Coulomb's Law Tutorial

Description: Use Coulomb's law to compute the (vector) force between two, and subsequently multiple, electric charges (both positive and negative). One charge acts at an angle of pi/4 radians relative to the given coordinate axes, and thus trigonometry is required to solve the problem.

Learning Goal:

To understand how to calculate forces between charged particles, particularly the dependence on the sign of the charges and the distance between them.

Coulomb's law describes the force that two charged particles exert on each other (by Newton's third law, those two forces must be equal and opposite). The force F_{21} exerted *by* particle 2 (with charge q_2) *on* particle 1 (with charge q_1) is proportional to the charge of each particle and inversely proportional to the square of the distance r between them:

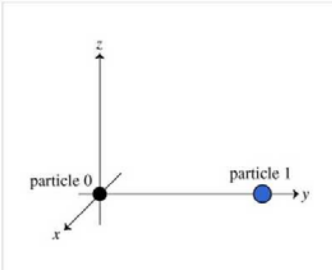
$$\vec{F}_{21} = \frac{k\,q_2\,q_1}{r^2}\,\hat{r}_{21},$$

where $k = \frac{1}{4\pi\epsilon_0}$ and $\hat{r}_{21}$ is the unit vector pointing *from* particle 2 *to* particle 1. The force vector will be parallel or antiparallel to the direction of $\hat{r}_{21}$, parallel if the product $q_1 q_2 > 0$ and antiparallel if $q_1 q_2 < 0$; the force is *attractive* if the charges are of opposite sign and *repulsive* if the charges are of the same sign.

Part A

Consider two positively charged particles, one of charge q_0 (particle 0) fixed at the origin, and another of charge q_1 (particle 1) fixed on the *y*-axis at $(0,\, d_1,\, 0)$. What is the net force $\vec{F}$ *on* particle 0 *due to* particle 1?

Express your answer (a vector) using any or all of k, q_0, q_1, d_1, $\hat{x}$, $\hat{y}$, and $\hat{z}$.



ANSWER:

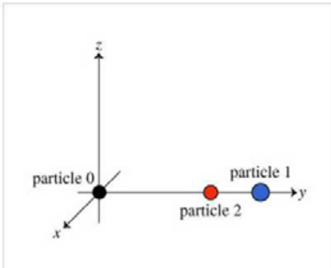
$\vec{F} = \frac{-kq_0q_1\hat{y}}{{d_1}^2}$

Also accepted: $\frac{\frac{-1}{4\pi\epsilon_0}q_0q_1\hat{y}}{{d_1}^2}$

Part B

Now add a third, negatively charged, particle, whose charge is $-q_2$ (particle 2). Particle 2 fixed on the *y*-axis at position $(0,\, d_2,\, 0)$. What is the new net force *on* particle 0, *from* particle 1 and particle 2?

Express your answer (a vector) using any or all of k, q_0, q_1, q_2, d_1, d_2, $\hat{x}$, $\hat{y}$, and $\hat{z}$.



ANSWER:

$\vec{F} = kq_0\left(\frac{-q_1}{{d_1}^2} + \frac{q_2}{{d_2}^2}\right)\hat{y}$

Also accepted: $\frac{1}{4\pi\epsilon_0}q_0\left(\frac{-q_1}{{d_1}^2} + \frac{q_2}{{d_2}^2}\right)\hat{y}$

Part C

Particle 0 experiences a repulsion *from* particle 1 and an attraction *toward* particle 2. For certain values of d_1 and d_2, the repulsion and attraction should balance each other, resulting in no net force. For what ratio d_1/d_2 is there no net force on particle 0?

Express your answer in terms of any or all of the following variables: k, q_0, q_1, q_2.

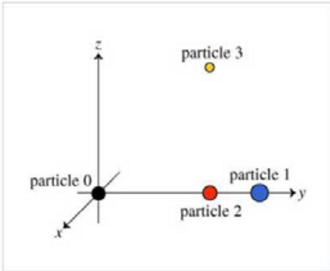
ANSWER:

$d_1/d_2 = \sqrt{\frac{q_1}{q_2}}$

Part D

Now add a fourth charged particle, particle 3, with positive charge q_3, fixed in the *yz*-plane at $(0,\ d_2,\ d_2)$. What is the net force $\vec{F}$ on particle 0 due *solely* to this charge?

Express your answer (a vector) using k, q_0, q_3, d_2, $\hat{x}$, $\hat{y}$, and $\hat{z}$. Include only the force caused by particle 3.



