MATH 264 — R E V I E W   E X A M   1

TOPICS COVERED (Chapters 13,14,15.1)

1. Vectors

add, subtract, multiply vectors, also graphically
magnitude of vectors
find resultant force if several forces are applied

2. Dot Product and Cross Product

compute dot and cross product of two vectors
magnitude and direction of cross product
what does $\mathbf{a} \cdot \mathbf{b} = 0$ *mean? what does* $\mathbf{a} \times \mathbf{b} = 0$ *mean?*
what is $\mathbf{a} \cdot \mathbf{a}$*? what is* $\mathbf{a} \times \mathbf{a}$*?*
know relation between dot/cross product and angle θ
find projections and orthogonal projections of $\mathbf{b}$ *onto* $\mathbf{a}$
applications: compute areas of triangles, parallelograms, find components of force in one direction, find vector normal to plane, etc

3. Lines and Planes

find equations for lines/planes
find intersections of lines and lines, lines and planes, planes and planes
find angle between lines and lines, lines and planes, planes and planes
determine when lines are parallel, skew, intersect
find distance from point to plane/line (dont memorize and apply formula)

4. Graphing in 3D

Graph any of the quadric surfaces in homework
Graph equations in cylindrical and spherical coordinates

5. Vector functions

Sketch elementary curves $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ *(circles, helices, ellipses, lines, curves which can be rewritten in nonparametric form)*
derivative $\mathbf{r}'(t)$ *and tangent vector*
differentiation rules
velocity, speed, acceleration
find arclength
find particle position given acceleration (integrate)

6. Functions of several variables

Read tables representing functions of two variables
Find domains of functions of two and three variables
Represent functions $z = f(x, y)$ of two variables by

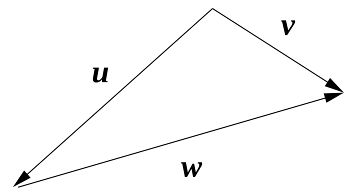
Graphing the surface $z = f(x, y)$ *in* $x - y - z$ *space*
Graphing contour (level) curves $f(x, y) = const$ *in the* $x - y$ *plane.*

STUDY PROBLEMS

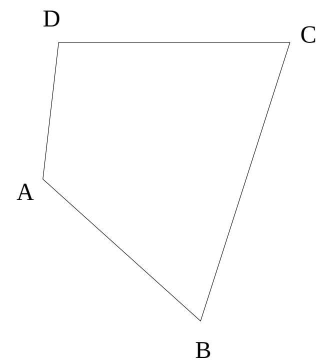
Here are some suggested study problems. Note that not all the topics are fully covered. You need to also go through the list of topics and do problems from the homework in those areas that are not extensively covered or that you feel a little weak in.

Vectors, Dot Product, Cross Product

1. Express $\mathbf{w}$ in terms of the vectors $\mathbf{u}$ and $\mathbf{v}$ in the figure.



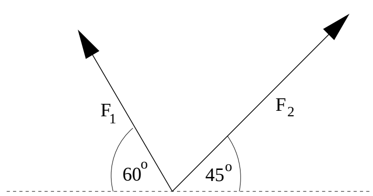
2. Consider a quadrilateral with vertices A,B,C,D, as shown. Let $\mathbf{a}_1 = \overline{AB}$, $\mathbf{a}_2 = \overline{BC}$, $\mathbf{a}_3 = \overline{CD}$, $\mathbf{a}_4 = \overline{DA}$.
   (a) Graph the vectors $\mathbf{b}_1 = \mathbf{a}_1 + \mathbf{a}_2$,
   $\mathbf{b}_2 = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$, $\mathbf{b}_3 = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4$,
   (b) Show that $\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4 = 0$ using the definition of $\mathbf{a}_1, \ldots, \mathbf{a}_4$.



3. §13.2, Exercise 4.

4. Let $\mathbf{F}_1$, $\mathbf{F}_2$ be as shown in Figure, where $|\mathbf{F}_1| = 3$ and $|\mathbf{F}_2| = 4$.
   (a) Graph the resultant force $\mathbf{F}$ in the figure.
   (b) Find the magnitude and elevation angle of the resultant force $\mathbf{F}$.

