COM307000 - Cryptography
Secret Sharing, Random Numbers & Info Hiding

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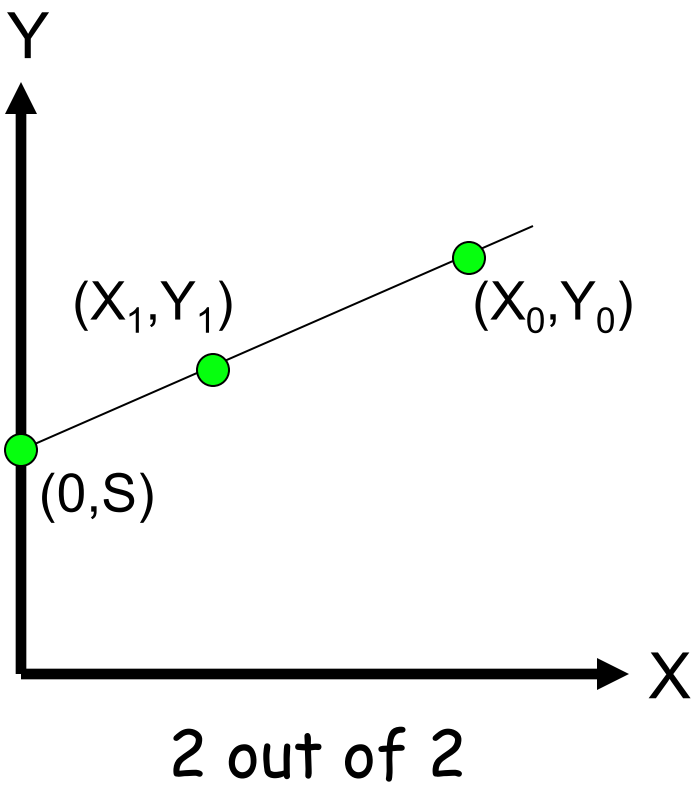
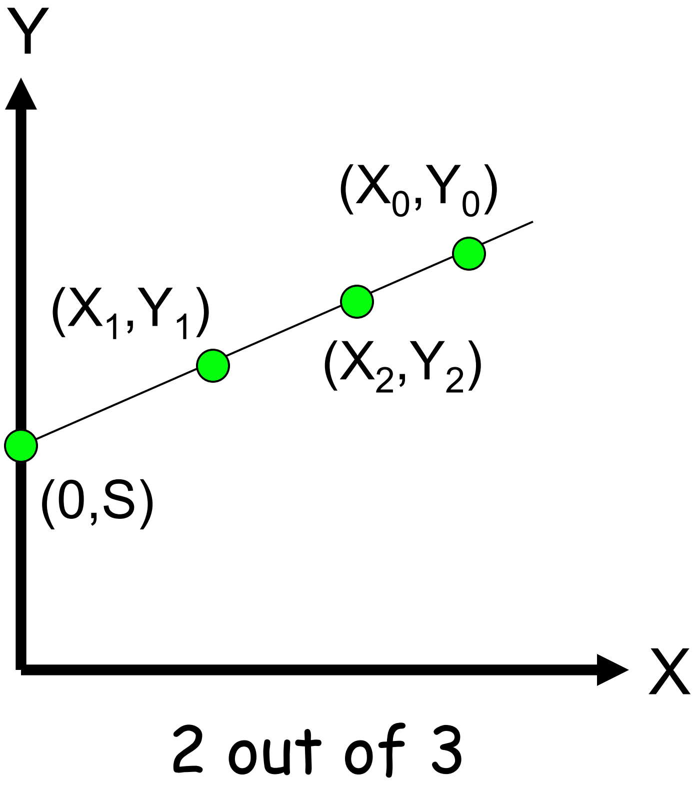
Secret Sharing

