#28 Newton's Second Law Redux Pre-class

Due: 11:00am on Monday, October 29, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Exercise 10.18

A thin, horizontal rod with length l and mass M pivots about a vertical axis at one end. A force with constant magnitude F is applied to the other end, causing the rod to rotate in a horizontal plane. The force is maintained perpendicular to the rod and to the axis of rotation.

Part A

Calculate the magnitude of the angular acceleration of the rod.

ANSWER:

$\frac{3F}{Ml}$

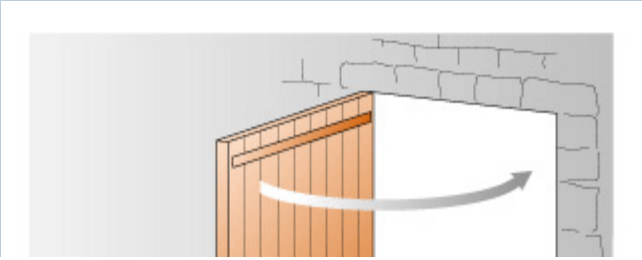
Correct

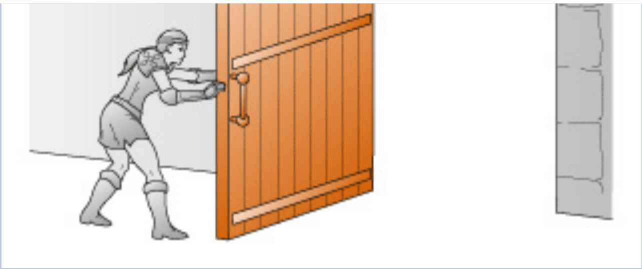
PSS 10.1 Rotational Dynamics for Rigid Bodies

Learning Goal:

To practice Problem-Solving Strategy 10.1 Rotational Dynamics for Rigid Bodies.

While exploring a castle, Exena the Exterminator is spotted by a dragon who chases her down a hallway. Exena runs into a room and attempts to swing the heavy door shut before the dragon gets to her . The door is initially perpendicular to the wall, so it must be turned through 90° to close it. The door is 3.00m tall and 1.25m wide, and it weighs 750 N . You can ignore the friction at the hinges. If Exena applies a force of 220N at the edge of the door and perpendicular to it, how much time does it take her to close the door?





Problem-Solving Strategy: Rotational dynamics for rigid bodies

IDENTIFY *the relevant concepts*:
In some cases you may be able to use an energy approach. However, if the target variable is a force, a torque, an acceleration, an angular acceleration, or an elapsed time, using $\Sigma\tau_z = I\alpha_z$ is almost always the best approach.

SET UP *the problem using the following steps*:

1. Sketch the situation and select the body or bodies to analyze.
2. Draw a free-body diagram for each body and label unknown quantities with algebraic symbols. Show the shape of the body accurately, with all dimensions and angles that you will need for calculations of torque.
3. Choose coordinate axes for each body, and indicate a positive sense of rotation for each rotating body.

EXECUTE *the solution as follows*:

1. Write an equation of motion for each body by applying $\Sigma\vec{F} = m\vec{a}$, $\Sigma\tau_z = I\alpha_z$, or both to each body.
2. There may be geometrical relationships between the motions of two or more bodies, as with a string that unwinds from a pulley while turning it. Express these relationships in algebraic form.
3. Check that the number of equations matches the number of unknown quantities. Then, solve the equations to find the target variable(s).

EVALUATE *your answer*:
Check that the algebraic signs of your results make sense. Whenever possible, check the results for special cases or extreme values of quantities. Ask yourself: “Does this result make sense?”

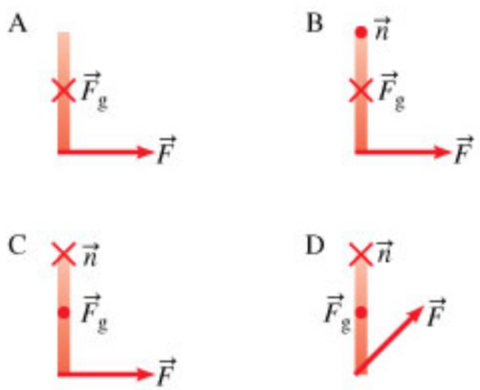
IDENTIFY the relevant concepts

You are asked to find the time it takes Exena to close the door (a rigid body that rotates about the hinges). Energy considerations do not easily provide information about time. Instead, applying $\Sigma\tau_z = I\alpha_z$ to describe the rotational motion of the door will be the best approach.

