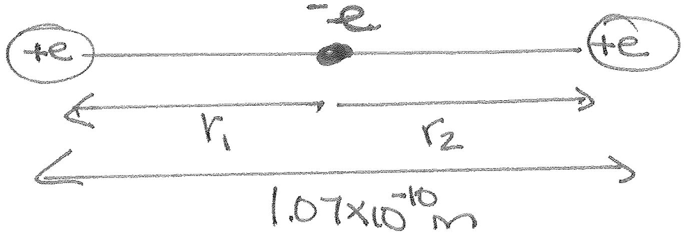
Physics 161, HW#3

#1



electron HALFWAY Between

a) FIND electron's potential Energy due to protons

Potential ENERGY ADDS $\Rightarrow U = U_1 + U_2$

Pot. Energy due to proton on left

proton on right

Point charges $\Rightarrow U_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_e}{r_1} = \frac{1}{4\pi\epsilon_0}\frac{(+e)(-e)}{r_1} = \frac{-1}{4\pi\epsilon_0}\frac{e^2}{r_1}$

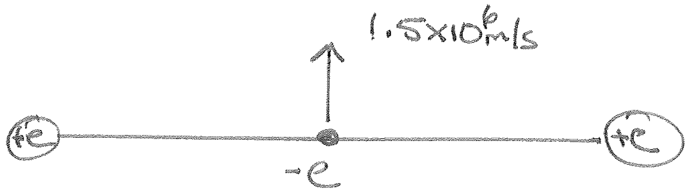
$r_1 = \frac{1}{2}(1.07\times10^{-10}\,m) = .535\times10^{-10}\,m$

$U_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2 q}{r_2} = \frac{-1}{4\pi\epsilon_0}\frac{e^2}{r_2}$ $\quad r_2 = r_1 = .535\times10^{-10}\,m$

$\Rightarrow U = 2\left(\frac{-1}{4\pi\epsilon_0}\right)\frac{e^2}{r_1} = -2(8.99\times10^9\,N\cdot m^2/C^2)\frac{(1.6\times10^{-19}\,C)^2}{.535\times10^{-10}\,m}$

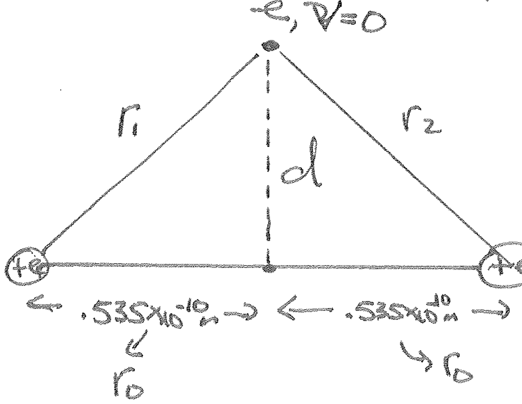
$\Rightarrow \boxed{U = -8.6\times10^{-18}\,J}$

b)



How FAR CAN Electron go?

electron moves until speed is ZERO



AT STOPPING point

$$U = \frac{1}{4\pi\epsilon_0}\frac{-e^2}{\sqrt{r_0^2+d^2}} + \frac{1}{4\pi\epsilon_0}\frac{-e^2}{\sqrt{r_0^2+d^2}}$$

$$= -\frac{2}{4\pi\epsilon_0}\frac{e^2}{\sqrt{r_0^2+d^2}}$$

CONSERVATION OF ENERGY: $K_1 + U_1 = K_2 + U_2$, $K = \frac{1}{2}mv^2$

$$\Rightarrow \frac{1}{2}(9.11\times10^{-31}\,kg)(1.5\times10^{6}\,m/s)^2 + -8.6\times10^{-18}\,J = 0 - \frac{2}{4\pi\epsilon_0}\frac{e^2}{\sqrt{r_0^2+d^2}}$$

