

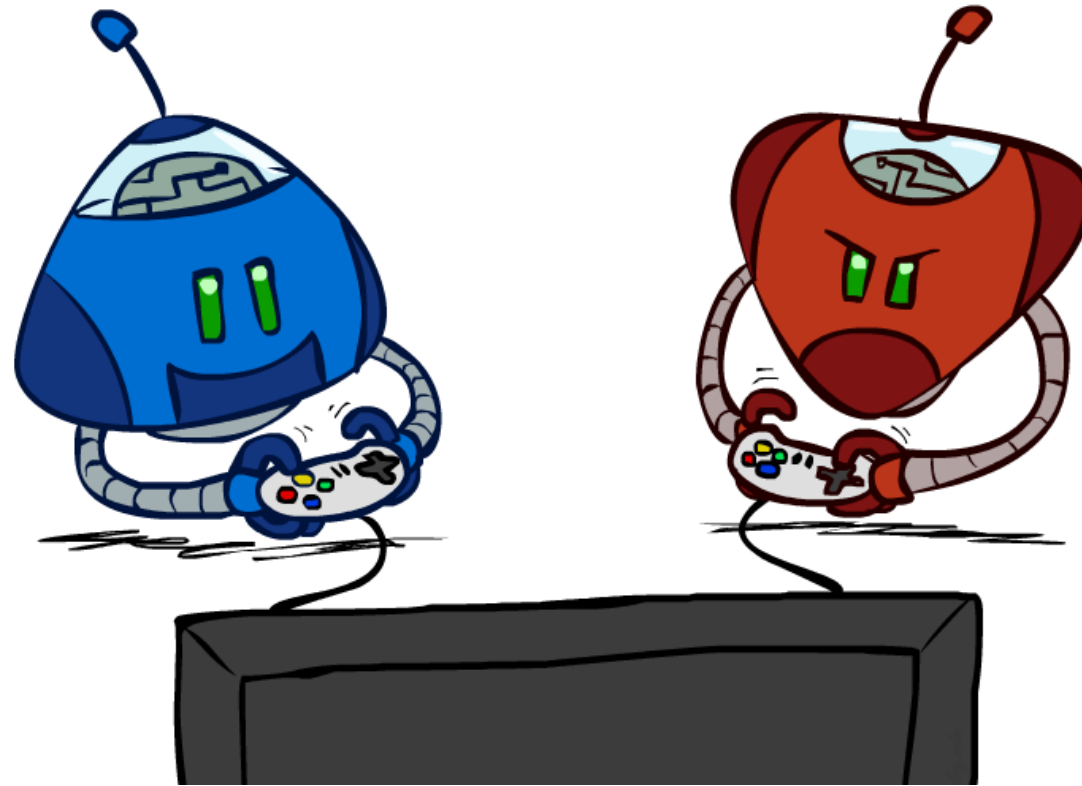
Dr. Seemab latif

Lecture 7

24th Oct 2024

