Problem Set #2 Solutions

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$\boxed{1}$ a) $\lim_{s \to 0} s \cdot G(s) \frac{1}{s}$

$$= \lim_{s \to 0} \frac{5(s+4)}{s^2+10s+50} = 20/50 = 2/5.$$

b) $Y(s) = G(s) \cdot R(s)$ for $R(s) = 1 = \mathcal{L}[\delta(t)]$

$$= \frac{5(s+4)}{(s+5)^2 + 5^2}$$

$$= A \frac{(s+5)}{(s+5)^2 + 5^2} + B \frac{5}{(s+5)^2 + 5^2}$$

$$\Rightarrow \quad As + 5A + 5B = 5s + 20$$

$$\therefore \ A = 5, \quad B = -1$$

$$y(t) = \left(5e^{-5t}\cos(5t) - 1 \cdot e^{-5t}\sin(5t)\right) \cdot \underline{u}(t)$$

$$= \left(5\cos(5t) - \sin(5t)\right) e^{-5t} \cdot \underline{u}(t)$$

$\boxed{2}$ a)

$$\dot{x}_1 = v_1$$
$$\dot{v}_1 = \frac{1}{m_1}\left(-k(x_1 - x_2) + f(t)\right)$$
$$\dot{x}_2 = v_2$$
$$\dot{v}_2 = \frac{1}{m_2}\left(+k(x_1 - x_2)\right)$$

b)

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/m_1 & 0 & +k/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ +k/m_2 & 0 & -k/m_2 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 1/m_1 \\ 0 \\ 0 \end{bmatrix} u \qquad (= Az + Bu)$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} z + 0 \cdot u \qquad (= Cz + Du)$$

$\boxed{3}$ $\dot{m}_A = -(k_{01} + k_{L1})m_A + k_{L2} m_V + k_{02} m_{SO} + k_{L4} m_S + u$

a) $\dot{m}_{SO} = k_{01} m_A - (k_{02} + k_{03}) m_{SO} + k_{04} m_{IDO}$

$\dot{m}_{IDO} = k_{03} m_{SO} - k_{04} m_{IDO}$

$\dot{m}_V = k_{L1} m_A - (k_{L2} + k_{L3}) m_V$

$\dot{m}_S = k_{L3} m_V - k_{L4} m_S$

b) $$\dot{x} = \underbrace{\begin{bmatrix} -(k_{01}+k_{L1}) & k_{02} & 0 & k_{L2} & k_{L4} \\ k_{02} & -(k_{02}+k_{03}) & k_{04} & 0 & 0 \\ 0 & k_{02} & -k_{04} & 0 & 0 \\ k_{L1} & 0 & 0 & -(k_{L2}+k_{L3}) & 0 \\ 0 & 0 & 0 & k_{L2} & -k_{L4} \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{B} u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}}_{C} x + \underbrace{0}_{D} \cdot u$$

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Problem Set #2
Problem 4

% Part (a)

sysA = ss(-5,3,7,0)

a =
        x1
   x1  -5

b =
        u1
   x1   3

c =
        x1
   y1   7

d =
        u1
   y1   0

Continuous-time state-space model.

sysB = ss(diag([-5 -1]),[3 1]',[7 0],0)

a =
        x1  x2
   x1  -5   0
   x2   0  -1

b =
        u1
   x1   3
   x2   1

c =
        x1  x2
   y1   7   0

d =
        u1
   y1   0

Continuous-time state-space model.
sysC = ss(diag([-5 -1]),[3 0]',[7 3],0)

a =
        x1  x2
   x1  -5   0
   x2   0  -1

b =
        u1
   x1   3
   x2   0

c =
        x1  x2
   y1   7   3

d =
        u1
   y1   0

Continuous-time state-space model.


% Part (b)

GA = tf(sysA)

Transfer function:
 21
-----
s + 5

GB = tf(sysB)

Transfer function:
 21
-----
s + 5


% Part(c)

[numC, denC]=tfdata(sysC,'v')

numC =

     0    21


denC =

     1     5
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% Part (d)

GC = tf(numC, denC)

Transfer function:
 21
-----
s + 5
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In part (b), tf takes a 'system' as input. In part (d), tf takes vectors as input. In part (b), tf transforms the system from state-space to transfer function form. In part (d), tf creates a system in transfer function form.