Problem Set #1 Solutions
ECE 345, Fall 2012
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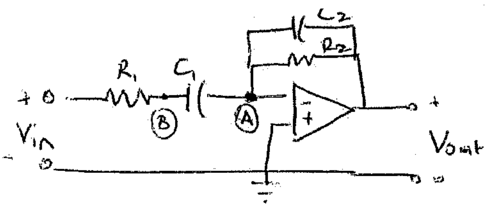


(a) KCL at A: $i_{IN} = i_{R2} + i_{C2} = C_2 \frac{d}{dt}(v_A - v_{out}) + \frac{(v_A - v_{out})}{R_2}$

$= -C_2 \dot{v}_{out} - \frac{1}{R_2} v_{out}$ ①

Since $v_A = 0$, $\quad \frac{v_{in} - v_B}{R_1} = C_1 \frac{d v_B}{dt}$ ②

and $C_1 \frac{d v_B}{dt} = i_{IN}$

Taking the Laplace transform,

② $V_{in}(s) - V_B(s) = R_1 C_1 s V_B(s)$

$V_{in}(s) = V_B(s)(R_1 C_1 s + 1)$

① $C_1 s V_B(s) = -s C_2 V_{out}(s) - \frac{1}{R_2} V_{out}(s)$

$s \cdot \frac{V_{in}(s)}{1 + R_1 C_1 s} \cdot R_2 C_1 = V_{out}(s)(-R_2 C_2 s - 1)$

$$\boxed{\frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_2 C_1 s}{(1 + R_1 C_1 s)(1 + R_2 C_2 s)}}$$

(b) Step response is $V_{out}(s)$ when $V_{in}(s) = \frac{1}{s}$

$$\boxed{V_{out}(s) = \frac{-1}{(1+2s)(1+s)}}$$

$\frac{-1}{(1+2s)(1+s)} = \frac{A}{1+s} + \frac{B}{1+2s}$ $= \frac{1}{1+s} + \frac{-2}{1+2s} = \frac{1}{1+s} - \frac{1}{\frac{1}{2}+s}$

$\therefore \boxed{v_{out}(t) = (e^{-t} - e^{-\frac{1}{2}t})u(t)}$

$-1 + 0 \cdot s = (A+B) + (2A+B)s$

$\therefore A + B = -1, \quad 2A + B = 0$

$A - 2A = -1 \quad 2A = -B$

$A = 1, \; B = -2$

$\boxed{12}$ $y(t) = e^{-t} - \frac{1}{4} e^{-2t} - \frac{3}{4} + \frac{1}{2}t, \quad t \geq 0$

(a) $Y(s) = G(s) \cdot R(s)$, and $R(s) = \frac{1}{s^2}$ for $r(t) = t u(t)$

$$= \frac{1}{s+1} - \frac{1}{4(s+2)} - \frac{3}{4s} + \frac{1}{2s^2}$$

$$= \frac{4(s+2)s^2 - (s+1)s^2 - 3s(s+1)(s+2) + 2(s+1)(s+2)}{4(s+1)(s+2)\cdot s^2}$$

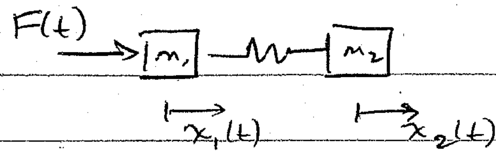
$$= \frac{4s^3 + 8s^2 - s^3 - s^2 - 3s^3 - 9s^2 - 6s + 2s^2 + 6s + 4}{4s^2(s+1)(s+2)}$$

$$= \frac{0\cdot s^3 + 0\cdot s^2 + 0\cdot s + 4}{4s^2(s+1)(s+2)}$$

$$\therefore \; G(s) = \frac{Y(s)}{1/s^2} = \frac{1}{(s+1)(s+2)}$$

(b) $Y(s) = G(s)\cdot 1 = G(s) = \frac{1}{(s+1)(s+2)}$

$\boxed{3}$ SMD System



"+" is $\rightarrow$.