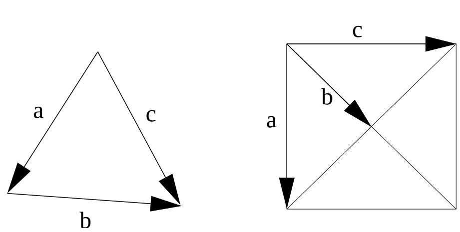
(Ans: $|\mathbf{F}|=\sqrt{25+6(\sqrt{6}-\sqrt{2})}$, $\theta=76.24^o$)



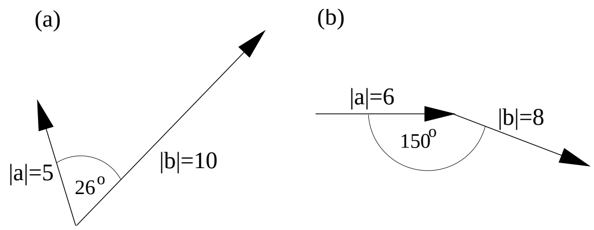
5. (Chain problem) §13.2, Exercise 34 (Ans: $30.09N$)

6. If two vectors enclose an acute angle, they point in the same general direction. If they enclose an obtuse angle, they point in generally opposite directions.
   Are the following pairs of vectors perpendicular to each other? If not, do they point in the same general direction or in generally opposite directions?
   (a) $\mathbf{a} = \langle 1, 2\rangle, \mathbf{b} = \langle -2, 4\rangle$ (b) $\mathbf{a} = \langle 0, 2, -2\rangle, \mathbf{b} = \langle 1, 0, 3\rangle$
   (c) $\mathbf{a} = \langle 2, -1, 4\rangle, \mathbf{b} = \langle 2, 0, -1\rangle$ (d) $\mathbf{a} = \langle 3, 4, 5\rangle, \mathbf{b} = \langle 2, -1, 1\rangle$

7. The following figures shows an equilateral triangle and a square, where $|\mathbf{a}| = 1$. Find $\mathbf{a}\cdot\mathbf{b}$ and $\mathbf{a}\cdot\mathbf{c}$, and $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}\times\mathbf{b}$ (both their magnitude and direction).



8. If $\mathbf{a}$ and $\mathbf{b}$ are the vectors shown in the figure, find $\mathbf{a}\cdot\mathbf{b}$ and $|\mathbf{a}\times\mathbf{b}|$. Determine whether $\mathbf{a}\times\mathbf{b}$ points into or out of the page.



9. Let $\mathbf{a} = 2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ and $\mathbf{b} = \mathbf{i}+6\mathbf{j}-2\mathbf{k}$. Draw a sketch showing the scalar and vector projection of $\mathbf{b}$ onto $\mathbf{a}$. Also show the vector which is the orthogonal projection of $\mathbf{b}$ onto $\mathbf{a}$. Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$. Explain all steps. (That is, do not refer to a formula in the book.) Find the orthogonal projection in terms of the vector projection.
(Ans: scalar projection $=-\frac{18}{\sqrt{14}}$, vector projection $-\frac{9}{7}\langle 2,-3,1\rangle$, orthogonal projection $\frac{5}{7}\langle 5,3,-1\rangle$)

10. Chapter 13 Review, Exercises: 2,4,5,6,7,11,12,13

Lines and Planes

1. Chapter 13 Review, Exercises: 15-20,21,23,24 (Ans to 24(b): 57.6^o)

2. (a) Describe a method to find the distance from a point to a plane. (Be clear and concise, so that someone else can use this method. Include a sketch. Do not refer to a formula in the book.)
   (b) Use your method to find the distance from the origin to the plane $4x - 6y + z = 5$.
   (Ans: $\frac{5}{\sqrt{53}}$)

3. (a) Describe a method to find the distance from a point to a line. (Be clear and concise, so that someone else can use this method. Include a sketch. Do not refer to a formula in the book.)
   (b) Use your method to find the distance from the point (1,2,3) to the line $\mathbf{r}(t) = \langle 2,2,0\rangle + t\langle 1,-3,5\rangle$. (Ans: $\sqrt{\frac{22}{5}}$)

