National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Calculus and Analytical Geometry	Course Code:	MT 1003
Degree Program:	BS (CS, SE & DS)	Semester:	Fall 2021
Exam Duration:	3 hours	Total Marks:	90
Paper Date:	10-01-22	Weight	50%
Section:	ALL	Page(s):	
Exam Type:	Final Exam		

Instruction/Notes: Attempt all questions. Programmable calculators are not allowed.

Question#1: CLO-3 [10+5]

a) For the given function

$$f(x)=\begin{cases} x^2-1, & -1\le x\le 0 \\ 2x, & 0<x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x<3 \end{cases}$$

1. Does $f(-1)$ exist?
2. Does $\lim_{x\to -1^+} f(x)$ exist?
3. Does $\lim_{x\to -1^+} f(x)=f(-1)$?
4. Is f continuous at $x=-1$?
5. Is f defined at $x=3$?
6. Is f continuous at $x=3$?
7. At what value of x, f is continuous?
8. What value should be assigned to $f(2)$ to make the extended function continuous at $x=2$?

b) Evaluate the limit using l'Hôpital's rule

$$\lim_{x\to\infty}(1+2x)^{\frac{1}{2\ln x}}$$

Question#2: CLO-5 [15]

Let $f'(x)=\dfrac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}$

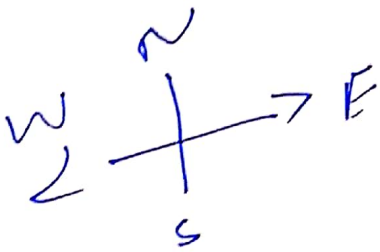
1. Find f''.
2. Find the critical points of f, if any, and identify the function's extreme values at each one.
3. Find intervals where the curve is increasing and where it is decreasing.
4. Find the points of inflection, if any occur, and determine the concavity of the curve.
5. Sketch the general shape of the curve with asymptotes (if any).

Question#3: CLO- 4[10+5]

a) A police cruiser, approaching a right angled intersection from the north is chasing a speeding car that has turned the corner and is now moving straight east. When cruiser is 6 mi north of intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

b) Solve the given inequality. Express the solution set as interval or union of intervals. Also show the solution set on real line.

$$\left|3-\frac{1}{x}\right| \geq \frac{1}{2}$$



Question#4: CLO-7 [10+5]

a) Find the area of the region enclosed by the curves given by

$$y = 7 - 2x^2 \quad \text{and} \quad y = x^2 + 4$$

b) Evaluate the integral using an appropriate substitution

$$\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx$$

Question#5 CLO-7 [7+8]

a) Use **DISK METHOD** to find the volume of the solid generated by revolving the region bounded by the curves about y-axis.

$$x = \frac{\sqrt{2y}}{y^2+1}, \qquad x = 0 \quad \text{and} \quad y = 1$$

b) Using Fundamental Theorem of Calculus find the length of the curve

$$y = \int_{-2}^{x} \sqrt{3t^4 - 1}\, dt, \qquad -2 \leq x \leq -1$$

Question#6 CLO-6. 8 [7+8]

Evaluate the improper integrals and determine whether the given improper integral converges or diverges

a) $\int_0^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}}\, d\theta$

b) $\int_0^\infty 2e^{-\theta}\sin\theta\, d\theta$