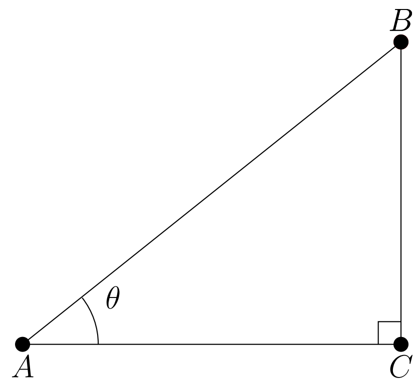
# Lecture 8 – Trigonometry Part I

Trigonometry is concerned with the study of the so called *Unit circle*. Before jumping in, lets recall some of the notation and terminology we'll be dealing with.

**Definition 8.1:** *Given a right triangle,*



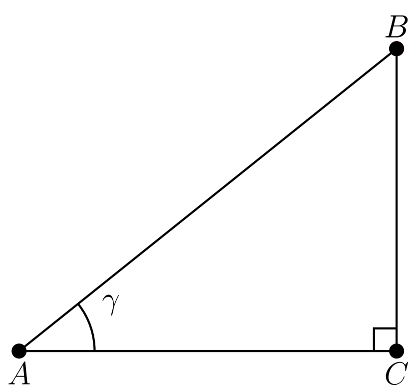

***Relative*** *to the angle $\theta$, we define the following sides of the triangle,*

- ***Adjacent side:*** *The side with both the angle $\theta$ and the right angle. (Side* $\boldsymbol{AC}$ *in fig)*
- ***Hypotenuse side:*** *The side opposite to the right angle. (Side* $\boldsymbol{AB}$ *in fig)*
- ***Opposite side:*** *The remaining side. (Side* $\boldsymbol{BC}$ *in fig)*

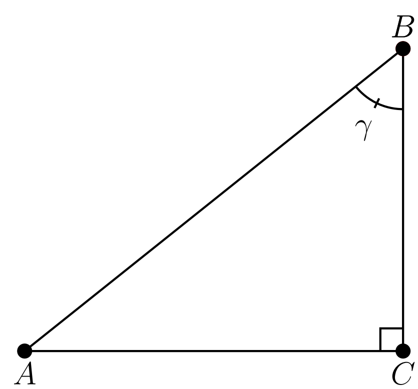
**Remark 8.1:** Notice how in the definition I emphasize that the way we label these sides are relative to the position of the angle $\theta$. How we label the adjacent, opposite and hypotenuse will change depending on the specified angle.

**Example 8.1:** Identify the Adj, Opp, Hyp sides for the following triangles *relative* to the angle $\gamma$.

a)




b) (**)



## 8.1 Primary Trigonometric Ratio's

**Definition 8.2:** *Given a right triangle and an inscribed angle $\theta$, we define the* ***primary trigonometric ratio's*** *to be,*

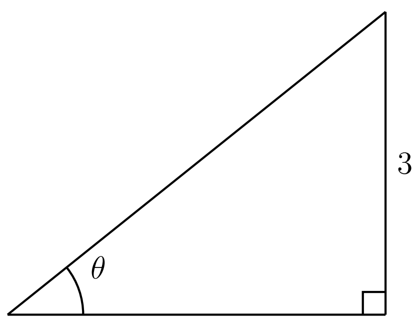
$$\sin\theta = \frac{\textbf{\textit{Adjacent}}}{\textbf{\textit{Hypotenuse}}} \qquad \cos\theta = \frac{\textbf{\textit{Opposite}}}{\textbf{\textit{Hypotenuse}}} \qquad \tan\theta = \frac{\textbf{\textit{Opposite}}}{\textbf{\textit{Adjacent}}}$$

**Theorem 8.1:** *(Pythagorean's Theorem) Given a right triangle, the following holds,*

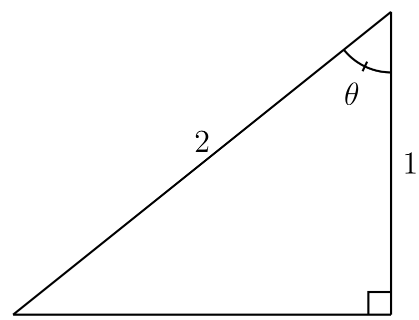
$$(\textbf{\textit{Hypotenuse}})^2 = (\textbf{\textit{Adjacent}})^2 + (\textbf{\textit{Opposite}})^2.$$

**Example 8.2:** Compute the primary trigonometric ratios of the angle $\theta$.

a)




b) (**)




We can also use the primary trigonometric ratios to help solve for unknown sides in a right triangle. Get **your calculator** ready. Henceforth I will be describing triangles more concisely using syntactical representations that conveniently preserve the semantics of these geometric objects while reducing my typing agony. You are encouraged to illustrate the triangles described in this fashion.

**Example 8.3:**

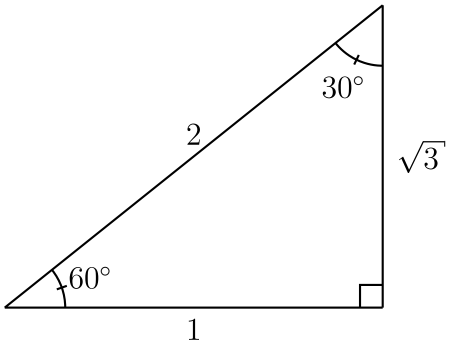
(a) Let $\triangle ABC$ be a **right triangle** with angle $\angle ACB = 40°$ and *opposite side* $AB = 4$. Determine the length of sides $CB$ and $CA$.

(b) Let $\triangle FGH$ be a **right triangle** with angle $\angle FHG = 35°$ and *hypotenuse* $FH = 7$. Determine the length of sides $FG$ and $HG$. (**)

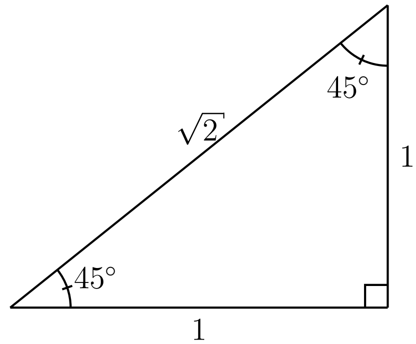
There are a group of triangles that form convenient trigonometric ratios that are referred to as the **special triangles**. Their convenience lies within the fact that you **don't** have to resort to your calculator because they form nice ratios.

**Theorem 8.2:** *We define the **special triangle's** to be,*

*a)*




*b)*




*And define the corresponding **special angles** to be,*

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \tan 45^\circ = 1$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{1}{2} \qquad \tan 60^\circ = \sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

**Example 8.4:**

(a) Let $\triangle PQR$ be a **right triangle** with angle $\angle PQR = 60^\circ$ and *hypotenuse* $PQ = 4$. Determine the length of the sides $QR$ and $PR$. `(**)`

(b) Determine the value of,

$$\sin 60^\circ \cdot \tan 60^\circ + \cos 60^\circ \cdot \sin 30^\circ.$$

### 8.1.1 Inverse trigonometric functions

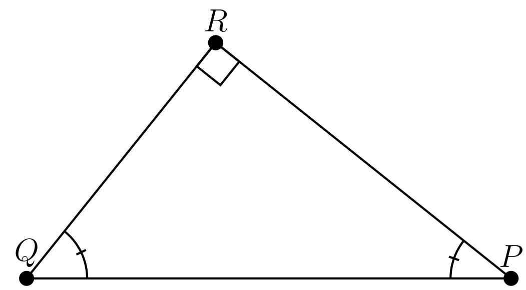
In turns out that each of $\sin\theta$, $\cos\theta$, $\tan\theta$ are invertible and have inverse functions that can give us the value of an inscribed angle provided we already have knowledge of any of the three primary corresponding ratio's.

**Definition 8.3:** *For each of* $\sin\theta$*,* $\cos\theta$*,* $\tan\theta$ *we define their inverse functions to be,*

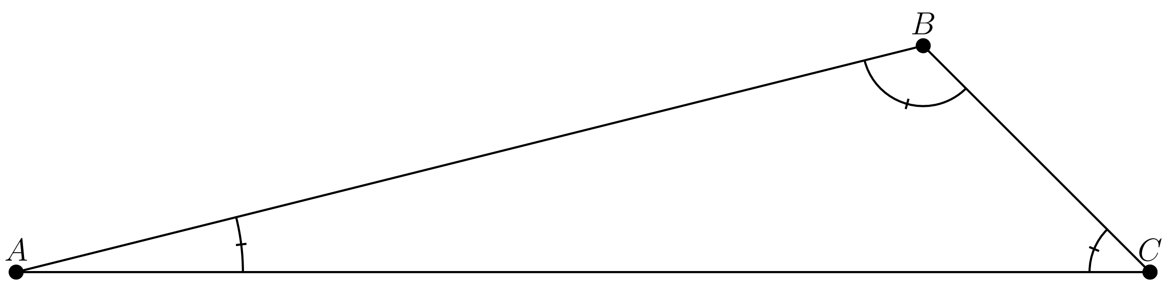
$$\theta = \sin^{-1}\left(\frac{\textbf{\textit{Adjacent}}}{\textbf{\textit{Hypotenuse}}}\right) \quad \theta = \cos^{-1}\left(\frac{\textbf{\textit{Opposite}}}{\textbf{\textit{Hypotenuse}}}\right) \quad \theta = \tan^{-1}\left(\frac{\textbf{\textit{Opposite}}}{\textbf{\textit{Adjacent}}}\right) \quad .$$

**Example 8.5:**

(a) Let $\triangle PQR$ be a **right triangle** with $PQ = 5$ and $QR = 3$. Determine the measure of the angle $\angle PQR$ and $\angle QPR$.




(b) Let $\triangle ABC$ be a triangle, not necessarily right angled, with sides $BC = 3\sqrt{2}$ and $AC = 30$. If the area of the triangle is $A_{\triangle} = 45$ units$^2$, determine the measure of the angle $\angle BAC$. (**)



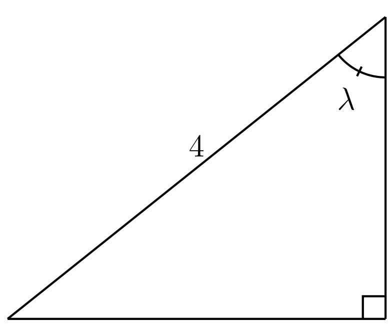

## 8.2 Reciprocal Trigonometric Ratio's

**Definition 8.4:** *Given a right triangle and an inscribed angle $\theta$, we define the* ***reciprocal trigonometric ratio's*** *to be,*

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\textbf{\textit{Hypotenuse}}}{\textbf{\textit{Adjacent}}} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{\textbf{\textit{Hypotenuse}}}{\textbf{\textit{Opposite}}} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\textbf{\textit{Adjacent}}}{\textbf{\textit{Opposite}}}$$

**Example 8.6:** Compute the reciprocal trigonometric ratios of the angle $\lambda$.

a)




b) (**)