Exercise 1

Consider the three degree-of-freedom planar gripper with one revolute joint θ_1 and two prismatic joints d_2, d_3 for the fingers shown in Fig. 1.1. The mass of the first link with length L is considered to be a point mass M placed at the middle of the link. The length of the fingers is equal to ℓ, while the length of the perpendicular segment at the end of the first link and between the fingers is d. The masses of the fingers are considered point masses m and are placed in the middle of each fingers link as shown in Fig. 1.1.

1. Compute the inertia/mass matrix of the manipulator. [2p]
2. Derive a constraint for d_2 and d_3 such that the coupling inertial forces due to acceleration are zero. Find the Coriolis torque on the revolute joint for d_2 and d_3 satisfying the constraint.[1p]

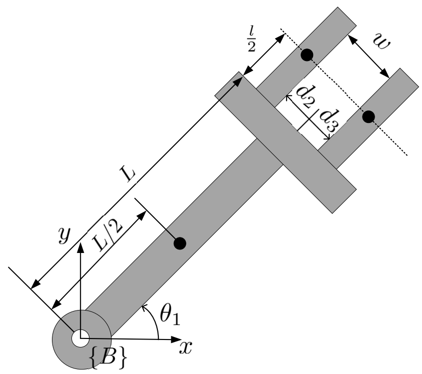


Figure 1.1

Exercise 2

Consider the 3 DoF planar manipulator with three revolute joints q_1, q_2, q_3 shown in Fig. 2.1; gravity acts along z axis. The lengths of the links are given by l_1, l_2 and l_3 respectively, while the length of the perpendicular segment at the end of the first link is given by $2d$. The masses of the links given by m_1, m_2, m_3 are considered point masses and are placed at the end of each link as show in Fig. 2.1.

1. Compute the inertia/mass matrix of the manipulator. [2p]
2. If $l_1 = l_2 = l_3 = d = 1$ m and the joint limits are $-\pi < q_i \leq \pi$, $i = 1, 2, 3$, what are the values of the joint variables for which the coupling inertial forces are zero? [1p]

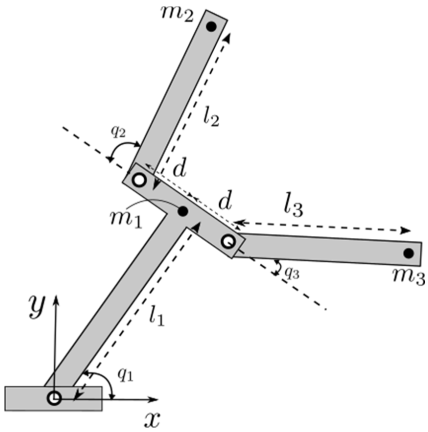


Figure 2.1

Exercise 3 Consider the PR robot of the Fig. 3.1; the gravitational force acts along z axis, as depicted in the figure. The robot has two actuators, one that drives the prismatic joint with a force f and the second one actuating the revolute joint with a torque τ. The kinematic and dynamic parameters of the robot, as well as the gravitational acceleration, are shown in Fig. 3.1

1. Find the robot's dynamic model of the PR robot. [3p]
2. Design a controller in the Joint Space that compensates for the gravitational forces using the models obtained in the previous tasks. [1.5p]

