# Summary of Required Proofs for Physics I Exam
## *PHYS001*

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## Contents



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# 1 Elasticity

Young's Modulus

$$\boxed{Y = \frac{F/A_\perp}{\Delta L/L_o}} \tag{1}$$

Shear's Modulus

$$\boxed{G = \frac{F/A_\parallel}{\Delta x/h}} \tag{2}$$

$$\tan(\gamma) = \gamma = \frac{\Delta x}{h} \qquad \tan\theta = \theta, \text{ for small } \theta \tag{3}$$

Bulk's Modulus

$$\boxed{B = -\frac{\Delta P}{\Delta V/V_o}} \tag{4}$$

# 2 Gravity

Kepler's Third Law (Orbital Period)

$$F_g = ma_r \tag{5}$$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}\frac{v^2}{r} \qquad \because v_{orbit} = \frac{2\pi r}{T} \tag{6}$$

$$\therefore \frac{GM}{r^2} = \frac{4\pi^2 r}{T^2} \implies \boxed{T^2 = \frac{4\pi^2}{GM} r^3} \tag{7}$$

$M$ is mass of the planet, $m$ mass of the satellite, $a_r$ centripetal acceleration, $T$ period.

Orbital Velocity

$$F_g = ma_r \tag{8}$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}} \tag{9}$$

$$\boxed{v = \sqrt{\frac{GM}{r}}} \tag{10}$$

**Energy in Orbits**

$$\boxed{U = -\frac{GMm}{r}}, \boxed{K.E_{orbit} = \frac{1}{2}\frac{GMm}{r}} \tag{11}$$
$$E_{orbit} = KE + U \tag{12}$$
$$\boxed{E_{orbit} = -\frac{1}{2}\frac{GMm}{r}} \tag{13}$$

**Launching & Escape Velocity**

*This proof is not required. Also, Eq. 17 (Launching Velocity) was explained in lecture but we are not sure if it is required.*

$$U_i + KE_i = U_f \tag{14}$$
$$\frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{r_i} = -\frac{GM\cancel{m}}{r_f} \tag{15}$$
$$v = \sqrt{2GM(\frac{1}{r_i} - \frac{1}{r_f})} \tag{16}$$
$$\boxed{v_{r_i \to r_f} = \sqrt{2GM(\frac{r_f - r_i}{r_i \cdot r_f})}} \tag{17}$$
$$\text{for } v_{escape}, r_f = \infty \implies v_{escape} = \sqrt{2GM(\frac{\cancel{r_f}}{r_i \cdot \cancel{r_f}})} \tag{18}$$
$$\boxed{v_{escape} = \sqrt{\frac{2GM}{r_i}}} \tag{19}$$

# 3 Fluid Dynamics

**Flow Rate Equation**

$$\boxed{Q = Av} \tag{20}$$

**Bernoulli's Equation**

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{Constant}} \tag{21}$$

,where $P$ is the pressure of the fluid, $\rho$ density of the fluid, $h$ difference of height from the initial condition (You can put the line $h = 0$ at any level you want).

For any two points in the same flow of a fluid, these points share the same constant.

## Continuity Equation

$$\boxed{A_1 v_1 = A_2 v_2} \tag{22}$$

## Torricelli's Approximation

First, set $h = 0$ level at the exit hole level $\implies h_2 = 0$

$$\because A_1 >> A_2 \qquad \therefore v_1 = \frac{A_2}{A_1} v_2 \approx 0 \tag{23}$$

$$P_1 + \cancel{\frac{1}{2}\rho v_1^2} + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g h_2} \tag{24}$$

$$v_2 = \sqrt{2(\frac{P_1 - P_2 + \rho g h_1}{\rho})} = \sqrt{\frac{2\Delta P}{\rho} + 2gh} \tag{25}$$

$$\because \text{Both holes are subjected to air} \qquad \therefore P_1 = P_2 = P_0 \tag{26}$$

$$\boxed{v_2 = \sqrt{2gh}} \tag{27}$$

## Venturi-meter

$$A_1 v_1 = A_2 v_2 \implies v_1 = \frac{A_2}{A_1} v_2 \tag{28}$$

$$P_1 + \frac{1}{2}\rho {v_1}^2 + \cancel{\rho g h} = P_2 + \frac{1}{2}\rho {v_2}^2 + \cancel{\rho g h} \tag{29}$$

$$\Delta P = \frac{1}{2}\rho({v_2}^2 - {v_1}^2) \qquad \because Eq.\ 28 \tag{30}$$

$$\therefore 2\Delta P = \rho(\frac{{A_1}^2{v_2}^2 - {A_2}^2{v_2}^2}{{A_1}^2}) \tag{31}$$

$$\boxed{v_2 = A_1\sqrt{\frac{2\Delta P}{\rho({A_1}^2 - {A_2}^2)}}} \qquad \because Eq.\ 20 \tag{32}$$

$$\therefore \boxed{Q = A_1 A_2\sqrt{\frac{2\Delta P}{\rho({A_1}^2 - {A_2}^2)}}} \tag{33}$$

