#27 Torque Pre-class

Due: 11:00am on Friday, October 26, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Calculating Torques Using Two Standard Methods

Learning Goal:

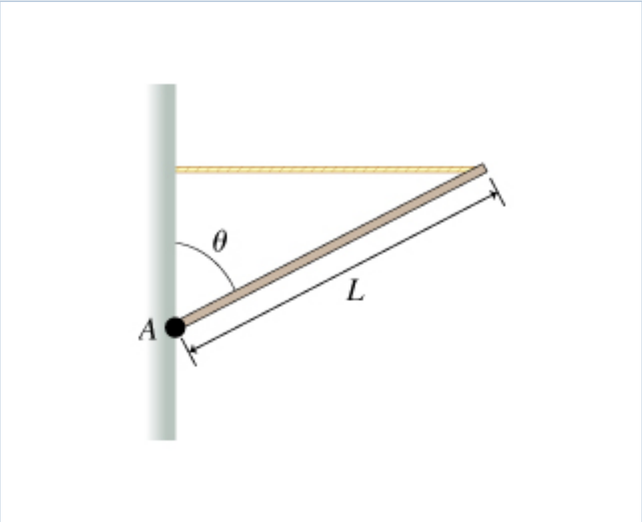
To understand the two most common procedures for finding torques when the forces and displacements are all in one plane: the moment arm method and the tangential force method.

The purpose of this problem is to give you further practice finding torques in two-dimensional situations. In this case it is overkill to use the full cross product definition of the torque because the only nonzero component of the torque is the component perpendicular to the plane containing the problem.

There are two common methods for finding torque in a two-dimensional problem: the *tangential force method* and the *moment arm method*. Both of these methods will be illustrated in this problem.

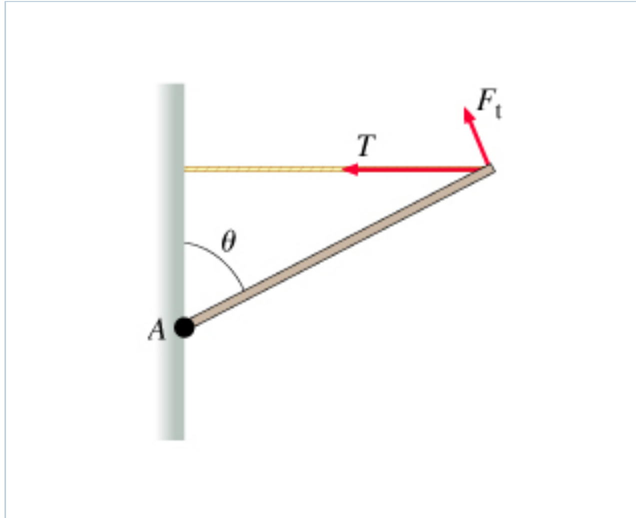
Throughout the problem, torques that would cause counterclockwise rotation are considered to be positive.

Consider a uniform pole of length L, attached at its base (via a pivot) to a wall. The other end of the pole is attached to a cable, so that the pole makes an angle θ with respect to the wall, and the cable is horizontal. The tension in the cable is T. The pole is attached to the wall at point A.



Tangential force method

The tangential force method involves finding the component of the applied force that is perpendicular to the displacement from the pivot point to where the force is applied. This perpendicular component of the force is called the *tangential force*.



Part A

What is F_t, the magnitude of the tangential force that acts on the pole due to the tension in the rope?

Express your answer in terms of T and θ.

ANSWER:

F_t = $T\cos\theta$

Correct

When using the tangential force method, you calculate the torque using the equation

$$\tau = F_t d,$$

where d is the distance from the pivot to the point where the force is applied. The sign of the torque can be determined by checking which direction the tangential force would tend to cause the pole to rotate (where counterclockwise rotation implies positive torque).

Part B

What is the magnitude of the torque τ on the pole, about point A, due to the tension in the rope?

Express your answer in terms of T, L, and θ.

