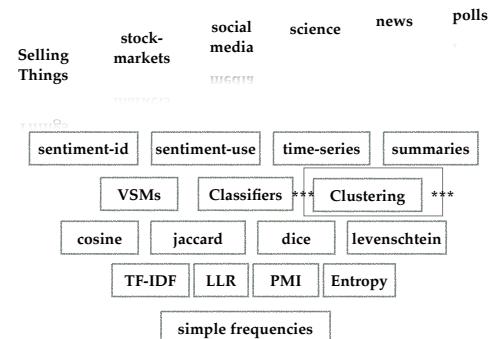


# Clustering

Lecture 6: Text Analytics for Big Data  
Mark Keane, Insight/CSI, UCD



## Outline

- ❖ Why Do This?
- ❖ Sometimes you have no labels, but rather want to *discover* the categories...
- ❖ Clustering (with Unsupervised Techniques)
- ❖ Tangent on Evaluation: Validating Clusters

## Unsupervised: Clustering

- ❖ K-Means
- ❖ Hierarchical Clustering
- ❖ Graph-Based Clustering

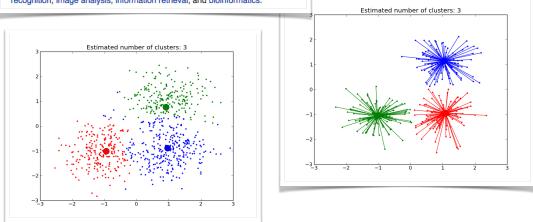
### Clustering Why We Do This?

## To Cluster...

- ❖ Clustering is used in text analytics to discover groups of items, to circumscribe...
- ❖ Do these text-items form a “natural” group that we find to be spam or non-spam?
- ❖ If these items have these features they group together; like birds of a feather...

## Why Clustering?

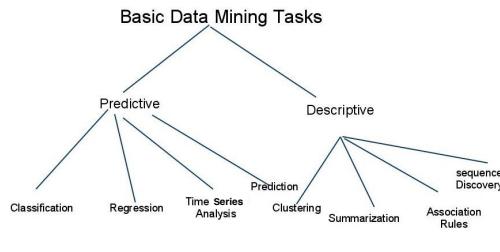
Cluster analysis or clustering is the task of grouping a set of objects in such a way that objects in the same group (called a cluster) are more similar (in some sense or another) to each other than to those in other groups (clusters). It is a main task of exploratory data mining, and a common technique for statistical data analysis, used in many fields, including machine learning, pattern recognition, image analysis, information retrieval, and bioinformatics.



## To Cluster...

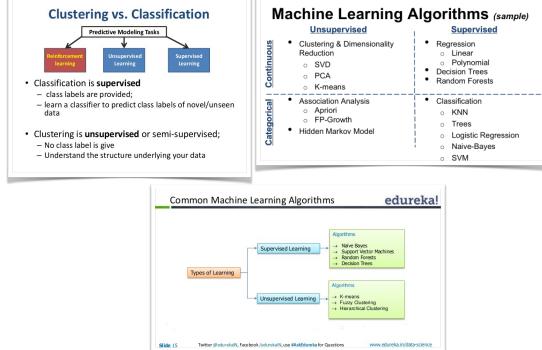
- ❖ Clustering is the unsupervised face of machine learning; when data is unlabelled
- ❖ It gives you a description of possible classes, based on how they group together
- ❖ As such, it may be a preliminary to classification; but it is a task in itself

## One View of Tasks...



<http://www.slideshare.net/KapilRode/data-mining-32694954>

## Clustering is Unsupervised...

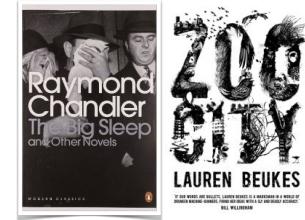


## Clustering is Ubiquitous

- ❖ We have seen similarity, as a way to compare individual items to one another (shopper1 is like shopper 2)
- ❖ Sometimes you want to find a group of things; so you can say  $x_1, x_2, x_3$  are from Group-X; *shopper1, shopper2, shopper3* are *BingeShoppers*, but *shopper4, shopper5, shopper6* are *YoungFamilyShoppers*
- ❖ X is a general class of things, a group, as is *BingeShopper*
- ❖ Many ML techniques use different clustering techniques; there are a wide variety of options

## Examples

- ❖ Activity profiles of Tweeters suggest 3 groups: lurkers, actives and celebs
- ❖ Company reports divide into negative or positive groups
- ❖ Urban-fantasy-crime-readers are a different group than traditional noir-crime-readers



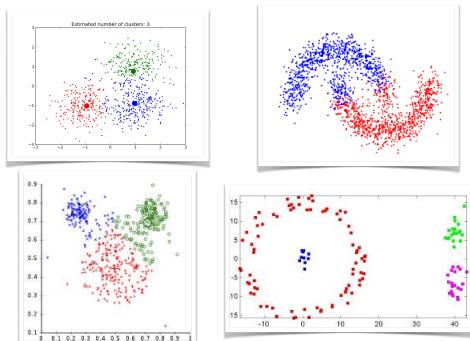
## Selecting Some...

- ❖ **Supervised Learning: Classification (last lecture)**
  - ❖ K Nearest Neighbours
  - ❖ Naive Bayes
  - ❖ Logistic Regression
  - ❖ Support Vector Machine
- ❖ **Unsupervised Learning: Clustering**
  - ❖ K-Means
  - ❖ Hierarchical Clustering
  - ❖ Graph-Based Clustering

<http://www.slideshare.net/mirjalil/clustering-on-database-systems-rkm-42588852>

## Clustering Unsupervised Techniques

## Clustering: Pics



## Unsupervised Techniques

### Unsupervised learning

From Wikipedia, the free encyclopedia

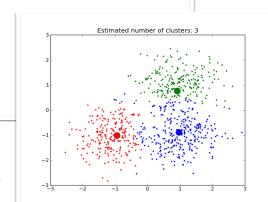
In machine learning, the problem of **unsupervised learning** is that of trying to find hidden structure in unlabeled data. Since the examples given to the learner are unlabeled, there is no error or reward signal to evaluate a potential solution. This distinguishes unsupervised learning from supervised learning and reinforcement learning.

Unsupervised learning is closely related to the problem of density estimation in statistics.<sup>[1]</sup> However unsupervised learning also encompasses many other techniques that seek to summarize and explain key features of the data. Many methods employed in unsupervised learning are based on data mining methods used to preprocess<sup>[2][3]</sup> data needed for learning.

