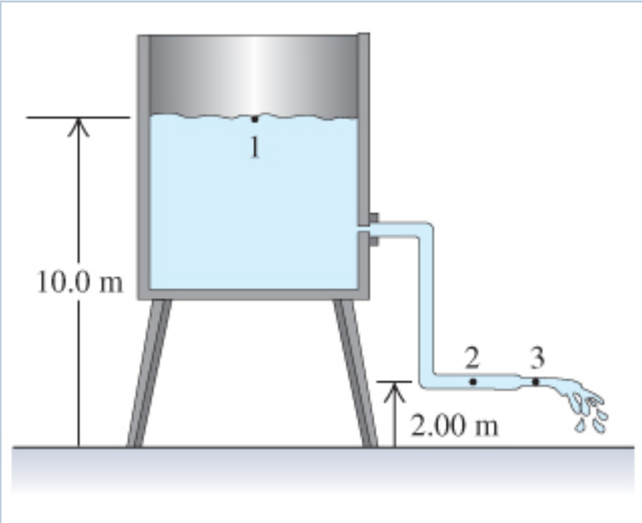
#34 Fluid Mechanics II Post-class

Due: 11:00am on Monday, November 12, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Water Flowing from a Tank

Water flows steadily from an open tank as shown in the figure. The elevation of point 1 is 10.0 meters, and the elevation of points 2 and 3 is 2.00 meters. The cross-sectional area at point 2 is 0.0480 square meters; at point 3, where the water is discharged, it is 0.0160 square meters. The cross-sectional area of the tank is very large compared with the cross-sectional area of the pipe.



Part A

Assuming that Bernoulli's equation applies, compute the discharge rate $\frac{dV}{dt}$.

Express your answer in cubic meters per second.

Hint 1. How to approach the problem

The discharge rate is the rate at which a given volume of water flows across the exit of the pipe per unit time. It is also defined as volume flow rate, and it depends on both the cross-sectional area of the pipe at the exit and the fluid speed at that point.

Hint 2. The volume flow rate

Consider a steadily moving incompressible fluid, and let A denote the cross-sectional area of a flow tube. The volume ΔV of fluid flowing across the cross section of area A at speed v during a small interval of time Δt is given by $Av\Delta t$. Therefore, the rate at which fluid volumes cross a portion of the flow tube is

$$\frac{dV}{dt} = Av.$$

Hint 3. Find the fluid speed at the end of the pipe

Assuming that Bernoulli's equation applies, find the speed v_3 of the water at point 3. Recall that the area of the tank is very large compared to the cross-sectional area of the pipe, and consequently, the velocity of water at a point on the surface of the water in the tank may be considered to be zero.

Express your answer in meters per second to three significant figures.

Hint 1. Apply Bernoulli's principle

Let p_a be the atmospheric pressure and ρ the density of water. Consider the entire volume of water as a single flow tube and apply Bernoulli's principle to point 3 and to point 1. Complete the expression below, where v_3 is the fluid speed at point 3.

Express your answer in terms of p_a, ρ, and g the free-fall acceleration due to gravity.

Hint 1. Bernoulli's principle

For the steady flow of an incompressible fluid with no internal friction, the pressure p and the flow speed v at depth H below the surface are linked by an important relationship, known as Bernoulli's principle. In particular, at any point at depth H along a flow tube, the following relation holds:

$$p + \rho g H + \frac{1}{2}\rho v^2 = \text{constant},$$

where ρ is the density of the fluid and g is the accereleration due to gravity.

Since Bernoulli's principle is valid at any point along a flow tube, it takes the form

$$p_1 + \rho g H_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g H_2 + \frac{1}{2}\rho v_2^2$$

when applied to two distinct points along a flow tube. The subscripts 1 and 2 refer to such points.

ANSWER:

$p_a + 2\rho g + \frac{1}{2}\rho v_3^2$ = $p_a + 10\rho g$

ANSWER:

v_3 = 12.5 $\mathrm{m/s}$

ANSWER:

$\frac{dV}{dt}$ = 0.200 $\mathrm{m^3/s}$

Correct

Part B

What is the gauge pressure at point 2?

Express your answer in pascals.

Hint 1. Definition of gauge pressure

Gauge pressure is defined as the excess pressure above atmospheric pressure. Let p_a be the atmospheric pressure and p the total pressure of a fluid. Then the gauge pressure is $p - p_{\rm a}$.

Hint 2. How to approach the problem

You can relate the fluid pressure at point 2 with the atmospheric pressure by applying Bernoulli's principle to point 2 and point 1, or alternatively to point 2 and point 3. To determine the fluid speed at point 2 you can use the continuity equation, using the fluid speed at the exit of the pipe found in Part A.

