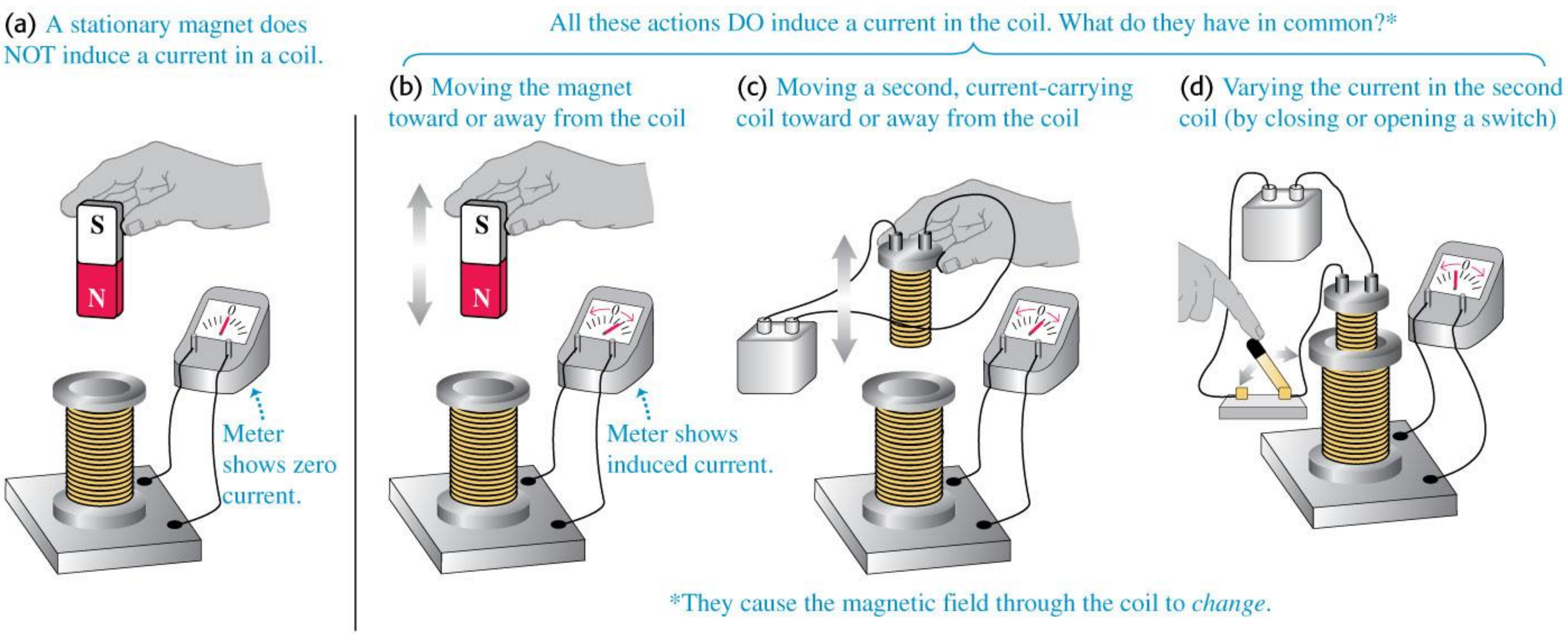
Lecture 32
(Induction & Faraday's Law)

