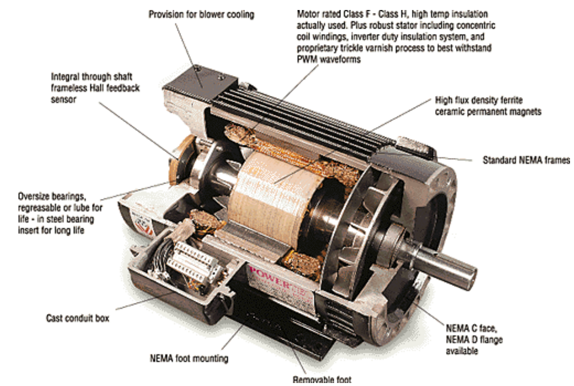
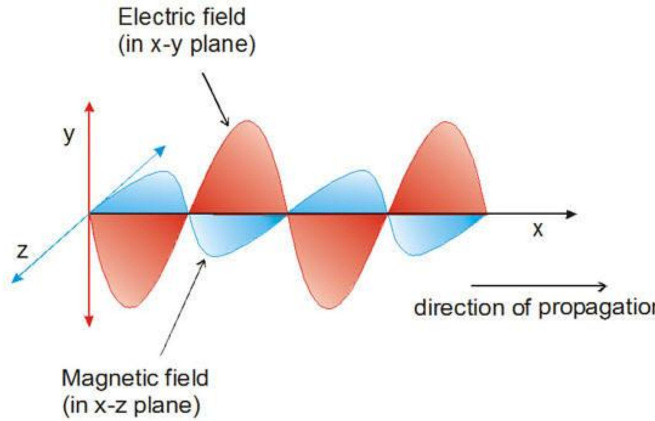
Lecture 28
(Magnetic Force & Torque)

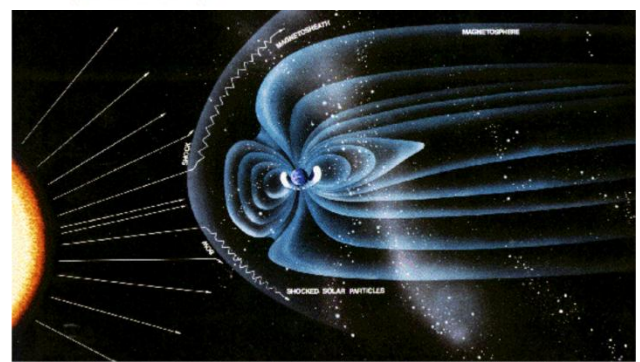
Physics 161-01 Spring 2012
Douglas Fields

Magnetic Forces

- Magnets attract and repel magnets.
- Magnets attract some metals.
- Magnets have some affect that can either turn electric current into work or turn work into electric current.
- What is the underlying, fundamental force that can describe all of these phenomena?





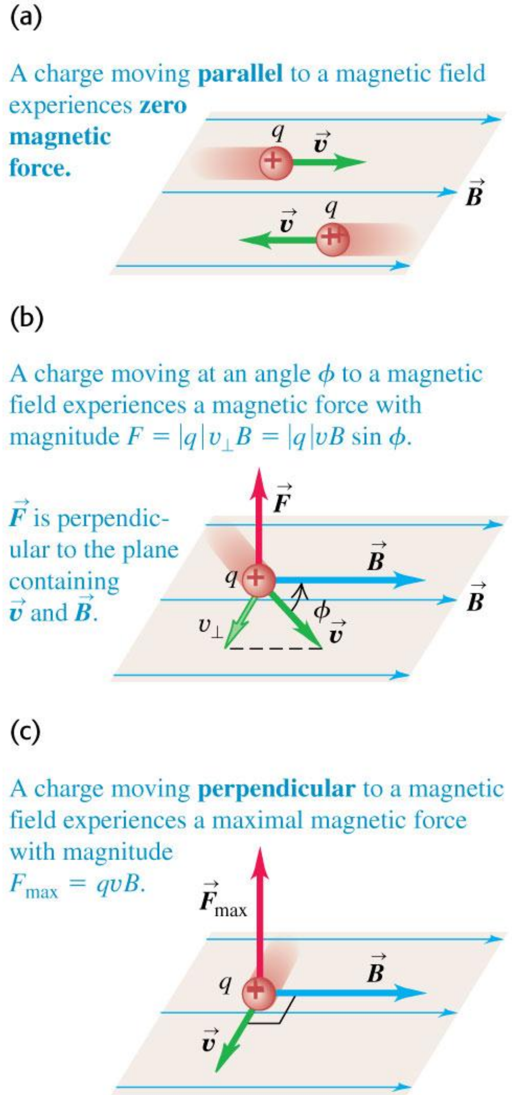


Magnetic Force

- Magnetic fields have an affect on *moving* charges.

$$\vec{F} = q\vec{v} \times \vec{B}$$

- Since it has been a while since we dealt with cross products, let’s take a few minutes to review what this means.
- There is no force on a charge moving in the same direction as the magnetic field.
- The force on a moving charge in a magnetic field is perpendicular to both the field direction and the direction of motion.
- The direction of the force on a negative charge is opposite to that on a positive charge moving in the same direction.
- Since the force is perpendicular to the direction of motion, no work is done by the magnetic force.