(a) $\boxed{m_1} \xrightarrow{F(t)} k(x_2-x_1)$ $\qquad$ $\xleftarrow{k(x_2-x_1)} \boxed{m_2}$ $\qquad$ assume $x_2 - x_1$ $\therefore$ spring is in tension

$\Sigma F = m_1\ddot{x}_1$ $\qquad$ $\Sigma F = m_2\ddot{x}_2$

(1) $F(t) + k(x_2 - x_1) = m_1\ddot{x}_1$ $\quad$ (2) $-k(x_2-x_1) = m_2\ddot{x}_2$

(b) $F(s) + k(X_2(s) - X_1(s)) = m_1 X_1(s)\cdot s^2$ $\qquad$ (1)

$$F(s) + k\,X_2(s) = X_1(s)\left(m_1 s^2 + k\right)$$

$$-k(X_2(s) - X_1(s)) = m_2 X_2(s)\cdot s^2 \qquad (2)$$

$$k\,X_1(s) = X_2(s)\left(m_2 s^2 + k\right)$$

$$\therefore \quad F(s) + k\,X_2(s) = X_2(s)\,\frac{(m_2 s^2 + k)}{k}(m_1 s^2 + k)$$

$$F(s) = \frac{X_2(s)}{k}\left(m_2 s^2 \cdot m_1 s^2 + k m_2 s^2 + k m_1 s^2 + \cancel{k^2} - \cancel{k^2}\right)$$

$$F(s) = X_2(s)\,\frac{s^2\left(m_1 m_2 s^2 + k(m_1+m_2)\right)}{k}$$

For $m_1 = m_2 = 1,\ k = 1$

$$F(s) = X_2(s)\, s^2(s^2+2)$$

$$\frac{X_2(s)}{F(s)} = \frac{1}{s^2(s^2+2)}$$

(c) $\lim_{t\to\infty} x_2(t) = \lim_{t\to\infty} s\,X_2(s)$

$$= \lim_{t\to\infty} s\cdot\frac{1}{s^2(s+2)}\cdot\frac{1}{s} = \infty$$

$\boxed{4}$ Linearization $\quad v_o(v_{in}) = 3.5\, v_{in}^3$

$$f(x) \simeq f(x_o) + \left.\frac{\partial f}{\partial x}\right|_{x=x_o} (x - x_o),$$

(a) Choose linearization point $v_{in} = 0$.

$$v_o(0) = 0.$$

$$v_o(v_{in}) \simeq 0 + \left.\frac{\partial v_o}{\partial v_{in}}\right|_{v_{in}=0} (v_{in} - 0), \quad \frac{\partial v_o}{\partial v_{in}} = 3 \cdot 3.5\, v_{in}^2$$

$$= 0 + 0 \cdot v_{in} \qquad\qquad = 10.5\, v_{in}^2$$

(1) $\quad v_o = 0.$

(b) Choose $v_{in} = 0.6. \Rightarrow v_o(0.6) = 0.756$

$$v_o(v_{in}) \simeq v_o(0.6) + 10.5\,(0.6)^2 (v_{in} - 0.6)$$

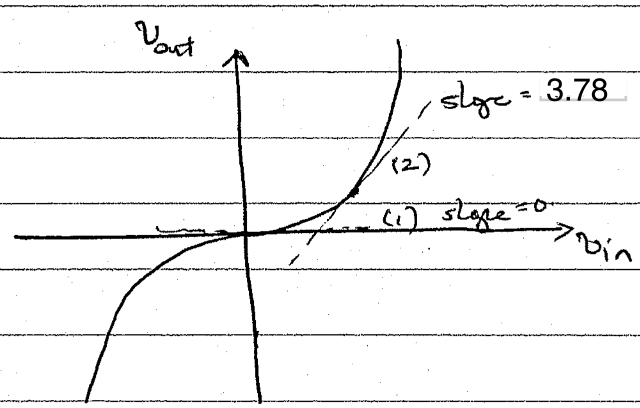
(2) $$\simeq 0.756 + 3.78\,(v_{in} - 0.6)$$

$\therefore \Delta v_o = v_o - 0.756$

$\Delta v_{in} = v_{in} - 0.6$

and $\quad \Delta v_o = 3.78\, \Delta v_{in}$. — Linear.

