Lecture 38
(Phasors & Alternating Current)

Physics 161-01 Spring 2012

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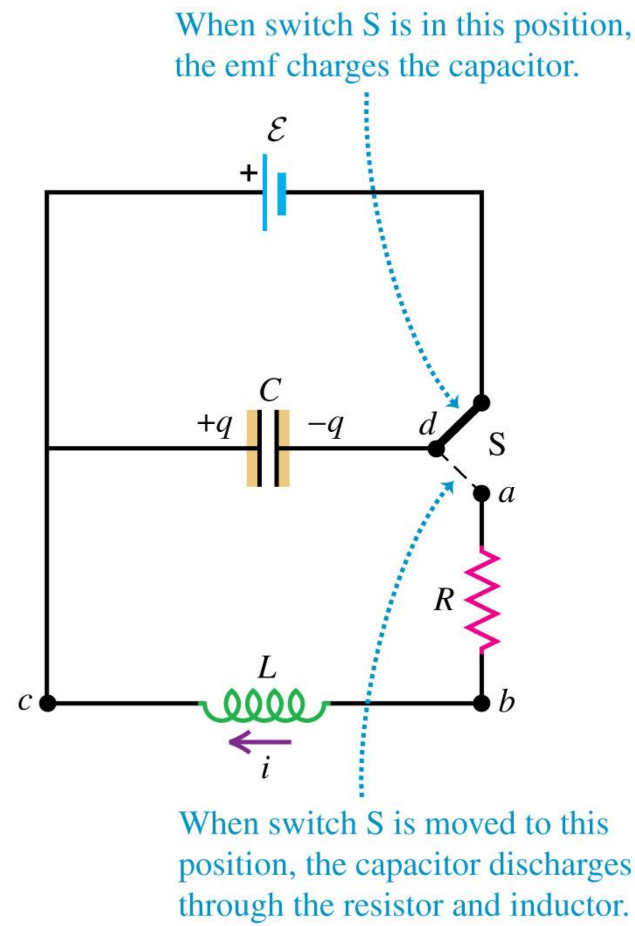
AC/DC

- Until now, we have dealt with circuits where the source of EMF (e.g., the battery) has a constant value.
- This is known as a direct current (DC) source.
- For many reasons however, much of the world's power is not delivered as a uni-directional EMF.

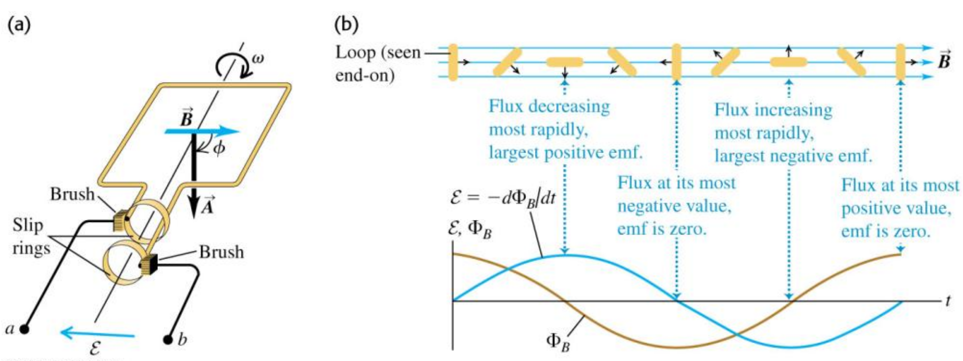


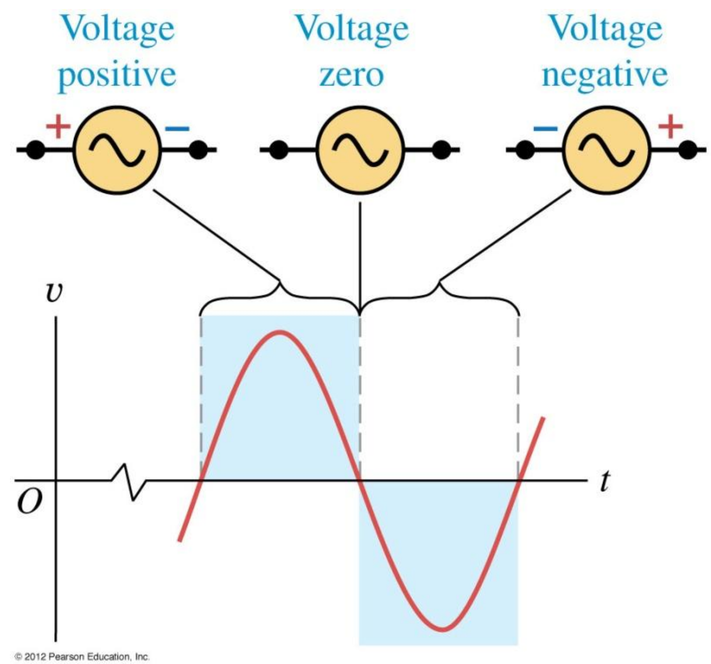
Alternating EMF

- So, now we want to examine how the circuit elements we have behave when they are driven with an alternating current (AC) source.
- An AC source supplies an EMF which follows a cosine dependency:

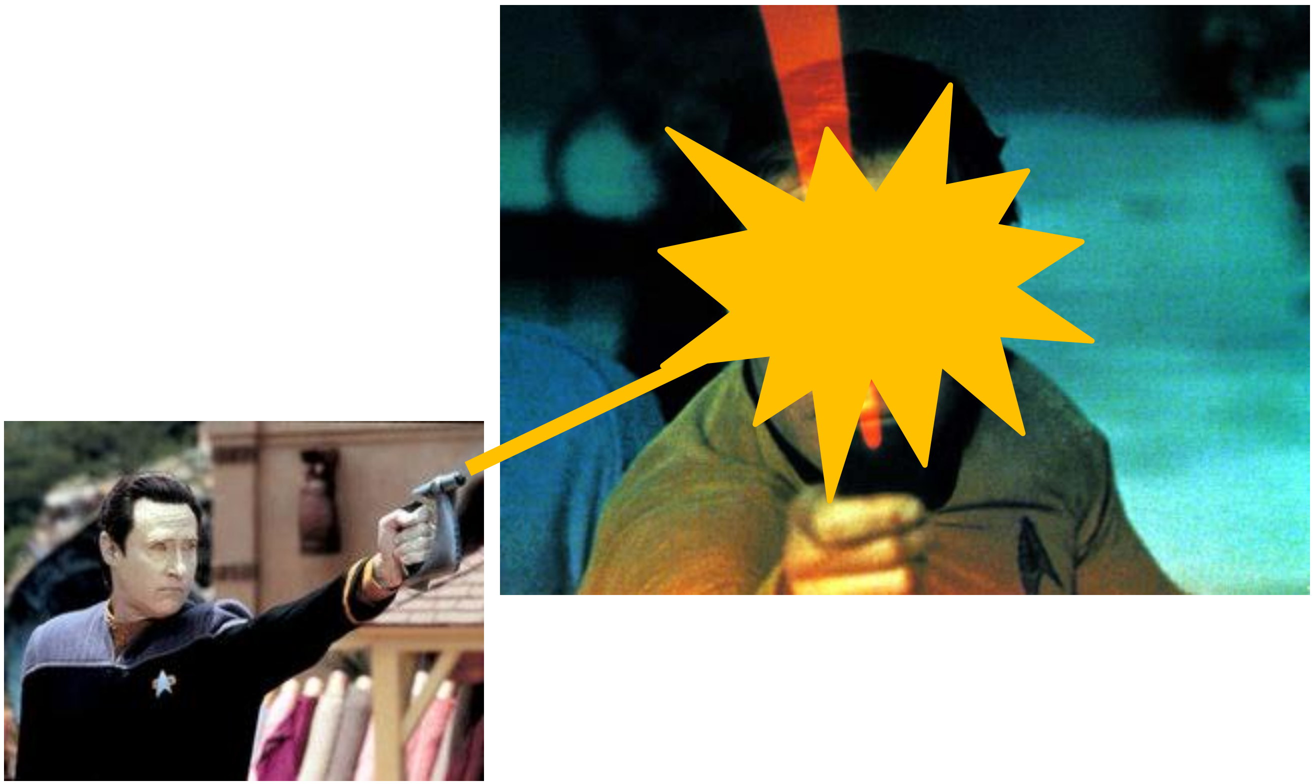
$$v(t) = V_{max}\cos(\omega t)$$

- Is this a random choice?
- No, remember what you get naturally from a generator?





