Chapter 3: Motion in Two or Three Dimensions

Due: 11:00pm on Tuesday, September 11, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Conceptual Problem about Projectile Motion

Learning Goal:

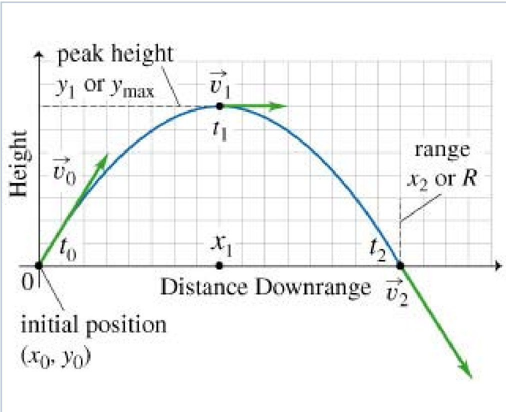
To understand projectile motion by considering horizontal constant velocity motion and vertical constant acceleration motion independently.

Projectile motion refers to the motion of unpowered objects (called projectiles) such as balls or stones moving near the surface of the earth under the influence of the earth's gravity alone. In this analysis we assume that air resistance can be neglected.

An object undergoing projectile motion near the surface of the earth obeys the following rules:

1. An object undergoing projectile motion travels horizontally at a constant rate. That is, the *x* component of its velocity, v_x, is constant.
2. An object undergoing projectile motion moves vertically with a constant downward acceleration whose magnitude, denoted by g, is equal to 9.80 $\mathrm{m/s^2}$ near the surface of the earth. Hence, the *y* component of its velocity, v_y, changes continuously.
3. An object undergoing projectile motion will undergo the horizontal and vertical motions described above from the instant it is launched until the instant it strikes the ground again. Even though the horizontal and vertical motions can be treated independently, they are related by the fact that they occur for exactly the same amount of time, namely the time t the projectile is in the air.

The figure shows the trajectory (i.e., the path) of a ball undergoing projectile motion over level ground. The time $t_0 = 0\ \mathrm{s}$ corresponds to the moment just after the ball is launched from position $x_0 = 0\ \mathrm{m}$ and $y_0 = 0\ \mathrm{m}$. Its launch velocity, also called the initial velocity, is $\vec{v}_0$.



Two other points along the trajectory are indicated in the figure.

- One is the moment the ball reaches the peak of its trajectory, at time t_1 with velocity $\vec{v}_1$. Its position at this moment is denoted by (x_1, y_1) or (x_1, y_{max}) since it is at its maximum height.
- The other point, at time t_2 with velocity $\vec{v}_2$, corresponds to the moment just before the ball strikes the ground on the way back down. At this time its position is (x_2, y_2), also known as (x_{max}, y_2) since it is at its maximum horizontal range.

Projectile motion is symmetric about the peak, provided the object lands at the same vertical height from which is was launched, as is the case here. Hence $y_2 = y_0 = 0\ \mathrm{m}$.

Part A

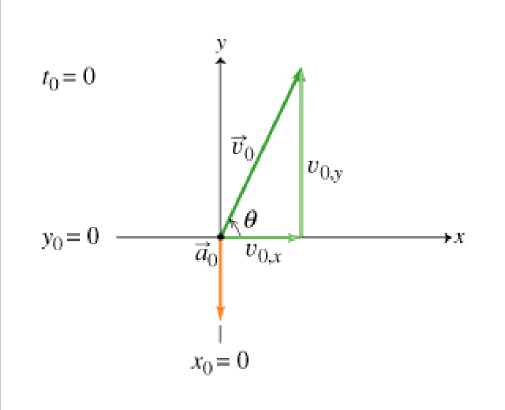
How do the speeds v_0, v_1, and v_2 (at times t_0, t_1, and t_2) compare?

ANSWER:

- ○ $v_0 = v_1 = v_2 > 0$
- ○ $v_0 = v_2 > v_1 = 0$
- ◉ $v_0 = v_2 > v_1 > 0$
- ○ $v_0 > v_1 > v_2 > 0$
- ○ $v_0 > v_2 > v_1 = 0$

Here v_0 equals v_2 by symmetry and both exceed v_1. This is because v_0 and v_2 include vertical speed as well as the constant horizontal speed.