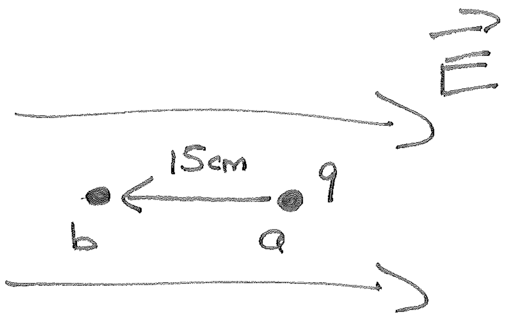
$$\Rightarrow -7.575\times10^{-18}\,J = \frac{-2(8.99\times10^{9}\,Nm^2/C^2)(1.6\times10^{-19}\,C)^2}{\sqrt{r_0^2+d^2}}$$

$$\Rightarrow \sqrt{r_0^2+d^2} = 6.0764\times10^{-11}\,m$$

$$\Rightarrow d^2 = (6.0764\times10^{-11}\,m)^2 - r_0^2 = (6.0764\times10^{-11}\,m)^2 - (.535\times10^{-10}\,m)^2$$

$$\Rightarrow \boxed{d = \sqrt{8.3\times10^{-22}\,m^2} = 2.881\times10^{-11}\,m}$$

#2



$q = -25\,nC = -25\times10^{-9}\,C$

q started from rest $\Rightarrow K_a = 0$

Other force does $W_{other} = 8\times10^{-5}\,J$

At b $K_b = 9.75\times10^{-5}\,J$

a) What is work done by Electric Field?

Electric Field and other force both do work

$\therefore\ W_{TOTAL} = \underbrace{W_{a\to b}}_{\text{work done by Electric Field}} + W_{OTHER}$

Work-Energy Theorem: $W_{TOTAL} = \Delta K = K_b - K_a$

$K_a = 0 \Rightarrow W_{TOTAL} = K_b - 0 = K_b$

$\therefore\ W_{a\to b} + W_{other} = K_b \Rightarrow W_{a\to b} = K_b - W_{other}$

$\Rightarrow W_{a\to b} = 9.75\times10^{-5}\,J - 8\times10^{-5}\,J \Rightarrow \boxed{W_{a\to b} = 1.75\times10^{-5}\,J}$

Negative charge would go faster from a to b so makes sense

b) What is potential of starting point with Respect to END Point? $\Rightarrow V_{ab} = ?$

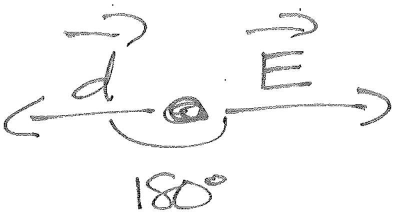
$$\boxed{V_{ab} = \frac{W_{a\to b}}{q} = \frac{1.75\times10^{-5}\,J}{-25\times10^{-9}\,C} = -700\,V}$$

Since $\vec{E}$ to Right b is at lower potential than a.

c) What is magnitude of E?

Uniform Field, ~~[illegible]~~ $\Rightarrow$

$V_{ab} = Ed\cos\phi$



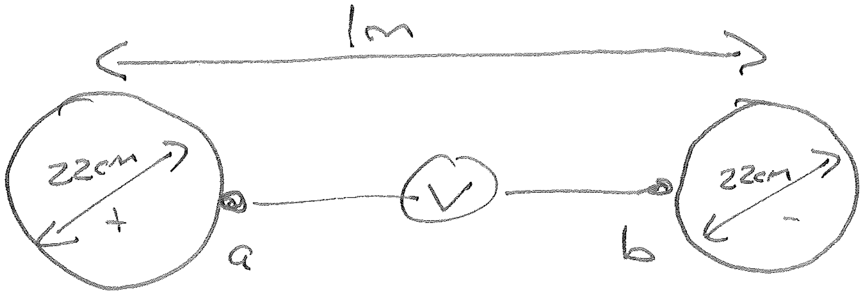
$d = 15\,cm$
$= .15\,m$

$\Rightarrow V_{ab} = Ed\cos 180° = -Ed$

$\therefore E = \frac{V_{ab}}{d} = \frac{-(-700V)}{0.15m} = 4666.6\ldots\ V/m$

$\therefore \boxed{E = +4670\ V/m}$

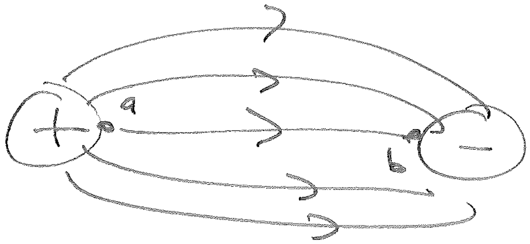
#3



a) which point is at higher Potential

Electric fields point Positive to Negative.

