#39 Damped and Forced Oscillations Post-class

Due: 11:00am on Monday, November 26, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

± PSS: Simple Harmonic Motion II: Energy

Learning Goal:

To practice Problem-Solving Strategy: Simple Harmonic Motion II: Energy.

A child's toy consists of a spherical object of mass 50g attached to a spring. One end of the spring is fixed to the side of the baby's crib so that when the baby pulls on the toy and lets go, the object oscillates horizontally with a simple harmonic motion. The amplitude of the oscillation is 6cm and the maximum velocity achieved by the toy is 3.2$\mathrm{m/s}$. What is the kinetic energy K of the toy when the spring is compressed 3.6cm from its equilibrium position?

Problem-Solving Strategy: **Simple Harmonic Motion II: Energy**

The energy equation, $E = \frac{1}{2}mv_x{}^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, is a useful alternative relationship between velocity and position, especially when energy quantities are also required. If the problem involves a relationship among position, velocity, and acceleration without reference to time, it is usually easier to use the equation for simple harmonic motion, $a_x = \frac{d^2x}{dt^2} = -\frac{k}{m}x$ (from Newton's second law) or the energy equation above (from energy conservation) than to use the general expressions for x, v_x, and a_x as functions of time. Because the energy equation involves x^2 and $v_x{}^2$, it cannot tell you the sign of x or of v_x; you have to infer the sign from the situation. For instance, if the body is moving from the equilibrium position toward the point of greatest positive displacement, then x is positive and v_x is positive.

IDENTIFY the relevant concepts

Energy quantities are required in this problem, therefore it is appropriate to use the energy equation for simple harmonic motion.

SET UP the problem using the following steps

Part A

The following is a list of quantities that describe specific properties of the toy. Identify which of these quantities are known in this problem.

Select all that apply.

ANSWER:

- [x] amplitude A
- [ ] total energy E
- [x] mass m
- [x] maximum velocity v_{max}
- [ ] force constant k
- [ ] potential energy U at x
- [ ] kinetic energy K at x
- [x] position x from equilibrium

Correct

Your target variable is the kinetic energy K of the toy at a distance 3.6cm from its equilibrium position.

EXECUTE the solution as follows

Part B

What is the kinetic energy of the object on the spring when the spring is compressed 3.6cm from its equilibrium position?

Express your answer in joules using three significant figures.

Hint 1. How to approach the problem

The energy equation,

$$E = \frac{1}{2}m{v_x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

shows that, for a body undergoing simple harmonic motion, the total energy E at a given distance x from the body's equilibrium position is equal to the sum of the body's kinetic energy $K = \frac{1}{2}m{v_x}^2$ and potential energy $U = \frac{1}{2}kx^2$.

Since the total energy is constant at any point, you can solve the energy equation for K. Before you do that, however, you will need to determine the value of the force constant k.

Hint 2. Determine an expression for the kinetic energy K

Write down an expression for the kinetic energy K of a harmonic oscillator at a position x from equilibrium and whose amplitude of motion and force constant are A and k, respectively.

Express your answer in terms of some or all of the variables k, A and x.

Hint 1. An expression for the total energy of an oscillating body

The total mechanical energy E of a body undergoing simple harmonic motion is directly related to the amplitude A of the motion. When the body reaches the point $x = A$, its maximum displacement from equilibrium, it momentarily stops as it turns back toward the equilibrium position. That is, when $x = A$ (or $-A$), $v_x = 0$. At this point the energy is entirely potential, and $E = \frac{1}{2}kA$. Because E is constant, it is equal to $\frac{1}{2}kA$ at any other point.

ANSWER:

K = $\frac{k\left(A^2 - x^2\right)}{2}$

Hint 3. Calculate the force constant k

Given that the maximum velocity of the toy is $v_{\rm max}$ = 3.2$\rm m/s$ and the amplitude of the motion is A = 6$\rm cm$, what is the force constant k of the spring?

Express your answer in newtons per meter using three significant figures.

Hint 1. How to calculate the force constant given the maximum velocity $v_{\rm max}$

When an oscillating object attached to a spring passes through the equilibrium position at $x = 0$, its energy is entirely kinetic because at that point the spring has zero potential energy. Therefore, at $x = 0$ the energy equation reduces to $E = K = \frac{1}{2}m{v_{\rm max}}^2$. Because the total energy can also be expressed as $E = \frac{1}{2}kA^2$, you can use the energy equation to write down an equation that relates the force constant to the mass and the maximum velocity of the object and the amplitude of the motion, and solve that equation for k.

ANSWER:

k = 142 $\rm N/m$

ANSWER:

K = 0.164 $\rm J$

Correct

EVALUATE your answer

Part C

What is the potential energy U of the toy when the spring is compressed 3.6$\rm cm$ from its equilibrium position?

Express your answer in joules using three significant figures.

ANSWER:

U = 9.22×10^{-2} J

Correct

The total energy in the system, E, remains constant as the toy oscillates on the spring, and is equal to the sum of the kinetic energy and the potential energy in the system at any given position x. To check your results for consistency, you should find that the sum of the kinetic and potential energy calculated in Parts B and C equal $E = \frac{1}{2}kA^2 = 0.256\,\mathrm{J}$.

± Damped Egg on a Spring

A 50.0-g hard-boiled egg moves on the end of a spring with force constant $k = 25.0\ \mathrm{N/m}$. It is released with an amplitude 0.300 m. A damping force $F_x = -bv$ acts on the egg. After it oscillates for 5.00 s, the amplitude of the motion has decreased to 0.100 m.

Part A

Calculate the magnitude of the damping coefficient b.

Express the magnitude of the damping coefficient numerically in kilograms per second, to three significant figures.

Hint 1. How damped is it?

The system described above is ___________.

Hint 1. How to determine damping

The key phrase is "it oscillates."

1. Underdamped corresponds to oscillatory motion with an exponential decay in amplitude.
2. Critically damped corresponds to simple decaying motion with at most one overshoot of the system's resting position.
3. Overdamped corresponds to simple exponentially decaying motion (without any oscillations).

ANSWER:

- critically damped
- overdamped
- underdamped

Hint 2. What formula to use

In this problem, the motion is described by the general equation for an underdamped oscillator,

$$x = Ae^{-bt/2m} \cos\left(\omega' t + \phi\right),$$

where

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}},$$

x is position, and t is time. The displacement is thus a product of a oscillating cosine term and a damping term $A_1\left(t\right)$. This equation is the solution to the damped oscillator equation $m\ddot{x} = -kx - b\dot{x}$.

Hint 3. Find the amplitude

What is $A_1\left(t\right)$, the amplitude as a function of time? Use A_0 for the initial displacement of the system and m for the mass of the egg.

Give your answer in terms of A_0, m, b, and t.

Hint 1. Initial amplitude

In the formula given in for the motion of the egg, A is the *initial* amplitude of the system. This means that A is a displacement at time $t = 0$.

ANSWER:

$A_1(t)$ = $A_0 e^{\frac{-bt}{2m}}$

Hint 4. Solving for y in e^y

If $e^y = C$ then $y = \ln C$.

ANSWER:

b = 2.20×10^{-2} kg/s

Correct

Score Summary:

Your score on this assignment is 104.2%.
You received 20.83 out of a possible total of 20 points.