ECE 345: Introduction to Control Systems
In-Class Exercise #3

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Due Thursday, October 18, 2012 at the end of class

1 Introduction

Hot air balloons are used in a variety of scientific missions and experiments. They gather data at different points in the atmosphere, provide initial testing of aeronautical and aerospace equipment, provide aerial surveys, and facilitate telecommunications and astronomical research. While in manned vehicles (e.g., for recreation), height control is accomplished through direct human control, in unmanned vehicles, automation is required to accomplish height control through feedback. While it is possible to simply send balloons into the atmosphere without any sophisticated feedback mechanisms, for tasks that require positioning the balloon at specific altitudes (which may be too high for human physiology), feedback is critical.

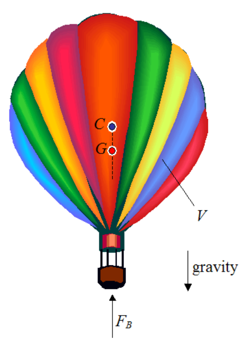


Figure 1: Forces acting a hot air balloon of volume V. The buoyancy force acts through the center of buoyancy C, and the force due to gravity acts through the center of mass G. Image reproduced from `http://www.real-world-physics-problems.com/hot-air-balloon-physics.html`.

We consider the problem of height control of a ballon of mass m. The forces acting on the balloon are gravity and lift, which is due to the buoyancy of the hot air trapped inside the balloon. Buoyancy occurs because the heated air inside the ballon is less dense than the cooler air outside. The input to the system is the net heat gained (through heat generated from a gas burner located below the balloon envelope) or lost (through a small opening at the top of the balloon canopy). The output of the system is the height of the balloon.

We model the lift as the sum of a steady-state force $F_{B_{ss}}$ (provided by a steady-state heat input q_{ss}) that is required to counter the force mg due to gravity, and the additional lift $F_B(t)$ that is required to make the balloon gain altitude. This allows us to essentially cancel the effect of gravity

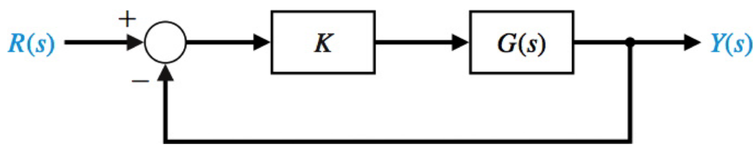


Figure 2: Closed-loop system under negative unitary feedback.

by removing the steady-state lift. We can then approximate the remaining dynamics of the balloon height $z(t)$ and temperature $T(t)$ with input $q(t)$. (Aside: Note when we remove the steady-state lift from the force equation, we also remove the steady-state input required to achieve steady-state lift. So that while the total input applied to the system is $q(t) + q_{ss}$, we focus for the remainder of the problem only on $q(t)$.)

$$\begin{aligned} m\ddot{z}(t) &= -\theta_1 \dot{z}(t) + \theta_2 T(t) \\ \dot{T}(t) &= -\theta_3 T(t) + \theta_4 q(t) \end{aligned} \qquad (1)$$

For a realistic hot-air balloon of a size similar to a recreational balloon, the constants are $\theta_1/m = \frac{m_g \rho g R}{p M_a m} = 0.0238$, $\theta_2/m = \frac{1}{2}\rho A C_D k/m = 0.2571$, $\theta_3 = 0.05$, and $\theta_4 = 0.5$ [1, 2, 3].

The goal of this exercise is to design a proportional feedback controller for the system and analyze the resulting closed-loop system's performance. *Each group should hand in the answers to the asterisk-marked problems (*).*

2 Pre-lecture work

1. Find the transfer function $G(s)$ in terms of the generic constants θ_i, $i \in \{1, \cdots, 4\}$.
2. Describe the asymptotic stability of the system, based on the location of the poles.
3. Use Matlab to plot the step response of the system.
4. What does the resulting plot tell you about the BIBO stability of the open-loop system?

3 In-class assignment

For all of the following questions in this Section, consider θ_i to be positive, known constants (but it is not necessary to evaluate with specific numerical values).

1. Show that the characteristic equation of the closed-loop system with generic gain K is

$$\Delta(s) = s^3 + \left(\frac{\theta_1}{m} + \theta_3\right) K \cdot s^2 + \frac{\theta_1}{m}\theta_3 \cdot s + \frac{\theta_2}{m}\theta_4 K \qquad (2)$$

We can rewrite this as

$$\Delta(s) = s^3 + a_2 s^2 + a_1 s + a_0 K \qquad (3)$$

where a_i are known and given, and K is as of yet unknown.

Since the third pole of a third-order system must always be on the real line (take a minute to convince yourself of this!), we can write the characteristic equation of a generic third-order system

$$0 = \Delta(s) = (s + \alpha)(s^2 + 2\zeta\omega_n s + \omega_n^2) \tag{4}$$

with the third pole located at $s = -\alpha$.

We wish to design a controller that does not require a great deal of actuator effort (to conserve energy for a long mission) and that is non-responsive to fast variations in sensor measurements of altitude (which can be erroneous). Hence the closed-loop system should have a response characterized by a damping ratio of $\zeta = 0.5$ for the complex pair of poles closest to the origin.