Consequences and Applications

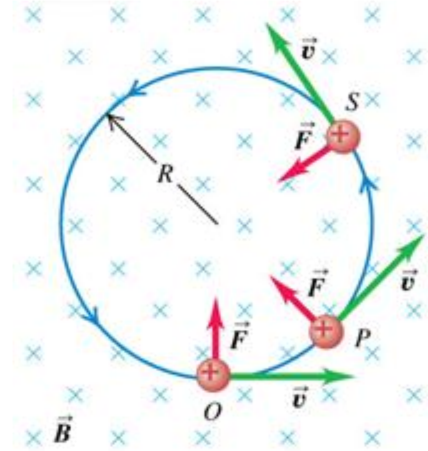
- A beam of charged particles will move in a circle at constant speed when they are sent into it perpendicular to a magnetic field.

$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a} \Rightarrow$$

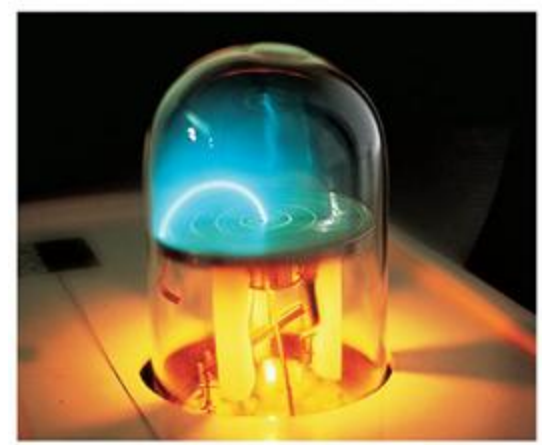
$$\left|\vec{a}\right| = \frac{\left|q\vec{v}\vec{B}\right|}{m} = \frac{v^2}{R} \Rightarrow$$

$$R = \frac{mv}{qB}$$

$$\left[\frac{kg \cdot \frac{m}{s}}{C \cdot T}\right] = \left[\frac{kg \cdot \frac{m}{s}}{C \cdot \frac{N}{A \cdot m}}\right] = \left[\frac{kg \cdot \frac{m}{s}}{C \cdot \frac{\frac{kg \cdot m}{s^2}}{\frac{C}{s} \cdot m}}\right] = [m]$$



(b) An electron beam (seen as a white arc) curving in a magnetic field

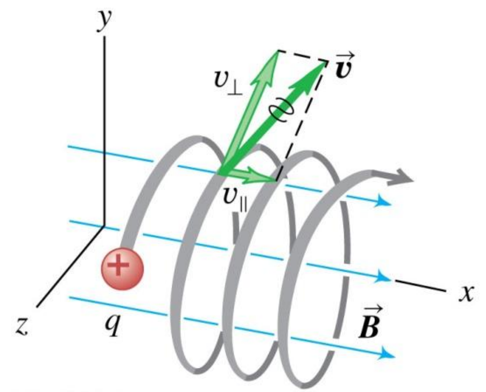


Consequences and Applications
- Magnetic Bottles

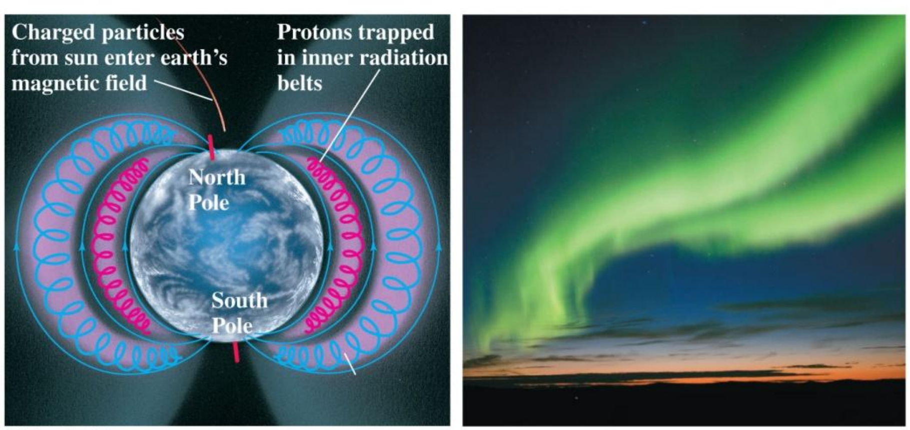
- If there is a component of the charged particle's velocity in the direction of the magnetic field, the path will be helical.
- By using a non-uniform field of the appropriate design, you can trap charged particles in a "magnetic bottle".

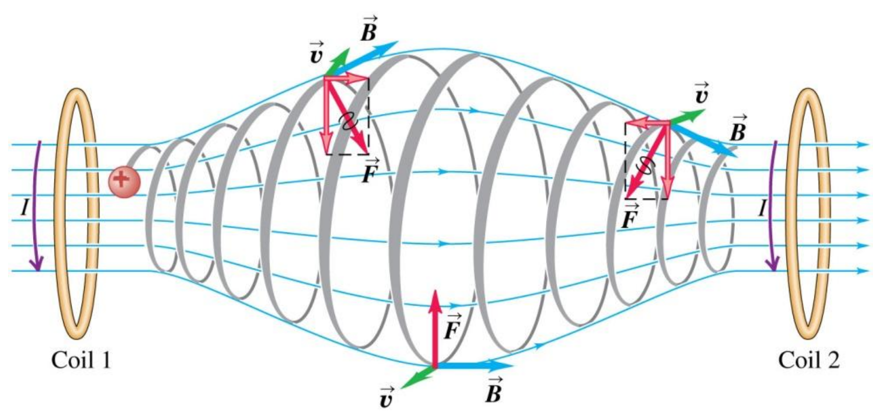
$$\vec{F} = q\vec{v} \times \vec{B}$$

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.



