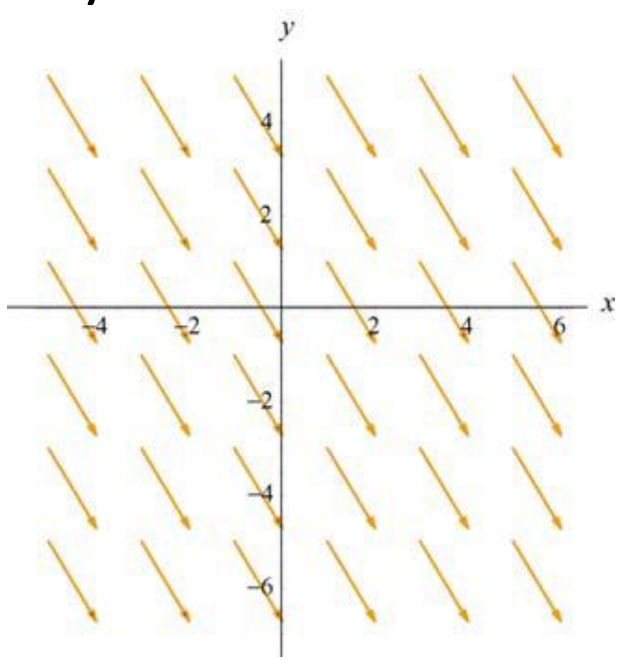
Lecture 13
(Electric Flux)

Physics 161-01 Spring 2012

Douglas Fields

Flux (Latin for “Flow”)

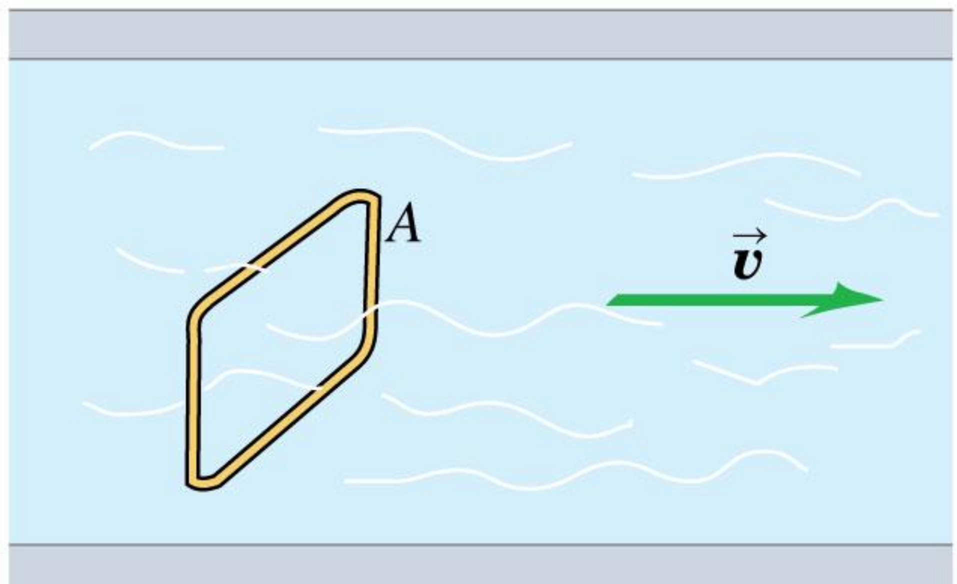
- For this, and following chapters, we will need to understand well the idea of flux.
- To do this, we need to return to the idea of a vector field, and, in particular, I will use the idea of fluid (water) flow.
- Let’s first consider a case of steady water flow (say, a deep river moving slowly with constant velocity throughout).



Flux (Latin for "Flow")

- Now, let's look at a surface, say a rectangular surface bounded by a wire, and ask "what is the volume fluid flow of water through the surface?"

$$\left.\frac{dV}{dt}\right|_{\text{Through surface}} = \frac{dx}{dt} A = vA$$

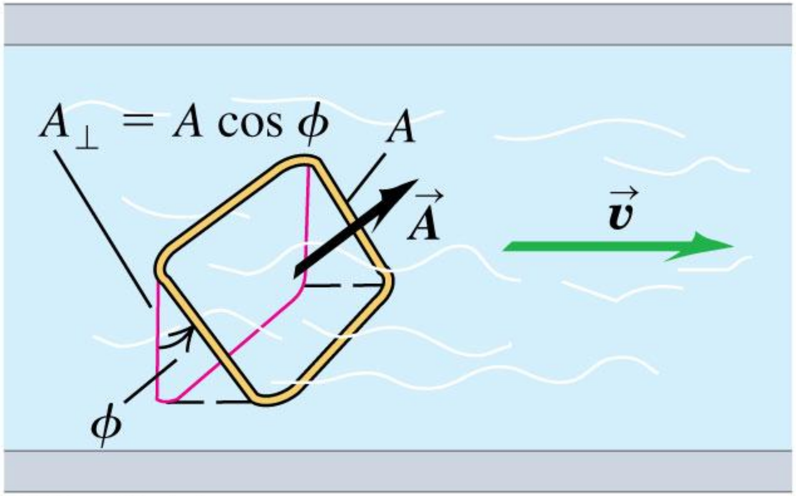


Flux in a Fluid Flow

- But, what if the wire boundary of the surface is tilted relative to the flow vectors?
- Then the volume flow through the surface is smaller and given by the bounds of the red rectangle in the figure below.

