Chapter 35 :The Celebrity problem.

In which we continue to apply Searching by Elimination.

Among N people, a celebrity is someone who is known to everyone but who doesn't know anyone. We are given a group of N people, which contains a celebrity, and asked to identify them. We will use the symmetric linear search algorithm.

We represent the problem domain using a 2 dimensional Boolean matrix

$$R[1..N, 1..N]$$

Where

$R.i.j \quad \equiv \quad$ "person i knows person j"

For a celebrity x we define

* (0) $F.x \equiv \langle \forall\ i : i \neq x : R.i.x \wedge \neg R.x.i \rangle$

We can now specify our problem as folows

Pre: $\langle \exists\ k : 1 \leq k \leq N :\ F.k \rangle$

Post : $F.x$

We now calculate the guards

$F.a \Rightarrow F.b$
$\Leftarrow$ {Predicate calculus, $\neg P \Rightarrow (P \Rightarrow Q)$}
$\neg F.a$
$=$ {definition of F}
$\neg \langle \forall\ i : i \neq a : R.i.a \wedge \neg R.a.i \rangle$
$=$ {deMorgan}
$\langle \exists\ i : i \neq a : \neg R.i.a \vee R.a.i \rangle$
$\Leftarrow$ {$a \neq b$, predicate calculus}
$\neg R.b.a \vee R.a.b$

Symmetrically, $(F.b \Rightarrow F.a) \Leftarrow \neg R.a.b \vee R.b.a$

This yields our finished program

```
a, b := 1, N
;do a ≠ b →     {M ≤ a < b ≤ N}

                  if ¬R.b.a ∨ R.a.b → a := a + 1
                  []¬R.a.b ∨ R.b.a → b := b – 1
                  fi

od
; x := a
```