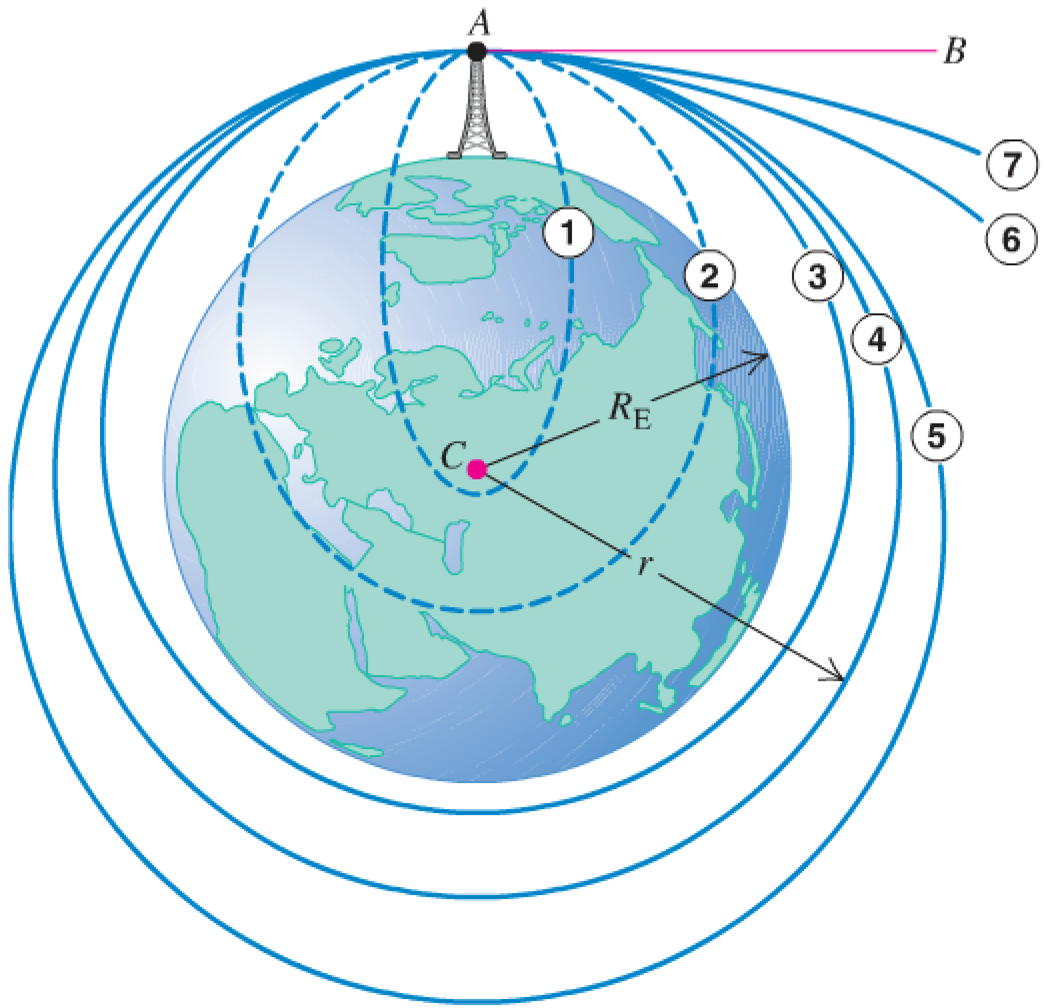
Lecture 33
(Kepler's Laws)

