#31 Fluid Mechanics I Pre-class

Due: 11:00am on Monday, November 5, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

± Playing with a Water Hose

Two children, Ferdinand and Isabella, are playing with a water hose on a sunny summer day. Isabella is holding the hose in her hand 1.0 meters above the ground and is trying to spray Ferdinand, who is standing 10.0 meters away.

Part A

Will Isabella be able to spray Ferdinand if the water is flowing out of the hose at a constant speed v_0 of 3.5 meters per second? Assume that the hose is pointed parallel to the ground and take the magnitude of the acceleration g due to gravity to be 9.81 meters per second, per second.

Hint 1. General approach: considerations on particle motion

Consider the water flowing out of the hose to be composed of independent particles in projectile motion. Recall that projectile motion is described as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration. Note that the hose is pointed parallel to the ground, so the initial velocity of the water is purely horizontal.

Hint 2. Projectile motion

A projectile is a body with an initial velocity that follows a path determined entirely by the effects of gravitational acceleration and air resistance. If the effects due to air resistance are ignored, the motion of a projectile can be analyzed as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.

Consider a particle undergoing projectile motion in the *x*-*y* coordinate plane, with the *x* axis horizontal and the *y* axis pointing vertically upward. Let v_x and v_y represent the components of the initial velocity of the particle. Let x_0 and y_0 represent the initial horizontal and vertical positions of the particle. The equations describing the position of the particle as a function of time t are then

$$x = x_0 + v_x t$$

$$y = y_0 + v_y t - \frac{1}{2} g t^2,$$

where g is the magnitude of the acceleration due to gravity.

ANSWER:

- Yes
- No

Correct

Part B

To increase the range of the water, Isabella places her thumb on the hose hole and partially covers it. Assuming that the flow remains steady, what fraction f of the cross-sectional area of the hose hole does she have to cover to be able to spray her friend?

Assume that the cross section of the hose opening is circular with a radius of 1.5 centimeters.

Express your answer as a percentage to the nearest integer.

Hint 1. General approach: considerations on fluid mechanics

By restricting the area across which the water is flowing, Isabella forces the water to flow out at a higher speed. After estimating the initial velocity that the water must have in order to reach a distance of 10 meters, you can determine the cross-sectional area of the hose hole that corresponds to such an initial velocity of the water by applying the continuity equation.

Hint 2. Find the outflow speed needed

Find the initial speed v that the water should have in order to reach a distance of 10 meters. Consider the water flowing out of the hose as composed of independent particles in projectile motion. Recall that projectile motion is described as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration. Note that the hose is pointed parallel to the ground, so the initial velocity of water is purely horizontal. Thus $v = v_x$.

Express your answer numerically in meters per second to three significant figures.

Hint 1. Projectile motion

If the effects due to air resistance are ignored, as they should be in this problem, the motion of a projectile can be analyzed as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.

Consider a particle undergoing projectile motion in the *x*-*y* coordinate plane, with the *x* axis horizontal and the *y* axis pointing vertically upward. Let v_x and v_y represent the components of the initial velocity of the particle. Let x_0 and y_0 represent the initial horizontal and vertical positions of the particle. The equations describing the position of the particle as a function of time t are then

$$x = x_0 + v_x t$$

$$y = y_0 + v_y t - \frac{1}{2} g t^2,$$

where g is the magnitude of the acceleration due to gravity. In this part, you are asked to find the outflow speed, which is horizontal, and thus corresponds to v_x is the above equation. Consider carefully what the values of the other constants-- x_0, y_0, and v_y--are.

ANSWER:

v_x = 22.1 $\mathrm{m/s}$

Hint 3. Find the cross-sectional area needed

Once you know the horizontal velocity of the water that would be needed for the emerging stream to reach a distance of 10 meters, find the cross-sectional area A that corresponds to that outflow speed.

Assume that the flow rate remains steady after the partial blockage of the hose hole, so that the continuity equation applies.

Express your answer numerically in centimeters squared to three significant figures.

Hint 1. Apply the continuity equation

In an incompressible fluid moving steadily, the mass of fluid flowing along a flow tube is constant. In particular, consider a flow tube between two stationary cross sections with areas A_1 and A_2. Let the fluid speeds at these sections be v_1 and v_2, respectively. Then conservation of mass takes the form

$$A_1 v_1 = A_2 v_2,$$

which is known as the continuity equation.

