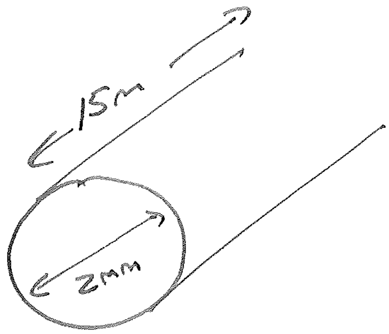
Physics 161, HW #4

#1



$R = 0.5\,\Omega$

Circular cross section of Diameter 2mm $\Rightarrow$ radius $r = 1\text{mm} = 1\times10^{-3}\text{m}$

$A = \pi r^2 = \pi(1\times10^{-3}\text{m})^2 = \pi\times10^{-6}\text{m}^2 = 3.14\times10^{-6}\text{m}^2$

a) What is Resistivity?

$$R = \rho\frac{L}{A} \Rightarrow \rho = \frac{RA}{L} = \frac{(0.5\,\Omega)(\pi\times10^{-6}\text{m}^2)}{15\text{m}} \Rightarrow \boxed{\rho = 1.05\times10^{-7}\,\Omega\cdot\text{m}}$$

b) If $E = 1.75\text{V/m}$, what is total Current?

$$E = \rho J \Rightarrow J = E/\rho = \frac{1.75\text{V/m}}{1.05\times10^{-7}\,\Omega\cdot\text{m}} = 1.67\times10^{7}\text{A/m}^2$$

$$J = \frac{I}{A} \Rightarrow I = JA = (1.67\times10^{7}\text{A/m}^2)(\pi\times10^{-6}\text{m}^2) \Rightarrow \boxed{I = 52.5\text{A}}$$

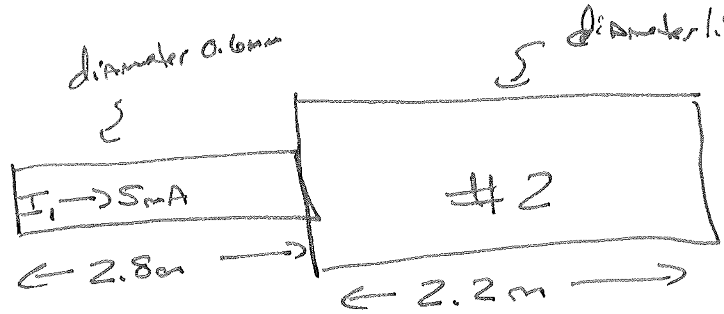
c) If $n = 9.5\times10^{28}$ free electrons/m³, Find Drift Velocity

$$J = nev_d \Rightarrow v_d = \frac{J}{ne} = \frac{1.67\times10^{7}\text{A/m}^2}{(9.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\text{C})}$$

$$\Rightarrow \boxed{v_d = 0.0011\text{ m/s} = 1.1\text{mm/s}}$$

Even for very large Current, Drift velocity is small.

#2



Let First section be #1 $\Rightarrow$

$l_1 = 2.8m,\ d_1 = 0.6mm,\ I_1 = 5mA$

#2 $\Rightarrow l_2 = 2.2m,\ d_2 = 1.5mm$

a) What is I_2? Every charge that leaves #1 must enter #2 (or vice versa) $\Rightarrow$ $\boxed{\text{#1 AND #2 ARE Connected in Series}}$ $\therefore \boxed{I_2 = 5mA}$

b) What is E_1? $E = \rho J$, Copper at 20°C $\Rightarrow \rho = 1.72\times10^{-8}\,\Omega\cdot m$

$$\therefore\ E_1 = \rho J_1 = \rho\left(I_1/A_1\right) \qquad A_1 = \pi r_1^2 = \pi\left(\frac{0.6}{2}\times10^{-3}m\right)^2 = \pi\left(0.3\times10^{-3}m\right)^2$$

radius!