## Venturi-meter with monometer

take two points (A,B) on the same level in each side of monometer

$$P_A = P_B, \ v_1 = v_2 = 0 \tag{34}$$

$$\rho_f g h + P_1 = \rho_{Hg} + P_2 \tag{35}$$

$$\boxed{\Delta P = (\rho_{Hg} - \rho_f) g h} \tag{36}$$

### Force on Airplane

Take two points of the same air flow, one above the wing and another one under it

$$P_1 + \frac{1}{2}\rho {v_1}^2 + \cancel{\rho g h} = P_2 + \frac{1}{2}\rho {v_2}^2 + \cancel{\rho g h} \qquad h \approx 0 \tag{37}$$

$$\boxed{\Delta P = \frac{1}{2}\rho({v_2}^2 - {v_1}^2)} \tag{38}$$

$$\boxed{F_w = \Delta P \cdot (2A_{wing})} \tag{39}$$

No ascending or descending: (40)

$$\sum F_y = 0 \implies F_w - mg = 0 \tag{41}$$

Ascending: (42)

(43)

$$\sum F_y = +ma \implies F_w - mg = ma \tag{44}$$

Descending: (45)

(46)

$$\sum F_y = -ma \implies F_w - mg = -ma \tag{47}$$

# 4 Oscillatory Motion

## Simple Harmonic Motion

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T}} \tag{48}$$

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x} \tag{49}$$

$$x(t) = A\cos(\omega t + \phi) \quad \text{OR} \quad x(t) = A\sin(\omega t + \phi) \tag{50}$$

$$\boxed{|v_{max}| = \omega A}, \ \boxed{|a_{max}| = \omega^2 A} \tag{51}$$

$V_{max}$ is at equilibrium point, while $a_{max}$ at turning points.

### Spring

*This proof was explained in the lecture, but we are not sure if required*

$$F_s = ma \tag{52}$$

$$-kx = m\frac{d^2x}{dt^2} \implies \tag{53}$$

$$\boxed{w^2 = \frac{k}{m}} \tag{54}$$

### Pendulum

$$F_t = ma_t = -mg\sin\theta \tag{55}$$

$$a_t = L\frac{d^2\theta}{dt^2} = -g\sin\theta \qquad \because \sin\theta = \theta \text{ for small } \theta \tag{56}$$

$$\therefore \frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \implies \text{Pendulum for small } \theta \text{ is performing SMH,} \tag{57}$$

$$\boxed{w^2 = \frac{g}{L}}, \ \boxed{\theta(t) = \theta_{max}\cos(\omega t = \phi)} \tag{58}$$

### Energy in Oscillation Motion

$$E = U + KE \tag{59}$$

$$\boxed{\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2} \tag{60}$$

Wave Motion

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}} \tag{61}$$

$$y(x,t) = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \phi\right) \tag{62}$$

$$\boxed{y(x,t) = A\sin(kx - \omega t + \phi)} \tag{63}$$

$$\boxed{k = \frac{2\pi}{\lambda}}, \quad \boxed{\omega = \frac{2\pi}{T}} \tag{64}$$

$$\boxed{v_{\text{wave speed}} = \lambda f = \frac{\omega}{k}} \tag{65}$$

$$v_y = \frac{\partial y}{\partial t} = \omega A\cos(kx - \omega t) \tag{66}$$

$$a_y = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx - \omega t) \tag{67}$$

$$\boxed{|v_{y,\,max}| = A\omega}, \quad \boxed{|a_{y,\,max}| = A\omega^2} \tag{68}$$

Wave on a String

$$\boxed{v_{\text{wave speed}} = \sqrt{\frac{T}{\mu}}} \qquad \because \mu = \frac{m}{L} \tag{69}$$

$$v = \sqrt{\frac{F}{\frac{m}{L}\cdot\frac{A}{A}}} \qquad \because \frac{F}{A} = Sress, \rho = \frac{m}{V} \tag{70}$$

$$\boxed{v = \sqrt{\frac{Stress}{\rho}}} \tag{71}$$

$$\boxed{K.E_\lambda = U_\lambda = \frac{1}{4}\mu\omega^2 A^2\lambda}, \quad \boxed{E_\lambda = \frac{1}{2}\mu\omega^2 A^2\lambda} \tag{72}$$

$$P = \frac{E}{T} = \frac{1}{2}\mu\omega^2 A^2\frac{\lambda}{T} \tag{73}$$

$$\boxed{P = \frac{1}{2}\mu\omega^2 A^2 v} \tag{74}$$

# 5 Thermodynamics: Ideal Gases

$$\boxed{Q = E_{int} + W} \tag{75}$$

$$\boxed{E_{int} = n\frac{fR}{2}\Delta T} \tag{76}$$

$f$ is degree of freedom. $f = 3$ for monoatomic, while $f = 5$ for diatomic.