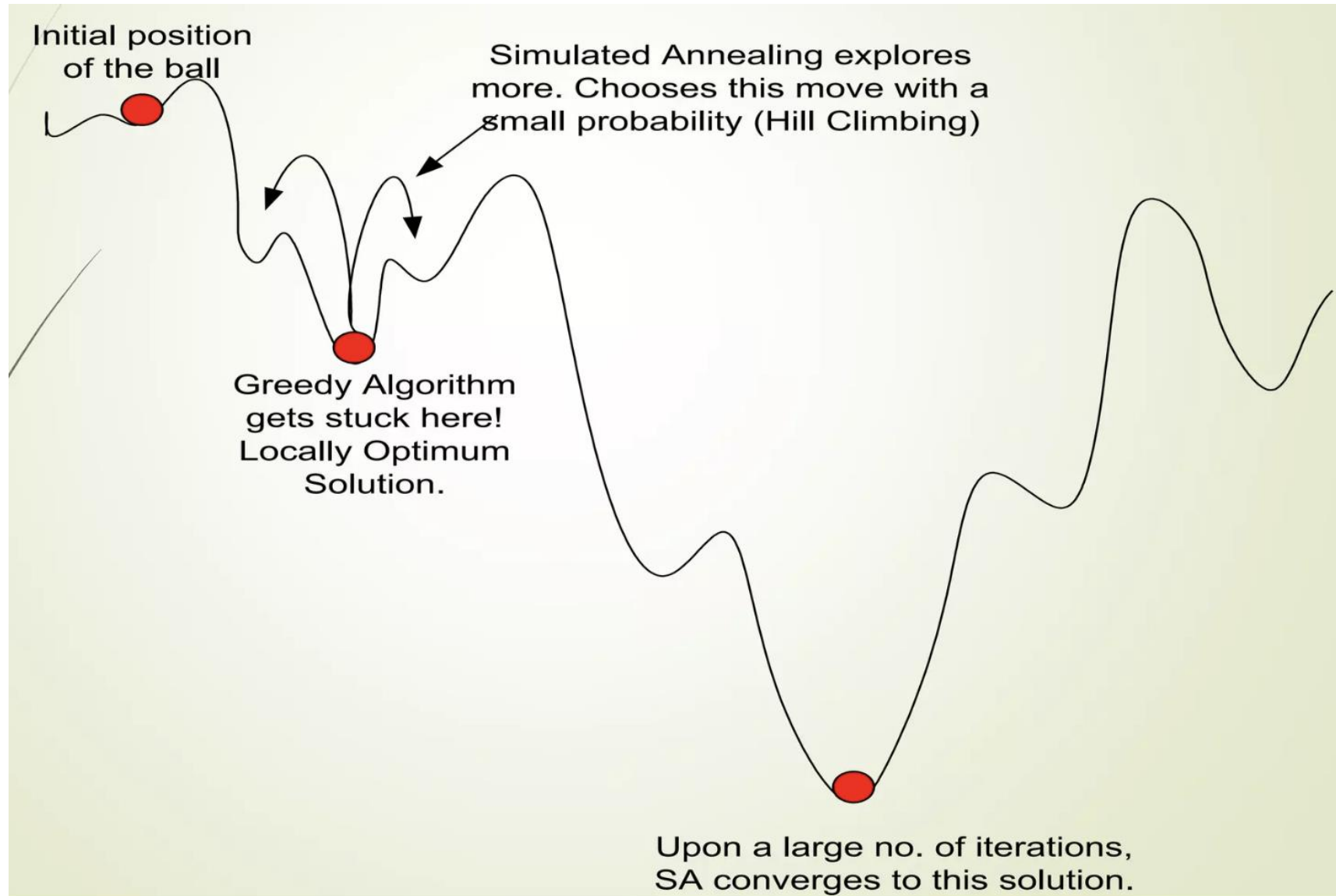
# AI: Representation and Problem Solving

