

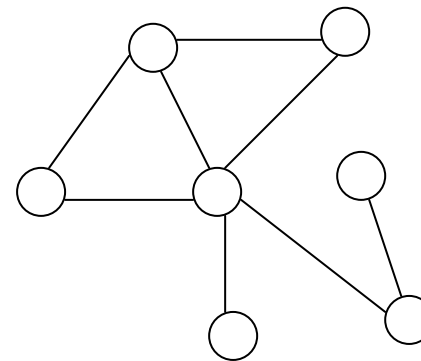
## Data Model

- Set of concepts and constructs used to describe and organize data and their relationships
- Basic feature: **structuring mechanism** (also: type constructor) as in programming languages
- *Example*: in the relational DB model, **relation** constructor organizes data as sets of homogeneous (same type) records
- Two main types of data model:
  - **Logical models**: used for organization of data at a level that abstracts from physical structures  
Examples: relational, network, hierarchical (traditional ones), object (more recent)
  - **Conceptual models**: used to describe data in a way that is completely independent of any system, with the goal of representing the concepts of the real world; used in the early stages of DB design  
Most popular: Entity-Relationship model

## Network Data Model

### Characteristics:

- Data represented as collection of *records*
- Binary relationships represented as *links* (also called *sets*, and implemented as pointers)
- The model is represented by means of graph structures where:
  - Nodes=records
  - Edges=links



## Relational Model

Characteristics:

- Data and relationships represented as values (relations)
- No explicit references, i.e., pointers as in the network model

=> higher level representation, while network model is closer to the physical structure of the DB

**EXAMPLE: A Relational Database****STUDENTS**

<b>RegNum</b>	<b>Surname</b>	<b>FirstName</b>	<b>BirthDate</b>
276545	Smith	Mary	25/11/1990
485745	Black	Anna	23/04/1991
200768	Verdi	Paolo	12/02/1991
587614	Smith	Lucy	10/10/1990
937653	Brown	Mavis	01/12/1990

**EXAMS**

<b>Student</b>	<b>Grade</b>	<b>Course</b>
276545	C	01
276545	B	04
937653	B	01
200768	B	04

**COURSES**

<b>Code</b>	<b>Title</b>	<b>Tutor</b>
01	Physics	Grant
03	Chemistry	Beale
04	Chemistry	Clark

## EXAMPLE: A Network Database

### STUDENTS

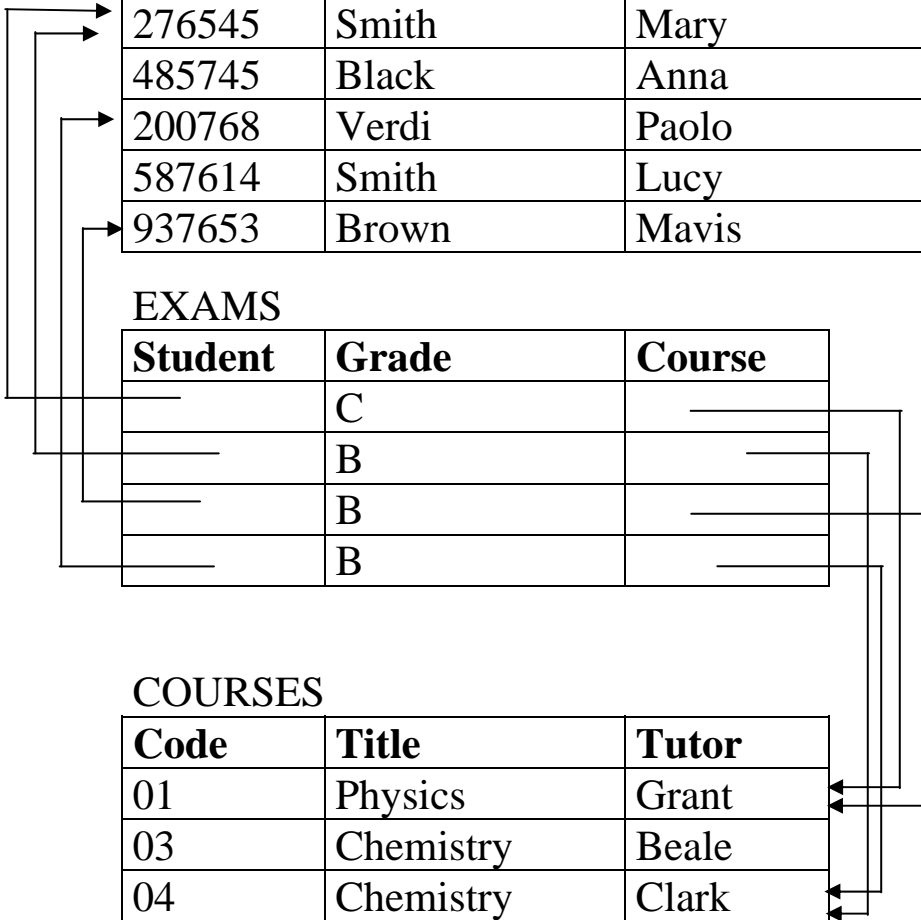
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### EXAMS