If initially the water is flowing through the hose of cross-sectional area A_0 at a speed v_0, and you want it to flow through a smaller area at a higher speed v, what should the new cross-sectional area A of the hose be?

Express your answer in terms of A_0, v_0, and v.

ANSWER:

$A = \frac{A_0 v_0}{v}$

ANSWER:

$A =$ 1.12 $\mathrm{cm^2}$

ANSWER:

$f =$ 84 %

Correct

Archimedes' Principle

Learning Goal:

To understand the applications of Archimedes' principle.

Archimedes' principle is a powerful tool for solving many problems involving equilibrium in fluids. It states the following:
When a body is partially or completely submerged in a fluid (either a liquid or a gas), the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.
As a result of the upward Archimedes force (often called the *buoyant force*), some objects may float in a fluid, and all of them appear to weigh less. This is the familiar phenomenon of *buoyancy*.

Quantitatively, the buoyant force can be found as

$$F_{\rm buoyant} = \rho_{\rm fluid} g V,$$

where $F_{\rm buoyant}$ is the force, $\rho_{\rm fluid}$ is the density of the fluid, g is the magnitude of the acceleration due to gravity, and V is the volume of the displaced fluid.

In this problem, you will be asked several qualitative questions that should help you develop a feel for Archimedes' principle.

An object is placed in a fluid and then released. Assume that the object either floats to the surface (settling so that the object is partly above and partly below the fluid surface) or sinks to the bottom. (Note that for Parts A through D, you should assume that the object has settled in equilibrium.)

Part A

Consider the following statement:
The magnitude of the buoyant force is equal to the weight of fluid displaced by the object.
Under what circumstances is this statement true?

Hint 1. Archimedes' principle

The statement of Part A is one way of expressing Archimedes' principle.

ANSWER:

- (selected) for every object submerged partially or completely in a fluid
- only for an object that floats
- only for an object that sinks
- for no object submerged in a fluid

Correct

Use Archimedes' principle to answer the rest of the questions in this problem.

Part B

Consider the following statement:
The magnitude of the buoyant force is equal to the weight of the amount of fluid that has the same total volume as the object.
Under what circumstances is this statement true?

Hint 1. Consider Archimedes' principle

Archimedes' principle deals with the displaced volume. When is the displaced volume equal to the total volume of the object?

ANSWER:

- for an object that is partially submerged in a fluid
- only for an object that floats
- (selected) for an object completely submerged in a fluid
- for no object partially or completely submerged in a fluid

Correct

Part C

Consider the following statement:
The magnitude of the buoyant force equals the weight of the object.

Under what circumstances is this statement true?

Hint 1. Forces and equilibrium

If the buoyant force and the weight of the object are equal, the object can be in equilibrium without any other forces acting on it.

ANSWER:

- ◯ for every object submerged partially or completely in a fluid
- ◉ for an object that floats
- ◯ only for an object that sinks
- ◯ for no object submerged in a fluid

Correct

Part D

Consider the following statement:
The magnitude of the buoyant force is less than the weight of the object.
Under what circumstances is this statement true?

ANSWER:

- ◯ for every object submerged partially or completely in a fluid
- ◯ for an object that floats
- ◉ for an object that sinks
- ◯ for no object submerged in a fluid

Correct

Now apply what you know to some more complicated situations.

Part E

An object is floating in equilibrium on the surface of a liquid. The object is then removed and placed in another container, filled with a denser liquid. What would you observe?

Hint 1. Density and equilibrium

If the second liquid is denser than the first one, the fluid would exert a greater upward force on the object if it were held submerged as deeply as in the first case.

ANSWER:

- ○ The object would sink all the way to the bottom.
- ○ The object would float submerged more deeply than in the first container.
- ◉ The object would float submerged less deeply than in the first container.
- ○ More than one of these outcomes is possible.

Correct

Part F

An object is floating in equilibrium on the surface of a liquid. The object is then removed and placed in another container, filled with a less dense liquid.

What would you observe?

