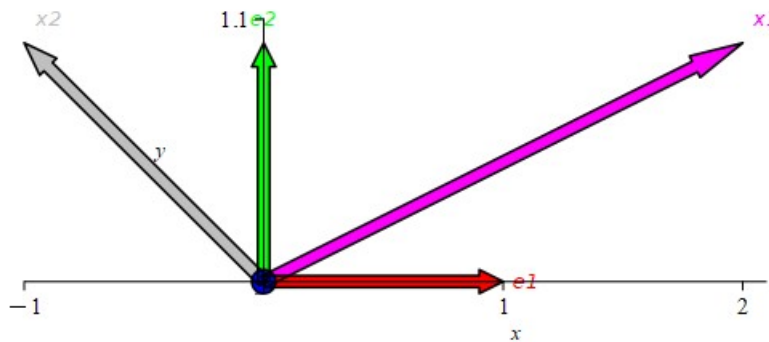


TP R5A12 Modélisation mathématique

Exercice 1

1.



2.

a) $A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

b) $P = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

c) $P^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

d) $A = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

$$\text{e) } p(X) = \begin{pmatrix} \frac{2x}{3} + \frac{2y}{3} \\ \frac{x}{3} + \frac{y}{3} \end{pmatrix}$$

$$\text{f) } p(x_1) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$p(x_2) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$p(x_3) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

3.

$$\text{a) } A_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{b) } A_2 = \begin{bmatrix} \frac{1}{3} & \frac{-2}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix}$$

$$\text{c) } q(X) = \begin{pmatrix} \frac{x}{3} - \frac{2y}{3} \\ \frac{-x}{3} + \frac{2y}{3} \end{pmatrix}$$

$$\text{d) } q(x_1) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$q(x_2) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$q(x_3) = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

4.

$$\text{a) } A_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{b) } A_4 = \begin{bmatrix} \frac{1}{3} & \frac{4}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$$

$$\text{c) } s(X) = \begin{pmatrix} \frac{x}{3} + \frac{4y}{3} \\ \frac{2x}{3} - \frac{y}{3} \end{pmatrix}$$

$$\text{d) } s(x_1) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$s(x_2) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$s(x_3) = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

5.



Exercise 2

1.

a)

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b)

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

c)

$$P^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \end{bmatrix}$$

$$\text{d) } A = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{e) } p(X) = \begin{pmatrix} \frac{x}{2} - \frac{y}{2} + \frac{z}{2} \\ \frac{-x}{2} + \frac{y}{2} + \frac{z}{2} \\ z \end{pmatrix}$$

$$\text{f) } p(x_1) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$p(x_2) = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$p(x_3) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$p(x_4) = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}$$

2.

$$\text{a) } A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{b) } A_2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ 1 & 1 & -1 \end{bmatrix}$$

$$\text{c) } q(X) = \begin{pmatrix} \frac{x}{2} + \frac{y}{2} - \frac{z}{2} \\ \frac{x}{2} + \frac{y}{2} - \frac{z}{2} \\ x + y - z \end{pmatrix}$$

$$d) q(x_1) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$q(x_2) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$q(x_3) = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$q(x_4) = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}$$

3.

$$a) A_5 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$b) A_4 = \begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 2 \\ -1 & -1 & 2 \end{bmatrix}$$

$$c) s(X) = \begin{pmatrix} y+2z \\ x+2z \\ -x-y+2z \end{pmatrix}$$

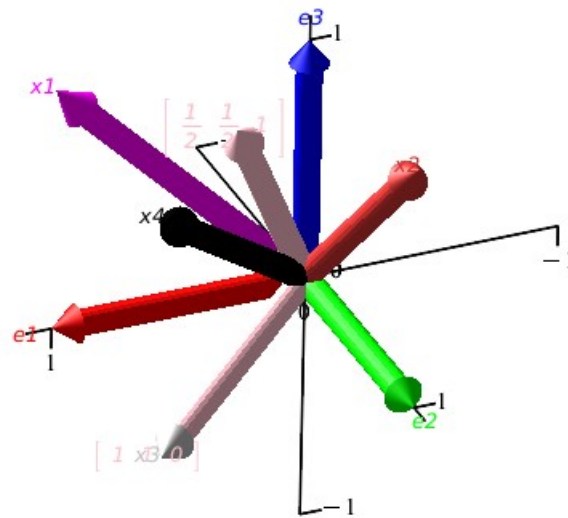
$$d) s(x_1) = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$s(x_2) = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$s(x_3) = \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$$

$$q(x_4) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

4.