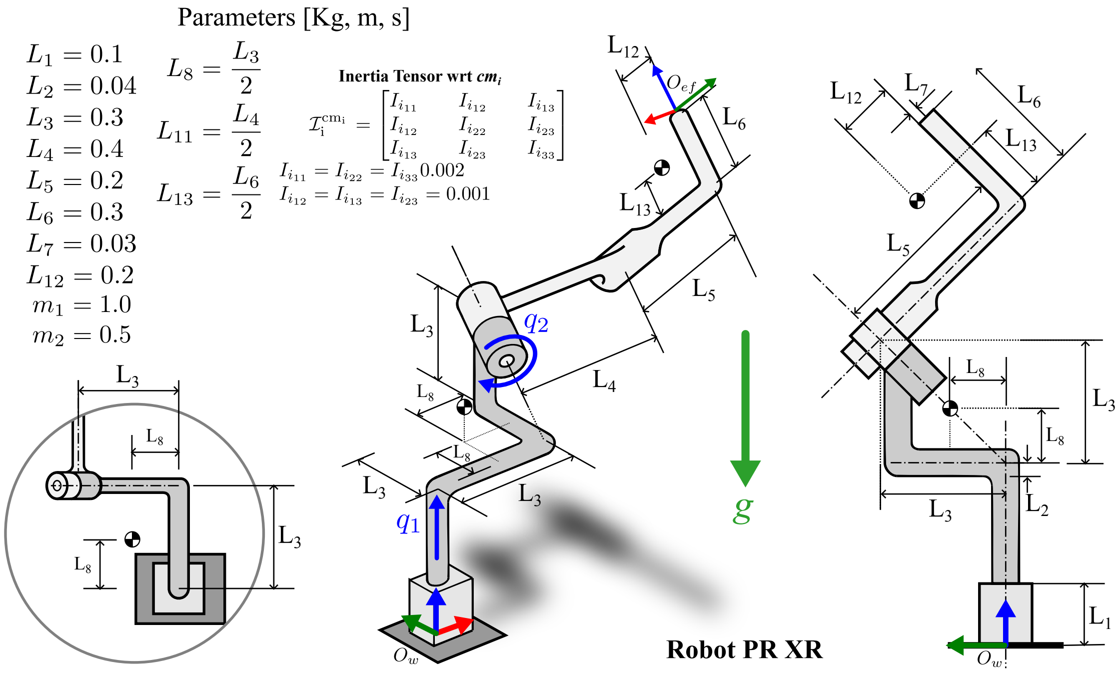


Figure 3.1

Exercise 4

Fig. 4.1 shows a 4DOF RPPR robot used for ankle rehabilitation. The robot consists of a serial kinematic chain with a revolute joint q_1, two prismatic joints q_2, q_3, and finishes with a revolute joint q_4. For a specific application, the manufacturer fixed the 2nd and 4th joints, i.e., $q_2 = q_4 = 0 \forall t$. This constraints generates a 2DOF RP robot. The two active joins are actuated by two motors which exert a force f_1 and a torque τ_2 in their corresponding joints. The gravitational acceleration acting on the robot is in the *negative* direction of z_w. All the robot's parameters are illustrated in Figure 4.1.
The dynamic parameters are [Kg, m, s]:

$$
\begin{aligned}
m_1 &= 0.2 \\
m_2 &= m_3 = 0.25 \\
m_4 &= 0.15 \\
\mathcal{I}_{\mathrm{i}}^{\mathrm{cm_i}} &= \begin{bmatrix} I_{i_{11}} & I_{i_{12}} & I_{i_{13}} \\ I_{i_{12}} & I_{i_{22}} & I_{i_{23}} \\ I_{i_{13}} & I_{i_{23}} & I_{i_{33}} \end{bmatrix} \\
I_{i_{11}} &= I_{i_{22}} = I_{i_{33}} 0.002 \\
I_{i_{12}} &= I_{i_{13}} = I_{i_{23}} = 0.001
\end{aligned}
$$

1. Find the robot's dynamic model of the RP robot. [3p]

2. If the robot is in a closed-loop with the controller $\tau = K_d S_q$, where K_d is a diagonal matrix with entries k_i, and S_q represents a simple PD-like controller in the Joint Space. Find the minimum threshold values for the gains k_i in order to preserve the passivity of the system. The joint limits are $q_1 = [0, 1.0]$ [m] and $q_2 = [-\pi, \pi]$ [rad]. [1.5p]

$[mm]$
$L_1 = 300$
$L_2 = 340$
$L_3 = 583$
$L_4 = 80$
$L_5 = 80$
$L_6 = 115$
$L_7 = 82$
$L_8 = 108$
$L_9 = 123$
$L_{10} = 167$
$L_{11} = 28$
$L_{12} = 50$
$c = d = 320$
$\theta_1 = 45°$
$\theta_2 = 30°$

