#20 Conservation of Energy Post-class

Due: 11:00am on Monday, October 8, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Work on a Sliding Box

A box of mass m is sliding along a horizontal surface.

Part A

The box leaves position $x = 0$ with speed v_0. The box is slowed by a constant frictional force until it comes to rest at position $x = x_1$.

Find F_f, the magnitude of the average frictional force that acts on the box. (Since you don't know the coefficient of friction, don't include it in your answer.)

Express the frictional force in terms of m, v_0, and x_1.

Hint 1. How to approach the problem

Use the work-energy theorem. As applied to this part, the theorem states that the work done by friction is equal to the change in kinetic energy of the box:

$W_f = \Delta K = K_f - K_i$.

Find K_i, K_f, and W_f (which will depend on F_f), then solve for F_f.

Hint 2. Find the initial kinetic energy

What is K_i, the kinetic energy of the box at position $x = 0$?

ANSWER:

K_i = $\frac{1}{2}m{v_0}^2$

Hint 3. Find the final kinetic energy

What is K_f, the kinetic energy of the box when it reaches position $x = x_1$?

ANSWER:

K_f = 0

Hint 4. Find the work done by friction

Find W_f, the work done by friction on the box. Note that the work done by friction is always negative (i.e., friction always dissipates energy).

Express your answer in terms of F_f and other given variables.

ANSWER:

W_f = $-F_f x_1$

ANSWER:

F_f = $\frac{1}{2}\frac{m{v_0}^2}{x_1}$

Correct

Part B

After the box comes to rest at position x_1, a person starts pushing the box, giving it a speed v_1.

When the box reaches position x_2 (where $x_2 > x_1$), how much work W_p has the person done on the box?

Assume that the box reaches x_2 after the person has accelerated it from rest to speed v_1.

Express the work in terms of m, v_0, x_1, x_2, and v_1.

Hint 1. How to approach the problem

Again, use the work-energy theorem. In this part of the problem, both the person and friction are doing work on the box:

$W_{\rm f} + W_{\rm p} = \Delta K = K_{\rm f} - K_{\rm i}$.

Hint 2. Find the work done by friction

What is $W_{\rm f}$, the total work done by friction on the box as the person pushes it from position x_1 to position x_2?

Answer in terms of given variables. (Your answer should not include $F_{\rm f}$.)

Hint 1. Finding the force of friction

The normal force on the box is unchanged from partA. Therefore, the force of friction is the same in this part as in part A.

ANSWER:

$W_{\rm f}$ = $\frac{-(m{v_0}^2)(x_2 - x_1)}{2x_1}$

Hint 3. Find the change in kinetic energy

What is ΔK, the change in kinetic energy of the box from the moment it is at position x_1 to the moment it is at position x_2?

ANSWER:

ΔK = $\frac{1}{2}m{v_1}^2$

ANSWER:

$W_p = \frac{1}{2}\frac{m{v_0}^2}{x_1}(x_2 - x_1) + \frac{1}{2}m{v_1}^2$

Correct

Exercise 7.24

In a "worst-case" design scenario, a 2000-kg elevator with broken cables is falling at 4.00 $\mathrm{m/s}$ when it first contacts a cushioning spring at the bottom of the shaft. The spring is supposed to stop the elevator, compressing 2.00 m as it does so. During the motion a safety clamp applies a constant 17000-N frictional force to the elevator.

Part A

What is the speed of the elevator after it has moved downward 1.00 m from the point where it first contacts a spring?

ANSWER:

v = 3.65 $\mathrm{m/s}$

Correct

Part B

When the elevator is 1.00 m below point where it first contacts a spring, what is its acceleration?

ANSWER:

a = 4.00 $\mathrm{m/s^2}$

Correct

A Mass-Spring System with Recoil and Friction

An object of mass m is traveling on a horizontal surface. There is a coefficient of kinetic friction μ between the object and the surface. The object has speed v when it reaches $x = 0$ and encounters a spring. The object compresses the spring, stops, and then recoils and travels in the opposite direction. When the object reaches $x = 0$ on its return trip, it stops.

Part A

Find k, the spring constant.

Express k in terms of μ, m, g, and v.

Hint 1. Why does the object stop?

Why does the object come to rest when it returns to $x = 0$?

Although more than one answer may be true of the system, you must choose the answer that explains *why* the object ultimately comes to a stop.

ANSWER:

- ○ When the object reaches $x = 0$ the second time all of its initial energy has gone into the compression and extension of the spring.
- ◉ When the object reaches $x = 0$ the second time all of its initial energy has been dissipated by friction.
- ○ $x = 0$ is an equilibrium position and at this point the spring exerts no force on the object.
- ○ At $x = 0$ the force of friction exactly balances the force exerted by the spring on the object.