This will be A Dipole-like field. It's probably a little more Complicated due to the Charge Distributions but, it will be the SAME BASIC SHAPE $\Rightarrow$



$\vec{E}$ from a to b.

$\vec{E}$ in DIRECTION OF decreasing Potential $\Rightarrow$

$\boxed{\text{a at Higher Potential than b}}$

So $V_{ab} = V_a - V_b$ will be positive

b) Assume Voltmeter connected to read V_{ab} to get a positive number.

Outside (or even on surface) of insulating sphere, Gauss's Law tells us Electric field is equivalent to a point charge

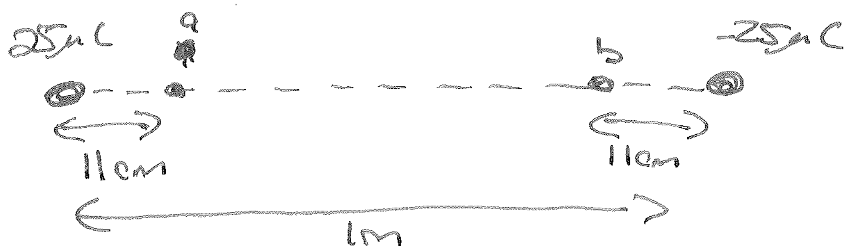
total charge enclosed $Q_{total} = \rho\left(\frac{4}{3}\pi\right)r_s^3$

radii of spheres $= \frac{22cm}{2}$

$= 11cm$

$\therefore\ Q_{TOTAL} = (4480\mu C/m^3)\left(\frac{4}{3}\pi\right)(0.11m)^3 = 24.977\mu C \approx 25\mu C$

So the equivalent picture looks like:

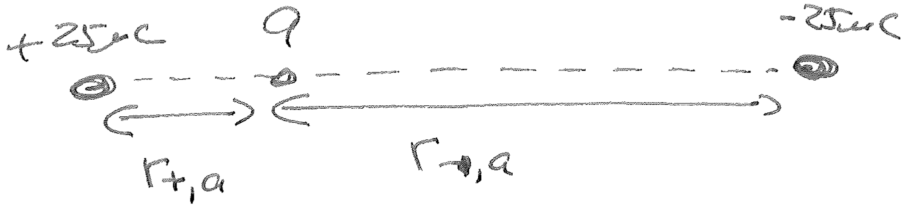


Point charge: $V = \frac{kq}{r}$

For two point charges:

$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$

For Point (a)



$$V_a = \frac{Kq_+}{r_{+,a}} + \frac{Kq_-}{r_{-,a}}$$

$q_+ = 25\mu C = 25\times10^{-6} C$

$r_{+,a} = 0.11m$

$q_- = -25\mu C$, $r_{-,a} = 1m - 0.11m$
$= 0.89m$

$$V_a = \frac{K\overset{+25\mu C}{q}}{r_{+,a}} + \frac{K(\overset{-25\mu C}{-q})}{r_{-,a}} = Kq\left(\frac{1}{r_{+,a}} - \frac{1}{r_{-,a}}\right) = \left(9\times10^{9}\frac{N.m^2}{C^2}\right)\left(25\times10^{-6}\right)\left(\frac{1}{0.11m} - \frac{1}{\underset{0.89m}{0.89}}\right)$$

$$\Rightarrow V_a = 1.79\times10^{6} V = 1.79MV$$

$$V_b = \frac{Kq_+}{r_{+,b}} + \frac{Kq_-}{r_{-,b}} = \frac{K(q)}{r_{+,b}} + \frac{K(-q)}{r_{-,b}} = Kq\left(\frac{1}{r_{+,b}} - \frac{1}{r_{-,b}}\right)$$

