National University of Computer and Emerging Sciences, Lahore Campus

	Course:	Computer Modeling and Simulation	Course Code:	CS-421
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	Paper Date:	10^{th} May, 2017	Weight:	50%
	Section:	All	Page(s):	3
	Exam:	Final		

NAME:____________________________ SECTION:________ ROLL#:______________

Instructions/Notes: Write answers clearly and precisely, if the answers are not easily readable then it will result in deduction of marks.

Question 1 (30 points): A ball dropped from a height h_0. The ball was initially at rest, and has its mass perfectly distributed all over its area. Meaning the density of the ball is uniform at each of its cross sections. The ball, when released, hits the ground, then bounces up, and then moves towards the ground once again. It continues to do so until its movement dies out. You have to do the following

1. Draw **three** Free Body Diagrams: One, when ball is moving down. Second, when ball touches the ground, and third, when ball moves upward.
2. For all **three** phases mention the parameters affecting the forces contributing to the movement of the ball.
3. Mention the function of each force and list the parameters. Also label the force on FBD.
4. Make **three tables**, one for each FBD (or phase), where you describe each and every symbol used in the model. The symbol table should contain the description of each force and the parameters affecting that force.
5. Clearly mention the order of all phases, and when a phase transitions into the other one.
6. Mention when the movement of the ball will die out. What physical factors play their part in rendering the ball still?

Take notice of the following

- Do **not** factor out anything from the model, for example air resistance. If you do not know how the parameters are related to each other in form of a differential equation, then omit the differential equation. Only mention an abstract representation of the function. For example, the equation $F_d = f_1(F_g, F_{ar})$ is used to say that the "downward force" (F_d) is a function (f_1) of "force of gravity" (F_g) and the "air resistance" (F_{ar}). Although it is quite obvious that when ball is moving down $F_d = F_g - F_{ar}$, still if you cannot figure it out, then resort to the first expression.
- You do **not** need to create a system of equations, where the number of unknowns should be equal to the number of equations, but everything should be in mathematical form as described above with an example of "downward force". Do not write plane sentences with long descriptions and explanations.
- The answer should be grouped intelligently; the FBD, the equations and the table of each phase should be grouped together. If they are scattered around and it is difficult to find their connections, then the marks will be deducted.

There are three stages as mentioned in the question.
At first stage, there are two forces acting on the ball, F_g and F_{air}, where F_g is the gravitational force, and F_{air} is the air resistance, acting in opposite direction.

At second stage, there are two forces acting on the ball, F_{recoil} and F_{damp}, where F_{recoil} is the reaction force, and F_{damp} is the damping force, which reduces the reaction force to some degree based on the material of ball and the surface to which the ball hits.

At third stage, there are three forces acting on the ball, F_{recoil}, F_{air} and F_g, where F_g is gravitational pull, F_{recoil} is the reaction force, and F_{air} is the air resistance.

Question 2 (20 points): Given below is the formula of Newton Iteration.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Write a pseudo code which finds the value x^* (the converged value). The first parameter of the function is x_0, which is the initial value. The second parameter is `double (*f) (double x)`, which is a pointer to the original function. When called, it returns the value calculated by the original function (same as $f(x_n)$ in the equation above). The third parameter is `double (*fder) (double x)`, which is a pointer to the derivative of the original function (same as $f'(x_n)$ in the equation above). Your code should return x^* .

The proper way to do it is to introduce a function which takes all parameters mentioned in the specific format. There is a loop inside that functions which evaluates the above expression in each iteration, assigning $x_n \leftarrow x_{n+1}$. It terminates when $x_n == x_{n+1}$

Question 3 (20 points): Given below is the formula of forward Euler's method that calculates the next state of a differential equation.

$$y'(t_n) = f(y(t_n), t_n)$$
$$y(t_{n+1}) = y(t_n + h) = y(t_n) + h \cdot y'(t_n)$$

Write a pseudo code which finds the value $y(t_{n+1})$. The parameters are following. The first parameters is t_0, which is the starting time. The second parameter is y_0, which is the initial condition. The third parameter is the function f in the above equation. The fourth parameter is the h, which is the time interval. The last parameter is t_{n+1}, which is the time when differential equation will have states y_{n+1} or $y(t_{n+1})$. Your code should return y_{n+1}.

Just put a loop over the instructions. The loop iterates over t which starts from t_0 and terminates when it reaches t_{n+1}.

Question 4 (20 points): Write pseudo code for generating Poisson random variables. The parameter to this function is λ. Each call to your function will return a randomly generated integer value. The value will be generated from the Poisson distribution with parameter λ.

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1. Generate a random number U.
2. $i = 0, p = e^{-\lambda}, F = p$
3. If $U < F$ then set $X = i$ and stop
4. $p = \lambda p/(i+1), F = F + p, i = i + 1$
5. Go to Step 3

Question 5 (3 points): Twenty sheets of aluminum alloy were examined for surface flaws. The frequency of the number of sheets with a given number of flaws per sheet was as follows:

Number of Flaws	Frequency
0	4
1	3
2	5
3	2
4	4
5	1
6	1

What is the probability of finding a sheet chosen at random which contains 3 or more surface flaws?

The average number of flaws for the 20 sheets is given by: $\mu = \frac{46}{20} = 2.3$
The required probability is:

$$
\begin{aligned}
P(X \geq 3) &= 1 - (P(X=0) + P(X=1) + P(X=2)) \\
&= 1 - \left(\frac{e^{-2.3} 2.3^0}{0!} + \frac{e^{-2.3} 2.3^1}{1!} + \frac{e^{-2.3} 2.3^2}{2!} \right)
\end{aligned}
$$

Question 6 (3 points): A company makes electric motors. The probability an electric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors?

The average number of defectives in 300 motors is $\mu = 0.01300 = 3$ The probability of getting 5 defective motors using **Poisson** Distribution is:

$$P(X) = \frac{e^{-3} \cdot 3^5}{5!} = 0.1008$$

The probability of getting 5 defective motors using **Binomial** Distribution is using $n = 300$, $x = 5$, $p = 0.01$ and $q = 0.99$:

$$P(X = 5) = C_5^{300} \cdot 0.001^5 \cdot 0.99^{295} = 0.10099$$

Question 7 (4 points): Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover?

This is a binomial distribution because there are only 2 outcomes (the patient dies, or does not).
Let X = number who recover. Here, $n = 6$ and $x = 4$. Let $p = 0.25$ (success, that is, they live), $q = 0.75$ (failure, i.e. they die) The probability that 4 will recover:

$$P(X) = C_x^n p^x q^{n-x} = C_4^6 (0.25)^4 (0.75)^2 = 15 \times 2.197 \times 10^{-3}$$