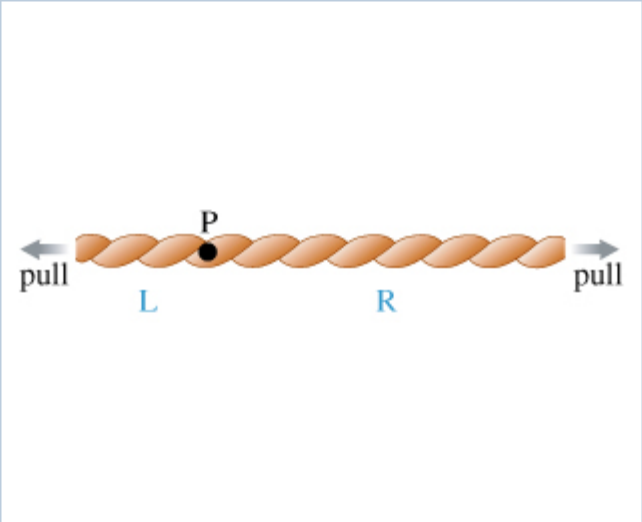
#11 Newton's 1st and 2nd Laws Post-class

Due: 11:00am on Monday, September 17, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Tension Definition

This problem concerns the concept of tension in a rope. Consider a rope subjected to a pulling force on its two ends as shown. The rope is stationary. An arbitrary point P divides the rope into a left-hand segment L and a right-hand segment R.



Part A

For segment R and segment L to hold together, they must exert forces on each other. What is the direction of the force exerted on segment R by segment L?

Hint 1. Look at the forces

The forces on segment R are

1. the force due to L and
2. the pull (to the right) at the right end of the rope.

What must be true about these two forces if the rope is to remain stationary?

ANSWER:

- left
- right

Correct

Part B

Assume that segment R exerts a force of magnitude T on segment L. What is the magnitude $F_{\rm RL}$ of the force exerted on segment R by segment L?

Give your answer in terms of T and other constants such as g.

Hint 1. How to approach the problem

Apply Newton's 3rd law.

ANSWER:

F_{RL} = T

Correct

The magnitude of the force exerted by one part of a rope on another at a certain point is called the *tension* at that point. Tension is often designated with the symbol T. The tension is also the magnitude of the force with which the rope pulls on whatever is attached to its ends.

Part C

Now imagine two points, Q and P, that divide the rope into segments L, M ,and R. The rope remains stationary. Assume that segment L exerts a force of magnitude F_{LM} on segment M.

What is the magnitude F_{RM} of the force exerted by segment R on segment M?

Give your answer in terms of F_{LM} and constants such as g.

Hint 1. How to approach the problem

Draw a force diagram for the section M of the rope.

Hint 2. Relative magnitudes of the two forces

The force exerted by segment L on segment M is ______ the force exerted by segment R on segment M?

ANSWER:

- less than
- greater than
- equal to

ANSWER:

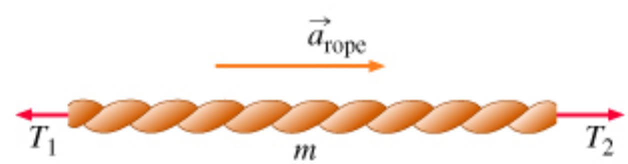
F_{RM} = F_{LM}

Correct

This answer implies that, in the absence of gravity, the tension in a stationary rope may be taken to be constant.

Part D

Now consider a rope that, unlike those usually studied in mechanics problems, actually has a significant mass m. The tension at the right end of this rope is T_2 and that at the left end is T_1. The rope has an acceleration a_{rope} to the right.



Complete the following equation for the acceleration of the section of the rope of mass m, taking the positive direction to be to the right.

Give your answer in terms of T_1, T_2, and constants such as g.

Hint 1. How to approach the problem

Use Newton's 2nd law of motion.

ANSWER:

$F_{\text{rope}} = ma_{\text{rope}}$ = $T_2 - T_1$

Correct

Part E

Which of the following phrases, if they appear in a problem, allow you to assume that $T_2 = T_1$ in a horizontally oriented rope?

