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# Comparing Bus Holding Methods With and Without Real-Time Predictions

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**Abstract**

On high-frequency routes, transit agencies hold buses at control points and seek to dispatch them with even headways to avoid bus bunching. This paper compares holding methods used in practice and recommended in the literature using simulated and historical data from Tri-Met route 72 in Portland, Oregon. We evaluated the performance of each holding method in terms of headway instability and mean holding time. We tested the sensitivity of holding methods to their parameterization and to the number of control points. We found that Schedule-Based methods require little holding time but are unable to stabilize headways even when applied at a high control point density. The Headway-Based methods are able to successfully control headways but they require long holding times. Prediction-Based methods achieve the best compromise between headway regularity and holding time on a wide range of desired trade-offs. Finally, we found the prediction-based methods to be sensitive to prediction accuracy, but using an existing prediction method we were able to minimize this sensitivity. These results can be used to inform the decision of transit agencies to implement holding methods on routes similar to TriMet 72.

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## 1. Introduction

On high frequency routes, there is a natural tendency for buses to *bunch* together. When a bus is traveling with a long headway[1], it has to pick up and drop off a relatively greater number of passengers, which slows down the bus even more. As the lagging bus becomes crowded, the following buses only have a few passengers to serve, which they can do relatively fast. Eventually the lead bus may get caught by one or several following buses and they start traveling as a platoon. Bus bunching is the product of unstable dynamics that cause delays to grow (Hickman, 2001). Even a small perturbation such as a traffic signal or a passenger paying in cash can destabilize the route and lead to bus bunching (Kittelson, 2003; Milkovits, 2008).

Unstable headway dynamics are a systemic problem that causes passenger wait and crowding. Fan and Machemehl (2009) showed that on routes where headways are less than 12 minutes, passengers tend to arrive randomly, even if a schedule is available. Because more passengers arrive during long headways than during short ones, gaps in service cause disutility to passengers in the form of undue waiting time and crowding (Newell and Potts, 1964; Milkovits, 2008). One way for transit agencies to stop the progression of instability among headways, is to provide control points, where buses with short headways can be held to absorb the delay of following buses.

Holding buses at control points can help reduce at-stop passenger waiting time, but it increases the wait of passengers who have already boarded. There is a trade-off between stabilizing headways and maintaining high operating speed (Furth et al., 2006; Furth and Wilson, 1981; Cats et al., 2011). This is why transit agencies value the benefit of headway reliability and the disadvantage of holding time differently.

[1] In this text, the term *headway* will be used as the time between the passing of two consecutive buses at a single location. Later, we will distinguish between the headway (or forward headway) and the backward headway, which is the time until the following vehicle will reach the current location.

In addition to selecting a holding method for their routes, transit agencies also need to decide how to implement it. Several holding methods in the literature require setting a parameter, which affects the trade-off between holding time and headway stability (Daganzo, 2009; Xuan et al., 2011; Bartholdi and Eisenstein, 2011; Daganzo and Pilachowski, 2011). Holding methods can also be applied at one or several control points along the route, which may impact the performance of each method. Understanding sensitivity of holding methods on the parameterization and number of control point is necessary to select the most adequate holding method based on route characteristics and desired trade-offs

Several methods are based on predictions for the arrival times of following buses (Bartholdi and Eisenstein, 2011; Daganzo and Pilachowski, 2011; Berrebi et al., 2015). The quality of the predictions may affect transit operators' ability to leverage headway stability from holding time. The required level of prediction accuracy and confidence can be burdensome for certain transit operators that may not need high-quality predictions for other applications. The ubiquity of prediction-based methods is therefore dependent on their sensitivity to prediction quality.

Research in the literature has compared holding methods, but there currently lacks a unified framework to evaluate the conflicting objectives of stabilizing headways and minimizing holding time. Xuan et al. (2011) and Berrebi et al. (2015) have case study sections to compare methods in the literature to their own. Cats et al. (2011) compare naive methods used in practice and a headway-based method similar to the method in Daganzo and Pilachowski (2011). There is a need for a sensitivity analysis to support the choice of holding methods and their parameterization based on route characteristics, including the number of control points on routes similar to Tri-Met 72.

In this paper, we investigate the holding trade-off of holding methods used in practice and recommended in the literature. To this end, we evaluate holding methods on a simulated bus route using historical data from Tri-Met Route 72 in Portland, Oregon. We use the prediction tool developed in Hans et al. (2015) to reproduce the predictions in a realistic setting. In the following section, we describe the holding methods used in practice and recommended in the literature. In Section 3, we discuss the simulation experiment, and particularly the methods evaluated. In Section 4 we compare the performance of each holding method. In Section 5, we investigate the impact of parameter choice, and number of control points on the trade-off between stabilizing headways and keeping short holding times. In Section 6 we test the sensitivity of prediction-based holding methods on the accuracy and confidence of predictions. Finally, we provide concluding remarks in Section 7.

## 2. Holding methods in the literature

Methods to hold buses at control points have been addressed for many decades. Osuana and Newell (1972) and Newell (1974) formulated the theoretical basis for holding mechanisms to minimize passenger waiting time on simple routes in the 1970's. Since then, two main approaches to the bus holding problem have been developed in the literature, mathematical optimization and analytical.

The first approach consists in optimizing a weighted function of passenger wait in mathematical programs that consider the dynamics of bus trajectories (analytically or by simulation). At each decision stage, the optimization tools model the future states of the system, and assign holds on a rolling horizon. Hickman (2001) developed a linear search optimization algorithm which considers holding decisions in isolation of each other based on a stochastic model for bus trajectories. In Eberlein et al. (2001) a heuristic algorithm is used to minimize the waiting time of passengers at stops in a quadratic program. Bukkapatnam and Dessouky (2003) developed an iterative model where buses and stations *negotiate* holding time to minimize marginal costs. The method in Zolfaghari et al. (2004) assigns all holding decisions simultaneously, while considering capacity constraints, using AVL data and perfect predictions. Delgado et al. (2009) and Delgado et al. (2012) developed a simulation-based optimization algorithm that reproduces stationary bus trajectories deterministically and minimizes a weighted function of wait time. Sánchez-Martínez et al. (2016, 2015) extended their methods to consider dynamic passenger arrival rates and travel time. Cortés et al. (2010) used a genetic algorithm to solve a multi-objective dynamic problem.

The second approach assigns holds as closed-form functions of bus arrival times (Daganzo, 2009; Daganzo and Pilachowski, 2011; Xuan et al., 2011; Bartholdi and Eisenstein, 2011; Berrebi et al., 2015). Buses are held with the objective of maintaining stable headways, and preventing bus bunching from the onset, which can minimize passenger waiting time globally and durably. Methods assign holds to buses as a function of the schedule, headways and, for some, predicted arrival times[2]. Unlike the mathematical programming approach, these methods do not consider the prediction model further than its output. Therefore, they can use any prediction model [3], which makes them much easier to implement. Closed-form holding methods will be the focus of this paper.

The notation used in this paper is consistent with Berrebi et al. (2015), and is shown in Table 1. The arrival time of the $i^{\text{th}}$ bus at the control point is $A_i$ (random variable) for a future arrival, and $a_i$ (realization of $A_i$) for a known arrival time. Once the $i^{\text{th}}$ bus arrives, it holds for time $h_i$, and is dispatched at time $d_i = a_i + h_i$. If the route runs according to a schedule, $\bar{d}_i$ is the scheduled departure time from the control point, and $H_i$ is its scheduled headway ($H_i = \bar{d}_i - \bar{d}_{i-1}$). The holding methods evaluated in this paper are described in their Eulerian version (as in Xuan et al. (2011)) in Table 2 with the information they require and their recommended holding times.

[2]In the remainder of this text, we refer to holding methods that consider schedules as "schedule-based" and methods that consider headways as their main input as "headway-based". Schedule-based methods include the Naive Schedule and the method recommended in Xuan et al. (2011). Headway based methods include the Naive Headway and the method recommended in Daganzo (2009).

[3]A discussion follows in Section 6.

Table 1: Summary of variable definitions.

| Variable | Definition |
|---|---|
| $A_i$ | Arrival time of bus $i$ (Random Variable) |
| $a_i$ | Arrival time of bus $i$ (Realization) |
| $h_i$ | Hold imposed on bus $i$ |
| $d_i$ | Departure time of bus $i$ (Realization) |
| $\bar{d}_i$ | Scheduled departure time of bus $i$ |
| $H_i$ | Scheduled headway of bus $i$ |
| $n_i$ | Number of following buses when bus $i$ reaches the control point |
| $CV^2$ | Headway Coefficient of Variation squared |
| $T[A_j]$ | Arrival time of a particle[4] of the $j^{\text{th}}$ bus |

### *2.1. Naive methods*

The simplest and most widely used methods to hold buses at control points are to plan scheduled departure times, $\bar{d}_i$, or scheduled headways, $H_i$, well in advance (Boyle et al., 2009; Abkowitz and Lepofsky, 1990; Van Oort et al., 2010). In this paper, we will consider the Naive Schedule and Naive Headway methods as holding buses until their departure time or headway reaches the planned threshold, as shown in Equations 1 and 2 of Table 2. The Naive Schedule method is easy to implement because it only requires information about the arrival time of the vehicle being controlled. The Naive Headway method requires the last departure time from the control point. This method never dispatches buses with short headways because it imposes a threshold headway, $H_i$. This feature allows the Naive Headway method to control *big gaps* in service that follow buses with small headways.

