Lecture 26
(Torque)

Physics 160-01 Fall 2012

Douglas Fields

Review

- Linear
  - For const a:

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

  - Kinetic energy:

$$KE = \frac{1}{2} M v^2$$

  - Comes from:

$$W = \int \vec{F} \cdot d\vec{s}$$

- Rotational
  - For const α:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

  - Kinetic energy:

$$KE = \frac{1}{2} I \omega^2$$

  - Comes from:

???

CPS Question 26-1

- **Two objects of equal mass and diameter roll without slipping down an incline, one is a hollow cylinder, the other a solid cylinder. Which one wins the race to the bottom of the incline?**

A) The solid cylinder.

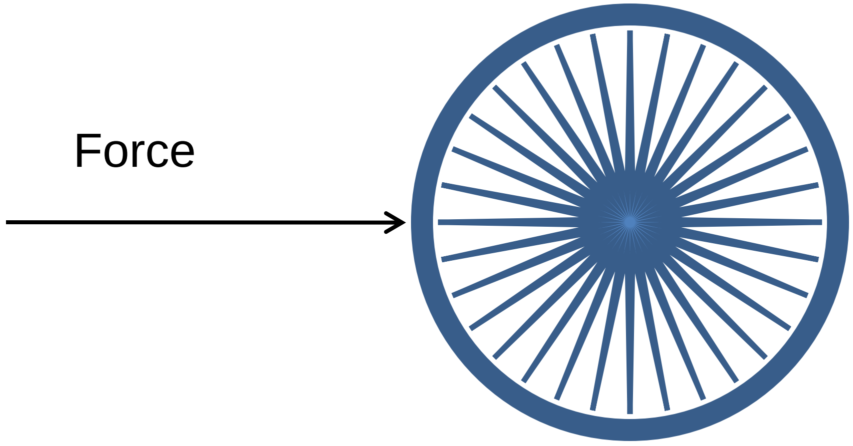
B) The hollow cylinder.

C) It's a tie.

D) Not enough information to solve.

How Do You Get Rotational KE?

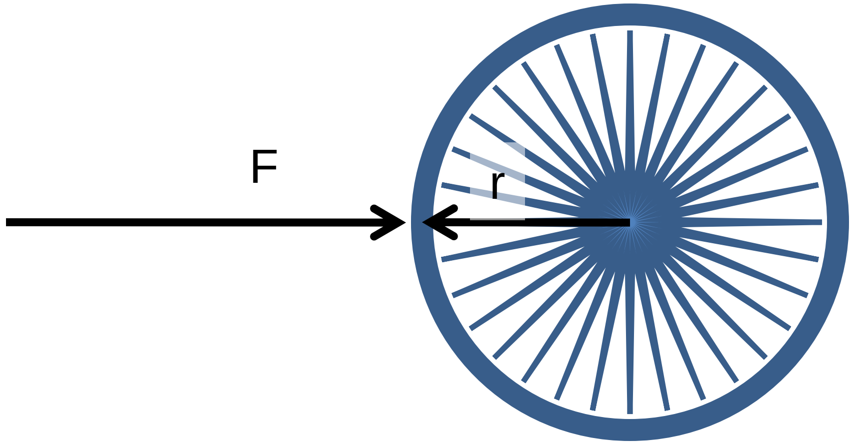
- You know it can’t just be a force:



- Getting the wheel to rotate depends upon *where* you apply the force!!

Torque

- The concept of torque takes into account both the magnitude of a force applied, and where it is applied.



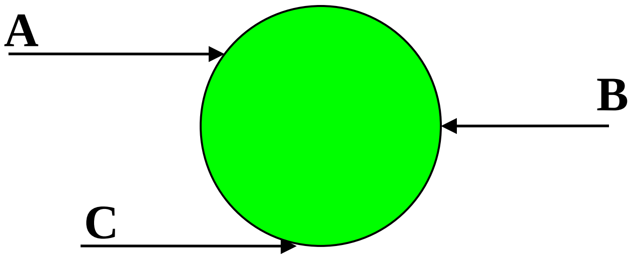
$$\vec{\tau} = \vec{r} \times \vec{F}$$

Direction from RHR

- Torque is maximum when the perpendicular distance to the rotation axis is maximum.

