Chapter 8: Momentum, Impulse, and Collisions

Due: 11:00pm on Tuesday, October 23, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Center of Mass and External Forces

Learning Goal:

Understand that, for many purposes, a system can be treated as a point-like particle with its mass concentrated at the center of mass.

A complex system of objects, both point-like and extended ones, can often be treated as a *point particle*, located at the system's *center of mass*. Such an approach can greatly simplify problem solving.

Before you use the center of mass approach, you should first understand the following terms:

- System: *Any* collection of objects that are of interest to you in a particular situation. In many problems, you have a certain freedom in choosing your system. Making a wise choice for the system is often the first step in solving the problem efficiently.
- Center of mass: The point that represents the "average" position of the entire mass of a system. The postion of the center of mass $\vec{r}_{\rm cm}$ can be expressed in terms of the position vectors $\vec{r}_i$ of the particles as

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}.$$

  The *x* coordinate of the center of mass $x_{\rm cm}$ can be expressed in terms of the *x* coordinates $(r_x)_i$ of the particles as

$$x_{cm} = \frac{\sum m_i (r_x)_i}{\sum m_i}.$$

  Similarly, the *y* coordinate of the center of mass can be expressed.
- Internal force: Any force that results from an interaction between the objects inside your system. As we will show, the internal forces do not affect the motion of the system's center of mass.
- External force: Any force acting on an object inside your system that results from an interaction with an object outside your system.

Consider a system of two blocks that have masses m_1 and m_2. Assume that the blocks are point-like particles and are located along the *x* axis at the coordinates x_1 and x_2 as shown . In this problem, the blocks can only move along the *x* axis.

Part A

Find the *x* coordinate $x_{\rm cm}$ of the center of mass of the system.

Express your answer in terms of m_1, m_2, x_1, and x_2.

ANSWER:

$$x_{\rm cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Part B

If $m_2 \gg m_1$, then the center of mass is located:

ANSWER:

- ○ to the left of m_1 at a distance much greater than $x_2 - x_1$
- ○ to the left of m_1 at a distance much less than $x_2 - x_1$
- ○ to the right of m_1 at a distance much less than $x_2 - x_1$
- ○ to the right of m_2 at a distance much greater than $x_2 - x_1$
- ○ to the right of m_2 at a distance much less than $x_2 - x_1$
- ◉ to the left of m_2 at a distance much less than $x_2 - x_1$

Part C

If $m_2 = m_1$, then the center of mass is located:

ANSWER:

- ○ at m_1
- ○ at m_2
- ◉ half-way between m_1 and m_2
- ○ the answer depends on x_1 and x_2

Part D

Recall that the blocks can only move along the *x* axis. The *x* components of their velocities at a certain moment are v_{1x} and v_{2x}. Find the x component of the velocity of the center of mass $(v_{cm})_x$ at that moment. Keep in mind that, in general: $v_x = dx/dt$.

Express your answer in terms of m_1, m_2, v_{1x}, and v_{2x}.

ANSWER:

$(v_{cm})_x$ = $\frac{m_1 v_{1x} + m_2 v_{2x}}{m_1 + m_2}$

Because v_{1x} and v_{2x} are the *x* components of the velocities of m_1 and m_2 their values can be positive or negative or equal to zero.

Part E

Suppose that v_{1x} and v_{2x} have equal magnitudes. Also, $\vec{v_1}$ is directed to the right and $\vec{v_2}$ is directed to the left. The velocity of the center of mass is then:

ANSWER:

- ○ directed to the left
- ○ directed to the right
- ○ zero
- ◉ the answer depends on the ratio $\frac{m_1}{m_2}$

Part F

Assume that the *x* components of the blocks' momenta at a certain moment are p_{1x} and p_{2x}. Find the *x* component of the velocity of the center of mass $(v_{cm})_x$ at that moment.

Express your answer in terms of m_1, m_2, p_{1x}, and p_{2x}.

ANSWER:

$(v_{cm})_x$ = $\frac{p_{1x} + p_{2x}}{m_1 + m_2}$

Part G

Suppose that $\vec{v}_{\text{cm}} = 0$. Which of the following must be true?

ANSWER:

- (selected) $|p_{1x}| = |p_{2x}|$
- $|v_{1x}| = |v_{2x}|$
- $m_1 = m_2$
- none of the above

Part H

Assume that the blocks are accelerating, and the *x* components of their accelerations at a certain moment are a_{1x} and a_{2x}. Find the *x* component of the acceleration of the center of mass $(a_{\text{cm}})_x$ at that moment. Keep in mind that, in general, $a_x = dv_x/dt$.

Express your answer in terms of m_1, m_2, a_{1x}, and a_{2x}.

ANSWER:

$(a_{\text{cm}})_x = \dfrac{m_1 a_{1x} + m_2 a_{2x}}{m_1 + m_2}$

Because a_{1x} and a_{2x} are the *x* components of the velocities of m_1 and m_2 their values can be positive or negative or equal to zero.