Student	Grade	Course
_____	C	_____
_____	B	_____
_____	B	_____
_____	B	_____

### COURSES

Code	Title	Tutor
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## Object-Oriented Data Model

### Characteristics:

- Newer model: based on objects, classes, etc.
- Attributes: describe the state of an object
- Methods (also: actions) describe the behaviour of an object
- The object encapsulates both state and behaviour
- Development of OODBMS: still research topic (ODMG: Object Database Management Group)
- No universally agreed data model

## Relational Model

- Proposed by E. F. Codd in 1970 in order to support data independence
- Used in almost all commercial DBMS since 1981
- It provides simple and declarative languages that are powerful and allow to express operations for access and manipulation of data
- It is based on the mathematical concept of **relation**;  
theoretical basis that allows to formally prove properties of data and operations

## Relational Model

- **Relation:** subset of the **Cartesian product** of a list of **domains**
  - **Domain:** a set (possibly infinite) of values;  
examples:
    - the set of integers is a domain;
    - the set of strings of characters with length=20 is a domain
    - $\{0,1\}$  is a domain
  - Let  $D_1, D_2, \dots, D_k$  be domains. The Cartesian product of such domains, denoted by
- is the set

$$D_1 \times D_2 \times \dots \times D_k$$
$$\{(v_1, v_2, \dots, v_k) \mid v_1 \in D_1, v_2 \in D_2, \dots, v_k \in D_k\}$$



## Relational Model

- Example:

let:  $k = 2$ ,  $D1 = \{0,1\}$ , and  $D2 = \{a,b,c\}$

$$D1 \times D2 = \{(0,a), (0,b), (0,c), (1,a), (1,b), (1,c)\}$$

- a **relation** is any subset of the Cartesian product of one or several domains. Example:

$\{(0,a), (0,c), (1,b)\}$  is a relation

$\{(1,b), (1,c)\}$  is a relation

- elements of a relation are called **tuples**.

With reference to previous example  $(0,a), (0,c), (1,b), (1,c)$  are tuples

- a relation that is the subset of a Cartesian product of  $k$  domains is said to have **degree**  $k$ .  
With reference to previous example: relations have degree 2

## Relational Model

- every tuple of a relation with degree  $k$  has  $k$  components. With reference to previous example: tuples have 2 components
- let  $r$  be a relation with degree  $k$ ;
  - let  $t$  be a tuple of  $r$
  - let  $i$  be an integer in  $\{1, \dots, k\}$
  - $t[i]$  is the  $i$ -th component of  $t$

Example:      let  $r = \{(0,a), (0,c), (1,b)\}$   
                   $t = (0,a)$  is a tuple of  $r$   
                   $t[2] = a$   
                   $t[1] = 0$

- the **cardinality** of a relation is the number of tuples belonging to the relation.

Example:      relation  $\{(0,a), (0,c), (1,b)\}$  has cardinality 3.

## Relational Model

Alternative (simpler) definition

- A relation can be seen as a table in which each row is a tuple and each column corresponds to a component
- In this definition, columns have associated names, called **attribute names**
  - the pair (attribute name, domain) is called an **attribute**
- The set of attributes of a relation is called **schema**
- If a relation has name ***R*** and attribute names  $A_1, A_2, \dots, A_k$ , the schema is often indicated by
$$\mathbf{R}(A_1, A_2, \dots, A_k)$$
- $UR = \{A_1, A_2, \dots, A_k\}$  is used to denote the set of all attribute names of relation ***R***

Example:

Relation ***Info\_City***

<i>City</i>	<i>Region</i>	<i>Population</i>
Roma	Lazio	3,000,000
Milano	Lombardia	1,500,000
Genova	Liguria	800,000
Pisa	Toscana	150,000

schema ***Info\_City***(*City*, *Region*, *Population*)

## Relational Model

An alternative (simpler) definition

- in this definition of the relational model, components of tuples are indicated by attribute names  
(notation by name vs notation by position)
- let  $R(A_1, A_2, \dots, A_k)$  be a relation schema, a tuple  $t$  on such a schema can be represented by the notation:

$[A_1 : v_1, A_2 : v_2, \dots, A_k : v_k]$       or by       $(v_1, v_2, \dots, v_k)$

where  $v_i$  is a value belonging to the domain of  $A_i$  (denoted  $dom(A_i)$ ) for  $i=1, \dots, k$

