

# Speech Recognition (DSAI 456)

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## Lecture 5

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# Lecture 4 Recap

- GMM: Mixture of  $k$  Gaussians
- EM algorithm
  - Expectation: Estimate the responsibilities
  - Maximization: Estimate the parameters of the  $k$  components
- Speaker Identification: one GMM for each speaker

# Agenda

- Motivation
- Mathematical formulation
- Fundamental Problems for HMM
- Forward algorithm
- Using HMMs in speech recognition

# Motivation

**Source:** The actual phonemes or spoken words being uttered by a speaker

**Observed:** while the acoustic signal features or audio measurements recorded by a microphone.

# Motivation

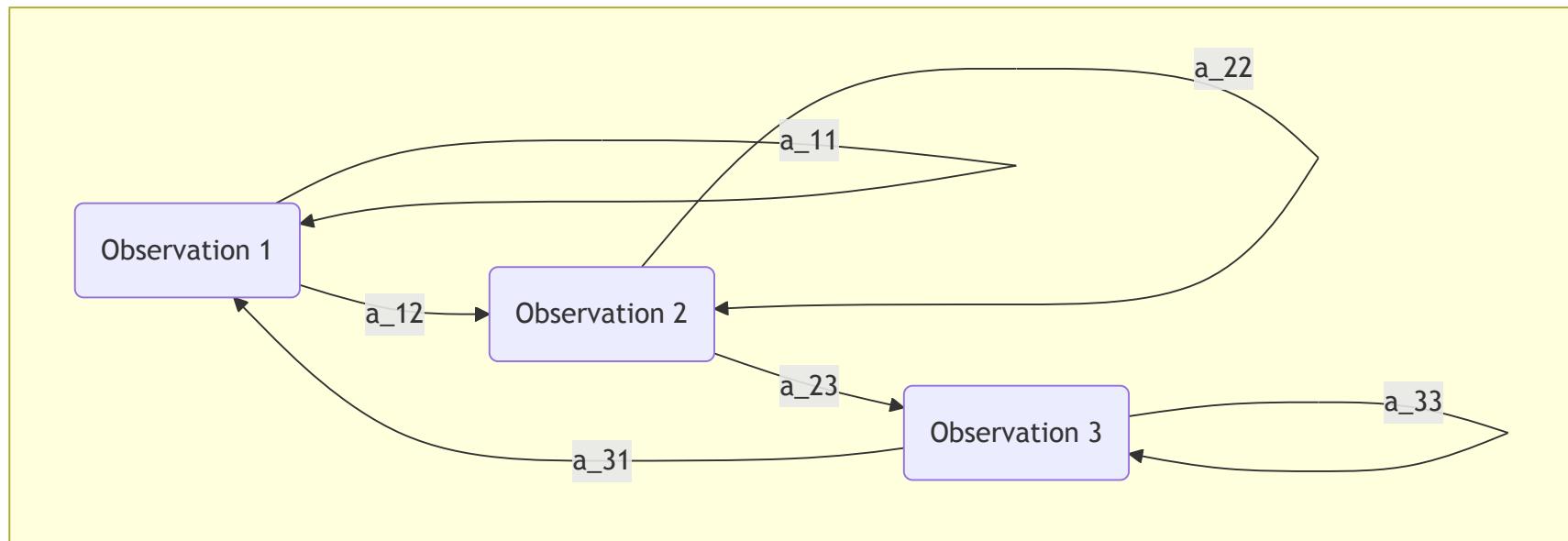
**Source:** The actual phonemes or spoken words being uttered by a speaker

**Observed:** while the acoustic signal features or audio measurements recorded by a microphone.

- The observed signal may have noise so we need to build a signal model for the source
  - to design a system to remove noise and any transmission distortion
  - to learn about the signal source (process that produced the signal) via simulation

