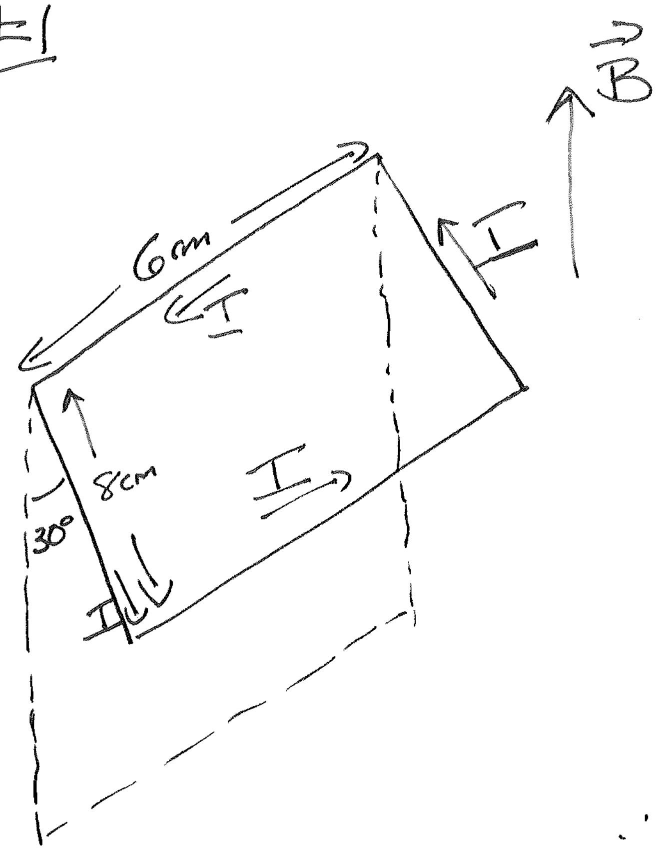
Physics 161,

HW #6

#1



Loop with .25g/cm

$I = 15A$

Find $\vec{B}$ such that Loop stays at 30°.

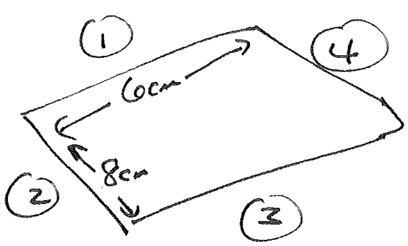
Gravity torques loop as does $\vec{B}$

When $\Sigma\vec{\tau} = 0$ Loop stays

$\therefore \vec{\tau}_g + \vec{\tau}_B = 0$

$\vec{\tau}_B = \vec{\mu} \times \vec{B}$ ⟵ Will eventually give us $\vec{B}$.

For gravity:



$M_1 = M_3 = 6cm\,(.25g/cm) = 1.5g$

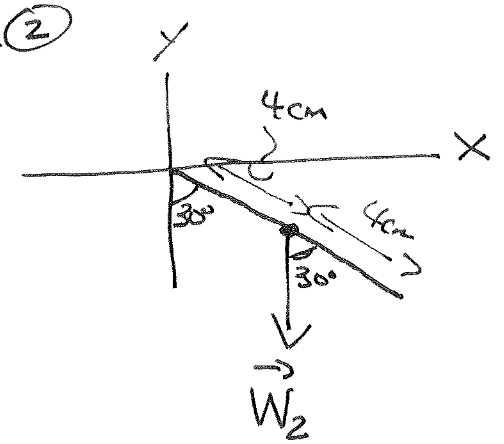
$\Rightarrow W_1 = W_3 = (.0015kg)(9.8m/s^2) = .0147N$

$M_2 = M_4 = 8cm\,(.25g/cm) = 2g$

$W_2 = W_3 = (.002kg)(9.8m/s^2) = .0196N$

Take each side separately and simplify by drawing in the x-y plane.

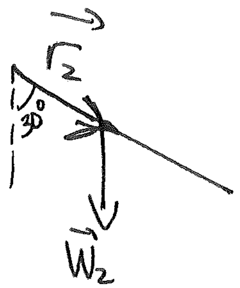
For 2



~~BAR~~ UNIFORM MASS $\Rightarrow$ Center OF gravity at the center

$\Rightarrow$ 4cm = 8cm/2

$$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \vec{\tau}_2 = \vec{r}_2 \times \vec{W}_2$$

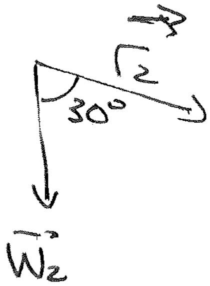


&

From origin to $\vec{W}_2$

$r_2 = 4\text{cm} = .04\text{m}$

$\Rightarrow$



$$\tau_2 = r_2 W_2 \sin 30^\circ$$

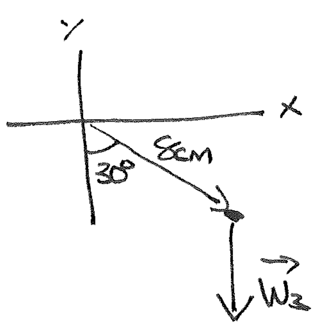
$$= (.04\text{m})(.0196\text{N}) \sin 30^\circ$$

$$= 3.92 \times 10^{-4}\ \text{N}\cdot\text{m}$$

From RHR, $\vec{\tau}_2 = 3.92 \times 10^{-4}\ \text{N}\cdot\text{m}$, x

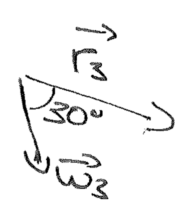
For 4, Picture Exactly THE SAME $\Rightarrow$ $\vec{\tau}_4 = 3.92 \times 10^{-4}\ \text{N}\cdot\text{m}$, x

For 3



NOTE THE Loop is going into AND out OF page HERE

$\vec{\tau}_3 = \vec{r}_3 \times \vec{W}_3$ $\quad r_3 = 8\text{cm} = .08\text{m}$



$\tau_3 = (.08\,m)(.0147\,N)\sin 30° = 5.88\times10^{-4}\,N\cdot m$

RHR $\Rightarrow$ $\vec{\tau}_3 = 5.88\times10^{-4}\,N\cdot m,\ \otimes$

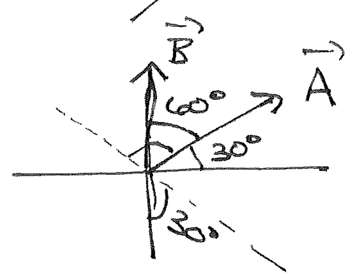
THE EASIEST IS (1) SINCE $r = 0$ (IT'S ON THE AXIS)

$\therefore\ \vec{\tau}_g = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_4 = 0 + 3.92\times10^{-4}\,N\cdot m, \otimes + 5.88\times10^{-4}\,N\cdot m, \otimes$
$+ 3.92\times10^{-4}\,N\cdot m, \otimes$

$\Rightarrow \vec{\tau}_g = 1.372\times10^{-3}\,N\cdot m,\ \otimes$

FOR MAGNETIC TORQUE, $\vec{\tau}_B = \vec{\mu}\times\vec{B}$, $\vec{\mu} = I\vec{A}$

GIVEN DIRECTION OF CURRENT:



$\vec{A}$ IS $60°$ FROM $\vec{B}$

$\Rightarrow \tau_B = IAB\sin 60° = (15A)(.06m)(.08m)B\sin60° = (.06235\,A\cdot m^2)B$

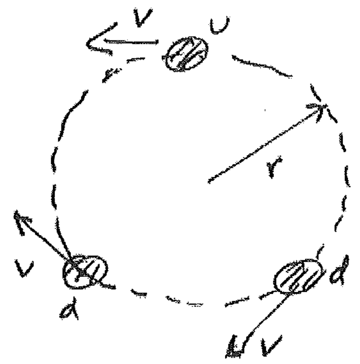
$\vec{\tau}_B = (.06235\,A\cdot m^2)B,\ \odot$

$\therefore$
$\vec{\tau}_g + \vec{\tau}_B = 0 \Rightarrow 1.372\times10^{-3}\,N\cdot m = (.06235\,A\cdot m^2)B \Rightarrow \boxed{B = 0.022T}$

$\Downarrow$

$\vec{\tau}_g = -\vec{\tau}_B$

#2



NEUTRON = 3 Quarks

u = up Quark, $+\frac{2}{3}e$

d = down Quark, $-\frac{1}{3}e$

a) DETERMINE current due TO u's CIRCULATION.

$I = \frac{dq}{dt}$ but WITH A SINGLE CHARGE, WE CAN USE $\frac{\Delta q}{\Delta t}$

u IS ROTATING WITH speed V $\Rightarrow$ For ANY cross-SECTION OF THE ORbit, A CHARGE $+\frac{2}{3}e$ PASSES THROUGH IT ONCE EVERY PERIOD, T.

