Chapter 6: Work and Kinetic Energy

Due: 11:00pm on Tuesday, October 2, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

± All Work and No Play

Learning Goal:

To be able to calculate work done by a constant force directed at different angles relative to displacement

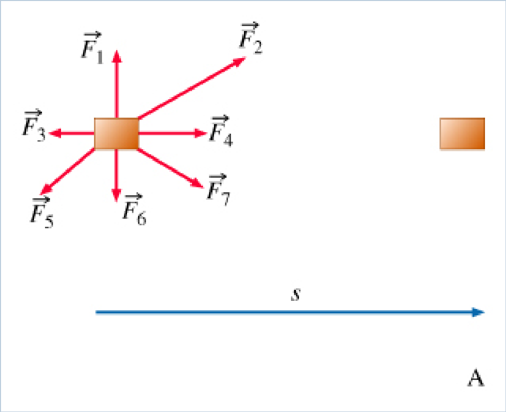
If an object undergoes displacement while being acted upon by a force (or several forces), it is said that *work is being done* on the object. If the object is moving in a straight line and the displacement and the force are known, the work done by the force can be calculated as

$$W = \vec{F}\cdot\vec{s} = \left|\vec{F}\right| |\vec{s}| \cos\theta,$$

where W is the work done by force $\vec{F}$ on the object that undergoes displacement $\vec{s}$ directed at angle θ relative to $\vec{F}$.

Note that depending on the value of $\cos\theta$, the work done can be positive, negative, or zero.

In this problem, you will practice calculating work done on an object moving in a straight line. The first series of questions is related to the accompanying figure.



Part A

What can be said about the sign of the work done by the force $\vec{F}_1$?

ANSWER:

- It is positive.
- It is negative.
- (selected) It is zero.
- There is not enough information to answer the question.

When $\theta = 90°$, the cosine of θ is zero, and therefore the work done is zero.

Part B

What can be said about the work done by force $\vec{F}_2$?

ANSWER:

- ◉ It is positive.
- ○ It is negative.
- ○ It is zero.

When $0° < \theta < 90°$, $\cos\theta$ is positive, and so the work done is positive.

Part C

The work done by force $\vec{F}_3$ is

ANSWER:

- ○ positive
- ◉ negative
- ○ zero

When $90° < \theta < 180°$, $\cos\theta$ is negative, and so the work done is negative.

Part D

The work done by force $\vec{F}_4$ is

ANSWER:

- ◉ positive
- ○ negative
- ○ zero

Part E

The work done by force $\vec{F}_5$ is

ANSWER:

- ○ positive
- ◉ negative
- ○ zero

Part F

The work done by force $\vec{F}_6$ is

ANSWER:

- ○ positive
- ○ negative
- ◉ zero

Part G

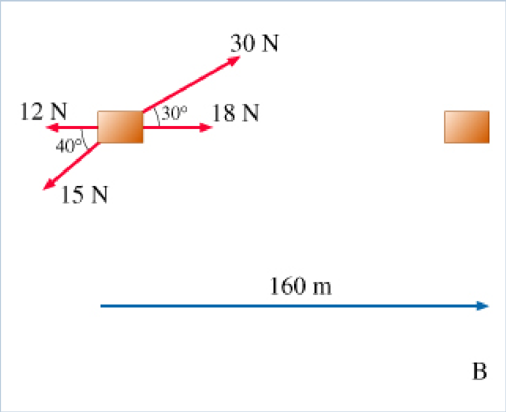
The work done by force $\vec{F}_7$ is

ANSWER:

- positive (selected)
- negative
- zero

In the next series of questions, you will use the formula $W = \vec{F} \cdot \vec{s} = \left|\vec{F}\right| |\vec{s}| \cos\theta$

to calculate the work done by various forces on an object that moves 160 meters to the right.



Part H

Find the work W done by the 18-newton force.

Use two significant figures in your answer. Express your answer in joules.

ANSWER:

W = 2900 J

Part I

Find the work W done by the 30-newton force.

Use two significant figures in your answer. Express your answer in joules.

ANSWER:

W = 4200 J

Part J

Find the work W done by the 12-newton force.

Use two significant figures in your answer. Express your answer in joules.

ANSWER:

W = -1900 J

Part K

Find the work W done by the 15-newton force.

Use two significant figures in your answer. Express your answer in joules.

ANSWER:

W = -1800 J

± The Power of One

Learning Goal:

To learn the definition of power and how power, force, and velocity are related.

The definition of work done by a force ($W = \vec{F} \cdot \vec{s}$) does not include time. For practical purposes, however, it is often important to know how fast work is being done. The rate at which work is being done is called power P. The *average power* $P_{\rm avg}$ can be calculated as

$$P_{\rm avg} = \frac{\Delta W}{\Delta t},$$

where ΔW is the amount of work done during the time interval Δt.

