Lecture 22
(Current and Resistance)

Physics 161-01 Spring 2012

Douglas Fields

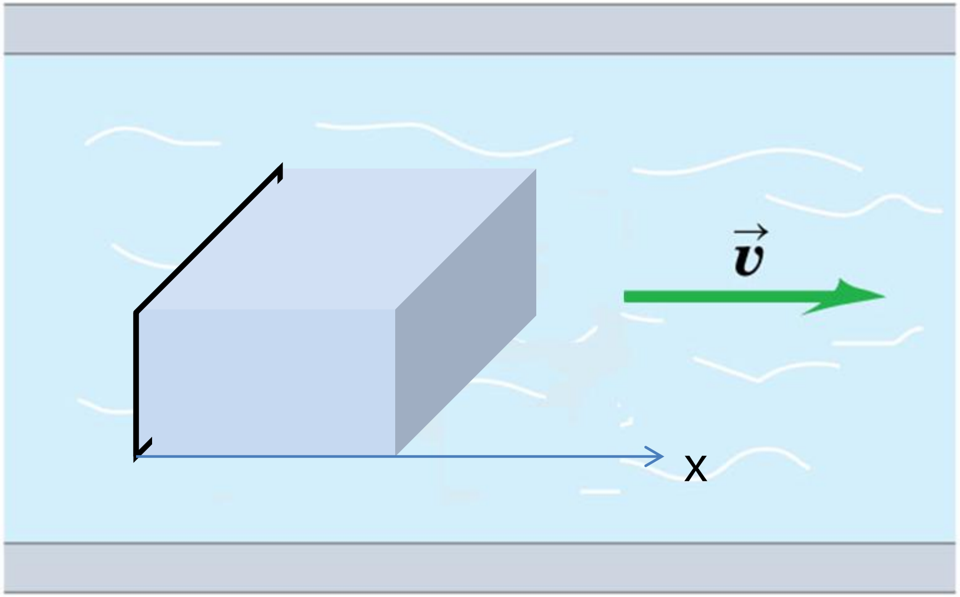
Attention

- Up until now, we have been studying electro*statics*, in other words no moving charges.
- From now on, we are going to allow (in fact, encourage) charges to move.
- So, I will have to slightly alter some of the things that I have told you, especially about conductors.

Current

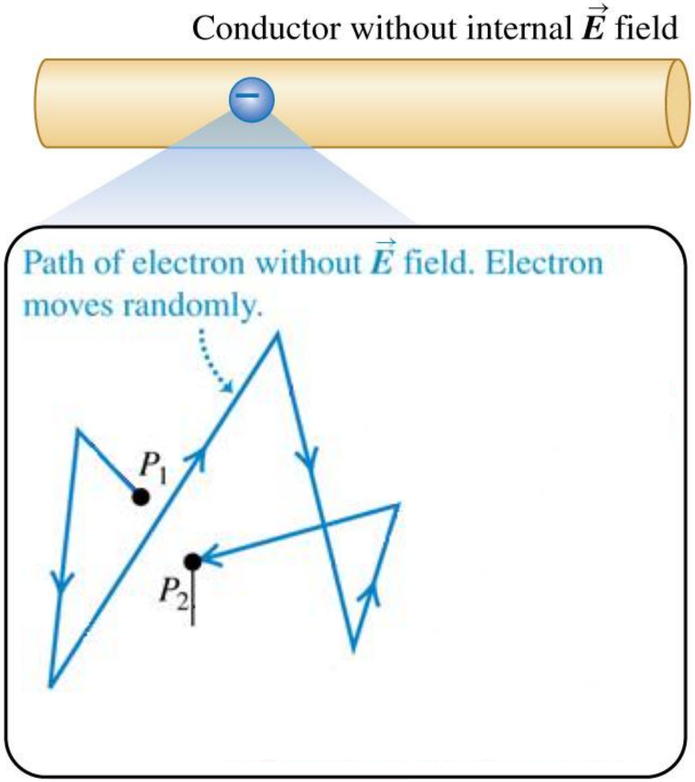
- When you use the term current, most people think of water current flowing down a river.
- I will use this analogy frequently, but you are now warned not to take it as a perfect analogy!
- How do we define current?

$$\left.\frac{dV}{dt}\right|_{\text{Through surface}} = \frac{dx}{dt}A = vA$$



Electric Current

- So, if we want to talk about electric current, we have to know something about the velocity of charges in a material.
- Without an electric field, charges feel no force, and hence just move with random motion (because of their thermal energy).
- But, if we apply an electric field in the conductor, there still is random motion, but on top of that, a net drift in the opposite direction of the field.
- This is known as the drift velocity, v_d.

