#23 Center of Mass and Rocket Equation Pre-class

Due: 11:00am on Wednesday, October 17, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Center of Mass and External Forces

Learning Goal:

Understand that, for many purposes, a system can be treated as a point-like particle with its mass concentrated at the center of mass.

A complex system of objects, both point-like and extended ones, can often be treated as a *point particle*, located at the system's *center of mass*. Such an approach can greatly simplify problem solving.

Before you use the center of mass approach, you should first understand the following terms:

- System: *Any* collection of objects that are of interest to you in a particular situation. In many problems, you have a certain freedom in choosing your system. Making a wise choice for the system is often the first step in solving the problem efficiently.
- Center of mass: The point that represents the "average" position of the entire mass of a system. The postion of the center of mass $\vec{r}_{\rm cm}$ can be expressed in terms of the position vectors $\vec{r}_i$ of the particles as

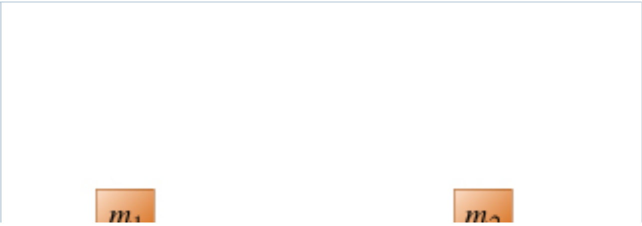
  $$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}.$$

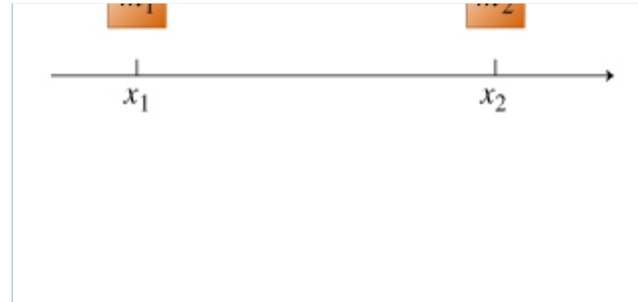
  The *x* coordinate of the center of mass $x_{\rm cm}$ can be expressed in terms of the *x* coordinates $(r_x)_i$ of the particles as

  $$x_{cm} = \frac{\sum m_i (r_x)_i}{\sum m_i}.$$

  Similarly, the *y* coordinate of the center of mass can be expressed.
- Internal force: Any force that results from an interaction between the objects inside your system. As we will show, the internal forces do not affect the motion of the system's center of mass.
- External force: Any force acting on an object inside your system that results from an interaction with an object outside your system.

Consider a system of two blocks that have masses m_1 and m_2. Assume that the blocks are point-like particles and are located along the *x* axis at the coordinates x_1 and x_2 as shown . In this problem, the blocks can only move along the *x* axis.





Part A

Find the *x* coordinate $x_{\rm cm}$ of the center of mass of the system.

Express your answer in terms of m_1, m_2, x_1, and x_2.

ANSWER:

$$x_{\rm cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

Correct

Part B

If $m_2 \gg m_1$, then the center of mass is located:

ANSWER:

- to the left of m_1 at a distance much greater than $x_2 - x_1$
- to the left of m_1 at a distance much less than $x_2 - x_1$
- to the right of m_1 at a distance much less than $x_2 - x_1$
- to the right of m_2 at a distance much greater than $x_2 - x_1$
- to the right of m_2 at a distance much less than $x_2 - x_1$
- to the left of m_2 at a distance much less than $x_2 - x_1$

Correct

Part C

If $m_2 = m_1$, then the center of mass is located:

ANSWER:

- at m_1
- at m_2
- half-way between m_1 and m_2
- the answer depends on x_1 and x_2

Correct

Part D

Recall that the blocks can only move along the *x* axis. The *x* components of their velocities at a certain moment are v_{1x} and v_{2x}. Find the x component of the velocity of the center of mass $(v_{cm})_x$ at that moment. Keep in mind that, in general: $v_x = dx/dt$.

Express your answer in terms of m_1, m_2, v_{1x}, and v_{2x}.

ANSWER:

$(v_{cm})_x = \dfrac{m_1 v_{1x} + m_2 v_{2x}}{m_1 + m_2}$

Correct

Because v_{1x} and v_{2x} are the *x* components of the velocities of m_1 and m_2 their values can be positive or negative or equal to zero.

Part E

Suppose that v_{1x} and v_{2x} have equal magnitudes. Also, $\vec{v_1}$ is directed to the right and $\vec{v_2}$ is directed to the left. The velocity of the center of mass is then:

ANSWER:

- ○ directed to the left
- ○ directed to the right
- ○ zero
- ◉ the answer depends on the ratio $\dfrac{m_1}{m_2}$

Correct

Part F

Assume that the *x* components of the blocks' momenta at a certain moment are p_{1x} and p_{2x}. Find the *x* component of the velocity of the center of mass $(v_{cm})_x$ at that moment.

Express your answer in terms of m_1, m_2, p_{1x}, and p_{2x}.