1. The rope is massless.
2. The rope is moving at constant speed.
3. The rope is stretched with negligible sag.

ANSWER:

- A only
- B only
- C only
- A or B
- A or C
- B or C
- A or B or C

Correct

The reason why a massless rope would cause $T_1 = T_2$ can be seen using Newton's 2nd law: $F = ma$. The force F is given by $T_2 - T_1$. Since $m = 0$ then $ma = 0$ for any value of a. This means that the force must be zero ($F = ma = 0$). Therefore, $T_1 = T_2$.

Understanding Newton's Laws

Part A

An object cannot remain at rest unless which of the following holds?

Hint 1. How to approach the problem

This problem describes a situation of static equilibrium (i.e., a body that remains at rest). Hence, it is appropriate to apply Newton's 1st law.

Hint 2. Newton's 1st law: a body at rest

According to Newton's 1st law, a body at rest remains at rest if the net force acting on it is zero.

ANSWER:

- (selected) The net force acting on it is zero.
- The net force acting on it is constant and nonzero.
- There are no forces at all acting on it.
- There is only one force acting on it.

Correct

If there is a net force acting on a body, regardless of whether it is a constant force, the body accelerates. If the body is at rest and the net force acting on it is zero, then it will remain at rest. The net force could be zero either because there are no forces acting on the body at all or because several forces are acting on the body but they all cancel out.

Part B

If a block is moving to the left at a constant velocity, what can one conclude?

Hint 1. How to approach the problem

This problem describes a situation of dynamic equilibrium (i.e., a body that moves at a constant velocity). Hence, it is appropriate to apply Newton's 1st law.

Hint 2. Newton's 1st law: a body in motion

According to Newton's 1st law, a body initially in motion continues to move with constant velocity if the net force acting on it is zero.

ANSWER:

- ○ There is exactly one force applied to the block.
- ○ The net force applied to the block is directed to the left.
- ◉ The net force applied to the block is zero.
- ○ There must be no forces at all applied to the block.

Correct

If there is a net force acting on a body, regardless of whether the body is already moving, the body accelerates. If a body is moving with constant velocity, then it is not accelerating and the net force acting on it is zero. The net force could be zero either because there are no forces acting on the body at all or because several forces are acting on the body but they all cancel out.

Part C

A block of mass $2\ \mathrm{kg}$ is acted upon by two forces: $3\ \mathrm{N}$ (directed to the left) and $4\ \mathrm{N}$ (directed to the right). What can you say about the block's motion?

Hint 1. How to approach the problem

This problem describes a situation of dynamic motion (i.e., a body that is acted on by a net force). Hence, it is appropriate to apply Newton's 2nd law, which allows you to relate the net force acting on a body to the acceleration of the body.

Hint 2. Newton's 2nd law

Newton's 2nd law states that a body accelerates if a net force acts on it. The net force is proportional to the acceleration of the body and the constant of proportionality is equal to the mass of the body. In other words,

$$F = ma,$$

where F is the net force acting on the body, and m and a are the mass and the acceleration of the body, respectively.

Hint 3. Relating acceleration to velocity

Acceleration is defined as the change in velocity per unit time. Keep in mind that both acceleration and velocity are vector quantities.

ANSWER:

- It must be moving to the left.
- It must be moving to the right.
- It must be at rest.
- It could be moving to the left, moving to the right, or be instantaneously at rest.

Correct

The acceleration of an object tells you nothing about its velocity--the direction and speed at which it is moving. In this case, the net force on (and therefore the acceleration of) the block is to the right, but the block could be moving left, right, or in any other direction.

Part D

A massive block is being pulled along a horizontal frictionless surface by a constant horizontal force. The block must be ___________.

Hint 1. How to approach the problem

This problem describes a situation of dynamic motion (i.e., a body that is acted on by a net force). Hence, it is appropriate to apply Newton's 2nd law, which allows you to relate the net force acting on a body to the acceleration of the body.

Hint 2. Newton's 2nd law

Newton's 2nd law states that a body accelerates if a net force acts on it. The net force is proportional to the acceleration of the body and the constant of proportionality is equal to the mass of the body. In other words,

$$F = ma,$$

where F is the net force acting on the body, and m and a are the mass and the acceleration of the body, respectively.

ANSWER:

- continuously changing direction
- moving at constant velocity
- moving with a constant nonzero acceleration
- moving with continuously increasing acceleration

Correct

Since there is a net force acting, the body does not move at a constant velocity, but it accelerates instead. However, the force acting on the body is constant. Hence, according to Newton's 2nd law of motion, the acceleration of the body is also constant.