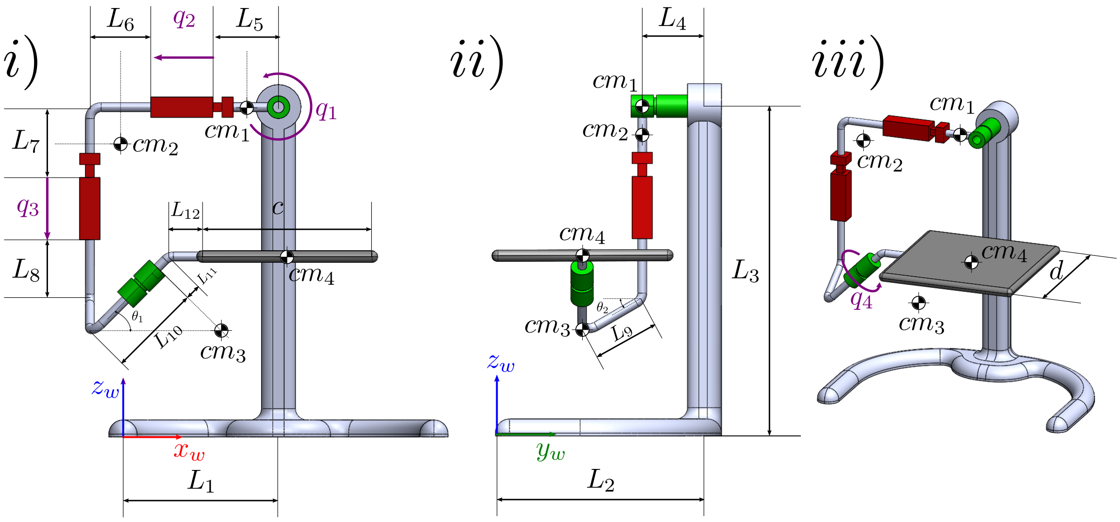


Figure 4.1

Exercise 5

The 2DOF PR robot shown in Fig. 5.1 is used in an industrial application. The two joints, prismatic q_1 and revolute q_2, are actuated with motors that exert a force f_1 and a torque τ_2 in each link, respectively. The kinematic and dynamic parameters of the robot, as well as the gravitational acceleration, are defined in the Fig. 5.1.

1. Find the robot's dynamic model of the PR robot. [3p]
2. Design a controller in the Joint Space that compensates for the internal robot's forces using the models obtained in the previous tasks.[2p]