Isovolumetric Process

$$dV = 0 \tag{77}$$

$$W = \int_{V_i}^{V_f} P\,dV = 0 \tag{78}$$

$$\boxed{Q = E_{int}}, \quad \boxed{Q = nC_v\Delta T} \tag{79}$$

## Isobaric Process

$$dP = 0 \implies \frac{\cancel{nR}T_i}{V_i} = \frac{\cancel{nR}T_f}{V_f} \tag{80}$$

$$W = \int_{V_i}^{V_f} P\, dV = P \int_{V_i}^{V_f} dV = P\Delta V \qquad \because PV = nRT \tag{81}$$

$$\therefore \boxed{W = nR\Delta T} \tag{82}$$

$$dQ = dE_{int} + dW \tag{83}$$

$$\boxed{nC_p\, \Delta T = nC_v\, \Delta T + nR\, \Delta T} \tag{84}$$

$$\boxed{C_p = C_v + R}, \ \boxed{C_p = \tfrac{fR+2R}{2}} \tag{85}$$

$$\boxed{\gamma = \tfrac{C_v+R}{C_v} = \tfrac{C_p}{C_v}}, \ \boxed{\gamma - 1 = \tfrac{R}{C_v}} \tag{86}$$

in Eq. 82 positive sign is used because gas needs more energy to be introduced, i.e., more $Q$.

## Isothermal Process

$$T_i = T_f \implies E_{int} = 0,\ PV = nRT = \text{Constant} \tag{87}$$

$$\boxed{Q = W} \tag{88}$$

$$P(V) = \frac{nRT}{V},\ W = nRT \int_{V_i}^{V_f} \frac{1}{V}\, dV \tag{89}$$

$$\boxed{Q = W = nRT \ln\left(\tfrac{V_f}{V_i}\right)}, \ \boxed{Q = W = PV \ln\left(\tfrac{V_f}{V_i}\right)} \tag{90}$$

## Adiabatic Process

$$dQ = 0 \implies dE_{int} + dW = 0 \tag{91}$$

$$nC_v\, dT + P\, dV = 0 \qquad \because PV = nRT \qquad \therefore \cancel{n}C_v\, dT + \cancel{n}RT\, \frac{dV}{V} = 0 \tag{92}$$

$$\int_{T_i}^{T_f} \cdot \frac{dT}{T} + \frac{R}{C_v} \int_{V_i}^{V_f} \frac{dV}{V} = 0 \qquad \because Eq.\ 86 \tag{93}$$

$$\therefore \ln\left(\frac{T_f}{T_i}\right)\left(\frac{V_f}{V_i}\right)^{(\gamma-1)} = 0 \tag{94}$$

$$\boxed{T_f V_f^{(\gamma-1)} = T_i V_i^{(\gamma-1)}} \qquad \because PV = nRT \quad \therefore \frac{P_f V_f}{\cancel{nR}} V_f^{(\gamma-1)} = \frac{P_i V_i}{\cancel{nR}} V_i^{(\gamma-1)} \tag{95}$$

$$\boxed{P_f V_f^{\gamma} = P_i V_i^{\gamma} = K} \text{ or (using same way) } \boxed{\left(\frac{T_i}{T_f}\right)^{\gamma} = \left(\frac{P_i}{P_f}\right)^{\gamma-1}} \tag{96}$$

$$W = \int_{V_i}^{V_f} P\, dV = K \int_{V_i}^{V_f} V^{-\gamma}\, dV \tag{97}$$

$$= \frac{K}{1-\gamma}[V_f^{(1-\gamma)} - V_i^{(1-\gamma)}] \quad = \frac{1}{1-\gamma}[(P_f V_f^{\gamma})V_f^{(1-\gamma)} - (P_i V_i^{\gamma})V_i^{(1-\gamma)}] \tag{98}$$

$$\boxed{W = \tfrac{P_i V_i - P_f V_f}{\gamma - 1}} \tag{99}$$

## Engines

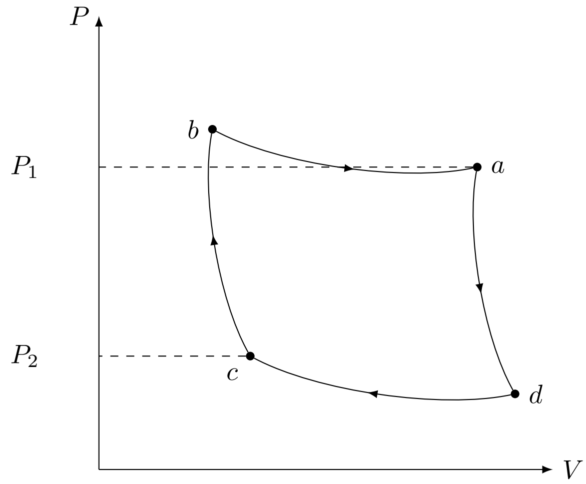
$$\boxed{W = \sum Q = Q_H + Q_C} \tag{100}$$

$$\boxed{\eta = \frac{W}{Q_H} = \frac{Q_H+Q_C}{Q_H} = 1 - \frac{|Q_C|}{Q_H}} \tag{101}$$

$Q_H$ is heat entering (Hot) , $Q_C$ is heat getting out (Cold).

