CS 7650 Midterm (Spring 2022)

April 11, 2022

Please submit your solutions on Gradescope.

1 Word Embeddings

In this question, we will investigate the training objectives used in the WORD2VEC algorithm. This question may require you to refer to Chapters 14.5, 14.6 of the Eisenstein readings.

Here is a sentence for which the algorithm will make a prediction for the missing word. The word embedding for each word in the context has been given.

Index Position	**Word**	**Embedding**
0	the	$[2, 1]$
1	quick	$[3, 2]$
2	brown	$[-2, 0]$
3	?	?
4	jumped	$[5, -1]$
5	over	$[2, -3]$
6	the	$[2, 1]$
7	lazy	$[-3, -1]$
8	dog	$[1, 2]$

Table 1: Word Embeddings for the Input Sentence.

1. Compute the Continuous Bag-of-Words (CBOW) vector representation of the missing word for a context window h of size 3. Show your work.

2. We've subset the vocabulary down to the words in Table 2. Fill in the scores of each word being the missing word in Table 2. Use the base-2 exponent and round to 2 decimal places.
Hint: use dot products for this, not traditional vector-space similarity.

3. Which word would be predicted by the CBOW algorithm to be the missing word?

Word	Embedding	Unnormalized Score	Normalized Score (P(Word))
dog	$[1, 2]$		
horse	$[3, 4]$		
motorcycle	$[0, -1]$		
leopard	$[3, 0]$		
wolf	$[4, 0]$		

Table 2: A subset of the vocabulary of the CBOW model.

2 LSTMs

$$\begin{aligned}
\boldsymbol{f}_{m+1} &= \sigma(\boldsymbol{\Theta}^{(h\to f)}\boldsymbol{h}_m + \boldsymbol{\Theta}^{(x\to f)}\boldsymbol{x}_{m+1} + \boldsymbol{b}_f) && \text{forget gate}\\
\boldsymbol{i}_{m+1} &= \sigma(\boldsymbol{\Theta}^{(h\to i)}\boldsymbol{h}_m + \boldsymbol{\Theta}^{(x\to i)}\boldsymbol{x}_{m+1} + \boldsymbol{b}_i) && \text{input gate}\\
\tilde{\boldsymbol{c}}_{m+1} &= \tanh(\boldsymbol{\Theta}^{(h\to c)}\boldsymbol{h}_m + \boldsymbol{\Theta}^{(w\to c)}\boldsymbol{x}_{m+1}) && \text{update candidate}\\
\boldsymbol{c}_{m+1} &= \boldsymbol{f}_{m+1}\odot \boldsymbol{c}_m + \boldsymbol{i}_{m+1}\odot \tilde{\boldsymbol{c}}_{m+1} && \text{memory cell update}\\
\boldsymbol{o}_{m+1} &= \sigma(\boldsymbol{\Theta}^{(h\to o)}\boldsymbol{h}_m + \boldsymbol{\Theta}^{(x\to o)}\boldsymbol{x}_{m+1} + \boldsymbol{b}_o) && \text{output gate}\\
\boldsymbol{h}_{m+1} &= \boldsymbol{o}_{m+1}\odot \tanh(\boldsymbol{c}_{m+1}) && \text{output.}
\end{aligned}$$

Figure 1: LSTM update equations (Eisenstein Ch. 6.33).

The update equations for a LSTM at timestep $m+1$ are given in Figure 1. Eisenstein Chapter 6.3 may be useful in answering this question.

1. In Table 3 we provide weight values and in Table 4 timestep inputs. We'll now compute the value of $\boldsymbol{h}_{m+1}$ using Table 3 and Table 4:

   $\boldsymbol{f}_{m+1} = \sigma(4+4+0) = 1.0$
   $\boldsymbol{i}_{m+1} = \sigma(-1+9+1) = 1.0$
   $\tilde{\boldsymbol{c}}_{m+1} = \tanh([4,-8,-4]^T + [-3,12,1]^T) = \tanh([1,4,-3]^T) = [0.76, 1.0, -1.0]^T$
   $\boldsymbol{c}_{m+1} = 1.0\odot[1,0,-4]^T + 1.0\odot[0.76,1.0,-1.0]^T = [1.76, 1.0, -5.0]^T$
   $\boldsymbol{o}_{m+1} = \sigma(2+2-1) = 1.0$
   $\boldsymbol{h}_{m+1} = 1.0\odot\tanh([1.76,1.0,-5.0]^T) = \mathbf{[0.94, 0.76, -1.0]^T}$

   The gates of this LSTM do not restrict the flow of any information. To effectively turn this LSTM into an Elman RNN at the current timestep, i.e., include **only** information from the current input and prior hidden state and **no** information from the prior memory cell in $\boldsymbol{h}_{m+1}$, describe the values that you would need to set the gates $\boldsymbol{f}_{m+1}, \boldsymbol{i}_{m+1}$ and $\boldsymbol{o}_{m+1}$ equal to.

2. Which variable from the list of intermediate variables in Figure 1 most closely resembles the hidden state of a standard Elman RNN? (Answer choices are $\boldsymbol{f}_{m+1}$, $\boldsymbol{i}_{m+1}$, $\tilde{\boldsymbol{c}}_{m+1}$, $\boldsymbol{c}_{m+1}$, $\boldsymbol{o}_{m+1}$, $\boldsymbol{h}_{m+1}$).

Weight	Value
$\boldsymbol{\Theta}^{(h\to f)}$	$[1, -2, -3]$
$\boldsymbol{\Theta}^{(x\to f)}$	$[0, -1, -2]$
$\boldsymbol{b}_f$	0
$\boldsymbol{\Theta}^{(h\to i)}$	$[0, 0, 1]$
$\boldsymbol{\Theta}^{(x\to i)}$	$[-1, -2, -2]$
$\boldsymbol{b}_i$	1
$\boldsymbol{\Theta}^{(h\to c)}$	$\begin{bmatrix} 0 & 1 & -3 \\ -3 & 1 & 0 \\ -2 & -1 & -3 \end{bmatrix}$
$\boldsymbol{\Theta}^{(w\to c)}$	$\begin{bmatrix} 1 & 0 & 0 \\ -2 & -3 & 0 \\ 1 & -1 & -2 \end{bmatrix}$
$\boldsymbol{\Theta}^{(h\to o)}$	$[1, 0, 1]$
$\boldsymbol{\Theta}^{(x\to o)}$	$[-1, 0, 1]$
$\boldsymbol{b}_o$	-1

Table 3: Weights for LSTM.

Vector	Value
$\boldsymbol{h}_m$	$[3, 1, -1]^T$
$\boldsymbol{c}_m$	$[1, 0, -4]^T$
$\boldsymbol{x}_{m+1}$	$[-3, -2, -1]^T$

Table 4: Input/intermediate variables for LSTM.

3. In this problem, all the LSTM gates are scalars. What changes would have to be made to Table 3 in order to create vector gates? (Specify which weights would change and what their new dimensions would be). What is the benefit of vector gates over scalars?

4. What two problems in RNNs does the inclusion of the memory cell $\boldsymbol{c}_{m+1}$ improve? What property of its computation allows it to do this?

3 Beam Search Decoding

Consider the following bigram language model. The bigram probabilities are given in table 5. Each probability is of the form $P(x_i|x_{i-1})$, where x_i corresponds to i^{th} word in a post / word sequence. Here, $\langle s\rangle$ denotes the start of a sentence.

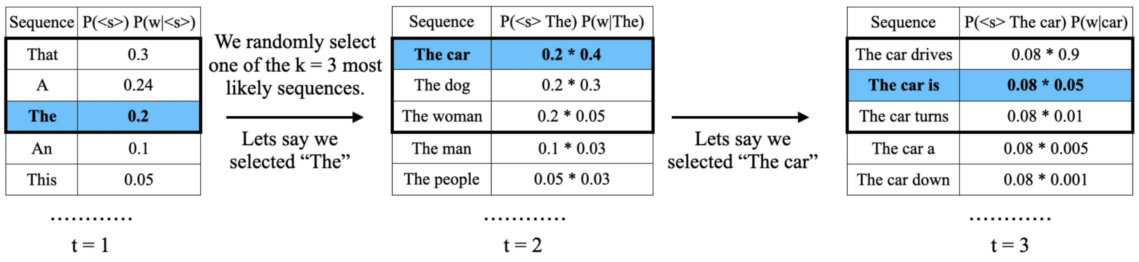
1. Given a prefix string "$\langle s\rangle$ *I know*", what are the next 3 possible tokens. Run beam search with width $k = 2$ and generate the next 3 tokens.

   Assume $P(\langle s\rangle\ I\ know) = 1$. Make sure you show the probabilities for each step. Also, show the word sequences in the beam at the end of each step.