Using this value of ζ, equate the desired characteristic equation (4) to the actual characteristic equation (3) to find the value of K required to achieve the desired response.

*2. The value of K that achieves the desired response is given by

   (a) $K = \left(\frac{a_1}{a_2}\right)^2 \left(a_2 - \frac{a_1}{a_2}\right)$

   (b) $K = \frac{1}{a_0} \left(\frac{a_1}{a_2}\right)^2 \left(a_2 - \frac{a_1}{a_2}\right)$

   (c) $K = a_0 \left(\frac{a_2}{a_1}\right)^2 \left(a_1 - \frac{a_2}{a_1}\right)$

   (d) $K \geq \frac{1}{a_0} \left(\frac{a_1}{a_2}\right) \left(a_1 - \frac{a_1}{a_2}\right)^2$

*3. Which of the following best describes the stability of the closed-loop system with the chosen value of K? *(More than one response may be correct.)*

   (a) Asymptotically stable

   (b) Marginally stable

   (c) Unstable

   (d) BIBO stable

Now consider your results regarding asymptotic stability from Pre-class Problem #2 and In-class Problem #3.

*4. Examine the natural response of the open-loop and closed-loop systems, respectively, in the below two figures, to the initial condition $z(0) = 0$, $\dot{z}(0) = 0$, $T(0) = 5$. Why does the open-loop system converge to a different (non-zero) value than the closed-loop system? (Keep your answer brief; 1–2 sentences at most.)

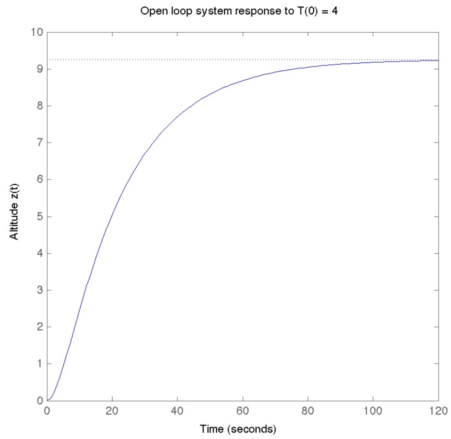
*5. For the closed-loop system that is BIBO stable, what would you expect for the step response?

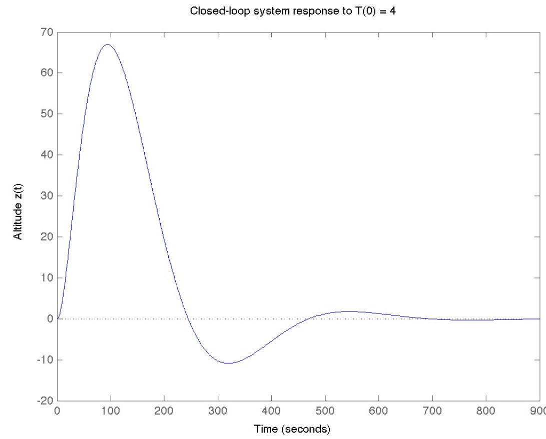
   (a) The balloon gains altitude, then levels out at some constant value

   (b) The balloon loses altitude, then levels out at some constant value

   (c) The balloon continues to gain altitude

   (d) The balloon initially loses altitude, then continues to gain altitude





*6. Compare the open-loop system with the closed-loop system with your chosen value of K. What is the effect feedback on system stability (asymptotic and BIBO)? In 1-2 sentences, compare the performance of the open-loop and closed-loop systems.

If your group finishes early...

Other points to consider (not necessary to hand in):

- Consider the case in which the pair of poles nearest to the origin are exactly on the imaginary axis, and the third pole is somewhere on the negative real line. Will the height of the balloon oscillate in a step response? Will the temperature oscillate? Why or why not?
- What values of K will result in poles in the RHP?
- Is it possible for the closed-loop system with gain K to have only one pole in the RHP?
- An alternative control strategy is to use a state-space approach to a design a full-state feedback controller $q(t) = -Kx(t)$, where $K \in \mathbb{R}^{1\times 3}$, and $x(t) = \begin{bmatrix} z(t) \\ \dot{z}(t) \\ T(t) \end{bmatrix}$. What value of K is required to meet the desired specifications?

References

[1] T. Das and R. Mukherjee, "Optimal trajectory planning for hot-air balloons in linear wind fields," in *Journal of Guidance, Control, and Dynamics*. Danvers, MA: AIAA, May–June 2003, vol. 26, no. 3, pp. 416–424.

[2] R. Furfaro, J. I. Lunine, A. Elfes, and K. Reh, "Wind-based navigation of a hot-air balloon on titan: A feasibility study," in *Proceedings of the SPIE: Space Exploration Technologies*, W. Fink, Ed., vol. 6960, 2008, pp. 69 600C–69 600C–13.

[3] T. Ellenrieder, "The modelling and validation of the vertical dynamics of a hot air balloon," in *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*. Institution of Mechanical Engineers, January 1999, vol. 213, no. 1, pp. 57–62.