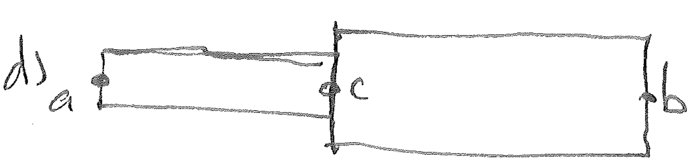
$$\therefore\ E_1 = \frac{(1.72\times10^{-8}\,\Omega\cdot m)(5\times10^{-3}A)}{\pi\,(0.3\times10^{-3}m)^2} \Rightarrow \boxed{E_1 = 3.04\times10^{-4}\,V/m}$$

c) What is E_2? $E_2 = \rho\dfrac{I_2}{A_2}$ $\quad A_2 = \pi r_2^2 = \pi\left(\frac{1.5}{2}\times10^{-3}m\right)^2 = \pi\left(0.75\times10^{-3}m\right)^2$

$$\therefore\ E_2 = \frac{(1.72\times10^{-8}\,\Omega\cdot m)(5\times10^{-3}A)}{\pi\,(0.75\times10^{-3}m)^2} \Rightarrow \boxed{E_2 = 4.87\times10^{-5}\,V/m}$$

Definitely UNEQUAL Electric Fields

d)



$V_{ab} = ?$

Let Connection Point be "c"

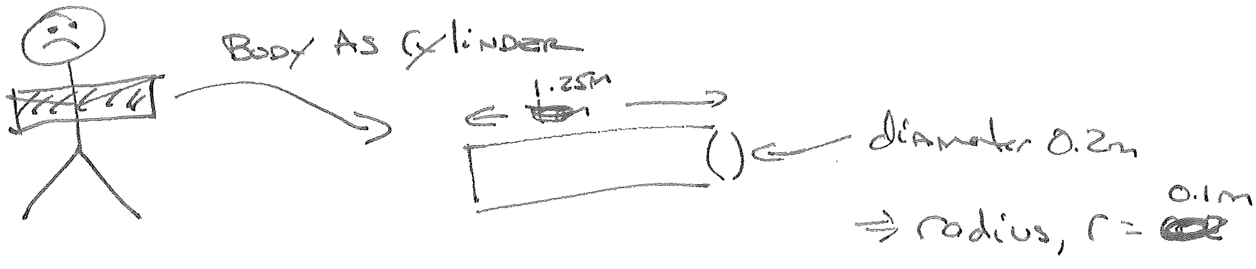
$V_{ac} = V_a - V_c,\ V_{cb} = V_c - V_b,\ V_{ab} = V_a - V_b$

$\Rightarrow \boxed{V_{ab} = V_{ac} + V_{cb}}$ Assuming uniform E_1 & E_2 $\Rightarrow V_{ac} = E_1 l_1,\ V_{cb} = E_2 l_2$

$\therefore V_{ac} = (3.04\times10^{-4}V/m)(2.8m) = 8.512\times10^{-4}V,\ V_{cb} = (4.87\times10^{-5}V/m)(2.2m) = 1.0714\times10^{-4}V$

$\therefore \boxed{V_{ab} = 9.58\times10^{-4}V = 0.958mV}$

#3



a) What is Resistance?

$$R = \rho \frac{L}{A} = \frac{\rho L}{\pi r^2}$$

Cross-sectional Area = πr^2

$$\therefore R = \frac{(5\,\Omega\cdot m)(1.25\,m)}{\pi (0.1\,m)^2} \Rightarrow \boxed{R = 199\,\Omega \approx 200\,\Omega}$$