Physics 160-01 Fall 2012

Douglas Fields

Motion of Satellites

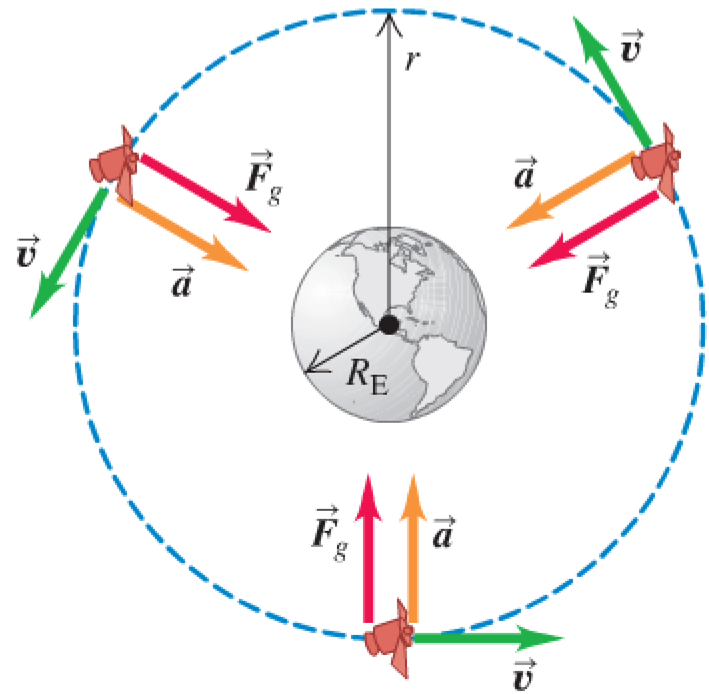


Motion of Satellites (Circular Orbit)

$$\sum \vec{F} = m\vec{a} \Rightarrow$$

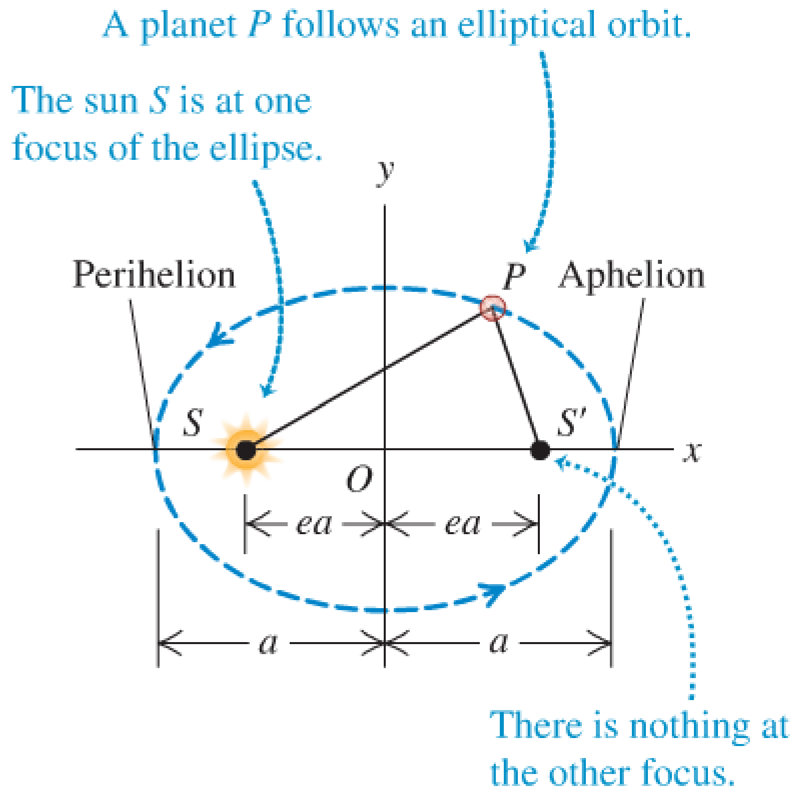
$$\frac{Gm_E m}{r^2} = m\frac{v^2}{r} \Rightarrow$$

$$v = \sqrt{\frac{Gm_E}{r}}$$



Kepler's Laws

1. **Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse.**
2. **A line from the sun to a given planet sweeps out equal areas in equal times.**
3. **The periods of the planets are proportional to the $\frac{3}{2}$ powers of the major axis lengths of their orbits.**



