#1 Units and Uncertainties Post-class

Due: 11:00am on Wednesday, August 22, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Consistency of Units

In physics, every physical quantity is measured with respect to a *unit*. Time is measured in seconds, length is measured in meters, and mass is measured in kilograms. Knowing the units of physical quantities will help you solve problems in physics.

Part A

Gravity causes objects to be attracted to one another. This attraction keeps our feet firmly planted on the ground and causes the moon to orbit the earth. The force of gravitational attraction is represented by the equation

$$F = \frac{Gm_1m_2}{r^2},$$

where F is the magnitude of the gravitational attraction on either body, m_1 and m_2 are the masses of the bodies, r is the distance between them, and G is the gravitational constant. In SI units, the units of force are $\mathrm{kg \cdot m/s^2}$, the units of mass are kg, and the units of distance are m. For this equation to have consistent units, the units of G must be which of the following?

Hint 1. How to approach the problem

To solve this problem, we start with the equation

$$F = \frac{Gm_1m_2}{r^2}.$$

For each symbol whose units we know, we replace the symbol with those units. For example, we replace m_1 with kg. We now solve this equation for G.

ANSWER:

- $\frac{kg^3}{m \cdot s^2}$
- $\frac{kg \cdot s^2}{m^3}$
- $\frac{m^3}{kg \cdot s^2}$
- $\frac{m}{kg \cdot s^2}$

Correct

Part B

One consequence of Einstein's theory of special relativity is that mass is a form of energy. This mass-energy relationship is perhaps the most famous of all physics equations:

$$E = mc^2,$$

where m is mass, c is the speed of the light, and E is the energy. In SI units, the units of speed are m/s. For the preceding equation to have consistent units (the same units on both sides of the equation), the units of E must be which of the following?

Hint 1. How to approach the problem

To solve this problem, we start with the equation

$$E = mc^2.$$

For each symbol whose units we know, we replace the symbol with those units. For example, we replace m with kg. We now solve this equation for E.

ANSWER:

- $\frac{\mathrm{kg \cdot m}}{\mathrm{s}}$
- $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^2}}$
- $\frac{\mathrm{kg \cdot s^2}}{\mathrm{m^2}}$
- $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s}}$

Correct

To solve the types of problems typified by these examples, we start with the given equation. For each symbol whose units we know, we replace the symbol with those units. For example, we replace m with kg. We now solve this equation for the units of the unknown variable.

Converting between Different Units

Unit conversion problems can seem tedious and unnecessary at times. However, different systems of units are used in different parts of the world, so when dealing with international transactions, documents, software, etc., unit conversions are often necessary.

Here is a simple example. The inhabitants of a small island begin exporting beautiful cloth made from a rare plant that grows only on their island. Seeing how popular the small quantity that they export has been, they steadily raise their prices. A clothing maker from New York, thinking that he can save money by "cutting out the middleman," decides to travel to the small island and buy the cloth himself. Ignorant of the local custom of offering strangers outrageous prices and then negotiating down, the clothing maker accepts (much to everyone's surpise) the initial price of 400 $\mathrm{tepizes/m^2}$. The price of this cloth in New York is 120 $\mathrm{dollars/yard^2}$.

Part A

If the clothing maker bought 500 $\mathrm{m^2}$ of this fabric, how much money did he lose? Use $1\ \mathrm{tepiz} = 0.625\ \mathrm{dollar}$ and $0.9144\ \mathrm{m} = 1\ \mathrm{yard}$.

Express your answer in dollars using two significant figures.

Hint 1. How to approach the problem

To find how much money the clothing maker loses, you must find how much money he spent and how much he would have spent in New York. Furthermore, since the problem asks how much he lost in dollars, you need to determine both in dollars. This will require unit conversions.

Hint 2. Find how much he paid

If the clothing maker bought 500 $\mathrm{m^2}$ at a cost of 400 $\mathrm{tepizes/m^2}$, then simple multiplication will give how much he spent in tepizes. Once you've found that, convert to dollars. How much did the clothing maker spend in dollars?

