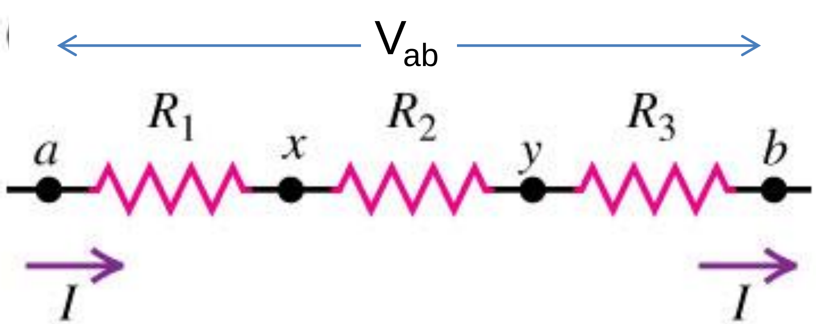
Lecture 24
(Resistors & Kirchhoff's Rules)

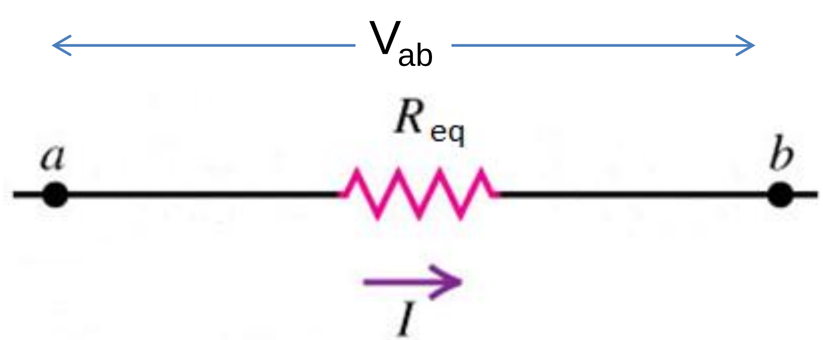
Physics 161-01 Spring 2012

Douglas Fields

Equivalent Resistance

- As we have said, resistors can be used to direct the flow of charge (current) in different ways in a circuit.
- Sometimes, combinations of resistors are used in order to get the required resistance.
- If we have some combination, and want to analyze the current through and voltage across each resistor, we need to simplify the combination by replacing the multiple resistors with a single equivalent (meaning that the nature of the circuit remains unchanged) resistor.

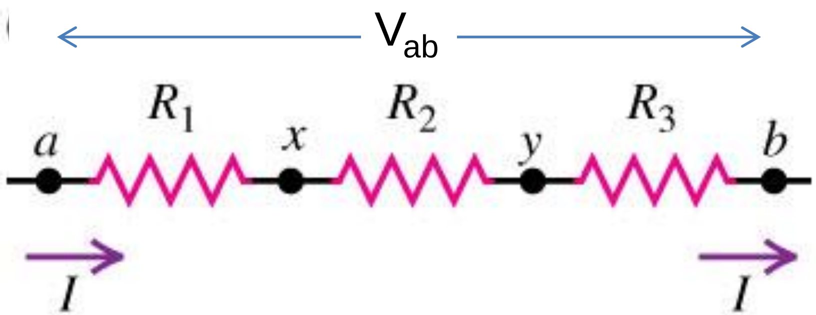


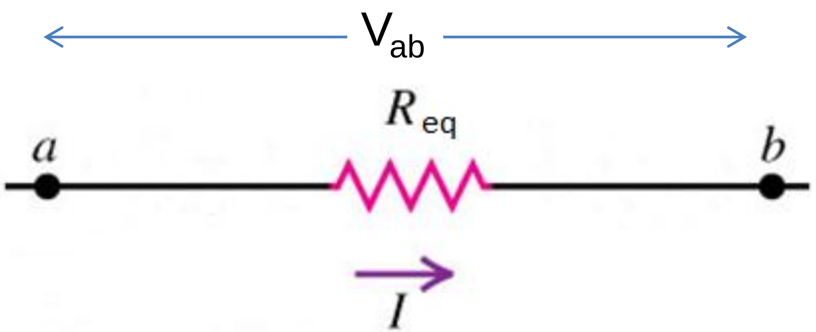


Series Resistors

- Since the current through each resistor is the same,

$$
\begin{aligned}
V_{ab} &= IR_1 + IR_2 + IR_3 \\
&= I\left(R_1 + R_2 + R_3\right) \\
&= IR_{eq} \Rightarrow \\
R_{eq} &= R_1 + R_2 + R_3
\end{aligned}
$$





