Lecture 10
(Electric Charge, Insulators and Conductors, Coulomb's Law)

Physics 161-01 Spring 2012

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Electricity and Magnetism

- http://ocw.mit.edu/courses/physics/8-02-electricity-and-magnetism-spring-2002/video-lectures/lecture-1-what-holds-our-world-together/

Electricity and Magnetism

- And you thought I was rude and arrogant.

Gravity

- What is the general form of the force due to gravity?

$$F_G = G\frac{m_1 m_2}{r^2}$$

- That is, two particles with mass will attract each other proportionately to their masses and inverse proportionately to the square of the distance between them
- The proportionality constant, G, is known as the universal gravitational constant.
- The gravitational force is very weak (?)…
- For two masses each of 1kg, separated by 1m:

$$F_G = G\frac{m_1 m_2}{r^2} = G\frac{1kg \cdot 1kg}{(1m)^2} = 6.6742 \times 10^{-11} N$$

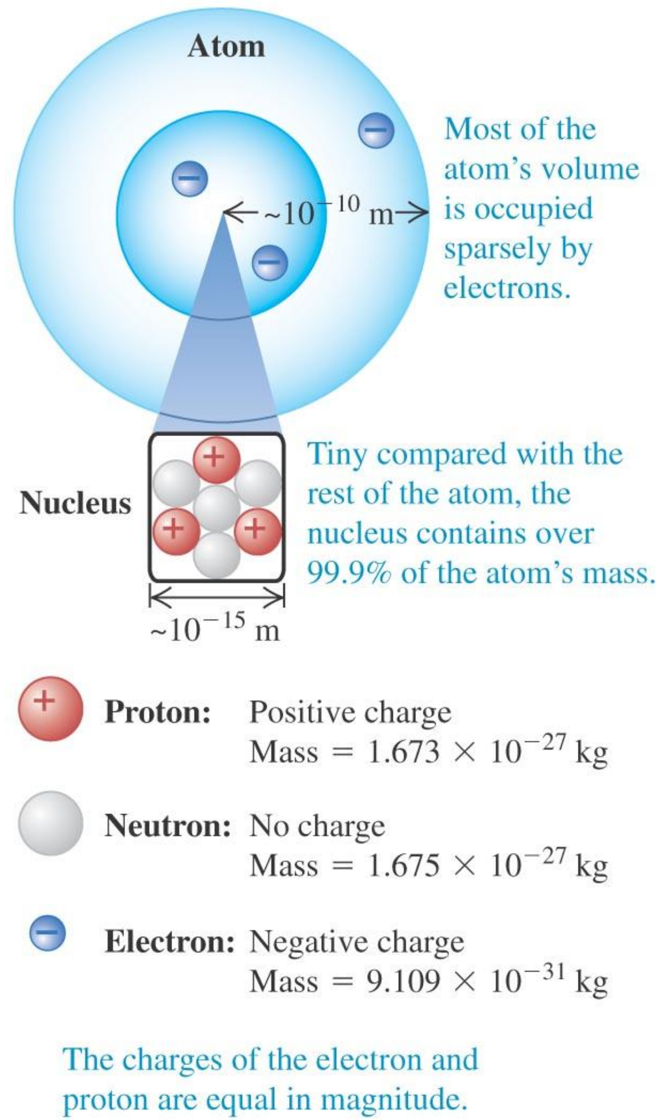
- That is, the proportionately constant, G = $6.6742 \mathrm{x} 10^{-11} Nm^2/kg^2$.

“Mass Charge” and “Electric Charge”

- Mass is a both fundamental property of matter, and a consequence of how fundamental particles are put together to forma a macroscopic object (when particles bind, the binding energy is reflected in the mass of the composite object).
- There is another fundamental property of matter which we will now investigate, called electric charge.
- In contrast to mass, it comes in two distinct forms: “positive charge” and “negative charge”. These are, of course, just names, but the connotation that they carry will prove useful.