$$\left.\frac{dV}{dt}\right|_{\text{Through surface}} = vA\cos\phi$$

$$= \vec{v} \cdot \vec{A}$$

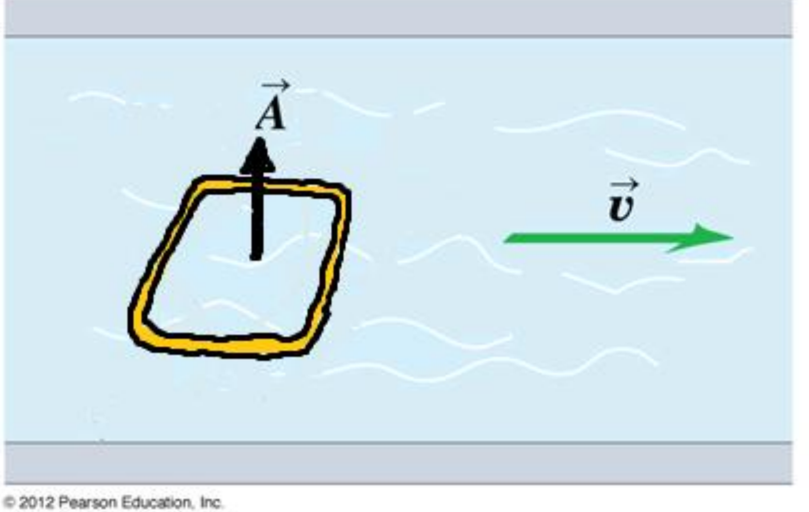


Flux in a Fluid Flow

- If the wire were titled perpendicular to the fluid flow, there would be no flow through the surface.
- Note that his doesn’t mean the fluid isn’t flowing.
- Also note that we didn’t define the shape of the surface, only it’s bound (the wire).

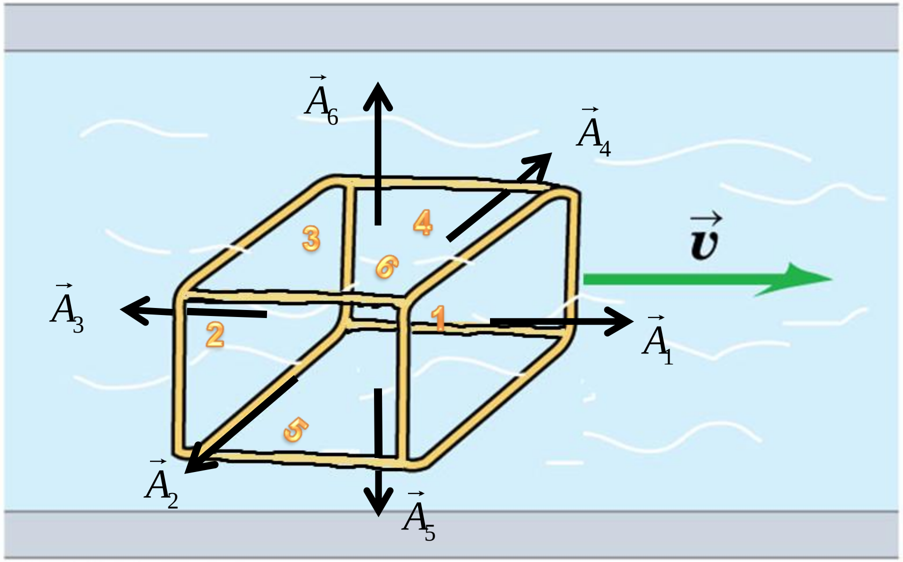
$$\frac{dV}{dt}_{\text{Through surface}} = vA\cos\phi$$

$$= \vec{v}\cdot\vec{A} = 0$$



Closed Surfaces

- Now, let us now consider a closed surface – a surface or combination of surfaces that completely enclose a volume.
- We now have to define away the ambiguity of the direction for the area vector:
  - The area vector of a closed surface points from inside the volume to outside.



Flux Through Closed Surfaces

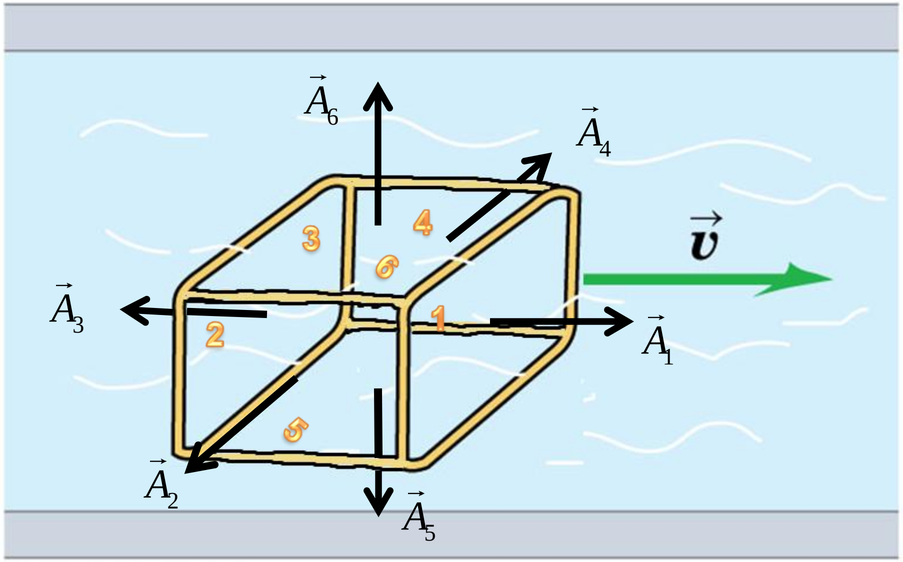
- We now ask "What is the NET flux through these surfaces?"
- We take each surface separately, find the flux, and add them together, remembering the direction of the area vectors, and hence the sign of the flux through them.

$$\left.\frac{dV}{dt}\right|_{\text{Net}} = \sum_{\text{surfaces}} \frac{dV}{dt}$$

$$= \sum_{i=1}^{6} \vec{v} \cdot \vec{A}_i$$

$$= vA_1 + 0 - vA_3 + 0 + 0 + 0$$

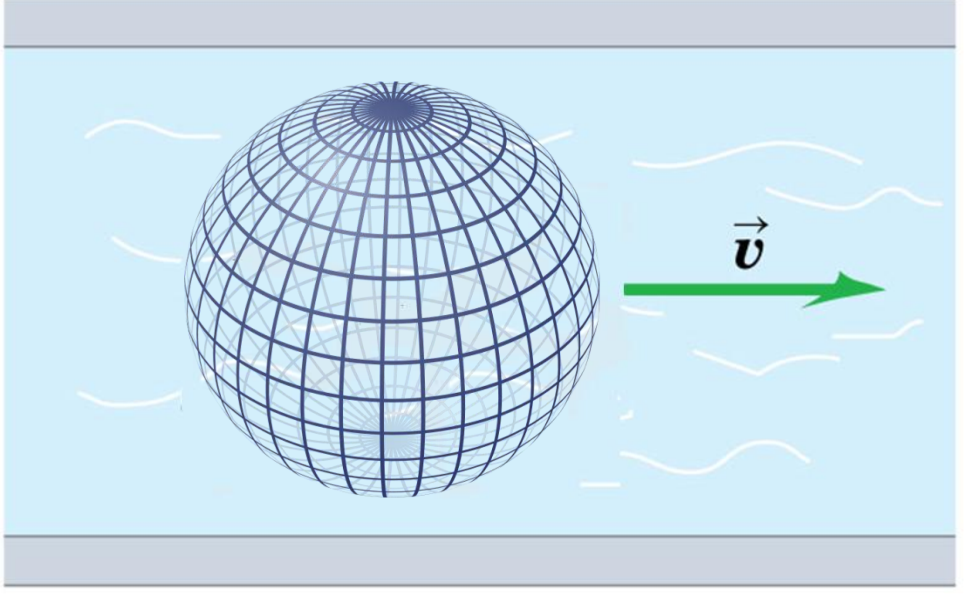
$$= 0$$



Flux Through Closed Surfaces

- This result turns out to be independent of the shape of the closed surface used.
- The volume flow INTO the surface equals the volume flow OUT of the surface.

$$\frac{dV}{dt}_{\text{Net}} = \sum_{\text{surfaces}} \frac{dV}{dt} = \sum_{i=1}^{1024} \vec{v} \cdot \vec{A}_i = 0$$

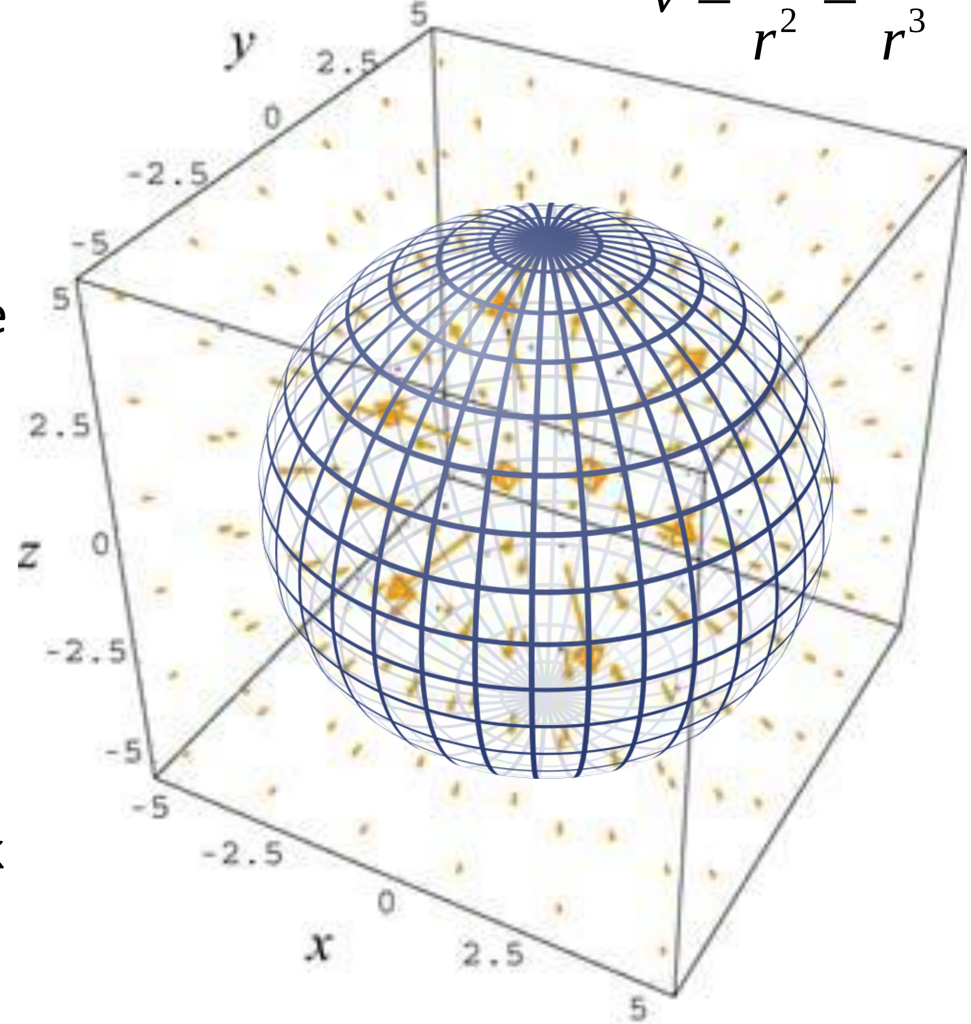


Can you show this using calculus?

Sources and Sinks

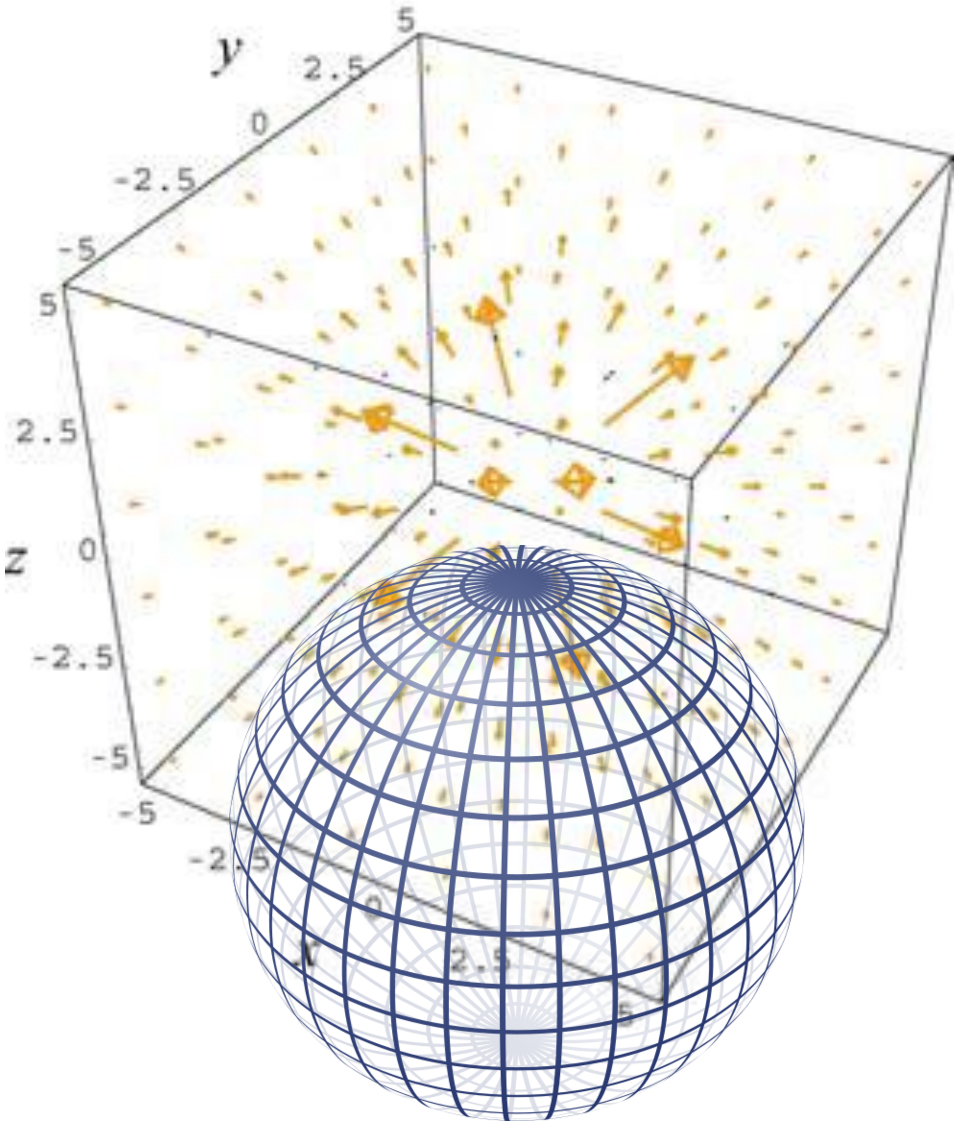
- Is this always true of the net flux?
- Consider the vector field we discussed last time, in the r-direction.
- This is a case of a vector field that has a singularity, in this case a source (think of a point water source with no gravity).
- Note that there will be a net flux, since all the vectors at the surface are pointing out of the surface, in the same direction as the area vectors for this surface.
- If you reversed the direction of all the vectors, it would be a sink (think of a water drain in three dimensions).