Parallel Resistors

- For resistors in parallel, the voltage across each is the same, so,

$$V_{ab} = I_1R_1 = I_2R_2 = I_3R_3$$

$$and$$

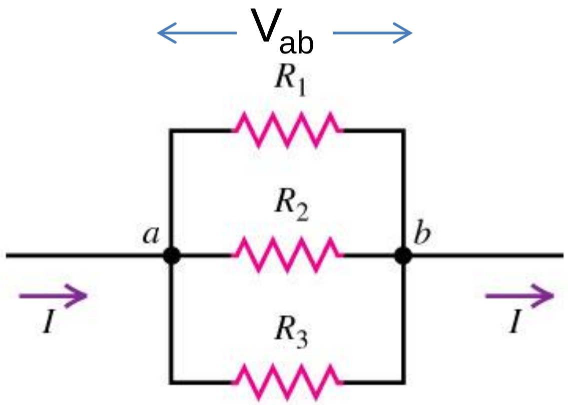
$$I = I_1 + I_2 + I_3$$

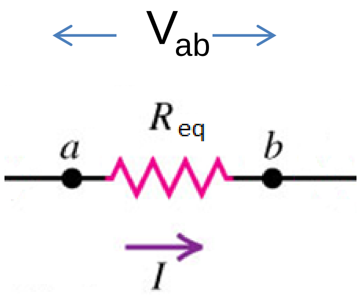
$$V_{ab} = IR_{eq} \Rightarrow$$

$$\frac{V_{ab}}{R_{eq}} = I = I_1 + I_2 + I_3$$

$$\frac{V_{ab}}{R_{eq}} = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3} \Rightarrow$$

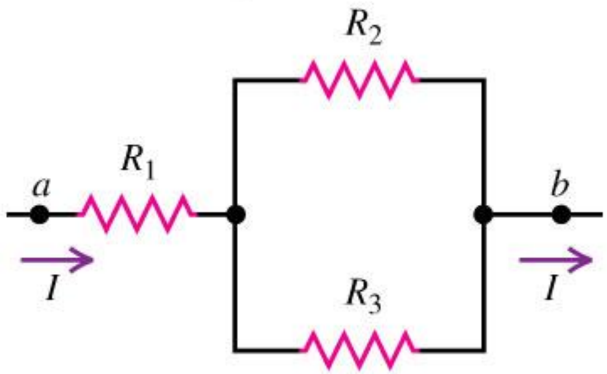
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