The power created by a force may be a constant; that is, work is being done at constant rate. However, this is not always the case. If the rate of performing work is changing, it makes sense to talk about the *instantaneous power*, defined as

$$P = \frac{dW}{dt}.$$

The SI unit of power is the watt (W). One watt is defined as the power created when one joule of work is done each second. In equation form, one writes

$$1\ \mathrm{W} = 1\ \mathrm{J/s}.$$

A commonly used unit of work is the kilowatt-hour (kW-hour). One kilowatt-hour is the amount of work done in one hour when the power is one kilowatt. In equation form, this is

$$1\ \mathrm{kW\text{-}hour} = 1\ \mathrm{kW} \cdot 1\ \mathrm{hour}$$
$$= 10^3\ W \cdot 3.6 \times 10^3\ s = 3.6\ MJ.$$

In this problem, you will answer several questions that will help familiarize you with power and enable you to derive a formula relating power, force, and velocity.

A sled of mass m is being pulled horizontally by a constant horizontal force of magnitude F. The coefficient of kinetic friction is $\mu_{\rm k}$. During time interval t, the sled moves a distance s, starting from rest.

Part A

Find the average power $P_{\rm avg}$ created by the force F.

Express your answer in terms of the given quantities and, if necessary, appropriate constants. You may or may not use all of the given quantities.

Hint 1. Find the work W done by the force F.

Find the work W done by the force F.

Express your answer in terms of the given quantities and, if necessary, appropriate constants. You may or may not use all of the given quantities.

ANSWER:

W = Fs

Note that all you have to do is to apply the definition of work; most of the quantities given in the problem are, in fact, irrelevant.

ANSWER:

$P_{\rm avg}$ = $\frac{Fs}{t}$

Part B

Find the average velocity $v_{\rm avg}$ of the sled during that time interval.

Express your answer in terms of the given quantities and, if necessary, appropriate constants. You may or may not use all of the given quantities.

ANSWER:

$v_{avg} = \frac{s}{t}$

Part C

Find the average power P_{avg} created by the force F in terms of the average speed v_{avg} of the sled.

Express your answer in terms of F and v_{avg}.

ANSWER:

$P_{avg} = Fv_{avg}$

You just obtained a very useful formula for the average power:

$$P = \frac{Fs}{t} = Fv.$$

If an object is moving at a constant speed, and the force F is also constant, this formula can be used to find the average power. If v is changing, the formula can be used to find the *instantaneous* power at any given moment (with the quantity v in this case meaning the *instantaneous* velocity, of course).

Part D

A sled of mass m is being pulled horizontally by a constant upward force of magnitude F that makes an angle θ with the direction of motion. The coefficient of kinetic friction is μ_k. The average velocity of the sled is v_{avg}.

Find the average power P_{avg} created by force F.

Express your answer in terms of F, θ, and v_{avg}.

Hint 1. Find the work done

When the directions of the force and the velocity vectors are the same, $W = Fs$. When the two vectors make an angle θ, which of the following formulas can be applied?

ANSWER:

- ○ $W = Fs\tan\theta$
- ○ $W = Fs\sin\theta$
- ◉ $W = Fs\cos\theta$
- ○ $W = \frac{Fs}{\cos\theta}$

ANSWER:

$P_{avg} = F\cos(\theta)\, v_{avg}$

Another way to express this formula is this

$$P = F_{||}v,$$

where $F_{||}$ is the component of force parallel to the velocity of the object.

Part E

Find the average power P_{avg} created by force $\vec{F}$.

Express your answer in watts to three significant figures.

Hint 1. Find the work W done by force $\vec{F}$.

Find the work W done by force $\vec{F}$.

Express your answer in joules to three significant figures.

Hint 1. Find the displacement of the sled

Which formula is useful in finding the displacement of the sled?

ANSWER:

- ○ $s = vt$
- ○ $s = \frac{v}{t}$
- ○ $s = \frac{at}{2}$
- ◉ $s = \frac{at^2}{2}$

ANSWER:

W = 8.64×10^4 J

ANSWER:

P_{avg} = 1440 W

Part F

Find the *instantaneous* power P created by force $\vec{F}$ at $t = 10 \text{ s}$.

Express your answer in watts to three significant figures.

Hint 1. Find the speed

Find the speed v of the sled after 10 s..

Express your answer in meters per second.

ANSWER:

v = 0.8 m/s

ANSWER:

P = 480 W

Part G

Find the instantaneous power P created by the normal force at $t = 10 \text{ s}$. The magnitude of the acceleration due to gravity is $g = 9.8 \text{ m/s}^2$.

Express your answer in watts to three significant figures.

Hint 1. Work done by the normal force

Recall that the normal force is directed upward, while the sled is moving horizontally. Given these directions, what is the angle between the force and the direction of motion? What does this tell you about how much work the normal force does?

ANSWER:

P = 0 W

When vectors $\vec{F}$ and $\vec{v}$ are perpendicular, the power created by force $\vec{F}$ is zero.

The Work-Energy Theorem

Learning Goal:

To understand the meaning and possible applications of the work-energy theorem.

In this problem, you will use your prior knowledge to derive one of the most important relationships in mechanics: the work-energy theorem. We will start with a special case: a particle of mass m moving in the *x* direction at constant acceleration a. During a certain interval of time, the particle accelerates from $v_{\rm initial}$ to $v_{\rm final}$, undergoing displacement s given by $s = x_{\rm final} - x_{\rm initial}$.

Part A

Find the acceleration a of the particle.

Express the acceleration in terms of $v_{\rm initial}$, $v_{\rm final}$, and s.

