How to Complete the Square

Lets say we are given some quadratic polynomial $y = ax^2 + bx + c$ and we would like to convert this into vertex form, $y = a(x-h)^2+k$, the technique to do so is called completing the square. Remember what we dicussed yesterday, if a polynomial is a perfect square, then its discrimnant ($d = b^2 - 4ac$) will be zero. (This idea will be used in the technique)

1. Find the t value such that
$$0 = b^2 - 4at.$$

2. Insert t into the equation as follows,
$$y = ax^2 + bx + t - t + c.$$

3. Seperate the equation into two parts,
$$y = (ax^2 + bx + t) + [c - t].$$

4. Factor out a from the round bracktes. **(IF $a = 1$ then nothing should change)**.
$$y = a\left(x^2 + \frac{b}{a}x + \frac{t}{a}\right) + [c - t].$$

5. **IF $a \cdot b > 0$, then replace the equation with the following**,
$$y = a\left(x + \sqrt{\frac{t}{a}}\right)^2 + [c - t].$$

6. **ELSE IF $a \cdot b < 0$, then replace the equation with the following**,
$$y = a\left(x - \sqrt{\frac{t}{a}}\right)^2 + [c - t].$$

7. Were done! (The new polynomial is now in vertex form if you look carefully)

Practice Problems:

Some of these problems are from the textbook, you can check your answers from there.

Question 1. Textbook: Question 1 a),c),e),f)

Question 2. Textbook: Question 2 b),d),f)