Chapter 27a : Different ways to parameterise lead to different results.

We are given f[0..M), g[0..N) of int. We are told that f is ascending and g is descending. We are asked to construct a program to compute the number of pairs f.i and g.j whose sum exceeds 37.

$$r = \langle + \, i,j : 0 \le i < M \wedge 0 \le j < N : h.(f.i).(g.j) \rangle$$

where

* (0) h.x.y	=	1	$\Leftarrow$	$x + y > 37$
* (1) h.x.y	=	0	$\Leftarrow$	$x + y \le 37$

We begin by modelling our domain.

* (2) C.m.n	=	$\langle + \, i,j : m \le i < M \wedge n \le j < N : h.(f.i).(g.j) \rangle$	
- (3) C.M.n	=	0	, $0 \le n \le N$
- (4) C.m.N	=	0	, $0 \le m \le M$
- (5) C.m.n	=	C.(m+1).n + D.n	, $0 \le m < M$
- (6) C.m.n	=	C.m.(n+1) + E.m	, $0 < n \le N$
* (7) D.n	=	$\langle + \, j : n \le j < N : h.(f.m).(g.j) \rangle$	
-*(8) E.m	=	$\langle + \, i : m \le i < M : h.(f.i).(g.n) \rangle$	
- (9) D.n	=	?	$\Leftarrow$ f.m + g.n > 37
- (10) D.n	=	0	$\Leftarrow$ f.m + g.n $\le$ 37
- (11) E.m	=	M-m	$\Leftarrow$ f.m + g.n > 37
- (12) E.m	=	?	$\Leftarrow$ f.m + g.n $\le$ 37

We note that (9), (10), (11) and (12) do allow us to cover all cases so the model is adequate for constructing a program. This is of course what was done in Chapter 27.

Now I want to look at the other possibilities. I am only going to record the theorems and not their proofs.

* (2) C.m.n	=	$\langle + i,j : m \le i < M \wedge 0 \le j < n : h.(f.i).(g.j) \rangle$	
- (3) C.M.n	=	0	, $0 \le n \le N$
- (4) C.m.0	=	0	, $0 \le m \le M$
- (5) C.m.n	=	C.(m+1).n + D.n	, $0 \le m < M$
- (6) C.m.n	=	C.m.(n-1) + E.m	, $0 < n \le N$
* (7) D.n	=	$\langle + j : 0 \le j < n : h.(f.m).(g.j) \rangle$	
-*(8) E.m	=	$\langle + i : m \le i < M : h.(f.i).(g.(n-1)) \rangle$	
- (9) D.n	=	n $\quad \Leftarrow f.m + g.(n-1) > 37$	
- (10) D.n	=	? $\quad \Leftarrow f.m + g.(n-1) \le 37$	
- (11) E.m	=	M-m $\quad \Leftarrow f.m + g.(n-1) > 37$	
- (12) E.m	=	? $\quad \Leftarrow f.m + g.(n-1) \le 37$	

(9), (10), (11) and (12) do not cover all cases so the model is not adequate.

* (2) C.m.n	=	$\langle + i,j : 0 \le i < m \wedge 0 \le j < n : h.(f.i).(g.j) \rangle$	
- (3) C.0.n	=	0	$, 0 \le n \le N$
- (4) C.m.0	=	0	$, 0 \le m \le M$
- (5) C.m.n	=	C.(m-1).n + D.n	$, 0 \le m < M$
- (6) C.m.n	=	C.m.(n-1) + E.m	$, 0 < n \le N$
* (7) D.n	=	$\langle + j : 0 \le j < n : h.(f.(m-1)).(g.j) \rangle$	
-*(8) E.m	=	$\langle + i : 0 \le i < m : h.(f.i).(g.(n-1)) \rangle$	
- (9) D.n	=	n $\quad \Leftarrow f.(m-1) + g.(n-1) > 37$	
- (10) D.n	=	? $\quad \Leftarrow f.(m-1) + g.(n-1) \le 37$	
- (11) E.m	=	? $\quad \Leftarrow f.(m-1) + g.(n-1) > 37$	
- (12) E.m	=	0 $\quad \Leftarrow f.(m-1) + g.(n-1) \le 37$	

(9), (10), (11) and (12) do cover all cases so the model is adequate.

$$* (2)\ C.m.n = \langle + i,j : 0 \le i < m \wedge n \le j < N : h.(f.i).(g.j) \rangle$$

$$- (3)\ C.0.n = 0 \qquad , 0 \le n \le N$$

$$- (4)\ C.m.N = 0 \qquad , 0 \le m \le M$$

$$- (5)\ C.m.n = C.(m\text{-}1).n + D.n \qquad , 0 \le m < M$$

$$- (6)\ C.m.n = C.m.(n\text{+}1) + E.m \qquad , 0 < n \le N$$

$$* (7)\ D.n = \langle + j : n \le j < N : h.(f.(m\text{-}1)).(g.j) \rangle$$

$$-*(8)\ E.m = \langle + i : 0 \le i < m : h.(f.i).(g.(n)) \rangle$$

$$- (9)\ D.n = ? \qquad \Leftarrow f.(m\text{-}1) + g.n > 37$$

$$- (10)\ D.n = 0 \qquad \Leftarrow f.(m\text{-}1) + g.n \le 37$$

$$- (11)\ E.m = ? \qquad \Leftarrow f.(m\text{-}1) + g.n > 37$$

$$- (12)\ E.m = 0 \qquad \Leftarrow f.(m\text{-}1) + g.n \le 37$$

(9), (10), (11) and (12) do not cover all cases so the model is not adequate.