Hint 1. Density and equilibrium

If the second liquid is less dense than the first one, the liquid would exert a smaller upward force on the object if it were held submerged as deeply as in the first case.

ANSWER:

- The object would sink all the way to the bottom.
- The object would float submerged more deeply than in the first container.
- The object would float submerged less deeply than in the first container.
- More than one of these outcomes is possible.

Correct

If the fluid in the second container is less dense than the object, then the object will sink all the way to the bottom. If the fluid in the second container is denser than the object (though less dense than the fluid in the original container), the object will still float, but its depth will be greater than it was in the original container.

Part G

Two objects, T and B, have identical size and shape and have uniform density. They are carefully placed in a container filled with a liquid. Both objects float in equilibrium. Less of object T is submerged than of object B, which floats, fully submerged, closer to the bottom of the container. Which of the following statements is true?

ANSWER:

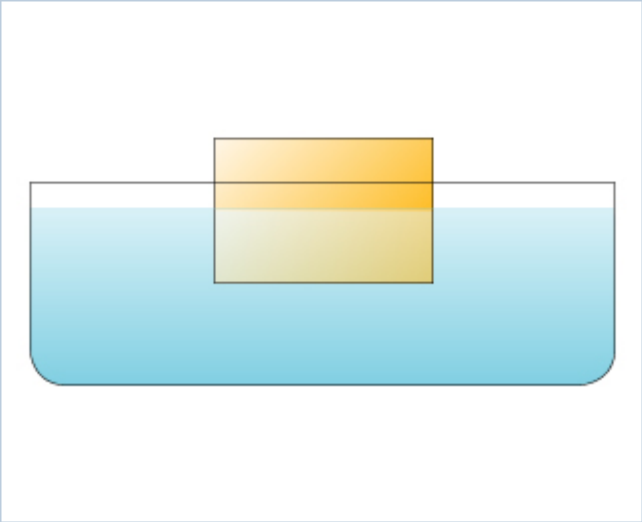
- Object T has a greater density than object B.
- Object B has a greater density than object T.
- Both objects have the same density.

Correct

Since both objects float, the buoyant force in each case is equal to the object's weight. Block B displaces more fluid, so it must be heavier than block T. Given that the two objects have the same volume, block B must also be denser. In fact, since the weight equals the buoyant force, and B is fully submerged, $\rho_{\rm B} V g = \rho_{\rm liquid} V g$, where all the symbols have their usual meaning. From this equation, one can see that the density of B must equal the density of the fluid.

± Buoyant Force Conceptual Question

A rectangular wooden block of weight W floats with exactly one-half of its volume below the waterline.



Part A

What is the buoyant force acting on the block?

Hint 1. Archimedes' principle

The upward buoyant force on a floating (or submerged) object is equal to the weight of the liquid displaced by the object. Mathematically, the buoyant force $F_{\rm buoyant}$ on a floating (or submerged) object is

$$F_{\rm buoyant} = \rho g V,$$

where ρ is the density of the fluid, V is the submerged volume of the object, and g is the acceleration due to gravity.

Hint 2. What happens at equilibrium

The block is in equilibrium, so the net force acting on it is equal to zero.

ANSWER:

- $2W$
- W
- $\frac{1}{2}W$
- The buoyant force cannot be determined.

Correct

Part B

The density of water is 1.00 $\rm g/cm^3$. What is the density of the block?

Hint 1. Applying Archimedes' principle

In Part A, you determined that the buoyant force, and hence the weight of the water displaced, is equal to the weight of the block. Notice, however, that the volume of the water displaced is one-half of the volume of the block.

Hint 2. Density

The density ρ of a material of mass m and volume V is

$$\rho = \frac{m}{V}.$$

ANSWER:

- 2.00 $\mathrm{g/cm^3}$
- between 1.00 and 2.00 $\mathrm{g/cm^3}$
- 1.00 $\mathrm{g/cm^3}$
- between 0.50 and 1.00 $\mathrm{g/cm^3}$
- 0.50 $\mathrm{g/cm^3}$ (selected)
- The density cannot be determined.

Correct

Part C

Masses are stacked on top of the block until the top of the block is level with the waterline. This requires 20 g of mass. What is the mass of the wooden block?