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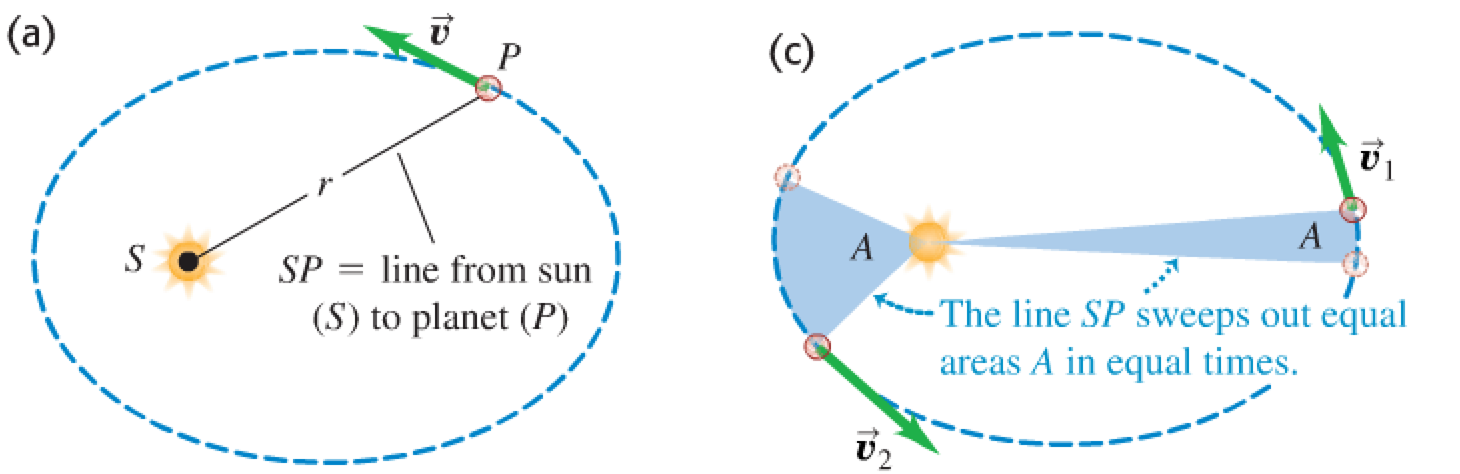
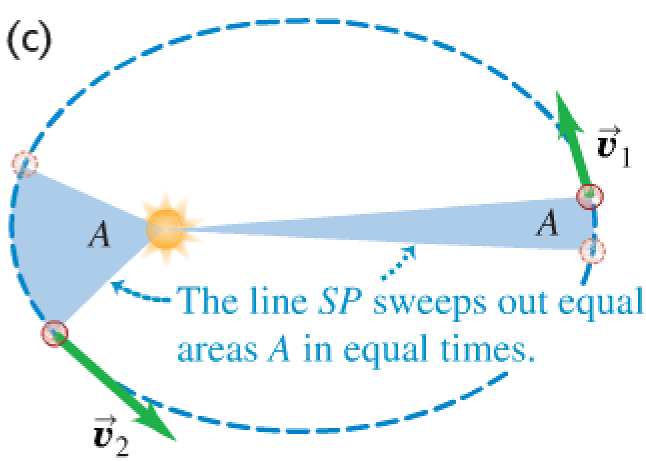
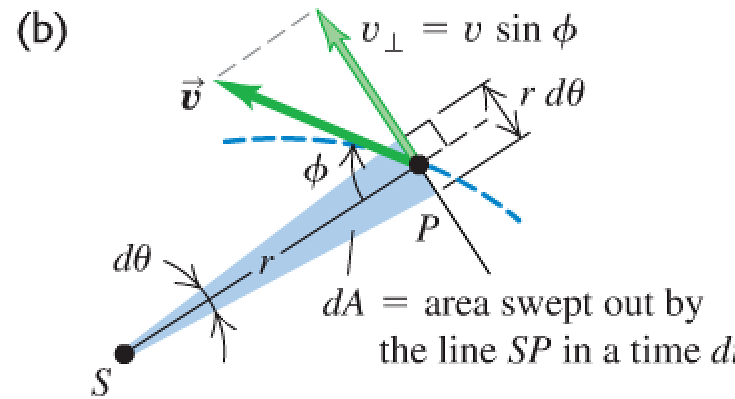


Kepler's Laws





$$dA = \frac{1}{2} r^2 d\theta \Rightarrow$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r \left(r \frac{d\theta}{dt} \right) = \frac{1}{2} r v_\perp = \frac{1}{2m} r m v_\perp = \frac{1}{2m} \vec{r} \times \vec{p} = \frac{1}{2m} \vec{L}$$

But, since the force acts along the line r, L is conserved, so dA/dt is constant!