Phasors



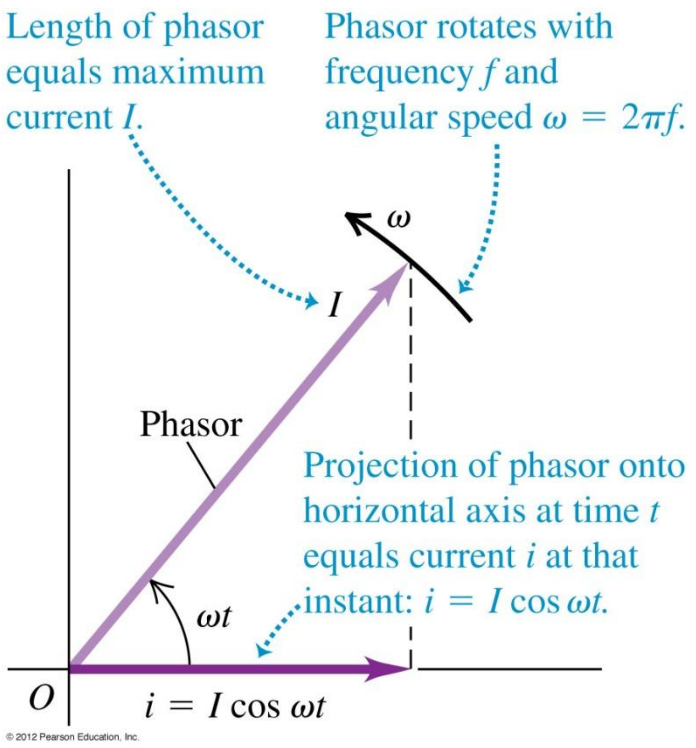
Phasors

- If we have some quantity that depends sinusoidally on time,

$$i(t) = I_{max} \cos(\omega t)$$
$$= I \cos(\omega t)$$

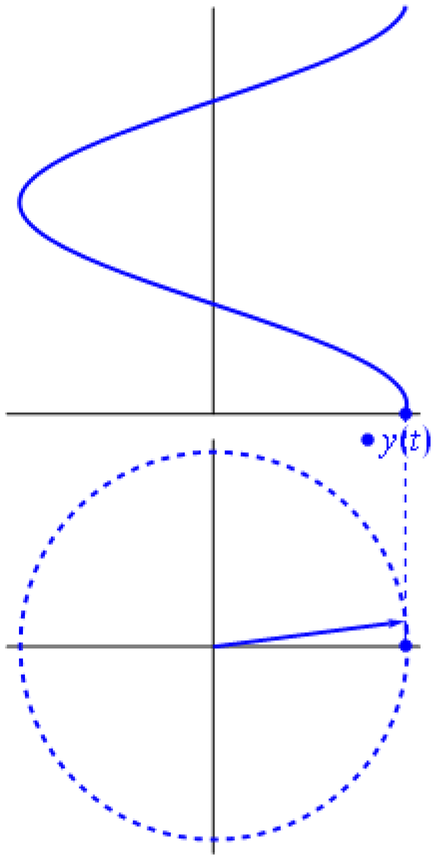
  we can represent that as a vector of length I, which rotates around the origin with an angular velocity ω.

- i(t) is now represented by the projection of the phasor onto the horizontal axis.



Phasors

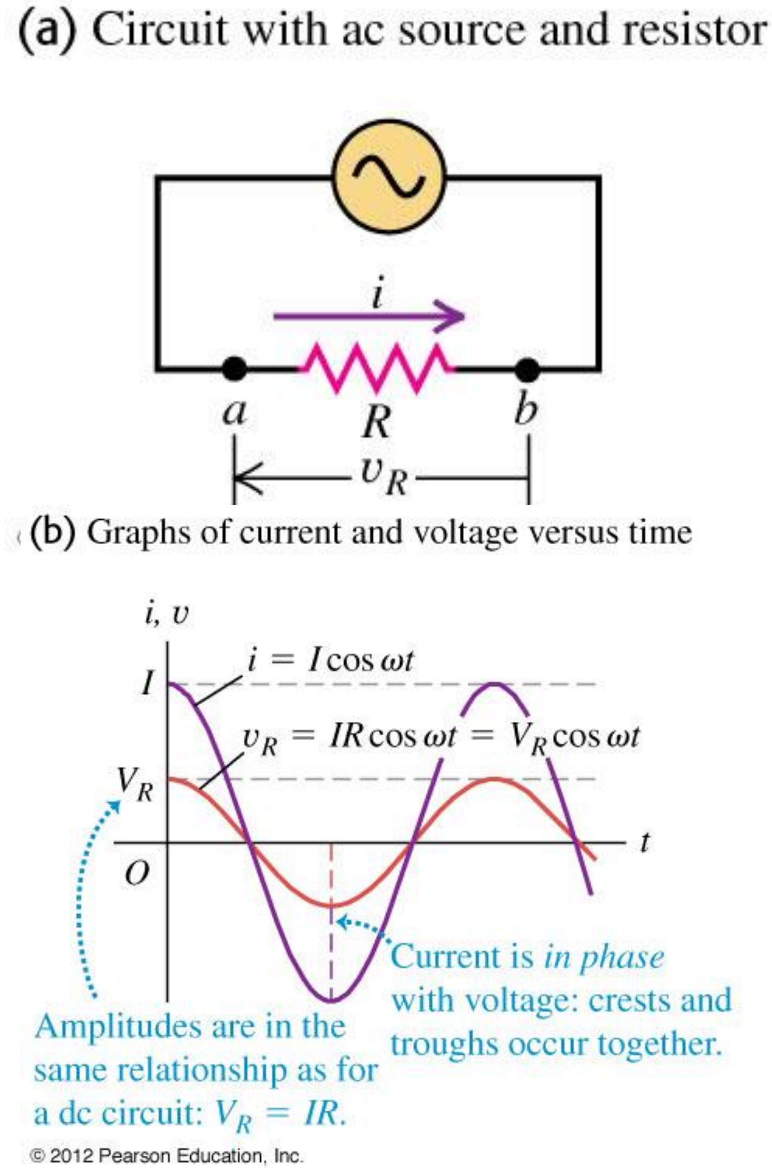
- Here is a nice representation of a moving phasor.
- You can find this at: http://en.wikipedia.org/wiki/Phasor
- You may be wondering: “How is this useful?”
- Hopefully, by the end of today’s lecture, you will see.



