

Lecture 6

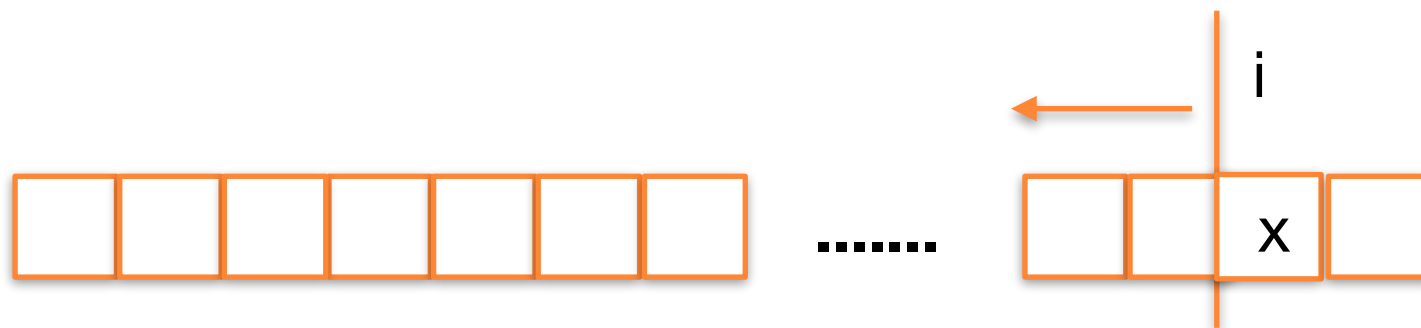
Linear Searches

Linear Searches

- Suppose you have an array `int f[20]` which already contains values.
- you also have a target value stored in the variable `x int`
- you know that this value is also stored somewhere in `f`
- you want to find the **smallest** index `i`, where `f[i] == x`

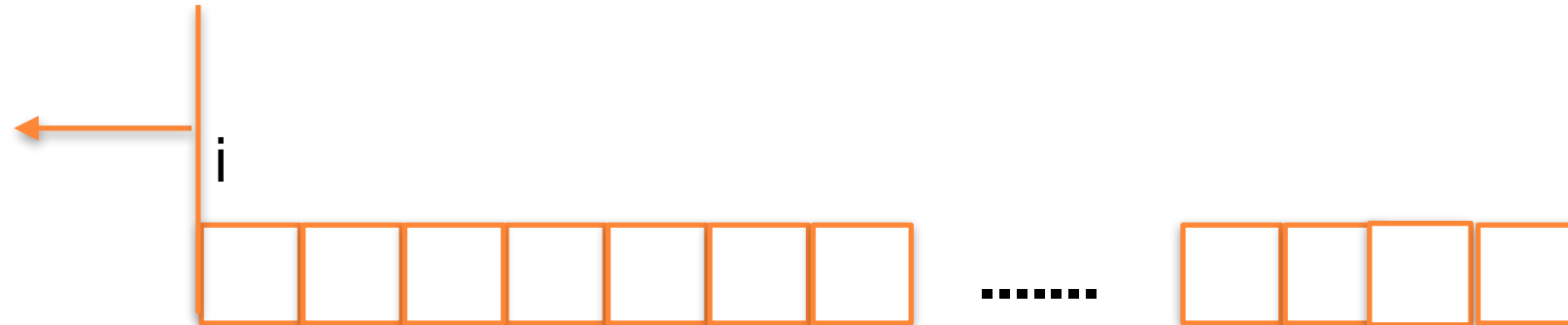
When we are finished we have a picture like this.

We haven't found x in any place to the left of the line, but we have found it in position i



At the start, before we begin, the line is at the left of the array.

We can say that the value x doesn't appear on the left of the line, because there are no values there



Our job is to keep moving that line to the right until we find the value x . We know that we will find it because we know it is in there somewhere.

```
// f[20] and x already have values
```

```
int i ;
```

```
i = 0 ;
```

```
while ( f[i] != x )
```

```
{
```

```
    i = i + 1 ;
```

```
}
```

```
// x is not in the part of f before index i
```

```
// and now f[i] == x
```

