REVIEWING MECHANICAL CONCEPTS OF ENERGY AND WORK

MEG 212 Week 3 Lecture

Lecture Learning Outcomes

At the end of this lecture, you will be able to,

- Discuss the relationship between work, kinetic energy and potential energy
- Identify the difference between the concept of conservation of energy in mechanics and thermodynamics
- Describe the thermodynamic definition of work

Lecture Learning Outcomes

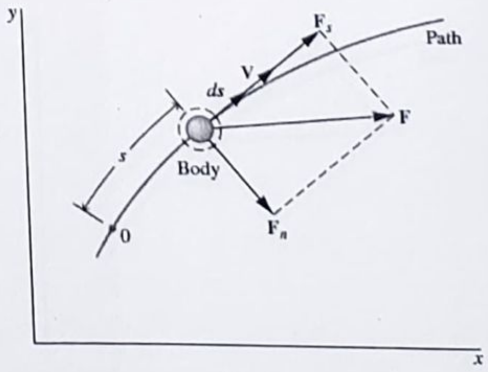
At the end of this lecture, you will be able to,

- Identify the sign convention and notation of work in thermodynamic analysis
- Model expansion and compression work
- Differentiate between actual processes and the idealized quasi-equilibrium process

Work and Kinetic Energy

- In the figure, the curved line is the path of a moving closed system
- The velocity of the center of mass is V
- The body is acted on by a resultant force F
- The effect of component F_s of the resultant force is to change the magnitude of the velocity while F_n is to change the direction of the velocity
- By Newton's second law of motion

$$F_s = m\frac{dV}{dt} = m\frac{dV}{ds}\frac{ds}{dt} = mV\frac{dV}{ds}$$



Forces acting on a moving system

Work and Kinetic Energy

- Integrating gives,

$$\int_{V_1}^{V_2} mVdV = \int_{S_1}^{S_2} F_s\, ds$$

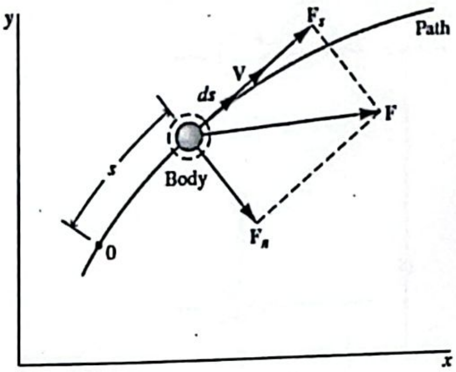
- For the LHS

$$\int_{V_1}^{V_2} mVdV = \frac{1}{2}m\left(V_2{}^2 - V_1{}^2\right)$$

Which gives the change in kinetic energy of the body

- For the RHS, we obtain the work of the resultant force on the body

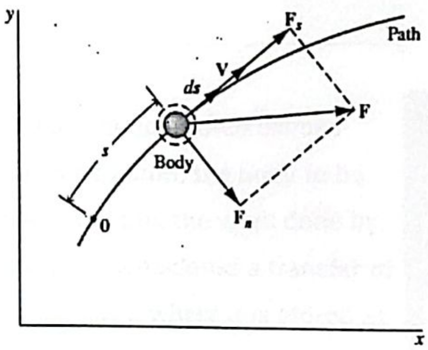
$$\int_{S_1}^{S_2} F_s\, ds = \int_{S_1}^{S_2} F.ds$$



Forces acting on a moving system

Work and Kinetic Energy

- Putting the two expressions on both sides together, the equation states that the work of the resultant force on the body equals the change in its kinetic energy
- The work done on the body can be considered a transfer of energy to the body where it is stored as kinetic energy
- Since kinetic energy is associated with the body as a whole, it is an extensive property



Forces acting on a moving system

Potential Energy

- The figure shows a body of mass m that moves vertically from elevation z_1 to z_2 relative to the earth's surface

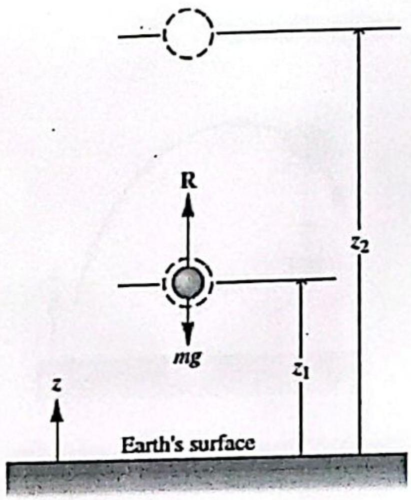
- In accordance to the previous equation, total work equals change in kinetic energy. Therefore,

$$\frac{1}{2}m({V_2}^2 - {V_1}^2) = \int_{z_1}^{z_2} R\,dz - \int_{z_1}^{z_2} mg\,dz$$

- Rewriting,

$$\frac{1}{2}m({V_2}^2 - {V_1}^2) + mg(z_2 - z_1) = \int_{z_1}^{z_2} R\,dz$$

Where $mg(z_2 - z_1)$ is potential energy associated with the force of gravity



Conservation of Energy in Mechanics

The previous equation states that the total work of all forces acting on the body from the surroundings, with the exception of gravitational force, equals the sum of the changes in the kinetic and potential energies of the body.

