

ENGINEERING CALCULUS III (GEG 217): [ASSIGNMENT 2]

November 28, 2023

ASSIGNMENT 2 PROBLEMS

1. If $|z + 1| = z + 2(1 + i)$, then find z .
2. If $\arg(z-1) = \arg(z + 3i)$, then find $x-1 : y$. where $z = x + iy$
3. Show that $\left| \frac{z-2}{z-3} \right| = 2$ represents a circle. Find its centre and radius.
4. If $\frac{z-1}{z+1}$ is a purely imaginary number ($z \neq -1$), then find the value of $|z|$.
5. z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then show that $z_1 = -\overline{z_2}$.
6. If $|z_1| = 1$ ($z \neq -1$) and $z_2 = \frac{z_1 - 1}{z_1 + 1}$, then show that the real part of z_2 is zero.
7. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then find $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$.
8. If $|z_1| = |z_2| = \dots = |z_n| = 1$, then show that $|z_1 + z_2 + z_3 + \dots + z_n| = \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots + \frac{1}{z_n} \right|$.
9. If for complex numbers z_1 and z_2 , $\arg(z_1) - \arg(z_2) = 0$, then show that $|z_1 - z_2| = |z_1| - |z_2|$.
10. Solve the system of equations $\operatorname{Re}(z^2) = 0, |z| = 2$.
11. Find the complex number satisfying the equation $z + \sqrt{2}|(z + 1)| + i = 0$.
12. Write the complex number $z = \frac{1-i}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$

13. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then show that $\bar{z}w = -i$.
14. Use de Moivre's theorem to prove the trig identities:
(a) $\sin 2\theta = 2\sin\theta\cos\theta$ (b) $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$
15. If $z = \cos\theta + i\sin\theta$, then use de Moivre's theorem to show that:
(a) $z + \frac{1}{z} = 2\cos\theta$ (b) $z^2 + \frac{1}{z^2} = 2\cos 2\theta$ (c) $z^n + \frac{1}{z^n} = 2\cos n\theta$.