Approaches to unsupervised learning include:

- clustering (e.g., k-means, hierarchical clustering),<sup>[2]</sup>
- Approaches for learning latent variable models such as
  - Expectation–maximization algorithm (EM)
  - Method of moments
  - Blind signal separation techniques, e.g.,
    - Principal component analysis,
    - Non-negative matrix factorization,
    - Bayesian value decomposition

Start from a set of items and analyse them to find clusters

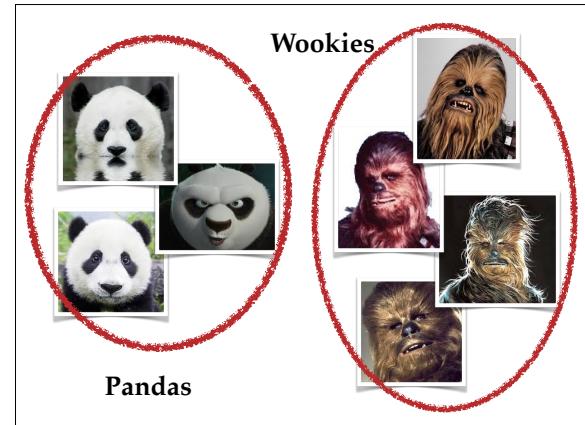


## Supervised: Clustering

- ◆ K-Means
- ◆ Hierarchical Clustering
- ◆ Graph-Based Clustering

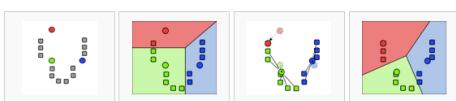
Clustering  
k-means

- ◆ Take set of animals described by their features and then group them on these features



## k-means: Idea

- ◆ Randomly pick some points in m-d space
- ◆ Get distance of all instances from these points, assign to group if closest to point
- ◆ Move the points to the *centre* of these groups
- ◆ Repeat until you're no longer moving points



[en.wikipedia.org/wiki/K-means\\_clustering](http://en.wikipedia.org/wiki/K-means_clustering)

## k-means: Formula

Given a set of observations  $(x_1, x_2, \dots, x_n)$ , where each observation is a  $d$ -dimensional real vector, k-means clustering aims to partition the  $n$  observations into  $k$  ( $n$ ) sets  $S = (S_1, S_2, \dots, S_k)$  so as to minimize the within-cluster sum of squares (WCSS). In other words, its objective is to find:

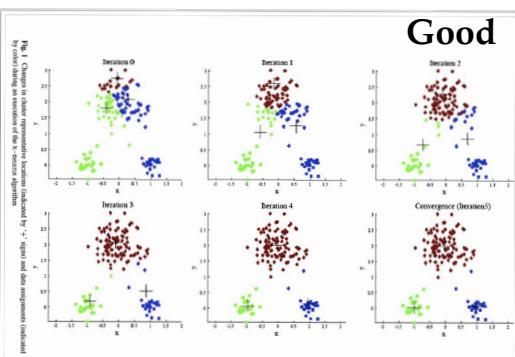
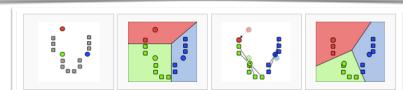
$$\arg \min_S \sum_{i=1}^n \sum_{x \in S_i} \|x - \mu_i\|^2$$

where  $\mu_i$  is the mean of points in  $S_i$ .

### Algorithm 1 The k-means algorithm

*Input:* set  $D$ , distance measure  $dist$ , number  $k$  of cluster  
*Output:* A partitioning  $\mathbb{P}$  of the set  $D$  of documents (i.e., a set  $\mathbb{P}$  of  $k$  disjoint subsets of  $D$  with  $\bigcup_{P \in \mathbb{P}} P = D$ ).

- 1: Choose randomly  $k$  data points from  $D$  as starting centroids  $t_{P_1} \dots t_{P_k}$ .
- 2: repeat
- 3:   Assign each point of  $P$  to the closest centroid with respect to  $dist$ .
- 4:   (Re-)calculate the cluster centroids  $t_{P_1} \dots t_{P_k}$  of clusters  $P_1 \dots P_k$ .
- 5: until cluster centroids  $t_{P_1} \dots t_{P_k}$  are stable
- 6: return set  $\mathbb{P} := \{P_1, \dots, P_k\}$  of clusters.



Wu, X., et al. (2008). Top 10 algorithms in data mining. Knowledge and Information Systems, 14(1), 1-37.

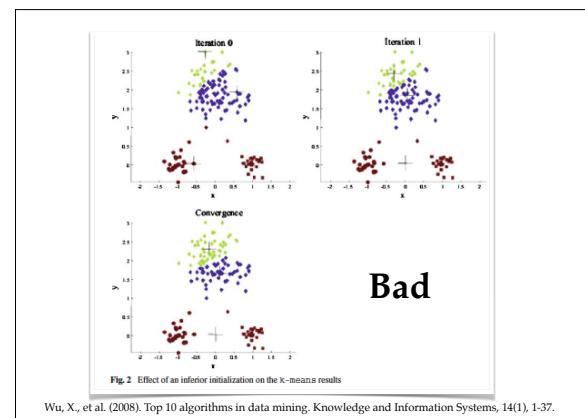


Fig. 2 Effect of an inferior initialization on the k-means results

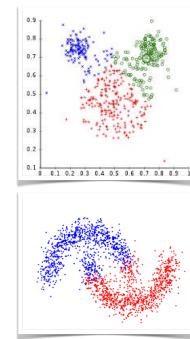
Wu, X., et al. (2008). Top 10 algorithms in data mining. Knowledge and Information Systems, 14(1), 1-37.

## k-means: Issues I

- May not find global optimum (random start points); though there are some heuristics to improve initial point choice
- May not find global optimum (no. of points/groups), but diagnostic methods find no. of groups and cost fn methods exist
- NP-hard Euclidean space, even for 2 clusters; but heuristics help
- Can use other distance-metrics but compromise convergence
- In practice, often do 1000 iterations of 2-points, 3-points, etc; and find the most persistent group that emerges running all night

## k-means: Issues II

- Data really needs to be reasonably separated, spherical clusters
- Poor if there are non-convex shaped clusters in the data



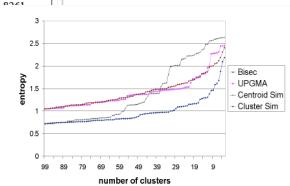
## k-means Eg: Text Classification

- Classic use is in text processing; recall, in VSMs, docs are vectors, so each is a point in an m-d space
- Euclidean distance is the metric between these points, then you can see how k-means can be used
- Use entropy and F measure can be used to determine goodness of clusters; entropy why?

Steinbach, M., Karypis, G., & Kumar, V. (2000, August). A comparison of document clustering techniques. In KDD workshop on text mining (Vol. 400, No. 1, pp. 525-526).

## k-means Eg: Text Classification

Table 1: Summary description of document sets.			
Data Set	Source	Documents	Classes
rcd	Reuters	1504	13
rcd	Reuters	1657	25
wap	WebACE	1560	20
tr31	TREC	927	7
tr45	TREC	690	10
fbis	TREC	2463	17
la1	TREC	3204	6
la2	TREC	3075	6

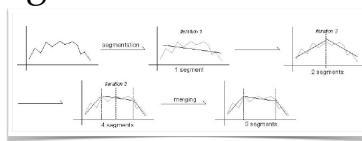


as Cluster-No decreases Entropy increases, why?

