Phys. 161 Ch. 18 HW

62.

$$\begin{aligned} T_1 &= 300\,\mathrm{K} \quad & T_2 &= 490\,\mathrm{K} \quad & T_3 &= T_1 \\ & & P_2 &= 1\,\mathrm{atm} & P_3 &= ? \\ & & n_2 &= ? & n_3 &= n_2 \end{aligned}$$

$$V = 1.50\,\mathrm{L} = 1.50\,(0.1\,\mathrm{m})^3 = 0.00150\,\mathrm{m}^3$$

a. After the gas is warmed to $T_2 = 490\,\mathrm{K}$, the amount left in the flask is $n_2 = {}^{P_2 V}/_{R T_2}$. When this quantity of gas is returned to $T_1 = 300\,\mathrm{K}$ without letting more gas in, the pressure is

$$P_3 = \frac{n_2 R T_1}{V} = \frac{P_2 T_1}{T_2} = (1\,\mathrm{atm})\,\frac{300\,\mathrm{K}}{490\,\mathrm{K}} = \boxed{0.612\,\mathrm{atm}} = 6.20 \times 10^4\,\mathrm{Pa}$$

b. The mass of the gas left in the flask is

$$M n_2 = M\,\frac{P_2 V}{R T_2} = (30.1\,\mathrm{g/mol})\,\frac{(1.013 \times 10^5\,\mathrm{Pa})\,(0.00150\,\mathrm{m}^3)}{(8.314\,\mathrm{J/mol \cdot K})\,(490\,\mathrm{K})} = \boxed{1.12\,\mathrm{g}}.$$

64.

$$\begin{aligned} V &= \pi r^2 h \\ r &= 10\,\mathrm{cm} = 0.1\,\mathrm{m} \\ P &= 0.500\,\mathrm{atm} = 0.500\,\mathrm{atm}\,\frac{1.013 \times 10^5\,\mathrm{Pa}}{1\,\mathrm{atm}} = 5.065 \times 10^4\,\mathrm{Pa} \\ T &= 20^\circ\mathrm{C} = 293\,\mathrm{K} \\ n &= 1.80\,\mathrm{mol} \end{aligned}$$

a.

$$m\,g = P A = \pi r^2 P$$

$$m_{\mathrm{piston}} = \pi\,\frac{r^2 P}{g} = \pi\,\frac{(0.1\,\mathrm{m})^2\,(5.065 \times 10^4\,\mathrm{Pa})}{9.80\,\mathrm{m/s^2}} = \boxed{162\,\mathrm{kg}}$$

b.

$$n R T = P V = \pi r^2 h P$$

$$h = \frac{n R T}{\pi r^2 P} = \frac{(1.80\,\mathrm{mol})\,(8.314\,\mathrm{J/mol \cdot K})\,(293\,\mathrm{K})}{\pi\,(0.1\,\mathrm{m})^2\,(5.065 \times 10^4\,\mathrm{Pa})} = \boxed{2.76\,\mathrm{m}}$$

76. **a.**

$$v_{\mathrm{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\,(1.381 \times 10^{-23}\,\mathrm{J/K})\,(5800\,\mathrm{K})}{1.67 \times 10^{-27}\,\mathrm{kg}}} = 11{,}200\,\mathrm{m/s} = \boxed{12.0\,\mathrm{km/s}}.$$

b.

$$v_{\mathrm{escape}} = \sqrt{2\,\frac{GM}{R}} = \sqrt{2\,\frac{(6.674 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2})\,(1.99 \times 10^{30}\,\mathrm{kg})}{6.96 \times 10^8\,\mathrm{m}}} = 6.18 \times 10^5\,\mathrm{m/s} = \boxed{618\,\mathrm{km/s}}.$$

c. Hydrogen cannot escape from the Sun in appreciable quantities because the typical speed of a hydrogen atom on the Sun is much less than the escape velocity from the Sun. Possibly, *no* hydrogen atoms can escape from the Sun, because a hydrogen atom is 1728 orders of magnitude less likely to have the escape velocity than to have the r.m.s. velocity, and there are surely fewer than 10^{1728} hydrogen atoms in the Sun. But, it is possible in principle even if it is very unlikely.

82. **a.** A diatomic molecule has two rotational degrees of freedom, so by equipartition of energy, the rotational kinetic energy per molecule is $\frac{2}{2} kT = kT$, and for n moles the total rotational kinetic energy is

$$n N_A k T = n R T = (1.00\,\mathrm{mol})\,(8.314\,\mathrm{J/mol \cdot K})\,(300\,\mathrm{K}) = \boxed{2.49(4)\,\mathrm{kJ}}.$$

b. The moment of inertia of one atom about the center of the molecule is $I_1 = m\,r^2 = m\,(^d/_2)^2$, where d is the distance between the atoms. The moment of inertia of the molecule is then

$$I = 2\,I_1 = m\,\frac{d^2}{2} = \frac{M\,d^2}{2\,N_A} = \frac{(16.0\,\mathrm{g/mol})\,(1.21\times 10^{-10}\,\mathrm{m})^2}{2\,(6.022\times 10^{23}\,1/\mathrm{mol})} = 1.945\times 10^{-43}\,\mathrm{g}\cdot\mathrm{m}^2 = \boxed{1.94(5)\times 10^{-46}\,\mathrm{kg}\cdot\mathrm{m}^2},$$

where M is the molar mass of the atoms.

c. The rotational kinetic energy of a rotating body is $K_{\mathrm{rot}} = \frac{1}{2}\,I\,\omega^2$. By equipartition of energy, the average rotational kinetic energy of a diatomic molecule is $(K_{\mathrm{rot}})_{\mathrm{av}} = \frac{2}{2}\,k\,T$ (since there are two rotational degrees of freedom)[1]; therefore $(\omega^2)_{\mathrm{av}} = {}^{kT}/_I$ and

$$\omega_{\mathrm{rms}} = \sqrt{(\omega^2)_{\mathrm{av}}} = \sqrt{2\,\frac{k\,T}{I}} = \sqrt{2\,\frac{(1.381\times 10^{-23}\,\mathrm{J/K})\,(300\,\mathrm{K})}{1.945\times 10^{-46}\,\mathrm{kg}\cdot\mathrm{m}^2}} = \boxed{6.53\times 10^{12}\,\mathrm{rad/s}}$$

Compared to $10{,}000\,\mathrm{rpm} = 1047\,^{\mathrm{rad}}/_{\mathrm{s}}$, this is very fast (over a billion times faster).

1. The wording of the problem is confusing because, while it makes sense to talk about the kinetic energy for just one degree of freedom, it doesn't make sense to talk about the angular velocity about just one axis. One could project the angular velocity vector onto one axis, but this would not mean that the molecule is completing fewer revolutions per second about this axis than about its axis of rotation.