Speech Recognition (DSAI 456)

Lecture 6

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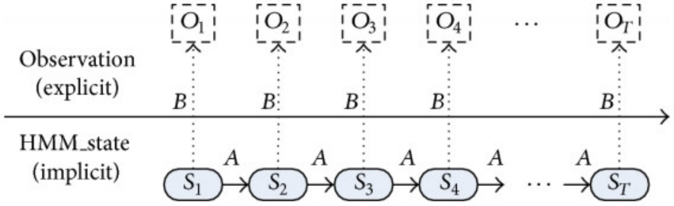
Lecture 5 Recap

- HMM : sequence of observed and hidden states
- HMM model $\lambda = (A, B, \pi)$
- Problems
  - Evaluation: $p(O \mid \lambda)$
  - Decoder: $p(Q|O, \lambda)$
  - Learning: $\arg\max_{\lambda} p(O|\lambda)$



Agenda

- Evaluation
- Decoder
- Learning

Evaluation Problem of HMM - Direct Computation

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There are N^T possible state sequences, each requires $2T$ calculations → exponential complexity

Can we find a faster algorithm to compute the likelihood?

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How to compute $\alpha_t(i)$? inductively

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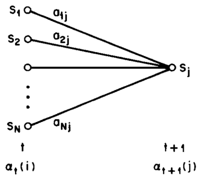
$$\alpha_{t+1}(j) = \left(\sum_{i=1}^{N} \alpha_t(i) a_{ij} \right) b_j(o_{t+1}), \quad 1 \le j \le N,\ 1 \le t \le T-1$$



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- Computing $\alpha_t(i)$ using above steps enables efficient likelihood evaluation with complexity $O(N^2T)$

Now we improved the time complexity of the likelihood evaluation $p(O \mid \lambda)$ from $N^T T$ to $N^2 T$ using Forward algorithm (induction trick)

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