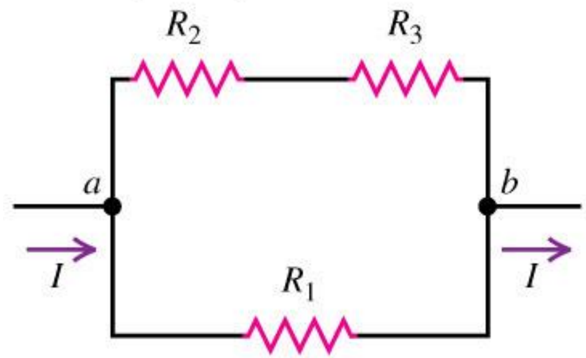


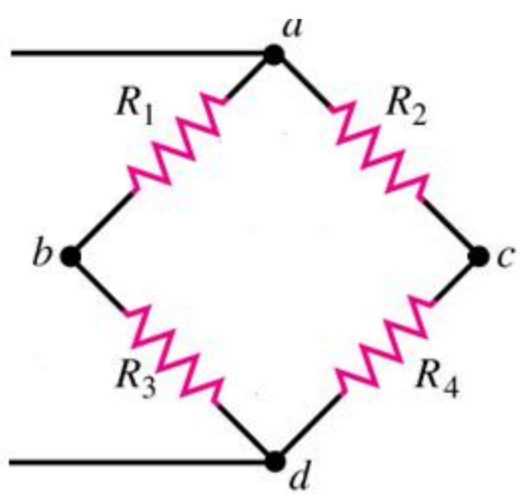
Mixed Resistors

(c) R_1 in series with parallel combination of R_2 and R_3



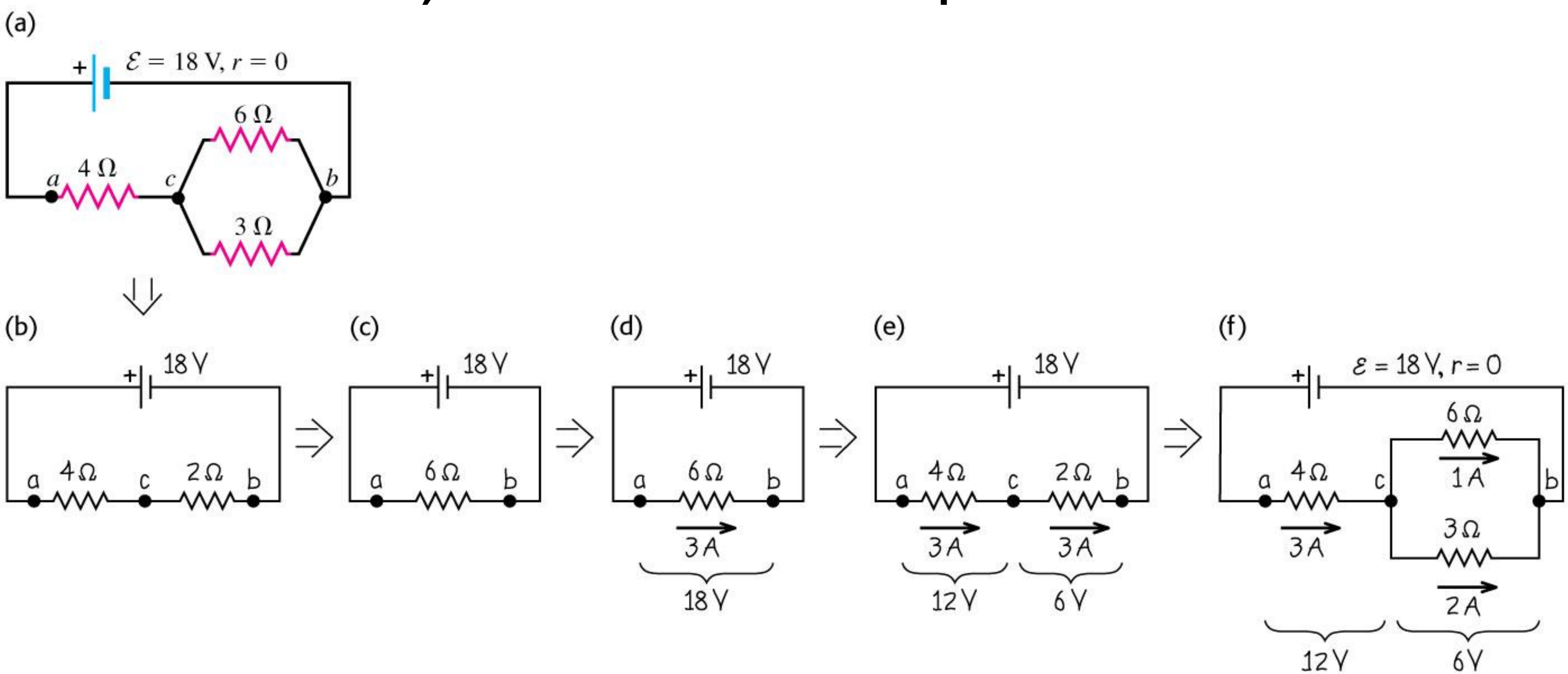
(d) R_1 in parallel with series combination of R_2 and R_3





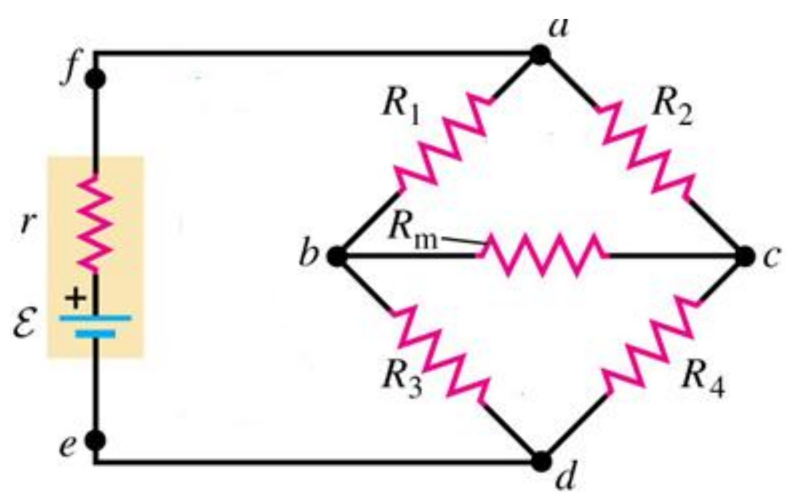
Analysis Steps

- If you are asked to determine the current through and potentials across several resistors in a circuit, follow these steps:



Caution!

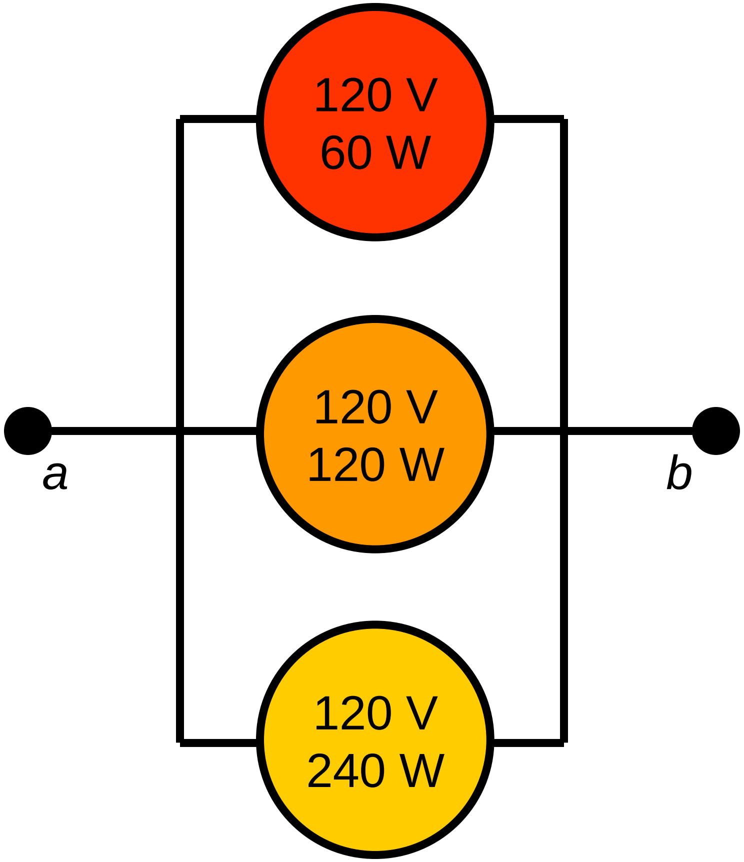
- Not all circuits can be analyzed in this way.
- In the diagram below, the resistors are neither in series or in parallel.
- We will develop another method for this analysis.



