ECE 345: Introduction to Control Systems
Problem Set #3

Dr. Oishi

Due Thursday, October 25, 2012 at the start of class

This homework is open note and open book. You are welcome to discuss the problems with other students, but your solutions and Matlab code *must be written independently.* Copying will not be tolerated.

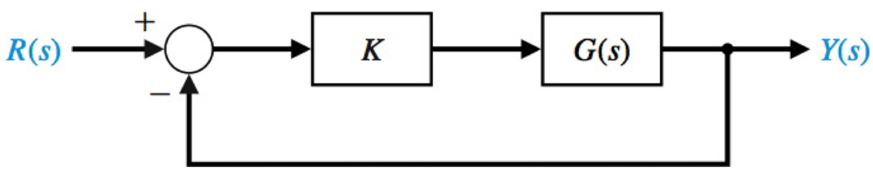

Figure 1: System under unitary negative feedback.

1. Consider the system shown in Figure 1, with *open-loop* transfer function $G(s) = \frac{s+1}{s^2+1}$ and gain $K = 1$.

   (a) Determine the BIBO stability of the open-loop system.

   (b) Find the transfer function $\frac{Y(s)}{R(s)}$ of the closed-loop system.

   (c) Are the poles of the open-loop system different from the poles of the closed-loop system?

   (d) Is the BIBO stability of the open-loop system different from the BIBO stability of the closed-loop system?

2. Determine the asymptotic stability of the following systems.

   (a) $G(s) = \frac{s-1}{(s+2)(s+1)}$

   (b) $\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x$

   (c) $G(s) = \frac{(s-1)(s+2)}{(s-1)(s^2+2s+1)}$

   (d) $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x$

3. Use a Routh table to find the values of $K > 0$ which will asymptotically stabilize the system shown in Figure 1, with

$$G(s) = \frac{s+2}{(s^2+1)(s+4)(s-1)}$$

4. Consider a state-space system $\dot{x} = Ax + Bu, y = Cx$ for which the control input is defined as $u = -Kx + r$, with $r(t)$ a reference input. This results in a closed-loop system

$$\begin{aligned} \dot{x} &= (A - BK)x(t) + Br(t) \\ y &= Cx \end{aligned} \tag{1}$$

with matrices

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ 3 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ K &= \begin{bmatrix} k_1 & k_2 \end{bmatrix} \end{aligned} \tag{2}$$

For this type of controller, $k_1, k_2 \in \mathbb{R}$ do not need to be restricted to positive numbers – any real number is fine.

(a) For what values of K is the closed-loop system asymptotically stable?

(b) For what values of K is the closed-loop system marginally stable?

Now consider certain performance criteria that the closed-loop system should meet. Ideally, the closed-loop system should have a) an overshoot of less than 5% and b) settling time $T_s < 4$ seconds.

(c) Sketch the location of all of the poles of the closed-loop system that will meet the desired performance criteria.

(d) Select a value of K such that the performance criteria is met.

BONUS: Plot the step response of the *output* of the closed-loop system in Matlab to demonstrate that the performance criteria and stability criteria has been met for your chosen value of K.