Assignment 1 Functions
Due Date: Thursday, December 16

November, 2021

1 Preamble

This assignment covers everything most of Unit 1. The solutions that you hand in should be **neat** and **legible**, this is an assignment, not a quiz, so I expect you to take your time and present thorough and detailed solutions.

2 Name and Date:

Print your name and todays date below;

Name	Date

Question 1. We define the **cardinality** of sets to be the size of the set, in other words the cardinality of a set is the number of elements in the set. For example if $S = \{3, 2, 1, \triangle\}$, then we say the that cardinality of S is 4, because S consists of four elements. The notation we use to describe the cardinality of a set is a pair of bars ($||$) similar to absolute values. So going back to our example with S, instead of saying the sentence; The cardinality of S is 4, we could instead simply write $|S| = 4$. In this question we will discover that for sets A, B, C, it is **not** always true that $|A + B| = |A| + |B|$.

Lets say we have the following sets,

- $\mathcal{H} = \{x \in \mathbb{Z} \mid -1 \leq x \leq 5\}$
- $\mathcal{T} = \{y \in \mathbb{Z} \mid 2 \leq y < 7\}$

(a) Write down the elements of both sets.

(b) Determine $|\mathcal{H}|$ and $|\mathcal{T}|$.

(c) Determine $|\mathcal{H} + \mathcal{T}|$. (Go back to Homework-1 if you forgot how we preform set addition).

(d) Explain why $|\mathcal{H} + \mathcal{T}| \neq |\mathcal{H}| + |\mathcal{T}|$.

Question 2. Let

$$f(x) = -2x^2 - 4x + 5$$
$$g(x) = \frac{3}{4}x - 4$$

(a) Compute $f(-2)$ **and** $g(3)$.

(b) Compute $f(g(f(0)))$.

(c) Provide a **sketch** of f on the axis sheet.

(d) Provide a **graph** of g on the axis sheet.

(e) Does the vertex of f represent a minimum or maximum? Explain your answer.

Question 3. Recall that when you divide two numbers, there will always be a remainder, sometimes the remainder is zero, other times it may not be zero, lets take a look at the following examples;

- $\frac{5}{3}$, the remainder is 2.
- $\frac{12}{2}$, the remainder is 0.
- $\frac{4}{7}$, the remainder is 4.

There is a better way to describe the remainder when dividing two numbers, that is to use the following notation,

$$\text{rem}(a, b).$$

The output of this is the remainder when dividing a by b, so if we repeat our previous examples again using this new notation, we would have,

- $\text{rem}(5, 3) = 2$
- $\text{rem}(12, 2) = 0$

- $\text{rem}(4, 7) = 4$

The set of all 'positive numbers' is written as,

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, \dots\}.$$

Now lets define the following function,

$$f \colon \mathbb{N} \to \mathbb{N}$$
$$f(n) = \text{rem}\,(n, 2) + 1$$

Lets take a look at some examples to see how the function behaves,

- $f(4) = \text{rem}\,(4, 2) + 1 = 0 + 1 = 0$
- $f(11) = \text{rem}\,(11, 2) + 1 = 1 + 1 = 2$

Determine the following,

(a) $f(6)$

(b) $f(15)$

(c) $f(f(1))$

(d) The range of f.
(Think carefully about this one, have you seen a pattern with the outputs so far?)

Question 4. Determine the Domain and Range of the following functions,

(a) $L(x) = -4x^2 + 8x + 4$

(b) $R(x) = -50|x - 3| - 7$

(c) $Q(x) = \frac{-3}{2x-4} + \frac{4}{5}$

(d) $g(x) = 11\sqrt{-3x + 12} + 1$

(e) $x^2 + (y + 11)^2 = 4.$

Question 5. Lets say we have the following functions,

- $f(x) = x^2 - 2x - 3$
- $g(x) = x - 4$

Lets define a new function,

$$h(x) = f(g(x)).$$

(a) How many solutions will $h(x)$ have? (**Hint:** Use the discriminant formula)

(b) Determine the solutions to,

$$h(x) = 0.$$

(c) Determine the solutions to,

$$f(x) = g(x).$$

(**Note!:** Leave answers in **exact** form, NO decimals).