Hint 2. How does friction affect the system?

Indicate which of the following statements regarding friction is/are true.

Check all that apply.

ANSWER:

- [x] Work done by friction is equal to $-mg\mu d$, where m is the mass of an object, g is the magnitude of the acceleration due to gravity, μ is the coefficient of kinetic friction, and d is the distance the object has traveled.
- [ ] Energy dissipated by friction is equal to $(1/2)\mu gmt^2$, where μ is the coefficient of friction, g is the acceleration due to gravity, m is the mass of the object, and t is the amount of time (since encountering the spring) the object has been moving.
- [ ] Friction is a conservative force.
- [x] Work done by friction is exactly equal to the negative of the energy dissipated by friction.

Hint 3. Energy stored in a spring

The potential energy stored in a spring having constant k that is compressed a distance d is

$$E_{\rm spring} = -\int F\,dx = \int_0^d kx\,dx = \frac{1}{2}kd^2.$$

Hint 4. Compute the compression of the spring

By what distance d does the object compress the spring?

Look at the initial condition when the object originally hits the spring and the final condition when the object returns to $x = 0$.

Express d in terms of v, μ, and g.

Hint 1. How to approach this question

Use the fact that

$$E_{\rm final} = E_{\rm initial} + W_{\rm nonconservative}$$

to solve for the distance the spring was compressed.

Hint 2. The value of $E_{\rm final}$

In its final position, the object is not moving. Also the spring is not compressed. Therefore $E_{\rm final} = 0$.

Hint 3. Find $E_{\rm initial}$

What is the value of $E_{\rm initial}$?

Express your answer in terms of some or all of the variables m, v, μ, and d and g, the acceleration due to gravity.

Hint 1. How to approach this part

Initially the spring is uncompressed, so the only contribution to the system's energy comes from the kinetic energy of the object.

ANSWER:

$E_{\rm initial}$ = $\frac{1}{2}mv^2$

Hint 4. Find $W_{\rm nonconservative}$

What is the value of $W_{\text{nonconservative}}$?

Express your answer in terms of some or all of the variables m, v, μ, and d and g, the acceleration due to gravity.

Hint 1. How to approach this part

The only nonconservative force in the system is the frictional force between the object and the surface it's on. If the object moves through a distance x, the work done by friction W_{friction} is

$$W_{\text{friction}} = \vec{f} \cdot \vec{s} = -\mu m g x$$

ANSWER:

$W_{\text{nonconservative}}$ = $-\mu m g\,(2d)$

ANSWER:

d = $\frac{v^2}{4\mu g}$

Hint 5. Putting it all together

In the previous part, at the two ends of the motion considered, the spring had no energy, so k was not part of the equation. However, you were able to find a relation for d in terms of the known quantities. To obtain an equation involving k, use conservation of energy again,

$$E_{\text{final}} = E_{\text{initial}} + W_{\text{nonconservative}},$$

but this time, take the initial condition to be the moment when the spring is at its maximum compression and the final condition to be the moment when the spring returns to $x = 0$. So now E_{initial} can be written in terms of k and other variables.

Hint 6. The value of E_{final}

The value of $E_{\rm final}$ is again zero.

Hint 7. Find $E_{\rm initial}$ for this part of the motion

What is the value of $E_{\rm initial}$ for this part of the motion?

Express your answer in terms of d and k, the spring constant, so that you end up with an equation containing k.

Hint 1. How to approach this part

Since the spring is at its maximum compression, the object must be momentarily at rest. So the only contribution to the energy is from the potential energy of the spring.

ANSWER:

$E_{\rm initial}$ = $\frac{1}{2}kd^2$

Hint 8. Find $W_{\rm nonconservative}$ for this part of the motion

What is the value of $W_{\rm nonconservative}$ for this part of the motion?

Express your answer in terms of m, μ, d, and g, the acceleration due to gravity.

Hint 1. How to approach this part

The only nonconservative force in the system is the frictional force between the object and the surface it's on. If the object moves through a distance x, the work done by friction $W_{\rm friction}$ is

$$W_{\rm friction} = \vec{f} \cdot \vec{s} = -\mu m g x.$$

ANSWER:

$W_{\rm nonconservative}$ = $-\mu m g d$

ANSWER:

k = $\frac{8\mu^2 g^2 m}{v^2}$

Correct

Score Summary:

Your score on this assignment is 105.5%.
You received 31.65 out of a possible total of 30 points.