#5 Motion With Constant Acceleration, Free Fall Pre-class

Due: 11:00am on Friday, August 31, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Tossing Balls off a Cliff

Learning Goal:

To clarify the distinction between speed and velocity, and to review qualitatively one-dimensional kinematics.

A woman stands at the edge of a cliff, holding one ball in each hand. At time t_0, she throws one ball straight up with speed v_0 and the other straight down, also with speed v_0.

For the following questions neglect air resistance. Pay particular attention to whether the answer involves "absolute" quantities that have only magnitude (e.g., speed) or quantities that can have either sign (e.g., velocity). Take upward to be the positive direction.

Part A

If the ball that is thrown downward has an acceleration of magnitude a at the instant of its release (i.e., when there is no longer any force on the ball due to the woman's hand), what is the relationship between a and g, the magnitude of the acceleration of gravity?

ANSWER:

- ○ $a > g$
- ◉ $a = g$
- ○ $a < g$

Correct

Part B

Which ball has the greater acceleration at the instant of release?

ANSWER:

- the ball thrown upward
- the ball thrown downward
- Neither; the accelerations of both balls are the same.

Correct

Part C

Which ball has the greater speed at the instant of release?

Hint 1. Consider the initial speeds

Both of the balls were given initial speed v_0.

ANSWER:

- the ball thrown upward
- the ball thrown downward
- Neither; the speeds are the same.

Correct

Part D

Which ball has the greater average speed during the 1-s interval after release (assuming neither hits the ground during that time)?

Hint 1. How to approach the problem

This question asks which ball has the greater average speed during the first second after release. You already know that the speed of the two balls is the same at the instant of release. However, the ball thrown downward starts to speed up due to gravity, whereas the ball thrown upward starts to slow down. So the instantaneous speed after the first second of flight is different for the two balls.

ANSWER:

- ○ the ball thrown upward
- ◉ the ball thrown downward
- ○ Neither; the average speeds of both balls are the same.

Correct

Part E

Which ball hits the ground with greater speed?

ANSWER:

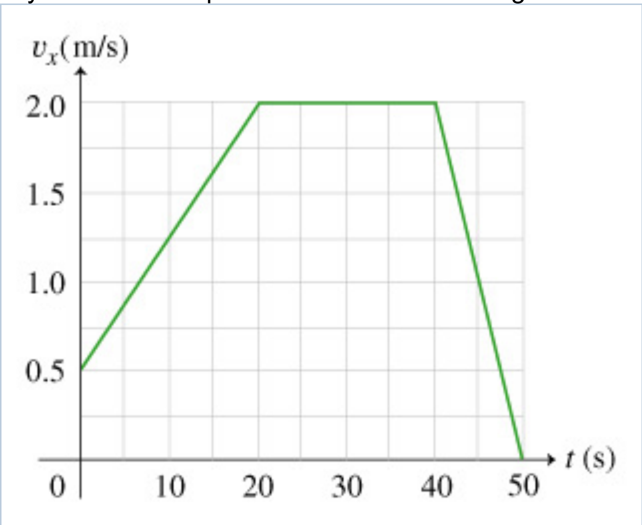
- ○ the ball thrown upward
- ○ the ball thrown downward
- ◉ Neither; the balls hit the ground with the same speed.

Correct

What Velocity vs. Time Graphs Can Tell You

A common graphical representation of motion along a straight line is the *v vs. t graph*, that is, the graph of (instantaneous) velocity as a function of time. In this graph, time t is plotted on the horizontal axis and velocity v on the vertical axis. Note that by definition, velocity and acceleration are vector quantities. In straight-line motion, however, these vectors have only a single nonzero component in the direction of motion. Thus, in this problem, we will call v the velocity and a the acceleration, even though they are really the components of the velocity and acceleration vectors in the direction of motion, respectively.

Here is a plot of velocity versus time for a particle that travels along a straight line with a varying velocity. Refer to this plot to answer the following questions.



Part A

What is the initial velocity of the particle, v_0?

Express your answer in meters per second.

