Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Referred/Improvement/Backlog Examination, Fall 2018 **Level-1 Term-II**
Course Code: CSE 1201 **Course Title: Discrete Mathematics**
Time: 03 (Three) hours **Full Marks: 210**

N.B. (i) Answer any three questions from each PART (ii) Use separate answer script for each PART
(iii) Marks allotted are indicated in the margin (iv) Special Instruction (if any)---------N/A----------

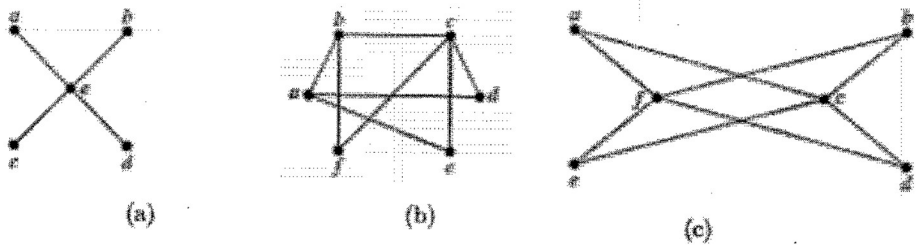
PART A

(Answer any three questions)

1. a) Define with example: Tree, Ancestors of a node, Descendant of a node, Depth of a node, Height of a tree, Binary Search Tree. **(9)**

   b) Differentiate between theory and theorem. Define injective function, surjective function, and bijective function with examples. **(3+8=11)**

   c) Show that a simple graph is bipartite if and only if it is a bi-chromatic graph. Determine whether graph (a),(b) and (c) is bipartite or not. **(6+9=15)**



(a) (b) (c)

2. a) Define: Universal and Existential quantifier. Let Q(x) be the statement "x < 2." What is the truth value of the quantification ∀x Q(x), where the domain consists of all real numbers? **(2+7=9)**

   b) Show that, $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$. Discuss whether → operator is associative or not. **(7+4=11)**

   c) i) Define Rules of inference, Modus Ponens and Modus Tollens with examples. **(6+9=15)**

   ii) Prove the statement "The square of an even number is even" using indirect proof technique.

3. a) Define identity function and inverse function with example. **(7)**

   b) i) Given, $f(x) = 3x - 2$ and $g(x) = \frac{x}{3} + \frac{2}{3}$, show that (f∘g)(x)=(g∘f)(x)=x. **(5+5=10)**

   ii) Given, $f(x) = \frac{x+4}{2x-5}$ find, $f^{-1}(x)$.

   c) Suppose , A = {x: x is an even number less than 10} ,B = {1, 3, 5, 7, 9}, C= { $x \in Z^+$ | x is divide by 3 and x < 10 } and U={ $x \in Z^+$, x < 10} then derive the below sets: **(3x6=18)**

   i) (AUB)∩(AUC)

   ii) $\overline{(AUB) \cap \overline{C}}$

   iii) Prove that : $\overline{(BUC)} = \overline{B} \cap \overline{C}$

4. a) State the converse, contrapositive, and inverse of each of these conditional statements. **(3x3=9)**

   i) If today is Friday, then today is my birth day.

   ii) I come to class whenever there is going to be a class test.

   iii) If a number has no divisors other than 1 and itself, then it is a prime number.

   b) What is spanning tree? Construct a decision tree that orders the elements of the list *a*, *b*, *c*. **(3+9=12)**

c) Form a binary search tree for the set A={12, 20, 6, 7, 10, 13, 15, 34, 5, 8}.Then answer the below questions: **(5+9=**

i) What is the depth of node 34 in the tree?

ii) What is the height of the tree?

iii) Write the pre-order, in-order, post-order traversal of the tree.

PART B

(Answer any three questions)

5. a) What is golden ratio and how can we get the value of this ratio using Fibonacci sequence and Geometric figure? **(2+6=8)**

b) Use mathematical induction to prove that the sum of the squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$. **(6)**

c) Find the sum of below series : **(3x4=12)**

i) $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots+\frac{1}{59049}$

ii) $1+4+9+16+\cdots+1024$

iii) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\infty$

d) Define recurrence relation. Find a recurrence relation and initial conditions for 1, 1, 2, 3, 5, 8….. . **(2+7=9)**

6. a) Define with example: Relation, Reflexive, Irreflexive, Symmetric, Asymmetric, Antisymmetric, and Transitive. **(7x2=14)**

b) Define the following: (i) Full m-ary tree (ii) Balanced m-ary tree (iii) Complete m-ary tree. Give one example of each type of tree of height 4. **(3x4=12)**

c) Define Binomial theorem. Expand the $(x+y)^{25}$ and then find the coefficient of $x^{13}y^{12}$ from the expansion. **(2+7=9)**

7. a) Define with figure: Directed Graph, Bipartite Graph, and Bichromatic Graph. **(3x2=6)**

b) What is the probability that when two dice are rolled, the sum of the numbers on the two dice is less than 6? **(6)**

c) How many vertices does a full 5-ary tree with 100 internal vertices have? **(6)**

d) Each of the letters of the word MATHEMATICS are written on separate pieces of paper that are then folded, put in a hat, and mixed thoroughly. One piece of paper is chosen (without looking) from the hat. What is the probability it is a 'M' or a 'T'? **(6)**

e) Throw a dice 2 times. What is the probability of getting two different sides? **(6)**

f) How many bit strings of length three will have two consecutive 1s? **(5)**

8. a) Define: Monoid, Ring and Field. **(6)**

b) Show that <Z,*> is NOT a group. Draw the table of a group of order 3 is |G|=3 is: G=<{e,a,b},*>. **(6+6=12)**

c) Define with example: Tautology, Contradiction and Contingency. How can this English sentence be translated into a logical expression? "You can't ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old." **(6+6=12)**

d) Prove that there must be infinitely many prime numbers. **(5)**