ECE340 Spring 2011

Homework-2 Solutions

Problems: 1-4.5, 1-4.7, 1-5.4(c,d), 1-5.5(a), 1-5.6, 1-7.1, 1-7.5, 1-7.6

1-4.5

Define E_{H1} as the event that the horsepower is 0.5.
Define E_{H2} as the event that the horsepower is 1.
Define E_{H3} as the event that the horsepower is 0.1.
Define E_{V1} as the event that the motor is designed for 240 V single phase.
Define E_{V2} as the event that the motor is designed for 240 V three phase.
Define E_{V3} as the event that the motor is designed for 120 V single phase.
Then we have:
a) $P(E_{H1})=(200+500+100)/3000 = 800/3000 = 4/15= 0.2667$
b) $P(E_{V1})=(400+500+200)/3000 = 1100/3000 = 11/30= 0.3667$
c) $P(E_{H2} \cap E_{V2})=600/3000 = 1/5 = 0.2$
d) $P(E_{H3} \cap E_{V3})=900/3000 = 3/10 = 0.3$

1-4.7

a) The probability of being bad equals 4/25 = 0.16.
b) When the first transistor tested is bad, we know that there are 3 bad transistors among 24 transistors in the box. Therefore, the probability that the second transistor will be bad equals (4-1)/(25-1) = 3/24 = 1/8 = 0.125
c) When the first transistor tested is good, we know that there are 4 bad transistors among 24 transistors in the box. Therefore, the probability that the second transistor will be bad equals 4/(25-1) = 4/24 = 1/6 = 0.1667

1-5.4

c) $A\cup(\overline{A}\cap B)=(A\cap\overline{A})\cap(A\cup B)=S\cap(A\cup B)=A\cup B$
d) $(A\cap B)\cup(A\cap\overline{B})\cup(\overline{A}\cap B)=((A\cap B)\cup(A\cap\overline{B})\cup(\overline{A}\cap B))$
$=(A\cap(B\cup\overline{B}))\cup(\overline{A}\cap B)=(A\cap S)\cup(\overline{A}\cap B)$
$=A\cup(\overline{A}\cap B)=A\cup B$

1-5.5

a) Since (A-B) and (B-A) have no common elements, it follows that $(A-B)\cap(B-A) = \emptyset$.
$(A-B)\cap(B-A) =(A\cap\overline{B})\cap(B\cap\overline{A})= (A\cap\overline{B}\cap B)\cap\overline{A})$
$=(A\cap(\overline{B}\cap B))\cap\overline{A}=(A\cap \emptyset)\cap\overline{A}= \emptyset\cap\overline{A}= \emptyset$.

1-5.6

a) $A \cup B$ ={a,c,d,e,f}
b) $A \cap B$ ={c,e}
c) (A-B)={a}
d) $\overline{A} \cap B$ ={d,f}
e) $A \cap \overline{B}$={a}
f) $(B-A) \cup A$={a,c,d,e,f}

1-7.1

Define T_0 as the event of a 0 being transmitted.
Define T_1 as the event of a 1 being transmitted.
Define R_0 as the event of a 0 being received.
Define R_1 as the event of a 1 being received.
Then we have:
$P(T_0)=0.4$, $P(T_1)=0.6$, $P(R_1|T_0)=0.08$, $P(R_0|T_1)=0.05$.

a) $P\,(T_0|R_0) = \frac{P(R_0|T_0)P_r(T_0)}{P(R_0|T_0)P(T_0)+P(R_0|T_1)P(T_1)} = \frac{(1-0.08)\times 0.4}{(1-0.08)\times 0.4+0.05\times 0.6} = 0.9246$

b) $P(T_1|R_1) = \frac{Pr(R_1|T_1)P_r(T_1)}{P(R_1|T_1)P\quad(T_1)+Pr(R_1|T_0)P_r(T_0)} = \frac{(1-0.05)\times 0.6}{(1-0.05)\times 0.6+0.08\times 0.4} = 0.9468$

c) The probability that any symbol is received in error equals

$(P(R_0|T_1)\,P(T_1)) + (P(R_1|T_0)\,P(T_0))=(0.05)(0.6) + (0.08)(0.4)$

1-7.5

Define E_{AB} as the event of being able to transmit a message from point A to point B.
Define E_{MF} as the event of the middle link/path fails.
Define E_{UF} as the event of the upper path fail.
Define E_{LF} as the event of the lower path fail.
Define E_F as the event of all paths fail.
A message can be transmitted if at least one of the following occurs:
The upper path works, the middle path works, or the lower path works. This event is the complement event of both three paths fail.
Since the links are independent, we have:
$P(E_F)=P(E_{UF}).\ P(E_{LF}).\ P(E_{MF})$
Also we have
$P(E_{UF})=1-0.9 \times 0.9$
$P(E_{LF})= 1-0.9 \times 0.9$
$P(E_{MF})=0.9$
Then
$P(E_F)=(1-0.9 \times 0.9)(1-0.9 \times 0.9)(0.9)$,
and
$P(E_{AB})=1- P(E_F)=1-(1-0.9 \times 0.9)(1-0.9 \times 0.9)(0.9)=0.99639$

1-7.6

Define A as the event of being selected from A.
Define B as the event of being selected from B.
Define C as the event of being selected from C.
Define $E_{B,A}$ as the event of being bad when selected from A.
Define $E_{B,B}$ as the event of being bad when selected from B.
Define $E_{B,C}$ as the event of being bad when selected from C.
Define E_B as the event of being bad.

a) We have:

$P(E_B)= P(E_{B,A})P(A)+ P(E_{B,B})P(B)+ P(E_{B,C})P(C) =0.1\times(1/3)+0.15\times(1/3)+0.05\times(1/3)=0.1$.

b) $P(B|E_B)=\frac{P(B,E_B)}{P(E_B)} = \frac{P(E_B|B)P(B)}{P(E_B)}$

$=\frac{(0.15\times(1/3))}{0.1\times(1/3)+0.15\times(1/3)+0.05\times(1/3)} = \frac{0.05}{0.1} = 0.5.$