Assignment № 1

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Problem 2.9

$$U(\theta,\phi) = \begin{cases} 1 & 0° \leq \theta < 20° \\ .342\csc(\theta) & 20° \leq \theta < 60° \\ 0 & 60° \leq \theta \leq 180° \end{cases}$$

$$D_o = \frac{U_{max}(\theta,\phi)}{P_{rad}}$$

$$P_{rad} = \oiint U(\theta,\phi)d\Omega$$

$$\implies \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) \implies \int_0^{2\pi}[\int_0^{\pi/9} \sin(\theta)d\theta + \int_{\pi/9}^{\pi/3} .342\csc(\theta)\sin(\theta)d\theta + \int_{\pi/3}^{\pi} 0\sin(\theta)d\theta]$$

$$\implies ([-\cos(\theta]\Big|_{\theta=0}^{\theta=\frac{\pi}{9}} + .342[\theta]\Big|_{\theta=\frac{\pi}{9}}^{\theta=\frac{\pi}{3}}) * 2\pi$$

$$\implies P_{rad} = 1.879$$

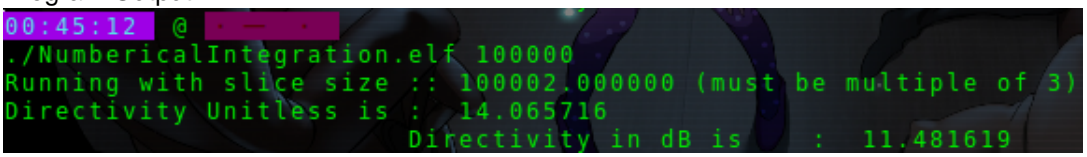
$$\implies D_o = \frac{4_{max}}{1.879} = \frac{4\pi}{1.879} \implies$$

$$\mathsf{D}_o = 6.687 \text{ no units}$$

$$D_{dBi} = 10log_{10}D_o \implies$$

$$\mathsf{D}_{dBi} = 8.253 \text{ dB}$$

Problem 2.26

1. C Program source code is in appendix.
   Program Output ::
   

2. Krauss Approxamation = $\frac{4\pi}{\Theta_{1r}\Theta_{2r}}$
   $\sqrt{\Theta}_{1r} = \sqrt{\Theta}_{2r} := U(\theta) = .5$
   $.5 = [\frac{\sin(\pi \sin(\theta))}{\pi \sin(\theta)}]^2 \implies \theta = .458$
   $\frac{4\pi}{.916^2} = 14.9768$ dimensionless
   $D_{dB} = 10log_{10}(14.9768) = 11.75$ dB

3. Tai-Peeira Approxamation =
   $\frac{22.181}{2(.916)^2} = 13.217$ dimensionless
   $D_{dB} = 10log_{10}(13.217) = 11.21$ dB

Problem 2.35

$$E_w^i = (\hat{a}x + j\hat{a}y)E_o e^{+jkz}$$
$$E_a = (\hat{a} + 2\hat{a}y)E_1 \frac{e^{-jkr}}{r}$$

- $\Delta_\phi = \phi_x - \phi_y = (2n+1)\frac{\pi}{2}$
  Circularly polarized because the y component is 90°out of phase with the x component due to its j amplitude. This will make the phase delta of x and y always be odd multiples of $\frac{\pi}{2}$ as well as E_x being equal to E_y.
- The rotation is clockwise because of the y component has a higher amplitude than the x component and will pull the rotation clockwise.
- $E_w^i = \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{5}}\sqrt{5}E_1\frac{e^{-jkz}}{z}$
  $\implies$ 2 components with 0° phase difference $\implies$ Linear polarization
- Since it is linear it has no rotation.
- $\hat{\rho}_w = \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}, \hat{\rho}_a = \frac{\hat{a}_x + 2\hat{a}_y}{\sqrt{5}}$
  PLF = $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 \implies \frac{|1+j|^2}{10} = \frac{5}{10}$
  PLF = .5 dimensionless
  $PLF_{dB} = 10log_{10}(.5) = -3$ dB

Problem 2.40

$\hat{\rho_a} = \frac{4\hat{a}_x + j\hat{a}_y}{\sqrt{17}}$
$\tau = 45°$
$PLF = |\frac{4\hat{a}_x + j\hat{a}_y}{\sqrt{17}} * \frac{\hat{a}_x + j\hat{a}_y}{\sqrt{2}}|^2$
$\implies \frac{|4+j|^2}{34} \implies .5$ dimensionless
$10log_{10}(.5) = -3.0103$ dB

