\}$, $\mathcal{R} = \{y \in \mathbb{R} \mid -2 \leq y \leq 2\}$.

Question 6. Solution:

(a) By the discriminant formula we have,

$$\begin{aligned} d &= b^2 - 4ac \\ &= (5)^2 - 4(2)(-3) \\ &= 25 + 24 \\ &= 49. \end{aligned}$$

Because $d > 0$ we conclude that $f(x)$ will have two **distinct** solutions.

(b) The two integers you should get are $p = -1, q = 6$. (How you label it is up to you), afterwards we need.

- $t = \gcd(|a|, |p|) = \gcd(|2|, |-1|) = \gcd(2, 1) = 1$.
- $k = \gcd(|q|, |c|) = \gcd(|6|, |-3|) = \gcd(6, 3) = 3$.

Since $a \cdot q = 2 \cdot 6 = 12$, we conclude that $a \cdot q > 0$ and hence,

$$f(x) = (tx + k)\left(\frac{a}{t}x + \frac{p}{t}\right) = (x + 3)(2x - 1).$$

(c) How you label them is up to you,

$$x_1 = -3, \quad x_2 = \frac{1}{2}.$$

(d) From part (c) we have the x-intercepts so we can skip step 1. Proceeding with calculating h,

$$h = \frac{x_1 + x_2}{2} = \frac{-3 + \frac{1}{2}}{2} = -\frac{5}{4}.$$

Now k,

$$\begin{aligned} k &= f(h) \\ &= f(-\frac{5}{4}) \\ &= 2\left(-\frac{5}{4}\right)^2 + 5\left(-\frac{5}{4}\right) - 3 \\ &= \frac{50}{16} - \frac{25}{4} - 3 = -\frac{98}{16}. \end{aligned}$$

Lastly a,

$$a = b \cdot m \cdot k = 1 \cdot 1 \cdot 2 = 2.$$

Finally we have $f(x)$ in vertex form,

$$f(x) = 2\left(x + \frac{5}{4}\right)^2 - \frac{98}{16}.$$

(e) Solution: