Lecture 6 – Radicals and Primes

6.1 Primes

Definition 6.1: *Let $x \in \mathbb{N}$. We say that x is **prime** if its only divisors are 1 and itself.*

Example 6.1: The following are examples of prime numbers,

$$2, 3, 5, 7, 11, 13, 17, 19.$$

There are actually an infinite number of primes, however I wont dwell upon this any further as, currently, it serves no benefit.

Theorem 6.1: *Let $x \in \mathbb{N}$, such that $x \geq 2$. Then we can factor x into a product of primes known as the **prime factorization of** x.*

This is actually a very, very, very important result. Mathematicians refer to this as the fundamental theorem of arithmetic.

Example 6.2: Preform a prime factorization for the following natural numbers.

(a) 28.

(b) 4.

(c) 5.

(d) 34 (**).

(e) 148 (**).

Theorem 6.2: *Let x be a prime number, then one of its prime factors will be less than $\sqrt{x}$.*

This should help narrow down your search when preforming prime factorizations.

6.2 Exponents

Properties of exponents 1:

Rule 1. $a^x \cdot a^y = a^{x+y}$.

Rule 2. $a^x / a^y = a^{x-y}$.

Rule 3. $(a^x)^y = a^{x \cdot y}$.

Rule 4. $a^{-x} = \frac{1}{a^x}$.

Rule 5. $\frac{1}{a^{-x}} = a^x$.

The aforementioned properties of exponents allow us to simplify exponential expressions.

Example 6.3: Simplify the following exponential expressions, reduce answers to positive exponents.

(a) $-x^2(-x^3)$.

(b) $(4^4)^{\frac{1}{2}}$.

(c) y^{-4}/y^2 (**).

(d) 2^{-2}.

(e) $(-y)^2(-y)^{-4}$ (**).

(f) $\frac{1}{5^{-3}}$ (**).

Properties of exponents 2:

Rule 5.

$$(a^x \cdot b^y)^z = a^{x \cdot z} \cdot b^{y \cdot z}.$$

Rule 6.

$$\left(\frac{a^x}{b^y}\right)^z = \frac{a^{x \cdot z}}{b^{y \cdot z}}.$$

Example 6.4: Simplify the following exponential expressions, reduce answers to positive exponents.

(a) $(x^{-2}y^4)^2$.

(b) $(4y^4x^{-3}x^6z^5)^2$ (**).

(c) $\left(\frac{x^{-4}}{y^2}\right)^{\frac{1}{2}}$.

6.3 Radicals

Radicals refer to expressions with square roots. We are mostly concerned with how to simplify and manipulate such expressions. They key here is to understand the following fact,

$$\sqrt[n]{x} = x^{\frac{1}{n}}.$$

So the n-th root of x is the same as raising x to the power of $1/n$. When $n = 2$, then we simply refer to it as the square root, and we don't normally write the 2 as you know. To be more explicit,

$$\sqrt[2]{x} = \sqrt{x} = x^{\frac{1}{2}}.$$

We refer to x as the **radicand**

Simplifying Radicals:

Step 1. If the radicand is a perfect square, then your done!

Step 2. Else divide the radicand by its *prime factors* until you get a perfect square.

Step 3. Simplify using exponent rules.

Example 6.5: Simplify each Radical expression. **(In class)**

(a) $\sqrt{50}$.

(b) $\sqrt{27}$ (**).

(c) $\sqrt{180}$.

Example 6.6: Simplify each Radical expression. **(In class)**

(a) $9\sqrt{7} - 4\sqrt{7}$.

(b) $4\sqrt{24} - 3\sqrt{6}$.

(c) $5\sqrt{2} + 3\sqrt{18}$ (**).

Example 6.7: Simplify each Radical expression. **(In class)**

(a) $\left(2\sqrt{3}\right)\left(3\sqrt{8}\right)$.

(b) $2\sqrt{3}\left(4 + 5\sqrt{3}\right)$.

(c) $\left(\sqrt{3} + 5\right)\left(2 - \sqrt{3}\right)$ (**).