Steinbach, M., Karypis, G., & Kumar, V. (2000, August). A comparison of document clustering techniques. In KDD workshop on text mining (Vol. 400, No. 1, pp. 525-526).

## k-means Eg: Stocks & News

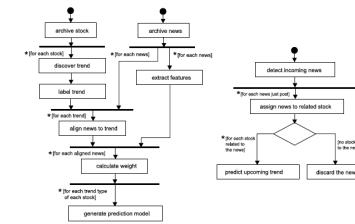
Traced rising/falling trends in stock prices



- Gathered articles within rising or falling period and clustered using k-means
- Compute similarities 'tween news-article group to relate group to rise/fall trend

Fung, G. P. C., Yu, J. X., & Lam, W. (2002). News sensitive stock trend prediction. In *Advances in knowledge discovery and data mining* (pp. 481-493). Springer Berlin Heidelberg.

## k-means #2: Stock Prediction



Fung, G. P. C., Yu, J. X., & Lam, W. (2002). News sensitive stock trend prediction. In *Advances in knowledge discovery and data mining* (pp. 481-493). Springer Berlin Heidelberg.

## k-means #2: Stock Prediction

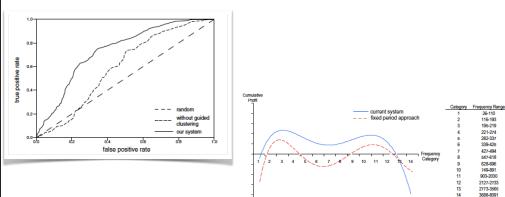
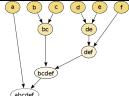


Fig. 6. The relationship between the frequency of news announced and the resulting profit

Fung, G. P. C., Yu, J. X., & Lam, W. (2002). News sensitive stock trend prediction. In *Advances in knowledge discovery and data mining* (pp. 481-493). Springer Berlin Heidelberg.

## Clustering Hierarchical Clustering (HCA-Hierarchical Cluster Analysis)

## HCA: The Idea



- ❖ Start with a set of items and a distance metric
- ❖ Find the (similarity) distance 'tween all items
- ❖ Merge the two shortest-distance items into a group; then get distance 'tween this group and other items
- ❖ Merge the items (group / singles) that have the shortest distance on this iteration
- ❖ Keep iterating ...

[http://en.wikipedia.org/wiki/Hierarchical\\_clustering](http://en.wikipedia.org/wiki/Hierarchical_clustering)

## HCA: Formula

A frequently used distance measure is the Euclidean distance. We calculate the distance between two text documents  $d_1, d_2 \in D$  as follows:

$$dist(d_1, d_2) = \sqrt{\sum_{k=1}^m |w(d_1, t_k) - w(d_2, t_k)|^2} . \quad (4)$$

However, the Euclidean distance should only be used for normalized vectors, since otherwise the different lengths of documents can result in a smaller distance between documents that share less words than between documents that have more words in common and should be considered therefore as more similar.

### Algorithmic steps for Agglomerative Hierarchical clustering

Let  $X = \{x_1, x_2, x_3, \dots, x_n\}$  be the set of data points.

- 1) Begin with the disjoint clustering having level  $L(0) = 0$  and sequence number  $m = 0$ .
- 2) Find the least distance pair of clusters in the current clustering, say pair  $(r), (s)$ , according to  $d((r),(s)) = \min d((i),(j))$  where the minimum is over all pairs of clusters in the current clustering.
- 3) Increment the sequence number:  $m = m + 1$ . Merge clusters  $(r)$  and  $(s)$  into a single cluster to form the next clustering. Set the level of this clustering to  $L(m) = d((r),(s))$ .
- 4) Update the distance matrix,  $D$ , by deleting the rows and columns corresponding to clusters  $(r)$  and  $(s)$  and adding a row and column corresponding to the newly formed cluster. The distance between the new cluster, denoted  $(r,s)$  and old cluster  $(k)$  is defined in this way:  $d((k), (r,s)) = \min [d((k),(r)), d((k),(s))]$ .
- 5) If all the data points are in one cluster then stop, else repeat from step 2).

Divisive Hierarchical clustering - It is just the reverse of Agglomerative Hierarchical approach.

<http://sites.google.com/site/dataclusteringalgorithms/hierarchical-clustering-algorithm>

## HCA: Types

- ❖ *Agglomerative HCA*: where you start bottom-up from individual items (eg docs) and merge them on similarity (shortest distance)
- ❖ *Divisive HCA\**: where you start top-down from one cluster and sub-divide it by difference (longest distance)

\* in practice, works poorly

C. D. Manning & H. Schütze. *Foundations of Statistical Natural Language Processing*. MIT Press, Camb., MA, 2001.

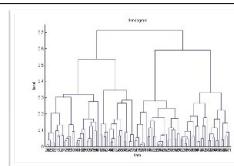
## HCA: Pros / Cons

### PROS:

- ❖ works well most of the time
- ❖ a priori number of clusters not required

### CONS:

- ❖ Correct no of clusters can be hard to identify
- ❖ Depending on  $dist$ : (i) sensitive to noise and outliers, (ii) breaking large clusters, (ii) handling different-sized clusters and convex shapes
- ❖ Complexity in memory and time



## HCA Eg: Text Clustering

- ❖ We saw earlier; dividing a set of documents into a number of classes

Table 1: Summary description of document sets.			
Data Set	Source	Documents	Classes
re0	Reuters	1504	13
re1	Reuters	1657	25
wap	WebAce	1560	20
tr31	TREC	927	7
tr45	TREC	690	10
fbis	TREC	2463	17
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Steinbach, M., Karypis, G., & Kumar, V. (2000, August). A comparison of document clustering techniques. In KDD workshop on text mining (Vol. 400, No. 1, pp. 525-526).

