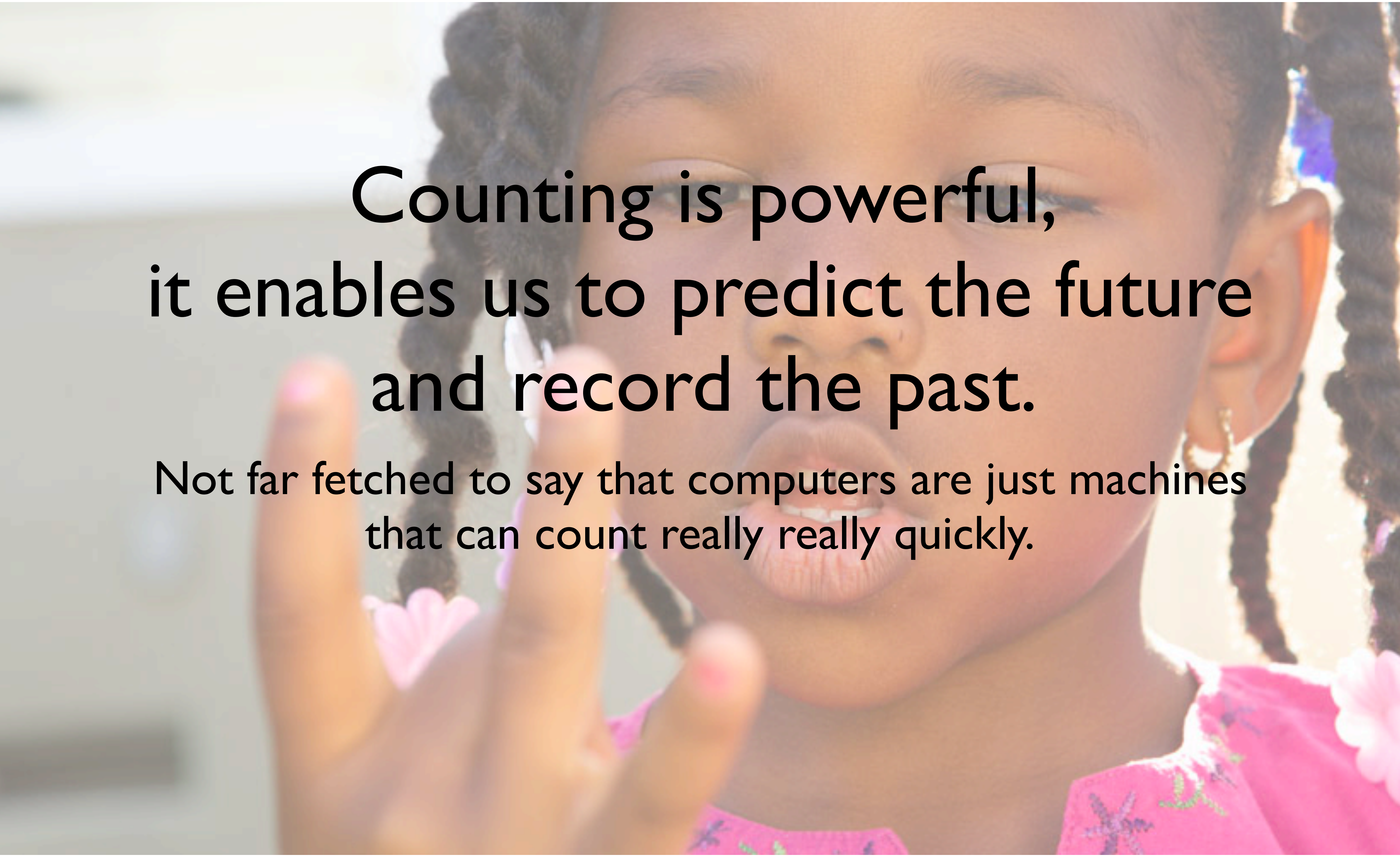
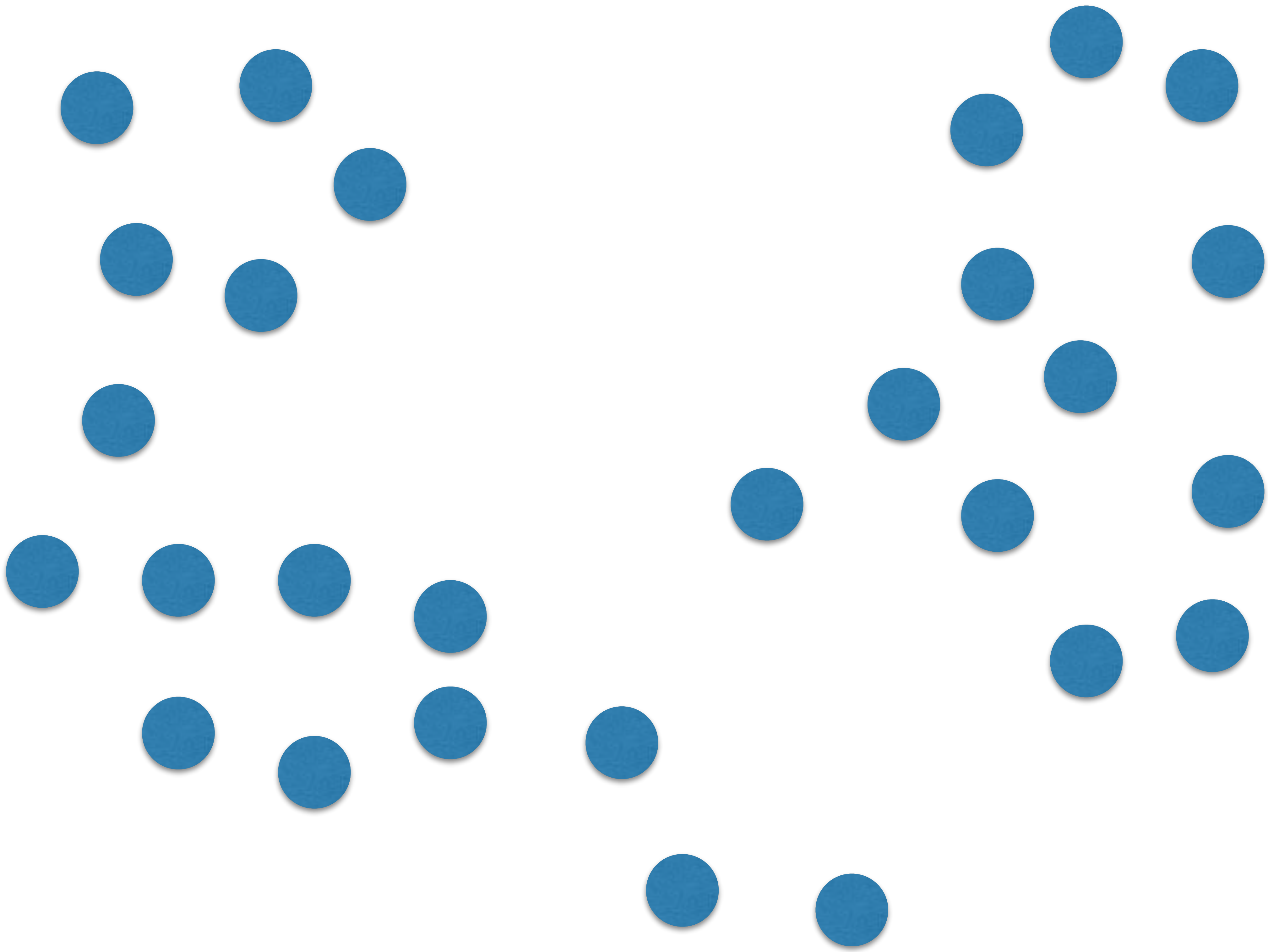
Number Systems