Hint 1. Some helpful relationships from kinematics

By definition for constant acceleration,

$$a = \frac{v_{\rm final} - v_{\rm initial}}{t}.$$

Furthermore, the average speed is

$$v_{\rm avg} = \frac{v_{\rm initial} + v_{\rm final}}{2},$$

and the displacement is

$$s = v_{\rm avg} t.$$

Combine these relationships to eliminate t.

ANSWER:

a = $\frac{(v_{\rm final})^2 - (v_{\rm initial})^2}{2s}$

Part B

Find the net force F acting on the particle.

Express your answer in terms of m and a.

Hint 1. Using Newton's laws

Which of Newton's laws may be helpful here?

ANSWER:

F = ma

Part C

Find the net work W done on the particle by the external forces during the particle's motion.

Express your answer in terms of F and s.

ANSWER:

W = Fs

Part D

Substitute for F from Part B in the expression for work from Part C. Then substitute for a from the relation in Part A. This will yield an expression for the net work W done on the particle by the external forces during the particle's motion in terms of mass and the initial and final velocities. Give an expression for the work W in terms of those quantities.

Express your answer in terms of m, $v_{\rm initial}$, and $v_{\rm final}$.

ANSWER:

W = $\frac{m\left((v_{\rm final})^2-(v_{\rm initial})^2\right)}{2}$

The expression that you obtained can be rearranged as

$$W = \frac{1}{2}m{v_{\rm final}}^2 - \frac{1}{2}m{v_{\rm initial}}^2.$$

The quantity $\frac{1}{2}mv^2$ has the same units as work. It is called the *kinetic energy* of the moving particle and is denoted by K. Therefore, we can write

$$K_{\rm initial} = \frac{1}{2}m{v_{\rm initial}}^2 \text{ and } K_{\rm final} = \frac{1}{2}m{v_{\rm final}}^2.$$

Note that like momentum, kinetic energy depends on both the mass and the velocity of the moving object. However, the mathematical expressions for momentum and kinetic energy are different. Also, unlike momentum, kinetic energy is a scalar. That is, it does not depend on the sign (therefore direction) of the velocities.

Part E

Find the net work W done on the particle by the external forces during the motion of the particle in terms of the initial and final kinetic energies.

Express your answer in terms of $K_{\rm initial}$ and $K_{\rm final}$.

ANSWER:

W = $K_{\rm final} - K_{\rm initial}$

This result is called the work-energy theorem. It states that the net work done on a particle equals the change in kinetic energy of that particle.
Also notice that if $K_{\rm initial}$ is zero, then the work-energy theorem reduces to

$$W = K_{\rm final}.$$

In other words, kinetic energy can be understood as the amount of work that is done to accelerate the particle from rest to its final velocity.

The work-energy theorem can be most easily used if the object is moving in one dimension and is being acted upon by a constant net force directed along the direction of motion. However, the theorem is valid for more general cases as well.

Let us now consider a situation in which the particle is still moving along the *x* axis, but the net force, which is still directed along the *x* axis, is no longer constant. Let's see how our earlier definition of work,

$$W = \vec{F}\cdot\vec{s},$$

needs to be modified by being replaced by an integral. If the path of the particle is divided into very small displacements dx, we can assume that over each of these small displacement intervals, the net force remains essentially constant and the work dW done to move the particle from x to $x + dx$ is

$$dW = F\,dx,$$

where F is the x component of the net force (which remains virtually constant for the small displacement from x to $x + dx$). The net work W done on the particle is then given by

$$W = \int_{x_{\text{initial}}}^{x_{\text{final}}} dW = \int_{x_{\text{initial}}}^{x_{\text{final}}} F\,dx.$$

Now, using

$$F = ma$$

and

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx},$$

it can be shown that

$$W = \int_{v_{\text{initial}}}^{v_{\text{final}}} mv\,dv.$$

Part F

Evaluate the integral $W = \int_{v_{\text{initial}}}^{v_{\text{final}}} mv\,dv$.

Express your answer in terms of m, v_{initial}, and v_{final}.

Hint 1. An integration formula

The formula for $\int_a^b t\,dt$ is

$$\int_a^b t\,dt = \frac{b^2 - a^2}{2}.$$

ANSWER:

W = $\frac{m\left((v_{\text{final}})^2 - (v_{\text{initial}})^2\right)}{2}$

The expression that you havejust obtained is equivalent to $W = K_{\text{final}} - K_{\text{initial}}$. Not surprisingly, we are back to the same expression of the work-energy theorem! Let us see how the theorem can be applied to problem solving.

Part G

A particle moving in the *x* direction is being acted upon by a net force $F(x) = Cx^2$, for some constant C. The particle moves from $x_{\text{initial}} = L$ to $x_{\text{final}} = 3L$. What is ΔK, the change in kinetic energy of the particle during that time?

Express your answer in terms of C and L.

Hint 1. Finding the work

Integrate $F(x)\,dx$ to calculate the work done on the particle.

Hint 2. An integration formula

The formula for $\int_a^b u^2\,du$ is

$$\int_a^b u^2\,du = \frac{b^3 - a^3}{3}.$$

ANSWER:

$\Delta K = \dfrac{26CL^3}{3}$

Also accepted: $8.67CL^3$

It can also be shown that the work-energy theorem is valid for two- and three-dimensional motion and for a varying net force that is not necessarily directed along the instantaneous direction of motion of the particle. In that case, the work done by the net force is given by the line integral

$$W = \int_{S_{\rm initial}}^{S_{\rm final}} \vec{F} \cdot d\vec{L},$$

where $S_{\rm initial}$ and $S_{\rm final}$ are the initial and the final positions of the particle, $d\vec{L}$ is the vector representing a small displacement, and $\vec{F}$ is the net force acting on the particle.