$t[A_i]$  indicates the value of the attribute named  $A_i$  of tuple  $t$

- Example:

$t = [\text{City} : \text{Roma}, \text{Region} : \text{Lazio}, \text{Population} : 3,000,000]$  or  $t = (\text{Roma}, \text{Lazio}, 3,000,000)$

is a tuple defined on schema **Info\_City**(City, Region, Population)

$t[\text{City}] = \text{Roma}$

## Relational Value

### Null Values

- sometimes no information is available on some components of entities represented in the DB  
i.e., no value is known for some attributes of some tuples
- special value (**null value**) denotes no value  
[often denoted '?']

## Relational Model: Key

- The **key** of a relation is the set of attributes that uniquely identifies tuples of the relation
- More precisely, a set  $X$  of attributes of a relation  $R$ , is a *key* of  $R$  if it satisfies the following properties:
  1. for each status of  $R$ , no pair of distinct tuples  $t'$  and  $t''$  exist in  $R$  such that  $t'$  and  $t''$  have same value for all attributes in  $X$ ;
  2. no proper subset (\*) of  $X$  satisfies property (1).
- In the previous example:  
     $\text{key}(\text{Info\_City}) = \{\text{City}\}$   
    *there cannot be multiple cities with same name*  
  
     $\text{key}(\text{Info\_City}) = \{\text{City}, \text{Region}\}$   
    *different cities with same name can exist but only in different regions*

-----  
(\*)  $S'$  is a proper subset of  $S$ , if it is a subset of  $S$  and  $S' \neq S$ .



## Relational Model: Key

- A key cannot have null values
- There can be more than one set  $X$  in a relation that satisfies the two properties (several possible keys)
- Sometimes it is necessary to choose one key if the system does not support multiple keys.
- **Primary key** is the selected key
- A possible selection criterion is to choose the key most frequently used in queries
- Another criterion: choose the key with least number of attributes

## Relational Model: Foreign Key

- Let  $R$  and  $S$  be two relations such that
  - $R$  has a set of attributes  $X$ ;
  - $S$  has a set  $Y$  of attributes as key;

$Y$  is **foreign key** of  $R$  on  $S$  if  $Y$  is a subset of  $X$

- In other words, if  $R$  has among its attributes a set  $Y$  of attributes that is key of a relation  $S$ , we say that  $Y$  is a **foreign** key of  $R$  on  $S$
- $S$  is said **referenced relation**
- Foreign keys allow to link tuples of different relations and provide a mechanism to model associations between entities
- A tuple  $t$  that references another tuple  $t'$  includes, among its attributes, one or more attributes whose value is the value of the key of  $t'$

### Relational Model: Example

We define two relations that contain information about employees of a company and the departments in which the company is organized

**Employees (Emp#, Name, Job, Start\_Date, Salary, Bonus, Dept#)**

**key(Employees) = {Emp#}**

**foreign-key(Employees) = {Dept#}**

**(referenced relation: Departments)**

**Departments (Dept#, Name\_Dept, Office#, Division#, Manager)**

**key (Departments) = {Dept#}**

### Example

#### Employees

<b>Emp#</b>	<b>Name</b>	<b>Job</b>	<b>Start_Date</b>	<b>Salary</b>	<b>Bonus</b>	<b>Dept#</b>
7369	Rossi	engineer	17-Dec-90	1600,00	500,00	20
7499	Andrei	technician	20-Feb-91	800,00	?	30
7521	Bianchi	technician	20-Feb-91	800,00	100,00	30
7566	Rosi	manager	02-Apr-91	2975,00	?	20
7654	Martini	secretary	28-Sep-91	800,00	?	30
7698	Blacchi	manager	01-May-91	2850,00	?	30
7782	Neri	engineer	01-Jun-91	2450,00	200,00	10
7788	Scotti	secretary	09-Nov-91	800,00	?	20
7839	Dare	engineer	17-Nov-91	2600,00	300,00	10
7844	Turni	technician	08-Sep-91	1500,00	?	30
7876	Adami	engineer	28-Sep-91	1100,00	500,00	20
7900	Gianni	engineer	03-Dec-91	1950,00	?	30
7902	Fordi	secretary	03-Dec-91	1000,00	?	20
7934	Milli	engineer	23-Jan-92	1300,00	150,00	10
7977	Verdi	manager	10-Dec-90	3000,00	?	10

#### Departments

<b>Dept#</b>	<b>Name_Dept</b>	<b>Office</b>	<b>Division</b>	<b>Manager</b>
10	Civil Engineering	1100	D1	7977
20	R&D	2200	D1	7566
30	Surveying	5100	D2	7698