(a) (b)

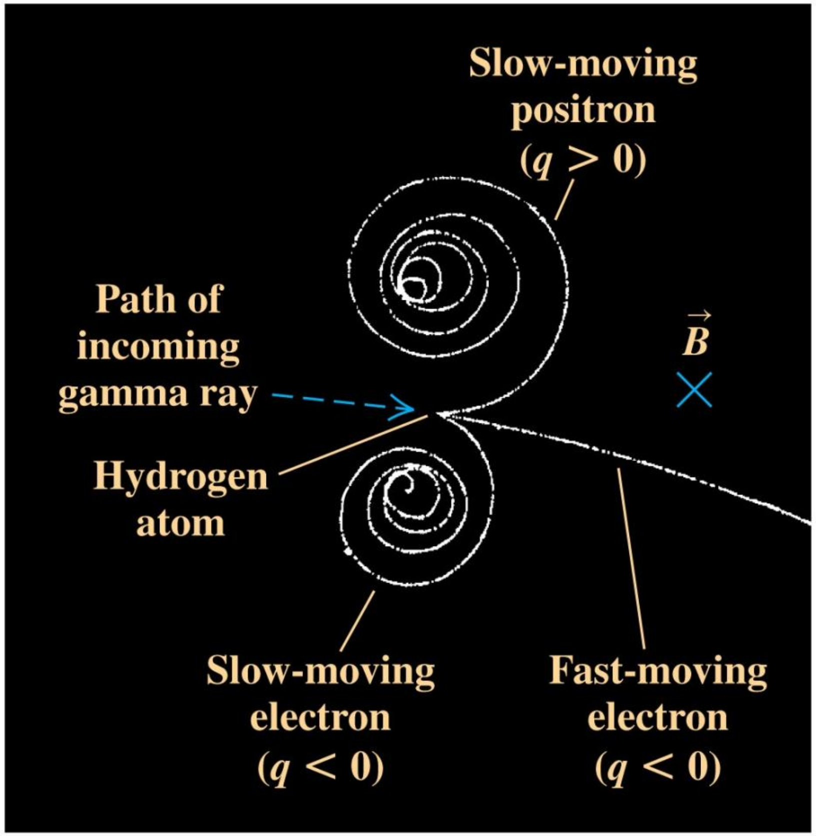




Consequences and Applications
- Momentum Determination

- In my line of work, magnetic fields are used to give information on the mass, charge and velocity of particles that are created when other particles collide together.

$$\vec{F} = q\vec{v} \times \vec{B}$$



Consequences and Applications

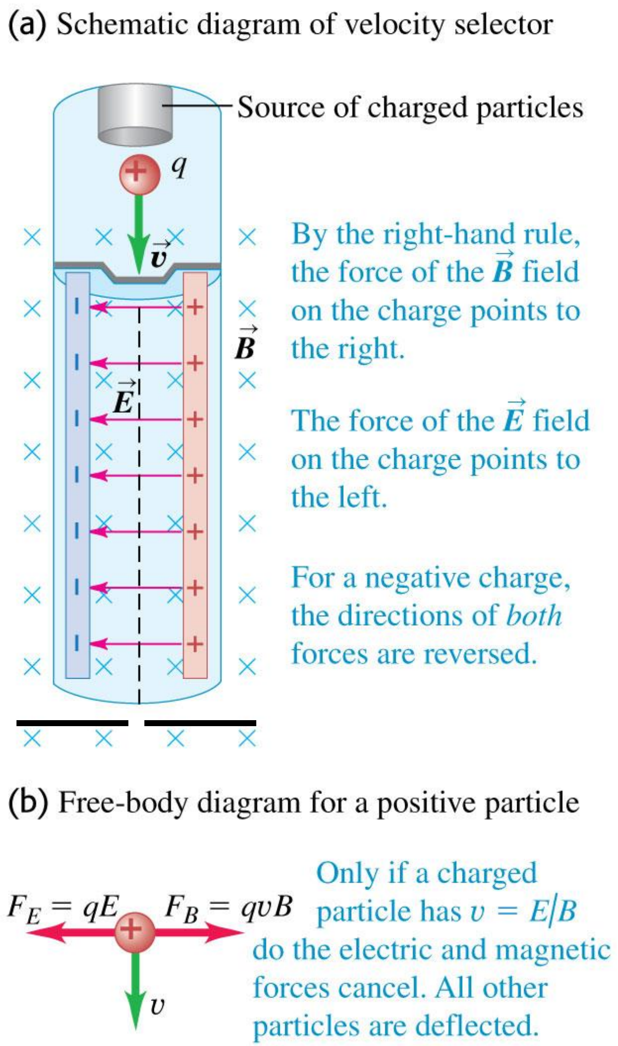
- Velocity Filter

- We can use the magnetic force in conjunction with the electric force to filter out particles of a certain velocity (or just determine velocity).

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{F}_E = q\vec{E}$$

- When the forces are equal, there is no deflection and v = E/B.
- By moving a slit that blocks particles except that go through the hole, you can pick out different velocities.



