Midterm Exam I, Fall 2008

Signals and Systems
University of New Mexico
Instructor: Balu Santhanam
Date Assigned: 10/09/2008
Duration: 2:00 - 3:15 PM

Instructions

1. Write clearly and legibly
2. Provide steps to obtain partial credit
3. It is assumed that you are aware of the UNM academic honesty policy. Needless to say copying will be dealt with seriously.

Problem # 1.0

Consider the following three systems:
(a) A full wave rectifier whose input-output map is given by

$$y(t) = L_1(x(t)) = |x(t)|,$$

(b) A double sideband amplitude modulation system whose input-output characteristics are given by:

$$y(t) = L_2(x(t)) = x(t)\cos(\omega_c t),$$

where ω_c is the carrier frequency,
(c) A system whose output is the average of the input over a period of time T given by:

$$y(t) = L_3(x(t)) = \frac{1}{T}\int_{t-T}^{t} x(\tau)d\tau.$$

For each of these maps, determine if the underlying system is: (a) linear, (b) time-invariant, (c) BIBO stable. Justify your answer properly.

Problem # 2.0

1. A rectangular pulse of unit amplitude and a duration of 3 seconds is input to a LTI system with an impulse response given by:

$$h(t) = \exp(-2t)\, u(t).$$

   Calculate the output of the system $y(t)$.

2. Calculate the convolution of the sequence $x_1[n] = u[n]$ with the sequence $x_2[n] = (0.5)^n u[n]$ via the reflect-slide method.

Problem # 3.0

Suppose an input voltage of $x(t)$ is applied on a series R-L-C circuit, and the output is tapped across the resistor.:

1. What is the differential equation satisfied by the input $x(t)$ and the output $y(t)$ of this system?
2. What is the frequency response $H(j\omega)$ of this system, i.e., the complex-gain experienced when a complex exponential is the input to this system.
3. What is the output of the system $y(t)$ when the input is sinusoidal, i.e., $x(t) = \cos(\omega_o t)$?