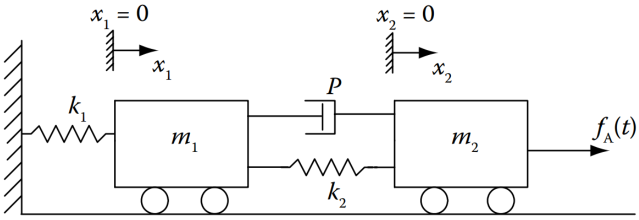
National University of Computer and Emerging Sciences, Lahore Campus

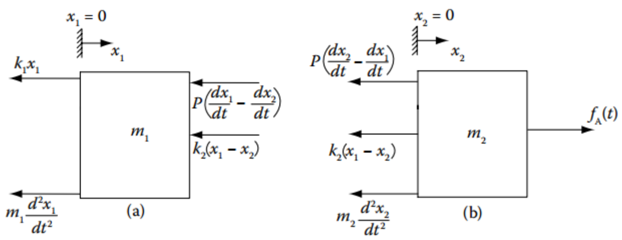
	Course:	Computer Modeling and Simulation	Course Code:	CS-421
	Program:	BS(Computer Science)	Semester:	Spring 2017
	Duration:	1 hour	Total Marks:	50
	Paper Date:	20^{th} February, 2017	Weight:	15%
	Section:	All	Page(s):	2
	Exam:	Mid-1	Roll No.	

Instructions/Notes: Write answers clearly and precisely, if the answers are not easily readable then it will result in deduction of marks.

Question 1 (30 points): In the following diagram a mechanical system is drawn. Develop a mathematical model of this system.

- Draw the relevant Free Body Diagram of the system.
- Each term you put in this mathematical model, explain its meaning and purpose. (Missing explanation of any term will lead to deduction of marks.)





$$\Sigma F_1 = m_1\frac{d^2x_1}{dt^2}$$ 1 equation 2 unknowns $[x_1, \Sigma F_1]$

$$\Sigma F_1 = F_{1,k_1} + F_{1,k_2} + F_{1,P} = -k_1x_1 - k_2(x_1 - x_2) - P(\frac{dx_1}{dt} - \frac{dx_2}{dt})$$ 2 equation 3 unknowns $[x_2]$

$$\Sigma F_2 = m_2\frac{d^2x_2}{dt^2}$$ 3 equation 4 unknowns $[\Sigma F_2]$

$$\Sigma F_2 = f_A(t) + F_{2,k_2} + F_{2,P} = f_A(t) - k_2(x_2 - x_1) - P(\frac{dx_2}{dt} - \frac{dx_1}{dt})$$ 4equation 4 unknowns

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Question 2 (8 points): If V is a vector space and $\mathbf{v}, \mathbf{w} \in V$ and s is a real number, then which of the following statements are true or false. Give **reason** for each

(a) $s\mathbf{v} + s\mathbf{w} \in V$ is True

(b) $s\mathbf{v} \times s\mathbf{w} \in V$ is False

(c) $s^2\mathbf{v} + s^2\mathbf{w} \in V$ is True

(d) $s\mathbf{v}^2 \in V$ is False

Question 3 (4 points): Suppose $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ are three vectors of a matrix **A**. All vectors are linearly independent. Now tell the size of its kernel space. Give **reasons**.

Matrices with linearly independent vectors have only one vector in their null space that is **0** vector.

Because $rank(A) + nullity(A) = n$ and here $rank(A) = n$

Question 4 (4 points): If matrix A is $n \times n$ identity matrix then eigenvectors of A form a basis of R^n, how and why?

Any thing which shows that $span(eigenvectos\ of\ I^n) = span(I^n) = R^n$

Question 5 (4 points): Following is a matrix A, where s is a real number. Tell one of its eigenvalues.

$$A = \begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{pmatrix}$$

One eigenvalue is $\lambda = s$, because if you factor out s, A will become an identity matrix. By definition identity matrices do not change anything in any vector, so s will only scale all the components of the vector, which is the definition of eigenvalues and eigenvectors.
