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Exercise 0.1

$$\begin{bmatrix} 3 & 2 & 4 \\ 1 & 5 & 3 \\ -1 & 8 & 2 \end{bmatrix} = A$$

for a singular matrix $|A| = 0$

$$\begin{vmatrix} 3 & 2 & 4 \\ 1 & 5 & 3 \\ -1 & 8 & 2 \end{vmatrix}$$

$$= 3\begin{vmatrix} 5 & 3 \\ 8 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} + 4\begin{vmatrix} 1 & 5 \\ -1 & 8 \end{vmatrix}$$

$= 3(-14) - 2(5) + 4(13)$

$= 0$, it is therefore a singular Matrix.

Exercise 0.2

$$\begin{bmatrix} 1 & 4 & 3 \\ 6 & 2 & 5 \\ 1 & 7 & 0 \end{bmatrix}$$

i $|A|$

$$= \begin{vmatrix} 1 & 4 & 3 \\ 6 & 2 & 5 \\ 1 & 7 & 0 \end{vmatrix}$$

$$1\begin{vmatrix} 4 & 3 \\ 2 & 5 \end{vmatrix} - 7\begin{vmatrix} 1 & 3 \\ 6 & 5 \end{vmatrix} + 0\begin{vmatrix} 1 & 4 \\ 6 & 2 \end{vmatrix}$$

$= 14 - 7(-13) + 0$

$= 105$

ii Adj A

Adj $A = C_T$ where C = cofactor

Minors.

$$M_{11} = \begin{vmatrix} 2 & 5 \\ 7 & 0 \end{vmatrix} = -35$$

$$M_{12} = \begin{vmatrix} 6 & 5 \\ 1 & 0 \end{vmatrix} = -5$$

$$M_{13} = \begin{vmatrix} 6 & 2 \\ 1 & 7 \end{vmatrix} = 40$$

$$M_{21} = \begin{vmatrix} 4 & 3 \\ 7 & 0 \end{vmatrix} = -21$$

$$M_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} = -3$$

$$M_{23} = \begin{vmatrix} 1 & 4 \\ 1 & 7 \end{vmatrix} = 3$$

$$M_{31} = \begin{vmatrix} 4 & 3 \\ 2 & 5 \end{vmatrix} = 14 \quad M_{32} = \begin{vmatrix} 1 & 3 \\ 6 & 5 \end{vmatrix} = -13$$

$$M_{33} = \begin{vmatrix} 1 & 4 \\ 6 & 2 \end{vmatrix} = -22$$

$$M = \begin{vmatrix} -35 & -5 & +40 \\ -21 & -3 & 3 \\ 14 & -13 & -22 \end{vmatrix}$$

Cofactor $= (-1)^{i+j}$ Minor.

$$= \begin{vmatrix} -35 & 5 & 40 \\ 21 & -3 & -3 \\ 14 & 13 & -22 \end{vmatrix}$$

$$\text{Adj} A = C_T = \begin{vmatrix} -35 & 21 & 14 \\ 5 & -3 & 13 \\ 40 & -3 & -22 \end{vmatrix}$$

Exercise 0.3

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 5 & 1 \\ 2 & 0 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}. A$$

$$|A| = \begin{vmatrix} 2 & 1 & 4 \\ 3 & 5 & 1 \\ 2 & 0 & 6 \end{vmatrix}$$

$$2\begin{vmatrix} 1 & 4 \\ 5 & 1 \end{vmatrix} - 0\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} + 6\begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix}$$

$-38 + 0 + 42 = 4.$

Minors:

$$M_{11} = \begin{vmatrix} 5 & 1 \\ 0 & 6 \end{vmatrix} = 30$$

$$M_{12} = \begin{vmatrix} 3 & 1 \\ 2 & 6 \end{vmatrix} = 16$$

$M_{13} = |3 \quad 5| = -10$

$M_{21} = \begin{vmatrix} 1 & 4 \\ 0 & 6 \end{vmatrix} = 6 \quad M_{23} = \begin{vmatrix} 2 & 1 \\ 2 & 0 \end{vmatrix} = -2$

