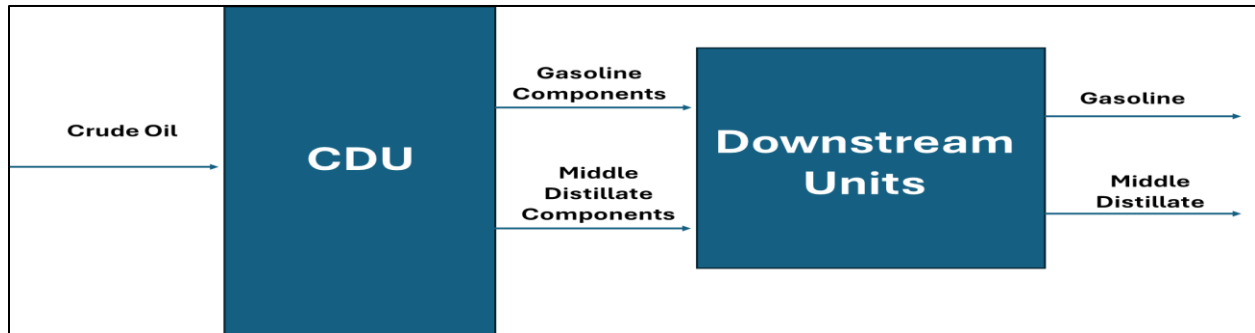


Q1. Refineries play a major role in the petroleum supply chain as it is the single processing point for Crude Oil to produce various petroleum products, mainly gasoline and middle distillate. Basically, the model shows how refinery is flexible to partially adjust its product yields (quota allocated for each product) by adjusting the operating philosophy depending on the capability of downstream units of Crude Distillation Unit (CDU). Thus, the objective of the model is to identify **how single refinery optimally set its Gasoline and Middle Distillate cuts to maximize the revenue generated, in which the prices of both products depend on Crude Prices, inventory levels, and demand seasonality coupled with market dynamic development.** The following figure simplifies the idea of refinery operations:



Model Construction:

Sets/ Indices:

- $i \in S$, and $S = [G, MD]$; G =Gasoline, MD =Middle Distillates (Diesel & Jet Fuel)

Parameters:

➤ Production:

Gasoline Production Rate: $S_G = S \cdot X$, Middle Distillate Production Rate: $S_{MD} = (1 - S) \cdot X$

Where:

- $X \geq 0$: Crude Throughput or Processing Capacity (Kbbl/day)
- S : Gasoline Cut Share

Variables:

➤ Quantity Demanded:

$$Q_G = \alpha_0 + \alpha_1 INV_G + \alpha_2 P_G + \alpha_3 EV + \alpha_4 W$$

$$Q_{MD} = \beta_0 + \beta_1 INV_{MD} + \beta_2 P_{MD} + \beta_3 (A\&A) + \beta_4 W$$

Where:

- Q_G and Q_{MD} are Gasoline and Middle Distillate quantity demanded respectively
- $\alpha_0, \alpha_1, \alpha_2, \alpha_3$, and α_4 are Gasoline demand coefficients
- $\beta_0, \beta_1, \beta_2, \beta_3$, and β_4 are Middle Distillate demand coefficients

- INV_G and INV_{MD} are Gasoline and Middle Distillate inventories, respectively.
- P_G and P_{MD} are Gasoline and Middle Distillate prices, respectively.
- EV is Electric vehicle adoption
- $(A\&A)$ is agriculture seasonality and aviation sector expansion
- W is Crude Price

➤ **Price (Inverse Demand):**

$$P_G = \frac{\alpha_0 + \alpha_1 INV_G + \alpha_3 EV + \alpha_4 W}{\alpha_2}, \quad P_{MD} = \frac{\beta_0 + \beta_1 INV_G + \beta_3 (A\&A) + \beta_4 W}{\beta_2}$$

➤ **Production Cost (C):**

$$K = S_G + S_{MD}, \quad C = C_0 + K W$$

Where:

C_0 is the processing cost, including administrative, power consumption, electricity generated, and labor (fixed cost), and W is Crude Oil Price (in which $K W$ is variable cost)

Demand elasticity gasoline and middle distillate can be expressed as:

$$\epsilon = \frac{P}{Q} \frac{dQ}{dP}, \text{ if } |\epsilon| > 1 \text{ demand is elastic and if } |\epsilon| < 1 \text{ demand is inelastic}$$