Hint 1. Initial velocity

The initial velocity is the velocity at $t = 0\ \mathrm{s}$.

Hint 2. How to read a v vs. t graph

Recall that in a graph of velocity versus time, time is plotted on the horizontal axis and velocity on the vertical axis. For example, in the plot shown in the figure, $v = 2.00\ \mathrm{m/s}$ at $t = 30.0\ \mathrm{s}$.

ANSWER:

v_0 = 0.5 $\mathrm{m/s}$

Correct

Part B

What is the total distance Δx traveled by the particle?

Express your answer in meters.

Hint 1. How to approach the problem

Recall that the area of the region that extends over a time interval Δt under the v vs. t curve is always equal to the distance traveled in Δt.

Thus, to calculate the total distance, you need to find the area of the entire region under the v vs. t curve. In the case at hand, the entire region under the v vs. t curve is not an elementary geometrical figure, but rather a combination of triangles and rectangles.

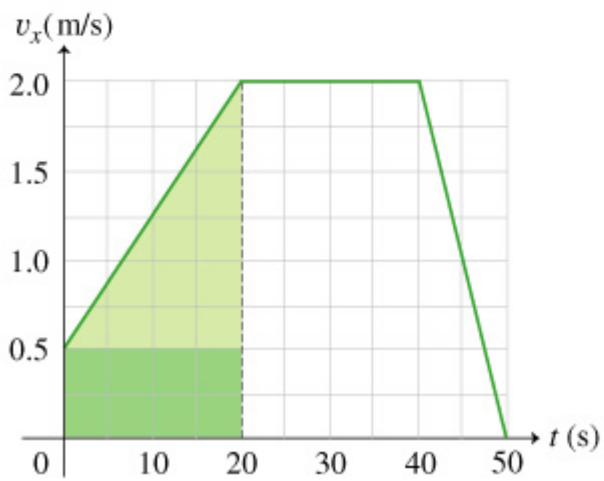
Hint 2. Find the distance traveled in the first 20.0 seconds

What is the distance Δx_1 traveled in the first 20 seconds of motion, between $t = 0.0\ \mathrm{s}$ and $t = 20.0\ \mathrm{s}$?

Express your answer in meters.

Hint 1. Area of the region under the v vs. t curve

The region under the v vs. t curve between $t = 0.0\ \mathrm{s}$ and $t = 20.0\ \mathrm{s}$ can be divided into a rectangle of dimensions $20.0\ \mathrm{s}$ by $0.50\ \mathrm{m/s}$, and a triangle of base $20.0\ \mathrm{s}$ and height $1.50\ \mathrm{m/s}$, as shown in the figure.



ANSWER:

Δx_1 = 25 m

Correct

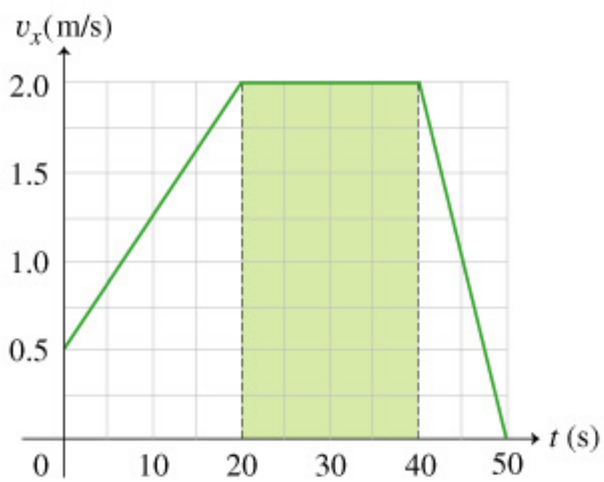
Hint 3. Find the distance traveled in the second 20.0 seconds

What is the distance Δx_2 traveled in the second 20 seconds of motion, from $t = 20.0 \text{ s}$ to $t = 40.0 \text{ s}$?

Express your answer in meters.