only one sig fig for such a calculation

b) $V = ?$ to Produce $I = 100\,mA = 0.1\,A$

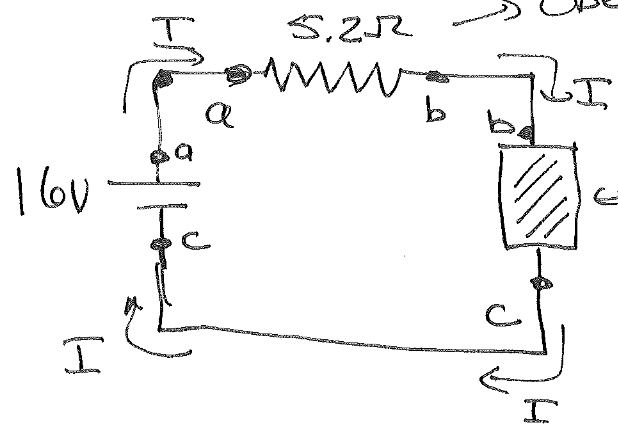
$$\boxed{V = IR = (.1\,A)(200\,\Omega) = 20\,V}$$

← much less than the 120V most appliances run on.

c) Power? $\boxed{P = IV = (0.1\,A)(20\,V) = 2\,watt}$

Not much.

#4



→ Obeys Ohm's Law ⇒ $V_R = I(5.2\Omega)$

← Thermistor with $V_T = \alpha I + \beta I^2$

$\alpha = 4\Omega$, $\beta = 1.8\Omega/A$

Label points (a), (b), (c) as above

$\Rightarrow V_{ac} = V_a - V_c = 16V$

$V_{ab} = V_a - V_b = V_R$

$V_{bc} = V_b - V_c = V_T$

$\Rightarrow V_{ab} + V_{bc} = V_{ac}$

$\Rightarrow V_R + V_T = 16V$

$\therefore I(5.2\Omega) + (4\Omega)I + (1.8\Omega/A)I^2 = 16V$

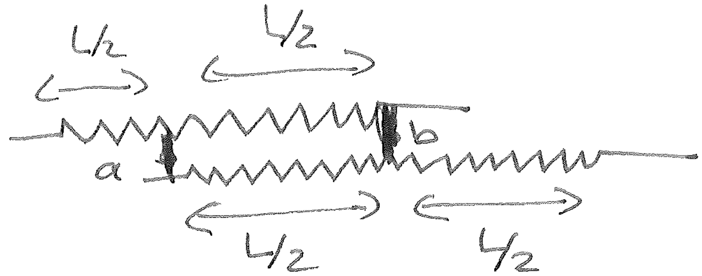
$\Rightarrow (1.8\Omega/A)I^2 + (9.2\Omega)I - 16V = 0$ ← Quadratic eqn.

$\Rightarrow I = \dfrac{-9.2\Omega \pm \sqrt{(9.2\Omega)^2 - 4(1.8\Omega/A)(-16V)}}{2(1.8\Omega/A)} = \dfrac{-9.2\Omega \pm \sqrt{199.84\Omega^2}}{3.6\Omega/A}$

$\Rightarrow I = 1.37A$ or $-6.48A$ ⇒ $\boxed{I = 1.37A}$

No

#5

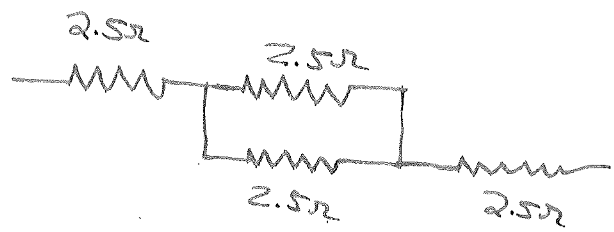


SOLDERED ⇒ Connected by Metal = EQUipotential SURFACE

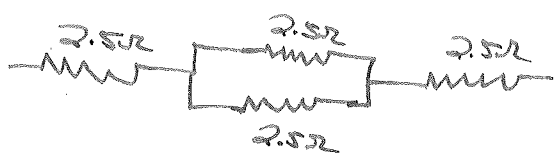
Label a & b as SHOWN ⇒ HALF of top Resistor AND HALF OF bottom Resistor have SAME V_{ab} ⇒ in Parallel with EACH other

