

$$P = 8 \quad N = 6 \quad \text{Total} = 14$$

$$\begin{aligned} \text{Entropy}(S) &= \frac{P}{P+N} \log\left(\frac{P}{P+N}\right) + \frac{n}{P+N} \log\left(\frac{n}{P+N}\right) \\ &= \frac{8}{14} \log\left(\frac{8}{14}\right) + \frac{6}{14} \log\left(\frac{6}{14}\right) = \underline{\underline{0.989}} \end{aligned}$$

\* Entropy for each attribute:-

→ Early registration

	P	n	Entropy
1	4	2	0.918
0	4	4	1

\* Calculate Average Information Entropy:-

$$\begin{aligned} I(\text{early registration}) &= \frac{P_1 + n_1}{P+n} \text{Entropy}(1) + \frac{P_2 + n_2}{P+n} \text{Entropy}(0) \\ &= \frac{4+2}{14} \times 0.918 + \frac{8}{14} \times 1 = \underline{\underline{0.965}} \end{aligned}$$

\* Calculate Gain

$$\begin{aligned} \text{Gain} &= \text{Entropy}(S) - I(\text{Attribute}) \\ &= 0.989 - 0.965 = \underline{\underline{0.024}} \end{aligned}$$

→ Finished homework II :-

	P	n	Entropy
1	5	2	0.863
0	3	4	0.985

$$I(\text{Finished homework II}) = \frac{5+2}{14} \times 0.863 + \frac{3+4}{14} \times 0.985 = 0.924$$

$$\text{Gain} = 0.989 - 0.924 = \underline{\underline{0.065}}$$

→ Senior :-

	P	n	Entropy
1	5	3	0.954
0	3	3	1

$$I(\text{Senior}) = \frac{5+3}{14} \times 0.954 + \frac{6}{14} \times 1 = 0.974$$

$$\text{Gain} = 0.989 - 0.974 = \boxed{0.015}$$

→ Likes Coffee :-

	P	n	Entropy
1	3	1	0.8113
0	5	5	1

$$I(\text{Likes Coffee}) = \frac{3+1}{14} \times 0.8113 + \frac{10}{14} \times 1 = 0.946$$

$$\text{Gain} = 0.989 - 0.946 = \boxed{0.043}$$

→ Liked the last homework

	P	n	Entropy
1	5	4	0.991
0	3	2	0.971

$$I(\text{Liked the last homework}) = \frac{5+4}{14} \times 0.991 + \frac{5}{14} \times 0.971 = 0.984$$

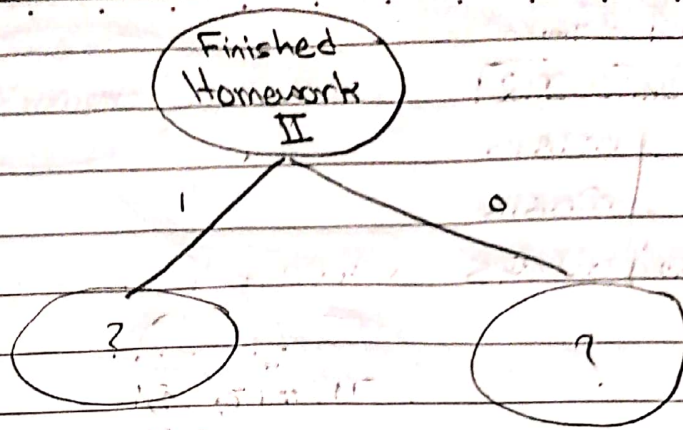
$$\text{Gain} = 0.989 - 0.984 = \boxed{0.005}$$

Attributes	Gain
Early registration	0.024
Finished homework II	0.065
Senior	0.015
Likes coffee	0.043
Liked the last homework	0.005

∴ Finished homework II  
will be the root node  
as it has the highest  
gain.







for (1):

$$P = 5 \quad N = 2 \quad \text{Total} = 7$$

$$\text{Entropy}(S) = -\frac{5}{7} \log\left(\frac{5}{7}\right) - \frac{2}{7} \log\left(\frac{2}{7}\right) = 0.863$$