## HCA: One Issue

- ❖ How to you compare a group to other item/group?
- ❖ use the centroid of the group
- ❖ use the average of the group
- ❖ use the max/min of distance between items in the group
- ❖ More complicated methods:
  - ❖ Decrease in variance of group being merged (*Ward's Criterion*)
  - ❖ Probability that groups come from same distribution

```
import numpy as np
import scipy.cluster.hierarchy as hac
import matplotlib.pyplot as plt

a = np.array([[0.1, 2.5],
              [1.5, .4],
              [0.3, 1 ],
              [1 , 0.8 ],
              [0.1, 0.1],
              [0.5, 0.5],
              [0.5, 0.5],
              [2.1, 1 ],
              [2.2, 3.1],
              [3 , 2 ],
              [3.2, 1.3]])

fig, axes23 = plt.subplots(2, 3)

for method in zip(['single', 'complete'], axes23):
    z = hac.linkage(a, method=method)

    # Plotting
    axes[0].plot(range(1, len(z)+1), z[:,1], 'k.-')
    knee = np.diff(z[:,1], 2, 2)
    axes[0].text(1.5, knee, 'knee')

    num_clust1 = knee.argmax() + 2
    num_clust2 = knee.argmax() + 2
    num_clust1 = knee.argmax() + 2
    num_clust2 = knee.argmax() + 2

    axes[0].text(num_clust1, z[-1, 2][num_clust1-1], 'possible(')

    part1 = hac.fcluster(z, num_clust1, 'maxclust')
    part2 = hac.fcluster(z, num_clust2, 'maxclust')

    clr = ['#2200CC', '#D9B07E', '#FF6600', '#AC6600', '#B099CC',
           '#9900CC', '#FFB0B0', '#FF9900', '#B0C0C0', '#B0B0CC']

    axes[1].text(1.5, 1.5, 'cosine')
    axes[1].text(1.5, 2.5, 'euclidean')
    axes[1].text(1.5, 3.5, 'manhattan')
    axes[1].text(1.5, 4.5, 'ward')

    axes[1].text(2.5, 1.5, 'single')
    axes[1].text(2.5, 2.5, 'complete')
    axes[1].text(2.5, 3.5, 'average')
    axes[1].text(2.5, 4.5, 'ward')

    axes[1].text(3.5, 1.5, 'maxclust')
    axes[1].text(3.5, 2.5, 'maxclust')
    axes[1].text(3.5, 3.5, 'maxclust')
    axes[1].text(3.5, 4.5, 'maxclust')

    axes[1].text(4.5, 1.5, 'single')
    axes[1].text(4.5, 2.5, 'complete')
    axes[1].text(4.5, 3.5, 'average')
    axes[1].text(4.5, 4.5, 'ward')

    axes[1].text(5.5, 1.5, 'maxclust')
    axes[1].text(5.5, 2.5, 'maxclust')
    axes[1].text(5.5, 3.5, 'maxclust')
    axes[1].text(5.5, 4.5, 'maxclust')

    axes[1].text(6.5, 1.5, 'single')
    axes[1].text(6.5, 2.5, 'complete')
    axes[1].text(6.5, 3.5, 'average')
    axes[1].text(6.5, 4.5, 'ward')

    axes[1].text(7.5, 1.5, 'maxclust')
    axes[1].text(7.5, 2.5, 'maxclust')
    axes[1].text(7.5, 3.5, 'maxclust')
    axes[1].text(7.5, 4.5, 'maxclust')

    axes[1].text(8.5, 1.5, 'single')
    axes[1].text(8.5, 2.5, 'complete')
    axes[1].text(8.5, 3.5, 'average')
    axes[1].text(8.5, 4.5, 'ward')

    axes[1].text(9.5, 1.5, 'maxclust')
    axes[1].text(9.5, 2.5, 'maxclust')
    axes[1].text(9.5, 3.5, 'maxclust')
    axes[1].text(9.5, 4.5, 'maxclust')

    axes[1].text(10.5, 1.5, 'single')
    axes[1].text(10.5, 2.5, 'complete')
    axes[1].text(10.5, 3.5, 'average')
    axes[1].text(10.5, 4.5, 'ward')

    axes[1].text(11.5, 1.5, 'maxclust')
    axes[1].text(11.5, 2.5, 'maxclust')
    axes[1].text(11.5, 3.5, 'maxclust')
    axes[1].text(11.5, 4.5, 'maxclust')

    axes[1].text(12.5, 1.5, 'single')
    axes[1].text(12.5, 2.5, 'complete')
    axes[1].text(12.5, 3.5, 'average')
    axes[1].text(12.5, 4.5, 'ward')

    axes[1].text(13.5, 1.5, 'maxclust')
    axes[1].text(13.5, 2.5, 'maxclust')
    axes[1].text(13.5, 3.5, 'maxclust')
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    axes[1].text(14.5, 3.5, 'average')
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    axes[1].text(21.5, 3.5, 'maxclust')
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    axes[1].text(26.5, 1.5, 'single')
    axes[1].text(26.5, 2.5, 'complete')
    axes[1].text(26.5, 3.5, 'average')
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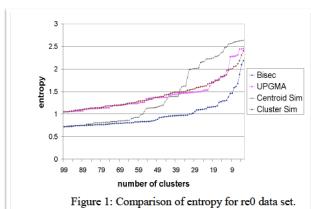
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    axes[1].text(93.5, 4.5, 'maxcl
```

## HCA Eg: Results

- ◆ Agglomerative Clustering different methods



Steinbach, M., Karypis, G., & Kumar, V. (2000, August). A comparison of document clustering techniques. In KDD workshop on text mining (Vol. 400, No. 1, pp. 525-526).

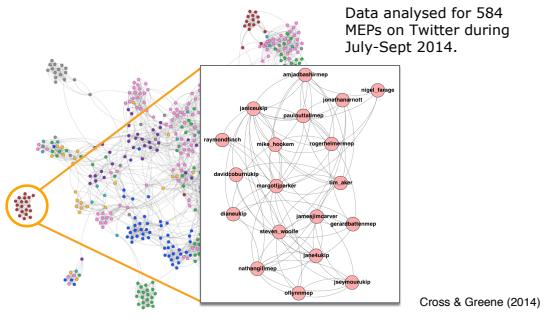
## Graphs: The Idea

- ◆ Many problems can be cast as graphs; entities and relations as nodes with links (vertices and edges)
  - ◆ So, we link up entities on the basis that they are related in some way (mailing, replying, sim docs)
  - ◆ Then the structure of that graph can tell us about clusters of entities; entities that are “close” or “highly connected” or “related”
  - ◆ Graph-based clustering uses many ideas from graph theory to define groups

# Clustering

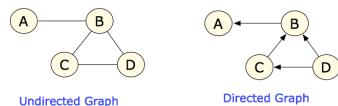
## Graph-Based Clustering

# Graphs: The Idea



# Graphs: Basics I

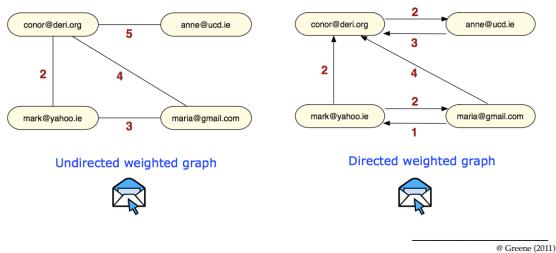
- **Graph:** a way of representing the relationships among a collection of objects.
  - Consists of a set of objects, called **nodes**, with certain pairs of these objects connected by links called **edges**.



- Two nodes are **neighbours** if they are connected by an edge.
  - **Degree** of a node is the number of edges ending at that node.
  - For a directed graph, the **in-degree** and **out-degree** of a node refer to numbers of edges incoming to or outgoing from the node.

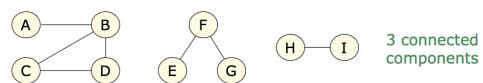
## Graphs: Basics II

- **Weighted graph**: numeric value is associated with each edge.
  - Edge **weights** may represent a concept such as similarity, distance, or connection cost.



