Mobile Robotics Winter term 2024/2025

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Submission instructions: see box
Submission and voting deadline in ILIAS: November 22, 2024, 14:00

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Submission Instructions: For the solution of theoretical tasks, use a header with your name and Matrikelnummer on each sheet and combine all files (pictures, scans, LaTeX-ed solutions) into a single `.pdf` document. For code solutions, if not stated otherwise, your code should execute correctly when called from a single `.m` or `.mlx` script (external functions are ok as long as they are called from the script). Each file you submit must include a header with your name and Matrikelnummer. Please add comments to make your code readable and to indicate to which task and subtask it refers to. For submission, all files should be included in a single `.zip` archive named as: `Ex03_YourLastname_Matrikelnummer.zip`. Remember to vote on the tasks that you solved and be ready to present them.

Exercise 03: Kinematic Analisys

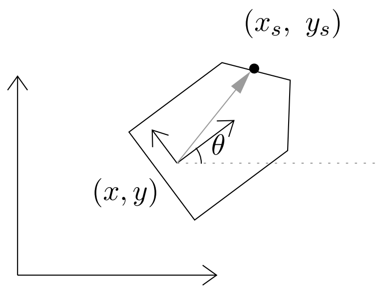
Exercise 03.1: Velocity of a sensor: (theoretical) Consider a robot represented with the canonical model:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}$$

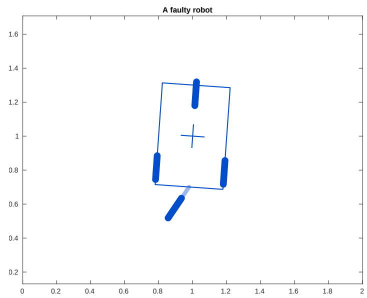
The robot has a sensor placed rigidly on its body at (x_s, y_s) with respect to the robot's own frame of reference. The position of the sensor in world coordinates is then:

$${}^{w}p_s = \begin{pmatrix}x\\ y\end{pmatrix} + \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{pmatrix}x_s\\ y_s\end{pmatrix}$$

Compute the velocity of the sensor position $(\dot{x}_s, \dot{y}_s)$ with respect to the kinematic inputs (v, ω). What happens when $x_s = 0$ and why?



Exercise 03.2: Faulty robot: (theoretical) Consider the following tricycle robot equipped with three fixed wheels and a caster wheel. Conduct a qualitative kinematic analysis and confirm that this is a *degenerate* wheeled mobile robot. What is the minimal fix to the wheel set of this robot that makes it *non-degenerate*? What is then the type classification (δ_m, δ_s)?



Exercise 03.3: Mobile Robot Classification: This exercise focuses on the kinematic analysis and modeling of wheeled mobile robots. In the provided code, a wheel will be represented as a struct with the following fields:

a) `type`: 0 for a fixed standard wheel, 1 for a orientable wheel, 2,3 for a caster wheel, 4 for a swedish wheel.

b) `pose`: a 3-D vector $[x_w, y_w, \theta_w]$ denoting the position of the wheel with respect to a reference point on the robot. Mind that $\theta_w = \mathtt{atan2}(y_w, x_w) + \beta_w$.

c) `params`: a vector containing, in order, wheel radius, width, offset (only for caster wheels), γ (only for swedish wheels).

Given a set of wheels:

a) Write a MATLAB function that generates the matrices of constraints J_1,J_2, arising from the pure rolling constraints, and the matrices C_1, C_2 arising from the no sliding constraint. The function also computes the type (δ_m, δ_s) of the robot .

Exercise 03.4: Instantaneous Center of Rotation:

a) Design a vehicle with three fixed wheels so that their axles intersect in a single point, the ICR. Create a plot on which you draw the smallest singular value of $C_1^*(\beta)$ when the angle of one wheel β is varied by small increments around the initial value.

   This should demonstrate how close are the wheels of a robot to be misaligned.

b) Given a set of wheels in a wheeled mobile robot, implement a MATLAB function that identifies the position (x, y) of the Instantaneous Center of Rotation (ICR). If the ICR is at infinity, the position has to be `[inf, inf]`. If the ICR does not exist, the position has to be `[NaN, NaN]`.

Exercise 03.5: The bicycle robot: (theoretical)

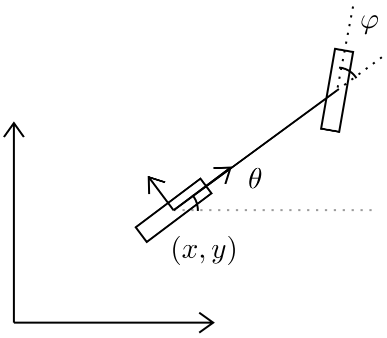
Considering the bicycle-like kinematic model illustrated in the picture:

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ \frac{\tan\psi}{l} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

a) Reason about this kinematic model: what happens to the robot when $\psi \to \frac{\pi}{2}$?

b) Consider the following input transformation: $v_f = \frac{v}{\cos\psi}$ and rewrite the model in the new input coordinates (v_f, ω). What happens to the robot when $\psi \to \frac{\pi}{2}$?

c) **Bonus**: Verify that v_f is equal to the *speed* of the front wheel $C \begin{pmatrix} x + l\cos\theta \\ y + l\sin\theta \end{pmatrix}$.



Exercise 03.6: Uneven wheels: (theoretical) Consider a differential drive robot (DDR) with two wheels of different radius, r_R and $r_L = \bar{k} \cdot r_R$.

a) Write down the kinematic model for this robot with the inputs u_R, u_L being the angular speeds of the right and left wheel.

b) Compare it with the model for a DDR both wheels of radius r_R. Commanding both models with the same inputs $u_R = u_L = \bar{u}$, compute the velocities $(\dot{x}, \dot{y}, \dot{\theta})$ in both models.

c) Find the input transformation matrix T such that the kinematic model with the inputs $T(\bar{k}) \begin{bmatrix} u_R \\ u_L \end{bmatrix}$ behaves the same as a DDR with equal wheels.