Logarithms Handout

CMPS 101 *Spring 2014*

This handout presents the definition and main properties of logarithm function, with particular focus on the logarithm with base 2.

Definition 1 *Given value $b > 0$ (usually b is one of 2, 10, or $e \approx 2.718$), and a number $x > 0$, the logarithm with base b of x is defined as*

$$\log_b(x) = y$$

for the unique y that satisfies

$$b^y = x.$$

We will call the logarithm with base e as the *natural logarithm* and denote as $\ln(x)$. In this class we will mostly use the logarithm with base 2, so in this case for brevity we may skip the base specification, and just say *logarithm of x*, and write $\log(x)$ instead of $\log_2(x)$.

Properties. The following properties hold for any base b (including $b = 2$).

1. Logarithm of a product:
$$\log_b(xy) = \log_b(x) + \log_b(y).$$
2. Logarithm of a quotient:
$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y).$$
3. Logarithm of a power:
$$\log_b\left(x^a\right) = a \cdot \log_b(x).$$
4. Logarithm of a root:
$$\log_b\left(\sqrt[q]{x}\right) = \frac{\log_b(x)}{q}.$$
5. Logarithm $\log_b(x)$ is a strictly increasing function, which is:
   (a) negative ($\log_b(x) < 0$), for $0 < x < 1$;
   (b) positive ($\log_b(x) > 0$), for $x > 1$, and

$$\log_b(1) = 0, \quad \log_b(b) = 1, \quad \log_b(b^n) = n.$$

Example 2 *Here is a simple expression that demonstrates all of the above properties (we use the default base $b = 2$):*

$$\log\left(\frac{\sqrt{5} \cdot 4^5}{10}\right) = \frac{\log(5)}{2} + 5 \cdot \log(4) - \log(10) = \frac{\log(5)}{2} + 5 \cdot 2 - \log(2) - \log(5) = 9 - \frac{\log(5)}{2}.$$

Change of base. If you want to change the base of your logarithm from c to b, then you use:

$$\log_c(x) = \frac{\log_b(x)}{\log_b(c)}.$$

In particular, to switch from base 2 logarithm to natural logarithm you would for example do

$$\log(5) = \frac{\ln(5)}{\ln(2)}.$$