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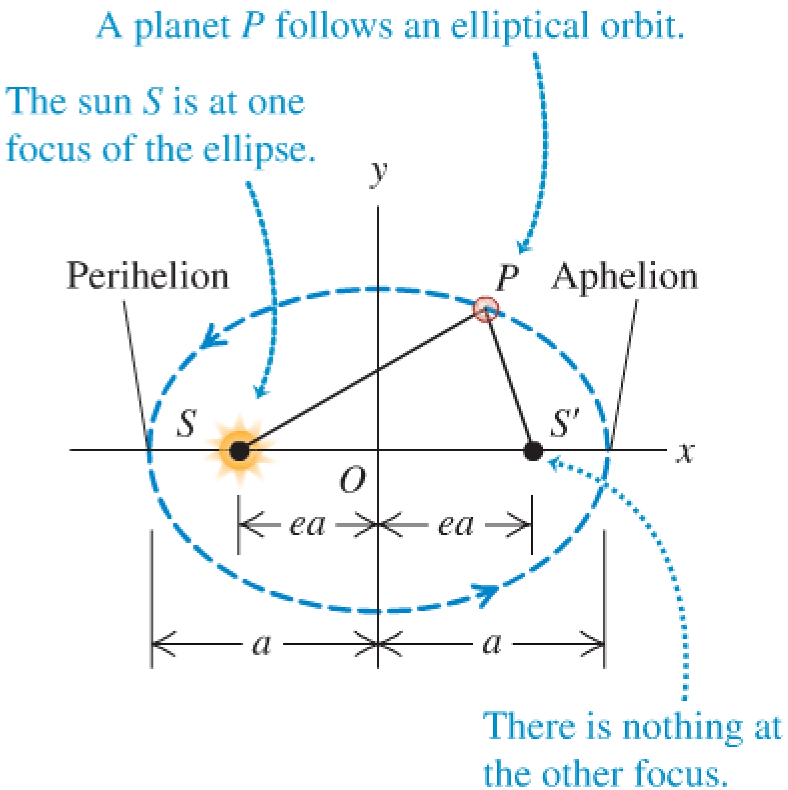