Physics 161-01 Spring 2012

Douglas Fields

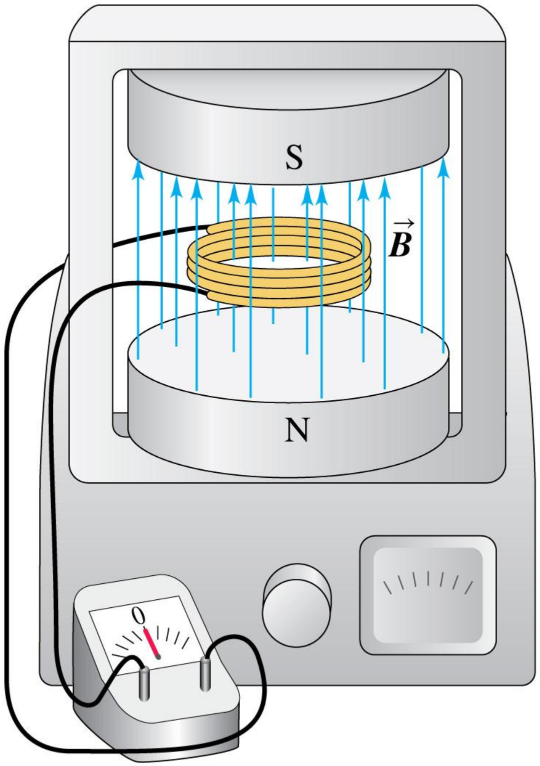
Induction Experiments

- It wasn't long after people started looking at electric charge and current that it was noticed that magnetic fields could also cause a current.
- But not just a steady magnetic field – a magnetic field with some change involved.



Induction Experiments

- Let's do a series of experiments:
  - When B = 0, there is no current in the loop.
  - When the field is turned on, there is a current in the loop while B is increasing, and then stops when B becomes constant.
  - If we squeeze the coil so to change its size, there is a current in the coil only during the process of deformation.
  - If we rotate the coil, there is a current during the rotation in one direction, but if we rotate it back, there is a current in the opposite direction.
  - If we pull the coil out of the B-field, there is a current in the same direction as when we squeezed the coil.
  - If we decrease the number of coils, again the is a current during that process in the same direction.
  - When the magnet is turned off, there is a current in the coil briefly and then stops.
  - The faster we do any of these things, the greater the current.



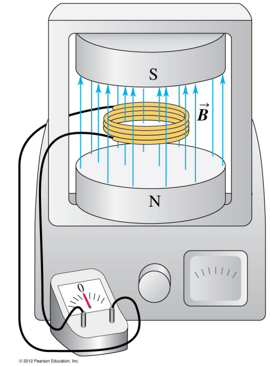
Faraday's Law

- We can encompass all of these experimental facts into one equation:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \qquad \Phi_B = \int_{\text{Surface}} \vec{B} \cdot d\vec{A}$$

- This is known as Faraday's Law.
- Let's check that all of the experimental results are reproduced in this equation.

Induction Experiments



$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \qquad \Phi_B = N\int_{\text{Surface}} \vec{B}\cdot d\vec{A} \Rightarrow$$

- Let's do a series of experiments:
  - When B = 0, there is no current in the loop.

    $$\mathcal{E} = -\frac{d}{dt}\left[N\int_{\text{Surface}} \vec{B}\cdot d\vec{A}\right] = -N\frac{dB}{dt}\cos\theta_{BA}A$$

  - When the field is turned on, there is a current in the loop while B is increasing, and then stops when B becomes constant.

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  - If we pull the coil out of the B-field, there is a current in the same direction as when we squeezed the coil.

    $$\mathcal{E} = -\frac{d}{dt}\left[N\int_{\text{Surface}} \vec{B}\cdot d\vec{A}\right] = -N\frac{dB}{dt}\cos\theta_{BA}A$$

  - If we decrease the number of coils, again the is a current during that process in the same direction.

    $$\mathcal{E} = -\frac{d}{dt}\left[N\int_{\text{Surface}} \vec{B}\cdot d\vec{A}\right] = -\frac{dN}{dt}B\cos\theta_{BA}A$$