$$\vec{v} = \frac{k\hat{r}}{r^2} = \frac{k\vec{r}}{r^3}$$



Sources and Sinks

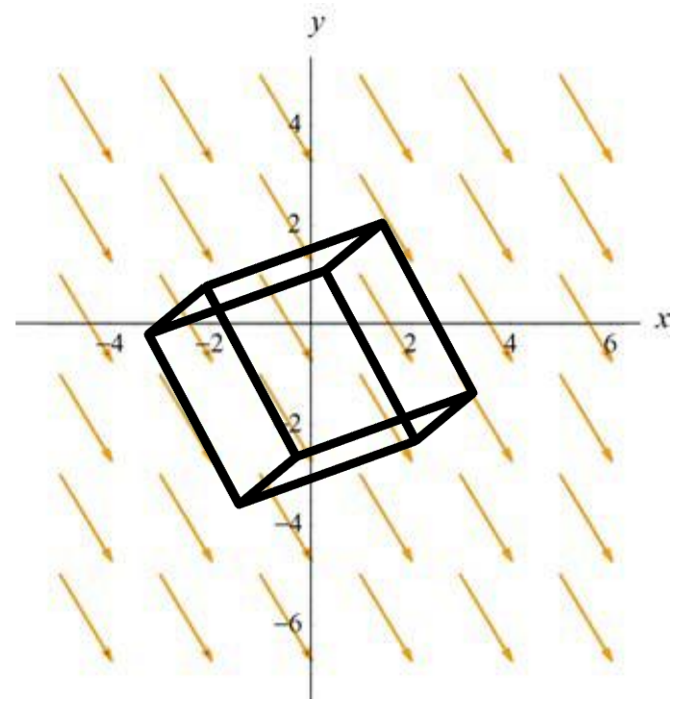
- Is this always true of the net flux with a source or sink?
- No! If the source (or sink) is not contained INSIDE the volume, then there are vectors that point into and out from the volume.
- It can be shown that the net flux in this situation is again zero.



Electric Flux

- So far I have used the analogy of water flow to describe flux in more intuitive terms.
- When we are considering the electric field flux, nothing is “flowing”, but since both are described by vector fields, the mathematics is the same.

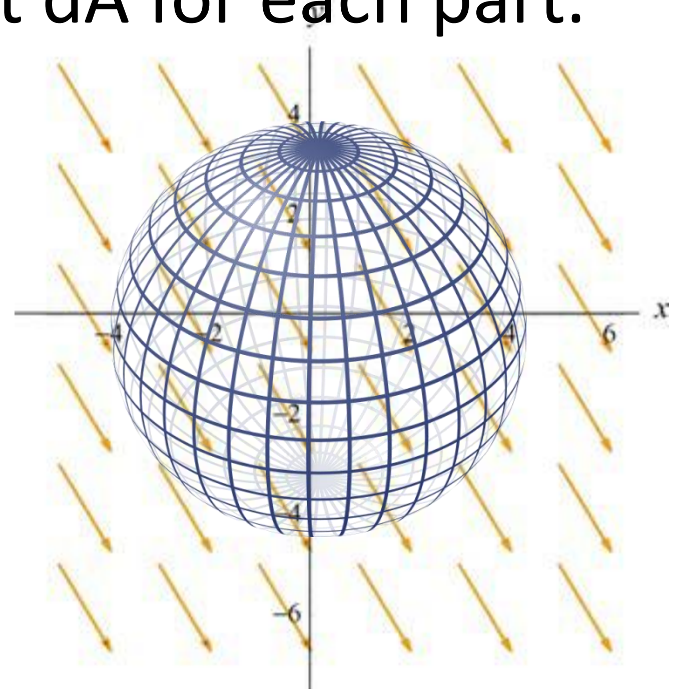
$$\Phi_{\mathrm{E,Net}} = \sum_{\mathrm{surfaces}} \vec{E} \cdot \vec{A}$$



Flux With Calculus

- So far we have worked with constant fields.
- How do we handle fields that vary over a surface (either in strength, or in direction)?
- We have to break up the surface into small parts, dA, and then add up E dot dA for each part.

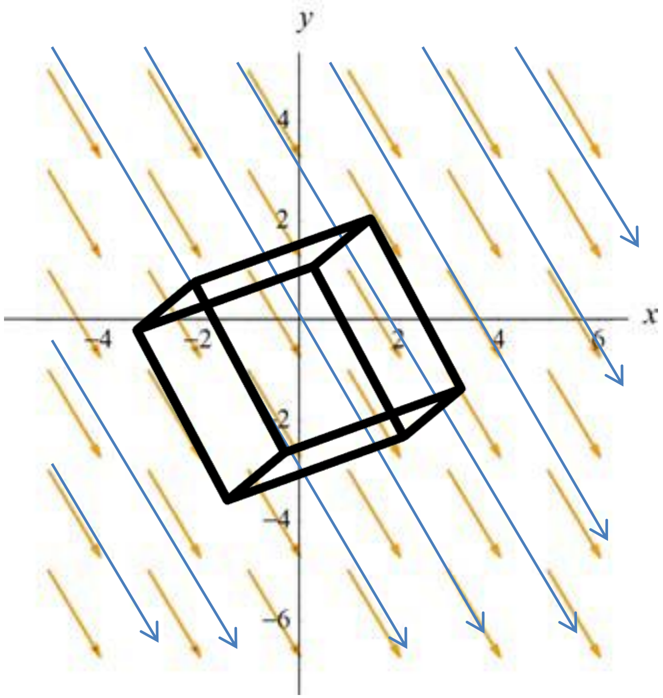
$$\Phi_{\mathrm{E,Net}} = \oint \vec{E} \cdot d\vec{A}$$



Electric Flux and Field Lines

- Since the relative spacing between field lines gives the relative strength of the field, and
- Since the field lines can be thought of as flow lines,
- The net flux through a closed surface is proportional to the net number of field lines entering/exiting the surface.