CPS 24-1

A 120-V, 60-W light bulb, a 120-V, 120-W light bulb, and a 120-V, 240-W light bulb are connected in parallel as shown.

The voltage between points *a* and *b* is 120 V. Through which bulb is there the greatest voltage drop?



A. the 120-V, 60-W light bulb

B. the 120-V, 120-W light bulb

C. the 120-V, 240-W light bulb

D. All three light bulbs have the same voltage drop.

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A 120-V, 60-W light bulb, a 120-V, 120-W light bulb, and a 120-V, 240-W light bulb are connected in parallel as shown.

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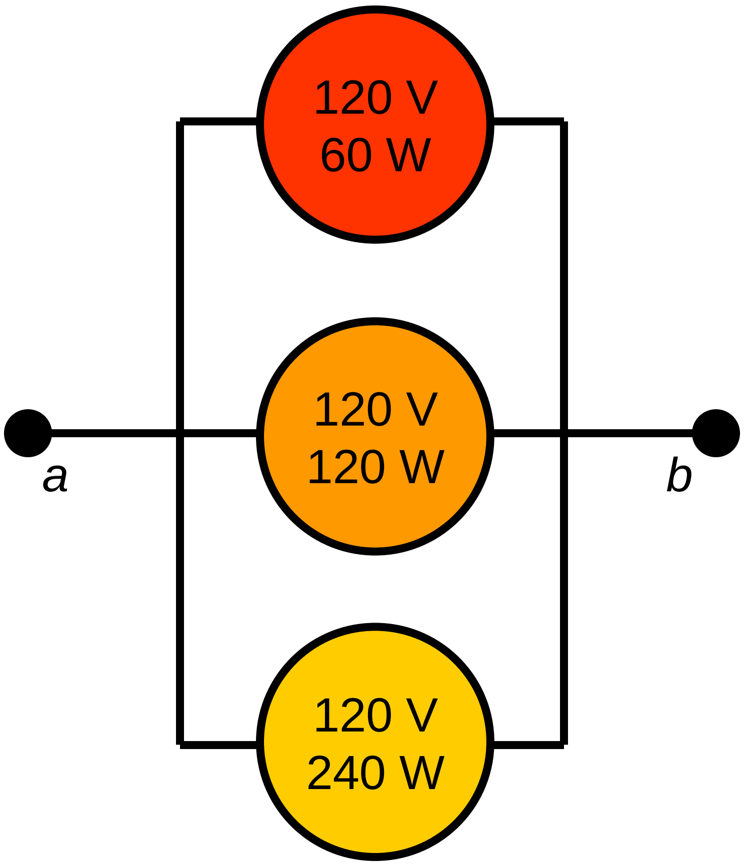


A. the 120-V, 60-W light bulb

B. the 120-V, 120-W light bulb

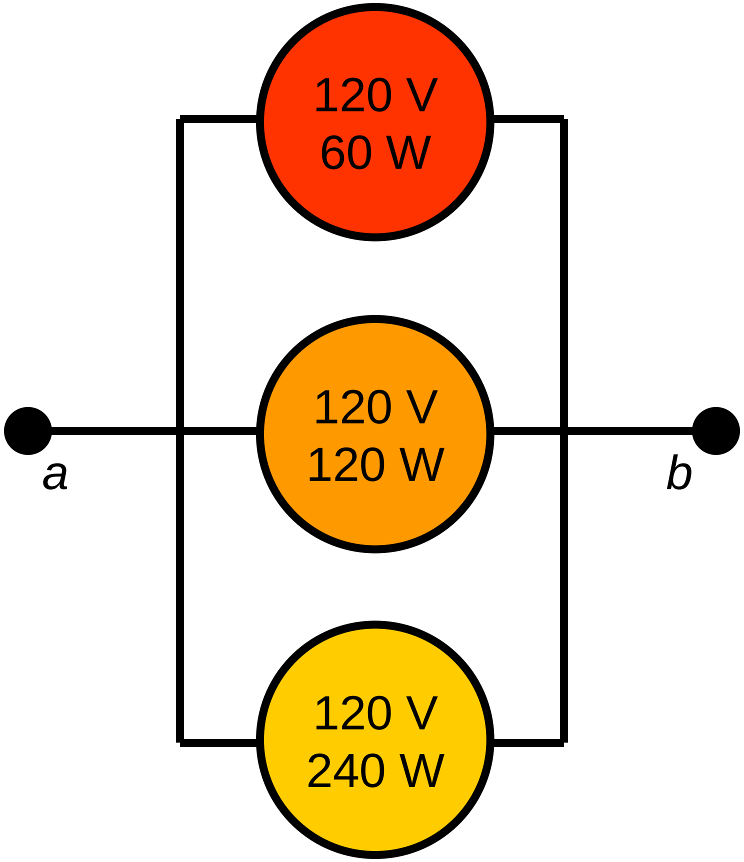
C. the 120-V, 240-W light bulb

D. All three light bulbs have the same voltage drop.

CPS 24-2

A 120-V, 60-W light bulb, a 120-V, 120-W light bulb, and a 120-V, 240-W light bulb are connected in parallel as shown.