### *2.2. Partial holding methods*

Daganzo (2009) developed a headway-based holding method that compensates for unstable headway dynamics. A dimensionless parameter $\beta$ accounts for the linear delay of vehicles resulting from a unit headway increase; values usually range between 0.01 and 0.1. When a bus arrives at a control point, its headway is readjusted to the scheduled headway, $H$, by a factor of $\alpha+\beta$, where $\alpha \in ]0,1[$. Equation 3 of Table 2 shows the hold imposed on a bus that arrives at a control point[5].

Xuan and collaborators then generalized this class of control, and developed a method that only considers the forward headway and deviation from schedule as shown in Equation 4 of Table 2 (Xuan et al., 2011). The method readjusts the headways with respect to the scheduled headway, $H$, by a factor $\beta$ and the off-schedule time, $a_i - \bar{d}_i$, by a factor $\alpha \in ]0,1[$. The authors showed that the holding mechanism was capable of maintaining a schedule in a stochastic environment.

The partial holding methods act like parametric versions of the Naive Schedule and Naive Headway, with the $\alpha$ term in place to reduce the holding time. Partial holding methods rely either on the scheduled departure, $\bar{d}_i$, or the scheduled headway, $H_i$, to stabilize operations as in Naive methods. Unlike the Naive methods, however, they only recommend holding for a portion of the thresholds to reduce the holding time [6]. Daganzo, Xuan, and collaborators showed that their methods can recover from bounded deviations from the schedule or scheduled headway in stationary operating conditions. In practice, running time can be highly stochastic and non-stationary, which can cause systematic deviations from the schedule or scheduled headway. The methods described thus far do not consider the predicted arrivals of following buses to adjust target headways.

[5]The forward headway in Daganzo (2009), Daganzo and Pilachowski (2011), and Xuan et al. (2011) is expressed in terms of inter-arrival time, $a_i - a_{i-1}$ without considering the hold imposed on the leading bus. To make these methods more robust, we have replaced the inter-arrival time by the time since last departure for the forward headway.

[6]To preserve the robustness of their method, and recover from perturbations, Daganzo (2009), Daganzo and Pilachowski (2011), and Xuan et al. (2011) recommend using slack time. Slack time leverages longer holding time to stabilize headways, as shown in Argote-Cabanero et al. (2015). We have found in a simulation, however, that adding slack time to the Route 72 schedule does not substantially affect the trade-off between headway stability and holding time This is why we decided to calibrate slack time to the historical schedule in this paper.

Table 2: Holding methods with their data requirements and recommended holding times.

| Holding method | Data requirement | Recommended holding time | Eq |
|---|---|---|---|
| Naive Schedule | Schedule | $\bar{d}_i - a_i$ | (1) |
| Naive Headway | Forward headway | $H - (a_i - d_{i-1})$ | (2) |
| Daganzo (2009) | Forward headway | $(\alpha + \beta)(H - (a_i - d_{i-1}))$ | (3) |
| Xuan et. Al (2011) | Forward headway | $\beta(H - (a_i - d_{i-1})) - \alpha(a_i - \bar{d}_i)$ | (4) |
| Bartholdi and Eisenstein (2011) | Predicted backward headway | $\text{Max}[H - (a_i - d_{i-1}), \alpha(E[A_{i+1}] - a_i)]$ | (5) |
| Daganzo and Pilachowski (2011) | Forward and predicted backward headway | $(\alpha + \beta)(H - (a_i - d_{i-1}))$ $-\alpha(H - (E[A_{i+1}] - a_i))$ | (6) |
| Berrebi et. Al (2015) | Joint probability distribution of next $n$ bus arrival times | $\frac{E\left[\max\limits_{r-i} \frac{A_r - a_i}{r-i} - (a_i - d_{i-1})\right]}{1 + E\left[\left(\arg\max\limits_{r-i} \frac{A_r - a_i}{r-i}\right)^{-1}\right]}$ | (7) |

### *2.3. Prediction-based holding methods*

A novel closed-form approach to the bus holding problem is to dispatch buses according to the predicted arrival times of following buses at the control point. Real-time prediction methods are becoming increasingly accurate and available, which allows the replacement of planned operations by the natural headway in current operating conditions. Today, the vast majority of public transportation agencies in the United-States are capable of tracking their vehicles in real-time (Grisby, 2013). Using real-time vehicle locations, increasingly sophisticated prediction algorithms have surfaced recently (Chien et al., 2002; Cathey and Dailey, 2003; Jeong and Rilett, 2004, 2005; Chen et al., 2005; Shalaby and Farhan, 2004; Gurmu and Fan, 2014; Sun et al., 2007; Mazloumi et al., 2011). Most notably, Hans et al. (2014, 2015) developed a prediction method specifically for the purpose of real-time control. Their prediction algorithm can generate probability distributions of arrival times. Based on these predictions, holding methods can consider following buses to equalize headways, without having to rely on planned operations.

Using real-time predictions, Bartholdi and Eisenstein (2011) developed a holding strategy that can stabilize headways without the need for planned operations. The method consists in holding each vehicle for the predicted time until the next arrival by a factor $\alpha \in ]0, 1[$ as in Equation 5 in Table 2. When several buses arrive in close succession, however, the method sends the middle few uncontrolled. To prevent this, Bartholdi and Eisenstein added a minimum forward headway[7]. Otherwise, the method can split the burden on a *big gap* between two buses: each vehicle leaves the control point with the weighted sum between its forward[8] and backward headways. Unlike the methods cited thus far, their method involves a mechanism that acts locally to scale headways to the rate of arriving vehicles.

Daganzo and Pilachowski used predictions on the next arrival time to blend the forward headway with the backward headway, as shown in Equation 6 of Table 2 (Daganzo and Pilachowski, 2011). The method considers the forward headway in the same way as Daganzo (2009), but it also subtracts the deviation of the expected time until the next arrival from the scheduled headway, $H - (E[A_{i+1}] - a_i)$, by a factor $\alpha \in ]0, \frac{1}{2}[$. The holding method can reduce the difference between the forward and backward headways by holding buses for a weighted sum of that difference.

More recently, Berrebi and collaborators developed a method that takes a global approach by considering every bus on the route. The method is a generalization of Daganzo and Pilachowski (2011), without $H$ and $\beta$, that considers $n$ buses. The vehicle that will arrive with the maximum relative delay, $\max\limits_{r-i} \frac{A_r - a_i}{r-i}$, is probabilistically identified and each preceding vehicle is held to absorb a share of that delay, as in Equation 7 in Table 2. When the lagging bus arrives at the control point, it can be dispatched with approximately the same headway as the leading few. The method can diffuse big gaps organically without the need

[7]To let the method act primarily on backward headways, we set the minimal forward headway as $H_i/2$.

[8]More exactly the backward headway of the following bus, a headway ago.

for schedules, scheduled headways, or any kind of explicit slack time.

## 3. Case study

### *3.1. The route*

Holding methods were compared by simulation using data on real bus trajectories from Tri-Met bus route 72 shown in Figure 1. Buses run in mixed traffic on the perpendicular 82$^{\text{nd}}$ Avenue and Killingsworth Street in Portland, Oregon. The scheduled headway alternates between seven and eight minutes in the afternoon peak. Historically, Route 72 had a bus bunching problem during peak hours (Berkow et al., 2007). Buses on route 72, however, are equipped with a Computer Aided Dispatching (CAD) system that would allow replacing the current schedule with real-time control. To evaluate the potential benefits and disadvantages of applying each real-time holding method described in the previous section, we have tested their performance in a case study.


Figure 1: Map of TriMet Route 72 TriMet (2016)

We used data available on the Portland Oregon Regional Transportation

Archive Listing (PORTAL)[9] to build the simulation framework. The online open platform provides Automatic Vehicle Location (AVL), Automatic Passenger Counts (APC), traffic signal settings, and loop-detector data for September $15^{\text{th}}$ to November $15^{\text{th}}$ 2011 on a part of route 72. The data covers the entire portion on $82^{\text{nd}}$ Avenue and ten blocks of Killingsworth Street (until $72^{\text{nd}}$ Avenue) towards Swan Island.

