#23 Conservation of Momentum Post-class

Due: 11:00am on Wednesday, October 17, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Exercise 8.17

On a frictionless, horizontal air table, puck *A* (with mass 0.245kg) is moving toward puck *B* (with mass 0.360kg), that is initially at rest. After the collision, puck *A* has a velocity of 0.124$\mathrm{m/s}$ to the left, and puck *B* has velocity 0.655$\mathrm{m/s}$ to the right.

Part A

What was the speed of puck *A* before the collision?

ANSWER:

v = 0.838 $\mathrm{m/s}$

Correct

Part B

Calculate the change in the total kinetic energy of the system that occurs during the collision.

ANSWER:

ΔK = -7.01×10^{-3} J

Correct

Exercise 8.19

The expanding gases that leave the muzzle of a rifle also contribute to the recoil. A .30 caliber bullet has mass $7.20\times10^{-3}\,\mathrm{kg}$ and a speed of $601\,\mathrm{m/s}$ relative to the muzzle when fired from a rifle that has mass $2.90\,\mathrm{kg}$. The loosely held rifle recoils at a speed of $1.95\,\mathrm{m/s}$ relative to the earth.

Part A

Find the momentum of the propellant gases in a coordinate system attached to the earth as they leave the muzzle of the rifle.

ANSWER:

p = 1.34 $\mathrm{kg\cdot m/s}$

Correct

Exercise 8.23

The nucleus of $^{214}\mathrm{Po}$ decays radioactively by emitting an alpha particle (mass $6.65\times10^{-27}\,\mathrm{kg}$) with kinetic energy $1.17\times10^{-12}\,\mathrm{J}$, as measured in the laboratory reference frame.

Part A

Assuming that the Po was initially at rest in this frame, find the magnitude of the recoil velocity of the nucleus that remains after the decay.

ANSWER:

v = 3.58×10^{5} $\mathrm{m/s}$

Correct

Exercise 8.16

You are standing on a sheet of ice that covers the football stadium parking lot in Buffalo; there is negligible friction between your feet and the ice. A friend throws you a ball of mass 0.400kg that is traveling horizontally at 11.5$\mathrm{m/s}$. Your mass is 70.0kg .

Part A

If you catch the ball, with what speed do you and the ball move afterwards?

ANSWER:

v = 6.53 $\mathrm{cm/s}$

Correct

Part B

If the ball hits you and bounces off your chest, so afterwards it is moving horizontally at 7.00$\mathrm{m/s}$ in the opposite direction, what is your speed after the collision?

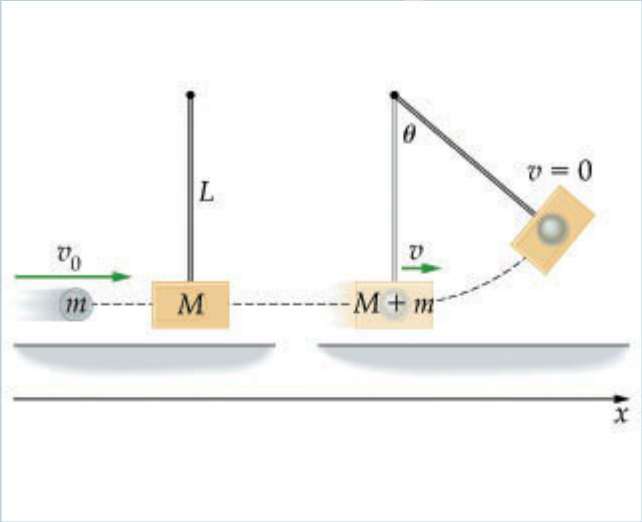
ANSWER:

v = 10.6 $\mathrm{cm/s}$

Correct

Ballistic Pendulum

In a ballistic pendulum an object of mass m is fired with an initial speed v_0 at a pendulum bob. The bob has a mass M, which is suspended by a rod of length L and negligible mass. After the collision, the pendulum and object stick together and swing to a maximum angular displacement θ as shown .



Part A

Find an expression for v_0, the initial speed of the fired object.

