LAB III: PHY-2020-1 (SPRING'23)

Measuring the Speed of Light

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Abstract

The speed of light is one of the fundamental constants of the universe. It is the maximum rate at which information can travel. The special theory of relativity has at its heart the postulate that the speed of light remains constant in any inertial reference frame. Light is one of the ways for us to gain information about the universe and its properties. Thus, light plays a fundamental role in the structure and advancement of all of physics. In this experiment I have attempted to measure the speed of light in air. I have done so by generating a time delay between two parts of the same original pulse of light by forcing them to travel different distances.

Theoretical Background

Electromagnetic Waves

James Clerk Maxwell was the first to predict the existence of electromagnetic waves through the famous Maxwell equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3}$$

$$\nabla \times \mathbf{B} = \frac{\mathbf{J}}{c^2 \epsilon_0} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \tag{4}$$

The speed of propagation of electromagnetic waves was found to be the same as the estimates of the speed of light of the time. This led him to predict that light is a kind of electromagnetic wave. This is considered to be one of the most important steps taken in the advancement of physics.

Consider a x-polarised electromagnetic wave travelling through a vacuum, we get the following coupled partial differential equations:

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_0}\frac{\partial H_y}{\partial z} \tag{5}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z} \tag{6}$$

From the equations above we can get by taking the appropriate partial derivatives,

$$\frac{\partial^2 E_x}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0}\frac{\partial^2 E_x}{\partial z^2} \tag{7}$$

$$\frac{\partial^2 H_y}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 H_y}{\partial z^2} \tag{8}$$

Where, $v = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ gives the phase velocity of the wave which in this case will be c

Historical Context

First Measurement of the Speed of Light:

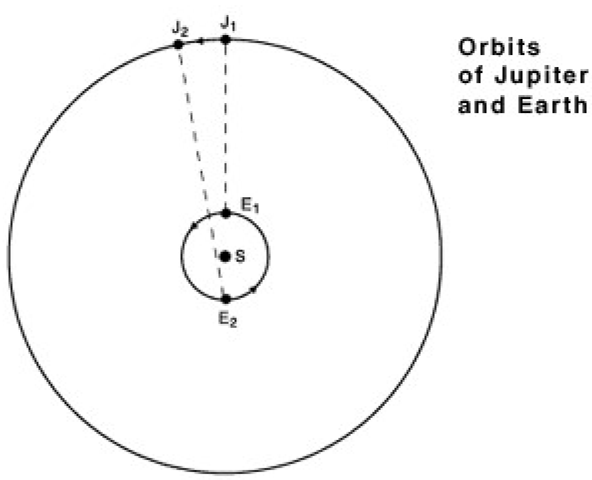


Figure 1: Schematic diagram illustrating the method used by Ole Roemer

The Danish astronomer Ole Roemer was the first to measure the speed of light. He was tracking the eclipses of Jupiter and its moon Lo. In each of its orbit around Jupiter, Lo is eclipsed once when seen from Earth. From the data collected, Roemer noticed That as the Earth moved through its orbit around the Sun, the distance between Jupiter and Earth changed and as the two plates came closer, the time interval between two successive eclipses kept on becoming shorter. He could thus predict that when the Earth was closest to Jupiter, the eclipse would occur eleven minutes earlier than predicted while when the Earth is at its farthest from Jupiter, the eclipse would occur eleven minutes before expected. To explain this he considered that the speed of light must be finite and thus the difference arises due to the light taking twenty two minutes to reach Earth. The value calculated from this method was, 2.66623×10^8 m/s.

