MUST

Wisdom & Virtue

MIRPUR UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARMENT OF SOFTWARE ENGINEERING

Propositional logic

(Lecture # 05)

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LECTURE CONTENTS

1. Rules of Inference and Logical Deductions
2. Logical Problem

- In propositional logic and predicate logic, inference rules are used to derive conclusions from premises.
- Two important types of rules are:

i. Introduction Rules → Used to introduce a logical connective (like ∧, ∨, →, ¬)

ii. Elimination Rules → Used to eliminate a connective to extract useful information.

Introduction

- These rules add a logical operator($\wedge$, $\vee$, $\rightarrow$, $\neg$) in the conclusion.

Examples:

(a) $\wedge$-Introduction (AND Introduction)

- If both statements are true, you can combine them with AND.

- Premise 1: It is raining (P)
- Premise 2: It is cold (Q)
- $\therefore$ It is raining and cold ($P \wedge Q$)

Introduction

- These rules add a logical operator($\wedge$, $\vee$, $\rightarrow$, $\neg$) in the conclusion.

Examples:

(b) $\vee$-Introduction (OR Introduction)

- If one statement is true, you can introduce OR with any other statement.

- Premise: It is raining (P)
- $\therefore$ It is raining or sunny ($P \vee Q$)

Introduction

- These rules add a logical operator($\wedge$, $\vee$, $\rightarrow$, $\neg$) in the conclusion.

Examples:

(c) →-Introduction (Implication Introduction)

If assuming P allows you to derive Q, then you can conclude $P \rightarrow Q$..

Assume P

... derive Q

$\therefore P \rightarrow Q$

Example

Assume: It is raining (P)

Then: The ground gets wet (Q)

∴ If it is raining, then the ground gets wet ($P \rightarrow Q$)

Elimination

Elimination Rules (∧, ∨, →, ¬)

These rules **remove** a logical connective to simplify or extract information.

Examples:

(a) ∧-Elimination (AND Elimination)

From **P ∧ Q**, we can take either part.

$P \land Q$

$\therefore P$

or

$\therefore Q$

Example:

Premise: It is raining and cold (P ∧ Q)

∴ It is raining (P)

Rules of Inference and Logical Deduction

Elimination

Elimination Rules (∧, ∨, →, ¬)

These rules **remove** a logical connective to simplify or extract information.

Examples:

(b) ∨-Elimination (OR Elimination)
If **P ∨ Q** is true, and each leads to R, then R is true.

P ∨ Q
P → R
Q → R
∴ R

Example:
Either it rains or it snows (P ∨ Q)
If it rains, the ground is wet (P → R)
If it snows, the ground is wet (Q → R)
∴ The ground is wet (R)

Rules of Inference and Logical Deduction

Elimination

Elimination Rules (∧, ∨, →, ¬)

These rules **remove** a logical connective to simplify or extract information.

Examples:

(c) →-Elimination (Modus Ponens)

If **P → Q** and **P** are true, you can conclude **Q**.

P → Q
P
∴ Q

Example:

If it rains, the ground gets wet (P → Q)
It rains (P)
∴ The ground gets wet (Q)

Does the Superman Exist?

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be incapable; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither incapable nor malevolent. Therefore Superman does not exist.

Inference and deduction

Superman exists	X
Superman is willing to prevent evil	W
Superman is able to prevent evil	A
Superman is malevolent	M
Superman is incapable	I
Superman prevents evil	E

Our objective is to prove the proposition:

$$((W \text{ and } A) \Rightarrow E)$$
$$\text{and } ((\text{not } A) \Rightarrow I)$$
$$\text{and } ((\text{not } W) \Rightarrow M)$$
$$\text{and } (\text{not } E)$$
$$\text{and } (X \Rightarrow \text{not } (I \text{ or } M))$$
$$\Rightarrow \text{not } X$$

References

[1]. Alagar, Vangalur S., and Kasilingam Periyasamy. *Specification of software systems*. Springer Science & Business Media, 2011.

Thanks