The voltage between points *a* and *b* is 120 V. Which bulb glows the brightest?



A. the 120-V, 60-W light bulb

B. the 120-V, 120-W light bulb

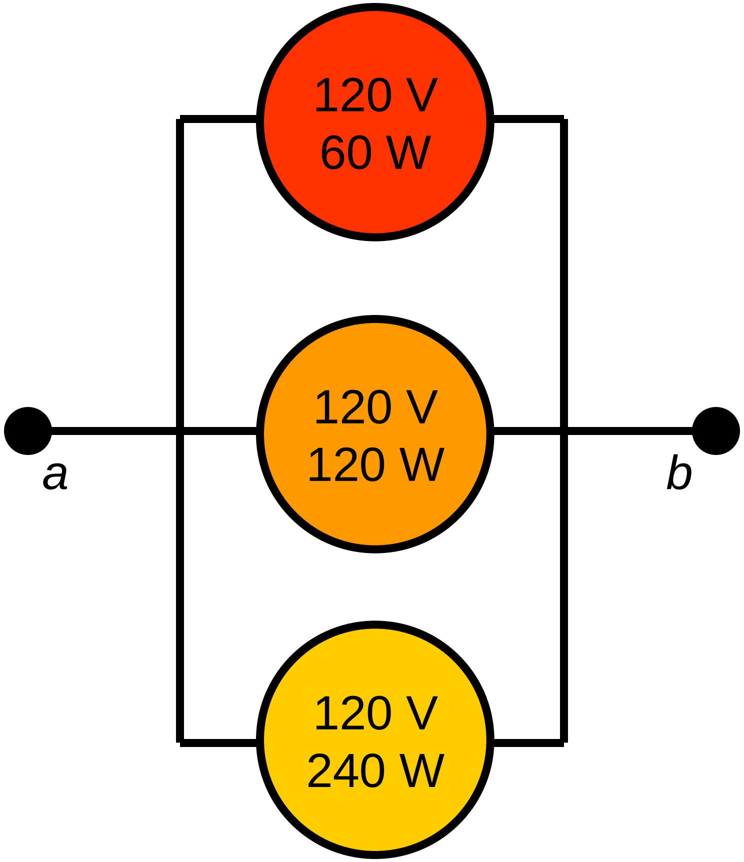
C. the 120-V, 240-W light bulb

D. All three light bulbs glow with equal brightness.

CPS 24-2

A 120-V, 60-W light bulb, a 120-V, 120-W light bulb, and a 120-V, 240-W light bulb are connected in parallel as shown.

The voltage between points *a* and *b* is 120 V. Which bulb glows the brightest?



A. the 120-V, 60-W light bulb

B. the 120-V, 120-W light bulb

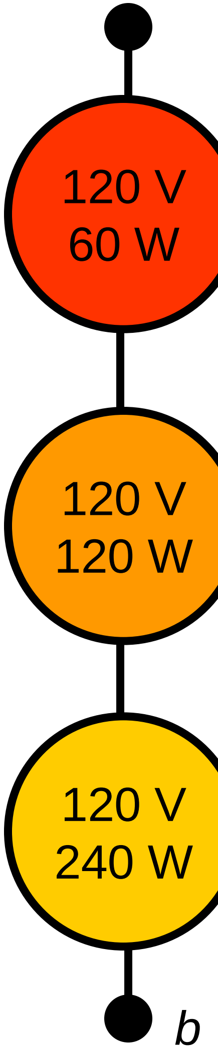
C. the 120-V, 240-W light bulb

D. All three light bulbs glow with equal brightness.

CPS 24-3

A 120-V, 60-W light bulb, a 120-V, 120-W light bulb, and a 120-V, 240-W light bulb are connected in series as shown.

The voltage between points *a* and *b* is 120 V. Through which bulb is there the greatest voltage drop?



A. the 120-V, 60-W light bulb

B. the 120-V, 120-W light bulb

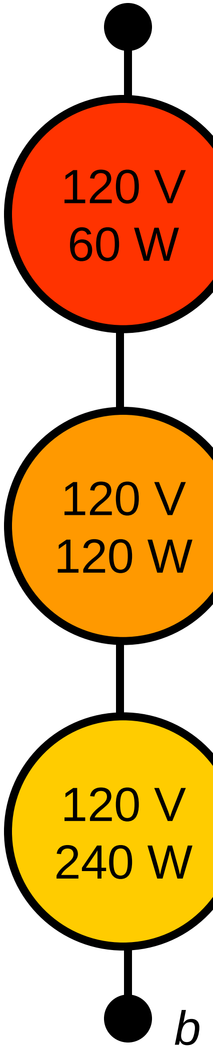
C. the 120-V, 240-W light bulb

D. All three light bulbs have the same voltage drop.

CPS 24-3

A 120-V, 60-W light bulb, a 120-V, 120-W light bulb, and a 120-V, 240-W light bulb are connected in series as shown.