Shamir's Secret Sharing



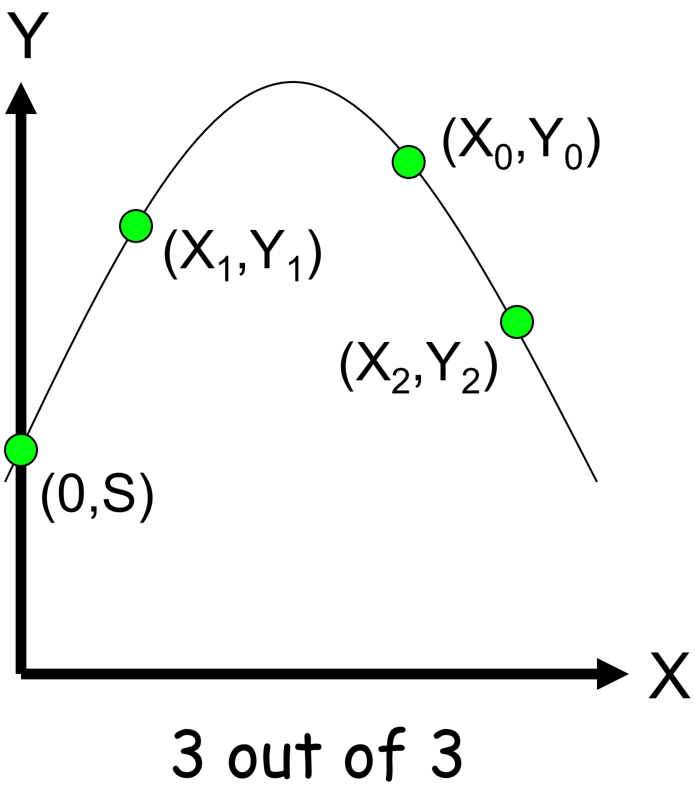
- ❑ Two points determine a line
- ❑ Give (X_0,Y_0) to Alice
- ❑ Give (X_1,Y_1) to Bob
- ❑ Then Alice and Bob must cooperate to find secret S
- ❑ Also works in discrete case
- ❑ Easy to make "m out of n" scheme for any $m \leq n$

Shamir's Secret Sharing



- Give (X_0,Y_0) to Alice
- Give (X_1,Y_1) to Bob
- Give (X_2,Y_2) to Charlie
- Then any ***two*** can cooperate to find secret S
- No ***one*** can determine S
- A "2 out of 3" scheme

Shamir's Secret Sharing

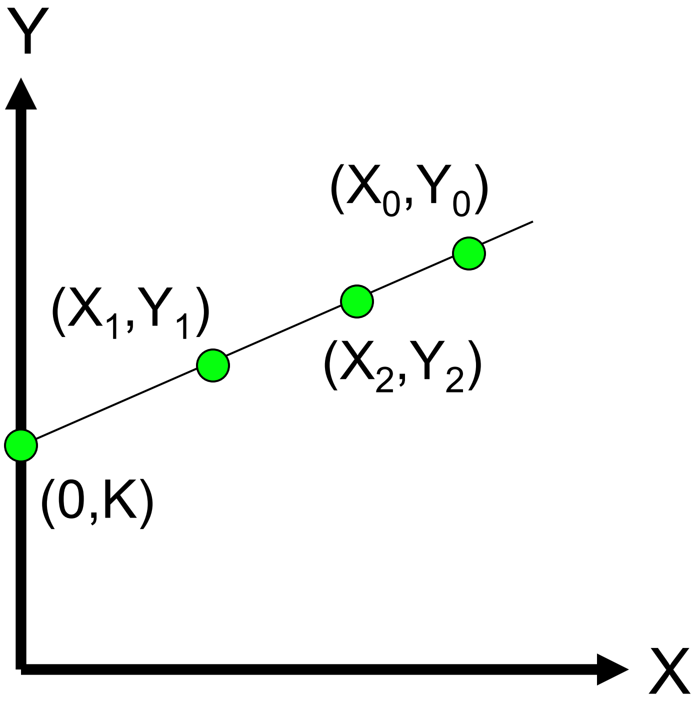


- Give (X_0,Y_0) to Alice
- Give (X_1,Y_1) to Bob
- Give (X_2,Y_2) to Charlie
- 3 pts determine parabola
- Alice, Bob, **and** Charlie must cooperate to find S
- A "3 out of 3" scheme
- What about "3 out of 4"?

