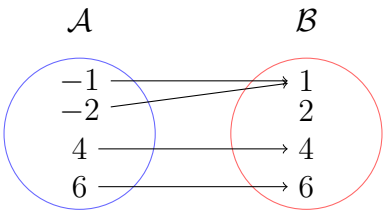
Lecture 3 - Homework

Question 1. Let $f\colon \mathbb{R} \to \mathbb{R}$, $f(x) = x$. Is f the identity function on $\mathbb{R}$? Justify your answer.

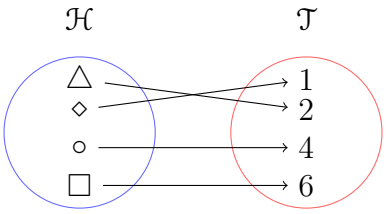
Question 2. For each of the following, you are given a function and its mapping diagram. For each question,

- State weather the function is invertible, if not then weather it is injective or surjective or neither, with justification.
- Determine the range of the function.

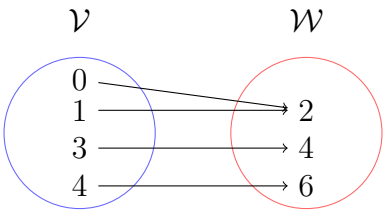
(a) $g\colon \mathcal{A} \to \mathcal{B}$,



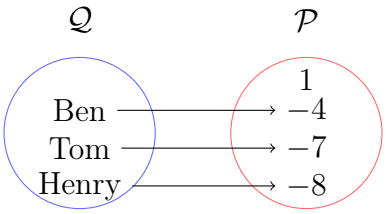
(b) $f\colon \mathcal{H} \to \mathcal{T}$,



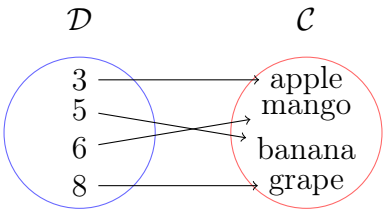
(c) $T\colon \mathcal{V} \to \mathcal{W}$,



(d) $S\colon \mathcal{Q} \to \mathcal{P}$,



(e) $P\colon \mathcal{D} \to \mathcal{C}$,



Question 3. Is the function defined in Example 3.2 an invertible function? Justify your answer.

Question 4. Let $\mathcal{A} = \{-1, 3, 4, 6, 7\}$ and $\mathcal{B} = \{6, -1, 4, 3, 7\}$ be sets. Come up with an invertible function between the two sets and prove that your function is invertible.

Question 5. Let $\mathcal{A}$ and $\mathcal{B}$ be sets. Let $T\colon \mathcal{A} \to \mathcal{B}$ be an invertible function. Let $\mathcal{R}_T$ be the range of T, is it true that $\mathcal{B} = \mathcal{R}_T$? Justify your answer. **(This is a very important question)**

Question 6. Let $\mathcal{X} = \{1, 4, 9, 36\}$ and $\mathcal{Y} = \{1, 2, 3, 6\}$ be sets, lets define the following function,

- $\mathcal{L}\colon \mathcal{A} \to \mathcal{B}$.
- $\mathcal{L}(x) = \sqrt{x}$.

Prove that $\mathcal{L}$ is invertible.

Question 7. Let $\mathcal{X} = \{-1, 0, 1, 2, 3\}$ and $\mathcal{Y} = \{10, 2, 5, 1\}$ be sets, lets define the following function,

- $\mathcal{P}\colon \mathcal{A} \to \mathcal{B}$.
- $\mathcal{P}(x) = x^2 + 1$.

Prove that $\mathcal{P}$ is **not** invertible.