

Tony Veale, UCD

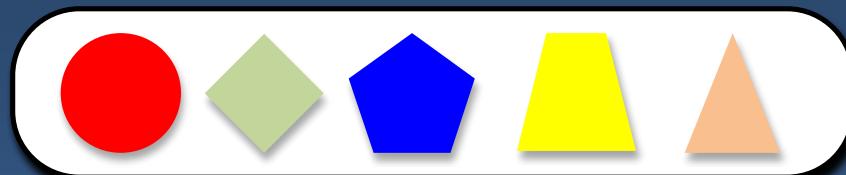
# Relational Algebra

**Relational Algebra  
offers a procedural  
basis for thinking  
about DB querying**



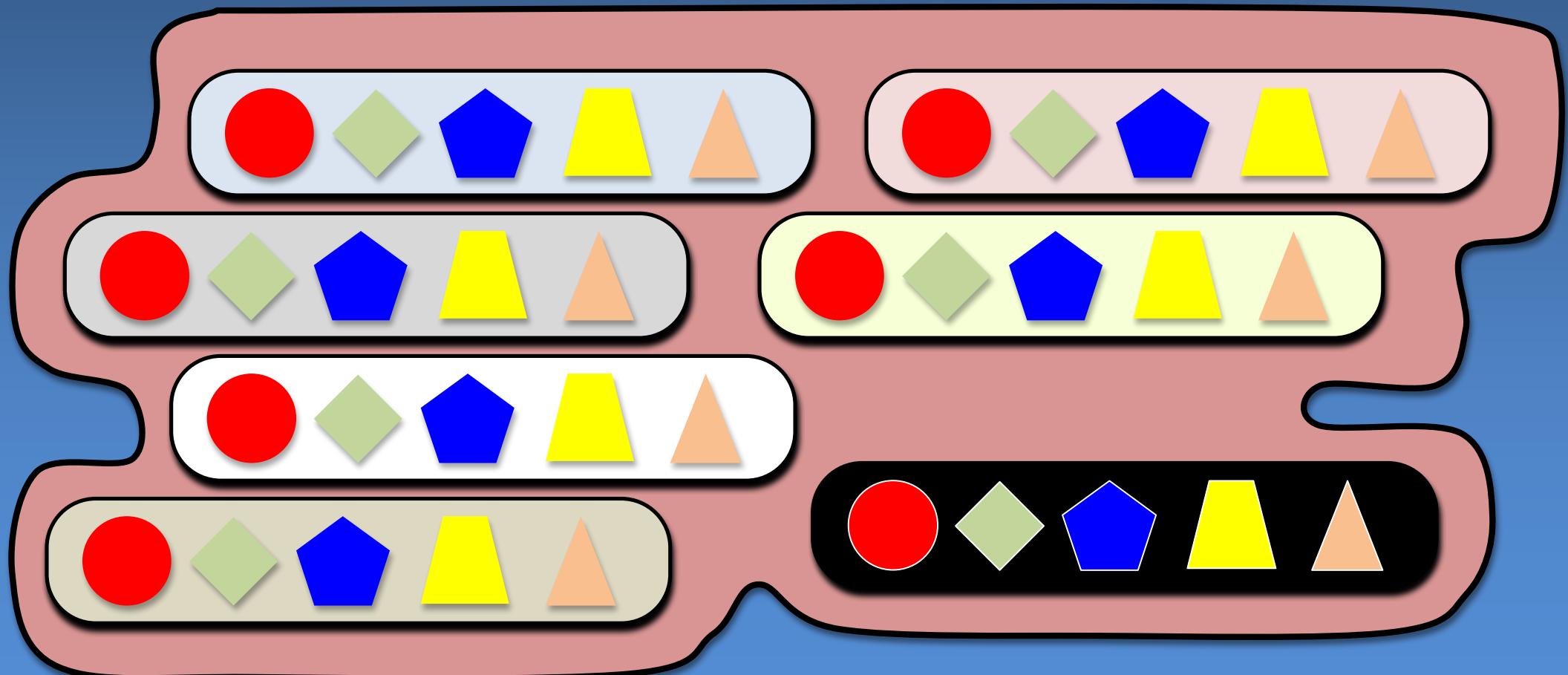
Select operation  
Project operation  
Union operation  
Set difference  
Cartesian product  
Rename as

A **tuple** is a structured grouping of associations:



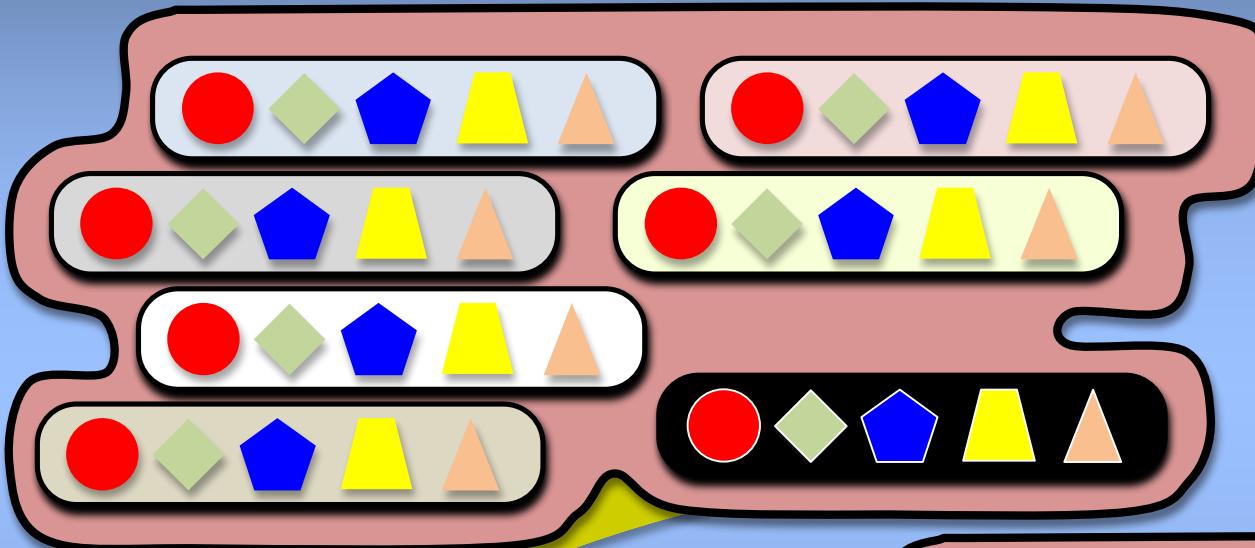
*(One tuple per entity)*

A **relation** is an exhaustive collection of tuples:



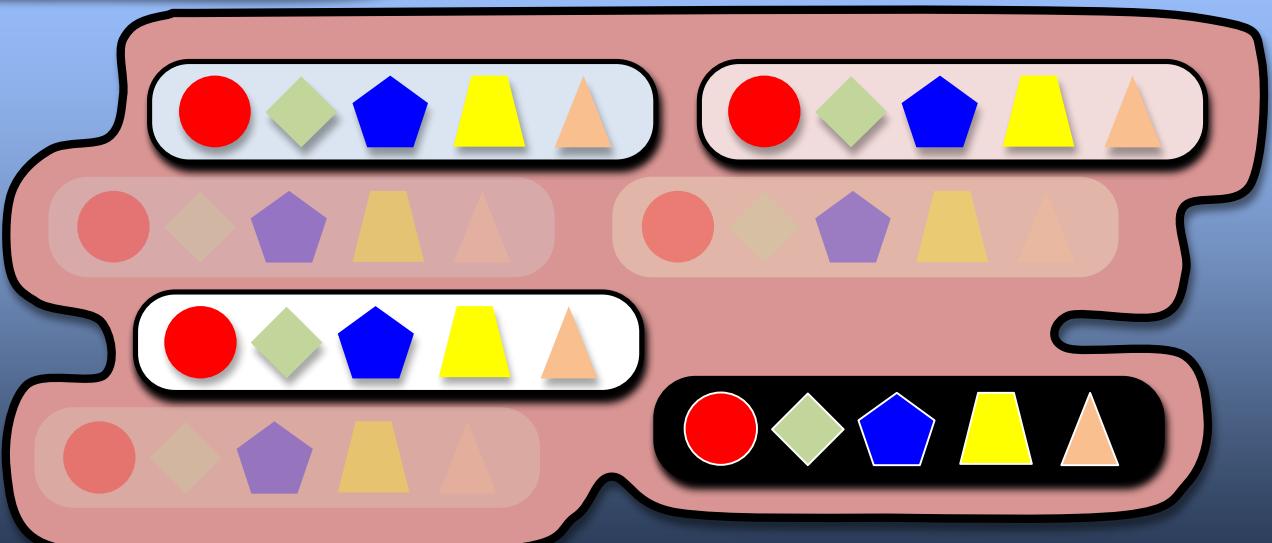
# Select Operation $\sigma_{\text{predicate}}$

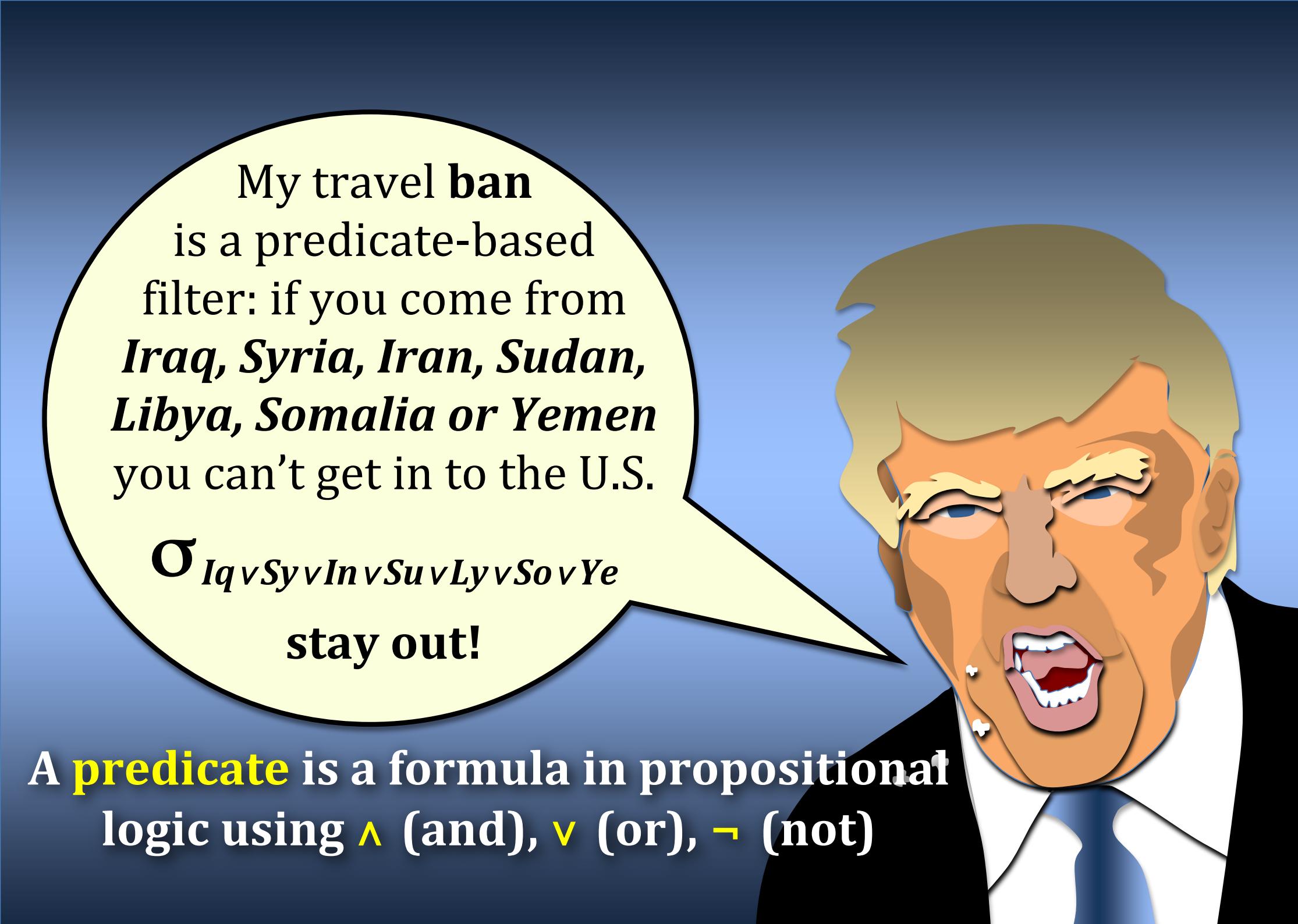
*Select only tuples that satisfy a given test or predicate*



$\sigma_{\text{predicate}}$

A **predicate** is a test comprising one or more constraints. It can be true or false



A caricature of Donald Trump with blonde hair and a blue suit, shouting with his mouth wide open. A yellow speech bubble is positioned in front of him, containing text.