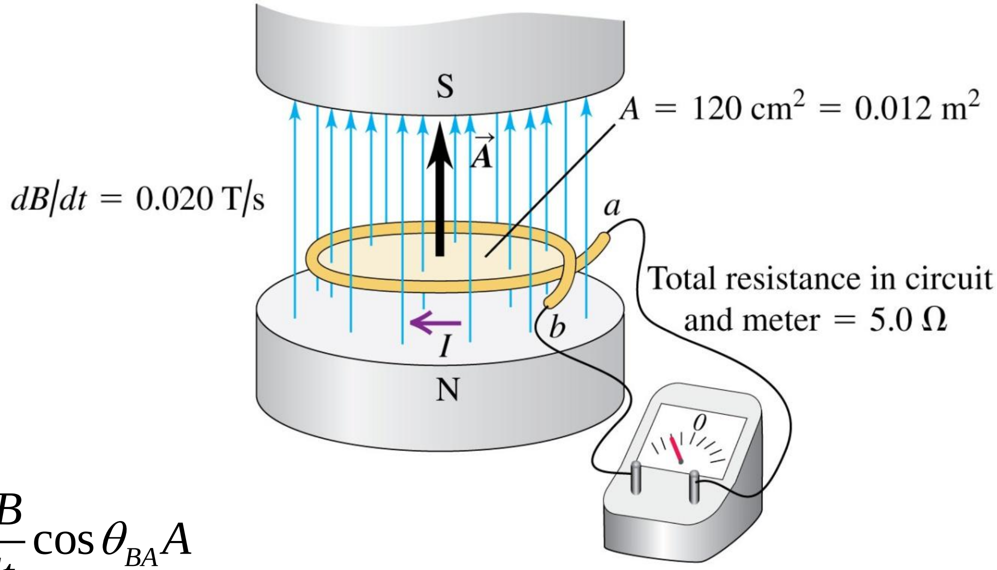
  - When the magnet is turned off, there is a current in the coil briefly and then stops.

    $$\mathcal{E} = -\frac{d}{dt}\left[N\int_{\text{Surface}} \vec{B}\cdot d\vec{A}\right] = -N\frac{dB}{dt}\cos\theta_{BA}A$$

  - The faster we do any of these things, the greater the current.

Example

- Let's put some numbers in to see how this might work:



$$\mathcal{E} = -\frac{d}{dt}\left[N\int_{\text{Surface}} \vec{B}\cdot d\vec{A}\right] = -N\frac{dB}{dt}\cos\theta_{BA}A$$

$$= -(1)(0.020\,T/s)(1)\left(0.012m^2\right) = 2.4\times10^{-4}\,\frac{Tm^2}{s}$$

$$I = \frac{\mathcal{E}}{R} = \frac{2.4\times10^{-4}\,\dfrac{Tm^2}{s}}{5.0\Omega} = 4.8\times10^{-4}\,\frac{Tm^2}{\Omega s}$$

Unit Check!!!

- Let's put some numbers in to see how this might work:

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$$d\vec{F} = Id\vec{l}\times\vec{B} \Rightarrow$$
$$N = AmT \Rightarrow$$
$$T = \frac{N}{Am}$$

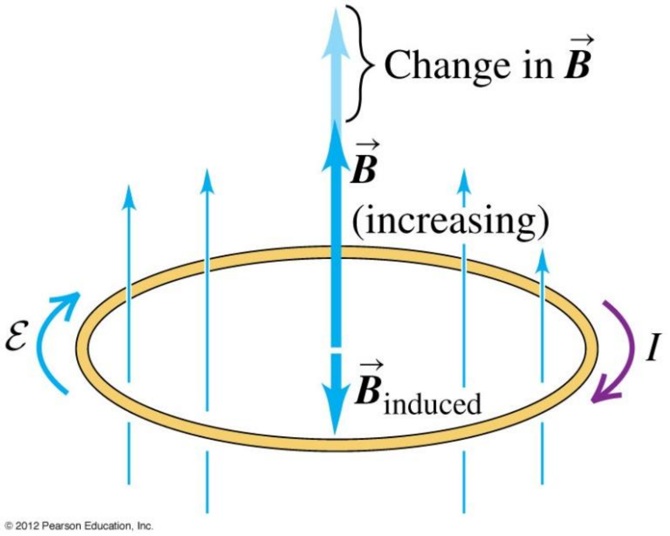
$$V = IR \Rightarrow$$
$$\frac{Nm}{C} = A\Omega \Rightarrow$$
$$\Omega = \frac{Nm}{AC}$$