## Graphs: Connectivity & Components

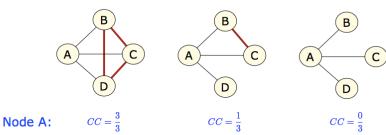
- A graph is **connected** if there is a path between every pair of nodes in the graph.
  - A **connected component** is a subset of the nodes where:
    1. A path exists between every pair in the subset.
    2. The subset is not part of a larger set with the above property.



- In many empirical social networks a larger proportion of all nodes will belong to a single **giant component**.

## Graphs: Clustering Coefficient

- The **neighbourhood** of a node is set of nodes connected to it by an edge, not including itself.
- The **clustering coefficient** of a node is the fraction of pairs of its neighbours that have edges between one another.

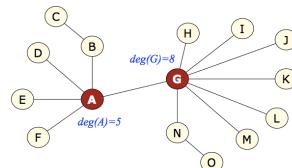


- Locally indicates how concentrated the neighbourhood of a node is, globally indicates level of clustering in a graph.
- Global score is average over all nodes:  $\bar{CC} = \frac{1}{n} \sum_{i=1}^n CC(v_i)$

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## Graphs: Community Detection

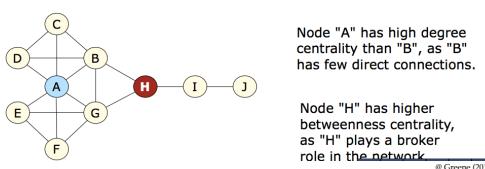
- A variety of different measures exist to measure the importance, popularity, or social capital of a node in a social network.
- Degree centrality** focuses on individual nodes - it simply counts the number of edges that a node has.
- Hub** nodes with high degree usually play an important role in a network. For directed networks, in-degree is often used as a proxy for popularity.



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## Graphs: Betweenness Centrality

- A **path** in a graph is a sequence of edges joining one node to another. The **path length** is the number of edges.
- Often want to find the **shortest path** between two nodes.
- A graph's **diameter** is the longest shortest path over all pairs of nodes.
- Nodes that occur on many shortest paths between other nodes in the graph have a high **betweenness centrality** score.



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## Graphs: Centrality

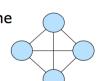
- The **eigenvector centrality** of a node proportional to the sum of the centrality scores of its neighbours.
- A node is important if it is connected to other important nodes.
- A node with a small number of influential contacts may outrank one with a larger number of mediocre contacts.
- Computation:**
  - Calculate the eigendecomposition of the pairwise **adjacency matrix** of the graph.
  - Select the eigenvector associated with largest eigenvalue.
  - Element  $i$  in the eigenvector gives the centrality of the  $i$ -th node.



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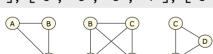
## Graphs: Cliques

- A **clique** is a social grouping where everyone knows everyone else (i.e. there is an edge between each pair of nodes).
- A **maximal clique** is a clique that is not a subset of any other clique in the graph.
- A clique with size greater than or equal to that of every other clique in the graph is called a **maximum clique**.



Find all maximal cliques in the specified graph:

```
>>> cl = list(nx.find_cliques(g))  
  
>>> print cl  
[['a', 'b', 'f'], ['c', 'e', 'b', 'f'], ['c', 'e', 'd']]
```

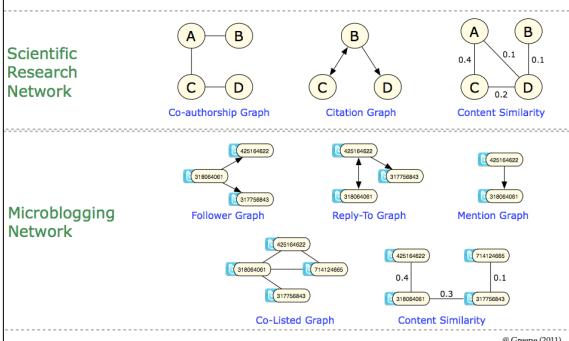


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## Graphs: Using Them...

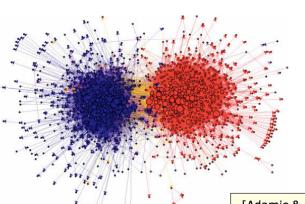
- So, these different properties of the graph are used to find clusters based on how :
  - the graph is structured
  - a node is connected to other nodes
  - one group of nodes differs from another
- Properties like (i) clique numbers, (ii) clique structure, (iii) centrality, (iv) in-/out-degrees, (v) weights/distance

## A Clique Means...



## Community Detection

- We will often be interested in identifying communities of nodes in a network...



[Adamic & Glance, 2005]

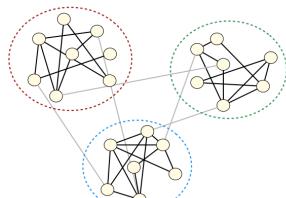
- Example: Two distinct communities of bloggers discussing 2004 US Presidential election.



## Social Media Community is...

## Graphs: Community Detection

- A variety of definitions of **community/cluster/module** exist:
    - A group of nodes which share common properties and/or play a similar role within the graph [Fortunato, 2010].
    - A subset of nodes within which the node-node connections are dense, and the edges to nodes in other communities are less dense [Girvan & Newman, 2002].



[Girvan & Newman, 2002]

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CTech: Modularity Optimisation

- Newman & Girvan [2004] proposed measure of partition quality....  
 ➔ Random graph shouldn't have community structure.  
 ➔ Validate existence of communities by comparing actual edge density with expected edge density in random graph.

$Q = (\text{number of edges within communities}) - (\text{expected number within communities})$

  - Apply agglomerative technique to iteratively merge groups of nodes to form larger communities such that modularity increases after merging.
  - Recently efficient greedy approaches to modularity maximisation have been developed that scale to graphs with up to  $10^9$  edges.

Louvain Method [Blondel et al, 2008] <http://findcommunities.googlepages.com>

## Issues for Community Detection:

- Total number of edges in graph controls the resolution at which communities are identified [Fortunato, 2010].
  - Is it realistic/useful to assign nodes to only a single community?

## Graphs: Clustering Methods

- ◆ So, there are many different methods that can be used to find interesting clusters in graphs:
    - ◆ computing centrality (hubs and authorities)
    - ◆ finding connected components (connected farms)
    - ◆ partition the graph according to some criteria
    - ◆ one group of nodes differs from another
  - ◆ Properties like (i) clique numbers, (ii) clique structure, (ii) centrality, (iii) cluster coefficients, (iv) in-/out-degrees, (v) weights/distance

## ClusterTech: Partitioning

- Goal:** Divide the nodes in a graph into a user-specified number of disjoint groups to optimise a criterion related to number of edges cut.

• **Min-cut** simply involves minimising number (or weight) of edges cut by the partition.

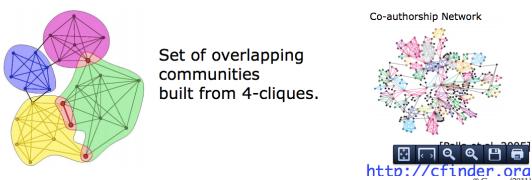
• Recent approaches use more sophisticated criteria (e.g. normalised cuts) and apply multi-level strategies to scale to large graphs.