We will now consider the effect of external and internal forces on the acceleration of the center of mass.

Part I

Consider the same system of two blocks. An *external* force $\vec{F}$ is now acting on block m_1. *No* forces are applied to block m_2 as shown . Find the *x* component of the acceleration of the center of mass $(a_{\text{cm}})_x$ of the system.

Express your answer in terms of the *x* component F_x of the force, m_1,and m_2.

Hint 1. Using Newton's laws

Find the acceleration of each block from Newton's second law and then apply the formula for $(a_{\text{cm}})_x$ found earlier.

ANSWER:

$(a_{\text{cm}})_x = \dfrac{F_x}{m_1 + m_2}$

Part J

Consider the same system of two blocks. Now, there are two forces involved. An *external* force $\vec{F_1}$ is acting on block m_1 and another *external* force $\vec{F_2}$ is acting on block m_2. Find the *x* component of the acceleration of the center of mass $(a_{\text{cm}})_x$ of the system.

Express your answer in terms of the *x* components F_{1x} and F_{2x} of the forces, m_1 and m_2.

ANSWER:

$(a_{cm})_x = \dfrac{F_{1x}+F_{2x}}{m_1+m_2}$

Note that, in both cases, the acceleration of mass can be found as

$$(a_{cm})_x = \frac{(F_{net})_x}{M_{total}}$$

where F_{net} is the net *external* force applied to the system, and M_{total} is the total mass of the system. Even though each force is only applied to *one* object, it affects the acceleration of the center of mass of the *entire system*.

This result is especially useful since it can be applied to a general case, involving *any* number of objects moving in *all* directions and being acted upon by *any* number of *external* forces.

Part K

Consider the previous situation. Under what condition would the acceleration of the center of mass be zero? Keep in mind that F_{1x} and F_{2x} represent *the components*, of the corresponding forces.

ANSWER:

- (selected) $F_{1x} = -F_{2x}$
- $F_{1x} = F_{2x}$
- $m_1 = m_2$
- $m_1 \ll m_2$

Part L

Consider the same system of two blocks. Now, there are two *internal* forces involved. An *internal* force $\vec{F}_{12}$ is applied to block m_1 by block m_2 and another *internal* force $\vec{F}_{21}$ is applied to block m_2 by block m_1 . Find the *x* component of the acceleration of the center of mass $(a_{cm})_x$ of the system.

Express your answer in terms of the *x* components F_{12x} and F_{21x} of the forces, m_1 and m_2.

ANSWER:

$(a_{cm})_x = \frac{F_{12x} + F_{21x}}{m_1 + m_2}$

Also accepted: 0, $zero$

Newton's 3rd law tells you that $|F_{12x}| = -|F_{21x}|$. From your answers above, you can conclude that $(a_{cm})_x = 0$.The *internal forces* do *not* change the velocity of the center of mass of the system. In the absence of any *external* forces, $(a_{cm})_x = 0$ and $(v_{cm})_x$ is constant.

You just demonstrated this to be the case for the two-body situation moving along the *x* axis; however, it is true in more general cases as well.

Conservation of Momentum in Inelastic Collisions

Learning Goal:

To understand the vector nature of momentum in the case in which two objects collide and stick together.

In this problem we will consider a collision of two moving objects such that after the collision, the objects stick together and travel off as a single unit. The collision is therefore completely inelastic.

You have probably learned that "momentum is conserved" in an inelastic collision. But how does this fact help you to solve collision problems? The following questions should help you to clarify the meaning and implications of the statement "momentum is conserved."

Part A

What physical quantities are conserved in this collision?

ANSWER:

- ○ the magnitude of the momentum only
- ◉ the net momentum (considered as a vector) only
- ○ the momentum of each object considered individually

Part B

Two cars of equal mass collide inelastically and stick together after the collision. Before the collision, their speeds are v_1 and v_2. What is the speed of the two-car system after the collision?

Hint 1. How to approach the problem

Think about how you would calculate the final speed of the two cars with the information provided and using the idea of conservation of momentum. Better yet, try the calculation out. What do you get?

ANSWER:

- ○ $v_1 + v_2$
- ○ $v_1 - v_2$
- ○ $v_2 - v_1$
- ○ $\sqrt{v_1 v_2}$
- ○ $\frac{v_1 + v_2}{2}$
- ○ $\sqrt{v_1^2 + v_2^2}$
- ◉ The answer depends on the directions in which the cars were moving before the collision.

Part C

Two cars collide inelastically and stick together after the collision. Before the collision, the magnitudes of their momenta are p_1 and p_2. After the collision, what is the magnitude of their combined momentum?

Hint 1. How to approach the problem

Think about how you would calculate the final momentum of the two cars using the information provided and the idea of conservation of momentum. Better yet, try the calculation out. What do you get? Keep in mind that momentum is a vector, but you are asked about the magnitude of the momentum, which is a scalar.