$\Rightarrow \Delta t = T = \frac{2\pi r}{V} \Rightarrow \boxed{I = \frac{\frac{2}{3}e}{2\pi r/V} = \frac{eV}{3\pi r}}$

b) Find $\mu = IA$ for up Quark

$A = \pi r^2 \leftarrow$ circle $\Rightarrow \mu = IA = \frac{eV}{3\pi r}(\pi r^2) = \frac{erV}{3} = \boxed{\frac{1}{3}evr = \mu}$

c) Find μ for 3quark system. $\vec{\mu} = I\vec{A}$

IN THIS CASE $\vec{A}$ IS ALONG THE AXIS OF ROTATION given BY THE RIGHT-HAND-RULE $\Rightarrow \vec{\mu}_u = IA, \odot = \frac{1}{3}evr, \odot$

up QUARK

$$\vec{\mu}_{TOTAl} = \vec{\mu}_u + 2\vec{\mu}_d$$

↳ DUE TO DOWN QUArks.

DOWN QUArks ROTATING Clockwise $\Rightarrow \vec{A}$ IS $\otimes$

BUT $I = \dfrac{-\frac{1}{3}e}{2\pi r/V} = \dfrac{-eV}{6\pi r} \Rightarrow \vec{\mu}_d = -\left(\dfrac{eV}{6\pi r}\right)\pi r^2, \otimes = \dfrac{eVr}{6}, \odot$

$$\Rightarrow \vec{\mu}_{TOTAl} = \tfrac{1}{3}evr, \odot + 2\left(\tfrac{1}{6}evr\right), \odot = \tfrac{2}{3}evr, \odot$$

$$\Rightarrow \boxed{\mu_{TOTAl} = \tfrac{2}{3}evr}$$

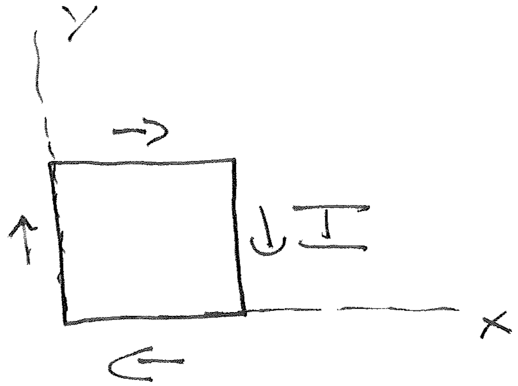
d) Find V SUCH THAT $\mu_{TOTAl} = 9.66\times10^{-27}\,A\cdot m^2$, $r = 1.2\times10^{-15}\,m$

$$\Rightarrow 9.66\times10^{-27}\,A\cdot m^2 = \tfrac{2}{3}(1.6\times10^{-19}\,C)\,V\,(1.2\times10^{-15}\,m)$$

$$\Rightarrow \boxed{V = 7.55\times10^{7}\,m/s}$$ ← about 25% THE SPEED OF LIGHT

UNIT: $A\cdot m^2 = C\cdot m^2/s = C\cdot(m/s)\cdot m \Rightarrow V = m/s$

#3

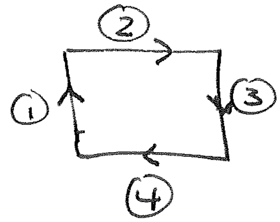


$$\vec{B} = \left(\frac{B_0 z}{L}\right)\hat{j} + \left(\frac{B_0 y}{L}\right)\hat{k}$$

FIND $\vec{F}$ FOR $L = 25\text{mm} = 2.5\times10^{-3}\text{m}$

$I = 2.5\text{A}$, $B_0 = 0.75\text{T}$

$d\vec{F} = I\,d\vec{\ell} \times \vec{B}$ so



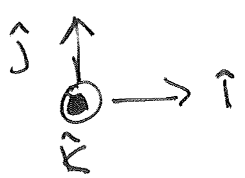
FOR 1 $d\vec{\ell}_1 = dy\,\hat{j}$ (UP, IN OTHER WORDS)

$$\therefore d\vec{F}_1 = I\,dy\,\hat{j} \times \left[\left(\frac{B_0 z}{L}\right)\hat{j} + \left(\frac{B_0 y}{L}\right)\hat{k}\right]$$

$$= \frac{I B_0 z\,dy}{L}(\hat{j}\times\hat{j}) + \frac{I B_0 y\,dy}{L}(\hat{j}\times\hat{k})$$