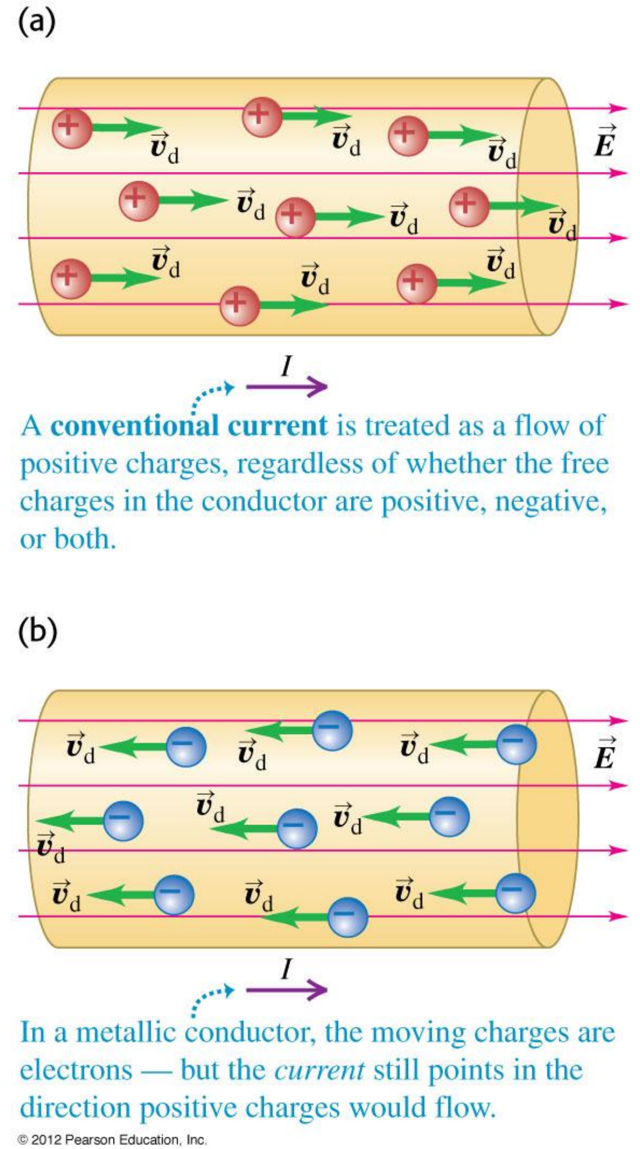


Electric Current

- Now, current, I, is defined as positive in the direction that positive charges move…

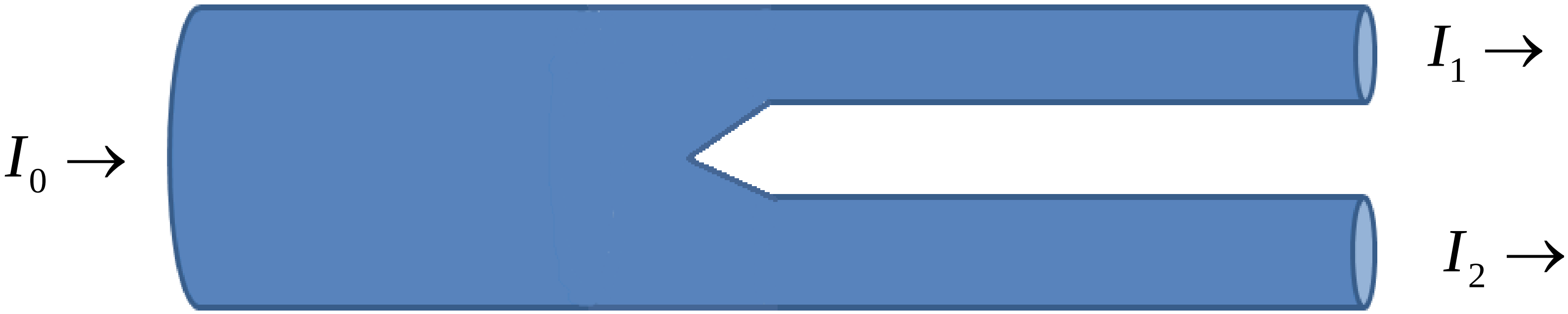
$$I = \frac{dq}{dt}\left[A \equiv \frac{C}{s}\right]$$

"Electric current is the amount of charge moving through a surface per time".

