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CMPS 102 — Winter 2019 – Homework 3

Two problems, 15 points, due 11:50 pm Wednesday Feb 13th, see the *Homework Guidelines*

1. (9 pts) Problem 20 in Chapter 4 (Greedy Algorithms). The input is a (undirected) graph where the nodes are towns and the edges represent the roads between them. Each edge is labeled with the maximum altitude along that road, and assume that all these altitudes are different. The height of a path is the maximum of the edge-altitudes over the edges in the path. For each pair of nodes, a winter-optimal path between them is a path from one to the other such that the path height is as small as possible (i.e. no other path between the two nodes has a lower path height). Since there may be many paths, there may be multiple paths with the same path height even though the edge altitudes are all different. Prove that the Minimum Cost Spanning Tree with edge costs equal to the edge altitudes provides winter-optimal paths between the vertices.

   You may use the fact that if a sequence of edges is a path between two vertices, then there is a simple path between the vertices using a subsequence of the edges. (If the path is already a simple path, the subsequence is the original path).

2. (6 points) The FFT can also be performed using modular arithmetic and integers rather than complex numbers. This problem will guide you along this path. Note that all calculations should be done modulo 7.

   - (1 pt) Find a number ω such that ω^0, ω^1, ω^2, ω^3, ω^4, and ω^5 are all different modulo 7 (i.e. these powers modulo 7 give some permutation of $\{1, 2, 3, 4, 5, 6\}$. The value of "n" will be 6.
   - (1 pt) Write the Fourier Matrix for this ω (see slide 12 of the FFT slides).
   - (2 pts) Use your Fourier Matrix to transform the sequence $\mathbf{a} = (0, 1, 2, 1, 5, 2)$ into the corresponding $\mathbf{y}$ sequence.
   - (2 pts) Finally, write down the inverse FFT matrix (G_n on slide 19 of the FFT slides) and compute the inverse FFT transform $G_n\,\mathbf{y}$ to recover the original sequence $\mathbf{a}$.

   Note that in mod 7 group, the value x^{-1} is that number z such that $x \cdot z \equiv 1 \pmod 7$, and don't forget the $1/n$ in the definition of G_n.