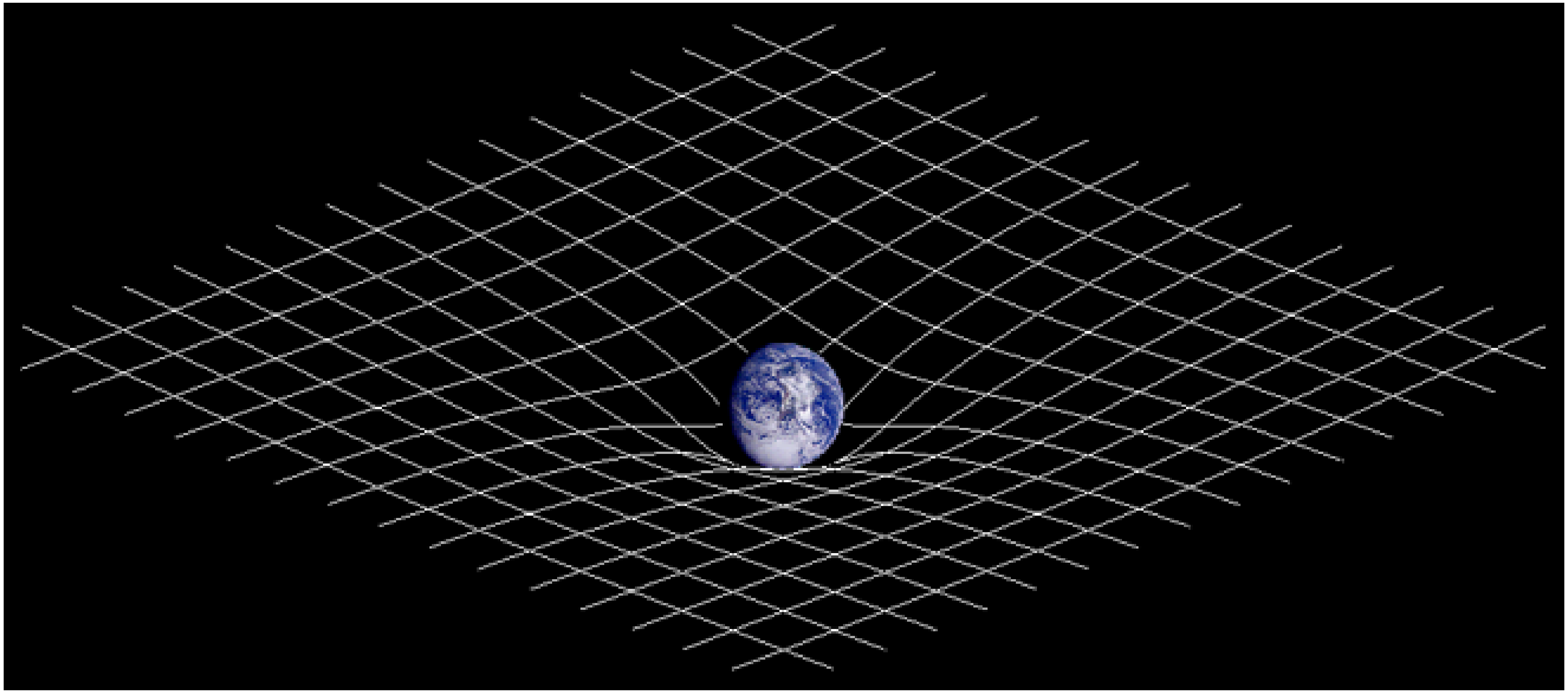
$$T = \frac{2\pi a^{3/2}}{\sqrt{G m_S}}$$

Einstein’s Gravity

- General relativity is a metric theory of gravitation. At its core are Einstein's equations, which describe the relation between the geometry of a four-dimensional, semi-Riemannian manifold representing spacetime on the one hand, and the energy-momentum contained in that spacetime on the other.[31] Phenomena that in classical mechanics are ascribed to the action of the force of gravity (such as free-fall, orbital motion, and spacecraft trajectories), correspond to inertial motion within a curved geometry of spacetime in general relativity; there is no gravitational force deflecting objects from their natural, straight paths. Instead, gravity corresponds to changes in the properties of space and time, which in turn changes the straightest-possible paths that objects will naturally follow.[32] The curvature is, in turn, caused by the energy-momentum of matter. Paraphrasing the relativist John Archibald Wheeler, spacetime tells matter how to move; matter tells spacetime how to curve.[33]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Space Time Curvature

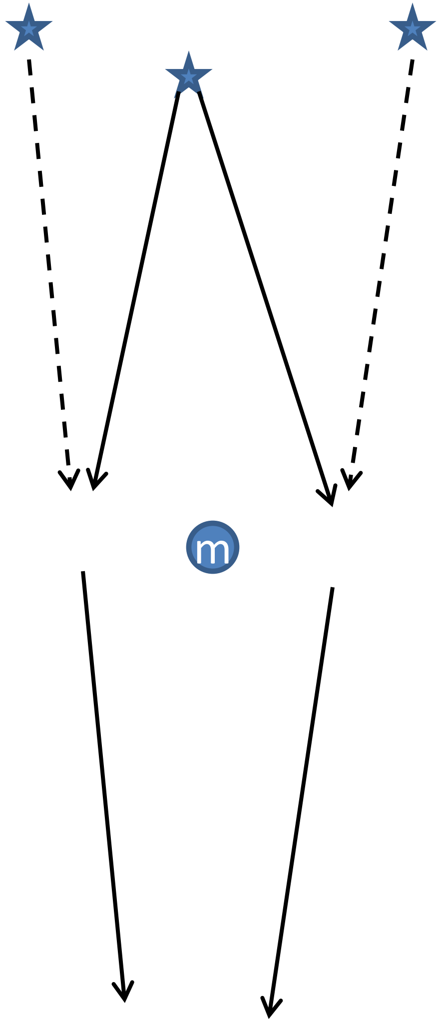


Gravity Affects Light Too!