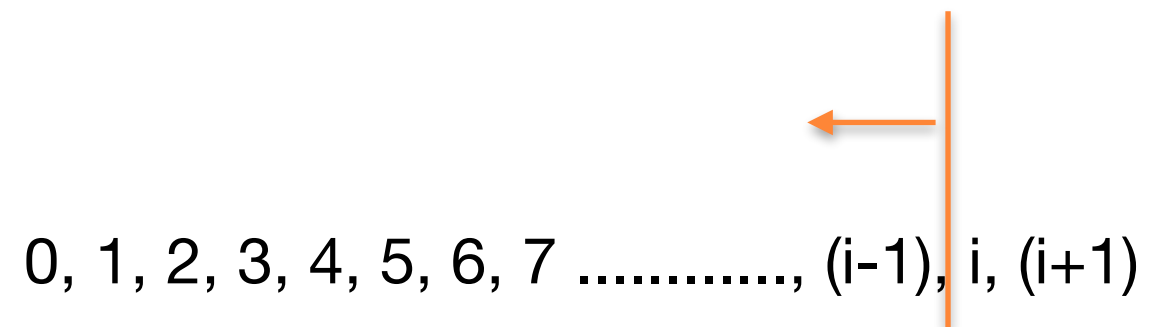
Linear Searches

- The integer square root of a natural number n , is the largest number i where $i * i \leq n$
- e.g. the integer square root of 126 is 11 because $11*11 \leq 126$, but $126 < 12*12$
- Given a natural number n find the integer square root of n

All of the values to the left of the line have the property that when you square them you get a value $\leq n$.

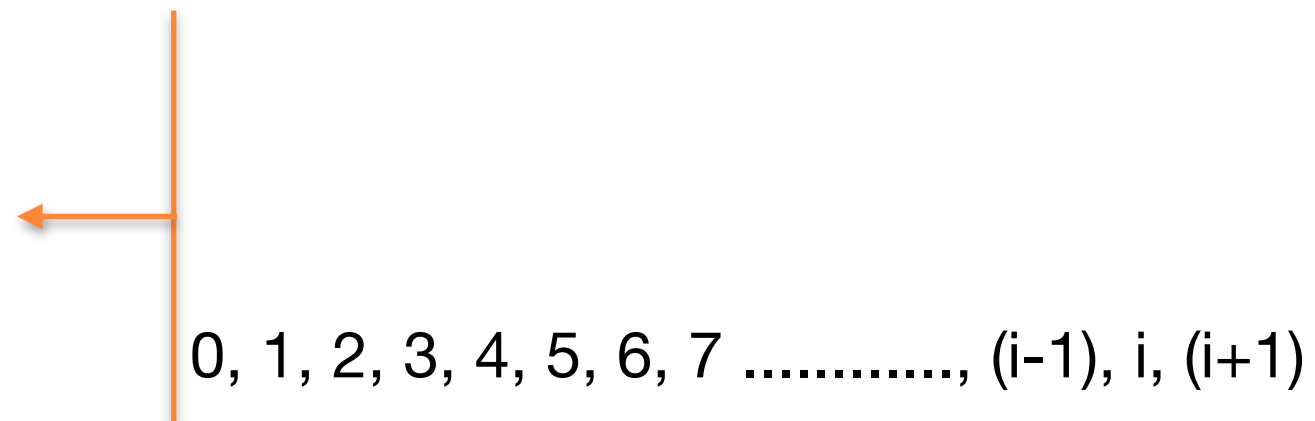
When we are finished, we have found the smallest value i , where $n < i^2$.

So, this will mean that the integer square root which we are looking for must be the value $i-1$



At the start all of the values to the left of the line have the property that when you square them you get a value $\leq n$.

Our job is to move that line to the right.




```
int i ;  
i = 0 ;  
  
while ( i*i <= n )  
{  
    i = i + 1 ;  
}
```