Understanding Work and Kinetic Energy

Learning Goal:

To learn about the Work-Energy Theorem and its basic applications.

In this problem, you will learn about the relationship between the work done on an object and the kinetic energy of that object.

The kinetic energy K of an object of mass m moving at a speed v is defined as $K = (1/2)mv^2$. It seems reasonable to say that the speed of an object–and, therefore, its kinetic energy–can be changed by performing *work* on the object. In this problem, we will explore the mathematical relationship between the work done on an object and the change in the kinetic energy of that object.

First, let us consider a sled of mass m being pulled by a constant, horizontal force of magnitude F along a *rough*, horizontal surface. The sled is speeding up.

Part A

How many forces are acting on the sled?

ANSWER:

- one
- two
- three
- four (selected)

Part B

The work done on the sled by the force of gravity is __________.

ANSWER:

- zero (selected)
- negative
- positive

Part C

The work done on the sled by the normal force is __________.

ANSWER:

- zero (selected)
- negative
- positive

Part D

The work done on the sled by the pulling force is __________.

ANSWER:

- ○ zero
- ○ negative
- ◉ positive

Part E

The work done on the sled by the force of friction is __________.

ANSWER:

- ○ zero
- ◉ negative
- ○ positive

Part F

The *net* work done on the sled is __________.

Hint 1. Which force is bigger?

In the situaiton described, which statement is true?

ANSWER:

- ◉ The magnitude of the pulling force is greater than that of the force of friction.
- ○ The magnitude of the pulling force is less than that of the force of friction.
- ○ The magnitude of the pulling force is the same as that of the force of friction.

ANSWER:

- ○ zero
- ○ negative
- ◉ positive

Part G

In the situation described, the kinetic energy of the sled __________.

ANSWER:

- ○ remains constant
- ○ decreases
- ◉ increases

Part H

Find the net force $F_{\rm net}$ acting on the sled.

Express your answer in terms of some or all of the variables m, s, v_1, and v_2.

Hint 1. How to approach the problem

According to Newton's 2nd law,

$$F_{\rm net} = ma.$$

Therefore, you need to simply find the acceleration of the sled. Once you've found that, multiply it by m to get the force. You can use kinematics to find the acceleration.

Hint 2. Find the acceleration

Find the acceleration a of the sled.

Express your answer in terms of some or all of the variables s, v_1, and v_2.

Hint 1. Some useful kinematics

The definition of acceleration is

$$a = \frac{v_2 - v_1}{t}.$$

If the acceleration is a constant, the average velocity can be found as

$$v_{avg} = \frac{v_1 + v_2}{2}.$$

Finally, the distance can be expressed as

$$s = v_{avg}t.$$

Combining these equations and eliminating t and v_{avg} gives the desired answer.

ANSWER:

$a = \frac{(v_2)^2 - (v_1)^2}{2s}$

ANSWER:

$F_{net} = \frac{m\left((v_2)^2 - (v_1)^2\right)}{2s}$

Part I

Find the net work W_{net} done on the sled.

Express your answer in terms of some or all of the variables F_{net} and s.

Hint 1. Work, force, and displacement

In general, the work done by a constant force $\vec{F}$ can be found as

$$W = Fs\cos(\theta),$$

where θ is the angle between vectors $\vec{F}$ and $\vec{s}$. However, when the net force and displacement have the same direction (as is the case here), $\cos(\theta) = 1$.

ANSWER:

$W_{net} = F_{net}s$

Part J

Use $W = Fs\cos(\theta)$ to find the net work W_{net} done on the sled.

Express your answer in terms of some or all of the variables m, v_1, and v_2.

ANSWER:

W_{net} = $\frac{m\left((v_2)^2-(v_1)^2\right)}{2}$

Your answer can also be rewritten as

$$W_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

or

$$W_{net} = K_2 - K_1,$$

where K_1 and K_2 are, respectively, the initial and the final kinetic energies of the sled. Finally, one can write

$$W_{net} = \Delta K.$$

This formula is known as the Work-Energy Theorem. The calculations done in this problem illustrate the applicability of this theorem in a particlar case; however, they should not be interpreted as a proof of this theorem.

Nevertheless, it can be shown that the Work-Energy Theorem is applicable in all situations, including those involving nonconstant forces or forces acting at an angle to the displacement of the object. This theorem is quite useful in solving problems, as illustrated by the following example.

Part K

A car of mass m accelerates from speed v_1 to speed v_2 while going up a slope that makes an angle θ with the horizontal. The coefficient of static friction is μ_s, and the acceleration due to gravity is g. Find the total work W done on the car by the external forces.

Express your answer in terms of the given quantities. You may or may not use all of them.

Hint 1. How to approach the problem

You are asked to find, in effect, the *net* work done on an object. Use the Work-Energy Theorem.

