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Homework Assignment 3

H 3.1 (Variational Methods) 10 Points

Instead of using the grey value constancy assumption, let us assume that the y-derivative of the image f remains constant over time, i.e.

$$f_y(x+u, y+v, t+1) = f_y(x, y, t) \tag{1}$$

(a) Linearise the constancy assumption (**??**) w.r.t the flow functions u and v.

(b) Write down an energy functional similar to Horn and Schunck based on the linearised constancy assumption from (a).

(c) Compute the Euler-Lagrange equations for this energy functional.

(d) Discretise the Euler-Lagrange equations from (c). (Write the discrete approximations of all derivatives explicitly such that the discretized version only depends on $f_{(i,j)}$)

(e) Starting from the discrete equations computed in (d), derive an iterative scheme that computes the minimiser.

H 3.2 (Stereo) 6 Points

Consider a camera with focal distance 2. Its image coordinate system is orthogonal with square pixels of size 1. The principal point in this coordinate system is located in $(2, 3)^\top$. Furthermore, the position of the world coordinate system relative to the camera coordinate system is given by a rotation around the z-axis by an angle of $90°$ and a translation by the vector $(5, 0, -1)^\top$.

(a) Compute the intrinsic matrix A_{int} (including the focal length f).

(b) Compute the extrinsic matrix A_{ext}.

(c) Compute the full projection matrix P.

P 3.3 (Horn and Schunck)

Please download the required file `cv24_ex03.zip` from ILIAS and unpack the data.

In the routine `flow`, supplement the missing code such that it computes one Jacobi iteration step. Make sure the image boundaries are treated correctly. To compile the program, type

```
gcc -O3 -o hsTemplate hsTemplate.c -lm
```

The filling-in effect that is characteristic for variational methods can be studied with the image pair `pig1.pgm`, `pig2.pgm`. To this end, investigate the result for different numbers of iterations. What is a good value for the regularisation parameter α in this case?

Submission:

Please note that up to four people can work and submit their solutions together. The theoretical problem(s) have to be submitted digitally (uploaded in ILIAS) before the deadline.

Deadline for Submission: see ILIAS.

Classroom Assignment 3

C 3.1 (Eigenvalue Analysis)

Let $\mathbf{J} \in \mathbb{R}^{n \times n}$ be a symmetric $(n \times n)$ matrix with real components. We consider its corresponding quadratic form given by

$$E : \mathbb{R}^n \longrightarrow \mathbb{R}, \qquad E(\mathbf{v}) = \mathbf{v}^\top \mathbf{J} \mathbf{v}$$

Show that among all vectors $\mathbf{v} \in \mathbb{R}^n$ with $|\mathbf{v}| = 1$, the function value $E(\mathbf{v})$ is minimal for the eigenvector of $\mathbf{J}$ corresponding to its smallest eigenvalue. What can we say about E if $\mathbf{J}$ is positive definite?