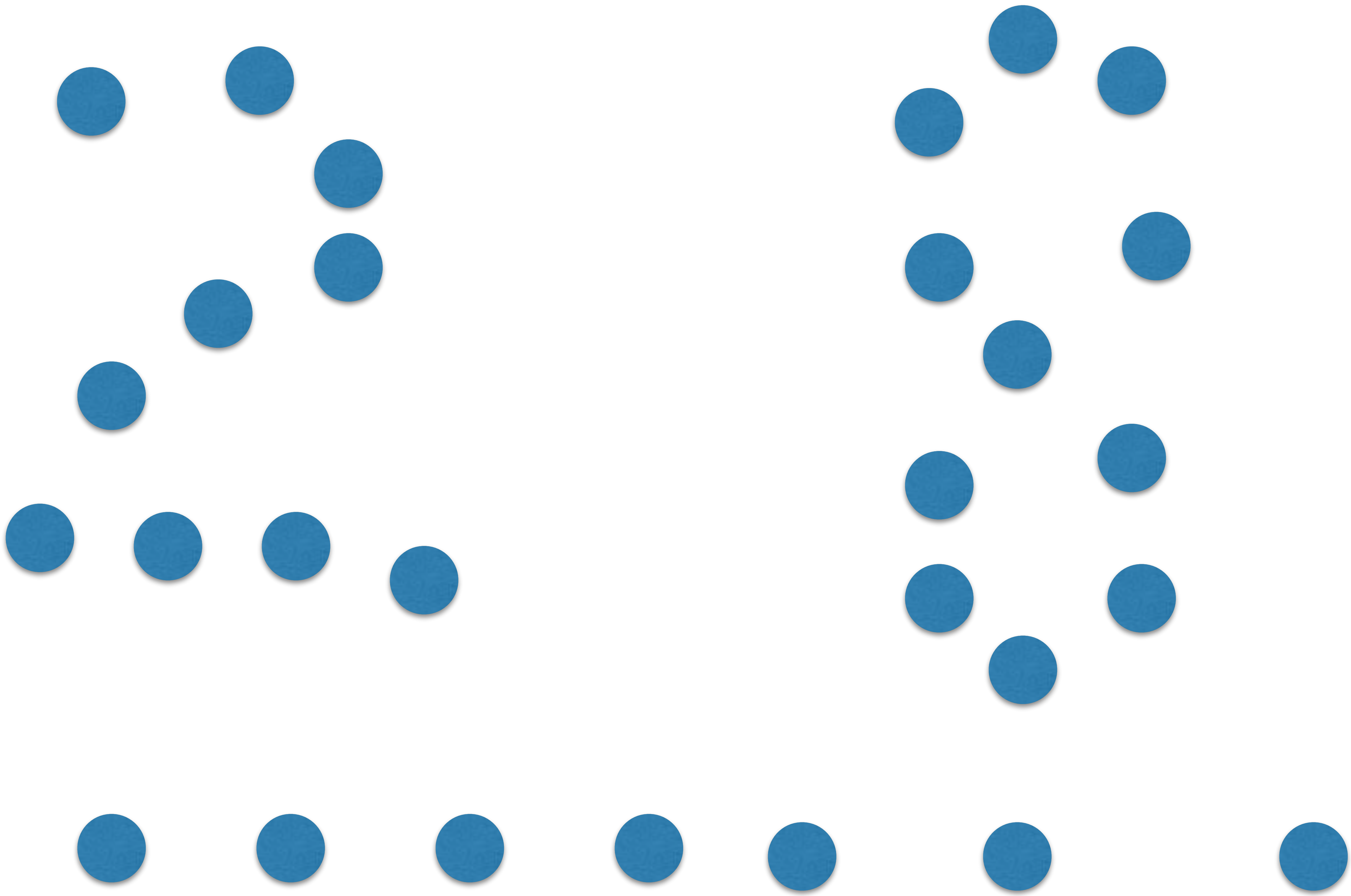
Number Systems

Learning Objectives, be able to:

- count in Digital, Binary, Octal and Hex (slow is ok)
- explain why computers are binary
- convert between digital and binary
- add binary numbers
- multiply binary numbers by 2,4 and 8


Counting is powerful,
it enables us to predict the future
and record the past.
Not far fetched to say that computers are just machines
that can count really really quickly.