My travel ban  
is a predicate-based  
filter: if you come from  
***Iraq, Syria, Iran, Sudan,  
Libya, Somalia or Yemen***  
you can't get in to the U.S.

$\sigma_{Iq \vee Sy \vee In \vee Su \vee Ly \vee So \vee Ye}$

stay out!

A **predicate** is a formula in propositional logic using  $\wedge$  (and),  $\vee$  (or),  $\neg$  (not)

$\sigma_{\text{Fictive}=\text{"yes"}(\text{Entities})}$  selects fictional entities

$\sigma_{\text{Gender}=\text{"male"}(\text{Entities})}$  selects male entities

$\sigma_{\text{Gender}=\text{"male"} \wedge \text{Fictive}=\text{"yes"}(\text{Entities})}$  selects fictional  
male entities

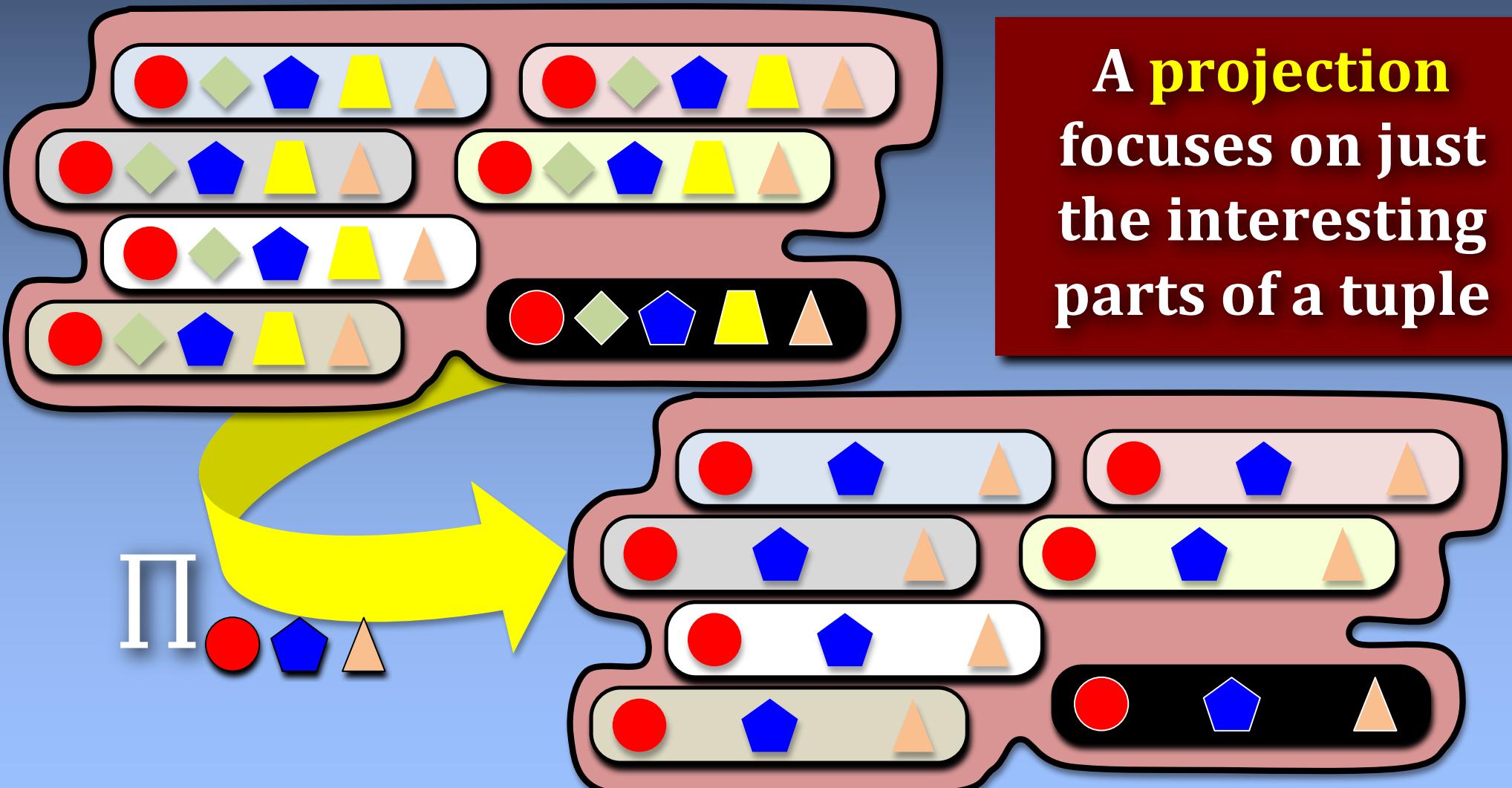
## Entities (relation)

Name	Gender	Fictive	Opponent
Adam Smith	<i>male</i>	<i>no</i>	<i>Karl Marx</i>
Princess Leia	<i>female</i>	<i>yes</i>	<i>Darth Vader</i>
Hillary Clinton	<i>female</i>	<i>no</i>	<i>Donald Trump</i>
Al Capone	<i>male</i>	<i>no</i>	<i>Eliot Ness</i>

The output is a selection of matching tuples

# Project Operation $\prod_{\text{associations}}$ (r)

*projects only **associations** from relation **r** into new tuple set*



$$\Pi_{\text{Name}, \text{Gender}, \text{Opponent}} (\text{Entities})$$

## Entities (minus Fictive)

Name	Gender	Opponent
Adam Smith	<i>male</i>	<i>Karl Marx</i>
Princess Leia	<i>female</i>	<i>Darth Vader</i>
Hillary Clinton	<i>female</i>	<i>Donald Trump</i>
Al Capone	<i>male</i>	<i>Eliot Ness</i>

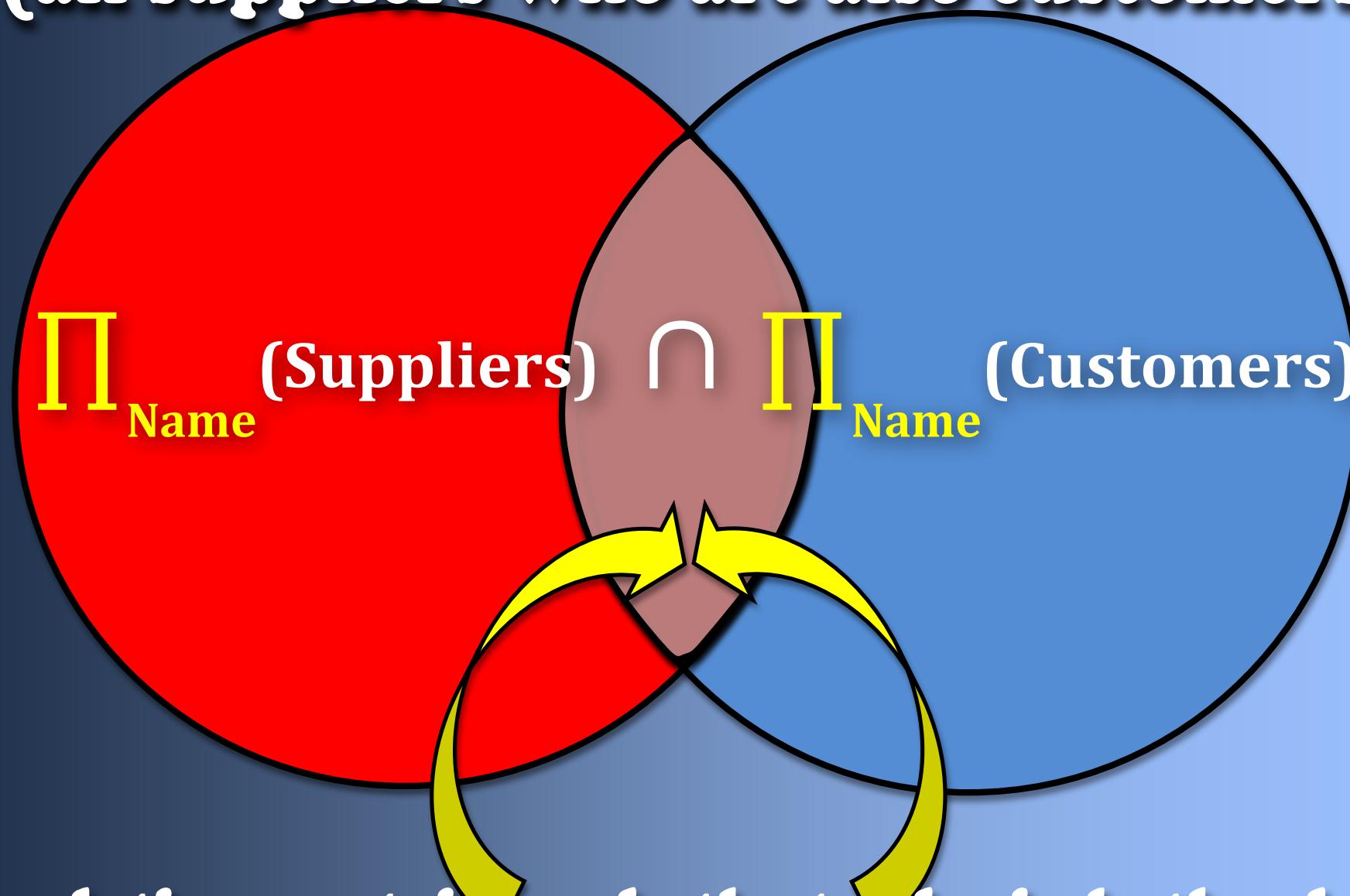
Output is all tuples minus unprojected associations

# Set Union of two relations (so duplicates are removed)

$$\Pi_{\text{Name}} (\text{Suppliers}) \cup \Pi_{\text{Name}} (\text{Customers})$$