AC Circuits With a Resistor

- Now, let's re-look at all of our circuit elements, now with an AC source.
- If we put a resistor in series with an AC source, the current through the resistor will just be given by Ohm's Law:

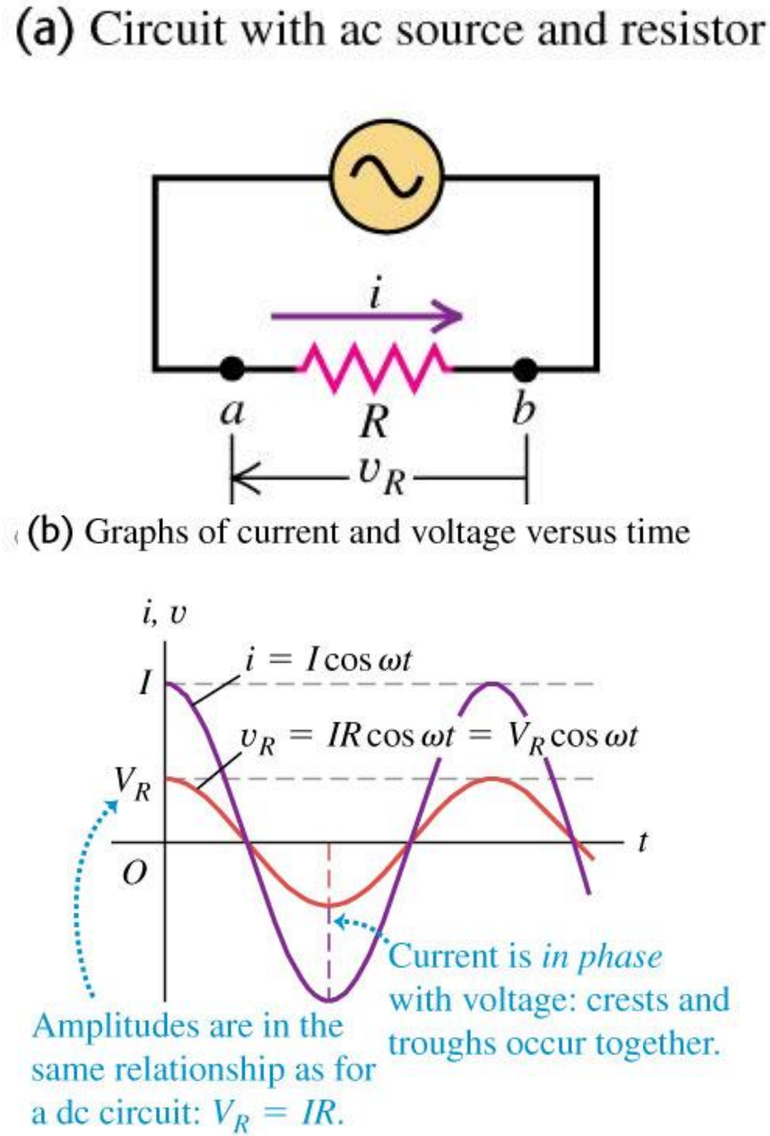
$$v_R = iR \Rightarrow$$
$$v_R = IR\cos(\omega t)$$



AC Circuits With a Resistor

- So we can represent the voltage across the resistor as a phasor, which ***is in phase with*** current phasor.