$\text{cut}(A, B) = 3$

**Issues:** Requirement to pre-specify number of partitions, cut criteria often make strong assumptions about cluster structure

## k-cliques & overlapping communities

- **CFinder**: algorithm based on the clique percolation method [Palla et al., 2005].
  - Identify *k-cliques*: a fully connected subgraph  $k$  nodes.
  - Pair of  $k$ -cliques are "adjacent" if they share  $k-1$  nodes.
  - Form overlapping communities from maximal union of  $k$ -cliques that can be reached from each other through adjacent  $k$ -cliques.



## Graphs: Random Networks

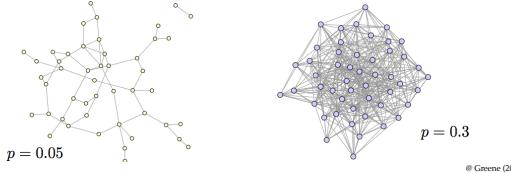
- Erdős-Rényi random graph model:
    - Start with a collection of  $n$  disconnected nodes.
    - Create an edge between each pair of nodes with a probability  $p$ , independently of every other edge.

```
g1 = networkx.erdos_renyi_graph(50, 0.05)
```

```
G2 = networkx.erdos_renyi_graph(50, 0.3)
```

```
g2 = networkx.erdos_renyi_graph(50, 0.3)
```

Specify number of nodes  
to create, and connection  
probability  $p$ .



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## Graphs: Caution

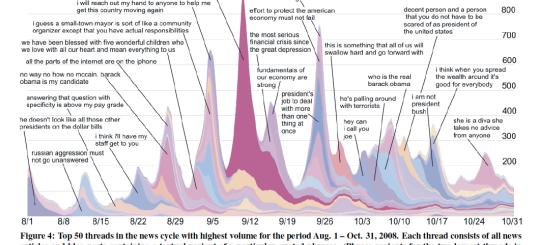
- ◆ Note that a lot of graph-research is not specifically or uniquely about text analytics
  - ◆ More about social relations; but there are cases where the graph relations are based on analysing the text of nodes (eg. MEP)
  - ◆ We consider eggs that are mainly textual

## EG1: Kleinberg's Memes

- ◆ Looks at how memes (quotes) rise and fall with news attention..."lipstick on a pig"...analyses 90M item corpus of quotes in news articles & blogs
  - ◆ Built graphs of the links between quote-bits (with frequencies) and partitioned them into components by deleting links in the graph
  - ◆ Then pictured changing occurrences over time

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articles and blog posts containing a textual variant of a particular quoted phrases. (Phrase variants for the two largest threads in each week are shown as labels pointing to the corresponding thread.) The data is drawn as a stacked plot in which the thickness of the strand corresponding to each thread indicates its volume over time. Interactive visualization is available at <http://memetracker.org>.

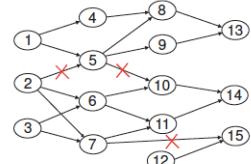
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## EG1: Kleinberg's Memes

- ◆ To find clusters of phrases from news, that should be quotes (“genetic signatures”); but note how they are not always identical !
  - ◆ Corpus 90M news articles and blogs from Q4 2008; 390GB; 122M quotes, cut down to 47M common ones
  - ◆ Build a phrase graph, where each node is a phrase and directed edges connect “related” phrases

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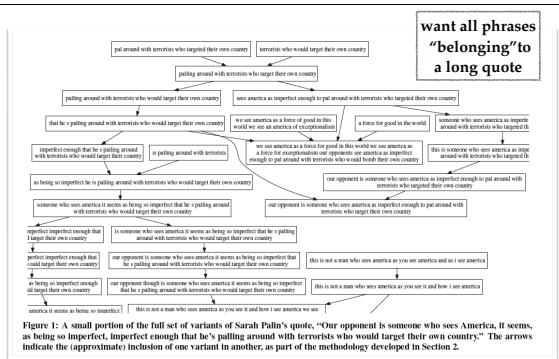
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**Figure 2: Phrase graph.** Each phrase is a node and we want to delete the least edges so that each resulting connected component has a single root node/phrase, a node with zero out-edges. By deleting the indicated edges we obtain the optimal solution.

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**Figure 1:** A small portion of the full set of variants of Sarah Palin's quote, "Our opponent is someone who sees America, it seems, as being so imperfect, imperfect enough that he's palling around with terrorists who would target their own country." The arrows indicate the (approximate) inclusion of one variant in another, as part of the methodology developed in Section 2.

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## EG1: Phase Selection

- ◆ Phrase is quoted string in a given article
  - ◆ All quotes: lower-bound,  $L$ , on phrase word-length
  - ◆ Lower-bound,  $M$ , on frequency of phrase in corpus
  - ◆ Eliminate phrases with  $e$  fraction of item occurrences from the same domain (aka Spammers)
  - ◆ Used  $e=25$ ,  $M = 10$ ,  $L = 4$

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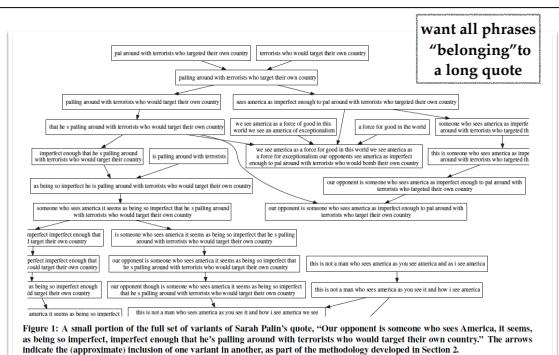
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## EG1: Graph Building & Cuts

- Build graph:  $p$  and  $q$  are related when  $p$  is contiguous subsequence of the words in phrase  $q$ , with tolerance for mismatching (edit distance = 1; 10 in-a-row)
  - All edges in this DAG, point from shorter  $p$ -phrases to longer  $q$ -phrases, with weights on edges that reflect edit-distance between  $p$  and  $q$  plus frequency of  $q$
  - Partition graph by cutting edges; phrase cluster should be a subgraph for which all paths ( $ps$  and minor  $qs$ ) terminate in a single root node ( $q$ ), delete edges of small total weight from phrase graph so it falls apart into disjoint parts
  - Clustering becomes a *DAG partitioning problem* turns out to be NP-hard for optimal solutions but there do a heuristic version that usually work

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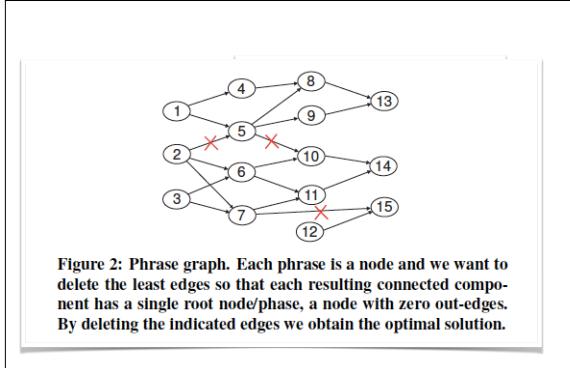
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**Figure 1:** A small portion of the full set of variants of Sarah Palin's quote, "Our opponent is someone who sees America, it seems, as being so imperfect, imperfect enough that he's palling around with terrorists who would target their own country." The arrows indicate the (approximate) inclusion of one variant in another, as part of the methodology developed in Section 2.