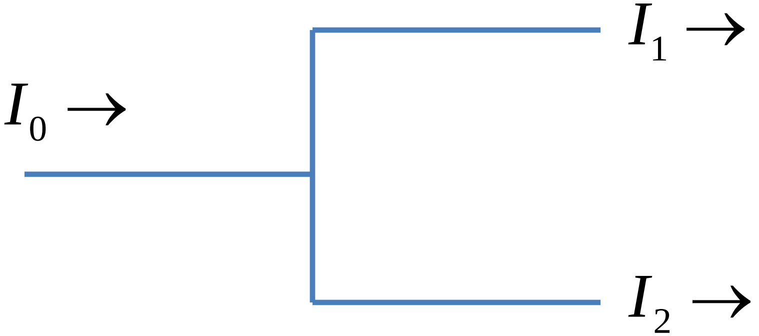


Electric Current

- Electric current, just like water current, is conserved:

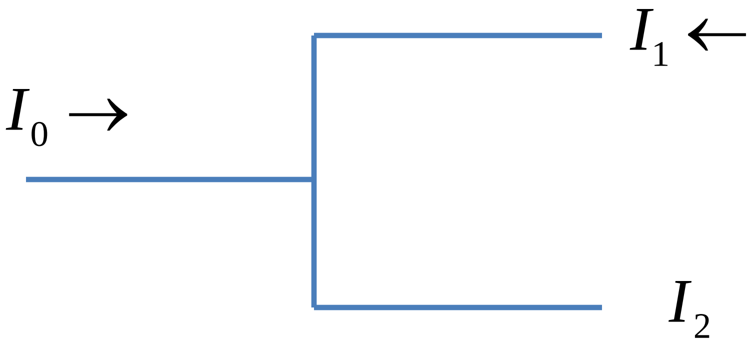


$$I_0 = I_1 + I_2$$



CPS 22-1

Given I_0 = 3A and I_1 = 1A, what is I_2 and which direction is it flowing?



A. 2A to the right.

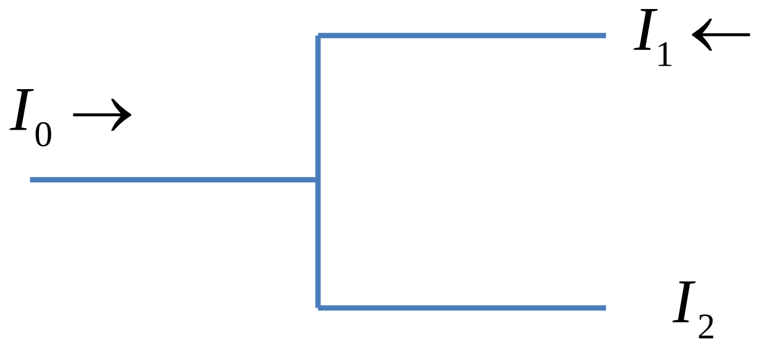
B. 2A to the left.

C. 4A to the left.

D. 4A to the right.

CPS 22-1

Given I_0 = 3A and I_1 = 1A, what is I_2 and which direction is it flowing?



