

# Functions Test 1

December 14, 2021

## 1 Preamble

This is a test covering what we have learnt so far in lecture. Student's must show all work to receive full marks.

## 2 Allowed Aids

The following aids are allowed on the Test

- Pencil, Pen, Eraser, Highlighter, Ruler, Protractor, Spare sheets of **blank** paper.
- Reference sheet (**Double sided paper prepared by student**)

## 3 Restrictions:

- **NO** calculator's.

## 4 Remarks:

- $\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$ .
- $\text{rem}(x, y)$  is the remainder when you divide  $x$  by  $y$ .

## 5 Name and Date:

Print your name and today's date below;

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Name

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Date

**Question 1.** (10 marks) Answer the following True/False questions,

1. Let  $\mathcal{R} = \{4, 5, 6, 7, 8\}$  and  $\mathcal{H} = \emptyset$ , then  $\mathcal{R} + \mathcal{H} = \emptyset$ .

Circle the correct answer:    **True**    **False**

2. Let  $S = \{3, 4, 5\}$ , then  $S + S = S$ .

Circle the correct answer:    **True**    **False**

3.  $(\sqrt{4} + \pi) \in \mathbb{Z}$ .

Circle the correct answer:    **True**    **False**

4. The vertex of

$$f(x) = 3(x + \pi)^2 - \sqrt{16}$$

is  $(-\pi, -8)$ .

Circle the correct answer:    **True**    **False**

5. The centre of the circle,

$$(x - 1)^2 + (y - 2)^2 = 4$$

is  $(-1, -2)$ .

Circle the correct answer:    **True**    **False**

6. The vertex of,

$$f(x) = -(x - 3)^2 - 4.$$

represents a maximum.

Circle the correct answer:    **True**    **False**

7. The Domain and Range of,

$$f(x) = -\frac{4}{2x + 1} + 8.$$

is  $\mathcal{D} = \{x \in \mathbb{R} \mid x \neq \frac{1}{2}\}$ ,  $\mathcal{R} = \{y \in \mathbb{R} \mid y \neq 8\}$ .

Circle the correct answer:    **True**    **False**

8. If  $\mathcal{V} = \{v \in \mathbb{N} \mid v^2 = -1\}$ , then  $\mathcal{V}$  is the empty set.

Circle the correct answer:    **True**    **False**

9. The x-intercepts of  $f(x) = x^2 - 5x + 6$  are  $x_1 = -2$  and  $x_2 = -3$ .

Circle the correct answer:    **True**    **False**

10. The vertex of  $f(x) = x^2 + 6x + 5$  is  $(-3, -4)$ .

Circle the correct answer:    **True**    **False**

**Hint:** Save time by going from factored form to vertex

**Question 2.** (4 marks) Write down the elements of the following sets.

(**Recall:**  $\mathbb{N} = \{1, 2, 3, 4, \dots\}$ )

(a)  $\mathcal{T} = \{a \in \mathbb{Z} \mid 3 < a < 7\}$ .

(b)  $\mathcal{X} = \{x \in \mathbb{N} \mid x \neq 1\}$ .

(c)  $\mathcal{Z} = \{y \in \mathbb{Z} \mid -3 \leq y \leq 0\} + \{i \in \mathbb{Z} \mid -2 \leq i \leq 1\}$

**Hint:** Add the two sets first.

(d)  $\mathcal{B} = \{x \in \mathbb{N} \mid \text{rem}(x, 2) = 0\}$ .

