Lecture 28
(Angular Momentum)

Physics 160-01 Fall 2012

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Review

- Linear
  - For const a:

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

  - Kinetic energy:

$$KE = \frac{1}{2} M v^2$$

  - Comes from:

$$W = \int \vec{F} \cdot d\vec{s}$$

  - Newton's 2nd Law

$$\sum \vec{F} = m\vec{a}$$

  - or

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

- Rotational
  - For const α:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

  - Kinetic energy:

$$KE = \frac{1}{2} I \omega^2$$

  - Comes from:

$$W = \int \tau_z \, d\theta$$

  - Newton's 2nd Law

$$\sum \vec{\tau} = I\vec{\alpha}$$

  - or

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

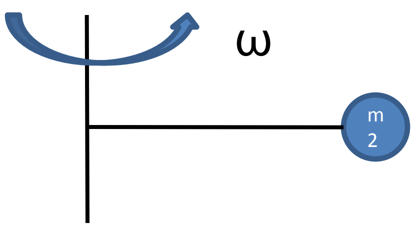
Wait, what the ^$^% is L?

- L is called the angular momentum, but what is its nature?
- Let's guess by the symmetry we have already explored:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \qquad \sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\vec{p} = m\vec{v} \qquad \vec{L} = I\vec{\omega}$$

Form 1



$$\left|\vec{L}\right| = I\omega = mr^2\omega = mr\left(r\omega\right) = rmv = rp$$

- And the direction can be gotten again from the RHR:

$$\vec{L} = \vec{r} \times \vec{p}$$

Form 2

Derivation…

- Let's derive the equation: $$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow$$

$$\frac{d}{dt}\left(\vec{L} = \vec{r} \times \vec{p}\right) \Rightarrow$$

$$\begin{aligned} \frac{d\vec{L}}{dt} &= \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \\ &= \vec{v} \times \vec{p} + \vec{r} \times \vec{F} \\ &= 0 + \vec{\tau} \end{aligned}$$

Conservation of Angular Momentum

- Remember that a system of particles which had no net outside force gave:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = 0$$

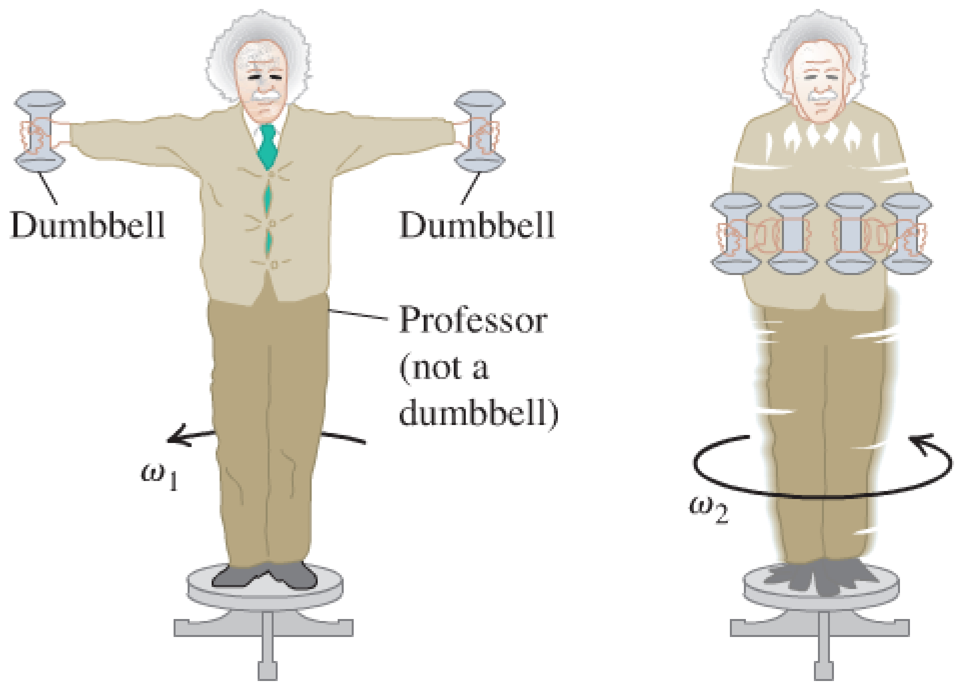
- Conservation of linear momentum.
- What if there are no net outside torques?