ANSWER:

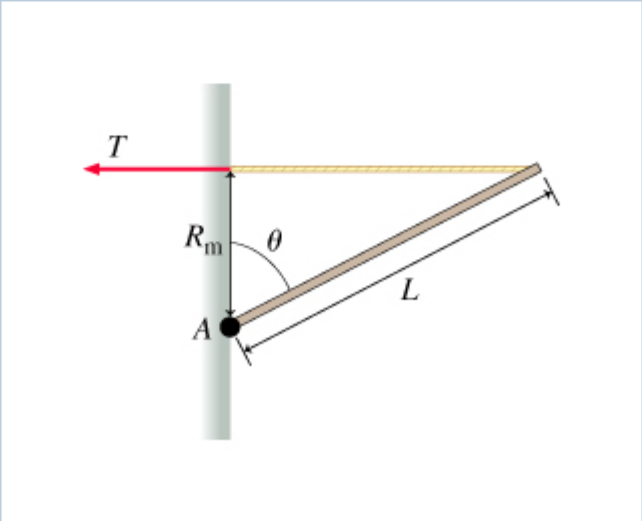
τ = $TL\cos\theta$

Correct

Moment arm method

The moment arm method involves finding the effective moment arm of the force. To do this, imagine a line parallel to the force, running through the point at which the force is applied, and extending off to infinity in either direction. You may shift the force vector anywhere you like along this line without changing the torque, provided you do not change the direction of the force vector as you shift it. It is generally most convenient to shift the force vector to a point where the displacement from it to the desired pivot point is perpendicular to its direction. This displacement is called the *moment arm*.

For example, consider the force due to tension acting on the pole. Shift the force vector to the left, so that it acts at a point directly above the point A in the figure. The moment arm of the force is the distance between the pivot and the tail of the shifted force vector. The magnitude of the torque about the pivot is the product of the moment arm and force, and the sign of the torque is again determined by the sense of the rotation of the pole it would cause.



Part C

Find R_{m}, the length of the moment arm of the force.

Express your answer in terms of L and θ.

ANSWER:

R_{m} = $L\cos(\theta)$

Answer Requested

To calculate the torque using the moment arm method, use the equation

$$\tau = FR_{\mathrm{m}},$$

where R_{m} is the moment arm perpendicular to the applied force.

Part D

Find the magnitude of the torque τ on the pole, about point A, due to the tension in the rope.

Express your answer in terms of T, L, and θ.

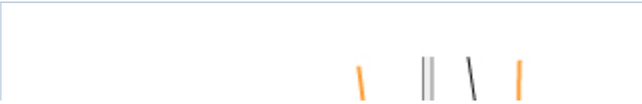
ANSWER:

τ = $TL\cos(\theta)$

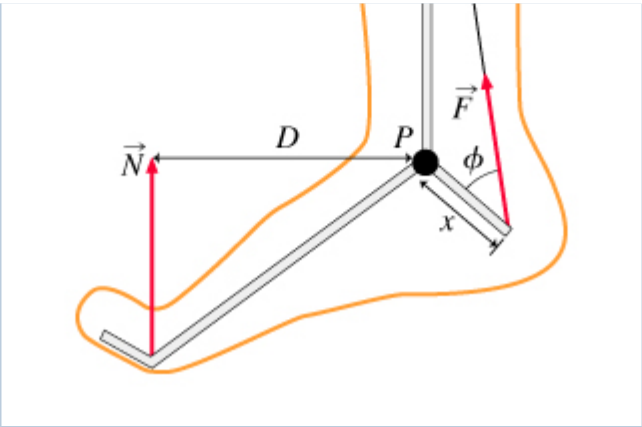
Answer Requested

For this problem, the two methods of finding torque involved nearly the same of amount of algebra, and either method could be used. Of course, both methods lead to the same final result.

Now consider a woman standing on the ball of her foot as shown . A normal force of magnitude N acts upward on the ball of her foot. The Achilles' tendon is attached to the back of the foot. The



tendon pulls on the small bone in the rear of the foot with a force F. This small bone has a length x, and the angle between this bone and the Achilles' tendon is ϕ. The horizontal displacement between the ball of the foot and the point P is D.