$$v_R = IR\cos(\omega t)$$

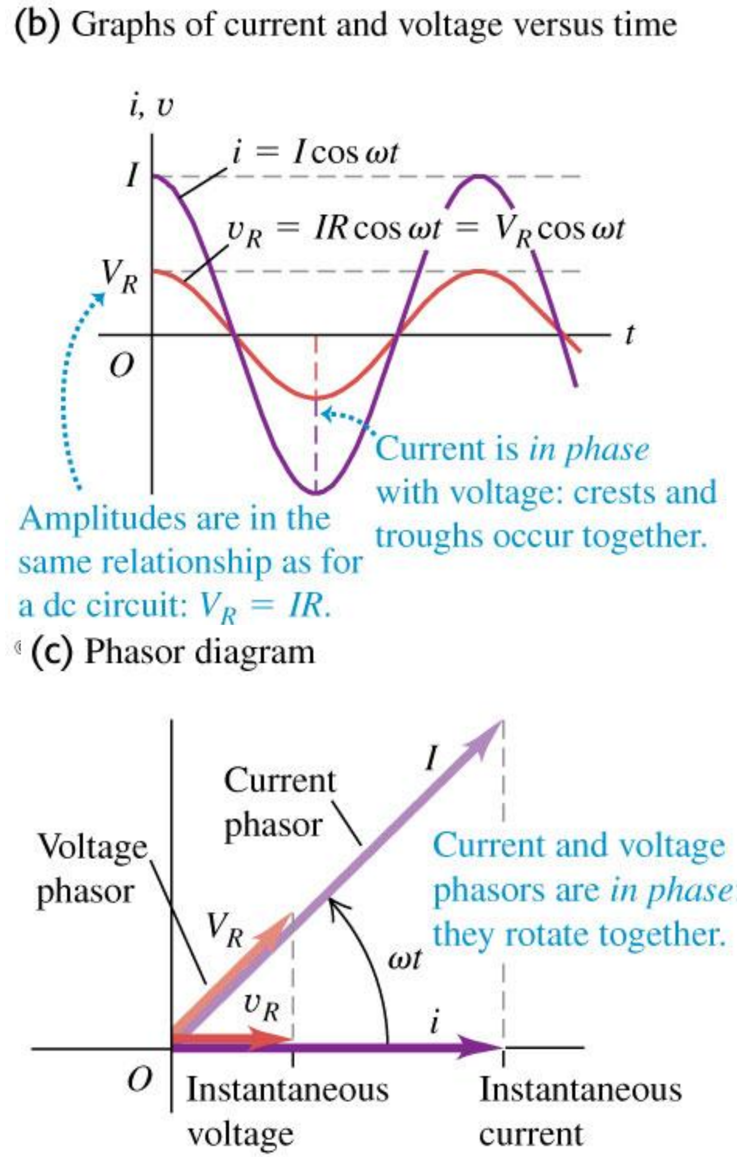


AC Circuits With a Resistor

- Notice that the magnitude of the phasor gives its maximum value, and we can write that maximum value as:

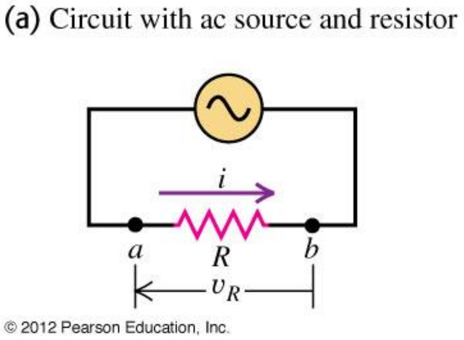
$$v_R = IR\cos(\omega t) = V_R\cos(\omega t)$$

$$V_R = IR$$



CPS 38-1

A resistor is connected across an ac source as shown. For this circuit, what is the relationship between the instantaneous current i through the resistor and the instantaneous voltage v_{ab} across the resistor?



A. i is maximum at the same time as v_{ab}.

B. i is maximum one-quarter cycle before v_{ab}.

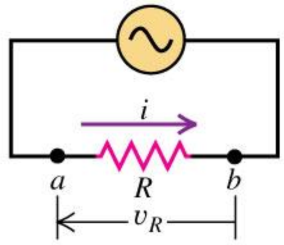
C. i is maximum one-quarter cycle after v_{ab}.

D. not enough information given to decide

CPS 38-1

A resistor is connected across an ac source as shown. For this circuit, what is the relationship between the instantaneous current i through the resistor and the instantaneous voltage v_{ab} across the resistor?

(a) Circuit with ac source and resistor



A. i is maximum at the same time as v_{ab}.

B. i is maximum one-quarter cycle before v_{ab}.

C. i is maximum one-quarter cycle after v_{ab}.

D. not enough information given to decide

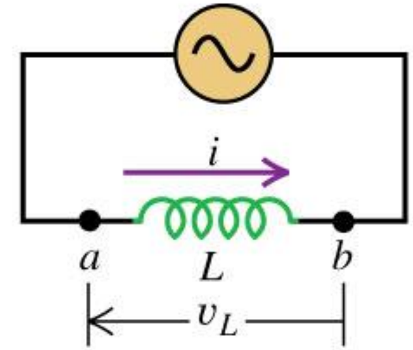
AC Circuits With an Inductor

- Now, let's look at what an inductor looks like when connected to an AC source.

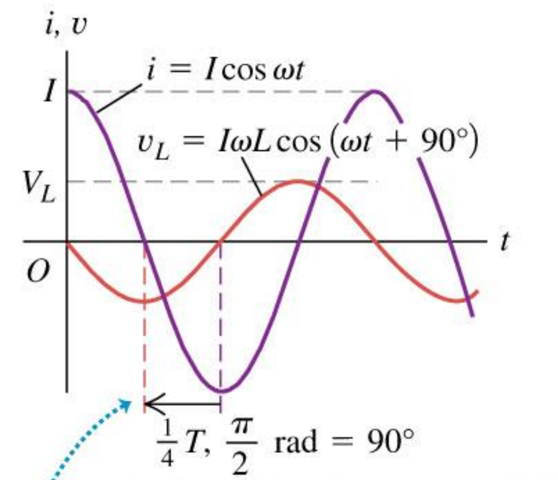
$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left(I\cos\left(\omega t\right)\right) = -I\omega L\sin\left(\omega t\right)$$
$$= I\omega L\cos\left(\omega t + 90°\right)$$

- So, the voltage across the inductor is out of phase with the current source…

(a) Circuit with ac source and inductor



(b) Graphs of current and voltage versus time



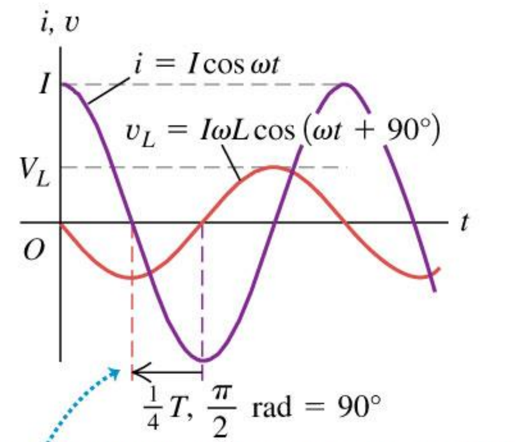
Voltage curve *leads* current curve by a quarter-cycle (corresponding to $\phi = \pi/2$ rad = 90°).