ANSWER:

W = $0.5m\left((v_2)^2-(v_1)^2\right)$

Exercise 6.4

A factory worker pushes a 30.7kg crate a distance of 4.6m along a level floor at constant velocity by pushing downward at an angle of 29° below the horizontal. The coefficient of kinetic friction between the crate and floor is 0.24.

Part A

What magnitude of force must the worker apply to move the crate at constant velocity?

Express your answer using two significant figures.

ANSWER:

F = $\frac{\mu_k m \cdot 9.80}{\cos(\alpha)-\mu_k \sin(\alpha)}$ = 95 N

Part B

How much work is done on the crate by this force when the crate is pushed a distance of 4.6m ?

Express your answer using two significant figures.

ANSWER:

$W = \frac{\mu_k m \cdot 9.80}{\cos(\alpha) - \mu_k \sin(\alpha)} s\cos(\alpha)$ = 380 J

Also accepted: $sigdig\left(\frac{\mu_k m \cdot 9.80}{\cos(\alpha) - \mu_k \sin(\alpha)}, 2\right) s\cos(\alpha)$ = 380

Part C

How much work is done on the crate by friction during this displacement?

Express your answer using two significant figures.

ANSWER:

$W_f = -s\mu_k\left(m \cdot 9.80 + \frac{\mu_k m \cdot 9.80}{\cos(\alpha) - \mu_k \sin(\alpha)} \sin(\alpha)\right)$ = -380 J

Also accepted: $-s\mu_k\left(m \cdot 9.80 + sigdig\left(\frac{\mu_k m \cdot 9.80}{\cos(\alpha) - \mu_k \sin(\alpha)}, 2\right) \sin(\alpha)\right)$ = -380

Part D

How much work is done by the normal force?

ANSWER:

W_{nf} = 0 J

Part E

How much work is done by gravity?

ANSWER:

W_g = 0 J

Part F

What is the total work done on the crate?

ANSWER:

W_{net} = 0 J

Exercise 6.6

Two tugboats pull a disabled supertanker. Each tug exerts a constant force of 1.6×10^{6} N , one an angle 14° west of north and the other an angle 14° east of north, as they pull the tanker a distance 0.60 km toward the north.

Part A

What is the total work they do on the supertanker?

Express your answer using two significant figures.

ANSWER:

$W = 2Fx\cos(\alpha)$ = 1.9×10^{9} J

Exercise 6.15

About 50000 years ago, a meteor crashed into the earth near present-day Flagstaff, Arizona. Recent (2005) measurements estimate that this meteor had a mass of about 1.4×10^{8} kg (around 150000 tons) and hit the ground at 12 km/s.

Part A

How much kinetic energy did this meteor deliver to the ground?

Express your answer using two significant figures.

ANSWER:

K = 1.0×10^{16} J

Part B

How does this energy compare to the energy released by a 1.0-megaton nuclear bomb? (A megaton bomb releases the same energy as a million tons of TNT, and 1.0 ton of TNT releases $4.184 \times 10^9 \mathrm{J}$ of energy.)

Express your answer using two significant figures.

ANSWER:

$\frac{E_{\mathrm{meteor}}}{E_{\mathrm{bomb}}}$ = 2.4

Exercise 6.24

A soccer ball with mass 0.410kg is initially moving with speed 2.20$\mathrm{m/s}$. A soccer player kicks the ball, exerting a constant force of magnitude 35.0N in the same direction as the ball's motion.

Part A

Over what distance must the player's foot be in contact with the ball to increase the ball's speed to 6.00 $\mathrm{m/s}$?

ANSWER:

s = $\frac{\frac{1}{2}m\left(36.0 - vi^2\right)}{F}$ = 0.183 m

Exercise 6.32

To stretch a spring 9.00cm from its unstretched length, 14.0J of work must be done.

Part A

What is the force constant of this spring?

ANSWER:

k = $\frac{2W}{l^2}$ = 3460 $\mathrm{N/m}$

If you need to use the value of the spring constant 'k' in subsequent parts, please use the unrounded full precision value and not the one you submitted for this part rounded using three significant figures.

Part B

What magnitude force is needed to stretch the spring 9.00cm from its unstretched length?

ANSWER:

F = $\frac{2W}{l}$ = 311 N

Part C

How much work must be done to compress this spring 4.00 cm from its unstretched length?

ANSWER:

$W = \frac{0.0016W}{l^2} = 2.77$ J

Part D

What force is needed to stretch it this distance?

ANSWER:

$F = \frac{0.08W}{l^2} = 138$ N

Exercise 6.44

Part A

Suppose you cut a massless ideal spring in half. If the full spring had a force constant k, what is the force constant of each half, in terms of k? (*Hint:* Think of the original spring as two equal halves, each producing the same force as the entire spring. Do you see why the forces must be equal?)

Express your answer using one significant figure.

ANSWER:

$k_h = 2\ k$

Part B

If you cut the spring into three equal segments instead, what is the force constant of each one, in terms of k?

Express your answer using one significant figure.

ANSWER:

$k_{seg} = 3\ k$

Exercise 6.46

An ingenious bricklayer builds a device for shooting bricks up to the top of the wall where he is working. He places a brick on a vertical compressed spring with force constant k = 450 N/m and negligible mass. When the spring is released, the brick is propelled upward.