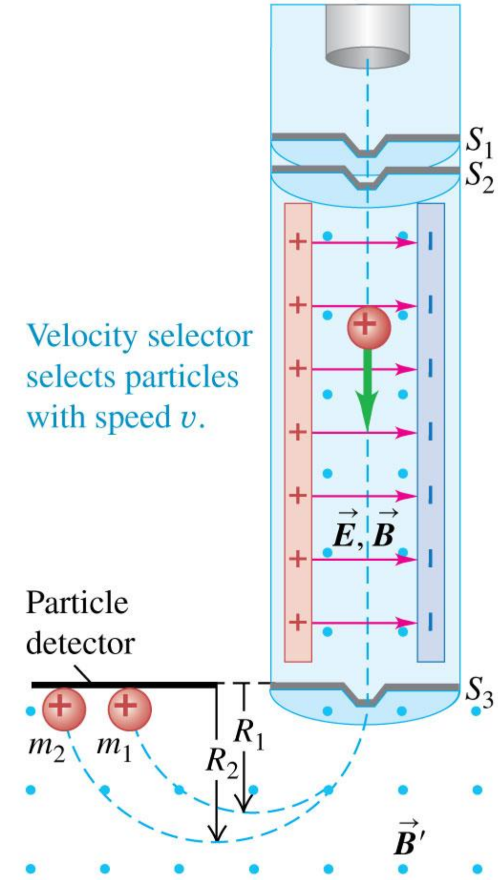
Consequences and Applications
- Mass Spectrometer

- By putting another magnetic field outside of the velocity filter, and then detecting the radius of curvature, one can separate particles out by mass.
- This is how a mass spectrometer works.

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{F}_E = q\vec{E}$$

$$R = \frac{mv}{qB'}$$



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

Consequences and Applications
- Thomson's e/m Apparatus

- In 1897, J.J. Thompson used a velocity filter to determine the ratio of the charge to mass of particles emitted from a cathode.
- He found that, regardless of cathode material, the ratio was always constant, and thus discovered the electron as a universal particle.

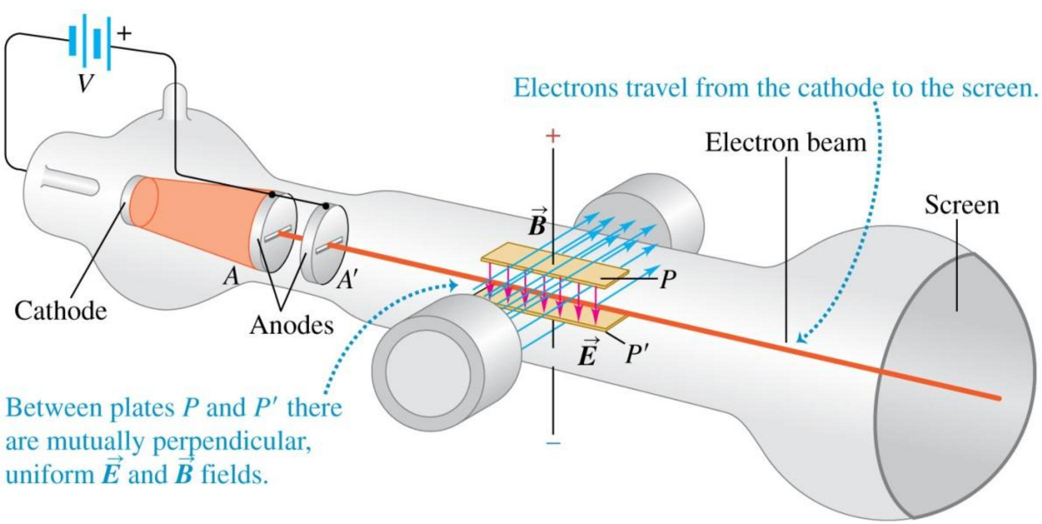
$$\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m}}$$

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{F}_E = q\vec{E}$$

$$v = \frac{E}{B} \Rightarrow$$

$$\frac{E}{B} = \sqrt{\frac{2eV}{m}} \Rightarrow \underline{\underline{\frac{e}{m} = \frac{E^2}{2B^2V}}}$$



CPS 28-1

A charged particle moves through a region of space that has both a uniform electric field and a uniform magnetic field. In order for the particle to move through this region at a constant velocity,

A. the electric and magnetic fields must point in the same direction.

B. the electric and magnetic fields must point in opposite directions.

C. the electric and magnetic fields must point in perpendicular directions.

D. The answer depends on the sign of the particle’s electric charge.

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Magnetic Force on a Current Element

- In electronics, we rarely deal with "beams" of charged particles, but rather deal with current in a wire.
- But current is just moving charged particles.

$$\vec{F}_B = q\vec{v}_d \times \vec{B}$$

- The total force on the wire segment of length *dl* is just the sum of the forces on all the moving charges:

$$d\vec{F} = n(V)q\vec{v}_d \times \vec{B}$$

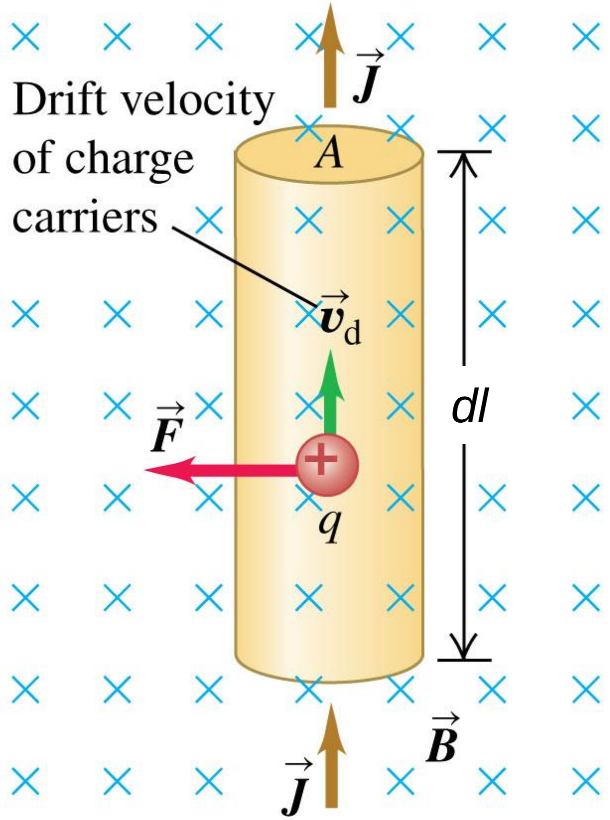
$$= n(Adl)q\vec{v}_d \times \vec{B}$$

- But nqv_d is just the current density, and the current density times the area is just the current:

$$d\vec{F} = n(Adl)q\vec{v}_d \times \vec{B}$$

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

- Where we have designated the direction of dl to be in the same direction as the current.



