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SAÉ Modélisation mathématique

Exercice 1

a)

$$\begin{bmatrix} 3 & 2 & -1 & 0 \\ -3 & 3 & -2 & 3 \\ 2 & -3 & 2 & -2 \end{bmatrix}$$

$L2 \leftarrow L2 - (\frac{-3}{3}) * L1$
$$\begin{bmatrix} 3 & 2 & -1 & 0 \\ 0 & 5 & -3 & 3 \\ 2 & -3 & 2 & -2 \end{bmatrix}$$

$L3 \leftarrow L3 - (\frac{-2}{3}) * L1$
$$\begin{bmatrix} 3 & 2 & -1 & 0 \\ 0 & 5 & -3 & 3 \\ 0 & \frac{-13}{3} & \frac{8}{3} & -2 \end{bmatrix}$$

$L3 \leftarrow L3 - (\frac{-5}{\frac{-13}{3}}) * L2$
$$\begin{bmatrix} 3 & 2 & -1 & 0 \\ 0 & 5 & -3 & 3 \\ 0 & 0 & \frac{1}{15} & \frac{3}{5} \end{bmatrix}$$

$L1 \leftarrow L1 - \frac{\frac{1}{15}}{-1} L3$
$$\begin{bmatrix} 3 & 2 & 0 & 9 \\ 0 & 5 & -3 & 3 \\ 0 & 0 & \frac{1}{15} & \frac{3}{5} \end{bmatrix}$$

$L2 \leftarrow L2 - \frac{-3}{\frac{1}{-15}} L3$
$$\begin{bmatrix} 3 & 2 & 0 & 9 \\ 0 & 5 & 0 & 30 \\ 0 & 0 & \frac{1}{15} & \frac{3}{5} \end{bmatrix}$$

$L1 \leftarrow L1 - \frac{2}{5} L2$
$$\begin{bmatrix} 3 & 0 & 0 & -3 \\ 0 & 5 & 0 & 30 \\ 0 & 0 & \frac{1}{15} & \frac{3}{5} \end{bmatrix}$$

$L1 \leftarrow \frac{1}{3} L1$
$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 5 & 0 & 30 \\ 0 & 0 & \frac{1}{15} & \frac{3}{5} \end{bmatrix}$$

$L1 \leftarrow \frac{1}{5} L2$
$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & \frac{1}{15} & \frac{3}{5} \end{bmatrix}$$

$$L3 \leftarrow \frac{1}{\frac{1}{15}} L3 \qquad \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 9 \end{bmatrix}$$

S={(-1, 6, 9)}

b)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & \frac{-8}{9} \\ 0 & 0 & 1 & 0 & \frac{1}{9} \\ 0 & 0 & 0 & 1 & \frac{5}{6} \end{bmatrix}$$

S={(1/3, -8/9, 1/9, 5/6)}

Exercice 2

a)

$$\begin{cases} 3x+z=0 \\ 3x+y+2z=0 \\ 3x+3y+2z=0 \end{cases}$$

$$\begin{bmatrix} 3 & 0 & 1 & 0 \\ 3 & 1 & 2 & 0 \\ 3 & 3 & 4 & 0 \end{bmatrix}$$

$$L2 \leftarrow L2-\frac{3}{3}L1 \qquad \begin{bmatrix} 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 3 & 3 & 4 & 0 \end{bmatrix}$$

$$L3 \leftarrow L3-\frac{3}{3}L1 \qquad \begin{bmatrix} 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & 3 & 0 \end{bmatrix}$$

$$L3 \leftarrow L3-\frac{3}{1}L2 \qquad \begin{bmatrix} 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L1 \leftarrow \frac{1}{3}L1 \qquad \begin{bmatrix} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

S	Nature de S	Dim S	Base de S	Type de sous espace vectoriel
$\left\{ \begin{pmatrix} \frac{-1}{3}z \\ -z \\ z \end{pmatrix}, z \epsilon R \right\}$	Vect$\left(\begin{pmatrix} \frac{-1}{3} \\ -1 \\ 1 \end{pmatrix}\right)$	1	$\left(\begin{pmatrix} \frac{-1}{3} \\ -1 \\ 1 \end{pmatrix}\right)$	Droite vectorielle dans R^3

b)

$$\begin{cases} x-y+z=0 \\ 2x-2y+2z=0 \\ -x+y-z=0 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ -1 & 1 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

S	Nature de S	Dim S	Base de S	Type de sous espace vectoriel
$\{\begin{pmatrix} y-z \\ y \\ z \end{pmatrix}, y, z \epsilon R\}$	Vect$(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix})$	2	$(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix})$	Plan de R^3

c)

$$\begin{cases} x+y+z+t=0 \\ x+y+2z+t=0 \\ x+2y+z+t=0 \\ 2x+y+z+2t=0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \\ 2 & 1 & 1 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