$R = \frac{\rho L}{A}$ ⇒ $\frac{1}{2}L$ of A 5Ω Resistor HAS $R = 2.5\Omega$

So we Basically have this:

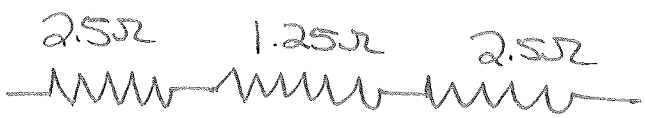


OR AS we'd More typically DRAW it:



For Parallel Resistors $R' = \frac{(2.5\Omega)(2.5\Omega)}{(2.5\Omega + 2.5\Omega)} = \frac{(\cancel{2.5\Omega})(2.5\Omega)}{\frac{\cancel{5\Omega}}{2}} = \frac{1}{2}(2.5\Omega) = 1.25\Omega$

two EQUAL Resistor in parallel have $R' = R/2$

⇒ 

← Series

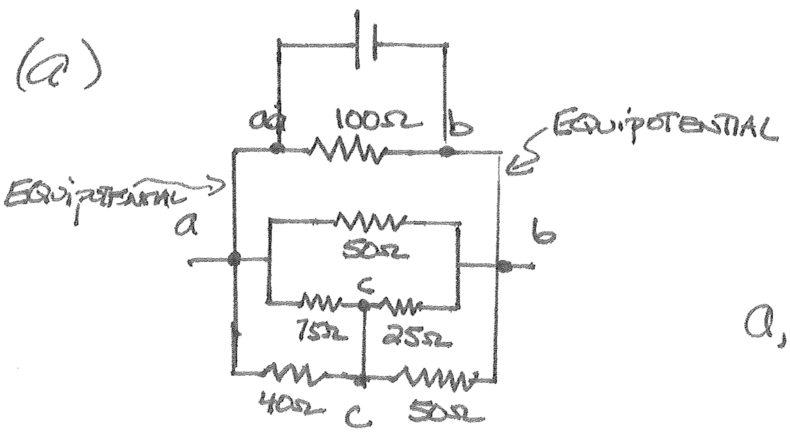
So $\boxed{R_{EQ} = 2.5\Omega + 1.25\Omega + 2.5\Omega = 6.25\Omega}$

#36

IF OHMMETER CONNECTED TO a & b, WHAT WILL IT READ?

OHMMETER APPLIES VOLTAGE ⇒

(a)

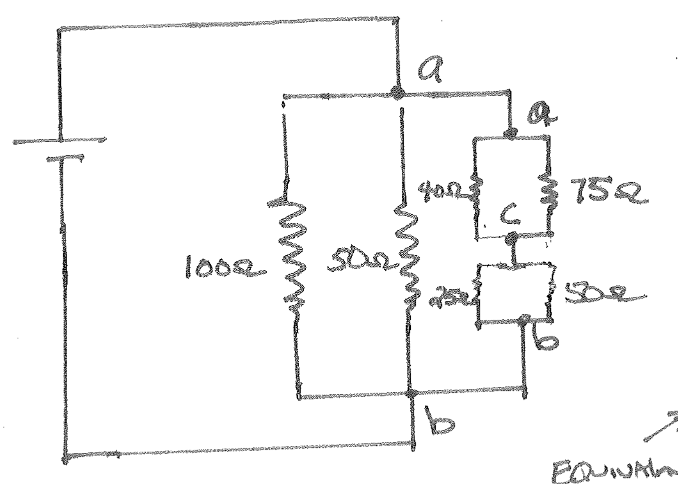


a, b ⇒ 100Ω, UPPER 50Ω, AND REST IN parallel.