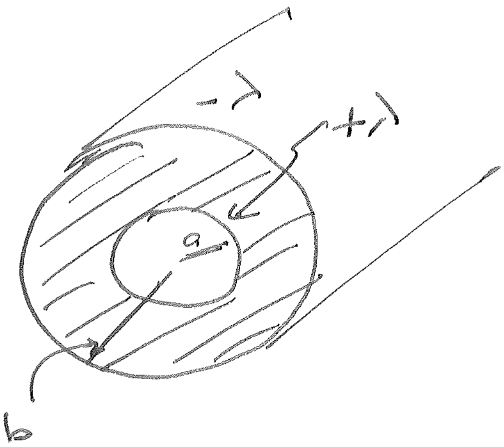
$r_{-,b} = 0.11m$

$r_{+,b} = 0.89m$

$\Rightarrow V_b = -1.79MV$

$$\therefore V_a - V_b = 1.79MV - (-1.79MV) \Rightarrow \boxed{V_a - V_b = V_{ab} = 3.58MV}$$

#4



From 0 to a: Conductor

From a to b: Insulator

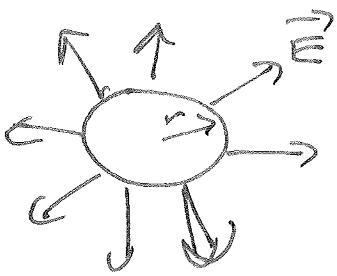
Small conductor at b

a) ~~Find V(r) take V = 0 at r = a~~

First Find Electric field

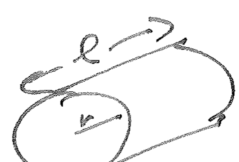
$0 < r < a$: Inside Conductor $\Rightarrow$ $\boxed{E = 0}$

$a < r < b$: Cylinder $\Rightarrow$ Radial Symmetry



E same magnitude at radius r

so for a Gaussian cylinder of radius r and length ℓ



$$\Phi_E = E(2\pi r \ell)$$

$0 < r < b$ $\Rightarrow$ only enclose inner conductor $\Rightarrow$

$$Q_{encl} = +\lambda(\ell) = +\lambda\ell$$

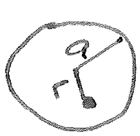
$$\therefore\ E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0} \Rightarrow \underline{\underline{E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}}} \quad a<r<b$$

for $r>b$ $\quad Q_{encl} = \underbrace{\lambda\ell}_{\text{inner conductor}} \underbrace{- \lambda\ell}_{\text{outer conductor}} = 0 \Rightarrow \underline{\underline{E=0}}$ again

b) Since Electric Field is Radial

$$V_1 - V_2 = \int_1^2 E\,dr$$

for $r<a$, $E=0 \Rightarrow V_r - V_a = 0 \Rightarrow V_r = V_a$

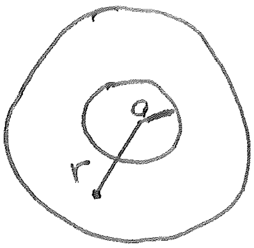


V_r: potential at distance r from center

but we're told to make $V_a = 0$

$$\Rightarrow \boxed{V_r = 0 \text{ for } r<a}$$

For $a<r<b$ $\quad E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$



$$V_r - V_a = \int_r^a \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\,dr$$

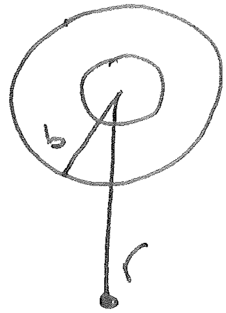
$$\Rightarrow V_r - 0 = \frac{\lambda}{2\pi\epsilon_0}\int_r^a \frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}(\ln r)\Big|_r^a$$

$$\Rightarrow V_r = \frac{\lambda}{2\pi\epsilon_0}(\ln a - \ln r) = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{a}{r}\right)$$

Since $r>a$, it might make more sense to write $V_r = \frac{-\lambda}{2\pi\epsilon_0}(\ln r - \ln a) = \frac{-\lambda}{2\pi\epsilon_0}\ln\left(\frac{r}{a}\right)$

Since otherwise the negative sign will be hidden and we know that potential is negative in this region since $\vec{E}$ is outwards and potential decreases in the direction of $\vec{E}$ so if V starts zero at $r=a$ and gets smaller, it must be negative.

