Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Final Examination, Fall 2018 — **Level-2 Term-II**
Course Code: CSE 2207 — **Course Title: Numerical Methods**
Time: 03 (Three) hours — **Full Marks: 210**

N.B. (i) Answer any three questions from each PART (ii) Use separate answer script for each PART
(iii) Marks allotted are indicated in the margin (iv) Symbols have their usual meanings

PART A

(Answer any three questions)

1. a) Briefly explain absolute, relative and percentage error. The report said the car-park held 220 cars but we counted 240 cars in the parking spaces. Find its percentage error and significance. **5+5**

b) Sum the following numbers: **10**

0.1532, 15.45, 0.000354, 305.1, 8.12, 143.3, 0.0212, 0.643 and 0.1734

Where in each of which all the given digits are correct. Determine its absolute error.

c) What are the differences between an open method and a breaking method? Obtain a root, of the following equations using the bisection method, $x^3 - 4x - 9 = 0$. Note that you must show the details calculation of the first iteration. **5+10**

2. a) Describe Newton-Raphson method. Find a root of the equation, $\mathbf{x \sin x + \cos x = 0}$ by initial guesses of $x_0=\pi$, using this Newton-Raphson method. **5+15**

b) Use the Secant Method to estimate the root of the equation, $x^2 - 4x - 10 = 0$, with the initial estimates of $x_1=4$ and $x_2=2$. **15**

3. a) Write the necessity for Pivoting. Given the system of equations **5+10**

$-3x_2+7x_3=2$
$x_1+2x_2-x_3=3$
$5x_1-2x_2=2$

Use Gauss elimination with pivoting to solve for the x's. Give reason for pivoting solution.

b) Solve the following system of equations, **20**

$x_1+7x_2-4x_3=-51$
$4x_1-4x_2+9x_3=62$
$12x_1-x_2+3x_3=8$

by the method of LU decomposition without pivoting. Where L is unit lower triangular and U is upper triangular matrix.

4. a) Find the best values of a_0 and a_1 by using the method of least squares to fit a straight line of the form $Y = a_0 + a_1 x$ to the following data: **10+5**

x_i	1	2	3	4	5
y_i	0.60	2.40	3.50	4.80	4.70

Also, determine the correlation coefficient (cc). Comment on the merit of the fitted model.

b) Briefly explain the necessity of Linearization. Show Linearization of the following nonlinear equations, **4+6**

i) $y = ax + \frac{b}{x}$ ii) $y = ab^x$ iii) $y = ae^{bx}$

c) Using the method of least squares find constants a and b such that the function $y = ae^{bx}$ fits the following data: (1.0, 2.473), (3.0, 6.722), (5.0, 18.274), (7.0, 49.673), (9.0, 135.026). Note that you can use linearization exponential equation of b (iii) directly. **10**

PART B

(Answer any three questions)

5. a) What is an Interpolation? Define Forward and Backward Differences. Why interpolation is needed? **7**

b) Estimate the value of Sin θ at θ =35° using **20**

a. Netwon's Forward Difference formula
b. Netwon's Backward Difference formula

with the help of the following table

θ	10	20	30	40	50
Sin θ	0.1736	0.3420	0.500	0.6428	0.7660

c) Find y(2) from the following data using Lagrange's Interpolation Formula. **8**

x	0	1	3	4	5
y	0	1	81	256	625

6. a) From the following table of values of x and y, obtain dy/dx and d^2y/dx^2 for x=1.2 **15**

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.953	6.0496	7.3891	9.025

b) Find the value of x for which y is minimum and find the minimum value of y from the following table: **10+10**

x	0.60	0.65	0.70	0.75
y	0.6221	0.6155	0.6138	0.6170

7. a) Derive the general formula for numerical integration from the Newton's forward difference formula. Also derive Trapezoidal rule by setting n=1 at this general formula. **10+5**

b) Calculate the value of the integral, $I = \int_0^6 \frac{1}{(1+x)^2} dx$ by taking six equidistant ordinates, using the following rules, **20**

(i) Trapezoidal rule,
(ii) Simpson's 1/3 rule,
(iii) Simpson's 3/8 rule and
(iv) Weddle's rule.

8. a) Given the differential equation, $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial condition y=0 when x=0, use Picard's method to obtain y for x=0.25, and 0.5. **15**

b) Applying Runge-Kutta method of (i) Second order and (ii) Fourth order to find an approximate value of $y(0.1)$ of $\frac{dy}{dx} = -y$, with the initial condition, $y(0) = 1$. Show steps. **20**