Part E

Suppose you were asked to find the torque about point P due to the normal force N in terms of given quantities. Which method of finding the torque would be the easiest to use?

ANSWER:

- tangential force method
- moment arm method

Correct

Part F

Find τ_N, the torque about point P due to the normal force.

Express your answer in terms of N and any of the other quantities given in the figure.

ANSWER:

$\tau_N = -ND$

Answer Requested

Part G

Suppose you were asked to find the torque about point P due to the force of magnitude F in the Achilles' tendon. Which of the following statements is correct?

ANSWER:

- ○ The tangential force method must be used.
- ○ The moment arm method must be used.
- ◉ Either method may be used.
- ○ Neither method can be used.

Correct

Part H

Find τ_F, the torque about point P due to the force applied by the Achilles' tendon.

Express your answer in terms of F, ϕ, and x.

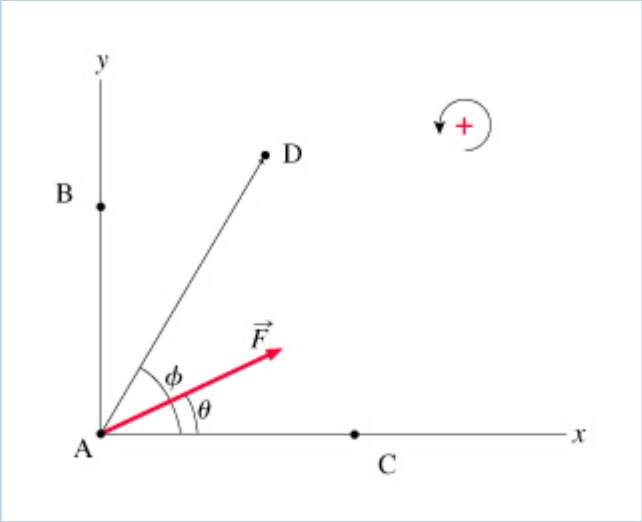
ANSWER:

τ_F = $Fx\sin(\phi)$

Answer Requested

Finding Torque

A force $\vec{F}$ of magnitude F making an angle θ with the *x* axis is applied to a particle located along axis of rotation A, at Cartesian coordinates $(0, 0)$ in the figure. The vector $\vec{F}$ lies in the *xy* plane, and the four axes of rotation A, B, C, and D all lie perpendicular to the *xy* plane.



A particle is located at a vector position $\vec{r}$ with respect to an axis of rotation (thus $\vec{r}$ points from the axis to the point at which the particle is located). The magnitude of the torque τ about this axis due to a force $\vec{F}$ acting on the particle is given by

$$\tau = rF\sin(\alpha) = (\text{moment arm}) \cdot F = rF_{\perp},$$

where α is the angle between $\vec{r}$ and $\vec{F}$, r is the magnitude of $\vec{r}$, F is the magnitude of $\vec{F}$, the component of $\vec{r}$ that is perpendicualr to $\vec{F}$ is the moment arm, and $F_{\perp}$ is the component of the force that is perpendicular to $\vec{r}$.

Sign convention: You will need to determine the sign by analyzing the direction of the rotation that the torque would tend to produce. Recall that negative torque about an axis corresponds to clockwise rotation.
In this problem, you must express the angle α in the above equation in terms of θ, ϕ, and/or π when entering your answers. Keep in mind that $\pi = 180 \text{ degrees}$ and $(\pi/2) = 90 \text{ degrees}$.

Part A

What is the torque τ_{A} about axis A due to the force $\vec{F}$?

Express the torque about axis A at Cartesian coordinates $(0, 0)$.

Hint 1. When force is applied at the pivot point

Does a force applied at a pivot point cause an object to rotate about that pivot?

ANSWER:

τ_{A} = 0

All attempts used; correct answer displayed

Part B

What is the torque τ_B about axis B due to the force $\vec{F}$? (B is the point at Cartesian coordinates $(0, b)$, located a distance b from the origin along the *y* axis.)