Hint 1. How to approach the problem

When you add the extra mass on top of the block, the buoyant force must change. Relating the mass to the new buoyant force will allow you to write an equation to solve for m.

Hint 2. Find the new buoyant force

With the 20-g mass on the block, twice as much of the block is underwater. Therefore, what happens to the buoyant force on the block?

ANSWER:

- ◉ The buoyant force doubles.
- ○ The buoyant force is halved.
- ○ The buoyant force doesn't change.

Hint 3. System mass

Before the 20-g mass is placed on the block, the mass of the system is just the mass of the block (m), and this mass is supported by the buoyant force. With the 20-g mass on top, the total mass of the system is now $m + 20\,\mathrm{g}$.

Hint 4. The mass equation

The original buoyant force was equal to the weight W of the block. If the new buoyant force F_b is twice the old buoyant force, then

$$F_\mathrm{b} = 2W = 2mg,$$

where m is the mass of the block.

The new buoyant force must support the weight of the block and the mass, so according to Newton's 2nd law

$$F_\mathrm{b} = (m + 20)g.$$

By setting these two expressions for F_b equal to each other, you can solve for m.

ANSWER:

- 40 g
- 20 g
- 10 g

Correct

Part D

The wooden block is removed from the water bath and placed in an unknown liquid. In this liquid, only one-third of the wooden block is submerged. Is the unknown liquid more or less dense than water?

ANSWER:

- more dense
- less dense

Correct

Part E

What is the density of the unknown liquid $\rho_{\rm unknown}$?

Express your answer numerically in grams per cubic centimeter.

Hint 1. Comparing densities

From Part C, the mass of the block is 20 g. In water, one-half of the block is submerged, so one-half of the volume of the block times the density of the water must be equivalent to 20 g. In the unknown substance, one-third of the block is submerged, so one-third of the volume of the block times the density of the unknown substance must be equivalent to 20 g. You can use these observations to write an equation for the density of the unknown substance.

Hint 2. Setting up the equation

The buoyant force on the block in water $F_{b,water}$ is the same as the buoyant force in the unknown liquid $F_{b,unknown}$,

$$F_{\rm b,water} = F_{\rm b,unknown},$$

so

$$\rho_{\rm water} g \left(\frac{V}{2}\right) = \rho_{\rm unknown} g \left(\frac{V}{3}\right),$$

where $\rho_{\rm water}$ is the density of water, $\rho_{\rm unknown}$ is the density of the unknown liquid, V is the volume of the entire block, and g is the acceleration due to gravity.

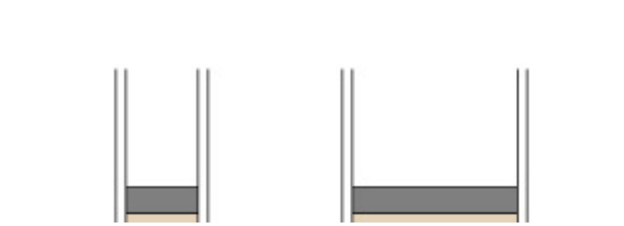
ANSWER:

$\rho_{\rm unknown}$ = 1.5 $\mathrm{g/cm^3}$

Correct

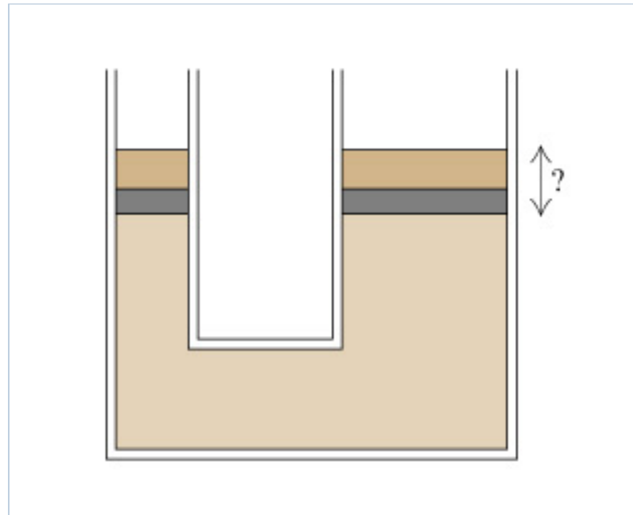
Fluid Pressure in a U-Tube

A U-tube is filled with water, and the two arms are capped. The tube is cylindrical, and the right arm has twice the radius of the left arm. The caps have negligible mass, are watertight, and can freely slide up and down the tube.



