INE308 Stochastic Operations Research **Technical Assignment** **Spring 2025**
Assignment Date: 21 March 2025 **Due Date:** 22 April 2025
Preparation: Form teams of no more than 5 students. You will loose 10% of the grade for each extra joined member.

The goal of this technical assignment is to analyze the operational dynamics of a bank using Markov Chains, where the state of the system is observed hourly during business hours (8:00 AM to 4:00 PM).

Part 1: Markov Chain Model

Consider the following states to be observed hourly in the bank: 0=No customers in queue, 1=Few customers(1-3), 2=Moderate queue (4-7 customers), 3=Long queue (>7 customers) (Special State), 4=Idle period (no transactions, staff handling paperwork). At the same time track the number of tellers serving clients at every hour of the day. Thus, depending on the maximum number of tellers that work in the bank, you will record how many are actually busy providing actual service to the bank clients. Do not include those tellers who are busy doing other tasks than direct service to the clients. Also measure how long each teller takes to complete a transaction.

Step 1: (20 points) Collect data hourly over several days (at least 5 working days) to estimate transition probabilities between states. The data can be recorded by direct observation, transaction logs, or employee reports. Then construct the transition probability matrix.

Step 2: (20 points) Using your model in the previous step, answer the following questions:
1- Compute the probability distribution that starting with an empty queue when the bank opens until it reaches a long queue state within 3 hours, 4 hours, 5 hours, etc. until the end of the day.
2- Calculate the probability of having a long queue at 12:00 PM; i.e., lunch time.
3- Determine the expected number of times a long queue is observed throughout the working hours in one day.
4- Compute the expected time spent in long queue state.
5- Provide the long-run probabilities of each state.
6- Calculate the expected number of steps before first attaining long queue from the initial state.
7- Find the expected time before returning to a long queue state after leaving it.

Part 2: Markov Process Model

Step 1: (10 points) Remodel the above system as Markov process by computing the rates of transition between the states using the formula below and then provide the transition rate matrix:

$$\lambda_{ij} = \frac{total\ number\ of\ transitions\ (i.e., jumps) from\ state\ i\ to\ state\ j}{total\ duration\ in\ hours\ staying\ at\ state\ i}$$

Step 2: (20 points) Using your model that you developed in the previous step, answer the following questions:
1- Find the steady state probabilities for all states.
2- Find the mean sojourn times for each state.
3- Find the mean first passage time from long queue state to each other state.

Part 3: Birth-Death Process:

In this model, the states represent the number of active tellers serving customers at any given moment, where transitions occur as customers arrive and complete their transactions. Define states based on the number of tellers currently occupied: 0=No tellers are occupied (idle state), 1=One teller occupied,2=Two tellers occupied, etc. up to N=all tellers occupied (maximum capacity), given that N is the maximum number of tellers who provide direct transaction to clients.

Step 1: (10 points) Measure how long each teller takes to complete a transaction and assume that the service times have an exponential distribution with the average service time per teller. Construct the transition rate matrix for analysis.

Step 2: (20 points) Compute transition rates based on observed data using similar formula as in Part 2.
1- Determine the long-run probabilities of each state to assess teller utilization.
2- Compute the likelihood that all tellers are busy at 12:00 PM.
3- Expected Time in Each State: Estimate how long the system stays in each state before transitioning.
4- Calculate the expected time before all tellers are occupied for the first time.
5- Analyze how increasing the number of tellers affects overall service efficiency and congestion probabilities.
6- Determine how often the system reaches full capacity during the working day.

Report Writing: Provide a detailed and well-organized report that includes:

1- Short description of how you executed each of the steps in each Part
2- Answers for all required questions
3- **Your own assessment, evaluation and judgement of all the results you obtained within realistic and factual perspectives**.