Magnetic Force on a Current

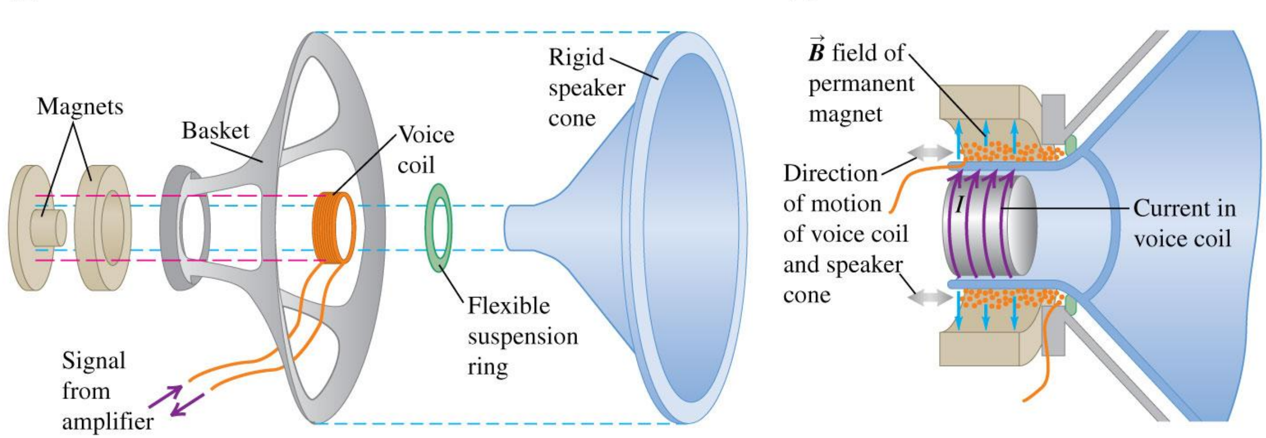
- One common use of this force is in speakers.

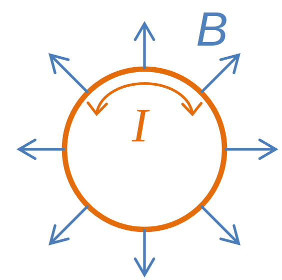
$$d\vec{F} = n(Adl)q\vec{v}_d \times \vec{B}$$

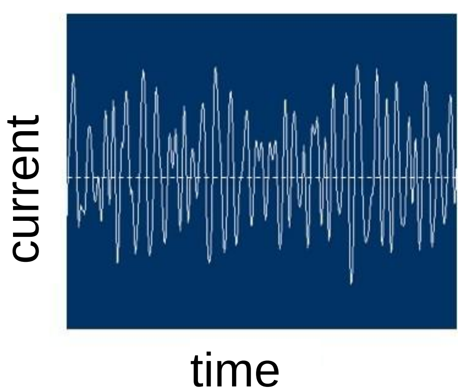
$$d\vec{F} = Id\vec{l} \times \vec{B} \Rightarrow$$

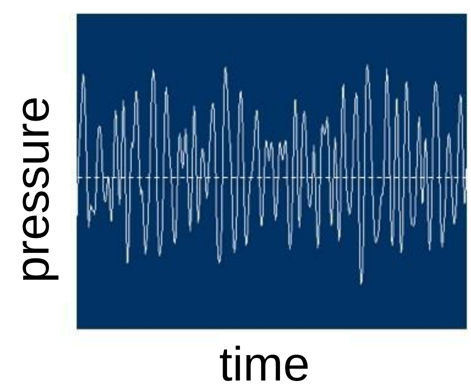
$$F_{Total} = IL_{Total}B$$

(a) (b)









Magnetic Force on a Current-Carrying Wire

- If the force isn't constant (either direction or magnitude), we must use our calculus tools:

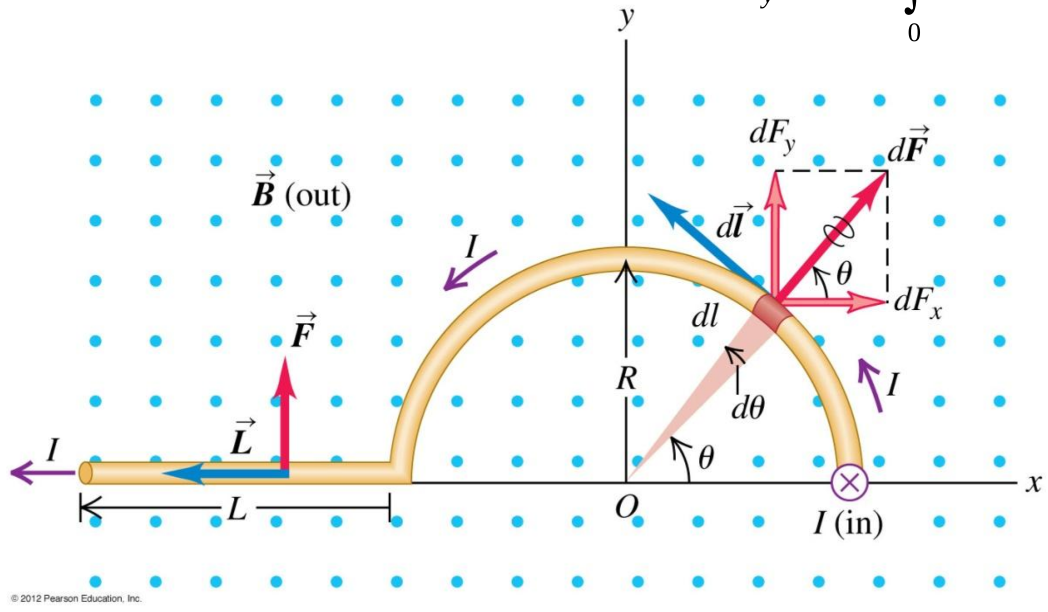
$$d\vec{F} = n\left(Adl\right)q\vec{v}_d \times \vec{B}$$