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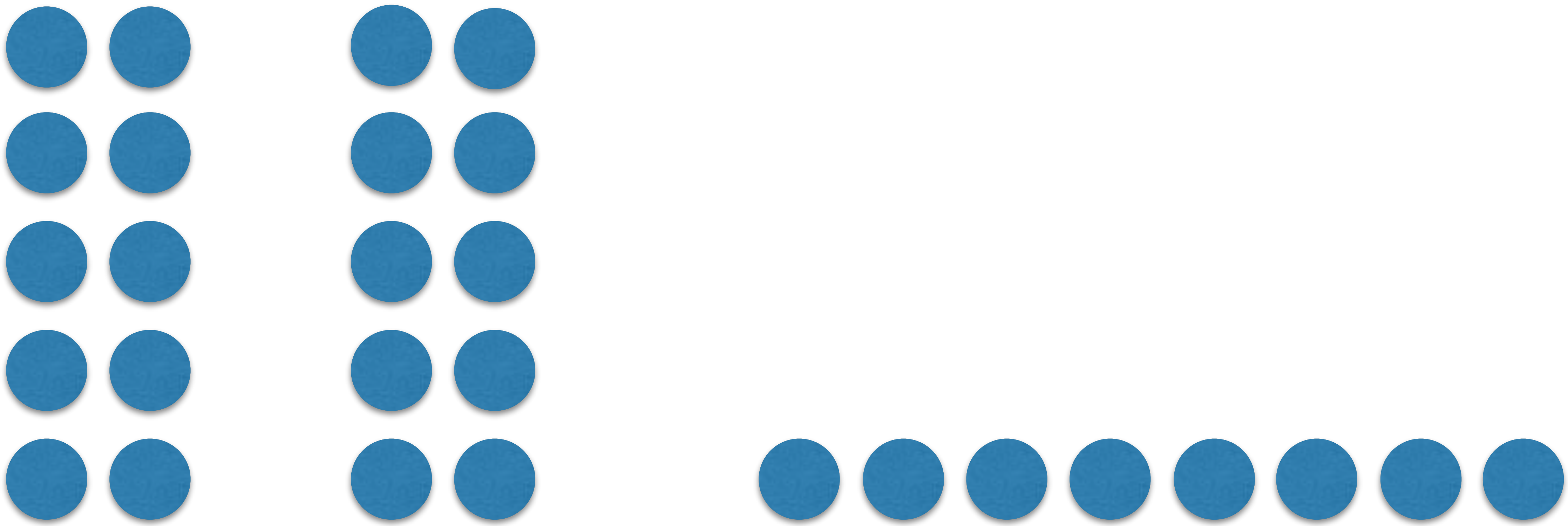
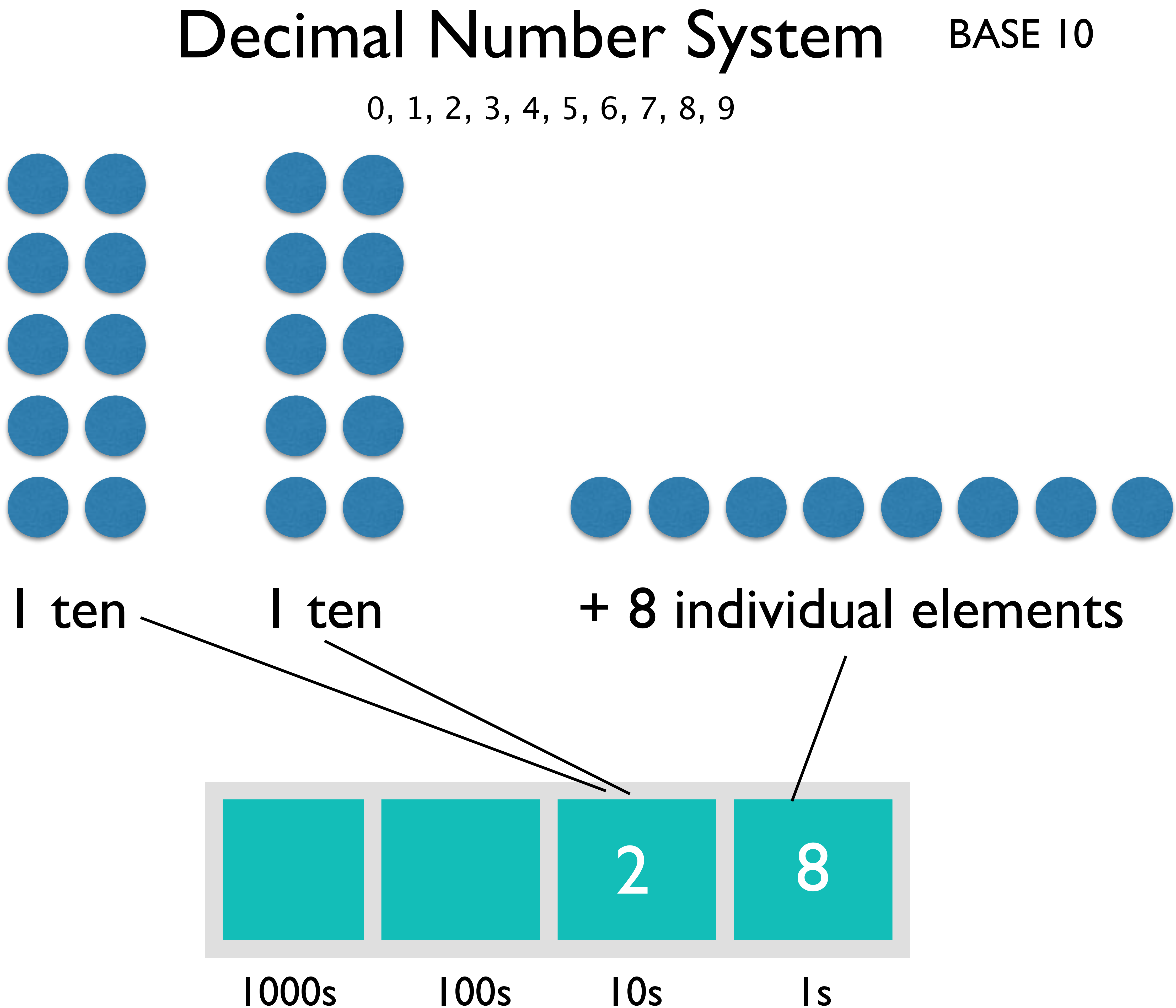
Why these groups?



1 ten   1 ten   + 8 individual elements


Decimal Number System
BASE 10
0, 1, 2, 3, 4, 5, 6, 7, 8, 9
1 ten
1 ten
+ 8 individual elements
2
8
1000s
100s
10s
1s

Decimal Number System

BASE 10

	1000s	100s	10s	1s
735		7	3	5

$$735 = 700 + 30 + 5$$
$$= 7\times10^2 + 3\times10^1 + 5\times10^0$$

Why do we count in tens (decimal)?

