#18 Gravitational Potential Energy Post-class

Due: 11:00am on Wednesday, October 3, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Where's the Energy?

Learning Goal:

To understand how to apply the law of conservation of energy to situations with and without nonconservative forces acting.

The law of conservation of energy states the following:
In an isolated system the total energy remains constant.
If the objects within the system interact through gravitational and elastic forces only, then the total *mechanical* energy is conserved.

The mechanical energy of a system is defined as the sum of *kinetic energy* K and *potential energy* U. For such systems where no forces other than the gravitational and elastic forces do work, the law of conservation of energy can be written as

$$K_i + U_i = K_f + U_f,$$

where the quantities with subscript "i" refer to the "initial" moment and those with subscript "f" refer to the final moment. A wise choice of initial and final moments, which is not always obvious, may significantly simplify the solution.

The kinetic energy of an object that has mass m and velocity v is given by

$$K = \frac{1}{2}mv^2.$$

Potential energy, instead, has many forms. The two forms that you will be dealing with most often in this chapter are the *gravitational* and *elastic* potential energy. Gravitational potential energy is the energy possessed by elevated objects. For small heights, it can be found as

$$U_g = mgh,$$

where m is the mass of the object, g is the acceleration due to gravity, and h is the elevation of the object above the *zero level*. The zero level is the elevation at which the gravitational potential energy is assumed to be (you guessed it) zero. The choice of the zero level is dictated by convenience; typically (but not necessarily), it is selected to coincide with the lowest position of the object during the motion explored in the problem.

Elastic potential energy is associated with stretched or compressed elastic objects such as springs. For a spring with a force constant k, stretched or compressed a distance x, the associated elastic potential energy is

$$U_e = \frac{1}{2}kx^2.$$

When all three types of energy change, the law of conservation of energy for an object of mass m can be written as

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2.$$

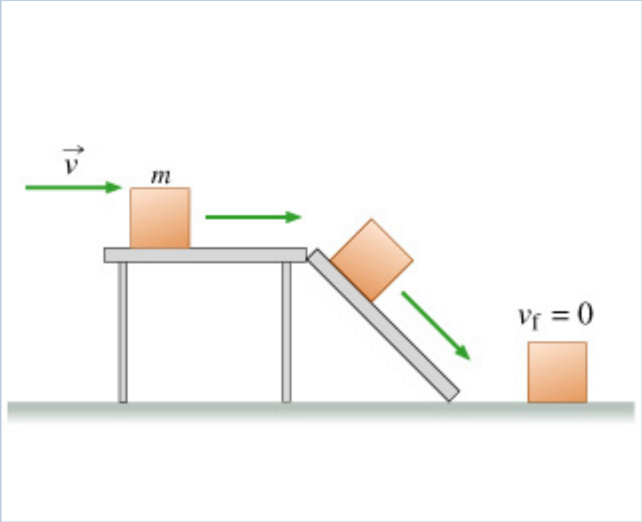
The gravitational force and the elastic force are two examples of conservative forces. What if nonconservative forces, such as friction, also act within the system? In that case, the total mechanical energy would change. The law of conservation of energy is then written as

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 + W_{nc} = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2,$$

where W_{nc} represents the work done by the nonconservative forces acting on the object between the initial and the final moments. The work W_{nc} is usually *negative*; that is, the nonconservative forces tend to decrease, or *dissipate*, the mechanical energy of the system.

In this problem, we will consider the following situation as depicted in the diagram : A block of mass m slides at a speed v along a horizontal, smooth table. It next slides down a smooth ramp, descending a height h, and then slides along a horizontal rough floor, stopping eventually. Assume that the block slides slowly enough so that it does not lose contact with the supporting surfaces (table, ramp, or floor).

You will analyze the motion of the block at different moments using the law of conservation of energy.



Part A

Which word in the statement of this problem allows you to assume that the table is frictionless?

ANSWER:

- straight
- smooth
- horizontal

Correct

Although there are no truly "frictionless" surfaces, sometimes friction is small enough to be neglected. The word "smooth" often describes such low-friction surfaces. Can you deduce what the word "rough" means?

Part B

Suppose the potential energy of the block at the table is given by $mgh/3$. This implies that the chosen zero level of potential energy is __________.