The voltage between points *a* and *b* is 120 V. Through which bulb is there the greatest voltage drop?



A. the 120-V, 60-W light bulb

B. the 120-V, 120-W light bulb

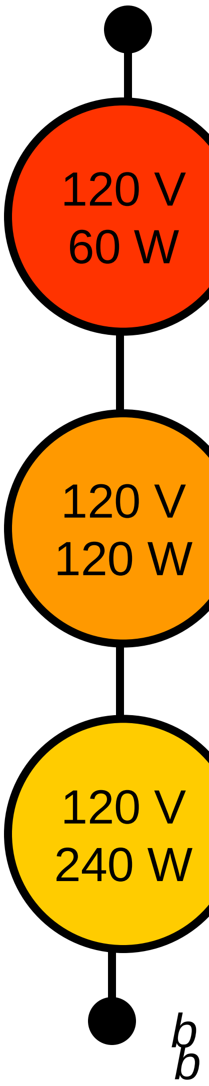
C. the 120-V, 240-W light bulb

D. All three light bulbs have the same voltage drop.

CPS 24-4

A 120-V, 60-W light bulb, a 120-V, 120-W light bulb, and a 120-V, 240-W light bulb are connected in series as shown.

The voltage between points *a* and *b* is 120 V. Which bulb glows the brightest?



b

A. the 120-V, 60-W light bulb

B. the 120-V, 120-W light bulb

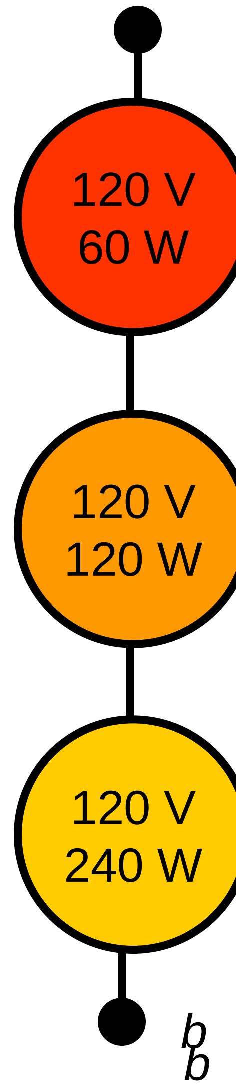
C. the 120-V, 240-W light bulb

D. All three light bulbs glow with equal brightness.

CPS 24-4

A 120-V, 60-W light bulb, a 120-V, 120-W light bulb, and a 120-V, 240-W light bulb are connected in series as shown.

The voltage between points *a* and *b* is 120 V. Which bulb glows the brightest?