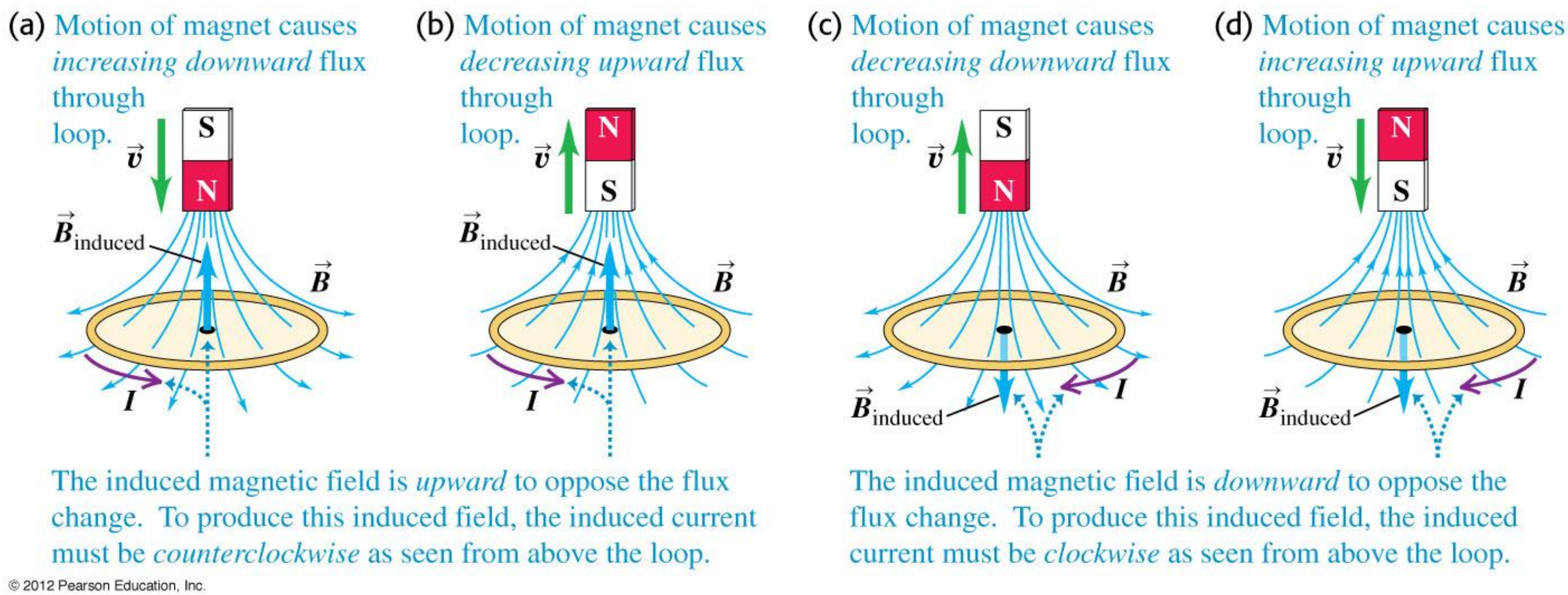
$$\Rightarrow \frac{Tm^2}{\Omega s} = \frac{\frac{N}{Am}m^2}{\frac{Nm}{AC}s} = \frac{C}{s} = A$$

Lenz's Law

- To get the direction of the induced EMF (and thus, the current in a circuit), remember:

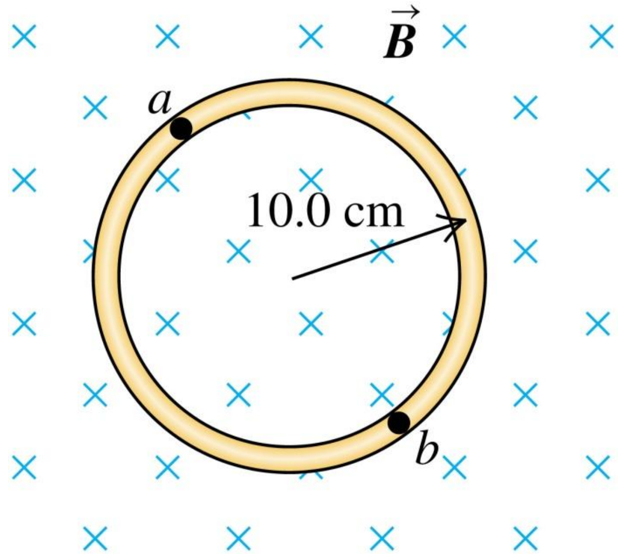
$$\mathcal{E} = -\frac{d}{dt}\Phi_B$$





CPS 32-1

A circular loop of wire is in a region of spatially uniform magnetic field. The magnetic field is directed into the plane of the figure. If the magnetic field magnitude is *decreasing*,