Secret Sharing Use?

- **Key escrow** — suppose it’s required that your key be stored somewhere
- Key can be “recovered” with court order
- But you don’t trust FBI to store your keys
- We can use secret sharing
  - Say, three different government agencies
  - Two must cooperate to recover the key

Secret Sharing Example



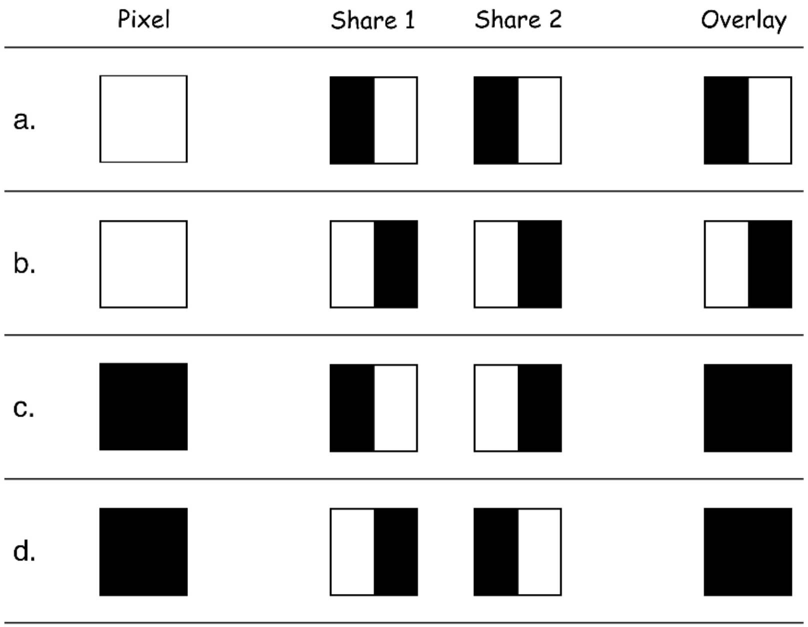
- ❑ Your symmetric key is K
- ❑ Point (X_0,Y_0) to FBI
- ❑ Point (X_1,Y_1) to DoJ
- ❑ Point (X_2,Y_2) to DoC
- ❑ To recover your key K, two of the three agencies must cooperate
- ❑ No one agency can get K

Visual Cryptography

- Another form of secret sharing…
- Alice and Bob “share” an image
- Both must cooperate to reveal the image
- Nobody can learn anything about image from Alice’s share or Bob’s share
    - That is, both shares are required
- Is this possible?

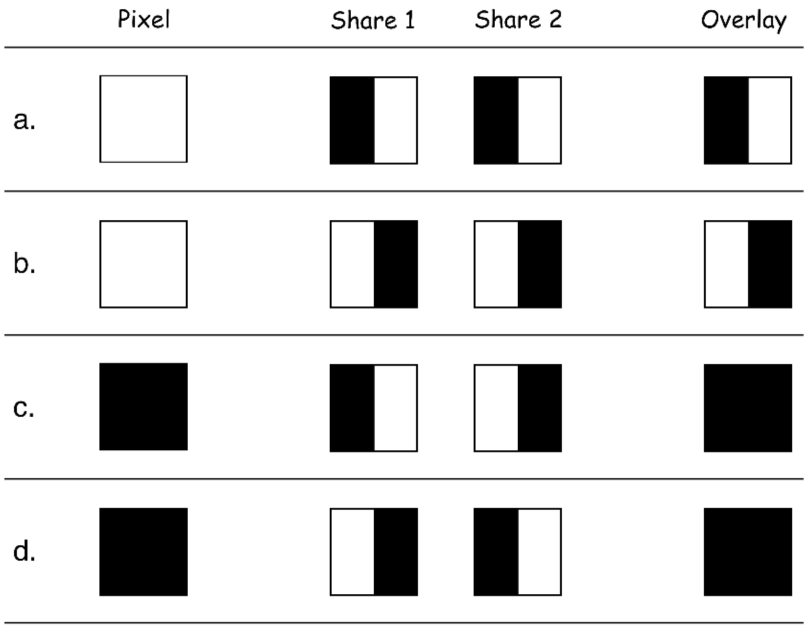
Visual Cryptography

- ❑ How to "share" a pixel?
- ❑ Suppose image is black and white
- ❑ Then each pixel is either black or white
- ❑ We split pixels as shown