Problem 2.70

1. Gain = Efficiency * Directivity
   $\varepsilon = \frac{A_{em}}{A_p} \implies$
   $e_o * \varepsilon = \frac{A_{em}}{A_p} * e_o \implies$
   $1 = \frac{\frac{\lambda^2}{4\pi} D_o}{10cm^2} e_o \implies$
   $10cm^2 = \frac{9cm^2}{4\pi} Gain \implies Gain = \frac{40\pi}{9}$
   $\implies 13.9626$ dimentionless
   $10log_{10}(13.9626) = 11.45 \implies$ Gain $= 11.45$ dB

2. $P_T = A_e * W_i$
   $P_T = 10\frac{(mW)}{(cm)^2}(10cm^2)(.5) \implies P_T = 50$ mW

Problem 2.80

$$U(\theta, \phi) = \begin{cases} \cos^4(\theta) & 0° \leq \theta < 90° \\ 0 & 90° \leq \theta \leq 180° \end{cases}$$

$$\begin{cases} 1 & 0° \leq \phi \leq 360° \end{cases}$$

$A_{em} = \frac{\lambda^2}{4\pi} D_o$

$P_{rad} = \int_0^{2\pi} \int_0^{2\pi} \cos^4(\theta)\sin(\theta) d\theta d\phi$

$\rightarrow$ calculator $\rightarrow$

$2\pi[.2] \implies P_{rad} = 1.256$

$D_o = 4\pi \frac{}{1.256 = 10.0051}$

$\implies \frac{9cm^2}{4\pi}(10.0051) = 7.165 * 10^{-2}$ m^2

Problem 2.86

$P_r = e_r * D_r * \frac{\lambda^2}{4\pi} * \frac{P_t D_t}{4\pi R^2}$

$\implies P_r = 20db \frac{10w * 20dB}{4\pi(50)^2\lambda^2} \frac{\lambda^2}{4\pi}$