CPS Question 25-2

- **Three forces of equal magnitude act on a sphere. Which force supplies the largest torque on the object?**



A) A

B) B

C) C

D) Not enough information to solve.

Then…

- Linear
  - For const a:

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

  - Kinetic energy:

$$KE = \frac{1}{2} M v^2$$

  - Comes from:

$$W = \int \vec{F} \cdot d\vec{s}$$

- Rotational
  - For const α:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

  - Kinetic energy:

$$KE = \frac{1}{2} I \omega^2$$

  - Comes from:

$$W = \int \tau_z d\theta$$

What Causes Acceleration?

- Linear
  - For const a:

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

  - Kinetic energy:

$$KE = \frac{1}{2} M v^2$$

  - Comes from:

$$W = \int \vec{F} \cdot d\vec{s}$$

  - Newton's 2nd Law

$$\sum \vec{F} = m\vec{a}$$

- Rotational
  - For const α:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

  - Kinetic energy:

$$KE = \frac{1}{2} I \omega^2$$

  - Comes from:

$$W = \int \tau_z d\theta$$

  - Newton's 2nd Law

$$\sum \vec{\tau} = I\vec{\alpha}$$

No New Physics!

- Start with F=ma…

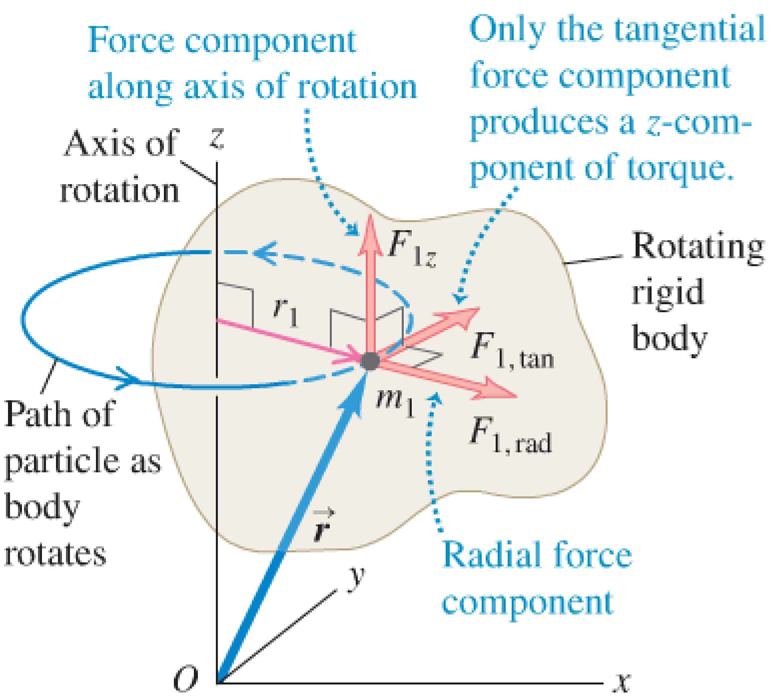
$$F_{1\tan} = m_1 a_{1\tan}$$

- But,

$$a_{1\tan} = r_1 \alpha_z$$

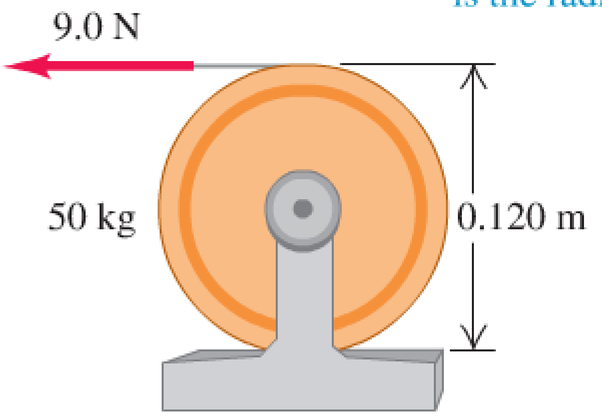
- And multiply both sides by r_1:

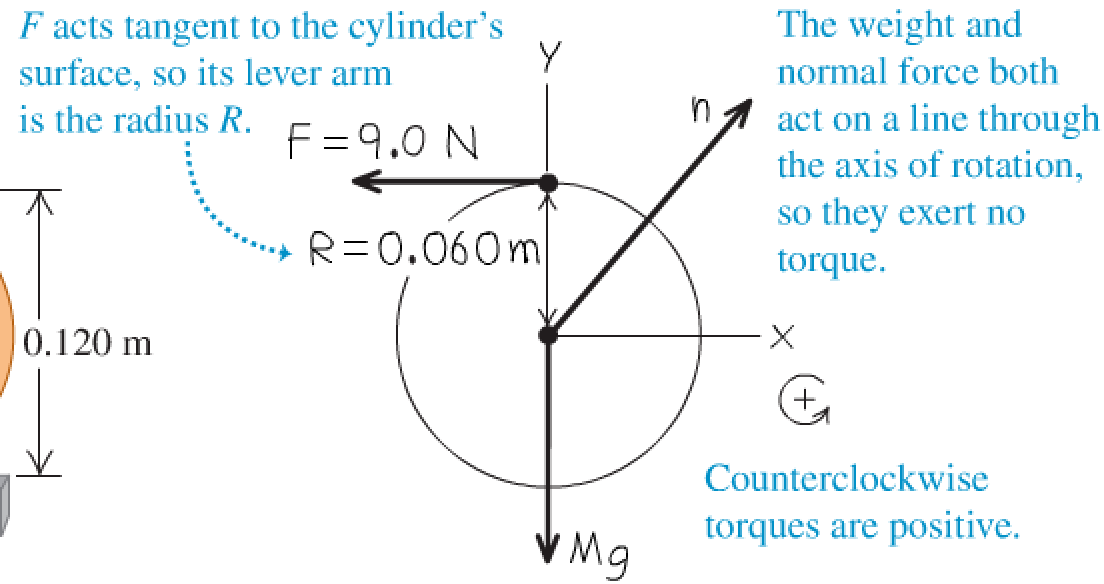
$$F_{1\tan} r_1 = m_1 r_1^2 \alpha_z$$



Example

Figure 10.9a shows the same situation that we analyzed in Example 9.8 (Section 9.4) using energy methods. A cable is wrapped several times around a uniform solid cylinder that can rotate about its axis. The cylinder has diameter 0.120 m and mass 50 kg. The cable is pulled with a force of 9.0 N. Assuming that the cable unwinds without stretching or slipping, what is its acceleration?