$$\sum \vec{\tau} = 0 = \frac{d\vec{L}}{dt}$$

- Conservation of angular momentum!

Example

- Professor on rotating stand



BEFORE

$$\vec{L}_1 = I_1\vec{\omega}_1$$

AFTER

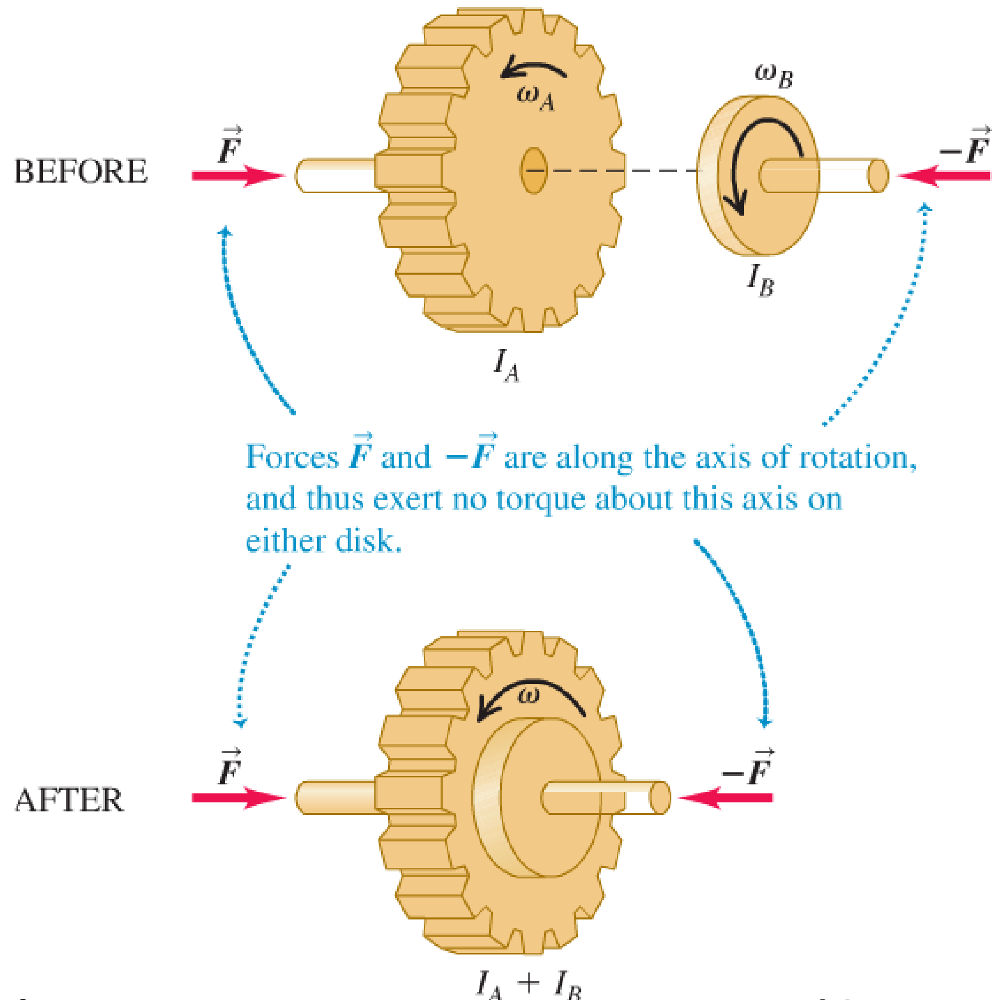
$$\vec{L}_2 = I_2\vec{\omega}_2$$

Example

- Rotational collision

$$\vec{L}_{Before} = I_A\vec{\omega}_A + I_B\vec{\omega}_B$$

$$\vec{L}_{After} = \left(I_A + I_B\right)\vec{\omega}$$



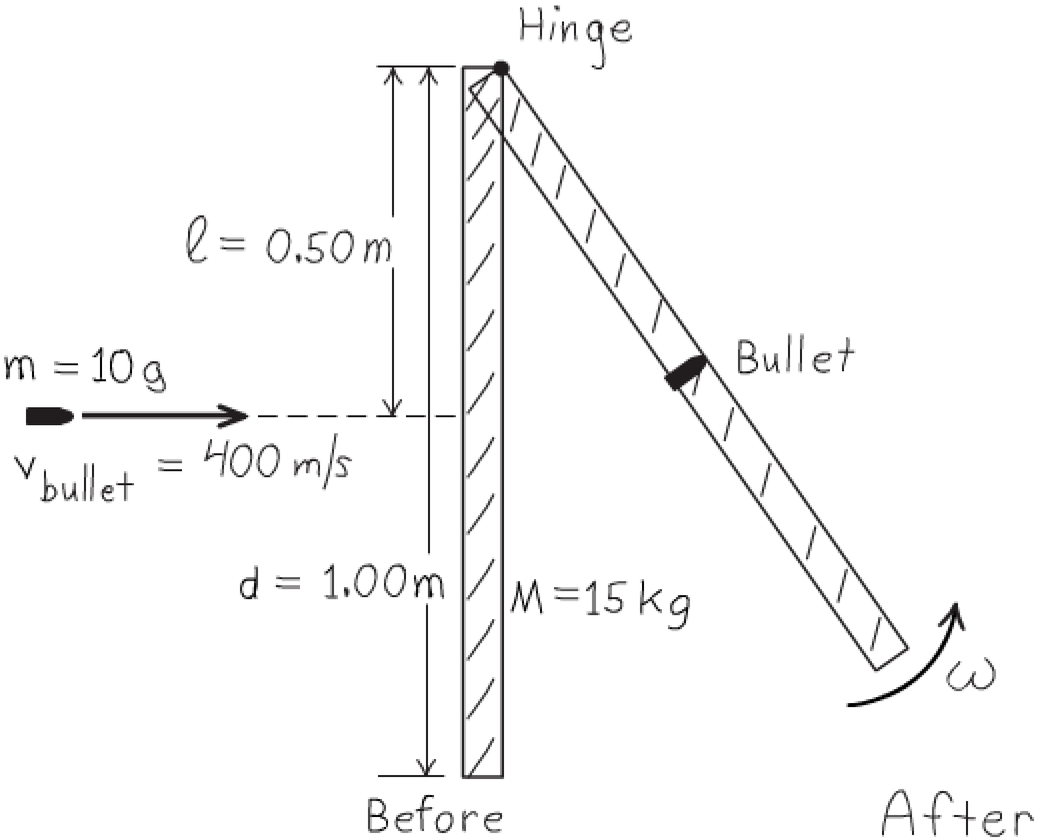
- Would you expect that kinetic energy is conserved?

Example

- Bullet hitting a door

$$\vec{L}_{Before} = \vec{r} \times \vec{p}_{bullet}$$

$$\vec{L}_{After} = \left(I_{bullet} + I_{door}\right)\vec{\omega}$$



CPS Question 28-1

- **In two situations, an object strikes a door. In one, a bullet, with initial momentum p imbeds into the door a distance d away from the hinges. In another, a basketball with the same initial momentum p strikes the door at the same distance from the hinges, but bounces back from the door. In which situation is the final angular velocity of the door the greatest?**

A) The bullet hitting the door.