Atomic Model Review

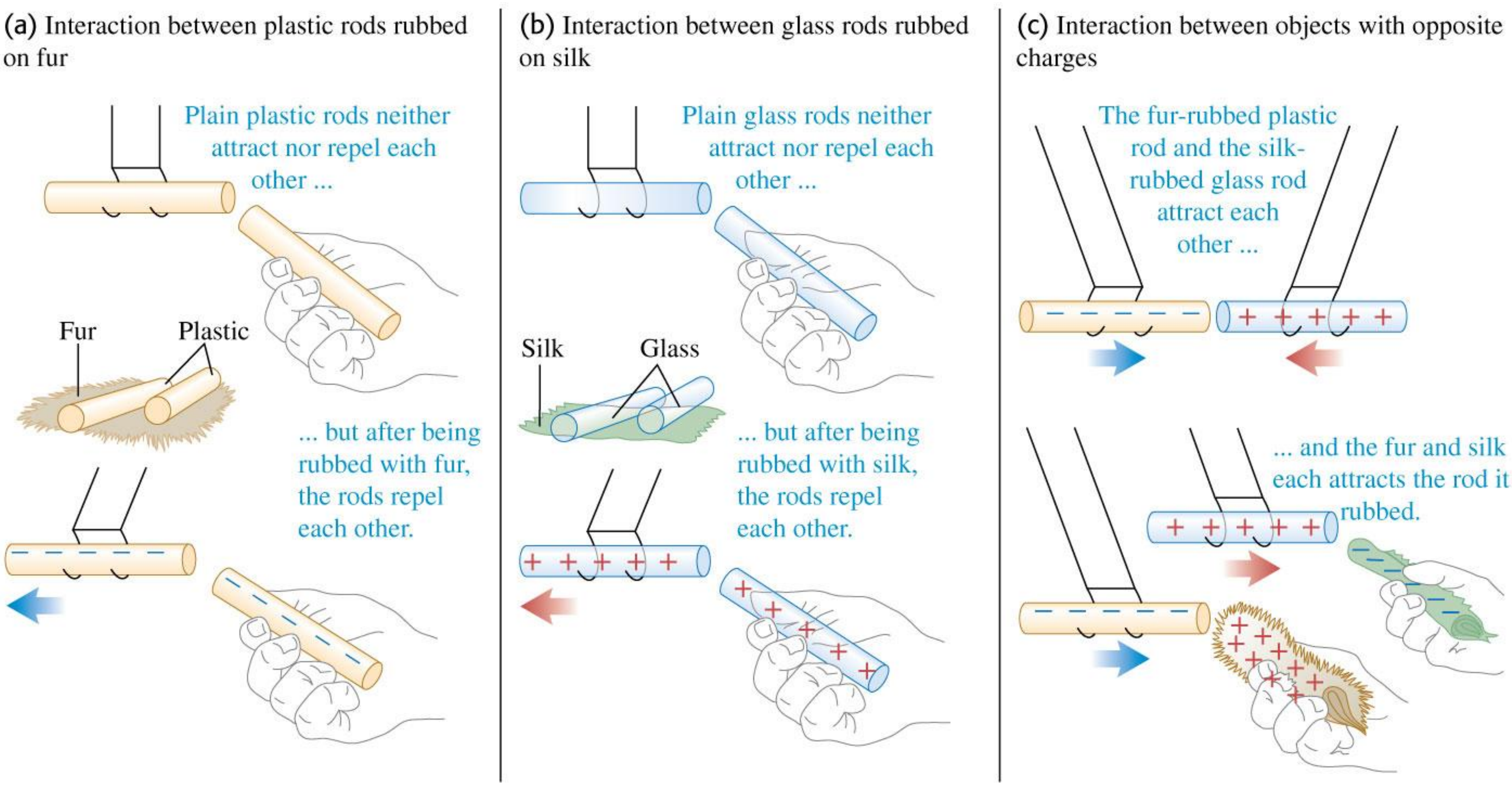
- Atoms consist of a nucleus made of protons and neutrons.
- Each element is designated by the number of protons, its atomic number "Z".
- Different isotopes of an element have different numbers of neutrons. The total number of nucleons is given by its mass number "N".
- In a neutral atom, the number of electrons equals the number of protons, since the electric charge of the electron is equal in magnitude, but opposite in sign of the proton's electric charge.
- An atom can either lose an electron or gain an electron to become a charged ion.



(Very) Basic E&M

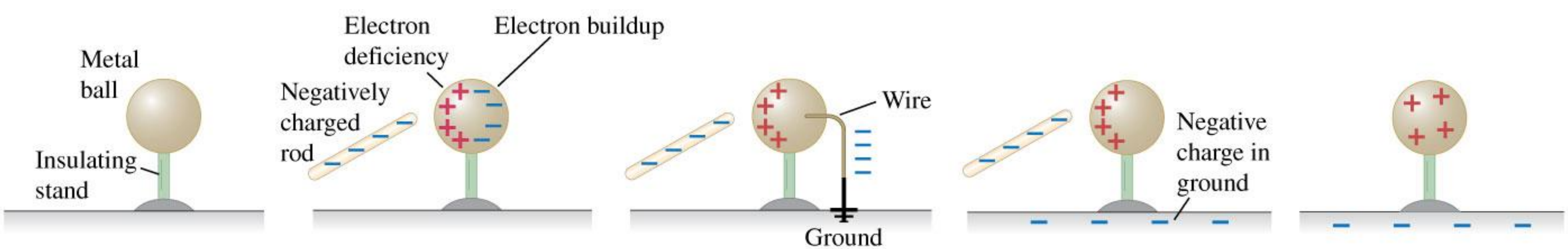
- There is a lot that can be understood about nature by just understanding the basic behavior of the interaction between electric charges and charges themselves. [Note: I will use the term “charge” to denote “electric charge”, although there are other types of charge in nature.]
  - Charge comes in a fundamental package or unit. It is the magnitude of charge on the electron (and proton). While some fundamental particles have charges that are fractions of this unit, one cannot isolate these fundamental particles in nature.
  - Charge is conserved – this means if you start with a neutral (sum of positive MINUS negative charges = 0) object, one can only give it a net positive or negative charge by either taking charge from it or putting charge on it from some other object.
  - Like-sign charges repel, unlike-sign charges attract. The electrons in an atom are attracted to the positively charged nucleus. The protons in the nucleus repel each other, but are held in place by an even stronger force (the so-called “strong force”).
  - Some materials allow electrons to move freely within them and on their surface. These are called conductors. Other materials prevent electrons from moving freely. These are called insulators.

Some Demonstrations



Inductive Charging

- Because of these properties, we can add a charge to an object without touching it:



(a) Uncharged metal ball

(b) Negative charge on rod repels electrons, creating zones of negative and positive **induced charge**.

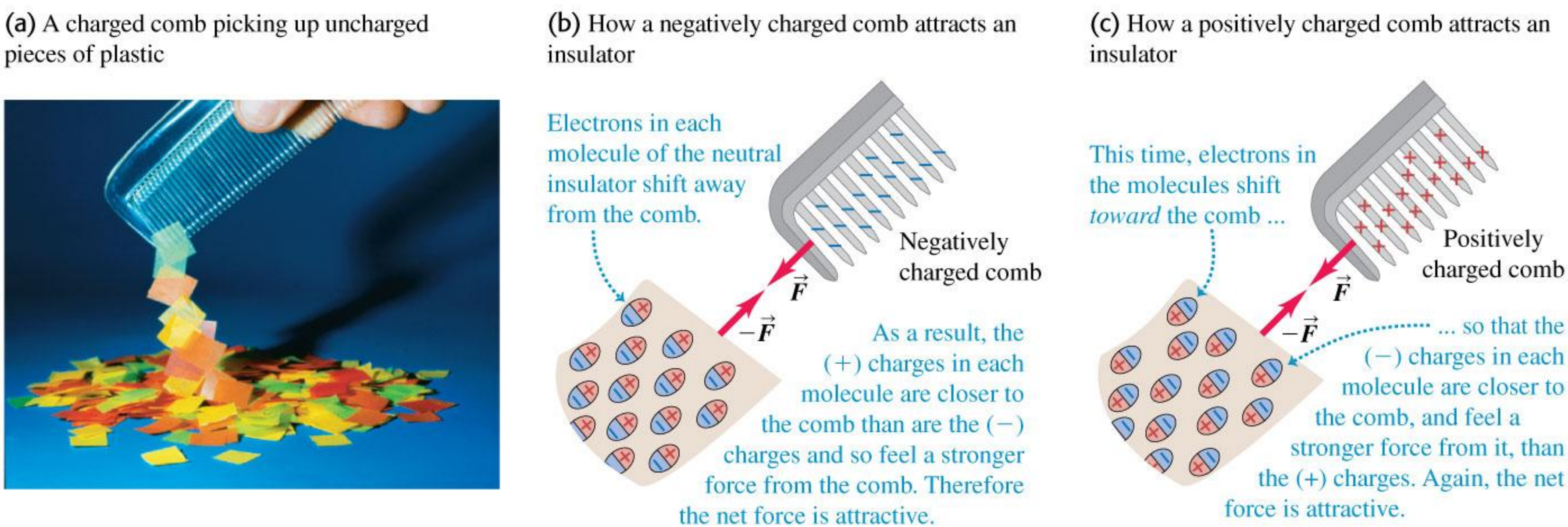
(c) Wire lets electron build-up (induced negative charge) flow into ground.

(d) Wire removed; ball now has only an electron-deficient region of positive charge.

(e) Rod removed; electrons rearrange themselves, ball has overall electron deficiency (net positive charge).

Electric Forces on Uncharged (Neutral) Objects

- Even if an object has no net charge, the charges can rearrange themselves so that there is a net attractive force:



Coulomb's Law

- The force law that dictates the behavior of electric charges is very similar to the gravitational force law: $$F_G = G\frac{m_1 m_2}{r^2}$$

- For two point charges, q_1 and q_2, separated by a distance r, the magnitude of the force between them is just: $$F_C = k\frac{|q_1 q_2|}{r^2}$$

Coulomb's Law

- Since we now have a new aspect of nature to deal with, we must also have a new unit in which to measure electric charge.
- We define that unit as a Coulomb [C], and relate its value to the fundamental charge of the electron as:

$$e = 1.602176487(40) \times 10^{-19} C$$

- Then, we re-write Coulombs law with a new proportionality constant:

$$F_C = k \frac{|q_1 q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2}$$

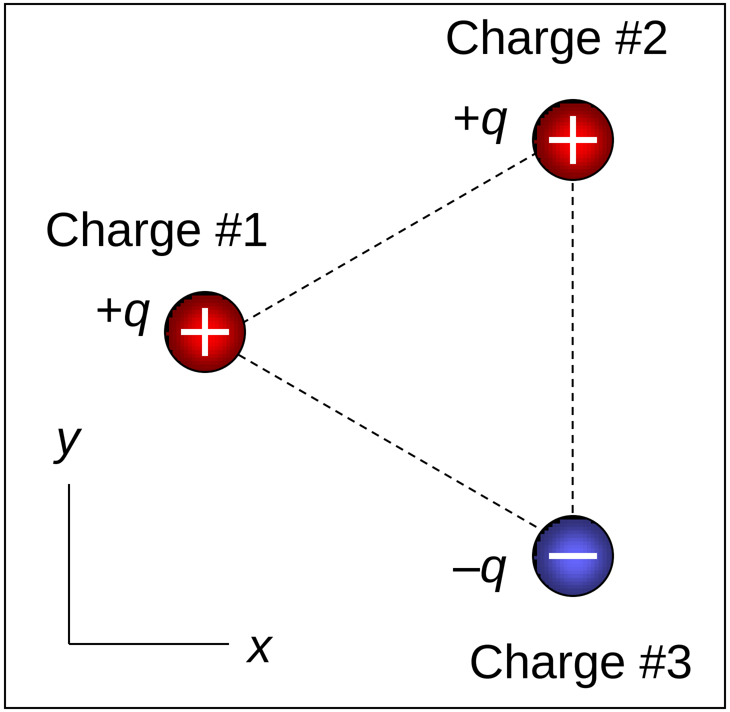
$$k = 8.987551787 \times 10^9 \, N \cdot m^2 / C^2$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \, C^2 / N \cdot m^2$$