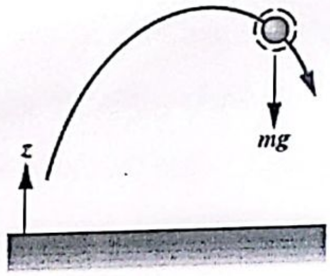
When the resultant force causes increase in elevation, the body to be accelerated, or both, the work done by the force can be considered a transfer of energy to the body, where it is stored as gravitational potential energy and/or kinetic energy

Conservation of Energy in Mechanics

- For a case where the only force acting is that due to gravity, then

$$\frac{1}{2}m({V_2}^2 - {V_1}^2) + mg(z_2 - z_1) = 0$$

$$\frac{1}{2}m{V_2}^2 + mgz_2 = \frac{1}{2}m{V_1}^2 + mgz_1$$



- Under these conditions, the sum of the kinetic and gravitational potential energies remain constant.
- The equation above also shows that energy can be converted from one form to another

Conservation of Energy in Mechanics

In the cases presented so far, discussion has been centered on systems on which applied forces affect only their overall velocity and position

Engineering systems interact with their surroundings in more complicated ways in which changes in other properties also occur

The concept of K.E and P.E alone is not sufficient to analyze such systems.

Also, the basic conservation of energy principle introduced does not suffice

Conservation of Energy in Thermodynamics

In thermodynamics, the concept of energy is broadened to account for other observable changes

The principle of conservation of energy is extended to include a wide variety of ways in which systems interact with their surroundings

Thermodynamic Definition of Work

Work done by, or on a system, obtained in terms of macroscopically observable forces and displacements is given by

$$W = \int_{s_1}^{s_2} F.ds$$

Thermodynamic definition of work says, Work is done by a system on its surrounding if the sole effect on everything external to the system could have been the raising of a weight

Thermodynamic Definition of Work

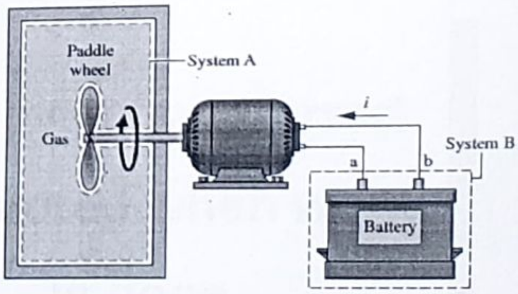
Note that the raising of a weight is in effect, a force acting through a distance

We see that the concept of work in thermodynamics is a natural extension of the concept of work in mechanics

This does not mean that the elevation of weight had taken place or that a force has actually acted through a distance BUT THAT THE SOLE EFFECT COULD HAVE BEEN AN INCREASE IN ELEVATION OF A WEIGHT

Thermodynamic Definition of Work

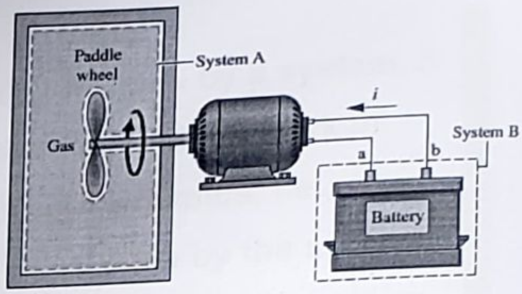
- In system A, a gas is stirred by a paddle wheel. Therefore the paddle wheel does work on the gas
- For system A, work could be evaluated in terms of forces and motions at the boundary between the paddle wheel and the gas.



Two examples of Work

Thermodynamic Definition of Work

- In system B, which includes only the battery, forces and motions are not evident.
- Rather there is an electric current i driven by an electrical potential difference existing across terminals a and b.
- This type of interaction, from the thermodynamic view can be classified as work: We can imagine the current is supplied to a hypothetical electric motor that lifts a weight in the surrounding



Two examples of Work

Thermodynamic Definition of Work

In Thermodynamics, Work is a means for transferring energy.

It does not refer to what is being transferred between systems or to what is being stored within systems

Energy is transferred and stored when work is done

Sign Convention and Notation

In Engineering Thermodynamics, we are frequently concerned with devices whose purpose is to do work, e.g. internal combustion engines, turbines etc.