Hint 3. Apply Bernoulli's principle

Consider the entire volume of water as a single flow tube. Let p_2 and v_2 be respectively the pressure and the fluid speed at point 2. Let the atmospheric pressure be p_a and the density of water ρ. Apply Bernoulli's principle to point 1 and point 2 and complete the expression below.

Express your answer in terms of v_2, ρ, and g, the free-fall acceleration.

Hint 1. Bernoulli's principle

For the steady flow of an incompressible fluid with no internal friction, the pressure p and the flow speed v at depth H below the surface are linked by an important relationship, known as Bernoulli's principle. In particular, at any point at depth H along a flow tube, the following relation holds:

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when applied to two distinct points along a flow tube. The subscripts 1 and 2 refer to such points.

ANSWER:

$p_2 - p_a$ = $-0.5\rho {v_2}^2 + 8g\rho$

Hint 4. Find the fluid speed at point 2

Find v_2, the speed of the water at point 2.

Express your answer in meters per second to three significant figures.

Hint 1. The continuity equation

In a steadily moving incompressible fluid, the mass of fluid flowing along a flow tube is constant. In particular, consider a flow tube between two stationary cross sections with areas A_1 and A_2. Let v_1 and v_2 be the fluid speeds at these sections, respectively. Then conservation of mass takes the form

$$A_1v_1 = A_2v_2,$$

which is known as the continuity equation.

ANSWER:

v_2 = 4.17 $\mathrm{m/s}$

Hint 5. Density of Water

Recall that the density of water is $1000\ \mathrm{kg/m^3}$

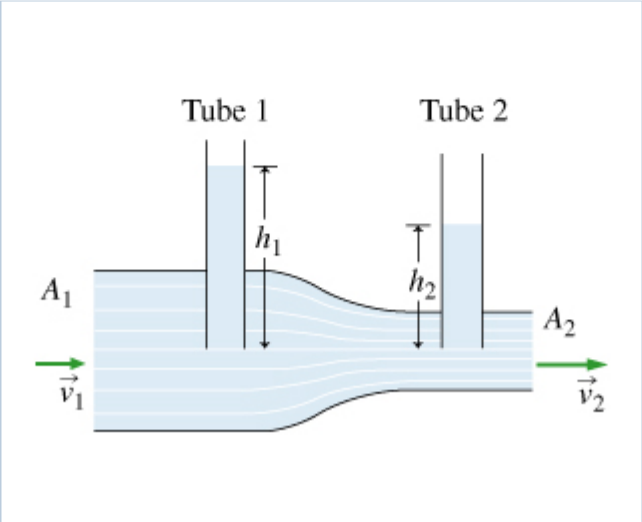
ANSWER:

6.98×10^4 Pa

Correct

Venturi Meter with Two Tubes

A pair of vertical, open-ended glass tubes inserted into a horizontal pipe are often used together to measure flow velocity in the pipe, a configuration called a *Venturi meter*. Consider such an arrangement with a horizontal pipe carrying fluid of density ρ. The fluid rises to heights h_1 and h_2 in the two open-ended tubes (see figure). The cross-sectional area of the pipe is A_1 at the position of tube 1, and A_2 at the position of tube 2.



Part A

Find p_1, the gauge pressure at the bottom of tube 1. (Gauge pressure is the pressure in excess of outside atmospheric pressure.)

Express your answer in terms of quantities given in the problem introduction and g, the magnitude of the acceleration due to gravity.

Hint 1. How to approach the problem

Use Bernoulli's law to compute the difference in pressure between the top and bottom of tube 1. The pressure at the top of the tube is defined to be atmospheric pressure. Note: Inside the tube, since the fluid is not flowing, the terms involving velocity in Bernoulli's equation can be ignored. Thus, Bernoulli's equation reduces to the formula for pressure as a function of depth in a fluid of uniform density.

Hint 2. Simplified Bernoulli's equation

For a fluid of uniform density ρ that is not flowing, the pressure p at a depth h below the surface is given by $p = p_0 + \rho g h$, where p_0 is the pressure at the surface and g is the magnitude of the acceleration due to gravity.

ANSWER:

p_1 = $\rho g h_1$

Correct

The fluid is pushed up tube 1 by the pressure of the fluid at the base of the tube, and not by its kinetic energy, since there is no streamline around the sharp edge of the tube. Thus energy is not conserved (there is turbulence at the edge of the tube) at the entrance of the tube. Since Bernoulli's law is essentially a statement of energy conservation, it must be applied separately to the fluid in the tube and the fluid flowing in the main pipe. However, the pressure in the fluid is the same just inside and just outside the tube.

Part B

Find v_1, the speed of the fluid in the left end of the main pipe.

