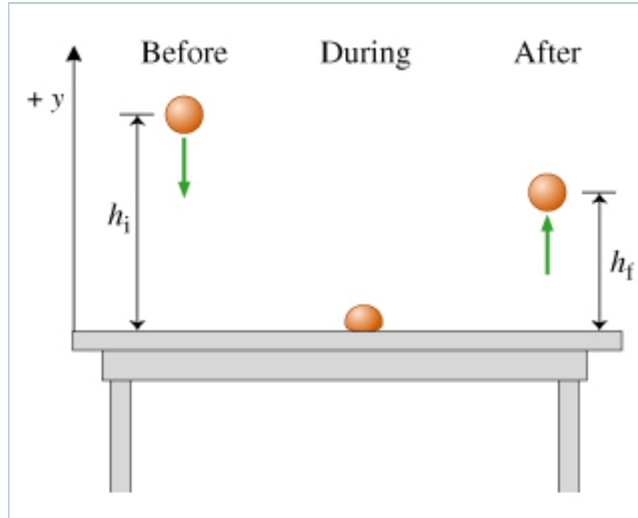
#22 Momentum and Impulse Post-Class

Due: 11:00am on Monday, October 15, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

± A Superball Collides Inelastically with a Table

As shown in the figure , a superball with mass m equal to 50 grams is dropped from a height of $h_i = 1.5\ \mathrm{m}$. It collides with a table, then bounces up to a height of $h_f = 1.0\ \mathrm{m}$. The duration of the collision (the time during which the superball is in contact with the table) is $t_c = 15\ \mathrm{ms}$. In this problem, take the positive *y* direction to be upward, and use $g = 9.8\ \mathrm{m/s^2}$ for the magnitude of the acceleration due to gravity. Neglect air resistance.



Part A

Find the *y* component of the momentum, $p_{\mathrm{before},y}$, of the ball immediately before the collision.

Express your answer numerically, to two significant figures.

Hint 1. How to approach the problem

To find the superball's *speed* immediately before the collision, use the fact that energy is conserved as the ball descends. Apply conservation of energy by setting the initial (potential) energy of the ball equal to its final (kinetic) energy at the moment it hits the table. Determine the velocity of the ball when it hits the table and use that velocity to compute the ball's momentum.

Hint 2. Find the speed of the ball

Find the speed v of the ball just before it hits the table.

Express your answer numerically in meters per second.

Hint 1. The speed of the ball

Using conservation of energy, you can show that the speed of the ball just before it hits the the table is $\sqrt{2gh_i}$, where h_i is the height it falls through, and g is the magnitude of the acceleration due to gravity.

ANSWER:

v = 5.42 $\mathrm{m/s}$

ANSWER:

$p_{\mathrm{before},y}$ = -0.27 $\mathrm{kg \cdot m/s}$

Correct

Part B

Find the *y* component of the momentum of the ball immediately after the collision, that is, just as it is leaving the table.

Express your answer numerically, to two significant figures.

Hint 1. How to approach the problem

Recall that the ball reaches a height of only $h_f = 1.0 \mathrm{m}$ on the rebound. To find the ball's velocity immediately after the collision (that is, just as it is leaving the table), use the fact that energy is conserved as the ball ascends. (Note: Energy is *not* conserved during the collision!) Apply conservation of energy by setting the initial (kinetic) energy of the ball when it leaves the table equal to its final (potential) energy when it reaches the maximum height of its bounce. Determine the velocity of the ball when it leaves the table and use that to compute the ball's momentum.

Hint 2. Find the speed of the ball

Find the speed v of the ball just as it is leaving the table.

Express your answer numerically in meters per second.

Hint 1. The speed of the ball

Using conservation of energy, you can show that the speed of the ball just before it leaves the table is $\sqrt{2gh_f}$, where h_f is the maximum height it rises to, and g is the magnitude of the acceleration due to gravity.

ANSWER:

v = 4.43 $\mathrm{m/s}$

ANSWER:

$p_{\mathrm{after},y}$ = 0.22 $\mathrm{kg \cdot m/s}$

Correct

Part C

Find J_y, the *y* component of the impulse imparted to the ball during the collision.

