National University of Computer and Emerging Sciences, Lahore Campus				
	Course:	Calculus and Analytical Geometry	Course Code:	MT 119
	Program:	BS(CS), BS(DS), BS(SE)	Semester:	Fall 2020
	Duration:	3 hours	Total Marks:	80
	Paper Date:	16-02-2021	Weight	50%
	Section:	All	Page(s):	02
	Exam:	Final Exam	Roll No:	[redacted]
Instruction/Notes:	Attempt all questions. Programable calculator are not allowed.			

Question # 1: CLO-3 [6+4]

(a). For what value of a and b is

$$f(x) = \begin{cases} ax + 2b & , \quad x \le 0, \\ x^2 + 3a - b & , \quad 0 < x \le 2, \\ 3x - 5 & , \quad x > 2 \end{cases}$$

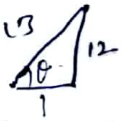
continuous at every x?

(b). Evaluate the indeterminate power $\lim_{x\to\infty} (1 + 2x)^{1/(2\ln x)}$.

$(1+2x)^{\frac{1}{2\ln x}}$

Question # 2: CLO-4 [10]

A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.



i) How fast is the top of the ladder sliding down the wall then?

ii) At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

iii) At what rate is the angle u between the ladder and the ground changing then?

Question # 3: CLO-5 [15]

Let $y = 4x^3 - x^4,$

1. Find the derivatives y' and y''.
2. Find the critical points of $y = f(x)$, if any, and identify the function's behavior at each one.
3. Find where the curve is increasing and where it is decreasing.
4. Find the points of inflection, if any, and determine the concavity of the curve.
5. Sketch the general graph of the curve.

1/2

$y = 192 - 256$ / 1024

$a = 48 - 32$

$-\frac{3}{2}$

$-9 + 6 = 2b$

PTO

$-1 = 12 + 4$

$3(-\frac{3}{2}) + 3 = b$

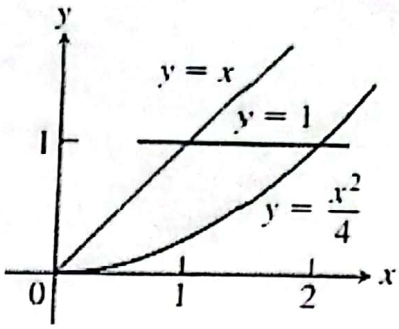
$-\frac{9}{2} + 3 = b$

$3 = 72 - 108$

$1 = 24 - 12$

Question # 4: CLO-6 [5+5]

a. Find the area of shaded region bounded by the curves $y = x, y = 1$ and $y = \frac{x^2}{4}$ for $x \in [0, 2]$.



b. Find $\frac{dy}{dx}$, if $y = \int_{secx}^{2} \frac{1}{t^2+1} dt$.

Question # 5: CLO-6 [10+5]

(a). Find the volume of the solid generated by revolving the region bounded by the curves $y = x^2, y = 2 - x, x = 0,$ for $x \geq 0$ about the $y -$ axis.

(b). Find the length of the curve given below

$$y = \left(\frac{3}{4}\right)x^{4/3} - \left(\frac{3}{8}\right)x^{2/3} + 5, \quad \text{where } x \in [1, 8].$$

Question # 6: CLO-7 [5+5]

a. Find the value of p for which integral $\int_1^2 \frac{1}{x(lnx)^p} dt$ converges.

b. Is the area under the curve $y = \frac{lnx}{x^2}$ from $x = 1$ to $x = \infty$ is finite? If so, what is its value?

Question # 7: CLO-7 [5+5]

a. Find the vector projection of $\boldsymbol{u} = 4\boldsymbol{i} + 6\boldsymbol{j} + 3\boldsymbol{k}$ onto $\boldsymbol{v} = 2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$ and the scalar component of $\boldsymbol{u}$ in the direction of $\boldsymbol{v}$.

b. Find the distance from the point $(1, 0, -1)$ to the plane $-4x + y + z = 4$.

(Good luck)

$x = 4$ $\quad t = \tan\theta,\ \theta = \tan^{-1}(t)$ $\quad \frac{1}{1+\tan^2\theta} \sec\theta\, d\theta$

$dx = du$ $\quad dt = \sec\theta\, d\theta$

2/ $\int \underset{I}{\ln x} \cdot \underset{II}{x^{-2}}\, dx$

$-\ln x \cdot \frac{1}{x} + \int \frac{1}{x} \cdot \frac{1}{x}\, dx$

$-\ln x \cdot \frac{1}{x} + \int \frac{1}{x^2}\, dx$

$\frac{\sec\theta}{\sec^2\theta} d\theta$

$\cos\theta\, d\theta$

$\sin\theta$

$\sin(\tan^{-1}t)$

$\sin(\tan^{-1}(2)) - \sin(\tan^{-1}(\sec x))$