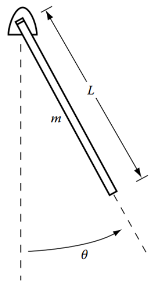
National University of Computer and Emerging Sciences, Lahore Campus

Course:	Computer Modeling and Simulation	Course Code:	CS-421
Program:	BS(Computer Science)	Semester:	Spring 2017
Duration:	1 hour	Total Marks:	40
Paper Date:	6^{th} April, 2017	Weight:	15%
Section:	All	Page(s):	2
Exam:	Mid-2	Roll No.	

Instructions/Notes: Write answers clearly and precisely, if the answers are not easily readable then it will result in deduction of marks.

Question 1 (15 points): The pendulum shown in the figure is a rod of length L and mass m. It is put into motion by rotating the rod to an angle θ_0 and releasing it from rest. Neglecting air resistance and frictional force around the pivot point of the pendulum, develop an expression for the position θ of the pendulum as a function of time.

- Draw Free Body Diagram
- Label the FBD with equation of forces
- Write the system in terms of system of equations.
- Write the meaning of each and every symbol you use in the equation in a separate table.



$$-\tau_g + J\frac{d^2\theta}{dt} = 0$$ 1 equation 3 unknowns $[J, \theta, \tau_g]$

$$\tau_g = mg\frac{L}{2}sin(\theta)$$ 2 equations 3 unknowns

$$J = m\frac{L^2}{3}$$ 3 equations 3 unknowns

Question 2 (5 points): Prove Markov's inequality, and state it in terms of probability of getting a random variable X greater than a constant a.

$$\begin{aligned} E[X] &= \sum_{i=0}^{n} x_i P(x_i) \\ &= \sum_{i=0}^{a} x_i P(x_i) + \sum_{i=a}^{n} x_i P(x_i) \\ &\geq \sum_{i=a}^{n} x_i P(x_i) \\ &\geq \sum_{i=a}^{n} aP(x_i) = a\sum_{i=a}^{n} P(x_i) = aP(X \geq a) \end{aligned}$$

Question 3 (5 points): Prove Chebyshev's inequality, and state it in terms of number of values of a random variable X, close to its means μ.

Since $(X-\mu)^2/\sigma^2$ is a positive random variable whose mean is

$$E\left[\frac{(X-\mu)^2}{\sigma^2}\right] = \frac{E[(X-\mu)^2]}{\sigma^2} = 1$$

Replacing X by $(X-\mu)^2/\sigma^2$ and $a = k^2$ in Markov's inequality we get following, which after a little rearrangement proves the theorem

$$P\left\{\frac{(X-\mu)^2}{\sigma^2} \geq k^2\right\} \leq \frac{1}{k^2}$$

Question 4 (5 points): Show that $(E \cup F)^c = E^c \cap F^c$

If $(E \cup F)^c$ occurs, then $E \cup F$ does not occur, and so E does not occur (and so E^c does); F does not occur (and so F^c does) and thus E^c and F^c both occur.

Question 5 (5 points): Suppose each of three persons tosses a coin. If the outcome of one of the tosses differs from the other outcomes, then the game ends. If not, then the persons start over and toss their coins again. Assuming fair coins, what is the probability that the game will end with the first round of tosses? If all three coins are biased and have probability $1/4$ of landing heads, what is the probability that the game will end at the first round ?

Let define probability to end as P_e, and probability to continue as P_c.

$$P_e = 1 - P_c = 1 - P(\{H,H,H\} \cup \{T,T,T\})$$

$$Fair \quad Coin: P_e = 1 - [1/2 \cdot 1/2 \cdot 1/2 + 1/2 \cdot 1/2 \cdot 1/2] = 3/4$$

$$Biased \quad Coin: P_e = 1 - [1/4 \cdot 1/4 \cdot 1/4 + 3/4 \cdot 3/4 \cdot 3/4] = 9/16$$

Question 6 (5 points): Consider n independent flips of a coin having probability p of landing heads. Say a changeover occurs whenever an outcome differs from the one preceding it. For instance, if the results of the flips are $HHTHTHHT$, then there are a total of five changeovers. If $p = 1/2$, what is the probability there are k changeovers?

Each flip after the first will, independently, result in a changeover with probability $1/2$. Therefore

$$P\{k \quad changeovers\} = C_k^{n-1}(1/2)^{n-1}$$
