

Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Referred/Improvement/Backlog Examination, Winter 2018-2019

Level-1 Term-I

Course Code: MATH 1141

Course Title: Math-I (Differential and Integral Calculus)

Time: 03 (Three) hours

Full Marks: 210

N.B. (i) Answer any three questions from each PART

(ii) Use separate answer script for each PART

(iii) Marks allotted are indicated in the margin

(iv) Symbols have their usual meanings

PART A

1. a) Define absolute value. If $x, y \in \mathbb{R}$, then prove that $|xy| = |x||y|$. 10
b) A function $f(x)$ is defined as follows: 10
$$f(x) = \begin{cases} x^2 & \text{when } x < 1 \\ 2.5 & \text{when } x = 1 \\ x^2 + 2 & \text{when } x > 1 \end{cases}$$
Does $\lim_{x \rightarrow 1} f(x)$ exist?
c) Find differential coefficients of $\sin x$ and a^x by using 1st principle rules. 15
2. a) Find the 5th derivative of the function $y = x^4 \log x$. 10
b) If $y = \log(x + \sqrt{a^2 + x^2})$, then show that $(a^2 + x^2)y_2 + xy_1 = 0$. 12
c) State Rolle's theorem. Justify Rolle's theorem of the function $f(x) = x^2$ in the interval $(-1, 1)$. 13
3. a) State L'Hospital's theorem. Evaluate $\lim_{x \rightarrow \infty} \frac{x^4}{e^x}$. 13
b) If $u = e^{xyz}$, then prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$. 10
c) If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$, then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. 12
4. a) Find the equation of the tangent at (x, y) to the curve $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 1$. 10
b) Show that the minimum value of $4e^{2x} + 9e^{-2x}$ is 12. 12
c) Define center of curvature. Find the center of curvature at any point (x, y) on the parabola $y^2 = 4ax$. 13

PART B

5. a) Integrate (i) $\int \cos^2 x \, dx$ and (ii) $\int \frac{\cos x \, dx}{(a + b \sin x)^2}$. 15
b) Prove that $\int e^{ax} \sin bx \, dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$. 10
c) Integrate $\int \frac{x^2 \, dx}{(x+1)^2(x+2)}$. 10
6. a) Obtain a reduction formula for $\int \sec^n x \, dx$. Also, evaluate $\int \sec^6 x \, dx$. 15
b) Show that $\int_0^{\log 2} \frac{e^x}{1+e^x} \, dx = \log \frac{3}{2}$. 10
c) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} \, dx = \frac{\pi}{8} \log 2$. 10

7. a) Prove that $\int_0^1 x^6 \sqrt{1-x^2} dx = \frac{5\pi}{256}$. 10
- b) Define gamma function. Prove that $\int_0^\infty e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$. 12
- c) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^\pi \cos(x+y) dx dy$. 13
- 8 a) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 10
- b) Find the length of the arc of the parabola $y^2 = 4ax$ measured from the vertex to one extremity of the latus rectum. 15
- c) Find the volume of the solid generated by revolving the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ about its base. 10