$\hat{j}\times\hat{j}$: $\uparrow \hat{j}, \hat{j}$ ← FOR PARALLEL VECTORS, CROSS-PRODUCT IS ZERO

$$\hat{j}\times\hat{j} = 0$$



$\hat{j}\times\hat{k} = \hat{i}$ SINCE $|\hat{j}\times\hat{k}| = (1)(1)\sin 90^\circ = 1$

AND RHR

↳ $\hat{k}$ = out of page

$$\therefore \quad d\vec{F}_1 = \frac{I B_0 y}{L}\, dy\ \hat{\imath}$$

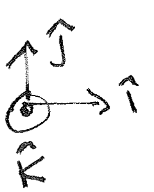
$$\Rightarrow dF_1 = \frac{I B_0 y}{L}\, dy \Rightarrow F_1 = \int_0^L \frac{I B_0}{L}\, y\, dy$$

$$= \frac{I B_0}{L}\int_0^L y\, dy = \frac{I B_0}{L}\left(\frac{y^2}{2}\Big|_0^L\right) = \frac{I B_0}{L}\left(\frac{L^2}{2}\right)$$

$$\Rightarrow \vec{F}_1 = \frac{1}{2} I L B_0 \,, \rightarrow$$

FOR (2) $\xrightarrow{d\vec{l}_2}$ $\Rightarrow d\vec{l}_2 = dx\ \hat{\imath}$

$$d\vec{F}_2 = I\, dx\ \hat{\imath} \times \vec{B} = \frac{I B_0 z\, dx}{L}(\hat{\imath}\times\hat{\jmath}) + \frac{I B_0 y}{L}\, dx\ (\hat{\imath}\times\hat{k})$$



$$\hat{\imath}\times\hat{\jmath} = \hat{k}\,, \quad \hat{\imath}\times\hat{k} = -\hat{\jmath}$$

$$\Rightarrow d\vec{F}_2 = \frac{I B_0 z\, dx}{L}\ \hat{k} - \frac{I B_0 y}{L}\, dx\ \hat{\jmath}$$

but LOOP IN PLANE $\Rightarrow Z = 0 \Rightarrow d\vec{F}_2 = -\frac{I B_0 y}{L} dx\, \hat{\jmath}$

ALong ② $y = L \Rightarrow d\vec{F}_2 = -\frac{I B_0 L}{L} dx\, \hat{j}$

$$= -I B_0\, dx\, \hat{j}$$

$$F_2 = \int_0^L -I B_0\, dx = -I B_0 \int_0^L dx = -I B_0 L$$

$$\therefore\ \vec{F}_2 = +I B_0 L, \downarrow$$

For ③ $\downarrow d\vec{\ell}_3$, $d\vec{\ell}_3 = -dy\, \hat{j}$

SAME CROSS-PRODUCTS but WITH NEGative

$$\Rightarrow d\vec{F}_3 = -\frac{I B_0 y}{L} dy\, \hat{i} \Rightarrow \vec{F}_3 = -\tfrac{1}{2} I L B_0\, \hat{i}$$

$$\Rightarrow \vec{F}_3 = \tfrac{1}{2} I L B_0, \leftarrow$$

So $\vec{F}_1$ AND $\vec{F}_3$ ARE EQUAL but opposite

For ④ $\overleftarrow{d\vec{\ell}_4}$ $d\vec{\ell}_4 = -dx\, \hat{i} \Rightarrow d\vec{F}_4 = +\frac{I B_0 y}{L} dx\, \hat{j}$

but ALong ④ $y = 0 \Rightarrow dF_4 = 0 \Rightarrow F_4 = 0$

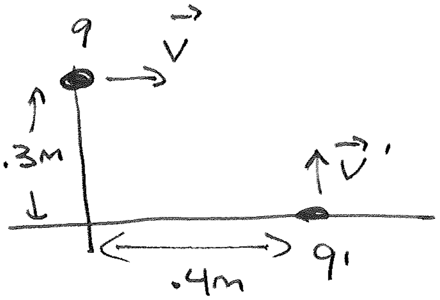
Finally, $\cancel{\vec{F}_1}^{\,0} + \vec{F}_2 + \cancel{\vec{F}_3} + \cancel{\vec{F}_4}^{\,0} = \vec{F}_2$

EQUAL but opposite

So Net Force on loop is $\boxed{\vec{F}_2 = I B_0 L, \downarrow}$

$$F_2 = (2.5A)(.75T)(2.5\times10^{-3}m) \Rightarrow \boxed{F_2 = 4.6875\times10^{-3}N}$$

#4



$q = 75\,\mu C = 75 \times 10^{-6}\,C$

$q' = -50\,\mu C = -50 \times 10^{-6}\,C$

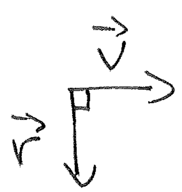
$V = 3 \times 10^{5}\,m/s, \quad V' = 6.5 \times 10^{5}\,m/s$

a) $\vec{B}_o$ at ORIGIN? BOTH q AND q' Create FIELD

$\Rightarrow \vec{B}_0 = \vec{B} + \vec{B}'$. MOVING CHARGES $\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{V} \times \vec{r}}{r^3}$

$$\vec{B}' = \frac{\mu_0}{4\pi} \frac{q\vec{V} \times \vec{r}'}{r'^3}$$

$\vec{r}$ = FROM q to ORIGIN $\Rightarrow$ $\downarrow .3m$



$\vec{V} \times \vec{r} = Vr \sin 90°$, ⊗

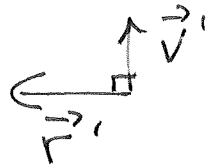
q is positive $\Rightarrow \vec{B}$ Also ⊗

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{qVr}{r^3} ⊗ = \frac{\mu_0}{4\pi} \frac{qV}{r^2}, ⊗ = \frac{(4\pi \times 10^{-7}\,T \cdot m/A)}{4\pi} \frac{(75 \times 10^{-6}\,C)(3 \times 10^{5}\,m/s)}{(.3m)^2}$$

$$= \frac{(1 \times 10^{-7}\,T \cdot m/A)(75 \times 10^{-6}\,C)(3 \times 10^{5}\,m/s)}{(.3m)^2}, ⊗$$

$$= 2.5 \times 10^{-5}\,T, ⊗$$

$\vec{r}\,'$ = FROM q' to ORigin $\Rightarrow$ $\xleftarrow[.4m]{}$



$\vec{v}\,' \times \vec{r}\,' = v' r' \sin 90°, \odot$

but q' Negative $\Rightarrow$ $\vec{B}\,' = \otimes$ Also

$$\vec{B}\,' = \frac{\mu_0}{4\pi}\frac{|q'|v'r'}{r'^3}, \otimes = \frac{\mu_0}{4\pi}\frac{|q'|v'}{r'^2}, \otimes = \frac{(1\times10^{-7}\,T\cdot m/A)(50\times10^{-6}\,C)(6.5\times10^{5}\,m/s)}{(.4m)^2}$$

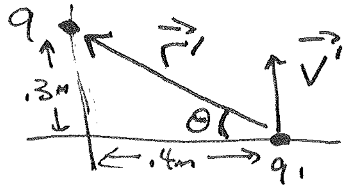
$$\Rightarrow \vec{B}\,' = 2.03125\times10^{-5}\,T, \otimes$$

$\vec{B}, \vec{B}\,'$ IN SAME DIRECTION $\Rightarrow B_0 = B + B' = (2.5\times10^{-5}\,T) + (2.03125\times10^{-5}\,T)$

$$\Rightarrow \boxed{\vec{B}_0 = 4.53125\times10^{-5}\,T = 4.53\times10^{-5}\,T, \otimes}$$

b) MAGNITUDE OF MAGNETIC FORCE THAT q' EXERTS ON q.

$\vec{F}_q = q\vec{v}\times\vec{B}\,'$ but NOW $\vec{B}\,'$ IS MAGNETIC FIELD CREATED by q' at q's LOCATION



$$r' = \sqrt{(.4m)^2 + (.3m)^2} = .5m$$

$$\theta = \tan^{-1}\left(\frac{.3}{.4}\right) = 36.87°$$

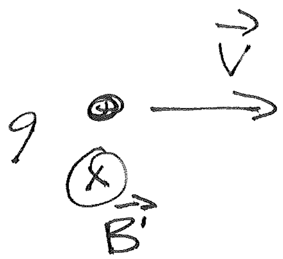
$$\vec{V}' \times \vec{r}' = V'r'\sin(90^\circ - \theta),\ \odot = V'r'\sin 53.13^\circ,\ \odot$$

$$= V'r'(.8),\ \odot$$

Again, q' Negative $\rightarrow \vec{B}' = \otimes$

$$\vec{B}' = \frac{\mu_0}{4\pi}\frac{|q'|V'\cancel{r'}(.8)}{r'^{\cancel{3}2}},\ \otimes = \frac{(1\times10^{-7}\,T\cdot m/A)(50\times10^{-6}\,C)(6.5\times10^{5}\,m/s)(.8)}{(.5\,m)^2},\ \otimes$$