## Local Search

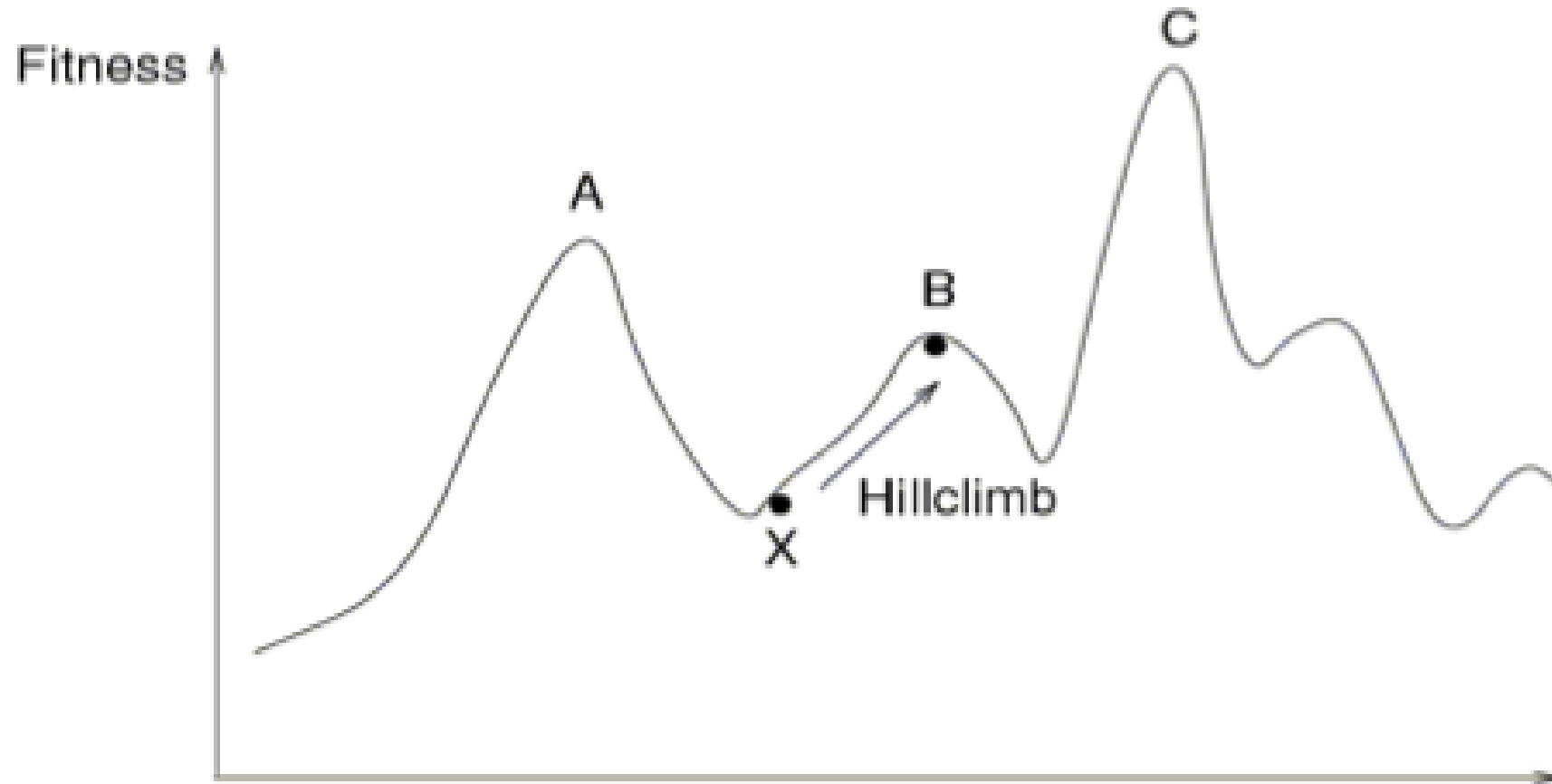


Slide credits: Pat Virtue, <http://ai.berkeley.edu>

# Hill climbing

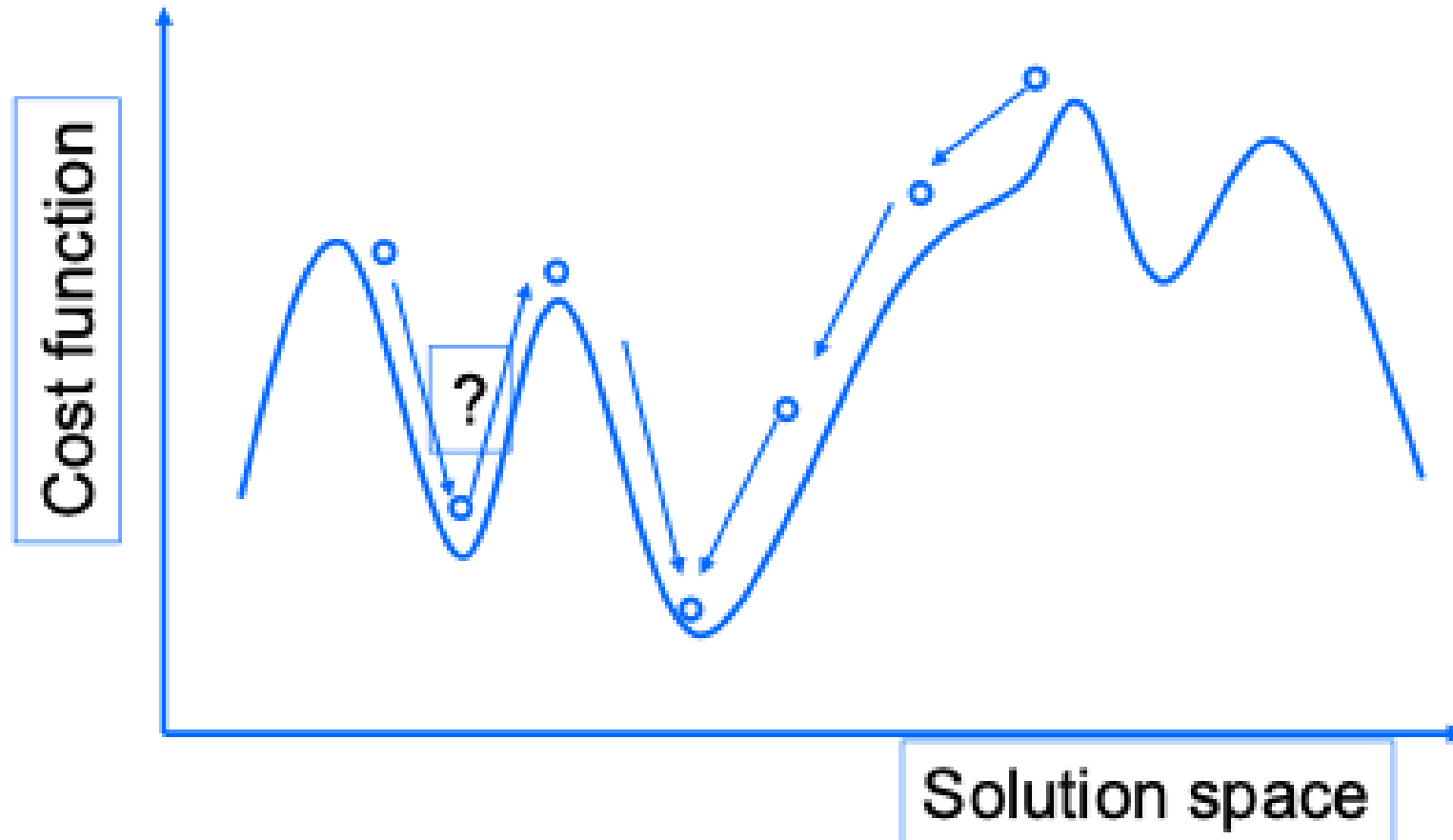


# Hill climbing



# Simulated Annealing

- Local Search



# Annealing

- Annealing is a thermal process for obtaining low energy states of a solid in a heat bath.
- The process contains two steps:
  - Increase the temperature of the heat bath to a maximum value at which the solid melts.
  - Decrease carefully the temperature of the heat bath until the particles arrange themselves in the ground state of the solid. Ground state is a minimum energy state of the solid.
- The ground state of the solid is obtained only if the maximum temperature is high enough and the cooling is done slowly.



# Simulated Annealing

- To apply simulated annealing with optimization purposes we require the following:
  - A successor function that returns a "close" neighboring solution given the actual one. This will work as the "disturbance" for the particles of the system.
  - A target function to optimize that depends on the current state of the system. This function will work as the energy of the system.
- The search is started with a randomized state. In a polling loop we will move to neighboring states always accepting the moves that decrease the energy while only accepting bad moves accordingly to a probability distribution dependent on the "temperature" of the system.



