

# Functions Final Exam

February 3, 2021

## 1 Preamble

This final exam covers everything we have learned in this course, with emphasize towards material after test 2. Student's **must show all work** to receive full marks.

## 2 Allowed Aids

The following aids are allowed on the Test

- Pencil, Pen, Eraser, Highlighter, Ruler, Protractor, Spare sheets of **blank** paper.
- Reference sheet (**Double sided paper preprepared by student**)

## 3 Restrictions:

- **NO** calculator's.

## 4 Remarks:

- $\sqrt{xy} = \sqrt{x} \sqrt{y}$ .
- $\sqrt{x} \sqrt{y} = \sqrt{xy}$ .

## 5 Name and Date:

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Name

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Date

## Part A - Multiple Choice

**Question 1.** Answer the following True/False questions,

1. Let  $\mathcal{S} = \{1, 2, 3\}$ , then  $\mathcal{S} + \mathcal{S} = \mathcal{S} + \mathcal{S} + \mathcal{S}$ .

Circle the correct answer:    **True**    **False**

2.  $(\sqrt{4} + \sqrt{64}) \notin \mathbb{N}$ .

Circle the correct answer:    **True**    **False**

3. The number 29 is a prime number.

Circle the correct answer:    **True**    **False**

4. Let  $T = \{x \in \mathbb{Z} \mid |x| = -1\}$ , then  $T$  is **not** empty.

Circle the correct answer:    **True**    **False**

5. The vertex of

$$g(x) = 3(x + \sqrt{4})^2 - 4^2$$

is  $(-4, -4)$ .

Circle the correct answer:    **True**    **False**

6. The vertex of,

$$H(x) = -(x + 2)^2 + 1.$$

represents a minimum.

Circle the correct answer:    **True**    **False**

7. Let  $f(x) = \sqrt{x}$ . Suppose we apply the following transformations to  $f$ ,

- Reflection across the x-axis.
- Vertical stretch by a factor of 2.
- Horizontal compression by a factor of 2.
- Horizontal shift, right by 2 units.
- Vertical shift, down by 4 units.

Then the corresponding transformation equation is  $h(x) = -2f(2x - 4) - 4$ .

Circle the correct answer:    **True**    **False**

8. Let  $f(x) = |x|$ , and let  $h(x) = -f(2x+4)-5$  be a transformation of  $f(x)$ , then the corresponding coordinate transformation of  $f$  is,

$$(x, f(x)) \longrightarrow \left(\frac{x+4}{2}, -f(x) - 5\right).$$

Circle the correct answer:    **True**    **False**

9. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = 2\sqrt{x} + 1$  be a function. Then  $f$  is not invertible.

**Hint:** Try using the Horizontal line test.

Circle the correct answer:    **True**    **False**

10. Let  $\mathcal{X} = \{45^\circ, 60^\circ, 240^\circ\}$  and  $\mathcal{Y} = \{0, 1, \sqrt{3}\}$  be sets, define the following function,

- $\omega: \mathcal{X} \rightarrow \mathcal{Y}$ .
- $\omega(x) = \tan(x)$ .

Then  $\omega$  is an invertible function.

Circle the correct answer:    **True**    **False**

11. Let  $\triangle PQR$  be a **right triangle** with angle  $\angle PQR = 60^\circ$  and *hypotenuse*  $PQ = 8$ . Then  $QR = 4$ .

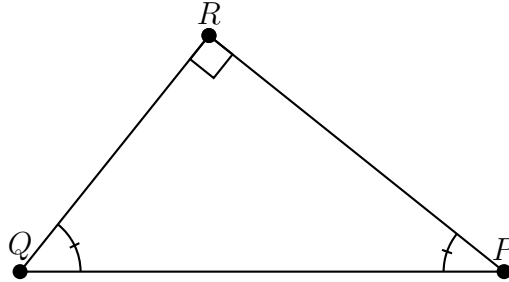
Circle the correct answer:    **True**    **False**

12. The exact value of

$$\sin 150^\circ \cdot \sec 240^\circ + \tan 240^\circ \tan 30^\circ = 0$$

Circle the correct answer:    **True**    **False**

13. Let  $\triangle PQR$  be a **right triangle** with  $PQ = 6$  and  $QR = 3\sqrt{3}$ . Then,  $\angle PQR = 60^\circ$ .



Circle the correct answer:    **True**    **False**

14.  $\sin 330^\circ = -0.5$ .

Circle the correct answer:    **True**    **False**

15. Suppose we have the standard coordinates  $\mathbf{P}(2, -\sqrt{12})$ , then the corresponding polar coordinates are  $\mathbf{P}(4, 60^\circ)$ .

Circle the correct answer:    **True**    **False**

16.  $\sqrt{4^4 \cdot 3^2 \cdot 2} = 48\sqrt{2}$ .