Consider a diagram of the ball at time t_0. Recall that t_0 refers to the instant just after the ball has been launched, so it is still at ground level ($x_0 = y_0 = 0 \mathrm{~m}$). However, it is already moving with initial velocity $\vec{v}_0$, whose magnitude is $v_0 = 30.0 \mathrm{~m/s}$ and direction is $\theta = 60.0$ degrees counterclockwise from the positive *x* direction.



Part B

What are the values of the intial velocity vector components $v_{0,x}$ and $v_{0,y}$ (both in $\mathrm{m/s}$) as well as the acceleration vector components $a_{0,x}$ and $a_{0,y}$ (both in $\mathrm{m/s^2}$)? Here the subscript 0 means "at time t_0."

Hint 1. Determining components of a vector that is aligned with an axis

If a vector points along a single axis direction, such as in the positive *x* direction, its *x* component will be its full magnitude, whereas its *y* component will be zero since the vector is perpendicular to the *y* direction. If the vector points in the negative *x* direction, its *x* component will be the negative of its full magnitude.

Hint 2. Calculating the components of the initial velocity

Notice that the vector $\vec{v}_0$ points up and to the right. Since "up" is the positive *y* axis direction and "to the right" is the positive *x* axis direction, $v_{0,x}$ and $v_{0,y}$ will both be positive.

As shown in the figure, $v_{0,x}$, $v_{0,y}$, and v_0 are three sides of a right triangle, one angle of which is θ. Thus $v_{0,x}$ and $v_{0,y}$ can be found using the definition of the sine and cosine functions given below. Recall that $v_0 = 30.0 \mathrm{~m/s}$ and $\theta = 60.0$ degrees and note that

$$\sin(\theta) = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{v_{0,y}}{v_0},$$

$$\cos(\theta) = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} = \frac{v_{0,x}}{v_0}.$$

What are the values of $v_{0,x}$ and $v_{0,y}$?

Enter your answers numerically in meters per second separated by a comma.

ANSWER:

15.0, 26.0 $\mathrm{m/s}$

ANSWER:

- ○ 30.0, 0, 0, 0
- ○ 0, 30.0, 0, 0
- ○ 15.0, 26.0, 0, 0
- ○ 30.0, 0, 0, -9.80
- ○ 0, 30.0, 0, -9.80
- ◉ 15.0, 26.0, 0, -9.80
- ○ 15.0, 26.0, 0, +9.80

Also notice that at time t_2, just before the ball lands, its velocity components are $v_{2,x} = 15\ \mathrm{m/s}$ (the same as always) and $v_{2,y} = -26.0\ \mathrm{m/s}$ (the same size but opposite sign from $v_{0,y}$ by symmetry). The acceleration at time t_2 will have components (0, -9.80 $\mathrm{m/s^2}$), exactly the same as at t_0, as required by Rule 2.

The peak of the trajectory occurs at time t_1. This is the point where the ball reaches its maximum height y_{max}. At the peak the ball switches from moving up to moving down, even as it continues to travel horizontally at a constant rate.

Part C

What are the values of the velocity vector components $v_{1,x}$ and $v_{1,y}$ (both in $\mathrm{m/s}$) as well as the acceleration vector components $a_{1,x}$ and $a_{1,y}$ (both in $\mathrm{m/s^2}$)? Here the subscript 1 means that these are all at time t_1.

ANSWER:

- ○ 0, 0, 0, 0
- ○ 0, 0, 0, -9.80
- ○ 15.0, 0, 0, 0
- ◉ 15.0, 0, 0, -9.80
- ○ 0, 26.0, 0, 0
- ○ 0, 26.0, 0, -9.80
- ○ 15.0, 26.0, 0, 0
- ○ 15.0, 26.0, 0, -9.80

At the peak of its trajectory the ball continues traveling horizontally at a constant rate. However, at this moment it stops moving up and is about to move back down. This constitutes a downward-directed change in velocity, so the ball is accelerating downward even at the peak.