Express your answer numerically, to two significant figures.

Hint 1. How to approach the problem

Impulse can be found either by calculating the change in momentum and using the impulse-momentum theorem (be careful; momentum is a vector!) or by using the definition of impulse as the force multiplied by the duration over which it is applied.

ANSWER:

J_y = 0.49 $\mathrm{kg \cdot m/s}$

All attempts used; correct answer displayed

Part D

Find the *y* component of the time-averaged force $F_{\mathrm{avg},y}$, in newtons, that the table exerts on the ball.

Express your answer numerically, to two significant figures.

Hint 1. How to approach the problem

By finding the velocity of the superball immediately before and immediately after the collision, you can find the ball's average acceleration while it is in contact with the table. Recall that the duration of the collision is given in the problem introduction. Also, be careful with the signs of the velocities.
Alternatively, you can use the equation

$$\vec{F}_{\rm avg} = \frac{\Delta \vec{p}}{\Delta t}.$$

ANSWER:

F_{avg_y} = 33 $\rm N$

Correct

Part E

Find $K_{\rm after} - K_{\rm before}$, the change in the kinetic energy of the ball during the collision, in joules.

Express your answer numerically, to two significant figures.

Hint 1. Find the kinetic energy before the collision

What is the ball's kinetic energy, in joules, immediately before the collision?

Express your answer numerically, to two decimal places.

ANSWER:

K_{before} = 0.74 J

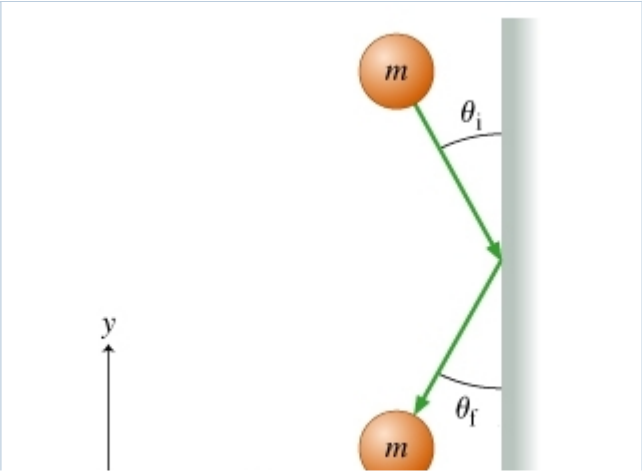
ANSWER:

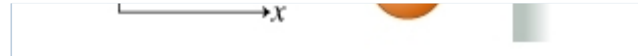
$K_{\mathrm{after}} - K_{\mathrm{before}}$ = -0.25 J

Correct

A Ball Hits a Wall Elastically

A ball of mass m moving with velocity $\vec{v}_{\mathrm{i}}$ strikes a vertical wall. The angle between the ball's initial velocity vector and the wall is θ_{i} as shown on the diagram, which depicts the situation as seen from above. The duration of the collision between the ball and the wall is Δt, and this collision is completely elastic. Friction is negligible, so the ball does not start spinning. In this idealized collision, the force exerted on the ball by the wall is parallel to the *x* axis.





Part A

What is the final angle θ_f that the ball's velocity vector makes with the negative *y* axis?

Express your answer in terms of quantities given in the problem introduction.

Hint 1. How to approach the problem

Relate the vector components of the ball's initial and final velocities. This will allow you to determine θ_f in terms of θ_i.

Hint 2. Find the *y* component of the ball's final velocity

What is v_{fy}, the y component of the final velocity of the ball?

Express your answer in terms of quantities given in the problem introduction and/or v_{ix} and v_{iy}, the x and y components of the ball's initial velocity.

Hint 1. How to approach this part

There is no force on the ball in the y direction. From the impulse-momentum theorem, this means that the change in the y component of the ball's momentum must be zero.

ANSWER:

v_{fy} = $-v_i\cos(\theta_i)$

Hint 3. Find the x component of the ball's final velocity

What is v_{fx}, the x component of the ball' final velocity?

Express your answer in terms of quantities given in the problem introduction and/or v_{ix} and v_{iy}, the x and y components of the ball's initial velocity.