Circle the correct answer:    **True**    **False**

## Part B

**Question 2.** Explain in your own words, what is a function?

**Question 3.** Given a function  $f: A \rightarrow B$ , explain in your own words, what is the definition of the range of  $f$ ,  $\mathcal{R}_f$ , what does it contain? Is it necessarily true that  $\mathcal{R}_f = B$ ?

**Question 4.** Given a function  $f: A \rightarrow B$ , explain in your own words, what do we mean when we say that  $f$  is invertible?

**Question 5.** Explain what the horizontal line test is as well as the vertical line test.

## Part C

**Question 6.** Let  $F(x) = x^3 + 1$ , and  $G(x) = 2x^2 + x - 1$  be functions,

(a) Compute  $F(-1)$ .

(b) Compute  $G(2)$ .

(c) Compute  $F(G(1))$ .

(d) Compute  $G(F(F(0)))$ .

**Question 7.** Let  $g(x) = 2x^2 - 4x + 4$ ,

(a) How many solutions will  $g(x)$  have?

(b) Convert  $g(x)$  into vertex form by completing the square.

(c) Does the vertex of  $g(x)$  represent a minimum or maximum, justify your answer.

**Question 8.** Determine the inverse function for the following functions,

(a)  $F(x) = -8x + 16$ .

(b)  $G(x) = 2\sqrt{x+8} - 4$ .

**Question 9.** Simplify the following exponential expression, leave your answer with positive exponents.

$$\frac{(2x^2x^4y^{-3}z^{-4})^2}{(8x^{-2}y^{-5}z^2)^2}$$

**Question 10.** Evaluate the following,

$$\left(16^{\frac{4}{4}}\right)\left(9^{\frac{3}{2}}\right)\left(4^{\frac{1}{2}}\right)\left(2^{-3}\right)$$



**Question 11.** Simply the following radical expressions.

(a)

$$2\sqrt{27} + 3\sqrt{3} - 2\sqrt{12} + \sqrt{48}$$

(b)

$$(2\sqrt{2} + \sqrt{3})(5\sqrt{3} + 3\sqrt{2})$$

**Question 12.** Simplify the following,

(a)

$$\frac{2x^2 - 8x}{x^2 - 11x + 18} \times \frac{2x^2 - 7x + 6}{x^2 - 5x + 4}$$

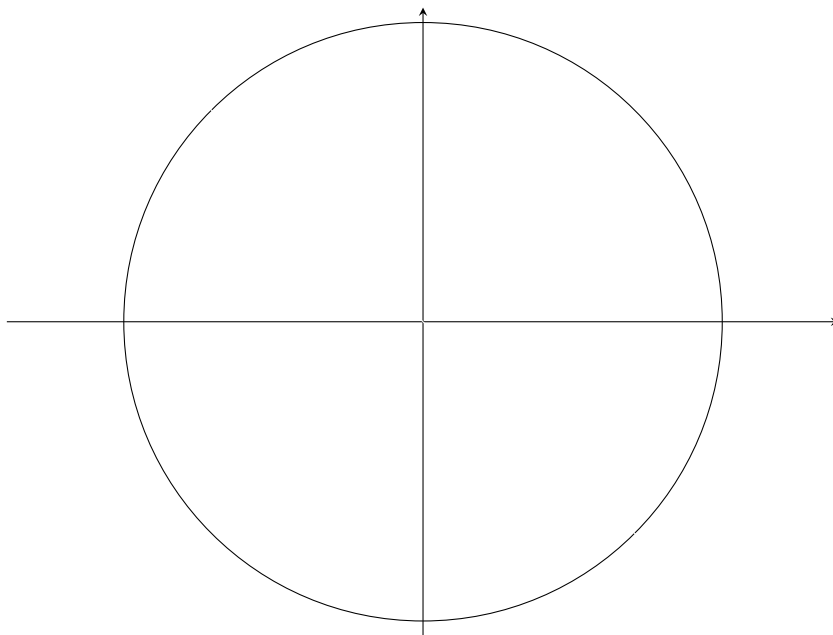
(b)

$$\frac{x}{x^2 - 5x + 6} - \frac{3}{x^2 - 4x + 4}$$

**Question 13.** For each of the following, you are given a trigonometric ratio, solve for  $\theta$ . Assume that each angle  $\theta$  lies in the **fourth** quadrant. (**You can use the circle below if it helps**).