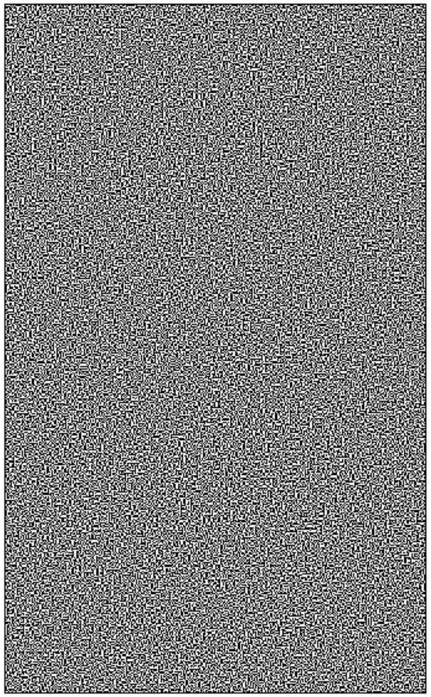
Sharing Black & White Image

- If pixel is white, randomly choose a or b for Alice’s/Bob’s shares
- If pixel is black, randomly choose c or d
- **No information** in one “share”

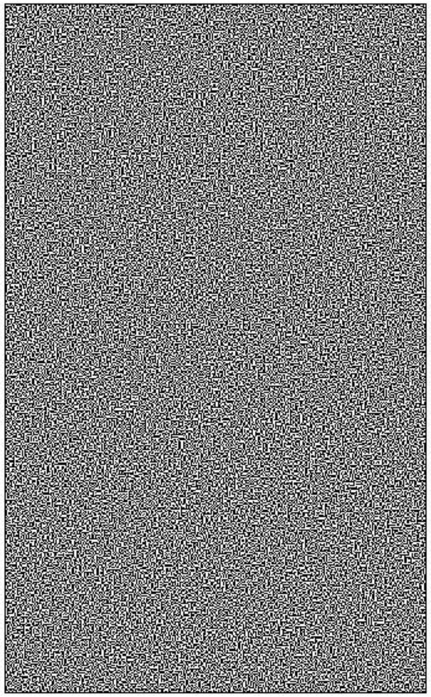


Visual Crypto Example

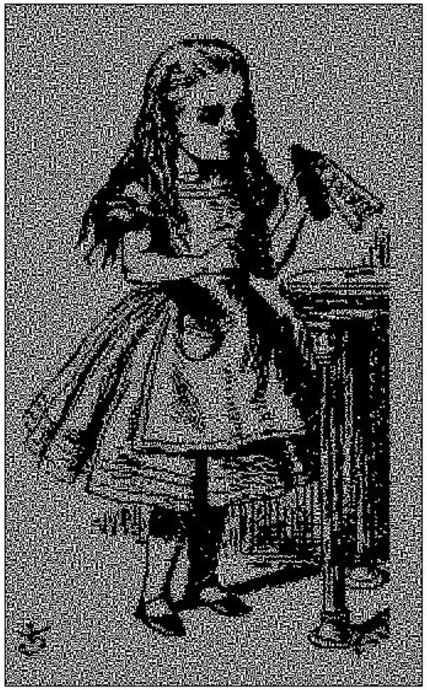
- Alice's share



- Bob's share



- Overlaid shares



Visual Crypto

- Visual crypto — no exhaustive search…
- How does visual crypto compare to crypto?
  - Visual crypto is "information theoretically" secure — also true of secret sharing schemes
  - With regular encryption, goal is to make cryptanalysis computationally infeasible
- Visual crypto an example of **secret sharing**
  - Not really a form of crypto, in the usual sense