ANSWER:

- ○ $p_1 + p_2$
- ○ $p_1 - p_2$
- ○ $p_2 - p_1$
- ○ $\sqrt{p_1 p_2}$
- ○ $\frac{p_1 + p_2}{2}$
- ○ $\sqrt{p_1{}^2 + p_2{}^2}$
- ◉ The answer depends on the directions in which the cars were moving before the collision.

Part D

Two cars collide inelastically and stick together after the collision. Before the collision, their momenta are $\vec{p}_1$ and $\vec{p}_2$. After the collision, their combined momentum is $\vec{p}$. Of what can one be certain?

Hint 1. Momentum is a vector

Momentum is a vector quantity, and conservation of momentum holds for two-dimensional and three-dimensional collisions as well as for one-dimensional collisions.

ANSWER:

- ◉ $\vec{p} = \vec{p_1} + \vec{p_2}$
- ○ $\vec{p} = \vec{p_1} - \vec{p_2}$
- ○ $\vec{p} = \vec{p_2} - \vec{p_1}$

You can decompose the vector equation that states the conservation of momentum into individual equations for each of the orthogonal components of the vectors.

Part E

Two cars collide inelastically and stick together after the collision. Before the collision, the magnitudes of their momenta are p_1 and p_2. After the collision, the magnitude of their combined momentum is p. Of what can one be certain?

Hint 1. How to approach the problem mathematically

Momentum is a vector quantity. It is impossible to make exact predictions about the direction of motion after a collision if nothing is known about the direction of motion before the collision. However, one can put some bounds on the values of the final momentum. Start with the expression for $\vec{p}$ from Part D:

$$\vec{p} = \vec{p_1} + \vec{p_2}.$$

Therefore,

$$|p| = |\vec{p_1} + \vec{p_2}| = \sqrt{|p_1|^2 + |p_2|^2 + 2\vec{p_1} \cdot \vec{p_2}} = \sqrt{|p_1|^2 + |p_2|^2 + 2|p_1||p_2| \cos\theta},$$

where θ is the angle between $\vec{p}_1$ and $\vec{p}_2$. (To derive the above, you would have to break each vector into components.) So the value of

$|p|$ is controlled by θ.

Hint 2. How to approach the problem empirically

Consider the directions for the initial momenta that will give the largest and smallest final momentum.

ANSWER:

- ○ $p_1 + p_2 \geq p \geq \sqrt{p_1 p_2}$
- ○ $p_1 + p_2 \geq p \geq \frac{p_1 + p_2}{2}$
- ◉ $p_1 + p_2 \geq p \geq |p_1 - p_2|$
- ○ $p_1 + p_2 \geq p \geq \sqrt{{p_1}^2 + {p_2}^2}$

When the two cars collide, the magnitude of the final momentum will always be at most $p_1 + p_2$ (a value attained if the cars were moving in the same direction before the collision) and at least $|p_1 - p_2|$ (a value attained if the cars were moving in opposite directions before the collision).

Momentum and Internal Forces

Learning Goal:

To understand the concept of total momentum for a system of objects and the effect of the internal forces on the total momentum.

We begin by introducing the following terms:

System: Any collection of objects, either pointlike or extended. In many momentum-related problems, you have a certain freedom in choosing the objects to be considered as your system. Making a wise choice is often a crucial step in solving the problem.

Internal force: Any force interaction between two objects belonging to the chosen system. Let us stress that both interacting objects must belong to the system.

External force: Any force interaction between objects at least one of which does not belong to the chosen system; in other words, at least one of the objects is external to the system.

Closed system: a system that is not subject to any external forces.

Total momentum: The vector sum of the individual momenta of all objects constituting the system.

In this problem, you will analyze a system composed of two blocks, 1 and 2, of respective masses m_1 and m_2. To simplify the analysis, we will make several assumptions:

1. The blocks can move in only one dimension, namely, along the *x* axis.
2. The masses of the blocks remain constant.
3. The system is closed.

At time t, the *x* components of the velocity and the acceleration of block 1 are denoted by $v_1(t)$ and $a_1(t)$. Similarly, the *x* components of the velocity and acceleration of block 2 are denoted by $v_2(t)$ and $a_2(t)$. In this problem, you will show that the total momentum of the system is not changed by the presence of internal forces.

Part A

Find $p(t)$, the *x* component of the total momentum of the system at time t.

Express your answer in terms of m_1, m_2, $v_1(t)$, and $v_2(t)$.

ANSWER:

$p(t) = m_1 v_1(t) + m_2 v_2(t)$

Part B

Find the time derivative $dp(t)/dt$ of the *x* component of the system's total momentum.

Express your answer in terms of $a_1(t)$, $a_2(t)$, m_1, and m_2.

Hint 1. Finding the derivative of momentum for one block

Consider the momentum of block 1: $p_1(t) = m_1 v_1(t)$. Take the derivative of this expression with respect to time, noting that velocity is a function of time, and mass is a constant:

$$\frac{dp_1(t)}{dt} = \frac{d\left(m_1 v_1(t)\right)}{dt} = m_1 \frac{dv_1(t)}{dt}.$$

Hint 2. The relationship between velocity and acceleration

Recall the definition of acceleration as $a(t) = \frac{dv(t)}{dt}$.