S	Nature de S	Dim S	Base de S	Type de sous espace vectoriel
$\{\begin{pmatrix} -t \\ 0 \\ 0 \\ t \end{pmatrix}, t \epsilon R\}$	Vect$(\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix})$	2	$(\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix})$	Droite vectorielle dans R^4

d)

$$\begin{cases} x+y+z+t=0 \\ x+y+2z+t=0 \\ x+2y+z+t=0 \\ 2x+4y+z-3t=0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \\ 2 & 4 & 1 & -3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

S	Nature de S	Dim S	Base de S	Type de sous espace vectoriel
$\{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\}$	Vect$(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix})$	0	-	Point dans R^4

e)

$$\begin{cases} 2x+3y+z+t=0 \\ x+z+t=0 \\ 3y-z-t=0 \\ 4x+3y+3z+3t=0 \end{cases}$$

$$\begin{bmatrix} 2 & 3 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 3 & -1 & -1 & 0 \\ 4 & 3 & 3 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & \frac{-1}{3} & \frac{-1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

S	Nature de S	Dim S	Base de S	Type de sous espace vectoriel
$\{\begin{pmatrix} -z-t \\ 1/3z+1/3t \\ z \\ t \end{pmatrix}, z, t \varepsilon R\}$	Vect$(\begin{pmatrix} -1 \\ \frac{1}{3} \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ \frac{1}{3} \\ 0 \\ 1 \end{pmatrix})$	2	$(\begin{pmatrix} -1 \\ \frac{1}{3} \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ \frac{1}{3} \\ 0 \\ 1 \end{pmatrix})$	Plan dans R^4

f)

$$\begin{cases} x-y+z-t=0 \\ 2x-2y+2z-2t=0 \\ -x+y-z+t=0 \\ -2x+2y-2z+2t=0 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 0 \\ 2 & -2 & 2 & -2 & 0 \\ -1 & 1 & -1 & 1 & 0 \\ -2 & 2 & -2 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

S	Nature de S	Dim S	Base de S	Type de sous espace vectoriel
$\{\begin{pmatrix} y-z+t \\ y \\ z \\ t \end{pmatrix}, z, t\ \varepsilon\ R\}$	Vect$(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix})$	3	$(\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix})$	Espace tridimensionnel dans R^4

Exercice 3

1.

$$V = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$Vect(V) = \{\begin{pmatrix} x \\ x \\ x \end{pmatrix}, x \in R\}$$

$$\begin{cases} y = x \\ z = x \end{cases}$$

$$\begin{cases} x - y = 0 \\ x - z = 0 \end{cases}$$

2.

$$V = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix}$$

$$Vect(V) = \{\begin{pmatrix} x \\ -x \\ 2x \\ 0 \end{pmatrix}, x \in R\}$$

$$\begin{cases} y & = & -x \\ z & = & 2x \\ t & = & 0 \end{cases}$$

$$\begin{cases} x+y & = & 0 \\ 2x-z & = & 0 \\ t & = & 0 \end{cases}$$

3.

$$V_1=\begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix} V_2=\begin{pmatrix} -1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$

$$Vect(V)=\{\begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix} a+\begin{pmatrix} -1 \\ 2 \\ 1 \\ 1 \end{pmatrix} b,(a,b)\in R^2\}$$

$$\begin{cases} x & = & a-b \\ y & = & 2b \\ z & = & -a+b \\ t & = & 2a+b \end{cases}$$

$$\begin{matrix} L1 \leftarrow 2L1 \\ L2 \leftarrow L2 \\ L3 \leftarrow 2L3 \\ L4 \leftarrow 2L4 \end{matrix} \qquad \begin{cases} 2x & = & 2a-2b \\ y & = & 2b \\ 2z & = & -2a+2b \\ 2t & = & 4a+2b \end{cases}$$

$$\begin{matrix} L1 \leftarrow L1+L2 \\ L2 \leftarrow L2 \\ L3 \leftarrow L3-L2 \\ L4 \leftarrow L4-L2 \end{matrix} \qquad \begin{cases} 2x+y & = & 2a \\ y & = & 2b \\ -y+2z & = & -2a \\ -y+2t & = & 4a \end{cases}$$

$$\begin{matrix} L1 \leftarrow L1 \\ L2 \leftarrow L2 \\ L3 \leftarrow L3+L1 \\ L4 \leftarrow L4-2L1 \end{matrix} \qquad \begin{cases} 2x+y & = & 2a \\ y & = & 2b \\ 2x+2z & = & 0 \\ -4x-3y+2t & = & 0 \end{cases}$$

$$\begin{matrix} L3 \leftarrow \frac{1}{2}L3 \\ L4 \leftarrow -L4 \end{matrix} \qquad \begin{cases} x+z & = & 0 \\ 4x+3y-2t & = & 0 \end{cases}$$