When work is done by a system, it is taken as positive work in thermodynamics, i.e.,

$W > 0$: Work done by the system

$W < 0$: Work done on the system

Sign Convention and Notation

The value of W depends on the details of interactions taking place between the system and surroundings during a process not just on the initial and final states.

Therefore, work is not a property of the system or surroundings

$$\int_1^2 \delta W = W$$

The differential of work, δW, is said to be inexact the integral cannot be evaluated without specifying details of the process

Sign Convention and Notation

But the differential of a property of a system is exact because the change in property between two particular states does not depend on the details of the process linking the two states

$$\int_{V_1}^{V_2} dV = V_2 - V_1$$

For example, change in volume between two states is an exact differential because we are not concerned with the details of the process

Sign Convention and Notation

Many thermodynamic systems are also concerned with the time rate at which energy transfer occurs

The rate of energy transfer by work is called power

$$\dot{W} = \boldsymbol{F}.\boldsymbol{V}$$

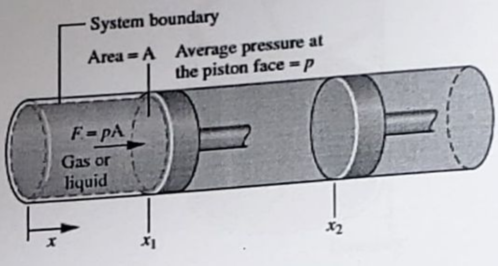
Power: The product of the force and the velocity at the point of application of force

$$W = \int_{t_1}^{t_2} \dot{W} dt = \int_{t_1}^{t_2} \boldsymbol{F}.\boldsymbol{V}\ dt$$

Modeling Expansion or Compression Work

- The figure shows a gas (or liquid) contained in a piston-cylinder assembly as the gas expands.
- During the process, the gas pressure exerts a normal force on the piston
- The force exerted by the gas on the piston is pA
- The work done by the system as the piston is displaced a distance dx is

$$\delta W = pAdx = pdV; W = \int_{V_1}^{V_2} p\, dV$$



Expansion or compression of a gas or liquid

Actual versus Quasi-equilibrium Processes

- There is no requirement that a system undergoing a process be in equilibrium during a process
- This means, some or all the intervening states may be non-equilibrium states
- In thermodynamic analysis, processes are sometimes modeled as an idealized type of process called a quasi-equilibrium or quasi-static process
- A quasi-equilibrium process is one in which the departure from thermodynamic equilibrium is at most infinitesimal

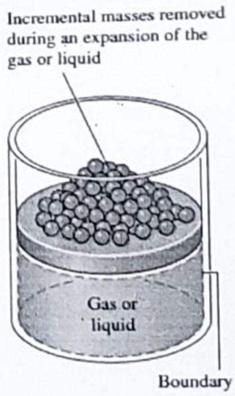


Illustration of a quasi-equilibrium expansion or compression

Actual versus Quasi-equilibrium Processes

- The figure shows a quasi-equilibrium process
- The system consisting of the gas is initially at an equilibrium state.
- The gas pressure is maintained uniform throughout by a small number of masses resting on the freely moving piston
- If one of the masses is removed, allowing the piston to move upward as the gas expands slightly, the state of the gas would depart only slightly from equilibrium

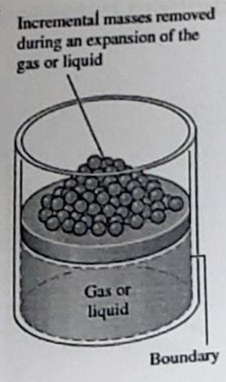


Illustration of a quasi-equilibrium expansion or compression

Actual versus Quasi-equilibrium Processes

- The system would eventually come to a new equilibrium state, where all the pressure and all other intensive properties would again be uniform in value
- If we replace the mass that was initially removed, the gas would be restored to its initial state, while again the departure from equilibrium would be very small
- A quasi-equilibrium compression process can be visualized in a similar way

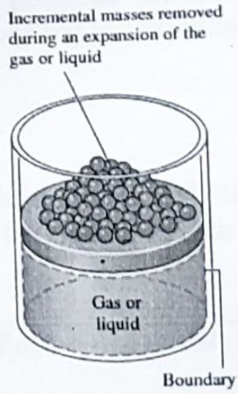
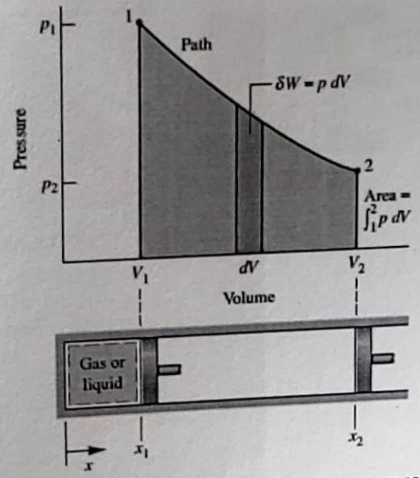


