EX1 Computer vision

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a

Given matrix f:

$$f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 5 & 5 & 0 & 0 & 0 \\ 5 & 5 & 0 & 0 & 0 \\ 5 & 5 & 0 & 0 & 0 \end{bmatrix}$$

Compute the Derivatives f_x and f_y for the Inner Pixels (matrix 3x3):
Using the formulas:

$$[f_x]_{i,j} = \frac{f_{i+1,j} - f_{i-1,j}}{2}$$

$$[f_y]_{i,j} = \frac{f_{i,j+1} - f_{i,j-1}}{2}$$

We calculate f_x and f_y for each inner pixel.

$$f_x = \begin{bmatrix} 0 & 0 & 0 \\ -2.5 & -2.5 & 0 \\ -2.5 & -2.5 & 0 \end{bmatrix}$$

$$f_y = \begin{bmatrix} 2.5 & 0 & 0 \\ 2.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b

Compute the Structure Tensor J_0 for Each Inner Pixel:

The structure tensor J_0 at each pixel (i, j) is defined as:

$$J_0 = \begin{pmatrix} [f_x]_{i,j}^2 & [f_x]_{i,j} \cdot [f_y]_{i,j} \\ [f_x]_{i,j} \cdot [f_y]_{i,j} & [f_y]_{i,j}^2 \end{pmatrix}$$

The structure tensor J_0 for each specified inner pixel is as follows:

$$J_{1,1} = \begin{bmatrix} 0 & 0 \\ 0 & 6.25 \end{bmatrix}$$

$$J_{1,2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{1,3} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{2,1} = \begin{bmatrix} 6.25 & -6.25 \\ -6.25 & 6.25 \end{bmatrix}$$

$$J_{2,2} = \begin{bmatrix} 6.25 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{2,3} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{3,1} = \begin{bmatrix} 6.25 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{3,2} = \begin{bmatrix} 6.25 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_{3,3} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

c

By looking at the eigenvalues of each tensor for each pixel we can classify them as edge, corner or flat area. If the λ_1 and λ_2 is ≈ 0 then it is flat area, If the $\lambda_1 > 0$ and λ_2 is ≈ 0 or the opposite then edge , otherwise it is corner

$$\begin{bmatrix} \text{Edge} & \text{Flat Area} & \text{Flat Area} \\ \text{Edge} & \text{Edge} & \text{Flat Area} \\ \text{Edge} & \text{Edge} & \text{Flat Area} \end{bmatrix}$$

d

Smoothed Structure Tensor for Central Pixel:

$$\begin{bmatrix} 1.1719 & -0.7812 \\ -0.7812 & 2.3438 \end{bmatrix}$$

Classification of Central Pixel after Convolution:

Corner

e

Norm of the sum of gradients at the central pixel is equal to Zero.

f

Joint Color Gradient at the central pixel:

3.5355

g

$$\text{tensor\_R} = \begin{bmatrix} 6.2500 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{tensor\_G} = \begin{bmatrix} 6.2500 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{tensor\_B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Joint Color Structure Tensor at the central pixel:

$$\begin{bmatrix} 12.5000 & 0 \\ 0 & 0 \end{bmatrix}$$

Expressions (f) and (g) are most useful for edge detection:

- (f) **Joint color gradient**: Measures gradient strength in each color channel separately, helping to detect edges across all colors.
- (g) **Joint color structure tensor**: Combines gradients from all channels, capturing spatial information to distinguish edges more effectively.