- Light is also affected by gravity and can get pulled into an massive object.

Gravitational Lensing (simulation)

Gravitational Lensing



Black Holes

- Remember that we discussed the escape velocity of an object from a massive body (planet, sun, etc.).

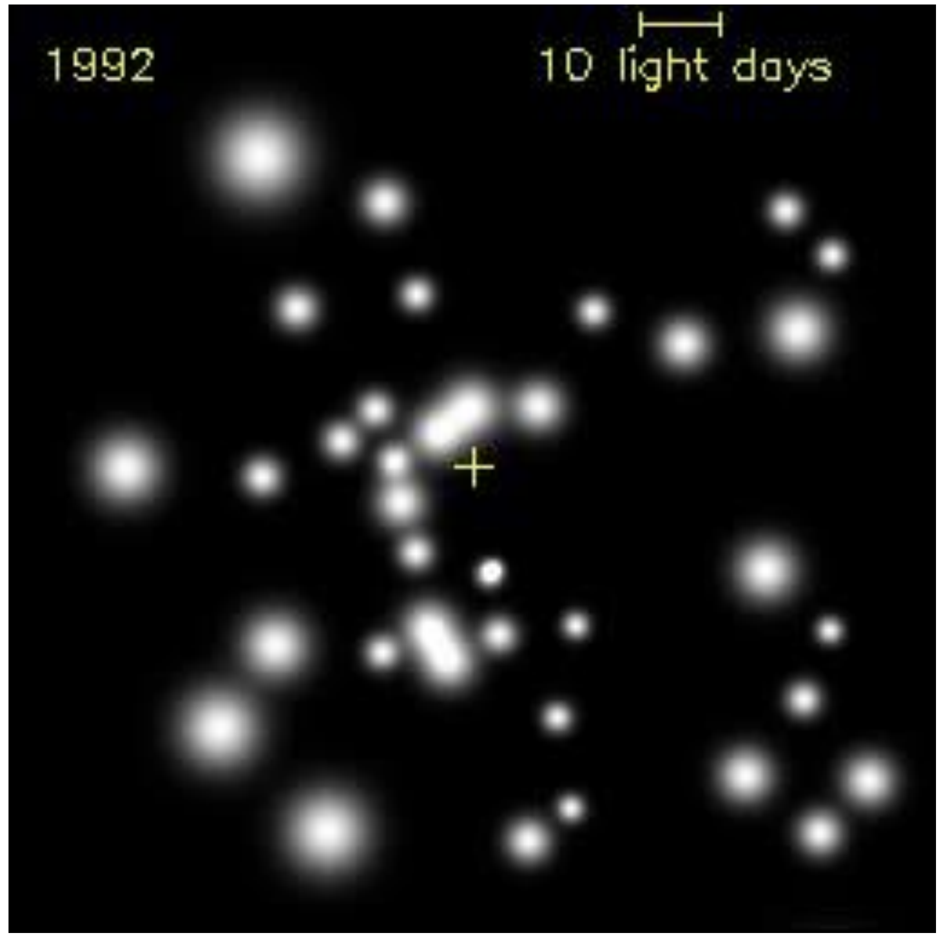
$$v = \sqrt{\frac{2Gm}{R}}$$

- We will learn later that light has a characteristic velocity, c = 3×10^8 m/s.
- What happens when the mass/radius of an object becomes so large that v > c?
- It's called a black hole.

Stars Around our Black Hole

- How do we know black holes exist?
- Look for objects that have orbits which require massive objects.

