Speech Recognition (DSAI 456)

Lecture 7

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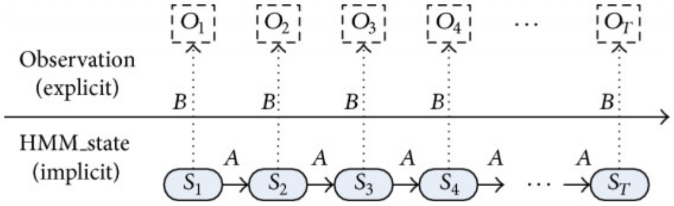
Recording is NOT allowed

Lecture 5 Recap

- HMM : sequence of observed and hidden states
- HMM model $\lambda = (A, B, \pi)$
- Problems
  - **Evaluation**: $p(O \mid \lambda)$
    - Direct Computation $O(TN^T)$
    - Forward Algorithm $O(TN^2)$
  - **Decoder**: $p(Q|O, \lambda)$
  - **Learning**: $\arg\max_\lambda p(O|\lambda)$



The Decoding Problem $p(Q|O, \lambda)$

Find the most likely sequence of hidden states that produced a given sequence of observations

The HMM Decoding Problem

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$$\delta_t(i) = \max_{q_1, \ldots, q_{t-1}} P(q_1, q_2, \ldots, q_{t-1}, q_t = i, o_1, o_2, \ldots, o_t \mid \lambda)$$

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You got it right! 👍 use Induction

Viterbi Algorithm

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- Initialization: the highest probability along a single path, at time 1, which accounts for the first observation o_1 and ends in state i

$$\delta_1(i) = \pi_i b_i(o_1), \quad 1 \leq i \leq N$$

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Where is the state sequence? 😢

We need to keep track of selected states along the path

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Backtrack from q_T^* to q_1^* using $\psi_t(i)$ to get the most probable state sequence

The Viterbi algorithm efficiently solves the decoding problem with complexity $O(N^2T)$

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