If the sequence of partial sums $\{S_n\}$ has limit S, the limit S is the sum of the series.

Geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = \begin{cases} \frac{a}{1-r} & |r|<1 \\ \text{diverges} & |r|\geq 1 \end{cases}$$

P-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \begin{cases} \text{converges} & p>1 \\ \text{diverges} & p\leq 1 \end{cases}$$

Comparison Test

If $0 \leq c_n \leq a_n$ for all n and $\sum a_n$ converges then $\sum c_n$ converges. If $c_n \geq d_n \geq 0$, for all n and $\sum d_n$ diverges, then $\sum c_n$ diverges.

Limit Comparison Test

If $0 \leq c_n$ and $0 < b_n$ and

$$\lim_{n\to\infty} \frac{c_n}{b_n} = l \quad 0 < l < \infty$$

then series $\sum c_n$ converges if $\sum b_n$ converges and diverges if $\sum b_n$ diverges.