Part A

If the brick has mass 1.80 kg and is to reach a maximum height of 3.6 m above its initial position on the compressed spring, what distance must the bricklayer compress the spring initially?

Express your answer using two significant figures.

ANSWER:

$d = 0.53$ m

Part B

The brick loses contact with the spring when the spring returns to its uncompressed length. Why?

ANSWER:

3642 Character(s) remaining

The spring will provide an upward force while the spring and the brick are in contact. When this force goes to zero,

Exercise 6.55

Your job is to lift 30-kg crates a vertical distance of 0.90 m from the ground onto the bed of a truck.

Exercise 6.46

Part B ANSWER:

The spring will provide an upward force while the spring and the brick are in contact. When this force goes to zero, the spring is at its uncompressed length.

Part A

How many crates would you have to load onto the truck in one minute for the average power output you use to lift the crates to equal 0.50 hp?

Express your answer using two significant figures.

ANSWER:

85 $1/\mathrm{min}$

Part B

How many crates for an average power output of 100 W?

Express your answer using two significant figures.

ANSWER:

23 $1/\mathrm{min}$

Exercise 6.57

A ski tow operates on a slope of angle 14.6° of length 290m . The rope moves at a speed of 11.5$\mathrm{km/h}$ and provides power for 46 riders at one time, with an average mass per rider of 66.0kg .

Part A

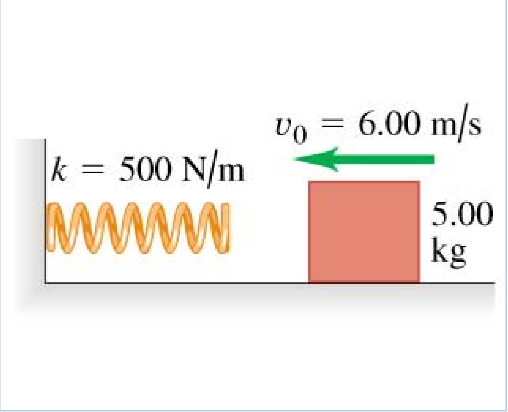
Estimate the power required to operate the tow.

ANSWER:

$P = Nmgv\sin(\theta)$ = 2.40×10^4 W

Problem 6.85

A 5.00-kg block is moving at 6.00 $\mathrm{m/s}$ along a frictionless, horizontal surface toward a spring with force constant k=500 $\mathrm{N/m}$ that is attached to a wall (the figure). The spring has negligible mass.



Part A

Find the maximum distance the spring will be compressed.

ANSWER:

x = 0.600 m

Part B

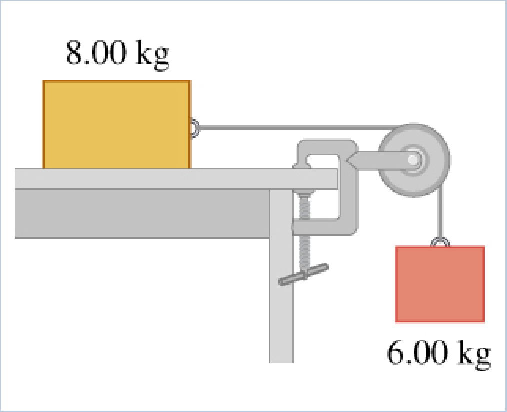
If the spring is to compress by no more than 0.500m , what should be the maximum value of v_0?

ANSWER:

v_0 = $10x$ = 5.00 $\mathrm{m/s}$

Problem 6.86

Consider the system shown in the figure. The rope and pulley have negligible mass, and the pulley is frictionless. The coefficient of kinetic friction between the 8.00-kg block and the tabletop is $\mu_k = 0.250$. The blocks are released from rest.



Part A

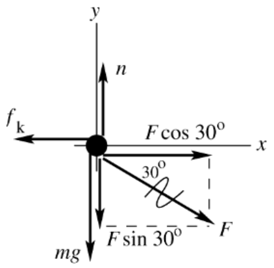
Use energy methods to calculate the speed of the 6.00-kg block after it has descended 1.50 m.

ANSWER:

v = 2.90 $\mathrm{m/s}$

6.4.IDENTIFY: The forces are constant so Eq. (6.2) can be used to calculate the work. Constant speed implies $a = 0$. We must use $\Sigma\vec{F} = m\vec{a}$ applied to the crate to find the forces acting on it.

(a) SET UP: The free-body diagram for the crate is given in Figure 6.4.



EXECUTE: $\Sigma F_y = ma_y$

$n - mg - F\sin 30° = 0$

$n = mg + F\sin 30°$

$f_k = \mu_k n = \mu_k mg + F\mu_k \sin 30°$

Figure 6.4

$\Sigma F_x = ma_x$

$F\cos 30° - f_k = 0$

$F\cos 30° - \mu_k mg - \mu_k \sin 30° F = 0$

$$F = \frac{\mu_k mg}{\cos 30° - \mu_k \sin 30°} = \frac{0.25(30.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 30° - (0.25)\sin 30°} = 99.2\text{ N}$$

(b) $W_F = (F\cos\phi)s = (99.2\text{ N})(\cos 30°)(4.5\text{ m}) = 387\text{ J}$

($F\cos 30°$ is the horizontal component of $\vec{F}$; the work done by $\vec{F}$ is the displacement times the component of $\vec{F}$ in the direction of the displacement.)