A. 2A to the right.

B. 2A to the left.

C. 4A to the left.

D. 4A to the right.

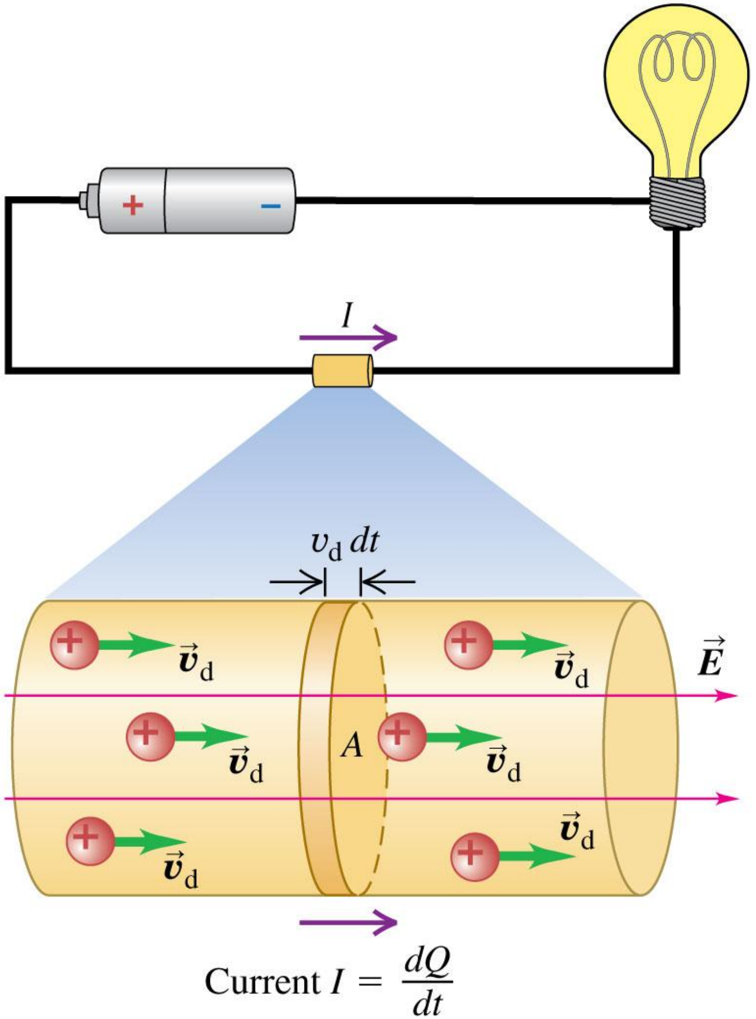
Electric Current

- Microscopic understanding

$$dq = \left(\frac{\text{number of charge carriers}}{\text{volume}}\right) \times (\text{volume}) \times (\text{charge of each carrier})$$
$$= (n) \times (A \cdot dL) \times (q_e)$$
$$= nAdLq_e$$

$$\text{Distance moved} = (\text{Drift velocity}) \times (\text{time})$$
$$dL = (v_d) \times (dt) \Rightarrow$$
$$dt = \frac{dL}{v_d}$$

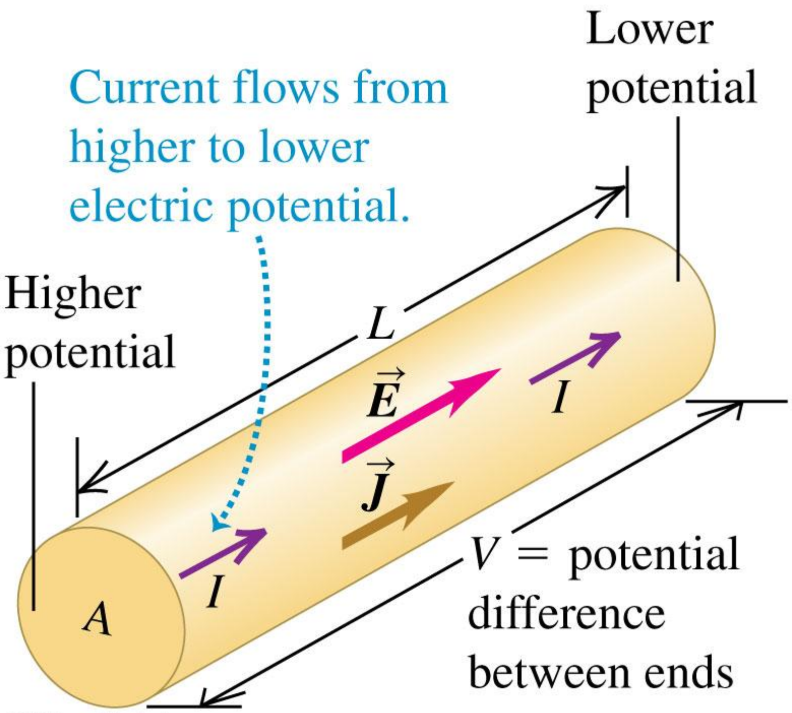
$$I = \frac{dq}{dt} = \frac{nAdLq_e}{dL/v_d} = nAv_d q_e$$



Current Density

- A useful variable is the current density, defined as the current per unit cross-sectional area.

