Q1 (i)
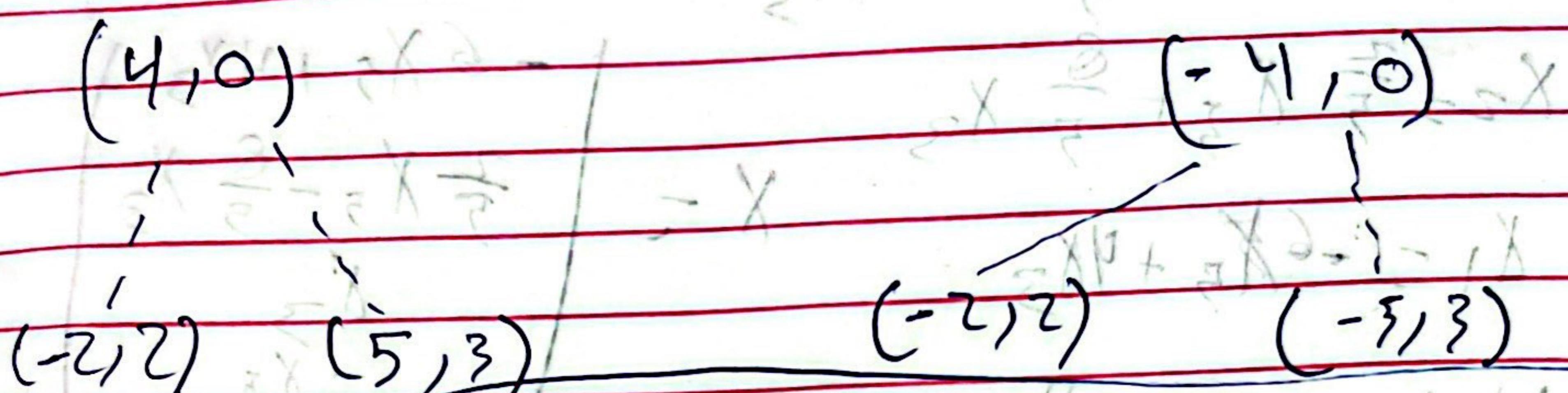
$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 2 & 5 \\ 0 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AA = \begin{bmatrix} -4 & -2 & -5 \\ 0 & 2 & 3 \end{bmatrix}$$

$(4,0)$ $(-4,0)$

$(-2,2)$ $(5,3)$ $(-2,2)$ $(-5,3)$

(ii) Transform

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\text{Scale} = \begin{vmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\text{rotation} = \begin{vmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Scale × rotation × transform
$$\begin{vmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{3}{2} & 0 & 4 \\ 0 & 0 \end{vmatrix}$$

Q2 (i)

$$\begin{vmatrix} 1 & 0 & 12 & -4 & -10 \\ -1 & 0 & 4 & 3 & 3 \\ -2 & -5 & 0 & 5 & 5 \\ 3 & 35 & 44 & -4 & -8 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & 12 & -4 & -10 \\ 0 & 1 & 6 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

2 Free variables

q = 2

P = 3

not subspace

(ii)

$$\begin{vmatrix} 1 & 0 & 12 & -4 & -10 & 1 \\ -1 & 0 & 4 & 3 & 3 & 1 \\ -2 & -5 & 0 & 5 & 5 & 1 \\ 3 & 35 & 44 & -4 & -8 & 1 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & 12 & -4 & -10 & 1 \\ 0 & 0 & 10 & -1 & -7 & 2 \\ 0 & 0 & -6 & 1 & 1 & -9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Rank A = 3

Rank A/B = 4

consistent

dim = 2

span R4

(iii)

$$Col(A) = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 10 \\ 0 \\ -3 \\ =35 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 5 \\ -4 \end{bmatrix} \right\} \quad dim = 3 \text{ in } R^4$$

(4) $-2x - \dfrac{3}{2}x_4 = \dfrac{8}{2}x_5$ $3x_4 = \dfrac{-9}{2}x_5 - 4$

$\therefore x_4 = 3x_5$

$10x_2 = 4x_5 + 16x_3$

$x_2 = \dfrac{2}{5}x_5 + \dfrac{8}{5}x_3$

$x_1 = -6x_5 + 4x_3$

$dim = 2 \text{ in } R^4$

$$x = \begin{bmatrix} -6x_5 + 4x_3 \\ \dfrac{2}{5}x_5 - \dfrac{8}{5}x_3 \\ x_3 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$Nul\, A = \left\{ \begin{bmatrix} 4 \\ -\dfrac{8}{5} \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} -6 \\ \dfrac{2}{5} \\ 0 \\ 3 \end{bmatrix} \right\}$$

(vi) $Row(A) = \begin{bmatrix} 1 & 0 & -1 & -\dfrac{2}{3} \\ 10 & 0 & 0 & \dfrac{1}{2} \\ 12 & 4 & 1 & 0 \\ -4 & 3 & 0 & 5 \\ 6 & 3 & 1 & 9 \end{bmatrix}$ dim is 3 in R^4

(b) $\begin{bmatrix} 1 & 4 & 3 & -3 & 2 \\ -1 & -5 & -1 & 6 & 6 \\ -2 & -7 & -8 & 3 & 0 \\ 5 & 16 & 23 & -3 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 9 & -14 \\ 0 & 1 & 0 & -3 & 4 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $dim\ Col A = 3$

$Row A = 3$

$x_3 = 0$

$x_2 = 3x_3 = 4x_5$

$x_1 = -9x_3 + 14x_5$

$dim = 2 \text{ in } R^4$

$$Nul\, A = \left\{ \begin{bmatrix} 9 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 14 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Q5 (i) ⟹ not subspace because relation between z & x, w is dependent so that it is subspace

(ii) not subspace, because it is dependent.

(iii) subspace because it is independent.

(iv) subspace, because it is independent.

Q2

$$(X)_B = \begin{bmatrix} 0 & 2 & -4 \\ -2 & 1 & -6 \end{bmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \end{vmatrix}$$

$$(X)_B = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$(y)_B = \begin{vmatrix} 0 & 2 & 4 \\ -2 & 1 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$(z)_B = \begin{vmatrix} 0 & 2 & -1 \\ -2 & 1 & 3.5 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & -2 \\ 0 & 1 & -\frac{1}{5} \end{vmatrix}$$

$$= \begin{vmatrix} -2 \\ -\frac{1}{2} \end{vmatrix}$$