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Date التاريخ

[1] $F(x_1,x_2) = 100(x_2 - x_1^2)^2 + (1-x_1)^2$

(a) $X_0 \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad S_1 = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$S_1 = \{4, 0\}^T$

$F(\alpha) = 100(1 - (-1+4\alpha)^2)^2 + (1 - (-1+4\alpha))^2$

$X_0 + \lambda S_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} -1+4\lambda \\ 1 \end{bmatrix}$

$= 25600\lambda^4 - 25600\lambda^3 + 6416\lambda^2 - 16\lambda + 4$

$\lambda = (0, 0.1)$

$\lambda^* = 0.00125 \rightarrow F(\lambda^*) = 3.989$

n	0	1	2	3	4	5	6	7	8
F_n	1	1	2	3	5	8	13	21	34

$F_n > \frac{b-a}{\varepsilon} = \frac{0.1-0}{0.01} = 10 \Rightarrow F_n = F_6 = 13 \ (6)$

$\delta_k = \gamma_k (b-a)$ where $\gamma_k = \frac{F_{5-k}}{F_{6-k}}$

$x_1 = b - \delta \qquad x_2 = a - \delta$

k	γ_k	δ_k	a	x_1	$F(x_1)$	x_2	$F(x_2)$	b
0	8/13	0.0615	0	0.038	11.47	0.06	21.713	0.1
1	5/8	0.0384	0	0.023	6.74	0.103	11.47	0.06
2	3/5	0.0230	0	0.015	5.18	0.02	6.74	0.03
3	2/3	0.0153	0	0.007	4.24	0.01	5.18	0.02
4	1/2	0.0076	0	0.007	4.24	0.007	4.24	0.1
5	1/1		0					0.007

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$\lambda = \frac{0+0.076}{2} \Rightarrow 0.038 \Rightarrow F(x^*) = 41.03$

interval $[0, 0.007]$ $\frac{L_6}{L_0} = \frac{0.007-0}{0.1-0} = 0.07$

$\frac{L_6}{L_0} = \frac{1}{F_6} = \frac{1}{13} = 0.07$

(b) Golden

$L_0 = [0, 0.1]$ $\phi = 0.618$

$\lambda_1 = a + (1-\phi)(b-a)$ $\lambda_2 = a + \phi(b-a)$

$\alpha = 0.1$

$\lambda_1 = 0 + (1-0.618)(0.1) = 0.038$ $\lambda_2 = 0.06$

$F(\lambda_1) = \lambda = 0.038$

$X_1 = -0.848$ $X_2 = 1$

Eval $\lambda_2 = 0.062$ $X_1 = -0.752$ $X_2 = 1$

$F(-0.752, 1) = 24.38$

Newton Method

$X_0 = [1, 1]^T$ $\nabla F = \begin{pmatrix} -400 X_1 (X_2 - X_1^2) - 2(1 - X_1) \\ 200 (X_2 - X_1^2) \end{pmatrix}$

$H = \begin{bmatrix} 1200 X_1^2 - 400 X_2 + 2 & -400 X_1 \\ -400 X_1 & 200 \end{bmatrix}$

$\nabla F[-1, 1] = \begin{bmatrix} -400(-1)(-1(-1)^2) - 2(1-(-1)) \\ 200(1-(-1)^2) \end{bmatrix} = \begin{bmatrix} 0 \\ -200 \end{bmatrix}$

$H[-1, 1] = \begin{bmatrix} 1200(-1)^2 - 400(1) + 2 & -400(-1) \\ -400(-1) & \end{bmatrix}$ $\begin{bmatrix} 802 & 400 \\ 400 & 200 \end{bmatrix}$

$X_1 = \lambda_0 - \lambda^{-1} \nabla F(X_0) = [-1.005, 1.008]$

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Quasi newton

$X_0 = [-1, 1]$ $B = I$ $\nabla F(X_0) = (0, 200)^T$

$X_1 = X_0 - B_0^{-1} \nabla F(X_0)$

$X_1 = (-1, 0.995)$

$X^* = (0,0)$ $S_1 = (1,0)$ $\nabla F = \begin{pmatrix} 8X_1 - 5X_2 - 8 \\ -5X_1 + 6X_2 \end{pmatrix}$

For X_0 $\nabla F \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

① $S_1 = (1,0)$ check it "oposite gradient"

② $F(X_1, 0) = 4X^2 - 8X$ $\frac{\partial F}{\partial X_1} = 8X_1 - 8$

$8X_1 - 8 = 0$ $X_1 = 1$ $X_1 = X_0 + \lambda S_1 = (0,0) + 1 \cdot (1,0) = (1,0)$

$F(X_1) = -4$

② $X_1 = (1, 0)$ direction $S_2 = (0, 1)$

$\nabla F = \begin{bmatrix} -8 \\ -5 \end{bmatrix}$

$F(1, X) = 4(1)^2 - 5X_2 + 3X_2^2 - 8$

$\boxed{F = -4 - 5X_2 + 3X_2^2}$

$\frac{\partial F}{\partial X_2} = -5 + 6X_2 \Rightarrow X_2 = \frac{5}{6}$ $X_2 = X_1 + \lambda S_2 = (1, \frac{5}{6})$

$F(X_2) = -9.72$

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③ $X_2 = (1, \frac{5}{6})$ $S_1 = (1, 0)$

$\nabla F = (1, \frac{5}{6}) = \begin{bmatrix} -8.33 \\ 0 \end{bmatrix}$

$F(x_1, \frac{5}{6}) = 4x_1^2 - 5x_1(\frac{5}{6}) + 3(\frac{5}{6})^2 - 8x_1$

$= 4x_1^2 - 8.33x_1 + 2.08$

$\partial F / \partial x_1 = 8x_1 - 8.33$

$X_3 = (2.67, \frac{5}{6})$ $F(X) = -12.89$

④ $X_3 = (2.67, \frac{5}{6})$ $S_2 = (0, 1)$

$\nabla F(2.67, \frac{5}{6}) = \begin{bmatrix} -1.67 \\ -5 \end{bmatrix}$

$S_2 = (0, 1)$ along X_1

$F(2.67, X_2) = 4(2.67)^2 - 5(2.67)X_2 + 3X_2^2 - 8(2.67)$

$F = 7.148 - 13.35X_2 + 3X_2^2$

$\partial F / \partial X_2 = -13.35 + 6X_2 \approx 0.33$

$X_4 = X_3 + \lambda S_2 = (2.67, \frac{5}{6}) + 0.33(0, 1) = (2.67, 1)$

$F(X_4) = -13.33$

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