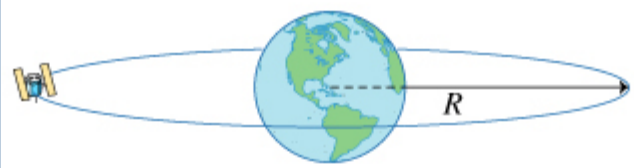
#35 Gravitation, Potential Energy and Gauss's Law Post-class

Due: 11:00am on Wednesday, November 14, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Geosynchronous Satellite

A satellite that goes around the earth once every 24 hours is called a *geosynchronous* satellite. If a geosynchronous satellite is in an equatorial orbit, its position appears stationary with respect to a ground station, and it is known as a *geostationary* satellite.



Part A

Find the radius R of the orbit of a geosynchronous satellite that circles the earth. (Note that R is measured from the center of the earth, not the surface.) You may use the following constants:

- The universal gravitational constant G is $6.67 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$.
- The mass of the earth is $5.98 \times 10^{24}\ \mathrm{kg}$.
- The radius of the earth is $6.38 \times 10^{6}\ \mathrm{m}$.

Give the orbital radius in meters to three significant digits.

Hint 1. Find the force on the satellite

If we just consider the earth-satellite system, then there is only one force acting on the satellite. Suppose the mass of the satellite is m, the mass of the earth is M, and the radius of the satellite's orbit is R. What is the magnitude of the force that acts on the satellite?

Answer in terms of m, M, R, and the universal gravitational constant G. (Use variables, not numerical values.)

ANSWER:

$F = \frac{GmM}{R^2}$

Hint 2. Angular frequency of satellite

The gravitational force on the satellite provides a centripetal acceleration that pulls the satellite inward, holding it in a circular orbit. A generic formula for the magnitude of the centripetal acceleration is $a = R\omega^2$, where ω is the angular frequency of the satellite's orbit.

What is the numerical value for ω in radians per second for a geosynchronous satellite?

Hint 1. How to calculate ω

Calculate ω using the definition of a geosynchronous orbit; the angular velocity should be such that the satellite makes one orbit per day. The equation relating the angular velocity ω and the time period T is

$$\omega = \frac{2\pi}{T}.$$

Hint 2. What is T?

Here the time period T is one day, but you are asked for the angular velocity in radians per second.

ANSWER:

ω = 7.27×10^{-5} radians/s

Hint 3. Find an expression for R

Type an expression for the radius R of the circular orbit of a satellite orbiting the earth.

Express your answer in terms of G, M (the mass of the earth), and ω, the angular velocity of the satellite.

Hint 1. Putting it all together

Using Newton's 2nd law, $F = ma$, gives us

$$\frac{GMm}{R^2} = m\omega^2 R.$$

Find R from this equation.

ANSWER:

R = $\sqrt[3]{\frac{GM}{\omega^2}}$

ANSWER:

R = 4.23×10^7 m

Correct

± Gravitational Force of Three Identical Masses

Three identical masses of 720kg each are placed on the *x* axis. One mass is at x_1 = -110cm , one is at the origin, and one is at x_2 = 410cm .

Part A

What is the magnitude of the net gravitational force F_{grav} on the mass at the origin due to the other two masses?

Take the gravitational constant to be G = 6.67×10^{-11}$\mathrm{N \cdot m^2/kg^2}$.

Express your answer in Newtons.

Hint 1. How to approach the problem

Calculate the force on the mass at the origin, including magnitude and direction, exerted by each of the other masses, then use vector addition to find the total force on it.

Hint 2. Calculate the gravitational force from the first mass

Calculate the gravitational force F_1 exerted on the mass at the origin by the mass at x_1 = -110cm .

Express your answer in Newtons.

Hint 1. Law of gravitation

Recall that Newton's law of gravitation states that the force exerted on a mass m_1 by a second mass m_2 is

$$F = G\frac{m_1 m_2}{r^2},$$

where r is the distance between the two masses and 6.67×10^{-11}$\mathrm{N \cdot m^2/kg^2}$ is the gravitational constant.

ANSWER:

F_1 = 2.86×10^{-5} N

Hint 3. Determine the direction of the gravitational force from the first mass

What is the direction of the gravitational force due to the mass at x_1 = -110cm ?

ANSWER:

- +*x* direction
- -*x* direction

Hint 4. Calculate the gravitational force from the second mass

Calculate the gravitational force F_2 exerted on the mass at the origin by the mass at x_2 = 410cm .

Express your answer in newtons.

Hint 1. Law of gravitation

Recall that Newton's law of gravitation states that the force exerted on a mass m_1 by a second mass m_2 is

$$F = G\frac{m_1 m_2}{r^2},$$

where r is the distance between the two masses and $6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2}$ is the gravitational constant.

ANSWER:

F_2 = 2.06×10^{-6} N

Hint 5. Determine the direction of the gravitational force from the second mass

What is the direction of the gravitational force due to the mass at x_2 = 410cm ?

ANSWER:

- +*x* direction
- -*x* direction

ANSWER:

F_{grav} = 2.65×10^{-5} N

Correct

Part B

What is the direction of the net gravitational force on the mass at the origin due to the other two masses?

ANSWER:

- +*x* direction
- -*x* direction

Correct

The closer together two masses are, the stronger is the gravitational attraction between them. Thus, the mass at the origin is more strongly attracted to the mass at x_1 = -110 cm than it is to the mass at x_2 = 410 cm . Thus, the net force on the mass at the origin is in the -*x* direction.