$$d\vec{F} = Id\vec{l} \times \vec{B} = I\left(Rd\theta\right)B\hat{r}$$

$$d\vec{F} = I\left(Rd\theta\right)B\left[\cos\theta\hat{i} + \sin\theta\hat{j}\right]$$

$$dF_y = IRB\sin\theta d\theta, \quad dF_x = IRB\cos\theta d\theta$$

$$F_y = IRB\int_0^{\pi}\sin\theta d\theta$$



Magnetic Force and Torque on a Current Loop

- Let's look at the Net force and net torque on a current loop:

$$d\vec{F} = Id\vec{l} \times \vec{B} \Rightarrow$$

$$F = IaB \quad (\text{top and bottom})$$

$$F = IbB \quad (\text{sides})$$

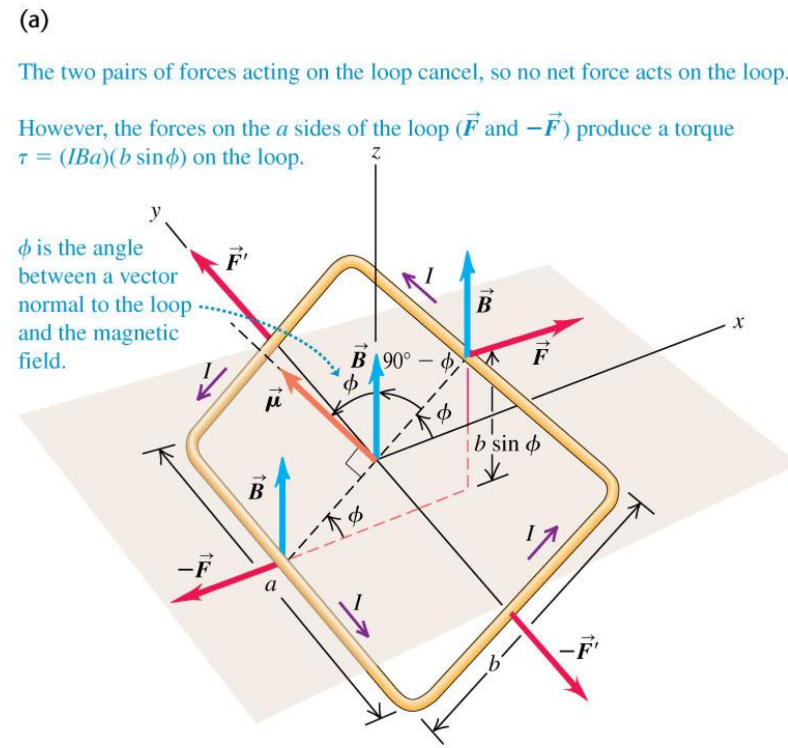
- But, the forces on opposite sides are opposing, so:

$$F_{Net} = 0$$

- Take the axis of rotation to be the y-axis, then the torque is:

$$|\vec{\tau}| = IaB\left(\frac{b}{2}\sin\phi\right) + IaB\left(\frac{b}{2}\sin\phi\right)$$

$$|\vec{\tau}| = IabB\sin\phi = IAB\sin\phi$$



Magnetic Torque on a Current Loop

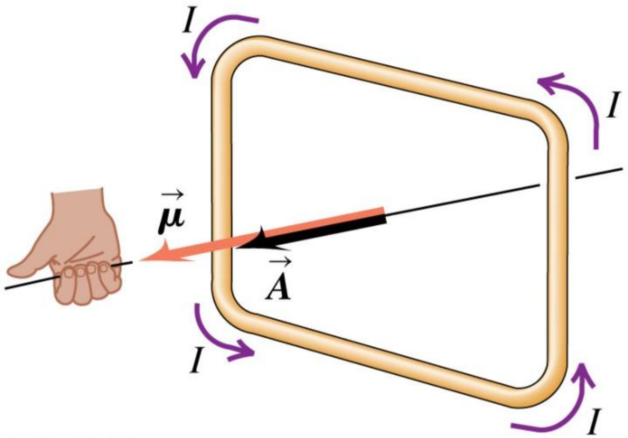
- We can rewrite this as:

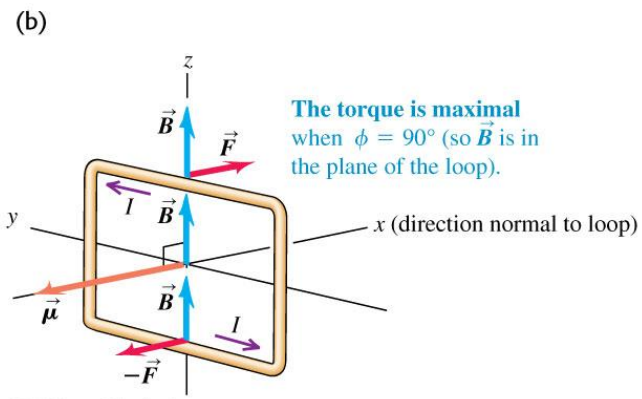
$$|\vec{\tau}| = IAB\sin\phi = \mu B\sin\phi$$