SET UP the problem using the following steps

Part A

In the following free-body diagrams, let $\vec{F}$ be the force exerted by Exena on the door, $\vec{F}_g$ the weight of the door, and n the magnitude of the normal force exerted by the hinges. The diagrams show a view of the door from above, with the edge of the door in contact with the hinges at the top of each diagram. Any force directed out of the plane of the figure is represented by a dot, and any force directed into the plane of the figure is represented by a cross. Which free-body diagram correctly describes the system under investigation?



ANSWER:

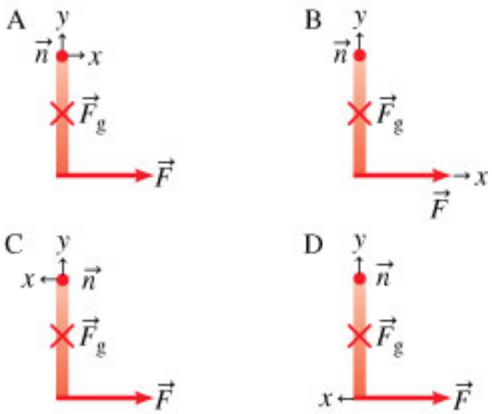
- diagram A
- diagram B
- diagram C
- diagram D

Correct

If we ignore the friction at the hinges, there are only three forces acting on the door: its weight, the forces exerted by Exena, and the normal force exerted by the hinges.

Part B

Which set of axes shown in the figure represents the best orientation for the coordinate axes? As in Part A, the diagrams show a view of the door from above, with the edge of the door in contact with the hinges at the top of each diagram.



ANSWER:

- set A
- set B
- set C
- set D

Correct

It is most convenient to take a point on the axis of rotation as the origin of the coordinate axes. Here, the axis of rotation is parallel to the door and passes through the hinges, so the *z* axis is directed out of the plane of the figure through the edge of the door at the top of the diagram. It also helpful to choose the positive *x* axis in the direction of the force that causes the rotation. Keep in mind that counterclockwise torque is positive.

EXECUTE the solution as follows

Part C

If Exena applies a force of 220 N at the edge of the door and perpendicular to it, how much time t does it take her to close the door?

Express your answer in seconds to three significant figures.

Hint 1. How to approach the problem

To find the time it takes Exena to shut the door, you need to first find the angular acceleration of the door, which can be found using the equation for rotational motion. You can then use the equations associated with rotational kinematics to solve for time.

Hint 2. Find the net torque acting on the door

Find the net torque τ_{net} around the *z* axis (i.e., the direction coming out of the plane of the diagram shown in Part B at the origin) acting on the door.

Express your answer in newton-meters to four significant figures.

Hint 1. The definition of torque

The tendency of a force to cause or change rotational motion around a chosen axis is measured by the *torque*, which is the product of the magnitude of the force F and the perpendicular distance l between the axis of rotation and the line of action of the force, or

$$\tau = Fl.$$

The distance l is also called the *moment arm* of the force F.

Hint 2. Determine which forces exert a torque

As shown in the free-body diagram in Part A, there are three forces acting on the door: the weight, the normal force, and the force applied by Exena. Which one of these forces exerts a nonzero torque on the door around the *z* axis (i.e., the direction coming out of the plane of the diagram at the origin)?

Check all that apply.

ANSWER:

- [ ] the weight
- [ ] the normal force
- [x] the force applied by Exena

Hint 3. Find the moment arm of the force applied by Exena

What is the moment arm l of the force applied by Exena? Recall that Exena applies a force perpendicular to the door and the door is 3.00m high and 1.25m wide.

Express your answer in meters to four significant figures.

ANSWER:

l = 1.250 m

ANSWER:

τ_{net} = 275.0 $\mathrm{N \cdot m}$

Hint 3. Find the moment of inertia

What is the moment of inertia I of the door with respect to an axis along its vertical edge?

Express your answer in kilogram-meters squared to four significant figures.

Hint 1. The moment of inertia of a rectangular plate

The moment of inertia I of a thin rectangular plate of width a and height h with respect to an axis along the edge of length h is

$$I = \frac{1}{3}Ma^2,$$

where M is the mass of the plate.

Hint 2. Find the mass of the door

You are given the weight of the door. What is the mass m of the door?

Express your answer in kilograms to four significant figures.

ANSWER:

m = 76.45 kg

ANSWER:

I = 39.82 $\mathrm{kg \cdot m^2}$

Hint 4. Find the angular acceleration of the door

What is α, the angular acceleration of the door?

Express your answer in radians per second per second to three significant figures.

Hint 1. The equation for rotational motion

As mentioned in the IDENTIFY step, to study the rotational motion of the door, it is most convenient to use the equation $\Sigma\tau_z = I\alpha_z$, which states that the net torque acting on a body is equal to the product of its angular acceleration α and its moment of inertia I.

ANSWER:

α = 6.91 $\mathrm{rad/s^2}$

Hint 5. Identify the rotational kinematics equation needed to find the time

As established in Part A, the door undergoes only rotational motion. Assume that Exena turns the door an angle θ in a time interval t. Let l be the width of the door; g the gravitational acceleration; v and a, respectively, the speed and acceleration of the center of mass of the door; and α its angular acceleration. Then, which of the following expressions will be most useful in solving this problem?

ANSWER:

- $\theta = \frac{1}{2}\alpha t^2$
- $l = \frac{1}{2}at^2$
- $\theta = \frac{1}{2}gt^2$
- $a = \alpha l$

ANSWER:

t = 0.674 s

Correct

EVALUATE your answer

Part D

How long would it take Exena to close the door if she doubled the magnitude of the force applied to the door?

Express your answer in seconds to three significant figures.

ANSWER:

t = 0.477 s

Correct

This makes sense. If she pushes twice as hard, it will take less time to shut the door.

Exercise 10.29

A 1.80 kg grinding wheel is in the form of a solid cylinder of radius 0.130 m .

Part A

What constant torque will bring it from rest to an angular speed of 1900 $\mathrm{rev/min}$ in 2.50 s ?

ANSWER:

τ = 1.21 $\mathrm{N \cdot m}$

Correct

Part B

Through what angle has it turned during that time?

ANSWER:

θ = 249 rad

Correct

Part C

Use equation $W = \tau_z(\theta_2 - \theta_1) = \tau_z \Delta\theta$ to calculate the work done by the torque.

ANSWER:

W = 301 J

Correct

Part D

What is the grinding wheel's kinetic energy when it is rotating at 1900 $\mathrm{rev/min}$?

ANSWER:

K = 301 J

Correct

Part E

Compare your answer in part (D) to the result in part (C).

ANSWER:

- (selected) The results are the same.
- The results are not the same.

Correct

Introduction to Rotational Work and Power

Learning Goal:

To understand work and power in rotational systems and to use the work-energy theorem to determine kinematics variables.

The variables used in standard linear mechanics (the study of objects that do not rotate) all have analogues in rotational mechanics (the study of objects that rotate). Here is a summary of variables used in kinematics and dynamics.

Linear variable	Rotational variable

linear position: x	angular position: θ
linear velocity: v	angular velocity: ω
linear acceleration: a	angular acceleration: α
linear inertia (*mass*): m	moment of inertia: I
force: F	torque: τ

The kinetic energy $K_{\rm rot}$ associated with a rotating object is defined as

$$K_{\rm rot} = \frac{1}{2}I\omega^2,$$

where ω is measured in $\rm rad/s$. Applying a torque can change the angular velocity of an object, and hence its kinetic energy. The torque is said to have done work on the object, just as a force applied over a distance can change the kinetic energy of an object in linear mechanics. In terms of rotational variables, the work W done by a constant torque τ is

$$W = \tau\Delta\theta,$$

where $\Delta\theta$ describes the total angle, measured in $\rm radians$, through which the object rotates while the torque is being applied.

Consider a motor that exerts a constant torque of 25.0$\rm N \cdot m$ to a horizontal platform whose moment of inertia is 50.0$\rm kg \cdot m^2$. Assume that the platform is initially at rest and the torque is applied for 12.0$\rm rotations$. Neglect friction.

Part A

How much work W does the motor do on the platform during this process?

Enter your answer in joules to four significant figures.

Hint 1. Convert rotations to radians

How many radians are there in 12.0$\rm rotations$?

Enter your answer in radians to four significant figures.

Hint 1. The number of radians in one revolution

One rotation is the same as one revolution. For each revolution the platform makes, it travels through 2π radians.

ANSWER:

$\Delta\theta$ = 75.40 rad

ANSWER:

W = 1885 J

Correct

Recall that the work-energy theorem states that the net work done on an object is responsible for its change in kinetic energy (as long as no other forms of energy change):

$$W = \Delta K = K_{\rm f} - K_{\rm i}.$$

This theorem can also be applied in rotational mechanics.

Part B

What is the rotational kinetic energy of the platform $K_{\rm rot,f}$ at the end of the process described above?

Enter your answer in joules to four significant figures.

Hint 1. How to approach the problem

We cannot use the definition $K_{\rm rot} = (1/2)I\omega^2$ since the value of ω is not known. Instead, apply the work-energy theorem. Since the platform starts at rest, its initial kinetic energy is easy to calculate.

ANSWER:

$K_{\rm rot,f}$ = 1885 $\rm J$

Correct

The net work done by the motor increases the kinetic energy of the platfrom from $K_{\rm rot,i} = 0$ to $K_{\rm rot,f}$.

Now that the final kinetic energy is known, the values of other kinematic variables at this final time may also be determined.

Part C

What is the angular velocity $\omega_{\rm f}$ of the platform at the end of this process?

