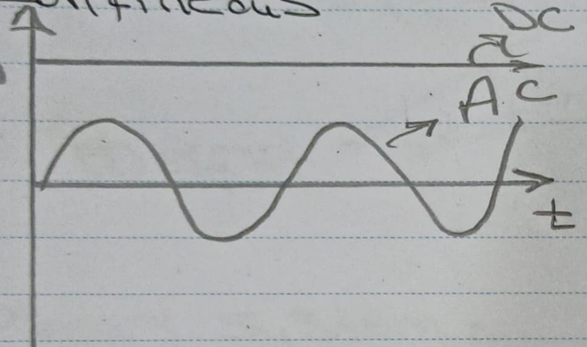
lec1: Numbering systems

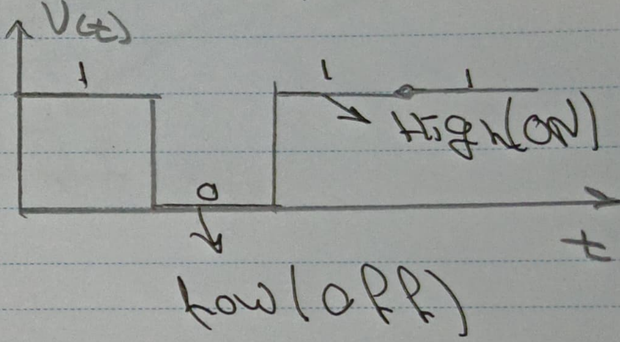
* **Digital Systems:** Systems has ability to represent & manipulate discrete elements

* Difference between analog systems & digital systems

=> Analog systems: Signal is contineous



=> Digital systems: Signal is discrete



=> Number systems:

there exist basic four systems

1. Decimal → Base 10
2. Binary → Base 2
3. Octal → Base 8
4. Hexadecimal → Base 16

In general: the magnitude of any number in decimal form is

$$\text{Magnitude} = \text{Sum of} \left(\text{digit value} * \text{Base}^{\text{Position}} \right)$$

$\text{Base}^{\text{Position}}$: Called $\boxed{\text{Weight}}$

[1] Decimal numbers:

⇒ Digits: 0, 1, 2, --- 9
⇒ Base = 10 (Base also called radix)

Ex $925 = 900 + 20 + 5$

$= 9*100 + 2*10 + 5*1$

$= 9 * \underline{10^2} + 2 * \underline{10^1} + 5 * \underline{10^0}$

Weight Digits

[2] Binary system:

⇒ Digits: 0, 1
⇒ Base = 2

Ex:

16	8	4	2	1
0	1	0	0	1
2^4	2^3	2^2	2^1	2^0

Magnitude $= 8 + 2 = \boxed{10}$

Ex: convert $(325)_{10}$ in binary

	Quotient	remainder				
325 /2	162	1	LSB	2/2	1	0
162 /2	81	0				
81/2	40	1		1/2	0	1
40/2	20	0				MSB
20/2	10	0				
10/2	5	0				
5/2	2	1				

LSB: Least Sinificat Bit (At right)
MSB: Most Sinificat Bit (At left)

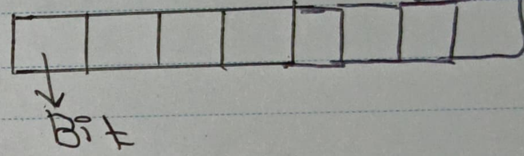
$\therefore (325)_{10} = (101000101)_2$

Notes:

① For n bits, the number of possible numbers is 2^n (from $\underline{0}$ to $\underline{2^n - 1}$)