$M_{22} = \begin{vmatrix} 2 & 4 \\ 2 & 6 \end{vmatrix} = 4$

$M_{31} = \begin{vmatrix} 1 & 4 \\ 5 & 1 \end{vmatrix} = -19 \quad M_{33} = \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix} = 7$

$M_{32} = \begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} = -10$

$\text{Minor} = M = \begin{vmatrix} 30 & 16 & -10 \\ 6 & 4 & -2 \\ -19 & -10 & 7 \end{vmatrix}$

cofactor $= (-1)^{i+j}$ Minor.

$C = \begin{vmatrix} 30 & -16 & -10 \\ -6 & 4 & 2 \\ -19 & 10 & 7 \end{vmatrix}$

$\text{Adj}\, A = \begin{vmatrix} 30 & -6 & -19 \\ -16 & 4 & 10 \\ -10 & 2 & 7 \end{vmatrix}$

$A^{-1} = \frac{1}{4} \begin{vmatrix} 30 & -6 & -19 \\ -16 & 4 & 10 \\ -10 & 2 & 7 \end{vmatrix}$

Exercise 0.4.

$A = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

i B^{-1}

$B^{-1} = \frac{1}{|B|} \text{Adj}. B.$

$|B| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3-4 = -1$

$B^{-1} = -1 \begin{vmatrix} 3 & -2 \\ -2 & 1 \end{vmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

ii AB

$\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1+1 & 2+1.5 \\ 0.5+2 & 1+3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3.5 \\ 2.5 & 4 \end{bmatrix}$

iii $B^{-1}A$.

$\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0.5 \\ 0.5 & 3 \end{bmatrix} = \begin{bmatrix} -3+1 & -1.5+6 \\ 2-0.5 & 1-3 \end{bmatrix}$

$= \begin{bmatrix} -2 & 4.5 \\ 1.5 & -2 \end{bmatrix}$

Exercise 0.5

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$

i $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{vmatrix}$

$= 1\begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ 1 & 7 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix}$

$= 1 - 20 + 21 = 2$

ii Adj A

Minors.

$M_{11} = \begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix} = 1 \quad M_{12} = \begin{vmatrix} 2 & 4 \\ 1 & 7 \end{vmatrix} = 10$

$M_{13} = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = 7 \quad M_{21} = \begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix} = -1$

$M_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 7 \end{vmatrix} = 4 \quad M_{23} = \begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} = 3$

$M_{31} = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1 \quad M_{32} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = -2$

$M_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$

$\text{Minor} = \begin{vmatrix} 1 & 10 & 7 \\ -1 & 4 & 3 \\ -1 & -2 & -1 \end{vmatrix}$

$\text{Cofactor} = \begin{vmatrix} 1 & -10 & 7 \\ +1 & 4 & -3 \\ -1 & 2 & -1 \end{vmatrix}$

$\text{Adj}\, A = \begin{vmatrix} 1 & 1 & -1 \\ -10 & 4 & 2 \\ 7 & -3 & -1 \end{vmatrix}$

$A^{-1} = \frac{1}{|A|} . \text{Adj}\, A$

$= \frac{1}{2} \begin{vmatrix} 1 & 1 & -1 \\ -10 & 4 & 2 \\ 7 & -3 & -1 \end{vmatrix}$

iii) Verify $A(Adj\,A)\,I = |A|\,I$.

$$\begin{bmatrix}1&2&3\\2&3&4\\1&5&7\end{bmatrix}\begin{bmatrix}1&1&-1\\-10&4&2\\7&-3&-1\end{bmatrix}\cdot I$$

$$= \begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix} \therefore LHS.$$

RHS.

