

How to determine Domain & Range

We first must identify which type of function we are dealing with before we proceed.

1 Linear Functions ($f(x) = mx + b$)

1. The Domain will always be,

$$\mathcal{D} = \mathbb{R}.$$

2. The Range will always be,

$$\mathcal{R} = \mathbb{R}.$$

2 Quadratic Functions ($f(x) = ax^2 + bx + c$)

1. First convert the quadratic into vertex form by completing the square. After your done, you should have something that looks like,

$$f(x) = a(x - h)^2 + k.$$

Where (h, k) is the vertex.

2. The Domain will always be,

$$\mathcal{D} = \mathbb{R}.$$

3. **IF** $a > 0$ (**a is positive**) then the Range will be,

$$\mathcal{R} = \{y \in \mathbb{R} \mid y \geq k\}.$$

4. **ELSE IF** $a < 0$ (**a is negative**) then the Range will be,

$$\mathcal{R} = \{y \in \mathbb{R} \mid y \leq k\}.$$

3 Absolute Value functions ($f(x) = a|mx + b| + k$)

1. The Domain will always be,

$$\mathcal{D} = \mathbb{R}.$$

2. **IF** $a > 0$ (**a is positive**) then the Range will be,

$$\mathcal{R} = \{y \in \mathbb{R} \mid y \geq k\}.$$

3. **ELSE IF** $a < 0$ (**a is negative**) then the Range will be,

$$\mathcal{R} = \{y \in \mathbb{R} \mid y \leq k\}.$$

4 Rational Functions ($f(x) = \frac{a}{mx+b} + k$)

1. Then the Domain will be,

$$\mathcal{D} = \{x \in \mathbb{R} \mid x \neq -\frac{b}{m}\}.$$

2. The Range will always be,

$$\mathcal{R} = \{y \in \mathbb{R} \mid y \neq k\}.$$

5 Radical Functions ($f(x) = a\sqrt{mx+b} + k$)

1. **IF** $m < 0$ (m is **negative**) then the Domain will be,

$$\mathcal{D} = \{x \in \mathbb{R} \mid x \leq -\frac{b}{m}\}.$$

2. **ELSE IF** $m > 0$ (m is **positive**) then the Domain will be,

$$\mathcal{D} = \{x \in \mathbb{R} \mid x \geq -\frac{b}{m}\}.$$

3. **IF** $a > 0$ (a is **positive**) then the Range will be,

$$\mathcal{R} = \{y \in \mathbb{R} \mid y \geq k\}.$$

4. **ELSE IF** $a < 0$ (a is **negative**) then the Range will be,

$$\mathcal{R} = \{y \in \mathbb{R} \mid y \leq k\}.$$

6 Cirlces $((x-a)^2 + (y-b)^2 = r)$

NOTE!: Circles are not functions! (**Explanation in Class**). We would still like to know the Domain and Range of circles (Just ignore how they are not functions).

1. First write down the centre of the circle (a, b) .

2. The Domain will be,

$$\mathcal{D} = \{x \in \mathbb{R} \mid a - \sqrt{r} \leq x \leq a + \sqrt{r}\}.$$

3. The Range will be,

$$\mathcal{R} = \{y \in \mathbb{R} \mid b - \sqrt{r} \leq y \leq b + \sqrt{r}\}.$$

Practice Problems:

Question 1. Determine the Domain and Range of the following functions,

(a) $L(x) = x$

(b) $f(x) = 2x - 1$

(c) $T(x) = 2$

(d) $f(x) = 2(x - 1)$

(e) $2y - 5x = 11$

(f) $y - 5x = 1$

Question 2. Determine the Domain and Range of the following functions,

(a) $L(x) = x^2$

(b) $f(x) = x^2 - x - 6.$

(c) $q(x) = -3x^2 - 2x + 1.$

(d) $p(x) = 4x^2 - 9.$ (Isnt this already in vertex form?)

(e) $r(x) = -2x^2 - 8x + 1.$

(f) $V(x) = -x^2 - \frac{9}{5}x + 1.$

Question 3. Determine the Domain and Range of the following functions,

(a) $L(x) = |x|$

(b) $f(x) = -2|2x + 1| - 1.$

(c) $g(x) = 3|-x - 1| + 2.$

(d) $m(x) = 5|-\frac{x}{2} - \frac{3}{2}| - \frac{5}{2}.$

(e) $\xi(x) = -|-x|.$

Question 4. Determine the Domain and Range of the following functions,

(a) $L(x) = \frac{1}{x}$

(b) $f(x) = \frac{-3}{-2x+1} - 3$

(c) $T(x) = \frac{3}{x-1} - \frac{4}{3}$

(d) $r(x) = \frac{1}{\frac{1}{3}x - \frac{5}{4}} - 16$

(e) $q(x) = \frac{1}{-x}$

Question 5. Determine the Domain and Range of the following functions,

(a) $L(x) = \sqrt{x}$.

(b) $f(x) = -2\sqrt{-5x + 7} - 1$

(c) $g(x) = 5\sqrt{-2x - 36} + 12$

(d) $H(x) = -\sqrt{-x}$

(e) $V(x) = \sqrt{x + 1} - 1$

Question 6. Determine the Domain and Range of the following circles,

(a) $(x + 7)^2 + (y - 2)^2 = 4$

(b) $x^2 + (y + 1)^2 = 9$

(c) $(x + 9)^2 + (y - 4)^2 = 1$

(d) $x^2 + y^2 = 1$

(e) $(x - 5)^2 + (x + 3)^2 = 25$