ANSWER:

$\vec{F}$ = $\frac{-kq_0q_3}{2{d_2}^2}\frac{\sqrt{2}}{2}(\hat{y}+\hat{z})$

Also accepted: $\frac{\frac{-1}{4\pi\epsilon_0}q_0q_3}{2{d_2}^2}\frac{\sqrt{2}}{2}(\hat{y}+\hat{z})$

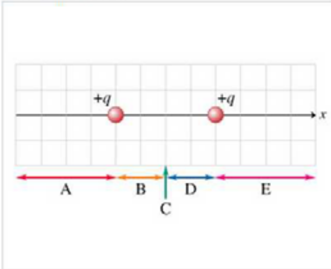
Electric Field Conceptual Question

Description: Simple conceptual question about identifying where on the x axis the electric field would be zero, given two charges.

Part A

For the charge distribution provided, indicate the region (A to E) along the horizontal axis where a point exists at which the net electric field is zero.

If no such region exists on the horizontal axis choose the last option (nowhere).



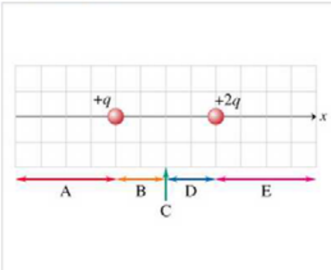
ANSWER:

- ○ A
- ○ B
- ◉ C
- ○ D
- ○ E
- ○ nowhere

Part B

For the charge distribution provided, indicate the region (A to E) along the horizontal axis where a point exists at which the net electric field is zero.

If no such region exists on the horizontal axis choose the last option (nowhere).



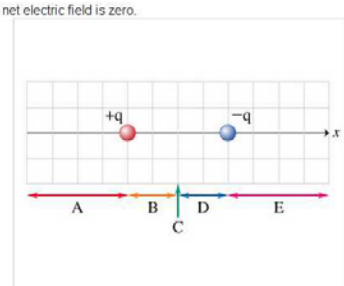
ANSWER:

- ○ A
- ◉ B
- ○ C
- ○ D
- ○ E
- ○ nowhere

Part C

For the charge distribution provided, indicate the region (A to E) along the horizontal axis where a point exists at which the net electric field is zero.

If no such region exists on the horizontal axis choose the last option (nowhere).

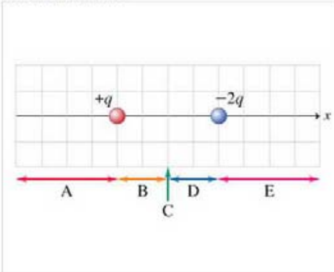


ANSWER:

- ○ A
- ○ B
- ○ C
- ○ D
- ○ E
- ◉ nowhere

Part D

For the charge distribution provided, indicate the region (A to E) along the horizontal axis where a point exists at which the net electric field is zero.



ANSWER:

- (selected) A
- B
- C
- D
- E
- Nowhere along the finite x axis

The Trajectory of a Charge in an Electric Field

Description: Simple two-dimensional kinematics of a charge moving in a uniform electric field.

An charge with mass m and charge q is emitted from the origin, $(x, y) = (0, 0)$. A large, flat screen is located at $x = L$. There is a target on the screen at *y* position y_h, where $y_h > 0$. In this problem, you will examine two different ways that the charge might hit the target. Ignore gravity in this problem.

Part A

Assume that the charge is emitted with velocity v_0 in the positive *x* direction. Between the origin and the screen, the charge travels through a constant electric field pointing in the positive *y* direction. What should the magnitude E of the electric field be if the charge is to hit the target on the screen?

Express your answer in terms of m, q, y_h, v_0, and L.

Hints (5)

ANSWER:

$E = 2\frac{m}{q}y_h\left(\frac{v_0}{L}\right)^2$

Part B

Now assume that the charge is emitted with velocity v_0 in the positive *y* direction. Between the origin and the screen, the charge travels through a constant electric field pointing in the positive *x* direction. What should the magnitude E of the electric field be if the charge is to hit the target on the screen?

Express your answer in terms of m, q, y_h, v_0, and L.

Hints (5)

ANSWER:

$E = 2\frac{m}{q}L\left(\frac{v_0}{y_h}\right)^2$

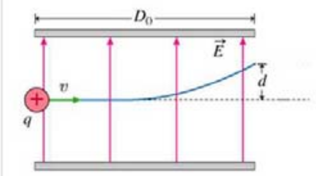
The equations of motion for this part are identical to the equations of motion for the previous part, with L and y_h interchanged. Thus it is no surprise that the answers to the two parts are also identical, with L and y_h interchanged.

Operation of an Inkjet Printer

Description: Find the charge that an ink droplet must be given to be deflected by a uniform electric field between two parallel plates.

In an inkjet printer, letters and images are created by squirting drops of ink horizontally at a sheet of paper from a rapidly moving nozzle. The pattern on the paper is controlled by an electrostatic valve that determines at each nozzle position whether ink is squirted onto the paper or not.