The flight time refers to the total amount of time the ball is in the air, from just after it is launched (t_0) until just before it lands (t_2). Hence the flight time can be calculated as $t_2 - t_0$, or just t_2 in this particular situation since $t_0 = 0$. Because the ball lands at the same height from which it was launched, by symmetry it spends half its flight time traveling up to the peak and the other half traveling back down. The flight time is determined by the initial vertical component of the velocity and by the acceleration. The flight time does not depend on whether the object is moving horizontally while it is in the air.

Part D

If a second ball were dropped from rest from height y_{max}, how long would it take to reach the ground? Ignore air resistance.

Check all that apply.

Hint 1. Kicking a ball of cliff; a related problem

Consider two balls, one of which is dropped from rest off the edge of a cliff at the same moment that the other is kicked horizontally off the edge of the cliff. Which ball reaches the level ground at the base of the cliff first? Ignore air resistance.

Hint 1. Comparing position, velocity, and acceleration of the two balls

Both balls start at the same height and have the same initial *y* velocity ($v_{0,y} = 0$) as well as the same acceleration ($\vec{a} = g$ downward). They differ only in their *x* velocity (one is zero, the other nonzero). This difference will affect their *x* motion but not their *y* motion.

ANSWER:

- ○ The ball that falls straight down strikes the ground first.
- ○ The ball that was kicked so it moves horizontally as it falls strikes the ground first.
- ◉ Both balls strike the ground at the same time.

The fact that one ball moves horizontally as it falls does not influence its vertical motion. Hence both balls are at the same height at all moments in time and thus they strike the ground at the same instant.
Now return to the original question, in which you are asked to compare the flight time for a ball that rises from the ground to a peak and then falls back down to the ground with the flight time for a second ball that only needs to fall from the peak height to the ground.

ANSWER:

- [ ] t_0
- [x] t_1
- [ ] t_2
- [x] $t_2 - t_1$
- [x] $\frac{t_2}{2}$

In projectile motion over level ground, it takes an object just as long to rise from the ground to the peak as it takes for it to fall from the peak back to the ground.

The *range* R of the ball refers to how far it moves horizontally, from just after it is launched until just before it lands. Range is defined as $x_2 - x_0$, or just x_2 in this particular situation since $x_0 = 0$.

Range can be calculated as the product of the flight time t_2 and the *x* component of the velocity v_x (which is the same at all times, so $v_x = v_{0,x}$). The value of v_x can be found from the launch speed v_0 and the launch angle θ using trigonometric functions, as was done in Part B. The flight time is related to the initial *y* component of the velocity, which may also be found from v_0 and θ using trig functions.

The following equations may be useful in solving projectile motion problems, but these equations apply only to a projectile launched over level ground from position ($x_0 = y_0 = 0$) at time $t_0 = 0$ with initial speed v_0 and launch angle θ measured from the horizontal. As was the case above, t_2 refers to the flight time and R refers to the range of the projectile.

$$\text{flight time: } t_2 = \frac{2v_{0,y}}{g} = \frac{2v_0\sin(\theta)}{g}$$

$$\text{range: } R = v_x t_2 = \frac{v_0^2\sin(2\theta)}{g}$$

In general, a high launch angle yields a long flight time but a small horizontal speed and hence little range. A low launch angle gives a larger horizontal speed, but less flight time in which to accumulate range. The launch angle that achieves the maximum range for projectile motion over level ground is 45 degrees.

Part E

Which of the following changes would increase the range of the ball shown in the original figure?

Check all that apply.

ANSWER:

- [x] Increase v_0 above 30 $\mathrm{m/s}$.
- [ ] Reduce v_0 below 30 $\mathrm{m/s}$.
- [x] Reduce θ from 60 $\mathrm{degrees}$ to 45 $\mathrm{degrees}$.
- [ ] Reduce θ from 60 $\mathrm{degrees}$ to less than 30 $\mathrm{degrees}$.
- [ ] Increase θ from 60 $\mathrm{degrees}$ up toward 90 $\mathrm{degrees}$.