Random Numbers in Cryptography

Random Numbers

- Random numbers used to generate **keys**
  - Symmetric keys
  - RSA: Prime numbers
  - Diffie Hellman: secret values
- Random numbers used for nonces
  - Sometimes a sequence is OK
  - But sometimes nonces must be random
- Random numbers also used in simulations, statistics, etc.
  - In such apps, need "statistically" random numbers

Random Numbers

- *Cryptographic random* numbers must be statistically random and **unpredictable**
- Suppose server generates symmetric keys
  - Alice: K_A
  - Bob: K_B
  - Charlie: K_C
  - Dave: K_D
- Alice, Bob, and Charlie don't like Dave…
- Alice, Bob, and Charlie, working together, must not be able to determine K_D

Non-random Random Numbers

- Online version of Texas Hold ‘em Poker
  - ASF Software, Inc.

1
2
3
4
Player's hand
Community cards in center of the table

- Random numbers used to shuffle the deck
- Program did not produce a random shuffle
- A serious problem, or not?

Card Shuffle

- There are 52! > 2^{225} possible shuffles
- The poker program used "random" 32-bit integer to determine the shuffle
  - So, only 2^{32} distinct shuffles could occur
- Code used Pascal pseudo-random number generator (PRNG): Randomize()
- Seed value for PRNG was function of number of milliseconds since midnight
- Less than 2^{27} milliseconds in a day
  - So, less than 2^{27} possible shuffles

Card Shuffle

- Seed based on milliseconds since midnight
- PRNG re-seeded with each shuffle
- By synchronizing clock with server, number of shuffles that need to be tested < 2^{18}
- Could then test all 2^{18} in real time
  - Test each possible shuffle against "up" cards
- Attacker knows **every card** after the first of five rounds of betting!

Poker Example

- Poker program is an extreme example
  - But common PRNGs are predictable
  - Only a question of how many outputs must be observed before determining the sequence
- Crypto random sequences not predictable
  - For example, keystream from RC4 cipher
  - But "seed" (or key) selection is still an issue!
- How to generate initial **random** values?
  - Keys (and, in some cases, seed values)

What is Random?

- True "random" hard to even define
- **Entropy** is a measure of randomness
- Good sources of "true" randomness
  - Radioactive decay — but, radioactive computers are not too popular
  - Hardware devices — many good ones on the market
  - Lava lamp — relies on chaotic behavior

Randomness

- Sources of randomness via software
  - Software is supposed to be deterministic
  - So, must rely on external "random" events
  - Mouse movements, keyboard dynamics, network activity, etc., etc.
- Can get **quality** random bits by such methods
- But **quantity** of bits is very limited
- Bottom line: "The use of pseudo-random processes to generate secret quantities can result in pseudo-security"

Information Hiding

Information Hiding

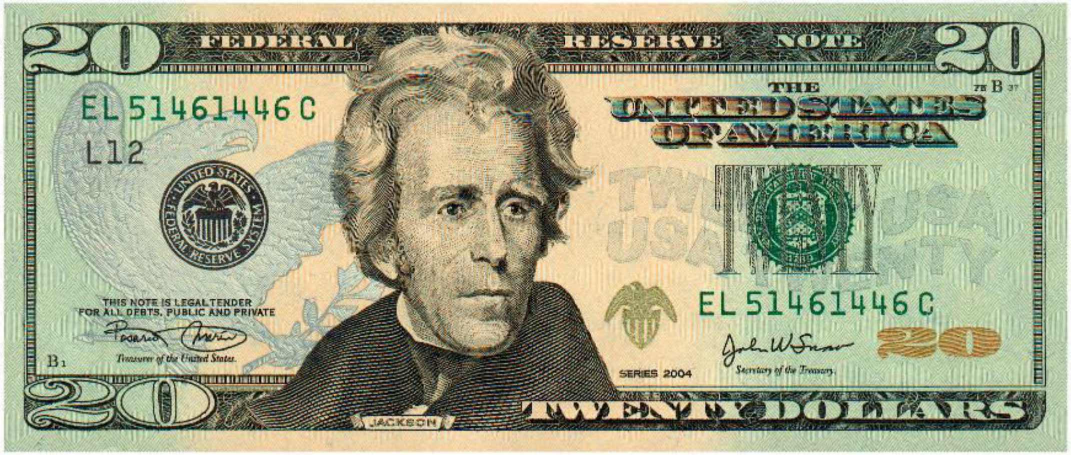
- **Digital Watermarks**
  - Example: Add "invisible" info to data
  - Defense against music/software piracy
- **Steganography**
  - "Secret" communication channel
  - Similar to a **covert channel**
  - Example: Hide data in an image file