Gravitational Acceleration inside a Planet

Consider a spherical planet of uniform density ρ. The distance from the planet's center to its surface (i.e., the planet's radius) is R_p. An object is located a distance R from the center of the planet, where $R < R_p$. (The object is located inside of the planet.)

Part A

Find an expression for the magnitude of the acceleration due to gravity, $g(R)$, inside the planet.

Express the acceleration due to gravity in terms of ρ, R, π, and G, the universal gravitational constant.

Hint 1. Force due to planet's mass outside radius R

From Newton's *Principia*, Proposition LXX, Theorem XXX:
If to every point of a spherical surface there tend equal centripetal forces decreasing as the square of the distances from those points, I say, that a corpuscle placed within that surface will not be attracted by those forces any way.
In other words, you don't have to worry about the portion of the planet's mass that is located outside of the radius R. The net gravitational force from this "outer shell" of mass will equal zero. You only have to worry about that portion of the planet's mass that is located within a radius R.

Hint 2. Find the force on an object at distance R

Suppose the object has a mass m. Find the magnitude of the gravitational force acting on this object when it is located a distance R from the center of the planet.

Express the force in terms of m, ρ, π, R, and G, the universal gravitational constant.

Hint 1. Find the mass within a radius R

Find the net mass of the planet located within the radius R. Remember, the volume of a sphere is $(4/3)\pi R^3$.

Express your answer in terms of ρ, π, and R.

ANSWER:

$M(R) = \frac{4}{3}\pi R^3\rho$

Hint 2. Magnitude of gravitational force

The general equation for the magnitude of the gravitational force is F_{\rm grav} = GMm/R^2.

ANSWER:

$F(R) = \frac{4}{3}\pi Gm\rho R$

Hint 3. Finding $g(R)$ from $F(R)$

According to Newton's 2nd law, the net force acting on an object is given by $F_{\rm net} = ma$. In this problem, $a = g(R)$ and $F_{\rm net} = F(r)$ since the only force acting on the object is the gravitational force. Therefore, $g(R) = F(R)/m$, where $F(R)$ is the force you found in the previous hint.

Note that in this usage, both $F(R)$ and a are magnitudes and hence are positive. By convention, g (or $g(R)$ in this case) is the magnitude of the gravitational field. This gravitational field is a vector, with direction downward.

ANSWER:

$g(R) = \frac{G4R\pi\rho}{3}$

Correct

Part B

Rewrite your result for $g(R)$ in terms of $g_{\rm p}$, the gravitational acceleration at the surface of the planet, times a function of R.

Express your answer in terms of $g_{\rm p}$, R, and $R_{\rm p}$.

Hint 1. Acceleration at the surface

Note that the acceleration at the surface should be equal to the value of the function $g(R)$ from Part A evaluated at the radius of the planet: $g_{\rm p} = g(R_{\rm p})$.

ANSWER:

$g(R)$ = $g_p \frac{R}{R_p}$

Correct

Notice that g increases linearly with R, rather than being proportional to $1/R^2$. This assures that it is zero at the center of the planet, as required by symmetry.

Part C

Find a numerical value for $\rho_{\rm earth}$, the average density of the earth in kilograms per cubic meter. Use $6378\ {\rm km}$ for the radius of the earth, $G = 6.67 \times 10^{-11} {\rm m^3/(kg \cdot s^2)}$, and a value of g at the surface of $9.8\ {\rm m/s^2}$.

Calculate your answer to four significant digits.

Hint 1. How to approach the problem

You already derived the relation needed to solve this problem in Part A:

$$g(R) = (4/3)\pi G\rho R.$$

At what distance R is $g\,(R)$ known so that you could use this relation to find ρ?

ANSWER:

ρ_{earth} = 5497 $\mathrm{kg/m^3}$

Correct

± Weight on a Neutron Star

Neutron stars, such as the one at the center of the Crab Nebula, have about the same mass as our sun but a *much* smaller diameter.

Part A

If you weigh 665N on the earth, what would be your weight on the surface of a neutron star that has the same mass as our sun and a diameter of 24.0 km ?

Take the mass of the sun to be m_s = 1.99×10^{30}kg , the gravitational constant to be G = 6.67×10^{-11}$\mathrm{N \cdot m^2/kg^2}$, and the acceleration due to gravity at the earth's surface to be g = 9.810$\mathrm{m/s^2}$.

Express your weight w_{star} in newtons.

Hint 1. How to approach the problem

"Weight" is the same as "the force of gravitational attraction." Use Newton's law of gravitation to calculate the gravitational force that would be exerted on you if you were standing on the surface of the star. Be careful when determining the values needed for the equation.

Hint 2. Law of universal gravitation

The gravitational force exerted on a mass m_1 by a second mass m_2 is

$$F = G\frac{m_1 m_2}{r^2},$$

where r is the distance between the two masses, and $6.67\times10^{-11}\,\mathrm{N \cdot m^2/kg^2}$ is the universal gravitational constant.

Hint 3. Calculate your mass

Calculate your mass m if you weigh 665N on earth.

Express your answer in kilograms.

ANSWER:

m = 67.8 kg

Hint 4. Calculate the distance between you and the star

Calculate your distance r from the center of the star if you are standing on its surface.

Express your answer in meters.

ANSWER:

r = 1.20×10^{4} m

ANSWER:

w_{star} = 6.25×10^{13} N

Correct

This is over 10^{11} times your weight on earth! You probably shouldn't venture there....

Score Summary:

Your score on this assignment is 105.6%.
You received 42.25 out of a possible total of 40 points.