At q: $\vec{B}' = 1.04\times10^{-5}\,T,\ \otimes$



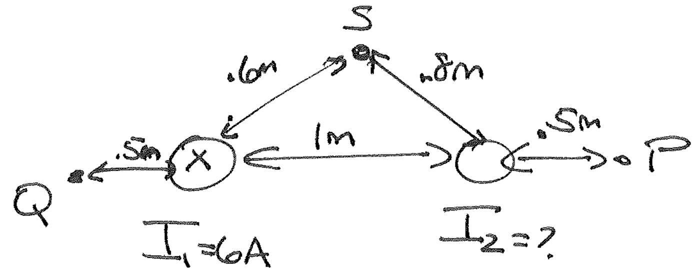
$$\vec{F}_q = q\vec{V}\times\vec{B}' \Rightarrow F_q = qVB'$$

$$F_q = (75\times10^{-6}\,C)(3\times10^{5}\,m/s)(1.04\times10^{-5}\,T)$$

$$= 2.34\times10^{-4}\,N$$

From RHR: $\boxed{\vec{F}_q = 2.34\times10^{-4}\,N,\ \uparrow}$

#5



INFINITE WIRES

$\Rightarrow B = \frac{\mu_0 I}{2\pi r} = (2\times10^{-7})\frac{I}{r}$

a) $I_2 = ?$ so $\sum \vec{B} = 0$ at P

$\sum \vec{B} = \vec{B}_1 + \vec{B}_2$ FROM RHR $\vec{B}_1 = \downarrow$ at P

$\Rightarrow \vec{B}_2$ MUST be $\uparrow$ $\Rightarrow$ $\boxed{I_2 \text{ MUST be } \odot}$

$\sum B_y = B_2 - B_1 = 0 \Rightarrow B_2 = B_1 \Rightarrow (2\times10^{-7})\frac{I_2}{r_2} = (2\times10^{-7})\frac{I_1}{r_1}$

$\Rightarrow \frac{I_2}{r_2} = \frac{I_1}{r_1}$ $r_2 = .5m$, $r_1 = 1.5m$

$\Rightarrow I_2 = \frac{r_2}{r_1} I_1 = \left(\frac{.5}{1.5}\right)(6A) \Rightarrow \boxed{I_2 = 2A}$

b) What is $\sum \vec{B}$ at Q?

FROM RHR, $\vec{B}_1 = \uparrow$, $B_2 = \downarrow$ at Q

$$\Rightarrow \sum B_y = B_1 - B_2 = \frac{(2\times10^{-7})I_1}{r_1} - \frac{(2\times10^{-7})I_2}{r_2}$$

$$= (2\times10^{-7})\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right)$$

$r_1 = .5m$, $r_2 = 1.5m$ at Q

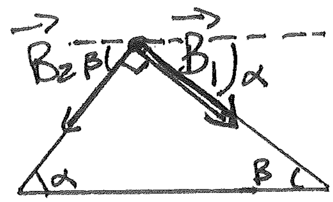
$$\Rightarrow \sum B_y = (2\times10^{-7}\,T\cdot m/A)\left(\frac{6A}{.5m} - \frac{2A}{1.5m}\right) = (2\times10^{-7}\,T\cdot m/A)\left(10.667\frac{A}{m}\right)$$

$$\Rightarrow \boxed{\sum B_y = 21.3\times10^{-7}\,T = 2.13\times10^{-6}\,T = 2.13\,\mu T}$$

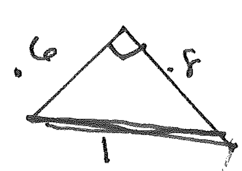
$$\boxed{\Rightarrow \vec{B} = 2.13\,\mu T, \uparrow}$$

c) What is $\sum \vec{B}$ at S?