Illustration of a quasi-equilibrium expansion or compression

Actual versus Quasi-equilibrium Processes

- The relationship between the pressure and volume can be represented graphically



Work of a quasi-equilibrium expansion or compression process

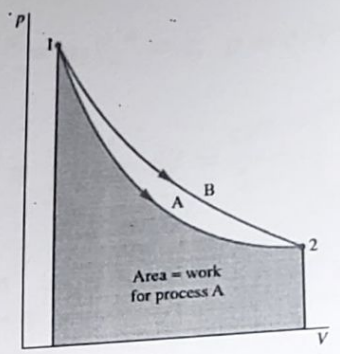


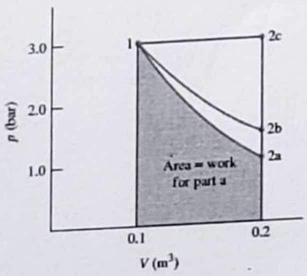
Figure showing that work depends on the process

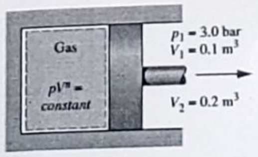
Worked Example

- A gas in a piston-cylinder assembly undergoes and expansion process for which the relationship between pressure and volume is given by a polytropic process

$$pV^n = constant$$

Where n is a constant. The initial pressure is 3 bar, the initial volume is 0.1 m³, and the final volume is 0.2 m³. Determine the work for the process in kJ for cases where (1) $n = 1.5$, (2) $n = 1.0$ and (3) $n = 0$

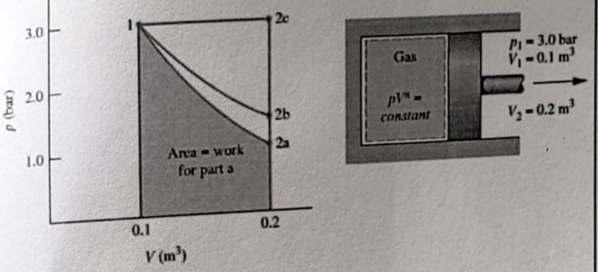




The Engineering Model

- ✓ The gas in the piston-cylinder assembly is a closed system
- ✓ The moving boundary is the only work mode
- ✓ The expansion is a polytropic process

Worked Example



Solution A

First, obtain the expression for work

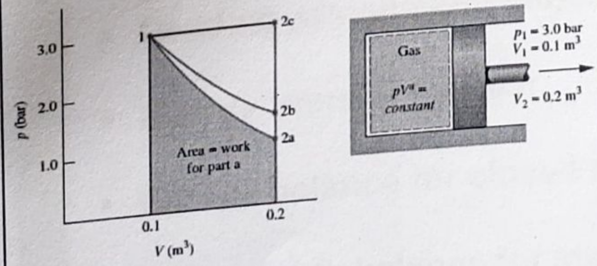
$$pV^n = c;\ p_1{V_1}^n = p_2{V_2}^n = c;\ p = c/V^n$$

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{c}{V^n}\,dV = \left.\frac{cV^{1-n}}{1-n}\right|_{V_1}^{V_2}$$

$$= \frac{c{V_2}^{1-n} - c{V_1}^{1-n}}{1-n} = \frac{p_2{V_2}^n{V_2}^{1-n} - p_1{V_1}^n{V_1}^{1-n}}{1-n}$$

$$= \frac{p_2V_2 - p_1V_1}{1-n}$$

Worked Example



Solution A

Next, you obtain the values of all the parameters need to evaluate work

$$p_1{V_1}^n = p_2{V_2}^n = c;\ p_2 = p_1\left(\frac{V_1}{V_2}\right)^n = 1.06\ bar$$

We can now determine the work for the process

$$W = \frac{p_2V_2 - p_1V_1}{1-n} = +17.6\ kJ$$

Class Exercise: Attempt to obtain the solutions for questions 2 and 3.

What trend do you observe with the different processes?

SUMMARY

1. Relationship between Work, K.E and P.E
2. Conservation of Energy in Mechanics
3. Conservation of Energy in Thermodynamics
4. Thermodynamic definition of Work
5. Sign Convention and Notation & Modeling Expansion or Compression Work
6. Actual versus Quasi-equilibrium process

Next Lecture

- Reading assignment: pages 56 to 75
- Energy transfer by Heat
- Energy balance for closed systems
- Energy rate balance for steady state operation
- Energy rate balance for transient state operation
- Energy balance for power, refrigeration and heat pump cycles