Physics 161, HW #7

#1

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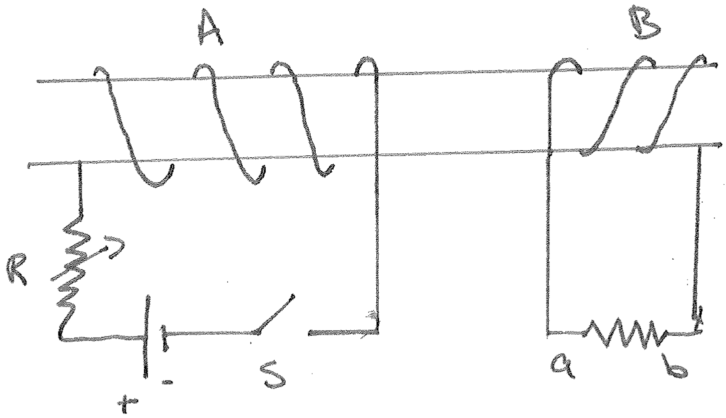
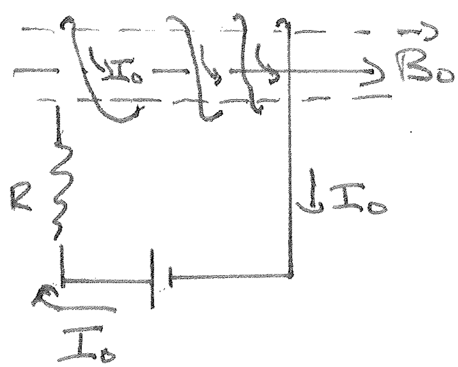
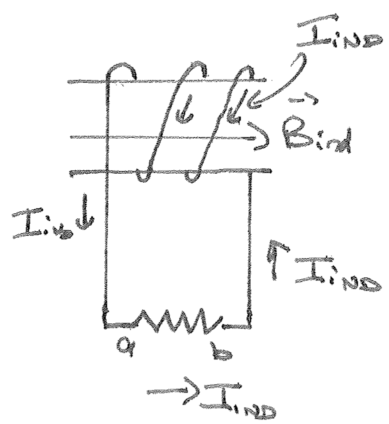


WITH SWITCH CLOSED: I_0 FLOWS FROM + to −



FROM RIGHT-HAND-RULE, $\vec{B}_0 = \rightarrow$

$\Rightarrow$ SAME $\vec{B}_0$ IN OTHER Loops



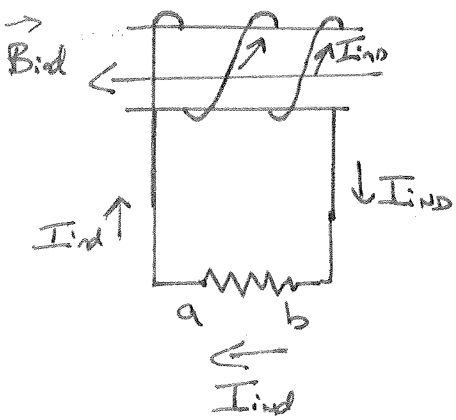
SWITCH OPENED $\Rightarrow$ B_0 decreases to ZERO

$\Rightarrow$ B_{IND} tries to MANTAIN $\Rightarrow$ $\vec{B}_{IND} = \rightarrow$

$\Rightarrow$ CURRENT FLOWS FROM a to b

b) Coil B brought TOWARDS A $\Rightarrow$ MAGNITUDE OF B_0 INCREASING IN B

$\Rightarrow$ FLUX INCREASING IN B $\Rightarrow$ $\vec{B}_{IND}$ tries to CANCEL $\Rightarrow$ $\vec{B}_{IND} = \leftarrow$



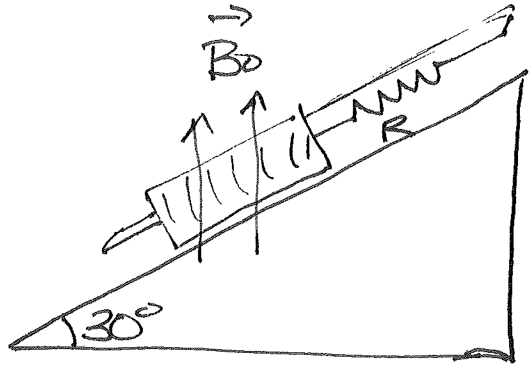
Current Flows FROM b to a.

c) RESISTANCE R IS DECREASED $\Rightarrow$ I_0 INCREASES & B_0 increases

$\Rightarrow$ INCREASING FLUX $\Rightarrow$ B_{IND} tries to CANCEL $\Rightarrow$ $\vec{B}_{ind} = \leftarrow$

$\Rightarrow$ Like part b $\Rightarrow$ Current FLOWS FROM b to a.

