# #35 Kepler's Law and Black Holes Pre-class

Due: 11:00am on Wednesday, November 14, 2012

***Note: You will receive no credit for late submissions.*** To learn more, read your instructor's Grading Policy

## Satellites and Kepler's Laws

This applet shows satellites orbiting a central body from various starting distances. The mass of the central body is much greater than the masses of the satellites. You should think of this applet as data obtained in an experiment (e.g., a set of videos made by compiling data of observations of satellites).

### Part A

Determine the orbital period $T_{\rm p}$ of the purple (innermost) satellite when it has an initial speed of 2.0. (For this problem, the units are chosen for convenience and may be ignored for the first few parts of the problem.) Note that simply watching one orbit and seeing how long it takes will not give you three significant figures. Try to think of a technique that will.

**Express your answer to three significant figures.**

**Hint 1.** How to approach the problem

The applet does not give an accurate enough accounting of the position and time for you to get three significant figures by watching just a single orbit. However, watching one orbit would allow you to measure the orbital period to two significant figures. Therefore, if you observe the amount of time to complete 10 orbits, you should have a time measurement with three significant figures. Dividing this by the number of orbits will give you the orbital period to three significant figures.

You can use the arrow buttons at the lower right-hand corner of the applet to move the time back and forth in 0.1-s steps to get an accurate measure of when the satellite finishes its 10th orbit.

ANSWER:

$T_{\rm p}$ = 3.16

**Correct**

## Part B

Determine the period $T_r$ of revolution for the red satellite if its initial speed is 1.5.

**Express your answer to three significant figures.**

**Hint 1.** Obtaining the desired accuracy

Finding the time for the red satellite to complete three orbits and dividing by 3 will give the orbital period to three significant figures.

ANSWER:

$T_r$ = 10.7

**Correct**

## Part C

Find the length of the major axis $2a_p$ of the orbit of the purple satellite with initial speed 2.0 and the length of the major axis $2a_r$ of the orbit of the red satellite with initial speed 1.5. Recall that the semi-major axis of an ellipse is usually denoted $a$; hence we use the notation $2a_p$ for the major axis of the purple satellite's orbit.

**Give the length of the major axis for the purple satellite followed by the length for that of the red satellite separated by a comma. Express your answers to two significant figures.**

**Hint 1.** How to approach the problem

You will need to find the length of the longest segment connecting two points on the ellipse, which passes through the two foci of the ellipse. (This is the definition of the major axis.) Since the initial speed at the top of the orbit is purely tangential, one axis will be vertical. The other must be perpendicular to the first, and so it is horizontal. Whichever is longer is the major axis.

ANSWER:

$(2a_p, 2a_r)$ = 2.0,4.6

**Correct**

## Part D

Now, use the periods (obtained in Parts A and B) and Kepler's laws to determine the ratio $a_r/a_p$ of the semi-major axes.

**Express your answer to three significant figures. Note that if you use your results from part C, your answer will have two significant figures instead of three.**

**Hint 1.** Kepler's 3rd law

Recall that Kepler's 3rd law states the following:
The periods of the planets are proportional to the 3/2 powers of the semi-major axis lengths of their orbits.

In other words, if you have an orbit of period $T$, then the length $a$ of the semi-major axis will be related to the period by $T = k(a)^{3/2}$, where $k$ is a constant of proportionality. If you set up this relation for two different orbits, then you can divide the equations to eliminate the unknown constant $k$.

ANSWER:

$\frac{a_r}{a_p}$ = 2.25

**Correct**

Notice that if you take your answer from Part C and calculate the ratio $a_r/a_p$, you will get 2.3, which is the same as your answer here to two significant figures.

**Now, assume that the speed is measured in kilometers per second and the distance is measured in hundreds of thousands of kilometers.** (e.g., A distance measured as 5 would be $5 \times 10^5$ km.)

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## Part E

Which of the following techniques could be used to determine the mass $m_c$ of the central object, using only information available in the applet?

**Check all that apply.**

ANSWER:

- [ ] Determine the initial potential energy of one of the satellites and use it to find the central mass.
- [x] Determine the escape speed of one of the satellites from its initial position, and use it to find the central mass.
- [x] Determine the speed for a circular orbit of one of the satellites and use it to find the central mass.

**Correct**

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## Part F

Use one of these techniques to determine the mass $m_c$ of the central object. Be sure to think about which satellite will give the most accurate information for the method you choose. Experiment with the applet to help you make this choice.

**Express your answer in kilograms to two significant figures and remember that speed is measured in kilometers per second and distance is**

**measured in hundreds of thousands of kilometers.**

### **Hint 1.** How to approach the problem using escape speed

You can determine the escape speed of a satellite, roughly, by increasing the speed in increments of 0.1 until you reach a speed where the satellite does not return to the screen before the applet runs out of time. That speed is the escape speed, at least to the accuracy with which the applet can determine the escape speed. Once you have determined the escape speed experimentally, then just use the formula for the escape speed to determine the mass of the central body. To get the best accuracy you should try to find one of the satellites that has a very eccentric orbit that just barely returns to the screen by the end of the applet to be sure that the next increment up in speed is close to the escape speed.

Which satellite gives the most accurate information for the escape velocity?