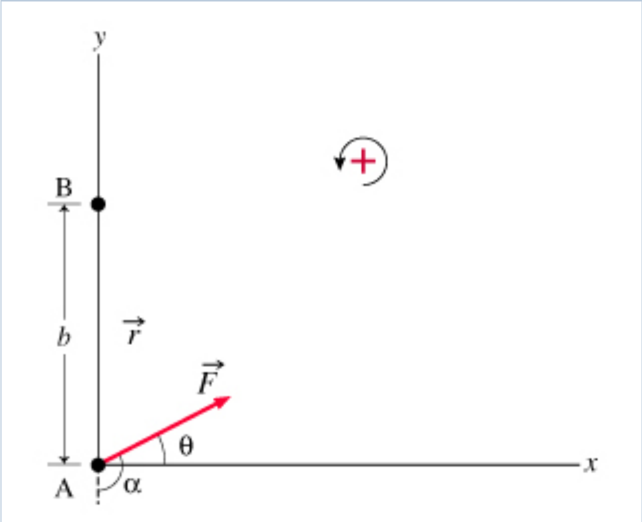
Express the torque about axis B in terms of F, θ, ϕ, π, and/or other given coordinate data.

Hint 1. Finding $\vec{r}$ with respect to an axis

The vector $\vec{r}$ should be drawn from the axis B to the point where $\vec{F}$ is being applied. Consider both the magnitude and direction of this vector.

Hint 2. A helpful figure

The figure shows $\vec{r}$ for this part of the problem. What is the value of α in terms of θ?



ANSWER:

- $\pi - \theta$
- $(\pi/2) - \theta$
- $(\pi/2) + \theta$ (selected)
- θ

ANSWER:

τ_B = $bF\sin\left(\frac{\pi}{2}+\theta\right)$

All attempts used; correct answer displayed

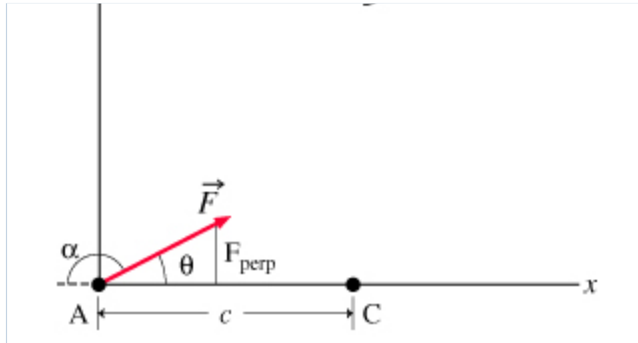
Part C

What is the torque τ_C about axis C due to $\vec{F}$? (C is the point at Cartesian coordinates $(c, 0)$, a distance c along the *x* axis.)

Express the torque about axis C in terms of F, θ, ϕ, π, and/or other given coordinate data.

Hint 1. A helpful figure

The figure shows $\vec{r}$ for this part of the problem. What is the value of $F_\perp$?



ANSWER:

- (selected) $F\sin\theta$
- $F\cos\theta$

Hint 2. Clockwise or counterclockwise?

Imagine a wheel of radius r with its axle passing through the axis of rotation (so that the particle at point A is on the rim of the wheel). Look at the direction of $\vec{F}$ in the figure. Which way do you think $\vec{F}$ will "try" to turn the wheel: clockwise or counterclockwise? Note that only the component of $\vec{F}$ that is tangent to the rim of the wheel (perpendicular to $\vec{r}$) generates torque.

ANSWER:

τ_C = $-cF\sin(\theta)$

All attempts used; correct answer displayed

Part D

What is the torque τ_D about axis D due to $\vec{F}$? (D is the point located at a distance d from the origin and making an angle ϕ with the *x* axis.)

Express the torque about axis D in terms of F, θ, ϕ, π, and/or other given coordinate data.