Part A

A one-inch depth of sand is poured onto the cap on each arm. After the caps have moved (if necessary) to reestablish equilibrium, is the right cap higher, lower, or the same height as the left cap?



Hint 1. Pressure at the surface

The pressure at the base of each arm depends on the pressure at the surface of each arm, not the weight at the surface of each arm.

Hint 2. Evaluate the weight of the sand

Compare the weight of the sand on the two caps. Which of the following is true?

ANSWER:

The weight of the sand on the right cap is
- (selected) greater than
- less than
- equal to

the weight of the sand on the left cap.

Hint 3. Evaluate the pressure applied by the sand

Now compare the pressure exerted by the sand on the two caps. Which of the following is true?

ANSWER:

The pressure exerted by the sand on the right cap is
- greater than
- less than
- (selected) equal to

the pressure exerted by the sand on the left cap.

ANSWER:

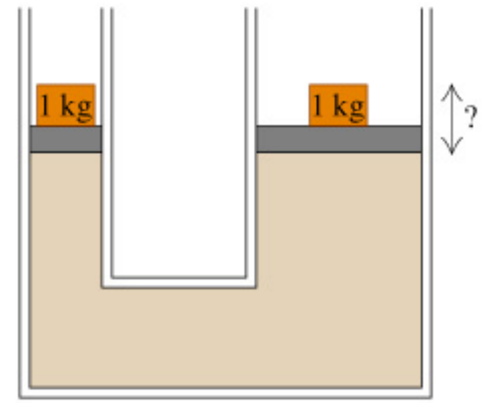
- higher
- lower
- (selected) the same height

Correct

Although one inch of sand on the right cap is much heavier than one inch of sand on the left cap, the pressures exerted by the sand are the same on both caps. Since the pressures exerted by the sand are equal, the pressures at the base of each arm due to the water must be equal. This requires equal heights of water in the two arms.

Part B

The sand is removed and a 1.0-kg-mass block is placed on each cap. After the caps have moved (if necessary) to reestablish equilibrium, is the right cap higher, lower, or the same height as the left cap?



Hint 1. Evaluate the pressure applied by the 1.0-kg blocks

Compare the pressure exerted on the water from the two 1-kg blocks. Which of the following is true?

ANSWER:

The pressure exerted on the right cap is
- greater than
- less than
- equal to

the pressure exerted on the left cap.

ANSWER:

- higher
- lower
- the same height

Correct

Although the masses of the blocks are equal, the pressures exerted by them on the caps are not equal. There is a greater pressure on the left cap, which results in a greater pressure at the base of the left arm. To compensate for this increased pressure, the height of the water column in the right arm will have to be greater than in the left arm.

Part C

If a 1.0-kg-mass block is on the left cap, how much total mass must be placed on the right cap so that the caps equilibrate at equal height?

Express your answer in kilograms.

Hint 1. Meaning of equal water levels

Equal water levels in the two arms require equal pressures on the two caps. Remember that pressure is force divded by area and that area is proportional to the *square* of the radius.

ANSWER:

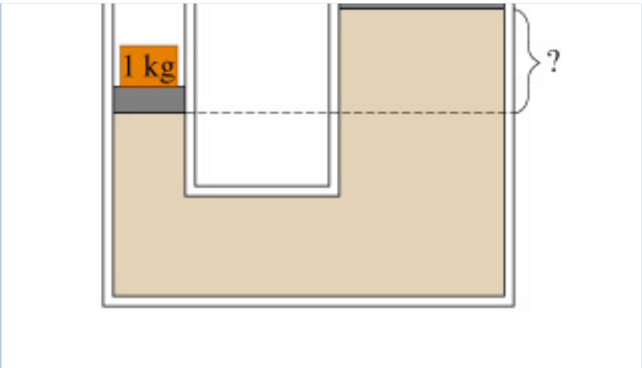
4.0 kg

Correct

Part D

The locations of the two caps at equilibrium are now as given in this figure. The dashed line represents the level of the water in the left arm.
What is the mass of the water located in the right arm between the dashed line and the right cap?