ANSWER:

$(v_{cm})_x$ = $\frac{p_{1x} + p_{2x}}{m_1 + m_2}$

Correct

Part G

Suppose that $\vec{v}_{cm} = 0$. Which of the following must be true?

ANSWER:

- (selected) $|p_{1x}| = |p_{2x}|$
- $|v_{1x}| = |v_{2x}|$
- $m_1 = m_2$
- none of the above

Correct

Part H

Assume that the blocks are accelerating, and the *x* components of their accelerations at a certain moment are a_{1x} and a_{2x}. Find the *x* component of the acceleration of the center of mass $(a_{cm})_x$ at that moment. Keep in mind that, in general, $a_x = dv_x/dt$.

Express your answer in terms of m_1, m_2, a_{1x}, and a_{2x}.

ANSWER:

$(a_{cm})_x$ = $\frac{m_1 a_{1x} + m_2 a_{2x}}{m_1 + m_2}$

Correct

Because a_{1x} and a_{2x} are the *x* components of the velocities of m_1 and m_2 their values can be positive or negative or equal to zero.

We will now consider the effect of external and internal forces on the acceleration of the center of mass.

Part I

Consider the same system of two blocks. An *external* force $\vec{F}$ is now acting on block m_1. *No* forces are applied to block m_2 as shown . Find the *x* component of the acceleration of the center of mass $(a_{cm})_x$ of the system.

Express your answer in terms of the *x* component F_x of the force, m_1,and m_2.

Hint 1. Using Newton's laws

Find the acceleration of each block from Newton's second law and then apply the formula for $(a_{\rm cm})_x$ found earlier.

ANSWER:

$(a_{\rm cm})_x$ = $\dfrac{F_x}{m_1+m_2}$

Correct

Part J

Consider the same system of two blocks. Now, there are two forces involved. An *external* force $\vec{F}_1$ is acting on block m_1 and another *external* force $\vec{F}_2$ is acting on block m_2. Find the *x* component of the acceleration of the center of mass $(a_{\rm cm})_x$ of the system.

Express your answer in terms of the *x* components F_{1x} and F_{2x} of the forces, m_1 and m_2.

ANSWER:

$(a_{\rm cm})_x$ = $\dfrac{F_{1x}+F_{2x}}{m_1+m_2}$

Correct

Note that, in both cases, the acceleration of mass can be found as

$$(a_{\rm cm})_x = \frac{(F_{\rm net})_x}{M_{\rm total}}$$

where $F_{\rm net}$ is the net *external* force applied to the system, and $M_{\rm total}$ is the total mass of the system. Even though each force is only applied to *one* object, it affects the acceleration of the center of mass of the *entire system*.

This result is especially useful since it can be applied to a general case, involving *any* number of objects moving in *all* directions and being acted upon by *any* number of *external* forces.

Part K

Consider the previous situation. Under what condition would the acceleration of the center of mass be zero? Keep in mind that $F_{\rm 1x}$ and $F_{\rm 2x}$ represent *the components*, of the corresponding forces.

ANSWER:

- (selected) $F_{1x} = -F_{2x}$
- $F_{1x} = F_{2x}$
- $m_1 = m_2$
- $m_1 \ll m_2$

Correct

Part L

Consider the same system of two blocks. Now, there are two *internal* forces involved. An *internal* force $\vec{F}_{12}$ is applied to block m_1 by block m_2 and another *internal* force $\vec{F}_{21}$ is applied to block m_2 by block m_1 . Find the *x* component of the acceleration of the center of mass $(a_{cm})_x$ of the system.

Express your answer in terms of the *x* components F_{12x} and F_{21x} of the forces, m_1 and m_2.

ANSWER:

$(a_{cm})_x$ = $\dfrac{F_{12x}+F_{21x}}{m_1+m_2}$

Correct

Newton's 3rd law tells you that $|F_{12x}| = -|F_{21x}|$. From your answers above, you can conclude that $(a_{cm})_x = 0$.The *internal forces* do *not* change the velocity of the center of mass of the system. In the absence of any *external* forces, $(a_{cm})_x = 0$ and $(v_{cm})_x$ is constant.

You just demonstrated this to be the case for the two-body situation moving along the *x* axis; however, it is true in more general cases as well.

Exercise 8.58

A rocket is fired in deep space, where gravity is negligible.

Part A

If the rocket has an initial mass of 6000 kg and ejects gas at a relative velocity of magnitude 2000 $\mathrm{m/s}$, how much gas must it eject in the first second to have an initial acceleration of 25.0 $\mathrm{m/s^2}$?

ANSWER:

m = 75.0 kg

Correct

Exercise 8.48

Part A

Find the position of the center of mass of the system of the sun and Jupiter. (Since Jupiter is more massive than the rest of the planets combined, this is essentially the position of the center of mass of the solar system.)

ANSWER:

x = 7.42×10^8 m from center of thr sun

Correct

Part B

Does the center of mass lie inside or outside the sun? The sun's radius is $6.96 \times 10^8\ \mathrm{m}$.

ANSWER:

- inside
- outside

Correct

Score Summary:

Your score on this assignment is 100%.
You received 15 out of a possible total of 15 points.