# Simulated Annealing

- Decrease the temperature slowly, accepting less bad moves at each temperature level until at very low temperatures the algorithm becomes a greedy hill-climbing algorithm.
- The distribution used to decide if we accept a bad movement is known as Boltzmann distribution.
- This distribution is very well known in solid physics and plays a central role in simulated annealing. Where  $\gamma$  is the current configuration of the system,  $E_\gamma$  is the energy related with it, and  $Z$  is a normalization constant.

$$P(\gamma) = \frac{e^{-E_\gamma/T}}{Z(T)},$$

$$Z(T) = \sum_{\gamma'} e^{-E_{\gamma'}/T},$$

# Annealing Process

- Raising the temperature up to a very high level (melting temperature, for example), the atoms have a higher energy state and a high possibility to re-arrange the crystalline structure.
- Cooling down slowly, the atoms have a lower and lower energy state and a smaller and smaller possibility to re-arrange the crystalline structure.

# Annealing – Analogy

## **Physical System**

## **Optimization Problem**

State (configuration) 

Solution

Energy 

Cost function

Ground State 

Optimal solution

Rapid Quenching 

Iteration improvement

Careful Annealing 

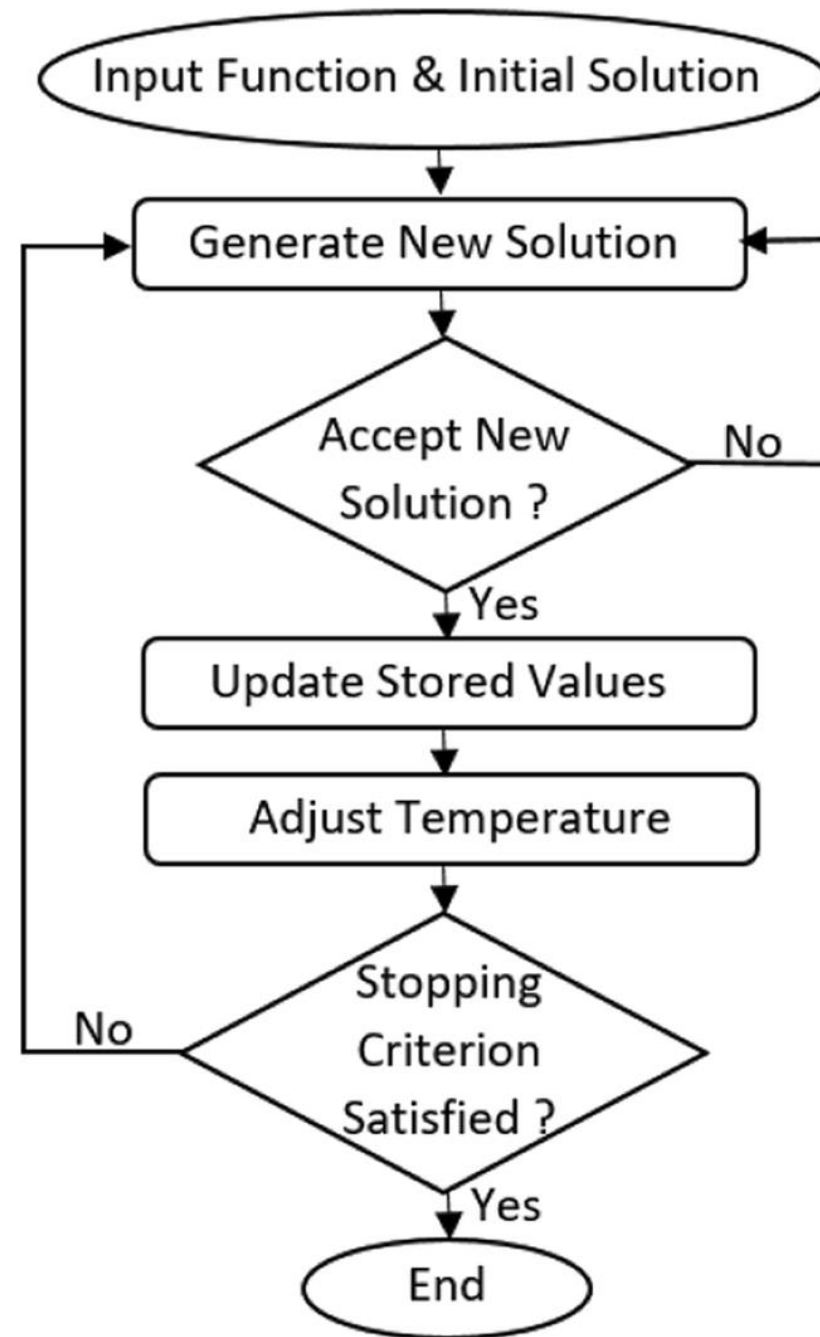
Simulated annealing

# Simulated Annealing - Analogy

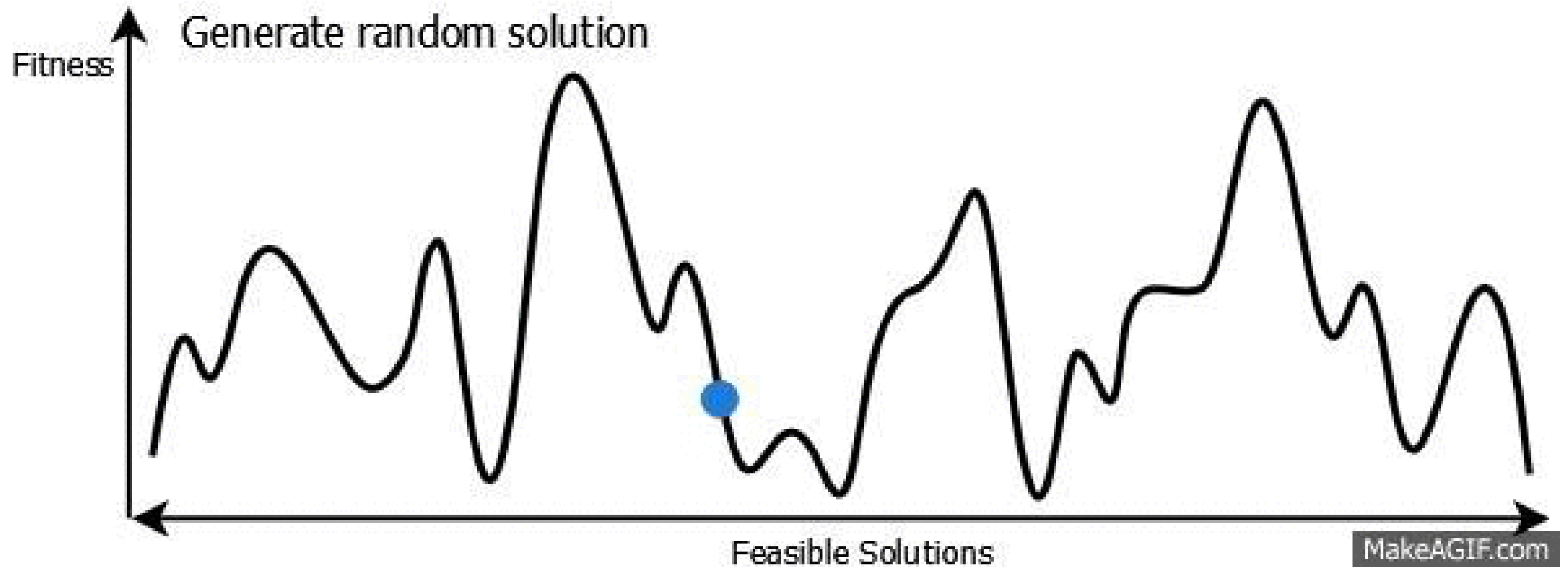
- Metal  $\leftrightarrow$  Problem
- Energy State  $\leftrightarrow$  Cost Function
- Temperature  $\leftrightarrow$  Control Parameter
- A completely ordered crystalline structure  
 $\leftrightarrow$  the optimal solution for the problem

