Lecture 4 - Homework

Question 1. Suppose we have the identity function on $\mathbb{R}$, $\mathrm{id}_{\mathbb{R}} \colon \mathbb{R} \to \mathbb{R}$, $\mathrm{id}_{\mathbb{R}}(x) = x$. Is it true that the inverse function of $\mathrm{id}_{\mathbb{R}}$ is itself? Justify your answer.

Question 2. Let $\mathcal{V} = \{1, 4, 9, 25, 49\}$ and $\mathcal{W} = \{5, 4, 7, 9, 3\}$ be sets, define the following function,

- $\mathcal{L} \colon \mathcal{V} \to \mathcal{W}$.
- $\mathcal{L}(v) = \sqrt{v} + 2$.

Prove that the function,

- $\mathcal{L}^{-1} \colon \mathcal{W} \to \mathcal{V}$.
- $\mathcal{L}^{-1}(w) = (w - 2)^2$.

is the inverse function for $\mathcal{L}$.

Question 3. Let $\mathcal{V} = \{-1, 0, 1, 5, 8\}$ and $\mathcal{W} = \{-3, 0, 1, 4, 5\}$ be sets, define the following function,

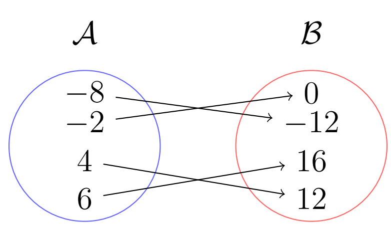
- $\mathcal{T} \colon \mathcal{V} \to \mathcal{W}$.
- $\mathcal{T}(v) = \frac{3x}{x-2}$.

Prove that the function,

- $\mathcal{T}^{-1} \colon \mathcal{W} \to \mathcal{V}$.
- $\mathcal{T}^{-1}(w) = \frac{2w}{w-3}$.

is the inverse function for $\mathcal{T}$.

Question 4. You are given the mapping diagram for a function $F \colon \mathcal{A} \to \mathcal{B}$. Your task is to determine the formula for the function as well as its inverse function based on the observed pattern.



Question 5. Given an invertible function, can you explain why it would be advantageous to determine its inverse function? Why is it a useful tool?

Question 6. Let $f \colon \mathbb{R} \to \mathbb{R}$, $f(x) = |x|$. Is f invertible? If it is, prove it using mapping tables. If it is not, then provide a counter example to show that f fails to be injective or surjective. (Its your choice) **(Important Question **)**.

Question 7. Let $G\colon \mathcal{A} \to \mathcal{B}$ be an invertible function. Suppose that the invertible function is,

$$G^{-1}(b) = b - 2.$$

Given the co-domain $\mathcal{B} = \{-2, -1, 0, 3, 7\}$ of G, recover the domain $\mathcal{A}$.

Question 8. Given binary strings $\mathbf{S} = 00101$ and $\mathbf{T} = 1101$, determine the following,

(a) $\mathbf{S} + \mathbf{T} + 0$.

(b) $1 + \mathbf{T} + 1$.

(c) $\epsilon + \mathbf{T} + \mathbf{S}$.

(d) $\mathbf{T} + \mathbf{T}$.

Question 9. Recall that for normal addition of integers, $a + b = b + a$. Is this also true for binary strings? In other words is it always true that $\mathbf{S} + \mathbf{T} = \mathbf{T} + \mathbf{S}$? If it is, then prove it. If not, then provide a counter example.

Question 10. Let $\mathbf{A} = 11011$. Determine,

(a) The substring $\mathbf{R} = \mathbf{a}_1\mathbf{a}_3\mathbf{a}_5$.

(b) The substring $\mathbf{Z} = \mathbf{a}_1\mathbf{a}_2$.

(c) The substring $\mathbf{X} = \mathbf{a}_2\mathbf{a}_4$.

Question 11. Let $\mathcal{S} = \{11, 01, 10, 00\}$ and $\mathcal{R} = \{10001, 10101, 11001, 11101\}$. Define the following function,

- $\phi\colon \mathcal{S} \to \mathcal{R}$.
- $\phi(\mathbf{S}) = 1 + \mathbf{S} + 0 + 1$.

Determine the inverse function ϕ^{-1}.
Hint: A mapping diagram may help.