a, c ⇒ 75Ω, 40Ω in parallel

b, c ⇒ 25Ω, 50Ω in parallel

⇒



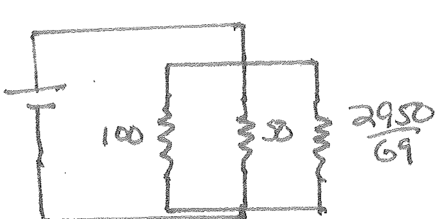
EQUIVALENT FOR 40Ω, 75Ω IN parallel

$$R' = \frac{75\,\Omega(40\,\Omega)}{(75\,\Omega + 40\,\Omega)} = \frac{3000}{115}\,\Omega = \frac{600}{23}\,\Omega = 26.0\ldots$$

EQUIVALENT for 25Ω, 50Ω

$$R'' = \frac{25\,\Omega(50\,\Omega)}{(25\,\Omega + 50\,\Omega)} = \frac{1250}{75}\,\Omega = \frac{50}{3}\,\Omega = 16.666\,\Omega$$

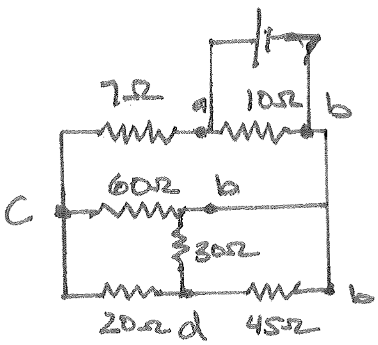
$$R' + R'' = \frac{2950}{69} = 22.753$$ ← R', R'' IN SERIES



$$\frac{1}{R_{EQ}} = \frac{1}{100} + \frac{1}{50} + \frac{1}{2950/69} \Rightarrow \boxed{R_{EQ} = 18.73\,\Omega}$$

#7

(b)



ab ⇒ 10Ω IN parallel to everything else

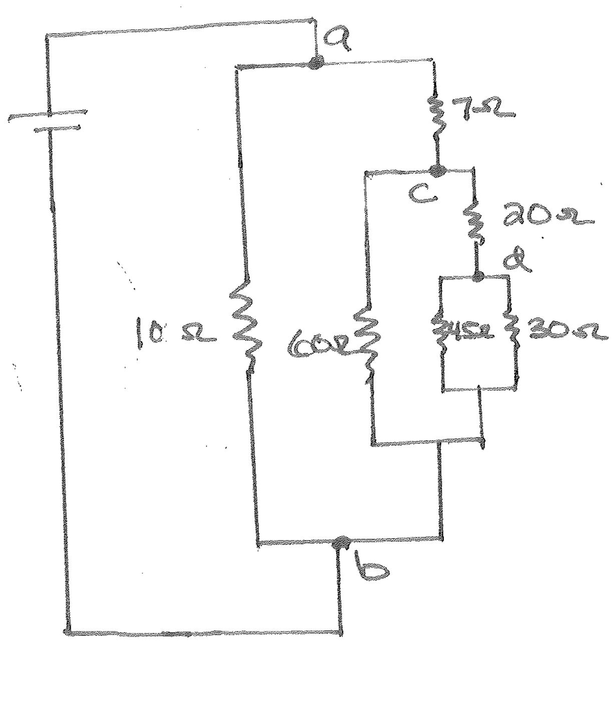
ac ⇒ 7Ω IN SERIES WITH 60, 20, 30, 45 COMBO