AC Circuits With an Inductor

- So we can represent the voltage across the inductor also as a phasor, which ***leads*** the current phasor by 90 degrees.

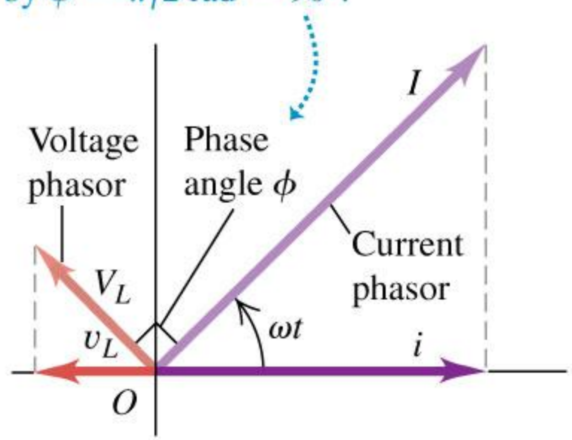
$$v_L = I\omega L\cos\left(\omega t + 90°\right)$$

(b) Graphs of current and voltage versus time



Voltage curve *leads* current curve by a quarter-cycle (corresponding to $\phi = \pi/2$ rad $= 90°$).

Voltage phasor *leads* current phasor by $\phi = \pi/2$ rad $= 90°$.



AC Circuits With an Inductor

- Notice that the magnitude of the phasor gives its maximum value, and we can write that maximum value as:

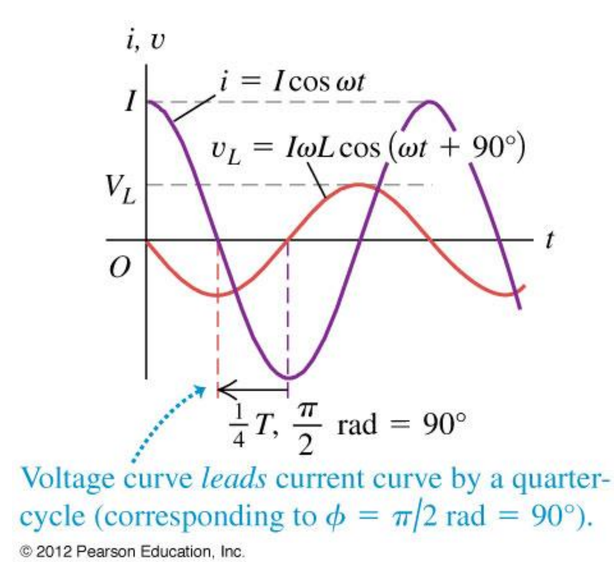
$$v_L = I\omega L\cos\left(\omega t + 90^\circ\right) = V_L\cos\left(\omega t + 90^\circ\right)$$

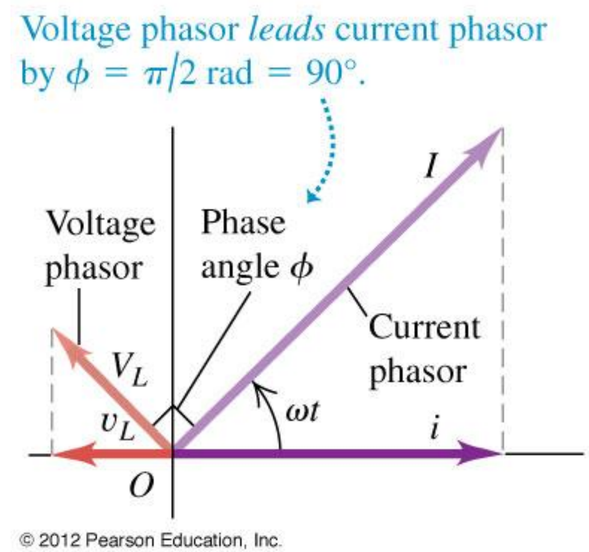
$$V_L = IX_L,$$

$$X_L = \omega L$$

- X_L is called the inductive reactance.