Enter your answer in radians per second to three significant figures.

Hint 1. How to approach the problem

Use your knowledge of the final rotational kinetic energy of the platform to determine its final rotational speed.

ANSWER:

$\omega_{\rm f}$ = 8.68 $\rm rad/s$

Correct

Part D

How long Δt does it take for the motor to do the work done on the platform calculated in Part A?

Enter your answer in seconds to three significant figures.

Hint 1. Determine how to approach the problem

This is a situation in which the rotational acceleration is constant, so Δt can be found using constant-acceleration kinematics relations. Which of the following lines of reasoning, all of which are correct, can be applied most directly in this situation to find Δt?

ANSWER:

- Use the equation $\omega_f = \omega_i + \alpha\Delta t$.
- Use the equation $\omega_{avg} = \Delta\theta/\Delta t$ where $\omega_{avg} = (1/2)(\omega_i + \omega_f)$.
- Use the equation \Delta \theta = (1/2) \alpha (\Delta t)^2.

ANSWER:

Δt = 17.4 s

Correct

Power refers to the rate at which work is done by a force or, in rotating situations, by a torque. The average power $P_{\rm avg}$ is the total work done divided by the total amount of time elapsed. Since the work done by a constant torque is $W = \tau\Delta\theta$, the average power during a short time interval Δt can be written as

$$P_{\rm avg} = \tau\frac{\Delta\theta}{\Delta t}.$$

Technically, $\Delta\theta/\Delta t$ is the average angular velocity $\omega_{\rm avg}$ during this time interval. As the time interval becomes shorter and shorter, this expression for power reduces to

$$P = \tau\omega,$$

where P, τ, and ω are all instantaneous quantities.

Part E

What is the average power $P_{\rm avg}$ delivered by the motor in the situation above?

Enter your answer in watts to three significant figures.

Hint 1. Find the average angular velocity

What is the average angular velocity of the platform? Keep in mind that the average velocity is given by the equation

$$\omega_{\rm avg} = \frac{\omega_{\rm i} + \omega_{\rm f}}{2}.$$

Enter your answer in radians per second to three significant figures.

ANSWER:

$\omega_{\rm avg}$ = 4.34 $\rm rad/s$

ANSWER:

P_{avg} = 109 W

Correct

Part F

Note that the instantaneous power P delivered by the motor is directly proportional to ω, so P increases as the platform spins faster and faster. How does the instantaneous power P_{f} being delivered by the motor at the time t_{f} compare to the average power P_{avg} calculated in Part E?

Hint 1. How to approach the problem

In part E you have already calculated the average power P_{avg}. Now calculate P_{f} using the appropriate values of τ and ω that apply at time t_{f} and compare the instantaneous power to the average power. Since the platform is spinning faster and faster in this situation, you can probably guess in advance which one will be larger.

ANSWER:

- $P = P_{\text{avg}}$
- $P = 2 * P_{\text{avg}}$
- $P = P_{\text{avg}}/2$
- none of the above

Correct

This makes sense! As another example, consider a constant-acceleration situation in which ω increases from rest at a rate of 1 $\mathrm{rad/s}$. During the first second, ω changes from 0 to 1 $\mathrm{rad/s}$. About 10 seconds later ω changes from 10 to 11 $\mathrm{rad/s}$, again in a single second. This requires about 20 times more work than the first change because kinetic energy depends on the square of ω rather than on ω alone. Since the work in both cases must be done in just one second, much more power is required to maintain the angular acceleration when the object is spinning faster.

Exercise 10.10

A cord is wrapped around the rim of a solid uniform wheel 0.220 m in radius and of mass 8.80 kg . A steady horizontal pull of 36.0 N to the right is exerted on the cord, pulling it off tangentially from the wheel. The wheel is mounted on frictionless bearings on a horizontal axle through its center.

Part A

Compute the angular acceleration of the wheel.

ANSWER:

|\alpha| = 37.2 $\mathrm{rad/s^2}$

Correct

Part B

Compute the acceleration of the part of the cord that has already been pulled off the wheel.

ANSWER:

$|a|$ = 8.18 $\mathrm{m/s^2}$

Correct

Part C

Find the magnitude of the force that the axle exerts on the wheel.

ANSWER:

F = 93.5 N

Correct

Part D

Find the direction of the force that the axle exerts on the wheel.

ANSWER:

φ = 67.3 $^\circ$ above the horizontal, away from the direction of the pull on the cord.

Correct

Part E

Which of the answers in parts (A), (B), (C) and (D) would change if the pull were upward instead of horizontal?

ANSWER:

- (A), (B), (C) and (D)
- (A) and (B)
- (C) and (D)
- (A) and (D)

Correct

Score Summary:

Your score on this assignment is 98.4%.
You received 24.6 out of a possible total of 25 points.