Hint 1. Area of the region under the v vs. t curve

The region under the v vs. t curve between $t = 20.0 \text{ s}$ and $t = 40.0 \text{ s}$ is a rectangle of dimensions 20.0 s by 2.00 m/s, as shown in the figure.



ANSWER:

Δx_2 = 40 m

Correct

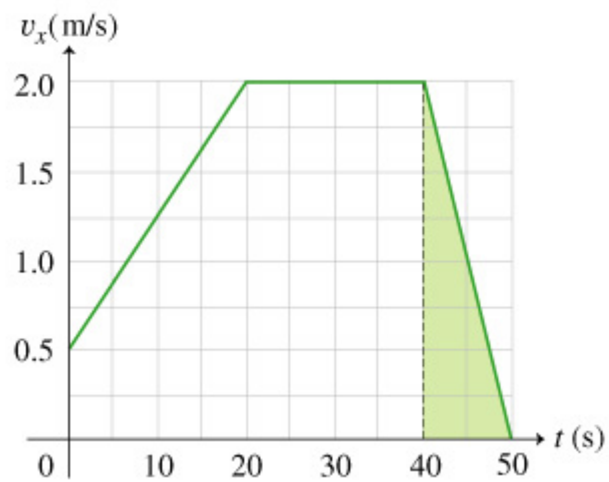
Hint 4. Find the distance traveled in the last 10.0 seconds

What is the distance Deltax_3 traveled in the last 10 seconds of motion, from $t = 40.0 \text{ s}$ to $t = 50.0 \text{ s}$?

Express your answer in meters.

Hint 1. Area of the region under the v vs. t curve

The region under the v vs. t curve between $t = 40.0 \text{ s}$ and $t = 50.0 \text{ s}$ is a triangle of base 10.0 s and height 2.00 m/s, as shown in the figure.



ANSWER:

Deltax_3 = 10 m

Correct

Now simply add the distances traveled in each time interval to find the total distance.

ANSWER:

Δx = 75 m

Correct

Part C

What is the average acceleration $a_{\rm av}$ of the particle over the first 20.0 seconds?

Express your answer in meters per second per second.

Hint 1. Definition and graphical interpretation of average acceleration

The average acceleration $a_{\rm av}$ of a particle that travels along a straight line in a time interval Δt is the ratio of the change in velocity Δv experienced by the particle to the time interval Δt, or

$$a_{\rm av} = \frac{\Delta v}{\Delta t}.$$

In a v vs. t graph, then, the average acceleration equals the slope of the line connecting the two points representing the initial and final velocities.

Hint 2. Slope of a line

The slope m of a line from point A, of coordinates $(x_{\rm A}, y_{\rm A})$, to point B, of coordinates $(x_{\rm B}, y_{\rm B})$, is equal to the "rise" over the "run," or

m=\frac{y_{\rm B}-y_{\rm A}}{x_{\rm B}-x_{\rm A}}.

ANSWER:

$a_{\rm av}$ = 0.075 $\mathrm{m/s^2}$

Correct

The average acceleration of a particle between two instants of time is the slope of the line connecting the two corresponding points in a v vs. t graph.

Part D

What is the instantaneous acceleration a of the particle at $t=45.0\;\rm s$?

Hint 1. Graphical interpretation of instantaneous acceleration

The acceleration of a particle at any given instant of time or at any point in its path is called the instantaneous acceleration. If the v vs. t graph of the particle's motion is known, you can directly determine the instantaneous acceleration at any point on the curve. The instantaneous acceleration at any point is equal to the slope of the line tangent to the curve at that point.