Watermark Examples

- Add **robust invisible** mark to digital music
  - If pirated music appears on Internet, can trace it back to original source of the leak
- Add **fragile invisible** mark to audio file
  - If watermark is unreadable, recipient knows that audio has been tampered with (integrity)
- Combinations of several types are sometimes used
  - E.g., visible plus robust invisible watermarks

Watermark Example (1)

- Non-digital watermark: U.S. currency


FEDERAL RESERVE NOTE
THE UNITED STATES OF AMERICA
EL 51461446 C
L12
THIS NOTE IS LEGAL TENDER FOR ALL DEBTS, PUBLIC AND PRIVATE
Treasurer of the United States.
Secretary of the Treasury.
SERIES 2004
JACKSON
TWENTY DOLLARS
20

- Image embedded in paper on rhs
  - Hold bill to light to see embedded info

Watermark Example (2)

- Add **invisible** watermark to photo
- Claim is that 1 inch^2 contains enough info to reconstruct entire photo
- If photo is damaged, watermark can be used to reconstruct it!

Steganography

- According to Herodotus (Greece 440 BC)
  - Shaved slave’s head
  - Wrote message on head
  - Let hair grow back
  - Send slave to deliver message
  - Shave slave’s head to expose a message warning of Persian invasion
- Historically, steganography used by military more often than cryptography

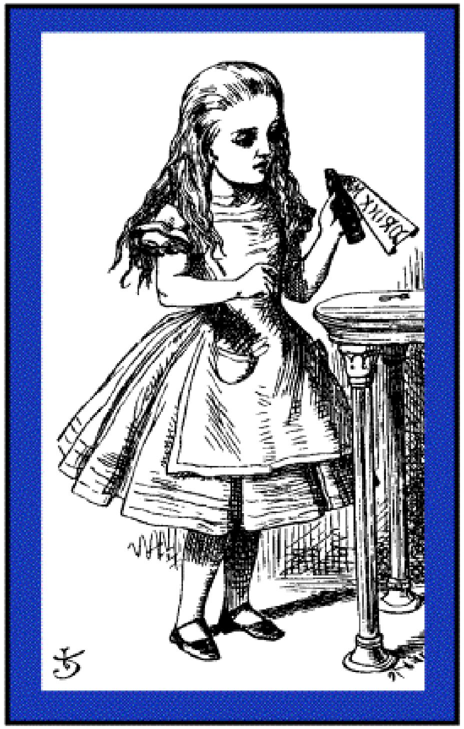
Images and Steganography

- Images use 24 bits for color: **RGB**
  - 8 bits for red, 8 for green, 8 for blue
- For example
  - **0x7E 0x52 0x90** is this color
  - **0xFE 0x52 0x90** is this color
- While
  - **0xAB 0x33 0xF0** is this color
  - **0xAB 0x33 0xF1** is this color
- Low-order bits don’t matter…

Images and Stego

- ❑ Given an uncompressed image file…
  - o For example, BMP format
- ❑ …we can insert information into low-order RGB bits
- ❑ Since low-order RGB bits don't matter, changes will be "invisible" to human eye
  - o But, computer program can "see" the bits