A. the 120-V, 60-W light bulb

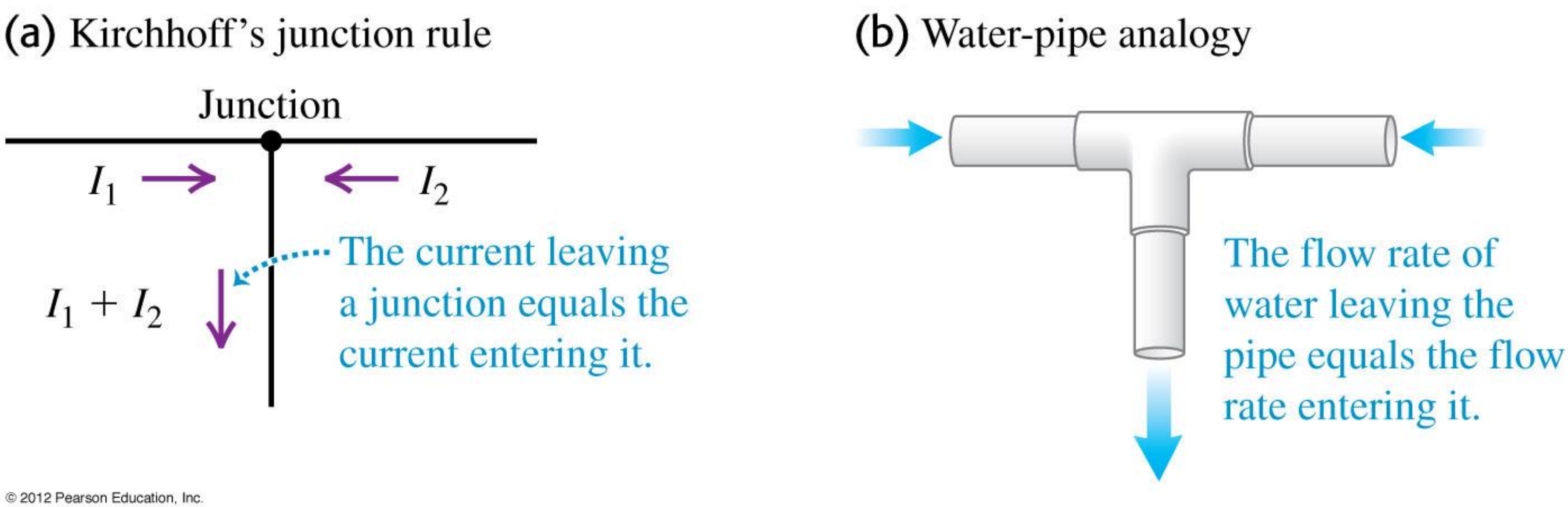
B. the 120-V, 120-W light bulb

C. the 120-V, 240-W light bulb

D. All three light bulbs glow with equal brightness.

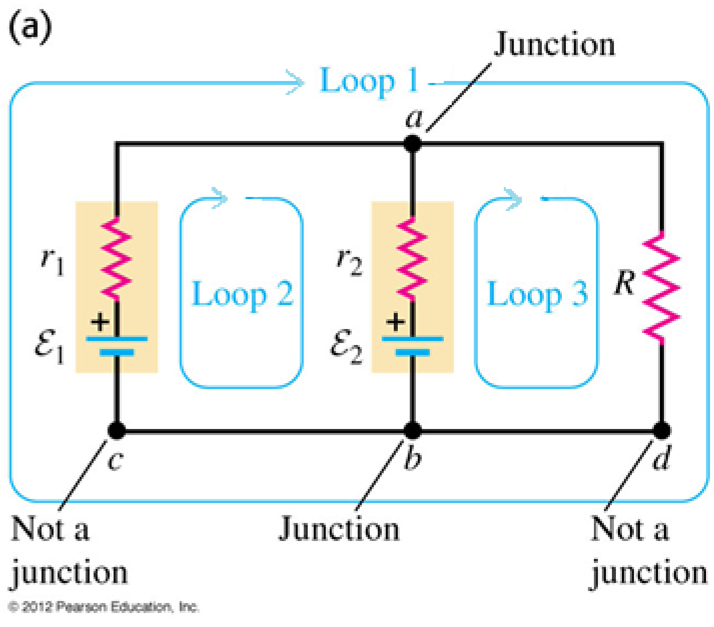
Kirchhoff’s Rules

- There are two rules for analyzing a circuit, that can be applied in general.
  - Junction Rule: For any junction of wires, the current is conserved (current leaving it = current entering it).



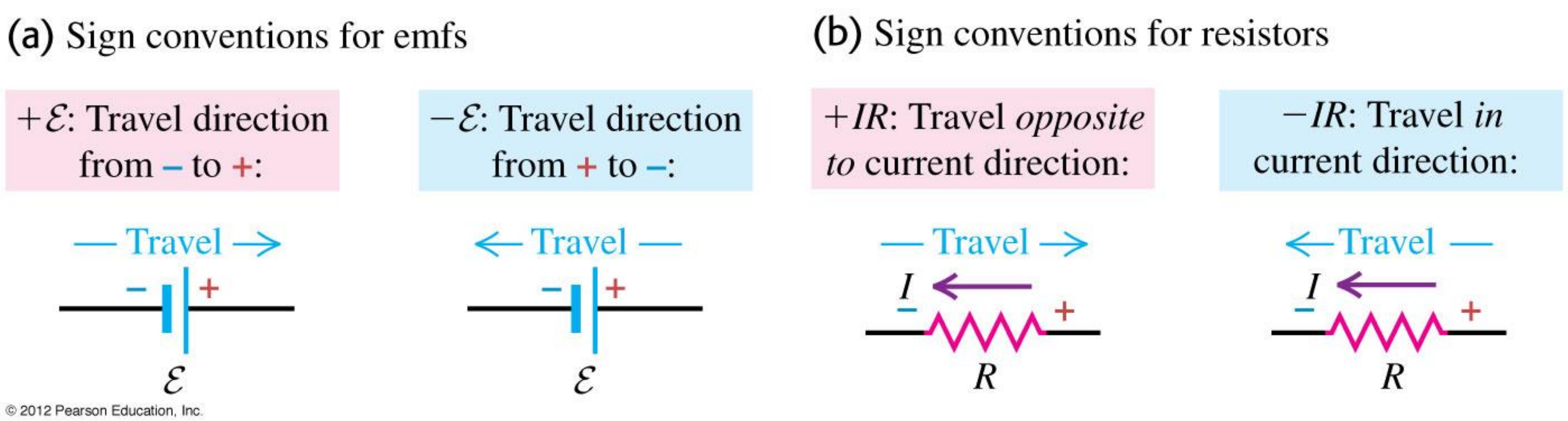
Kirchhoff's Rules

- There are two rules for analyzing a circuit, that can be applied in general.
  - Loop Rule: The net voltage drop around any complete loop is zero (the voltage must return to its original value when you return to the same place).



Kirchhoff's Rules

- For the Loop Rule, you must pay attention to the sign of the voltage change when crossing an element (EMF, resistor, capacitor).