The new relation contains every Customer and Supplier

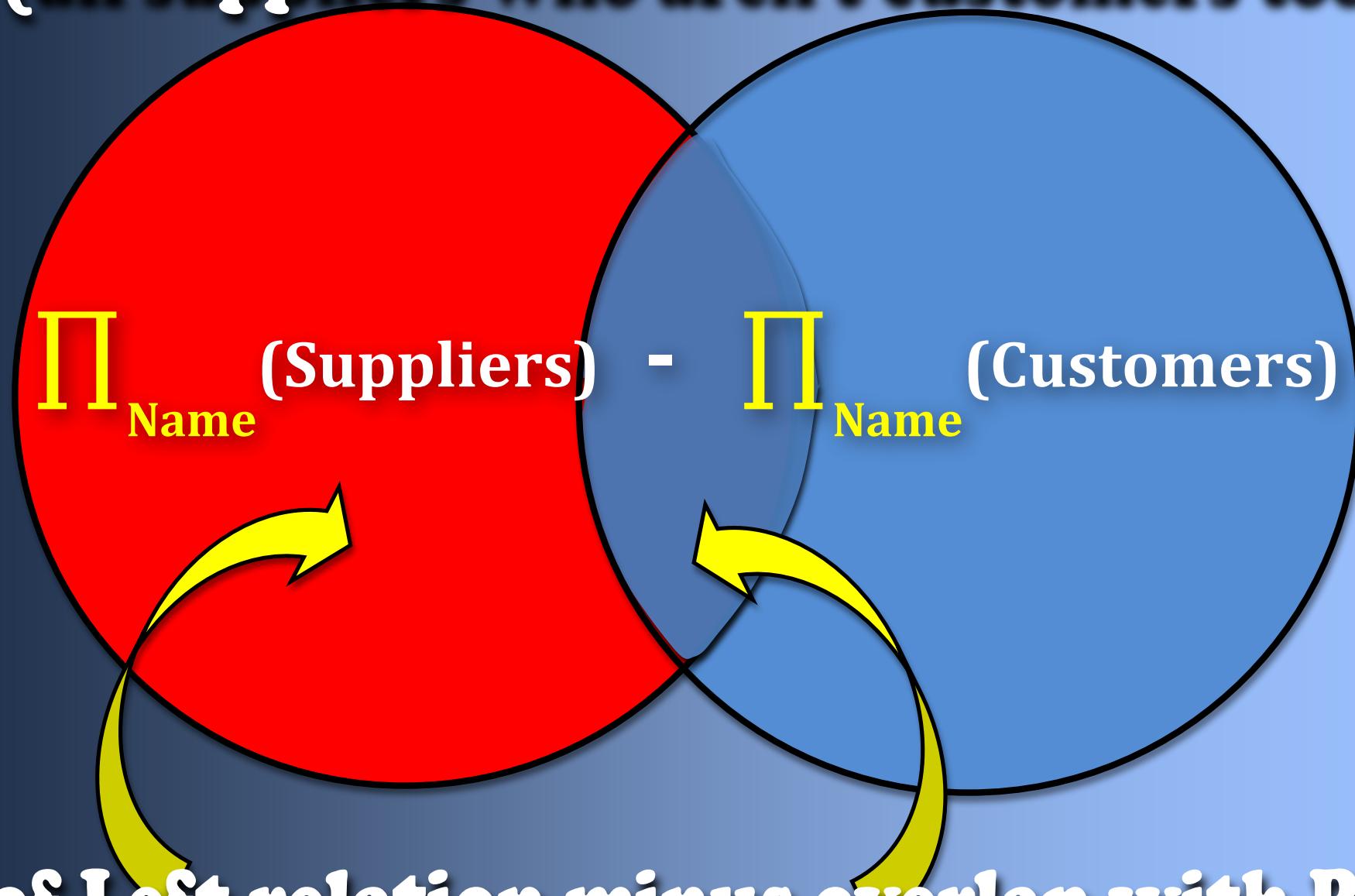
# SET Intersection of Two Relations (all suppliers who are also customers)



New relation contains only the tuples in both relations

# **SET Difference of Two Relations**

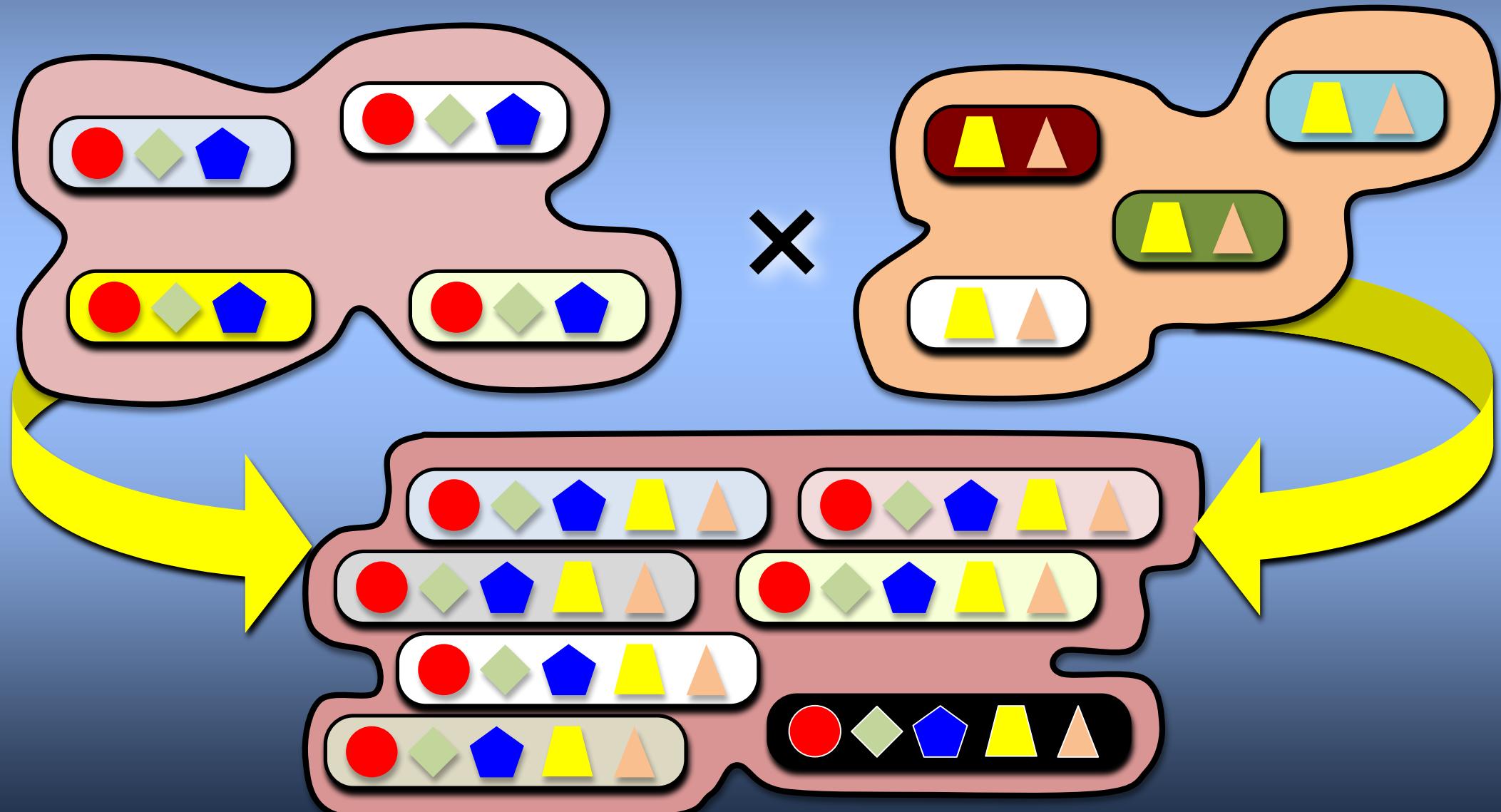
(all suppliers who aren't customers too)



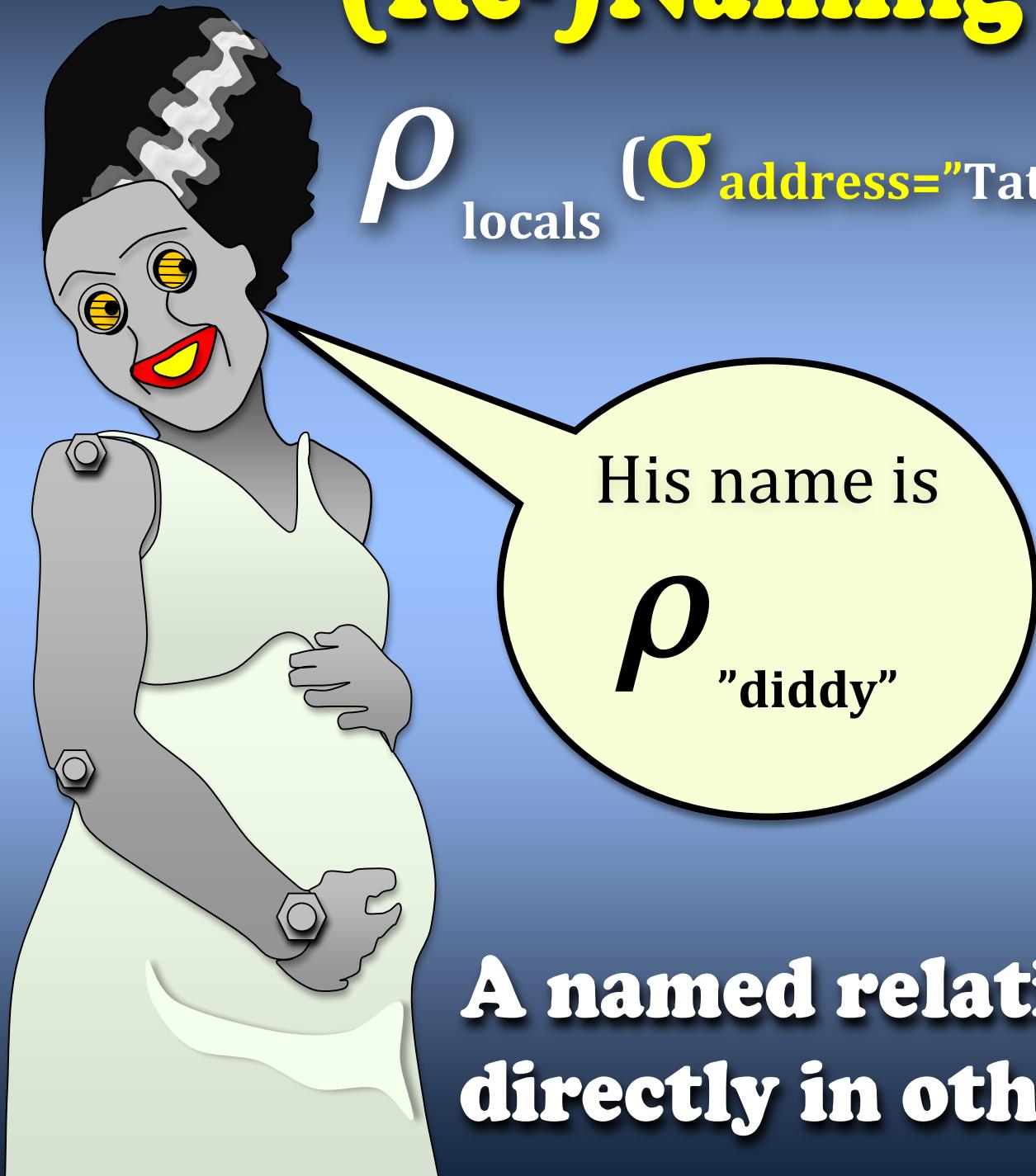
All of Left relation minus overlap with Right

# Cartesian Product of Two Relations

E.g.,  $\sigma_{\text{order\_id}=\text{"ORD3753"}}$  (Customers x Orders)