Express your answer in kilograms.



Hint 1. Pressure at the dashed line

The pressure at the location of the dashed line must be equal in the two arms. Therefore, the total mass above the line in the left arm, divided by the area of the left arm, must equal the total mass above the line in the right arm, divided by the area of the right arm.

ANSWER:

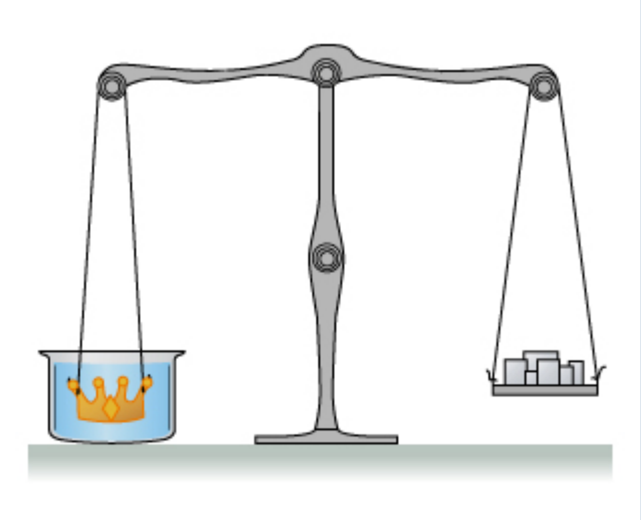
3.0 kg

Correct

Crown of Gold?

According to legend, the following challenge led Archimedes to the discovery of his famous principle: Hieron, king of Syracuse, was suspicious that a new crown that he had received from the royal goldsmith was not pure gold, as claimed. Archimedes was ordered to determine whether the crown was in fact made of pure gold, with the condition that only a nondestructive test would be allowed. Rather than answer the problem in the politically most expedient way (or perhaps extract a bribe from the goldsmith), Archimedes thought about the problem scientifically. The legend relates that when Archimedes stepped into his bath and caused it to overflow, he realized that he could answer the challenge by comparing the volume of water displaced by the crown with the volume of water displaced by an amount of pure gold equal in weight to the crown. If the crown was made of pure gold, the two volumes would be equal. If some other (less dense) metal had been substituted for some of the gold, then the crown would displace more water than the pure gold.

A similar method of answering the challenge, based on the same physical principle, is to compute the ratio of the actual weight of the crown, $W_{\rm actual}$, and the apparent weight of the crown when it is submerged in water, $W_{\rm apparent}$. See whether you can follow in Archimedes' footsteps. The figure shows what is meant by weighing the crown while it is submerged in water.



Part A

Take the density of the crown to be $\rho_{\rm c}$. What is the ratio of the crown's apparent weight (in water) $W_{\rm apparent}$ to its actual weight $W_{\rm actual}$?

Express your answer in terms of the density of the crown $\rho_{\rm c}$ and the density of water $\rho_{\rm w}$.

Hint 1. Find an expression for the actual weight of the crown

Assume that the crown has volume V. Find the actual weight $W_{\rm actual}$ of the crown.

Express $W_{\rm actual}$ in terms of V, g (the magnitude of the acceleration due to gravity), and $\rho_{\rm c}$.

ANSWER:

$W_{\rm actual}$ = $\rho_{\rm c} V g$

Hint 2. Find an expression for the apparent weight of the crown

Assume that the crown has volume V, and take the density of water to be $\rho_{\rm w}$. Find the apparent weight $W_{\rm apparent}$ of the crown submerged in water.

Express your answer in terms of V, g (the magnitude of the acceleration due to gravity), $\rho_{\rm w}$, and $\rho_{\rm c}$.

Hint 1. How to approach the problem

The apparent weight of the crown when it is submerged in water will be less than its actual weight (weight in air) due to the buoyant force, which opposes gravity.

Hint 2. Find an algebraic expression for the buoyant force.

Find the magnitude $F_{\rm buoyant}$ of the buoyant force on the crown when it is completely submerged in water.

Express your answer in terms of $\rho_{\rm w}$, V, and the gravitational acceleration g.