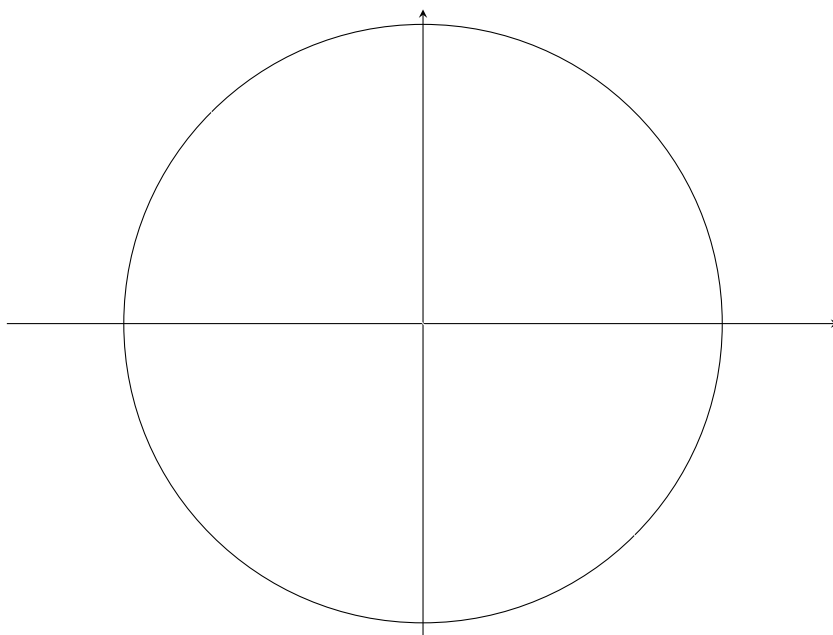
(a)

$$\cos \theta_1 = \frac{1}{2}.$$



(b)

$$\tan \theta_2 = -\frac{1}{\sqrt{3}}.$$



## Part D - Solve one of the two problems

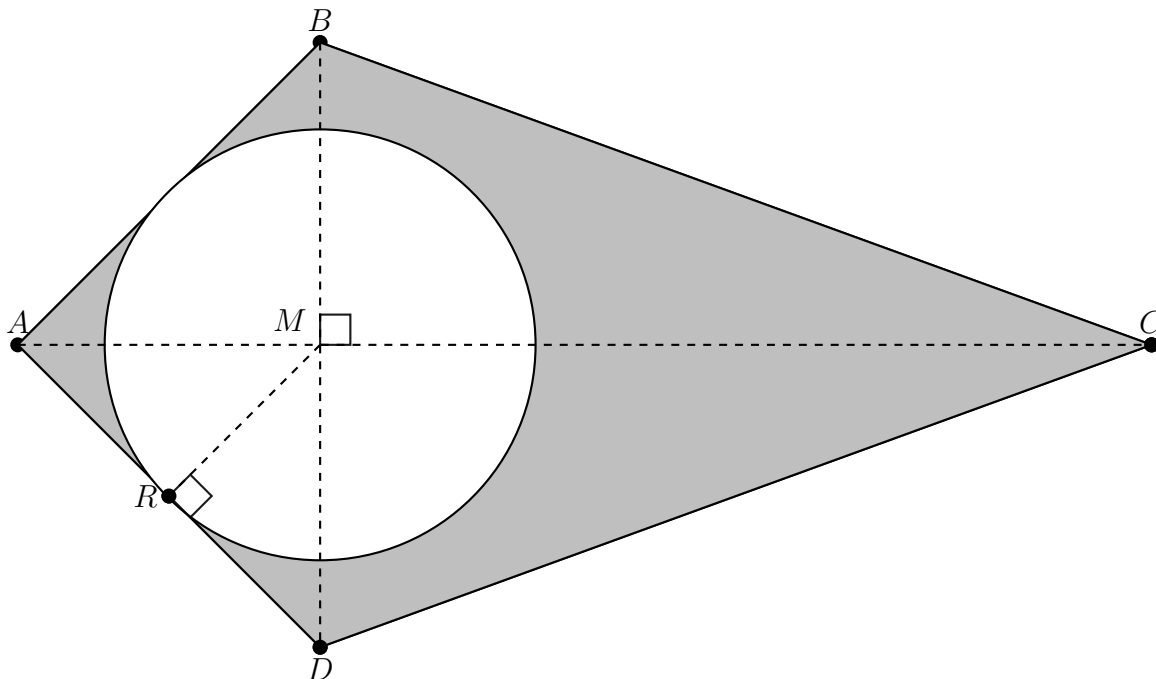
**Question 14.** Suppose we have two standard coordinates  $\mathbf{P}(x_1, y_1)$  and  $\mathbf{Q}(x_2, y_2)$ . Recall that the distance between these two points is given by the following formula,

$$\text{dist} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Given polar coordinates  $\mathbf{R}(2, 150^\circ)$  and  $\mathbf{T}(2, 330^\circ)$ , compute the distance between them.

**Question 15.** The figure below is composed of a kite and a circle. The radius of the circle is 4 and the point  $M$  represents the midpoint of the circle.  $\angle ADM = 30^\circ$  and  $MC = 20$ . If the area of the circle is  $A_c = 48$ , then determine the area of the shaded region in exact form.

(**Hint:** A kite has unique symmetric relationships which assert that  $\angle ADM = \angle ABM$ ,  $\angle MDC = \angle MBC$ ,  $DM = MB$ ,  $AD = AB$ ,  $BC = DC$ ).



**CHOOSE AND SOLVE ON NEXT PAGE**

Question \_\_\_\_.

(You can use the circle below if it helps)