$|A| = 2 \quad I = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$|A|\,I = \begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$

LHS = RHS.

iv) A^{-1}

$A^{-1} = \frac{1}{|A|} Adj\,A$

$= \frac{1}{2}\begin{bmatrix}1&1&-1\\-10&4&2\\7&-3&-1\end{bmatrix}$

Exercise 0.6

$A = \begin{bmatrix}x^2&3\\1&3x\end{bmatrix} \quad B = \begin{bmatrix}3&6\\2&x\end{bmatrix}$

$AB = \begin{bmatrix}x^2&3\\1&3x\end{bmatrix}\begin{bmatrix}3&6\\2&x\end{bmatrix}$

$= \begin{bmatrix}3x^2+6&6x^2+3x\\3+6x&6+3x^2\end{bmatrix}$

$BA = \begin{bmatrix}3&6\\2&x\end{bmatrix}\begin{bmatrix}x^2&3\\1&3x\end{bmatrix}$

$= \begin{bmatrix}3x^2+6&9+18x\\2x^2+x&6+3x^2\end{bmatrix}$

comparing

$6x^2+3x = 9+18x$

$6x^2-18x+3x-9=0$

$6x^2-15x-9=0$

$2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$x = -\frac{1}{2}$ or 3.

to check: $2x^2+x = 3+6x$

$2(3)^2+3 = 3+6(3) \quad 21=21$

$x = -\frac{1}{2}$ or 3

Exercise 0.7

$A^2 = mA + nA$.

$m\begin{bmatrix}3&2\\-4&1\end{bmatrix} + n\begin{bmatrix}3&2\\-4&1\end{bmatrix} = \begin{bmatrix}1&8\\-16&-7\end{bmatrix}$

$3m+3n=1 \cdots (i) \quad 3m+2n=8 \cdots (ii)$

$-4m-4n=-16 \cdots (iii) \quad n+m=-7 \cdots (iv)$

No solution to m, and n.

Exercise 0.8

a) $(I-A)^{-1} = I+A+A^2+A^3$ if $A^4=0$.

$\therefore AA^{-1}=I \quad AI=A$.

$(I-A)(I-A)^{-1} = (I-A)(I+A+A^2+A^3)$

$I = I+AI+A^2I+A^3I-A-A^2-A^3-A^4$

$I = I+A+A^2+A^3-A-A^2-A^3-A^4$

$I = I-A^4 \quad A^4=0$.

$I = I$

$\therefore (I-A)^{-1} = I+A+A^2+A^3$ if $A^4=0$.

b) $(I-A)^{-1} = I+A+A^2+A^3+\dots+A^n$ if $A^{n+1}=0$

$AA^{-1}=I \quad AI=A$

$(I-A)(I-A)^{-1} = (I-A)(I+A+A^2+A^3+\dots A^n)$

$I = I+AI+A^2I+A^3I+\dots+A^nI-A-A^2$
$-A^3-A^4-\dots-A^{n+1}$

$I = I+A+A^2+A^3+\dots+A^n-A-A^2-A^3-A^4$
$-\dots-A^{n+1}$

$I = I-A^{n+1} \quad A^{n+1}=0$

$I = I$

$\therefore (I-A)^{-1} = I+A+A^2+A^3+\dots+A^n$ if $A^{n+1}=$

Exercise 0.9

$A=\begin{bmatrix}1 & 4\\ -2 & 3\\ 1 & -2\end{bmatrix}$ $B=\begin{bmatrix}2 & 0 & 0\\ 0 & 1 & -1\end{bmatrix}$ $C=\begin{bmatrix}8 & 6 & -6\\ 6 & -1 & 1\\ -4 & 0 & 0\end{bmatrix}$

$AKB=C$

Since $A=3\times2$ $B=2\times3$

K has to be a 2×2 for the matrix multiplication to be possible.

Set $K=\begin{bmatrix}m & n\\ p & q\end{bmatrix}$

$AKB=A(KB)=(AK)B$ Associative.