→ Early registration =

	P	n	Entropy
1	3	0	0
0	2	2	1

$$I(\text{Early registration}) = \frac{3}{7} \times 0 + \frac{2+2}{7} \times 1 = 0.571$$

$$\text{Gain} = 0.863 - 0.571 = \underline{\underline{0.292}}$$

→ Senior =

	P	n	Entropy
1	3	2	0.9709
0	2	0	0

$$I(\text{Senior}) = \frac{5}{7} \times 0.9709 + 0 = 0.6935$$

$$\text{Gain} = 0.863 - 0.6935 = \underline{\underline{0.1695}}$$

→ Likes Coffee :-

	P	n	Entropy
1	1	1	1
0	4	1	0.7219

$$I(\text{Likes Coffee}) = \frac{2}{7} \times 1 + \frac{5}{7} \times 0.7219 = 0.8014$$

$$\text{Gain} = 0.863 - 0.8014 = \underline{\underline{0.0616}}$$

→ Likes the last homeworks

	P	n	Entropy
1	3	2	0.9709
0	2	0	0

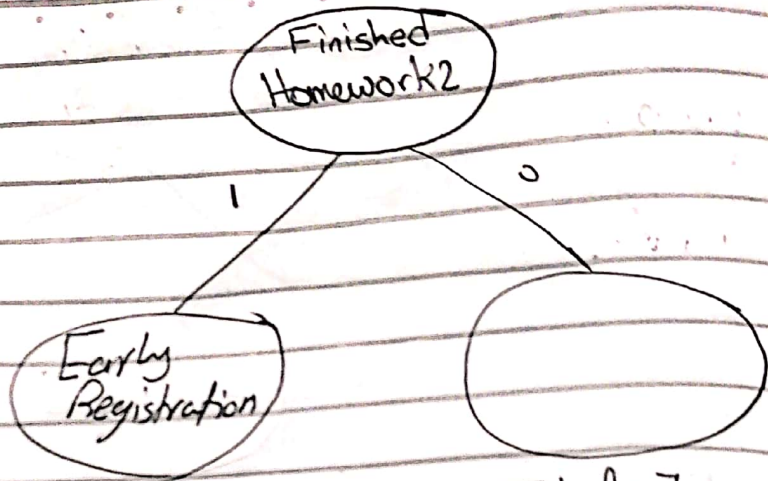
$$I(\text{LLH}) = \frac{5}{7} \times 0.9709 + 0 = 0.6935$$

$$\text{Gain} = 0.863 - 0.6935$$

$$= \underline{\underline{0.1695}}$$

ROX

Attribute	Gain
Early registration	0.202
Senior	0.1695
Likes Coffee	0.0616
Likes last homework	0.1695



for Finished Homeworks II = 0 :  $P=3$   $n=4$  Total = 7

→ Early registration:

Entropy(S) = 0.985

	P	n	Entropy
1	1	2	0.918
0	2	2	1

$$I(ER) = \frac{3}{7} \times 0.918 + \frac{4}{7} \times 1 = 0.9668$$

$$\text{Gain} = 0.985 - 0.9668 = \boxed{0.0202}$$

→ Senior

	P	n	Entropy
1	2	1	0.918
0	1	3	0.8113

$$I(\text{Senior}) = \frac{3}{7} \times 0.918 + \frac{4}{7} \times 0.8113 = 0.857$$

