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Wisdom & Virtue

MIRPUR UNIVERSITY OF SCIENCE AND TECHNOLOGY (MUST), MIRPUR

DEPARTMENT OF SOFTWARE ENGINEERING

Formal Methods in Software Engineering

Lecture [6]: Predicate Calculus, Modus Ponen, Modus Tollen

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Topics discussed in Today’s Lectures

- Predicate Calculus
- Truth Tables
- Modus Ponens or Direct Reasoning
- Modus Tollens or Indirect Reasoning

Predicate Calculus

- To generate new knowledge from old ones, one can use the truth table, which provides basic definition of predicate calculus. Table 1-3 (a), (b), and (c) shows the truth table:

Table 1-3. Truth Table of: (a) ¬p, (b) p∨q, and (c) p∧q.

(a) Negation

p	¬p
T	F
F	T

(b) Disjunction

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

(c) Conjunction

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Predicate Calculus - Conditional and Bidirectional

- Conditional and bidirectional are other forms of predicate calculus. For example, if p, then q can also be represented as: **p implies q**, and we write $p \Rightarrow q$.
- This can be represented as $(\neg p) \vee q$. Table 1-4 shows the truth table of such logic

Table 1-4. $(\neg p) \vee q$.

p	q	$p \Rightarrow q$	$\neg p$	$(\neg p) \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Predicate Calculus – Conditional Statements

Table 1-5. Examples of Conditional Statements

If p, then q. p implies q.
q follows from p. Not p unless q.
q if p. p only if q.
Whenever p, q. q whenever p.
p is sufficient for q. q is necessary for p.
p is a sufficient condition for q. q is a necessary condition for p.

Predicate Calculus – Biconditional Statements

The biconditional p⇔q, which we read "p if and only if q" or "p is equivalent to q," is defined by the truth table, shown in table 1-6.

Table 1-6. Biconditional Statements.

p	q	p⇔q
T	T	T
T	F	F
F	T	F
F	F	T

These primitive statements and predicate calculus can be used for rules inference.

Predicate Calculus

- **Predicate Calculus** (also called **First-Order Logic**) is a branch of logic that deals with **statements involving objects and their properties or relationships**.
- It uses:

•**Predicates** → to describe properties or relations (e.g., *Student(x), Cares(x, y)*)

•**Quantifiers** →

  - **∀ (for all)** — universal quantifier
  - **∃ (there exists)** — existential quantifier

- It helps represent logical statements precisely for reasoning, AI, databases, and computer programs.

Predicate Calculus

Practical Examples 1.

- **Medical Diagnosis System**
  - **Statement:** "If a person has a fever and cough, then they may have flu."
  - **Predicate Form:** $\forall x\ ((Fever(x) \land Cough(x)) \rightarrow HasFlu(x))$
  - **Use:** Medical expert systems use such logic rules to infer possible diseases.

Predicate Calculus

Practical Examples 2

- **Student Grading System**
  - **Statement:** "Every student who scores more than 50 passes the exam."
  - **Predicate Form:** $\forall x\ ((Student(x) \land Score(x) > 50) \rightarrow Pass(x))$
  - **Use:** Automated grading software or academic databases apply such logic for decision-making.

Predicate Calculus

Practical Examples 3

- **Online Shopping Recommendation**
  - **Statement:** “If a customer buys a laptop, they might also buy a mouse.”
  - **Predicate Form:** $\forall x\ (Buys(x, Laptop) \rightarrow LikelyToBuy(x, Mouse))$
  - **Use:** E-commerce recommendation systems use such logic to suggest related products.

Modus Ponens (Rule of Affirmation)

- *If one thing is true, then the other thing that follows from it must also be true. i.e. If the condition is true, then the result must be true.*

Form:

If $P \rightarrow Q$

P

$\therefore Q$

Meaning:

If P implies Q, and P is true, then Q must also be true.

Examples:

Example 1:If it rains, the ground gets wet.($P \rightarrow Q$)

It is raining. (P)

∴ The ground gets wet. (Q)

Modus Ponens (Rule of Affirmation)

Example 2:

- If a student studies hard, he will pass the exam
- The student studies hard.

  ∴ The student will pass the exam.

Example 3:

- If a person is a bachelor, then he is unmarried.
  - Ali is a bachelor.
  - ∴ Ali is unmarried.

Modus Tollens (Rule of Denial)

- If one thing is supposed to cause another, and the result didn't happen, then the first thing didn't happen either. (If the result is false, then the condition must also be false.)

Form:

If $P \rightarrow Q$

$\neg Q$

$\therefore \neg P$

Meaning:

If P implies Q, and Q is false, then P must also be false.

Example 1:

If it rains, the ground gets wet. ($P \rightarrow Q$)

The ground is not wet. ($\neg Q$)

$\therefore$ It did not rain. ($\neg P$)

Modus Tollens (Rule of Denial)

Example 2:

If the car has fuel, it will start.

The car did not start.

∴ The car has no fuel.

Example 3:

If a person is a teacher, they work in a school.

Ali does not work in a school.

∴ Ali is not a teacher.

THANKS