The ink drops have a mass m = $1.00\times10^{-11}\,\mathrm{kg}$ each and leave the nozzle and travel horizontally toward the paper at velocity v = $18.0\,\mathrm{m/s}$. The drops pass through a charging unit that gives each drop a positive charge q by causing it to lose some electrons. The drops then pass between parallel deflecting plates of length D_0 = $2.00\,\mathrm{cm}$, where there is a uniform vertical electric field with magnitude E = $8.15\times10^{4}\,\mathrm{N/C}$.



Part A

If a drop is to be deflected a distance d = $0.350\,\mathrm{mm}$ by the time it reaches the end of the deflection plate, what magnitude of charge q must be given to the drop? Assume that the density of the ink drop is $1000\,\mathrm{kg/m^3}$, and ignore the effects of gravity.

Express your answer numerically in coulombs.

Hints (5)

ANSWER:

$q = \frac{m\cdot 2dv^2}{D_0{}^2E}$ = 6.96×10^{-14} C

Here is something to think about: Is it reasonable to ignore the effect of gravity on the droplet in our calculations? For an average inkjet printer, the magnitude of the acceleration due to the electric field will be over ten times larger than the magnitude of the acceleration due to gravity. However, gravity will still cause a small deflection of the droplet and hence should not be ignored if the accuracy of the placement of the ink droplet is particularly important.

Exercise 21.54

Description: A straight, nonconducting plastic wire l long carries a charge density of lambda distributed uniformly along its length. It is lying on a horizontal tabletop. (a) Find the magnitude and direction of the electric field this wire produces at a point x...

A straight, nonconducting plastic wire 9.00 cm long carries a charge density of 125 $\mathrm{nC/m}$ distributed uniformly along its length. It is lying on a horizontal tabletop.

Part A

Find the magnitude and direction of the electric field this wire produces at a point 5.50 cm directly above its midpoint.

ANSWER:

$$E = \frac{2 \cdot 9 \cdot 10^9 \lambda}{x\left(\sqrt{1+\left(\frac{2x}{l}\right)^2}\right)} = 2.59 \times 10^4 \ \mathrm{N/C}$$

Part B

ANSWER:

- (selected) electric field is directed upward
- electric field is directed downward

Part C

If the wire is now bent into a circle lying flat on the table, find the magnitude and direction of the electric field it produces at a point 5.50 cm directly above its center.

ANSWER:

$$E = \frac{9 \cdot 10^9 \lambda l x}{\left(x^2+\left(\frac{l}{2\pi}\right)^2\right)^{\frac{3}{2}}} = 3.03 \times 10^4 \ \mathrm{N/C}$$

Part D

ANSWER:

- (selected) electric field is directed upward
- electric field is directed downward

Exercise 21.56

Description: The ammonia molecule (NH)_3 has a dipole moment of 5.0 * 10^(- 30) (C) * m. Ammonia molecules in the gas phase are placed in a uniform electric field E_vec with magnitude E. (a) What is the change in electric potential energy when the dipole...

The ammonia molecule (NH_3) has a dipole moment of $5.0 \times 10^{-30} C \cdot m$. Ammonia molecules in the gas phase are placed in a uniform electric field $\vec{E}$ with magnitude $1.6 \times 10^6 N/C$.

Part A

What is the change in electric potential energy when the dipole moment of a molecule changes its orientation with respect to $\vec{E}$ from parallel to perpendicular?

Express your answer using two significant figures.

ANSWER:

$\Delta U = 5.0 \cdot 10^{-30} E = 8.0 \times 10^{-24}$ J

Part B

At what absolute temperature T is the average translational kinetic energy $\frac{3}{2}kT$ of a molecule equal to the change in potential energy calculated in part (a)? (*Note:* Above this temperature, thermal agitation prevents the dipoles from aligning with the electric field.)

Express your answer using two significant figures.

ANSWER:

$T = \frac{2 \cdot 5.0 \cdot 10^{-30} E}{3 \cdot 1.381 \cdot 10^{-23}} = 0.39$ K

21.64. **IDENTIFY:** Apply $F=\frac{k|qq'|}{r^2}$ to find the force of each charge on $+q.$ The net force is the vector sum of the individual forces.

SET UP: Let $q_1=+2.50\ \mu\text{C}$ and $q_2=-3.50\ \mu\text{C}.$ The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge $q_1,$ since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of $q_1.$ Let $+q$ be a distance d to the left of $q_1,$ so it is a distance $d+0.600\text{ m}$ from $q_2.$

EXECUTE: $F_1=F_2$ gives $\frac{kq|q_1|}{d^2}=\frac{kq|q_2|}{(d+0.600\text{ m})^2}$.