ANSWER:

$dp(t)/dt$ = $m_1 a_1(t) + m_2 a_2(t)$

Why did we bother with all this math? The expression for the derivative of momentum that we just obtained will be useful in reaching our desired conclusion, if only for this very special case.

Part C

The quantity ma (mass times acceleration) is dimensionally equivalent to which of the following?

ANSWER:

- ○ momentum
- ○ energy
- ◉ force
- ○ acceleration
- ○ inertia

Part D

Acceleration is due to which of the following physical quantities?

ANSWER:

- ○ velocity
- ○ speed
- ○ energy
- ○ momentum
- ◉ force

Part E

Since we have assumed that the system composed of blocks 1 and 2 is closed, what could be the reason for the acceleration of block 1?

Hint 1. Force and acceleration

Since the system is closed, the only object that can affect block 1 is the other block in the system, block 2.

ANSWER:

- the large mass of block 1
- air resistance
- Earth's gravitational attraction
- (selected) a force exerted by block 2 on block 1
- a force exerted by block 1 on block 2

Part F

What could be the reason for the acceleration of block 2?

ANSWER:

- a force exerted by block 2 on block 1
- (selected) a force exerted by block 1 on block 2

Part G

Let us denote the *x* component of the force exerted by block 1 on block 2 by F_{12}, and the *x* component of the force exerted by block 2 on block 1 by F_{21}. Which of the following pairs equalities is a direct consequence of Newton's second law?

ANSWER:

- (selected) $F_{12} = m_2 a_2$ and $F_{21} = m_1 a_1$
- $F_{12} = m_1 a_1$ and $F_{21} = m_2 a_2$
- $F_{12} = m_1 a_2$ and $F_{21} = m_2 a_1$
- $F_{12} = m_2 a_1$ and $F_{21} = m_1 a_2$

Note that both F_{12} and F_{21} are internal forces.

Part H

Let us recall that we have denoted the force exerted by block 1 on block 2 by F_{12}, and the force exerted by block 2 on block 1 by F_{21}. If we suppose that m_1 is greater than m_2, which of the following statements about forces is true?

Hint 1. Which of Newton's laws is useful here?

Newton's third law!

ANSWER:

- $|F_{12}| > |F_{21}|$
- $|F_{21}| > |F_{12}|$
- (selected) Both forces have equal magnitudes.

Newton's third law states that forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are equal in magnitude and opposite in direction. Therefore, their *x* components are related by $F_{12} = -F_{21}$

Part I

Now recall the expression for the time derivative of the *x* component of the system's total momentum: $dp_x(t)/dt = F_x$. Considering the information that you now have, choose the best alternative for an equivalent expression to $dp_x(t)/dt$.

Hint 1. What is F_x?

$$F_x = F_{12} + F_{21},$$

the total (internal) force on the *system* (as a whole). Use the information from the last part to simplify the right-hand side of the above equation.

ANSWER:

- (selected) 0
- nonzero constant
- kt
- kt^2

The derivative of the total momentum is zero; hence the total momentum is a constant function of time. We have just shown that for the special case of a closed two-block system, the internal forces do not change the total momentum of the system. It can be shown that in any system, the internal forces do not change the total momentum: It is conserved. In other words, total momentum is always conserved in a closed system of objects.

± The Impulse-Momentum Theorem

Learning Goal:

To learn about the impulse-momentum theorem and its applications in some common cases.

Using the concept of momentum, Newton's second law can be rewritten as

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt}, \ (1)$$

where $\Sigma\vec{F}$ is the *net* force $\vec{F}_{\rm net}$ acting on the object, and $\frac{d\vec{p}}{dt}$ is the rate at which the object's momentum is changing.

If the object is observed during an interval of time between times t_1 and t_2, then integration of both sides of equation (1) gives

$$\int_{t_1}^{t_2} \Sigma\vec{F}dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}dt. \ (2)$$

The right side of equation (2) is simply the change in the object's momentum $\vec{p}_2 - \vec{p}_1$. The left side is called the *impulse of the net force* and is denoted by $\vec{J}$. Then equation (2) can be rewritten as

$$\vec{J} = \vec{p}_2 - \vec{p}_1.$$

This equation is known as the *impulse-momentum theorem*. It states that the change in an object's momentum is equal to the impulse of the net force acting on the object. In the case of a constant *net* force $\vec{F}_{\rm net}$ acting along the direction of motion, the impulse-momentum theorem can be written as

$$F(t_2 - t_1) = mv_2 - mv_1. \ (3)$$

Here F, v_1, and v_2 are the *components* of the corresponding vector quantities along the chosen coordinate axis. If the motion in question is two-dimensional, it is often useful to apply equation (3) to the *x* and *y* components of motion separately.

The following questions will help you learn to apply the impulse-momentum theorem to the cases of constant and varying force acting along the direction of motion. First, let us consider a particle of mass m moving along the *x* axis. The net force F is acting on the particle along the *x* axis. F is a constant force.