CPS 10-1

Three point charges lie at the vertices of an equilateral triangle as shown. All three charges have the same magnitude, but charges #1 and #2 are positive ($+q$) and charge #3 is negative ($-q$).

The net electric force that charges #2 and #3 exert on charge #1 is in



A. the $+x$-direction.

B. the $-x$-direction.

C. the $+y$-direction.

D. the $-y$-direction.

E. none of the above

CPS 10-1

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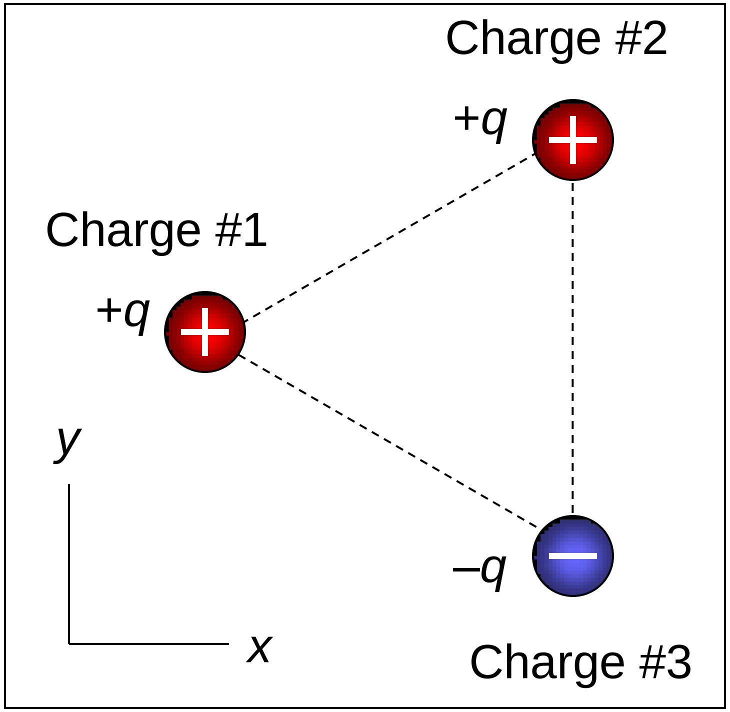


A. the +x-direction.

B. the –x-direction.

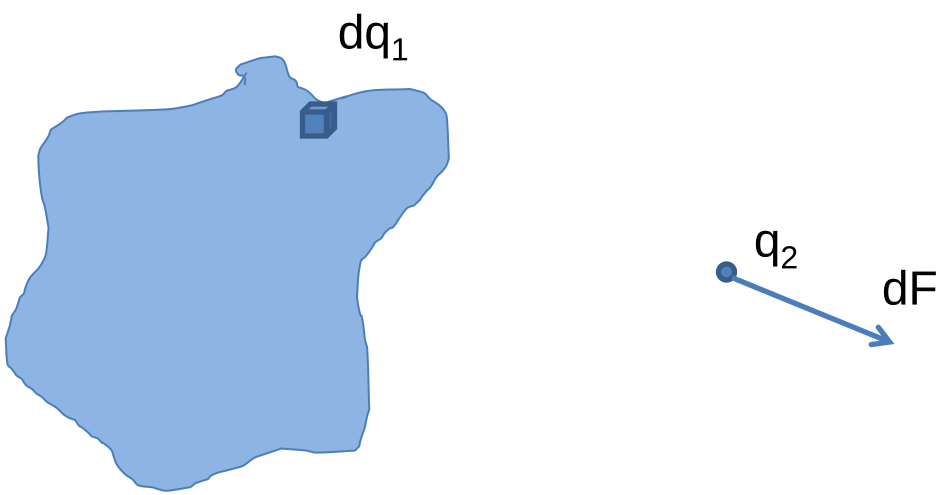
C. the +y-direction.