For $r > b$ $E = 0$ ~~$\rightarrow V = V_b$~~



$$\int_r^b E\,dr = 0 \Rightarrow V_r - V_b = 0$$

$$\Rightarrow V_r = V_b$$

~~Potential Must Be Continuous~~

Using Previous Expression for $a < r < b$

$$V(b) = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$$

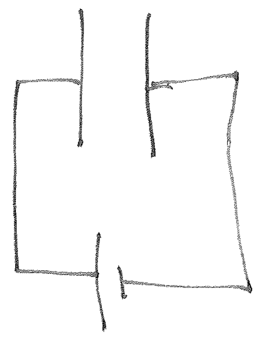
So for $r > b$ $V_r = \frac{-\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$

c) Show $V_{ab} = \frac{+\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$

$$V_{ab} = V_a - V_b = 0 - \left(\frac{-\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)\right) = \underline{\underline{\frac{+\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)}}$$

You should CHECK that in the ORIGINAL BOOK Problem, you were supposed to set $V = 0$ at $r = b$. Notice that the V_{ab} expression is the same. V_{ab} is independent of your choice of zero.

#5



$C = 22.5\mu F = 22.5\times10^{-6} F$

Dielectric MATERIAL WITH $K = 5.2$ PUT BETWEEN PLATES

a) HOW MUCH STORED ENERGY BEFORE AND AFTER?

POWER SUPPLY ENSURES THAT $V_{ab} = 16V$ BOTH BEFORE AND AFTER.

$\Rightarrow$ USE $U = \frac{1}{2} C V_{ab}^2$

BEFORE: $U_1 = \frac{1}{2} C_1 V_{ab}^2 = \frac{1}{2}(22.5\times10^{-6}F)(16V)^2 \Rightarrow \boxed{U_1 = .00288J = 2.88 mJ}$

mili

AFTER: INSERTING DIELECTRIC INCREASES CAPACITANCE

to $C_2 = K C_1 = 5.2(22.5\times10^{-6}F) = 117\times10^{-6}F$

$\therefore U_2 = \frac{1}{2} C_2 V_{ab}^2 = \frac{1}{2}(117\times10^{-6}F)(16V)^2 \Rightarrow \boxed{U_2 = .014976J = 14.976 mJ}$

b) $\Delta U = ?$ $\boxed{\Delta U = 14.976mJ - 2.88mJ = 12.096mJ}$ $\leftarrow$ INCREASED

NOTE: ENERGY INCREASES BECAUSE IT WOULD REQUIRE WORK to be done to ~~CAPACITOR~~ INSERT DIELECTRIC.

#6 FLASH LASTS FOR $t = \frac{1}{675}$ S

WITH Power $= 3.1\times10^5$ Watt, AND 89% EFFICIENCY.

a) HOW MUCH ENERGY STORED FOR ONE FLASH?

SINCE ONLY 89% EFFICIENT, CAPACITOR Needs to deliver a Power

$\cancel{P} = \frac{3.1\times10^5 \text{ Watt}}{.89} = 3.483\times10^5$ Watt

Capacitor Loses Energy $\Rightarrow$ ~~P = 3.483×10^5 Watt~~, $P = \frac{W}{\Delta t} = \frac{-\Delta U}{\Delta t}$

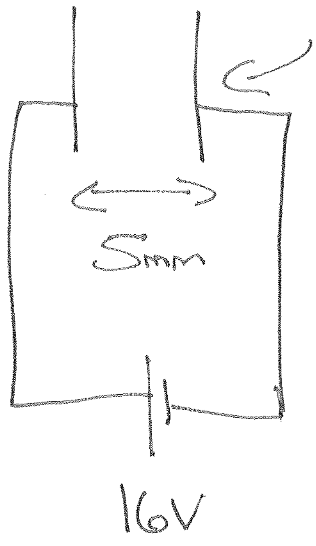
$\Delta U = U_2 - U_1$. $U_2 = 0$, $U_1 = U = ?$ $\Rightarrow P = \frac{-(0-U)}{\Delta t} = \frac{U}{\Delta t}$