Measuring the Speed of Light

The setup for the measurement of the speed tries to generate a path difference between two light pulses which are generated at the same time. The basic setup for the measurement is as follows:

A pulsed laser is used as the source of light, the pulses generated by it are then split into two pulses of the same intensity using a beam splitter angled at 45°. One of the beams is incident directly onto a photo-detector placed directly in line with the laser source. The other part of the pulse is reflected repeatedly using mirrors placed in a zig-zag manner and is then made to fall on another photo-detector after it has reflected several times. Thus the two parts of the same original pulse travel different distances at a constant speed (the speed of light) and are thus incident on the photo-detector at different times. The two photo-detectors are connected to a DSO which records the pulse incident on the photo-detector thus enabling us to get the difference in the times of the incidence of the peaks. Using this time difference and the difference in the distances travelled by the two pulses we can get the value of the speed of light using the simple formula:

$$c = \frac{\Delta x}{\Delta t} \tag{9}$$

Wave Pulses

A wave pulse is a wave localised in position. Such a pulse can be represented as the superposition of sinusoidal waves multiplied by particular amplitude terms. The amplitudes these terms are multiplied by can be given by a function such as a Gausian.

$$\psi(x,t) = \int_{-\infty}^{\infty} C(k)\cos(\omega t - kx)dk \tag{10}$$

Where,

$$C(k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{(k-k_0)^2}{2\sigma_k}\right)$$

This represents a wave pulse as shown in the image below:

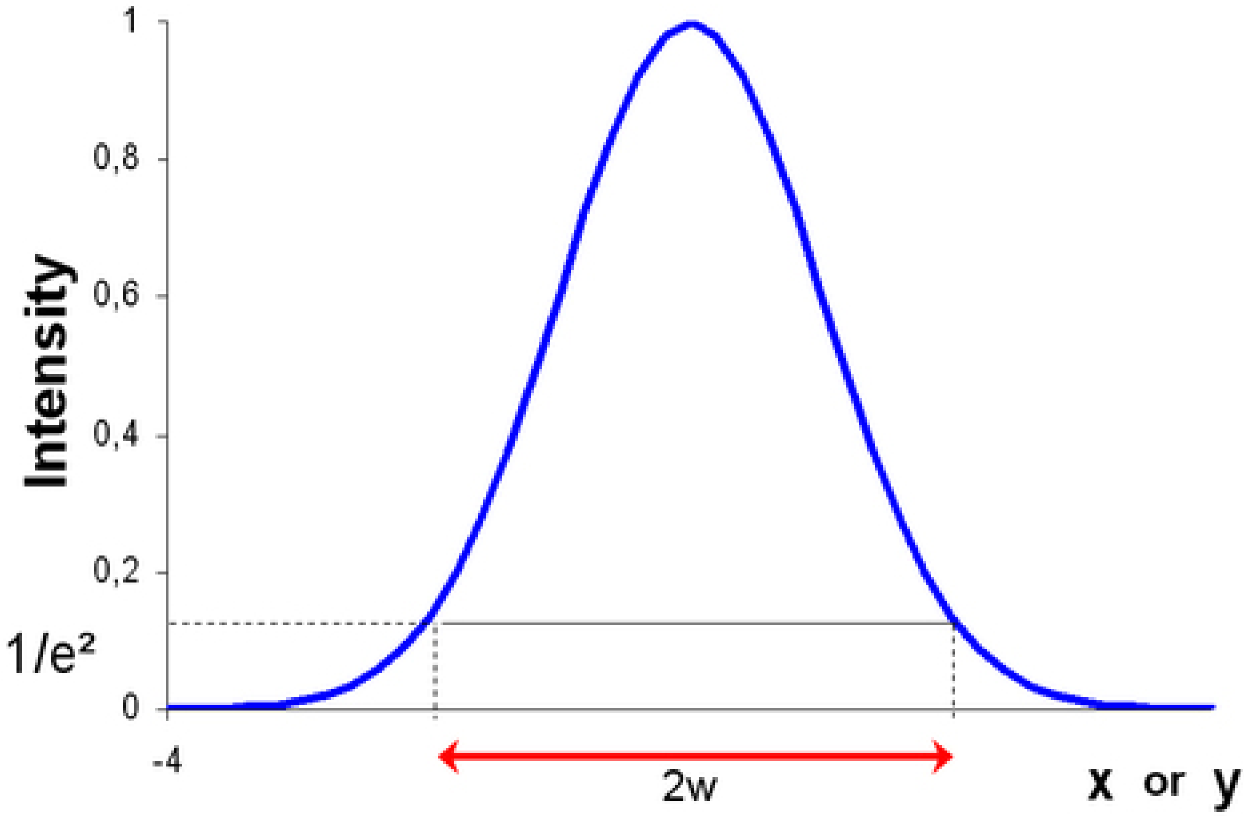


Figure 2: Gausian wave pulse

Here, use was made of a pulsed laser to get such pulses in the time domain. The value of voltage spiked with time every time a pulse of the laser was incident on the photo-detector showing a pulse such as the one above. The two photo-detectors gave two different pulses. The difference between the peaks of which enabled the measurement of the time delay between the two.

Apparatus

1. Pulsed Laser source
2. Laser source
3. Beam Splitter
4. Photodiodes
5. Photo-detector assembly
6. Mirrors (As many as possible)
7. Digital Oscilloscope
8. BNC cables
9. Optical bench
10. Rods and Stands
11. Measuring tape

Set-up and Analysis

Set-up 1

In the first attempt the pulsed laser was not used. In its stead, the continuous beam of the laser was split into pulses using a plastic fan as an opaque obstacle. The fan spun when the batteries were put in, giving rise to pulses. Along with this, in this setup two photodiodes were used instead of the photo-detector assembly. In this attempt four mirrors available in the lab with fine adjustment stands were used to reflect one part of the pulse. The pulses thus observed on the DSO were obtained.

Set-up 2

In this setup, new mirrors, made in the lab using large pieces of scrap back-coated mirrors found in the basement were used in addition to the fine adjustment mirrors available in the lab.

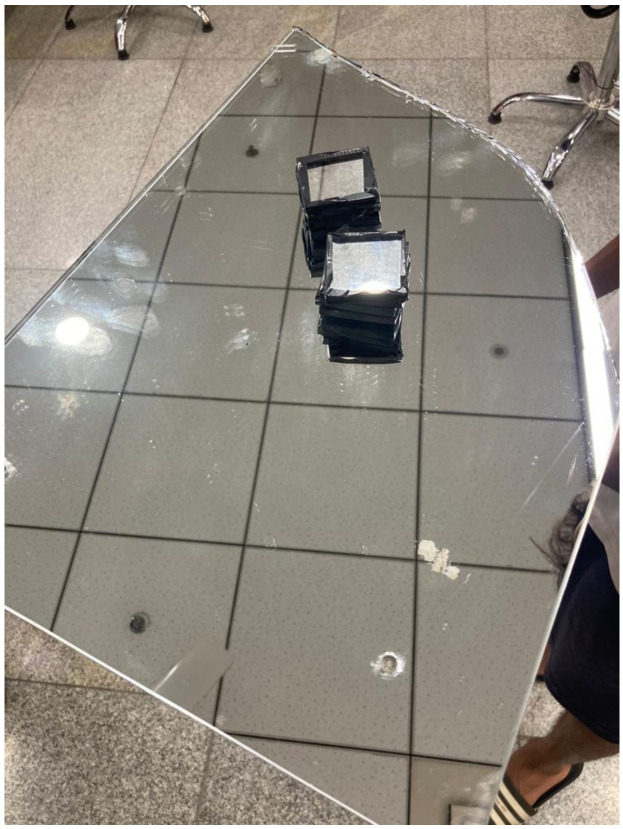


Figure 3: Picture of the mirrors made from the scrap found in the basement

This made the pulse reflect more times thus travelling a larger distance which in turn gives rise to a larger path difference ergo a larger time lag. The same two photodiodes as well as the continuous laser-plastic fan set-up were used to detect and generate pulses respectively. The pulses obtained on the DSO were as follows:

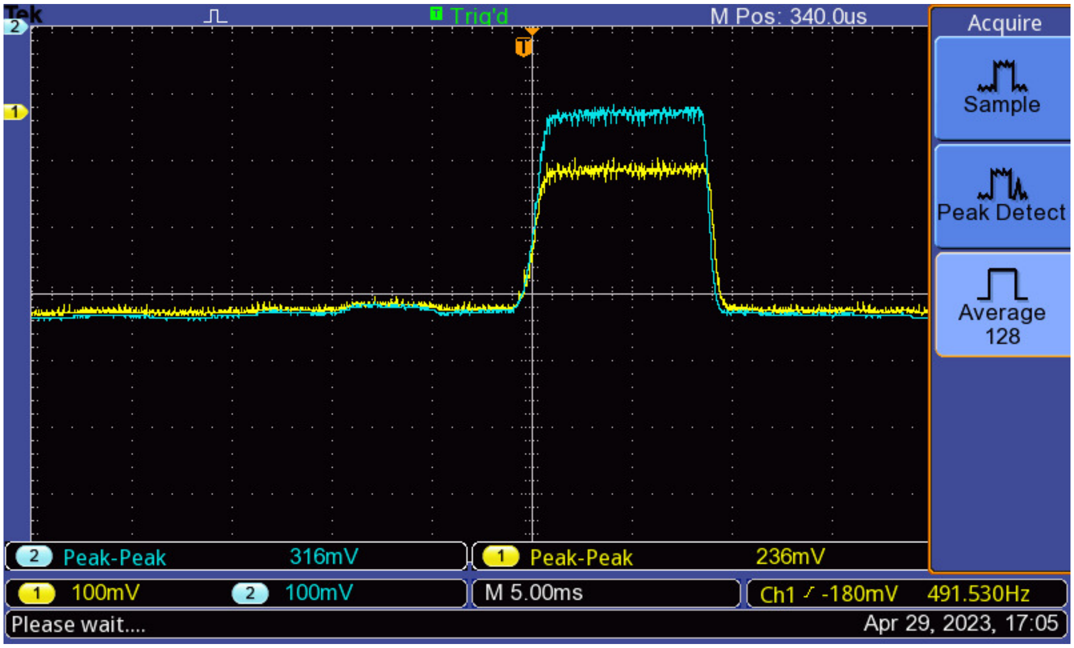


Figure 4: Pulses obtained in attempt 2

Set-up 3

In this setup, again both the fine adjustment mirrors and the newly made mirrors were used alond with the continuous laser, plastic fan and the two photodiodes. The change made to the set-up was that one line of mirrors in the zig-zag assembly was moved to the edge of the same optical table as the laser and the phodiodes. This significantly increased the distance the pulse travelled before falling on the photodiode. Thus generating a greater path difference. The pulses obtained on the DSO were as follows:

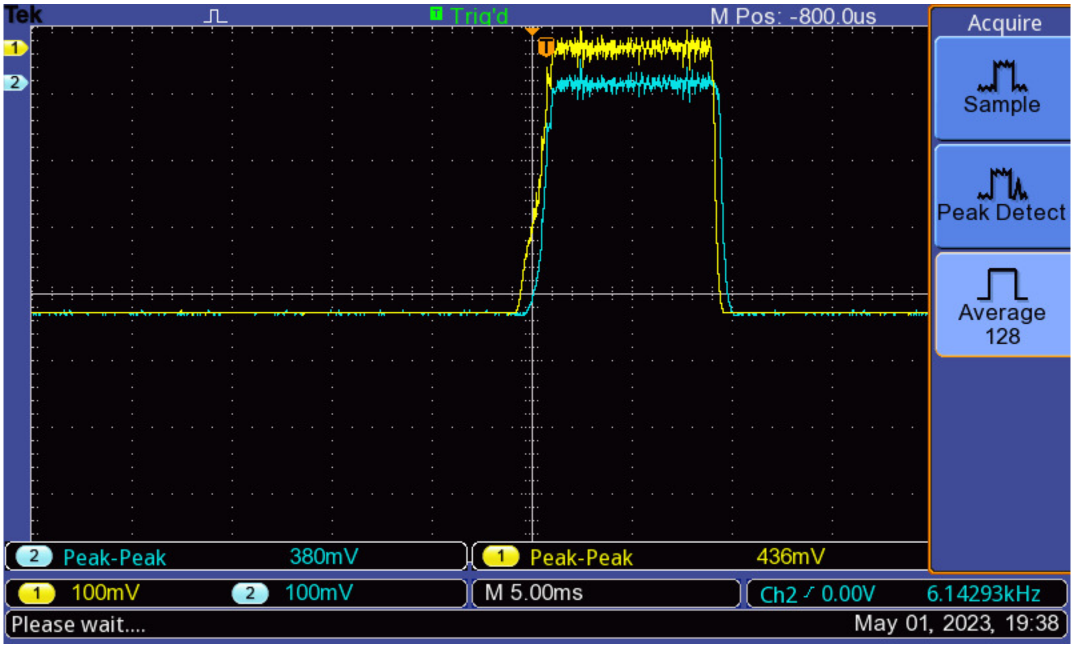


Figure 5: Pulses obtained in attempt 3

Set-up 4

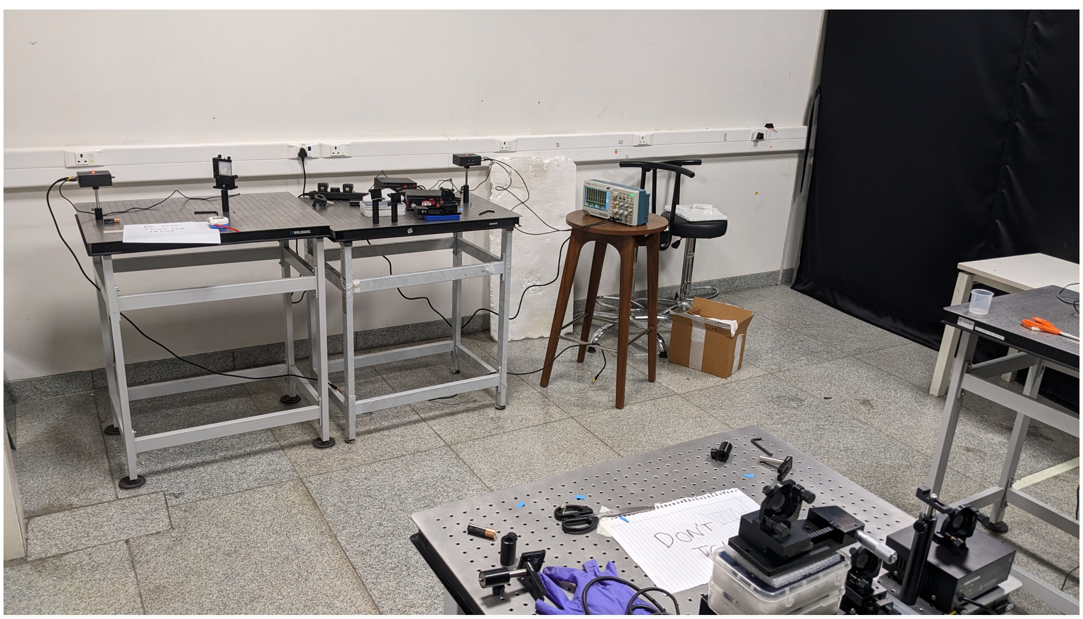
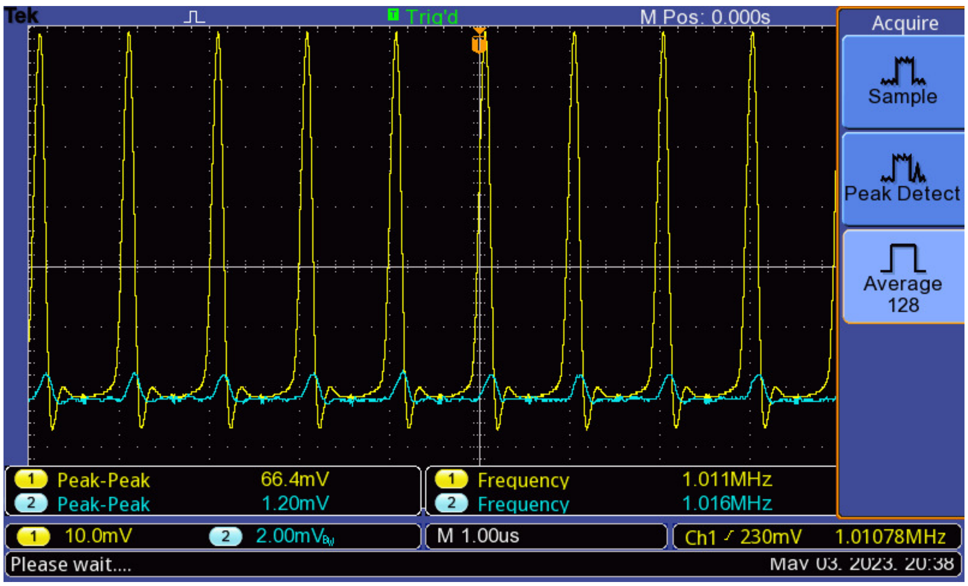
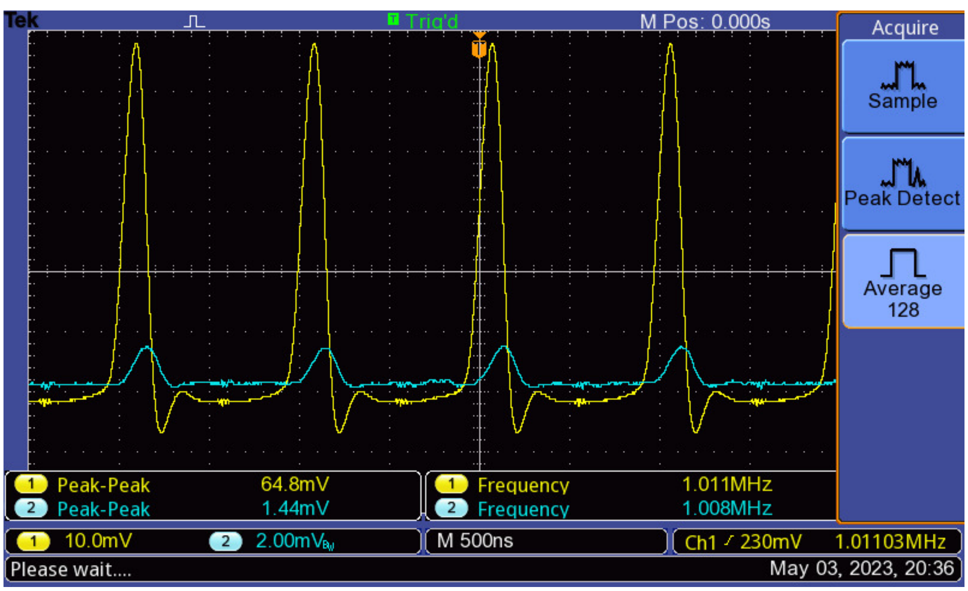


Figure 6: Picture of the set-up for attempt 4

In this attempt, the continuous laser was replaced by a pulsed laser and the two photodiodes were replaced by the photo-detector assembly. similar to Set-up 3, both the fine adjustment as well as the newly made mirrors were used and kept on the distant optical tables as before. This helped in getting well defined pulses when compared to the pulses obtained when the beam was being cut by the fan. The pulses obtained on the DSO were as follows:





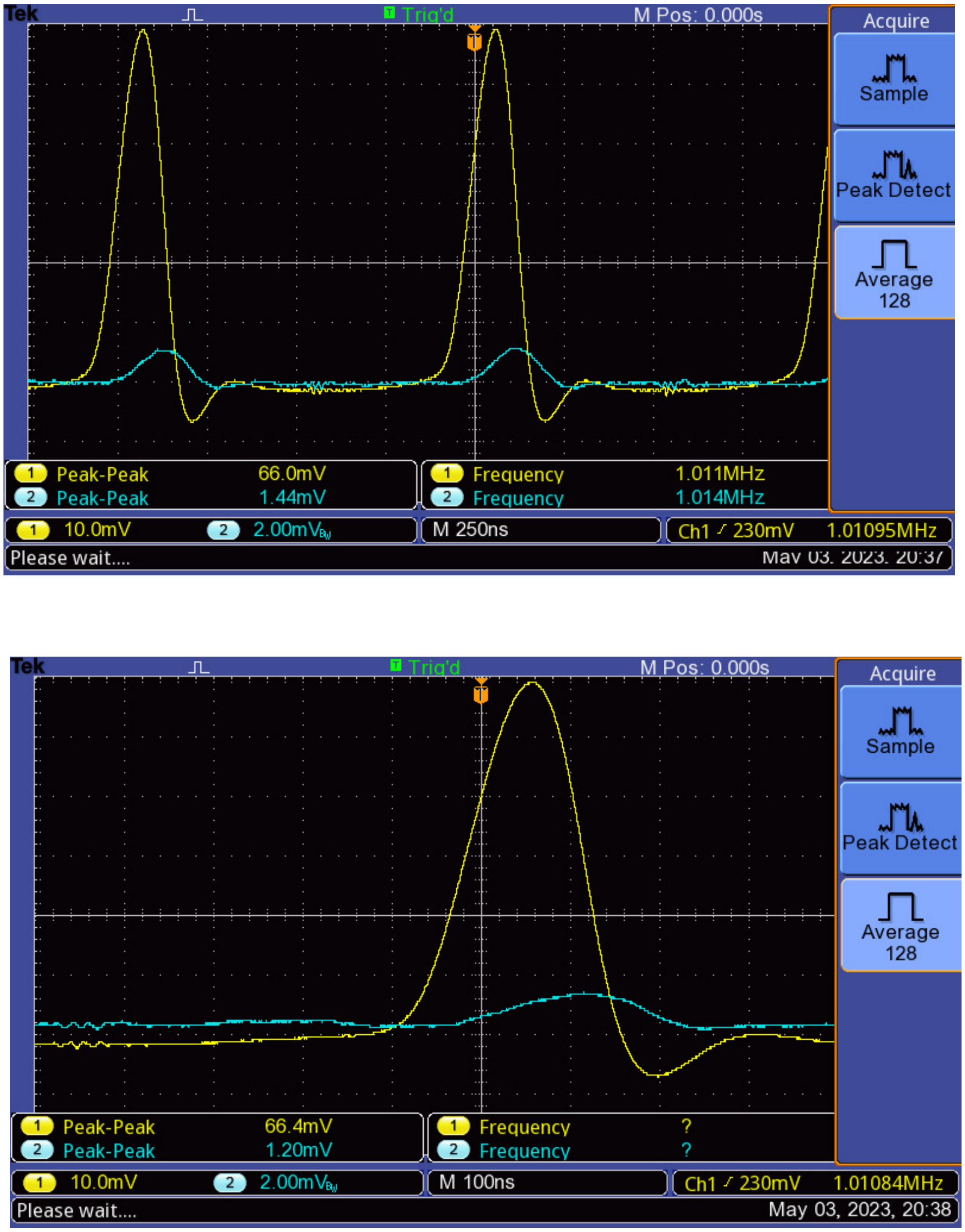


Figure 7: Screenshots from the DSO showing two pulses with a time delay between their respective peaks

Analysis

- Least Count of the Measuring Tape = 1 mm
- Least count of Length Measurement in ImageJ software = 0.26×10^{-3}mm