A solid understanding of the concepts of projectile motion will take you far, including giving you additional insight into the solution of projectile motion problems numerically. Even when the object does not land at the same height from which is was launched, the rules given in the introduction will still be useful.
Recall that air resistance is assumed to be negligible here, so this projectile motion analysis may not be the best choice for describing things like frisbees or feathers, whose motion is strongly influenced by air. The value of the gravitational free-fall acceleration g is also assumed to be constant, which may not be appropriate for objects that move vertically through distances of hundreds of kilometers, like rockets or missiles. However, for problems that involve relatively dense projectiles moving close to the surface of the earth, these assumptions are reasonable.

Position, Velocity, and Acceleration

Learning Goal:

To identify situations when position, velocity, and /or acceleration change, realizing that change can be in direction or magnitude.

If an object's position is described by a function of time, $\vec{r}(t)$ (measured from a nonaccelerating reference frame), then the object's velocity is described by the time derivative of the position, $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$, and the object's acceleration is described by the time derivative of the velocity, $\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2}$.

It is often convenient to discuss the average of the latter two quantities between times t_1 and t_2:

$$\vec{v}_{\text{avg}}(t_1, t_2) = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

and

$$\vec{a}_{\text{avg}}(t_1, t_2) = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}.$$

Part A

You throw a ball. Air resistance on the ball is negligible. Which of the following functions change with time as the ball flies through the air?

Hint 1. Newton's 2nd Law

Newton's 2nd Law ($\vec{F} = m\vec{a}$) says that every acceleration is caused by a force. What force acts on the ball after it leaves your hand?

Does this force change during the flight of the ball, or is it constant through time?

ANSWER:

- ○ only the position of the ball
- ○ only the velocity of the ball
- ○ only the acceleration of the ball
- ◉ the position and velocity of the ball
- ○ the position and the velocity and acceleration of the ball

Part B

You are driving a car at 65 mph. You are traveling north along a straight highway. What could you do to give the car a nonzero acceleration?

Hint 1. What constitutes a nonzero acceleration?

The velocity of the car is described by a vector function, meaning it has both magnitude (65 mph) and direction (north). The car experiences a nonzero acceleration if you change either the magnitude of the velocity or the direction of the velocity.

ANSWER:

- Press the brake pedal.
- Turn the steering wheel.
- Either press the gas or the brake pedal.
- (selected) Either press the gas or the brake pedal or turn the steering wheel.

Part C

A ball is lodged in a hole in the floor near the outside edge of a merry-go-round that is turning at constant speed. Which kinematic variable or variables change with time, assuming that the position is measured from an origin at the center of the merry-go-round?

Hint 1. Change of a vector

A vector quantity has both magnitude and direction. The vector changes with time if *either* of these quantities changes with time.

ANSWER:

- the position of the ball only
- the velocity of the ball only
- the acceleration of the ball only
- both the position and velocity of the ball
- (selected) the position and velocity and acceleration of the ball

Part D

For the merry-go-round problem, do the magnitudes of the position, velocity, and acceleration vectors change with time?

Hint 1. Change of magnitude of a vector

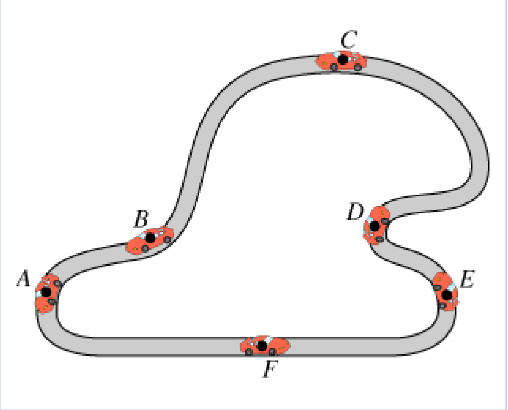
A vector quantity has both magnitude and direction. The magnitude of a vector changes with time *only* if the length changes with time.

ANSWER:

- yes
- (selected) no

Accelerating along a Racetrack

A road race is taking place along the track shown in the figure . All of the cars are moving at constant speeds. The car at point F is traveling along a straight section of the track, whereas all the other cars are moving along curved segments of the track.



Part A

Let $\vec{v}_A$ be the velocity of the car at point A. What can you say about the acceleration of the car at that point?