digit

Origin

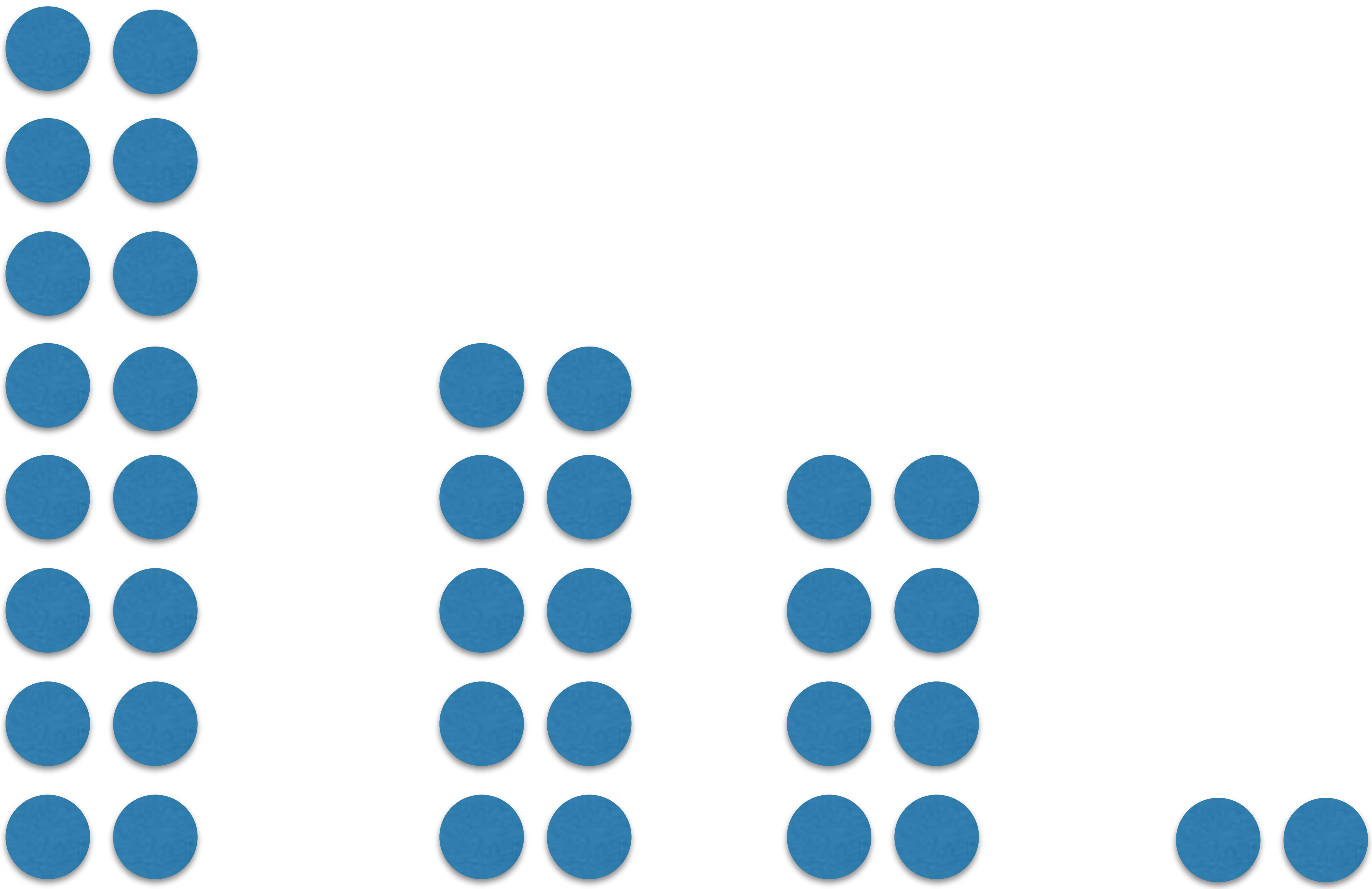
LATIN

digitus → digit

finger, toe — late Middle English

late Middle English: from Latin *digitus* 'finger, toe'; digit (sense 1) arose from the practice of counting on the fingers.

What is the base?



16 10 8 2

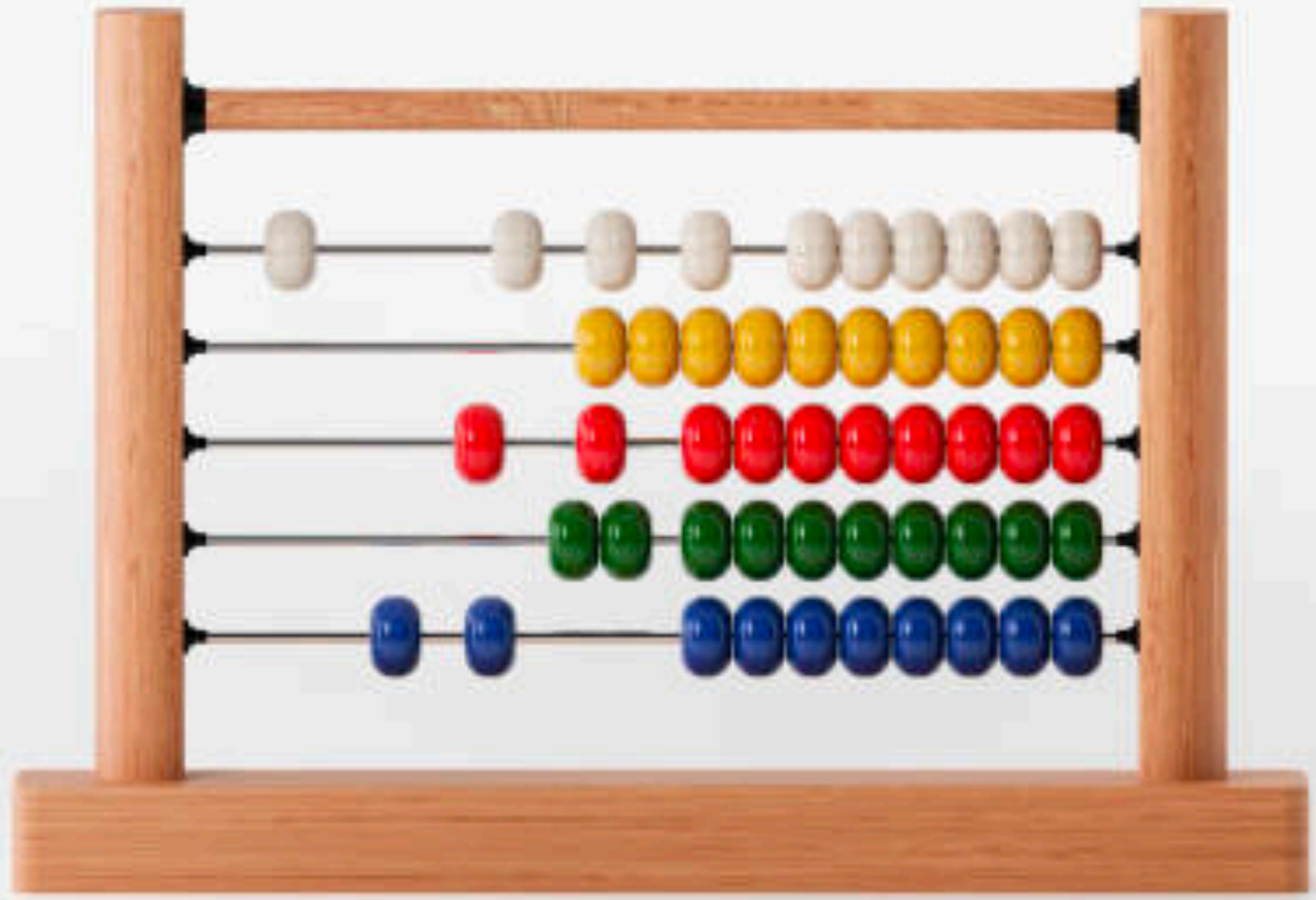
What do they use in Springfield?

Decimal Number System

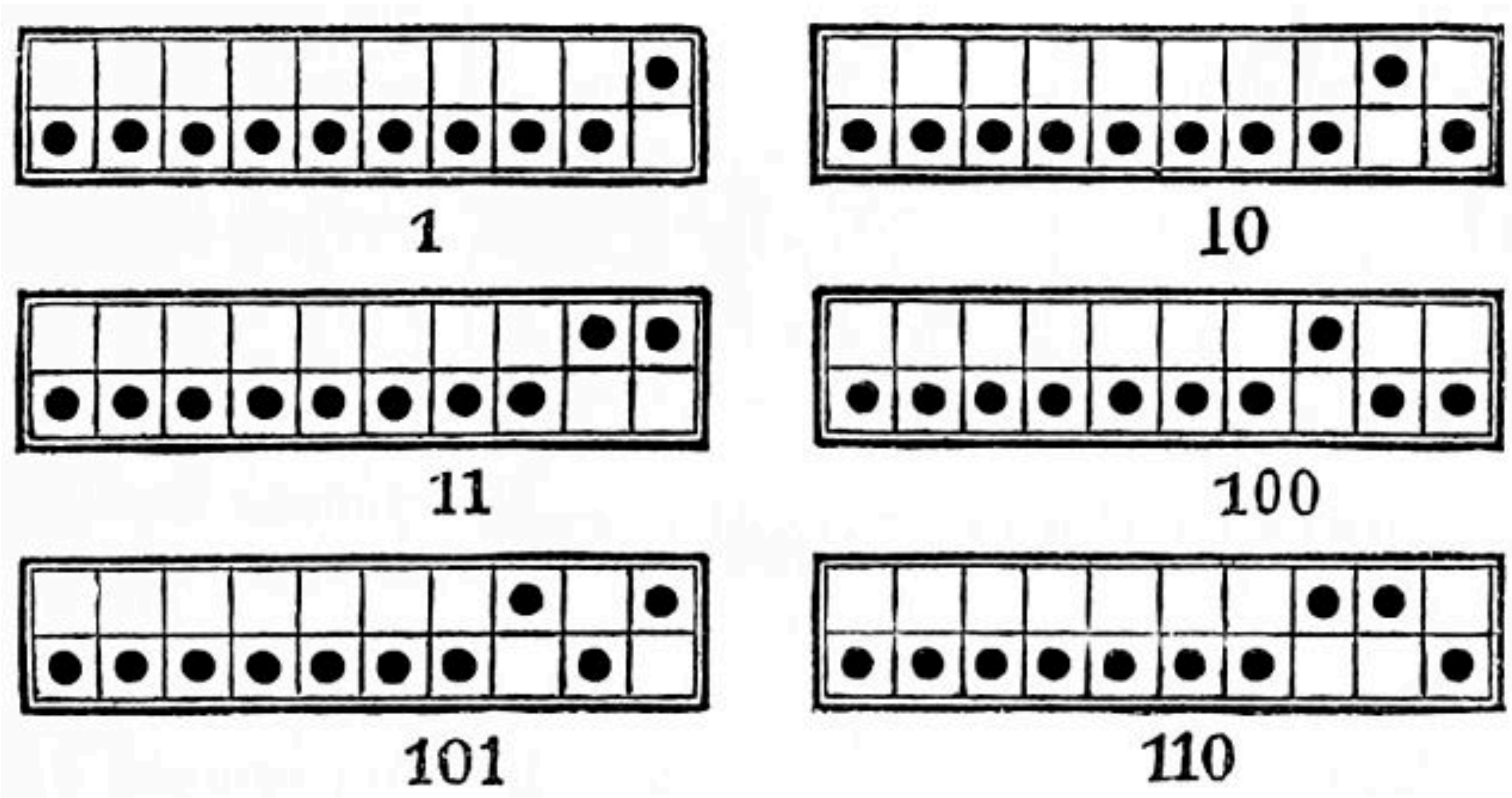
BASE 10

	1000s	100s	10s	1s
How many is: Decimal 28?			2	8
Decimal 154?		1	5	4
Decimal 1074?	1	0	7	4

Abacus: tool for addition



There is a binary abacus



Binary Number System BASE 2

Only two symbols: 0, 1 called **bits***

1	1	0	1
8s	4s	2s	1s

What is Binary 1101 in decimal?

$$= 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$$
$$= 8 + 4 + 0 + 1$$
$$= 13$$