B) The ball hitting the door.

C) They are both the same.

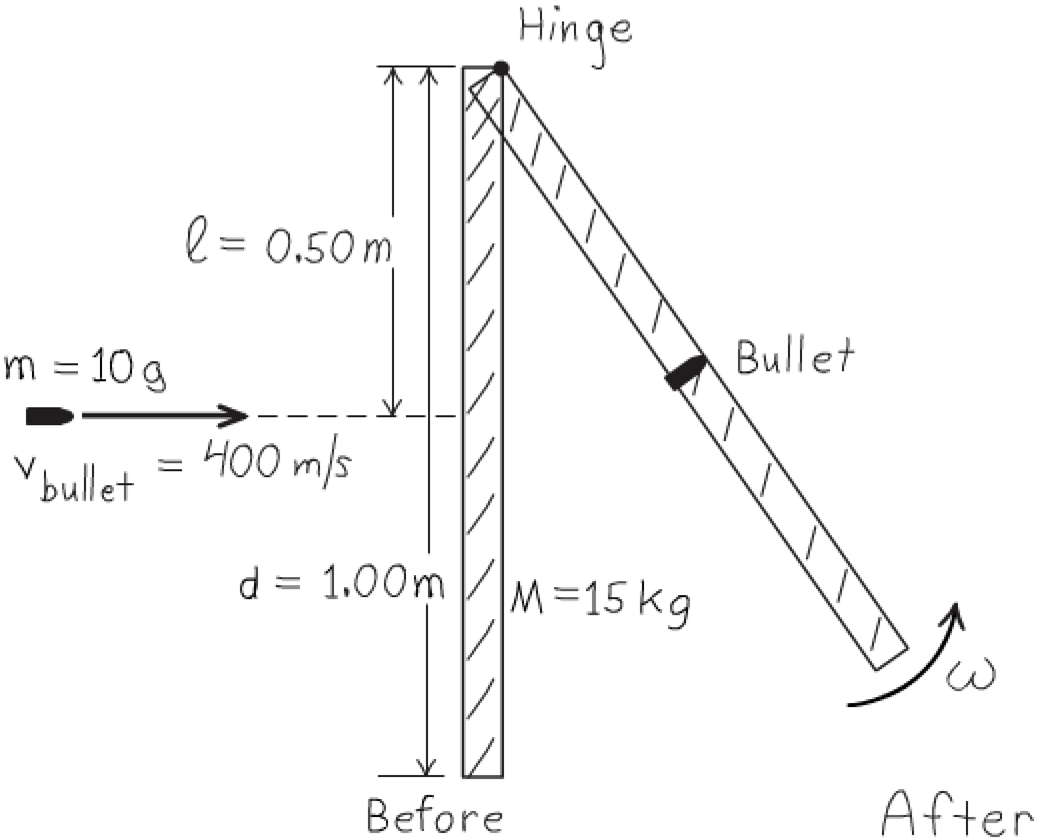
D) Not enough information to solve.

Example

- Bullet hitting a door

$$\vec{L}_{Before} = \vec{r} \times \vec{p}_{bullet}$$

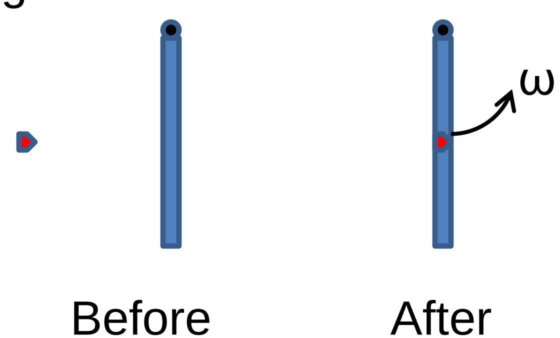
$$\vec{L}_{After} = \left(I_{bullet} + I_{door}\right)\vec{\omega}$$