$AK=\begin{bmatrix}1 & 4\\ -2 & 3\\ 1 & -2\end{bmatrix}\begin{bmatrix}m & n\\ p & q\end{bmatrix}$

$=\begin{bmatrix}m+4p & n+4q\\ -m+3p & -2n+3q\\ m-2p & n-2q\end{bmatrix}$

$(AK)B=\begin{bmatrix}m+4p & n+4q\\ -m+3p & -2n+3q\\ m-2p & n-2q\end{bmatrix}\begin{bmatrix}2 & 0 & 0\\ 0 & 1 & -1\end{bmatrix}$

$=\begin{bmatrix}2(m+4p) & n+4q & -(n+4q)\\ 2(-m+3p) & -2n+3q & 2n-3q\\ 2(m-2p) & n-2q & -n+2q\end{bmatrix}$

Equating $AKB=C$:

$2(m+4p)=8$ ...... (i)

$n+4q=6$ ...... (ii)

$2(-m+3p)=6$ ...... (iii)

$-2n+3q=-1$ ...... (iv)

$n-2q=0$ ...... (v)

from eqn (v)

$n=2q$ Put this value in eqn (iv)

$-2(2q)+3q=-1$

$-4q+3q=-1$

$-q=-1$

$q=1$ $n=2q=2(1)=2$

eqn (1) + eqn (3)

$8p+6p=8+6$

$14p=14$ $p=1$

from eqn (1) put $p=1$

$2(m+4(1))=8$

$2m+8=8$ $2m=0$ $m=0$.

$K=\begin{bmatrix}m & n\\ p & q\end{bmatrix}=\begin{bmatrix}0 & 2\\ 1 & 1\end{bmatrix}$

Exercise 10.

$x_1 \neq x_2$

(x_2, x_1)

$L\begin{pmatrix}u_1\\ u_2\end{pmatrix}=A\begin{pmatrix}u_1\\ u_2\end{pmatrix}$

Reflection on line (x_2, x_1)

$A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

reflection

$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\begin{bmatrix}u_2\\ u_1\end{bmatrix}$

shifting 4 unit to the left.

$=\begin{bmatrix}u_2-4\\ u_1\end{bmatrix}$

Exercise 0.11

$L\begin{pmatrix}1\\ 1\end{pmatrix}=A\begin{pmatrix}1\\ 1\end{pmatrix}$

clock wise rotation of $180°$

$A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ $\theta=180°$

$A=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

rotation

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Shifting 3 unit to the right.

$$= \begin{bmatrix} -1+3 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Exercise 0.12

reflection about the x-axis

(u_1, u_2)

$$L\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = A\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

reflection with respect to x-axis.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

reflection. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ -u_2 \end{bmatrix}$

shift 2 unit to the right and 2 unit up.

$$= \begin{bmatrix} u_1 + 2 \\ -u_1 + 2 \end{bmatrix}$$

Exercise 0.13.

$$\begin{bmatrix} a & -2 \\ b & 7 \end{bmatrix} = \begin{bmatrix} 2 & c \\ 3 & 2c+d \end{bmatrix}$$

comparing/equating.

$a = 2$ $c = -2$ $b = 3$

$2c + d = 7$ $(c = -2)$

$2(-2) + d = 7$

$-4 + d = 7$ $d = 11$

$a = 2, b = 3, c = -2, d = 11$

Exercise 0.14.

$$\begin{bmatrix} 2x+3y & a-2b \\ 2a+b & 3x-2y \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 6 & 8 \end{bmatrix}$$

solution.

$2x + 3y = 3$ ... (i)

$a - 2b = 8$ ... (ii)

$2a + b = 6$ ... (iii)

$3x - 2y = 8$ ... (iv)

eqn (i) and eqn (iv)

$2x + 3y = 3$ ... (i) $\times 3$

$3x - 2y = 8$ ... (iv) $\times 2$

$6x + 9y = 9$

$6x - 4y = 16$

$13y = -7$ $y = -7/13$

$x = 30/13$

eqn (ii) and eqn (iii).