ANSWER:

- the purple one
- the red one
- the white one

### **Hint 2.** Formula for escape speed

The escape speed $v_{\rm escape}$ of an object at distance $R$ from the center of a large body of mass $M$ is given by the formula

$$v_{\rm escape} = \sqrt{\frac{2GM}{R}}.$$

### **Hint 3.** How to approach the problem using circular orbits

The first step would be to adjust the speed until the satellite follows a circular orbit. Different satellites will take variously shaped orbits, so for this technique it is best to use the satellite for which you can make the orbit the most nearly circular, rather than averaging all three with some of them having less than ideally circular orbits. Once you find the speed of the satellite for the most nearly circular orbit, use the formula for speed in a circular orbit to determine the mass of the central object.

Which satellite can you get into the most nearly circular orbit?

ANSWER:

- the purple one
- the red one
- the white one

**Hint 4.** Formula for the speed of a satellite in a circular orbit

The speed $v$ of a satellite in a circular orbit of radius $r$ around a large body of mass $M$ is given by the formula

$$v = \sqrt{\frac{GM}{r}}.$$

ANSWER:

$m_e$ = 5.87×10$^{24}$ $\mathrm{kg}$

**Correct**

## Part G

Using only information available in the applet to determine the initial potential energy of the red satellite, can you find the red satellite's mass?

ANSWER:

- yes
- no

**Correct**

## Part H

Using only information available in the applet to determine the escape speed of the red satellite from its initial position, can you find the red satellite's mass?

ANSWER:

- ○ yes
- ◉ no

**Correct**

### Part I

Using only information available in the applet to determine the speed for a circular orbit of the red satellite, can you find the red satellite's mass?

ANSWER:

- yes
- no

**Correct**

None of the techniques will work and hence the mass of an orbiting satellite cannot be found using information in the applet. Interestingly, the mass of the satellite cancels out of the expressions for the speed in a circular orbit and the escape speed. The potential energy could be used to determine the satellite's mass, but the potential energy cannot be determined directly from the applet.

## At the Galaxy's Core

Astronomers have observed a small, massive object at the center of our Milky Way galaxy. A ring of material orbits this massive object; the ring has a diameter of about 15 light years and an orbital speed of about 200 $\mathrm{km/s}$.

### Part A

Determine the mass $M$ of the massive object at the center of the Milky Way galaxy.

Take the distance of one light year to be $9.461 \times 10^{15}\ \mathrm{m}$.

**Express your answer in kilograms.**

**Hint 1.** How to approach the problem

Each small piece of material in the ring can be considered a satellite orbiting the massive object. Since the orbit is assumed to be circular, the centripetal force keeping the ring in orbit must be equal to the gravitational force that the massive object at the center must exert on the ring. Solve the resulting equation for the mass of the object. Also, be careful about the units in your calculations.

**Hint 2.** Find an equation for the velocity of an orbiting satellite

The gravitational force on the ring is $GMm_{\rm ring}/r^2$, where $G$ is the gravitational constant, $M$ is the mass of the object in the center, $m_{\rm ring}$ is the mass of a small piece of the ring, and $r$ is the radius of the ring. The centripetal force on the orbiting ring is $m_{\rm ring}v^2/r$, where $v$ is the orbital speed of the ring.

What is the equation for the orbital speed $v$ of the ring?

**Express your answer in terms of the gravitational constant $G$, $M$, and $r$.**

ANSWER:

$v = \sqrt{\frac{GM}{r}}$

ANSWER:

$M$ = $4.26 \times 10^{37}$ $\rm kg$

**Correct**

## Part B

Express your answer in solar masses instead of kilograms, where one solar mass is equal to the mass of the sun, which is $1.99 \times 10^{30}\ \rm kg$.

ANSWER:

$M$ = $2.14\times10^{7}$ solar masses

**Correct**

## Part C

Observations of stars, as well as theories of the structure of stars, suggest that it is impossible for a single star to have a mass of more than about 50 solar masses. Can this massive object be a single, ordinary star?

ANSWER:

- yes
- no (selected)

**Correct**

## Part D

Many astronomers believe that the massive object at the center of the Milky Way galaxy is a black hole. If so, what is its Schwarzschild radius $R_S$?

**Express your answer in meters.**

**Hint 1.** Equation for the Schwarzschild radius

Einstein's theory of relativity gives $R_S = 2GM/c^2$ as the Schwarzschild radius, where $G$ is the gravitational constant, $M$ is the mass of the object, and $c$ is the speed of light.

ANSWER:

$R_S$ = $6.31 \times 10^{10}$ m

**Correct**

The Schwarzschild radius of an object is the distance within which nothing, not even light, can escape its gravitational attraction. The sphere surrounding a black hole whose radius is the Schwarzschild radius is also called the *event horizon*.

### Part E

Would a black hole of this size fit inside the earth's orbit around the sun? The mean distance from the sun to the earth is $1.5 \times 10^{11}$ m.

ANSWER:

- (selected) yes
- no

**Correct**

In other words, it would be possible for the earth to orbit the black hole at the same distance that it is from the sun without falling into the event horizon. However, since the black hole is much more massive than the sun, the speed of the earth's orbit would be incredibly high. In fact, if our sun were replaced by the black hole, it would make one earth year (the time to make one complete orbit) equal to just a few hours!

## Score Summary:

Your score on this assignment is 101.5%.
You received 10.15 out of a possible total of 10 points.