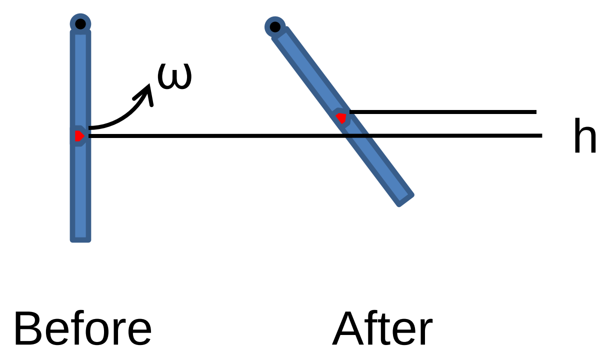
Problem 10.89

10.89. A target in a shooting gallery consists of a vertical square wooden board, 0.250 m on a side and with mass 0.750 kg, that pivots on a horizontal axis along its top edge. The board is struck face-on at its center by a bullet with mass 1.90 g that is traveling at 360 m/s and that remains embedded in the board. (a) What is the angular speed of the board just after the bullet's impact? (b) What maximum height above the equilibrium position does the center of the board reach before starting to swing down again? (c) What minimum bullet speed would be required for the board to swing all the way over after impact?

Angular Momentum Conserved



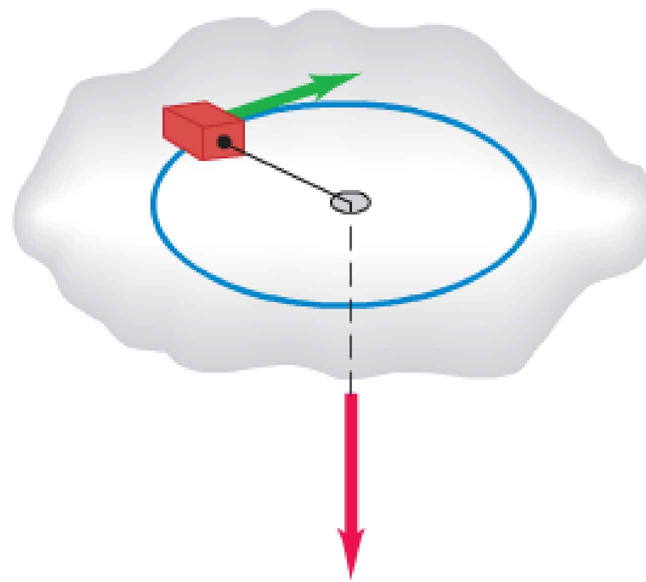
Energy Conserved



Problem 10.40

10.40. A small block on a frictionless, horizontal surface has a mass of 0.0250 kg. It is attached to a massless cord passing through a hole in the surface (Fig. 10.48). The block is originally revolving at a distance of 0.300 m from the hole with an angular speed of 1.75 rad/s. The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.150 m. Model the block as a particle. (a) Is angular momentum of the block conserved? Why or why not? (b) What is the new angular speed? (c) Find the change in kinetic energy of the block. (d) How much work was done in pulling the cord?

Figure 10.48 Exercise 10.40, Problem 10.92, and Challenge Problem 10.103.



Problem 10.98

10.98. Center of Percussion. A baseball bat rests on a frictionless, horizontal surface. The bat has a length of 0.900 m, a mass of 0.800 kg, and its center of mass is 0.600 m from the handle end of the bat (Fig. 10.64). The moment of inertia of the bat about its center of mass is 0.0530 kg · m^2. The bat is struck by a baseball traveling perpendicular to the bat. The impact applies an impulse $J = \int_{t_1}^{t_2} F\,dt$ at a point a distance x from the handle end of the bat. What must x be so that the handle end of the bat remains at rest as the bat begins to move? [*Hint:* Consider the motion of the center of