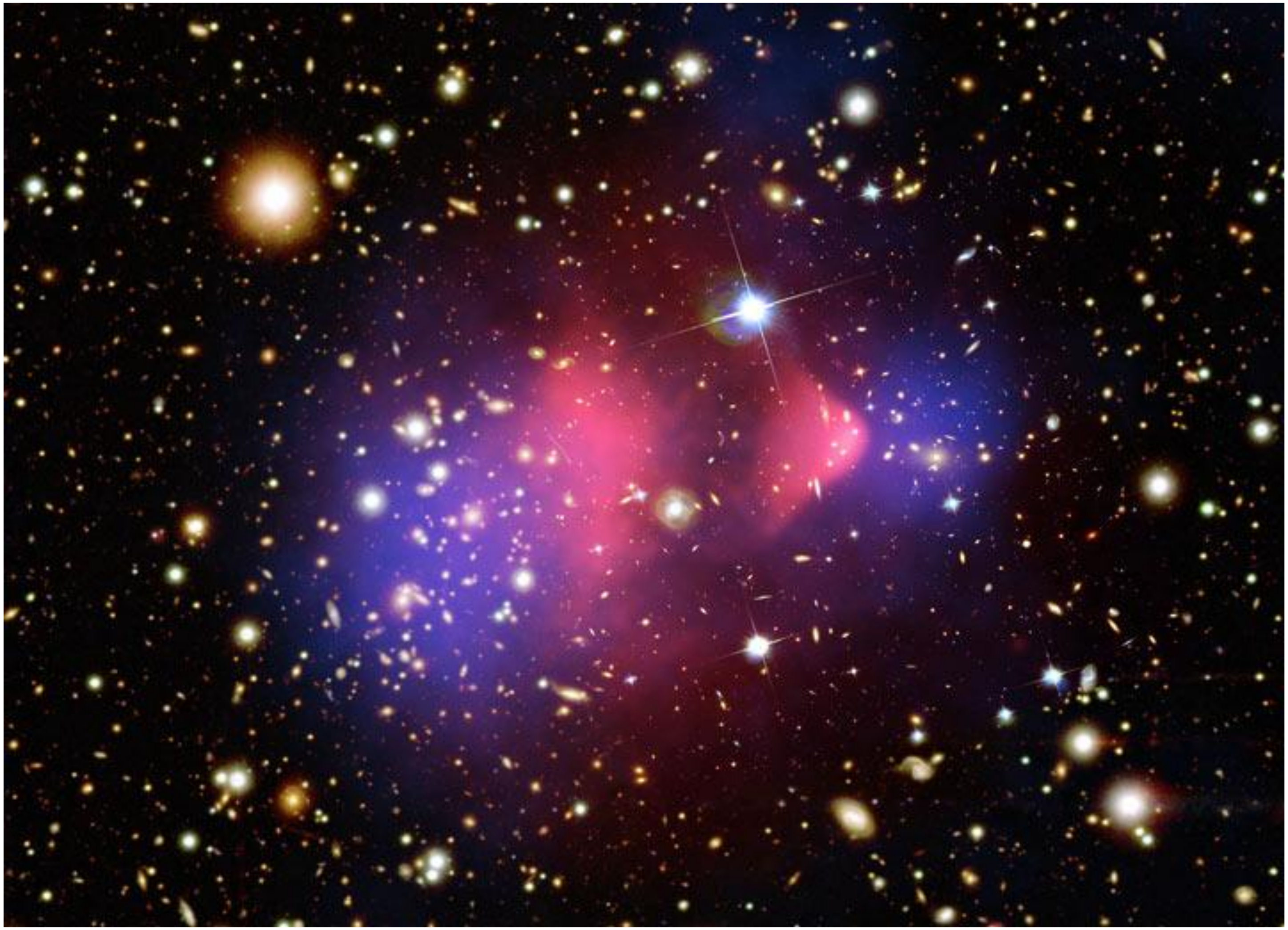
$$T = \frac{2\pi a^{3/2}}{\sqrt{Gm}}$$



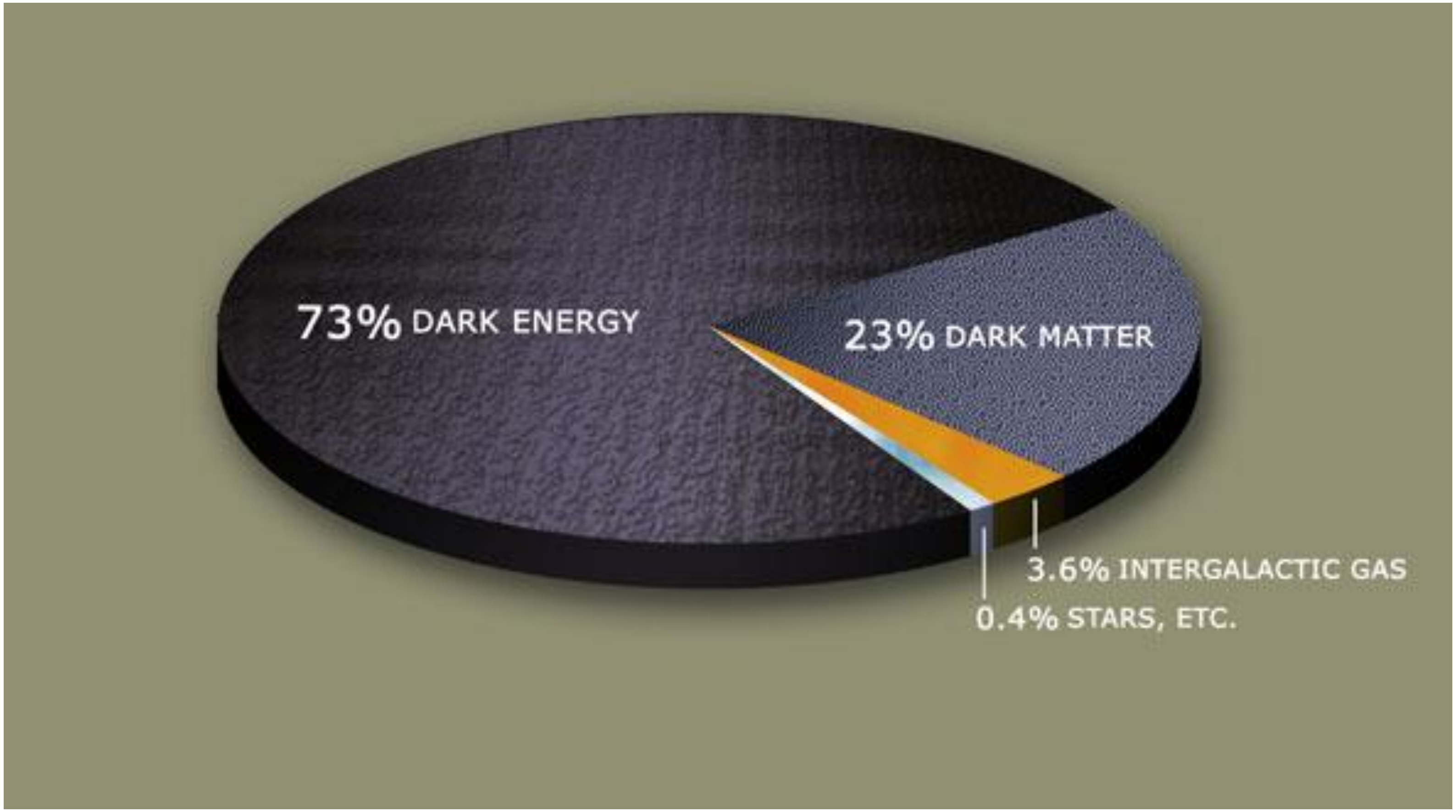
M = 3.7million times the mass of the sun!!!

Dark Matter

Dark Matter



Dark Energy


73% DARK ENERGY
23% DARK MATTER
3.6% INTERGALACTIC GAS
0.4% STARS, ETC.