D. the –y-direction.

E. none of the above

Coulomb's Law With Extended Objects

- Don't forget that Coulomb's Law is only applicable for point charges, just like the gravitational law is only applicable for point masses.
- If we are dealing with extended objects, we have to break them up into tiny pieces and use calculus to find the net force.



$$F_C = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{r^2}dq_1$$

Coulomb's Law With Extended Objects

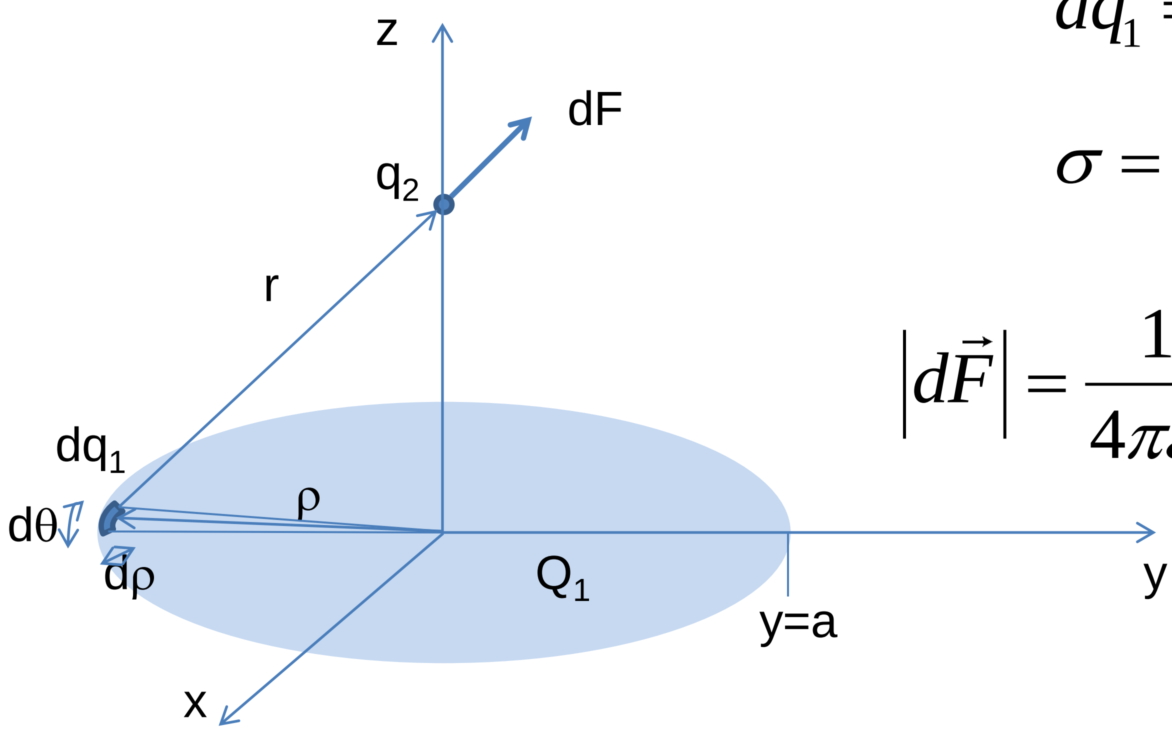
- Example: finite disk.

