# #26 Energy in Rotational Motion and Moments of Inertia Pre-class

Due: 11:00am on Wednesday, October 24, 2012

***Note: You will receive no credit for late submissions.*** To learn more, read your instructor's Grading Policy

## Scaling of Moments of Inertia

**Learning Goal:**

To understand the concept of moment of inertia and how it depends on mass, radius, and mass distribution.

In rigid-body rotational dynamics, the role analogous to the mass of a body (when one is considering translational motion) is played by the body's moment of inertia. For this reason, conceptual understanding of the motion of a rigid body requires some understanding of moments of inertia. This problem should help you develop such an understanding.

The moment of inertia of a body about some specified axis is $I = cmr^2$, where $c$ is a dimensionless constant, $m$ is the mass of the body, and $r$ is the perpendicular distance from the axis of rotation. Therefore, if you have two similarly shaped objects of the same size but with one twice as massive as the other, the more massive object should have a moment of inertia twice that of the less massive one. Furthermore, if you have two similarly shaped objects of the same mass, but one has twice the size of the other, the larger object should have a moment of inertia that is four times that of the smaller one.

### Part A

Two spherical shells have their mass uniformly distrubuted over the spherical surface. One of the shells has a diameter of 2 meters and a mass of 1 kilogram. The other shell has a diameter of 1 meter. What must the mass $m$ of the 1-meter shell be for both shells to have the same moment of inertia about their centers of mass?

ANSWER:

$m$ = 4 kg

**Correct**

It is important to understand how the dimensionless constant $c$ in the moment of inertia formula given in the problem introduction is determined. Consider a disk and a thin ring, both having the same outer radius $r$ and mass $m$. The moment of inertia of the disk is $\frac{1}{2}mr^2$, while the moment of inertia of the ring is

$mr^2$. (For each object, the axis is perpendicular to the plane of the object and passes through the object's center of mass.)

The factor of $\frac{1}{2}$ for the disk gives an indication of how the mass is distributed in that object. If both the disk and ring are spinning with the same angular velocity $\omega$, the ring should have a greater kinetic energy, because all of the mass of the ring has linear speed $\omega r$, whereas the linear speeds of the different parts of the disk vary, depending on how far the part is from the center, and these speeds vary from zero to $\omega r$.

In general, the value of $c$ reflects the distribution of mass within the object. A number close to $1$ indicates that most of the mass is located at a distance from the center of mass close to $r$, while a number much less than $1$ indicates that most of the mass is located near the center of mass.

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## Part B

Consider the moment of inertia of a solid uniform disk, versus that of a solid sphere, about their respective centers of mass. Assume that they both have the same mass and outer radius, that they have uniform mass distributions, and that the disk is rotated about an axis perpendicular to its face. What is the relation between the moment of inertia of the disk $I_{\rm disk}$ and that of the sphere $I_{\rm sphere}$?

**Hint 1.** How to approach the problem

Draw a figure of each object and the axis. Consider two "slices" parallel to the axis at different radii and the ratio of masses inside these slices. Which object has a greater percentage of the mass closer to the axis?

ANSWER:

- (selected) $I_{\rm disk} > I_{\rm sphere}$
- $I_{\rm sphere} > I_{\rm disk}$

**Correct**

# The Rotational Kinetic Energy of the Earth

The Earth can be approximated as a sphere of uniform density, rotating on its axis once a day. The mass of the Earth is $5.97 \times 10^{24}\ \mathrm{kg}$ , the radius of the Earth is $6.38 \times 10^{6}\ \mathrm{m}$ , and the period of rotation for the Earth is $24.0\ \mathrm{hrs}$ .

## Part A

What is the moment of inertia of the Earth? Use the uniform-sphere approximation described in the introduction.

**Express your answer in kilogram meters squared to three significant figures.**

**Hint 1.** The moment of inertia for a uniform sphere

The moment of inertia of a uniform sphere is

$$I = \frac{2}{5} M R^2,$$

where $M$ is the mass of the sphere and $R$ is the radius of the sphere.