cb ⇒ 60Ω IN parallel to 20, 30, 45Ω COMBINATION

db ⇒ 30, 45 IN parallel

cd ⇒ 20 IN SERIES WITH 30, 45

⇒

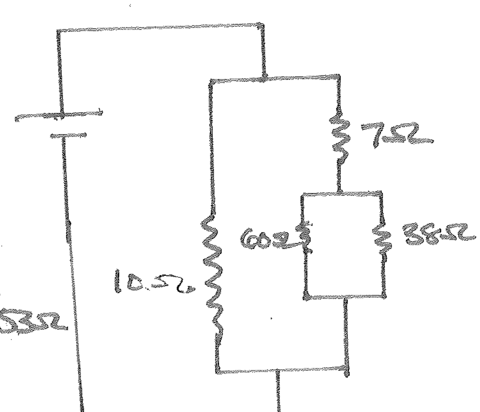


$R' = 30\Omega, 45\Omega$ IN parallel

$\Rightarrow R' = \frac{30\Omega(45\Omega)}{(30\Omega + 45\Omega)} = 18\Omega$

$20\Omega + 18\Omega = 38\Omega$

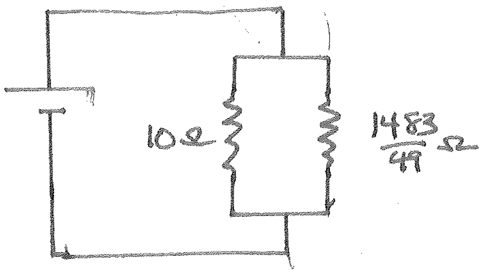
⇒



60, 38 IN parallel $\Rightarrow R'' = \frac{60\Omega(38\Omega)}{98\Omega} = \frac{1140}{49}\Omega$

$= 23.26531\Omega$

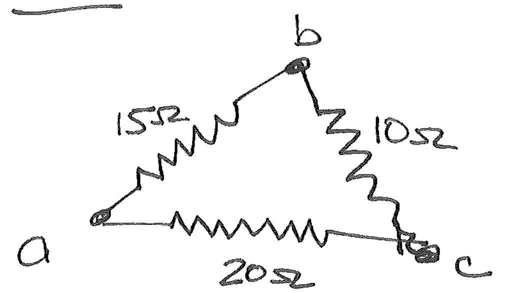
7 IN SERIES $\Rightarrow 7 + \frac{1140}{49} = \frac{1483}{49} = 30.2653\Omega$



$$R_{EQ} = \frac{10\Omega\left(\frac{1483}{49}\Omega\right)}{\left[10\Omega + \frac{1483}{49}\Omega\right]}$$

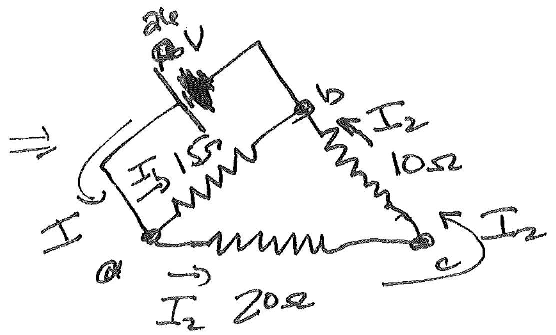
$$\Rightarrow R_{EQ} = 7.5165 = 7.52\Omega$$

#8



WHAT CURRENT FROM 26V battery IF Connected to:

(a) ab ⇒



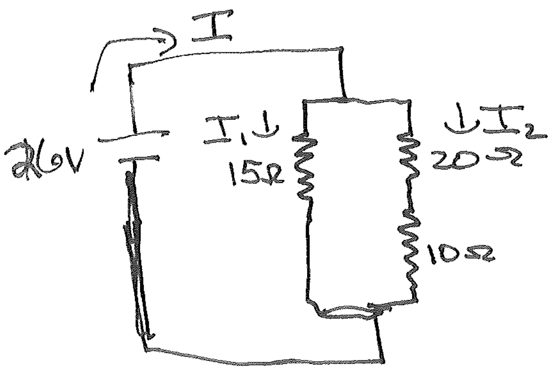
Current splits at a

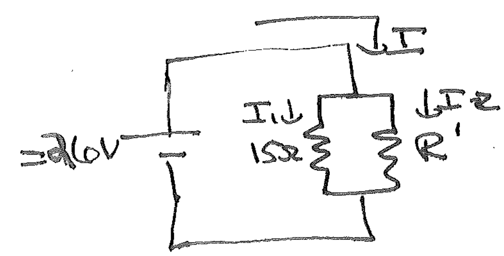
I_1 goes through 15Ω then back to battery