(c) We have an expression for f_k from part (a):

$$f_k = \mu_k(mg + F\sin 30°) = (0.250)[(30.0\text{ kg})(9.80\text{ m/s}^2) + (99.2\text{ N})(\sin 30°)] = 85.9\text{ N}$$

$\phi = 180°$ since f_k is opposite to the displacement. Thus

$W_f = (f_k\cos\phi)s = (85.9\text{ N})(\cos 180°)(4.5\text{ m}) = -387\text{ J}$

(d) The normal force is perpendicular to the displacement so $\phi = 90°$ and $W_n = 0.$ The gravity force (the weight) is perpendicular to the displacement so $\phi = 90°$ and $W_w = 0.$

(e) $W_{tot} = W_F + W_f + W_n + W_w = +387\text{ J} + (-387\text{ J}) = 0$

EVALUATE: Forces with a component in the direction of the displacement do positive work, forces opposite to the displacement do negative work and forces perpendicular to the displacement do zero work. The total work, obtained as the sum of the work done by each force, equals the work done by the net force. In this problem, $F_{net} = 0$ since $a = 0$ and $W_{tot} = 0,$ which agrees with the sum calculated in part (e).

6.6.IDENTIFY and **SET UP:** $W_F = (F\cos\phi)s,$ since the forces are constant. We can calculate the total work by

summing the work done by each force. The forces are sketched in Figure 6.6.

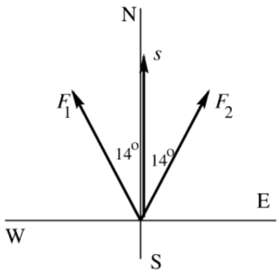


Figure 6.6

EXECUTE: $W_1 = F_1 s\cos\phi_1$

$W_1 = (1.80\times10^6\ \text{N})(0.75\times10^3\ \text{m})\cos14°$

$W_1 = 1.31\times10^9\ \text{J}$

$W_2 = F_2 s\cos\phi_2 = W_1$

$W_{\text{tot}} = W_1 + W_2 = 2(1.31\times10^9\ \text{J}) = 2.62\times10^9\ \text{J}$

EVALUATE: Only the component $F\cos\phi$ of force in the direction of the displacement does work. These components are in the direction of $\vec{s}$ so the forces do positive work.

6.15. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Since the meteor comes to rest the energy it delivers to the ground equals its original kinetic energy.

SET UP: $v = 12\ \text{km/s} = 1.2\times10^4\ \text{m/s}$. A 1.0 megaton bomb releases $4.184\times10^{15}\ \text{J}$ of energy.

EXECUTE: **(a)** $K = \frac{1}{2}(1.4\times10^8\ \text{kg})(1.2\times10^4\ \text{m/s})^2 = 1.0\times10^{16}\ \text{J}$.

(b) $\dfrac{1.0\times10^{16}\ \text{J}}{4.184\times10^{15}\ \text{J}} = 2.4$. The energy is equivalent to 2.4 one-megaton bombs.

EVALUATE: Part of the energy transferred to the ground lifts soil and rocks into the air and creates a large crater.

6.24. **IDENTIFY** and **SET UP:** Use Eq. (6.6) to calculate the work done by the foot on the ball. Then use Eq. (6.2) to find the distance over which this force acts.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.420\ \text{kg})(2.00\ \text{m/s})^2 = 0.84\ \text{J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.420\ \text{kg})(6.00\ \text{m/s})^2 = 7.56\ \text{J}$

$W_{\text{tot}} = K_2 - K_1 = 7.56\ \text{J} - 0.84\ \text{J} = 6.72\ \text{J}$

The 40.0 N force is the only force doing work on the ball, so it must do 6.72 J of work.

$W_F = (F\cos\phi)s$ gives that

$$s = \frac{W}{F\cos\phi} = \frac{6.72\ \text{J}}{(40.0\ \text{N})(\cos 0)} = 0.168\ \text{m}$$

EVALUATE: The force is in the direction of the motion so positive work is done and this is consistent with an increase in kinetic energy.

6.32. IDENTIFY: The work that must be done to move the end of a spring from x_1 to x_2 is $W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$. The force required to hold the end of the spring at displacement *x* is $F_x = kx$.

SET UP: When the spring is at its unstretched length, $x = 0$. When the spring is stretched, $x > 0$, and when the spring is compressed, $x < 0$.

EXECUTE: **(a)** $x_1 = 0$ and $W = \tfrac{1}{2}kx_2^2$. $k = \dfrac{2W}{x_2^2} = \dfrac{2(12.0\text{ J})}{(0.0300\text{ m})^2} = 2.67 \times 10^4$ N/m.

(b) $F_x = kx = (2.67 \times 10^4\text{ N/m})(0.0300\text{ m}) = 801$ N.

(c) $x_1 = 0$, $x_2 = -0.0400$ m. $W = \tfrac{1}{2}(2.67 \times 10^4\text{ N/m})(-0.0400\text{ m})^2 = 21.4$ J.

$F_x = kx = (2.67 \times 10^4\text{ N/m})(0.0400\text{ m}) = 1070$ N.

EVALUATE: When a spring, initially unstretched, is either compressed or stretched, positive work is done by the force that moves the end of the spring.