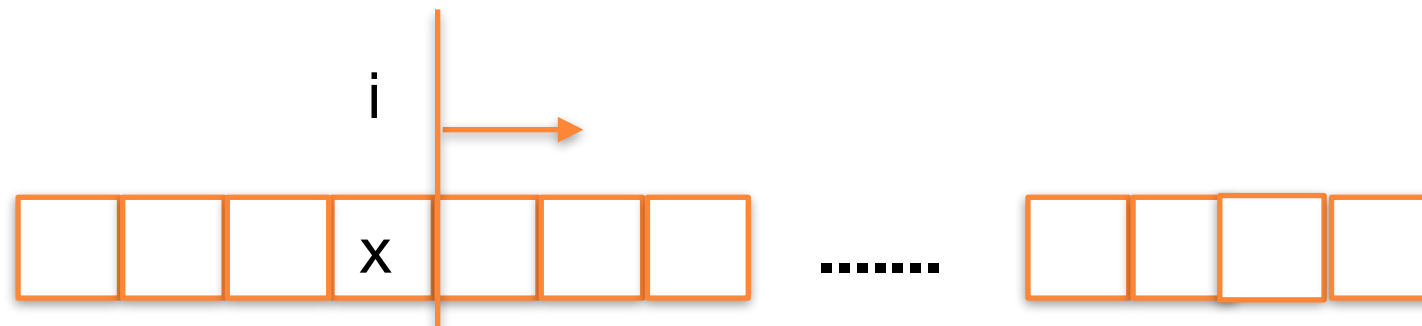
// all of the numbers from 0 to i-1 have the property
// that when they are squared they are $\leq n$.
// but $n < i*i$, so (i-1) is the integer square root of n.

Linear Searches

- Suppose you have an array `int f[20]` which already contains values.
- you also have a target value stored in the variable `x int`
- you know that this value is also stored somewhere in `f`
- you want to find the **largest** index `i`, where `f[i] == x`

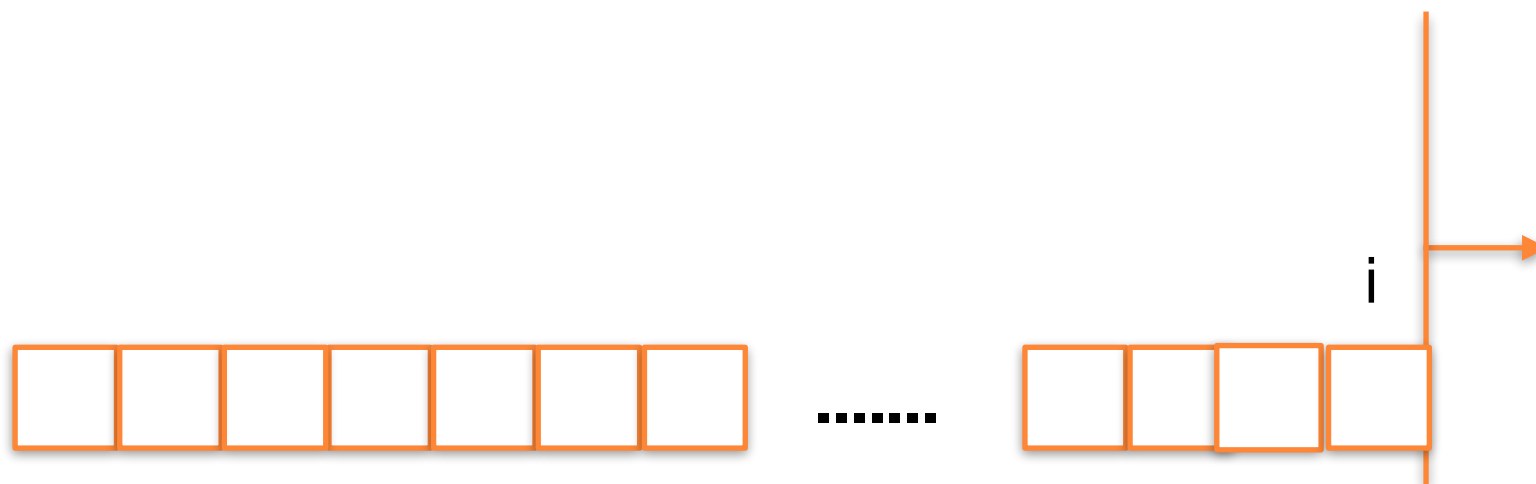
When we are finished we have a picture like this.

We haven't found x in any place to the **right** of the line, but we have found it in position i



At the start, before we begin, the line is at the **right** of the array.

We can say that the value x doesn't appear on the **right** of the line, because there are no values there



Our job is to keep moving that line to the left until we find the value x . We know that we will find it because we know it is in there somewhere.

```
// f[20] and x already have values
```

```
int i ;
```

```
i = 19 ;
```

```
while ( f[i] != x )
```

```
{
```

```
    i = i - 1 ;
```

```
}
```

```
// x is not in the part of f after index i
```

```
// and now f[i] == x
```

Linear Searches

- In the examples which we have seen so far we knew that the value we were searching for existed.
- In the examples using arrays this prevented us from falling off the end of the array. In the number example it prevented us from going on forever.
- What would we need to change if we were not sure that the value we were searching for could be found?