I_2 goes through 20Ω then 10Ω then back to battery

⇒ 20Ω + 10Ω IN SERIES. By "Finger Test" 15Ω in parallel to 20Ω/10Ω COMBINATION

⇒

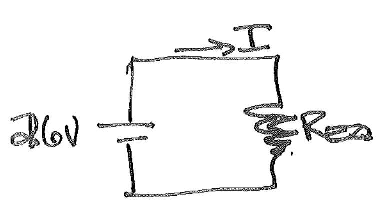




$R' = 20\Omega + 10\Omega$

$= 30\Omega$

||

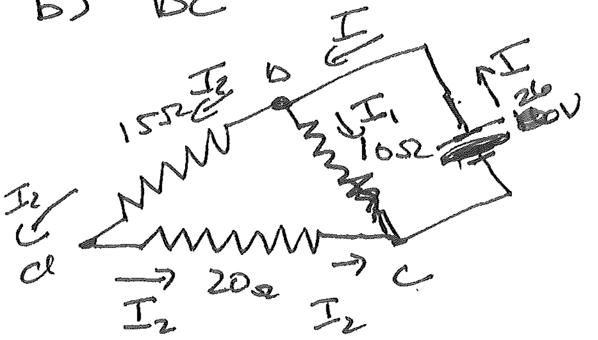


$R_{EQ} = \frac{15\Omega(30\Omega)}{(15\Omega + 30\Omega)}$

$= 10\Omega$

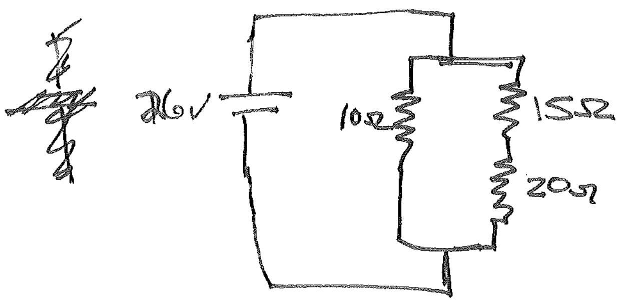
$\Rightarrow \boxed{I_0 = \frac{26V}{10\Omega} = 2.6A}$

b) bc



Now when Current splits, I_1 goes Through 10Ω then back to battery. I_2 goes through 15Ω then 20Ω

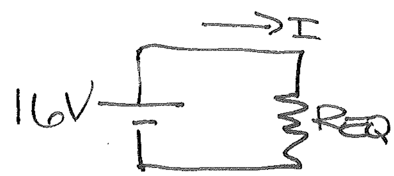
⇒ 15Ω AND 20Ω IN SERIES, 10Ω IN parallel to Combination.



SAME CAlculation for R_{EQ} but

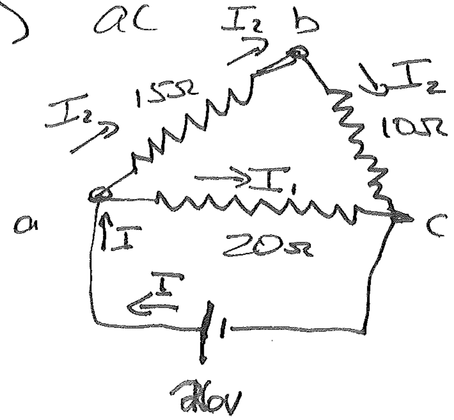
$$R' = 15\Omega + 20\Omega = 35\Omega$$

$$\Rightarrow R_{EQ} = \frac{(10\Omega)(35\Omega)}{(10\Omega + 35\Omega)} = 7.77\ldots\Omega$$