#2



Solenoid: $r = 4cm = .04m$

$N = 1500$

$R_S = 1000\Omega$

$R = 600\Omega$

$$B_0 = \begin{cases} 0 & t<0 \\ .25T(1-\cos\pi t) & 0<t<3s \\ .5T & t>3s \end{cases}$$

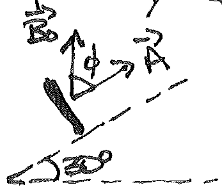
a) Find Plot of I vs t. $\Rightarrow$ Induced Current vs. t

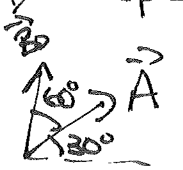
$$I_{IND} = \frac{\mathcal{E}_{IND}}{R_{EQ}} \qquad R_{EQ} = R_S + R = 1000\Omega + 600\Omega = 1600\Omega$$

Faraday's Law: $\mathcal{E}_{IND} = -N\dfrac{d\Phi_B}{dt}$

Since $\vec{B_0}$ is uniform spatially $\leftarrow$ B_0 only depends on time

$\Phi_B = \vec{B_0} \cdot \vec{A}$





$\phi = 60°$

$$\Rightarrow \Phi_B = B_0 A \cos 60° = B_0 A (1/2) = \frac{1}{2} B_0 A$$

Solenoid Cross-Section is circular $\Rightarrow A = \pi r^2 = \pi(.04m)^2 = \pi(.0016

$$\Rightarrow \Phi_B = \tfrac{1}{2} B_0 \pi (.0016\,m^2) = \begin{cases} 0 & t<0 \\ \tfrac{1}{2}(.25T)\pi(.0016\,m^2)(1-\cos\pi t) & 0<t<3 \\ \tfrac{1}{2}(.5T)\pi(.0016\,m^2) & t>3s \end{cases}$$

$\therefore$ Φ_B is Constant for $t<0$, $t>3s$ $\Rightarrow$ $\frac{d\Phi_B}{dt} = 0$

$\Rightarrow I_{IND} = 0$ for $t<0$, $t>3s$

For $0<t<3s$ $\Phi_B = \tfrac{1}{2}(.25T)\pi(.0016\,m^2)(1-\cos\pi t) = 6.283\times10^{-4}\,Wb\,(1-\cos\pi t)$

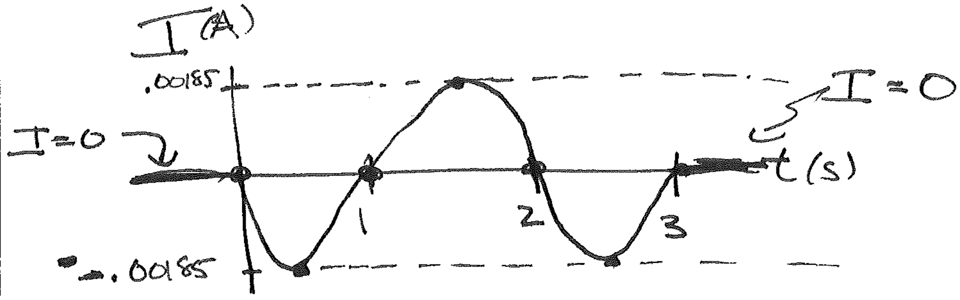
$$\Phi_B = 6.283\times10^{-4}\,Wb\,(1-\cos\pi t)$$

$$\Rightarrow \frac{d\Phi_B}{dt} = 6.283\times10^{-4}\,Wb\left(0-(-\sin\pi t)\pi/s\right) = 1.97\times10^{-3}\,V\,(\sin\pi t)$$

Unit must be s^{-1} to make πt have no unit

$$\mathcal{E}_{IND} = -N\frac{d\Phi_B}{dt} = -1500(1.97\times10^{-3}V)\sin\pi t = (2.96V)\sin\pi t$$

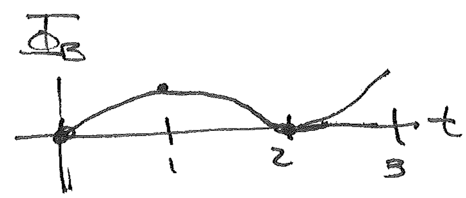
$$I_{IND} = \frac{\mathcal{E}_{IND}}{R_{EQ}} = -\frac{(2.96V)}{1600\Omega}\sin\pi t = -(.00185A)\sin\pi t$$



b) Looking Down table;

