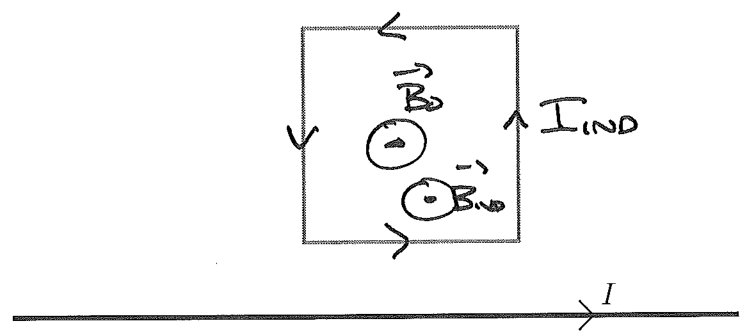
PHYSICS 161 TEST 7

A very long, straight wire has current I flowing through it in the direction shown in the figure below. Same distance above the wire is a square conducting loop. In all problems assume all sides of the square loop are in the same plane.

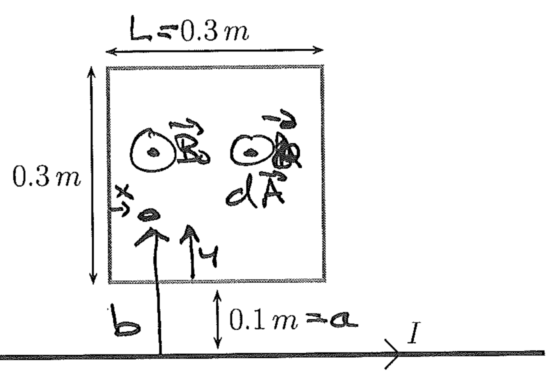


(a) If the current I is decreasing, will the induced current in the square loop be clockwise or counter-clockwise? For full credit, you must give a detailed explanation of the direction of the original magnetic field, whether the flux is increasing or decreasing, the direction of the induced magnetic field, and finally, the direction of the induced current. *(3pts)*

FROM RHR $\vec{B}_o$ IS $\odot$ at all points IN SQUARE Loop.

FOR WIRE $\Rightarrow B_o = \frac{\mu_o I}{2\pi b} \Rightarrow$ DECREASING Current MEANS Decreasing B_o AND FLUX. LENZ'S LAW $\Rightarrow \vec{B}_{IND}$ Will try to MAINTAIN by Also being $\odot$, so by OTHER RHR, I_{IND} will be COUNTER-clockwise.

(b) Assume the square loop has 0.3 m-long sides and is 0.1 m from the straight wire. Find the flux through the square loop as a function of the current I. **Hint:** The original magnetic field of the straight wire is not uniform throughout the square loop; therefore, you must integrate to find the flux. *(4pts)*



At any point $\vec{B}_0$ and $d\vec{A}$ are both ⊙ $\Rightarrow \Phi_B = \int B_0 dA$

$= \int \frac{\mu_0 I}{2\pi b} dA$ using Cartesian coordinates (x,y) we see that

B_0 is constant for constant y $\Rightarrow dA = L\,dy$ where $L = .03$m is x-distance. $b = a + y$ where $a = .1$m

$$\Rightarrow \Phi_B = \int \frac{\mu_0 I L\,dy}{2\pi(a+y)} = \frac{\mu_0 I L}{2\pi} \int_0^L \frac{dy}{a+y} \quad \text{Let } u = a+y,\ du = dy$$

$$\Rightarrow \Phi_B = \frac{\mu_0 I L}{2\pi} \int_a^{L+a} \frac{du}{u} = \frac{\mu_0 I L}{2\pi} \ln\left(\frac{L+a}{a}\right) = \boxed{\left(\frac{\mu_0}{2\pi}\right) L \ln\left(\frac{L+a}{a}\right) I}$$

Numerical answer on next page

(c) If the current through the straight wire obeys the equation $I = 1000\,(1 - t^2)$ (where I is in Amps when t is in seconds), find the amount of induced current in the 25-Ω square loop at $t = 0.75\,s$. *(3pts)*

$$\Phi_B = \left(\frac{\mu_0}{2\pi}\right) L \ln\left(\frac{L+a}{a}\right) I = (2\times10^{-7}\,T\cdot m/A)(.3m)\ln\left(\frac{.4m}{.1m}\right) I$$

$$= (6\times10^{-8}\,T\cdot m^2/A)\ln 4\ I = \underline{\underline{8.3178\times10^{-8}\,T\cdot m^2/A\ I}}$$

Ampere's Law : $\mathcal{E}_{IND} = -N\dfrac{d\Phi_B}{dt}$, $I_{IND} = \dfrac{\mathcal{E}_{IND}}{R}$

$I = 1000(1-t^2)\,A \Rightarrow \Phi_B = (6\times10^{-8})\ln 4\,(1000)(1-t^2)\ Wb$

overall unit

$$\frac{d\Phi_B}{dt} = (6\times10^{-5})\ln 4(-2t) \quad N = 1 \text{ here} \Rightarrow \mathcal{E}_{IND} = -\frac{d\Phi_B}{dt}$$

$\Rightarrow \mathcal{E}_{IND} = (6\times10^{-5})\ln 4(+2t)$ at $t = .75s$, $\mathcal{E}_{IND} = (6\times10^{-5})\ln 4\,(1.5)$

$\Rightarrow \mathcal{E}_{IND} = .000125V \Rightarrow \boxed{I_{IND} = \dfrac{.000125V}{25\,\Omega} = 4.99\times10^{-6}A \approx 5\mu A}$