Page: Assignment 2 Linear Algebra Date: 25/7/2023

Q1 (i)

$$\begin{vmatrix} 1 & 5 & -6 \\ 1 & 6 & 5 \\ -2 & -4 & 7 \end{vmatrix} \sim \begin{vmatrix} 1 & 5 & -6 \\ 0 & 1 & 11 \\ 0 & 6 & 5 \end{vmatrix} \sim \begin{vmatrix} 1 & 5 & -6 \\ 0 & 1 & 11 \\ 0 & 0 & -61 \end{vmatrix}$$

$|A| = -61$

(ii)

$$\begin{vmatrix} 1 & -2 & -1 & 0 & -2 \\ 0 & 3 & 8 & 5 & -7 \\ -2 & 4 & 2 & 5 & -1 \\ 1 & -5 & 5 & -9 & -7 \\ 0 & 3 & 8 & 10 & 4 \end{vmatrix} \sim \begin{vmatrix} 1 & -2 & -1 & 0 & -2 \\ 0 & 3 & 8 & 5 & -7 \\ 0 & 0 & 0 & 5 & -5 \\ 0 & -3 & 6 & -9 & 5 \\ 0 & 3 & 6 & 10 & -4 \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & -2 & -1 & 0 & -2 \\ 0 & 3 & 8 & 5 & -7 \\ 0 & 0 & 14 & -4 & -2 \\ 0 & 0 & 0 & 5 & -9 \\ 0 & 0 & 0 & 0 & 8 \end{vmatrix} = 1680$$

1-2 $|A \cdot BC| = |I|$

$|A| \neq 0$

$|A| \cdot |B| \cdot |C| = 1$

$|A| \cdot |B| \cdot |C| = 1 \neq 0$

Q2 (i)

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \end{array}\right]$$

(ii) $$A^3 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 2 & 2 & 3 & -1 \end{vmatrix} \qquad |A| = -1 \qquad 2^4 X' - 1 = -16$$

$$A = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix}, \quad A^T = \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

$$\underbrace{\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{vmatrix}}_{A} \underbrace{\begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix}}_{X} \underbrace{=}_{=} \underbrace{\begin{vmatrix} 1 \\ 2 \\ 3 \\ 5 \end{vmatrix}}_{B} \qquad (A^T)^{-1} \begin{vmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{vmatrix} \begin{vmatrix} 1 \\ 2 \\ 3 \\ 5 \end{vmatrix}$$

$$X = \begin{bmatrix} 7 \\ -6 \\ 8 \\ -5 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & A & -1 \end{bmatrix} \quad \text{(i) Find } Tr(C + 2C^{T})$$

$$C^{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & A \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad C + 2C^{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & A & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & A \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & A \\ 0 & 0 & A & -2 \end{bmatrix}$$

$$Tr(C + 2C^{T}) = 4 \longrightarrow \boxed{1}$$

$$C^{-1} = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & A & -1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & A & -1 \end{array}\right] \Rightarrow C^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & A & -1 \end{bmatrix} \rightarrow \boxed{2}$$

$$(5C^{-1}) = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 5A & -5 \end{bmatrix}, \quad (5C)^{-1} = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 5A \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

$$|(5C^{-1})| = 5\begin{vmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 5A & -5 \end{vmatrix} = 25(-25 - 0) = -625 \rightarrow \boxed{3}$$

$$|(5C)^{-1}| = 5\begin{vmatrix} 5 & 0 & 0 \\ 0 & 5 & 5A \\ 0 & 0 & -5 \end{vmatrix} = 25(-25) = -625$$

Q4

i) False, Reduce Row echlon Form doesn't mean the matrix is invertible

ii) True, if ABC = I and A, B, C Are square matrix is invertible

iii) True

iv) True

v) False, (-1)'

vi) False, No Relation

vii) False, interchange changes the sign

viii) True

ix) True

(A)

$$A = \begin{bmatrix} 3 & -5 & -3 \\ -3 & 2 & 2 \\ 9 & -6 & -7 \end{bmatrix}$$

i)

$$A^{-1} = \left[\begin{array}{ccc|ccc} 3 & -5 & -3 & 1 & 0 & 0 \\ -3 & 2 & 2 & 0 & 1 & 0 \\ 9 & -6 & -7 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 3 & -5 & -5 & 1 & 0 & 0 \\ 0 & -3 & -1 & 1 & 1 & 0 \\ 0 & 9 & 2 & -3 & 0 & 1 \end{array}\right]$$

$3R_2 + R_3$

$$\left[\begin{array}{ccc|ccc} 3 & -5 & -5 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -\frac{5}{3} & -\frac{5}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -3 & -1 \end{array}\right]$$

$$\cancel{A^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & -3 & -1 \end{bmatrix}}$$

$-\frac{1}{3}R_3 + R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -\frac{5}{3} & -\frac{5}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & -3 & -1 \end{array}\right]$$

$\frac{5}{3}R_2 + R_1$, $\frac{5}{3}R_3 + R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{9} & \frac{35}{9} & -\frac{10}{9} \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & -3 & -1 \end{array}\right]$$

