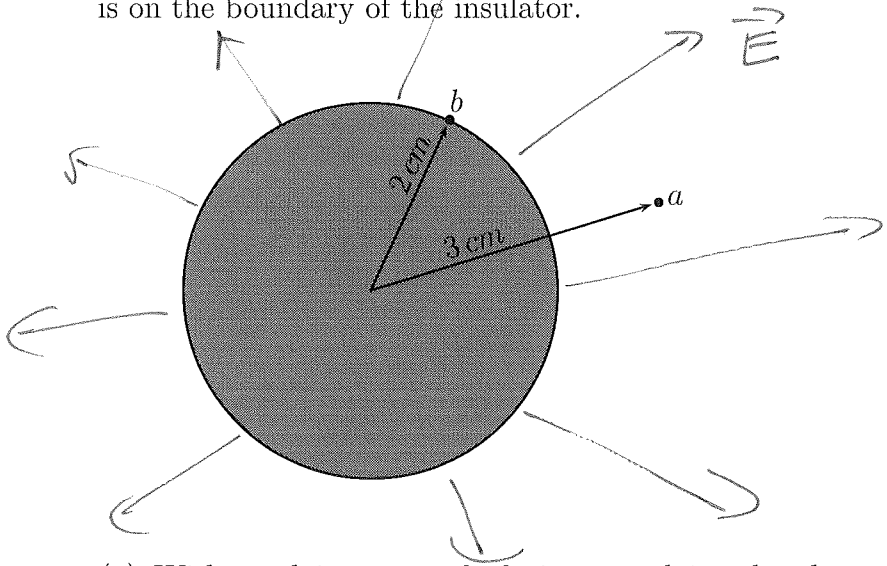
Physics 161 Test 3

An insulating sphere of radius 2.0 cm contains total charge $+75\,\mu C$. Two points of interest are a which is 3.0 cm from the insulator's center and b which is on the boundary of the insulator.



$\vec{E}$

POSITIVELY CHARGED SPHERE, SO RADIALLY OUTWARD FIELD

(a) Without doing any calculations, explain why the potential of point a relative to b is a negative number. *(2pts)*

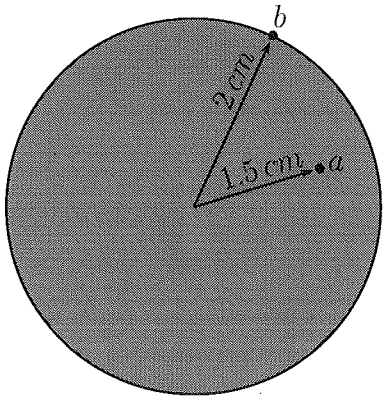
$\vec{E}$ points in direction of decreasing potential so a at lower potential than b. $V_{ab} = V_a - V_b$ $\Rightarrow V_{ab} < 0$

(b) What is the numerical value of the potential of point a relative to b? *(3pts)*

Outside, sphere's electric field identical to point charge $Q = 75\mu C$ located at $r = 0$ $\Rightarrow V = \frac{KQ}{r}$ at both a and b. $V_{ab} = V_a - V_b = \frac{KQ}{r_a} - \frac{KQ}{r_b} = KQ\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$

$V_{ab} = \left(9\times10^9 N\cdot m^2/C^2\right)(75\times10^{-6} C)\left(\frac{1}{.03m} - \frac{1}{.02m}\right)$ $\Rightarrow$ $V_{ab} = -11.25\times10^6 V$ $= -11.25 MV$

(c) If point a is $1.5\,cm$ from the center (in other words *inside* the insulator) what is the potential of point a relative to b? **Hint:** Inside a spherical insulator, the electric field's magnitude is given by $E = \frac{\rho r}{3\epsilon_o}$. *(5pts)*



INSIDE, FIELD IS NOT LIKE A point charge AND NOT CONSTANT $\Rightarrow$

$$V_{ab} = \int_a^b \vec{E} \cdot d\vec{\ell}$$

by SYMMETRY, $\vec{E}$ IS RADIAL $\Rightarrow \vec{E} \cdot d\vec{\ell} = E\,dr$

$$\therefore V_{ab} = \int_{r_a}^{r_b} E\,dr = \int_{r_a}^{r_b} \frac{\rho}{3\epsilon_0} r\,dr = \frac{\rho}{3\epsilon_0} \int_{r_a}^{r_b} r\,dr = \frac{\rho}{3\epsilon_0}\left(\frac{r^2}{2}\right)\Big|_{r_a}^{r_b}$$

$$\therefore V_{ab} = \frac{\rho}{6\epsilon_0}\left(r_b^2 - r_a^2\right)$$ Uniformly distributed charge over

Sphere of RADIUS $r_b = .02m \Rightarrow \rho = \frac{Q}{\frac{4}{3}\pi r_b^3} = \frac{75\times10^{-6}C}{\frac{4}{3}\pi(.02m)^3} = 2.238 C/m^3$

$$\therefore V_{ab} = \frac{2.238 C/m^3}{6(8.85\times10^{-12} C^2/N\cdot m^2)}\left((.02m)^2 - (.015m)^2\right) = 7.376\times10^6 V \Rightarrow \boxed{V_{ab} = 7.376 MV}$$

Units: $\frac{C}{m^3} \cdot \frac{N\cdot m^2}{C^2} \cdot m^2 = \frac{C\cdot N\cdot m^4}{m^3\cdot C^2} = \frac{N\cdot m}{C} = J/C = V$