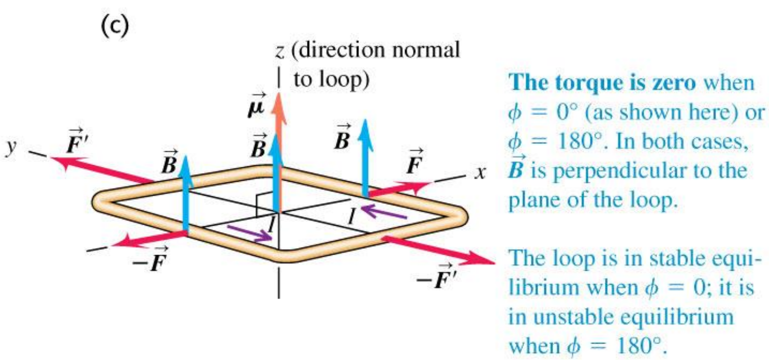
$$or$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

- Where the direction of the magnetic moment, μ, is given by the right-hand rule.







CPS 28-2

A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the *net magnetic force* on the loop is

A. perpendicular to the plane of the loop, in a direction given by a right-hand rule.

B. perpendicular to the plane of the loop, in a direction given by a left-hand rule.

C. in the same plane as the loop.

D. zero.

E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.

CPS 28-2

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CPS 28-3

A circular loop of wire carries a constant current. If the loop is placed in a region of uniform magnetic field, the *net magnetic torque* on the loop

A. tends to orient the loop so that its plane is perpendicular to the direction of the magnetic field.

B. tends to orient the loop so that its plane is edge-on to the direction of the magnetic field.

C. tends to make the loop rotate around its axis.

D. is zero.

E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.

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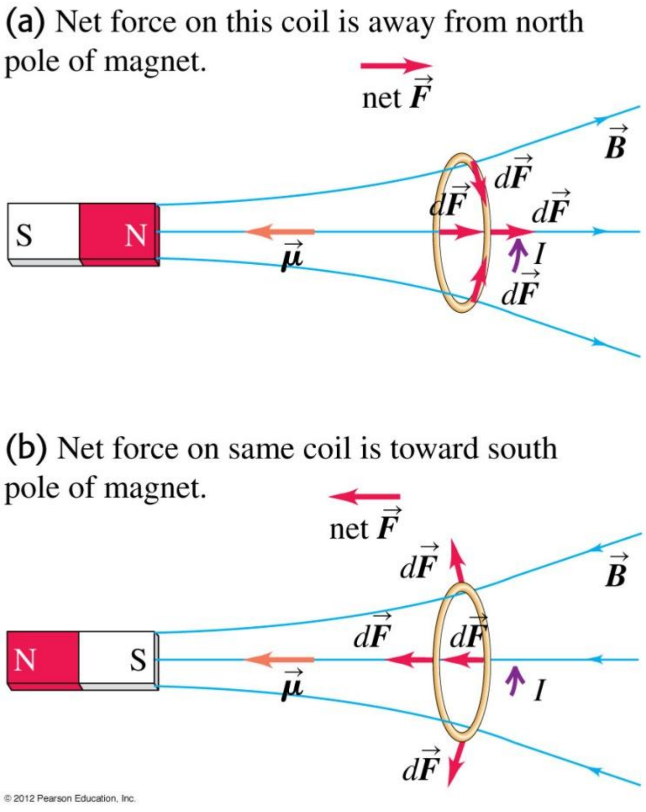
E. The answer depends on the magnitude and direction of the current and on the magnitude and direction of the magnetic field.

Non-Uniform Magnetic Field Force on a Current Loop

- Let’s return to the question of how magnets attract each other and also non-magnetized metals.
- If the magnetic field is not uniform, then there CAN be a net force on a current loop (read: magnetic moment).
- The net force attracts if the magnetic moment aligns with the field direction, and repels if it is in the opposite direction.

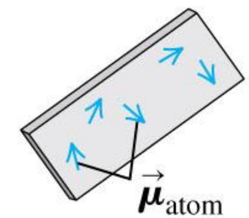
$$d\vec{F} = Id\vec{l} \times \vec{B}$$



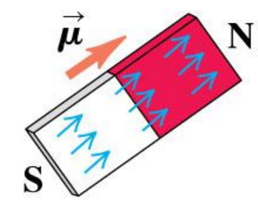
Magnetic Force on a Magnetic Moment

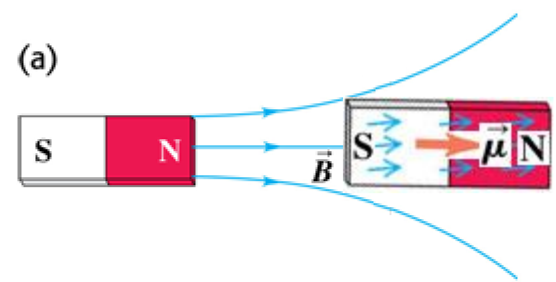
- Now, the atoms of iron (and other metals) have a magnetic moment because of the current loops of the electrons.
- In non-magnetized objects, they are randomly oriented.
- But in a magnet, they are (mostly) aligned with each other, giving a net total magnetic moment.

(a) Unmagnetized iron: magnetic moments are oriented randomly.