$\Phi_B = \frac{1}{2} B_0 A$

$B_0 = .25(1-\cos\pi t)$ ⇒



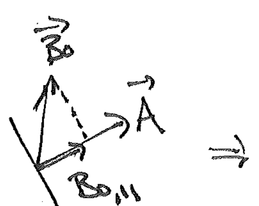
Flux increases for 0<t<1s

Decreases for 1s<t<2s

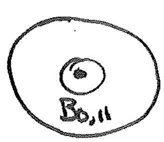
Then increases for 2s<t<3s

Looking down the table means

Area Vector is out of page



⇒



0<t<1s, increasing Flux ⇒ $\vec{B}_{IND}$ = ⊗ ⇒ clockwise I_{IND}

1s<t<2s, Decreasing Flux ⇒ $\vec{B}_{IND}$ = ⊙ ⇒ counter-clockwise

2s<t<3s, increasing Flux ⇒ clockwise

#3

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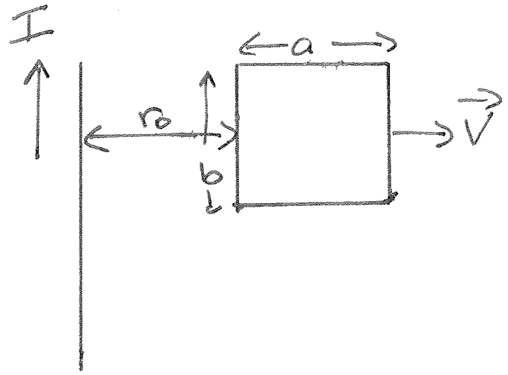


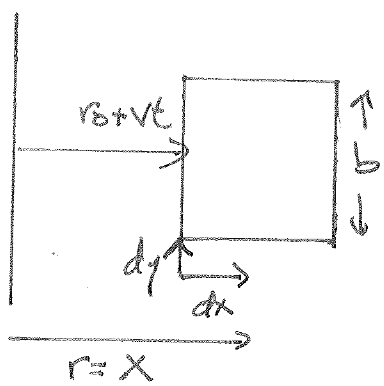
Call INITIAL DISTANCE FROM wire r_0 INSTEAD OF BOOK's r.

a) CALCULATE NET EMF IN LOOP USING FARADAY's LAW OF INDUCTION

$$\mathcal{E}_{IND} = \frac{-d\Phi_B}{dt}$$

WIRE CREATES $\vec{B} = \frac{\mu_0 I}{2\pi r}, \otimes \Rightarrow$ decreasing WITH r

$$\Phi_B = \int \vec{B}\cdot d\vec{A}, \quad d\vec{A} \text{ Also } \otimes \Rightarrow \int B\,dA$$



For given value of x, B constant

$$\Rightarrow dA = dx\,dy = dx\,(b)$$

$$\Rightarrow \Phi_B = \int_{r_0+Vt}^{r_0+Vt+a} \frac{\mu_0 I b}{2\pi X}\,dx = \frac{\mu_0 I b}{2\pi}\int_{r_0+Vt}^{r_0+Vt+a} \frac{dx}{X} = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{r_0+Vt+a}{r_0+Vt}\right)$$

$$\Rightarrow \Phi_B = \frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{r_0+vt}\right)$$

$$\mathcal{E}_{IND} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 I b}{2\pi}\left(\frac{1}{1+\frac{a}{r_0+vt}}\right)\left(\frac{a(-1)v}{(r_0+vt)^2}\right) = +\frac{\mu_0 I b a v}{2\pi}\left(\frac{1}{\frac{r_0+vt+a}{r_0+vt}}\right)\left(\frac{1}{(r_0+vt)^2}\right)$$

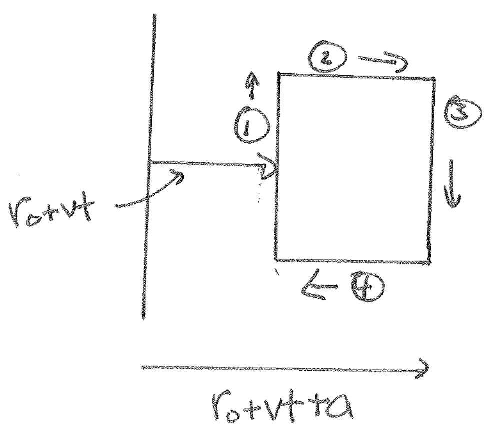
$$\Rightarrow \mathcal{E}_{IND} = \frac{\mu_0 I a b v}{2\pi}\left(\frac{1}{r_0+vt+a}\right)\left(\frac{1}{r_0+vt}\right)$$

