SSY281 MPC assignment 2

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February 2024

1 Question 1: Set-point tracking

a

The system has two inputs and two outputs witch means that we should consider set point tracking. The following equation shows the condition in order to fullfill the steady state for set point tracking :

$$\begin{pmatrix} I-A & -B \\ C & 0 \end{pmatrix} \begin{pmatrix} x_s \\ u_s \end{pmatrix} = \begin{pmatrix} 0 \\ y_{\text{sp}} \end{pmatrix}$$

By takin the invers of the big matrix and solve for the x_s and u_s matrix with inserted values of the A, B, C and y_{sp}

$$\begin{pmatrix} x_s \\ u_s \end{pmatrix} = \begin{pmatrix} I-A & -B \\ C & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ y_{\text{sp}} \end{pmatrix}$$

$$x_s = \begin{pmatrix} 0.1745 \\ 0.0722 \\ -3.1416 \\ 0.0008 \end{pmatrix}$$

$$u_s = \begin{pmatrix} -0.0153 \\ -0.1427 \end{pmatrix}$$

b

In this case the outputs is two but the inputs is only one therfore we need to minimizet the following exprtession:

$$\left(|Cx_s - y_{sp}|^2 \, Q_s \right)$$

Whit "quadprog" function in matlab can we calculate the expression and get the following values:

$$x_s = \begin{pmatrix} 0.0000 \\ 0.0000 \\ -3.1416 \\ 0.0000 \end{pmatrix}$$

$$u_s = -1.2096 \times 10^{-13}$$

The set point that required is $y_s = [\frac{\pi}{18} - \pi]^T$. With the input $u_s = -1.2096 \times 10^{-13}$ the only set point state that can reach is $y_s = [0 - \pi]^T$ therefor this control is not reliable.

c

In this case we have instead two inputs but only one output witch meas that there could be more then only one solution that minimize $(|u_s - u_{sp}|^2_{R_s} + |C_{xs} - y_{sp}|^2_{Q_s})$. Using again "quadprog" in matlab to solve it, the results is:

$$x_s = \begin{pmatrix} 0.1745 \\ 0.0722 \\ 0.0000 \\ 0.0008 \end{pmatrix} u_s = \begin{pmatrix} -0.0153 \\ -0.1427 \end{pmatrix}$$

2 Question 1: Set-point tracking

a

To determine if the augmented systems can be detected, we need to meet two criteria. First, the rank of the matrix $[I-A \quad -Bd; C \quad C_d]$ should be equal to the sum of the number of states and the number of disturbances (denoted by $n+nd$). Second the rank of the matrix formed by combining matrices A and C should be equal to the number of states (denoted by n). In simpler terms, the original system must be detectable, and the augmented system must have full rank.

$$\operatorname{rank}\begin{pmatrix} I-A & -B_d \\ C & C_d \end{pmatrix} = n + n_d \quad and \quad \operatorname{rank}\mathcal{O}(A,C) = n,$$

Only the first and third system that fulfill the above conditions. The second system is not detectable because of rank deficient. The augmented system is provided in MATLAB code with the numerical values.

b

The following equation are for solving the estimation erro covariance P and the optimal observer gain for Kalman filter.

$$\begin{aligned} P &= A_a P A_a^\top - A_a P C_a^\top (C_a P C_a^\top + R)^{-1} C_a P A_a^\top + Q \\ L &= C_a P C_a^\top (C_a P C_a^\top + R)^{-1} \end{aligned}$$

Where A_a and C_a are the A and C matrices for the augmented system and. The disturbance covariance Q and measurement noise covariance R:

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The results:

$$Le_{1\_2b} = \begin{pmatrix} 1.1835 & 9.7171 & 0.4284 & 0.4089 & -0.3508 \\ -0.4358 & -3.5104 & 0.2323 & -0.1008 & 0.4763 \end{pmatrix}$$

$$Le_{3\_2b} = \begin{pmatrix} 1.3594 & 11.5489 & 0.0140 & 0.3197 & -0.4815 & -0.0102 \\ 0.0128 & 0.3198 & 0.2478 & 2.0989 & -0.0090 & 0.5109 \end{pmatrix}$$

c

Rewriting the expression in lecture notes in 5.2, where $Z_{sp} = 0$ we obtain the following expression where M_{ss} is the right matrices together without $\hat{d}$:

$$\begin{pmatrix} x_s \\ u_s \end{pmatrix} = \begin{pmatrix} I-A & -B \\ HC & 0 \end{pmatrix}^{-1} \begin{pmatrix} B_d \\ -HC_d \end{pmatrix} \hat{d}$$

The M_{ss} for both system is calculated to :

$$Mss_{1\_2c} = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$Mss_{3\_2c} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & -1 \end{pmatrix}$$

d

In this question We will designe RHC for augmented system 1 and 3 since only they are detectable.

