PS#4 Solutions
EECE 345
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$\boxed{1}$

a) BIBO unstable due to poles at $\pm j$
(Bounded input $u(t) = \sin(t)$ generates an unbounded output)

b). $$\frac{Y(s)}{R(s)} = \frac{kG(s)}{1+kG(s)} = \frac{s+1}{s^2+1+s+1} = \frac{s+1}{s^2+s+2}$$

c). poles of $G(s)$: $s = \pm j$

poles of $\frac{Y(s)}{R(s)}$: $s = -\frac{1}{2} \pm \frac{\sqrt{7}}{2} j$

$s^2+s+2 \Rightarrow$
$$s = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2}}{2}$$

$\therefore$ poles are different.

d) The closed-loop system is asymptotically stable (poles in open LHP) $\Rightarrow$ it is also BIBO stable.

$\therefore$ Stability is different in open-loop & closed-loop systems.

$\boxed{2}$

a) poles at $s = -1, -2 \Rightarrow$ asymptotically stable

b) poles at $\begin{vmatrix} s & -1 \\ -1 & s \end{vmatrix} = 0$.

$s^2 = -1$

$s = \pm j \Rightarrow$ marginally stable

c) poles at $s = -1, -1, +1 \Rightarrow$ unstable.

d) poles at $\begin{vmatrix} s & 1 \\ 0 & s \end{vmatrix} = 0 \Rightarrow s^2 = 0 \Rightarrow$ unstable (repeated poles at $s=0$)

$\boxed{13}$

$$\frac{Y(s)}{R(s)} = \frac{KG(s)}{1+KG(s)} = \frac{KN(s)}{D(s)+KN(s)}$$

$$= \frac{K\cdot(s+2)}{\underbrace{(s^2+1)(s+4)(s-1)+K(s+2)}}$$

$$= (s^2+1)(s^2+3s-4) + Ks + 2K$$

$$= s^4 + 3s^3 - 4s^2$$
$$+ s^2 + 3s - 4$$
$$1Ks + 2K$$

$$= s^4 + 3s^3 - 3s^2 + (3+K)s + (2K-4)$$

s^4	1	- 3	2K - 4
s^3	3	K+ 3	0
s^2	$\frac{-(K+12)}{3}$	2K - 4	0
s^1	$\frac{K(K+33)}{K+12}$	0	0
s^0	2K - 4	0	0

Conditions state that $K < -12$, $K > 2$, and $K > -33$. These conditions cannot be met simultaneously.

System is not stable for any value of K.

$\boxed{4}$ $A - BK = \begin{bmatrix} 0 & 1 \\ 3 & -3 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \end{bmatrix} [k_1 \ \ k_2] = \begin{bmatrix} 0 & 1 \\ 3-2k_1 & -3-2k_2 \end{bmatrix}$

a) $0 = |\lambda I - (A - BK)|$

$= \begin{vmatrix} \lambda & -1 \\ -(3-2k_1) & \lambda + (3+2k_2) \end{vmatrix}$

$= \lambda^2 + \underbrace{(3+2k_2)}_{>0}\lambda \underbrace{- (3-2k_1)}_{>0}$

$\Rightarrow 2k_2 > -3$ $\quad \Rightarrow -3 + 2k_1 > 0$

$\boxed{k_2 > -3/2}$ $\quad \boxed{k_1 > +3/2}$

b) marginally stable for

$k_1 = 3/2$ and $k_2 > -3/2$

OR $k_1 > 3/2$ and $k_2 = -3/2$

(cannot have both $k_1 = 3/2$ and $k_2 = -3/2$ simultaneously $\Rightarrow$ 2 poles at $s = 0$)

c) $M_p = 1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}} \leq 1.05$

$-\zeta\pi/\sqrt{1-\zeta^2} \leq \ln(0.05)$

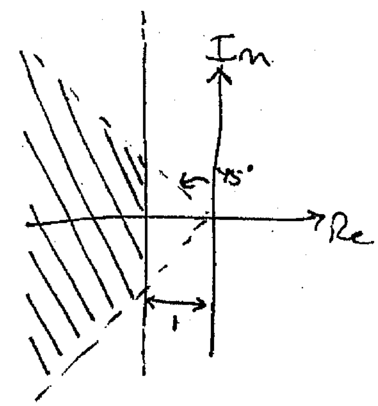
$\zeta^2\pi^2 \geq (1-\zeta^2)(\ln(0.05))^2$

$\zeta^2(\pi^2 + \ln(1/20)^2) \geq \ln(1/20)^2$

$\zeta \geq \dfrac{|\ln(1/20)|}{\sqrt{\pi^2 + \ln(1/20)^2}} \simeq 0.7$

$T_s \simeq \dfrac{4}{\zeta\omega_n} \leq 4$

$1 \leq \zeta\omega_n$



d) $0 = s^2 + 2\zeta\omega_n s + \omega_n^2$

Choose $\zeta = 1/\sqrt{2}$, $\omega_n = \sqrt{2}$

$= s^2 + 2s + 2$ (desired characteristic eqn)

$= s^2 + (3+2k_2)s + (-3+2k_1)$ (actual char. eqn)

$\Rightarrow 2 = 3 + 2k_2$, $\quad -3 + 2k_1 = 2$

$\boxed{-1/2 = k_2}$ $\quad \boxed{k_1 = \frac{5}{2}}$.

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diary off
K = [5/2 -1/2];
A = [0 1; 3 -3]; B = [0; 2]; C = [1 0];D = 0;
sys = ss(A-B*K, B,C,D);
step(sys)
```

Bonus: