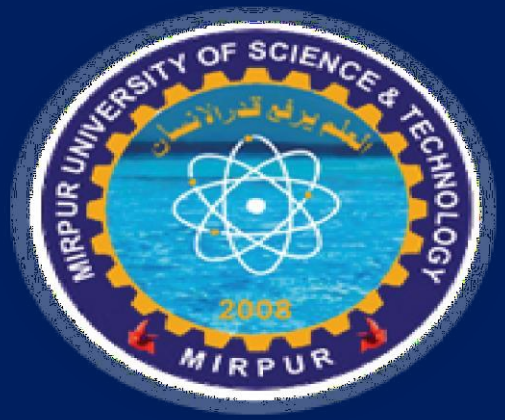

MUST

Wisdom & Virtue

MIRPUR UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARMENT OF SOFTWARE ENGINEERING

Propositional Logic

(Lecture # 04)

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(Lecturer)

LECTURE CONTENTS

1. Dijkstra's Game Mathematical Model and Solution
2. Propositional Logic

Mathematical Model

$$f(b,w) = \begin{cases} \text{(2 black out, 1 black in)} & b\text{-}2+1, w \equiv b\text{-}1, w \\ \text{(2 white out, 1 black in)} & b+1, w\text{-}2 \\ \text{(1 of each out, 1 white in)} & b\text{-}1, w\text{-}1+1 \equiv b\text{-}1, w \end{cases}$$

- Total number of balls is reduced by exactly one in each move.
- Parity of the white ball does not change

Solution:

- As Parity of the white ball does not change so if white balls are even these shall stay even till the end and least even number is 0 while in case of Odd 1 is the least number.

Implication

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

- This means “If P is true, then Q must also be true.”
- Here: P is called the premise or antecedent.
- Q is called the conclusion or consequent.

- If it rains (P), then the ground gets wet (Q).

So:

- If it rains → ground gets wet ✅

- If it doesn’t rain → we don’t care whether the ground is wet or not (the implication is still considered true)

Implication

- **Implication** is used to describe conditions, properties, or requirements in software and system specifications
  - ensuring that whenever certain conditions hold, certain results must also hold.

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

Example 1:If a user is authenticated, then access is granted. Formal notation:

$Authenticated(user) \Rightarrow AccessGranted(user)$

Implication

Example 2:
If the input value is negative, then the system should display an error message.
Formal notation:

$$Input < 0 \Rightarrow DisplayError = true$$

Implication in formal methods expresses cause-and-effect or conditional relationships between system properties, helping define how a system should behave under specific conditions.

Example 3:
If the temperature exceeds 100°C, then the alarm must turn on.
Formal notation:

$$Temperature > 100 \Rightarrow Alarm = ON$$

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

Bi-conditional – if and only if

- A **bi-conditional** means both statements are **true together** or **false together**.
- It is true when both P and Q have the same truth value.

P	Q	P ⇔ Q
T	T	T
T	F	F
F	F	T
F	T	F

Bi-conditional – if and only if

Example 1

- You will pass the test if and only if you study hard.
- Meaning: You pass when you study, and you don't pass when you don't study.

P	Q	P ⇔ Q
T	T	T
T	F	F
F	F	T
F	T	F

Bi-conditional – if and only if

Example 2

- A figure is a square if and only if it has four equal sides and four right angles
- **Meaning:** Being a square and having four equal sides with right angles always go together.

P	Q	P ⇔ Q
T	T	T
T	F	F
F	F	T
F	T	F

Bi-conditional – if and only if

Example 3

- You can drive a car if and only if you have a valid driving license.
- Meaning: You are allowed to drive exactly when you have a license — no other case.

P	Q	P ⇔ Q
T	T	T
T	F	F
F	F	T
F	T	F

Logic problem for the day

Someone asks person A, “Are you a knight?” He replies, “If I am a knight then I’ll eat my hat”. Prove that A has to eat his hat.

Thanks