By testing the system, we observe the outcome depicted in Figure 1. The disturbance initiates at the marked red point on the graphs (when $k = 50$). It is evident that the controller encounters difficulty in rectifying the deviation for the third state governing the wheel angle θ_2. Subsequent to the disturbance, the wheel angle deviates from its intended position and fails to recover. This predicament arises from the controller's inadequate management of B_d and C_d parameters. Since these parameters remain unknown, designers must make assumptions regarding their values. In this instance, it was assumed that $B_d = [0\ 0\ 0\ 0]^T$ and $C_d = [1\ 1]^T$, indicating that the disturbance solely impacts measurements, not the actual process. However, this assumption is erroneous because the actual system incorporates both process disturbances $B_pP(k)$ and measurement noise $C_pP(k)$, as delineated in equations (1a-1b) within the assignment. Consequently, the controller fails to rectify the deviation in the third state. Nonetheless, despite this limitation, as the controller successfully addresses the deviation in the first state, ball angle θ_1, it can still effectively maintain balance with the ball atop the wheel.

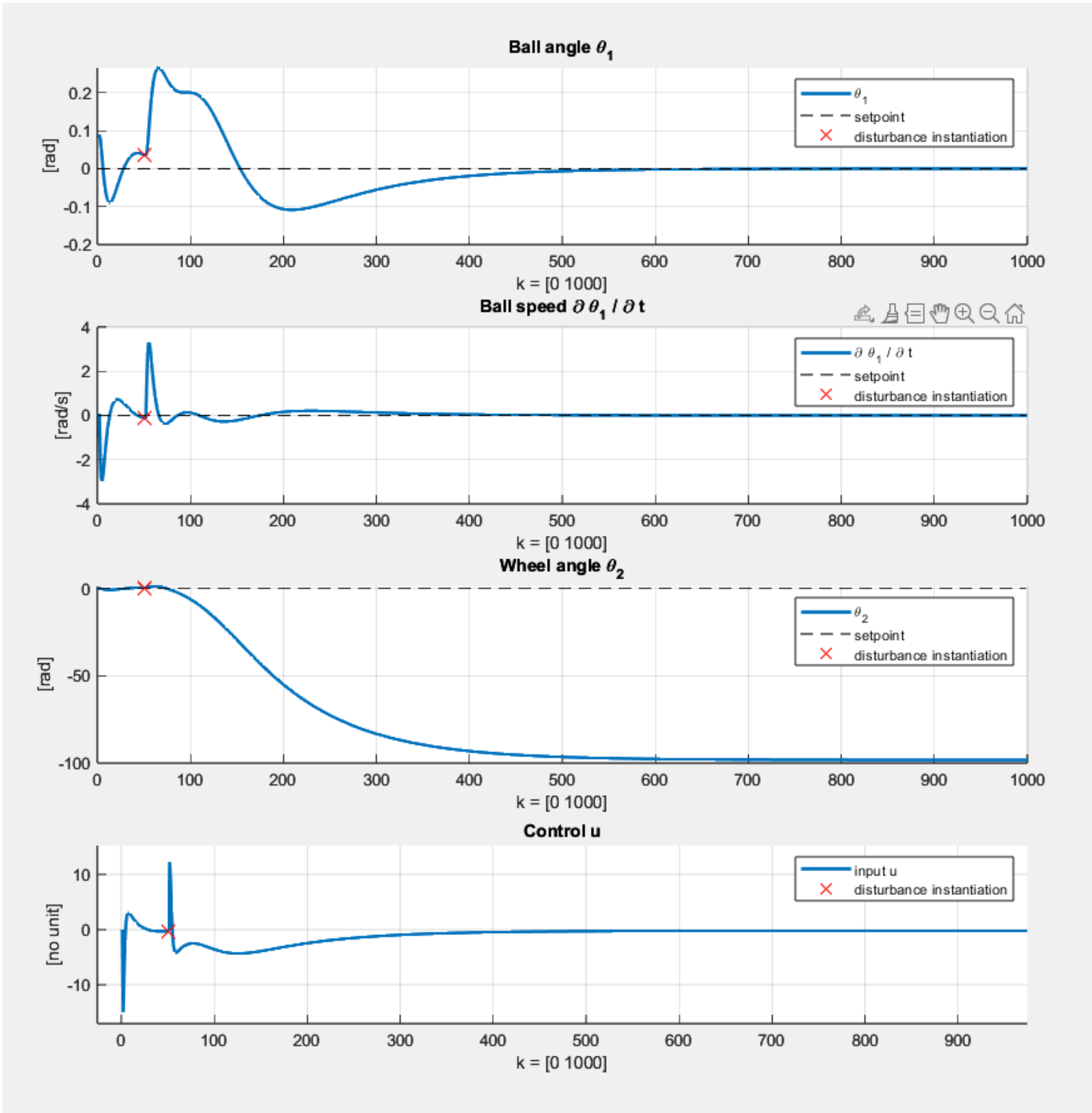

Figure 1: System 1

By simulate system 3, we obtain the results depicted in Figure 2. Once again, the disturbance is introduced at the marked red point on the plots (when $k = 50$). In comparison to the previous model, we can now successfully eliminate the deviation in the third state θ_2, allowing the wheel angle to reach its desired setpoint of 0.