$$\left|\vec{J}\right| = \frac{I}{A} = \frac{nAv_d q_e}{A} = nv_d q_e$$

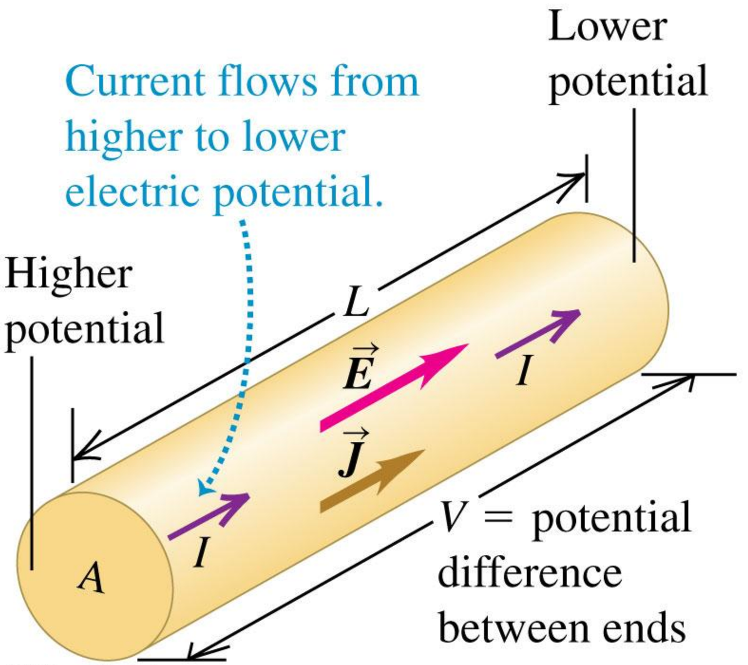


Current Density

- But what makes the charges drift again?
- The electric field inside the conductor.
- Note that the current density is a vector, in the direction of the electric field that causes charge motion.
- If the charge is positive, the drift velocity is in the direction of the E-field.
- If it is negative, it's in the opposite direction.
- Either way, the current density is in the direction of the E-field!

$$\left|\vec{J}\right| = \frac{I}{A} = \frac{nAv_d q_e}{A} = nv_d q_e$$

$$\vec{J} = n\vec{v}_d q_e$$

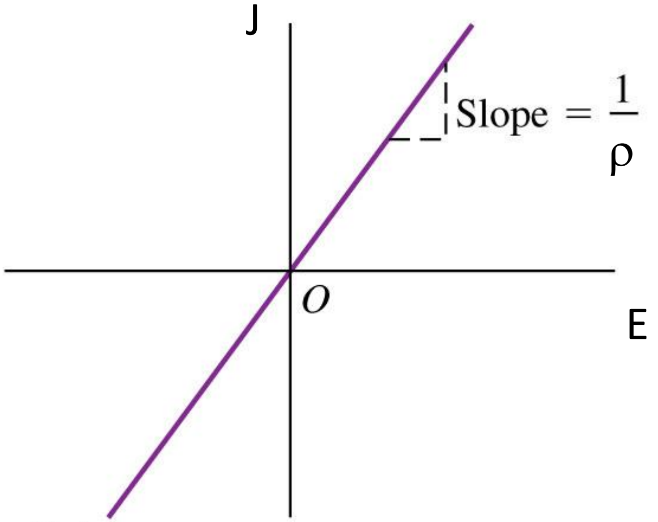


Ohm’s Law and Resistivity

- If there is an electric field, the charges feel a force.
- A force will cause them to accelerate: F=ma.
- But if there is a constant drift velocity, what causes the force that tends to slow them down?
- The resistivity of a material! OK, the resistivity is actually due to collisions…
- An “ohmic” material follows a proportionality between the electric field and the current density:

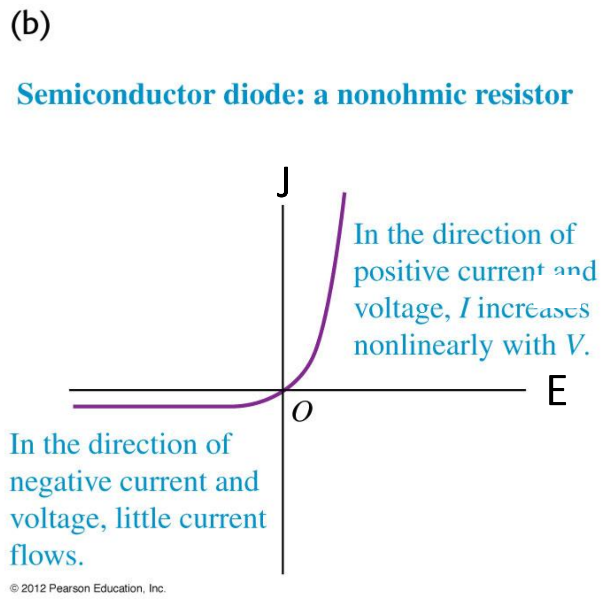
$$J = \frac{E}{\rho} \Rightarrow$$

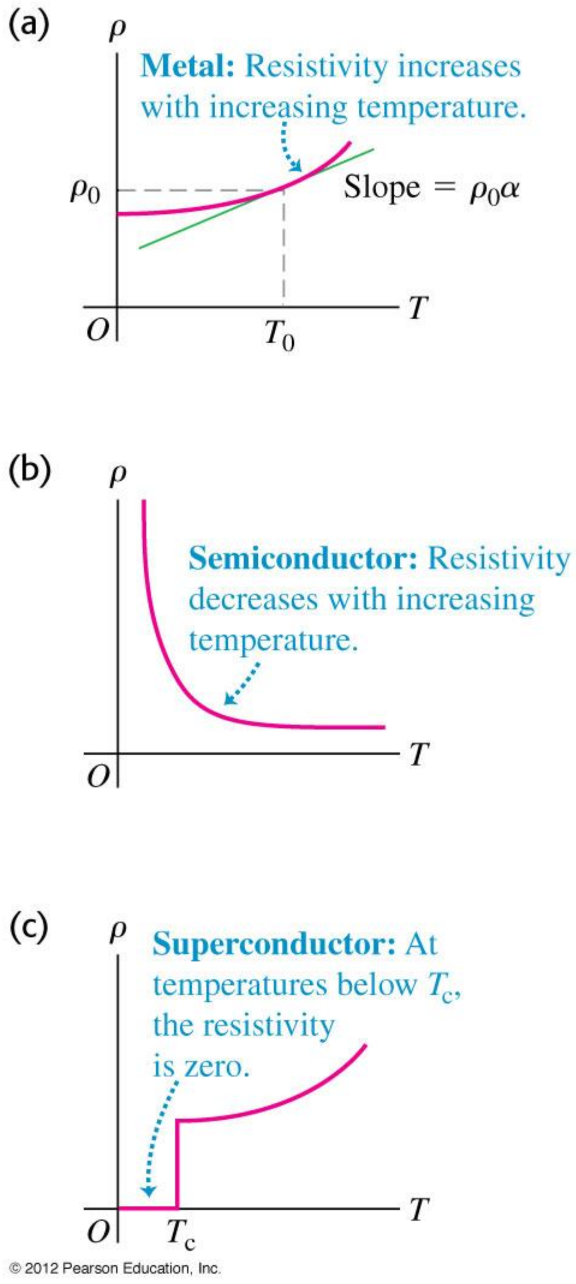
$$\rho = \frac{E}{J}$$ Definition of resistivity



Caveats

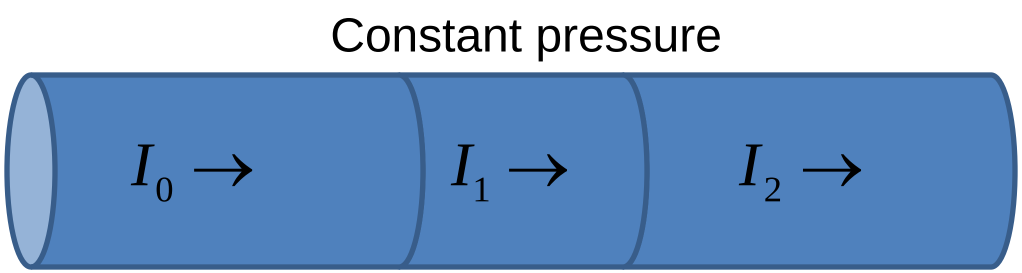
- The resistivity of materials in general depends on temperature.
- Not all materials are ohmic.