Express your answer in dollars to three significant figures.

Hint 1. Find how much he paid in $\mathrm{tepizes}$

If the clothing maker bought 500 $\mathrm{m^2}$ at a cost of 400 $\mathrm{tepizes/m^2}$, then how much did he pay in total, in $\mathrm{tepizes}$?

Express your answer in $\mathrm{tepizes}$.

ANSWER:

2.00×10^5 $\mathrm{tepizes}$

ANSWER:

1.25×10^5 $\mathrm{dollars}$

Hint 3. Find the price in New York

You know that the price of the fabric in New York is 120 $\mathrm{dollars/yard^2}$. Thus, you need only to find the number of square yards that the clothing maker purchased and then multiply to find the price in New York. What would it have cost him to buy the fabric in New York?

Express your answer in dollars to three significant figures.

Hint 1. Determine how much cloth he bought in $\mathrm{yard^2}$

You are given that $0.9144\ \mathrm{m} = 1\ \mathrm{yard}$. Squaring both sides, you would get that $0.8361\ \mathrm{m^2} = 1\ \mathrm{yard^2}$. How much is $500\ \mathrm{m^2}$?

Express your answer in $\mathrm{yard^2}$ to three significant figures.

ANSWER:

598 $\mathrm{yard^2}$

ANSWER:

7.18×10^4 $\mathrm{dollars}$

ANSWER:

5.3×10^4 $\mathrm{dollars}$

Correct

Still think that unit conversion isn't important?

Here is a widely publicized, true story about how failing to convert units resulted in a huge loss. In 1998, the Mars Climate Orbiter probe crashed into the surface of Mars, instead of entering orbit. The resulting inquiry revealed that NASA navigators had been making minor course corrections in SI units, whereas the software written by the probe's makers implicitly used British units. In the United States, most scientists use SI units, whereas most engineers use the British, or Imperial, system of units. (Interestingly, British units are *not* used in Britain.) For these two groups to be able to communicate to one another, unit conversions are necessary.

The unit of force in the SI system is the newton (N), which is defined in terms of basic SI units as $1\ \mathrm{N} = 1\ \mathrm{kg \cdot m/s^2}$. The unit of force in the British system is the pound (lb), which is defined in terms of the slug (British unit of mass), foot (ft), and second (s) as $1\ \mathrm{lb} = 1\ \mathrm{slug \cdot ft/s^2}$.

Part B

Find the value of 15.0 N in pounds. Use the conversions $1\ \mathrm{slug} = 14.59\ \mathrm{kg}$ and $1\ \mathrm{ft} = 0.3048\ \mathrm{m}$.

Express your answer in pounds to three significant figures.

Hint 1. How to approach the problem

When doing a unit conversion, you should begin by comparing the units you are starting with and the units you need to finish with. In this problem, we have the following:

Starting units	Final units
$\frac{\mathrm{kg \cdot m}}{\mathrm{s^2}}$	$\frac{\mathrm{slug \cdot ft}}{\mathrm{s^2}}$

Notice that both have seconds squared in the denominator. You will only have to change the units in the numerator. Match up the units that measure the same quantity (e.g., kilograms and slugs both measure mass). Once you've done this, create a fraction (e.g., $1\ \mathrm{hour}/60\ \mathrm{minutes}$)

based on conversion factors such that the old unit is canceled out of the expression and the new unit appears in the position (i.e., numerator or denominator) of the old unit. In this problem, there are two pairs within the starting and final units that must be converted in this way (i.e., kilograms/slugs and meters/feet).

Hint 2. Calculate the first conversion

The first step is to eliminate kilograms from the expression for newtons in favor of slugs. What is the value of 15 $\mathrm{kg \cdot m/s^2}$ in $\mathrm{slug \cdot m/s^2}$?