THIS GIVES INDUCED EMF AT ALL TIMES. Book wants $t=0$

$$\Rightarrow \boxed{\mathcal{E}_{IND} = \frac{\mu_0 I a b v}{2\pi}\left(\frac{1}{r_0+a}\right)\left(\frac{1}{r_0}\right)}$$

ii USE MOTIONAL EMF.

FOR CHANGING $\vec{B}$ AND DIFFERENT $d\vec{l}$'s, WE USE $\mathcal{E}_{IND} = \oint (\vec{v}\times\vec{B})\cdot d\vec{l}$



NOTE: WE HAVE TO INTEGRATE clockwise SO THAT THE AREA VECTOR POINTS INTO PAGE AGAIN.

$$\Rightarrow \mathcal{E}_{IND} = \int_{①} (\vec{v}\times\vec{B})\cdot d\vec{l}_1 + \int_{②} (\vec{v}\times\vec{B})\cdot d\vec{l}_2 + \int_{③} (\vec{v}\times\vec{B})\cdot d\vec{l}_3 + \int_{④} (\vec{v}\times\vec{B})\cdot d\vec{l}_4$$

$d\vec{l}_1 = dy, \uparrow$, $d\vec{l}_2 = dx, \rightarrow$, $d\vec{l}_3 = dy, \downarrow$, $d\vec{l}_4 = dx, \leftarrow$

$\vec{V} = \rightarrow$

AT All POINTS ON LOOP, $\vec{B} = \otimes$, $\Rightarrow$ $\vec{V}\times\vec{B} = \uparrow$ & $|\vec{V}\times\vec{B}| = VB\sin 90^\circ = VB$

$\cos 90^\circ = 0 \Rightarrow (\vec{V}\times\vec{B})\cdot d\vec{\ell}_2 = (\vec{V}\times\vec{B})\cdot d\vec{\ell}_4 = 0$

$\cos 0^\circ = 1 \Rightarrow (\vec{V}\times\vec{B})\cdot d\vec{\ell}_1 = VB\,dy$

$\cos 180^\circ = -1 \Rightarrow (\vec{V}\times\vec{B})\cdot d\vec{\ell}_3 = -VB\,dy$

AT All POINTS ON ① $B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi}\frac{1}{(r_0+vt)}$ = Constant

$$\Rightarrow \int_{①} (\vec{V}\times\vec{B})\cdot d\vec{\ell}_1 = \int_0^b \frac{V\mu_0 I}{2\pi}\frac{1}{(r_0+vt)}\,dy = \frac{V\mu_0 I}{2\pi}\frac{1}{(r_0+vt)}\,b$$

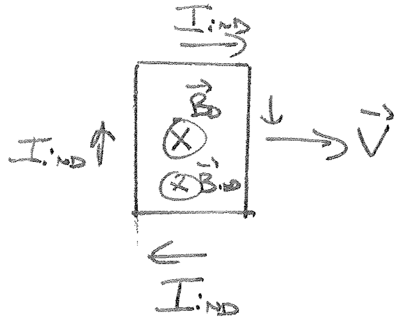
AT All POINTS ON ③ $B = \frac{\mu_0 I}{2\pi}\frac{1}{r_0+vt+a} \Rightarrow \int_{③} (\vec{V}\times\vec{B})\cdot d\vec{\ell}_3 = -\frac{V\mu_0 I}{2\pi}\frac{b}{(r_0+vt+a)}$

$$\Rightarrow \mathcal{E}_{IND} = \frac{\mu_0 I b v}{2\pi}\left(\frac{1}{r_0+vt} - \frac{1}{r_0+vt+a}\right) = \frac{\mu_0 I b v}{2\pi}\left[\frac{(r_0+vt+a)-(r_0+vt)}{(r_0+vt)(r_0+vt+a)}\right]$$

$$= \frac{\mu_0 I b v}{2\pi}\left[\frac{a}{(r_0+vt)(r_0+vt+a)}\right]$$ ← SAME ANSWER AS BEFORE

SO $\mathcal{E}_{ind}\big|_{t=0} = \frac{\mu_0 I a b v}{2\pi}\left(\frac{1}{r_0+a}\right)\left(\frac{1}{r_0}\right)$

b) FIND DIRECTION OF CURRENT USING LENZ'S LAW



AS LOOP GETS FARTHER AWAY FROM WIRE THE WIRE'S FIELD (NOW CALLED $\vec{B}_0$) IS DECREASING $\Rightarrow$ Φ_B IN LOOP DECREASING.