$a - 2b = 8$ ... (ii)

$2a + b = 6$ ... (iii)

$a = 4, b = -2$

$a = 4, b = -2, x = \frac{30}{13}$ $y = -\frac{7}{13}$

Exercise 0.15

$$2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

solution.

$2x + 1 = 5$ $8 + y = 0$

$2x = 4$ $y = -8$

$x = 2, y = -8$

Exercise 0.16

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$$

$$2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$2M = \begin{bmatrix} 8 & 2 \\ +2 & -12 \end{bmatrix}$$

$$M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$$

Exercise

Exercise 0·18

$A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$A + X = 2B + C$

$X = 2B + C - A$

$= 2\begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} -7 & 5 \\ 6 & 2 \end{bmatrix}$

Exercise 0·19.

$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$

$\begin{bmatrix} 6x - 2 \\ -2x + 4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

Equating.

$6x - 2 - 8 = 8$ ---- (i)

$-2x + 4 + 10 = 4y$ ---- (ii)

from eqn (i)

$6x - 10 = 8$

$6x = 18$

$x = 3$

put $x = 3$ in eqn (ii)

$-6 + 4 + 10 = 4y$

$8 = 4y$ $y = 2$.

$x = 3, y = 2$

Exercise 0·20.

$A\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ 14 & 22 \end{bmatrix}$

Note: $AB = C$

$A = CB^{-1}$

let $B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$ $B^{-1} = \frac{1}{|B|} \text{Adj}\cdot B$.

$|B| = 10$ $\text{Adj}\,B = \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix}$

$B^{-1} = \frac{1}{10}\begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix}$

$A = \frac{1}{10}\begin{bmatrix} 9 & 2 \\ 14 & 22 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix}$

$A = \frac{1}{10}\begin{bmatrix} 20 & -30 \\ 50 & 10 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$

Exercise 0·21

$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} x & 1 \\ 4 & -1 \end{bmatrix}$

$(A+B)^2 = A^2 + B^2$

$\cancel{A^2 + B^2} + 2AB = \cancel{A^2 + B^2}$

$2AB = 0$

wher 0 is a zero matrix.

$A + B = \begin{bmatrix} 1+x & 0 \\ 6 & -2 \end{bmatrix}$

$(A+B)^2 = (A+B)(A+B)$

$= \begin{bmatrix} 1+x & 0 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} 1+x & 0 \\ 6 & -2 \end{bmatrix}$

$= \begin{bmatrix} (1+x)^2 & 0 \\ 6(1+x) - 12 & 4 \end{bmatrix}$ LHS

A^2+B^2

$A^2=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$

$=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

$B^2=\begin{bmatrix}x & 1\\ 4 & -1\end{bmatrix}\cdot\begin{bmatrix}x & 1\\ 4 & -1\end{bmatrix}$

$=\begin{bmatrix}x^2+4 & x-1\\ 4x-4 & 5\end{bmatrix}$

$A^2+B^2=\begin{bmatrix}x^2+3 & x-1\\ 4x-4 & 4\end{bmatrix}$ RHS.

LHS = RHS.

$\begin{bmatrix}(1+x)^2 & 0\\ 6x-6 & 4\end{bmatrix}=\begin{bmatrix}x^2+3 & x-1\\ 4x-4 & 4\end{bmatrix}$

comparing/equating

$x-1=0 \quad x=1$

Exercise 0.22

Exercise 0.23.

i $\begin{bmatrix}1 & 2 & 1\\ 3 & 1 & -2\\ 0 & 1 & -1\end{bmatrix}$

for a singular Matrix Det. A = 0

$\begin{vmatrix}1 & 2 & 1\\ 3 & 1 & -2\\ 0 & 1 & -1\end{vmatrix}$.

