Lecture 4 - Homework

Question 1. Suppose we have the identity function on $\mathbb{R}$, $\mathrm{id}_\mathbb{R} \colon \mathbb{R} \to \mathbb{R}$, $\mathrm{id}_\mathbb{R}(x) = x$. Is it true that the inverse function of $\mathrm{id}_\mathbb{R}$ is itself? Justify your answer.

Question 2. Let $\mathcal{V} = \{1, 4, 9, 25, 49\}$ and $\mathcal{W} = \{5, 4, 7, 9, 3\}$ be sets, define the following function,

- $\mathcal{L} \colon \mathcal{V} \to \mathcal{W}$.
- $\mathcal{L}(v) = \sqrt{v} + 2$.

Prove that the function,

- $\mathcal{L}^{-1} \colon \mathcal{W} \to \mathcal{V}$.
- $\mathcal{L}^{-1}(w) = (w - 2)^2$.

is the inverse function for $\mathcal{L}$.

Question 3. Let $\mathcal{V} = \{-1, 0, 1, 5, 8\}$ and $\mathcal{W} = \{-3, 0, 1, 4, 5\}$ be sets, define the following function,
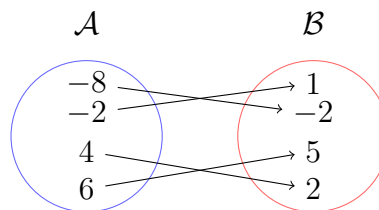
- $\mathcal{T} \colon \mathcal{V} \to \mathcal{W}$.
- $\mathcal{T}(v) = \frac{3x}{x-2}$.

Prove that the function,

- $\mathcal{T}^{-1} \colon \mathcal{W} \to \mathcal{V}$.
- $\mathcal{T}^{-1}(w) = \frac{2w}{w-3}$.

is the inverse function for $\mathcal{T}$.

Question 4. You are given the mapping diagram for a function $F \colon \mathcal{A} \to \mathcal{B}$. Your task is to determine the formula for the function as well as its inverse function based on the observed pattern.

Question 5. Given an invertible function, can you explain why it would be advantageous to determine its inverse function? Why is it a useful tool?

Question 6. Given binary strings $\mathbf{S} = 00101$ and $\mathbf{T} = 1101$, determine the following,

(a) $\mathbf{S} + \mathbf{T} + 0$.

(b) $1 + \mathbf{T} + 1$.

(c) $\epsilon + \mathbf{T} + \mathbf{S}$.

(d) $\mathbf{T} + \mathbf{T}$.

Question 7. Recall that for normal addition of integers, $a + b = b + a$. Is this also true for binary strings? In other words is it always true that $\mathbf{S} + \mathbf{T} = \mathbf{T} + \mathbf{S}$? If it is, then prove it. If not, then provide a counter example.

Question 8. Let $\mathbf{A} = 11011$. Determine,

(a) The substring $\mathbf{R} = \mathbf{a_1 a_3 a_5}$.

(b) The substring $\mathbf{Z} = \mathbf{a_1 a_2}$.

(c) The substring $\mathbf{Z} = \mathbf{a_2 a_4}$.

Question 9. Let $\mathcal{S} = \{11, 01, 10, 00\}$ and $\mathcal{R} = \{10001, 10101, 11001, 11101\}$. Define the following function,

- $\phi \colon \mathcal{S} \to \mathcal{R}$.

- $\phi(\mathbf{T}) = 1 + \mathbf{T} + 0 + 1$.

Determine the inverse function ϕ^{-1}.
Hint: A mapping diagram may help.