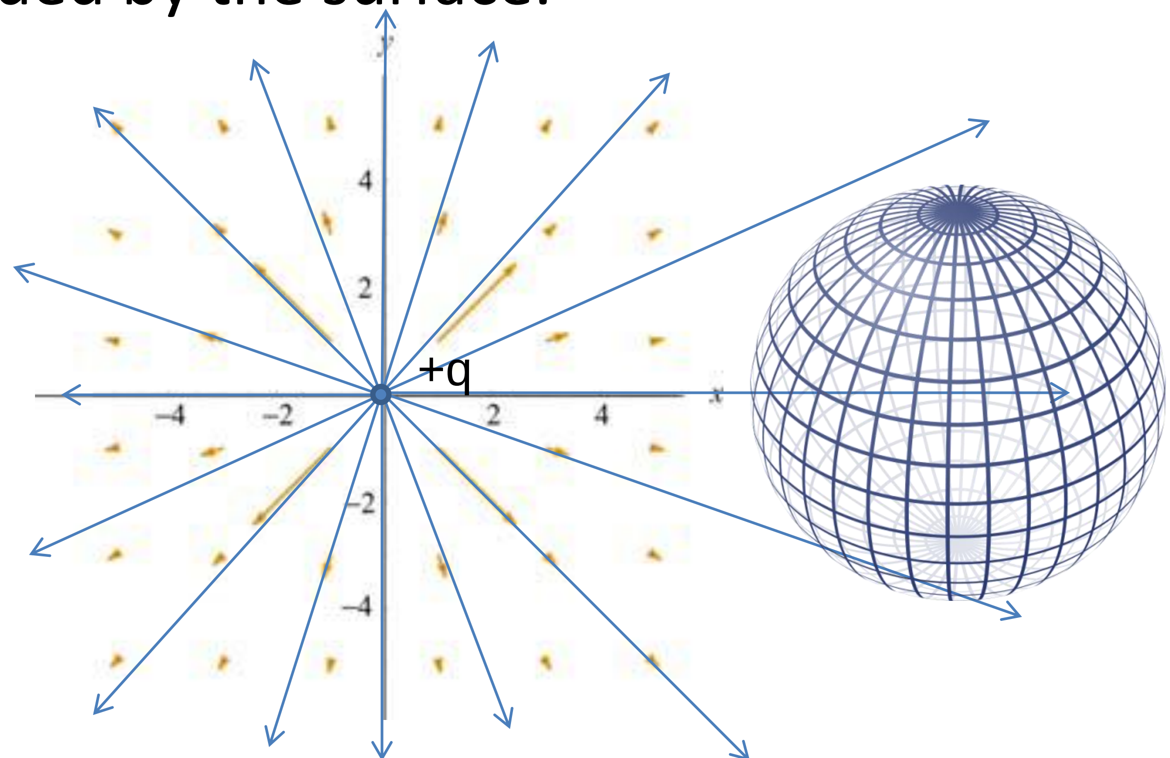
$$\Phi_{\mathrm{E,Net}} = \sum_{\mathrm{surfaces}} \vec{E} \cdot \vec{A}$$



Electric Field Sources and Sinks

- So, using the electric field line way of thinking about electric flux, the net flux through a closed surface is always zero (ever field line that enters, also exits), unless there is a source or sink enclosed within the volume bounded by the surface.

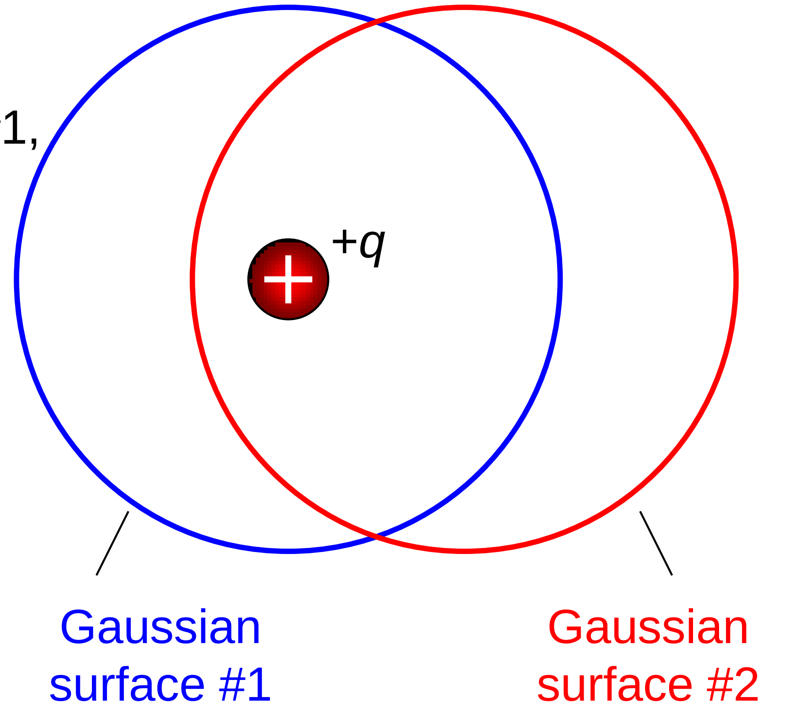
$$\vec{E} = k\frac{q}{r^2}\hat{r} = k\frac{q}{r^3}\vec{r}$$



CPS 13-1

A spherical Gaussian surface (#1) encloses and is centered on a point charge $+q$. A second spherical Gaussian surface (#2) of the same size also encloses the charge but is not centered on it.