$=0\begin{vmatrix}2 & 1\\ 1 & -2\end{vmatrix}-1\begin{vmatrix}1 & 1\\ 3 & -2\end{vmatrix}-1\begin{vmatrix}1 & 2\\ 3 & 1\end{vmatrix}$

$=5+5=10$ Non-Singular

ii) $\begin{vmatrix}1 & 2 & 1\\ -3 & 4 & 5\\ -4 & 2 & 6\end{vmatrix}$

$1(14)-2(2)+1(10)$

$=20$ Non-singular.

iii $\begin{vmatrix}1 & 1 & -2\\ 3 & -1 & 1\\ 3 & 3 & -6\end{vmatrix}$

$1(3)-1(-21)-2(12)$

$=0$ Singular.

Exercise 0.24

i $\begin{bmatrix}k & 6\\ 4 & 3\end{bmatrix}$ Singular. Det. = 0.

$\begin{vmatrix}k & 6\\ 4 & 3\end{vmatrix}=0$

$3k-24=0$

$3k=24 \quad k=8$

ii $\begin{vmatrix}1 & 2 & -1\\ -3 & 4 & k\\ -4 & 2 & 6\end{vmatrix}=0$.

$+3\begin{vmatrix}2 & -1\\ 2 & 6\end{vmatrix}+4\begin{vmatrix}1 & -1\\ -4 & 6\end{vmatrix}-k\begin{vmatrix}1 & 2\\ -4 & 2\end{vmatrix}=0$

$42+8-10k=0$

$50 - 10k = 0$

$10k = 50$

$k = 5$

iii $\begin{vmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ k & 3 & -6 \end{vmatrix} = 0$

$k\begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - 6\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = 0$

$k(-1) - 3(7) - 6(-4) = 0$

$-k - 21 + 24 = 0$

$-k = -3$

$k = 3$

Exercise 0.24

i. $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A$

$|A| = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -7$

$\text{Adj } A = \begin{vmatrix} -1 & -3 \\ -2 & 1 \end{vmatrix}$

$A^{-1} = \frac{1}{-7}\begin{vmatrix} -1 & -3 \\ -2 & 1 \end{vmatrix}$

it does exist.

ii $\begin{vmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{vmatrix}$

$2\begin{vmatrix} 1 & 3 \\ -1 & -2 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ -1 & -2 \end{vmatrix}$

$2(1) - 3(1) = -1$

$\text{Adj } A = C_T$

Minors:

$M = \begin{vmatrix} 0 & 1 & 1 \\ -2 & -3 & -2 \\ 3 & 3 & 2 \end{vmatrix}$

$C = \begin{vmatrix} 0 & -1 & 1 \\ 2 & -3 & 2 \\ 3 & -3 & 2 \end{vmatrix}$

$\text{Adj } A = \begin{vmatrix} 0 & 2 & 3 \\ -1 & -3 & -3 \\ 1 & 2 & 2 \end{vmatrix}$

$A^{-1} = \begin{vmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{vmatrix}$

it exist.

iii $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{vmatrix}$

$0\begin{vmatrix} 2 & 3 \\ 0 & 4 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ -\cancel{0} & 4 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix}$

$-14 + 4 = -10$

Adj A

$M = \begin{vmatrix} -8 & -2 & -2 \\ -2 & 2 & 2 \\ 8 & 7 & 2 \end{vmatrix}$

$C = \begin{vmatrix} -8 & 2 & -2 \\ 2 & 2 & -2 \\ 8 & -7 & 2 \end{vmatrix}$

$\text{Adj } A = C_T$

$\begin{vmatrix} -8 & 2 & 8 \\ 2 & 2 & -7 \\ -2 & -2 & 2 \end{vmatrix}$

$A^{-1} = \frac{-1}{10}\begin{vmatrix} -8 & 2 & 8 \\ 2 & 2 & -7 \\ -2 & -2 & 2 \end{vmatrix}$

iv $\begin{vmatrix} 1 & 2 & -1 \\ -3 & 4 & 5 \\ -4 & 2 & 6 \end{vmatrix}$

$= 1(14) - 2(2) - 1(10)$

$= 0$

this is a singular matrix it does not have an inverse, so inverse does not exist.