Bigram	Prob.	Bigram	Prob.
P(, | know)	0.4	P(important | the)	0.3
P(the | know)	0.6	P(response | correct)	0.6
P(I | ,)	0.8	P(answer | correct)	0.25
P(it | ,)	0.2	P(problems | correct)	0.15
P(will | I)	0.4	P(response | important)	0.5
P(know | I)	0.6	P(answer | important)	0.5
P(was | it)	1.0	P(answer | exact)	1.0
P(correct | the)	0.5	P(exact | the)	0.2

Table 5: Bigram Language Model probabilities for Beam Search.

2. Lets introduce some randomness into the standard beam search method used in question 3.1. At the end of each step, we randomly choose one of the top-k beam candidates and discard the rest. In other words, only the randomly chosen top-k candidate is expanded in the next step instead of all the top-k candidates. Figure 2 illustrates this sampling approach.



Sequence	P(<s>) P(w|<s>)
That	0.3
A	0.24
The	**0.2**
An	0.1
This	0.05

Sequence	P(<s> The) P(w|The)
The car	**0.2 * 0.4**
The dog	0.2 * 0.3
The woman	0.2 * 0.05
The man	0.1 * 0.03
The people	0.05 * 0.03

Sequence	P(<s> The car) P(w|car)
The car drives	0.08 * 0.9
The car is	**0.08 * 0.05**
The car turns	0.08 * 0.01
The car a	0.08 * 0.005
The car down	0.08 * 0.001

....
t = 1
We randomly select one of the k = 3 most likely sequences.
Lets say we selected "The"
....
t = 2
Lets say we selected "The car"
....
t = 3

Figure 2: Example of top-k random sampling. At each timestep t, the model generates the probability of the possible next word. Then, we randomly sample from the k most likely candidates from this distribution. Here, we consider $k = 3$. **Bold** sequences represent the sampled candidate at each timestep.

Once again, consider the bigram language model in Table 5.

(a) Given the prefix string "$\langle s \rangle$ *I know*", run the top-k sampling approach for the next 3 tokens. Let S be the set of output sequences that this approach could possibly generate at the end of 3 steps. What is the size of S? Assume $k = 2$ and $P(\langle s \rangle\ I\ know) = 1$. You can just report the number.

(b) What is the sequence with maximum probability in S? Report the sequence and the probability of the sequence.

(c) What is the sequence with minimum probability in S? Report the sequence and the probability of the sequence.

4 Evaluation

1. Consider the following sentence with gold and predicted named entity tags.

Predicted NER Tags

Barack	and	Michelle	Obama	attend	the	WHCD	event	at	the	Hilton	Hotel	in	Washington
O	O	B-PER	I-PER	O	O	B-LOC	O	O	O	B-LOC	O	O	B-LOC

Gold NER Tags

Barack	and	Michelle	Obama	attend	the	WHCD	event	at	the	Hilton	Hotel	in	Washington
B-PER	O	B-PER	I-PER	O	O	O	O	O	O	B-LOC	I-LOC	O	B-LOC

Table 6: Predicted and gold NER tag sequences.

Compute the overall precision, recall, and F1 scores for the predicted entity tag sequence. Assume that the prediction is correct only when there is an exact match between the predicted entity and gold spans. You don't need to differentiate between the entity types. True positives will be the entity spans that match with the gold. False positives will be the entity spans in the predicted sequence that do not match or are not contained in the gold sequence. False negatives will be the entity spans in the gold sequence that do not match or are not contained in the predicted sequence.

Hint: You need to consider only the tags starting with B and I as they are the tags that represent entities; You should not be considering O as a class.

2. *BLEU* score is the most common automatic evaluation metric for machine translation. Given a candidate translation **c** and human-written reference translations $\mathbf{r_1}, ..., \mathbf{r_k}$, we compute the *BLEU* score of **c** as follows:

We first compute the modified n-gram precision p_n of **c** with $n = 1, 2, 3, 4$. Here n is the size of ngram:

$$p_n = \frac{\sum_{ngram \in c} min\Big(Max\_Ref\_Count(ngram), Count_c(ngram)\Big)}{\sum_{ngram \in c} Count_c(ngram)}$$
$$Max\_Ref\_Count(ngram) = max_{i=1..k} Count_{r_i}(ngram)$$

For each of the n-grams in **c**, we count the maximum number of times it appears in any one reference translation. Then, we clip this count by the number of times it appears in **c**. We divide these clipped counts by the number of ngrams in **c**.