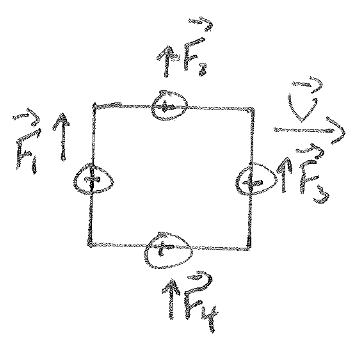
$\Rightarrow$ INDUCED EMF WILL TRY MAINTAIN.

$\vec{B}_0 = \otimes$ SO $\vec{B}_{IND} = \otimes$

↳ INDUCED MAGNETIC FIELD

FROM RIGHT-HAND-RULE, INDUCED CURRENT MUST FLOW CLOCKWISE TO CREATE INTO PAGE $\vec{B}_{ND}$.

ii FIND DIRECTION FROM MAGNETIC FORCE



FOR A POSITIVE CHARGE WITH VELOCITY TO RIGHT, $\rightarrow$ IN A $\vec{B}_0 = \otimes$, $\vec{F} = q\vec{v} \times \vec{B}_0 \Rightarrow \vec{F} = \uparrow$

$\Rightarrow$ 4 UPWARD FORCES, WE'RE TOLD THAT LOOP TRAVELS TO RIGHT SO THERE MUST BE A DOWNWARD FORCE ON ② & ③ (PROBABLY THE WIRE ITSELF).

SINCE ① IS ALWAYS CLOSER TO THE WIRE THAN ③, $\vec{B}_0$ at ① Larger than $\vec{B}_0$ at ③

$\Rightarrow$ $\vec{F}_1$ LARGER THAN $\vec{F}_3$ $\Rightarrow$ CHARGES MOVE UPWARDS through ① which pushes charge to right through ②, DOWN through ③, left through ④ $\Rightarrow$ Clockwise

c) CHECK IF Loop stationary $\Rightarrow V=0$

$$\mathcal{E}_{IND} = \frac{\mu_0 I a b V}{2\pi}\left(\frac{1}{r_0+a}\right)\left(\frac{1}{r_0}\right). \quad V=0 \Rightarrow \mathcal{E}_{IND}=0 \checkmark$$

ii) Loop very thin $\Rightarrow a\to 0 \Rightarrow \mathcal{E}_{IND} = \frac{\mu_0 I a b V}{2\pi}\left(\frac{1}{r_0^2}\right) \to 0$ as $a\to 0$ ✓

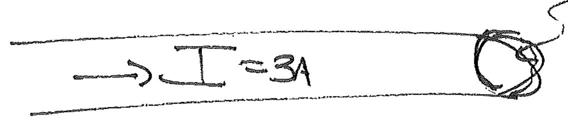
iii) Lopp gets very far AWAY $\Rightarrow t\to\infty$

HERE HAVE TO USE $\mathcal{E}_{IND} = \frac{\mu_0 I a b V}{2\pi}\left(\frac{1}{r_0+Vt+a}\right)\left(\frac{1}{r_0+Vt}\right)$

as $t\to\infty$ $\mathcal{E}_{IND} = \frac{\mu_0 I a b V}{2\pi}\left(\frac{1}{Vt}\right)\left(\frac{1}{Vt}\right) = \frac{\mu_0 I a b}{2\pi}\left(\frac{1}{Vt^2}\right)$

as $t\to\infty \Rightarrow \mathcal{E}_{IND}\to 0$ ✓ as IT SHOULD

#4



$A = 1.5\,mm^2 \times \frac{1\,m^2}{(1000\,mm)^2} = 1.5\times10^{-6}\,m^2$

$\rho = 2\times10^{-8}\,\Omega\cdot m$

a) $E = ?$ $\quad E = \rho J = \rho\left(\frac{I}{A}\right) = \rho\frac{I}{A}$

$\Rightarrow E = \frac{(2\times10^{-8}\,\Omega\cdot m)(3A)}{1.5\times10^{-6}\,m^2} \Rightarrow \boxed{E = .04\ V/m}$

b) $\frac{dI}{dt} = 6.6\times10^{7}\,A/s$ WHAT IS $\frac{dE}{dt}$

$E = \rho\frac{I}{A}$ SINCE ρ AND A ARE Constant, $\frac{dE}{dt} = \frac{\rho}{A}\frac{dI}{dt}$

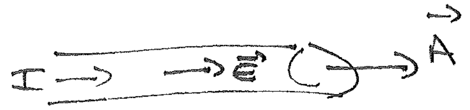
$\frac{dE}{dt} = \frac{(2\times10^{-8}\,\Omega\cdot m)}{1.5\times10^{-6}\,m^2}(6.6\times10^{7}\,A/s) \Rightarrow \boxed{\frac{dE}{dt} = 880000\ V/m\cdot s = 8.8\times10^{5}\ V/m\cdot s}$

THis CHANGING Electric Field creates a MAGNetic Field.

C) WHAT IS DISPLACEMENT CURRENT DENSITY?

$j_d = ?$ $\quad j_d = \dfrac{i_d}{A}$ where $i_d = \epsilon \dfrac{d\Phi_E}{dt}$

$K \approx 1 \Rightarrow i_d = \epsilon_0 \dfrac{d\Phi_E}{dt}$



ASSUMING UNIFORM FIELD $\Rightarrow \Phi_E = \vec{E} \cdot \vec{A}$

$\vec{E}$ IN SAME DIRECTION AS CURRENT $\Rightarrow \Phi_E = EA\cos 0° = EA$

A constant $\Rightarrow \dfrac{d\Phi_E}{dt} = \dfrac{d}{dt}(EA) = A\left(\dfrac{dE}{dt}\right)$

$$\boxed{i_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 A \frac{dE}{dt}}$$ ← We'll need this in part d

$$j_d = \frac{i_d}{A} = \frac{\epsilon_0 A}{A}\frac{dE}{dt} = \epsilon_0 \frac{dE}{dt} = \left(8.85\times10^{-12}\frac{C^2}{N\cdot m^2}\right)\left(8.8\times10^{5}\,V/m\cdot s\right)$$

$$\Rightarrow \boxed{j_d = 7.788\times10^{-6}\,A/m^2 = 7.79\times10^{-6}\,A/m^2}$$

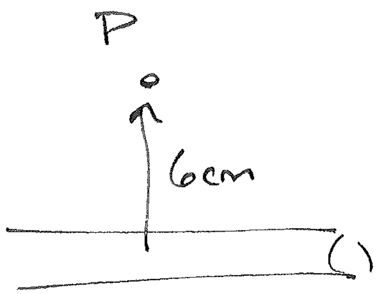
Unit: $\dfrac{C^2}{N\cdot m^2} = \dfrac{C}{\left(\frac{N\cdot m}{C}\right)\cdot m} = \dfrac{C}{\left(\frac{J}{C}\right)\cdot m} = \dfrac{C}{V\cdot m} \Rightarrow \dfrac{C^2}{N\cdot m^2}\cdot\dfrac{V}{m\cdot s} = \dfrac{C}{\cancel{V}\cdot m}\cdot\dfrac{\cancel{V}}{m\cdot s} = \dfrac{C}{s}\cdot\dfrac{1}{m^2}$

$= A/m^2$

d) WHAT IS MAGNITUDE OF B 6cm FROM WIRE?

Must be outside wire since

$A = 1.5\,mm^2 \Rightarrow r = 1.22\,mm$



Ampere's LAW: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (i_c + i_d)_{encl}$

$i_c = I = 3A$

$i_d = J_d A = \epsilon_0 A \frac{dE}{dt} = \left(8.85\times10^{-12} \frac{C^2}{N \cdot m^2}\right)(1.5\times10^{-6} m^2)(8.8\times10^{5})$

$= 1.1682\times10^{-11} A = 1.17\times10^{-11} A$

(OK, WE could HAVE Also just used $i_d = J_d A$

$= (7.788\times10^{-6} A/m^2)(1.5\times10^{-6} m^2)$

$= 1.17\times10^{-11} A$)