Part E

Two forces, of magnitude $4\ \mathrm{N}$ and $10\ \mathrm{N}$, are applied to an object. The relative direction of the forces is unknown. The net force acting on the object

__________.

Check all that apply.

Hint 1. How to approach the problem

By definition, the net force is the vector sum of all forces acting on the object. To find the magnitude of the net force you need to add the components of the two forces acting. Try adding the two forces graphically (by connecting the head of one force to the tail of the other). The directions of the two forces are arbitrary, but by trying different possibilities you should be able to determine the maximum and minimum net forces that could act on the object.

Hint 2. Find the net force when the two forces act on the object in opposite directions

Find the magnitude of the net force if both the forces acting on the object are horizontal and the 10-N force is directed to the right, while the 4-N force is directed to the left.

Express your answer in newtons.

Hint 1. Vector addition

The magnitude of the vector sum of two parallel forces is the sum of the magnitudes of the forces. The magnitude of the vector sum of two antiparallel forces is the absolute value of the difference in magnitudes of the forces.

ANSWER:

6.0 N

Hint 3. Find the direction of the net force when the two forces act in opposite directions

If both the forces acting on the object are horizontal and the 10-N force is directed to the right, while the 4-N force is directed to the left, the net force is horizontal and directed ___________.

ANSWER:

- (selected) in the same direction as the 10-N force
- in the opposite direction to the 10-N force

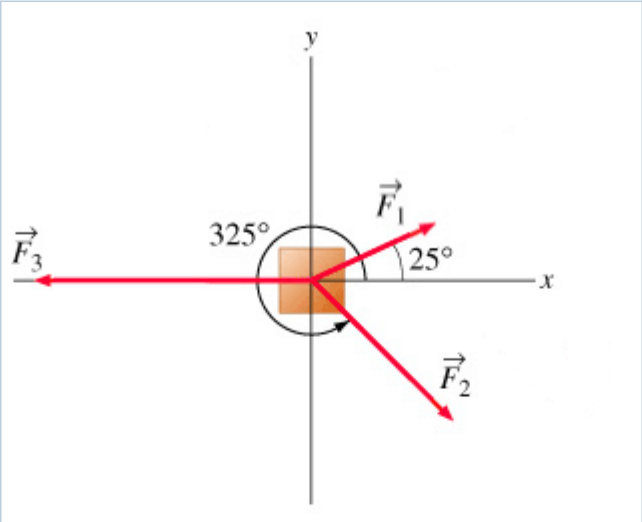
ANSWER:

- [x] cannot have a magnitude equal to $5\ \mathrm{N}$
- [ ] cannot have a magnitude equal to $10\ \mathrm{N}$
- [ ] cannot have the same direction as the force with magnitude $10\ \mathrm{N}$
- [ ] must have a magnitude greater than $10\ \mathrm{N}$

Correct

± Motion of a Block with Three Forces

The diagram below shows a block of mass $m = 2.00\ \mathrm{kg}$ on a frictionless horizontal surface, as seen from above. Three forces of magnitudes $F_1 = 4.00\ \mathrm{N}$, $F_2 = 6.00\ \mathrm{N}$, and $F_3 = 8.00\ \mathrm{N}$ are applied to the block, initially at rest on the surface, at angles shown on the diagram. In this problem, you will determine the resultant (total) force vector from the combination of the three individual force vectors. All angles should be measured counterclockwise from the positive *x* axis (i.e., all angles are positive).



Part A

Calculate the magnitude of the total resultant force $\vec{F}_r = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ acting on the mass.

Express your answer in Newtons to three significant figures.

Hint 1. Definition of resultant force

When several forces are applied to an object, the vector sum is often called the *resultant* or the *resultant force*.

Hint 2. How to find the resultant

When working with vectors, the general rule is to *think geometrically* but to *calculate using components*. Thus to add vectors one estimates the sum by imagining the tail of the second vector to be placed at the point of the first, the tail of the third to be placed at the second, etc. But to calculate the vector sum each vector is represented by components in a convenient coordinate system and these components are added to find the components of the sum.

Hint 3. Find the components of $\vec{F_1}$

What are the *x* component and *y* component of $\vec{F_1}$?

Express your answer as an ordered pair of numbers, separated by a comma, to three significant figures.

