Assignment 3

1)

$$\min c^T x$$

$$\text{s.t } Ax \geq b$$

$$x \geq 0$$

$$L(x,y) = c^T x + y^T (b - Ax) - s^T x$$

$$q(y,s) = \inf[c^T x - y^T (Ax - b) - s^T x]$$

$$q(y,s) = b^T y + \inf[(c^T - y^T A - s^T) x]$$

$(c^T - y^T A - s^T) x$ is bounded if and only if

$c - A^T y - s = 0$ ⇒ stat cond

$A^T y + s = c \quad s, y \geq 0$

$q(y,s) = b^T y \quad , A^T y \leq c ; y \geq 0$

Dual: $\max b^T y$ s.t $A^T y \leq c , y \geq 0$

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[2] 1)

P: $\min c^T x$ s.t $Ax = b$, $x \geq 0$

D: $\max b^T y$ s.t $A^T y \leq c$, y unrestricted

we can make dual of dual by the same steps to build dual,

in D, $\max b^T y$ with constrains, in DD: $\min c^T x$

$L(\lambda)$: $\min = -\max$

in D, $A^T y \leq c$, but in DD it will be $Ax = b$

	primal	dual	DD
ObJ:	$c^T x$	$b^T y$	$c^T x$
Const:	$Ax = b$,	$A^T y \leq c$,	$Ax = b$
	$x \geq 0$	y vector unrestricted	$x \geq 0$

DD: $\min c^T x$ s.t $Ax = b$, $x \geq 0$

$L(x, \lambda) = -b^T x + \lambda^T (A^T x - c)$

$q(\lambda) = \inf \{ -b^T x + \lambda^T (A^T x - c) \}$

$\inf \{ (b - A^T \lambda)^T x + c^T \lambda \}$

$b - A^T \lambda \geq 0 \Rightarrow \lambda$

$q(\lambda) = c^T \lambda$

$D^* \leq P^* \rightarrow$ weak duality

$P^* = d^* \rightarrow$ strong duality

$\therefore P^* - d^* = 0$

when strong duality holds

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Q3 : min $F(x) = x^2+1$

s.t $g(x) = x^2 - 6x + 8 \leq 0$

① primal: $g(x) = x^2 - 6x + 8 \leq 0$

$x^2 - 6x + 8 = (x-2)(x-4) \leq 0$

$F(x) = x^2 + 1$ $\qquad x \in [2, 4]$

$F(x)$ increase when $x > 0$; it's min in $x = 2$ $\downarrow$ x^*

$\therefore F(2) = 5$ opt value p^*

② $L(x, \lambda) = x^2 + 1 + \lambda(x^2 - 6x + 8)$

$= (1+\lambda)x^2 - 6\lambda x + (1+8\lambda)$

$g(\lambda) = \inf L(x, \lambda)$

lower bound $\therefore p^* \geq \inf L(x, \lambda)$, $\lambda \geq 0$

$\frac{\partial L}{\partial x} = 2(1+\lambda)x - 6\lambda = 0 \quad \therefore x = \frac{3\lambda}{1+\lambda}$

③ Max $g(\lambda)$ s.t $\lambda \geq 0$ $\rightarrow$ concave

x^2+1 s.t $x^2 - 6x + 8 \leq 0$

$L(x, \lambda) = (1+\lambda)x^2 - 6\lambda x + (1+8\lambda)$

$\frac{\partial L}{\partial x} = 2(1+\lambda)x - \lambda \Rightarrow x = \frac{3\lambda}{1+\lambda}$ $\qquad \frac{18\lambda}{1+\lambda}$

$\frac{9\lambda^2}{1+\lambda}$

$L\left(\frac{3\lambda}{1+\lambda}, \lambda\right) = (1+\lambda)\left(\frac{3\lambda}{1+\lambda}\right)^2 - 6\lambda\left(\frac{3\lambda}{1+\lambda}\right) + (1+8\lambda)$

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$g(\lambda) = -\frac{9\lambda^2}{1+\lambda} + 1 + 8\lambda \rightarrow \lambda^* = 2$

$\rightarrow$ concave $\frac{-2}{8} \leq 0$

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4) [1] $P: AX = b,\ X \geq 0$

$D: A^T y + s = c,\ s \geq 0$

slackness: $X^T s = 0$

$\nabla_x L(x,y,s) = 0 \ \& \ \nabla_y L(x,y,s) = 0$

$$\frac{\partial L}{\partial x} = c - A^T y - s = 0 \Rightarrow c = A^T y + s$$

$$\frac{\partial L}{\partial y} = -(AX - b) = 0 \Rightarrow AX = b$$

$$\frac{\partial L}{\partial s} = -X = 0 \Rightarrow X \geq 0$$

[2]

$$c^T X - b^T y = 0$$

$$X^T s = 0 \Rightarrow \sum X_i s_i = 0$$

$$s = c - A^T y \Rightarrow X^T (c - A^T y) = 0$$

$$X^T c - X^T (A^T y) = 0$$

$$X^T A^T = b^T$$

$$X^T c - b^T y = 0$$

$$c^T X = b^T y$$

$$\therefore c^T X - b^T y = 0$$

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4) (3)

$$F(x,y,s) = \begin{bmatrix} Ax - b \\ A^T y + s - c \\ XSe \end{bmatrix} = 0$$

$$J\,\nabla z = F(x,y,s)$$

$$\therefore \Delta x = -X^{-1}(XSe)$$