Hint 1. Definition of U

Gravitational potential energy is given by

$$U_{\rm g} = mgh,$$

where h is the height relative to the zero level. Note that $h > 0$ when the object is *above* the chosen zero level; $h < 0$ when the object is *below* the chosen zero level.

ANSWER:

- a distance $h/3$ above the floor
- a distance $h/3$ below the floor
- a distance $2h/3$ above the floor
- a distance $2h/3$ below the floor
- on the floor

Correct

Part C

If the zero level is a distance $2h/3$ above the floor, what is the potential energy U of the block on the floor?

Express your answer in terms of some or all the variables m, v, and h and any appropriate constants.

ANSWER:

U = $-\frac{2h}{3}mg$

Correct

Part D

Considering that the potential energy of the block at the table is $mgh/3$ and that on the floor is $-2mgh/3$, what is the *change* in potential energy ΔU of the block if it is moved from the table to the floor?

Express your answer in terms of some or all the variables m, v, and h and any appropriate constants.

Hint 1. Definition of ΔU

By definition, the change in potential energy is given by \Delta U = U_{\rm f} - U_{\rm i}. In general, *change* is always defined as the "final" quantity minus the "initial" one.

ANSWER:

ΔU = $-mgh$

Correct

As you may have realized, this choice of the zero level was legitimate but not very convenient. Typically, in such problems, the zero level is assumed to be on the floor. In solving this problem, we will assume just that: *the zero level of potential energy is on the floor*.

Part E

Which form of the law of conservation of energy describes the motion of the block when it slides from the top of the table to the bottom of the ramp?

Hint 1. How to approach the problem

Think about these questions:

- Are there any nonconservative forces acting on the block during this part of the trip?
- Are there any objects involved that can store elastic potential energy?
- Is the block changing its height?
- Is the block changing its speed?

ANSWER:

- $\frac{1}{2}mv_{\rm i}^2+mgh_{\rm i}+W_{\rm nc}=\frac{1}{2}mv_{\rm f}^2+mgh_{\rm f}$
- \frac{1}{2}mv_{\rm i}^2+\frac{1}{2}kx_{\rm i}^2=\frac{1}{2}mv_{\rm f}^2+\frac{1}{2}kx_{\rm f}^2
- $\frac{1}{2}mv_{\rm i}^2+mgh_{\rm i}=mgh_{\rm f}+\frac{1}{2}kx_{\rm f}^2$
- \frac{1}{2}mv_{\rm i}^2+mgh_{\rm i}=\frac{1}{2}mv_{\rm f}^2+mgh_{\rm f}
- $\frac{1}{2}mv_{\rm i}^2+mgh_{\rm i}+\frac{1}{2}kx_{\rm i}^2+W_{\rm nc}=\frac{1}{2}mv_{\rm f}^2+mgh_{\rm f}+\frac{1}{2}kx_{\rm f}^2$

Correct

Part F

As the block slides down the ramp, what happens to its kinetic energy K, potential energy U, and total mechanical energy E?

ANSWER:

- K decreases; U increases; E stays the same
- K decreases; U increases; E increases
- K increases; U increases; E increases
- K increases; U decreases; E stays the same

Correct

Part G

Using conservation of energy, find the speed v_b of the block at the bottom of the ramp.

Express your answer in terms of some or all the variables m, v, and h and any appropriate constants.

Hint 1. How to approach the problem

Use the equation for the law of conservation of energy that describes the motion of the block as it slides down the ramp. Then substitute in all known values and solve for the unknown.

ANSWER:

v_b = \sqrt{v^{2}+2gh}

Correct

Part H

Which form of the law of conservation of energy describes the motion of the block as it slides on the floor from the bottom of the ramp to the moment it stops?

Hint 1. How to approach the problem

Think about these questions:

- Are there any nonconservative forces acting on the block during this part of the trip?
- Are there any objects involved that can store elastic potential energy?
- Is the block changing its height?
- Is the block changing its speed?