ANSWER:

$F_{\rm buoyant}$ = $\rho_w V g$

ANSWER:

$W_{\rm apparent}$ = $(\rho_c - \rho_w)\, gV$

ANSWER:

$$\frac{W_{\rm apparent}}{W_{\rm actual}} = 1 - \frac{\rho_w}{\rho_c}$$

Correct

Part B

Imagine that the apparent weight of the crown in water is $W_{\rm apparent} = 4.50 \ {\rm N}$, and the actual weight is $W_{\rm actual} = 5.00 \ {\rm N}$. Is the crown made of pure (100%) gold? The density of water is $\rho_{\rm w} = 1.00$ grams per cubic centimeter. The density of gold is $\rho_{\rm g} = 19.32$ grams per cubic centimeter.

Hint 1. Find the ratio of weights for a crown of pure gold

Given the expression you obtained for $\frac{W_{\rm apparent}}{W_{\rm actual}}$, what should the ratio of weights be if the crown is made of pure gold?

Express your answer numerically, to two decimal places.

ANSWER:

$\frac{W_{\rm apparent}}{W_{\rm actual}}$ = 0.95

ANSWER:

- Yes
- No

Correct

For the values given, $\frac{W_{\rm apparent}}{W_{\rm actual}} = 4.50/5.00 = 0.90$, whereas for pure gold, $\frac{W_{\rm apparent}}{W_{\rm actual}} = 1 - \frac{\rho_{\rm w}}{\rho_{\rm g}} = 0.95$. Thus, you can conclude that the the crown is not pure gold but contains some less-dense metal. The goldsmith made sure that the crown's (true) weight was the same as that of the amount of gold he was allocated, but in so doing was forced to make the volume of the crown slightly larger than it would otherwise have been.

Pressure in the Ocean

The pressure at 10 $\rm m$ below the surface of the ocean is about $2.00\times10^{5}\,\rm Pa$.

Part A

Which of the following statements is true?

Hint 1. How to approach the problem

Since pressure is defined as force per unit area, the pressure at a given depth below the surface of the ocean is the normal force on a small area at that depth divided by that area. Thus, given the pressure at 10 $\rm m$ you can determine the normal force on a unit area at that depth. Also recall that, in general, the pressure in a fluid varies with height, but it also depends on the external pressure applied to the fluid. In the case of the ocean, the external pressure applied at its surface is the atmospheric pressure.

ANSWER:

- The weight of a column of seawater 1 m^2 in cross section and 10 m high is about 2.00×10^5 N.
- The weight of a column of seawater 1 m^2 in cross section and 10 m high plus the weight of a column of air with the same cross section extending up to the top of the atmosphere is about 2.00×10^5 N.
- The weight of 1 m^3 of seawater at 10 m below the surface of the ocean is about 2.00×10^5 N.
- The density of seawater is about 2.00×10^5 times the density of air at sea level.

Correct

The pressure at a given level in a fluid is caused by the weight of the overlying fluid *plus* any external pressure applied to the fluid. In this case, the external pressure is the atmospheric pressure. Tropical storms are formed by low-pressure systems in the atmosphere. When such a storm forms over the ocean, both the atmospheric pressure and the fluid pressure in the ocean decrease.

Part B

Now consider the pressure 20 m below the surface of the ocean. Which of the following statements is true?

Hint 1. Variation of pressure with height

If the pressure at a point in a fluid is p_0, then, at a distance h below this point, the pressure p in the fluid is

$$p = p_0 + \rho g h,$$

where ρ is the density of the fluid and g is the acceleration due to gravity. This means that the pressure at a depth h below a reference height of pressure p_0 is *greater* than p_0 by an amount $\rho g h$. Note that the quantity $\rho g h$ is related to the weight of a column of fluid of height h and 1 m^2 in cross section.

ANSWER:

- The pressure is twice that at a depth of 10 m.
- The pressure is the same as that at a depth of 10 m.
- The pressure is equal to that at a depth of 10 m plus the weight per 1 m^2 cross sectional area of a column of seawater 10 m high.
- The pressure is equal to the weight per 1 m^2 cross sectional area of a column of seawater 20 m high.

Correct

Score Summary:

Your score on this assignment is 99.4%.
You received 29.81 out of a possible total of 30 points.