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Q1) Consider the unconstrained minimization problem.

$$f: R^2 \longrightarrow R$$

$$f(x) = (x_1 - x_2)^2 + (x_{\cancel{3}} - 2x_3)^2 + (2x_3 - 1)^2$$

a. Find the local minimizer

b. Is the function convex? What this signifies ~~d~~ for Part a?

$$f(x) = x_1^2 - 2x_1x_2 + \underbrace{x_2^2 + x_2^2}_{2x_2^2} - 4x_2x_3 + \underbrace{4x_3^2 - 4x_3^2}_{8x_3^2} + \cancel{4}\,1$$

(a) Ans:

$$\nabla f(x) = \begin{array}{c} \text{with respect } x_1 \\ x_2 \\ x_3 \end{array} \begin{bmatrix} 2x_1 - 2x_2 \\ -2x_1 + 4x_2 - 4x_3 \\ -4x_2 + \overset{16}{\cancel{8}}x_3 - 4 \end{bmatrix}$$

min: $x_1 = x_2$, $2x_1 = 2x_2 \Rightarrow$ meaning $x_1 = x_2 = 1$

$$-4x_2 + 8x_3 - 4 \Rightarrow -4(1) + \overset{16}{\cancel{8}}x_3 - 4$$

$$= -8 + 16x_3 \Rightarrow -8 + 16(0.5) = 0$$

So, $\boxed{x_1 = 1,\ x_2 = 1,\ x_3 = 0.5}$

b)

$$Q = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \begin{bmatrix} 2 & -2 & 0 \\ -2 & 4 & -4 \\ 0 & -4 & 16 \end{bmatrix} \quad 2 > 0$$

$$\begin{vmatrix} 2 & -2 \\ -2 & 4 \end{vmatrix} = (8 - 4) > 0$$

$$|Q| = 2(64 - 16) + 2(-32 - 0) + 0 = \boxed{32}$$

$$32 > 0$$

Hence convex!

Q2) Consider the function f of the variable x

$$f: \mathbb{R}^2 \to \mathbb{R}$$

$$f(x) = (x_1-4)^2 + (x_2+1)^2 - c(x_1+4)(x_2-1)$$

a. Find the length of the gradient at $[-4 \quad -2]^T \Rightarrow x^*$

Ans: $\nabla f(x) = \begin{bmatrix} 2(x_1-4) - c(x_2-1) \\ 2(x_2+1) - c(x_1+4 \end{bmatrix}$

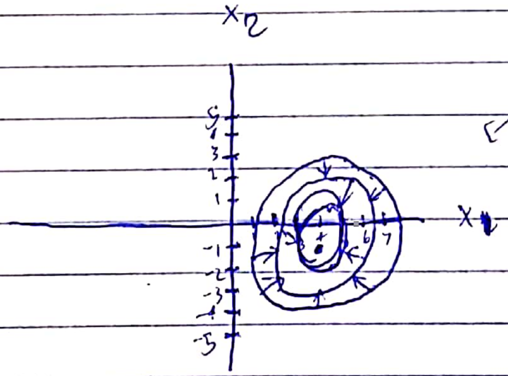
$$\nabla f(x^*) = \begin{bmatrix} -16 - c(-3) \\ -2 - c(0) \end{bmatrix} \Rightarrow \begin{bmatrix} -16 + 3c \\ -2 \end{bmatrix}$$

$$\|\nabla f(x^*)\| = \sqrt{(-16+3c)^2 + (-2)^2}$$

$\cancel{\sqrt{256}}$

b. considering $c=0$, Draw the ~~contours~~ direction of the steepest descent at the point: $[3 \quad -2]^T$

Ans $x^* = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$



this is a circles

the direction is towards the minmizer

Q3) Consider the function to be minimized:

$$f : R^2 \to R$$
$$f(x) = (x_1-4)^2 + (x_2+2)^2 - 1.75(x_1+4)(x_2-3)$$

Starting at $x^{(0)} = [6\ \ -4]^T$

a- Using steepest descent method, find the value of x^1. Can we say anything about the new point.

(a) Ans: $\cancel{f(x) = x_1^2 - 8x_1 + 16 + x_2^2 + 4x_2 + 4}$

$$x^1 = x^0 - \alpha \nabla f(x^0)$$
$$\alpha = \frac{\|\nabla f(x^0)\|^2}{\|\nabla f(x^0)\|_Q^2}$$

$$\nabla f(x) = \begin{array}{c|c} x_1 & \multirow{2}{*}{} \\ x_2 & \end{array} \begin{bmatrix} 2(x_1-4) - 1.75(x_2-3) \\ 2(x_2+2) - 1.75(x_1+4) \end{bmatrix}$$

$$\nabla f(x^0) = \begin{bmatrix} 4 + 12.25 \\ -4 - 17.5 \end{bmatrix} = \begin{bmatrix} 16.25 \\ -21.5 \end{bmatrix}$$

$$\alpha = \frac{[16.25\ \ -21.5] \begin{bmatrix} 16.25 \\ -21.5 \end{bmatrix}}{[16.25\ \ -21.5] \begin{bmatrix} 2 & -1.75 \\ -1.75 & 2 \end{bmatrix} \begin{bmatrix} 16.25 \\ -21.5 \end{bmatrix}}$$

$$\alpha = \frac{726.3125}{[70.125\ \ -71.4375] \begin{bmatrix} 16.25 \\ -21.5 \end{bmatrix}} = \frac{726.3125}{2675.4375} \approx 0.2715$$

$$x^1 = \begin{bmatrix} 6 \\ -4 \end{bmatrix} - 0.2715 \begin{bmatrix} 16.25 \\ -21.5 \end{bmatrix} = \begin{bmatrix} 6 - 4.412 \\ -4 + 5.837 \end{bmatrix} = \begin{bmatrix} 1.588 \\ 1.837 \end{bmatrix}$$

we can see that x_1 is getting far from the minimizer and x_2 is getting closer in this iteration.

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