$$\left|d\vec{F}\right| = \frac{1}{4\pi\varepsilon_0}\frac{\left|q_2\right|}{r^2}dq_1$$

$$dq_1 = \sigma dA = \sigma\rho d\theta d\rho$$

$$\sigma = \frac{Q_1}{A} = \frac{Q_1}{\pi a^2}$$

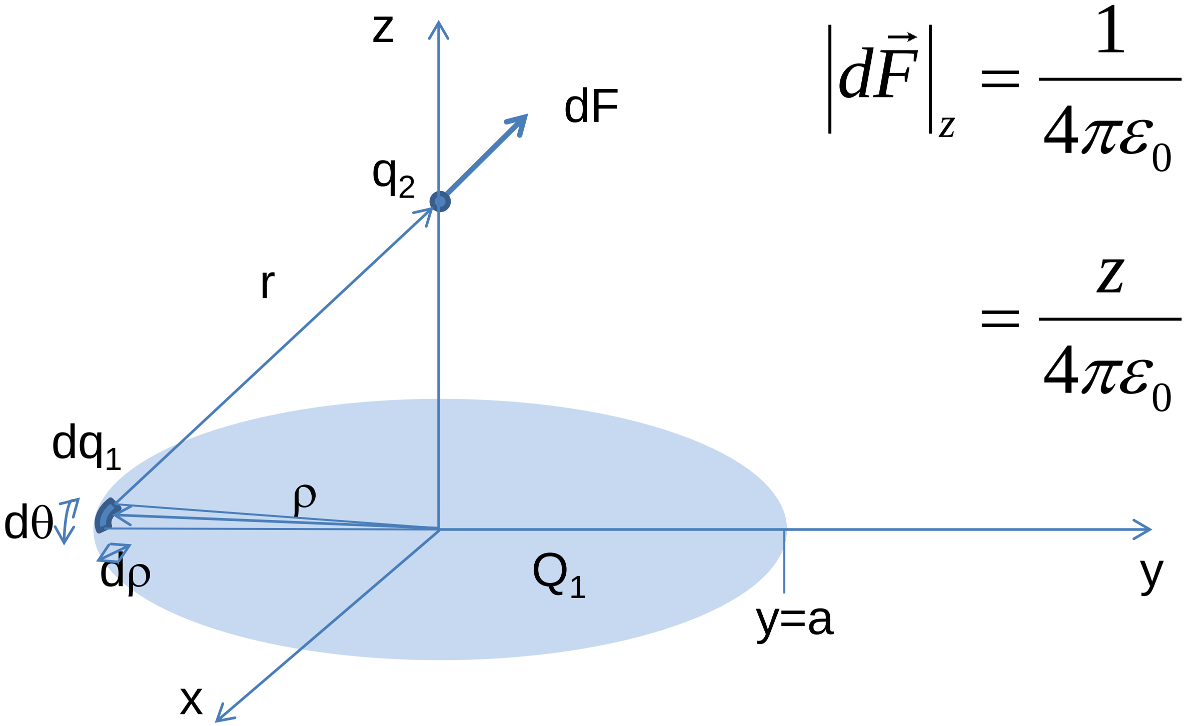
$$\left|d\vec{F}\right| = \frac{1}{4\pi\varepsilon_0}\frac{\left|q_2\right|}{\left(\rho^2 + z^2\right)}\sigma\rho d\theta d\rho$$



Coulomb's Law With Extended Objects

- Example: finite disk.

$$\left|d\vec{F}\right| = \frac{1}{4\pi\varepsilon_0}\frac{\left|q_2\right|}{\left(\rho^2+z^2\right)}\sigma\rho d\theta d\rho$$