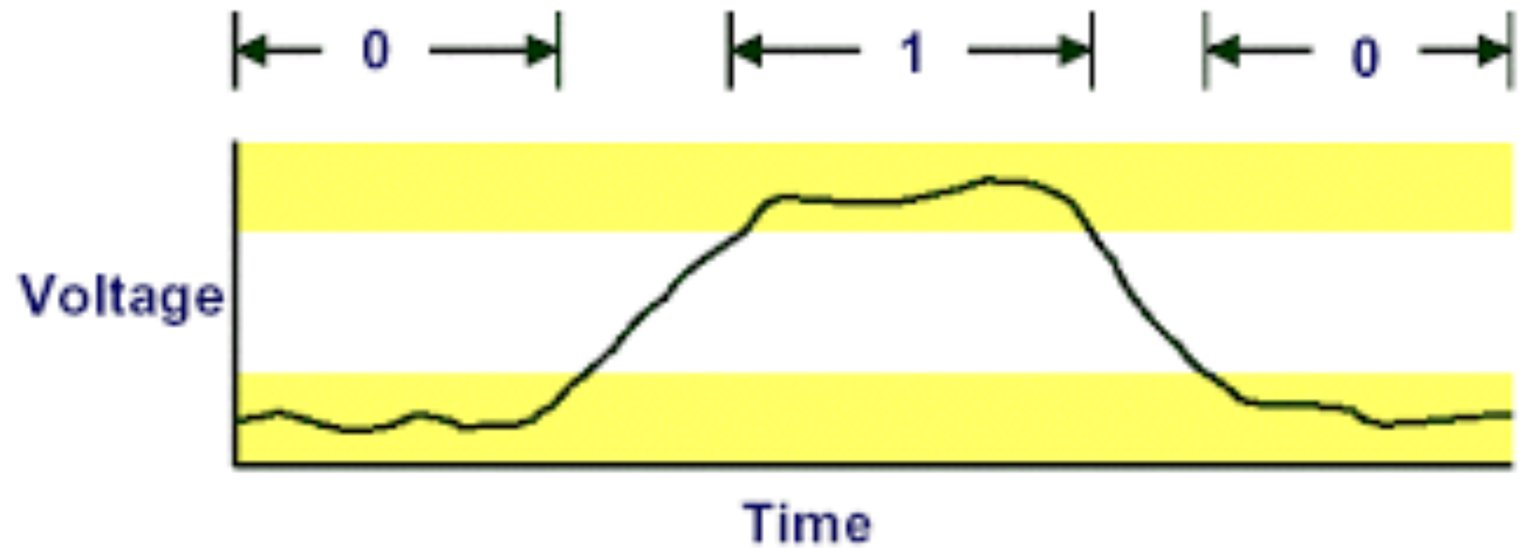
*8 bits are called a **byte**

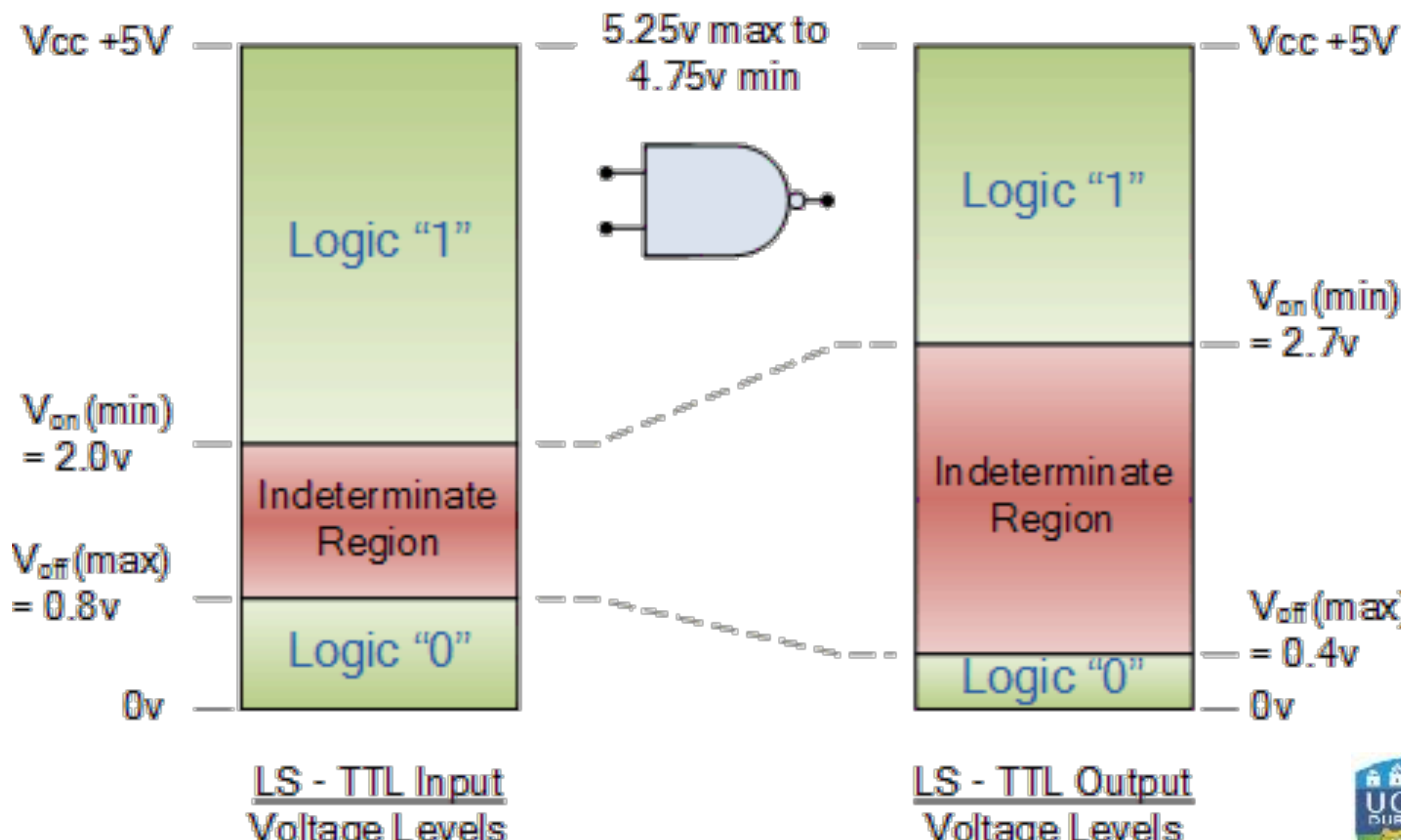
Why not decimal computers?