Express your answer in terms of some or all of the variables m, M, L, and θ and the acceleration due to gravity, g.

Hint 1. How to approach the problem

There are two distinct physical processes at work in the ballistic pendulum. You must treat the collision and the following swing as two separate events. Identify which physical law or principle applies to each event, write an expression to describe the collision, write an expression to describe the swing, and then relate the two expressions to find v_0.

Hint 2. Determine which physical laws and principles apply

Which of the following physical laws or principles can best be used to analyze the collision between the object and the pendulum bob? Which can best be used to analyze the resulting swing?

1. Newton's first law
2. Newton's second law
3. Newton's third law
4. Conservation of mechanical energy
5. Conservation of momentum

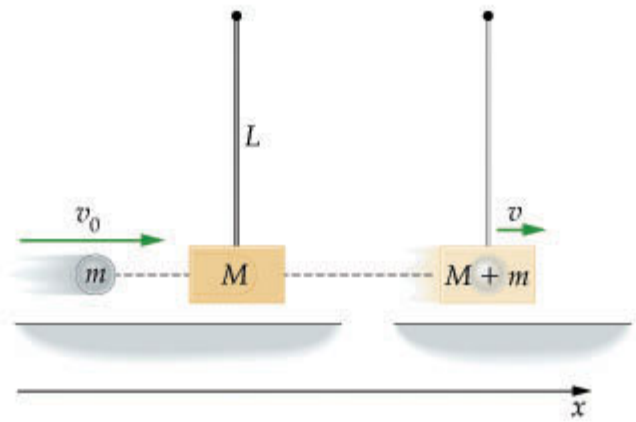
Enter the letters corresponding to the correct answers for the collision and the swing separated by a comma.

ANSWER:

E,D

Hint 3. Describe the collision

Compose an expression that describes the collision between the object and the pendulum bob. Put this expression in the form $v_0 = \cdots$.



Express your answer in terms of some or all of the variables m, M, v_0, v, L, and θ and the acceleration due to gravity, g.

Hint 1. Identify the type of collision

Is the collision between the object and the pendulum bob an elastic or inelastic collision?

ANSWER:

- elastic
- inelastic

Hint 2. Find the momentum before the collision

Compose an expression for p_{before}, the momentum of the object and pendulum bob before the collision when the object moves with speed v_0.

Express your answer in terms of some or all of the variables m, M, v_0, v, L, and θ and the acceleration due to gravity, g.

Hint 1. Momentum

The momentum of an object of mass m moving with speed v is given by mv.

ANSWER:

p_{before} = mv_0

Hint 3. Find the momentum after the collision

Compose an expression for p_{after}, the momentum of the object and pendulum bob after the collision when they move with speed v.

Express your answer in terms of some or all of the variables m, M, v_0, v, L, and θ and the acceleration due to gravity, g.

ANSWER:

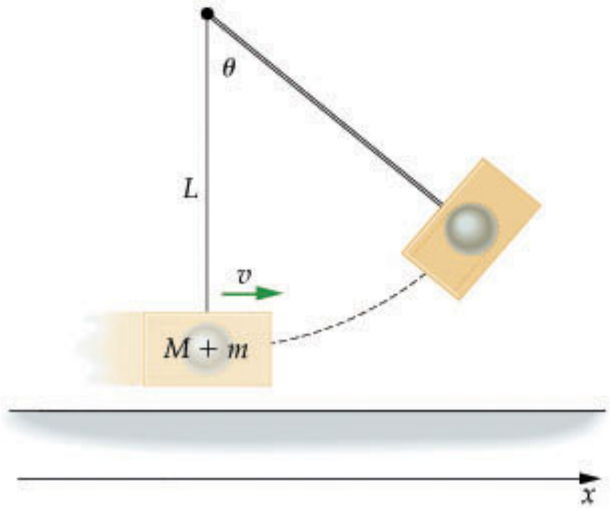
p_{after} = $(m+M)\,v$

ANSWER:

v_0 = $\frac{m+M}{m}v$

Hint 4. Describe the swing

Compose an expression that describes the motion of the object and the pendulum bob after the collision. Put this expression in the form $v = \cdots$.