Part A

The particle starts from rest at $t = 0$. What is the magnitude p of the momentum of the particle at time t? Assume that $t > 0$.

Express your answer in terms of any or all of m, F, and t.

ANSWER:

$p = Ft$

Part B

The particle starts from rest at $t = 0$. What is the magnitude v of the velocity of the particle at time t? Assume that $t > 0$.

Express your answer in terms of any or all of m, F, and t.

ANSWER:

$v = \frac{Ft}{m}$

Part C

The particle has momentum of magnitude p_1 at a certain instant. What is p_2, the magnitude of its momentum Δt seconds later?

Express your answer in terms of any or all of p_1, m, F, and Δt.

ANSWER:

$p_2 = p_1 + F\Delta t$

Part D

The particle has momentum of magnitude p_1 at a certain instant. What is v_2, the magnitude of its velocity Δt seconds later?

Express your answer in terms of any or all of p_1, m, F, and Δt.

ANSWER:

$v_2 = \frac{p_1 + F\Delta t}{m}$

Let us now consider several two-dimensional situations.

A particle of mass m is moving in the positive *x* direction at speed v. After a certain constant force is applied to the particle, it moves in the positive *y* direction at speed $2v$.

Part E

Find the magnitude of the impulse J delivered to the particle.

Express your answer in terms of m and v. Use three significant figures in the numerical coefficient.

Hint 1. How to approach the problem

This is a two-dimensional situation. It is helpful to find the components J_x and J_y separately and then use the Pythagorean theorem to find J.

Hint 2. Find the change in momentum

Find Δp_x, the *magnitude* of the change in the *x* component of the momentum of the particle.

Express your answer in terms of m and v.

ANSWER:

$\Delta p_x = mv$

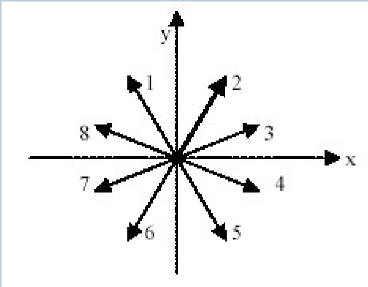
Similarly, you can find Δp_y. Once you have obtained both Δp_x and Δp_y, you can find the magnitude of the delivered impulse J.

ANSWER:

J = $2.24mv$

Part F

Which of the vectors below best represents the direction of the impulse vector $\vec{J}$?

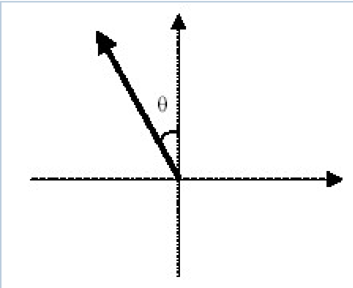


ANSWER:

- (selected) 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8

Part G

What is the angle θ between the positive *y* axis and the vector $\vec{J}$ as shown in the figure?



ANSWER:

- (selected) 26.6 degrees
- 30 degrees
- 60 degrees
- 63.4 degrees

Part H

If the magnitude of the net force acting on the particle is F, how long does it take the particle to acquire its final velocity, $2v$ in the positive *y* direction?

Express your answer in terms of m, F, and v. If you use a numerical coefficient, use three significant figures.

ANSWER:

t = $\frac{2.24mv}{F}$

So far, we have considered only the situation in which the magnitude of the net force acting on the particle was either irrelevant to the solution or was considered constant. Let us now consider an example of a *varying* force acting on a particle.

Part I

A particle of mass $m = 5.00$ kilograms is at rest at $t = 0.00$ seconds. A varying force $F(t) = 6.00t^2 - 4.00t + 3.00$ is acting on the particle between $t = 0.00$ seconds and $t = 5.00$ seconds. Find the speed v of the particle at $t = 5.00$ seconds.

Express your answer in meters per second to three significant figures.

Hint 1. Use the impulse-momentum theorem

In this case, $v_1 = 0$ and $v_2 = v$. Therefore,

$$\int_{0.00}^{5.00} F\,dt = \Delta mv.$$

Hint 2. What is the correct antiderivative?

Which of the following is an antiderivative $\int (6.00t^2 - 4.00t + 3.00)dt$?

ANSWER:

- ○ $6.00t^3 - 4.00t^2 + 3.00t$
- ○ $6.00t - 4.00$
- ◉ $2.00t^3 - 2.00t^2 + 3.00t$
- ○ $12.00t - 4.00$

ANSWER:

v = 43 m/s

Also accepted: 43.0

Exercise 8.9

A 0.160-kg hockey puck is moving on an icy, frictionless, horizontal surface. At $t = 0$ the puck is moving to the right at 2.92$\mathrm{m/s}$.

Part A

Calculate the magnitude of the velocity of the puck after a force of 24.5N directed to the right has been applied for 6.0×10^{-2}s .

Express your answer using two significant figures.

