3. (20 points) Determine the width of the depletion region and the junction capacitance of a PN^+ junction when it is connected to a 1V reverse bias potential. The doping concentration of P and N+ are $N_A=10^{15}$ cm^{-3} and $N_D=10^{19}$ cm^{-3}, respectively, and the junction area is 0.25μmx0.25μm. The relative permittivity of silicon is 11.9, V_{th}=25.9 mV at 300ºK, $n_i=1.45\times10^{10}$ cm^{-3} at 300ºK, and $\varepsilon_0=8.854\times10^{-14}$ F/cm.

Hint: You may use the following equations:

$$V_{bi} = V_{th} Ln\left(\frac{N_A \cdot N_D}{n_i^2}\right) \qquad W = \sqrt{\frac{2\varepsilon_{si}}{q}\frac{N_A + N_D}{N_A \cdot N_D}(V_{bi} - V_D)}$$

$$V_{bi} = 25.9^{mV} \times Ln\left(\frac{10^{15}\times 10^{19}}{(1.45\times10^{10})^2}\right) \Rightarrow V_{bi} = 0.815^{V}$$

$$W = \sqrt{\frac{2\times 11.9\times 8.854\times10^{-14}}{1.602\times10^{-19}}\ \frac{10^{19}+10^{15}}{10^{19}\times10^{15}}(0.815-(-1))} = 1.545\times10^{-4}\ cm$$

$$\Rightarrow \underline{W = 1.545^{\mu m}}$$

$$C = \frac{\epsilon A}{d} = \frac{\epsilon_{si} A}{W} = \frac{11.9\times 8.854\times10^{-14}\times(0.25\times10^{-4})^2}{1.545\times10^{-4}} \Rightarrow \underline{C = 4.26\times10^{-18}\ F}$$

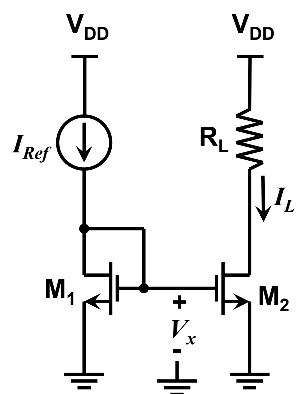
6. (20 points) The following circuit is called "current mirror". Assume that the physical parameters of M_2 is the same as M_1, except that M_2 is N times larger than M_1 (i.e. $(W/L)_2 = N*(W/L)_1$)

(a) Determine the region of operation for M_1.

(b) Based on your answer to part (a), find V_x as a function of K'_n, I_{Ref}, V_T, $(W/L)_1$.

(c) Assume that R_L is chosen such that M_2 is in saturation region. Find I_L as a function of V_x, then use your V_x equation form part (b) to simplify the result.

(d) Determine the range of R_L that guarantees saturation region for M_2.



a) M1 in saturation

b) $I_{DS} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS} - V_T)^2$

$\Rightarrow I_{Ref} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_1 (V_x - V_T)^2$

$\Rightarrow V_x = V_T + \sqrt{\frac{2 I_{Ref}}{K_n' \left(\frac{W}{L}\right)_1}}$

c) $I_{DS} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_2 (V_{GS} - V_T)^2$

$\Rightarrow I_L = \frac{K_n'}{2}\left(\frac{W}{L}\right)_2 (V_x - V_T)^2 = \frac{K_n'}{2}\left(\frac{W}{L}\right)_2 \left[\cancel{V_T} + \sqrt{\frac{2 I_{Ref}}{K_n' \left(\frac{W}{L}\right)_1}} - \cancel{V_T}\right]^2$

$\Rightarrow I_L = \cancel{\frac{K_n'}{2}}\left(\frac{W}{L}\right)_2 * \frac{\cancel{2} I_{Ref}}{\cancel{K_n'}\left(\frac{W}{L}\right)_1} = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} \cdot I_{Ref} = N I_{Ref}$

$\Rightarrow I_L = N I_{Ref}$

d) In the worst case: $V_{DS_2} = V_{GS_2} - V_T \Rightarrow V_{DD} - I_L R_L = V_x - V_T$

$\Rightarrow R_{L,max} = \frac{V_{DD} - V_x + V_T}{I_L} = \frac{V_{DD} - V_x + V_T}{N I_{Ref}}$

$\Rightarrow R_{L,max} = \frac{1}{N I_{Ref}} \cdot \left[V_{DD} - \sqrt{\frac{2 I_{Ref}}{K_n' \left(\frac{W}{L}\right)_1}}\right]$