Hint 2. Slope of a line

The slope m of a line from point A, of coordinates $(x_{\rm A}, y_{\rm A})$, to point B, of coordinates $(x_{\rm B}, y_{\rm B})$, is equal to the "rise" over the "run," or

$$m=\frac{y_{\rm B}-y_{\rm A}}{x_{\rm B}-x_{\rm A}}.$$

ANSWER:

a =

- 1 $\mathrm{m/s^2}$
- 0.20 $\mathrm{m/s^2}$
- -0.20 $\mathrm{m/s^2}$
- 0.022 $\mathrm{m/s^2}$
- -0.022 $\mathrm{m/s^2}$

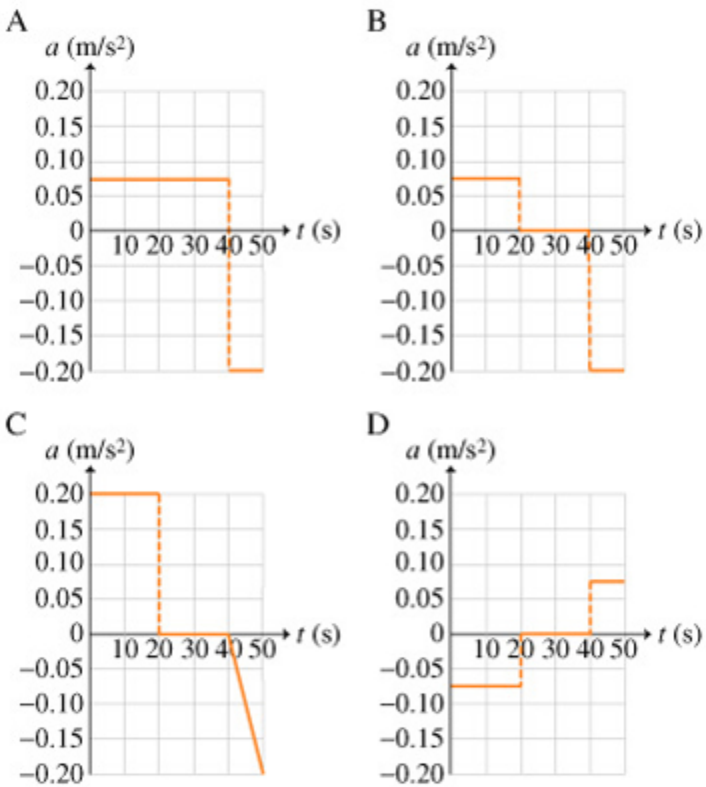
Correct

The instantaneous acceleration of a particle at any point on a v vs. t graph is the slope of the line tangent to the curve at that point. Since in the last 10 seconds of motion, between $t = 40.0\ \mathrm{s}$ and $t = 50.0\ \mathrm{s}$, the curve is a straight line, the tangent line is the curve itself. Physically, this means that the instantaneous acceleration of the particle is *constant* over that time interval. This is true for any motion where velocity increases linearly with time. In the case at hand, can you think of another time interval in which the acceleration of the particle is constant?

Now that you have reviewed how to plot variables as a function of time, you can use the same technique and draw an acceleration vs. time graph, that is, the graph of (instantaneous) acceleration as a function of time. As usual in these types of graphs, time t is plotted on the horizontal axis, while the vertical axis is used to indicate acceleration a.

Part E

Which of the graphs shown below is the correct acceleration vs. time plot for the motion described in the previous parts?



Hint 1. How to approach the problem

Recall that whenever velocity increases linearly with time, acceleration is constant. In the example here, the particle's velocity increases linearly with time in the first 20.0 s of motion. In the second 20.0 s , the particle's velocity is constant, and then it decreases linearly with time in the last 10 s. This means that the particle's acceleration is constant over each time interval, but its value is different in each interval.

Hint 2. Find the acceleration in the first 20 s

What is a_1, the particle's acceleration in the first 20 s of motion, between $t = 0.0\ \mathrm{s}$ and $t = 20.0\ \mathrm{s}$?

Express your answer in meters per second per second.

Hint 1. Constant acceleration

Since we have already determined that in the first 20 s of motion the particle's acceleration is constant, its constant value will be equal to the average acceleration that you calculated in Part C.

ANSWER:

a_1 = 0.075 $\mathrm{m/s^2}$

Hint 3. Find the acceleration in the second 20 s

What is a_2, the particle's acceleration in the second 20 s of motion, between $t = 20.0\ \mathrm{s}$ and $t = 40.0\ \mathrm{s}$?