RHR $\Rightarrow$



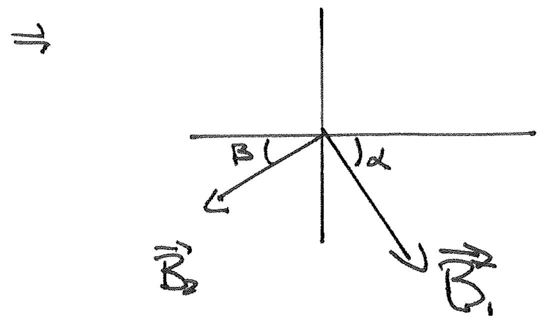
NOTICE THAT THIS PROBLEM IS NOT THAT HARD SINCE



$.6^2 + .8^2 = 1$

$\Rightarrow$ RIGHT TRIANGLE

$\Rightarrow$



$$B_1 = \frac{(2\times10^{-7})I_1}{r_1} = \frac{(2\times10^{-7}\,T\cdot m/A)(6A)}{.6m}$$

$$= 20\times10^{-7}\,T$$

$$B_2 = \frac{(2\times10^{-7})I_2}{r_2} = \frac{(2\times10^{-7}\,T\cdot m/A)(2A)}{.8m}$$

$$= 5\times10^{-7}\,T$$

$$\cos\alpha = \frac{.6}{1} = .6 \Rightarrow \alpha = \cos^{-1}(.6) = 53.13°$$

OR Better yet: $\cos\alpha = .6$, $\sin\alpha = \sqrt{1-.6^2} = \sqrt{.64} = .8$

$$\cos\beta = \frac{.8}{1} = .8 \Rightarrow \beta = \cos^{-1}(.8) = 36.87°$$

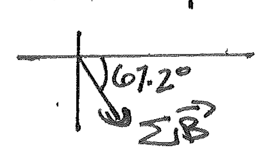
OR Better: $\cos\beta = .8$, $\sin\beta = .6$

$$\Sigma B_x = B_1\cos\alpha - B_2\cos\beta = (20\times10^{-7}T)(.6) - (5\times10^{-7}T)(.8) = 8\times10^{-7}T$$

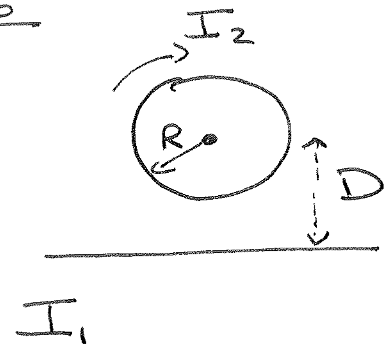
$$\Sigma B_y = B_1\sin\alpha + B_2\sin\beta = (20\times10^{-7}T)(.8) + (5\times10^{-7}T)(.6) = 19\times10^{-7}T$$

$$\Rightarrow \Sigma B = \sqrt{(8\times10^{-7}T)^2 + (19\times10^{-7}T)^2} = 20.6\times10^{-7}T = 2.06\times10^{-6}T = 2.06\,\mu T$$

BY THE WAY $\theta = \tan^{-1}\left(\frac{19}{8}\right) = 67.2°$



#6



What Direction is I_1 if $\vec{B}_{TOTAL} = 0$ at Center

For Current loop, $\vec{B}_2 = \otimes$ at Center For Clockwise I_2

$\vec{B}_{TOTAL} = \vec{B}_1 + \vec{B}_2$. $B_{TOTAL} = 0 \Rightarrow \vec{B}_1 = -\vec{B}_2$ so Field from wire needs to be $\odot$. For long wire, $\boxed{I_1 = \longrightarrow}$ gives $\odot$ $\vec{B}_1$ above wire

b) Current Loop: $B_2 = \frac{\mu_0 I_2}{2R}$ Long wire: $B_1 = \frac{\mu_0 I_1}{2\pi D}$

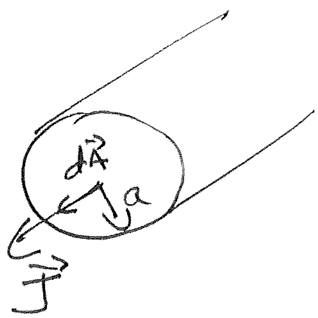
$B_1 = B_2$ For $B_{TOTAL} = 0$ $\Rightarrow \frac{\cancel{\mu_0} I_2}{\cancel{2}R} = \frac{\cancel{\mu_0} I_1}{\cancel{2}\pi D}$

$$\Rightarrow \boxed{I_1 = \pi \left(\frac{D}{R}\right) I_2}$$

c) $\boxed{I_1 = \pi \left(\frac{2\,cm}{0.5\,cm}\right)(3A) = (12\pi)A = 37.7A}$

#7

$$\vec{J} = \begin{cases} \frac{3I_0}{\pi a^2}\left(1-\frac{r}{a}\right)\hat{k} & r \le a \\ 0 & r \ge a \end{cases}$$