ANSWER:

$I$ = $9.72 \times 10^{37}$ $\mathrm{kg \cdot m^2}$

**Correct**

By studying the Earth and its interactions with other celestial bodies, scientists have concluded that the actual moment of inertia for the Earth is about twenty percent smaller than the moment of inertia you calculated in the previous part of the problem.

## Part B

Consider the following statements, all of which are actually true, and select the one that best explains why the moment of inertia of the Earth is actually smaller than the moment of inertia you calculated.

ANSWER:

- The Earth is an oblate spheroid rather than a perfect sphere. For an oblate spheroid, the distance from the center to the equator is a little larger than the distance from the center to the poles. This is a similar shape to a beach ball resting on the ground, being pushed on from above.
- The Earth does not have uniform density. As the planet formed, the densest materials sank to the center of the Earth. This created a dense iron core. Meanwhile, the lighter elements floated to the surface. The crust of the Earth is considerably less dense than the core.
- While the Earth currently has a period of 24 hours, it is in fact slowing down. Once it was rotating much faster, giving days that were closer to 20 hours than 24 hours. In the future, it is expected that days will become longer.

### Correct

The moment of inertia of the Earth is smaller than predicted by the assumptions of the problem because the Earth does not have a uniform density. When the mass is concentrated near the center of a spinning object, the moment of inertia is smaller than if the mass is concentrated near the rim. Engineers keep this in mind as they design wheels for cars. A good car wheel will concentrate as much mass as possible near the axle, which gives it the smallest moment of inertia possible. A small moment of inertia makes the wheel easier to speed up or slow down, thereby increasing the fuel efficiency of a car.

## Part C

What is the rotational kinetic energy of the Earth? Use the moment of inertia you calculated in Part A rather than the actual moment of inertia given in Part B.

**Express your answer in joules to three significant figures.**

**Hint 1.** How to approach the problem

The equation for rotational kinetic energy is

$$KE_{\rm rot} = \frac{1}{2} I \omega^2,$$

where $I$ is the moment of inertia and $\omega$ is the angular speed of the Earth. You will need to calculate the angular speed before you can calculate the rotational kinetic energy.

**Hint 2.** Find the angular speed of the Earth.

Find the angular speed $\omega$ of the Earth.

**Express your answer in radians per second to three significant figures.**

**Hint 1.** Angular speed

The angular speed is defined as the change in angular displacement, in radians, divided by the change in time:

$$\omega = \frac{2\pi}{T},$$

where $T$ is the period in seconds. Recall that the period is the time it takes for the object to rotate on its axis once and $2\pi$ is the angular displacement for an object that has made a complete rotation.

ANSWER:

$\omega$ = $7.27\times10^{-5}$ $\mathrm{rad/s}$

ANSWER:

$KE_{\rm rot}$ = $2.57\times10^{29}$ $\mathrm{J}$

**Correct**

Recall that while energy can change forms, it is always conserved. In other words, if you start with a certain amount of energy, you must end with the same amount of energy. In Part C, you calculated how much rotational kinetic energy the Earth now has. By conservation of energy, that energy had to come from somewhere.

### Part D

Where did the rotational kinetic energy of the Earth come from?

**Select the option that best explains where the Earth's rotational kinetic energy came from.**

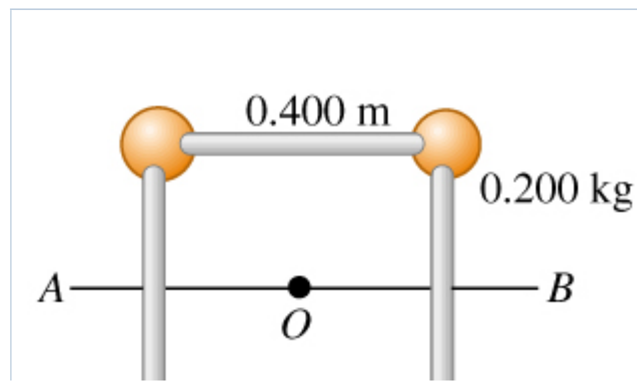
ANSWER:

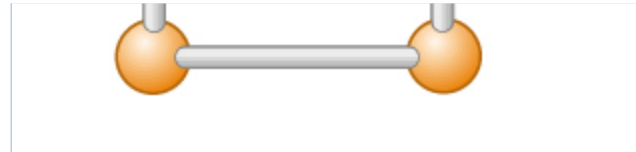
- (selected) The solar system formed from a massive cloud of gas and dust, which was slowly rotating. As the cloud collapsed under its own gravitational pull, the cloud started to spin faster, just as an ice skater pulling his arms in will spin faster. Because all of the material that accreted to form the planet was rotating, the planet was rotating as well.
- As the Earth formed, it experienced a series of collisions with asteroids and comets. These asteroids and comets hit the ball of rock that was forming into the planet off-center. Over time, the off-center collisions gradually caused the planet to rotate faster.
- As the Moon orbits around the Earth, it creates tides on the Earth. Over time the tides have caused the Earth to rotate faster and faster.

**Correct**

## Exercise 9.34

Four small spheres, each of which you can regard as a point of mass 0.200 $\mathrm{kg}$, are arranged in a square 0.400 $\mathrm{m}$ on a side and connected by light rods .





## Part A

Find the moment of inertia of the system about an axis through the center of the square, perpendicular to its plane (an axis through point O in the figure).

ANSWER:

$I$ = $6.40 \times 10^{-2}$ $\mathrm{kg \cdot m^2}$

**Correct**

## Part B

Find the moment of inertia of the system about an axis bisecting two opposite sides of the square (an axis along the line AB in the figure).

ANSWER:

$I$ = $3.20 \times 10^{-2}$ $\mathrm{kg \cdot m^2}$

**Correct**

## Part C

Find the moment of inertia of the system about an axis that passes through the centers of the upper left and lower right spheres and through point O.

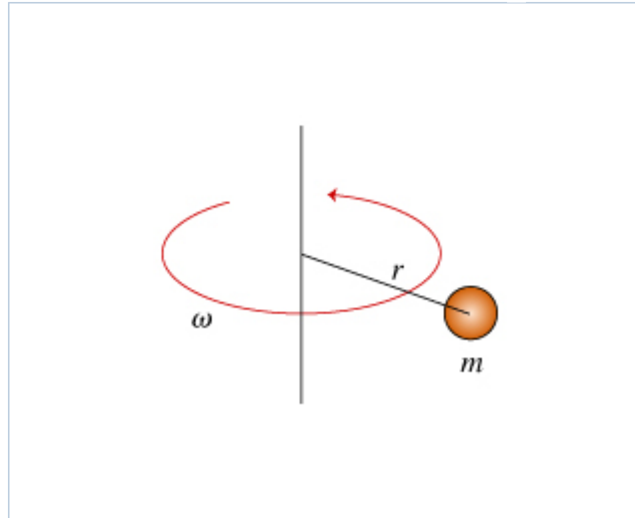
ANSWER:

$I$ = $3.20 \times 10^{-2}$ $\mathrm{kg \cdot m^2}$

**Correct**

## Kinetic Energy and Moment of Inertia

Consider a particle of mass $m$ that is revolving with angular speed $\omega$ around an axis. The perpendicular distance from the particle to the axis is $r$.




### Part A

Find the kinetic energy $K$ of the rotating particle.

**Express your answer in terms of $m$, $r$, and $\omega$.**

**Hint 1.** Find the speed of the particle

What is the magnitude $v$ of the velocity of the particle?

**Express your answer in terms of $\omega$ and $r$.**

ANSWER:

$v$ = $\omega r$

ANSWER:

$K$ = $\frac{1}{2}m(\omega r)^2$

**Correct**

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**Part B**

The kinetic energy of a rotating body is generally written as $K = \frac{1}{2}I\omega^2$, where $I$ is the moment of inertia. Find the moment of inertia of the particle described in the problem introduction with respect to the axis about which it is rotating.

