#16 Work, Energy and Power Pre-class

Due: 11:00am on Friday, September 28, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

± The Power of One

Learning Goal:

To learn the definition of power and how power, force, and velocity are related.

The definition of work done by a force ($W = \vec{F} \cdot \vec{s}$) does not include time. For practical purposes, however, it is often important to know how fast work is being done. The rate at which work is being done is called power P. The *average power* $P_{\rm avg}$ can be calculated as

$$P_{\rm avg} = \frac{\Delta W}{\Delta t},$$

where ΔW is the amount of work done during the time interval Δt.

The power created by a force may be a constant; that is, work is being done at constant rate. However, this is not always the case. If the rate of performing work is changing, it makes sense to talk about the *instantaneous power*, defined as

$$P = \frac{dW}{dt}.$$

The SI unit of power is the watt (W). One watt is defined as the power created when one joule of work is done each second. In equation form, one writes

$$1 \ {\rm W} = 1 \ {\rm J/s}.$$

A commonly used unit of work is the kilowatt-hour (kW-hour). One kilowatt-hour is the amount of work done in one hour when the power is one kilowatt. In equation form, this is

$$1 \ {\rm kW\text{-}hour} = 1 \ {\rm kW} \cdot 1 \ {\rm hour}$$
$$= 10^3 \ W \cdot 3.6 \times 10^3 \ s = 3.6 \ MJ.$$

In this problem, you will answer several questions that will help familiarize you with power and enable you to derive a formula relating power, force, and velocity.

A sled of mass m is being pulled horizontally by a constant horizontal force of magnitude F. The coefficient of kinetic friction is $\mu_{\rm k}$. During time interval t, the sled moves a distance s, starting from rest.

Part A

Find the average power $P_{\rm avg}$ created by the force F.

Express your answer in terms of the given quantities and, if necessary, appropriate constants. You may or may not use all of the given quantities.

Hint 1. Find the work W done by the force F.

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ANSWER:

$W = Fs$

ANSWER:

$P_{\rm avg} = \frac{Fs}{t}$

Correct

Part B

Find the average velocity $v_{\rm avg}$ of the sled during that time interval.

Express your answer in terms of the given quantities and, if necessary, appropriate constants. You may or may not use all of the given

quantities.

ANSWER:

v_{avg} = $\frac{s}{t}$

Correct

Part C

Find the average power P_{avg} created by the force F in terms of the average speed v_{avg} of the sled.

Express your answer in terms of F and v_{avg}.

ANSWER:

P_{avg} = Fv_{avg}

Correct

You just obtained a very useful formula for the average power:

$$P = \frac{Fs}{t} = Fv.$$

If an object is moving at a constant speed, and the force F is also constant, this formula can be used to find the average power. If v is changing, the formula can be used to find the *instantaneous* power at any given moment (with the quantity v in this case meaning the *instantaneous* velocity, of course).

Part D

A sled of mass m is being pulled horizontally by a constant upward force of magnitude F that makes an angle θ with the direction of motion. The coefficient of kinetic friction is μ_k. The average velocity of the sled is v_{avg}.

Find the average power P_{avg} created by force F.

Express your answer in terms of F, θ, and v_{avg}.

Hint 1. Find the work done

When the directions of the force and the velocity vectors are the same, $W = Fs$. When the two vectors make an angle θ, which of the following formulas can be applied?

ANSWER:

- ○ $W = Fs\tan\theta$
- ○ $W = Fs\sin\theta$
- ◉ $W = Fs\cos\theta$
- ○ $W = \frac{Fs}{\cos\theta}$

ANSWER:

P_{avg} = $F\cos\theta v_{avg}$

Correct

Another way to express this formula is this

$$P = F_{||}v,$$

where $F_{||}$ is the component of force parallel to the velocity of the object.

Let us now consider several questions that include numeric data.

A sled is being pulled along a horizontal surface by a horizontal force $\vec{F}$ of magnitude 600 N. Starting from rest, the sled speeds up with acceleration 0.08 m/s^2 for 1 minute.

Part E

Find the average power P_{avg} created by force $\vec{F}$.

Express your answer in watts to three significant figures.

Hint 1. Find the work W done by force $\vec{F}$.

Find the work W done by force $\vec{F}$.

Express your answer in joules to three significant figures.

Hint 1. Find the displacement of the sled

Which formula is useful in finding the displacement of the sled?

ANSWER:

- ○ $s = vt$
- ○ $s = \frac{v}{t}$
- ○ $s = \frac{at}{2}$
- ◉ $s = \frac{at^2}{2}$

ANSWER:

W = 8.64×10^4 J

ANSWER:

P_{avg} = 1440 W

Correct

Part F

Find the *instantaneous* power P created by force $\vec{F}$ at $t = 10 \text{ s}$.

Express your answer in watts to three significant figures.

Hint 1. Find the speed

Find the speed v of the sled after 10 s..

Express your answer in meters per second.

ANSWER:

v = 0.8 m/s

ANSWER:

P = 480 W

Correct

Part G

Find the instantaneous power P created by the normal force at $t = 10\ \mathrm{s}$. The magnitude of the acceleration due to gravity is $g = 9.8\ \mathrm{m/s^2}$.

Express your answer in watts to three significant figures.

Hint 1. Work done by the normal force

Recall that the normal force is directed upward, while the sled is moving horizontally. Given these directions, what is the angle between the force and the direction of motion? What does this tell you about how much work the normal force does?

ANSWER:

P = 0 W

Correct

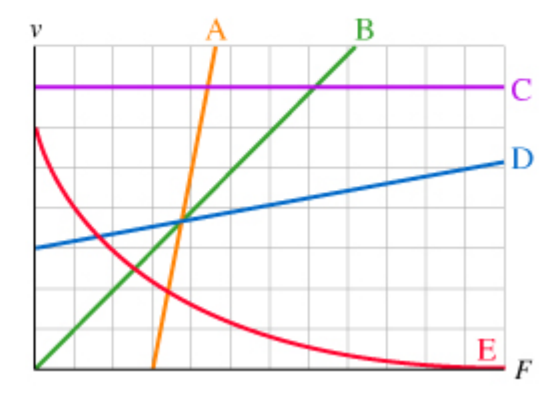
When vectors $\vec{F}$ and $\vec{v}$ are perpendicular, the power created by force $\vec{F}$ is zero.

Hill's Law Conceptual Question

Imagine that you're loading a pickup truck with bags of groceries. You notice that the smaller the weight you attempt to lift, the quicker you can lift it. However, you also notice that there is a limit to how quickly you can lift even very small weights, and that above a certain weight, you can no longer lift the weight at all. The detailed relationship between the contraction velocity of a muscle (the speed with which you can lift something) and the weight you are attempting to lift, is known as Hill's law.

Part A

Based on this description, which of the following graphs of velocity vs. force is a possible representation of Hill's law?



Hint 1. Maximum weight

Hill's law states that there exists a maximum force that a muscle can exert, and thus a maximum weight that a muscle can lift. Only one graph has a limit to the maximum force that can be produced.

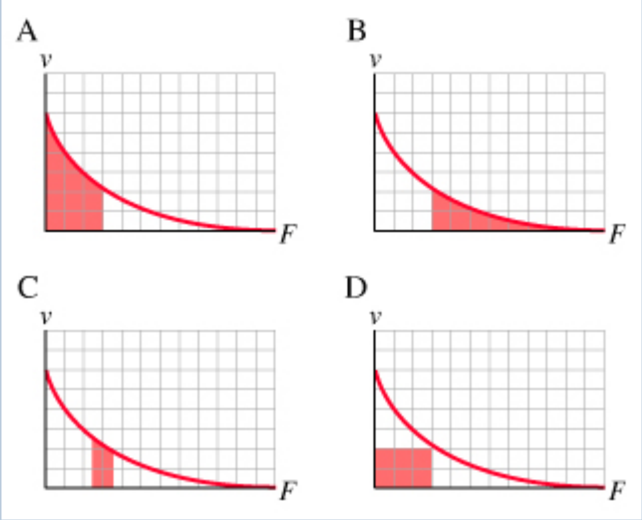
ANSWER:

- A
- B
- C
- D
- E

Correct

Part B

The *power*exerted by a muscle is the product of the force exerted and the velocity of contraction. The area of which of these shaded regions represents the power exerted while a weight is lifted at maximum speed?



Hint 1. How to approach the problem

In this example, power is the product of the force exerted by the muscles and the contraction velocity of the muscles. In lifting any given weight, our muscles have a single maximum contracting velocity; we assume, in this case, that the weight is lifted at this velocity. In lifting a given weight at a constant velocity, your muscles exert a constant force. Therefore, in this example, both the force and the contraction velocity are constant for a given weight. How can the product of two constant numbers be shown graphically?

ANSWER:

- A
- B
- C
- D
- None of the above

Correct

The power produced by a muscle is represented by the area of the rectangle formed by the two coordinate axes and the point on the Hill's law graph representing the weight being lifted. Notice that if you lift a very large weight (near the limit of the maximum force your muscle can produce), the area of this "long and skinny" rectangle can be quite small. If you lift a very small weight, the area of this "tall and skinny" rectangle can also be quite small. However, if you lift a weight near the middle of your weightlifting range, the area of the rectangle, and hence the power produced by your muscle, is a maximum.

± Hooke's Law

Learning Goal:

To understand the use of Hooke's law for a spring.

Hooke's law states that the *restoring force* $\vec{F}$ on a spring when it has been stretched or compressed is proportional to the displacement $\vec{x}$ of the spring from its equilibrium position. The equilibrium position is the position at which the spring is neither stretched nor compressed.

Recall that $\vec{F} \propto \vec{x}$ means that $\vec{F}$ is equal to a constant times $\vec{x}$. For a spring, the proportionality constant is called the *spring constant* and denoted by k.

The spring constant is a property of the spring and must be measured experimentally. The larger the value of k, the stiffer the spring.

In equation form, Hooke's law can be written

$$\vec{F} = -k\vec{x}.$$

The minus sign indicates that the force is in the opposite direction to that of the spring's displacement from its equilibrium length and is "trying" to *restore* the spring to its equilibrium position. The magnitude of the force is given by $F = kx$, where x is the magnitude of the displacement.

In Haiti, public transportation is often by taptaps, small pickup trucks with seats along the sides of the pickup bed and railings to which passengers can hang on. Typically they carry two dozen or more passengers plus an assortment of chickens, goats, luggage, etc. Putting this much into the back of a pickup truck puts quite a large load on the truck springs.
A truck has springs for each wheel, but for simplicity assume that the individual springs can be treated as one spring with a spring constant that includes the effect of all the springs. Also for simplicity, assume that all four springs compress equally when weight is added to the truck and that the equilibrium length of the springs is the length they have when they support the load of an empty truck.

Part A

A 62kg driver gets into an empty taptap to start the day's work. The springs compress $1.9\times10^{-2}\mathrm{m}$. What is the effective spring constant of the spring system in the taptap?

Enter the spring constant numerically in newtons per meter. Express your answer using two significant figures.

Hint 1. How to approach the problem

The compression of the springs is governed by Hooke's law. The amount the springs are compressed when the driver climbs into the truck is given in the problem statement. The force that acts to compress the springs is the force caused by the driver getting into the truck.

ANSWER:

k = 3.2×10^{4} $\mathrm{N/m}$

Correct

If you need to use the spring constant in subsequent parts, use the full precision value you calculated, only rounding as a final step before submitting your answer.

Part B

After driving a portion of the route, the taptap is fully loaded with a total of 24 people including the driver, with an average mass of 62kg per person. In addition, there are three 15-kg goats, five 3-kg chickens, and a total of 25 kg of bananas on their way to the market. Assume that the springs have somehow not yet compressed to their maximum amount. How much are the springs compressed?

Enter the compression numerically in meters. Express your answer using two significant figures.

Hint 1. How to find the compression of the spring

The spring compression is governed by Hooke's law. Use the spring constant you calculated to full precision in Part A prior to rounding your answer. To find the force add the total weight of the load on the truck. Only round as a final step before submitting your answer.

ANSWER:

x = 0.48 m

Correct

Part C

Whenever you work a physics problem you should get into the habit of thinking about whether the answer is physically realistic. Think about how far off the ground a typical small truck is. Is the answer to Part B physically realistic?

Select the best choice below.

ANSWER:

- ◉ No, typical small pickup truck springs are not large enough to compress 0.48m .
- ○ Yes, typical small pickup truck springs can easily compress 0.48m .

Correct

The answer to Part B is not physically realistic because the springs of a typical light truck will compress their maximum amount (typically about 10 cm) before the total weight of all the passengers and other cargo given in Part B is added to the truck. When this maximum compression is reached, the springs will bottom out, and the ride will be very rough.

Part D

Now imagine that you are a Haitian taptap driver and want a more comfortable ride. You decide to replace the springs with new springs that can handle the typical heavy load on your vehicle. What spring constant do you want your new spring system to have?

ANSWER:

The new springs should have a spring constant that is

- (selected) substantially larger
- slightly larger
- slightly smaller
- substantially smaller

than the spring constant of the old springs.

Correct

A spring constant with a large value is a stiff spring. It will take more force to compress (or stretch) a stiff spring. On a taptap, stiffer springs are less likely to bottom out under a heavy load. However, with a lighter load, for most vehicles, very stiff springs will not compress as much for a bump in the road. Hence very stiff springs will give a better ride with a very heavy load, but less-stiff springs (lower spring constant) will give a smoother ride with a light load. This is why larger vehicles need stiffer springs than smaller vehicles.

Score Summary:

Your score on this assignment is 101.3%.
You received 15.19 out of a possible total of 15 points.