## Relational Model: Referential Integrity Constraints

- imposed to guarantee that values refer to actual values in the referenced relation
- if a tuple  $t$  references  $v_1, \dots, v_n$  as values of a foreign key, there must be a tuple  $t'$  in the referenced relation with key values  $v_1, \dots, v_n$
- relations Employees and Departments verify this property
- consider the following tuple and assume it is inserted in relation Employees

[Emp#: 7899, Name: Smith, Job: technician,  
Start\_Date\_A:03-Dec-91, Salary:2000,  
Bonus: 100, Dept#: 50]

this tuple violates referential integrity as there is no department in relation Departments with Dept# = 50

- DB languages (SQL) allow the user to specify for which relations and attributes it is necessary to preserve referential integrity (and what to do when there is violation)

## Query Languages for Relational DB

- Operations on DB:
  1. queries: read from the DB
  2. updates: change the content of the DB
- Both types of operations can be modeled as functions from DB to DB
- Formalization with reference to query languages:
  - *relational algebra*: a “procedural” language
  - *relational calculus*: a “declarative” language
- Later, we will see SQL: practical language for queries and updates

## Operations in Relational Model

Two basic formalisms

- 1) **Relational Algebra:** queries are expressed by applying operators to relations
- 2) **Relational Calculus:** queries are expressed by means of logical formulas that must be satisfied by the tuples obtained as result of the query

Theoretical Result: the two formalisms have same expressive power (under certain assumptions).

## Relational Algebra

- 5 basic operations:
  - *union*
  - *difference*
  - *Cartesian product*
  - *projection*
  - *selection*
- these operations completely define relational algebra
- every operation returns a relation as result; it is then possible to apply an operation to the result of another operation (closure property)
- there are additional operations that can be expressed in terms of the 5 basic operations
- these operations do not add expressive power to the set of basic operations but they are useful shortcuts and they are called *derived* operations
- the most important derived operation: *join*
- *renaming*: to modify names of attributes



## Union

- Union of two relations R and S, indicated  $R \cup S$ :

set of tuples that are in R, or in S, or in both

- Union of two relations is possible only if the two relations have same degree; also: the first attribute of R must be compatible with the first attribute of S, the second attribute of R must be compatible with the second attribute of S and so on.
- if the two relations have different attribute names, in the returned relation by convention the names from the first relation (in this case R) are used, unless renaming is applied
- duplicate tuples are eliminated
- the degree of the returned relation is the same as the degree of the two original relations

## Union

### Example

A	B	C
a	b	c
d	a	f
c	b	d

relation R

D	E	F
b	g	a
d	a	f

relation S

A	B	C
a	b	c
d	a	f
c	b	d
b	g	a

$R \cup S$

## Difference

- Difference of two relations R and S, indicated  $R - S$ :

set of tuples that are in R, but not in S

- difference (like union) of two relations is possible only if the two relations have same degree and attributes are compatible
- if the two relations have different attribute names, in the returned relation by convention the names from the first relation (in this case R) are used, unless renaming is applied
- the degree of the returned relation is the same as the degree of the two original relations

## Difference

### Example

A	B	C
a	b	c
d	a	f
c	b	d

relation R

D	E	F
b	g	a
d	a	f

relation S

A	B	C
a	b	c
c	b	d

$R - S$

## Cartesian Product

- Cartesian product of two relations R and S, with degree  $k_1$  and  $k_2$ , respectively, indicated

$$R \times S$$

is a relation with degree  $k_1 + k_2$  composed of all possible tuples such that:

- their first  $k_1$  components are tuples of R, and
  - their last  $k_2$  components are tuples of S
- 
- in the returned relation, the names of the first  $k_1$  attributes are the names of attributes of relation R and the names of the last  $k_2$  attributes are the names of the attributes of relation S
- 
- if the two relations have attributes with same name it is necessary to rename those attributes in one of the two relations (more on renaming later)

## Example

<u>A</u>	<u>B</u>	<u>C</u>
a	b	c
d	a	f
c	b	d

relation R

<u>D</u>	<u>E</u>	<u>F</u>
b	g	a
d	a	f

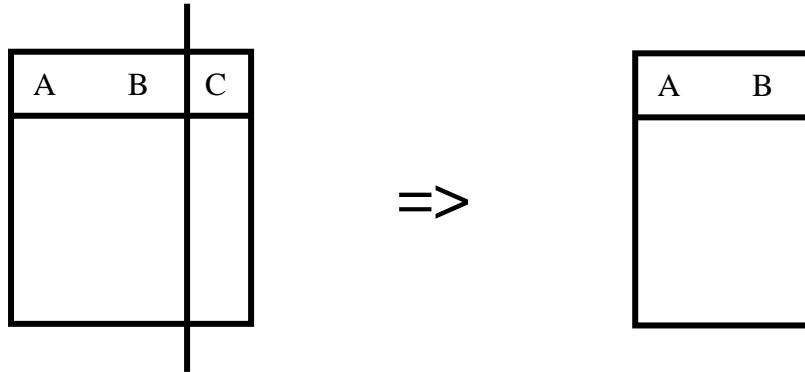
relation S

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>	<u>F</u>
a	b	c	b	g	a
a	b	c	d	a	f
d	a	f	b	g	a
d	a	f	d	a	f
c	b	d	b	g	a
c	b	d	d	a	f