(c)



15 (a) $$G_1(s) = \frac{20s^4 + 380s^3 + 2480s^2 + 6480s + 5760}{s^5 + 41s^4 + 613s^3 + 3975s + 9450}$$

(b) roots of denom. of $G_2(s)$: $-1 \pm 3j$
$G_3(s)$: $-3 \pm j$

(c) see next page

(d) $G_2(s)$ has a step response with less damped oscillations. In addition, in comparison to $G_3(s)$, the system step response takes longer to settle down to the steady-state value. Lastly, the overshoot in $G_2(s)$ far exceeds that in the step response of $G_3(s)$

```
G1_num = 20*conv( conv([1 2],[1 3]), conv([1 6],[1 8]) )

G1_num =

          20          380         2480         6480         5760

G1_den = conv( conv( conv([1 7],[1 9]), conv([1 10],[1 15]) ), [1 0] )

G1_den =

           1           41          613         3975         9450            0



roots([1 2 10])

ans =

  -1.0000 + 3.0000i
  -1.0000 - 3.0000i

roots([1 6 10])

ans =

  -3.0000 + 1.0000i
  -3.0000 - 1.0000i

sys2 = tf(5*[1 2],[1 2 10])

Transfer function:
   5 s + 10
--------------
s^2 + 2 s + 10

sys3 = tf(5*[1 2],[1 6 10])

Transfer function:
   5 s + 10
--------------
s^2 + 6 s + 10

figure; step( sys2 )
print -djpeg95 ex1_p5_sys2.jpg
figure(gcf+1); step(sys3)
print -djpeg95 ex1_p5_sys3.jpg
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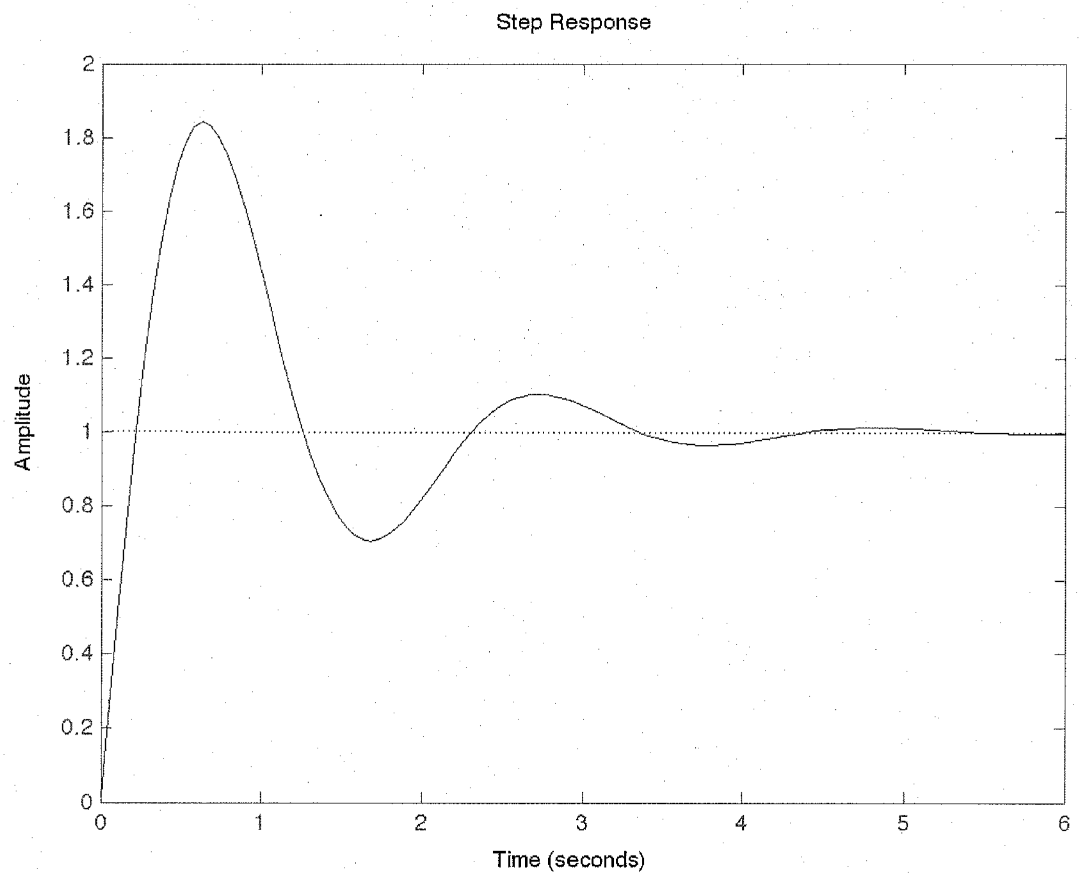
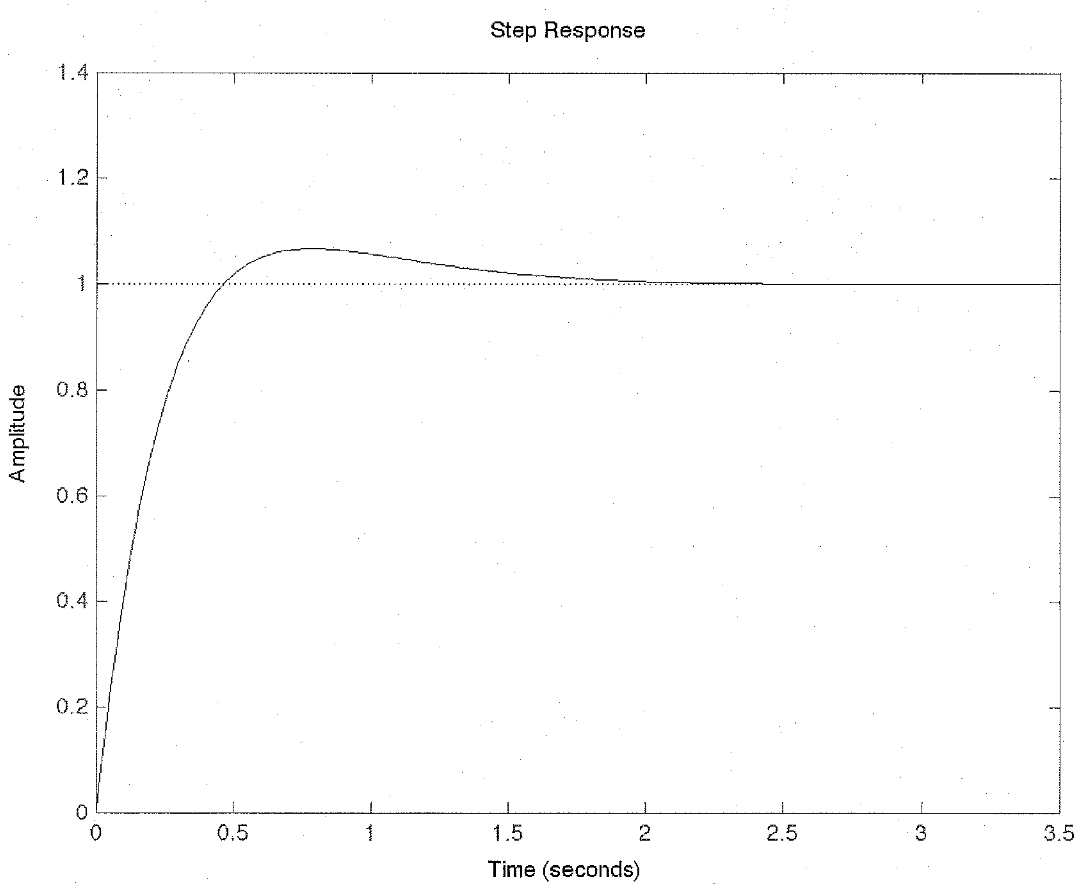


Step Response
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
1
2
3
4
5
6



Step Response
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.5
1
1.5
2
2.5
3
3.5