In the study, we used historical data leading up to the first control point [10]. Each method had the opportunity to hold vehicles at that point to stabilize operations and mitigate bus bunching. Whenever the boarding and alighting times exceeded the hold recommended by a control method, buses were dispatched after they finish loading and unloading. The headways of buses leaving the control point were recorded to evaluate the performance of the dispatching strategy. We used simulated data downstream from the control point to take into account the impacts of each holding method on headway dynamics.

In this paper, we considered a single value of $H_i$: its historical value. In the historical data, vehicles arrive at the control point(s) according to a set scheduled frequency of service. The scheduled frequency at a control point determines the scheduled headway, because the frequency of departure cannot be greater than the frequency of arrival. If we reduced the scheduled headway, the holding methods would be unable to control buses at all. If we increased the scheduled headway, a continually accumulating queue of buses would form. This is why we did not test holding methods with varying values of $H_i$.

### *3.2. Prediction*

Once buses arrive at the control point, each method can recommend a holding time based on the last departure time (recommended by the method) and the predicted next arrivals. The methods recommended in Daganzo and Pilachowski (2011) and Bartholdi and Eisenstein (2011) need the expected next

[9] `http://portal.its.pdx.edu/Portal/index.php/fhwa`

[10] Several holding point are considered in Section 5.2.

arrival time, $E[A_{i+1}]$. The method recommended in Berrebi et al. (2015) needs the probability distribution of each future arrival to infer the maximum relative delay, $\max \frac{A_r - a_i}{r-i}$, and its corresponding index, $\arg\max \frac{A_r - a_i}{r-i}$.

The prediction tool used in our simulation is a particle filter combined with an event-based mesoscopic model developed in Hans et al. (2014) and Hans et al. (2015). The prediction tool is capable of generating simulated trajectories solely using vehicle location data. The predicted arrival times of buses at the control point are generated iteratively as a function of their dwell and travel times, as shown in Table 3. The dwell time is estimated as the sum of boarding, alighting and door operation time, assuming passengers board and alight through the same door. The number of passengers boarding are assumed to follow a Poisson distribution, with no capacity constraints. The share of passengers alighting follows a binomial distribution with passenger loads estimated using historical headways. Travel times between stations are generated as a Gamma distribution. To ensure that the particle filter accurately reproduces operations on Route 72, the parameters of dwell and travel times, such as the rate of arriving passengers and the mean travel time between stops were calibrated on historical data from the route. Odd days were used for the calibration, and even days were kept for the simulation.

Table 3: Particle generation

| Equation | Variable | Definition |
|---|---|---|
| $A_i = \sum_{s=s_{\text{last}}}^{s_{\text{ctrl}-1}} \delta_{i,s} + \pi_{i,s}$ | $s_{\text{ctrl}-1}$ | Upstream stop from the control point |
| | $s_{\text{last}}$ | Last stop visited by bus $i$ |
| | $\delta_{i,s}$ | Dwell time of bus $i$ at stop $s$ |
| | $\pi_{i,s}$ | Travel time of bus $i$ between stop $s$ and $s+1$ |
| $\delta_{i,s} = aN_A + bN_B + c$ | $a$ | Individual alighting time |
| | $b$ | Individual boarding time |
| | $c$ | Time lost in door opening and closing |
| | $N_A$ | Number of alighting passengers |
| | $N_B$ | Number of boarding passengers |
| $N_B \sim \mathrm{P}(d_s h_{i,s})$ | $h_{i,s}$ | Headway of bus $i$ at stop $s$ |
| | $d_s$ | Demand rate at stop $s$ |
| $N_A \sim \mathrm{Bin}(L_{i,s}, \mu_s)$ | $L_{i,s}$ | Load of bus $i$ at its departure from stop $s$ |
| | $\mu_s$ | Alighting proportion at stop $s$ |
| $L_{i,s} = (1-\mu_s)L_{i,s-1} + d_s h_{i,s}$ | | |
| $L_{i,0} = 0$ | | |
| $\pi_{i,s} \sim \mathrm{Gam}(\text{mean}_s, \text{stdev}_s)$ | $\text{mean}_s$ | Mean travel time between stop $s$ and $s+1$ |
| | $\text{stdev}_s$ | Standard deviation of travel time between $s$ and $s+1$ |

When a bus arrives at the control point, 100 particles (simulated bus trajectories) are generated for each following bus traveling on the route.[11] The arrival times of following buses at the control point are then aggregated in a histogram, which are treated as probability distributions. Figure 2 shows a time-space diagram with the trajectories of following vehicles in part (a), and the associated histogram of bus arrival times in part (b). At each station, the particles in part

[11]We chose to generate 100 particles as a compromise between the computational time and the resolution of the histogram.

(a) tend to divert away from their mean because the prediction accounts for the delay accumulation caused by unstable headway dynamics. In part (b), the arrival time of the current and leading buses are represented by vertical lines because their arrival times are known. The arrival times of following vehicles are random and their distributions widen with the horizon of their prediction.

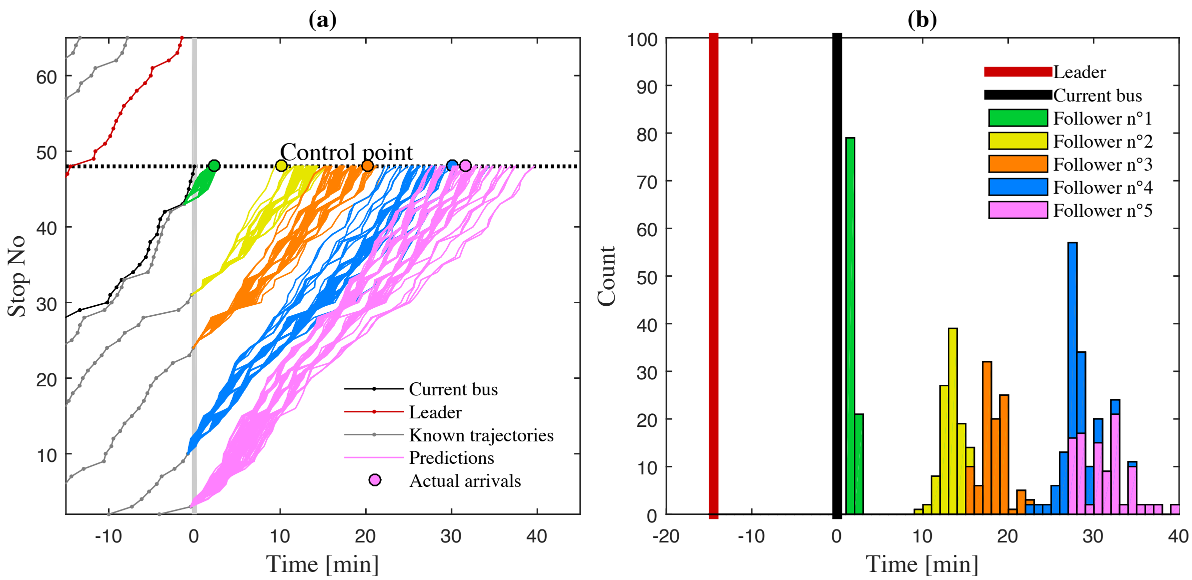


Figure 2: (a) Generation of particles to simulate bus trajectories. (b) Histograms of simulated bus arrival times treated as probability distributions.

The particle filter recommended in Hans et al. (2014) and Hans et al. (2015) is particularly suitable for real-time control. Unlike regression and machine learning prediction methods, the particle filter is capable of generating probability distributions, which is necessary for the method in Berrebi et al. (2015). The model is simple to calibrate, and it is compatible with any bus route and any data format. The tool can consider traffic congestion and traffic signal data if available, but in this study, we assumed that only vehicle locations were available for the prediction

*3.3. Performance indicators*

Performance indicators allow transit agencies to identify and address gaps in service. On high frequency routes, a measure of instability is the squared coefficient of variation of headways, denoted $CV^2$ as shown in Equation 8 (Kittelson, 2003). The variable measures the extent of headway instability. When $CV^2 = 0$, buses are evenly spaced, and when $CV^2 = 1$, Average Passenger Wait (APW) is equal to H. Assuming Poisson arrivals, APW is directly proportional to $CV^2$ (Newell and Potts, 1964), as shown in Equation 9.

$$CV^2[\text{headway}] = \frac{Var[\text{headway}]}{E[\text{headway}]^2} \tag{8}$$

$$\text{APW} = \frac{E[\text{headway}]\left(1 + CV^2[\text{headway}]\right)}{2} \tag{9}$$

Variation in headways tends to increase along the route unless buses are controlled (Hickman, 2001). Therefore, unstable headway dynamics can have lasting effects on the system and on passenger experience if headways are not stabilized. We choose to evaluate headway stability in terms of $CV^2$ because it is a dimensionless parameter that directly relates to at-stop passenger wait and allows extrapolation of results to other routes.

Holding methods can stabilize headways by trading off holding time. Holding time too has a cost. It causes passengers who are already aboard the vehicle undue waiting time. In addition, holding time reduces bus operating speed. Finally, holding at a control point can disrupt surrounding traffic, depending on its location. Since transit agencies may value the benefit of headway stability and the cost of holding time differently depending on the route, we chose to keep these measures of performance separate.