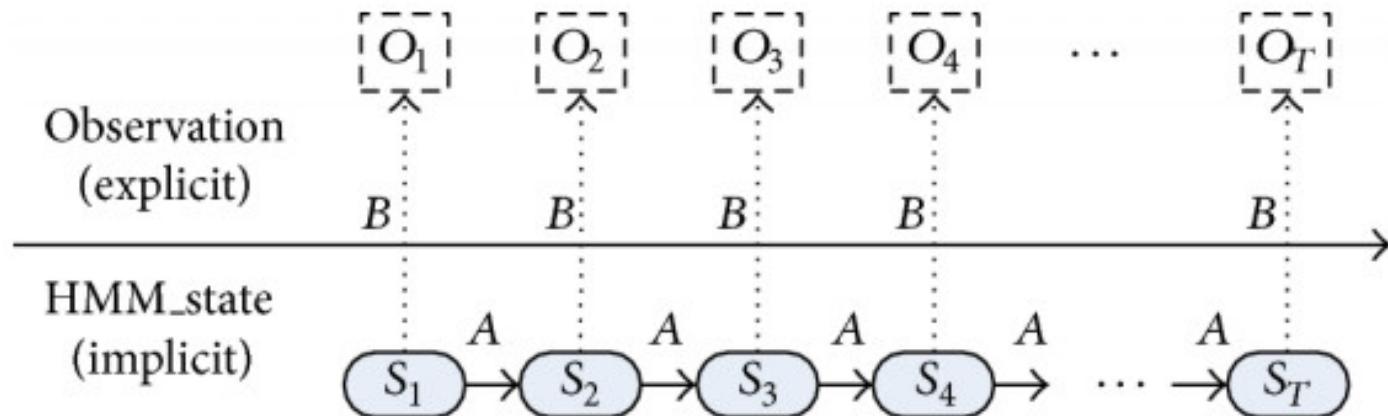
# (Observable) Markov Models

- model is the set of states and transition probability matrix
- we can compute  $p(O|model)$ , where  $O$  is the sequence of observation

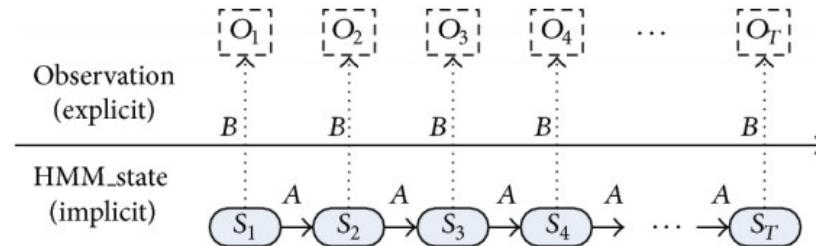


# Hidden Markov Models (HMM)

- Double stochastic processes (one is hidden which is responsible for generating the process which is observable)



# Elements of HMM



- states (hidden)  $S$
- alphabets (observation symbols)  $O$
- transition probability distribution  $A$
- observation symbol probability (emission probability)  $B$
- initial state distribution  $\pi$

# HMM Fundamental Problems: Speech Recognition Example

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## 1. Evaluation Problem

- Given an HMM model for a word (states = phonemes, probabilities learned)
- Given an observed acoustic sequence  $o$  (MFCC features)
- Compute likelihood  $p(o | \text{model})$  that the word model generated the audio

Evaluate models on  
audio input to score  
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## 3. Learning Problem

- Given training acoustic sequences without phoneme labels
- Estimate transition and emission probabilities to maximize data likelihood (Baum-Welch)

Learn model  
parameters from  
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## 2. Decoding Problem

- Given model and observed audio features
- Find most likely phoneme state sequence that produced the audio (Viterbi)

Decode best phoneme path to recognize speech content

## 3. Learning Problem

- Given training acoustic sequences without phoneme labels
- Estimate transition and emission probabilities to maximize data likelihood (Baum-Welch)

Learn model parameters from training data to improve accuracy

# Fundamental Problems for HMM

1. **Evaluation problem:** Compute  $p(o|\lambda)$  the probability of the observation sequence given the model  $\lambda = \{A, B, \pi\}$
2. **Decoding problem:** Estimate the state sequence given observation sequence and the model
3. **Learning problem:** Learn model parameters, i.e.  $\arg \max_{\lambda} p(o|\lambda)$

# Evaluation Problem of HMM

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# Evaluation Problem of HMM - Definition

## Input

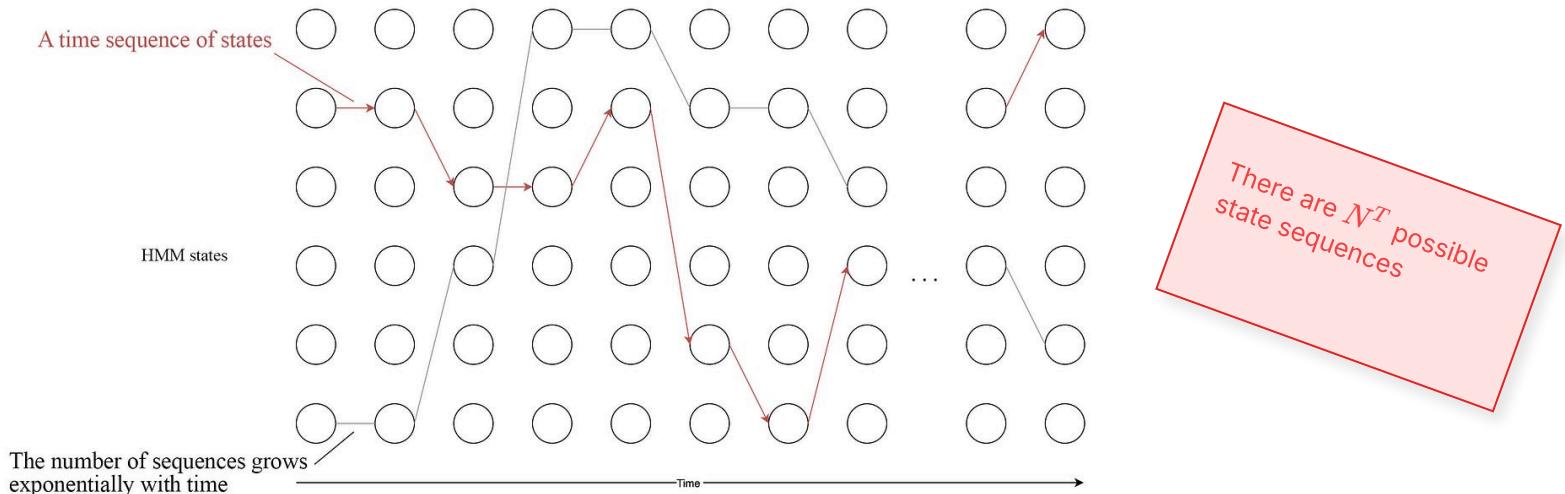
- Observation sequence  $O = (o_1, o_2, \dots, o_T)$
- HMM model  $\lambda = (A, B, \pi)$  where
  - $A = \{a_{ij}\}$  transition probabilities
  - $B = \{b_j(o_t) = p(o_t | s_j)\}$  emission probabilities
  - $\pi = \{\pi_i\}$  initial state distribution