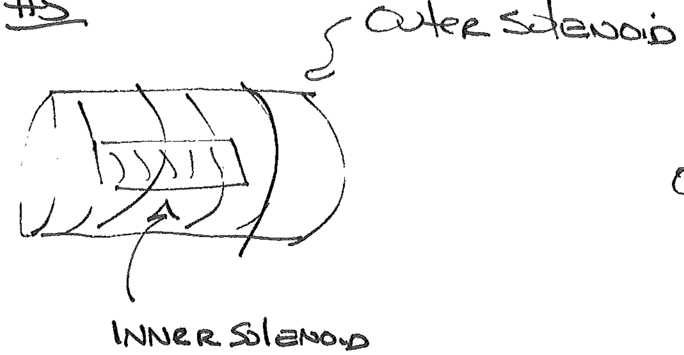
$i_c + i_d = 3A + 1.17\times10^{-11} A = 3A$ ← i_d NOT SIGNIFICANT

Outside WIRES $\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) \Rightarrow B = \frac{\mu_0 i_c}{2\pi r} = (2\times10^{-7} T \cdot m/A)\left(\frac{3A}{.06m}\right)$

$\Rightarrow \boxed{B = 2\times10^{-5} T}$

NOTICE THAT $E = .04\,V/m$ AND $\frac{dE}{dt} = 8.8\times10^{5}\,V/m \cdot s$ ARE QUITE LARGE, but the displacement current they created WAS SMALL. SO WE USUALLY DON'T HAVE to include this effect IN current-carrying WIRES.

#5



OUTER: $l_1 = 75cm = .75m$

$N_1 = 6750$

INNER: $l_2 = 12cm = .12m$, diameter $= 1.4cm$

$N_2 = 1200$

Current in outer solenoid changing at $49.2A/s \Rightarrow \frac{di_1}{dt} = 49.2A/s$

a) FIND MUTUAL INDUCTION

$$M = \frac{N_2 \Phi_{B2}}{i_1}$$

Φ_{B2} = FLUX THROUGH ONE TURN OF INNER SOLENOID $\Rightarrow \Phi_{B2} = B_1 A_2$ since magnetic field created by outer solenoid

For outer solenoid: $B_1 = \mu_0 \left(\frac{N_1}{l_1}\right) i_1 \Rightarrow \Phi_{B2} = \mu_0 \left(\frac{N_1}{l_1}\right) A_2 i_1$

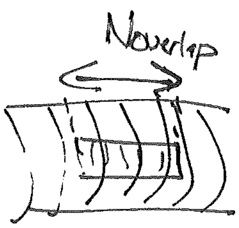
$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2 \mu_0 \left(\frac{N_1}{l_1}\right) A_2 i_1}{i_1} \Rightarrow M = \frac{\mu_0 N_1 N_2 A_2}{l_1}$$

<u>Note:</u> We could also use $M = \frac{N_1 \Phi_{B1}}{i_2}$ but it would be slightly trickier. Because $l_1 < l_2$ not all turns of the outer solenoid have a magnetic field passing through them.

$\Rightarrow$ Some turns have so flux, so we can't say $M = \frac{N_1 \Phi_{B1}}{i_2}$

Instead, we'd have to use $M = \frac{N_{overlap} \Phi_{B1}}{i_2}$

$N_{overlap}$ = # of outer solenoid's turns that overlap with inner solenoid



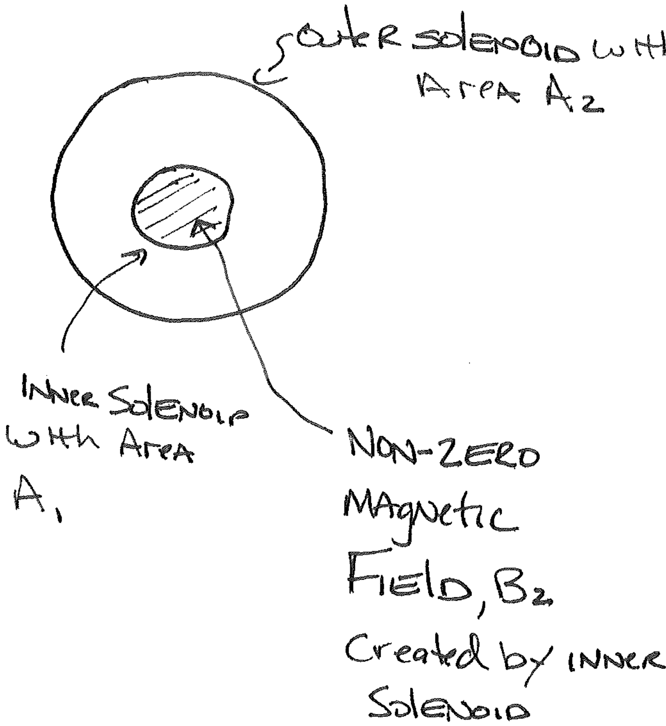
$$N_{overlap} = \underbrace{\left(\frac{N_1}{l_1}\right)}_{\text{\# of turns per length on outer solenoid}} l_2$$

length of inner solenoid

$$\Rightarrow M = \frac{\left(\frac{N_1}{l_1}\right) l_2 \Phi_{B1}}{i_2}$$

Φ_{B1} = Flux through outer solenoid created by inner solenoid.