Hint 1. How to approach this problem

Since energy is conserved in this collision, the final *speed* of the ball must be equal to its initial *speed*.

ANSWER:

v_{fx} = $-v_i \sin(\theta_i)$

Hint 4. Putting it together

Once you find the vector components of the final velocity in terms of the initial velocity, use the geometry of similar triangles to determine θ_f in terms of θ_i.

ANSWER:

θ_f = θ_i

Correct

Part B

What is the magnitude F of the average force exerted on the ball by the wall?

Express your answer in terms of variables given in the problem introduction and/or v_{ix}.

Hint 1. What physical principle to use

Use the impulse-momentum theorem, $\vec{J} = \vec{p}_{\rm f} - \vec{p}_{\rm i}$, along with the definition of impulse, $\vec{J} = \sum \vec{F} \Delta t$. In this case, only one force is acting, so $\left|\vec{J}\right| = F\Delta t$. Putting everything together, $F = \frac{\vec{p}_{\rm f} - \vec{p}_{\rm i}}{\Delta t}$.

Hint 2. Change in momentum of the ball

The fact that $\theta_{\rm f} = \theta_{\rm i}$ implies that the y component of the ball's momentum does not change during the collision. What is Δp_x, the magnitude of the change in the ball's x momentum?

Express your answer in terms of quantities given in the problem introduction and/or $v_{\rm ix}$.

ANSWER:

Δp_x = $2mv_i \sin(\theta_i)$

ANSWER:

F = $2mv_i \frac{\sin(\theta_f)}{\Delta t}$

Correct

± The Impulse-Momentum Theorem

Learning Goal:

To learn about the impulse-momentum theorem and its applications in some common cases.

Using the concept of momentum, Newton's second law can be rewritten as

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt}, \text{ (1)}$$

where $\Sigma \vec{F}$ is the *net* force $\vec{F}_{\text{net}}$ acting on the object, and $\frac{d\vec{p}}{dt}$ is the rate at which the object's momentum is changing.

If the object is observed during an interval of time between times t_1 and t_2, then integration of both sides of equation (1) gives

$$\int_{t_1}^{t_2} \Sigma \vec{F} dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt} dt. \text{ (2)}$$

The right side of equation (2) is simply the change in the object's momentum $\vec{p_2} - \vec{p_1}$. The left side is called the *impulse of the net force* and is denoted by $\vec{J}$. Then equation (2) can be rewritten as

$$\vec{J} = \vec{p_2} - \vec{p_1}.$$

This equation is known as the *impulse-momentum theorem*. It states that the change in an object's momentum is equal to the impulse of the net force acting on the object. In the case of a constant *net* force $\vec{F}_{\text{net}}$ acting along the direction of motion, the impulse-momentum theorem can be written as

$$F(t_2 - t_1) = mv_2 - mv_1. \text{ (3)}$$

Here F, v_1, and v_2 are the *components* of the corresponding vector quantities along the chosen coordinate axis. If the motion in question is two-dimensional, it is often useful to apply equation (3) to the *x* and *y* components of motion separately.

The following questions will help you learn to apply the impulse-momentum theorem to the cases of constant and varying force acting along the direction of motion. First, let us consider a particle of mass m moving along the *x* axis. The net force F is acting on the particle along the *x* axis. F is a constant force.

Part A

The particle starts from rest at $t = 0$. What is the magnitude p of the momentum of the particle at time t? Assume that $t > 0$.

Express your answer in terms of any or all of m, F, and t.

ANSWER:

p = Ft

Correct

Part B

The particle starts from rest at $t = 0$. What is the magnitude v of the velocity of the particle at time t? Assume that $t > 0$.

Express your answer in terms of any or all of m, F, and t.

ANSWER:

v = $\frac{Ft}{m}$

Correct

Part C

The particle has momentum of magnitude p_1 at a certain instant. What is p_2, the magnitude of its momentum Δt seconds later?

Express your answer in terms of any or all of p_1, m, F, and Δt.