# (Re-)Naming a Relation

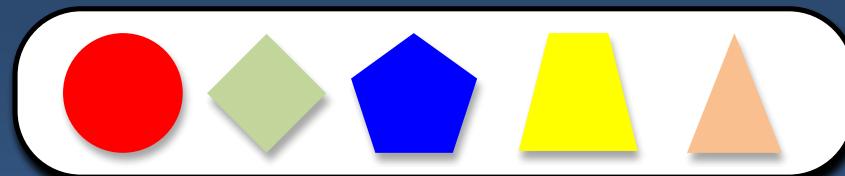


Here we name a selected subset of the **Customers** relation as **Locals**

A named relation can be used directly in other formulations

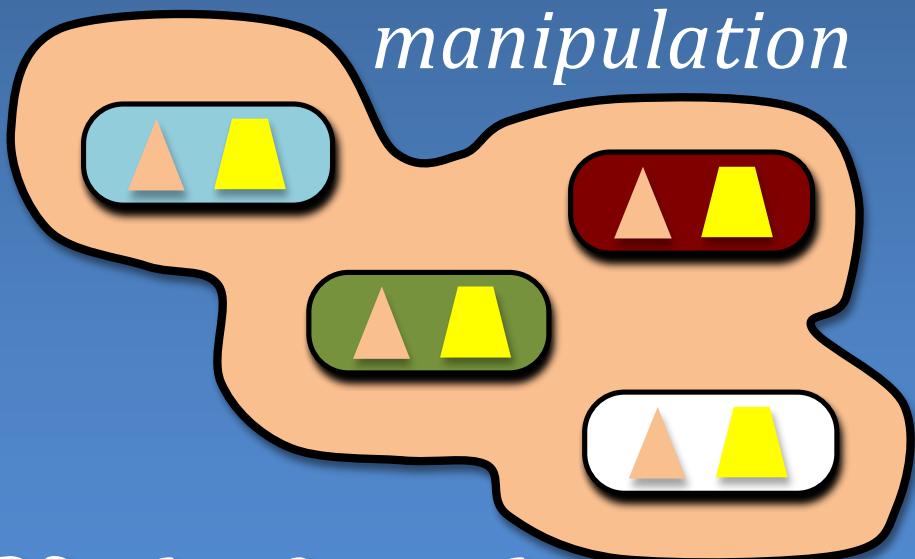
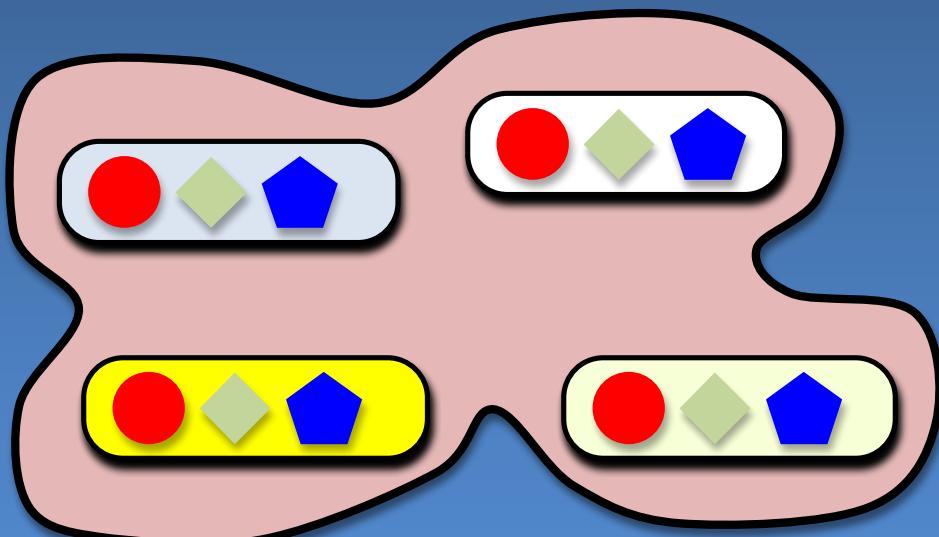
A DB relates specific **entities** to specific **relations**

*whether in tuples ...*



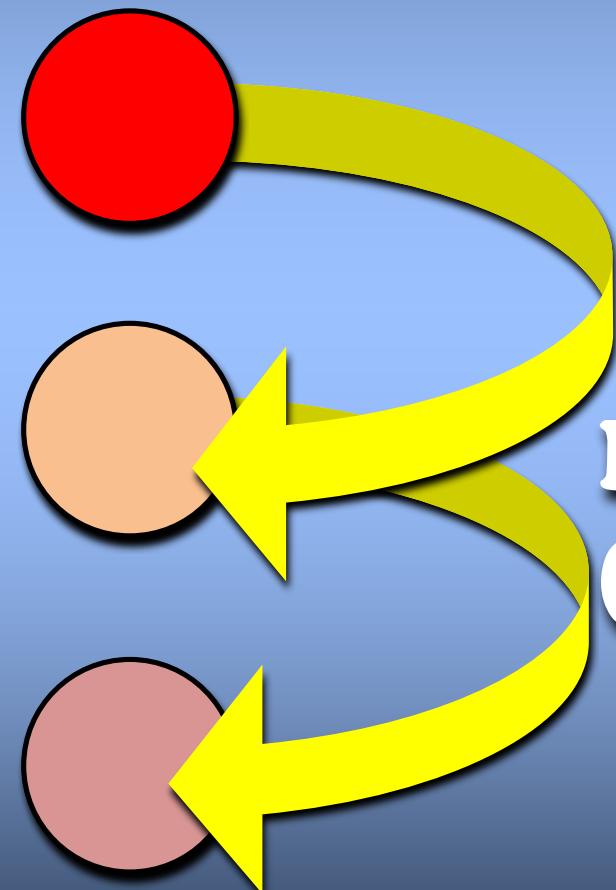
*Or in sets of tuples ...*

*... which are subject to  
set-theoretic  
manipulation*



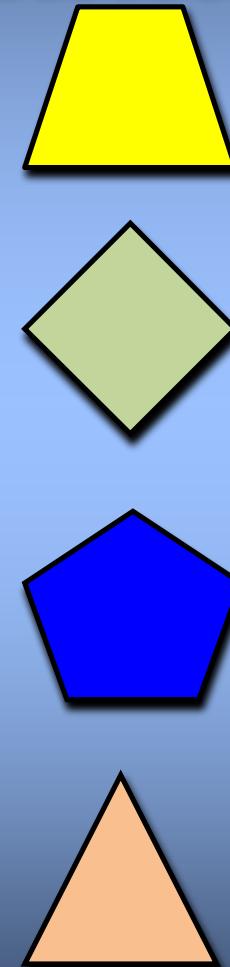
We model the **(E)entity-(R)elational**  
**structure of a DB with ER-graphs**

**Entities**  
**(things in  
the world)**



**Relations**  
**(between  
entities)**

**Attributes**  
**(properties  
of entities)**



# **Simple Attributes (indivisible properties)**



**For example,**

**Student IDs**

**First Names**

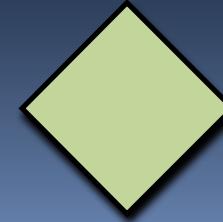
**Telephone Numbers**

**Surnames**

**ZIP Codes**

**Order Numbers**

# **Composite Attributes (Multiple Parts combined)**



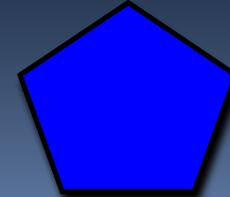
**For example,**

**Customer Name** **(First and Last Names)**

**Address** **(Street, Number, City, Country)**

**Order** **(Item, Quantity, Price, Delivery Options)**

# Derived Attributes (Functions of other Attributes)



For example,

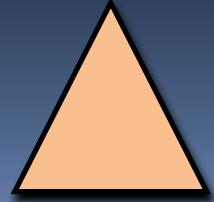
**MAX(Salary)**      (Salary of best-paid entity)

**MIN(Age)**      (Age of youngest entity)

**AVG(Hours)**      (Mean hours worked per entity)

# **Single-Value Attributes**

**(Just one value per entity  
but may be composite)**



**For example,**

**Salary**

**(Just one salary per entity)**

**Tax Number**

**(Just one tax number each)**

**Date-Of-Birth**

**(Everybody is born ONCE)**

# **Multi-Value Attributes**

**(Multiple values per entity)**



**For example,**

**Email address    (Possibly many per entity)**

**Phone Number    (Home, Work, Mobile, etc.)**

**Department    (Work for several bosses)**

# Super-Keys

A **Super-Key** is a set of 1 or more attributes that uniquely identifies every entity in an entity set.

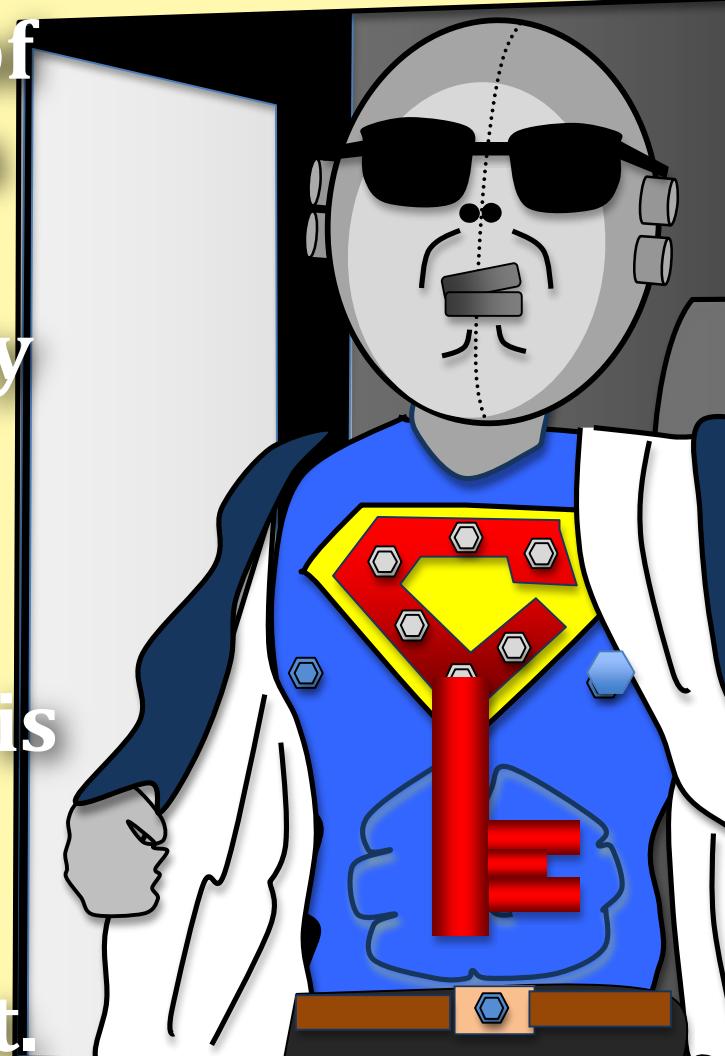
A **Minimal Super-Key** is one that does NOT contain a smaller Super-Key as a subset.

This is a **Candidate Key**

Telephone

Telephone

→ Primary Key





A relationship is an association between specific entities. E.g, ***married\_to*** is a relationship between two people.

A **relationship set** is a set of relationships of the same type. E.g. ***Marriage*** can be defined as the set of all ***married\_to*** instances.

**Monogamous** marriage  
is a ***binary*** relationship  
between two entities  
(husband and wife).

A cartoon illustration of a man's face with a wide, joyful smile showing his teeth. He has dark hair and wrinkles around his eyes. A white speech bubble is positioned above his head, containing text.

