Lecture 2 – Introduction to Functions

Definition 2.1: Let $\mathcal{A}$ and $\mathcal{B}$ be sets. A **function**, f, is rule that acts on each element in $\mathcal{A}$, and assigns it to some **unique** element in the set $\mathcal{B}$. We say that,

- $\mathcal{A}$ is the **domain** of f.
- $\mathcal{B}$ is the **co-domain** of f.

(I will define the word 'domain' in more detail later on)

Its best to visualize this using the concept of diagrams I introduced to you about sets form last lecture **(IN class)**.

Notation 2.1: Since a function f assigns elements from the set $\mathcal{A}$ to elements in the set $\mathcal{B}$, we write this symbolically as,

$$f \colon \mathcal{A} \to \mathcal{B}.$$

You can read this as, "f goes from the set $\mathcal{A}$ to $\mathcal{B}$".

Notation 2.2: When f will act on some element $a \in \mathcal{A}$, then symbolically we write

$$f(a).$$

You can also read this as "f at a" or "f evaluated at a", etc. **This is NOT multiplication, just notation.**

Example 2.1: Lets take a look at our first function. Let $\mathcal{A} = \{2, 3, 4\}, \mathcal{B} = \{3, 4, 5\}$ be sets, lets define a function f,

- $f \colon \mathcal{A} \to \mathcal{B}$. (Must define this)
- **rule of f:** Take each element in $\mathcal{A}$, and add 1.

Remark 2.1: **Note!** We do not require a function that goes from $\mathcal{A}$ to $\mathcal{B}$ to map to <u>every</u> element in B.

Example 2.2: Lets take another function. Let $\mathcal{H} = \{2, 3, 4\}, \mathcal{T} = \{3, 4, 5\}$ be sets, lets define a function g,

- $g \colon \mathcal{H} \to \mathcal{T}$.
- **rule of f:** Take each element in $\mathcal{H}$, and subtract 1.

Remark 2.2: **Notice** that the definition of a function requries that each element in $\mathcal{A}$ is mapped to a <u>unique</u> element in $\mathcal{B}$. **(Explanation in class)**

Notation 2.3: How to write the rule of a function symbolically.
When rules of functions begin to get more complex, we need to switch over to writing rules of functions symbolically. So when we say something like "Take each element of $\mathcal{H}$ and subtract one", how can we write this symbolically? We can say "Take each element $h \in \mathcal{H}$ and preform,

$$f(h) = h - 1.$$

This translates to, take each element in $\mathcal{H}$ and subtract that element by one. Now we have a much better may of defining the rules of functions.

Example 2.3: Translate the following symbolical rules to plain English.

(a) **rule of Q:** For every element $a \in \mathcal{A}$, evaluate $Q(a) = a + 1$.

(b) **rule of M:** For every element $x \in \mathcal{X}$, evaluate $M(x) = \frac{x}{2}$.

(c) **rule of Z:** For every element $b \in \mathcal{B}$, evaluate $Z(b) = 2b$.

(d) **rule of R:** For every element $c \in \mathcal{C}$, evaluate $R(c) = \sqrt{c}$.

(e) **rule of S:** For every element $y \in \mathcal{Y}$, evaluate $S(y) = y^2 - 2$.

Example 2.4: Let $\mathcal{A} = \mathbb{R}$, and $\mathcal{B} = \mathbb{R}$, lets define a function f,

- $f : \mathcal{A} \to \mathcal{B}$.
- **rule of f:** For every element $a \in \mathcal{A}$, evaluate $f(a) = a^2 - 5a + 6$.

With this definition, determine the following outputs

(a) $f(-1)$

(b) $f(\frac{2}{3})$

(c) $f(f(3))$

Example 2.5: (Class Example) Let $\mathcal{X} = \mathbb{R}$, and $\mathcal{Y} = \mathbb{R}$, lets define <u>two</u> functions f,g,

- $f : \mathcal{X} \to \mathcal{Y}$, $g : \mathcal{X} \to \mathcal{Y}$.
- **rule of f:** For every element $x \in \mathcal{X}$, evaluate $f(x) = x^2 - 4$.
- **rule of g:** For every element $x \in \mathcal{X}$, evaluate $g(x) = x - 1$.

With this definition, determine the following outputs

(a) $f(-2)$

(b) $g(0)$

(c) $f(g(3))$

2.1 Graphing/Sketching Functions

Lets take both functions from our previous example, $g(x) = x - 1$ and $f(x) = x^2 - 4$. Notice that the right hand side of g is a simple linear function. Lets graph it **(In class)**.

Notice that the right hand side of f is a quadratic function, lets sketch it as well **(In class)**.

Remark 2.3: There is something called the **vertical line test** that helps you determine wether or not a graph is a function or not. You use it by drawing a straight line through your graph (until it touches it everywhere possilbe). If your vertical line intersects the graph at two or more distinct points, then the graph is not a function. Else, it is a function.

Example 2.6: Use the vertical line test to show that $x^2 + y^2 = 4$ is not a function.

Example 2.7: Use the vertical line test to show that $f(x) = x^2 - 4$ is a function.

2.2 Domain and Range of Functions

Definition 2.2: The **domain** of a function is the set of all values that the function is allowed to take as input. (We usually label this set with the capital letter $\mathcal{D}$).

Example 2.8: Let $f(x) = \sqrt{x}$ be a function. Determine the domain of f. **(In class)**.

Example 2.9: Let $T(x) = x^2 - 4$ be a function. Determine the domain of T. **(In class)**.

Definition 2.3: The **range** of a function is the set that contains ALL of the output values of the function. (We usually label this set with the capital letter $\mathcal{R}$).

Example 2.10: Let $g(x) = x^2$ be a function. Determine the range of g. **(In class)**

Example 2.11: Let $r(x) = 2x + 1$ be a function. Determine the range of r. **(In class)**

Example 2.12: We will define a function as a class, then we will determine the range of our function.

- Define the domain $\mathcal{D} = \cdots$
- **rule of g:** For every element $x \in \mathcal{D}$, ...
- The range of our function is $\mathcal{R} = \cdots$

2.3 Maximum's and Minimums

Lets say we have some function,

$$f(x) = a(x-h)^2 + k$$

that outputs some quadratic equation in vertex form. Then,

- **IF** $a < 0$ **(**a **is negative)** then we say that the vertex represents a <u>maximum</u>.
  - This makes sense because if $a < 0$, the parabola points down, and the highest point of the parabola must have been the vertex.
- **ELSE IF** $a > 0$ **(**a **is positive)** then we say that the vertex represents a <u>minimum</u>.
  - This makes sense because if $a > 0$, the parabola points up, and the lowest point of the parabola must have been the vertex.