ANSWER:

- $\frac{1}{2}mv_{\rm i}^2+mgh_{\rm i}+W_{\rm nc}=\frac{1}{2}mv_{\rm f}^2+mgh_{\rm f}$
- \frac{1}{2}mv_{\rm i}^2=\frac{1}{2}mv_{\rm f}^2
- \frac{1}{2}mv_{\rm i}^2+W_{\rm nc}=\frac{1}{2}mv_{\rm f}^2
- \frac{1}{2}mv_{\rm i}^2+mgh_{\rm i}=\frac{1}{2}mv_{\rm f}^2+mgh_{\rm f}
- $\frac{1}{2}mv_{\rm i}^2+mgh_{\rm i}+\frac{1}{2}kx_{\rm i}^2+W_{\rm nc}=\frac{1}{2}mv_{\rm f}^2+mgh_{\rm f}+\frac{1}{2}kx_{\rm f}^2$

Correct

Part I

As the block slides across the floor, what happens to its kinetic energy K, potential energy U, and total mechanical energy E?

ANSWER:

- K decreases; U increases; E decreases
- K increases; U decreases; E decreases
- K decreases; U stays the same; E decreases
- K increases; U stays the same; E decreases
- K decreases; U increases; E stays the same
- K increases; U decreases; E stays the same

Correct

Part J

What force is responsible for the decrease in the mechanical energy of the block?

ANSWER:

- tension
- gravity
- friction
- normal force

Correct

Part K

Find the amount of energy E dissipated by friction by the time the block stops.

Express your answer in terms of some or all the variables m, v, and h and any appropriate constants.

Hint 1. How to approach the problem

Use the equation for the law of conservation of energy that you selected as the most appropriate for the block sliding on the floor. Then substitute in all known values and solve for the unknown. You will need to use the value for v_b that you found earlier in Part G, as your initial speed.

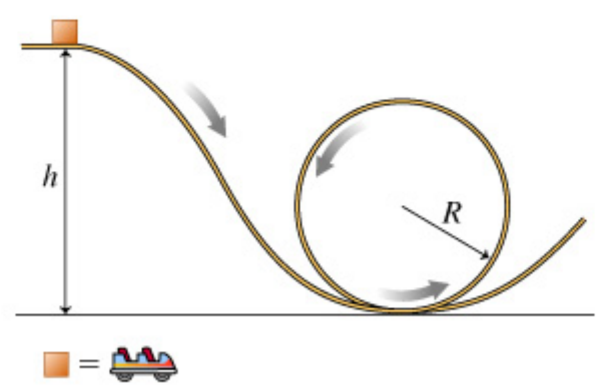
ANSWER:

E = \frac{1}{2}mv^{2}+mgh

Correct

Loop the Loop

A roller coaster car may be approximated by a block of mass m. The car, which starts from rest, is released at a height h above the ground and slides along a frictionless track. The car encounters a loop of radius R, as shown. Assume that the initial height h is great enough so that the car never loses contact with the track.



Part A

Find an expression for the kinetic energy of the car at the top of the loop.

Express the kinetic energy in terms of m, g, h, and R.

Hint 1. Find the potential energy at the top of the loop

What is the potential energy of the car when it is at the top of the loop? Define the gravitational potential energy to be zero at $h = 0$.

Express your answer in terms of R and other given quantities.

ANSWER:

U_top = m*g*(2*R)

ANSWER:

K = mg\left(h-2R\right)

Correct

Part B

Find the minimum initial height h at which the car can be released that still allows the car to stay in contact with the track at the top of the loop.

Express the minimum height in terms of R.

Hint 1. How to approach this part

Meaning of "stay in contact"

For the car to *just* stay in contact through the loop, without falling, the normal force that acts on the car when it's at the top of the loop must be zero (i.e., $N = 0$).

Find the velocity at the top such that the remaining force on the car i.e. its weight provides the necessary centripetal acceleration. If the velocity were any greater, you would additionally require some force from the track to provide the necessary centripetal acceleration. If the velocity were any less, the car would fall off the track.

Use the above described condition to find the velocity and then the result from the above part to find the required height.

Hint 2. Acceleration at the top of the loop

Assuming that the speed of the car at the top of the loop is v_{top}, and that the car stays on the track, find the acceleration of the car. Take the positive *y* direction to be upward.

Express your answer in terms of v_{top} and any other quantities given in the problem introduction.

ANSWER:

a_top = -v_top^2/R

Hint 3. Normal force at the top of the loop

Suppose the car stays on the track and has speed v_{top} at the top of the loop. Use Newton's 2nd law to find an expression for N, the magnitude of the normal force that the loop exerts on the car when the car is at the top of the loop.