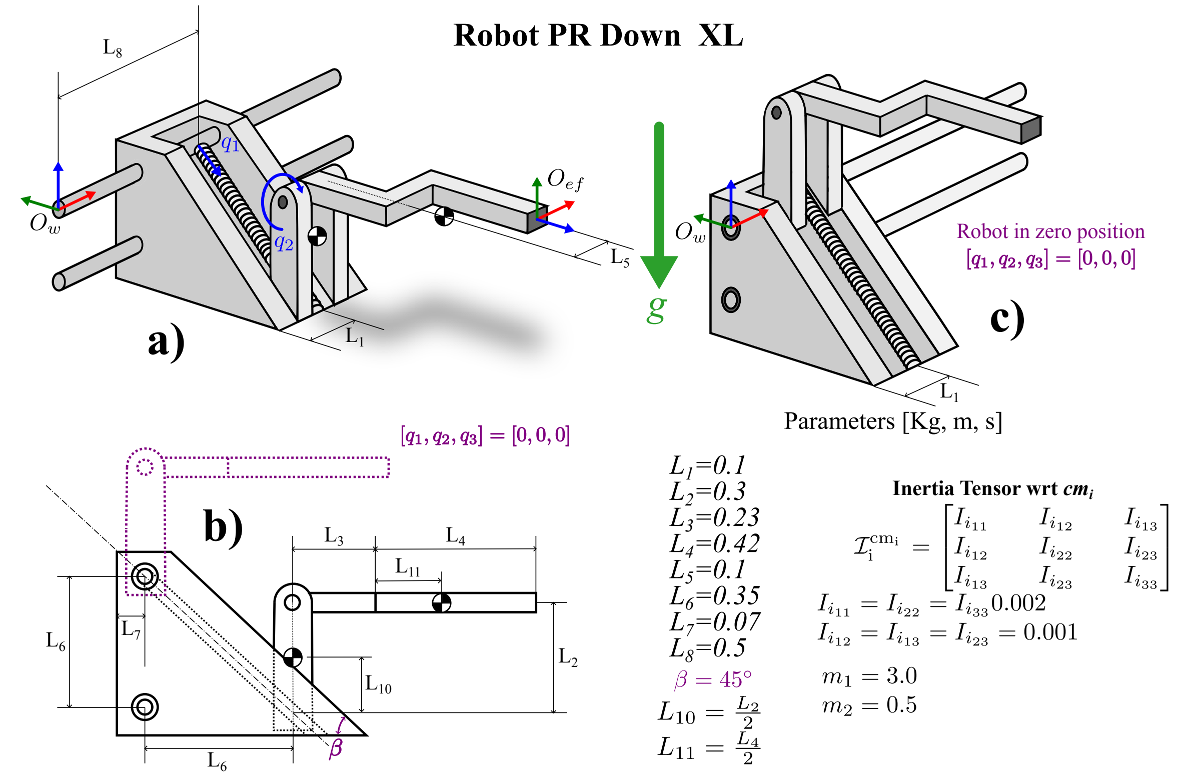


Figure 5.1

Exercise 6

Fig. 4.1 shows a 4DOF RPPR robot used for ankle rehabilitation. The robot consists of a serial kinematic chain with a revolute joint q_1, two prismatic joints q_2, q_3, and finishes with a revolute joint q_4. For a ankle inversion/eversion rehabilitation, the robot fixes the prismatic joints $q_2 = 0, q_3 = 0$. This mode generates a 2DOF RR robot. The two active joins are actuated by two motors which exert torques τ_1 and τ_2 in their corresponding joints. The gravitational acceleration acting on the robot is in the *positive* direction of z_w. All the robot's parameters are illustrated in the figure.

1. Find the robot's dynamic model of the RP robot. [3p]
2. Design a controller in the Joint Space to compensate for the forces induced by the gravity and external forces produced by the weight of a leg (f_l). Assume a force $f_l = 10N$ applied to the center of the platform with dimensions ($c \times d$) mounted in the end-effector. The leg force is always normal to the platform. [2p]

[mm]
$L_1 = 300$
$L_2 = 340$
$L_3 = 583$
$L_4 = 80$
$L_5 = 80$
$L_6 = 115$
$L_7 = 82$
$L_8 = 108$
$L_9 = 123$
$L_{10} = 167$
$L_{11} = 28$
$L_{12} = 50$
$c = d = 320$
$\theta_1 = 45°$
$\theta_2 = 30°$

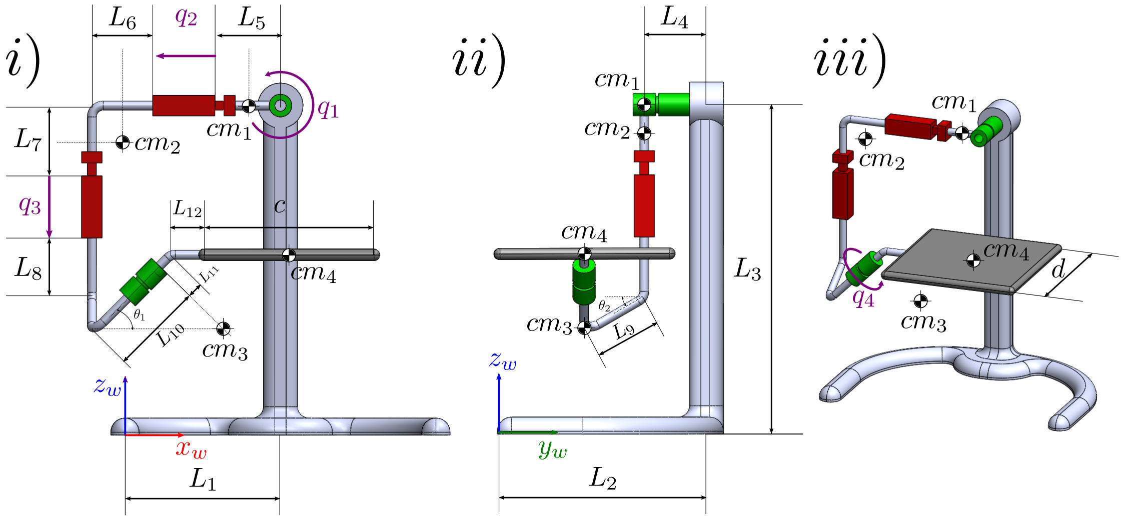
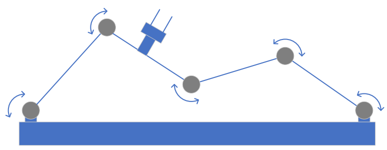