## 4. Cross-comparison

We simulated the decision process of each holding method described in Table 2 in the afternoon peak hour, when scheduled headways, $H_i$, oscillate between seven and eight minutes. In the simulation, buses were held at the 48$^{\text{th}}$ station, which is seven miles away from the departure point, at the intersection with the light-rail, MAX, on Banfield Expressway. We chose this station because it is used as a schedule recovery point in historical data. The station also has by far the greatest alighting proportion and boarding demand on Route 72. These considerations are important because the overall cost of holding time is proportional to the number of passengers who ride through the holding point and the overall benefit of stabilizing headways is proportional to the number of passengers waiting at downstream stops. Therefore, holding at station 48 inconveniences few passengers and benefits many.

Every method was parameterized with $\beta$ calculated at each arrival as per Daganzo (2009), Daganzo and Pilachowski (2011), and Xuan et al. (2011), and $\alpha = \frac{1}{2}$ to provide a middle-ground basis of comparison with the Naive Schedule and Headway methods. We discuss the choice of the $\alpha$ parameter in the next section.

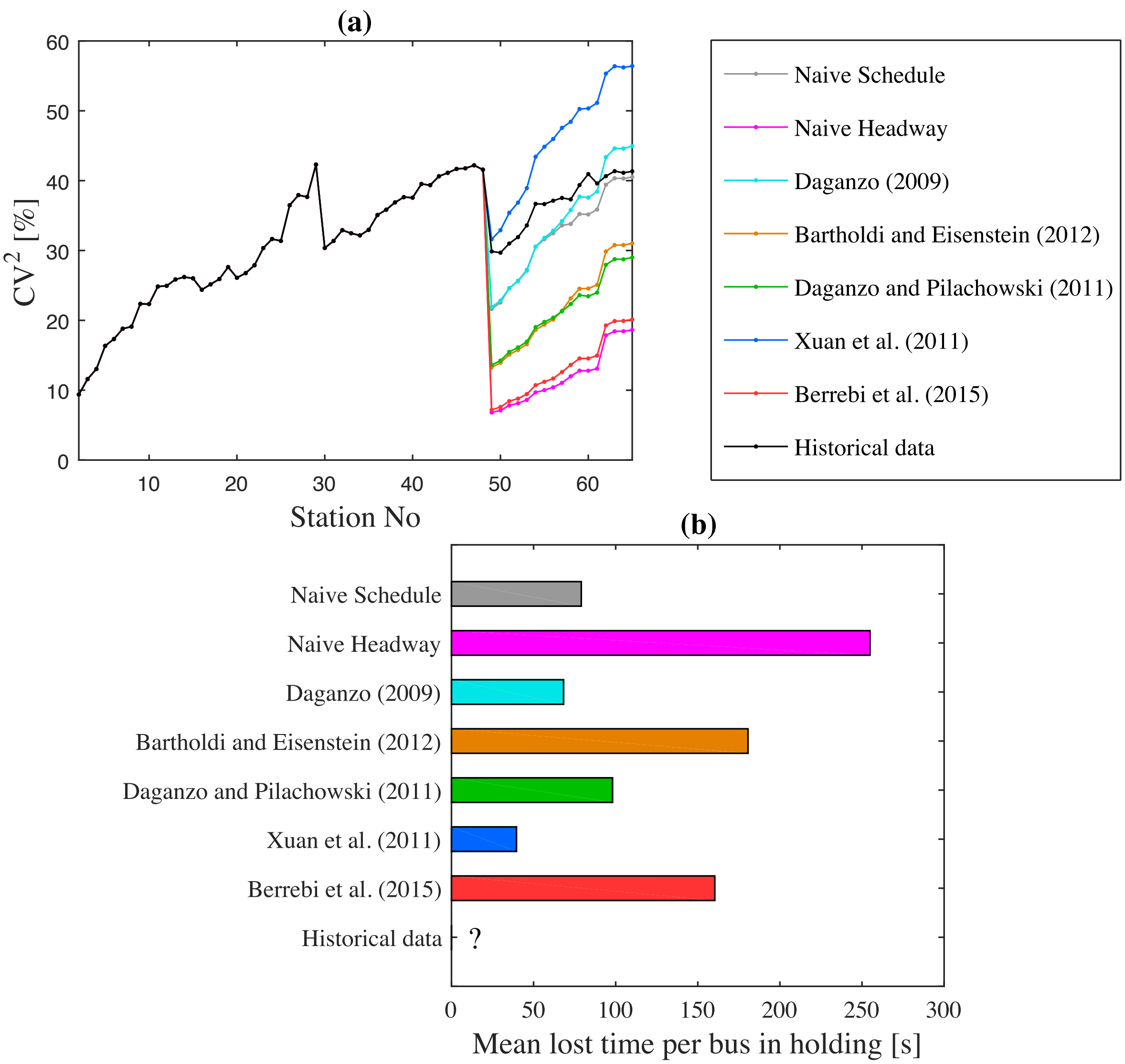


Figure 3: Performance of holding methods in terms of (a) squared headway coefficient of variation, ($CV^2$) and (b) mean holding time past boarding and alighting (lost time).

The results of our simulation are shown in Figure 3, with squared headway Coefficient of Variation, $CV^2$ in part (a) and mean holding time past boarding and alighting (lost time) in part (b) for each holding method. This figure and all figures following are based on 50 simulation runs. In part (a), values of $CV^2$ upstream of the control point come from historical data, and values downstream of the control point are simulated, except for the *historical data* curves in black.

In historical data, buses start the route with close to even headways, but get destabilized along the route, leading to the control point. The $CV^2$ increases in a saw tooth pattern, with small drops corresponding to mid-route control points. When buses arrive at the main control point, $CV^2$ is 42%, which, according to the second edition of the Transit Capacity and Quality of Service, corresponds to a Level of Service (LOS) E, and is denoted as *Frequent bunching* (Kittelson, 2003). In historical data, $CV^2$ is reduced to 30%, which is still LOS E. We did not display the mean lost time in *historical data* because we could not differentiate the share that was conscious holding from boarding, alighting, and other dwell operations.

The holding method applied in historical data, and the method recommended in Xuan et al. (2011) are both based on the Naive Schedule method, but they produce different results. The Naive Schedule method[12] reduces $CV^2$ to 22% (LOS D, *Irregular headways, with some bunching*) with a mean lost time holding of 79 seconds. The holding method in Xuan et al. (2011) sends buses with greater variability than the Naive Schedule, scoring LOS E, but only holds buses for 40 seconds on average.

The Naive Headway method is capable of sending buses at regular intervals, but requires long holding times. The Naive Headway method was effective at stabilizing headways with 7% of $CV^2$ (LOS B, *vehicles slightly off headway*). The method, however, holds buses for 255 seconds on average. The holding times tend to accumulate because each late bus pushes back the dispatching time of all upstream buses. The method proposed in Daganzo (2009) dispatches buses with much more erratic headways than the Naive Headway, which causes longer wait for passengers at a stop, but the method keeps mean holding time at 40

[12] Sometimes overlapping with Daganzo (2009) in parts (a) and (b).

seconds.

The prediction-based holding methods consider the predicted arrival times of following buses to diffuse big gaps. The method in Bartholdi and Eisenstein (2011) dispatched buses at $CV^2 = 0.13$ (LOS C, *vehicles often off headway*). The method, however, holds buses for 180 seconds on average. The method in Daganzo and Pilachowski (2011) produced almost the same results as in Bartholdi and Eisenstein (2011), but only required 98 seconds of holding time. The method recommended in Berrebi et al. (2015) considers the arrival time of every following bus on the route. The holding method reduces $CV^2$ to 7% (LOS B, *Vehicles slightly off headway*) with 160 seconds of mean holding time.

The reduction in headway variability at the control point has lasting effects downstream from the control point. The more a method is able to reduce headway variability at the control point, the less headways tend to destabilize further down the route. Conversely, the rate of increase of $CV^2$ is highest for methods that are the least able to stabilize headways. For example, between stops 49 and 60, $CV^2$ increased three times more for the method in Xuan et al. (2011) than the method in Berrebi et al. (2015), which dispatched buses with far more stable headways. This trend is not seen in historical data because it benefits from mid-route control points, which we have omitted in the simulation.

## 5. Sensitivity

The performance of control strategies can vary depending on how they are applied. It is important to understand the implications of parameter choice and the number of control points. In this section, we investigate how the performance of each method is affected by these factors.