Hint 1. *x* component of $\vec{F_1}$

The *x* component of $\vec{F_1}$ is $F_1 \cos(\theta)$, where θ is the angle between the positive *x* axis and the vector's direction.

ANSWER:

F_{1x}, F_{1y} = 3.63,1.69 N

Hint 4. Find the components of $\vec{F_2}$

Find the *x* and *y* components of the vector $\vec{F_2}$.

Express your answer as an ordered pair to three significant figures.

ANSWER:

F_{2x}, F_{2y} = 4.91,-3.44

Hint 5. Find the components of $\vec{F_r}$

Now find the x and y components of the resultant (sum) vector, $\vec{F}_r$. (Don't forget to include $\vec{F}_3$.)

Express your answer as an ordered pair to three significant figures.

ANSWER:

F_{rx}, F_{ry} = 0.540,-1.75

Hint 6. Magnitude of $\vec{F}_r$

The magnitude of $\vec{F}_r$ in terms of its *x* and *y* components F_{rx} and F_{ry} is given by

$$|\vec{F}_r| = \sqrt{{F_{rx}}^2 + {F_{ry}}^2}.$$

ANSWER:

$|\vec{F}_r|$ = 1.83 N

Correct

Part B

What angle does $\vec{F}_r$ make with the positive *x* axis?

Express your answer in degrees to two significant figures.

Hint 1. Find the angle symbolically

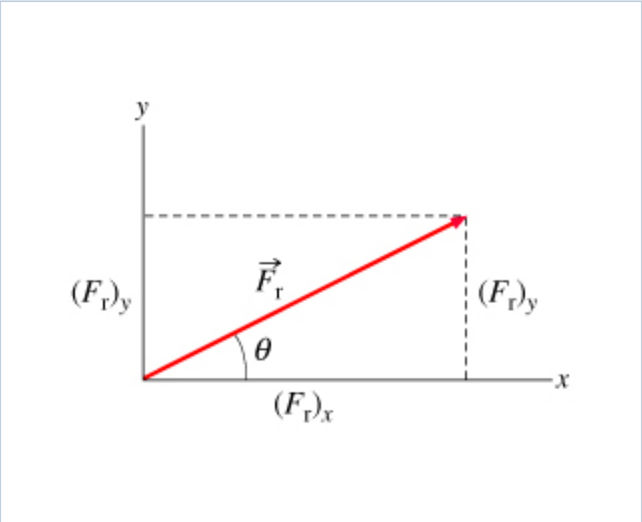
The angle that $\vec{F}_r$ makes with the *x* axis can be determined if you know its *x* and *y* components, which you should have from your calculation for

Part A. What is the angle that $\vec{F}_r$ makes with the positive *x* axis?

Answer symbolically in terms of F_{rx} and F_{ry}.

Hint 1. How to approach this problem

When developing a general formula for the angle, you can choose the vector to lie in any quadrant. It is easiest to have the vector lie in the first quadrant.



ANSWER:

$\mathrm{atan}\left(\frac{F_{ry}}{F_{rx}}\right)$

ANSWER:

290 degrees

Correct

Part C

What is the magnitude of the mass's acceleration vector, $\vec{a}$?

Express your answer to two significant figures.

Hint 1. Newton's 2nd law

Recall that $\vec{F} = m\vec{a}$, so you should be able to find $|\vec{a}|$ fairly easily here.

ANSWER:

$|\vec{a}|$ = 0.92 $\mathrm{m/s^2}$

Correct

Part D

What is the direction of $\vec{a}$? In other words, what angle does this vector make with respect to the positive *x* axis?

Express your answer in degrees to two significant figures.

Hint 1. Relation between the direction of $\vec{a}$ and $\vec{F}$

Is there any reason why the direction of $\vec{a}$ should be different from $\vec{F}$?

ANSWER:

290 degrees

Correct

Part E

How far (in meters) will the mass move in 5.0 s?

Express the distance d in meters to two significant figures.

Hint 1. Displacement with constant acceleration

Remember that we have constant acceleration here, so you can use the equation:

$$d(t) = d_0 + v_0 t + \frac{1}{2} a t^2,$$

where $d(t)$ is the displacement at time t.

ANSWER:

d = 11 m

Correct

Part F

What is the magnitude of the velocity vector of the block at $t = 5.0\ \mathrm{s}$?