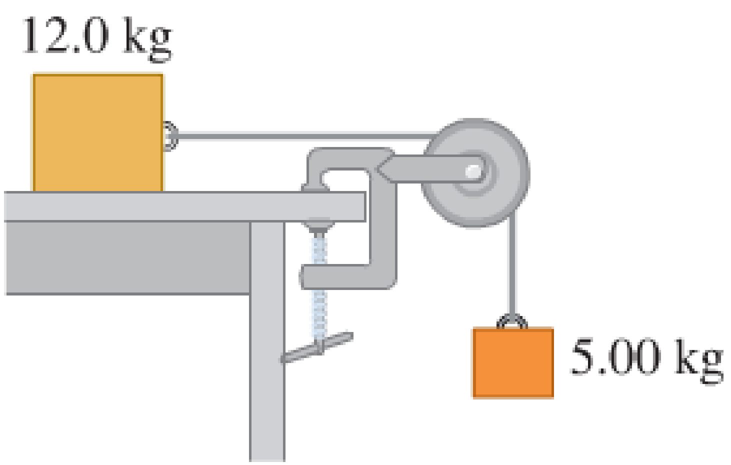




Problem 10.16

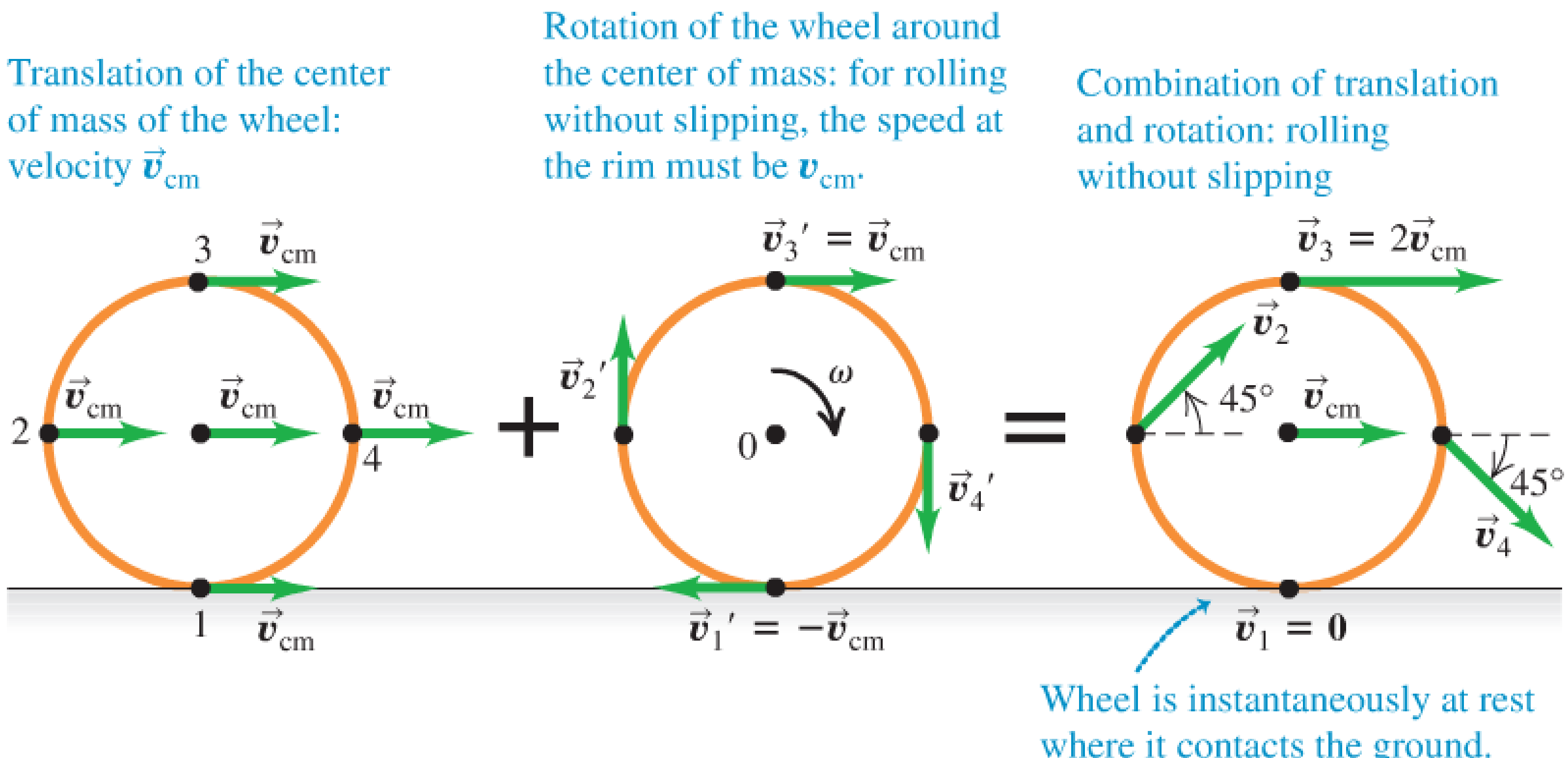
10.16. A 12.0-kg box resting on a horizontal, frictionless surface is attached to a 5.00-kg weight by a thin, light wire that passes over a frictionless pulley (Fig. 10.44). The pulley has the shape of a uniform solid disk of mass 2.00 kg and diameter 0.500 m. After the system is released, find (a) the tension in the wire on both sides of the pulley, (b) the acceleration of the box, and (c) the horizontal and vertical components of the force that the axle exerts on the pulley.