Because it is easier to develop binary logic circuits:

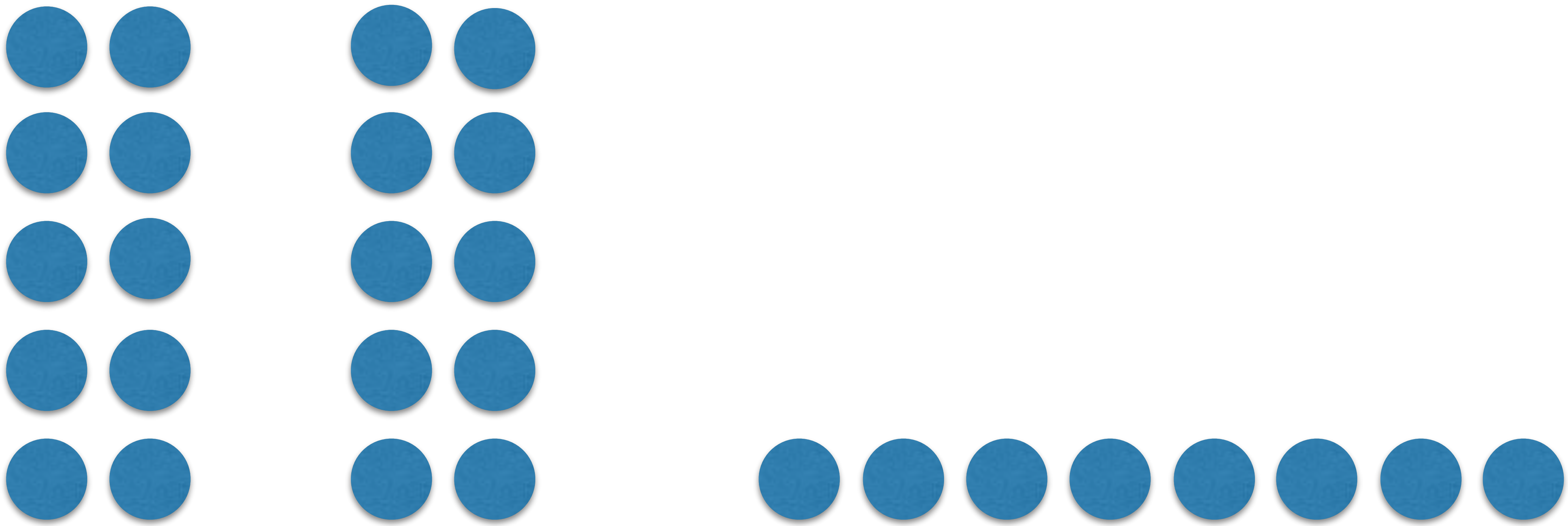
- On or off
- Transistors are either open or closed
- Low voltage or high voltage

Also more noise resistant in data communication





What would this look like in twos?



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Only two symbols: 0, 1

128s	64s	32s	16s	8s	4s	2s	1s
			1	1	1	0	0

Binary Number System

BASE 2

How many is:

	8s	4s	2s	1s
Decimal 1?				
Decimal 7?				
Decimal 15?				

Converting Binary to Decimal

Decimal numbers

$37_{10} = 3\times10^1 + 7\times10^0$

$403_{10} = 4\times10^2 + 0\times10^1 + 3\times10^0$

128s	64s	32s	16s	8s	4s	2s	1s
		1	0	0	1	0	1

Binary works the same

$100101_2 = 1\times2^5 + 0\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$100101_2 = 32_{10} + 0_{10} + 0_{10} + 4_{10} + 0_{10} + 1_{10}$

$100101_2 = 37_{10}$

Converting Decimal to Binary

Recursive division

Record the remainder

2	37	1
2	18	0
2	9	1
2	4	0
2	2	0
2	1	1
	0	

Read from bottom

100101_2

OR

37_{10}

Binary Addition

Same as decimal addition

easy for positive integers

```
5    0101
3    0011
--   ----
8    1000

19   0001 0011
15   0000 1111
--   ---------
34   0010 0010

33   ???? ????
17   ???? ????
--------------
??   ???? ????
```

Binary Addition Table

+	0	1
0	0	1
1	1	10

128	64	32	16	8	4	2	1

Binary Multiplication

Multiplying decimal numbers by 10, 100 is easy

	x10	x100
5	50	500
3	30	300

Multiplying binary numbers by 2,4,8 is also easy

Try this with:

		x2
5	0101	1010
3	0011	0110

To multiply by the base you simply shift the numbers to the left

Octal Number System BASE 8

0, 1, 2, 3, 4, 5, 6, 7

How many?
Octal 17?

512s	64s	8s	1s

Hexadecimal Number System BASE 16

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Hex 17?
Hex A2?

4096s	256s	16s	1s

What is special about hex (and octal)?

4 bits encode a single hex digit

8 bits in a byte <-> 2 hex
easy mapping binary and hex
e.g.
10101111 is AF
00111011 is 3B

What is 00100011 in hex?

What is 34_{16} in binary?

Replace each hex digit by the
4 equivalent bits

A3C5H = 1010 0011 1100 0101B
102AH = 0001 0000 0010 1010B

BINARY	HEX	Octal	Decimal
0000	0	0	0
0001	1	1	1
0010	2	2	2
0011	3	3	3
0100	4	4	4
0101	5	5	5
0110	6	6	6
0111	7	7	7
1000	8	10	8
1001	9	11	9
1010	A	12	10
1011	B	13	11
1100	C	14	12
1101	D	15	13
1110	E	16	14
1111	F	17	15

MAC Addresses

Unique address for a network device:

e.g.

54:88:0e:0e:7d:49

bc:8c:cd:e7:41:59

90:ef:68:a0:e5:46

See Network under "System Preferences"

a0:99:9b:1a:51:cf 1010 0000 1001 1001 1001 1011 0001 1010 0101 0001 1100 1111

How many bits in these MAC addresses?