(b) Graphs of current and voltage versus time





AC Circuits With a Capacitor

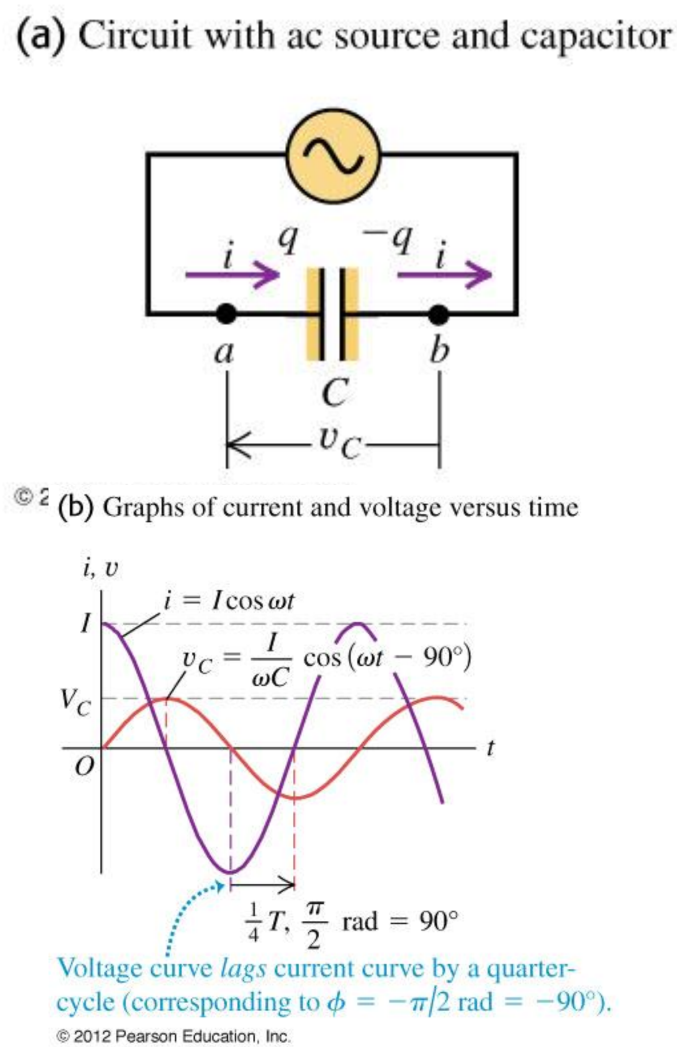
- Now, let's see what a capacitor looks like when connected to an AC source.

$$v_C = \frac{q}{C}$$

$$q = \int_0^q i\,dt = \int_0^q I\cos(\omega t)\,dt = \frac{I}{\omega}\sin(\omega t) \Rightarrow$$

$$v_C = \frac{I}{\omega C}\sin(\omega t) = \frac{I}{\omega C}\cos\left(\omega t - 90^\circ\right)$$

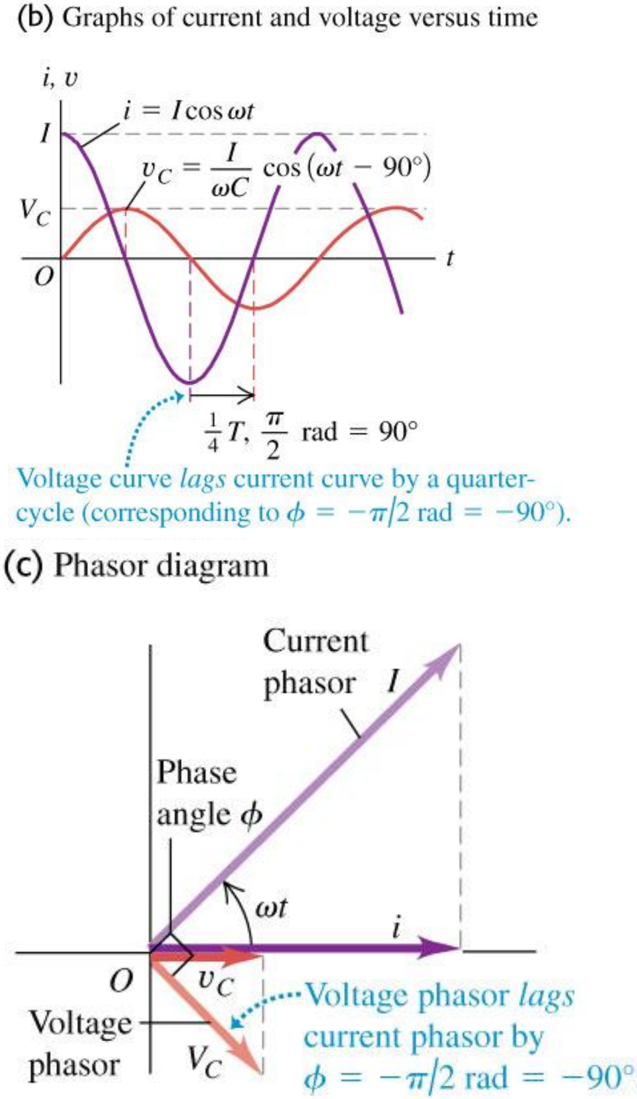
- So, the voltage across the capacitor is out of phase with the current source…



AC Circuits With a Capacitor

- So we can represent the voltage across the inductor also as a phasor, which ***lags*** the current phasor by 90 degrees.