Kirchhoff’s Rules

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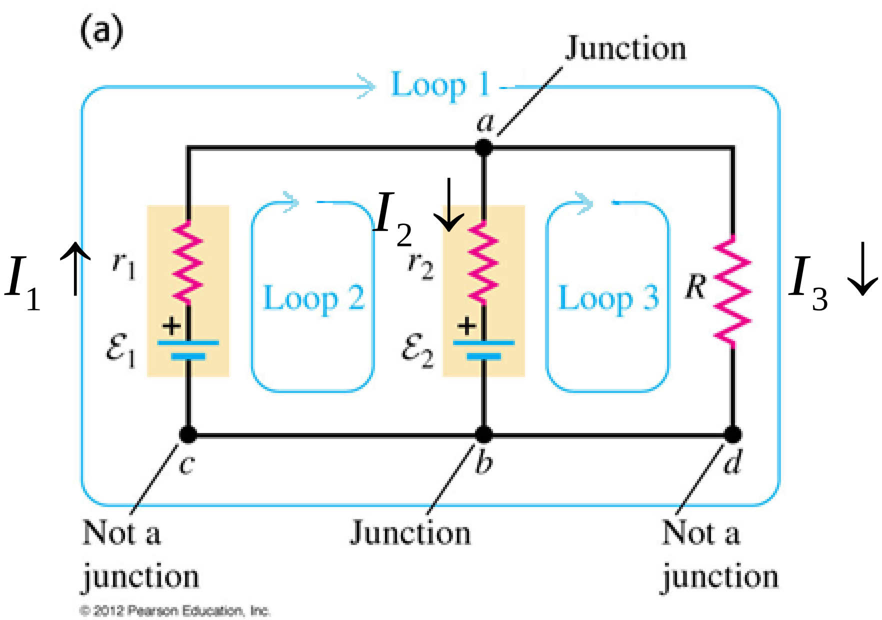


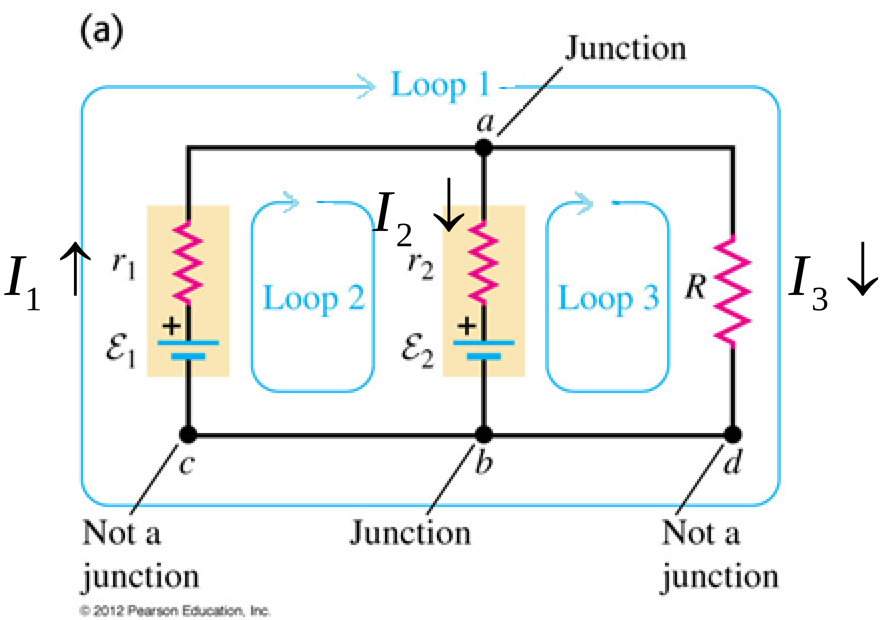
$$\text{Loop 1)} \sum \Delta V = \mathcal{E}_1 - I_1 r_1 - I_3 R = 0$$

$$\text{Loop 2)} \sum \Delta V = \mathcal{E}_1 - I_1 r_1 - I_2 r_2 - \mathcal{E}_2 = 0$$

$$\text{Loop 3)} \sum \Delta V = \mathcal{E}_2 + I_1 r_1 - I_3 R = 0$$

Kirchhoff’s Rules

- You can then combine the two rules to solve for the unknown currents.
- You will generally find that you have more equations than unknowns (over-determined), so you will not have to use every equation.
- Generally use the junction rule results to eliminate one or more of the currents from the loop rule equations.



$$\text{Loop 1)} \sum \Delta V = \mathcal{E}_1 - I_1 r_1 - I_3 R = 0$$

$$\text{Loop 2)} \sum \Delta V = \mathcal{E}_1 - I_1 r_1 - I_2 r_2 - \mathcal{E}_2 = 0$$

$$\text{Loop 3)} \sum \Delta V = \mathcal{E}_2 + I_1 r_1 - I_3 R = 0$$

$$\text{Junction a)} \sum I = I_1 - I_2 - I_3 = 0$$

$$\text{Junction b)} \sum I = I_2 - I_1 + I_3 = 0$$

Kirchhoff's Rules

- Example:

$$\text{Loop 1)} \sum \Delta V = 13V - I_1(1\Omega) - (I_1 - I_3)(1\Omega) = 0$$
$$\text{Loop 2)} \sum \Delta V = 13V - I_2(1\Omega) - (I_2 + I_3)(2\Omega) = 0$$
$$\text{Loop 3)} \sum \Delta V = -I_1(1\Omega) - I_3(1\Omega) + I_2(1\Omega) = 0$$