$\Rightarrow U = P\Delta t = (3.483\times10^5 \text{ Watt})(\frac{1}{675}\text{ S}) = 516\text{ J}$

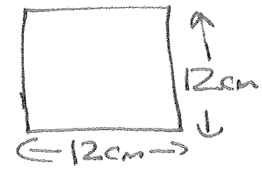
b) $V_{ab} = 125\text{V}$, $C = ?$ $U = \frac{1}{2}CV_{ab}^2 \Rightarrow C = \frac{2U}{V_{ab}^2}$

$$\Rightarrow \boxed{C = \frac{2(516\text{J})}{(125\text{V})^2} = .066\text{F}}$$

#7



Plates 12cm SQUARE



$A = (.12M)(.12M)$
$= .0144M^2$

a) WHAT IS CAPACITANCE?

$$C = \epsilon_0 A/d = \frac{(8.85\times10^{-12}\ C^2/N\cdot M^2)(.0144M^2)}{(5\times10^{-3}M)} = 2.5488\times10^{-11}F$$

Air-Filled

$$\Rightarrow \boxed{C = 2.55\times10^{-11}F}$$

$$\text{Unit: } \frac{C^2\cdot M^2}{N\cdot M^2\cdot M} = \frac{C^2}{N\cdot M} = \frac{C^2}{J} = \frac{C}{J/C} = \frac{C}{V} = F$$

b) WHAT IS CHARGE? $C = \frac{Q}{V_{ab}}$ battery ensures $V_{ab} = 16V$

$$\therefore\ Q = CV_{ab} = (2.5488\times10^{-11}F)(16V) \Rightarrow \boxed{Q = 4.078\times10^{-10}C = 4.08\times10^{-10}C}$$

c) WHAT IS E between plates?

ASSUME UNIFORM FIELD $\Rightarrow V = Ed \Rightarrow E = V/d$

$$\boxed{E = \frac{16V}{5\times10^{-3}m} = 3200\,V/m}$$

d) WHAT ENERGY IS STORED? $U = \frac{1}{2}CV^2 = \frac{1}{2}(2.5\overset{488}{\ }\times10^{-11}F)(16V)^2$

$$\Rightarrow \boxed{U = 3.26\times10^{-9}J}$$

e) THE Battery IS DISCONNECTED AND d IS INCREASED to 7.4mm. Repeat parts (a)-(d).

MOST IMPORTANTLY, DISCONNECTING battery MEANS ~~NO~~ CHARGE CAN'T LEAVE THE PLATES $\Rightarrow \boxed{Q = 4.08\times10^{-10}C \text{ still}}$ THE POTENTIAL BETWEEN THE PLATES will CHANGE!

$$C = \epsilon_0\frac{A}{d} = (8.85\times10^{-12}C^2/Nm^2)\left(\frac{.0144m^2}{7.5\times10^{-3}m}\right) \Rightarrow \boxed{C = 1.699\times10^{-11}F = 1.7\times10^{-11}F}$$

$$C = \frac{Q}{V_{ab}} \Rightarrow V_{ab} = \frac{Q}{C} = \frac{4.08\times10^{-10}C}{1.699\times10^{-11}F} \Rightarrow \boxed{V_{ab} = 24V}$$

$$E = \frac{V}{d} = \frac{24V}{7.5\times10^{-3}m} \Rightarrow \boxed{E = 3200\,V/m}$$ ← UNCHANGED!

ANOTHER WAY to SEE THIS IS to REMEMBER $E = \sigma/\epsilon_0$. No change in CHARGE OR AREA OF plates $\Rightarrow$ SAME σ $\Rightarrow$ UNCHANGED E.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(1.699\times10^{-11}F)(24V)^2 \Rightarrow \boxed{U = 4.89\times10^{-9}J}$$ ← REQUIRES WORK to pull plates APART