### *5.1. Parameterization*

The methods recommended in Daganzo (2009), Daganzo and Pilachowski (2011), Bartholdi and Eisenstein (2011), and Xuan et al. (2011) all require setting an $\alpha$ parameter, but their interpretation of the parameter is different. For the methods prescribed in Daganzo (2009) and Xuan et al. (2011), $\alpha$ corresponds to the fraction of the deviation from headway or schedule that is to be recovered by the hold. When $\alpha$ is close to zero, buses are barely controlled, and for values $\alpha$ close to one, the holding methods are similar to the Naive Schedule and Headway. For the method recommended in Bartholdi and Eisenstein (2011), $\alpha$ is the fraction of the predicted backward headway that buses should hold for. The mean holding time is $\alpha$ times the average headway. In Daganzo and Pilachowski (2011) the $\alpha$ parameter weights the difference between the forward and backward headway. For values of $\alpha$ close to zero, more importance is given to the forward headway, and for values close to one, more importance is given to the backward headway.

Figure 4 shows (a) $CV^2$ as a function of $\alpha$, (b) mean lost time as a function of $\alpha$ and (c) $CV^2$ as a function of mean lost time immediately downstream of the station 48 control point[13]. Solid lines represent the range of $\alpha$ parameter recommended by the authors of the holding methods, and dashed lines represent values of $\alpha$ outside the recommended range. Note that when $\alpha$ is greater than its recommended range, Daganzo (2009), Daganzo and Pilachowski (2011) and Xuan et al. (2011) compensate for perturbations excessively by dispatching buses past $\bar{a}_i$ or $H_i$. In the last section, we set $\alpha = \frac{1}{2}$ for each parametric method. The solid dots on Figure 4 show the performance of each holding method as parameterized in the previous section.

[13]Note that the interpretation of $\alpha$ differs for each method.

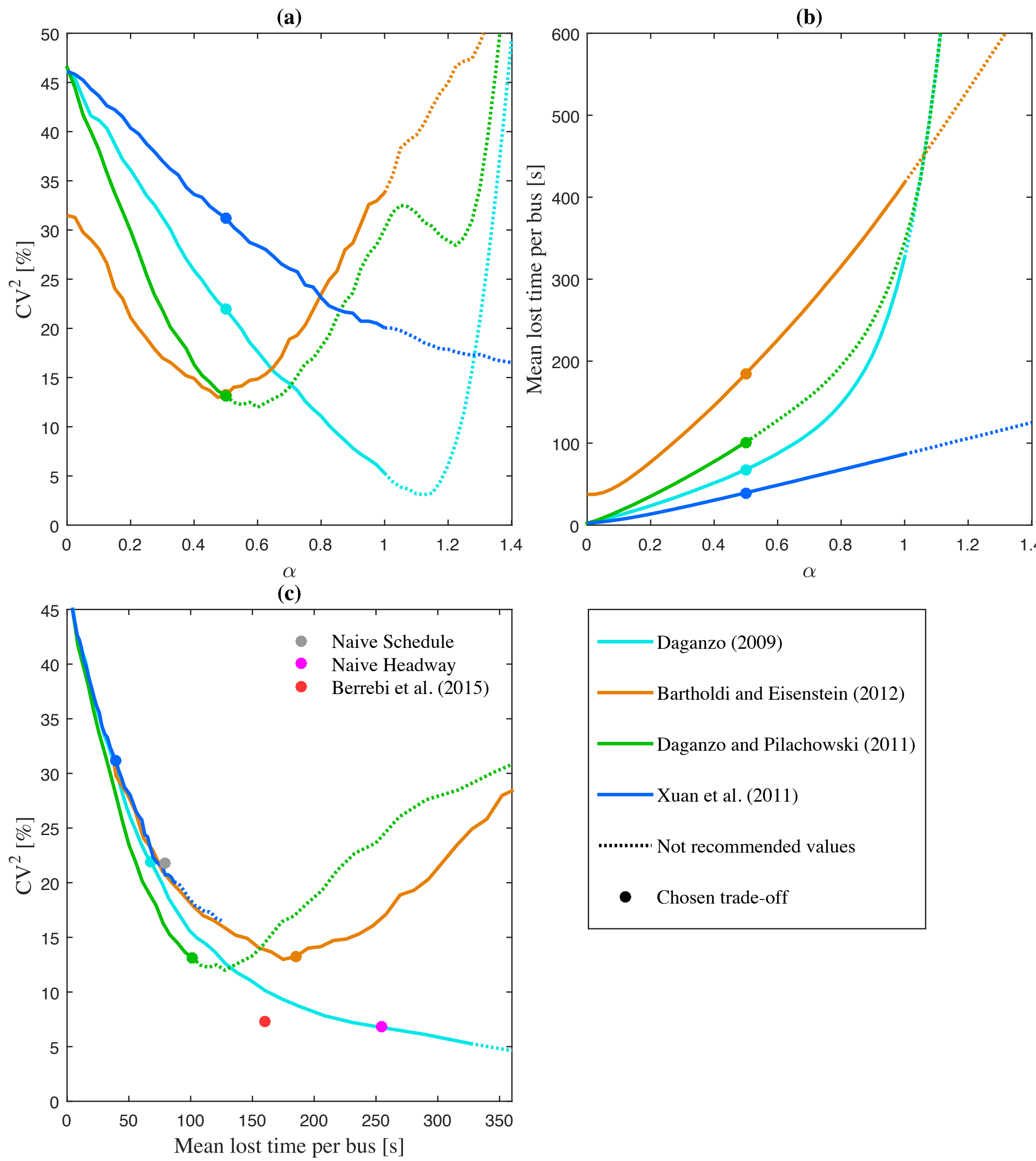


Figure 4: (a) $CV^2$ as a function of $\alpha$, (b) mean lost time as a function of $\alpha$ and (c) $CV^2$ as a function of mean lost time at station 49.

The choice of $\alpha$ affects the amount of holding time recommended by each method, and ultimately the $CV^2$ and average passenger wait. Part (a) of Figure 4 shows that $CV^2$ for the partial holding methods decreases monotonically with values of $\alpha$ within their recommended range. Part (a) also shows that the parametric prediction-based methods have convex shapes and attain their lowest $CV^2$ close to $\alpha = \frac{1}{2}$, with a slight deviation due to the $\beta$ parameter. In part (b) of Figure 4, the holding time of each parametric method grows with $\alpha$.

Part (c) of Figure 4 shows the trade-off between $CV^2$ and mean lost time attained by each method. The naive methods and the method recommended in Berrebi et al. (2015) are represented as single points because they are not parametric. The $CV^2$ in Xuan et al. (2011) and Daganzo (2009) decrease monotonically as a function of mean lost time. In both methods, the chosen trade-offs ($\alpha = 0.5$) yield far greater $CV^2$ than the naive methods from which they are derived. For the method in Xuan et al. (2011), the rate of decay remains high until the Naive Schedule trade-off is reached, whereas the method in Daganzo (2009) requires less than a third of Naive Headway's mean lost time.

Prediction-based methods achieve the best compromise between headway regularity and holding time in a wide range of settings. The method in Daganzo and Pilachowski (2011) can be parameterized to yield a lower $CV^2$ than any other method for any holding time up to 130 seconds. The method in Berrebi et al. (2015) can dispatch buses with 7% of $CV^2$ and 160 seconds of holding time, making it the preferable method for holding times over 130 seconds.

### *5.2. Multiple Control Points*

Whereas methods thus far have been applied at a single control point, we now test them at several control points along the route. Applying holds at several control points can help maintain stable headways throughout, but it requires more frequent holding. In this section, we evaluate the trade-offs between headway stability and mean holding time for each method based on the number of control points. This analysis can support the decision of transit agencies to implement the holding method most adequate for their route and objectives.

The methods in Berrebi et al. (2015) and Bartholdi and Eisenstein (2011) are designed to dispatch buses with one or few control points. The method in Berrebi et al. (2015) considers the predicted arrival times of each bus on the route. It cannot be applied at a close succession of control points because holds would interfere with predictions. The method in Bartholdi and Eisenstein (2011) holds vehicles for $\alpha H_i$ on average. Holding time would therefore accumulate proportionally to the number of control points. Both methods, however, can be paired with on-route holding methods that only consider the local headway dynamics. We introduce two hybrid methods that apply the method in Bartholdi and Eisenstein (2011) (Hybrid #1) and Berrebi et al. (2015) (Hybrid #2) at a "main control point" (stop 29) and the method in Daganzo and Pilachowski (2011) at all other control points.

The Hybrid Methods require the predicted arrivals of one or several upstream buses. In this simulation, the arrival times of vehicles at early stops were not predictable due to unavailable data. The Hybrid methods could only be applied on the second half of the route. In order to test non-Hybrid methods in the most favorable and realistic conditions, we allowed there methods to apply holding time progressively form the beginning of the route. Although the route characteristics including congestion levels, stop spacing, and passenger demand, are relatively stable throughout the route, the non-Hybrid methods may encounter slightly different operating conditions on the first 29 stops. This is why, Hybrid and non-Hybrid methods are plotted separately.