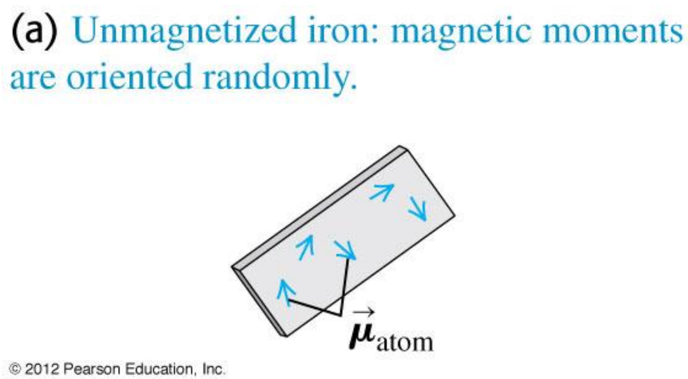
(b) In a bar magnet, the magnetic moments are aligned.

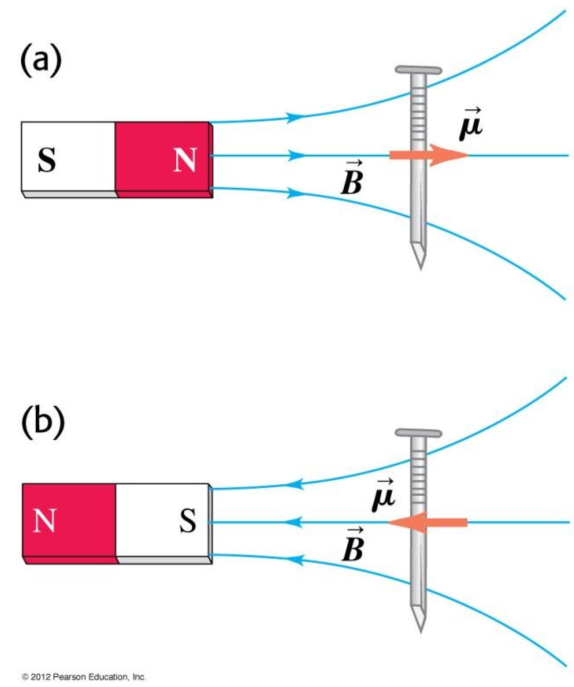




Magnetic Force on a Magnetic Moment

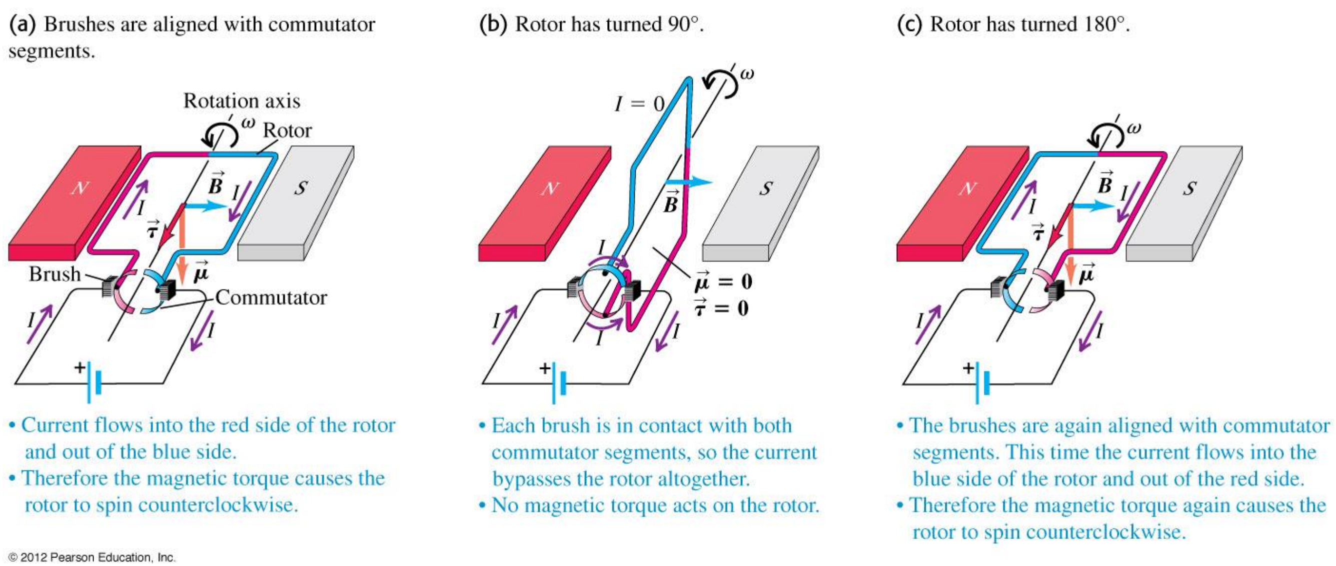
- For a non-magnetized object, like a nail, the field polarizes (rotates) the moments in the direction of the field.
- Then the objects has a net magnetic moment and is attracted to the magnet.
- When the field is removed, the atoms randomize their magnetic moments again, and the object returns to be non-magnetized.
- UNLESS, you heat and then cool it, or shock it by hitting it with a hammer…





Application – DC Motor

- We can now understand how a DC motor works.
- We put a current loop between the poles of a magnet.
- The torque on the loop causes rotation.
- We have to switch the direction of the current every time it rotates to keep it rotating in one direction.



Application – Hall Probe

- If we want to measure the magnetic field in a certain location, we can put a current through some (well understood) piece of conductor.
- The magnetic forces of the charges will tend to build up charges on opposite ends of the conductor.
- This charging will stop when the electric field force on the moving charges counters the magnetic force.
- By measuring the potential difference across the conductor in the z-direction, you can measure that charge, and thus, the field strength.

(a) Negative charge carriers (electrons)