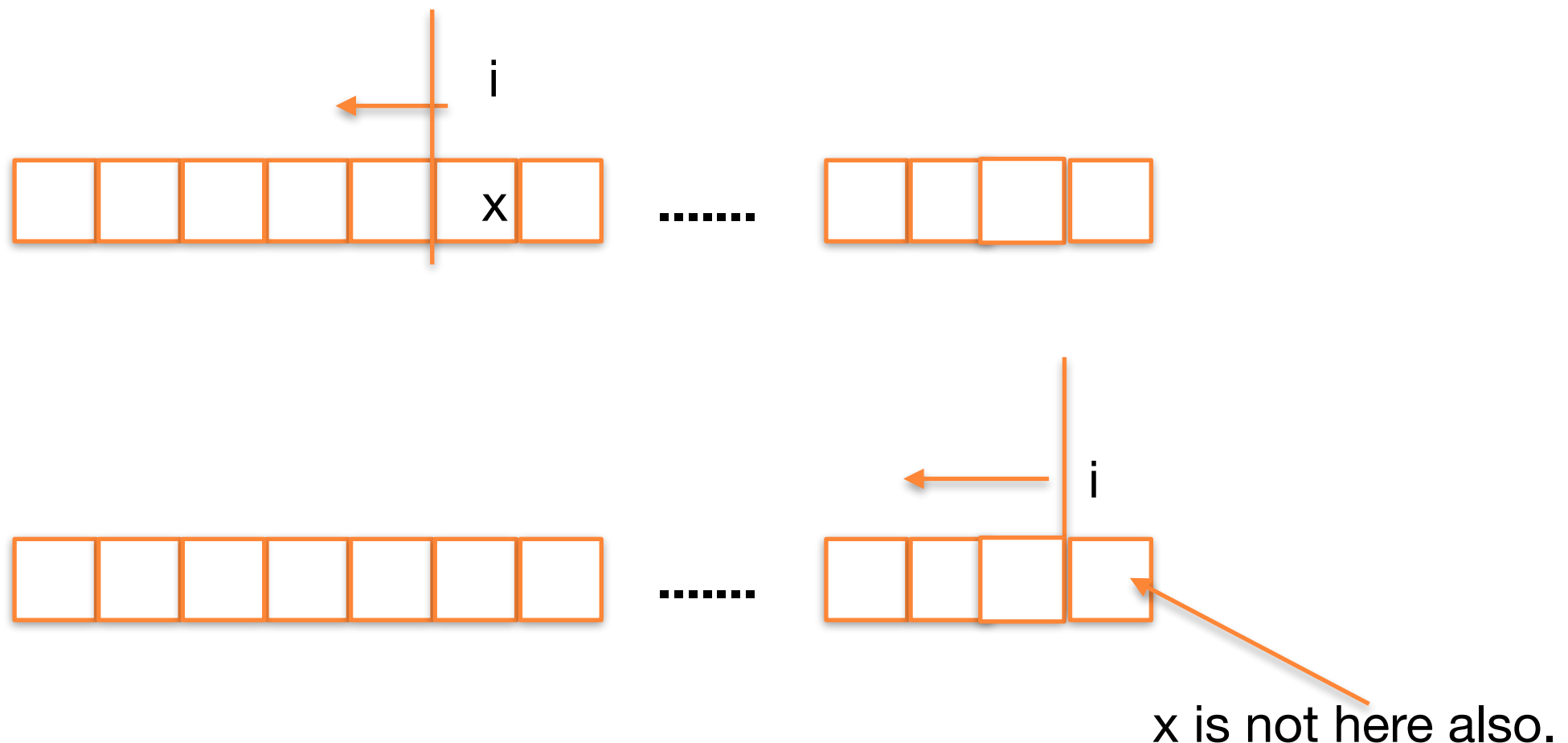
Linear Searches

- Suppose you have an array `int f[20]` which already contains values.
- you also have a target value stored in the variable `x int`
- you want to find the smallest index `i`, where `f[i] == x`, if such an index actually exists.

Now, when we are finished we have a picture like this.

We haven't found x in any place to the left of the line, but we have found it in position i .

Or, we haven't found x in any place to the left of the line and we are at the last place we could look and we don't find it there either.



