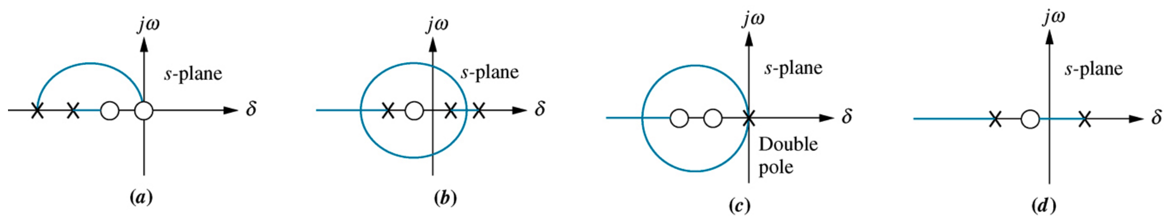
ECE 345: Introduction to Control Systems
Problem Set #5

Dr. Oishi

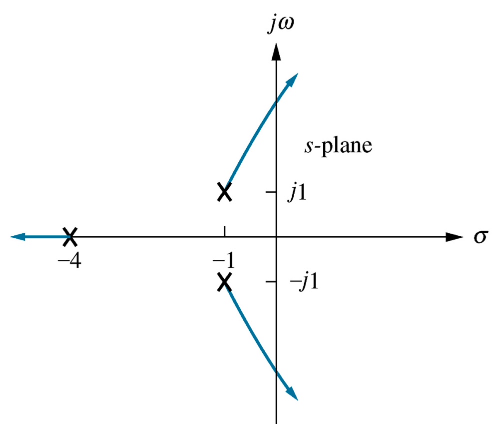
Due Tuesday, November 20, 2012 at the start of class

This homework is open note and open book. You are welcome to discuss the problems with other students, but your solutions and Matlab code *must be written independently*. Copying will not be tolerated.

1. For each of the root loci shown below, indicate whether the sketch is a feasible root locus, and if not, describe *all* reasons that the plot is not feasible.



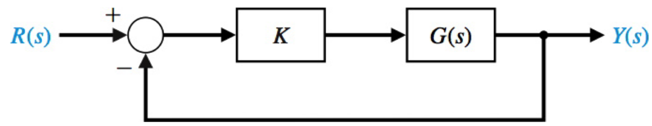
2. Given the root locus below,



   (a) Find the value of gain K that will make the system marginally stable.
   (b) Find the value of gain K for which the closed-loop transfer function will have a pole on the real axis at -5.

3. For the unity feedback system shown below, with

$$G(s) = \frac{s+10}{s^2 - 20s + 200}$$



   (a) Sketch the root locus by hand. As relevant, calculate (i) asymptote angles and centroid, (ii) breakaway / break-in points, and (iii) departure / arrival angles.
   (b) Find the range of gain K that makes the system stable.
   (c) Find the value of gain K that yields closed-loop critically damped dominant poles.
   (d) Find the value of gain K that yields a damping ratio of $1/\sqrt{2} \approx 0.707$ for the system's closed-loop dominant poles.

4. Consider the same system as in the above problem.

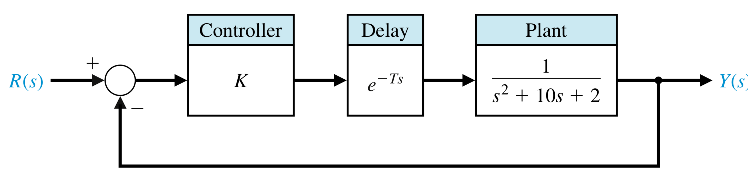
   (a) Use Matlab's `rlocus` to numerically plot the root locus for $K > 0$.
   (b) Use `rlocfind` to numerically find the value of K that yields a closed-loop system with critically damped dominant poles. Print out the resulting plot.
   (c) Use `rlocfind` in conjunction with `grid` to find the value of K that yields a closed-loop system with damping ratio of approximately 0.707. Print out the resulting plot (can be overlaid on the plot from the previous step).
   (d) Hand in a print-out of the commands (at the command line or in an m-file) used to generate answers to the above questions.

5. BONUS: In many industrial processes, there can be delays in the control loop which can affect system stability. A delay of T has Laplace transform of e^{-sT}, which we approximate as

$$e^{-sT} \approx \frac{-\frac{T}{2}s + 1}{\frac{T}{2}s + 1} \tag{1}$$

   via a *Pade approximation.*

   Consider the system below with delay T and plant $G(s) = \frac{1}{s^2+10s+2}$.



   (a) Sketch the root locus of the delay-free system (as the figure at the top of the page).
   (b) For what values of gain K will the delay-free system be stable?
   (c) Use Matlab to plot the root locus of the system with delay $T = 1$ in Matlab, and to find the values of K for which the system is stable.
   (d) Examine the Matlab plot of the root locus carefully. Is the root locus what you would have expected, had you done the sketch by hand? In what ways is it different?