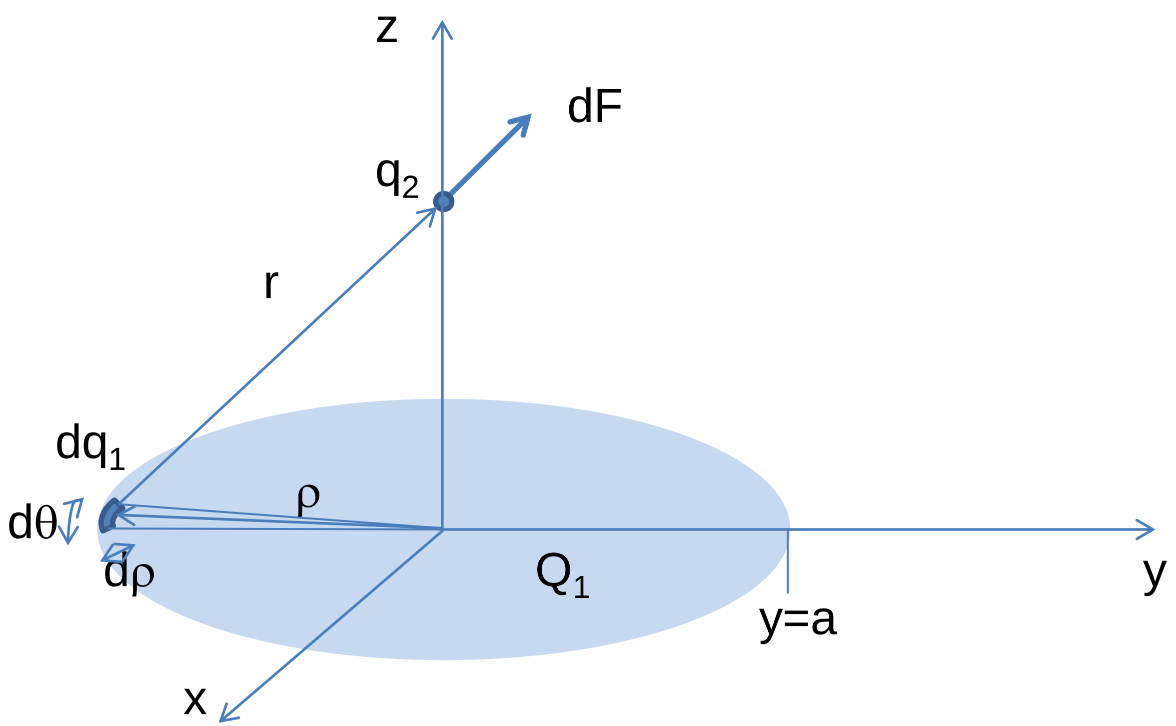


$$\left|d\vec{F}\right|_z = \frac{1}{4\pi\varepsilon_0}\frac{\left|q_2\right|}{\left(\rho^2+z^2\right)}\frac{z}{r}\sigma\rho d\theta d\rho$$

$$= \frac{z}{4\pi\varepsilon_0}\frac{\left|q_2\right|}{\left(\rho^2+z^2\right)^{3/2}}\sigma\rho d\theta d\rho$$

Coulomb's Law With Extended Objects

- Example: finite disk.



$$\left|d\vec{F}\right|_z = \frac{z}{4\pi\varepsilon_0}\frac{|q_2|}{\left(\rho^2+z^2\right)^{3/2}}\sigma\rho\, d\theta\, d\rho$$

$$F_z = \frac{z|q_2|\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^a \frac{\rho}{\left(\rho^2+z^2\right)^{3/2}}d\rho\, d\theta$$

$$= \frac{z|q_2|\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}d\theta\int_0^a \frac{\rho}{\left(\rho^2+z^2\right)^{3/2}}d\rho$$

$$= \frac{z|q_2|\sigma}{4\pi\varepsilon_0}2\pi\frac{1}{2}\int_0^a \frac{2\rho}{\left(\rho^2+z^2\right)^{3/2}}d\rho$$

$$= \frac{z|q_2|\sigma}{2\varepsilon_0}\left[\frac{1}{\sqrt{\rho^2+z^2}}\right]_0^a$$

$$= \frac{|q_2|\sigma}{2\varepsilon_0}\left[1-\frac{1}{\sqrt{a^2/z^2+1}}\right]$$