4. Show that the planes $3x + y - 4z = 24$ and $8z - 2y - 6x = -4$ are parallel. Find the distance between the planes making use of your methods described in the previous problems.
(Ans: $\frac{22}{\sqrt{26}}$)

5. Determine whether the following pairs of lines are parallel, skew, or intersecting. If they intersect, find the point of intersection. If they are parallel, determine whether they are identical or distinct lines.
   (a) $\mathbf{r}_1(t) = \langle 1,1,0\rangle + t\langle 1,-1,2\rangle$, $\mathbf{r}_2(t) = \langle 2,0,4\rangle + t\langle -1,1,0\rangle$
   (b) $\mathbf{r}_1(t) = \langle 1,0,2\rangle + t\langle 1,1,-5\rangle$, $\mathbf{r}_2(t) = \langle 1,-2,-1\rangle + t\langle 1,1,-1\rangle$
   (c) $\mathbf{r}_1(t) = \langle 1,0,2\rangle + t\langle 1,1,-5\rangle$, $\mathbf{r}_2(t) = \langle 1,-2,-1\rangle + t\langle 2,2,-10\rangle$
(Ans: (a) intersect at (3,-1,4), (b) skew,non-intersecting, (c) parallel,distinct)

6. (a) Derive the vector and scalar equations of the straight line through (x_o, y_o, z_o) in the direction of the vector $\mathbf{v} = \langle a, b, c \rangle$. Make a sketch using vectors.
   (b) Derive the vector and scalar equations of the plane containing the point (x_o, y_o, z_o) and normal to the vector $\mathbf{n} = \langle a, b, c \rangle$ Make a sketch using vectors.

Graphing in 3D

1. Chapter 13 Review, Exercises: 1,26-34

Vector functions

1. Chapter 14 Review, Exercises: 1,4,5,6,8,9,16,17,18,19
   (Ans to 4: $r(t){=}\langle 1{,}1{,}1{,}\rangle{+}t\langle 2{,}4{,}3\rangle$)
   (Ans to 6: (a)$\langle \frac{15}{8}{,}0{,}-ln\,2\rangle$, (b)$r(t){=}\langle 1{,}1{,}0{,}\rangle{+}t\langle -3{,}2{,}1\rangle$, (c)$-3(x-1){+}2(y-1){+}z{=}0$)
   (Ans to 8: $\frac{2}{27}(13^{3/2}-8)$)

2. The curvature of a curve $\mathbf{r}(t)$ is defined as $\kappa = |d\mathbf{T}/ds|$, where $\mathbf{T}$ is the unit tangent vector to the curve and s is arclength. Curves are not typically parametrized with respect to arclength, making this formula difficult to apply. Instead, they are parametrized with respect to an arbitrary parameter t. Derive an alternative formula for κ that may be easier to apply.
   (Ans: $\kappa = \frac{|dT/dt|}{ds/dt}$, where $ds/dt = |r'(t)|$).

3. Consider the curve $\mathbf{r}(t) = \langle \frac{t^3}{3}, t^2, 2t \rangle$.
   (a) Find the unit tangent vector at the point P$(\frac{8}{3}, 4, 4)$. (Ans: $\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$)
   (b) Use the formula you derived in Problem 2 to find the curvature of the curve at P. (Ans: $\frac{1}{18}$)

4. Find the value of a for which the parabola $y = ax^2$ has curvature 4 at the origin, as follows.
   (a) Parametrize the parabola by a parameter t.
   (b) Use the formula you derived in Problem 2 to find the curvature at the origin in terms of a. (Ans: $2a$)
   (c) Answer the question. (Ans: $a = 2$)
   (d) Check your answer: using your calculator, draw a graph of the resulting parabola ($y = 2x^2$) and the osculating circle of curvature 4 at the origin.

Functions of several variables

1. Chapter 15 Review, Concept Check: 1, Exercises: 1-8.

Other

1. Chapter 13 Review, Concept Check: 1,3,5,7,8,17
2. Chapter 13 Review, True-False 1-14