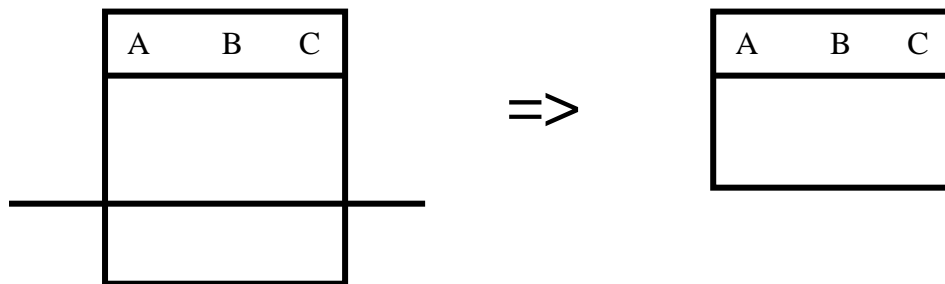
R X S

## Projection and Selection

Projection = vertical decomposition



Selection = horizontal decomposition



## Projection

- projection of a relation  $R$  on a set  $A=\{A_1, A_2, \dots, A_m\}$  of attributes, indicated

$$\Pi_{A_1, A_2, \dots, A_m}(R)$$

is a relation of degree  $m$  whose tuples have only attributes specified in  $A$

- projection operation generates a set  $T$  of  $m$ -tuples (i.e., tuples with  $m$  attributes)

let  $t = [A_1:v_1, A_2:v_2, \dots, A_m:v_m]$  be a  $m$ -tuple in  $T$

$t$  is such that there exists a tuple  $t'$  in  $R$  such that:

$$\forall A_j \in A \quad t[A_j] = t'[A_j]$$

- projection generates, from a given relation, a relation containing only a subset of attributes
- in the returned relation attributes are ordered according to the order specified in  $A$



## Example

<u>A</u>	<u>B</u>	<u>C</u>
a	b	c
d	a	f
c	b	d

Relation R

<u>A</u>	<u>C</u>
a	c
d	f
c	d

$\Pi_{A,C}(R)$

<u>B</u>	<u>A</u>
b	a
a	d
b	c

$\Pi_{B,A}(R)$

## Selection: predicates

- a predicate  $F$  on a relation can be one of the following:
  - simple predicate
  - Boolean combination of simple predicates by means of logical connectives  
 $\wedge$  (AND),  $\vee$  (OR),  $\neg$  (NOT)

- a *simple predicate* can be
  - (i)  $A \text{ op } \text{constant}$
  - (ii)  $A \text{ op } A'$

where  $A$  and  $A'$  are attributes of  $R$ ;

$op$  is a comparison operator:  $<$ ,  $>$ ,  $\leq$ ,  $\geq$ ,  $=$ , etc.

*constant* is a constant value compatible with the domain of  $A$

- examples:

$B=b$	simple predicate (i)
$A=C$	simple predicate (ii)
$B=b \vee A=C$	Boolean combination
$B=b \wedge A=C$	Boolean combination
$\neg B=b$	Boolean combination

## Selection

- Selection on a relation  $R$ , given a predicate  $F$ , indicated  $\sigma_F(R)$   
is a relation that contains all tuples satisfying predicate  $F$
- the degree of the returned relation is the same as the degree of the original relation; the names of its attributes are the same as the name of the original relation
- if no tuple of  $R$  satisfies  $F$ , the result is an empty relation (indicated 0 or  $\emptyset$ )
- if  $k$  is the degree of  $R$ , selection generates a set  $T$  of  $k$ -tuples

let  $t = [A_1:v_1, A_2:v_2, \dots, A_k:v_k]$  be a  $k$ -tuple in  $T$   
 $t$  is such that:

$F(A_1/t[A_1], A_2/t[A_2], \dots, A_k/t[A_k])$  is true,

where  $A_i/t[A_i]$ ,  $i=1, \dots, k$

denotes the substitution in  $F$  of the name of attribute  $A_i$  (if such name appears in  $F$ ) with the value of the attributes named  $A_i$  in  $t$