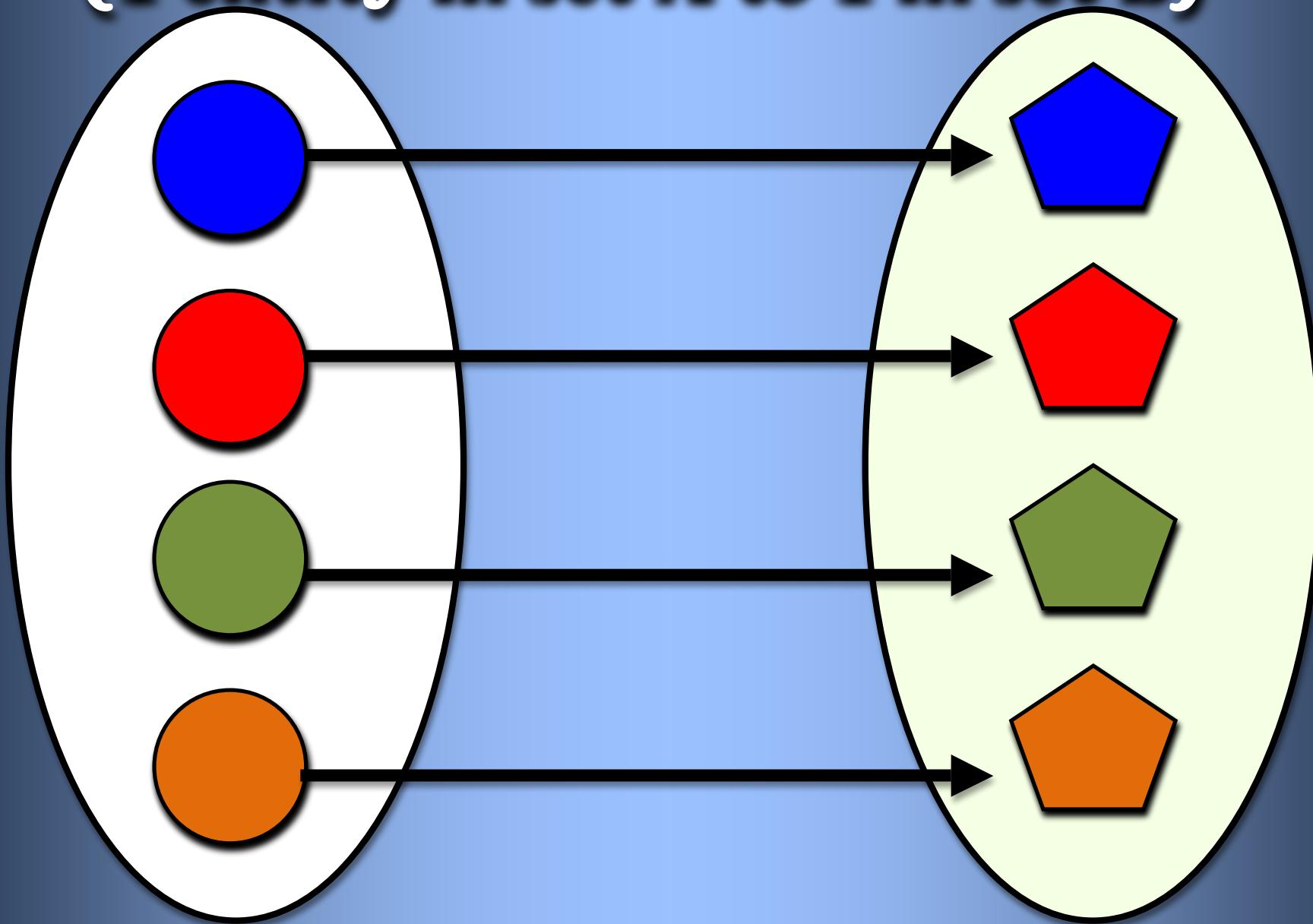
I prefer a  
good ternery  
relationship  
myself.

A ***ternery*** relationship  
is a relationship  
between **three** entities

An **n-ary** relationship  
is a relationship  
between **n** different  
entities

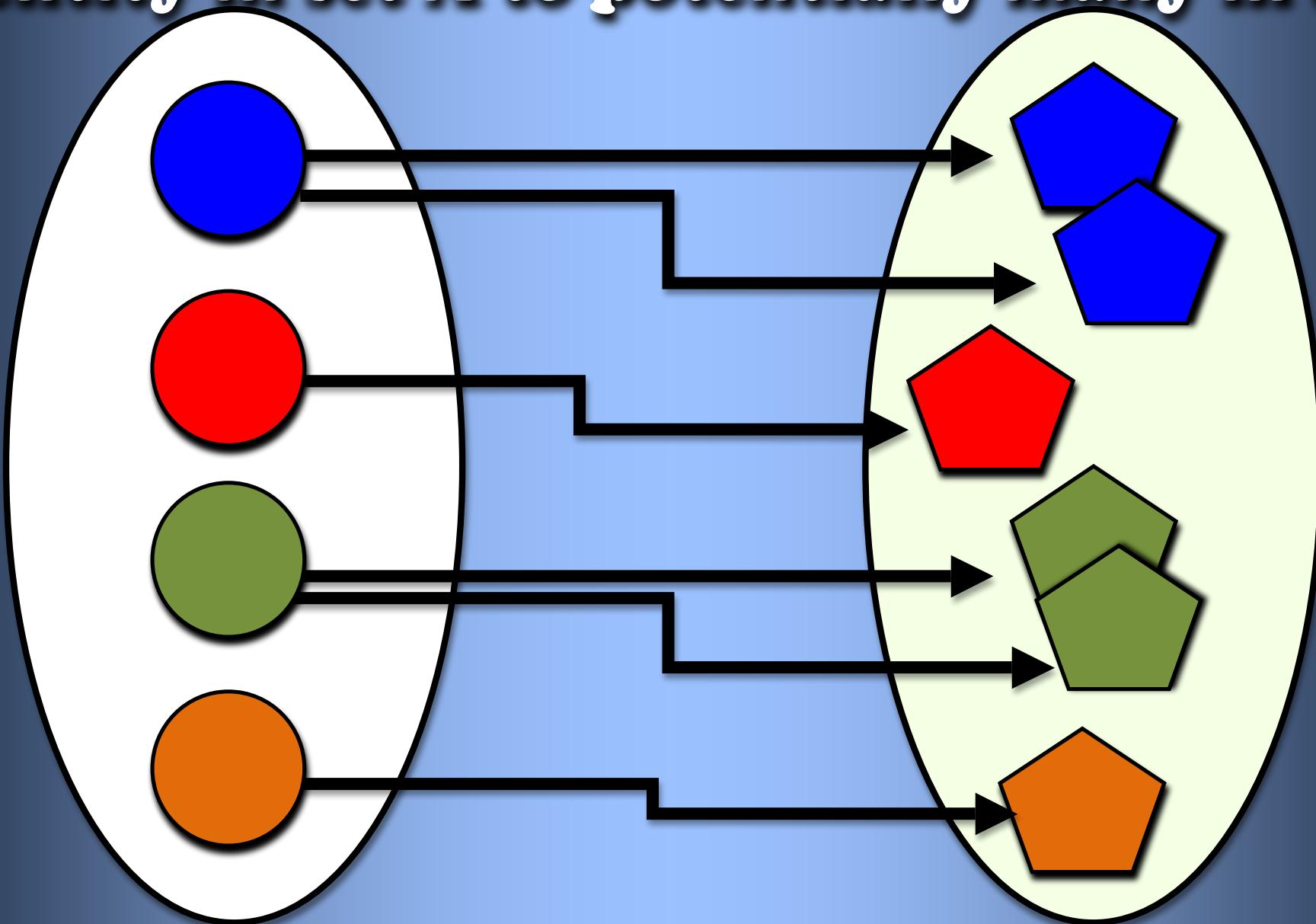
# 1-to-1 Relationships

(1 entity in set A to 1 in set B)



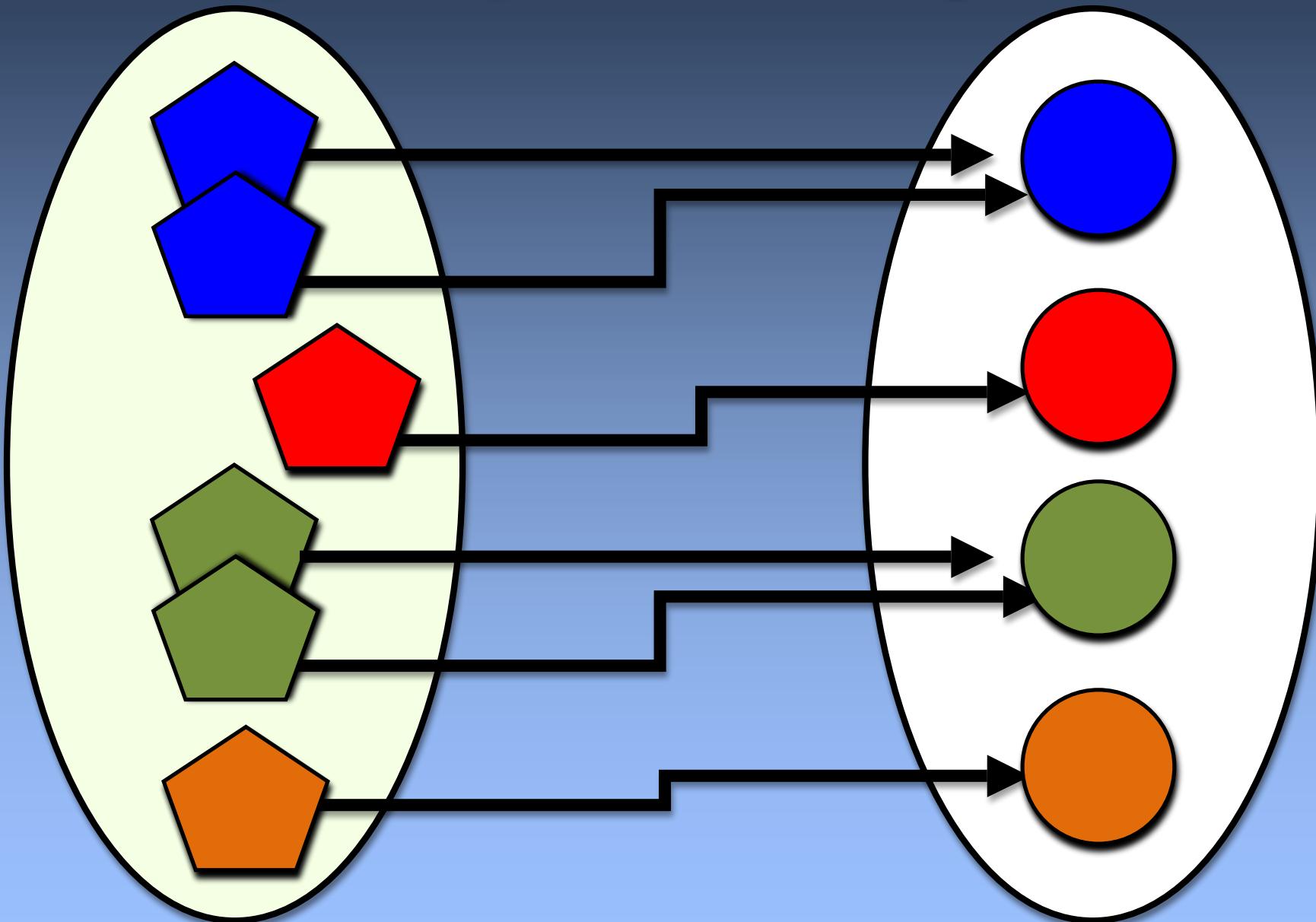
# 1-to-Many Relationships

(1 entity in set A to potentially many in set B)