Express your answer in meters per second per second.

Hint 1. Constant velocity

In the second 20 s of motion, the particle's velocity remains unchanged. This means that in this time interval, the particle does not accelerate.

ANSWER:

a_2 = 0 $\mathrm{m/s^2}$

Hint 4. Find the acceleration in the last 10 s

What is a_3, the particle's acceleration in the last 10 s of motion, between $t = 40.0\ \mathrm{s}$ and $t = 50.0\ \mathrm{s}$?

Express your answer in meters per second per second.

Hint 1. Constant acceleration

Since we have already determined that in the last 10 s of motion the particle's acceleration is constant, its constant value will be equal to the instantaneous acceleration that you calculated in Part D.

ANSWER:

a_3 = -0.20 $\mathrm{m/s^2}$

ANSWER:

- Graph A
- Graph B (selected)
- Graph C
- Graph D

Correct

In conclusion, graphs of velocity as a function of time are a useful representation of straight-line motion. If read correctly, they can provide you with all the information you need to study the motion.

One-Dimensional Kinematics with Constant Acceleration

Learning Goal:

To understand the meaning of the variables that appear in the equations for one-dimensional kinematics with constant acceleration.

Motion with a constant, nonzero acceleration is not uncommon in the world around us. Falling (or thrown) objects and cars starting and stopping approximate this type of motion. It is also the type of motion most frequently involved in introductory kinematics problems.

The kinematic equations for such motion can be written as

$$x(t) = x_{\rm i} + v_{\rm i}t + \frac{1}{2}at^2,$$

$$v(t) = v_{\rm i} + at,$$

where the symbols are defined as follows:

- $x(t)$ is the position of the particle;
- x_i is the *initial* position of the particle;
- $v(t)$ is the velocity of the particle;
- v_i is the *initial* velocity of the particle;
- a is the acceleration of the particle.

In anwering the following questions, assume that the acceleration is constant and nonzero: $a \neq 0$.

Part A

The quantity represented by x is a function of time (i.e., is not constant).

ANSWER:

- (selected) true
- false

Correct

Part B

The quantity represented by x_i is a function of time (i.e., is not constant).

ANSWER:

- true
- false

Correct

Recall that x_i represents an initial value, not a variable. It refers to the position of an object at some initial moment.

Part C

The quantity represented by v_i is a function of time (i.e., is not constant).

ANSWER:

- true
- false

Correct

Part D

The quantity represented by v is a function of time (i.e., is not constant).

ANSWER:

- true
- false

Correct

The velocity v always varies with time when the linear acceleration is nonzero.

Part E

Which of the given equations is not an explicit function of t and is therefore useful when you don't know or don't need the time?

ANSWER:

- ○ x=x_{\rm i}+v_{\rm i}t+\frac{1}{2}at^2
- ○ v=v_{\rm i}+at
- ◉ v^2=v_{\rm i}^2+2a(x-x_{\rm i})

Correct

Part F

A particle moves with constant acceleration a. The expression v_{\rm i}+at represents the particle's velocity at what instant in time?

ANSWER:

- ○ at time $t = 0$
- ○ at the "initial" time
- ◉ when a time t has passed since the particle's velocity was v_i

Correct

More generally, the equations of motion can be written as

$$x(t) = x_{\rm i} +v_{\rm i}\;\Delta t + \frac{1}{2}a \;(\Delta t)^2$$

and

$$v(t) = v_{\rm i} + a \;\Delta t.$$

Here Δt is the time that has elapsed since the beginning of the particle's motion, that is, $\Delta t = t - t_{\rm i}$, where t is the current time and t_i is the time at which we start measuring the particle's motion. The terms x_i and v_i are, respectively, the position and velocity at $t=t_{\rm i}$. As you can now see, the equations given at the beginning of this problem correspond to the case $t_{\rm i} = 0$, which is a convenient choice if there is only one particle of interest.