A. the induced emf is clockwise.

B. the induced emf is counterclockwise.

C. the induced emf is zero.

D. The answer depends on the strength of the field.

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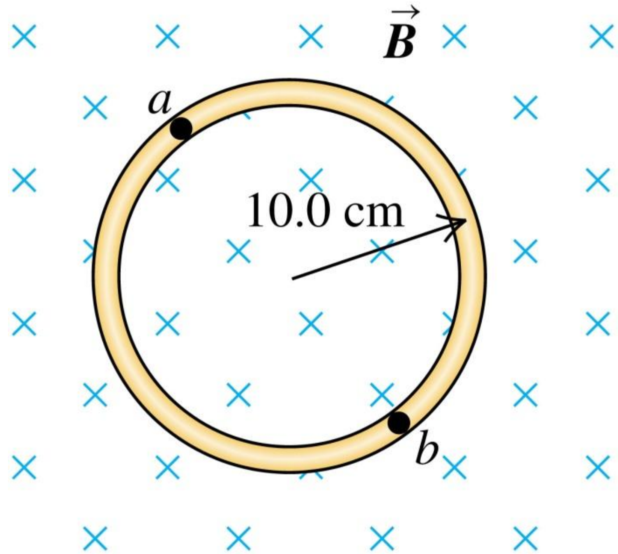

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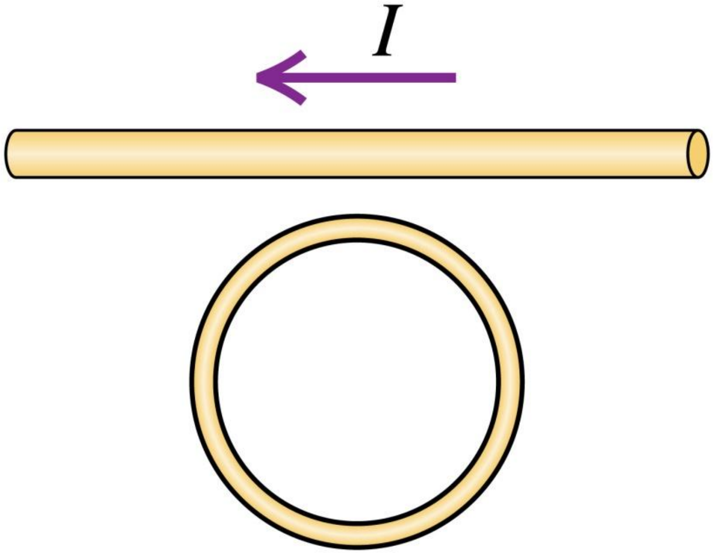
CPS 32-2

A circular loop of wire is placed next to a long straight wire. The current *I* in the long straight wire is increasing. What current does this induce in the circular loop?



A. a clockwise current

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C. zero current

D. not enough information given to decide

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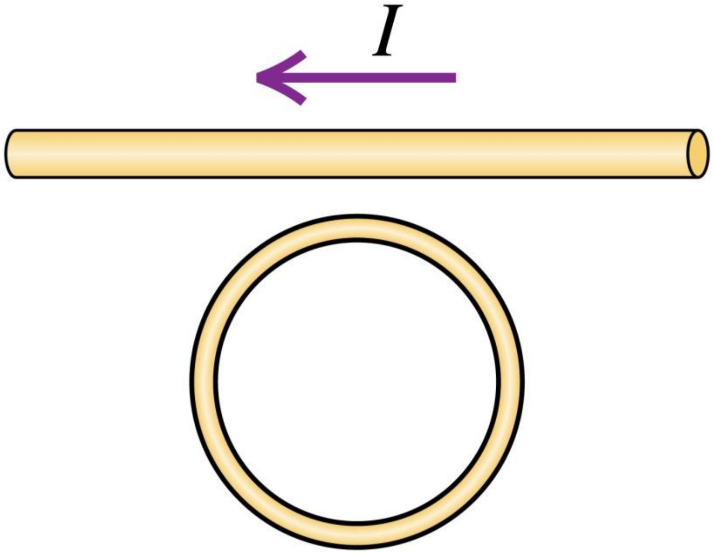