6.44. IDENTIFY: $F_x = kx$

SET UP: When the spring is in equilibrium, the same force is applied to both ends of any segment of the spring.

EXECUTE: **(a)** When a force *F* is applied to each end of the original spring, the end of the spring is displaced a distance *x*. Each half of the spring elongates a distance x_h, where $x_h = x/2$. Since *F* is also the force applied to each half of the spring, $F = kx$ and $F = k_h x_h$. $kx = k_h x_h$ and $k_h = k\left(\dfrac{x}{x_h}\right) = 2k$.

(b) The same reasoning as in part (a) gives $k_{seg} = 3k$, where k_{seg} is the force constant of each segment.

EVALUATE: For half of the spring the same force produces less displacement than for the original spring. Since $k = F/x$, smaller *x* for the same *F* means larger *k*.

6.46. IDENTIFY: Apply $W_{tot} = K_2 - K_1$ to the brick. Work is done by the spring force and by gravity.

SET UP: At the maximum height, $v = 0$. Gravity does negative work, $W_{grav} = -mgh$. The work done by the spring is $\tfrac{1}{2}kd^2$, where *d* is the distance the spring is compressed initially.

EXECUTE: The initial and final kinetic energies of the brick are both zero, so the net work done on the brick by the spring and gravity is zero, so $(1/2)kd^2 - mgh = 0$, or

$d = \sqrt{2mgh/k} = \sqrt{2(1.80\text{ kg})(9.80\text{ m/s}^2)(3.6\text{ m})/(450\text{ N/m})} = 0.53\text{ m}$. The spring will provide an upward force while the spring and the brick are in contact. When this force goes to zero, the spring is at its uncompressed length. But when the spring reaches its uncompressed length the brick has an upward velocity and leaves the spring.

EVALUATE: Gravity does negative work because the gravity force is downward and the brick moves upward. The spring force does positive work on the brick because the spring force is upward and the brick moves upward.

6.55.IDENTIFY: $P_{av} = \dfrac{\Delta W}{\Delta t}$. The work you do in lifting mass m a height h is mgh.

SET UP: $1\text{ hp} = 746\text{ W}$

EXECUTE: **(a)** The number per minute would be the average power divided by the work (mgh) required to lift one box, $\dfrac{(0.50\text{ hp})(746\text{ W/hp})}{(30\text{ kg})(9.80\text{ m/s}^2)(0.90\text{ m})} = 1.41/\text{s}$, or $84.6/\text{min}$.

(b) Similarly, $\dfrac{(100\text{ W})}{(30\text{ kg})(9.80\text{ m/s}^2)(0.90\text{ m})} = 0.378/\text{s}$, or $22.7/\text{min}$.

EVALUATE: A 30-kg crate weighs about 66 lbs. It is not possible for a person to perform work at this rate.

6.57.IDENTIFY: To lift the skiers, the rope must do positive work to counteract the negative work developed by the component of the gravitational force acting on the total number of skiers,

$F_{rope} = Nmg\sin\alpha.$

SET UP: $P = F_{\|}v = F_{rope}v$

EXECUTE: $P_{rope} = F_{rope}v = [+Nmg(\cos\phi)]v.$

$$P_{rope} = [(50\text{ riders})(70.0\text{ kg})(9.80\text{ m/s}^2)(\cos 75.0)]\left[(12.0\text{ km/h})\left(\frac{1\text{ m/s}}{3.60\text{ km/h}}\right)\right].$$

$P_{rope} = 2.96\times10^4\text{ W} = 29.6\text{ kW}.$

EVALUATE: Some additional power would be needed to give the riders kinetic energy as they are accelerated from rest.

6.85. **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$ to the blocks.

SET UP: If X is the distance the spring is compressed, the work done by the spring is $-\tfrac{1}{2}kX^2$.

At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy and $x_2 = 0$.

EXECUTE: **(a)** The work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\tfrac{1}{2}kX^2 = \tfrac{1}{2}mv_0^2$ and $X = \sqrt{\dfrac{m}{k}}v_0 = \sqrt{\dfrac{5.00\text{ kg}}{500\text{ N/m}}}(6.00\text{ m/s}) = 0.600\text{ m}.$

(b) Solving for v_0 in terms of a known X, $v_0 = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500 \text{ N/m}}{5.00 \text{ kg}}}(0.150 \text{ m}) = 1.50 \text{ m/s}$.

EVALUATE: The negative work done by the spring removes the kinetic energy of the block.

6.86.IDENTIFY: Apply $W_{\text{tot}} = K_2 - K_1$ to the system of the two blocks. The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table).

SET UP: Let h be the distance the 6.00 kg block descends. The work done by gravity is (6.00 kg)gh and the work done by friction is $-\mu_k(8.00 \text{ kg})gh$.

EXECUTE: $W_{\text{tot}} = (6.00 \text{ kg} - (0.25)(8.00 \text{ kg}))(9.80 \text{ m/s}^2)(1.50 \text{ m}) = 58.8 \text{ J}$. This work increases the kinetic energy of both blocks: $W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2$, so $v = \sqrt{\frac{2(58.8 \text{ J})}{(14.00 \text{ kg})}} = 2.90 \text{ m/s}$.

EVALUATE: Since the two blocks are connected by the rope, they move the same distance h and have the same speed v.