Express your answer in meters per second to two significant figures.

Hint 1. Velocity with constant acceleration

Remember, we have constant acceleration, and because the object starts from rest, the velocity vector will be parallel to the acceleration vector. Therefore, $v_\mathrm{f} = v_\mathrm{i} + at$.

ANSWER:

$|\vec{v}(5)|$ = 4.6 $\mathrm{m/s}$

Correct

Part G

In what direction is the mass moving at time $t = 5.0\ \mathrm{s}$? That is, what angle does the velocity vector make with respect to the positive *x* axis?

Express your answer in degrees to two significant figures.

Hint 1. Relationship between the direction of $\vec{v}$ and $\vec{a}$

The mass starts at rest and is accelerated in one direction. Therefore, it must have velocity in the direction of the acceleration, and that direction only.

ANSWER:

290 degrees

Correct

Free-Body Diagrams and Newton's Laws

When solving problems involving forces and Newton's laws, the following summary of things to do will start your mind thinking about getting involved in the problem at hand.

Problem Solving: Free-Body Diagrams and Newton's Laws

1. Draw a sketch of the situation.
2. Consider only one object (at a time), and draw a free-body diagram for that body, showing all the forces acting on that body. Do not show any forces that the body exerts on other bodies. If several bodies are involved, draw a free-body diagram for each body separately, showing all the forces acting on that body.
3. Newton's second law involves vectors, and it is usually important to resolve vectors into components. Choose an *x* and *y* axis in a way that simplifies the calculation.
4. For each body, Newton's second law can be applied to the x and y components separately. That is the *x* component of the net force on that body will be related to the *x* component of that body's acceleration: $\Sigma F_x = ma_x$, and similarly for the *y* direction.
5. Solve the equation or equations for the unknown(s).

Apply these steps

Use the steps outlined above to find the magnitude of the acceleration a of a chair and the magnitude of the normal force F_N acting on the chair: Yusef pushes a chair of mass m = 55.0kg across a carpeted floor with a force $\vec{F}_\mathrm{p}$ (the subscript 'p' here is lowercase and throughout the question) of magnitude F_p = 152N directed at θ = 35.0$\mathrm{degrees}$ below the horizontal . The magnitude of the kinetic frictional force between the carpet and the chair is F_k = 106N .

Part A

Identify and sketch all the external forces acting on the chair. Because the chair can be represented as a point particle of mass m, draw the forces with their tails centered on the black dot in the middle of the chair. Be certain to draw your forces so that they have the correct orientation.

Draw the vectors starting at the black dot. The location and orientation of the vectors will be graded. The length of the vectors will not be graded.

ANSWER:

Correct

Part B

Which set of coordinate axes is the most convenient to use in this problem?

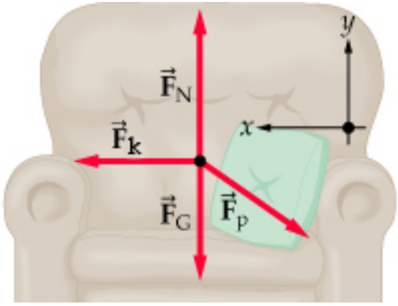
Hint 1. Determine the direction of the acceleration

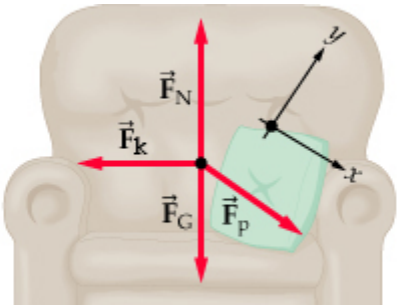
Indicate the direction of the chair's acceleration $\vec{a}$ on the figure below.

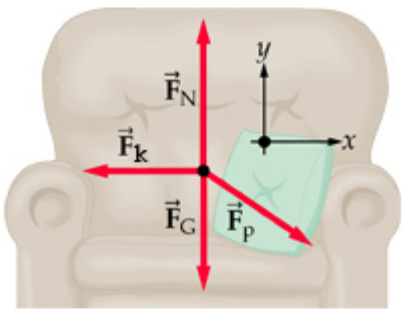
The orientation of the vector will be graded. The location and length of the vector will not be graded.