ANSWER:

v = $v + \frac{F_1 \Delta t}{0.160}$ = 12 $\mathrm{m/s}$

Part B

What is the direction of the velocity of the puck after a force of 24.5N directed to the right has been applied for 6.0×10^{-2}s .

ANSWER:

- ◉ to the right
- ○ to the left

Part C

If instead, a force of 11.3N directed to the left is applied from $t = 0$ to $t = 6.0\times10^{-2}\mathrm{s}$, what is the magnitude of the final velocity of the puck?

Express your answer using two significant figures.

ANSWER:

$v = \frac{F_2 \Delta t}{0.160} - v$ = 1.3 $\mathrm{m/s}$

Part D

What is the direction of the final velocity of the puck in this case?

ANSWER:

- ◯ to the right
- ◉ to the left

Exercise 8.12

A bat strikes a 0.145-kg baseball. Just before impact, the ball is traveling horizontally to the right at 40.0$\mathrm{m/s}$, and it leaves the bat traveling to the left at an angle of 30° above horizontal with a speed of 55.0$\mathrm{m/s}$. The ball and bat are in contact for 1.85ms .

Part A

Find the horizontal component of the average force on the ball. Take the *x*-direction to be positive to the right

Express your answer using two significant figures.

ANSWER:

$F_x = \frac{-0.145\,(v1 + v2\cos(\theta))}{t}$ = -6900 N

Part B

Find the vertical component of the average force on the ball.

Express your answer using two significant figures.

ANSWER:

$F_y = \frac{0.145\,(v2\sin(\theta))}{t}$ = 2200 N

Exercise 8.26

An atomic nucleus suddenly bursts apart (fissions) into two pieces. Piece A, of mass m_A, travels off to the left with speed v_A. Piece B, of mass m_B, travels off to the right with speed v_B.

Part A

Use conservation of momentum to solve for v_B in terms of m_A , m_B, and v_A.

ANSWER:

$v_B = \frac{m_A}{m_B} v_A$

Part B

Use the results of part A to show that $K_A/K_B = m_B/m_A$, where K_A and K_B are the kinetic energies of the two pieces.

ANSWER:

3723 Character(s) remaining

K_A/K_B=1/2*m_A*v_A^2/(1/2*m_B*v_B^2)=m_A*v_A^2/(m_B*(m_A*v_A/m_B)^2)=m_B/m_A

Exercise 8.28

You are standing on a large sheet of frictionless ice and are holding a large rock. In order to get off the ice, you throw the rock so it has velocity relative to the earth of 12.3$\mathrm{m/s}$ at an angle of 34.0° above the horizontal.

Part A

If your mass is 73.0kg and the rock's mass is 15.8kg , what is your speed after you throw the rock?

ANSWER:

$v = \frac{m_2 v\cos(\theta)}{m_1}$ = 2.21 $\mathrm{m/s}$

Exercise 8.32

Two skaters collide and grab on to each other on frictionless ice. One of them, of mass 67.0kg , is moving to the right at 2.00$\mathrm{m/s}$, while the other, of mass 69.0kg , is moving to the left at 2.50$\mathrm{m/s}$.

Part A

What are the magnitude of the velocity of these skaters just after they collide?

ANSWER:

$v = \frac{m2v2 - m1v1}{m1 + m2}$ = 0.283 $\mathrm{m/s}$

Part B

What are the direction of the velocity of these skaters just after they collide?

ANSWER:

- (selected) to the left
- to the right

Exercise 8.50

You are at the controls of a particle accelerator, sending a beam of 4.20×10^7 $\mathrm{m/s}$ protons (mass m) at a gas target of an unknown element. Your detector tells you that some protons bounce straight back after a collision with one of the nuclei of the unknown element. All such protons rebound with a speed of 3.90×10^7 $\mathrm{m/s}$. Assume that the initial speed of the target nucleus is negligible and the collision is elastic.

Part A

Find the mass of one nucleus of the unknown element. Express your answer in terms of the proton mass m .

ANSWER:

$\frac{v_1 + v_2}{v_1 - v_2}$ = 27.0 m

Part B

What is the speed of the unknown nucleus immediately after such a collision?

ANSWER:

$\frac{v_1 \cdot 2}{1 + \frac{v_1 + v_2}{v_1 - v_2}}$ = 3.00×10^6 $\mathrm{m/s}$

Exercise 8.52

Part A

Find the position of the center of mass of the system of the sun and Jupiter. (Since Jupiter is more massive than the rest of the planets combined, this is essentially the position of the center of mass of the solar system.)

ANSWER:

x = 7.42×10^8 m from center of thr sun

Part B

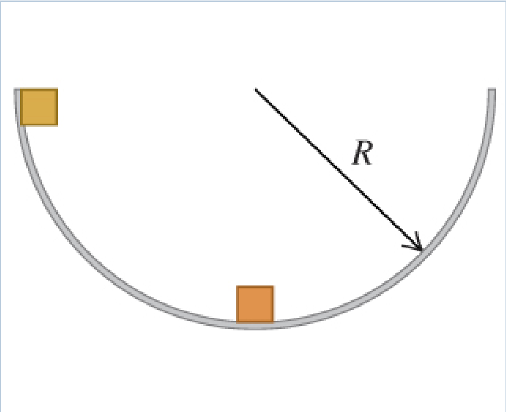
Does the center of mass lie inside or outside the sun? The sun's radius is 6.96×10^8 m.