Express your answer in terms of h_1, h_2, g, and either A_1 and A_2 or γ, which is equal to $\frac{A_1}{A_2}$.

Hint 1. How to approach the problem

Energy is conserved along the streamlines in the main flow. This means that Bernoulli's law can be applied to obtain a relationship between the fluid pressure and velocity at the bottom of tube 1, and the fluid pressure and velocity at the bottom of tube 2.

Hint 2. Find p_2 in terms of h_2

What is p_2, the pressure at the bottom of tube 2?

Express your answer in terms of h_2, g, and any other given quantities.

Hint 1. Recall Part A

Obtain the solution for this part in the same way that you found an expression for p_1 in terms of h_1 in Part A of this problem.

ANSWER:

p_2 = $\rho g h_2$

Hint 3. Find p_2 in terms of given quantities

Find p_2, the fluid pressure at the bottom of tube 2.

Express your answer in terms of p_1, v_1, ρ, A_1, and A_2.

Hint 1. Find the pressure at the bottom of tube 2

Find p_2, the fluid pressure at the bottom of tube 2.

Express your answer in terms of p_1, v_1, and v_2.

ANSWER:

p_2 = $p_1 + \frac{1}{2}\rho\left({v_1}^2 - {v_2}^2\right)$

Hint 2. Find v_2 in terms of v_1

The fluid is incompressible, so you can use the continuity equation to relate the fluid velocities v_1 and v_2 in terms of the geometry of the pipe. Find v_2, the fluid velocity at the bottom of tube 2, in terms of v_1.

Your answer may include A_1 and A_2, the cross-sectional areas of the pipe.

ANSWER:

$v_2 = \frac{A_1}{A_2} v_1$

ANSWER:

$p_2 = p_1 + \frac{1}{2}\rho\left(v_1{}^2\right)\left(1 - \left(\frac{A_1}{A_2}\right)^2\right)$

ANSWER:

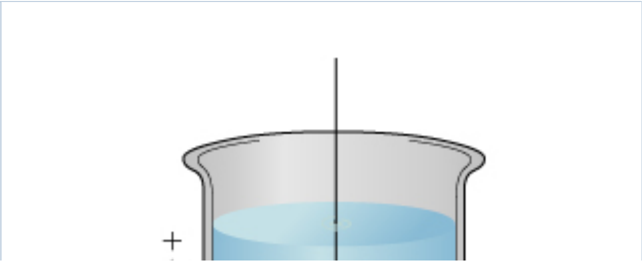
$v_1 = \sqrt{2g\left(\frac{h_1 - h_2}{(\gamma)^2 - 1}\right)}$

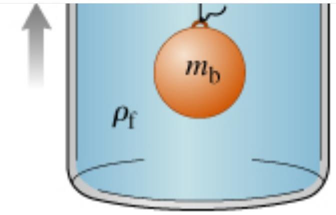
Correct

Note that this result depends on the difference between the heights of the fluid in the tubes, a quantity that is more easily measured than the heights themselves.

A Submerged Ball

A ball of mass m_b and volume V is lowered on a string into a fluid of density ρ_f. Assume that the object would sink to the bottom if it were not supported by the string.





Part A

What is the tension T in the string when the ball is fully submerged but not touching the bottom, as shown in the figure?

Express your answer in terms of any or all of the given quantities and g, the magnitude of the acceleration due to gravity.

Hint 1. Equilibrium condition

Although the fact may be obscured by the presence of a liquid, the basic condition for equilibrium still holds: The net force on the ball must be zero. Draw a free-body diagram and proceed from there.

Hint 2. Find the magnitude of the buoyant force

Find F_{buoyant}, the magnitude of the buoyant force.

Express your answer in terms of any or all of the variables ρ_f, V, m_b, and g.

Hint 1. Archimedes' principle

Quantitatively, the buoyant force can be found as

$$F_{\mathrm{buoyant}} = \rho_{\mathrm{fluid}} g V,$$

where F_{buoyant} is the force, ρ_{fluid} is the density of the fluid, g is the magnitude of the acceleration due to gravity, and V is the volume of the displaced fluid.

Hint 2. Find the mass of the displaced fluid

Compute the mass $m_{\rm f}$ of the fluid displaced by the object when it is entirely submerged.

Express your answer in terms of any or all of the variables $\rho_{\rm f}$, V, $m_{\rm b}$, and g.

ANSWER:

$m_{\rm f}$ = $\rho_f V$

ANSWER:

$F_{\rm buoyant}$ = $\rho_f V g$

ANSWER:

T = $m_b g - \rho_f V g$

Correct

Score Summary:

Your score on this assignment is 105.7%.
You received 31.7 out of a possible total of 30 points.