$$\text{Gain} = 0.985 - 0.857 = \boxed{0.128}$$

→ Likes Coffee

	P	n	Entropy
1	2	0	0
0	1	4	0.7219

$$I(LC) = \frac{2}{7} \times 0 + \frac{5}{7} \times 0.7219 = 0.515$$

$$\text{Gain} = 0.985 - 0.515 = \boxed{0.47}$$

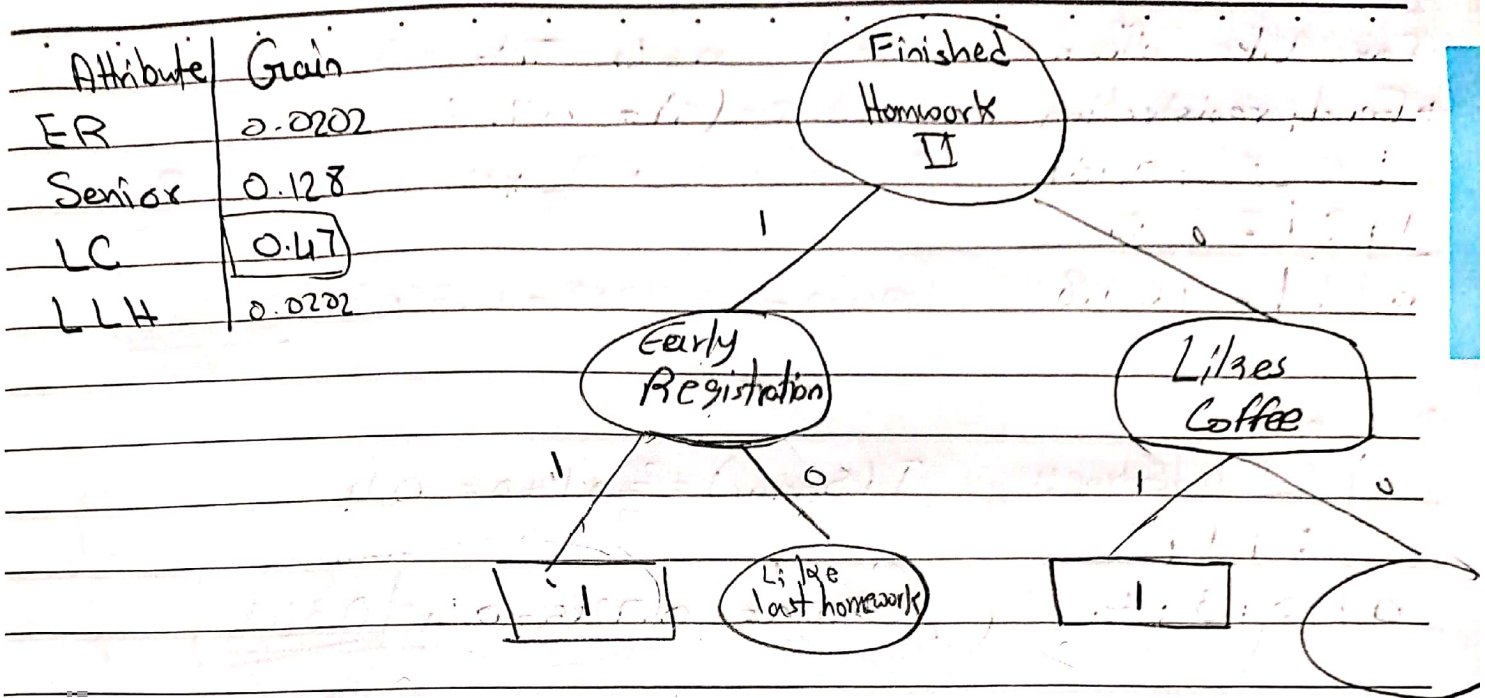
→ Likes Last Homework

	P	n	Entropy
1	2	2	1
0	1	2	0.918

$$I(LLH) = \frac{4}{7} \times 1 + \frac{3}{7} \times 0.918 = 0.9668$$

$$\text{Gain} = 0.985 - 0.9668 = \boxed{0.0202}$$





for ER = 0 :  $P = 2$   $n = 2$  Total = 4 Entropy(s) = 1.