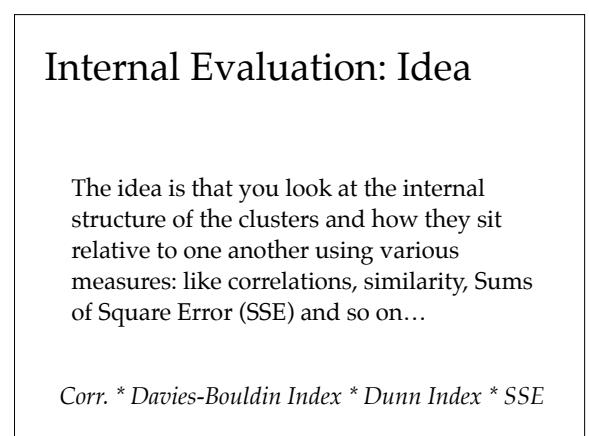
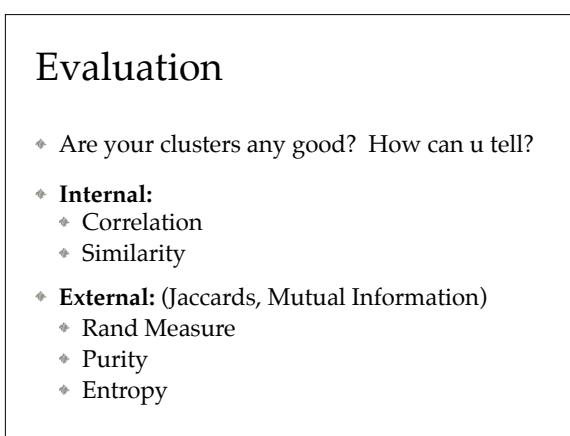
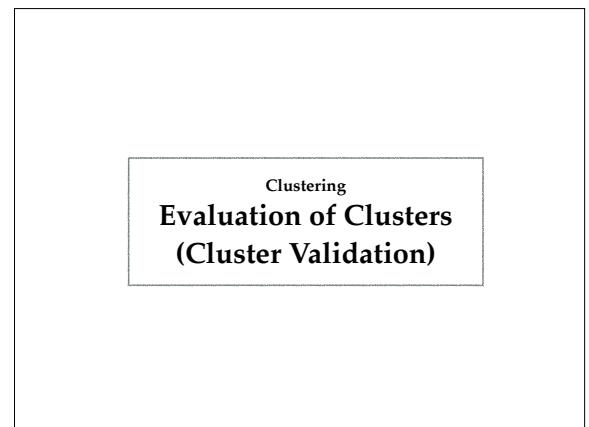
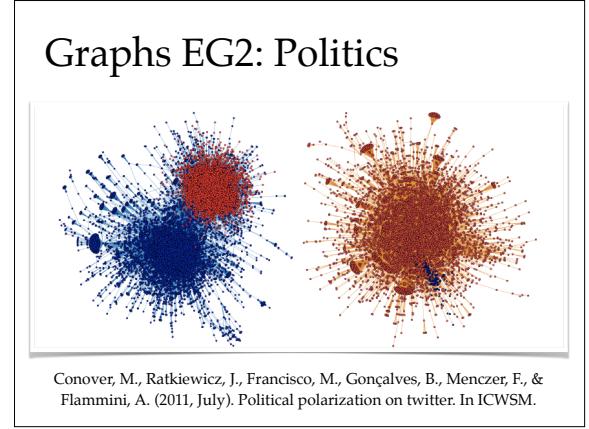
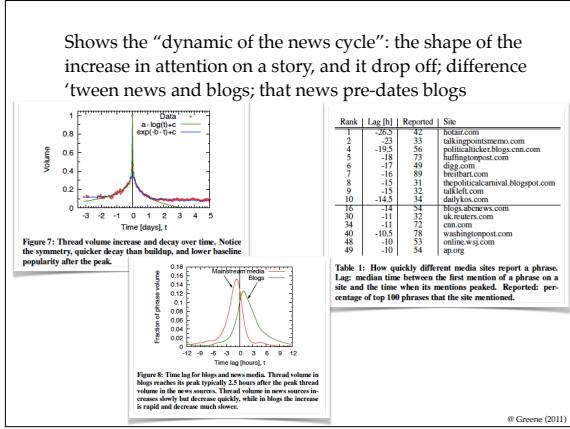
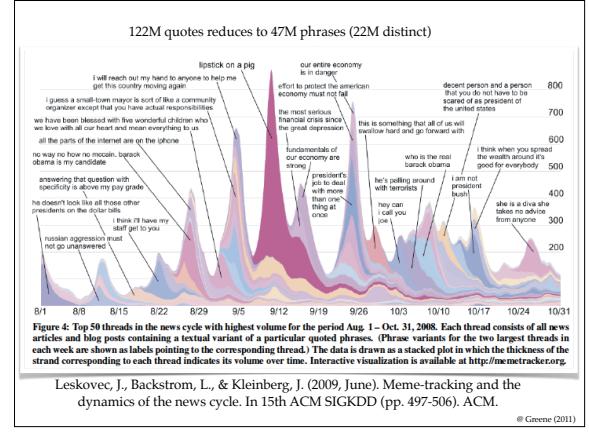
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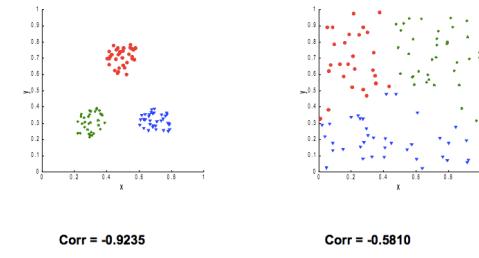


## Internal: Correlation

- >Create two matrices:
  - Distance*: a square matrix, with  $i$  rows and  $j$  columns; where a given  $i,j$  cell has an entry for similarity of these two items to one another, or the distance they are apart
  - Incidence*: an aligned matrix, same  $ij$  items, where (a) one row, one column for each data point; (b) entry is 1 if the associated pair of points belong to the same cluster, (c) entry is 0 if the associated pair of points belongs to different clusters
- Compute the correlation between the two matrices: only  $n(n-1)/2$  entries need to be calculated, as they are symmetric
- High correlation shows points in same cluster are close to each other