### Carnot's Cycle

Carnot's Cycle contains 2 adiabatic (cb & ad) and 2 isothermal processes (ba & dc).




*Adiabatic paths:*

$$T_H V_a^{\gamma-1} = T_C V_d^{\gamma-1} \;/\; T_H V_b^{\gamma-1} = T_C V_c^{\gamma-1} \tag{102}$$

$$\frac{V_a}{V_b} = \frac{V_d}{V_c} \tag{103}$$

*Isothermal paths:*

$$Q_H = nRT_H \ln\frac{V_a}{V_b} \tag{104}$$

$$|Q_C| = -nRT_C \ln\frac{V_c}{V_d} = nRT_C \ln\frac{V_d}{V_c} \tag{105}$$

$$\eta = 1 - \frac{|Q_C|}{Q_H} = 1 - \frac{\cancel{nR}T_H \cancel{\ln\frac{V_a}{V_b}}}{\cancel{nR}T_C \cancel{\ln\frac{V_d}{V_c}}} \tag{106}$$

$$\boxed{\eta_{carnot} = 1 - \frac{T_C}{T_H}} \tag{107}$$

$T_C$ Cold Temperature (getting out), $T_H$ Hot temperature entering.

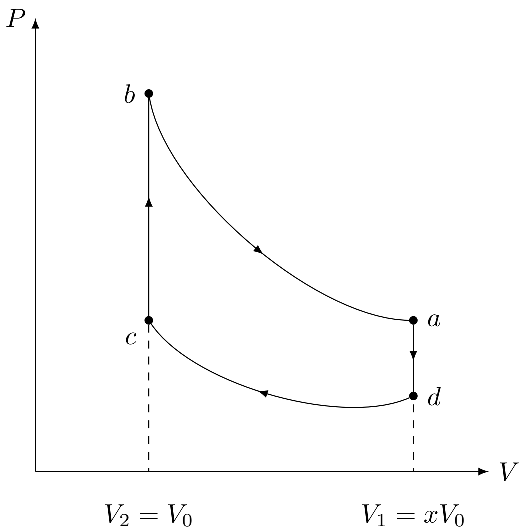
$$\eta \leq \eta_{carnot}$$

because $\eta_{carnot}$ is the maximum efficiency.

Continue: Engines

Otto's Cycle

Otto's cycle contains 2 adiabatic (ba & dc) and 2 isovolumetric processes (ad & bc). **Two** Volumes only




$$\frac{1}{x} = \frac{V_c}{V_d} = \frac{V_b}{V_a} \tag{108}$$

$$\eta = 1 - (\frac{V_c}{V_d})^{\gamma-1} = 1 - (\frac{V_b}{V_a})^{\gamma-1}, \qquad T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1} \tag{109}$$

$$\boxed{\eta = 1 - \frac{1}{x^{\gamma-1}} = 1 - \frac{T_d}{T_c} = 1 - \frac{T_a}{T_b}} \tag{110}$$

a way to remember which temperature points to use. Take one adiabatic path and get two points, one with high $V$ and one with low $V$. The temperature of the high volume divided the temperature of the low volume.

**Entropy**

$$\boxed{dS = \frac{dQ}{T}} \tag{111}$$

$$Q = \int T\ dS \implies Q = \text{Area under } T - S \text{ diagram} \tag{112}$$

$$\tag{113}$$

**General:**

$$dQ = mC\ dT \implies S = \int_{T_i}^{T_f} \frac{mC}{T}\ dT \tag{114}$$

$$\boxed{S = mC \ln \frac{T_f}{T_i}} \tag{115}$$

**Ideal Gasses Entropy**

*Isothermal:*

$$\boxed{S = \frac{Q}{T}} \tag{116}$$

*Isovolumetric:*

$$S = \int_{T_i}^{T_f} \frac{nC_v}{T}\ dT \tag{117}$$

$$\boxed{S = nC_v \ln \frac{T_f}{T_i}} \tag{118}$$

*Isobaric:*

$$\boxed{S = nC_p \ln \frac{T_f}{T_i}} \tag{119}$$

*Adiabatic:*

$$\boxed{S = 0} \tag{120}$$