ANSWER:

τ_D = $dF\sin(\phi - \theta)$

All attempts used; correct answer displayed

You could have found τ_D by using any of the three equations listed at the top of the page,

$$\tau = rF\sin(\alpha) = (\text{moment arm}) \cdot F = rF_\perp,$$

and making the following associations:

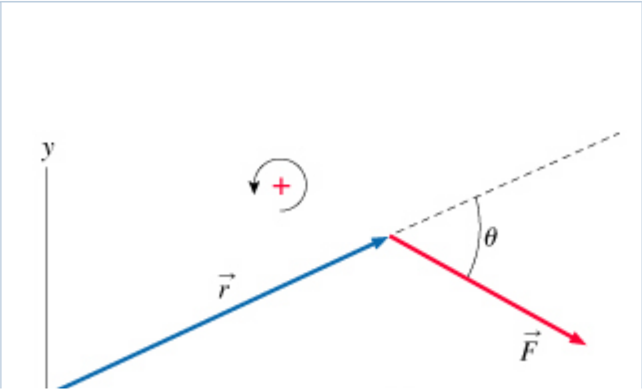
- $r = d$ and $\sin(\alpha) = \sin(\pi - \phi + \theta) = \sin(\phi - \theta)$
- $\text{moment arm} = r_\perp = d\sin(\phi - \theta)$
- $F_\perp = F\sin(\phi - \theta)$.

Torque about the z Axis

Learning Goal:

To understand two different techniques for computing the torque on an object due to an applied force.

Imagine an object with a pivot point p at the origin of the coordinate system shown . The force vector $\vec{F}$ lies in the *xy* plane, and this force of magnitude F acts on the object at a point in the *xy* plane. The vector $\vec{r}$ is the position vector relative to the pivot point p to the point where $\vec{F}$ is applied.



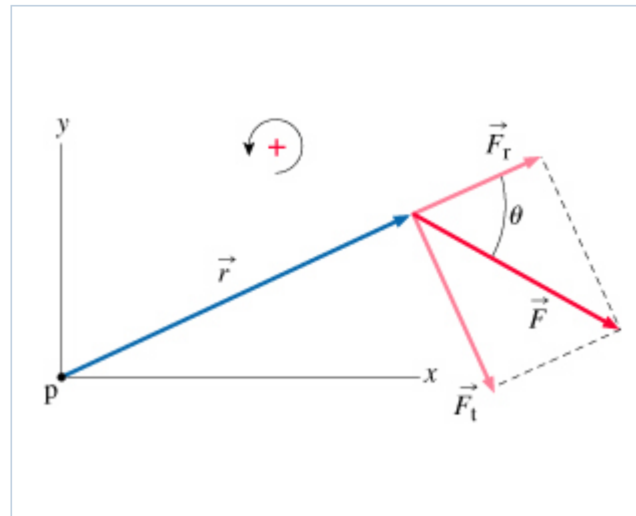
The torque on the object due to the force $\vec{F}$ is equal to the cross product $\vec{\tau} = \vec{r} \times \vec{F}$. When, as in this problem, the force vector and lever arm both lie in the *xy* plane of the paper or computer screen, only the *z* component of torque is nonzero.

When the torque vector is parallel to the z axis ($\vec{\tau} = \tau\hat{z}$), it is easiest to find the magnitude and sign of the torque, τ, in terms of the angle θ between the position and force vectors using one of two simple methods: the *Tangential Component of the Force* method or the *Moment Arm of the Force* method.

Note that in this problem, the positive *z* direction is perpendicular to the computer screen and points toward you (given by the right-hand rule $\hat{x} \times \hat{y} = \hat{z}$), so a positive torque would cause counterclockwise rotation about the *z* axis.

Tangential component of the force

Part A



Decompose the force vector $\vec{F}$ into radial (i.e., parallel to $\vec{r}$) and tangential (perpendicular to $\vec{r}$) components as shown. Find the magnitude of the radial and tangential components, F_r and F_t. You may assume that θ is between zero and 90 degrees.