## Output

The likelihood  $P(O | \lambda)$  which is the probability that model  $\lambda$  produces  $O$

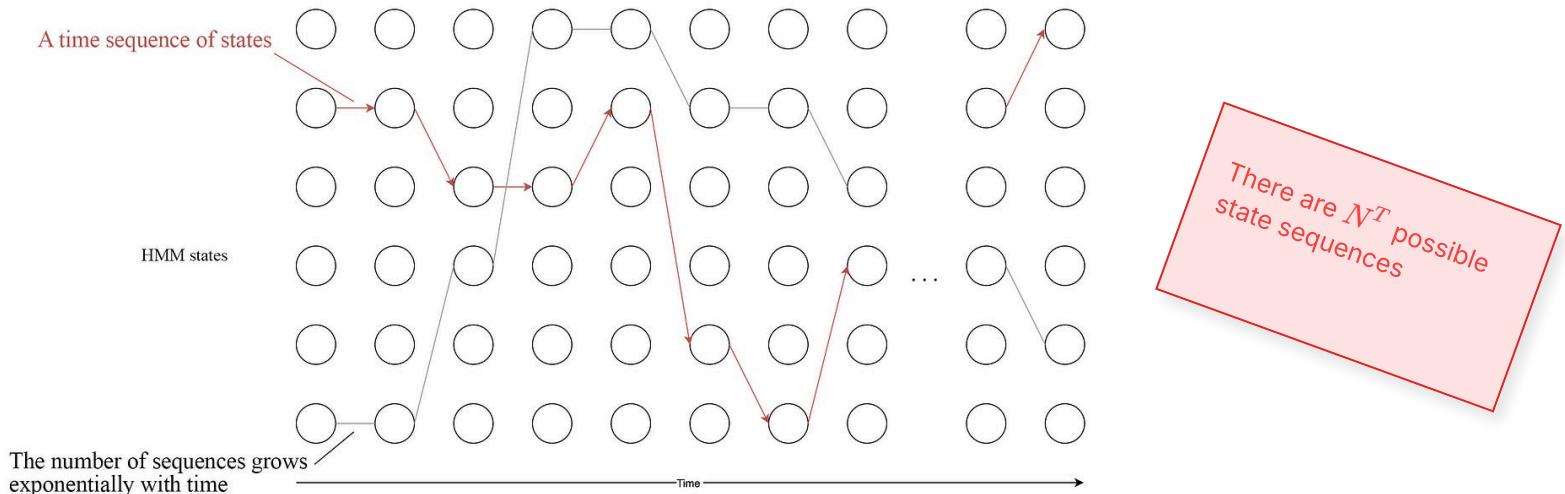
# Evaluation Problem of HMM - Direct Computation

- The observation could be generated from **any** possible state sequence  $Q = (q_1, q_2, \dots, q_T)$



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$$P(O | Q, \lambda) = b_1(o_1) \cdot b_2(o_2) \cdot b_3(o_3) \dots b_T(o_T)$$

$$P(Q | \lambda) = \pi_1 \cdot a_{12} \cdot a_{23} \cdot a_{34} \dots a_{(T-1)T}$$

$$\text{Then } P(O, Q | \lambda) = \pi_1 b_1(o_1) \prod_{t=2}^T a_{(t-1)t} b_t(o_t)$$

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- Initially we are at time 1, we are in state  $q_1$  with probability  $\pi_1$
  - Generate the symbol  $o_1$  with probability  $b_1(o_1)$
  - Make a transition from state  $q_1$  to state  $q_2$  with probability  $a_{12}$
  - Generate the symbol  $o_2$  with probability  $b_2(o_2)$
  - Continue until the last state
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There are  $N^T$  possible state sequences, each requires  $2T$  calculations → exponential complexity

**Can we find a faster algorithm to compute the likelihood?**

## Learn More

[Course Homepage](#)