## Example

A	B	C
a	b	c
d	a	f
c	b	d

relation R

A	B	C
a	b	c
c	b	d

$\sigma_{B=b}(R)$

A	B	C
d	a	f

$\sigma_{\neg(B=b)}(R)$

A	B	C
a	b	c
c	b	d

$\sigma_{B=b \vee A=C}(R)$

$\sigma_{B=b \wedge A=C}(R) = \emptyset$

### Example

#### Employees

<b>Emp#</b>	<b>Name</b>	<b>Job</b>	<b>Start_Date</b>	<b>Salary</b>	<b>Bonus</b>	<b>Dept#</b>
7369	Rossi	engineer	17-Dec-90	1600,00	500,00	20
7499	Andrei	technician	20-Feb-91	800,00	?	30
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7977	Verdi	manager	10-Dec-90	3000,00	?	10

#### Departments

<b>Dept#</b>	<b>Name_Dept</b>	<b>Office</b>	<b>Division</b>	<b>Manager</b>
10	Civil Engineering	1100	D1	7977
20	R&D	2200	D1	7566
30	Surveying	5100	D2	7698

## EXAMPLES

- **Q1: find the name of employees that have salary greater than 2000**

$\Pi_{\text{Name}}(\sigma_{\text{Salary} > 2000}(\text{Employees}))$

<u>Name</u>
Rosi
Blacchi
Neri
Dare
Verdi

- **Q2: find the name and numbers of department of employees that are engineers and have salary greater than 2000**

$\Pi_{\text{Name}, \text{Dept\#}}(\sigma_{\text{Salary} > 2000 \wedge \text{Job} = \text{'engineer'}}(\text{Employees}))$

<u>Name</u>	<u>Dep#</u>
Neri	10
Dare	10

- **Q3: find the employee number of employees that:** (a) work in department 30 and (b) are engineers or technicians

$\Pi_{\text{Emp\#}}(\sigma_{\text{Dept\#=30} \wedge (\text{Job= 'engineer'} \vee \text{Job= 'technician'})}(\text{Employees}))$

<u>Emp#</u>
7499
7521
7844
7900

## Renaming

Renaming of a relation R with respect to a list of pairs of names of attributes

$(A_1, B_1), (A_2, B_2), \dots, (A_m, B_m)$

such that  $A_i (i=1, \dots, m)$  is a name of an attribute in R, is denoted

$$\rho_{A_1, A_2, \dots, A_m \leftarrow B_1, B_2, \dots, B_m}(R)$$

and renames attribute named  $A_i (i=1, \dots, m)$  with name  $B_i$

Renaming is correct if the attributes of the new schema of relation R all have distinct names

*Example:*

$R(A, B, C)$

$$\rho_{A, B, C \leftarrow AA, BB, CC}(R)$$

modifies the schema of relation R to  $R(AA, BB, CC)$



## Basic Operations: Semantics

Let  $R = (A_1, \dots, A_k)$  be a relation schema, where  $A_i$  is a name of an attribute with domain  $S_i$ , with  $i = 1 \dots k$ .

We indicate  $\mathfrak{R}(R)$  the set of all relations on that schema

- $\_ \cup \_ : \mathfrak{R}(R) \times \mathfrak{R}(R) \rightarrow \mathfrak{R}(R)$   
 $r_1 \cup r_2 = \{t \mid t \in r_1 \vee t \in r_2\}$
- $\_ - \_ : \mathfrak{R}(R) \times \mathfrak{R}(R) \rightarrow \mathfrak{R}(R)$   
 $r_1 - r_2 = \{t \mid t \in r_1, t \notin r_2\}$
- $\_ \times \_ : \mathfrak{R}(R_1) \times \mathfrak{R}(R_2) \rightarrow \mathfrak{R}(R_1 \cdot R_2)$   
 $r_1 \times r_2 = \{t_1 \cdot t_2 \mid t_1 \in r_1, t_2 \in r_2\}$

- $\pi_{R'}_- : \mathfrak{R}(R) \rightarrow \mathfrak{R}(R')$  with  $R \supset R'$   
 $\pi_{R'}(r) = \{t[R'] \mid t \in r\}$
- $\sigma_{F_-} : \mathfrak{R}(R) \rightarrow \mathfrak{R}(R)$   
 $\sigma_F(r) = \{t \mid t \in r, F(t)\}$

### Derived operations : Join

- join of two relations R and S on attributes A of R and A' of S, indicated

$$R \bowtie_{A \theta A'} S$$

is defined as  $\sigma_{A \theta A'} (R \times S)$

- join is a Cartesian product followed by a selection;  $A \theta A'$  is called *join predicate*
- the degree of the resulting relation is the sum of the degrees of the original relations