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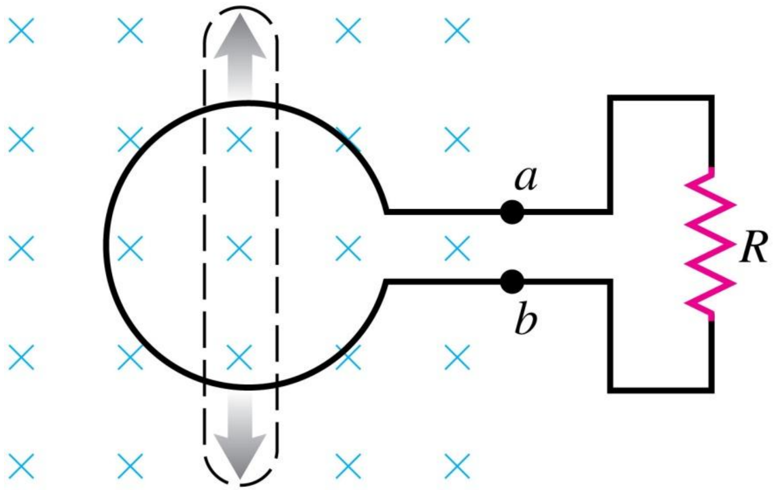
CPS 32-3

A flexible loop of wire lies in a uniform magnetic field of magnitude *B* directed into the plane of the picture. The loop is pulled as shown, reducing its area. The induced current



A. flows downward through resistor *R* and is proportional to *B.*

B. flows upward through resistor *R* and is proportional to *B.*

C. flows downward through resistor *R* and is proportional to B^2.

D. flows upward through resistor *R* and is proportional to B^2.

E. none of the above

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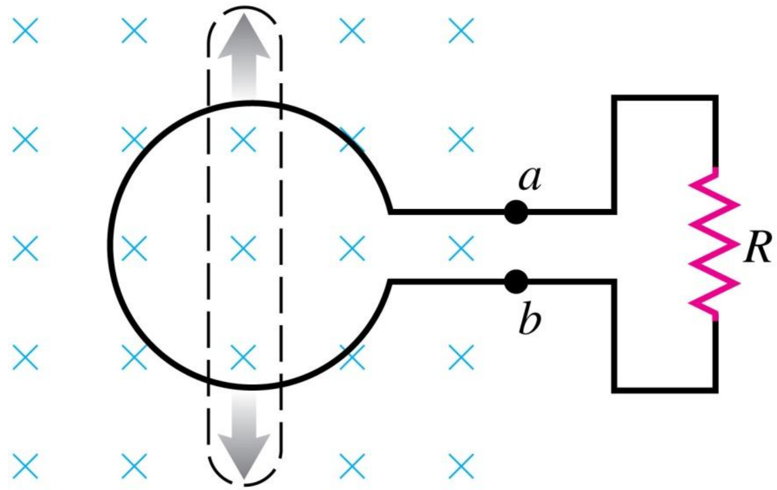

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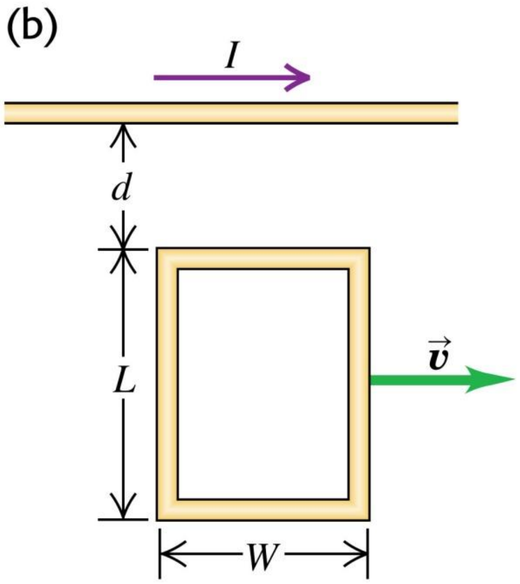
The rectangular loop of wire is being moved to the right at constant velocity. A constant current *I* flows in the long straight wire in the direction shown. The current induced in the loop is