$$v_C = \frac{I}{\omega C}\cos\left(\omega t - 90^\circ\right)$$



AC Circuits With a Capacitor

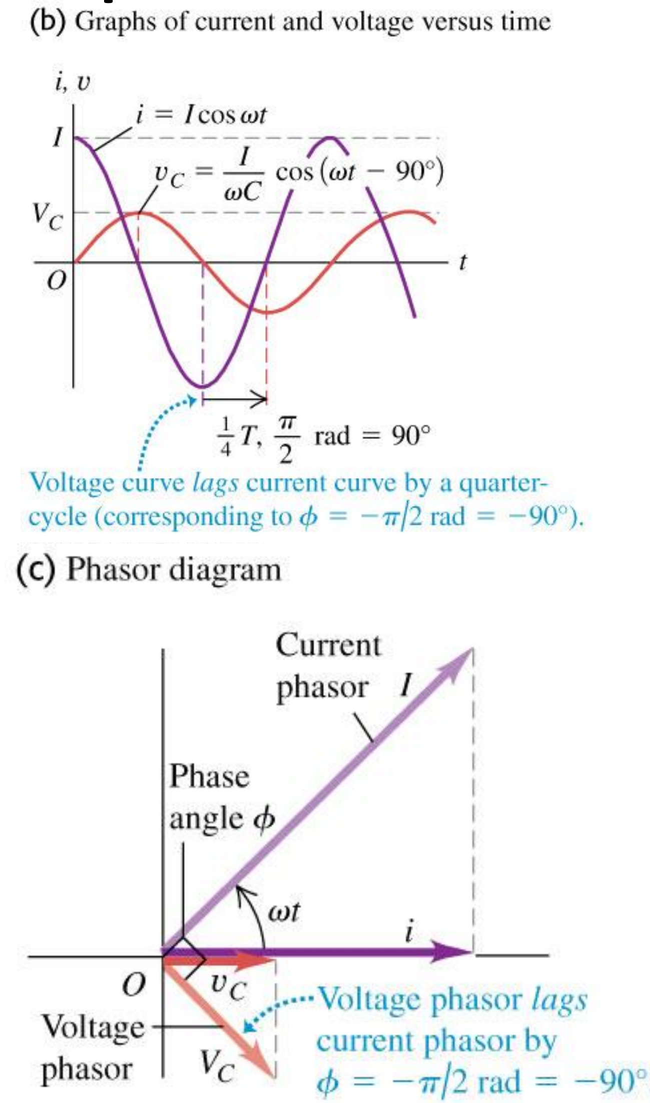
- Notice that the magnitude of the phasor gives its maximum value, and we can write that maximum value as:

$$v_C = \frac{I}{\omega C}\cos\left(\omega t - 90^\circ\right) = V_C \cos\left(\omega t - 90^\circ\right)$$

$$V_C = IX_C,$$

$$X_C = \frac{1}{\omega C}$$

- X_C is called the capacitive reactance.

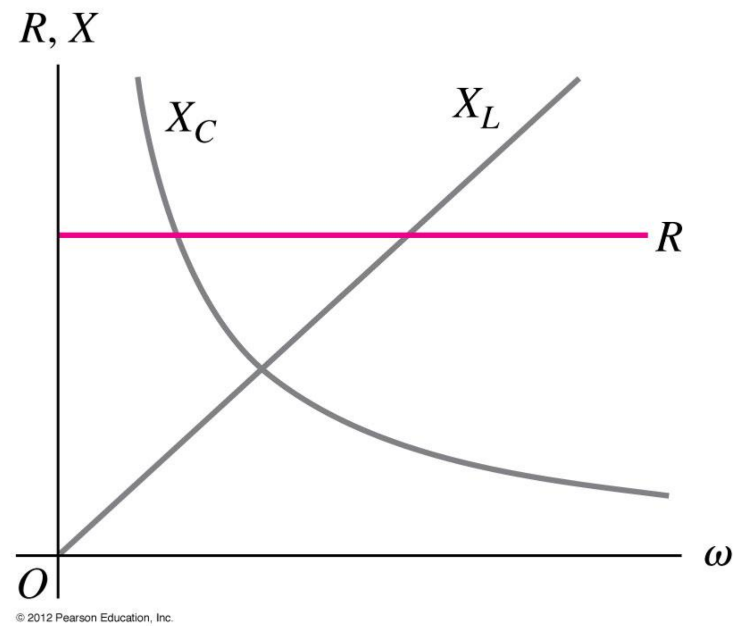


Usefulness of Reactances

Table 31.1 Circuit Elements with Alternating Current

Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	In phase with i
Inductor	$V_L = IX_L$	$X_L = \omega L$	Leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = 1/\omega C$	Lags i by 90°

- Notice that the reactances are dependent on the angular frequency, (the resistance is not).
- As $\omega \rightarrow 0$ (DC), there is no inductive effect – X_L goes to zero and current is passed through the inductor, while no current is passed through the capacitor – X_C diverges.
- As ω gets large, X_C goes to zero and current is passed through the capacitor, while no current is passed through the inductor – X_L gets large because of the quickly changing current.



Usefulness of Reactances

- We can use these properties to create frequency filters.
- Inductors are used as “low-pass” filters.
- Capacitors are used as “high-pass” filters.
- In combination, you can create a “cross-over” circuit.