**Give your answer in terms of $m$ and $r$, *not* $K$.**

**Hint 1.** How to approach the problem

Set the kinetic energy that you found in Part B equal to $\frac{1}{2}I\omega^2$, and then solve for $I$.

ANSWER:

$I$ = $mr^2$

## Correct

Consider a system of several point masses $m_i$ all rotating about the same axis. The total kinetic energy of the system is the sum of the kinetic energies of all of the masses. For $n$ point masses,

$$K = \frac{1}{2}(m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2)\omega^2.$$

Comparing this to the expression for the kinetic energy given in Part C, we see that the moment of inertia is

$$I = m_1r_1^2 + m_2r_2^2 + \cdots + m_nr_n^2.$$

In other words, the total moment of inertia is the sum of the moments of inertia of each mass.

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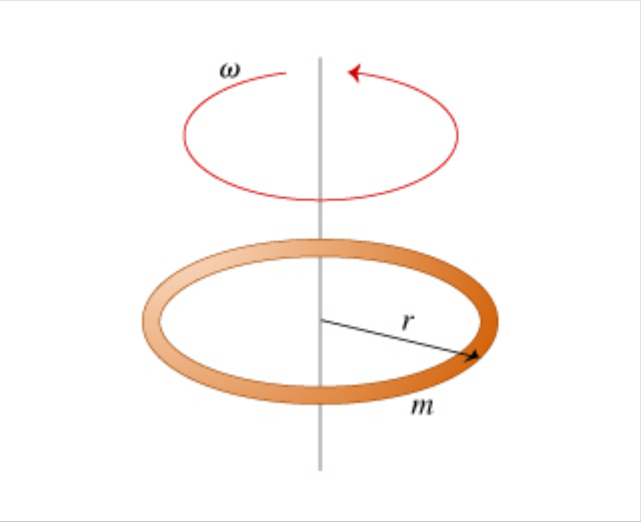
### Part C

Find the moment of inertia $I_{\text{hoop}}$ of a hoop of radius $r$ and mass $m$ with respect to an axis perpendicular to the hoop and passing through its center.

**Express your answer in terms of $m$ and $r$.**

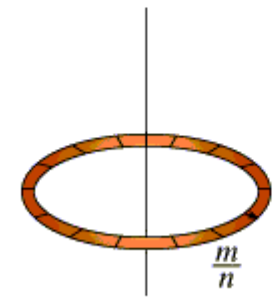


**Hint 1.** Find the moment of inertia of a segment of the hoop

Imagine that this hoop is cut into $n$ identical segments, as shown in the figure.

Each of these segments can be considered a separate particle of mass $m/n$ located at a distance $r$ from the axis. What is the moment of inertia $I_{\rm segment}$ of one such segment?

**Express your answer in terms of $m$, $n$, and $r$.**




ANSWER:

$I_{\rm segment}$ = $\frac{m}{n}r^2$

ANSWER:

$I_{\rm hoop}$ = $r^2m$

**Correct**

## Exercise 9.37

A uniform bar has two small balls glued to its ends. The bar is 2.00 $\mathrm{m}$ long and with mass 4.00 $\mathrm{kg}$, while the balls each have mass 0.500 $\mathrm{kg}$ and can be treated as point masses.

### Part A

Find the moment of inertia of this combination about an axis perpendicular to the bar through its center.

ANSWER:

$I$ = 2.33 $\mathrm{kg \cdot m^2}$

**Correct**

### Part B

Find the moment of inertia of this combination about an axis perpendicular to the bar through one of the balls.

ANSWER:

$I$ = 7.33 $\mathrm{kg \cdot m^2}$

**Correct**

### Part C

Find the moment of inertia of this combination about an axis parallel to the bar through both balls.

ANSWER:

$I$ = 0 $\mathrm{kg \cdot m^2}$

**Correct**

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**Part D**

Find the moment of inertia of this combination about an axis parallel to the bar and 0.500 m from it.

ANSWER:

$I$ = 1.25 $\mathrm{kg \cdot m^2}$

**Correct**

## Score Summary:

Your score on this assignment is 97.9%.
You received 24.48 out of a possible total of 25 points.