② In Computer Work, 1 Byte = 8 bit

⇒ kilo (k) = 2^{10}
Mega (M) = 2^{20}
Giga (G) = 2^{30}
Tera (T) = 2^{40}

Byte ⟹ 

⇒ fraction decimal to binary: $(0.625)_{10} = (?)_2$

	integer +	fraction
$0.625 * 2$	1 → MSB	0.25
$0.25 * 2$	0	0.5
$0.5 * 2$	1 → LSB	.00

$$\therefore \boxed{(0.625)_{10} = (0.101)_2}$$

3 Octal Systems

⇒ Digits: 0, 1, 2, ... 7
⇒ Base = 8

⇒ **Ex** $(127.4)_8 = (\ ?\ )_{10}$

We have

$$\begin{array}{lcccccc} & 1 & 2 & 7 & . & 4 \\ & \downarrow & \downarrow & \downarrow & & \downarrow \\ \text{Weight:-} & 8^2 & 8^1 & 8^0 & & 8^{-1} \end{array}$$

∴ Magnitude $= (1*8^2 + 2*8^1 + 7*8^0 + 4*8^{-1})_{10}$
$= (87.5)_{10}$

⇒ Decimal to Octal: $(175)_{10} = (\ ?\ )_8$

	integer	Remainder
175 / 8	21	7 → a_0
21 / 8	2	5
2 / 8	0	2 → a_2

∴ $(175)_{10} = (257)_8$

⇒ Fraction to Octal: $(0.3125)_{10} = (\ ?\ )_8$

	integer	+ fraction
0.3125 * 8	2 → a_{-1}	0.5
0.5 * 8	4 → a_{-2}	0.00

∴ $(0.3125)_{10} = (0.24)_8$

$\Rightarrow$ Octal in binary representation (in 3-bits)

	4	2	1
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

Ex: Express $(10110.01)_2$ in octal

$\Rightarrow$ $010 \quad 110 \,.\, 010$

$= (2 \quad 6 \,.\, 2)_8$

Ex: Express $(36.5)_8$ in binary

$\Rightarrow$ $3 \quad 6 \,.\, 5$

$= (011 \quad 110 \,.\, 101)_2$

[4] Hexadecimal systems:

⇒ Digits : 0, 1, 2, ... 9, A, B, C, D, E, F
⇒ Base = 16

⇒ A = 10
B = 11
C = 12
D = 13
E = 14
F = 15

in decimal

⇒ 1 Hex Place has 16 different Combinations
⇒ in Binary Stored in 4 bits

Hex	8	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
A	1	0	1	0
B	1	0	1	1
C	1	1	0	0
D	1	1	0	1
E	1	1	1	0
F	1	1	1	1

Binary

Ex : ① $(10110.01)_2 = (?)_{16}$
② $= (?)_{10}$

⇒ 0001 0110 . 0100
$= (1 \quad 6 . 4)_{16}$

② $(16.4)_{16} = 1 * 16^1 + 6 * 16^0 + 0.4 * 16^{-1}$

$= (22.025)_{10}$

1) List the numbers from 8 to 28 in base 12.

* Solution:

Decimal	Base 12
8	8
9	9
10	A
11	B
12	10
13	11
14	12
15	13
16	14
17	15

Decimal	Base 12
18	16
19	17
20	18
21	19
22	1A
23	1B
24	20
25	21
26	22
27	23
28	24

2) What is the largest binary number that can be expressed with 16 bits? What are the equivalent decimal and hexadecimal numbers?

* Solution: $n=16$

⇒ Largest number: $(1111\ 1111\ 1111\ 1111)_2$

⇒ in Decimal: Magnitude $= 2^{16}-1 = (65535)_{10}$

⇒ in Hexadecimal $\equiv (FFFF)_{16}$

3) How many bits needed to represent 205 in binary? (guess number of bits without conversion)

* Solution:

$$\Rightarrow \quad 2^n - 1 \geq 205$$

$$2^n \geq 206 \Rightarrow \ln 2^n \geq \ln 206$$

$$n \geq \frac{\ln 206}{\ln 2} \simeq \boxed{8 \text{ bits}}$$

4) What is the largest number (in decimal) that can be obtained with

a. 7 bits binary

b. 3 bits hexadecimal

* Solution:

$\boxed{a}$ $Max = 2^n - 1 = 2^7 - 1 = \boxed{127}$

$\boxed{b}$ $Max = 16^n - 1 = 16^3 - 1 = \boxed{4095}$

5) Convert the following numbers with the indicated bases to decimal:

a. $(10110.0101)_2$	b. $(121)_3$	c. $(345)_6$	d. $(77.7)_8$
e. $(435)_8$	f. $(198)_{12}$	g. $(AC5)_{16}$	h. $(16.5)_{16}$

* Solution

a) $(10110.0101)_2 = 1*2^4 + 1*2^2 + 1*2^1 + 1*2^{-2} + 1*2^{-4}$

$= \boxed{(22.3125)_{10}}$

b) $(121)_3 = 1*3^2 + 2*3^1 + 1*3^0 = \boxed{(16)_{10}}$

c) $(345)_6 = 3*6^2 + 4*6^1 + 5*6^0 = \boxed{(137)_{10}}$

d) $(77.7)_8 = 7*8^1 + 7*8^0 + 7*8^{-1} = \boxed{(63.125)_{10}}$

e) $(435)_8 = 4*8^2 + 3*8^1 + 5 = \boxed{(285)_{10}}$

f) $(198)_{12} = 1*12^2 + 9*12^1 + 8 = \boxed{(260)_{10}}$

g) $(AC5)_{16} = 10*16^2 + 12*16 + 5 = \boxed{(2757)_{10}}$

h) $(16.5)_{16} = 1*16 + 6*16^0 + 5*16^{-1} = \boxed{(22.3125)_{10}}$

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6) perform the following conversions

a. $(28.125)_{10}$ to binary

b. $(157.128)_{10}$ to hexadecimal

c. $(67.45)_{10}$ to octal

d. $(2AC5)_{16}$ to octal (without converting to decimal)

* Solution

a) $(28.125)_{10} = (28)_{10} + (0.125)_{10}$

	integer + remainder			integer + fraction	
28/2	14	LSB 0	0.125*2	0 → MSB	0.25
14/2	7	0	0.25*2	0	0.5
7/2	3	1	0.5*2	1 → LSB	0.00
3/2	1	1			
1/2	0	MSB 1			

$\therefore (28)_{16} = (11100)_2$ $\therefore (0.125)_{10} = (0.001)$

$\therefore (28.125)_{10} = (11100.001)_2$

b) $(157.128)_{10} = (\ ?\ )_{16}$

$\Rightarrow (157.128)_{10} = (157)_{10} + (0.128)_{16}$

	integer	Remainder
157/16	9	13 = D
9/16	0	9

$\therefore (157)_{10} = (9D)_{16}$

	integer	fraction
0.128*16	2	0.048
0.048*16	0	0.768
0.768*16	C	0.288

$\therefore (0.128)_{10} = (0.20C)_{16}$

$\therefore (157.128)_{10} = (9D.20C)_{16}$

c) $(67.46)_{10} = (\ ?\ )_8$

	integer	Remainder
67/8	8	3 → LSB
8/8	1	0
1/8	0	1 → MSB

$(67)_{10} = (103)_8$

	integer	fraction
0.46*8	3 MSB	0.68
0.68*8	5	0.44
0.44*8	3	0.52
0.52*8	4	0.16
0.16*8	1	0.28

$(0.46)_{10} = (0.353)_8$

$\therefore (67.46)_{10} = (103.353)_8$

d) $(2AC5)_{16} = (\ ?\ )_8$

$\Rightarrow$ $(2 \quad A \quad C \quad 5)_{16}$

$0010 \quad 1010 \quad 1100 \quad 0101$

$= \cancel{00} \quad 010 \quad 101 \quad 011 \quad 000 \quad 101$

$= (2\ 5\ 3\ 0\ 5)_8$