Enter your answer as an ordered pair. Express F_t and F_r in terms of F and θ.

Hint 1. Magnitude of $\vec{F}_r$

Use the given angle between the force vector $\vec{F}$ and its radial component $\vec{F}_r$ to compute the magnitude F_r.

ANSWER:

(F_r, F_t) = $F\cos(\theta), F\sin(\theta)$

All attempts used; correct answer displayed

Part B

Is the following statement true or false?

The torque about point p is proportional to the length r of the position vector $\vec{r}$.

ANSWER:

- true
- false

Correct

Part C

Is the following statement true or false?

Both the radial and tangential components of $\vec{F}$ generate torque about point p.

ANSWER:

- true
- false

Correct

Part D

Is the following statement true or false?

In this problem, the tangential force vector would tend to turn an object clockwise around pivot point p.

ANSWER:

- (selected) true
- false

Correct

Part E

Find the torque τ about the pivot point p due to force $\vec{F}$. Your answer should correctly express both the magnitude and sign of τ.

Express your answer in terms of F_t and r or in terms of F, θ, and r.

ANSWER:

τ = $-rF_t$

Answer Requested

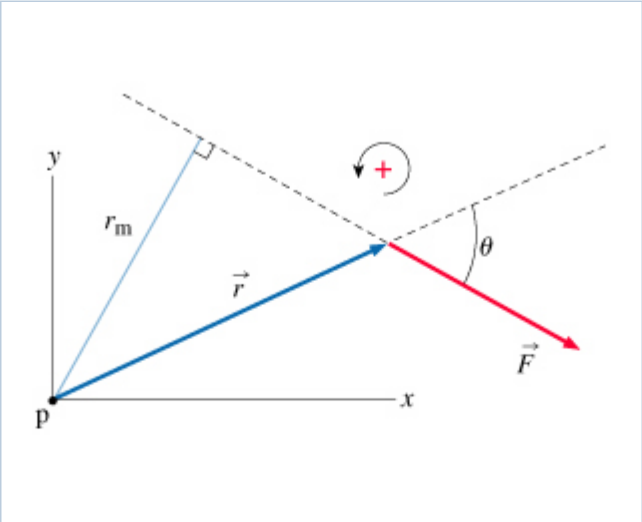
Moment arm of the force

In the figure, the dashed line extending from the force vector is called the line of action of $\vec{F}$. The perpendicular distance $r_{\rm m}$ from the pivot point p to the line of action is called the moment arm of the force.

Part F

What is the length, $r_{\rm m}$, of the moment arm of the force $\vec{F}$ about point p?

Express your answer in terms of r and θ.



ANSWER:

$r_{\rm m}$ = $r\sin(\theta)$

Answer Requested

Part G

Find the torque τ about p due to $\vec{F}$. Your answer should correctly express both the magnitude and sign of τ.

Express your answer in terms of $r_{\rm m}$ and F or in terms of r, θ, and F.

ANSWER:

τ = $-r_m F$

Answer Requested

Three equivalent expressions for expressing torque about the *z* axis have been discussed in this problem:

1. Torque is defined as the cross product between the position and force vectors. When both $\vec{F}$ and $\vec{r}$ lie in the *xy* plane, only the *z* component of torque is nonzero, and the cross product simplifies to:

$$\vec{\tau} = \vec{r} \times \vec{F} = r * F * \sin(\theta)\hat{z} = \tau\hat{z}.$$

Note that a positive value for τ indicates a counterclockwise direction about the *z* axis.

2. Torque is generated by the component of $\vec{F}$ that is tangential to the position vector $\vec{r}$ (the tangential component of force):

$$\tau = r * F_t = r * F\sin(\theta).$$

3. The magnitude of torque is the product of the force and the perpendicular distance between the *z* axis and the line of action of a force, $r_{\rm m}$, called the moment arm of the force:

$$\tau = r_m * F = r * \sin(\theta) * F.$$

Score Summary:

Your score on this assignment is 0%.
You received 0 out of a possible total of 15 points.