ANSWER:

- ○ inside
- ◉ outside

Problem 8.86

Two identical masses are released from rest in a smooth hemispherical bowl of radius R, from the positions shown in the figure . You can ignore friction between the masses and the surface of the bowl.



Part A

If they stick together when they collide, how high above the bottom of the bowl will the masses go after colliding?

ANSWER:

y = 0.25 R

8.9. IDENTIFY: Use Eq. 8.9. We know the initial momentum and the impulse so can solve for the final momentum and then the final velocity.

SET UP: Take the *x*-axis to be toward the right, so $v_{1x} = +3.00$ m/s. Use Eq. 8.5 to calculate the impulse, since the force is constant.

EXECUTE: **(a)** $J_x = p_{2x} - p_{1x}$

$$J_x = F_x(t_2 - t_1) = (+25.0\text{ N})(0.050\text{ s}) = +1.25\text{ kg}\cdot\text{m/s}$$

Thus $p_{2x} = J_x + p_{1x} = +1.25\text{ kg}\cdot\text{m/s} + (0.160\text{ kg})(+3.00\text{ m/s}) = +1.73\text{ kg}\cdot\text{m/s}$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{1.73\text{ kg}\cdot\text{m/s}}{0.160\text{ kg}} = +10.8\text{ m/s (to the right)}$$

(b) $J_x = F_x(t_2 - t_1) = (-12.0\text{ N})(0.050\text{ s}) = -0.600\text{ kg}\cdot\text{m/s}$ (negative since force is to left)

$$p_{2x} = J_x + p_{1x} = -0.600\text{ kg}\cdot\text{m/s} + (0.160\text{ kg})(+3.00\text{ m/s}) = -0.120\text{ kg}\cdot\text{m/s}$$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{-0.120\text{ kg}\cdot\text{m/s}}{0.160\text{ kg}} = -0.75\text{ m/s (to the left)}$$

EVALUATE: In part (a) the impulse and initial momentum are in the same direction and v_x increases. In part (b) the impulse and initial momentum are in opposite directions and the velocity decreases.

8.12. IDENTIFY: Apply Eq. 8.9 to relate the change in momentum to the components of the average force on it.

SET UP: Let $+x$ be to the right and $+y$ be upward.

EXECUTE: $J_x = \Delta p_x = mv_{2x} - mv_{1x} = (0.145\text{ kg})(-[65.0\text{ m/s}]\cos 30° - 50.0\text{ m/s}) = -15.4\text{ kg}\cdot\text{m/s}.$

$$J_y = \Delta p_y = mv_{2y} - mv_{1y} = (0.145\text{ kg})([65.0\text{ m/s}]\sin 30° - 0) = 4.71\text{ kg}\cdot\text{m/s}$$

The horizontal component is $15.4\text{ kg}\cdot\text{m/s}$, to the left and the vertical component is $4.71\text{ kg}\cdot\text{m/s}$, upward.

$$F_{\text{av-}x} = \frac{J_x}{\Delta t} = \frac{-15.4\text{ kg}\cdot\text{m/s}}{1.75\times10^{-3}\text{ s}} = -8800\text{ N}. \quad F_{\text{av-}y} = \frac{J_y}{\Delta t} = \frac{4.71\text{ kg}\cdot\text{m/s}}{1.75\times10^{-3}\text{ s}} = 2690\text{ N}.$$

The horizontal component is 8800 N, to the left, and the vertical component is 2690 N, upward.

EVALUATE: The ball gains momentum to the left and upward and the force components are in these directions.

8.26. IDENTIFY: Assume the nucleus is initially at rest. $K = \frac{1}{2}mv^2$.

SET UP: Let $+x$ be to the right. $v_{A2x} = -v_A$ and $v_{B2x} = +v_B$.

EXECUTE: **(a)** $P_{2x} = P_{1x} = 0$ gives $m_A v_{A2x} + m_B v_{B2x} = 0$. $v_B = \left(\frac{m_A}{m_B}\right)v_A$.

(b) $\dfrac{K_A}{K_B} = \dfrac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \dfrac{m_A v_A^2}{m_B(m_A v_A/m_B)^2} = \dfrac{m_B}{m_A}.$

EVALUATE: The lighter fragment has the greater kinetic energy.

8.28. **IDENTIFY** and **SET UP:** Let the $+x$-direction be horizontal, along the direction the rock is thrown. There is no net horizontal force, so P_x is constant. Let object *A* be you and object *B* be the rock.

EXECUTE: $0 = -m_A v_A + m_B v_B \cos 35.0°$

$$v_A = \frac{m_B v_B \cos 35.0°}{m_A} = 2.11 \text{ m/s}$$

EVALUATE: P_y is not conserved because there is a net external force in the vertical direction; as you throw the rock the normal force exerted on you by the ice is larger than the total weight of the system.