a) SHOW TOTAL Current is I_0

FOR NON-UNIFORM Current $J = \frac{I}{A}$ Becomes $J = \frac{dI}{dA}$

$\Rightarrow dI = J\,dA$ FOR A circle $dA = 2\pi r\,dr$

&

AND SYMMETRICAL J

$\Rightarrow dI = J\,2\pi r\,dr \Rightarrow I_{encl} = \int J\,2\pi r\,dr$

For total current in wire: $I_{encl} = \int_0^a J\,2\pi r\,dr$

For $0 < r < a$ $J = \frac{3I_0}{\pi a^2}\left(1-\frac{r}{a}\right)$

$$\Rightarrow I_{encl} = \int_0^a \frac{3I_o}{\pi a^2}\left(1-\frac{r}{a}\right) 2\pi r\,dr = \frac{3I_o}{\pi a^2} 2\pi \int_0^a \left(r - \frac{r^2}{a}\right) dr$$

$$= \frac{6I_o}{a^2}\int_0^a \left(r - \frac{r^2}{a}\right) dr = \frac{6I_o}{a^2}\left[\frac{r^2}{2} - \frac{r^3}{3a}\right]\Big|_0^a$$

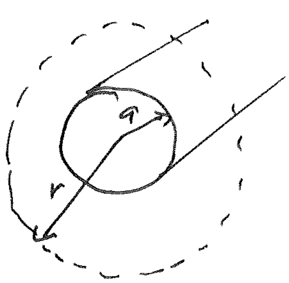
$$= \frac{6I_o}{a^2}\left[\frac{a^2}{2} - \frac{a^3}{3a}\right] = \frac{6I_o}{a^2}\left[\frac{a^2}{2} - \frac{a^2}{3}\right] = \underline{\underline{\frac{6I_o}{a^2}\left[\frac{a^2}{6}\right]}}$$

$$\Rightarrow \underline{\underline{I_{encl} = I_o}}$$

b) USE Ampere's LAW: FOR B outside WIRE

Ampere's LAw: $\oint \vec{B}\cdot d\vec{\ell} = \mu_o I_{encl}$

Since $\vec{J}$ is symmetric $\oint \vec{B}\cdot d\vec{\ell} = B(2\pi r)$



outside wire $I_{encl} = I_o$

$$\Rightarrow B(2\pi r) = \mu_o I_o \Rightarrow \boxed{B = \frac{\mu_o I_o}{2\pi r}}$$

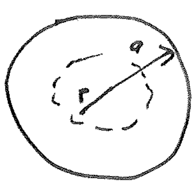
SAME AS INFINITE WIRE

c) Find I_{encl} for $r \le a$

$$I_{encl} = \int_0^r J(2\pi r)\,dr = \int_0^r \frac{3I_0}{\pi a^2}\left(1-\frac{r}{a}\right)2\pi r\,dr = \frac{6I_0}{a^2}\int_0^r \left(r-\frac{r^2}{a}\right)dr$$

$$= \frac{6I_0}{a^2}\left[\frac{r^2}{2}-\frac{r^3}{3a}\right]\Big|_0^r \Rightarrow \boxed{I_{encl} = \frac{6I_0}{a^2}\, r^2\left[\frac{1}{2}-\frac{r}{3a}\right]}$$

d)



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

$$\oint \vec{B}\cdot d\vec{l} = B(2\pi r) \text{ still}$$

$$\Rightarrow B(2\pi r) = \mu_0 \frac{6I_0}{a^2} r^2\left[\frac{1}{2}-\frac{r}{3a}\right]$$

$$\Rightarrow B = \frac{\mu_0}{2\pi}\frac{6I_0}{a^2}\frac{r^2}{r}\left[\frac{1}{2}-\frac{r}{3a}\right] \Rightarrow \boxed{B = \frac{\mu_0}{2\pi}\frac{6I_0 r}{a^2}\left[\frac{1}{2}-\frac{r}{3a}\right]}$$

at $r=a$ $\quad B = \frac{\mu_0}{2\pi}\frac{6I_0 a}{a^2}\left[\frac{1}{2}-\frac{a}{3a}\right] = \frac{\mu_0}{2\pi}\frac{6I_0}{a}\left[\frac{1}{6}\right] = \frac{\mu_0 I_0}{2\pi a}$

$B = \frac{\mu_0 I_0}{2\pi r} = \frac{\mu_0 I_0}{2\pi a}$ So THE SAME