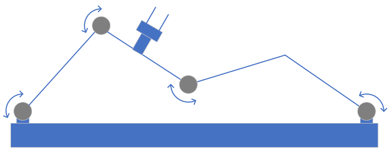
Figure 6.1

Questions

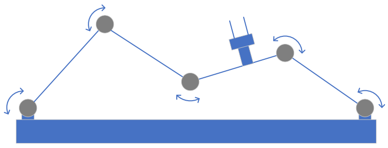
Q1 In the following Figures (a-f), planar constrained robotic mechanisms with revolute joints are shown. What is the number of degrees of freedom for the end-effector? What is the number of degrees of freedom of the structure? Explain your answers by subtracting the number of constraints from the total number of DoFs if the mechanisms were unconstrained. [0.5+0.5=1p]



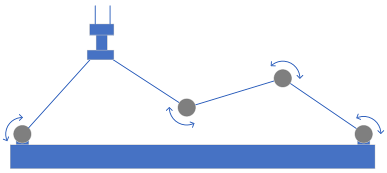
(a)



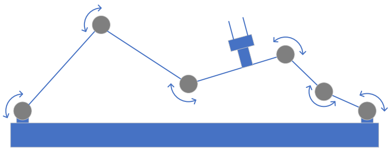
(b)



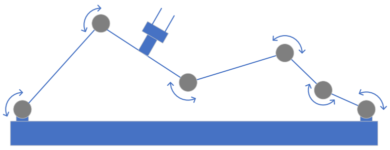
(c)



(d)



(e)

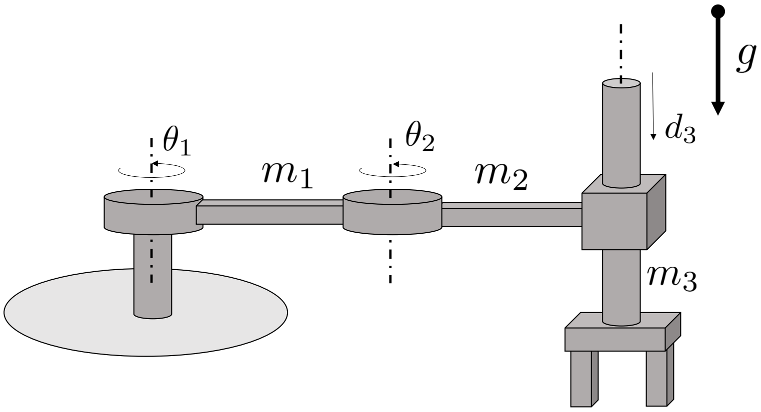


(f)

Q2 A SCARA manipulator shown in the figure is typically used in industrial pick and place applications and its dynamic model has the following form:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = 0$$

with $\boldsymbol{g} = [g_1 \quad g_2 \quad g_3]^T$, and $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$.



- **a)** Assume that a camera of mass m_c is attached on the second link. Find $\boldsymbol{g}$. [0.5p]
- **b)** Assume that a camera of mass m_c is attached on the second link and that the robot is grasping an object of m_o. Find g_1, g_3. [0.5p]
- **c)** Assume that a camera of mass m_c is attached on the gripper (eye on hand setting) and that the robot is grasping an object of m_o. Find g_1, g_3. [0.5p]
- **d)** An elevating mechanism consisting of one prismatic joint is added on the base of the robot and the dimension (DOF) of the model increased by 1. Assume that a camera of mass m_c is attached on the second link and that the robot is grasping an object of m_o. Find g_1 corresponding to the elevating mechanism. [0.5p]

Q3 The kinetic energy of robotic manipulator structures with one prismatic and one revolute joint is given below:

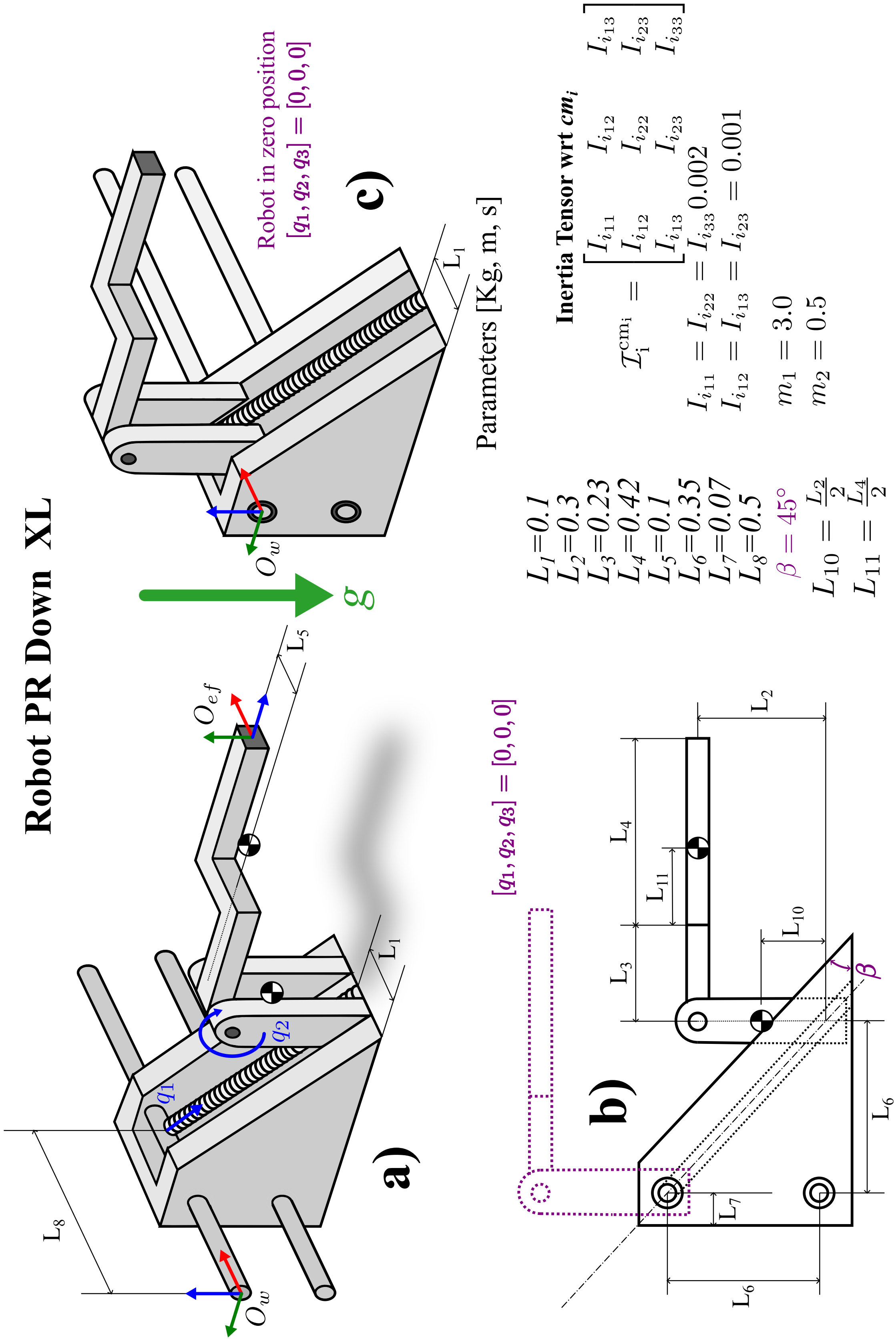
$$\mathcal{T} = \frac{1}{2}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}^T \begin{bmatrix} m_1 + m_2 & -m_2\ell_2\cos\theta \\ -m_2\ell_2\cos\theta & m_2\ell_2^2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

where q_1 (displacement) and q_2 (angle) are the generalized coordinates and m_1, m_2, ℓ_2 are constants.