Inner solenoid's magnetic field only exists within the inner solenoid itself (we always assume $B = 0$ outside), so



$$\Phi_{B1} = B_2 A_2 + 0(A_1 - A_2)$$

$$\Phi_{B1} = B_2 A_2$$

$$\Rightarrow M = \frac{\left(\frac{N_1}{\ell_1}\right) \ell_2 B_2 A_2}{i_2}$$

Finally, $B_2 = \mu_0 \left(\frac{N_2}{\ell_2}\right) i_2$

$$\Rightarrow M = \frac{\left(\frac{N_1}{\ell_1}\right) \cancel{\ell_2} \mu_0 \left(\frac{N_2}{\cancel{\ell_2}}\right) \cancel{i_2} A_2}{\cancel{i_2}} = \frac{\mu_0 N_1 N_2 A_2}{\ell_1}$$

← SAME RESULT. A LOT MORE WORK

$A_2 = \pi r_2^2 = \pi(.007m)^2$ ← diameter $= 1.4cm \Rightarrow r_2 = .7cm$
$= .007m$

$$\therefore M = \frac{(4\pi \times 10^{-7} T \cdot m/A)(6750)(1200)\pi(.007m)^2}{.75m} \Rightarrow \boxed{M = .00209H = .0021H}$$

b) WHAT IS INDUCED EMF IN THE INNER SOLENOID?

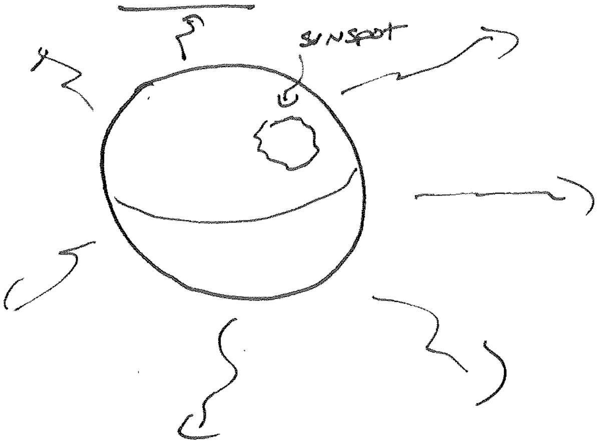
$\mathcal{E}_2 = ?$

$$\mathcal{E}_2 = -M\frac{di_1}{dt} = -(.00209H)(49.2A/s)$$

$$\Rightarrow \mathcal{E}_2 = -.103V \Rightarrow \boxed{|\mathcal{E}_2| = .103V}$$

DIRECTION OF i_2

#6



THE SUN!

$B = .4T$

$r = 25{,}000\,km$

density, $\rho = 3\times10^{-4}\,Kg/m^3$

IF 100% of MAGNETIC POTENTIAL ENERGY CONVERTED TO Kinetic, HOW FAST would SUNSPOT MATERIAL BE ejected

FIND KE of $1m^3$ OF MATERIAL since $u = \frac{B^2}{2\mu_0} = \frac{U}{V}$ $V \to$ volume

$\Rightarrow U = uV = K = \frac{1}{2}mv^2$

$\uparrow$ 100% conversion

density, $\rho = \frac{m}{V} \Rightarrow m = \rho V$

$\Rightarrow u\cancel{V} = \frac{1}{2}\rho \cancel{V} v^2 \Rightarrow u = \frac{1}{2}\rho v^2$

$u = \frac{B^2}{2\mu_0} = \frac{(.4T)^2}{2(4\pi\times10^{-7}\,T\cdot m/A)} = 6.366\times10^4\,J/m^3 = \frac{1}{2}(3\times10^{-4}\,kg/m^3)v^2$

Unit: $\frac{T^2}{T\cdot m/A} = \frac{T\cdot A}{m} = \frac{N/(A\cdot m)\cdot A}{m} = \frac{N}{m^2} = \frac{N\cdot m}{m^3}$

$= J/m^3$

$\Rightarrow \boxed{v = 2.06\times10^4\,m/s}$

NOT QUITE ENOUGH TO ESCAPE, BUT IMPRESSIVE.