Express your answer in terms of some or all of the variables m, M, v_0, v, L, and θ and the acceleration due to gravity g.

Hint 1. Identify the energy at the bottom of the swing

What is the mechanical energy $E_{\rm bottom}$ of the object and pendulum bob just after the collision but while they are still located at the bottom of the swing? Assume that the height of the pendulum bob and object is zero at this location.

Express your answer in terms of some or all of the variables m, M, v_0, v, L, and θ and the acceleration due to gravity, g.

Hint 1. Mechanical energy

The mechanical energy of a system is the total kinetic and gravitational potential energy of the system. The kinetic energy K of an object of mass m moving with speed v is

$$K = \frac{1}{2}mv^2.$$

The gravitational potential energy U of an object of mass m a height y above some reference point is

$$U = mgy.$$

Hint 2. Find the gravitational potential energy at the bottom of the swing

What is the gravitational potential energy U_{bottom} of the object and pendulum bob at the bottom of the swing? Keep in mind that the height of the pendulum bob and object is zero at this location.

ANSWER:

- ○ $U_{\text{bottom}} = mgL$
- ◉ $U_{\text{bottom}} = 0$
- ○ $U_{\text{bottom}} = -mgL$

Hint 3. Find the kinetic energy at the bottom of the swing

What is the kinetic energy K_{bottom} of the object and pendulum bob at the bottom of the swing?

Express your answer in terms of some or all of the variables m, M, v_0, v, L, and θ and the acceleration due to gravity, g.

ANSWER:

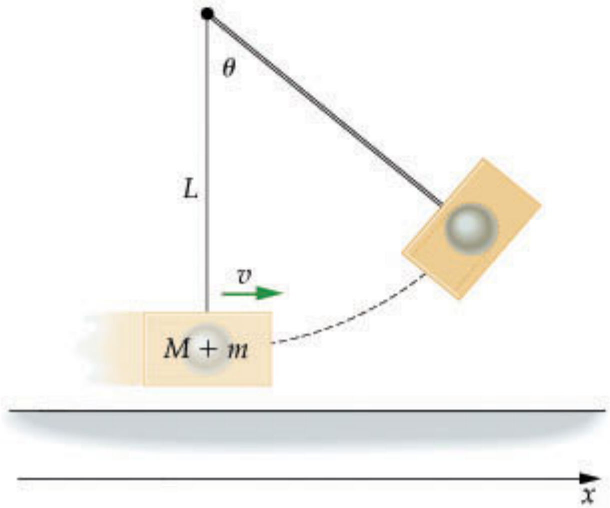
K_{bottom} = $.5\,(m+M)\,v^2$

ANSWER:

E_{bottom} = $.5\,(m+M)\,v^2$

Hint 2. Identify the energy at the top of the swing

What is the mechanical energy E_{top} of the object and pendulum bob at the top of the swing when it has reached its maximum angular displacement?



Express your answer in terms of some or all of the variables m, M, v_0, v, L, and θ and the acceleration due to gravity, g.

Hint 1. Mechanical energy

The mechanical energy of a system is the total kinetic and gravitational potential energy of the system. The kinetic energy K of an object of mass m moving with speed v is

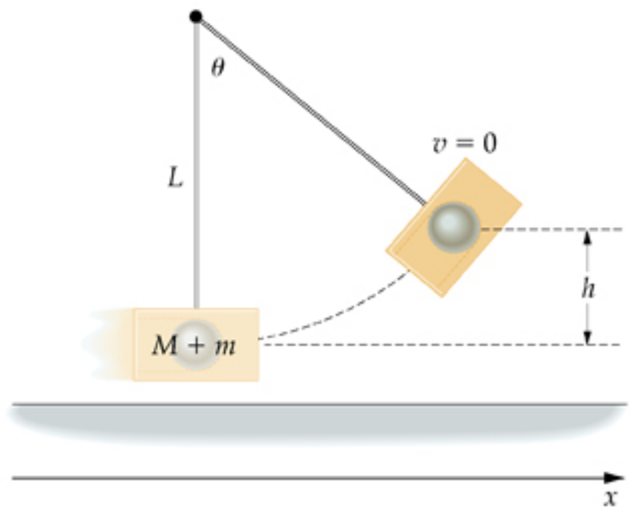
$$K = \frac{1}{2}mv^2.$$

The gravitational potential energy U of an object of mass m a height y above some reference point is