To illustrate the use of these more general equations, consider the motion of two particles, A and B. The position of particle A depends on time as $x_{\rm A}(t)=x_{\rm i}+v_{\rm i}t+(1/2)at^2$. That is, particle A starts moving at time $t=t_{\rm iA}=0$ with velocity $v_{\rm iA}=v_{\rm i}$, from $x_{\rm iA}=x_{\rm i}$. At time $t=t_1$, particle B has twice the acceleration, half the velocity, and the same position that particle A had at time $t=0$.

Part G

What is the equation describing the position of particle B?

Hint 1. How to approach the problem

The general equation for the distance traveled by particle B is

$$x_{\rm B}(t) = x_{\rm iB} + v_{\rm iB}\Delta t + \frac{1}{2}a_{\rm B}(\Delta t)^2,$$

or

$$x_{\rm B}(t) = x_{\rm B}(t=t_1) + v_{\rm B}(t=t_1)(t-t_1) + \frac{1}{2}a_{\rm B}(t-t_1)^2,$$

since $\Delta t = t - t_1$ is a good choice for B. From the information given, deduce the correct values of the constants that go into the equation for $x_B(t)$ given here, in terms of A's constants of motion.

ANSWER:

- x_{\rm B}(t)=x_{\rm i}+2v_{\rm i}t+\frac{1}{4}at^2
- x_{\rm B}(t)=x_{\rm i}+0.5v_{\rm i}t+at^2
- x_{\rm B}(t)=x_{\rm i}+2v_{\rm i}(t+t_1) +\frac{1}{4}a(t+t_1)^2
- x_{\rm B}(t)=x_{\rm i}+0.5v_{\rm i}(t+t_1)+a(t+t_1)^2
- x_{\rm B}(t)=x_{\rm i}+2v_{\rm i}(t-t_1) +\frac{1}{4}a(t-t_1)^2
- x_{\rm B}(t)=x_{\rm i}+0.5v_{\rm i}(t-t_1)+a(t-t_1)^2

Correct

Part H

At what time does the velocity of particle B equal that of particle A?

Hint 1. Velocity of particle A

Type an expression for particle A's velocity as a function of time.

Express your answer in terms of t and some or all of the variables x_i, v_i, and a.

Hint 1. How to approach this part

Look at the general expression for $v(t)$ given in the problem introduction.

ANSWER:

v_A(t) = v_i+a*t

Hint 2. Velocity of particle B

Type an expression for particle B's velocity as a function of time.

Express your answer in terms of t and some or all of the variables t_1, x_i, v_i, and a.

Hint 1. How to approach this part

The general expression for v_B(t) is

v_{\rm B}(t) = v_{\rm B}(t=t_1) + a_{\rm B}(t-t_1).

From the information given, deduce the correct values of the constants that go into this equation in terms of particle A's constants of motion.

ANSWER:

v_B(t) = 0.5*v_i+2*a*(t-t_1)

ANSWER:

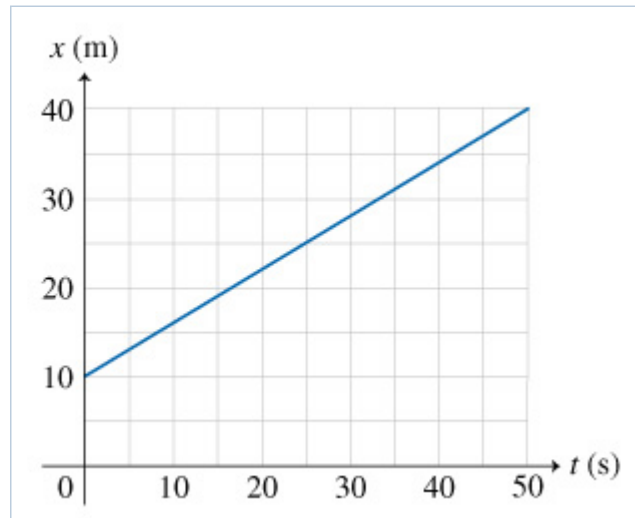
- t=t_1+\frac{v_{\rm i}}{4a}
- t=2t_1+\frac{v_{\rm i}}{2a}
- t=3t_1+\frac{v_{\rm i}}{2a}
- The two particles never have the same velocity.