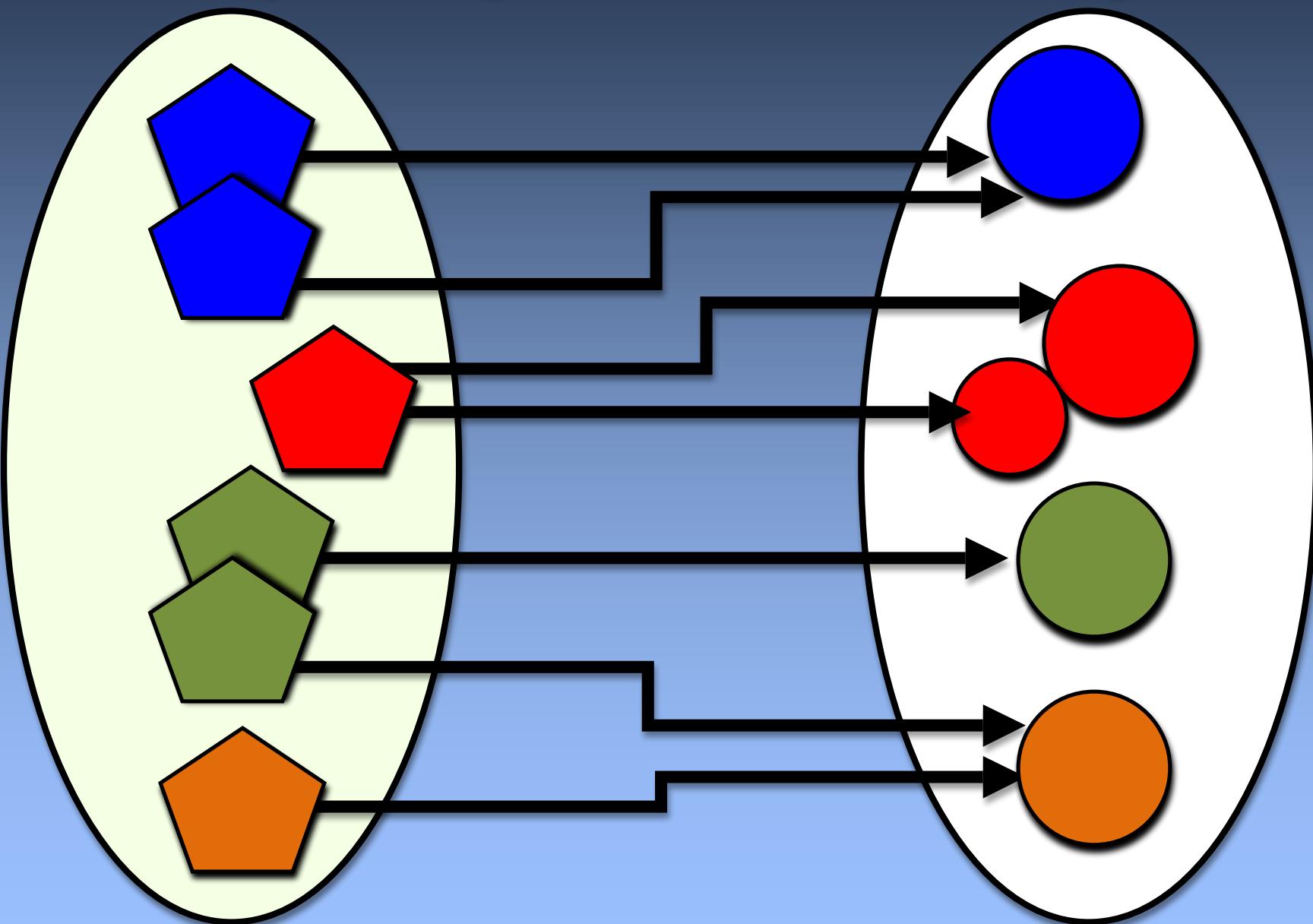
# Many-to-1 Relationships

(Potentially many in set A to just 1 in set B)



# Many-to-Many Relationships

(Potentially many in set A to many in set B)



Mentor

Intern

Department

Name

Date-of-Birth

We depict  
entities in our  
E-R graphs as  
*boxes*

... and depict  
attributes in  
our E-R graphs  
as *ovals*



Attributes may  
be composite ...