8.32. **IDENTIFY:** There is no net external force on the system of the two skaters and the momentum of the system is conserved.

SET UP: Let object *A* be the skater with mass 70.0 kg and object *B* be the skater with mass 65.0 kg. Let $+x$ be to the right, so $v_{A1x} = +2.00$ m/s and $v_{B1x} = -2.50$ m/s. After the collision the two objects are combined and move with velocity $\vec{v}_2$. Solve for v_{2x}.

EXECUTE: $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B) v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(70.0 \text{ kg})(2.00 \text{ m/s}) + (65.0 \text{ kg})(-2.50 \text{ m/s})}{70.0 \text{ kg} + 65.0 \text{ kg}} = -0.167 \text{ m/s}.$$

The two skaters move to the left at 0.167 m/s.

EVALUATE: There is a large decrease in kinetic energy.

8.50.IDENTIFY: Elastic collision. Solve for mass and speed of target nucleus.

SET UP: **(a)** Let *A* be the proton and *B* be the target nucleus. The collision is elastic, all velocities lie along a line, and *B* is at rest before the collision. Hence the results of Eqs. 8.24 and 8.25 apply.

EXECUTE: Eq. 8.24: $m_B(v_x + v_{Ax}) = m_A(v_x - v_{Ax})$, where v_x is the velocity component of *A* before the collision and v_{Ax} is the velocity component of *A* after the collision. Here,

$v_x = 1.50 \times 10^7$ m/s (take direction of incident beam to be positive) and $v_{Ax} = -1.20 \times 10^7$ m/s

(negative since traveling in direction opposite to incident beam).

$$m_B = m_A\left(\frac{v_x - v_{Ax}}{v_x + v_{Ax}}\right) = m\left(\frac{1.50 \times 10^7 \text{ m/s} + 1.20 \times 10^7 \text{ m/s}}{1.50 \times 10^7 \text{ m/s} - 1.20 \times 10^7 \text{ m/s}}\right) = m\left(\frac{2.70}{0.30}\right) = 9.00m.$$

(b) Eq. 8.25: $v_{Bx} = \left(\dfrac{2m_A}{m_A + m_B}\right)v = \left(\dfrac{2m}{m + 9.00m}\right)(1.50 \times 10^7 \text{ m/s}) = 3.00 \times 10^6 \text{ m/s}.$

EVALUATE: Can use our calculated v_{Bx} and m_B to show that P_x is constant and that $K_1 = K_2$.

8.52.IDENTIFY: Calculate x_{cm}.

SET UP: Apply Eq. 8.28 with the sun as mass 1 and Jupiter as mass 2. Take the origin at the sun and let Jupiter lie on the positive *x*-axis.

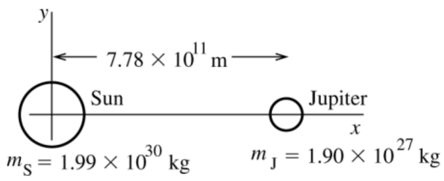


Figure 8.52

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

EXECUTE: $x_1 = 0$ and $x_2 = 7.78 \times 10^{11}$ m

$$x_{cm} = \frac{(1.90 \times 10^{27}\ \text{kg})(7.78 \times 10^{11}\ \text{m})}{1.99 \times 10^{30}\ \text{kg} + 1.90 \times 10^{27}\ \text{kg}} = 7.42 \times 10^{8}\ \text{m}$$

The center of mass is 7.42×10^{8} m from the center of the sun and is on the line connecting the centers of the sun and Jupiter. The sun's radius is 6.96×10^{8} m so the center of mass lies just outside the sun.

EVALUATE: The mass of the sun is much greater than the mass of Jupiter so the center of mass is much closer to the sun. For each object we have considered all the mass as being at the center of mass (geometrical center) of the object.

8.86. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision and apply conservation of momentum to the collision.

SET UP: Let *v* be the speed of the mass released at the rim just before it strikes the second mass. Let each object have mass *m*.

EXECUTE: Conservation of energy says $\frac{1}{2}mv^2 = mgR$; $v = \sqrt{2gR}$.

SET UP: This is speed v_1 for the collision. Let v_2 be the speed of the combined object just after the collision.

EXECUTE: Conservation of momentum applied to the collision gives $mv_1 = 2mv_2$ so

$v_2 = v_1/2 = \sqrt{gR/2}$.

SET UP: Apply conservation of energy to the motion of the combined object after the collision. Let y_3 be the final height above the bottom of the bowl.

EXECUTE: $\frac{1}{2}(2m)v_2^2 = (2m)gy_3$.

$$y_3 = \frac{v_2^2}{2g} = \frac{1}{2g}\left(\frac{gR}{2}\right) = R/4.$$

EVALUATE: Mechanical energy is lost in the collision, so the final gravitational potential energy is less than the initial gravitational potential energy.