Exercise 3

1.

a)

$$A_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

b)

$$P = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 2 & 1 \\ 2 & 2 & -1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

c)

$$P^{-1} = \begin{bmatrix} \frac{9}{2} & \frac{-1}{2} & \frac{-3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ 2 & 0 & -1 & 0 \\ -3 & 0 & 1 & 1 \end{bmatrix}$$

$$\text{d) } A = \begin{bmatrix} 7 & 1 & -1 & -3 \\ 18 & 2 & -3 & -7 \\ 6 & 2 & 0 & -4 \\ 19 & 1 & -4 & -6 \end{bmatrix}$$

$$\text{e) } p(X) = \begin{pmatrix} 7x+y-z-3t \\ 18x+2y-3z-7t \\ 6x+2y-4t \\ 19x+y-4z-6t \end{pmatrix}$$

$$\text{f) } p(x_1) = \begin{pmatrix} 2 \\ 5 \\ 2 \\ 5 \end{pmatrix}$$

$$p(x_2) = \begin{pmatrix} 4 \\ 9 \\ 6 \\ 7 \end{pmatrix}$$

$$p(x_3) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$p(x_4) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$p(x_5) = \begin{pmatrix} -6 \\ -15 \\ -6 \\ -15 \end{pmatrix}$$

2.

$$\text{a) } A_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\text{b) } A_2 = \begin{bmatrix} -6 & 0 & 1 & 4 \\ -18 & 0 & 5 & 8 \\ -4 & 0 & -1 & 6 \\ -18 & 0 & 5 & 8 \end{bmatrix}$$

$$\text{c) } q(X) = \begin{pmatrix} -6x+z+4t \\ -18x+5z+8t \\ -4x-z+6t \\ -18x+5z-8t \end{pmatrix}$$

$$\text{d) } q(x_1) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$q(x_2) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$q(x_3) = \begin{pmatrix} 3 \\ 3 \\ 7 \\ 3 \end{pmatrix}$$

$$q(x_4) = \begin{pmatrix} 4 \\ 8 \\ 6 \\ 8 \end{pmatrix}$$

$$q(x_5) = \begin{pmatrix} 13 \\ 29 \\ 17 \\ 29 \end{pmatrix}$$

3.

$$\text{a) } A_5 = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 2 & -2 & -1 \\ 2 & 2 & 1 & -2 \\ 1 & 1 & -1 & -2 \end{bmatrix}$$

$$\text{b) } A_4 = \begin{bmatrix} 13 & 1 & -2 & -7 \\ 36 & 2 & -8 & -15 \\ 10 & 2 & 1 & -10 \\ 37 & 1 & -9 & -14 \end{bmatrix}$$

$$\text{c) } s1(X) = \begin{pmatrix} 13x+y-2z-7t \\ 36x+2y-8z+15t \\ 10x+2y+z-10t \\ 37x+y-9z-14t \end{pmatrix}$$

$$d) s1(x_1) = \begin{pmatrix} 2 \\ 5 \\ 2 \\ 5 \end{pmatrix}$$

$$s1(x_2) = \begin{pmatrix} 4 \\ 9 \\ 6 \\ 7 \end{pmatrix}$$

$$s1(x_3) = \begin{pmatrix} -3 \\ -3 \\ -7 \\ -3 \end{pmatrix}$$

$$s1(x_4) = \begin{pmatrix} -4 \\ -8 \\ -6 \\ -8 \end{pmatrix}$$

$$s1(x_5) = \begin{pmatrix} -19 \\ -44 \\ -23 \\ -44 \end{pmatrix}$$

4.

$$a) A_7 = \begin{bmatrix} -1 & -1 & 0 & 1 \\ 0 & -2 & 2 & 1 \\ -2 & -2 & -1 & 2 \\ -1 & -1 & 1 & 2 \end{bmatrix}$$

$$b) A_6 = \begin{bmatrix} -13 & -1 & 2 & 7 \\ -36 & -2 & 8 & 15 \\ -10 & -2 & -1 & 10 \\ -37 & -1 & 9 & 14 \end{bmatrix}$$

$$c) s2(X) = \begin{pmatrix} -13x - y + 2z + 7t \\ -36x - 2y + 8z + 15t \\ -10x - 2y - z + 10t \\ -37x - y + 9z + 14t \end{pmatrix}$$

$$d) s2(x_1) = \begin{pmatrix} -2 \\ -5 \\ -2 \\ -5 \end{pmatrix}$$

$$s2(x_2) = \begin{pmatrix} -4 \\ -9 \\ -6 \\ -7 \end{pmatrix}$$

$$s_2(x_3)=\begin{pmatrix}3\\3\\7\\3\end{pmatrix}$$

$$s_2(x_4)=\begin{pmatrix}4\\8\\6\\8\end{pmatrix}$$

$$s(x_5)=\begin{pmatrix}19\\44\\23\\44\end{pmatrix}$$