Compared to the electric flux through surface #1, the flux through surface #2 is



A. greater.

B. the same.

C. less, but not zero.

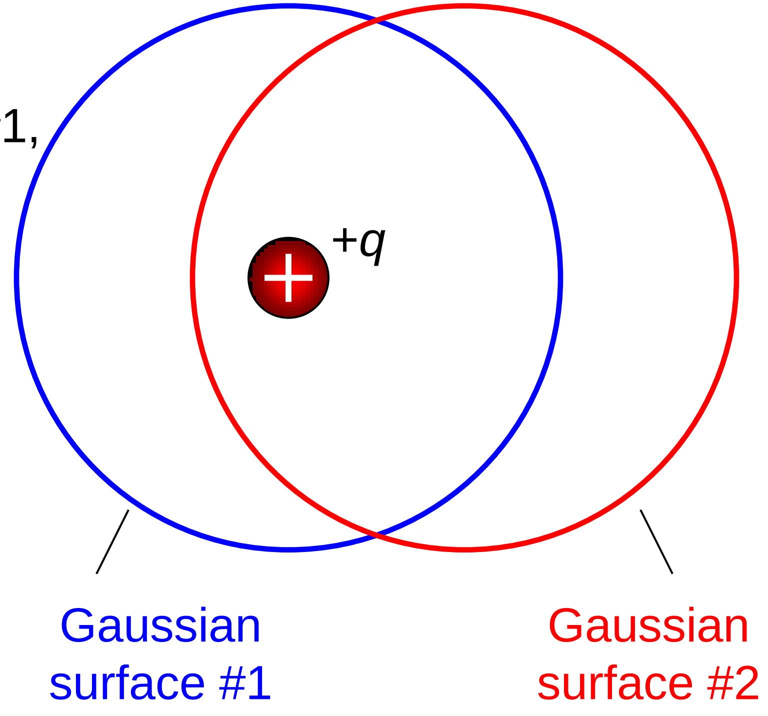
D. zero.

E. not enough information given to decide

CPS 13-1

A spherical Gaussian surface (#1) encloses and is centered on a point charge +*q*. A second spherical Gaussian surface (#2) of the same size also encloses the charge but is not centered on it.

Compared to the electric flux through surface #1, the flux through surface #2 is



A. greater.

B. the same.

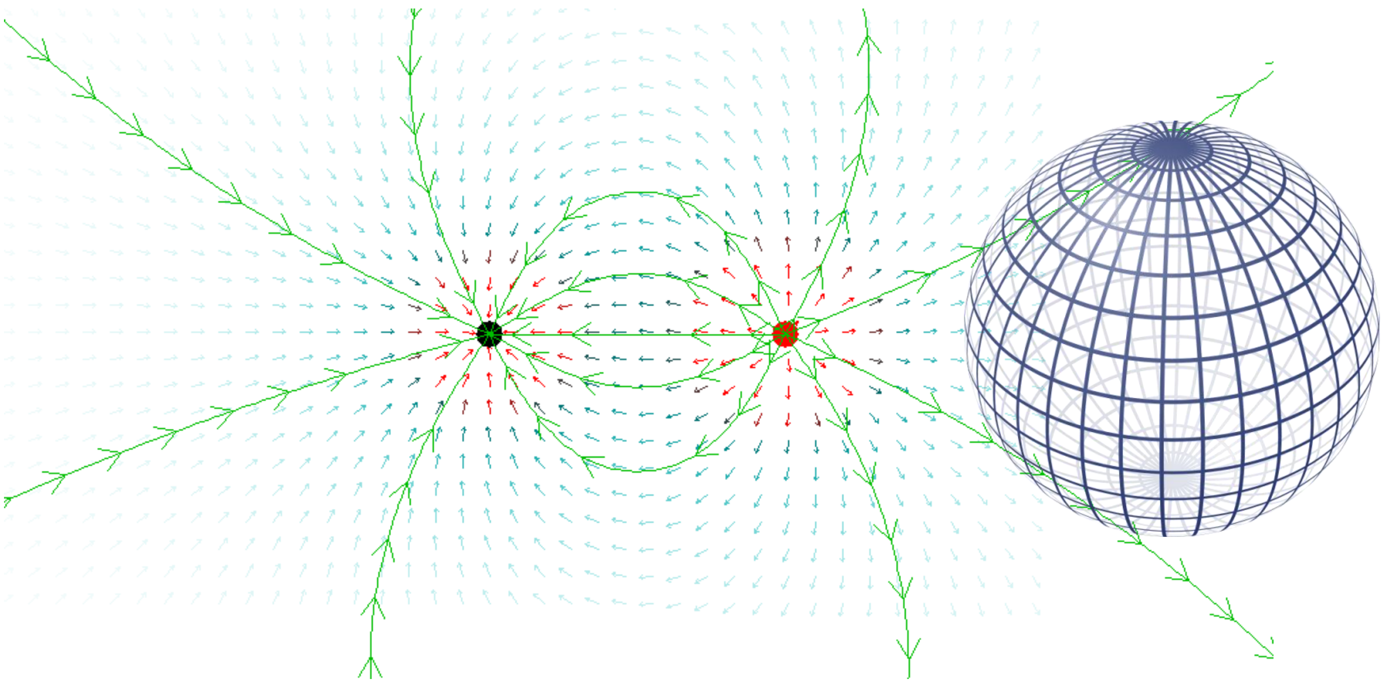
C. less, but not zero.

