

$$\textcircled{1} f = \alpha x^{\alpha-1} e^{-x^\alpha}$$

$$L(\alpha) = \alpha^n \sum_{i=1}^n x_i^{\alpha-1} e^{-\sum_{i=1}^n x_i^\alpha}$$

$$\ln L(\alpha) = \ln \left(\alpha^n \sum_{i=1}^n x_i^{\alpha-1} e^{-\sum_{i=1}^n x_i^\alpha} \right)$$

$$= n \ln(\alpha) + (\alpha-1) \sum_{i=1}^n \ln(x_i) - \sum_{i=1}^n x_i^\alpha$$

$$\frac{\partial}{\partial \alpha} \ln L(\alpha) = \frac{n}{\alpha} + \sum \ln x_i - \sum x_i^\alpha \ln(x_i) = 0$$

$$\frac{n}{\alpha} + \sum \ln(x_i) = \sum x_i^\alpha \ln(x_i)$$

$$\sum x_i^\alpha = \frac{n}{\alpha} \quad \therefore \alpha = \frac{\sum x_i^\alpha}{n}$$

$$f(x) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^\alpha}$$

$$L = \prod_{i=1}^n \frac{\alpha}{\beta^\alpha} x_i^{\alpha-1} e^{-\left(\frac{x_i}{\beta}\right)^\alpha}$$

$$\log(L) = \sum_{i=1}^n \left(\ln \alpha - \ln \beta + (\alpha-1) \ln x_i - \left(\frac{x_i}{\beta}\right)^\alpha \right)$$

$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} - n \ln \beta + \sum \ln x_i - \sum \left(\frac{x_i}{\beta}\right)^\alpha \ln \left(\frac{x_i}{\beta}\right)$$

$$\frac{\partial L}{\partial \beta} = \frac{n\alpha}{\beta} - \alpha \sum \left(\frac{x_i}{\beta}\right)^{\alpha-1} \frac{x_i}{\beta^2}$$

$$\frac{n\alpha}{\beta} = \alpha \sum \left(\frac{x_i}{\beta}\right)^{\alpha-1} \frac{x_i}{\beta^2}$$

$$\textcircled{1} \frac{1}{\alpha} - \ln \beta + \frac{1}{n} \sum \ln x_i = \frac{1}{n} \sum \left(\frac{x_i}{\beta}\right)^\alpha \ln \left(\frac{x_i}{\beta}\right)$$

$$\textcircled{2} \beta = \left(\frac{1}{n} \sum x_i^\alpha \right)^{\frac{1}{\alpha}}$$