4.

$$V_1=\begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \\ 1 \end{pmatrix} V_2=\begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$Vect(V)=\{\begin{pmatrix}1\\0\\1\\2\\1\end{pmatrix}a+\begin{pmatrix}1\\2\\1\\1\\1\end{pmatrix}b,(a,b)\in R^2\}$$

$$\begin{cases} x &= a+b\\ y &= 2b\\ z &= a+b\\ t &= 2a+b\\ w &= a+b\end{cases}$$

$$\begin{matrix} L1\leftarrow 2L1\\ L2\leftarrow L2\\ L3\leftarrow 2L3\\ L4\leftarrow 2L4\\ L5\leftarrow 2L5\end{matrix} \qquad \begin{cases} 2x &= 2a+2b\\ y &= 2b\\ 2z &= 2a+2b\\ 2t &= 4a+2b\\ 2w &= 2a+2b\end{cases}$$

$$\begin{matrix} L1\leftarrow L1-L2\\ L2\leftarrow L2\\ L3\leftarrow L3-L2\\ L4\leftarrow L4-L2\\ L5\leftarrow L5-L2\end{matrix} \qquad \begin{cases} 2x-y &= 2a\\ y &= 2b\\ -y+2z &= 2a\\ -y+2t &= 4a\\ -y+2w &= 2a\end{cases}$$

$$\begin{matrix} L1\leftarrow L1\\ L2\leftarrow L2\\ L3\leftarrow L3-L1\\ L4\leftarrow L4-2L1\\ L5\leftarrow L5-L1\end{matrix} \qquad \begin{cases} 2x-y &= 2a\\ y &= 2b\\ -2x+2z &= 0\\ -4x+y+2t &= 0\\ -2x+2w &= 0\end{cases}$$

$$\begin{matrix} L3\leftarrow -\frac{1}{2}L3\\ L4\leftarrow -L4\\ L5\leftarrow -\frac{1}{2}L5\end{matrix} \qquad \begin{cases} x-z &= 0\\ 4x-y-2t &= 0\\ x-w &= 0\end{cases}$$

5.

$$V_1=\begin{pmatrix}1\\0\\1\\1\\1\end{pmatrix} V_2=\begin{pmatrix}1\\1\\0\\1\\1\end{pmatrix} V_3=\begin{pmatrix}1\\1\\1\\0\\1\end{pmatrix}$$

$$Vect(V)=\{\begin{pmatrix}1\\0\\1\\1\\1\end{pmatrix}a+\begin{pmatrix}1\\1\\0\\1\\1\end{pmatrix}b+\begin{pmatrix}1\\1\\1\\0\\1\end{pmatrix}c,(a,b)\in R^2\}$$

$$\begin{cases} x &= a+b+c \\ y &= b+c \\ z &= a+c \\ t &= a+b \\ w &= a+b+c \end{cases}$$

$$\begin{matrix} L1 \leftarrow L1-L2 \\ L2 \leftarrow L2 \\ L3 \leftarrow L3 \\ L4 \leftarrow L4 \\ L5 \leftarrow L5-L4 \end{matrix} \qquad \begin{cases} x-y &= a \\ y &= b+c \\ z &= a+c \\ t &= a+b \\ w-t &= c \end{cases}$$

$$\begin{matrix} L1 \leftarrow L1 \\ L2 \leftarrow L2-L5 \\ L3 \leftarrow L3-L1 \\ L4 \leftarrow L4-L1 \\ L5 \leftarrow L5 \end{matrix} \qquad \begin{cases} x-y &= a \\ y+t-w &= b \\ -x+z+y &= c \\ -x+y+t &= b \\ -t+w &= c \end{cases}$$

$$\begin{matrix} L1 \leftarrow L1 \\ L2 \leftarrow L2 \\ L3 \leftarrow L3-L5 \\ L4 \leftarrow L4-L2 \\ L5 \leftarrow L5 \end{matrix} \qquad \begin{cases} x-y &= a \\ y+t-w &= b \\ -x+y+z+t-w &= 0 \\ -x+w &= 0 \\ -t+w &= c \end{cases}$$

$$\begin{matrix} L3 \leftarrow L3-L4 \\ L4 \leftarrow -L4 \end{matrix} \qquad \begin{cases} y+z+t-2w &= 0 \\ x-w &= 0 \end{cases}$$