<u>Number Theory</u>

Rule 1. If the prime factorization of a number **N** can be defined by the following equation,

$N = (p_1)^{x_1} * (p_2)^{x_2} * (p_3)^{x_3} * \ldots\ldots * (p_n)^{x_n}$; where $p_1, p_2, p_3 \ldots\ldots p_n$ are primes.

Then the **number of all the divisors** of N, which can be expressed as **n**, can be defined by the following equation,

$\mathbf{n = (x_1+1) * (x_2+1) * (x_3+1) * \ldots\ldots * (x_n+1)}$ ________________ **(i)**

Example : Prime factorization of 18 is,

$18 = 2^1 * 3^2$

and 18 = 1 * 18 = 2 * 9 = 3 * 6 ; which means,

18 has n = **6** *Divisors*.

From **Equation (i)**, we get, **n** = (1+1) * (2+1) = 2 * 3 = **6**.

So, the results are matched.

Rule 2. If the prime factorization of a number **N** can be defined by the following equation,

$N = (p_1)^{x_1} * (p_2)^{x_2} * (p_3)^{x_3} * \ldots\ldots * (p_n)^{x_n}$; where $p_1, p_2, p_3 \ldots\ldots p_n$ are primes.

Then the ***sum of all the divisors*** of N, which can be expressed as **S**, can be defined by the following equation,

$\mathbf{S = ((p_1^{(x_1+1)}-1)/(p_1-1)) * ((p_2^{(x_2+1)}-1)/(p_2-1)) * ((p_3^{(x_3+1)}-1)/(p_3-1)) * \ldots\ldots * ((p_n^{(x_n+1}-1^{)})/(p_n-1))}$ ________________ **(ii)**

Example : Prime factorization of 18 is,

$18 = 2^1 * 3^2$

and 18 = 1 * 18 = 2 * 9 = 3 * 6 ; which means,

18 has n = 6 *Divisors*

and *Sum* of them = 1 + 2 + 3 + 6 + 9 + 18 = **39**.

From **Equation (ii)**, we get, **S** = $((2^{(1+1)}-1)/(2-1)) * ((3^{(2+1)}-1)/(3-1)) = ((2^2-1)/1) * ((3^3-1)/2) = ((4-1)/1) * ((27-1)/2) = (3/1) * (26/2) = 3 * 13 =$ **39**.

So, the results are matched.

Rule 3. If a number can be expressed as in the form, n = 10^x, x ≥ 0;

Then the ***Number of all the Divisors*** of n is $(x+1)^2$ or d^2 where d is the number of total digits of n.

Example : 10^0 = 1 has $(0+1)^2$ = 1 divisor (1); 10^1 = 10 has $(1+1)^2$ = 4 divisors (1,2,5,10); 10^3 = 100 has $(2+1)^2$ = 9 divisors (1,2,4,5,10,20,25,50,100) and so on.

Rule 4. The number of digits of the factorial of an integer n, can be expressed as,

S = floor (log (n!)) + 1

We know, log (a*b) = log (a) + log (b).

So, for log (n!) = S = log (1) + log (2) + log (3) + ……….. + log (n)

Since, the value can be fractional, we take the ceiling value of the result then increment it by 1.

Example : Suppose, we have to find the digits of 5! , which is 120.

So, log (5!) = log(1) + log(2) + log(3) + log (4) + log (5) = 2.079181

Finally, S = floor (2.079181) + 1 = 3.

Rule 5. The number of digits of the factorial of an integer n in base b.

The number of digits is $\lfloor \log_b N! \rfloor + 1$. This can be rewritten as

$$\left\lfloor \sum_{r=1}^{N} \log_b r \right\rfloor + 1.$$

That sum should be possible to calculate on computer even for relatively large N. However, you can also approximate it using Stirling's formula:

$$\ln N! \approx (N + 1/2) \ln N - N + \ln \sqrt{2\pi}.$$

To convert this to what you want, use the fact that $\log_b x = \frac{\ln x}{\ln b}$.

This approximation is pretty good. For $\log_2 5!$ it gives 6.883, whereas the actual value is 6.907.

Rule 6. The number of bases of N from 2 to infinite that has at least one trailing zero(es) = (The number of divisors of N) – 1.

Rule 7. If x be a prime number, then

$$(a^n + b^n + c^n + \dots) \bmod x = (a^{n \bmod \Phi(n)} + b^{n \bmod \Phi(n)} + c^{n \bmod \Phi(n)} + \dots) \bmod x = (a^{n \bmod \Phi(x)} + b^{n \bmod \Phi(x)} + c^{n \bmod \Phi(x)} + \dots) \bmod x$$

Example : Suppose, n = 4 and x = 5.

So, L.H.S = $(1^4 + 2^4 + 3^4) \bmod 5 = (1 + 16 + 81) \bmod 5 = 98 \bmod 5 = 3$.

and we know that, $\Phi(5) = 4$, so $n \bmod \Phi(x) = 4 \bmod 4 = 0$.

R.H.S = $(1^0 + 2^0 + 3^0) \bmod 5 = (1 + 1 + 1) \bmod 5 = 3 \bmod 5 = 3$.

Therefore, L.H.S = R.H.S.