<http://www.cs.kent.edu/~jin/DM08/ClusterValidation.pdf>

## Internal: Correlation



<http://www.cs.kent.edu/~jin/DM08/ClusterValidation.pdf>

## Internal: Similarity

- Often used to compare competing algorithms; best is the one with highest within-cluster similarity & lowest between-cluster similarity
- Do tests to determine the internal similar of items in the cluster and check whether they are closer than ones in other clusters

## Internal: Similarity

### Davies-Bouldin index

The Davies-Bouldin index can be calculated by the following formula:

$$DB = \frac{1}{n} \sum_{x=1}^n \max_{j \neq i} \left( \frac{\sigma_x + \sigma_j}{d(c_i, c_j)} \right)$$

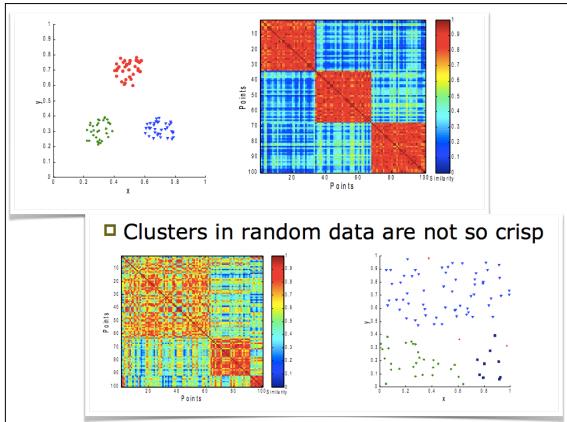
where  $n$  is the number of clusters,  $c_x$  is the centroid of cluster  $x$ ,  $\sigma_x$  is the average distance of all elements in cluster  $x$  to centroid  $c_x$ , and  $d(c_i, c_j)$  is the distance between centroids  $c_i$  and  $c_j$ . Since algorithms that produce clusters with low intra-cluster distances (high intra-cluster similarity) and high inter-cluster distances (low inter-cluster similarity) will have a low Davies-Bouldin index, the clustering algorithm that produces a collection of clusters with the smallest Davies-Bouldin index is considered the best algorithm based on this criterion.

### Dunn index

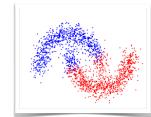
The Dunn index aims to identify dense and well-separated clusters. It is defined as the ratio between the minimal inter-cluster distance to maximal intra-cluster distance. For each cluster partition, the Dunn index can be calculated by the following formula [31]

$$D = \frac{\min_{1 \leq i < j \leq n} d(i, j)}{\max_{1 \leq k \leq n} d'(k)}$$

where  $d(i, j)$  represents the distance between clusters  $i$  and  $j$ , and  $d'(k)$  measures the intra-cluster distance of cluster  $k$ . The inter-cluster distance  $d(i, j)$  between two clusters may be any number of distance measures, such as the distance between the centroids of the clusters. Similarly, the intra-cluster distance  $d'(k)$  may be measured in a variety ways, such as the maximal distance between any pair of elements in cluster  $k$ . Since internal criterion seek clusters with high intra-cluster similarity and low inter-cluster similarity, algorithms that produce clusters with high Dunn index are more desirable.



## Internal: Some Points



- Note, these internal methods are never conclusive; eg., the fact that k-means can be screwed by convex clusters, will not be found
- Note, many methods build in the evaluation metric, so it gets circular
- Probably, best for inter-algorithm comparison

## External Evaluation: Idea

Idea is you go outside to some external source of “right” answers; then its a comparison between what this gold standard says and what we have found: either wrt individual items or the class (item is  $x$ , or item is in class- $y$  not class- $x$ )

Rand \* Jaccard \* F-measure \* Purity \* Entropy

## External: Rand & Jaccard

### Rand measure (William M. Rand 1971)[38]

The Rand index computes how similar the clusters (returned by the clustering algorithm) are to the benchmark classifications. One can also view the Rand index as a measure of the percentage of correct decisions made by the algorithm. It can be computed using the following formula:

$$RI = \frac{TP + TN}{TP + FP + FN + TN}$$

where  $TP$  is the number of true positives,  $TN$  is the number of true negatives,  $FP$  is the number of false positives, and  $FN$  is the number of false negatives. One issue with the Rand index is that false positives and false negatives are equally weighted. This may be an undesirable characteristic for some clustering applications. The F-measure addresses this concern, as does the chance-corrected adjusted Rand index.

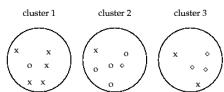
### Jaccard index

The Jaccard index is used to quantify the similarity between two datasets. The Jaccard index takes on a value between 0 and 1. A index of 1 means that the two dataset are identical, and an index of 0 indicates that the datasets have no common elements. The Jaccard index is defined by the following formula:

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|} = \frac{TP}{TP + FP + FN}$$

This is simply the number of unique elements common to both sets divided by the total number of unique elements in both sets.

## External: Purity



► Figure 16.1 Purity as an external evaluation criterion for cluster quality. Majority class and number of members of the majority class for the three clusters are: x, 5 (cluster 1); o, 4 (cluster 2); and x, 3 (cluster 3). Purity is  $(1/17) \times (5 + 4 + 3) = 0.71$ .

To compute purity, each cluster is assigned to the class which is most frequent in the cluster, and then the accuracy of this assignment is measured by counting the number of correctly assigned documents and dividing by  $N$ . Formally:

$$\text{purity}(\Omega, C) = \frac{1}{N} \sum_k \max_j |\omega_k \cap c_j| \quad (182)$$

where  $\Omega = \{\omega_1, \omega_2, \dots, \omega_K\}$  is the set of clusters and  $C = \{c_1, c_2, \dots, c_J\}$  is the set of classes. We interpret  $\omega_k$  as the set of documents in  $\omega_k$  and  $c_j$  as the set of documents in  $c_j$  in Equation 18.2.

We present an example of how to compute purity in Figure 16.4. Bad clusterings have purity values close to 0, a perfect clustering has a purity of 1. Purity is compared with the other three measures discussed in this chapter in Table 16.2.

<http://nlp.stanford.edu/IR-book/html/htmledition/evaluation-of-clustering-1.html>

## External: Entropy

- ◆ Or, you can use entropy; if the items in a cluster are “the same” then entropy is low, if they are “different” then entropy is high

Clustering  
Conclusions

## Selecting Some...

- ◆ Supervised Learning: Classification
  - ◆ k Nearest Neighbours
  - ◆ Naive Bayes
  - ◆ Logistic Regression
  - ◆ Support Vector Machine
- ◆ Unsupervised Learning: Clustering (next lecture)
  - ◆ K-Means
  - ◆ Hierarchical Clustering
  - ◆ Graph-based Clustering

<http://www.slideshare.net/mirjalil/clustering-on-database-systems-rkm-42588852>

End Bits

## Unsupervised: Clustering

- ◆ K-Means
- ◆ Hierarchical Clustering
- ◆ Graph-Based Clustering
- ◆ Expectation Maximisation
- ◆ (Blind Separation & Latent)