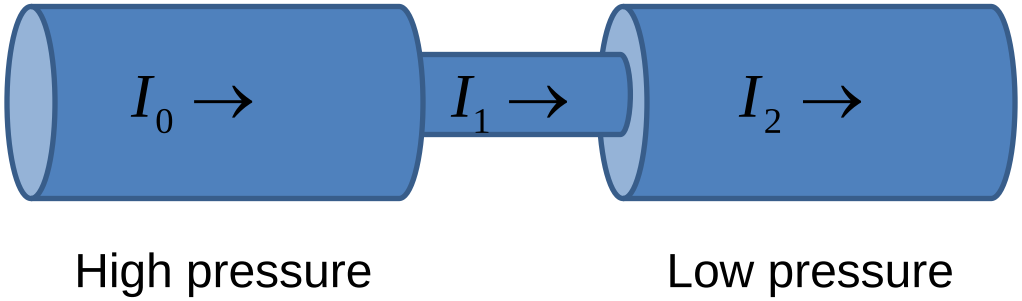


Resistors

- A resistor is an object (not a material) that is used to restrict the current.
- Let’s use the water analogy again…
- First, we have a pipe with water flowing through it.

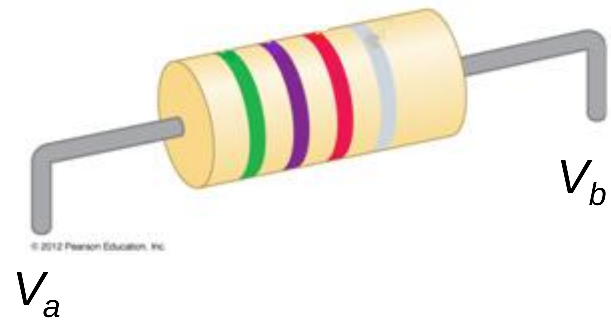


- Then, we add a restriction in the pipe.



Making a Resistor

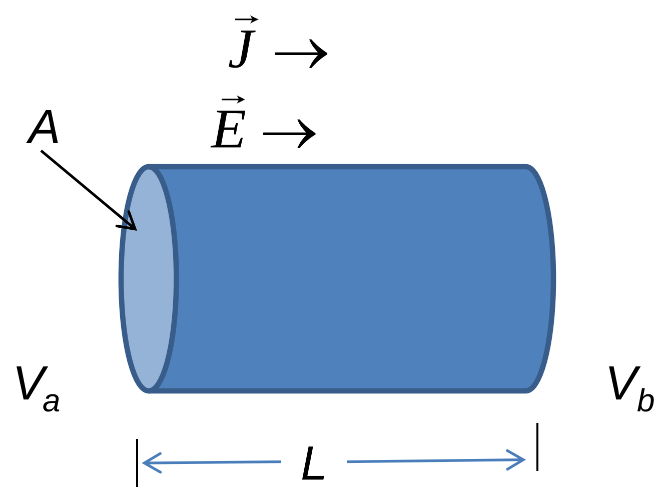
- Resistivity determines the current density in a material with a given electric field.
- We will find it useful to use this property to control the flow of current in circuits.
- So we now need to make a "resistor" out of some material with resistivity ρ.
- In this case, we want to control the entire current through the resistor.



$$I = JA$$

$$= \frac{E}{\rho} A$$

$$I = \frac{V}{L\rho} A$$



Ohm's Law $\quad V = IR, \quad R \equiv \frac{L\rho}{A}$

$$V \equiv V_a - V_b = E \cdot L$$

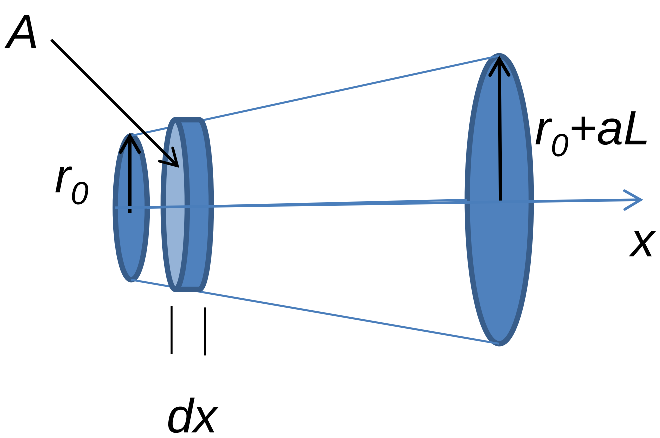
More Complicated

- Since you MUST know how to apply calculus to move on in your coursework…
- Let, $r = r_0 + ax \Rightarrow$

$$A = \pi r^2 = \pi\left(r_0 + ax\right)^2$$

$$R = \frac{L\rho}{A} \Rightarrow$$

$$dR = \frac{\rho(x)}{A(x)}dx$$



$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C \qquad (\text{for } n \neq -1)$$