**First Name**

**Surname**

**Gender**

**Name**

**Intern**

**Date-of-Birth**

# Connect Entities to Attributes

Double-ring multi-value attributes

First Name

Surname

Name

Phone Number

Gender

Intern

Date-of-Birth



Derived attributes  
get *dashed* ovals



First Name

Surname

Name

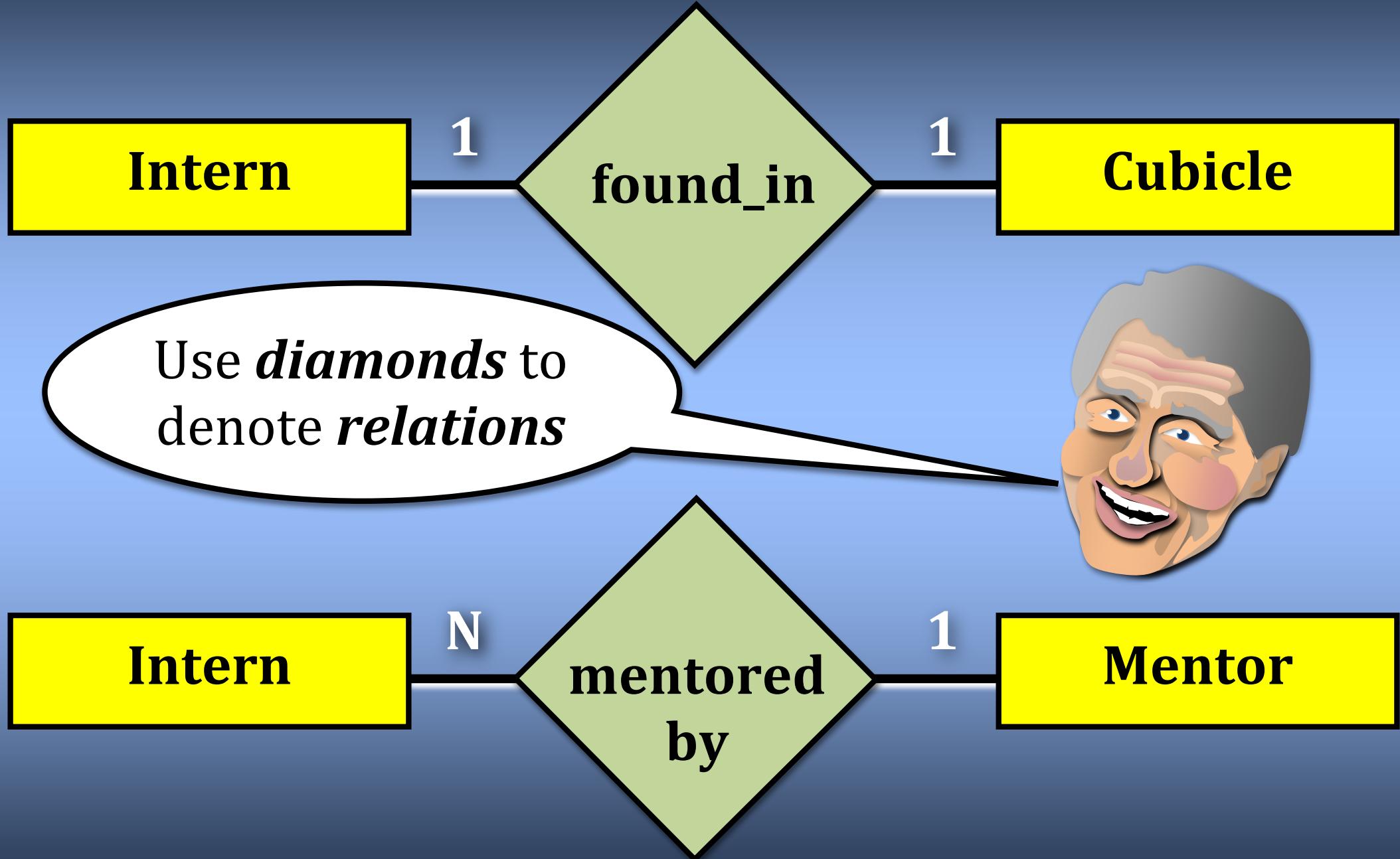
Age

Phone Number

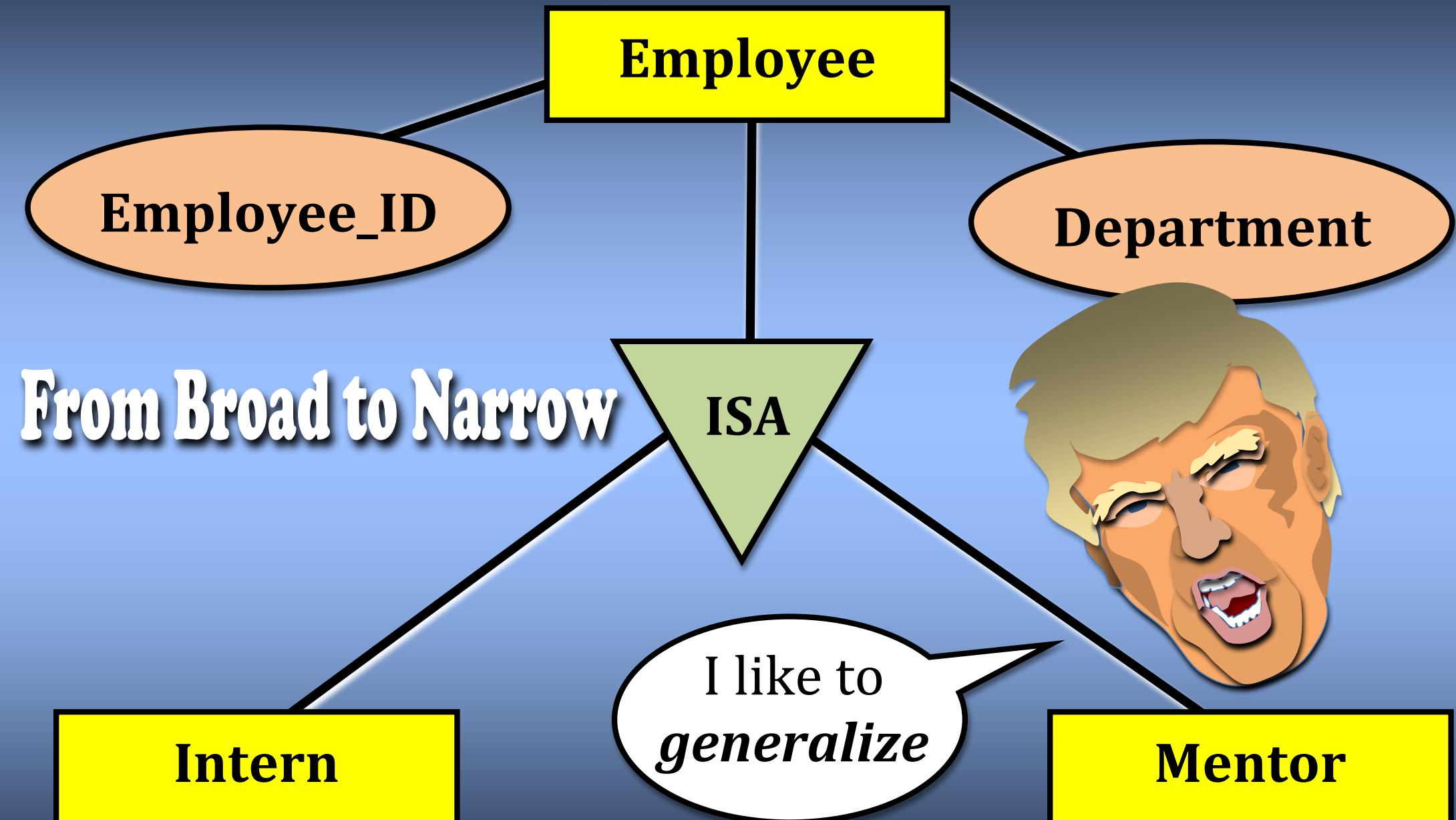
Intern

Gender

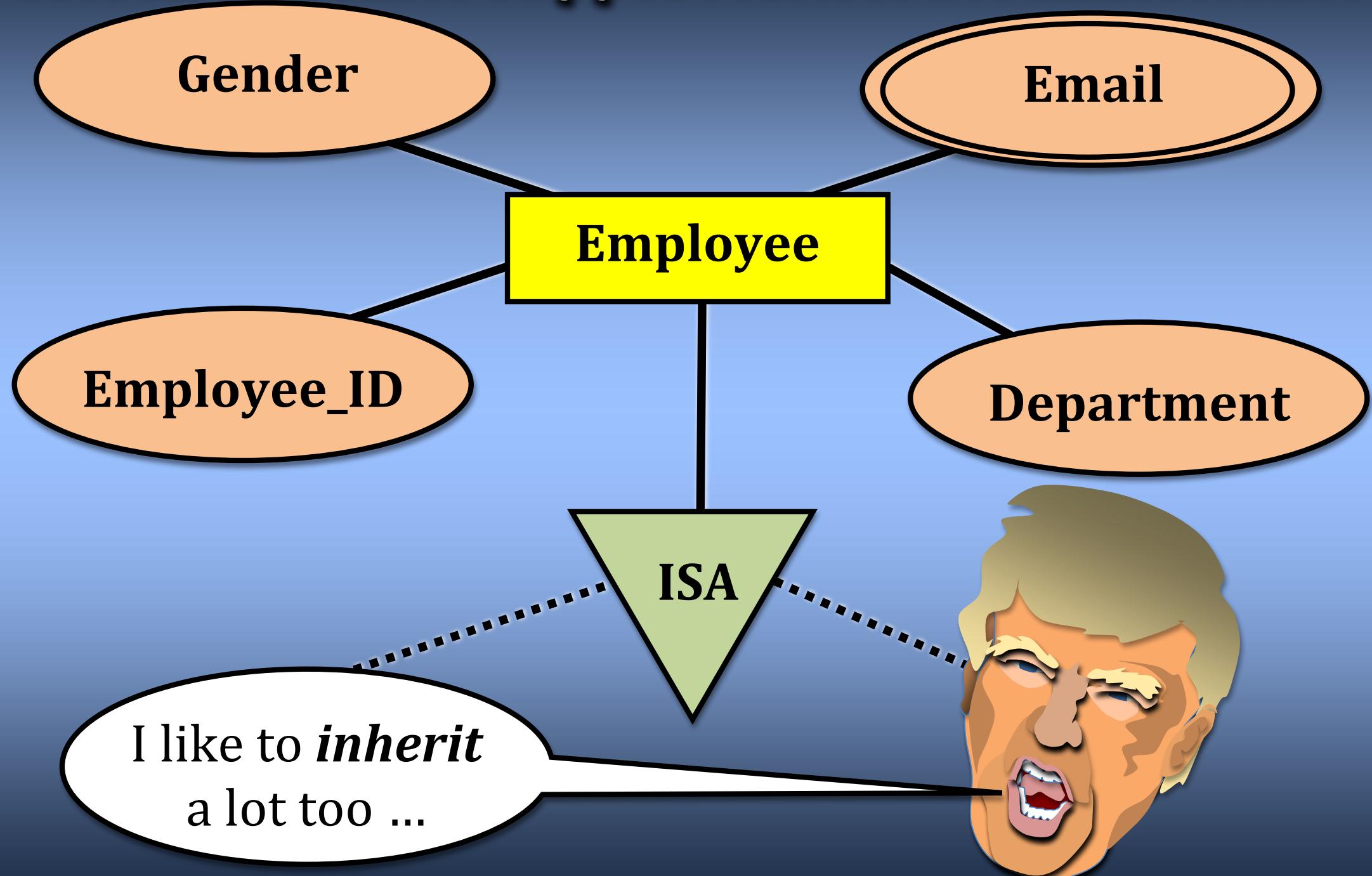
Date-of-Birth



# ISA Hierarchies generalize Entities



# ISA Hierarchies support Attribute Inheritance



# Turning schemes into Reality

**It's now time to  
deliver on the  
promise of our  
ER diagrams.**

Map Entities onto Tables, Attributes onto Columns  
and Assign a Primary Key

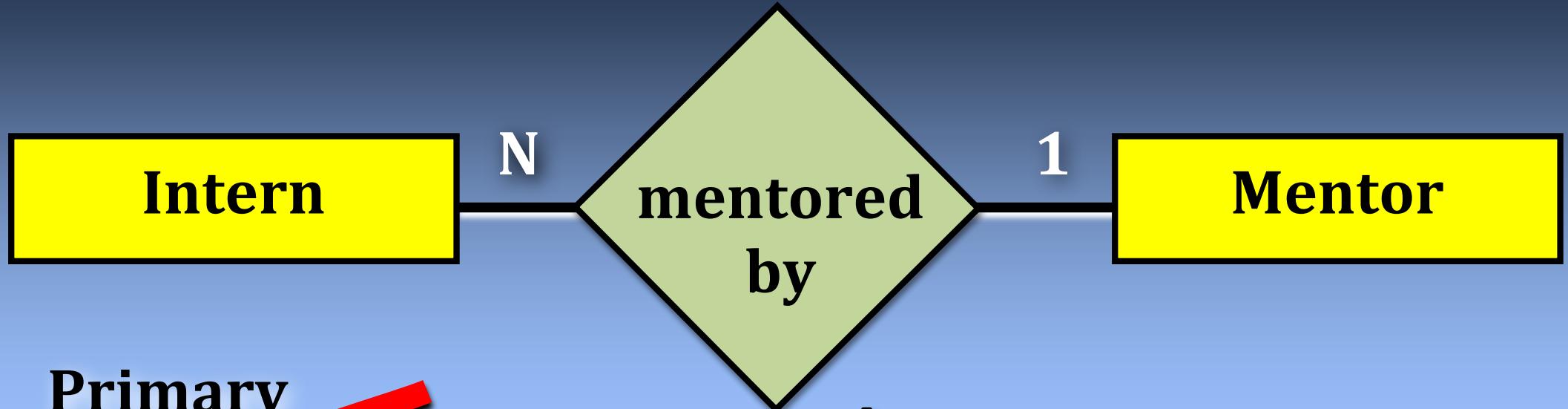
# Mentors

Staff_ID	Name	Date Of Birth	Gender	Email
THX1138	Darth Vader	28.01.0024	Male	<i>Darth.V@AOL.com</i>
REX2910	Sheev Palpatine	01.09.0000	Male	<i>Uggo@hotmail.com</i>
OBK1204	Obiwan Kenobi	07.10.0012	Male	<i>OB1@gmail.com</i>
PAD4908	Padmé Amidala	14.12.0020	Female	<i>Padm@outlook.com</i>

# Interns

Student_ID	Name	Date Of Birth	Gender	Email
8721138	Anakin Skywalker	28.01.0024	Male	<i>Darth.V@AOL.com</i>
3872614	Luke Skywalker	21.05.0042	Male	<i>Luke@hotmail.com</i>
3861638	Rey Skywalker	18.03.0066	Female	<i>Rey.S@gmail.com</i>

Map Entities onto Tables, Attributes onto Columns  
and Assign a Primary Key

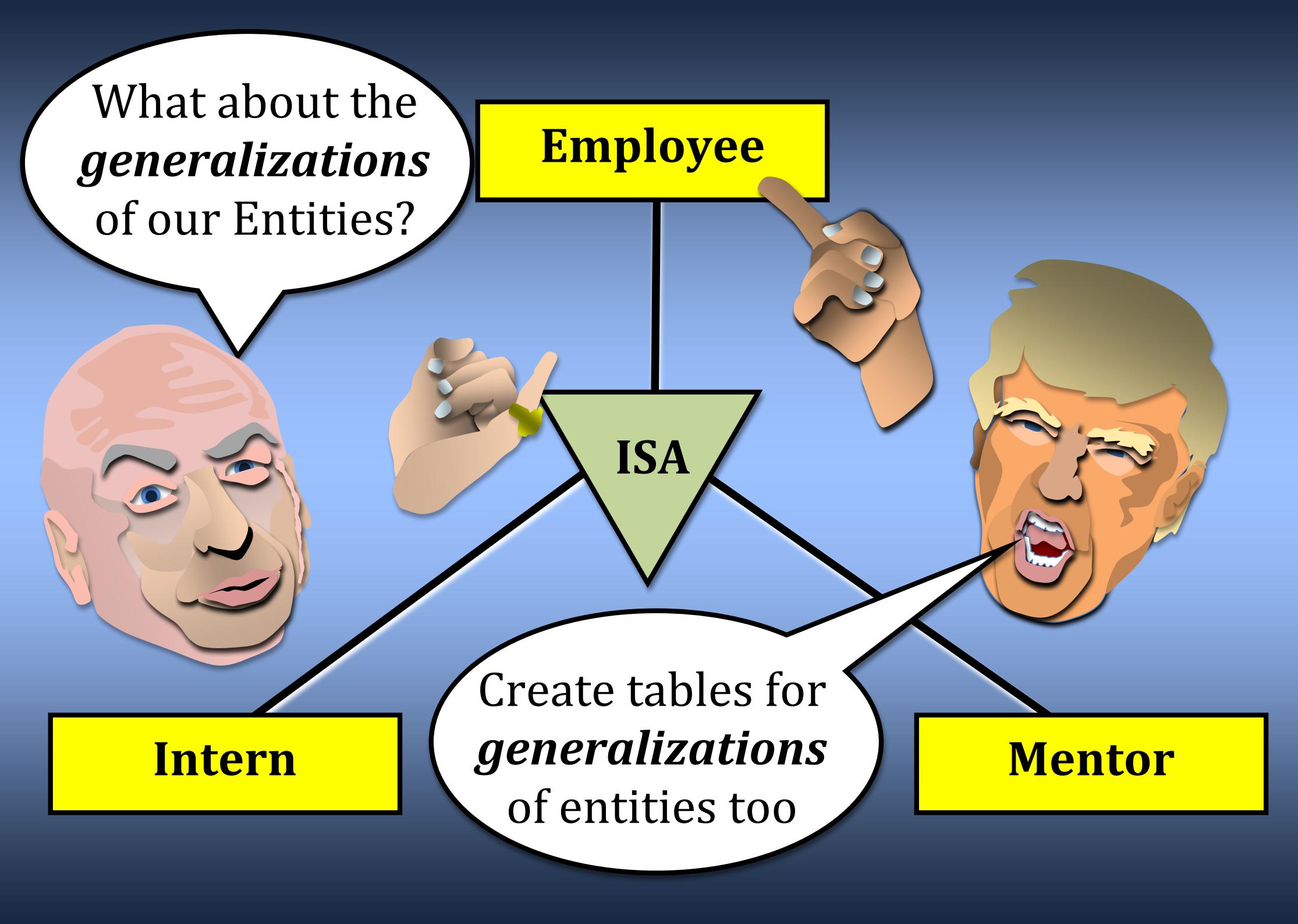


Primary  
Key

## Mentorships

Staff_ID	Student_ID	Start Date	End Date	Department
OBK1204	8721138	19.07.0036	01.04.0042	Jedi Affairs
OBK1204	3872614	11.08.0061	21.04.0061	Rebel Alliance
REX2910	3861638	01.04.0042	11.06.0065	Old Republic
LKS2614	8721138	24.06.0099		New Republic

Map Relations onto Tables to connect Entity Tables



What about the  
*generalizations*  
of our Entities?

Employee

Intern

Create tables for  
*generalizations*  
of entities too

Mentor

# Employees

Staff_ID	Name	Date Of Birth	Gender	Email
THX1138	Darth Vader	28.01.0024	Male	<i>Darth.V@AOL.com</i>
REX2910	Sheev Palpatine	01.09.0000	Male	<i>Uggo@hotmail.com</i>
OBK1204	Obiwan Kenobi	07.10.0012	Male	<i>OB1@gmail.com</i>
PAD4908	Padmé Amidala	14.12.0020	Female	<i>Padm@outlook.com</i>
LKS2614	Luke Skywalker	21.05.0042	Male	<i>Luke@hotmail.com</i>
RYS1638	Rey Skywalker	18.03.0066	Female	<i>Rey.S@gmail.com</i>



When we create the EMPLOYEES Table, we define the inheritable attributes at this level. There is no need to define these attributes as columns in the MENTORS & INTERNS tables (but now, we must give Staff IDs to Interns)

# In Specialization tables, put specialized attributes

Staff_ID	Skill Level	Skill Set
THX1138	4	speed racing, sabre duelling, choke holds
REX2910	10	manipulation, politics, leadership
OBK1204	9	sabre duelling, voice control
PAD4908	4	politics, leadership, youth activism

Staff_ID	Student_ID	Education
THX1138	8721138	High School, Bsc, Msc
LKS2614	3872614	High School, MA(Rebellion)
RYS1638	3861638	High School

# Inherited attributes are accessible via Table Joins



Now that Mentors and Interns are both specializations of Employee and both kinds of people have STAFF\_IDs, we can simplify our table for the relation Mentorships between Interns and Mentors.

## Mentorships

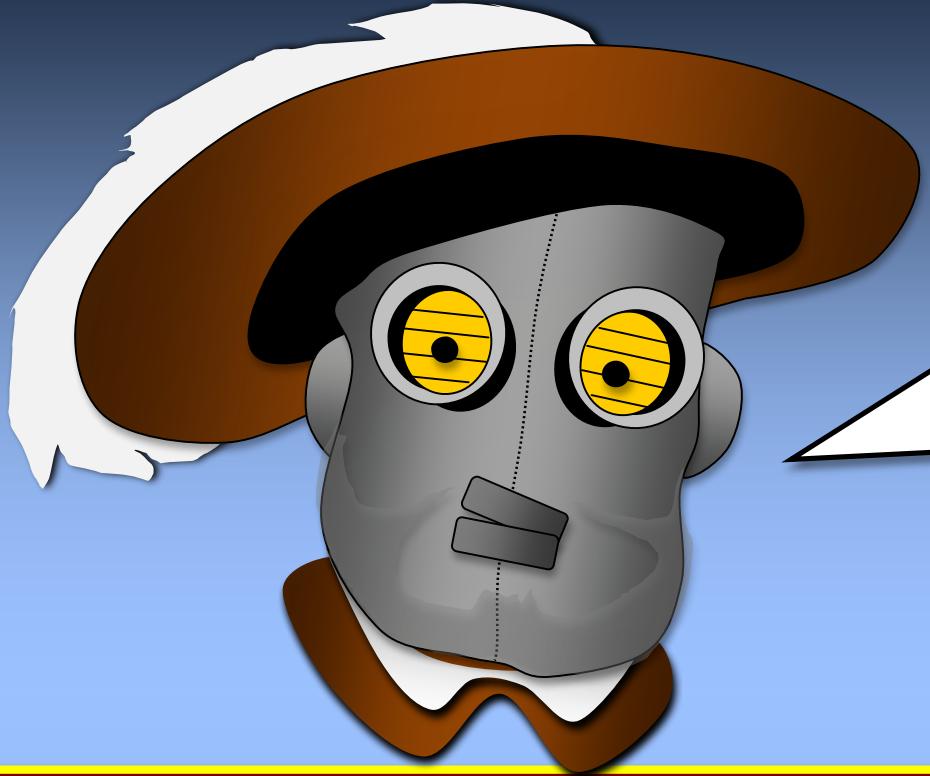
Mentor	Intern	Start Date	End Date	Department
OBK1204	THX1138	19.07.0036	01.04.0042	<i>Jedi Affairs</i>
OBK1204	LKS2614	11.08.0061	21.04.0061	<i>Rebel Alliance</i>
REX2910	THX1138	01.04.0042	11.06.0065	<i>Old Republic</i>
LKS2614	RYS1638	24.06.0099		<i>New Republic</i>

The Primary Key for this M-to-N relation is Two-fold



A **determinant** is any attribute (*simple or composite*) on which an other attribute is fully functionally dependent.

A relation is in **Boyce Codd Normal Form** (or **BCNF**) if, and only if, every determinant is a candidate key.



Let us consider the nature of **determination** in the abstract before exploring a concrete example.

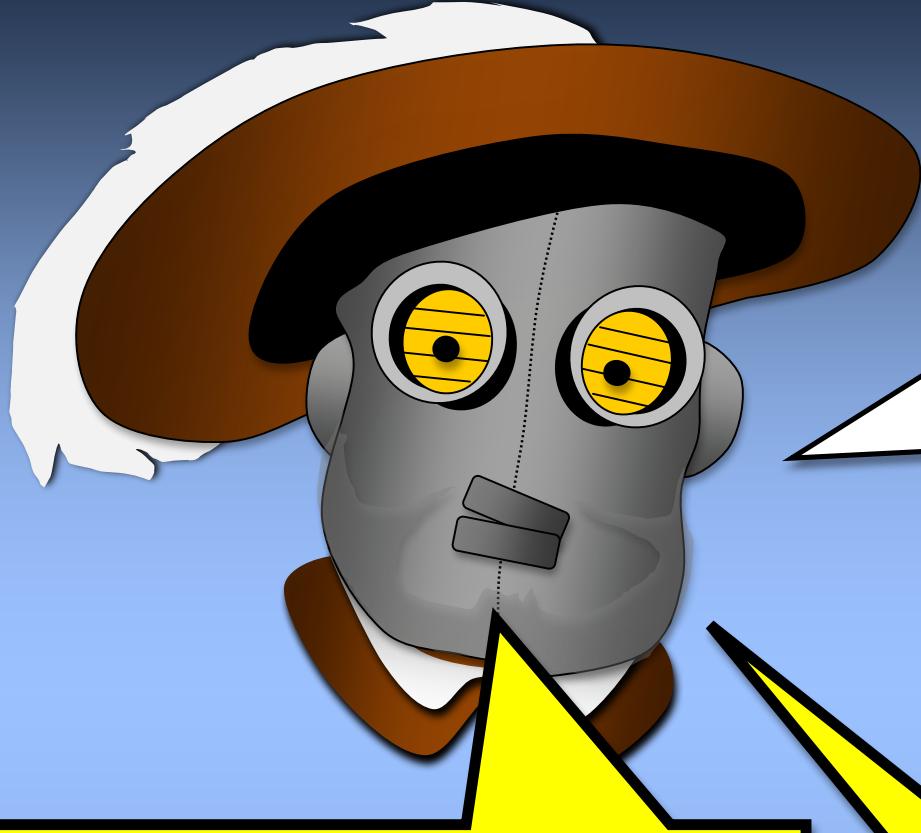
Let **R** be a ternary relation between four things, **A, B, C** and **D**. So let  $R(a, b, c, d)$  be any instance of the relation **R**.

Let us suppose that:  
**a + c determines b + d**  
and  
**a + d determines b**  
for every instance of **R**

Let us suppose that:  
 **$a + c$  determines  $b + d$**   
and  
 **$a + d$  determines  $b$**   
for every instance of  $R$

This pattern of determination suggests that  **$a,c$**  is a candidate key for  **$R$**  but  **$a,d$**  is not (because  **$c$**  is not determined by  **$a,d$** )





We have at least two determinants for **R**:  
 $a+c \rightarrow b+d$  and  
 $a+d \rightarrow b$ .  
They *overlap*!

A relation or table is in **BCNF (Boyce-Codd Normal Form)** when no determinants overlap (*no competing systems of determination*).

A relation or table can be in **3<sup>rd</sup> Normal Form (3NF)** but not be in BCNF (*Boyce-Codd Normal Form*). So further normalization is needed.



I run an evil clinic where patients have four stages of appointments (0 ... 3). Each stage is scheduled at different times, and with doctors with appropriate training.

Patient_Num	Patient_Name	Appointment_id	Time	Doctor
1	Mordo	0	09:00	Strange
2	Scott	0	09:00	Evil
3	Dormammu	1	10:00	Strange
4	Austin	0	13:00	Evil
5	Christine	1	14:00	Strange

What are the determinants here? First, ***Patient\_Num* → *Patient\_Name***. Second, ***Patient\_Num + Appointment\_id* → *Time + Doctor***. Third, ***Time* → *Appointment\_id***.



So what should be the *primary key* of my clinic's appointment book? Should it be **Patient\_Num, Appointment\_id** or should it be **Patient\_Num, Time** ?

Patient_Num	Patient_Name	Appointment_id	Time	Doctor
1	Mordo	0	09:00	Strange
2	Scott	0	09:00	Evil
3	Dormammu	1	10:00	Strange
4	Austin	0	13:00	Evil
5	Christine	1	14:00	Strange

Let's choose **Patient\_Num, Appointment\_id** for primary key. Now let's put the database into 1NF, then 2NF, then 3NF and then BCNF. The above table is already in 1NF (we have a key that is unique for every row).

**2NF:** Factor out the partial key dependencies, e.g.:

**Patient\_Num → Patient\_Name**



Patient_Num	Appointment_id	Time	Doctor
1	0	09:00	Strange
2	0	09:00	Evil
3	1	10:00	Strange
4	0	13:00	Evil
5	1	14:00	Strange

Patient_Num	Patient_Name
1	Mordo
2	Scott
3	Dormammu
4	Austin
5	Christine

**Remember, Patient\_NUM is not the whole primary key, so the dependency of Patient\_Name on this attribute is a partial key dependency.**

**3NF:** Factor out transitive dependencies. Here there are: **None**



Let's look at the refactored main table. Is it in BCNF? What are its determinants?

**Patient\_Num + Appointment\_id**

**→ Time + Doctor ? That's the primary key**

Patient_Num	Appointment_id	Time	Doctor
1	0	09:00	Strange
2	0	09:00	Evil
3	1	10:00	Strange
4	0	13:00	Evil
5	1	14:00	Strange

What about **Time → Appointment\_id** ? It seems that the appointment type is dependent upon the time, but time is not a candidate key! So this table is **not** in BCNF.

## Patients

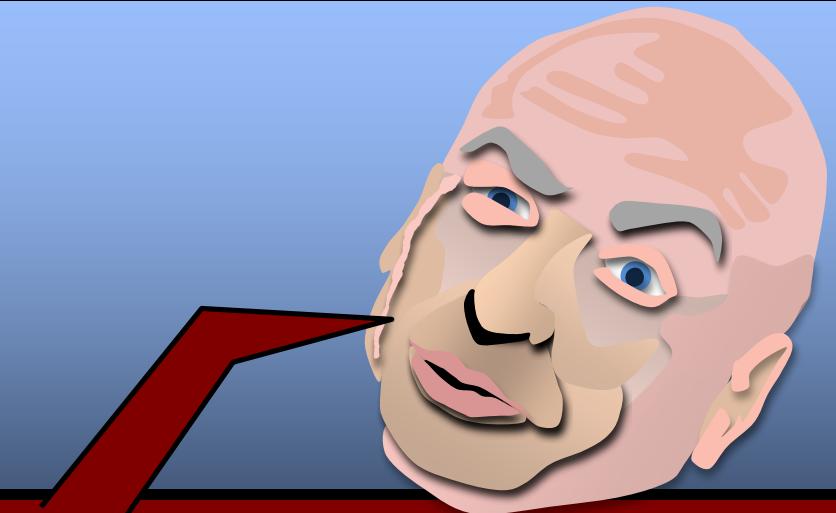
Patient_Num	Patient_Name
1	Mordo
2	Scott
3	Dormammu
4	Austin
5	Christine

Time	Appointment_id
09:00	0
09:00	0
10:00	1
13:00	0
14:00	1

## Time Allocation

## Patient Schedule

Patient_Num	Time	Doctor
1	09:00	Strange
2	09:00	Evil
3	10:00	Strange
4	13:00	Evil
5	14:00	Strange



**BCNF: Factor out non-candidate-key dependencies: All Gone!**