Prof. Dr.-Ing. A. Bruhn
Institute for Visualization and Interactive Systems
Department Intelligent Systems
University of Stuttgart

Homework Assignment 2

H 2.1 (Cooccurrence Matrices) 8 Points

Compute the cooccurrence matrix of the 4×4 image

0	3	2	1
1	1	3	2
0	0	2	1
3	0	1	0

with $\mathbf{d} = (-1, -1)^\top$. Assume that the x-axis points to the right and the y-axis downwards.

a) Determine the highest probability as well as the contrast of the cooccurrence matrix.

b) Assume $\mathbf{d} = (1, 1)^\top$. Would this yield the same tuple with the highest probability as (a)? Would it yield the same contrast? You may either compute the solution or answer with logical arguments.

H 2.2 (Lucas and Kanade) 4 Points

Minimising the local energy that corresponds to the approach of Lucas and Kanade requires to solve the following linear system of equations:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} ,$$

with the abbreviations

$$\begin{aligned} a_{11} &= \int_{B_\rho} f_x^2 \, dxdy , & b_1 &= -\int_{B_\rho} f_x f_z \, dxdy , \\ a_{12} &= \int_{B_\rho} f_x f_y \, dxdy , & b_2 &= -\int_{B_\rho} f_y f_z \, dxdy , \\ a_{22} &= \int_{B_\rho} f_y^2 \, dxdy . \end{aligned}$$

Derive closed form solutions for the unknowns u and v, i.e. come up with formulae how to compute u and v from $a_{11}, a_{12}, a_{22}, b_1$ and b_2.

Hint: Use Cramer's rule: For a linear system $\mathbf{Ax} = \mathbf{b}$, the components of $\mathbf{x}$ are given by

$$x_i = \frac{\det(\mathbf{A}_{i \to \mathbf{b}})}{\det(\mathbf{A})} ,$$

where $\mathbf{A}_{i \to \mathbf{b}}$ is obtained by replacing column i in matrix $\mathbf{A}$ by vector $\mathbf{b}$.

P 2.3 (Lucas and Kanade)

Please download the required file `cv24_ex02.zip` from ILIAS and unpack the data.

The programme `lkTemplate.c` should be extended to the method of Lucas and Kanade. Starting the program with two frames of a sequence yield two output images: the magnitude of the optic flow and a flow classification. This classification distinguishes three cases:

no information (black) – only normal flow (grey) – full flow (white)

(a) In the method `create_eq_systems`, supplement the missing code such that it computes the entries of the linear system of equations solved in the Lucas/Kanade approach.

(b) The aim of the method `lucas_kanade` is to reuse the entries calculated before and to solve the linear system of equations. The method should distinguishe the three cases given above. The normal flow or the full flow should be calculated if possible, otherwise u and v should be set to zero. Supplement the missing code. You can use the result from Problem 2 to compute the full flow, if possible.

(c) Compile the program using `gcc -O3 -o lkTemplate lkTemplate.c -lm` and test the program with the image pairs `pig1,2.pgm` and `sphere1,2.pgm`. What is the influence of the integration scale ρ? You can use a threshold $\varepsilon = 0.1$ for testing.

Submission:

Please note that up to four people can work and submit their solutions together. The theoretical problem(s) have to be submitted digitally (uploaded in ILIAS) before the deadline.

Deadline for Submission: see ILIAS.

Classroom Assignment 2

C 2.1 (Affine Lucas and Kanade)

Derive the matrix $J_0 = \mathbf{r}\,\mathbf{r}^\top$ for the affine Lucas and Kanade model.