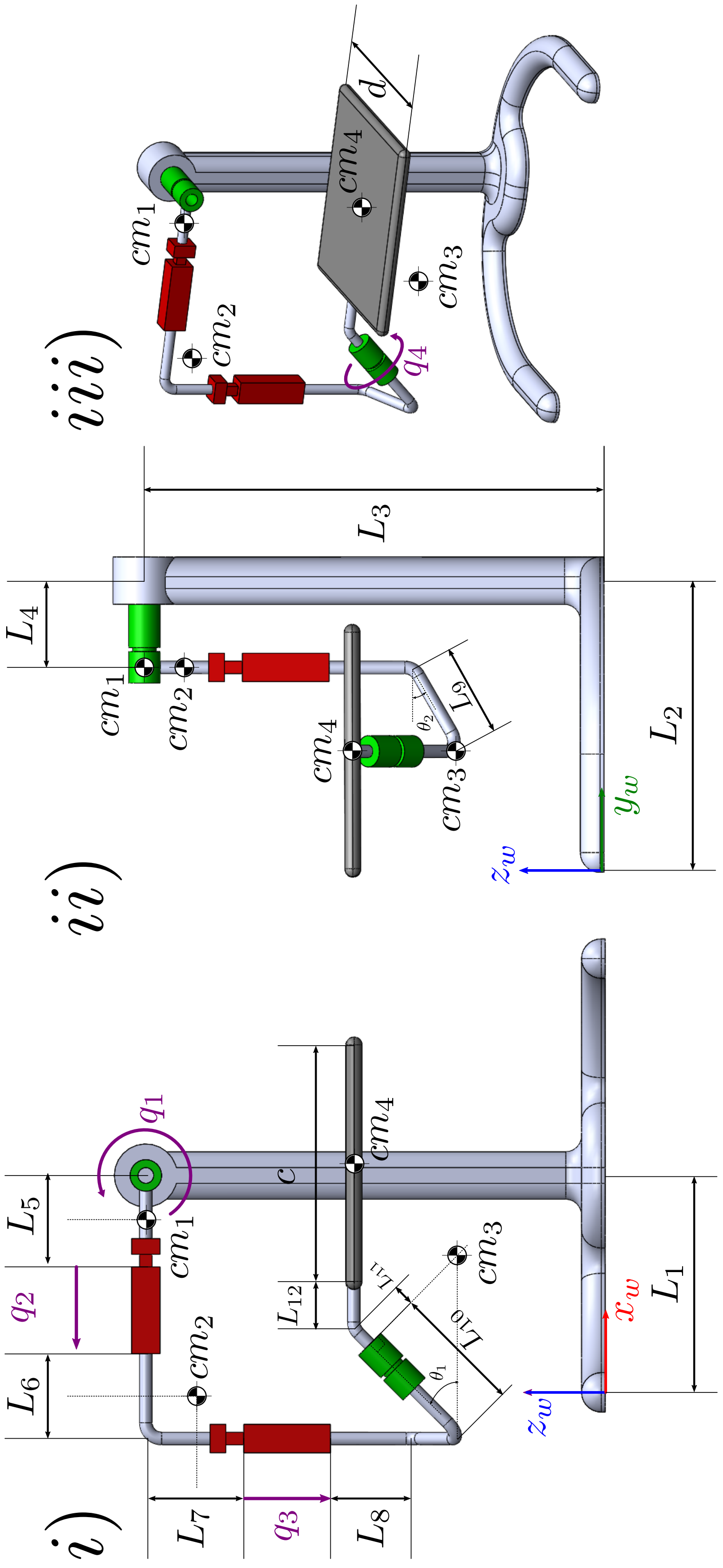
- **a)** Find the generalized force of the prismatic joint due to the accelerations of both joints. [0.5p]
- **b)** Find the generalized force of the revolute joint due to the accelerations of both joints. [0.5p]