ANSWER:

p_2 = $p_1 + F\Delta t$

Correct

Part D

The particle has momentum of magnitude p_1 at a certain instant. What is v_2, the magnitude of its velocity Δt seconds later?

Express your answer in terms of any or all of p_1, m, F, and Δt.

ANSWER:

$v_2 = \dfrac{p_1 + F\Delta t}{m}$

Correct

Let us now consider several two-dimensional situations.

A particle of mass m is moving in the positive *x* direction at speed v. After a certain constant force is applied to the particle, it moves in the positive *y* direction at speed $2v$.

Part E

Find the magnitude of the impulse J delivered to the particle.

Express your answer in terms of m and v. Use three significant figures in the numerical coefficient.

Hint 1. How to approach the problem

This is a two-dimensional situation. It is helpful to find the components J_x and J_y separately and then use the Pythagorean theorem to find J.

Hint 2. Find the change in momentum

Find Δp_x, the *magnitude* of the change in the *x* component of the momentum of the particle.

Express your answer in terms of m and v.

ANSWER:

$\Delta p_x =$ mv

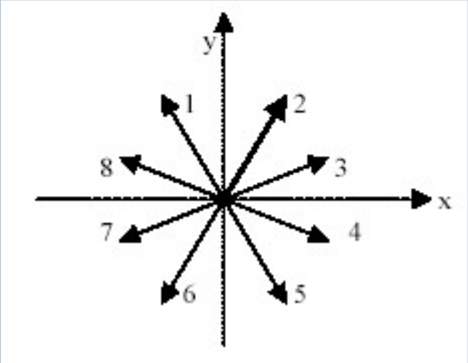
ANSWER:

$J =$ $2.24mv$

Correct

Part F

Which of the vectors below best represents the direction of the impulse vector $\vec{J}$?



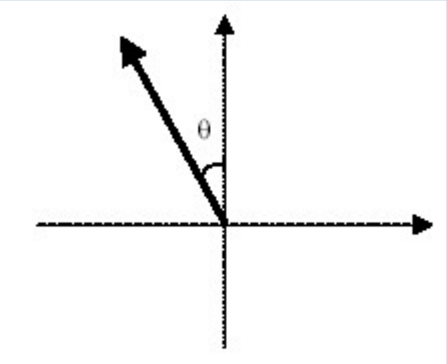
ANSWER:

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8

Correct

Part G

What is the angle θ between the positive *y* axis and the vector $\vec{J}$ as shown in the figure?



ANSWER:

- 26.6 degrees
- 30 degrees
- 60 degrees
- 63.4 degrees

Correct

Part H

If the magnitude of the net force acting on the particle is F, how long does it take the particle to acquire its final velocity, $2v$ in the positive *y* direction?

Express your answer in terms of m, F, and v. If you use a numerical coefficient, use three significant figures.

ANSWER:

$t = \frac{2.24mv}{F}$

Correct

So far, we have considered only the situation in which the magnitude of the net force acting on the particle was either irrelevant to the solution or was considered constant. Let us now consider an example of a *varying* force acting on a particle.

Part I

A particle of mass $m = 5.00$ kilograms is at rest at $t = 0.00$ seconds. A varying force $F(t) = 6.00t^2 - 4.00t + 3.00$ is acting on the particle between $t = 0.00$ seconds and $t = 5.00$ seconds. Find the speed v of the particle at $t = 5.00$ seconds.

Express your answer in meters per second to three significant figures.

Hint 1. Use the impulse-momentum theorem

In this case, $v_1 = 0$ and $v_2 = v$. Therefore,

$$\int_{0.00}^{5.00} F\,dt = \Delta mv.$$

Hint 2. What is the correct antiderivative?

Which of the following is an antiderivative $\int (6.00t^2 - 4.00t + 3.00)dt$?

ANSWER:

- ○ $6.00t^3 - 4.00t^2 + 3.00t$
- ○ $6.00t - 4.00$
- ◉ $2.00t^3 - 2.00t^2 + 3.00t$
- ○ $12.00t - 4.00$

ANSWER:

v = 43.0 m/s

Correct

Score Summary:

Your score on this assignment is 96.2%.
You received 28.87 out of a possible total of 30 points.