D. zero.

E. not enough information given to decide

More Complex

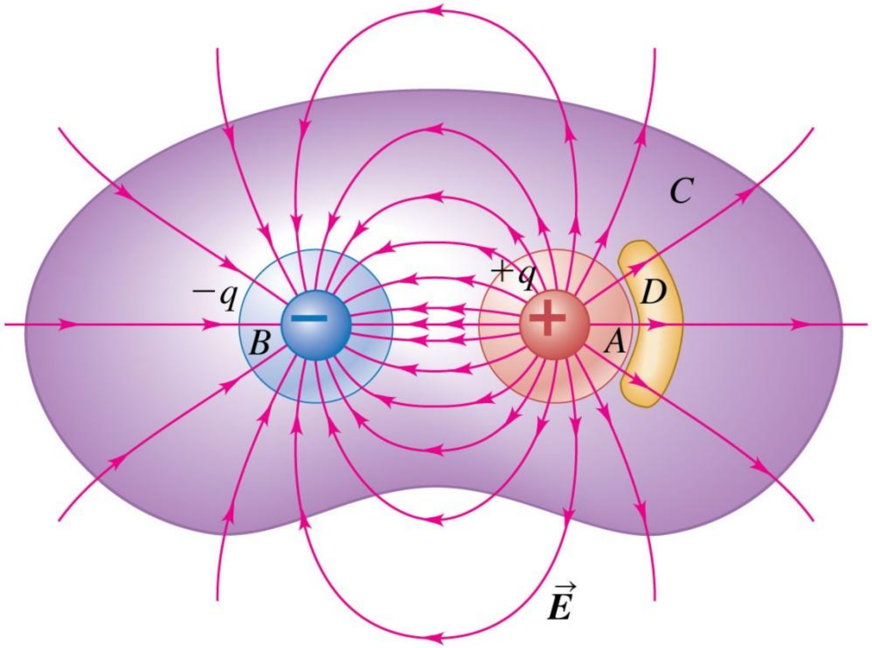
- A dipole is two charges of equal magnitude and opposite sign.



CPS 13-2

Two point charges, $+q$ (in red) and $-q$ (in blue), are arranged as shown.

Through which closed surface(s) is the net electric flux equal to zero?



A. surface *A*

B. surface *B*

C. surface *C*

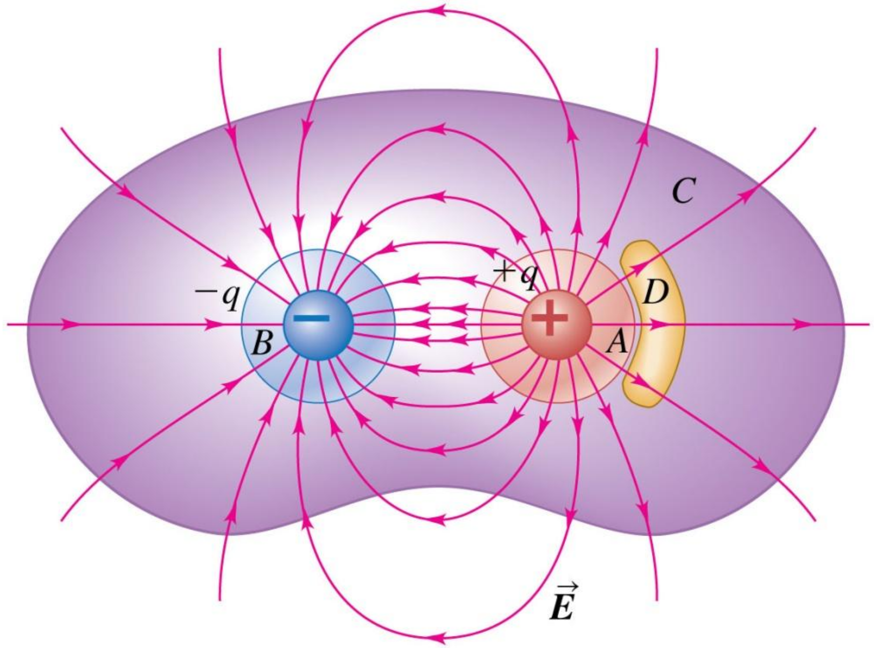
D. surface *D*

E. both surface *C* and surface *D*

CPS 13-2

Two point charges, +*q* (in red) and –*q* (in blue), are arranged as shown.

Through which closed surface(s) is the net electric flux equal to zero?



A. surface *A*

B. surface *B*

C. surface *C*

D. surface *D*

E. both surface *C* and surface *D*