Express your answer in slug-meters per second squared to four significant figures.

ANSWER:

1.028

Answer Requested

Follow the same procedure to replace meters with feet.

ANSWER:

15.0 N = 3.37 lb

Correct

Thus, if the NASA navigators believed that they were entering a force value of 15 N (3.37 lb), they were actually entering a value nearly four and a half times higher, 15 lb. Though these errors were only in tiny course corrections, they added up during the trip of many millions of kilometers.

In the end, the blame for the loss of the 125-million-dollar probe was placed on the lack of communication between people at NASA that allowed the units mismatch to go unnoticed. Nonetheless, this story makes apparent how important it is to carefully label the units used to measure a number.

Significant Figures Conceptual Question

In the parts that follow select whether the number presented in statement A is greater than, less than, or equal to the number presented in statement B. Be sure to follow all of the rules concerning significant figures.

Part A

- Statement A: 2.567 km, to two significant figures.
- Statement B: 2.567 km, to three significant figures.

Determine the correct relationship between the statements.

Hint 1. Rounding and significant figures

Rounding to a different number of significant figures changes a number. For example, consider the number 3.4536. This number has five significant figures. The following table illustrates the result of rounding this number to different numbers of significant figures:

Four significant figures	$\Longrightarrow$	3.454
Three significant figures	$\Longrightarrow$	3.45
Two significant figures	$\Longrightarrow$	3.5
One significant figure	$\Longrightarrow$	3

Notice that, when rounding 3.4536 to one significant figure, since 0.4536 is less than 0.5, the result is 3, even though if you first rounded to two significant figures (3.5), the result would be 4.

ANSWER:

Statement A is
- (●) greater than
- ( ) less than
- ( ) equal to

Statement B.

Correct

Part B

- Statement A: (2.567 km + 3.146 km), to two significant figures.
- Statement B: (2.567 km, to two significant figures) + (3.146 km, to two significant figures).

Determine the correct relationship between the statements.

ANSWER:

Statement A is
- greater than
- less than
- equal to

Statement B.

Correct

Evaluate statement A as follows: (2.567 km + 3.146 km) = 5.713 km to two significant figures is 5.7 km . Statement B evaluates as 2.6 km + 3.1 km = 5.7 km . Therefore, the two statements are equal.

Part C

- Statement A: Area of a rectangle with measured length = 2.536 m and width = 1.4 m.
- Statement B: Area of a rectangle with measured length = 2.536 m and width = 1.41 m.

Since you are not told specific numbers of significant figures to round to, you must use the rules for multiplying numbers while respecting significant figures. If you need a reminder, consult the hint.

Determine the correct relationship between the statements.

Hint 1. Significant figures and multiplication

When you multiply two numbers, the result should be rounded to the number of significant figures in the less accurate of the two numbers. For instance, if you multiply 2.413 (four significant figures) times 3.81 (three significant figures), the result should have three significant figures: $2.413 \times 3.81 = 9.19$. Similarly, $2 \times 7.664323 = 20$, when significant figures are respected (i.e., 15.328646 rounded to one significant figure).

ANSWER:

Statement A is
- (selected) greater than
- less than
- equal to

Statement B.

Correct

Evaluate statement A as follows: (2.536 m) (1.4 m) = 3.5504 m^2 to two significant figures is 3.6 m^2 . Statement B evaluates as (2.536 m) (1.41 m) = 3.57576 m^2 to three significant figures is 3.58 m^2 . Therefore, statement A is greater than statement B.

Converting Units: The Magic of 1

Learning Goal:

To learn how to change units of physical quantities.

Quantities with physical dimensions like length or time must be measured with respect to a *unit*, a standard for quantities with this dimension. For example, length can be measured in units of meters or feet, time in seconds or years, and velocity in meters per second.