(which directly corresponds to the measurement of the time interval)

The values of the time lag is calculated using the difference between the times of two peaks of the two pulses that are detected in the DSO. This is done using the ImageJ software for image analysis. Using the values of Δt thus obtained are used for the calculation of the speed of light in air. The extra distance travelled by pulse (Δx) was measured to be 18.327 m

Δt (ns)	Speed of Light (in air) c ($10 \times^8$ ms^{-1})
55.91	3.28
50.17	3.65
52.94	3.46
64.24	2.85
65.19	2.81
60.00	3.05
56.68	3.23
66.45	2.76
71.01	2.58
75.43	2.43
70.87	2.59
75.43	2.43
75.43	2.43

Table 1: Table of the values of thetime lag between the two pulses and the corresponding value of c

Each value in the table represents the value of the speed of light calculated using each pair of pulses on the DSO. The measurement of the path travelled was done by calculating the path travelled by the pulse as it represented the diagonal of the triangle formed by the net vertical and horizontal displacement over the course of a reflection (expanded on in the discussion section).

Error Analysis

The propagated relative error in the calculation of the speed of light in air will be given by,

$$\frac{\Delta c}{c} = 2\frac{\Delta l}{l} + \frac{\Delta t}{t} \tag{11}$$

The factor of 2 appears the length travelled is calculated by squaring the actual measurements of lengths.

$$\frac{\Delta c}{c} = 2\frac{\Delta 0.001}{2.29} + \frac{\Delta 0.00026}{64.60}$$

$$\frac{\Delta c}{c} = 0.00088$$

$$\Delta c = 2.63 \times 10^5 \mathrm{ms}^{-1}$$

The mean value of c and the standard deviation are respectively 2.89×10^8 ms^{-1} and 0.40×10^8 ms^{-1}

The percentage error in the value obtained compared to the known value is:

$$\frac{3.00 - 2.89}{3.00} \times 100 = 3.67\%$$

This shows that the instruments used have a sufficiently low least count and thus are not the major contributors to the error in the values of the speed of light. The most significant source of error is the error introduced while conducting the image analysis where the exact start and end points of the length to be measured are specified with errors.

Result

The value of the speed of light in air (c) was found out to be:

$$c = 2.89 \pm 0.40 \times 10^8 \mathrm{~ms}^{-1}$$

Discussion

- The mirrors made from the scrap found in the basement were of a poor reflectivity thus the more the pulse was reflected, the dimmer it got. Thus the amplitude of the reflected pulse by the DSO was very low compared to the directly incident pulse as seen by the images.

- The mirrors made later did not have the fine adjustment screws etc. Thus it was difficult to exactly control where the reflected pulse would be incident. As in some cases the angle at which these mirrors were set would change as they would lower under their own weight. A way to bypass this was to place the mirrors with and without fine adjustment in an alternating manner in the zig-zag arrangement. Thus every other reflection could be adjusted to a very high degree. Along with this it was made sure that the first mirror to "catch" the pulse coming from the beam splitter and the mirror which made the last reflection sending the pulse to the photo-detector were ones with fine adjustment

- The plastic fan was a far inferior way of generating pulses than using a pulsed laser, as the speed of the fan was limited thus as well as the fact that it varied with time. Th fan got slower the longer it ran. The slower the pulses were, the more spread out the peaks became thus, the fan gave very imprecise and spread out peaks while those obtained using the pulsed lase were sharp.

- The mirrors used in the setup were not set up perpendicular to the ground level as the heights of the stands of the laser, the beam splitter and the mirrors were different. The mirrors were thus adjusted to various angles depending on where they were wanted to be incident.In order to dit a better length measurement, length was measured using the vertical height of a particular reflection by measuring the change in the vertical position of the dot of the laser pulse over the reflection under consideration and then measuring the horizontal distance travelled over the course of the reflection and then finding the path length travelled by the beam using the Pythagoras' theorem as the path was the diagonal of the right angle triangle thus formed.

References

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