A. clockwise and proportional to *I.*

B. counterclockwise and proportional to *I.*

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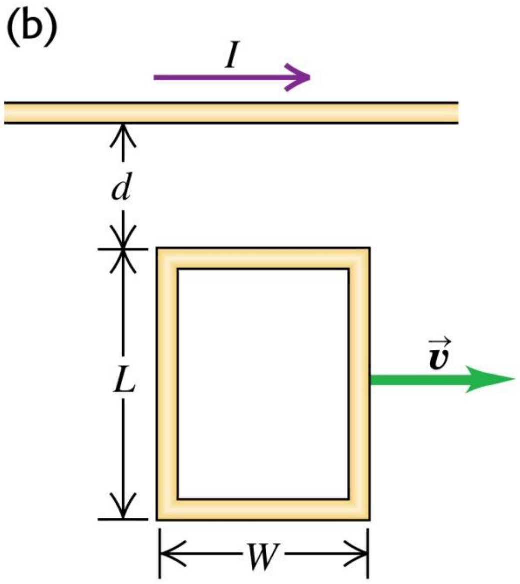

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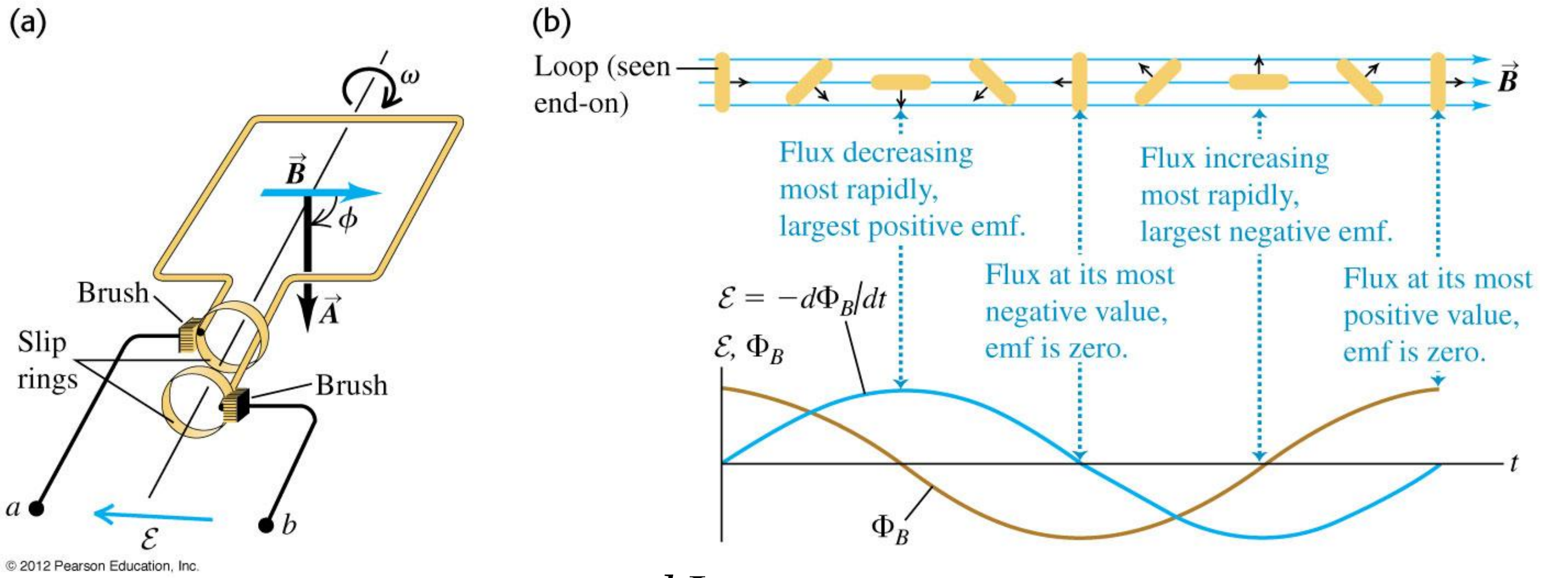
Applying Faraday's Law

- Let's make an AC Generator!



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$$= -NBA\sin\theta_{BA}\frac{d\theta_{BA}}{dt} = -NBA\sin\theta_{BA}\omega$$

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