Section A: Solve the following recurrence:

$T_n = 3T_{n-1} + 2$

Subject to $T_1 = 1$

$T_n = 3(3T_{n-2} + 2) + 2 = 3^2T_{n-2} + 3x2 + 2$

$T_n = 3^2(3T_{n-3} + 2) + 3x2 + 2 = 3^3T_{n-3}+3^2x2 + 3x2 + 2$

In general:

$T_n = 3^kT_{n-k} + 3^{k-1}2 + 3^{k-2}2 + \ldots + 3^12 + 3^02$

$T_n = 3^kT_{n-k} + (3^{k-1} + 3^{k-2} + \ldots + 3^1 + 3^0)2$

$T_n = 3^kT_{n-k} + 2(3^k-1) / (3-1) = 3^kT_{n-k} + 3^k-1$

For the base case n-k = 1, or k = n – 1, which gives:

$T_n = 3^{n-1}T_1 + 3^n- 1 = 3^{n-1}T_1 + 3^n$

Section B: Solve the following recurrence:

$S_n = 2S_{n-1}$

subject to $S_1 = 1$

$S_n = 2(2S_{n-2}) =2^2S_{n-2}$

$S_n = 2^2(2S_{n-3}) = 2^3S_{n-3}$

In general:

$S_n = 2^kS_{n-k}$

For the base case n – k = 1, or k = n – 1

$S_n = 2^{n-1} S_1 = 2^{n-1}$