$$U = mgy.$$

Hint 2. Find the height at the top of the swing

What is the height h of the object and pendulum bob at the top of the swing, when they have reached their maximum displacement? Keep in mind that the pendulum has a length L and swings through an angle θ.



Express your answer in terms of some or all of the variables L and θ.

ANSWER:

h = $L\left(1-\cos\left(\theta\right)\right)$

Hint 3. Find the gravitational potential energy at the top of the swing

What is the gravitational potential energy U_{top} of the object and pendulum bob at the top of the swing?

Express your answer in terms of some or all of the variables m, M, v_0, v, L, and θ and the acceleration due to gravity, g.

ANSWER:

U_{top} = $\left(m+M\right)gL\left(1-\cos\left(\theta\right)\right)$

Hint 4. Find the kinetic energy at the top of the swing

What is the kinetic energy K_{top} of the object and pendulum bob at the top of the swing?

ANSWER:

- $K_{top} = \frac{1}{2}mv_0^2$
- $K_{top} = \frac{1}{2}mv^2$
- $K_{top} = \frac{1}{2}(m+M)v^2$
- $K_{top} = 0$

ANSWER:

E_{top} = $(m+M)\,gL\,(1-\cos(\theta))$

ANSWER:

v = $\sqrt{2gL\,(1-\cos(\theta))}$

Hint 5. Relating the two physical processes

By applying conservation of momentum to the collision, you found an expression for v_0:

$$v_0 = \left(\frac{m+M}{m}\right)v.$$

By applying conservation of mechanical energy to the swing, you found an expression for v:

$$v = \sqrt{2gL[1-\cos(\theta)]}.$$

Combine these two expression into just one expression for v_0.

ANSWER:

v_0 = $\frac{M+m}{m}\sqrt{2gL(1-\cos\theta)}$

Correct

The ballistic pendulum was invented during the Napoleonic Wars to aide the British Navy in making better cannons. It has since been used by ballisticians to measure the velocity of a bullet as it leaves the barrel of a gun. In Part B you will use your expression for v_0 to compare the initial speeds of bullets fired from 9-mm and .44-caliber handguns.

Part B

An experiment is done to compare the initial speed of bullets fired from different handguns: a 9 mm and a .44 caliber. The guns are fired into a 10-kg pendulum bob of length L. Assume that the 9-mm bullet has a mass of 6 g and the .44-caliber bullet has a mass of 12 g . If the 9-mm bullet causes the pendulum to swing to a maximum angular displacement of 4.3° and the .44-caliber bullet causes a displacement of 10.1° , find the ratio of the initial speed of the 9-mm bullet to the speed of the .44-caliber bullet, $(v_0)_9/(v_0)_{44}$.

Express your answer numerically.

Hint 1. How to approach the problem

Use your expression from Part A to set up the ratio $(v_0)_9/(v_0)_{44}$. Try to cancel as many terms as possible before plugging in your numbers to solve for a numeric answer.

ANSWER:

$(v_0)_9/(v_0)_{44}$ = 0.852

Correct

Police officers in the United States commonly carry 9-mm handguns because they are easier to handle, having a shorter barrel than typical .44-caliber guns. Not only does the .44-caliber bullet have more mass than the 9-mm one, its passage through a longer gun barrel means that it also moves faster as it leaves the barrel, which makes the .44-caliber Magnum a particularly powerful handgun. A .44-caliber bullet can travel at speeds over 1000 $\mathrm{miles\ per\ hour}$ (1600 $\mathrm{kilometers\ per\ hour}$).

Score Summary:

Your score on this assignment is 101.3%.
You received 50.65 out of a possible total of 50 points.