$d=\pm\sqrt{\frac{|q_1|}{|q_2|}}(d+0.600\text{ m})=\pm(0.8452)(d+0.600\text{ m}).$ d must be positive, so

$d=\frac{(0.8452)(0.600\text{ m})}{1-0.8452}=3.27\text{ m}.$ The net force would be zero when $+q$ is at $x=-3.27\text{ m}.$

EVALUATE: When $+q$ is at $x=-3.27\text{ m},$ $\vec{F}_1$ is in the $-x$ direction and $\vec{F}_2$ is in the x direction.

21.80. **IDENTIFY:** We can treat the protons as point-charges and use Coulomb's law.

SET UP: **(a)** Coulomb's law is $F=(1/4\pi\epsilon_0)|q_1q_2|/r^2.$

EXECUTE: $F=(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2/(2.0\times10^{-15}\text{ m})^2=58\text{ N }=13\text{ lb},$ which is certainly large enough to feel.

(b) EVALUATE: Something must be holding the nucleus together by opposing this enormous repulsion. This is the strong nuclear force.

21.82. **IDENTIFY:** The positive sphere will be deflected in the direction of the electric field but the negative sphere will be deflected in the direction opposite to the electric field. Since the spheres hang at rest, they are in equilibrium so the forces on them must balance. The external forces on each sphere are gravity, the tension in the string, the force due to the uniform electric field and the electric force due to the other sphere.

SET UP: The electric force on one sphere due to the other is $F_\text{C}=k\frac{|q^2|}{r^2}$ in the horizontal direction, the force on it due to the uniform electric field is $F_E=qE$ in the horizontal direction, the gravitational force is *mg* vertically downward and the force due to the string is *T* directed along the string. For equilibrium $\sum F_x=0$ and $\sum F_y=0.$

EXECUTE: **(a)** The positive sphere is deflected in the same direction as the electric field, so the one that is deflected to the left is positive.

(b) The separation between the two spheres is $2(0.530 \text{ m})\sin 25^{\text{o}} = 0.4480 \text{ m}$.

$F_{\text{C}} = k\dfrac{|q^2|}{r^2} = \dfrac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(72.0\times10^{-9} \text{ C})^2}{(0.4480 \text{ m})^2} = 2.322\times10^{-4} \text{ N}$. $F_E = qE$. $\sum F_y = 0$ gives

$T\cos 25^{\text{o}} - mg = 0$ and $T = \dfrac{mg}{\cos 25^{\text{o}}}$. $\sum F_x = 0$ gives $T\sin 25^{\text{o}} + F_C - F_{\text{E}} = 0$.

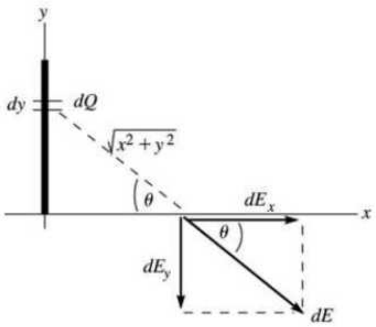
$mg\tan 25^{\text{o}} + F_C = qE$. Combining the equations and solving for E gives

$$E = \frac{mg\tan 25^{\text{o}} + F_{\text{C}}}{q} = \frac{(6.80\times10^{-6} \text{ kg})(9.8 \text{ m/s}^2)\tan 25^{\text{o}} + 2.322\times10^{-4} \text{ N}}{72.0\times10^{-9} \text{ C}} = 3.66\times10^3 \text{ N/C}.$$

EVALUATE: Since the charges have opposite signs, they attract each other, which tends to reduce the angle between the strings. Therefore if their charges were negligibly small, the angle between the strings would be greater than 50°.

21.90. **IDENTIFY:** Use Eq. (21.7) to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.10. Use Eq. (21.3) to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.90.



$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

Figure 21.90

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{dQ}{x^2+y^2}\right) = \dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{dy}{x^2+y^2}\right)$

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{(x^2+y^2)^{3/2}}\right)$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{(x^2+y^2)^{3/2}}\right)$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right)$$

(b) $\vec{F} = q_0\vec{E}$

$$F_x = -qE_x = \frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\; F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right)$$

(c) For $x \gg a$, $\dfrac{1}{\sqrt{x^2+a^2}} = \dfrac{1}{x}\left(1+\dfrac{a^2}{x^2}\right)^{-1/2} = \dfrac{1}{x}\left(1-\dfrac{a^2}{2x^2}\right) = \dfrac{1}{x} - \dfrac{a^2}{2x^3}$

$$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2},\; F_y \approx \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x} - \frac{1}{x} + \frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\epsilon_0 x^3}$$

EVALUATE: For $x \gg a$, $F_y \ll F_x$ and $F \approx |F_x| = \dfrac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x \gg a$ the charge distribution Q acts like a point charge.