Assignment 4

Q1 (i) $A = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix}$, $V_1 = \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$ $V_2 = \begin{vmatrix} 0 \\ 1 \\ -1 \end{vmatrix}$

$$\begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 4 \\ 2 \\ 4 \end{vmatrix}$$

V_1 not eigen Vector

V_2 is eigenVector

eigenValue $= 2$

eigenspace $= \text{span}\left\{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right\}$

$$\begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 1 \\ -1 \end{vmatrix} = \begin{vmatrix} 0 \\ 2 \\ -2 \end{vmatrix} = 2V_2$$

(ii), (iii) $\begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{vmatrix} \Rightarrow \begin{vmatrix} 3-\lambda & 1 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 1 & 3-\lambda \end{vmatrix}$

$(3-\lambda)(3-\lambda)(3-\lambda) - 1 - (3-\lambda) - 1 + 1 - (3-\lambda) \Rightarrow$

$27 - 3\lambda^2 - 9\lambda - \lambda^2 - 1 - 3 - \lambda - 41 - 3 - \lambda$

$(3-\lambda)\{(3-\lambda)(3-\lambda) - 1\} - [(3-\lambda) - 1] + [1 - (3-\lambda)]$

$(3-\lambda)(\lambda^2 - 6\lambda + 8) - 2 + \lambda + \lambda - 2$

$-\lambda^3 + 9\lambda^2 - 24\lambda + 2 \Rightarrow \lambda = 5, \lambda = 2, \lambda = 2$

$\lambda = 2$ $\qquad\qquad$ $\lambda = 5$

$v = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\qquad\qquad$ $v = \begin{vmatrix} 2 \\ 2 \\ 1 \end{vmatrix}$

$P = \begin{vmatrix} 0 & 0 & 2 \\ 1 & 1 & 2 \\ -1 & -1 & 1 \end{vmatrix}$ $\qquad$ $D = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{vmatrix}$

$A = P \cdot D \cdot P^{-1}$

$B = A^3 = P \cdot D^3 \cdot P^{-1} = \begin{vmatrix} 0 & 0 & 2 \\ 1 & 1 & 2 \\ -1 & -1 & 1 \end{vmatrix} \cdot \begin{vmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{vmatrix} \cdot \begin{vmatrix} 0 & 0 & 2 \\ 1 & 1 & 2 \\ -1 & -1 & 1 \end{vmatrix}^{-1}$

B is singular $\qquad$ A simitric

(iv) $A^{-1} = \begin{vmatrix} 0 & 0 & 2 \\ 1 & 1 & 2 \\ -1 & -1 & 1 \end{vmatrix} \cdot \begin{vmatrix} \frac{1}{8} & 0 & 0 \\ 0 & \frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{8} \end{vmatrix} \cdot \begin{vmatrix} 0 & 0 & 2 \\ 1 & 1 & 2 \\ -1 & -1 & 1 \end{vmatrix}^{-1}$

(Q2) $\lambda = 2$ $\quad \begin{vmatrix} 3 & 2 & 0 \\ 2 & 4 & 2 \\ 0 & 2 & 5 \end{vmatrix}$ $\quad$ A ⇒ diagonalizable ⇒ semitric

$(3-\lambda)((4-\lambda)(5-\lambda)-4) - 2(2(5-\lambda))$

$= (3-\lambda)(20-4\lambda-5\lambda+\lambda^2-4) - 20+4\lambda = 60 - 72\lambda - 75\lambda + \lambda$

$= -\lambda^3 + 9\lambda^2 - 94\lambda + 28$ $\qquad$ $b = 1$

$P = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$

Q3 $B \in M_{3\times 3}$ $\quad B\underset{\uparrow}{X_1} = \underset{\uparrow}{0} \cdot X_1$ $\quad X_2 \Rightarrow \lambda_2 = 1$

eigenvalue $\lambda = 0$ $\quad X_3 \to \lambda_3 = -1$

$B \Rightarrow \overset{x_1}{\lambda = 0}, \overset{x_2}{\lambda = 1}, \overset{x_3}{\lambda = -1}$

$P = \begin{vmatrix} | & | & | \\ x_1 & x_2 & x_3 \\ | & | & | \end{vmatrix}$

(i) NO, $\lambda_1 = 0$, (ii) yes, there was 3 different eigenvalue

(iii) $B^2 = \begin{vmatrix} | & | & | \\ x_1 & x_2 & x_3 \\ | & | & | \end{vmatrix} \cdot \begin{vmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \cdot \begin{vmatrix} | & | & | \\ x_1 & x_2 & x_3 \\ | & | & | \end{vmatrix}^{-1}$

Q4 $A = \begin{vmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{vmatrix}$ $\quad V = \begin{vmatrix} 5 \\ 0 \\ 1 \end{vmatrix}$ $\quad (2-\lambda)[(-1-\lambda)(-\lambda) - 12] + 2(2\lambda - 6) + 3(-4 - (-1-\lambda))$

$\Rightarrow -\lambda^3 + \lambda^2 + 18\lambda = 0$

$\lambda = 0$ $\quad \lambda = -4$ $\quad \lambda = 5$

$\begin{vmatrix} 2 \\ 1 \\ 0 \end{vmatrix}$ $\quad \begin{vmatrix} -1 \\ 1 \\ 0 \end{vmatrix}$ $\quad \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix}$

$v = 5$, $k = -1$, eigenspace $= \text{span}\left\{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$

$P = \begin{vmatrix} 2 & -1 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$