1 Appendix A: Figures

1.a Robot Exercise 3



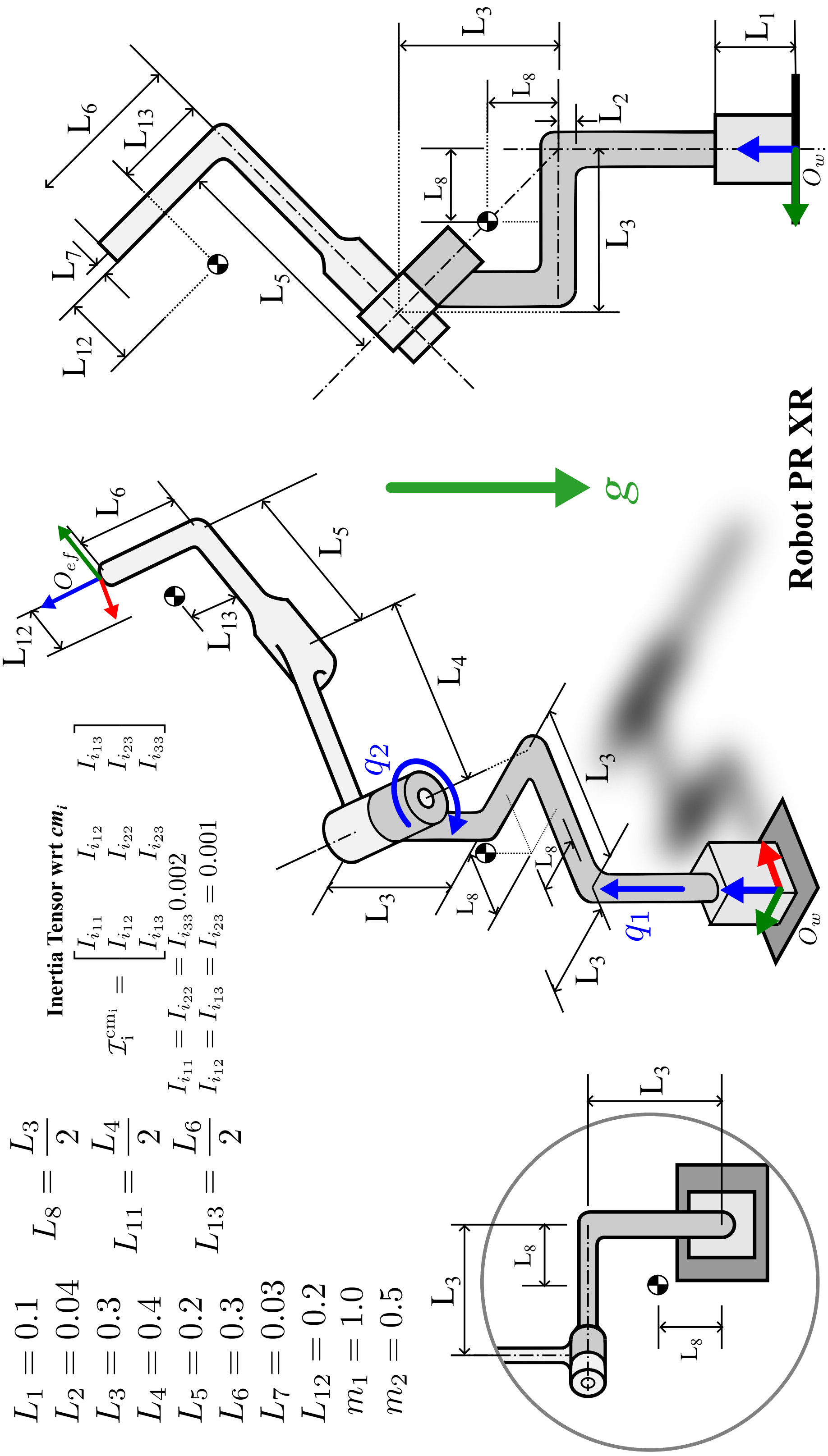
1.b Robot Exercise 2, 4, 6, and 8



$[mm]$

$L_1 = 300$
$L_2 = 340$
$L_3 = 583$
$L_4 = 80$
$L_5 = 80$
$L_6 = 115$
$L_7 = 82$
$L_8 = 108$
$L_9 = 123$
$L_{10} = 167$
$L_{11} = 28$
$L_{12} = 50$
$c = d = 320$
$\theta_1 = 45°$
$\theta_2 = 30°$

1.c Robot Exercise 3



Parameters [Kg, m, s]

$L_1 = 0.1$

$L_2 = 0.04$

$L_3 = 0.3$

$L_4 = 0.4$

$L_5 = 0.2$

$L_6 = 0.3$

$L_7 = 0.03$

$L_{12} = 0.2$

$m_1 = 1.0$

$m_2 = 0.5$

$$L_8 = \frac{L_3}{2}$$

$$L_{11} = \frac{L_4}{2}$$

$$L_{13} = \frac{L_6}{2}$$

Inertia Tensor wrt cm_i

$$\mathcal{I}_i^{cm_i} = \begin{bmatrix} I_{i11} & I_{i12} & I_{i13} \\ I_{i12} & I_{i22} & I_{i23} \\ I_{i13} & I_{i23} & I_{i33} \end{bmatrix}$$

$I_{i11} = I_{i22} = I_{i33} 0.002$

$I_{i12} = I_{i13} = I_{i23} = 0.001$