In this systen, the assumptions regarding B_d and C_d are better. For this disturbance modeling, it is assumed that there exists a noise acting as a process disturbance. Specifically, it is assumed that $B_d = [0\ 0\ 0\ 0\ B_p]$ and $C_d = [1\ 0; 0\ 1]$, which provides a more accurate representation of how the disturbances affect the actual system. Consequently, this controller can track the deviation in all outputs, resulting in each state reaching its setpoint of 0.2.

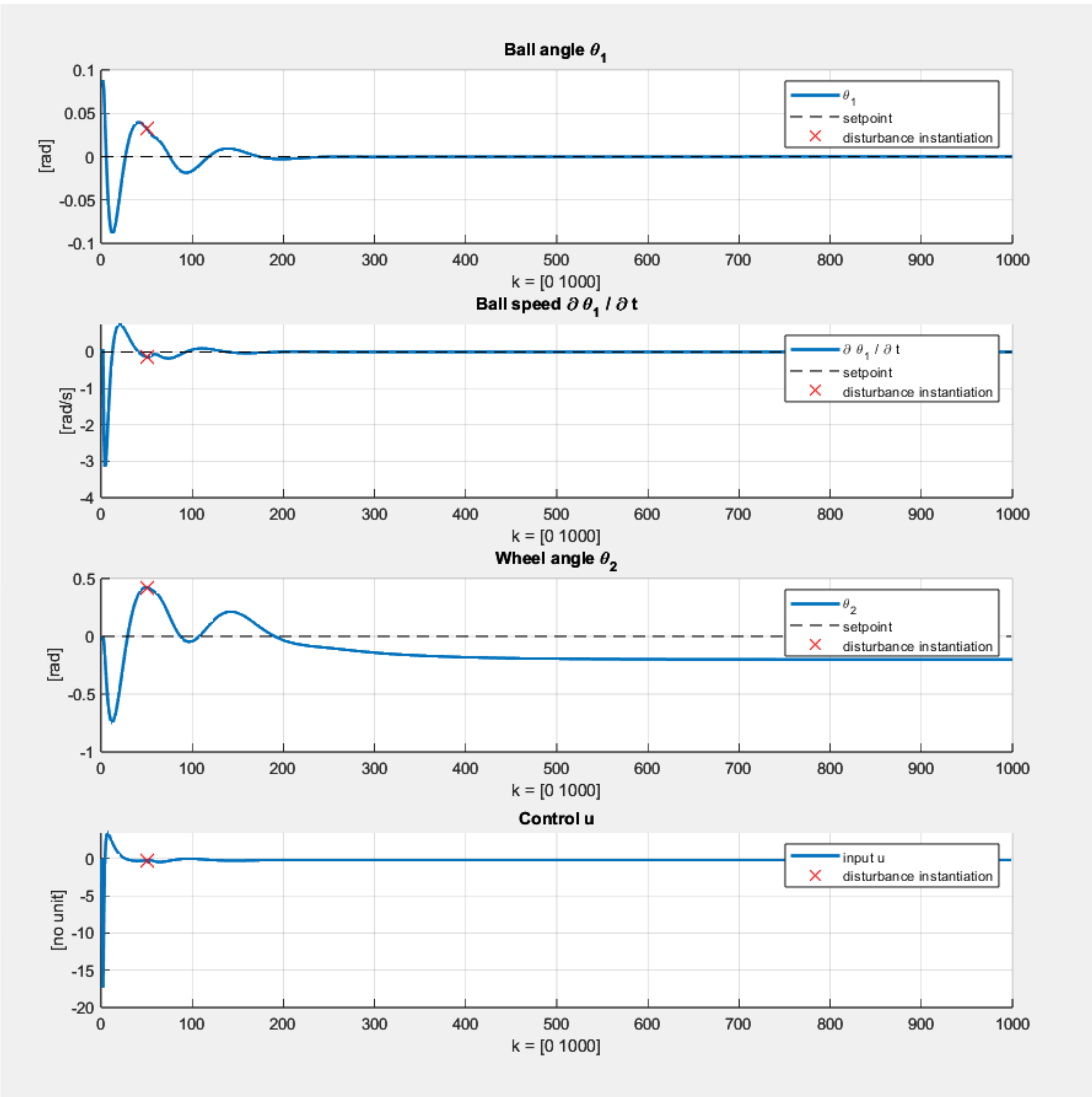

Figure 2: System 3