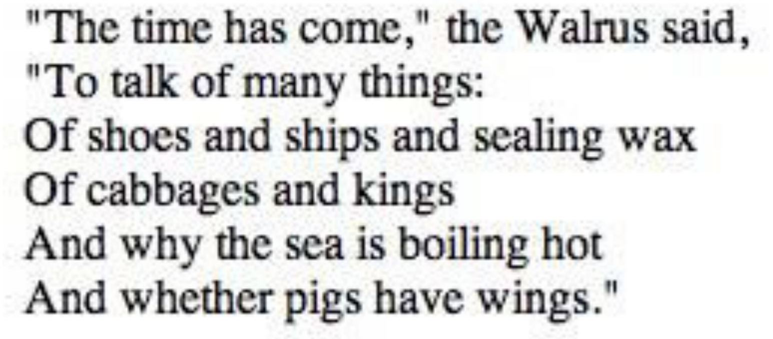
Stego Example 1



- Left side: plain Alice image
- Right side: Alice with entire *Alice in Wonderland* (pdf) “hidden” in the image

Non-Stego Example

- Walrus.html in web browser



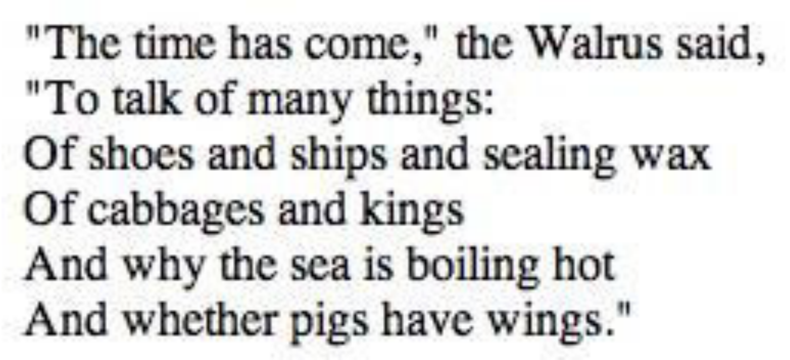
"The time has come," the Walrus said,
"To talk of many things:
Of shoes and ships and sealing wax
Of cabbages and kings
And why the sea is boiling hot
And whether pigs have wings."

- "View source" reveals:

```
<font color=#000000>"The time has come," the Walrus said,</font><br>
<font color=#000000>"To talk of many things: </font><br>
<font color=#000000>Of shoes and ships and sealing wax </font><br>
<font color=#000000>Of cabbages and kings </font><br>
<font color=#000000>And why the sea is boiling hot </font><br>
<font color=#000000>And whether pigs have wings." </font><br>
```

Stego Example 2

- stegoWalrus.html in web browser



"The time has come," the Walrus said,
"To talk of many things:
Of shoes and ships and sealing wax
Of cabbages and kings
And why the sea is boiling hot
And whether pigs have wings."

- "View source" reveals:

```
<font color=#000101>"The time has come," the Walrus said,</font><br>
<font color=#000100>"To talk of many things: </font><br>
<font color=#010000>Of shoes and ships and sealing wax </font><br>
<font color=#010000>Of cabbages and kings </font><br>
<font color=#000000>And why the sea is boiling hot </font><br>
<font color=#010001>And whether pigs have wings." </font><br>
```

- "Hidden" message: **011 010 100 100 000 101**

Steganography

- Some formats (e.g., image files) are more difficult than html for **humans** to read
  - But easy for computer programs to read…
- Easy to hide info in **unimportant bits**
- Easy to damage info in unimportant bits
- To be ***robust,*** must use **important bits**
  - But stored info must not damage data
  - Collusion attacks are also a concern
- Robust steganography is tricky!

Information Hiding: The Bottom Line

- Not-so-easy to hide digital information
  - “Obvious” approach is **not** robust
  - **Stirmark:** tool to make most watermarks in images unreadable without damaging the image
  - Stego/watermarking are active research topics
- If information hiding is suspected
  - Attacker may be able to make information/watermark unreadable
  - Attacker may be able to read the information, given the original document (image, audio, etc.)