Case 1.

“From index 0 to index $i-1$, we didn't find x “ and “ $f[i] == x$ ”

Case 2.

“From index 0 to index $i-1$, we didn't find x ” and “ $i = 19$ and $f[i] != x$ ”

We can combine these to get

“From index 0 to index i-1, we didn’t find x”

&&

(f[i] == x) || (i == 19 && f[i] != x)

This describes what will be true when we finish

$$(f[i] == x) \parallel (i == 19 \ \&\& \ f[i] != x)$$

It can be simplified to

$$(f[i] == x) \parallel (i == 19)$$

Now if this describes when we are finished

$$(f[i] == x) \parallel (i == 19)$$

Then the opposite of this describes when we are not finished

$$(f[i] != x) \&\& (i != 19)$$

So this will be the guard on our loop.

```
// f[20] and x already have values
```

```
int i ;
```

```
i = 0 ;
```

```
while (( f[i] != x) && ( i != 19 ))
```

```
{
```

```
    i = i + 1;
```

```
}
```

```
// none of the locations before index i contain the value x.
```

```
// at this stage f[i] == x or i = 19.
```

```
// Now we check to see if we found x
```

```
if ( f[i] == x)
```

```
    { printf (" the value was found at location %d ", i ) ; }
```

```
else
```

```
    { printf("The value was not found in the array") ; }
```

We made use of some Logical Laws in solving that last problem.

$$[(P \parallel (\text{Not}.P \ \&\& \ Q)) == (P \parallel Q)]$$

$$[\text{Not}.(P \ \&\& \ Q) == \text{Not}.P \parallel \text{Not}.Q]$$

$$[\text{Not}. (P \parallel Q) == \text{Not}.P \ \&\& \ \text{Not}.Q]$$

Linear Searches

- Suppose we have an array `int f[100]` which already contains values.
- We want to write a program to determine if the 2nd half of the array is an exact copy of the first half.

Linear Searches

- If the 2nd half is an exact copy of the 1st half then for each value $f[j]$ in the 1st half it will be true that $f[j] == f[j+50]$
- If the 2nd half is not an exact copy of the 1st half then there must be some value $f[i]$ in the 1st half where we find that $f[i] != f[i+50]$
- I am going to try to find a value that doesn't appear 50 places further on, if I find one I conclude they aren't exact copies, if I can't find one and I get to the end I conclude they are exact copies.

Our picture at the end tells us that

“ each of the values in f to the left of index i appear again 50 places to the right”

and either

“ at the current index i , $f[i] \neq f[i+50]$ ”

or

“ we are at the last position in the 1st half of f , $i == 49$, and here too $f[i] == f[50+i]$ ”

We can write this more precisely as

```
(( f[i] != f[i+50] ) || (( i == 49) && ( f[i] == f[50+i] )))
```

This can be simplified to

```
( f[i] != f[i+50] ) || (i == 49))
```

The **opposite** of this then becomes the **loop guard**

```
( f[i] == f[i+50] ) && (i != 49))
```

```
// f[100] already has values
```

```
int i ;
```

```
i = 0 ;
```

```
while (( f[i] == f[i+50) && ( i != 49))
```

```
{
```

```
    i = i + 1 ;
```

```
}
```

```
// Each of the values from the start up to position i
```

```
// are repeated 50 places further on.
```

```
// Now either f[i] != f[i+50] or i == 49. Let us find check.
```

```
if ( f[i] != f[i+50])
```

```
    { printf(" 2nd half is not a copy of first half") ; }
```

```
else
```

```
    { printf(" 2nd half is an exact copy of 1st half"); }
```

Linear Searches

- In the last 2 examples we were searching for something without knowing whether we could find it.
- When we don't have a guarantee that we will find the value we call our search a Bounded Linear Search.

Further problems

- Given int f[100] which already contains values, construct programs to do the following
 - Find out if all of the values in f are positive
 - Find out if any of the values in f are even
 - Find out if f is a palindrome.
 - Find out if f is sorted in ascending order

A Challenge

Suppose we have `f[5]` of `int` which contains single digit values.
We can read it like a single number.

f

1	2	5	4	3
---	---	---	---	---

 12543

Suppose we decide to rearrange it so it reads like the next highest number that we could make using these digits.

1	3	2	4	5
---	---	---	---	---

 13245

How would we do this?