I A^{-1}

ii) $|A| = 3\overset{-6}{(-14+12)} + 5\overset{15}{(21-18)} - 3\overset{0}{(18-18)} = 9$

$$\frac{1}{9}\begin{bmatrix} \begin{vmatrix} 2 & 2 \\ -6 & -7 \end{vmatrix} & -\begin{vmatrix} -3 & 2 \\ 9 & -7 \end{vmatrix} & +\begin{vmatrix} -3 & 2 \\ 9 & -6 \end{vmatrix} \\ -\begin{vmatrix} -5 & -3 \\ -6 & -7 \end{vmatrix} & +\begin{vmatrix} 3 & -3 \\ 9 & -7 \end{vmatrix} & -\begin{vmatrix} 3 & -5 \\ 9 & -6 \end{vmatrix} \\ \begin{vmatrix} -5 & -3 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & -3 \\ -3 & 2 \end{vmatrix} & +\begin{vmatrix} 3 & -5 \\ -3 & 2 \end{vmatrix} \end{bmatrix}^T = \frac{1}{9}\begin{bmatrix} -2 & -3 & 0 \\ +17 & 6 & -17 \\ -4 & 3 & -9 \end{bmatrix}^T$$

$$\frac{1}{9}\begin{bmatrix} -2 & 17 & -4 \\ -3 & 6 & 3 \\ 0 & -17 & -9 \end{bmatrix} \sim \begin{bmatrix} -\frac{2}{9} & \frac{17}{9} & -\frac{4}{9} \\ -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & -3 & -1 \end{bmatrix}$$

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(D) $A = \begin{bmatrix} 3 & -5 & -3 \\ -3 & 2 & 2 \\ 9 & -6 & -7 \end{bmatrix}$, $B \begin{bmatrix} 1 \\ 6 \\ 0 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 3 & -5 & -3 & 1 \\ -3 & 2 & 2 & 6 \\ 9 & -6 & -7 & 8 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -2 & -3 & 1 \\ 3 & -9 & 0 & -9 & 6 \\ -1 & 2 & 4 & 7 & 0 \\ -3 & -6 & 26 & 2 & 3 \end{array}\right] \sim L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ -3 & -3 & -2 & 1 \end{bmatrix}, U = \begin{bmatrix} 1 & -2 & -2 & -3 \\ 0 & -3 & 6 & 0 \\ 0 & 0 & -5 & -10 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

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(A)

$$A = \begin{bmatrix} -3 & -5 & -3 \\ -3 & 2 & 2 \\ 9 & -6 & -7 \end{bmatrix}$$

(i)

$$A^{-1} = \left[\begin{array}{ccc|ccc} -3 & -5 & -3 & 1 & 0 & 0 \\ -3 & 2 & 2 & 0 & 1 & 0 \\ 9 & -6 & -7 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 3 & -5 & -5 & 1 & 0 & 0 \\ 0 & -3 & -1 & 1 & 1 & 0 \\ 0 & 9 & 2 & -3 & 0 & 1 \end{array}\right]$$

$3R_2+R_3$

$$\left[\begin{array}{ccc|ccc} 3 & -5 & -5 & 1 & 0 & 0 \\ 0 & -3 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -\frac{5}{3} & -\frac{5}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -3 & -1 \end{array}\right]$$

$$\cancel{A^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & -3 & -1 \end{bmatrix}}$$

$-\frac{1}{3}R_3+R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -\frac{5}{3} & -\frac{5}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & -3 & -1 \end{array}\right] \xrightarrow{\frac{5}{3}R_2+R_1,\ \frac{5}{3}R_3+R_1} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{-9} & -\frac{35}{9} & -\frac{10}{9} \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & -3 & -1 \end{array}\right]$$

I $\quad A^{-1}$

(ii)

$$|A| = 3(-14+12)+5(21-18)-3(18-18) = 9$$

$$\frac{1}{9}\begin{bmatrix} \begin{vmatrix} 2 & 2 \\ -6 & -7 \end{vmatrix} & -\begin{vmatrix} -3 & 2 \\ 9 & -7 \end{vmatrix} & +\begin{vmatrix} -3 & 2 \\ 9 & -6 \end{vmatrix} \\ -\begin{vmatrix} -5 & -3 \\ -6 & -7 \end{vmatrix} & +\begin{vmatrix} 3 & -3 \\ 9 & -7 \end{vmatrix} & -\begin{vmatrix} 3 & -5 \\ 9 & -6 \end{vmatrix} \\ \begin{vmatrix} -5 & -3 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & -3 \\ -3 & 2 \end{vmatrix} & +\begin{vmatrix} 3 & -5 \\ -3 & 2 \end{vmatrix} \end{bmatrix}^T = \frac{1}{9}\begin{bmatrix} -2 & -3 & 0 \\ +17 & 6 & -17 \\ -4 & 3 & -9 \end{bmatrix}^T$$

$$\frac{1}{9}\begin{bmatrix} -2 & 17 & -4 \\ -3 & 6 & 3 \\ 0 & -17 & -9 \end{bmatrix} \sim \begin{bmatrix} -\frac{2}{9} & \frac{17}{9} & -\frac{4}{9} \\ -\frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & -\frac{17}{9} & -1 \end{bmatrix}$$