Name: ______________________________ **Roll no.:** ______________________________

QUESTION 1: (MCQs, 20 Marks, 20min)

1. The order of differential equation $y'' + xy\,(y')^2 = 0$ is ________.

a. 0 b. 1 c. 2 d. 3

2. The order of differential equation $\left[1 + (y')^2\right]^{1/2} = (y'')^2$ is ________.

a. 0 b. 1 c. 2 d. 3

3. The degree of differential equation $y' + (y')^2 + (y')^3 = 0$ is _____.

a. 0 b. 1 c. 2 d. 3

4. The differential equation $(y^2 + 2xy)dx + x^2 dy = 0$ is _______.

a. Exact b. Homogeneous c. Separable d. None

5. The differential equation $y''' + 4y'' - 5y' + 3y^2 = \cos x$ is _______.

a. Exact b. Homogeneous c. Linear d. Non Linear

6. The generalized form of Fourier Transformation is

a. $G\{F\}=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega t}dt$ b. $G\{F\}=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i2\pi f t}dt$ c. Both a&b d. None

7. Fourier Transformation is used to

a. Represent Wave Form b. Decompose wave form c. Both a&b d. None

8. The generalized form of Inverse Fourier Transformation is

a. $F^{-1}\{G(t)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty} G(F)e^{-i\omega t}dt$ b. $F^{-1}\{G(t)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty} G(F)e^{i\omega t}dt$ c. Both a&b d. None

9. Euler's Theorem is

a. $e^{it} = \cos t + i\sin t$ b. $e^{iat} = \cos at + i\sin at$ c. Both a&b d. None

10. Laplace Transformation is

a. $L\{F(t)\} = \int_{-\infty}^{\infty} e^{-st}$dt b. $L\{F(t)\} = \int_{0}^{\infty} e^{-st}$dt c. Both a&b d. None

QUESTION 2: (Short Questions, 14 marks)

1. Show that $L\,[\mathrm{Cos}\,h(at)] = \frac{s}{s^2-a^2}$. **(2 Marks)**

2. Apply Laplace transformation on $\mathrm{Sin}\,at$. **(2 Marks)**

3. Find $L\,\{\mathrm{Cos}^2\,at\}$. **(2 Marks)**

4. Using Shift Property Find Fourier Transformation of the given function, when t_0=4 towards left $f(x) = f(t) = \begin{cases} 1, & 3 \le t \le 5 \\ 0, & otherwise \end{cases}$. **(3 Marks)**

5. Find Fourier Transformation of function $f(t) = \begin{cases} 4, & -3 \le t \le 3 \\ 0, & otherwise \end{cases}$. **(3 Marks)**

6. Obtain $H_2(x)$ by means of Rodrigue Formula. **(2 Marks)**

QUESTION 3: (Long Questions, 16 marks)

1. Solve the differential Equation $y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$ by separating variables. **(6 Marks)**
2. Derive 2nd order Hermite Differential equation in terms of Generating Function. **(10 Marks)**

11. Generating Function Formula is

a. $e^{t^2+2tx} = \sum_0^\infty H_n(x)\frac{t(n)}{n!}$ b. $e^{-t^2+2tx} = \sum_0^\infty H_n(x)\frac{t(n)}{n!}$ c. Both a&b d. None

12. Differential Formula for Hermite Polynomial is

a. $\frac{d}{dx}\left(H_n(x)\right) = 2nH_{n-1}(x)$ b. $\frac{d}{dx}\left(H_{n-1}(x)\right) = 2nH_{n-1}(x)$ c. Both a&b d. None

13. According to change of scale property of laplace $L\{f(at)\} =$

a. $F\left(\frac{s}{a}\right)$ b. $\frac{1}{2a}F\left(\frac{s}{a}\right)$ c. $\frac{1}{2\pi}F\left(\frac{s}{a}\right)$ d. $\frac{1}{a}F\left(\frac{s}{a}\right)$

14. According to first shifting theorem of laplace $L[e^{at}f(t)] =$

a. $F(s+a)$ b. $F(s-a)$ c. $\frac{1}{2\pi}F\left(\frac{s}{a}\right)$ d. $\frac{1}{a}F\left(\frac{s}{a}\right)$

15. $L[\sin h(at)] =$

a. $\frac{a}{s^2+a^2}$ b. $\frac{a}{s^2-a^2}$ c. $\frac{a}{s^2-2a^2}$ d. $\frac{2a}{s^2+a^2}$

16. The integrating factor of $\frac{dy}{dx}+\frac{1}{x}y = 3x$ is

a. ex b. ey c. x d. y

17. The integrating factor of $\frac{dy}{dx}+y = 2x$ is

a. ex b. ey c. x d. y

17. The integrating factor of $-ydx+(x-y)dy = 0$ is

a. x^2 b. y2 c. $\frac{1}{x^2}$ d. $\frac{1}{y^2}$

18. $L\{f'(t)\} =$

a. $SL\{f(t)\} - f(1)$ b. $SL\{f(t)\} - f(t)$ c. $SL\{f'(t)\} - f(t)$ d. $SL\{f(t)\} - f(0)$

19. $L\{f''(t)\} =$

a. $S^2L\{f(t)\} - Sf(t) - f'(t)$ b. $S^2L\{f(t)\} - Sf''(t) - f'(t)$

c. $S^2L\{f(t)\} - Sf(0) - f'(0)$ d. $SL\{f(t)\} - f(0)$

20. $L(t^n) =$

a. $\frac{n!}{s^{n-1}}$ b. $\frac{n!}{s^n}$ c. $\frac{n!}{s^{n+1}}$ d. $\frac{n!}{s^{n+2}}$