Next, we compute the brevity penalty BP. Let $len(\mathbf{c})$ be the length of c and let $len(\mathbf{r})$ be the length of the shortest reference translation.

$$BP = \begin{cases} 1 & \text{if } len(\mathbf{c}) \geq len(\mathbf{r}) \\ exp(1 - \frac{len(\mathbf{r})}{len(\mathbf{c})}) & \text{otherwise} \end{cases}$$

Lastly, the $BLEU$ score for candidate c with respect to $r_1, ..., r_k$ is:

$$BLEU = BP \ \exp\left(\sum_{n=1}^{N} w_n \log p_n\right) \tag{1}$$

where w_1, w_2, w_3, w_4 are weights that sum to 1. The log here is natural log.

Compute the $BLEU$ score for the following two candidate translations $\mathbf{c_1}, \mathbf{c_2}$ against two human-written references $\mathbf{r_1}, \mathbf{r_2}$:

Candidate Translation $\mathbf{c_1}$: fruits are good for health

Candidate Translation $\mathbf{c_2}$ vegetables are very important for good health

Reference Translation $\mathbf{r_1}$: eating fruits is good for health

Reference Translation $\mathbf{r_2}$: fruits and vegetables are essential for good health

Let $w_i = \frac{1}{3}$ for $i \in 1, 2, 3$ and $w_4 = 0$. In other words, we do not compute 4-grams. Show the computation of p_1, p_2, p_3, BP, and the final $BLEU$ score in your answer.

5 Transformer Self Attention

This section deals with transformer self attention. It may be helpful to read section 9.7 (Self-Attention Networks: Transformers) in the Speech and Language Processing textbook while answering these questions. **For all questions in this section, unless otherwise stated work must be shown in the form of matrix multiplications to receive full credit** (i.e. $C = AB^T$). For performing the computations, using Excel or other software is recommended to avoid computation errors. When writing your answers please round to 2 decimal places. You may use scientific notation to represent your answers if necessary.

Word			
Attention	1	2	4
is	-1	0	2
all	3	1	3
you	5	0	0
need	2	-2	-1

Table 7: Word embeddings

$$W^q = \begin{bmatrix} 1 & 1 \\ -3 & 1 \\ -2 & 3 \end{bmatrix}, \ W^k = \begin{bmatrix} -1 & 3 \\ -2 & -5 \\ -1 & -2 \end{bmatrix}, \ W^v = \begin{bmatrix} 3 & 0 \\ 2 & -4 \\ 4 & 0 \end{bmatrix}$$

$$W^o = \begin{bmatrix} -5 & 4 & -5 \\ 2 & -1 & 2 \\ 1 & -4 & 4 \\ 0 & 0 & 0 \\ 3 & 3 & -1 \\ -4 & -3 & -1 \\ 3 & 1 & 5 \\ 1 & 3 & 3 \end{bmatrix}$$

1. We will first consider a single attention head. Given the set of word embeddings, projection matrices, and a normalization factor of 48 instead of $\sqrt{d^k}$, fill out this table with the normalized query-key score for each possible pair of words.

Word					
Attention					
is					
all					
you					
need					

Table 8: Normalized Scores

2. Given the normalized scores, calculate the attention weights for each word with respect to the other words in the input sentence and fill in the table with your results. You do not need to show work for this question. Please do not mask any attention values.

Word					
Attention					
is					
all					
you					
need					

Table 9: Attention Values

3. Given the embeddings, the previously calculated attention values, and the value projection matrix, calculate the output embeddings of this attention head. Fill in the table with your results.

Word		
Attention		
is		
all		
you		
need		

Table 10: Self Attention Head Outputs

4. The outputs of three other self attention heads have been computed for you. Combine these values with the self attention embedding you calculated earlier in this question, and find the final output of this self attention layer using the output weight matrix. Fill in the table with your results.

Word		
Attention	1.5	-2.5
is	-4.78	0.15
all	1.75	-1.97
you	-3.96	-2.9
need	-0.53	4.61

Word		
Attention	1.51	0.07
is	-4.95	-3.47
all	2.33	-4.81
you	0.05	0.68
need	2.85	-1.91

Word		
Attention	-3.59	-3.18
is	3.38	-1.85
all	3.77	4.21
you	-0.15	1.46
need	-1.65	1.51

Table 11: Other Attention Head Outputs

Word			
Attention			
is			
all			
you			
need			

Table 12: Output of self attention layer

6 Byte-Pair Encoding

Consider the training corpus containing only one sentence - **fresh french fries**

1. Apply byte-pair encoding on this corpus until no bigram appears more than once.

   Hint: The corpus should first be white-space-separated to get a set of strings, each corresponding to the characters of a word, plus a special end-of-word symbol _

   Training corpus: **fresh_**, **french_**, **fries_**

   Clearly provide the sequence of merge-operations. In case of any ties, feel free to choose any option.