ANSWER:

ANSWER:







Correct

Now that you have selected a coordinate system, you should resolve the forces into *x* and *y* components so that you can apply Newton's second law to each coordinate direction independently.

Part C

Use the component form of Newton's second law to write an expression for the *x* component of the net force, ΣF_x.

Express your answer in terms of some or all of the variables: F_G, F_N, F_p, θ, and F_k .

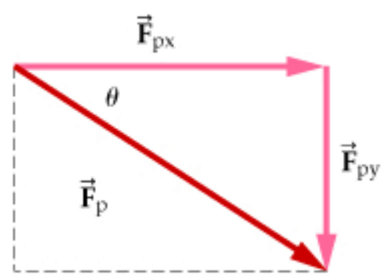
Hint 1. Find the *x* component of the pushing force

What is F_px, the *x* component of the force $\vec{F}_\mathrm{p}$ exerted by Yusef?

Express your answer in terms of the variables F_p and θ.

Hint 1. Right angle triangle trigonometry

Keep in mind that the cosine of an angle is defined as the length of the adjacent leg divided by the length of the hypoteneuse. The sine of an angle is the length of the opposite leg divided by the length of the hypoteneuse. Apply these definitions to the triangle formed by $\vec{F}_\mathrm{p}$, θ , F_px ,and F_py.



ANSWER:

F_px = $F_p \cos(\theta)$

ANSWER:

ΣF_x = $F_p \cos\theta - F_k$ $= ma_x$

Correct

Part D

Use the component form of Newton's second law to write an expression for the *y* component of the net force, ΣF_y.

Express your answer in terms of some or all of the variables: F_G, F_N, $F_{\rm p}$, θ, and $F_{\rm k}$.

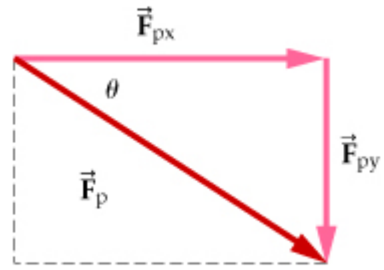
Hint 1. Find the *y* component of the pushing force

What is $F_{\rm py}$, the *y* component of the force $\vec{F}_{\rm p}$ exerted by Yusef?

Express your answer in terms of the variables $F_{\rm p}$ and θ.

Hint 1. Right angle triangle trigonometry

Keep in mind that the cosine of an angle is defined as the length of the adjacent leg divided by the length of the hypoteneuse. The sine of an angle is the length of the opposite leg divided by the length of the hypoteneuse. Apply these definitions to the triangle formed by $\vec{F}_{\rm p}$, θ, $F_{\rm px}$,and $F_{\rm py}$.



ANSWER:

F_{py} = $-F_p \sin(\theta)$

ANSWER:

ΣF_y = $-F_G - F_p \sin\theta + F_N \quad = ma_y$

Correct

You have created two equations that describe the motion of the chair:

$$\Sigma F_x = F_p \cos\theta - F_k = ma_x$$

and

$$\Sigma F_y = F_N - F_G - F_p \sin\theta = ma_y$$

Solve these equations to find F_N and a.

Part E

What is the magnitude of the acceleration a of the chair? What is the magnitude of the normal force F_N acting on the chair?

Express your answers, separated by a comma, in meters per second squared and newtons to three significant figures.

Hint 1. Find the component of the acceleration in the *y* direction

The chair only moves in the postive *x* direction. What is a_y?

Express your answer in meters per second squared.

ANSWER:

a_y = 0 $\mathrm{m/s^2}$

Hint 2. Find the weight of the chair

What is the weight of the chair?

Express the weight in newtons to three significant figures.

Hint 1. An equation for weight

Recall that the weight of an object of mass m is given by mg, where 9.8$\mathrm{m/s^2}$ is the acceleration due to gravity.

ANSWER:

F_G = 539 N

ANSWER:

a, F_N = 0.337,626 $\mathrm{m/s^2}$, N

Correct

A free-body diagram is a useful way to begin all problems involving forces. This drawing will help you to easily identify the most appropriate coordinate axes and to resolve any 2 dimensional vectors into components. Then you can apply Newton's second law to each coordinate direction to set up equations which will allow you to solve for any unknown quantities.

Score Summary:

Your score on this assignment is 102.6%.
You received 41.03 out of a possible total of 40 points.