**Question 3.** (8 marks) Determine the Domain and Range of the following functions,

(a)  $\mathcal{Y}(x) = -2\sqrt{5x - 10} - 8.$

(b)  $x^2 + (y + 4)^2 = 16.$

(c)  $\mathcal{L}(x) = -5|x + 1| - 3.$

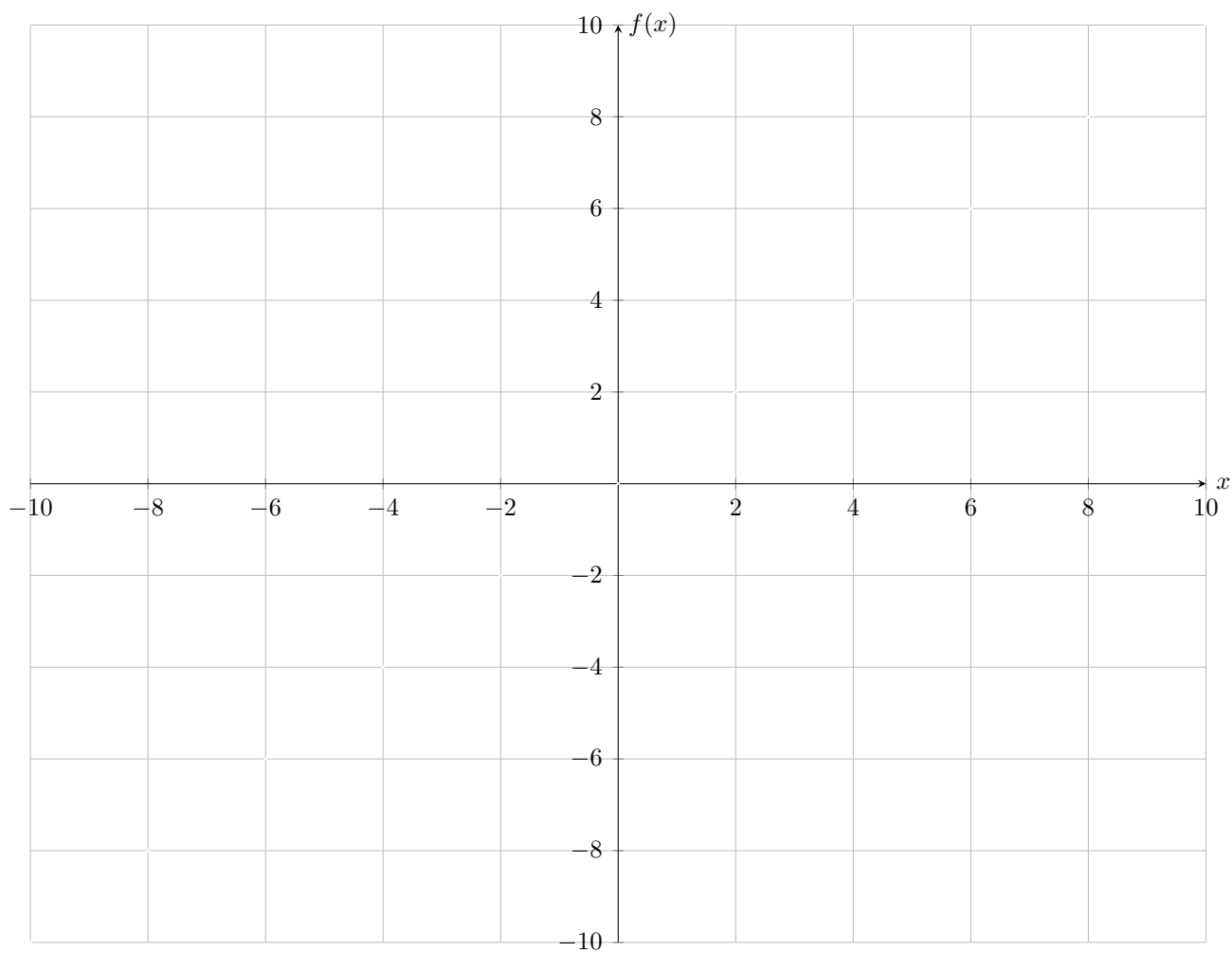
(d)  $\mathcal{E}(x) = -\frac{5}{2x-10} + 5.$

**Question 4.** Lets define the following function,

$$f: \mathbb{R} \rightarrow \mathbb{R}$$
$$f(x) = -x^2 + 4x + 3$$

(a) (4 marks) Convert  $f(x)$  into vertex form by completing the square.

(b) (3 marks) Sketch the function on the plot below. **(Label y-intercept and vertex)**



**Question 5.** (2 marks) We call a function idempotent if  $f(f(x)) = f(x)$ . Is the function  $f(x) = x$  idempotent? Justify your answer.

**Question 6.** (7 marks) Factor the following quadratic functions,

(a)  $f(x) = -x^2 + 9x - 20$ .

(b)  $f(x) = 4x^2 - 1$ .

(c)  $f(x) = 2x^2 - 4x - 16$ .

**Hint:** This can be factored simply.

**Question 7.** (6 marks) Let  $g(x) = 5x^2 + 14x - 3$ ,

(a) How many solutions will  $g(x)$  have? (**Note:**  $14^2 = 196$ )

(b) Factor  $g(x)$ .

**Question 8.** (6 marks) Let  $T(x) = -\frac{1}{2}(2x - 22)(x + 1)$ . Convert  $T(x)$  into vertex form.

**Question 9.** (6 marks) A function is nilpotent if there exists some number  $t$  such that  $f(f(t)) = 0$ . Let  $T(x) = x^2 - 1$ .

(a) Determine  $T(T(1))$ .

(b) Determine  $T(T(0))$ .

(c) Is  $T(x)$  nilpotent? Justify your answer.

**Question 10.** (6 marks) Let  $f(x) = x - 1$  and  $g(x) = x + 1$ . We call  $f$  and  $g$  mutual inverses of each other if **both** of the following conditions hold,

- For every number  $b$ ,  $f(g(b)) = b$ .
- For every number  $a$ ,  $g(f(a)) = a$ .

(a) Determine  $f(g(1))$ .



(b) Determine  $g(f(2))$ .

(c) Based on your answers from part (a) and (b), do you think  $f$  and  $g$  are inverses of each other?

**Question 11.** (6 marks) Let  $S = \{x \in \mathbb{R} \mid 8x^2 + 2x - 3 = 0\}$ . Write down the elements of  $S$ .

**Hint:** Try factoring first.