Bangladesh Army University of Science and Technology

Department of Computer Science and Engineering

Final Examination, Winter 2018-2019 **Level-1 Term-I**
Course Code: MATH 1141 **Course Title: Math-I(Differential and Integral Calculus)**
Time: 03 (Three) hours **Full Marks: 210**

N.B. (i) Answer any three questions from each PART (ii) Use separate answer script for each PART
(iii) Marks allotted are indicated in the margin (iv) Symbols have their usual meanings

PART A

1. a) What is a function? If $f(x) = x^2 + 3$ and $g(x) = \sqrt{x}$, then find $f \circ g(x)$ and $g \circ f(x)$. 10

b) Define limit of a function. A function $f(x)$ is defined as follows: 10

$$f(x) = \begin{cases} x^2 & when\, x < 1 \\ 2.5 & when\, x = 1 \\ x^2 + 2 & when\, x > 1 \end{cases}.$$

Does $\lim\limits_{x \to 1} f(x)$ exist?

c) Define continuity of a function. If $f(x) = \begin{cases} x & when\, 0 \le x < \frac{1}{2} \\ 1 - x & when\, \frac{1}{2} \le x < 1 \end{cases}$, then show that $f(x)$ is continuous but not differentiable at $x = \frac{1}{2}$. 15

2. a) Find the 5th derivative of the function $y = x^4 \log x$. 10

b) State Leibnitz's theorem. If $y = \log(x + \sqrt{a^2 + x^2})$, then show that $(a^2 + x^2)y_2 + xy_1 = 0$. 12

c) State Mean Value theorem. Discuss the geometrical interpretation of Mean Value theorem. 13

3. a) State L'Hospital's theorem. Evaluate $\lim\limits_{x \to 0} (\frac{\tan x}{x})^{\frac{1}{x}}$. 13

b) If $u = e^{xyz}$, then prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 + y^2 + z^2)e^{xyz}$. 10

c) State Euler's theorem. If $u = \sin^{-1}(\frac{x^2+y^2}{x+y})$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. 12

4 a) Define tangent. Find the equation of the tangent at the point (x, y) to the curve $y = f(x)$. 10

b) Find the maximum and minimum values of u, where $u = \frac{4}{x} + \frac{36}{y}$ and $x + y = 2$. 12

c) Define center of curvature. Find the center of curvature at any point (x, y) on the parabola $y^2 = 4ax$. 13

PART B

5. a) Integrate (i) $\int \frac{e^x dx}{e^{2x} + 2e^x + 5}$ and (ii) $\int \tan x \, dx$. 15

b) Prove that $\int e^{ax} \sin bx\, dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2+b^2}$. 10

c) Integrate $\int \frac{dx}{(x-1)^3(x+1)}$. 10

6. a) Obtain a reduction formula for $\int \sec^n x\, dx$. Also, evaluate $\int \sec^6 x\, dx$. 15

b) Evaluate $\int_0^a \sqrt{a^2 - x^2} dx$. 10

c) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$. 10

7. a) Prove that $\int_0^{\frac{\pi}{2}} \cos^3 x \sqrt[4]{\sin x}\, dx = \frac{32}{65}$. 10

b) Define beta and gamma function. Prove that $\int_0^\infty e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$. 12

c) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^b (y^2 + z^2) dz dy dx$. 13

8 a) Find the area above the x-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$. 10

b) Find the length of the arc of the parabola $y^2 = 4ax$ measured from the vertex to one extremity of the latus rectum. 15

c) Find the volume of the solid generated by revolving the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ about its base. 10