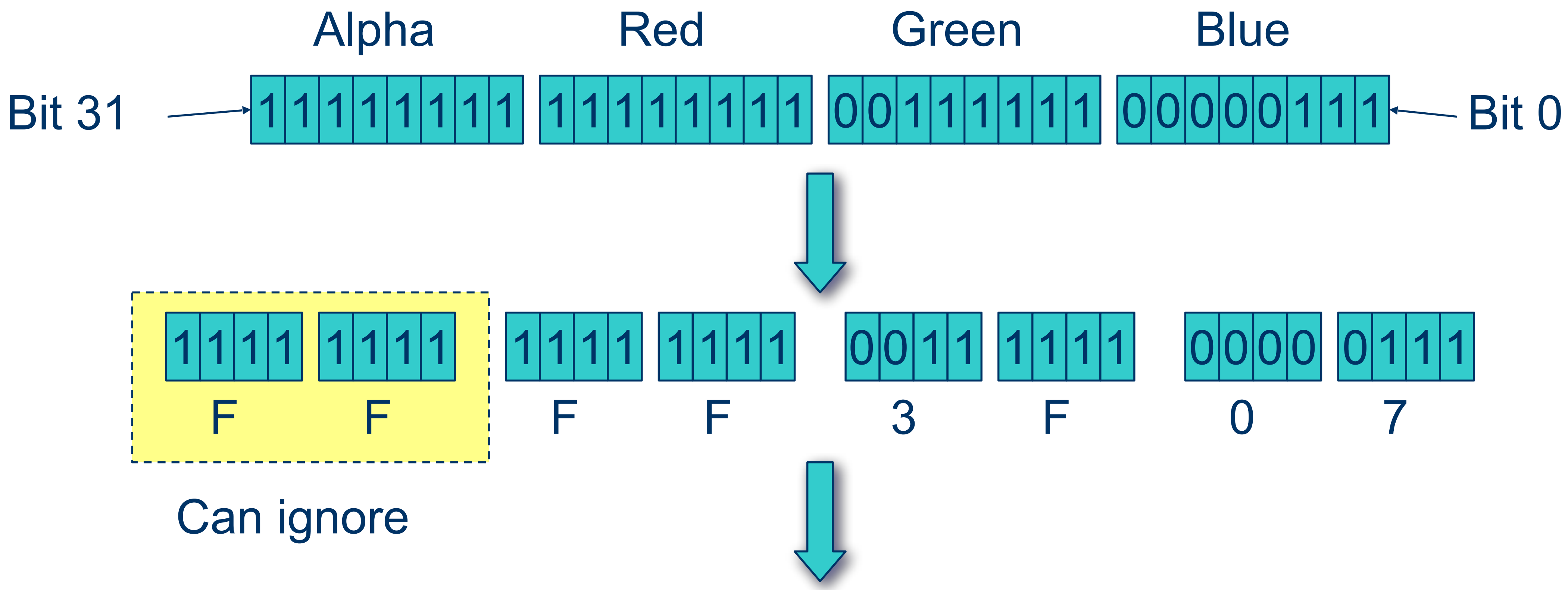
How many unique devices does that allow?

2^{48} or 281,474,976,710,656 possible MAC addresses

Hexadecimal

Hex is a convenient way to express pixel values

RGBA example (https://en.wikipedia.org/wiki/RGBA_color_space)




```
color = 0xFF3F07;
```

Hexadecimal

Each colour channel (R, G, B) has 8 bits

Therefore 256 levels = {0, 255} = 2^8

50% grey = all colours at 50%:

{50%, 50%, 50%}

$\{128_{10}, 128_{10}, 128_{10}\}$ ← note rounding

$\{10000000_2, 10000000_2, 10000000_2\}$

$\{80_{16}, 80_{16}, 80_{16}\}$

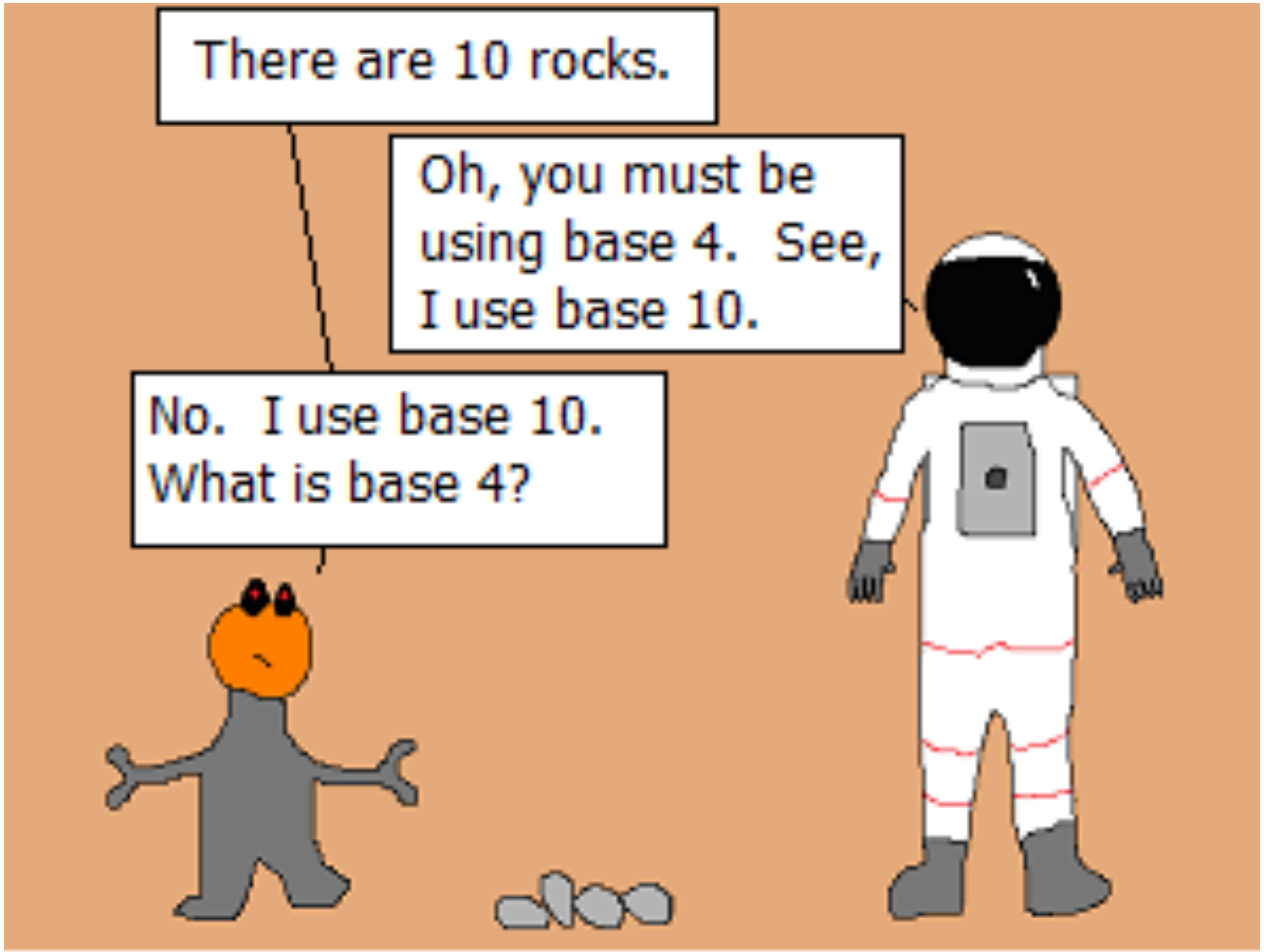
```
int color = 0x808080;
```

Blue:

```
int color = 0x0000FF;
```

Purple:

```
int color = 0xFF00FF;
```



Every base is base 10.

First practical / tutorial

Getting familiar with binary and hex

Converting between number systems

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References

see Chapter 3 Number Systems, The Architecture of Computer Hardware….

for more in depth reading on this see The Language of Mathematics