## Examples

<u>A</u>	<u>B</u>	<u>C</u>
1	2	3
4	5	6
7	8	9

relation R

<u>D</u>	<u>E</u>
3	1
6	2

relation S

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>
1	2	3	3	1

$R \bowtie S$   
A=E

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>
1	2	3	3	1
1	2	3	6	2
4	5	6	6	2

$R \bowtie S$   
B<D

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>
1	2	3	3	1
1	2	3	6	2
4	5	6	3	1
4	5	6	6	2
7	8	9	3	1
7	8	9	6	2

R X S

<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>
1	2	3	3	1

$\sigma_{A=E} (R \times S)$

$R \bowtie_{A=E} S$

## Natural Join

- Natural join is a particular case of join
- Example: “find the name of all employees and the office in which they work”

We can express this query by joining Employees and Departments based on the predicate:

$\text{Employees.Dept\#} = \text{Departments.Dept\#}$

- this particular case of join is based on the equality of all attributes common to the two relations
- joins based on equality of attributes are very frequently used
- in this case we can omit the predicate

## Natural Join

We can express the previous query as:  $\Pi_{\text{Name, Office}} (\text{Employees} \bowtie \text{Departments})$

### Definition

- let  $R$  and  $S$  be relations
- let  $\{A_1, A_2, \dots, A_k\} = U_R \cap U_S$  be the set of attributes common both to the schema of  $R$  and the schema of  $S$
- let  $\{I_1, I_2, \dots, I_m\} = U_R \cup U_S$  be the union of attributes in the schema of  $R$  and in the schema of  $S$

the expression that defines natural join is

$$R \bowtie S = \Pi_{I_1, I_2, \dots, I_m} (\sigma_C (R \times (\rho_{A_1, A_2, \dots, A_k \leftarrow S.A_1, S.A_2, \dots, S.A_k} (S))))$$

where  $C$  is a predicate  $A_1=S.A_1 \text{ AND } A_2=S.A_2 \text{ AND } \dots \text{ AND } A_k=S.A_k$

- natural join performs a join based on the equality of attributes common to the two relations and then eliminates all duplicate attributes ie. in our example only one of the columns Dept# appears in the result (and there is no need to use Renaming)





## Derived Operations: Semantics

Let  $R = (A_1, \dots, A_k)$  be a relation schema, with  $A_i$  name of attribute with domain  $S_i$ ,  $i = 1 \dots k$ .

We indicate with  $\mathfrak{R}(R)$  the set of all relations on such schema

- $\_ \cap \_ : \mathfrak{R}(R) \times \mathfrak{R}(R) \rightarrow \mathfrak{R}(R)$   

$$r_1 \cap r_2 = r_1 - (r_1 - r_2) = \{t \mid t \in r_1, t \in r_2\}$$
- $\_ \bowtie_F \_ : \mathfrak{R}(R_1) \times \mathfrak{R}(R_2) \rightarrow \mathfrak{R}(R_1 \cdot R_2)$   

$$r_1 \bowtie_F r_2 = \sigma_F(r_1 \times r_2) =$$

$$\{t_1 \cdot t_2 \mid t_1 \in r_1, t_2 \in r_2, F(t_1, t_2)\}$$

- $\_ \bowtie \_ : \mathfrak{R}(R1) \times \mathfrak{R}(R2) \rightarrow \mathfrak{R}(R1 \cup R2)$   
 $r1 \bowtie r2 =$   
 $\{t \mid t[R1] \in r1, t[R2] \in r2\}$   
  
- if  $R1 \cap R2 = \emptyset$      $r1 \bowtie r2 = r1 \times r2$   
- if  $R1 = R2$           $r1 \bowtie r2 = r1 \cap r2$

## Relational Calculus

- Relational Algebra is a "procedural" language: to specify an algebraic expression, we indicate operations that must be performed to generate the query result
- Relational Calculus: we provide a formal description of the result without specifying how to obtain it ("declarative" language)
- two alternatives:
  - tuple relational calculus (TRC) = variables represent tuples (we study this version)
  - domain relational calculus (DRC) = variables represent domains

## Relational Calculus

In TRC a query is an expression:

$$\{t: U \mid P(t)\}$$

ie. It is defined as the set of tuples  $t$  on a set  $U$  of attributes such that  $t$  satisfies predicate  $P$

Notation  $t[A]$  indicates the value of attribute  $A$  in  $t$

(example:  $t[\text{Name}]$ )

$t \in R$  indicates that tuple  $t$  is in relation  $R$

### Examples:

- **find all employees whose salary is greater than 2000**

$$\{t: U_{\text{Employees}} \mid t \in \text{Employees} \wedge t[\text{Salary}] > 2000\}$$