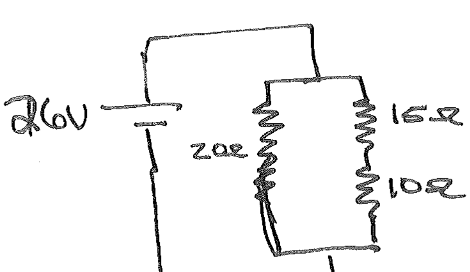


$$\boxed{I = \frac{26V}{7.77\Omega} = 3.34A}$$

c) ac



15Ω AND 10Ω IN SERIES, 20Ω IN Parallel WITH Combination



$$R' = 15\Omega + 10\Omega = 25\Omega$$

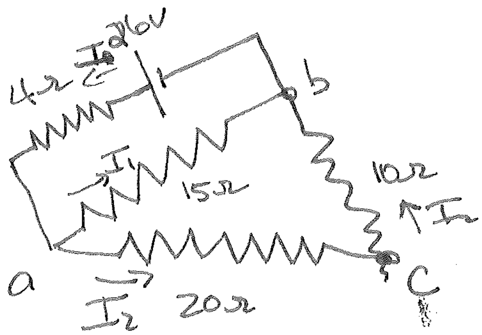
$$R_{EQ} = \frac{(20\Omega)(25\Omega)}{(20\Omega + 25\Omega)}$$

$$= 11.111\ldots\ \Omega$$

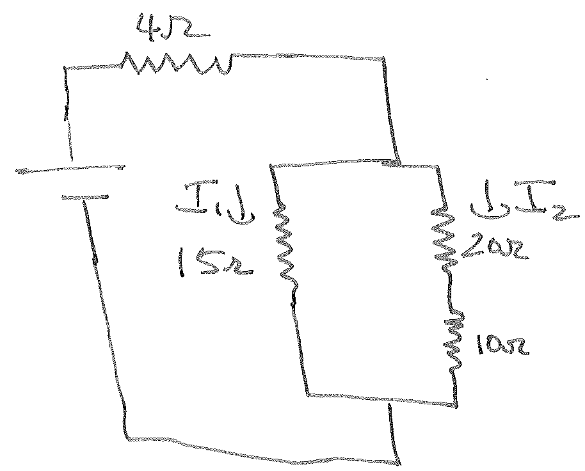
$$\Rightarrow \boxed{I = \frac{26V}{11.111\Omega} = 2.34\ A}$$

d) IF Battery HAS 4Ω of internal Resistance, what Current if Connected across ab?

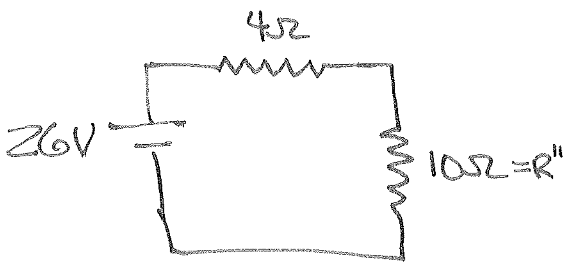
Simply ADD 4Ω is Series with battery.



Still Have 20Ω and 10Ω in Series, 15Ω in parallel with combination.



Like Before the 20Ω, 10Ω And 15Ω Behave like $R'' = \frac{(15\Omega)(30\Omega)}{(15\Omega + 30\Omega)} = 10\Omega$



4Ω AND R" in Series $\Rightarrow R_{EQ} = 4\Omega + 10\Omega$

$\Rightarrow R_{EQ} = 14\Omega$

$$\boxed{I = \frac{26V}{14\Omega} = 1.857A = 1.86A}$$

IN SUMMARY:

	ab	bc	ac	ab with Int. R
I	2.6A	3.34A	2.34A	1.86A