$\implies \frac{400dB * 10w}{16\pi^2 2500} = 10$mW

APPENDIX

```
1
2  /*************************************************************************
3      This program performs a numerical integration of
4      [ (sin( pi sin(theta) ) / (pi sin(theta)) ] ^2
5      between 0 and PI. It then uses the result to find the
6      directivity of an antenna with the U(theta, phi)
7      characteristics.
8
9      Operations are therded to speed up calculations
10     as a signficant number of slices must be chosen to
11     obtain accuracy. An example is 1000 for fair accurasy.
12
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14
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17 the Free Software Foundation, either version 3 of the License, or
18 (at your option) any later version.
19
20 This program is distributed in the hope that it will be useful,
21 but WITHOUT ANY WARRANTY; without even the implied warranty of
22 MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
23 GNU General Public License for more details.
24
25 You should have received a copy of the GNU General Public License
26 along with this program.  If not, see <http://www.gnu.org/licenses/>.
27 *************************************************************************/
28 #include <stdio.h>
29 #include <math.h>
30 #include <pthread.h>
31 #include <unistd.h>
32 #include <time.h>
33 #include <signal.h>
34 #include <stdlib.h>
35 #include <sys/wait.h>
36
37
38 #define PI 3.14159265358979323846264338327950288419716939937510582
39
40 float h_val, x_i, a_bound, b_bound, n_slices;
41 long double temp_vals[4];
42 int first, second = 0;
43
44 void * threes (void *t)
45 {
46     long double a_n;
47     long double seq_threes;
48     long double x_i;
```

```
49      long tid = (long)t;
50      long double threes_sum = 0;
51      for(int i = 1; i < n_slices; i++)
52      {
53          a_n = .25*((6*i)-(pow((-1),i))-3);
54          if (fmodl(a_n, i) == 0)
55          {
56              x_i = a_n * (PI / n_slices);
57          //  printf("Value of i :: %d  Value of x_i :: %Lf \n", i, x_i);
58              seq_threes = 3*(pow(((sin(PI*sin(x_i)))/(PI*sin(x_i))),2))*sin(x_i);
59              threes_sum = threes_sum + seq_threes;
60          }
61      }
62
63      temp_vals[0] = threes_sum;
64      pthread_exit((void*) t);
65      return EXIT_SUCCESS;
66  }
67
68  void * twos (void *t)
69  {
70      long double a_n;
71      long double seq_twos;
72      long double x_i;
73      long double twos_sum = 0;
74      long tid = (long)t;
75      for(int i = 3; i < n_slices; i++)
76      {
77          a_n = 3*i;
78          if (fmodl(a_n, i) == 0)
79          {
80              x_i = a_n * (PI / n_slices);
81          //  printf("Value of i :: %d  Value of x_i :: %Lf \n", i, x_i);
82              seq_twos = 2*(pow(((sin(PI*sin(x_i)))/(PI*sin(x_i))),2))*sin(x_i);
83              twos_sum = twos_sum + seq_twos;
84          }
85      }
86
87      temp_vals[1] = twos_sum;
88      pthread_exit((void*) t);
89      return EXIT_SUCCESS;
90  }
91
92  void* ends(void *t)
93  {
94      long double a_n, a_0;
95      long double x_n = n_slices;
96      long double x_0 = 0;
97      long tid = (long)t;
98      pid_t end_to_end;
99      //printf("End to ends:: x_n ==%Lf \n",x_n);
```

```
100
101     end_to_end = fork();
102     if (end_to_end == 0)
103     {
104         x_0 = 0 * (PI / n_slices);
105         a_0 = 1;
106     //  printf("a_0 :: %Lf :: x_0 :: %Lf\n",a_0, x_0);
107         exit(1);
108     }
109     else
110     {
111         x_n = n_slices * (PI / n_slices);
112         a_n = pow(((sin(PI*sin(x_n)))/(PI*sin(x_n))),2)*sin(x_i);
113     //  printf("a_n :: %Lf || x_n :: %Lf\n",a_n, x_n);
114         wait(NULL);
115     }
116
117     temp_vals[2] = a_0 + a_n;
118     pthread_exit((void*) t);
119     return EXIT_SUCCESS;
120 }
121
122
123 int main(int argc, char *argv[])
124 {
125
126     pid_t id;
127     pthread_t ends_thread, twos_and_threes[3];
128     pthread_mutex_t thread_locker;
129     void * status;
130     long t;
131
132     n_slices =(atof(argv[1])) ;
133     n_slices = ((n_slices + 3 - 1) / 3) * 3 ;
134
135     printf("Running with slice size :: %lf (must be multiple of 3)\n", n_slices);
136     pthread_attr_t attr;
137     pthread_attr_init(&attr);
138     pthread_mutex_init(&thread_locker, NULL);
139     pthread_attr_setdetachstate(&attr, PTHREAD_CREATE_JOINABLE);
140     //Child Process
141     if(pthread_create(&twos_and_threes[0], &attr,ends,(void*) t))
142         return EXIT_FAILURE;
143     if(pthread_create(&twos_and_threes[1],&attr,twos,(void*)t))
144         return EXIT_FAILURE;
145     if(pthread_create(&twos_and_threes[2],&attr,threes,(void*)t))
146         return EXIT_FAILURE;
147
148     pthread_attr_destroy(&attr);
149     for(int i = 0 ; i < 3 ; i++)
150         if(pthread_join(twos_and_threes[i], &status))
```

```
151         {
152             fprintf(stderr, "Catastrophic Failure");
153             return EXIT_FAILURE;
154         }
155     /*
156     twos();
157     threes();
158     ends();
159     */
160 
161     temp_vals[3] = temp_vals[0]+temp_vals[1] +temp_vals[2];
162     //printf( "Temp vals :: %Lf :: %Lf :: %Lf :: %Lf\n",\
163             temp_vals[0],temp_vals[1],temp_vals[2],temp_vals[3]);
164 
165     long double h_val = ((PI-0) / n_slices);
166     long double result = temp_vals[3] * (3*h_val) * (.125) ;
167     //printf("Result == %Lf \n",result);
168 
169     long double P_rad = result*2*PI;
170     long double directivity = (2*PI*1) / (P_rad);
171 
172     printf("Directivity Unitless is :  %Lf \n \
173             Directivity in dB is    :  %f \n", directivity, (10*log10f(directivity↩
                 )));
174 
175 
176     pthread_exit(NULL);
177 }
```

*Makefile for compiling the above code

```
#
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#
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#  Foundation, Inc., 51 Franklin Street, Fifth Floor, Boston,
#  MA 02110-1301, USA.
#
#


CC= gcc
CC2= clang
LD= ld -r
CFLAGS= -std=gnu99 -g -O3 -lm -pthread
FL2=
FL3=
TGTS=
OBJ=
RM= /bin/rm -f

all : NumbericalIntegration

NumbericalIntegration:
    $(CC2) $(CFLAGS) $@.c -o $@.elf
clean :
    $(RM) *.elf *.txt
```