Objective Function:

$$\pi_{max} = P_G \cdot S_G + P_{MD} \cdot S_{MD} - C_0 - K W$$

Constraints:

$$X \geq K, \quad K = S_G + S_{MD}, \quad 0.3 \geq S \geq 0.7$$

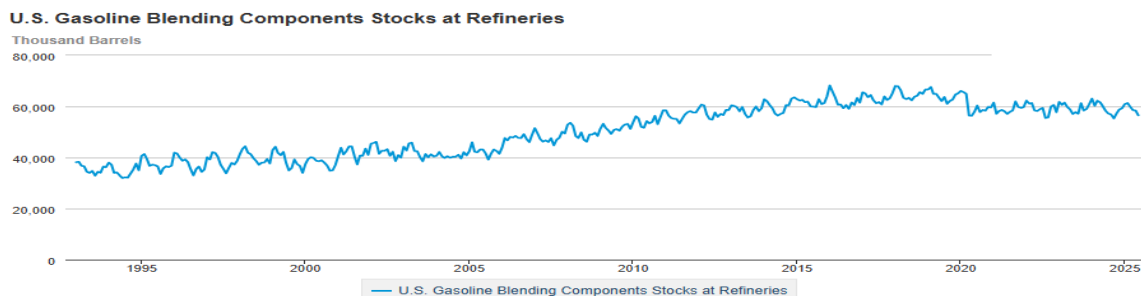
Generally, two constraints control the model. First, crude throughput (refinery processing capacity) shall be at least as high as total quantity produced of gasoline and middle distillates. Second, each product has a minimum cut percentage of 30% and maximum cut percentage of 70% of total crude oil processed. Such cut percentages are allocated by the model to maximize the profit while maintaining safe and smooth operations. Thorough refinery's Crude Distillation Unit (first unit in the refinery) optimization keeps the allocated cuts the most optimum, minimizing the giveaway, and eliminating the contaminations to produce on-specification products with higher recovery rate of Gasoline and Middle Distillate products. Simple code is attached with the homework to show how the refinery optimizes its process.

Simple code has been provided that shows how model allocates the highest possible cut for the highest priced product to maximize the profit. Later, we can expand the model to include elasticity, consumer and producer surplus, and other related optimization equations. Coefficients for G/MD inventory, Crude Price, G/MD demand can be estimated by running Ordinary Least Squares Regression.

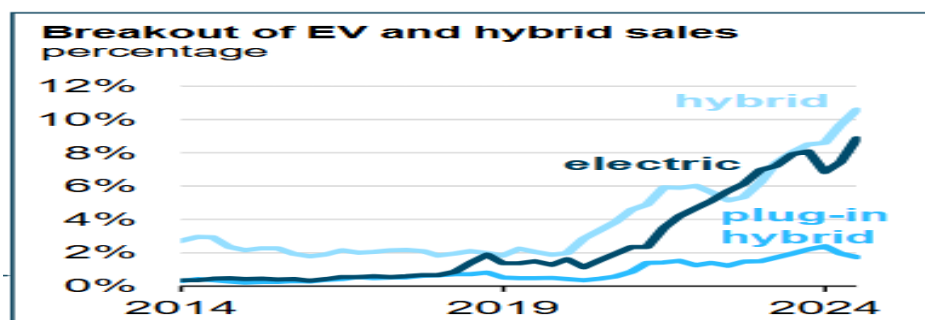
Q2. The model requires relevant data that captures the changes in crude oil, gasoline, and middle distillate prices as well as factors affecting the demand outlook. The following table shows each data source along with the proper utilization of it.

Source	Usage/Utilization
1- EIA Weekly Petroleum Status Report: Weekly Petroleum Status Report - U.S. Energy Information Administration (EIA)	Obtaining crude oil, gasoline, and middle distillate price since 1986.
2- EIA Weekly U.S. Ending Stocks of Petroleum Products: Weekly U.S. Ending Stocks of Total Gasoline (Thousand Barrels)	Obtaining gasoline and middle distillate weekly closing inventory.
3- USDA Crop Progress Report: USDA Economics, Statistics and Market Information System	Providing an insight into agriculture statistics that could be related to diesel demand.
4- FAA/BTS Aviation Data: Airlines, Airports, and Aviation Bureau of Transportation Statistics	Providing an insight into flight occupancy and passengers counts statistics that could be related to jet fuel demand.
5- U.S. share of electric and hybrid vehicle sales: U.S. share of electric and hybrid vehicle sales reached a record in the third quarter - U.S. Energy Information Administration (EIA)	Shows the trend of EV percentage among other vehicles that could lead to lower gasoline demand

U.S. gasoline components inventory is shown below for the period of 1990-2025:



The following graph shows EV sales percentage between 2014-2024 that indicates growth.



The philosophy of operating the refinery adopts the concept of making the CDU the highest processing capacity among other units and installing 2-3 crude distillation trains, in which the maintenance activity of those trains should not overlap to avoid single-point shutdown, maximize crude processing, and ensure sustainable production across the year where those factors contribute positively to the profit recognized. This concept is aligned with ABB Measurement & Analytics- Crude Distillation Unit Optimization Paper ([ABB-CDU-Unit-A4-US.indd](#))