- **find the name of all employees whose salary is greater than 2000**

$$\{t: \{\text{Name}\} \mid (\exists s) (s \in \text{Employees} \wedge s[\text{Salary}] > 2000 \wedge s[\text{Name}] = t[\text{Name}])\}$$

t represents a variable that indicates tuples belonging to a relation with schema = {Name}  
 notation  $(\exists t)(Q(t))$  indicates that there exists a tuple t such that Q(t) is true

- **find names and offices of employees whose salary is greater than 2000**

$$\{t: \{\text{Name}, \text{Office}\} \mid (\exists s) (s \in \text{Employees} \wedge s[\text{Salary}] > 2000 \wedge s[\text{Name}] = t[\text{Name}] \wedge (\exists u) (u \in \text{Departments} \wedge s[\text{Dept\#}] = u[\text{Dept\#}] \wedge u[\text{Office}] = t[\text{Office}]))\}$$

- **find names of employees that either have a salary greater than 2000 or work in a department of division D1**

$$\{t: \{\text{Name}\} \mid (\exists s) (s \in \text{Employees} \wedge s[\text{Name}] = t[\text{Name}] \wedge (s[\text{Salary}] > 2000 \vee (\exists u) (u \in \text{Departments} \wedge s[\text{Dept\#}] = u[\text{Dept\#}] \wedge u[\text{Division}] = \text{"D1"}))))\}$$

## Relational Calculus

Operations of relational algebra are expressed as:

- $R \cup S$                       **Union**  
 $\{t : U_R \mid t \in R \vee t \in S\}$
- $R - S$                       **Difference**  
 $\{t : U_R \mid t \in R \wedge t \notin S\}$

- $R \times S$  **Cartesian Product**

with  $U_R = \{A1, A2, \dots, A_n\}$   
 $U_S = \{A1', A2', \dots, A_{m'}\}$

$$\{t: \{U_R \cup U_S\} \mid (\exists x) (\exists y) (x \in R \wedge y \in S \wedge$$

$$x[A1] = t[A1] \wedge x[A2] = t[A2] \wedge \dots \wedge x[An] = t[An] \wedge$$

$$y[A1'] = t[A1'] \wedge y[A2'] = t[A2'] \wedge \dots \wedge y[Am'] = t[Am'])\}$$

## Relational Calculus

- $\Pi_{A_1, A_2, \dots, A_k}(R)$  **Projection**  
 $\{t : \{A_1, A_2, \dots, A_k\} \mid (\exists x) (x \in R \wedge$   
 $x[A_1] = t[A_1] \wedge x[A_2] = t[A_2] \wedge \dots \wedge x[A_k] = t[A_k])\}$

- $\sigma_F(R)$  **Selection**

$$\{t: U_R \mid t \in R \wedge F'\}$$

where  $F'$  is formula  $F$  where each attribute  $A$  has been replaced by  $t[A]$



## Relational Calculus

Expressions in relational calculus are also called FORMULAS and they are of the form:

$\{t: U \mid P(T)\}$  set of tuples on a schema U that satisfy predicate P

In other words, an answer tuple is an assignment of constant values to variables that make the formula evaluate true

## UNSAFE QUERIES AND EXPRESSIVE POWER

Possible to write syntactically correct calculus queries with infinite number of answers.

Such queries are called UNSAFE.

Example:  $\{t: U_R \mid \neg (t \in R)\}$

However, it has been shown that every query that can be expressed in relational algebra can be expressed as a safe query in relational calculus (TRC/DRC); the converse is also true.

=> same expressive power.

*Relational Completeness:* Relational query languages (e.g., SQL) can express every query that can be expressed in relational algebra/calculus.

## Semantic Integrity Constraints

A constraint is a property that a set of data must satisfy. One possible classification of constraints:

- immediate: verified immediately after each modification of the DB
- deferred: verified only at the end of a series of operations (transaction)
- constraints can also be classified depending on the objects they access:
  - on a single relation
    - (i) on a single tuple:
      - \* attribute constraints
      - \* multiple attribute constraints
    - (ii) on multiple tuples of the same relation
      - \* functional dependencies
      - \* cardinality constraints
    - (iii) aggregation constraints
  - on multiple relations:   referential integrity

### Examples:

- on a single attribute:  
salary of an employee must be between 500 and 1000
- on multiple attributes:  
bonus of an employee must always be less than the salary
- cardinality constraints:  
there must be at least 3 technicians (i.e., 3 employees whose job = “technician”)
- aggregation constraints:  
the average salary for a technician must be greater than 500
- constraints on multiple relations:  
the sum of salaries of employees that work on project P must be less than the budget for P