Figure 5 shows mean $CV^2$ and total lost time applied and recorded throughout the route in Part (a) and applied and recorded in the second half of the route in Part (b). All holding methods were implemented at 1,2,4, and 8 holding points. The Schedule-Based methods were also held at 16 and 32 holding points because they are the only ones that do not consider headways. All other methods use either the forward or the backward headway, which would be affected by holds imposed at intermediate bus stops. The $\alpha$ parameter was set to

0.5 for all parametric methods, including both Hybrid Methods. [14] [15] In every case, control points were placed at regular intervals. [16]

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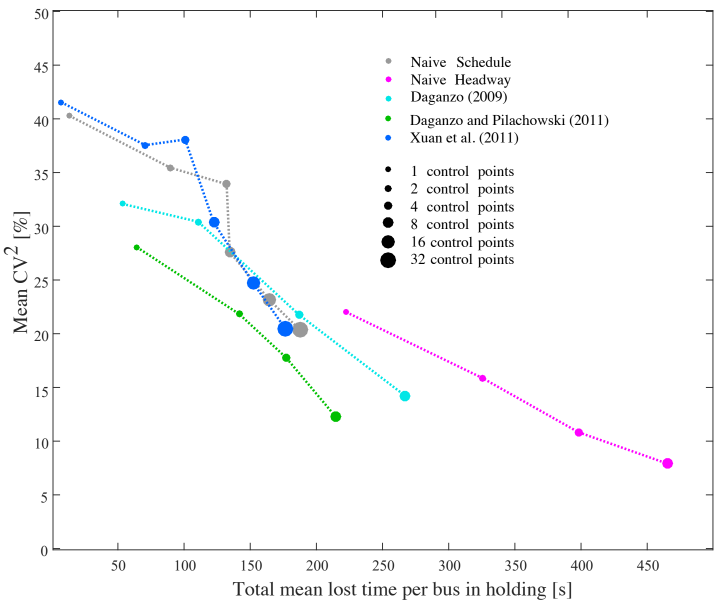
[14] Testing was done with various values of $\alpha$. We found that

[15] Note that the $\beta$ parameter is accumulated over the number of stops between control points for the method in Xuan et al. (2011). We did not, however, use a cumulative $\beta$ when applied at a single control point.

[16] Control points are {30}, {30, 45}, {30, 38, 46, 54}, {30, 34, 38, 42, 46, 50, 54, 58} for the Hybrid Methods and {30}, {20, 40}, {12, 24, 36, 48}, {7, 14, 21, 28, 35, 42, 49, 56}, {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64}, {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 68, 60, 62, 64} for other methods.

**(a)**




**(b)**

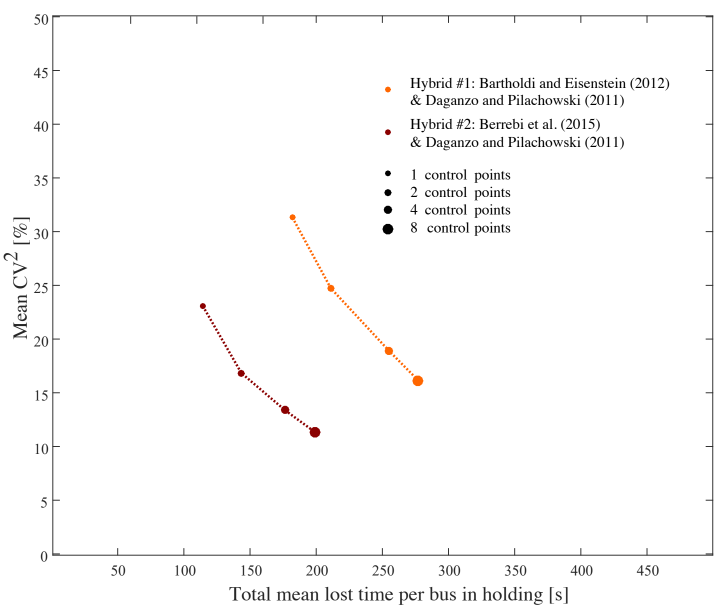



Figure 5: Mean $CV^2$ and total lost time applied throughout the route in Part (a) and applied in the second half of the route in Part (b)

The Schedule-Based Methods are similarly affected by the number of control points. The Naive Schedule and the method in Xuan et al. (2011) are unable to stabilize the route when applied sparsely. When applied at many stops (16 or 32) the methods require between 152 and 188 seconds to maintain a $CV^2$ between 20% and 25%, which is more than the methods in Daganzo and Pilachowski (2011) ~~and Berrebi et al. (2015)~~ for the same level of stability. As expected, the method in Xuan et al. (2011) yields more unstable headways but requires less holding time than the Naive Schedule method for any number of control points.

For any number of control points, the method applied in Daganzo (2009) requires much less holding time than the Naive Headway, but it also dispatches vehicles with greater $CV^2$. The method recommended in Daganzo (2009) requires the same amount of holding time as the methods in Daganzo and Pilachowski (2011) ~~and Berrebi et al. (2015)~~ but yields greater $CV^2$. When applied at 1,2, or 4 control points, the Naive Headway method yields the same level of stability as methods in Daganzo and Pilachowski (2011) ~~and Berrebi et al. (2015)~~ but requires far greater holding time. ~~The Naive Headway can exceed the method in Berrebi et al. (2015) by 4%, when applied at 8 control point, at the cost of 266 additional seconds of holding time.~~ The method recommended in Daganzo and Pilachowski (2011) yields the lowest $CV^2$ of any non-Hybrid method for any value of holding times greater than 65 and less than 398 seconds.

In Part (b) of Figure 5, the Hybrid #2 consistently yields more stable headways with less holding time than the Hybrid #1 for any number of holding points. Although, the Hybrid methods were applied starting at stop 29, whereas non-Hybrid methods were applied throughout, it is worth noting that the Hybrid #2 method yields the lowest $CV^2$ out of all holding methods for any value of holding time greater than 118 and less than 398 seconds. These results are consistent with those obtained in section 4. We expect that these results would remain consistent in a live-implementation if the two sets of methods were tested on the same route portion due to the resemblance of the first 30 stop of the route with the following 35.

Figure 5 shows that for any value of holding time less than 177 seconds, the combination of method and number of holding points yielding the lowest $CV^2$, (a) has the greatest number of holding points the method can support under the set holding time, and (b) has the lowest number of holding point of any method under the set holding time. In other words, the best compromise between headway stability and holding time is always the method with the most concentrated holding time. ~~In particular, the Hybrid #2 yields the best compromise between the two objectives but concentrates holding time in few holding points located in the second half of the route.~~ The concentration of holding time disproportionately impacts the passengers riding through the few holding points. Transit agencies should therefore be mindful of finding holding points where many passengers board and alight or selecting methods that dilute holding time, even at the cost of more holding and instability.

~~The number of control points affects the nature of the trade-off between holding time and headway stability. When applied at a single control point, the holding time is a riding cost imposed on passengers riding through. When applied at many control points, holding time can be approximated as a cost per distance. In any case, the at-stop waiting time due to uneven headways is a boarding cost for any passenger getting on the vehicle.~~

~~There is a discrepancy between Prediction-Based methods. The Hybrid # 1 yields both greater $CV^2$ and greater holding time than Partial Methods and other Prediction-Based methods for any number of control points.~~

~~and the Hybrid # 2, on the other hand, The Hybrid # 2 dispatches vehicles with more stable headways than the method in Daganzo and Pilachowski (2011) for any number of control points and requires less holding time when applied at four or eight control points.~~

## 6. Prediction

For transit agencies that wish to take advantage of bus dispatching methods using real-time predictions such as Daganzo and Pilachowski (2011), Bartholdi and Eisenstein (2011), and Berrebi et al. (2015), it is important to know what type of prediction is required and how accuracy will affect their performance. Although real-time vehicle tracking and prediction technologies are widely available among transit agencies, many of these systems were designed for passenger information rather than operational control. On these systems, using inadequate predictions could negatively impact the quality of dispatching mechanisms. Conversely, implementing a separate prediction system for control would duplicate efforts. To help transit agencies decide on the most appropriate prediction model, with respect to the performance of each holding method and to the costs of acquiring high-quality predictions, we tested the sensitivity of holding methods to the accuracy of predictions.

To evaluate the sensitivity of prediction-based holding methods to the prediction accuracy, we simulated their performance with synthetic predictions. Each time a bus arrived at the control point, a synthetic distribution of arrival times of the following buses was generated with errors on the distribution mean, $\Delta_1$, and within the distribution, $\Delta_2$. The systematic error $\Delta_1$ affects each prediction-based holding method because it biases the expected arrival time of each bus. The shape parameter on the other hand should not affect the methods in Daganzo and Pilachowski (2011) and Bartholdi and Eisenstein (2011) because they only require expected arrival times, but it may affect the method in Berrebi et al. (2015), which uses probability distributions.