Figure 10.44 Exercise 10.16.



Rolling Without Slipping

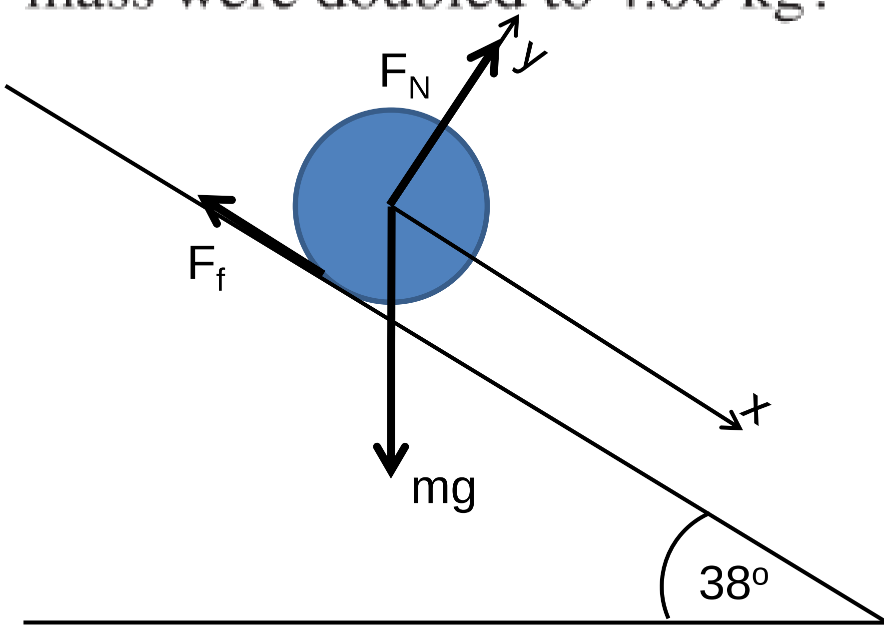
- An important case of a combination of translational and rotational motion:



- So, must have: $v_{cm} = R\omega$
- If v_{cm} changes with time, then also must have: $a_{cm} = R\alpha$

Problem 10.22

10.22. A hollow, spherical shell with mass 2.00 kg rolls without slipping down a 38.0° slope. (a) Find the acceleration, the friction force, and the minimum coefficient of friction needed to prevent slipping. (b) How would your answers to part (a) change if the mass were doubled to 4.00 kg?



$$\sum F_x = ma_x$$

$$\sum F_y = ma_y = 0$$

$$\sum \tau_z = I\alpha_z$$

$$a_{cm} = R\alpha$$

Problem 10.83

10.83. A uniform, solid cylinder with mass M and radius $2R$ rests on a horizontal tabletop. A string is attached by a yoke to a frictionless axle through the center of the cylinder so that the cylinder can rotate about the axle. The string runs over a disk-shaped pulley with mass M and radius R that is mounted on a frictionless axle through its center. A block of mass M is suspended from the free end of the string (Fig. 10.62). The string doesn't slip over the pulley surface, and the cylinder rolls without slipping on the tabletop. Find the magnitude of the acceleration of the block after the system is released from rest.

Figure 10.62 Problem 10.83.