Correct

What x vs. t Graphs Can Tell You

To describe the motion of a particle along a straight line, it is often convenient to draw a graph representing the position of the particle at different times. This type of graph is usually referred to as an *x vs. t graph.* To draw such a graph, choose an axis system in which time t is plotted on the horizontal axis and position x on the vertical axis. Then, indicate the values of x at various times t. Mathematically, this corresponds to plotting the variable x as a function of t. An example of a graph of position as a function of time for a particle traveling along a straight line is shown below. Note that an x vs. t graph like this does *not* represent the path of the particle in space.

Now let's study the graph shown in the figure in more detail. Refer to this graph to answer Parts A, B, and C.



Part A

What is the total distance Δx traveled by the particle?

Express your answer in meters.

Hint 1. Total distance

The total distance Δx traveled by the particle is given by the difference between the initial position x_0 at $t = 0.0\ \mathrm{s}$ and the position x at $t = 50.0\ \mathrm{s}$. In symbols,

\Delta x= x-x_0.

Hint 2. How to read an x vs. t graph

Remember that in an x vs. t graph, time t is plotted on the horizontal axis and position x on the vertical axis. For example, in the plot shown in the figure, x=16.0\;\rm m at t=10.0\;\rm s.

ANSWER:

Δx = 30 m

Correct

Part B

What is the average velocity v_av of the particle over the time interval \Delta t=50.0\;\rm s ?

Express your answer in meters per second.

Hint 1. Definition and graphical interpretation of average velocity

The average velocity v_av of a particle that travels a distance Δx along a straight line in a time interval Δt is defined as

v_{\rm av}=\frac{\Delta x}{\Delta t}.

In an x vs. t graph, then, the average velocity equals the slope of the line connecting the initial and final positions.

Hint 2. Slope of a line

The slope m of a line from point A, with coordinates $(x_{\rm A}, y_{\rm A})$, to point B, with coordinates $(x_{\rm B}, y_{\rm B})$, is equal to the "rise" over the "run," or

m=\frac{x_{\rm B}-x_{\rm A}}{y_{\rm B}-y_{\rm A}}.

ANSWER:

v_av = 0.600 $\rm m/s$

Correct

The average velocity of a particle between two positions is equal to the slope of the line connecting the two corresponding points in an x vs. t graph.

Part C

What is the instantaneous velocity v of the particle at t=10.0\;\rm s?

Express your answer in meters per second.

Hint 1. Graphical interpretation of instantaneous velocity

The velocity of a particle at any given instant of time or at any point in its path is called instantaneous velocity. In an x vs. t graph of the particle's motion, you can determine the instantaneous velocity of the particle at any point in the curve. The instantaneous velocity at any point is equal to the slope of the line tangent to the curve at that point.

ANSWER:

v = 0.600 $\mathrm{m/s}$

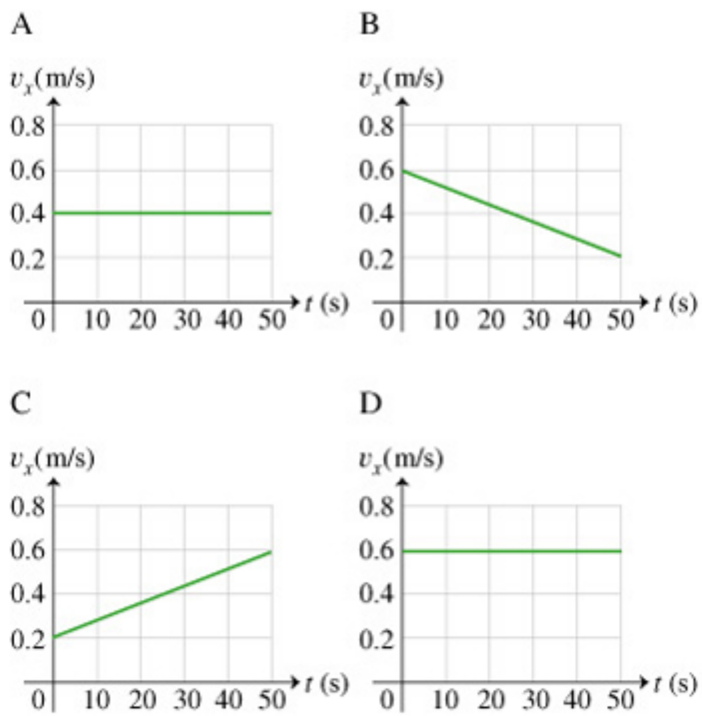
Correct

The instantaneous velocity of a particle at any point on its x vs. t graph is the slope of the line tangent to the curve at that point. Since in the case at hand the curve is a straight line, the tangent line is the curve itself. Physically, this means that the instantaneous velocity of the particle is *constant* over the entire time interval of motion. This is true for any motion where distance increases linearly with time.

Another common graphical representation of motion along a straight line is the *v vs. t graph*, that is, the graph of (instantaneous) velocity as a function of time. In this graph, time t is plotted on the horizontal axis and velocity v on the vertical axis. Note that by definition, velocity and acceleration are vector quantities. In straight-line motion, however, these vectors have only one nonzero component in the direction of motion. Thus, in this problem, we will call v the velocity and a the acceleration, even though they are really the components of the velocity and acceleration vectors in the direction of motion.

Part D

Which of the graphs shown is the correct v vs. t plot for the motion described in the previous parts?



Hint 1. How to approach the problem

Recall your results found in the previous parts, namely the fact that the instantaneous velocity of the particle is constant. Which graph represents a variable that always has the same constant value at any time?

ANSWER:

- Graph A
- Graph B
- Graph C
- Graph D

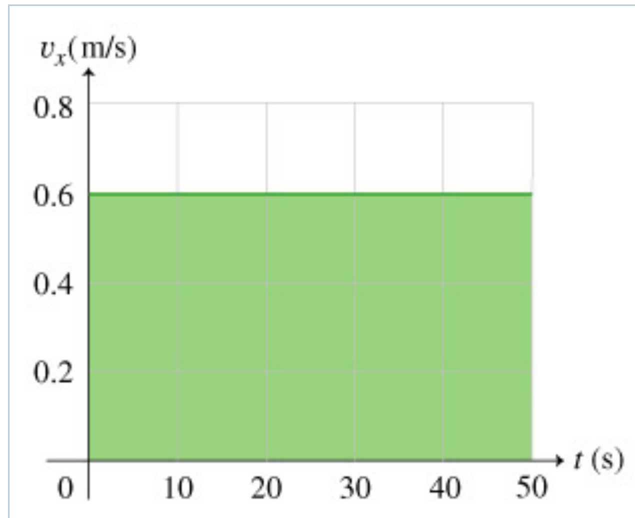
Correct

Whenever a particle moves with constant nonzero velocity, its x vs. t graph is a straight line with a nonzero slope, and its v vs. t curve is a horizontal line.

Part E

Shown in the figure is the v vs. t curve selected in the previous part. What is the area A of the shaded region under the curve?

Express your answer in meters.



Hint 1. How to approach the problem

The shaded region under the v vs. t curve is a rectangle whose horizontal and vertical sides lie on the t axis and the v axis, respectively. Since the area of a rectangle is the product of its sides, in this case the area of the shaded region is the product of a certain quantity expressed in seconds and another quantity expressed in meters per second. The area itself, then, will be in meters.

ANSWER:

A = 30 m

Correct

Compare this result with what you found in Part A. As you can see, the area of the region under the v vs. t curve equals the total distance traveled by the particle. This is true for any velocity curve and any time interval: The area of the region that extends over a time interval Δt under the v vs. t curve is always equal to the distance traveled in Δt.

Score Summary:

Your score on this assignment is 95.4%.
You received 19.07 out of a possible total of 20 points.