Express your answer in terms of m, g, R, and v_{top}.

Hint 1. Find the sum of forces at the top of the loop

Find the sum of the forces acting on the car at the top of the loop. Remember that the positive *y* direction is *upward*.

Express your answer in terms of N, m, and g .

ANSWER:

\sum F_{\rm top} = -N-m*g

ANSWER:

N = m*(v_top^2/R-g)

Hint 4. Solving for h

The requirement to stay in contact results in an expression for v_{top}^2 in terms of R and g. Substitute this into your expression for kinetic energy, found in Part A, to determine a relation between h and R.

ANSWER:

h_min = \frac{5R}{2}

Correct

For h > 2.5 \; R the car will still complete the loop, though it will require some normal reaction even at the very top.

For h < R the car will just oscillate. Do you see this?

For R < h < 2.5 \; R, the cart will lose contact with the track at some earlier point. That is why roller coasters must have a *lot* of safety features. If you like, you can check that the angle at which the cart loses contact with the track is given by \theta = \arcsin\left(\frac{2}{3} \left(\frac{h}{R} - 1\right)\right). Where θ is the angle measured counterclockwise from the horizontal positive *x*-axis, where the origin of the *x*-axis is at the center of the loop.

± Graphing Gravitational Potential Energy

A 1.00 kg ball is thrown directly upward with an initial speed of 16.0 $\mathrm{m/s}$.

A graph of the ball's gravitational potential energy vs. height, U_{\rm g}(h), for an *arbitrary* initial velocity is given in Part A. The zero point of gravitational potential energy is located at the height at which the ball leaves the thrower's hand.
For this problem, take g=10.0\;\rm {m/s^2} as the acceleration due to gravity.

Part A

Draw a line on the graph representing the total energy E of the ball.

Hint 1. How to approach the problem

The total energy is the sum of the kinetic energy and potential energy. You can compute the total energy at any point in the ball's trajectory, but the simplest method is to add the initial kinetic and potential energies just as the ball is thrown.

Hint 2. Find the initial kinetic energy

When the ball first leaves the thrower’s hand, what is its kinetic energy K?

ANSWER:

K = 128 J

Hint 3. Find the initial potential energy

What is the potential energy U_{\rm g}(0) of the ball when it first leaves the thrower's hand?

ANSWER:

U_{\rm g}(0) = 0 J

Hint 4. Shape of the total energy graph

As the ball ascends, does its total energy increase, decrease, or stay the same?

ANSWER:

- increase
- decrease
- stay the same

ANSWER:

Correct

Part B

Using the graph, determine the maximum height reached by the ball.

Express your answer to one decimal place.

Hint 1. Maximum height

The ball reaches its maximum height when its velocity (and therefore kinetic energy) is zero, so all of its energy is potential. This occurs at the height at which the total energy and potential energy graphs intersect. The ball does not have enough energy to rise above this point on the potential energy graph.

ANSWER:

12.8 m

Correct

The ball reaches its maximum height when its velocity (and therefore kinetic energy) is zero, so all of its energy is potential. This occurs at the height at which the total energy and potential energy graphs intersect.

Part C

Draw a new gravitational potential energy vs. height graph to represent the gravitational potential energy if the ball had a mass of 2.00 kg. The graph for a 1.00-kg ball with an *arbitrary* initial velocity is provided again as a reference.

Take g=10.0\;\rm {m/s^2} as the acceleration due to gravity.

Hint 1. Slope

The gravitational potential energy is defined by

U_{\rm g}(h) = mgh.

In a graph of potential energy vs. height, mg is the slope.

Hint 2. Determine the new gravitational potential energy

What is the gravitational potential energy for a 2.00-kg ball at a height of h = 5.00\; \rm m?

Take g=10.0\;\rm {m/s^2} as the acceleration due to gravity and express your answer to three decimal places.

ANSWER:

U_{\rm g}(5 {\rm m}) = 100 J

ANSWER:

Correct

For a ball with twice the mass, you should expect the plot of potential energy vs. height to have twice the slope.

Score Summary:

Your score on this assignment is 103.8%.
You received 31.13 out of a possible total of 30 points.