Equations 10, 11, and 12 show the probability distribution of a synthetic trajectory of the $j^{\text{th}}$ bus, denoted $T[A_j]$, at time $a_i$. The distribution is centered around $a_j + \Delta_1$, which is a uniformly distributed random variable, with interval length proportional to the horizon, $a_j - a_i$, and to the accuracy indicator, $\epsilon$. The random variable $\Delta_2$ is the error of trajectories around $\Delta_1$, which is normally distributed with scale parameter proportional to the horizon and to the confidence parameter, $\sigma$. The choice of Uniform and Normal distributions with the horizon, $a_j - a_i$ as parameters is consistent with the distribution of errors on the particle filter used in Sections 4 (Hans et al., 2015). Note, however, that the random variable $\Delta_1$ is determined once for all synthetic simulated arrival times of the same bus, whereas $\Delta_2$ is re-determined for each particle.

$$T[A_j] = a_j + \Delta_1 + \Delta_2 \tag{10}$$

$$\Delta_1 \sim U[-\epsilon(a_j - a_i), \epsilon(a_j - a_i)] \tag{11}$$

$$\Delta_2 \sim N[0, \sigma(a_j - a_i)] \tag{12}$$

Figure 6 shows the sensitivity of (a) $CV^2$ and (b) mean lost time holding to the error terms, $\epsilon$ and $\sigma$. The methods recommended in Bartholdi and Eisenstein (2011) and Daganzo and Pilachowski (2011) are shown with $\sigma = 0$ because they only require $E[A_j]$. The method proposed in Berrebi et al. (2015) is shown with $\sigma = \{0, 0.2, 0.3, 0.4\}$[17]. For each value of $\epsilon$, solid lines show the case where $\sigma = 0$, i.e. all trajectories equal $a_j + \Delta_1$. These lines describe the simulated performance of each holding method that would result if transit agencies used expected arrival times instead of probability distributions. The dashed lines describe the outcome of considering uncertainty, $\sigma$, surrounding the expected synthetic trajectories, which contain error distributed with accuracy parameter, $\epsilon$.

[17]We did not include $\sigma = 0.1$ because it closely overlapped with $\sigma = 0$.

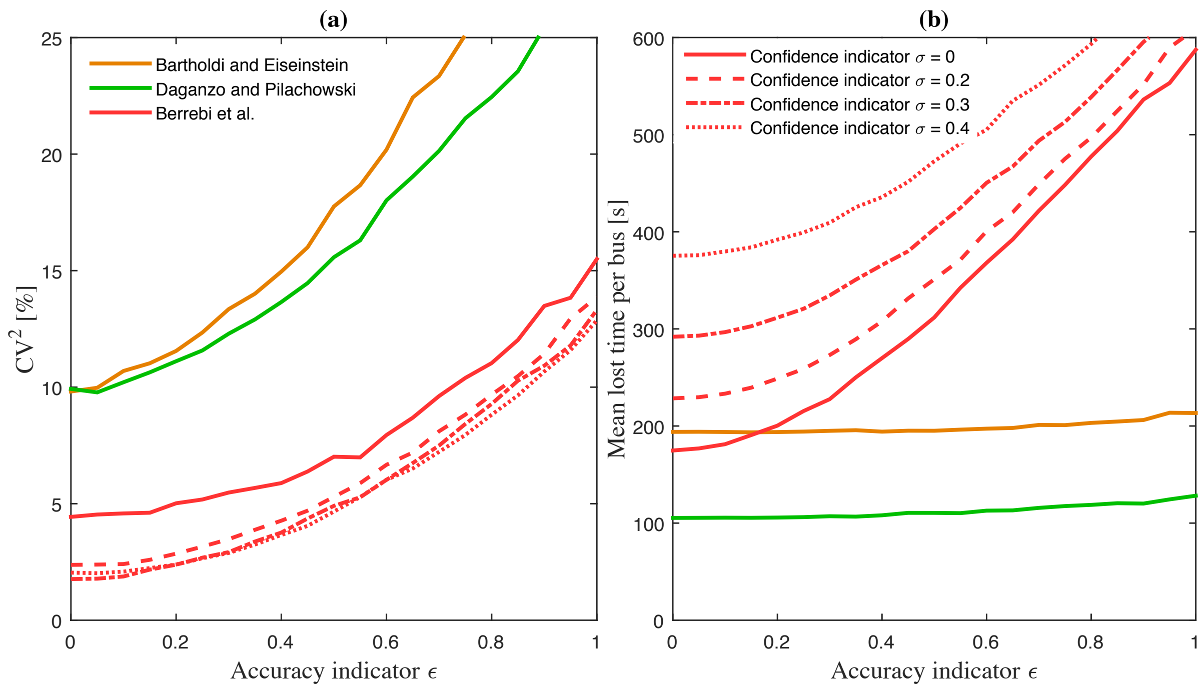



Figure 6: Sensitivity of (a) $CV^2$ and (b) Mean holding time to the maximal error in prediction

The methods recommended in Daganzo and Pilachowski (2011) and in Bartholdi and Eisenstein (2011) are affected by prediction accuracy in similar ways. When $\epsilon$ is null (perfect predictions), they dispatch buses with almost exactly the same $CV^2$. As $\epsilon$ increases, they both destabilize at a fast rate (although $CV^2$ grows at a slightly greater rate for the method in Bartholdi and Eisenstein (2011)). The error in accuracy, $\epsilon$, causes these methods to dispatch buses too soon sometimes and too late other times. The method in Daganzo and Pilachowski (2011) can dispatch buses with roughly half of the mean lost time in Bartholdi and Eisenstein (2011). The mean lost time is only slightly affected by increasing $\epsilon$ because the methods consider the expected arrival time, whose error is centered around the true mean.

For every value of $\epsilon$ and $\sigma$, the method in Berrebi et al. (2015) can dispatch buses with less than half the $CV^2$ of the other two methods, but it requires more holding time as $\epsilon$ and $\sigma$ increase. The method in Berrebi et al. (2015) can dispatch buses with slightly more stable headways when it considers the uncertainty around its prediction, $\sigma$, but considering more uncertainty requires much more holding time. The reason for this increase in lost time is that the expected maximum of random variables is a monotonous increasing function of their variability. The confidence $\sigma$ also causes holding time, especially when $\epsilon$ is small, for the same reasons. These results indicate that it may be adequate to replace the joint probability distribution of bus arrival times by their expectations to sacrifice headway stability for holding time and computational simplicity.

## 7. Conclusion

In this paper, we compared the performance of closed-form bus holding methods used in practice and recommended in the literature on Tri-Met route 72 in Portland, Oregon. We applied control at one and several control points along the route and tested how each method holds vehicles to stabilize headways, reduce passenger waiting time, and prevent bus bunching. We used a new prediction tool developed in Hans et al. (2015) to simulate the performance of real-time holding methods, which was essential to apply several holding methods studied. In addition, we coupled the methods in Bartholdi and Eisenstein (2011) and Berrebi et al. (2015) with the method in Daganzo and Pilachowski (2011) to produce hybrid methods that can be applied at several control points along the route.

In the simulation, we found the trade-offs between headway stability and holding time for each holding method The Schedule-Based methods can reduce the need for holding time but they are unable to stabilize headways. The Headway-Based methods can be parametrized and applied at several holding points to yield a wide range of holding times. The methods, however, are not competitive in terms of headway stability for any level of holding time.

We found that Prediction-Based holding methods coupled with the prediction method in Hans et al. (2015) achieved the best compromises between headway regularity and holding time on a wide range of desired trade-offs. In particular, the method in Daganzo and Pilachowski (2011) was the most effective for stops where little holding can be implemented at once and the method in Berrebi et al. (2015) was the most effective when vehicles could be held for longer periods of time. A Hybrid between the two methods, applied at one or several holding points produced the lowest headway $CV^2$ for a wide range of possible holding times. The latter analysis, however was made comparing holding methods on different route portions due to the lack of data on previous vehicle trips. In order to model the effects of cyclical vehicle trips and to compare holding methods in a more similar framework, future research should test and compare holding methods on a real bus route, and include optimization-based holding methods in the analysis.

~~In particular, a hybrid between the method from Berrebi et al. (2015) and Daganzo and Pilachowski (2011), applied at one or several holding points, achieved the best compromises between headway regularity and holding time on a wide range of desired trade-offs.~~

When evaluating the impact of prediction accuracy on the performance of holding mechanisms, we found that prediction errors increased headway instability of real-time holding methods. Inaccurate predictions substantially increase mean lost time holding for the method in Berrebi et al. (2015). Using high quality predictions such as those in Section 4 for real-time holding methods is necessary to maintain stable headways and short holding times. The uncertainty considered in the prediction for the method in Berrebi et al. (2015) helps reduce $CV^2$ at a high cost of holding time, especially for highly accurate predictions. Replacing the probability distribution of expected arrival times by their mean could help reduce the mean lost time holding.

