Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Final Examination, Fall 2018 **Level-2** **Term-I**

Course Code: MATH 2145 **Course Title: Math-III(Vector Analysis, Matrices and Fourier Analysis)**

Time: 03 (Three) hours **Full Marks: 210**

N.B. (i) Answer any three questions from each PART (ii) Use separate answer script for each PART
(iii) Marks allotted are indicated in the margin (iv) Symbols have their usual meanings

PART A

1. a) Define cross product. Prove that the area of a parallelogram with sides $\vec{A}$&$\vec{B}$ is $|\vec{A}\times\vec{B}|$. 10

   b) Find the volume of parallelepiped if $\vec{a}=2\hat{\imath}-4\hat{\jmath}-2\hat{k}$, $\vec{b}=4\hat{\imath}+6\hat{\jmath}+2\hat{k}$ and $\vec{c}=10\hat{\jmath}-4\hat{k}$ are the three co-terminus edges of the parallelopiped. 10

   c) If $\vec{a}$ and $\vec{b}$ are non-collinear vectors where, $\vec{A}=(x+4y)\vec{a}+(2x+y+1)\vec{b}$ and $\vec{B}=(y-2x+2)\vec{a}+(2x-3y-1)\vec{b}$, then find x & y such that $2\vec{A}=3\vec{B}$. 15

2. a) Find the directional derivative of $\frac{1}{|\vec{r}|}$ in the direction $\vec{r}$, where $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$. Find the greatest rate of increase of $\frac{1}{|\vec{r}|}$. 20

   b) If a vector field is given by $\vec{F}=(x^2-y^2+x)\hat{\imath}-(2xy+y)\hat{\jmath}$. Is this field irrotational? If so, find its scalar potential. 15

3. a) A particle moves so that its position vector is given by $\vec{r}=\cos\omega t\,\hat{\imath}+\sin\omega t\,\hat{\jmath}$ where, ω is a constant. Show that (i) the velocity $\vec{V}$of the particle is perpendicular to $\vec{r}$.(ii) the acceleration $\vec{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin. (iii) $\vec{r}\times\vec{V}$is a constant vector. 20

   b) If $\vec{V}=\vec{\omega}\times\vec{r}$ then prove that $\vec{\omega}=\frac{1}{2}curl\vec{V}$ where, $\vec{\omega}$ is a constant vector. 8

   c) If a force $\vec{F}=2x^2y\hat{\imath}+3xy\hat{\jmath}$ displaces in the xy plane from $(0,0)$ to $(1,4)$ along a curve $y=4x^2$ find the work done. 7

4. a) Apply divergence theorem to evaluate $\iint_s \vec{F}\cdot\hat{n}\,ds$, where $\vec{F}=4x^3\hat{\imath}-x^2y\hat{\jmath}+x^2z\hat{k}$ and s is the surface bounded by the region $x^2+y^2=a^2, z=0$ and $z=b$. 20

   b) State Green's theorem. Apply Green's theorem to evaluate $\oint_c[(2x^2-y^2)dx+(x^2+y^2)dy]$ where, c is the boundary of the area enclosed by the x-axis and the upper half of circle $x^2+y^2=a^2$. 15

PART B

5. a) Define orthogonal matrix. Determine the values of α,β,γ when $\begin{bmatrix}0 & 2\beta & \gamma\\ \alpha & \beta & -\gamma\\ \alpha & -\beta & \gamma\end{bmatrix}$ is orthogonal. 15

   b) Find the inverse of the matrix, $A=\begin{bmatrix}1 & 2 & 3\\ 5 & 6 & 7\\ 1 & 0 & 2\end{bmatrix}$. 20

6. a) Define matrix with example. The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number we get 12.Then find the numbers. 2

b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ -1 & -3 & -4 & -3 \end{bmatrix}$. 15

7. a) Determine the Fourier constant and co-efficient $a_0,\ a_n,\ b_n$ within the period 2π in the interval $(-\pi,\ \pi)$. 15

b) Find the Fourier series for the function $F(x) = x^2, -\pi \leq x \leq \pi$. 20

8. a) If $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$, then prove that $\int_0^\infty \frac{(\sin x - x\cos x)}{x^3} \cos\frac{x}{2}\, dx = \frac{3\pi}{16}$ 20

b) Define Fourier transform. Find the Fourier cosine transform of $F(x)$ where, 15

$$F(x) = \begin{cases} \cos x, & 0 < x < a \\ 0, & x > a \end{cases}$$