**Global optimal solution can be achieved as long as the cooling process is slow enough.**

# Simulated Annealing Flow Chart



# How SA explores the solution space



# Conclusion

- SA is a global optimization technique.
- SA distinguishes between different local optima.
- SA is a memory less algorithm, the algorithm does not use any information gathered during the search
- SA is motivated by an analogy to annealing in solids.
- Simulated Annealing - an iterative improvement algorithm

Solving  
Sudoku  
using SA

3			8		1			2
2		1		3		6		4
			2		4			
8		9				1		6
	6						5	
7		2				4		9
			5		9			
9		4		8		7		5
6			1		7			3



# Steps for solving Sudoku

## 1- Generating random states

5	2	4	4	3	7	6	8	9
6	7	9	1	5	9	2	7	3
8	1	3	6	8	2	4	1	5
4	3	1	6	7	5	4	8	1
5	8	6	1	8	9	9	3	2
7	9	2	2	3	4	7	6	5
2	8	9	7	1	5	8	4	2
6	4	5	2	9	3	9	1	3
3	1	7	8	6	4	7	5	6

# Steps for solving Sudoku

Simulated Annealing Explained By Solving  
Sudoku - Artificial Intelligence

1- Generating random states

2- Writing a cost function

1	5	2	4	4	3	7	6	8	9
2	6	9	7	1	5	9	2	7	3
2	8	1	3	6	8	2	4	1	5
1	4	3	1	6	7	5	4	8	1
2	5	8	6	1	8	9	9	3	2
2	7	9	2	2	3	4	7	6	5
2	2	8	9	7	1	5	8	4	2
2	6	4	5	2	9	3	9	1	3
2	3	1	7	8	6	4	7	5	6
	2	3	1	3	2	3	3	2	3

Total cost = 38

# Steps for solving Sudoku

- 1- Generating random states
- 2- Writing a cost function
- 3- Selecting starting temperature

1	5	2	4	4	3	7	6	8	9
2	6	9	7	1	5	9	2	7	3
2	8	1	3	6	8	2	4	1	5
1	4	3	1	6	7	5	4	8	1
2	5	8	6	1	8	9	9	3	2
2	7	9	2	2	3	4	7	6	5
2	2	8	9	7	1	5	8	4	2
2	6	4	5	2	9	3	9	1	3
2	3	1	7	8	6	4	7	5	6
	2	3	1	3	2	3	3	2	3

Total cost = 38

## Steps for solving Sudoku

- 1- Generating random states
- 2- Writing a cost function
- 3- Selecting starting temperature
- 4- Calculating iterations per T

1	5	2	4	4	3	7	6	8	9
2	6	9	7	1	5	9	2	7	3
2	8	1	3	6	8	2	4	1	5
1	4	3	1	6	7	5	4	8	1
2	5	8	6	1	8	9	9	3	2
2	7	9	2	2	3	4	7	6	5
2	2	8	9	7	1	5	8	4	2
2	6	4	5	2	9	3	9	1	3
2	3	1	7	8	6	4	7	5	6
	2	3	1	3	2	3	3	2	3

Total cost = 38

# Steps for solving Sudoku

- 1- Generating random states
- 2- Writing a cost function
- 3- Selecting starting temperature
- 4- Calculating iterations per T
- 5- Choosing a cooling rate

Simulated Annealing Explained By Solving  
Sudoku - Artificial Intelligence

1	5	2	4	4	3	7	6	8	9
2	6	9	7	1	5	9	2	7	3
2	8	1	3	6	8	2	4	1	5
1	4	3	1	6	7	5	4	8	1
2	5	8	6	1	8	9	9	3	2
2	7	9	2	2	3	4	7	6	5
2	2	8	9	7	1	5	8	4	2
2	6	4	5	2	9	3	9	1	3
2	3	1	7	8	6	4	7	5	6
	2	3	1	3	2	3	3	2	3

Total cost = 38