Review on Factoring

1 Difference of Squares factoring $(f(x) = ax^2 + c)$

IF $c < 0$ then you can factor the quadratic as follows,

$$f(x) = \left(\sqrt{a}\,x + \sqrt{|c|}\right)\left(\sqrt{a}\,x - \sqrt{|c|}\right).$$

ELSE IF $a < 0$ then you can factor the quadratic as follows,

$$f(x) = -\left(\sqrt{|a|}\,x + \sqrt{c}\right)\left(\sqrt{|a|}\,x - \sqrt{c}\right).$$

2 Simple Factoring $(f(x) = x^2 + bx + c)$

IF $a = 1$, then the factoring is known as 'simple' because the procedure is quite straight forward. As such we call quadratics with $a = 1$ simple trinomials. So assume that we are working with a quadratic function with $a = 1$,

$$f(x) = x^2 + bx + c.$$

Step 1. Find the integers p, q such that,

$$\begin{aligned} p + q &= b \\ p \cdot q &= c. \end{aligned}$$

Step 2. Factor the quadratic as follows,

$$f(x) = (x + q)(x + p).$$

3 Non-Simple Factoring (FIXED) $(f(x) = ax^2 + bx + c)$

If $a \neq 1$ then proceed with the following steps,

Step 1. IF b **and** c are divisible by a, then factor a out of the polynomial. Then apply simple factoring to the polynomial leftover and your done!

Step 2. ELSE, find the integers p, q such that,

$$\begin{aligned} p + q &= b \\ p \cdot q &= ac. \end{aligned}$$

Step 3. Then ,

- Find the $\gcd(|a|, |p|)$, let that integer be t.
- Find the $\gcd(|q|, |c|)$, let that integer be k.

Step 4. IF $a \cdot q > 0$, then complete the factorization by writing,

$$f(x) = (tx + k)\Big(\frac{a}{t}x + \frac{p}{t}\Big).$$

Step 5. ELSE IF $a \cdot q < 0$, then complete the factorization by writing,

$$f(x) = (tx - k)\Big(\frac{a}{t}x + \frac{p}{t}\Big).$$

Practice Problems:

Completely factor the following quadratics. (I will typeset the solutions to these problems soon, just ask me in class if you feel uncertain about an answer).

1. $f(x) = x^2 + 16x + 64$
2. $g(x) = -x^2 - 15x - 26$
3. $f(x) = 9x^2 - 49$
4. $r(x) = 6x^2 + x - 2$
5. $g(x) = -9x^2 + 6x - 1$
6. $Y(x) = 21x^2 + 27x + 6$
7. $I(x) = 12x^2 - 26x - 16$ (Make life easy by first factoring out a 4)
8. $A(x) = 2x^2 + 7x + 5$
9. $f(x) = 4x^2 + 16x + 16$
10. $r(x) = -x^2 + 13x - 22$
11. $f(x) = 121 - 100x^2$
12. $f(x) = 7x^2 + 15 - 18$
13. $L(x) = 4 - 9x^2$
14. $\Gamma(x) = 12x^2 + 17x + 6$
15. $\Lambda(x) = -2x^2 + 8x + 24$
16. $\Phi(x) = -5x^2 + 2x + 7$
17. $\Theta(x) = -3x^2 - 9x + 30$
18. $\xi(x) = x^2 + 4x - 77$
19. $\alpha(x) = 2x^2 - 3x - 14$
20. $S(x) = 6x^2 + 5x - 6$