This paper compares holding methods assuming Poisson distributed arrivals, but this assumption is not always true (Luethi et al., 2006). The performance metrics do not reflect the value that schedules offer for both passengers and operators. In addition, on certain routes, schedule or real-time information coordinated arrivals may impact the performance of holding methods compared. Future research should compare holding methods based on different passenger arrival distributions.

In this paper, TriMet Route 72 is used as a test bed for holding methods used in practice and recommended in the literature. There are routes resembling Tri-Met Route 72 in terms of passenger demand, traffic congestion, and land-use in almost every metro area in the United States. Route 72 is a typical high-frequency route (7-8 minute headways in peak hours) that faces the issue of bus bunching. We considered Route 72 as generally as possible, often applying sensitivity analyses. The analysis presented in this paper can therefore provide a basis for transit agencies to decide on a closed-form holding method on their high frequency routes. Every holding method presented here strikes a different balance between the conflicting objectives to stabilize headways and dispatch buses with little holding time.

As transit agencies look to implement innovative holding methods, further testing and simulation should be done for cases of unique passenger loading rules, severe passenger overflow, and other route characteristics that substantially deviate from TriMet Route 72. Future research should explore how route characteristics such as travel patterns, route instability, and perception of waiting time affect the desired trade-offs between these conflicting objectives.

## 8. Acknowledgment

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## Bibliography


Abkowitz, M.D., Lepofsky, M., 1990. Implementing headway-based reliability control on transit routes. Journal of Transportation Engineering 116, 49–63.

Argote-Cabanero, J., Daganzo, C.F., Lynn, J.W., 2015. Dynamic control of complex transit systems. Transportation Research Part B: Methodological 81, 146–160.

Bartholdi, J.J., Eisenstein, D.D., 2011. A self-coordinating bus route to resist bus bunching. Transportation Research Part B: Methodological .

Berkow, M., Chee, J., Bertini, R., Monsere, C., 2007. Transit Performance Measurement and Arterial Travel Time Estimation Using Archived AVL Data .

Berrebi, S., Watkins, K., Laval, J., 2015. A real-time bus dispatching policy to minimize passenger wait on a high frequency route. Transportation Research Part B: Methodological doi:10.1016/j.trb.2015.05.012.

Boyle, D., Pappas, J., Boyle, P., Nelson, B., Sharfarz, D., Benn, H., 2009. REPORT 135 Controlling System Costs : Basic and Advanced Scheduling Manuals and Contemporary Issues in Transit Scheduling. Technical Report. The National Academies Press. Washington, D.C.

Bukkapatnam, S., Dessouky, M., 2003. Distributed architecture for real-time coordination of bus holding in transit networks. IEEE Transactions on Intelligent Transportation Systems 4, 43–51. URL: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=1200899, doi:10.1109/TITS.2003.809769.

Cathey, F., Dailey, D., 2003. A prescription for transit arrival/departure prediction using automatic vehicle location data. Transportation Research Part C: Emerging Technologies 11, 241–264.

Cats, O., Larijani, A., Koutsopoulos, H., Burghout, W., 2011. Impacts of holding control strategies on transit performance: Bus simulation model analysis. Transportation Research Record: Journal of the Transportation Research Board , 51–58.

Chen, M., Liu, X., Xia, J., 2005. Dynamic Prediction Method with Schedule Recovery Impact for Bus Arrival Time. Transportation Research Record 1923, 208–217. doi:10.3141/1923-22.

Chien, S.I.J., Ding, Y., Wei, C., 2002. Dynamic bus arrival time prediction with artificial neural networks. Journal of Transportation Engineering 128, 429–438.

Cortés, C.E., Sáez, D., Milla, F., Núñez, A., Riquelme, M., 2010. Hybrid predictive control for real-time optimization of public transport systems' operations based on evolutionary multi-objective optimization. Transportation Research Part C: Emerging Technologies 18, 757–769.

Daganzo, C.F., 2009. A headway-based approach to eliminate bus bunching: Systematic analysis and comparisons. Transportation Research Part B: Methodological 43, 913–921.

Daganzo, C.F., Pilachowski, J., 2011. Reducing bunching with bus-to-bus cooperation. Transportation Research Part B: Methodological 45, 267–277. doi:10.1016/j.trb.2010.06.005.

Delgado, F., Muñoz, J.C., Giesen, R., Cipriano, A., 2009. Real-Time Control of Buses in a Transit Corridor Based on Vehicle Holding and Boarding Limits. Transportation Research Record: Journal of the Transportation Research Board 2090, 59–67.

Delgado, F., Munoz, J.C., Giesen, R., 2012. How much can holding and/or limiting boarding improve transit performance? Transportation Research Part B: Methodological 46, 1202–1217.

Eberlein, X.J., Wilson, N.H., Bernstein, D., 2001. The holding problem with real–time information available. Transportation science 35, 1–18.

Fan, W.D., Machemehl, R.B., 2009. Do transit users just wait for buses or wait with strategies? Transportation Research Record: Journal of the Transportation Research Board 2111, 169–176.

Furth, P., Hemily, B., Muller, T., Strathman, J., 2006. TCRP Report 113: Using Archived AVL-APC Data to Improve Transit Performance and Management. Technical Report. The National Academies Press. Washington, D.C.

Furth, P.G., Wilson, N.H., 1981. Setting frequencies on bus routes: theory and practice. Transportation Research Record 818, 1–7.

Grisby, D., 2013. APTA Surveys Transit Agencies on Providing Information and RealTime Arrivals to Customers , 1–13.

Gurmu, Z.K., Fan, W.D., 2014. Dynamic travel time prediction models for buses using only gps data, in: Transportation Research Board 93rd Annual Meeting.

Hans, E., Chiabaut, N., Leclercq, L., 2014. Investigating the irregularities of bus routes. Journal of Advanced Transportation 45, 117–128. doi:10.1002/atr.

Hans, E., Chiabaut, N., Leclercq, L., Bertini, R., 2015. Real-time bus route state forecasting using particle filter. Transportation Research Part C: Emerging Technologies .

Hickman, M.D., 2001. An Analytic Stochastic Model for the Transit Vehicle Holding Problem. Transportation Science 35, 215–237.

Jeong, R., Rilett, L., 2005. Prediction Model of Bus Arrival Time for Real-Time Applications. Transportation Research Record 1927, 195–204. doi:10.3141/1927-23.

Jeong, R., Rilett, L.R., 2004. Artificial Neural Network Model. IEEE Intelligent Transportation Systems Conference , 988–993.

Kittelson, 2003. Transit capacity and quality of service manual. volume 100. Transportation Research Board.

Luethi, M., Weidmann, U., Nash, A., Weidmann, U., Weidmann, U., 2006. Passenger arrival rates at public transport stations. Institute for Transport Planning and Systems, ETH Zurich.

Mazloumi, E., Rose, G., Currie, G., Moridpour, S., 2011. Prediction intervals to account for uncertainties in neural network predictions: Methodology and application in bus travel time prediction. Engineering Applications of Artificial Intelligence 24, 534–542. doi:10.1016/j.engappai.2010.11.004.

Milkovits, M.N., 2008. Simulating Service Reliability of a High Frequency Bus Route Using Automatically Collected Data by Master of Science in Transportation .

Newell, F., Potts, R.B., 1964. Maintaining a bus schedule. volume 2.

Newell, G., 1974. Control of Pairing Vehicles on a public Transportation Route, Two Vehicles, One Control Point .

Osuana, E., Newell, G., 1972. Control Strategies for an Idealized Public Transportation System , 52–72.

Sánchez-Martínez, G., Koutsopoulos, H., Wilson, N., 2016. Real-time holding control for high-frequency transit with dynamics. Transportation Research Part B: Methodological 83, 1–19.

Sánchez-Martínez, G.E., Koutsopoulos, H.N., Wilson, N.H., 2015. Event-driven holding control for high-frequency transit. Transportation Research Record: Journal of the Transportation Research Board , 65–72.

Shalaby, A., Farhan, A., 2004. Prediction model of bus arrival and departure times using avl and apc data. Journal of Public Transportation 7, 41–62.

Sun, D., Luo, H., Fu, L., Liu, W., Liao, X., Zhao, M., 2007. Predicting Bus Arrival Time on the Basis of Global Positioning System Data. Transportation Research Record 2034, 62–72.

TriMet, 2016. Line 72 route map. URL: `http://trimet.org/schedules/r072.htm#map`.

Van Oort, N., Wilson, N.H., Van Nes, R., 2010. Reliability improvement in short headway transit services. Transportation Research Record: Journal of the Transportation Research Board 2143, 67–76.

Xuan, Y., Argote, J., Daganzo, C.F., 2011. Dynamic bus holding strategies for schedule reliability: Optimal linear control and performance analysis. Transportation Research Part B: Methodological 45, 1831–1845.

Zolfaghari, S., Azizi, N., Jaber, M.Y., 2004. A model for holding strategy in public transit systems with real-time information. International Journal of Transport Management 2, 99–110.