→ Senior

P	n	Entropy
1	2	0.918
0	0	0

$$I(\text{Senior}) = \frac{3}{4} \times 0.918 + 0 = 0.6885$$

$$\text{Gain} = 1 - 0.6885 = 0.3115$$

→ Likes Coffee

P	n	Entropy
1	1	1
0	1	1

$$I(\text{LC}) = \frac{2}{4} \times 1 + \frac{2}{4} \times 1 = 1$$

$$\text{Gain} = 1 - 1 = 0$$

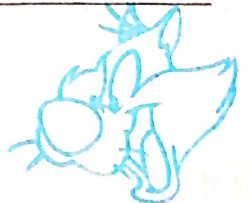
→ Likes last homework

P	n	Entropy
1	2	0.918
0	0	0

$$I(\text{LLH}) = 0.6885$$

$$\text{Gain} = 1 - 0.6885 = 0.3115$$

Attribute	Gain
Senior	0.3115
LC	0
LLH	0.3115



for Like Coffee = 0  $P=1$   $n=4$  Total = 5

→ Early registration

	P	n	Entropy
1	0	2	0
0	1	2	0.918

$$Entropy(S) = 0.7219$$

$$I(ER) = 0 + \frac{3}{5} \times 0.918 = 0.5508$$

$$Gain = 0.7219 - 0.5508 = \boxed{0.1711}$$

→ Senior

	P	n	Entropy
1	1	1	1
0	0	3	0

$$I(Senior) = \frac{2}{5} \times 1 + 0 = 0.4$$

$$Gain = 0.7219 - 0.4 = \boxed{0.3219}$$

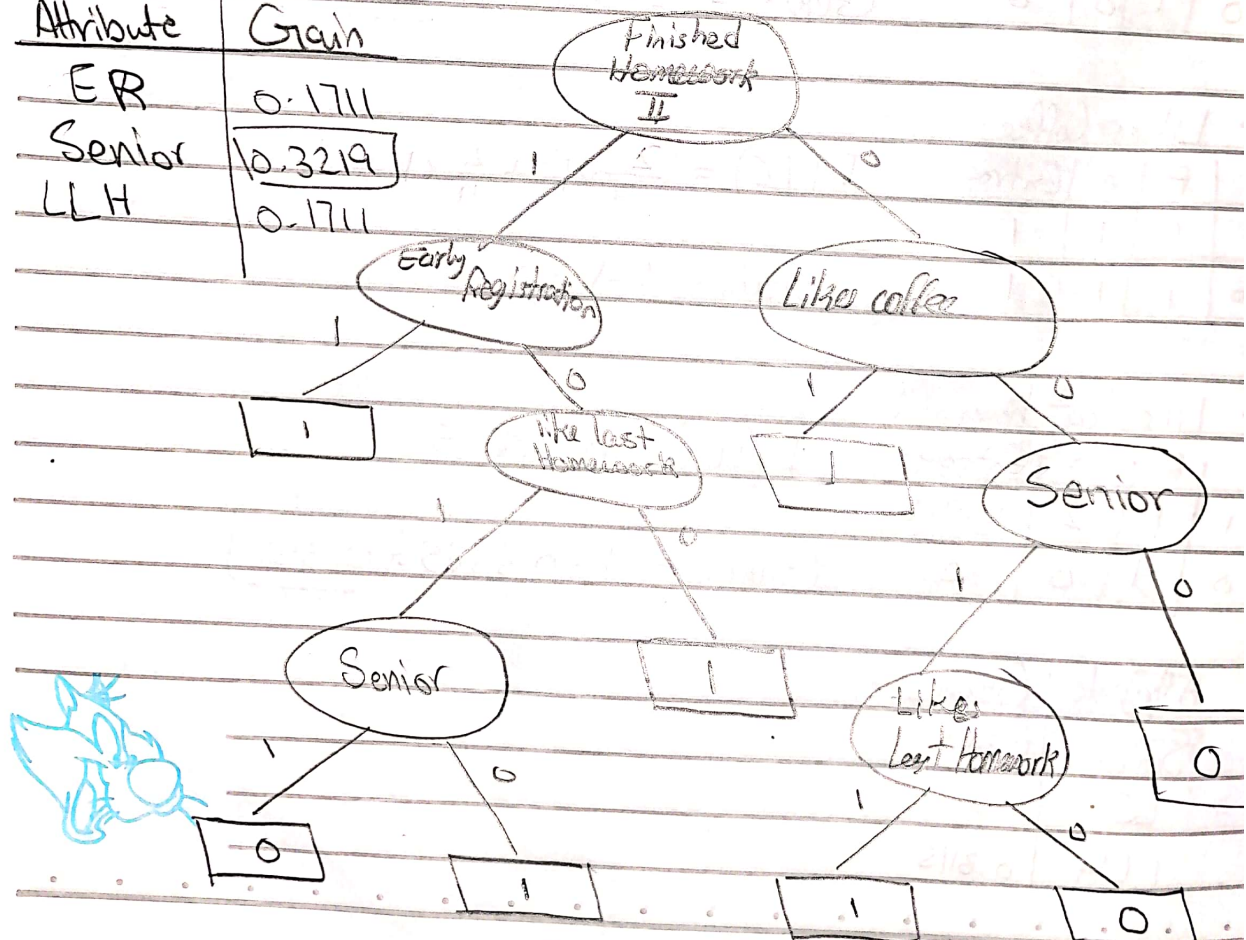
→ Likes Last homework

	P	n	Entropy
1	1	2	0.918
0	0	2	0

$$I(ER) = \frac{3}{5} \times 0.918 = 0.5508$$

$$Gain = \boxed{0.1711}$$

Attribute	Gain
ER	0.1711
Senior	<u>0.3219</u>
LLH	0.1711





for like last homework = 1  $P=1$   $n=2$  Total = 3 Entropy(s) = 0.918

Senior

	P	n	Entropy
1	0	2	0
0	1	0	0

$$I(\text{Senior}) = 0$$

$$\text{Gain} = 0.918 - 0 = 0.918$$

Like coffee

	P	n	Entropy
1	1	1	1
0	0	1	0

$$I(LC) = \frac{2}{3} = 0.67$$

$$\text{Gain} = 0.918 - 0.67 = \underline{\underline{0.248}}$$

for Senior = 1  $P=1$   $n=1$  Total = 2 Entropy(s) = 1

Early registration

	P	n	Entropy
1	0	0	0
0	1	1	1

$$I(ER) = \frac{2}{2} \times 1 = 1$$

$$\text{Gain} = 1 - 1 = 0$$

Like last Homework

	P	n	Entropy
1	1	0	0
0	0	1	0

$$I(LLH) = 0$$

$$\text{Gain} = 1 - 0 = \underline{\underline{1}}$$