When solving problems in physics, it is necessary to use a consistent system of units such as the International System (abbreviated SI, for the French Système International) or the more cumbersome English system. In the SI system, which is the preferred system in physics, mass is measured in kilograms, time in seconds, and length in meters. The necessity of using consistent units in a problem often forces you to convert some units from the given system into the system that you want to use for the problem.

The key to unit conversion is to multiply (or divide) by a ratio of different units that equals one. This works because multiplying any quantity by one doesn't change it. To illustrate with length, if you know that $1\ \mathrm{inch} = 2.54\ \mathrm{cm}$, you can write

$$1 = \frac{2.54\ \mathrm{cm}}{1\ \mathrm{inch}}.$$

To convert inches to centimeters, you can multiply the number of inches times this fraction (since it equals one), cancel the inch unit in the denominator with the inch unit in the given length, and come up with a value for the length in centimeters. To convert centimeters to inches, you can divide by this ratio and cancel the centimeters.

For all parts, notice that the units are already written after the answer box; don't try to write them in your answer also.

Part A

How many centimeters are there in a length 42.3inches ?

Express your answer in centimeters to three significant figures.

ANSWER:

107 cm

Correct

Sometimes you will need to change units twice to get the final unit that you want. Suppose that you know how to convert from centimeters to inches and from inches to feet. By doing both, in order, you can convert from centimeters to feet.

Part B

Suppose that a particular artillery piece has a range R = 7220yards . Find its range in miles. Use the facts that $1\ \mathrm{mile} = 5280\ \mathrm{ft}$ and $3\ \mathrm{ft} = 1\ \mathrm{yard}$.

Express your answer in miles to three significant figures.

Hint 1. Convert yards to feet

The first step in this problem is to convert from yards to feet, because you know how to then convert feet into miles. Convert 7220yards into feet. Use

$$1 = \frac{3\ \mathrm{ft}}{1\ \mathrm{yard}}.$$

Express your answer in feet to three significant figures.

ANSWER:

2.17×10^4 ft

ANSWER:

$7220\ \mathrm{yards}$ = 4.10 miles

Correct

Often speed is given in miles per hour (mph), but in physics you will almost always work in SI units. Therefore, you must convert mph to meters per second ($\mathrm{m/s}$).

Part C

What is the speed of a car going $v = 1.000\ \mathrm{mph}$ in SI units? Notice that you will need to change from miles to meters and from hours to seconds. You can do each conversion separately. Use the facts that $1\ \mathrm{mile} = 1609\ \mathrm{m}$ and $1\ \mathrm{hour} = 3600\ \mathrm{s}$.

Express your answer in meters per second to four significant figures.

Hint 1. Convert miles to meters

In converting $1.000\ \mathrm{mph}$ into meters per second, you will need to multiply by

$$1 = \frac{1609\ \mathrm{m}}{1\ \mathrm{mile}}.$$

When you do this, the miles will cancel to leave you with a value in meters per hour. You can then finish the conversion. What is $v = 1.000\ \mathrm{mph}$ in meters per hour?

Express your answer in meters per hour to four significant figures.

ANSWER:

v = 1609 $\mathrm{m/hour}$

Hint 2. Convert hours to seconds

Which of the following would you multiply $1609\ \mathrm{m/hours}$ by to convert it into meters per second ($\mathrm{m/s}$)?

ANSWER:

- $3600\ \mathrm{s}$
- $1\ \mathrm{hour}$
- $\frac{3600\ \mathrm{s}}{1\ \mathrm{hour}}$
- $\frac{1\ \mathrm{hour}}{3600\ \mathrm{s}}$

ANSWER:

v = 0.4469 $\mathrm{m/s}$

Correct

Notice that by equating the two values for v, you get $1.000\ \mathrm{mph} = 0.4469\ \mathrm{m/s}$. It might be valuable to remember this, as you may frequently need to convert from miles per hour into more useful SI units. By remembering this relationship in the future, you can reduce this task to a single conversion.

Score Summary:

Your score on this assignment is 70.7%.
You received 28.28 out of a possible total of 40 points.