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(2) $\begin{bmatrix}1&2&3&5\\3&4&7&9\\5&7&9&1\end{bmatrix} \sim \begin{bmatrix}1&3&5&7\\0&-4&-6&-12\\5&7&9&1\end{bmatrix} \sim \begin{bmatrix}1&3&5&7\\0&-4&-6&-12\\0&-8&-16&-30\end{bmatrix}$

$\sim \begin{bmatrix}1&3&5&7\\0&1&2&3\\0&0&0&10\end{bmatrix} \sim \begin{bmatrix}1&0&-1&-2\\0&1&2&3\\0&0&0&1\end{bmatrix} \sim \begin{bmatrix}1&0&-1&0\\0&1&2&0\\0&0&0&1\end{bmatrix}$ inconsistent, No solution

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(2.1) $\begin{bmatrix}1&-6&4&2\\0&6&7&-7\\3&18&-12&-2\end{bmatrix} \sim \begin{bmatrix}1&0&11&-5\\0&6&7&-7\\0&0&0&4\end{bmatrix} \sim \begin{bmatrix}1&0&11&-5\\0&1&\frac{7}{6}&-\frac{7}{6}\\0&0&0&4\end{bmatrix}$

No Solution "inconsistent", B isn't a linear combination of A

(3.1) $\left[\begin{array}{ccc:c}0&0&6&A\\0&-6&-5&B\\-3&6&9&C\end{array}\right] \sim \left[\begin{array}{ccc:c}-3&6&9&C\\0&-6&-5&B\\0&0&6&A\end{array}\right]$

$C_3=\frac{A}{6}$, $C_2=-\frac{6+5A}{6}$

$C_1=-C=-\frac{9A}{18}=-\frac{1}{2}A$

(3.2) $\left[\begin{array}{cc:c}2&2&h\\-1&1&19\end{array}\right] \sim \left[\begin{array}{cc:c}2&2&h\\0&2&h+k\end{array}\right]$

$C_2=h+k$

$C_1=\frac{k}{2}-\frac{h-2k}{2}$

vector $\begin{bmatrix}h\\19\end{bmatrix}$ span$\left\{\begin{bmatrix}2\\-1\end{bmatrix},\begin{bmatrix}2\\1\end{bmatrix}\right\}$

(3.3) $\left[\begin{array}{ccc:c}2&0&6&10\\-1&8&5&3\\1&-2&1&3\end{array}\right] \sim \left[\begin{array}{ccc:c}2&0&6&10\\0&8&8&8\\0&-2&-2&-2\end{array}\right] \sim \left[\begin{array}{ccc:c}2&0&6&10\\0&8&8&8\\0&0&0&0\end{array}\right]$

C_3 Free variable

(B) Yes, B in span $C_1\begin{bmatrix}2\\0\\0\end{bmatrix}+C_2\begin{bmatrix}0\\8\\0\end{bmatrix}+C_3\begin{bmatrix}6\\8\\0\end{bmatrix}=\begin{bmatrix}2C_1+6C_3\\8C_2+8C_3\\0\end{bmatrix}$

(A) No, B isn't in span$\{a_1, a_2, a_3\}$

First

(4) $\begin{bmatrix} 4 & 4 & 8 & 16 \\ -12 & -12 & -24 & -48 \\ 0 & -7 & -21 & 14 \end{bmatrix} \sim \begin{bmatrix} 4 & 4 & 8 & 16 \\ 0 & 0 & 0 & 0 \\ 0 & -7 & -21 & 14 \end{bmatrix}$

$\begin{bmatrix} 4 & 4 & 8 & 16 \\ 0 & -7 & -21 & 14 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $x_2 = -2 + 3x_3$

$x_1 = 4 - 2x_3 - (-2 + 3x_3)$

$x_1 = 6 - 5x_3$

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 - 5x_3 \\ -2 + 3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 3 \\ 1 \end{bmatrix}$

second

$\begin{bmatrix} 4 & 4 & 8 & 0 \\ -12 & -12 & -24 & 0 \\ 0 & -7 & -21 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $x_2 = 3x_3$

$x_1 = -7x_3$

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7x_3 \\ 3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix}$

Assignment 1 Linear Algebra

1.1

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 5 & 6 & 7 & 8 \end{bmatrix} \xrightarrow[-5R_1+R_3]{-2R_1+R_2} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -4 & -8 & -12 \end{bmatrix} \xrightarrow{-4R_2+R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & -3 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{matrix} x_3 = 2 \\ x_2 = -1 \\ x_1 = 4 - 6 + 2 = 0 \end{matrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$

1.2

$$\begin{bmatrix} 1 & -6 & 4 & 2 \\ 0 & 6 & 7 & -7 \\ 3 & 18 & -12 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 25 & F \\ C & \delta & g \end{bmatrix} \begin{matrix} x_1 = F - 25x_2 \\ \delta x_2 = g - Cx_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 25 & F \\ 0 & \delta - 25C & g - CF \end{bmatrix}$$

inconsistent if $\delta - 25C \neq 0$ AND $g - CF \neq 0$

1.3 d

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 6 & 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & -5 & -10 & -15 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

consistent infinite num of sols