Hint 1. Acceleration along a curved path

Since acceleration is a vector quantity, an object moving at constant speed along a curved path has nonzero acceleration because the direction of its velocity $\vec{v}$ is changing, even though the magnitude of its velocity (the speed) is constant. Moreover, if the speed is constant, the object's acceleration is always perpendicular to the velocity vector $\vec{v}$ at each point along the curved path and is directed toward the center of curvature of the path.

ANSWER:

- ○ The acceleration is parallel to $\vec{v}_A$.
- ◉ The acceleration is perpendicular to $\vec{v}_A$ and directed toward the inside of the track.
- ○ The acceleration is perpendicular to $\vec{v}_A$ and directed toward the outside of the track.
- ○ The acceleration is neither parallel nor perpendicular to $\vec{v}_A$.
- ○ The acceleration is zero.

Part B

Let $\vec{v}_C$ be the velocity of the car at point C. What can you say about the acceleration of the car at that point?

Hint 1. Acceleration along a curved path

Since acceleration is a vector quantity, an object moving at constant speed along a curved path has nonzero acceleration because the direction of its velocity $\vec{v}$ is changing, even though the magnitude of its velocity (the speed) is constant. Moreover, if the speed is constant, the object's acceleration is always perpendicular to the velocity vector $\vec{v}$ at each point along the curved path and is directed toward the center of curvature of the path.

ANSWER:

- ○ The acceleration is parallel to $\vec{v}_C$.
- ◉ The acceleration is perpendicular to $\vec{v}_C$ and pointed toward the inside of the track.
- ○ The acceleration is perpendicular to $\vec{v}_C$ and pointed toward the outside of the track.
- ○ The acceleration is neither parallel nor perpendicular to $\vec{v}_C$.
- ○ The acceleration is zero.

Part C

Let $\vec{v}_D$ be the velocity of the car at point D. What can you say about the acceleration of the car at that point?

Hint 1. Acceleration along a curved path

Since acceleration is a vector quantity, an object moving at constant speed along a curved path has nonzero acceleration because the direction of its velocity $\vec{v}$ is changing, even though the magnitude of its velocity (the speed) is constant. Moreover, if the speed is constant, the object's acceleration is always perpendicular to the velocity vector $\vec{v}$ at each point along the curved path and is directed toward the center of curvature of the path.

ANSWER:

- ( ) The acceleration is parallel to $\vec{v}_D$.
- ( ) The acceleration is perpendicular to $\vec{v}_D$ and pointed toward the inside of the track.
- (•) The acceleration is perpendicular to $\vec{v}_D$ and pointed toward the outside of the track.
- ( ) The acceleration is neither parallel nor perpendicular to $\vec{v}_D$.
- ( ) The acceleration is zero.

Part D

Let $\vec{v}_F$ be the velocity of the car at point F. What can you say about the acceleration of the car at that point?

Hint 1. Acceleration along a straight path

The velocity of an object that moves along a straight path is always parallel to the direction of the path, and the object has a nonzero acceleration only if the magnitude of its velocity changes in time.

ANSWER:

- ( ) The acceleration is parallel to $\vec{v}_F$.
- ( ) The acceleration is perpendicular to $\vec{v}_F$ and pointed toward the inside of the track.
- ( ) The acceleration is perpendicular to $\vec{v}_F$ and pointed toward the outside of the track.
- ( ) The acceleration is neither parallel nor perpendicular to $\vec{v}_F$.
- (•) The acceleration is zero.

Part E

Assuming that all cars have equal speeds, which car has the acceleration of the greatest magnitude, and which one has the acceleration of the least magnitude?

Use A for the car at point A, B for the car at point B, and so on. Express your answer as the name the car that has the greatest magnitude of acceleration followed by the car with the least magnitude of accelation, and separate your answers with a comma.

Hint 1. How to approach the problem

Recall that the magnitude of the acceleration of an object that moves at constant speed along a curved path is inversely proportional to the radius of curvature of the path.

ANSWER:

D,F

Part F

Assume that the car at point A and the one at point E are traveling along circular paths that have the same radius. If the car at point A now moves twice as fast as the car at point E, how is the magnitude of its acceleration related to that of car E.

Hint 1. Find the acceleration of the car at point E

Let r be the radius of the two curves along which the cars at points A and E are traveling. What is the magnitude a_E of the acceleration of the car at point E?

Express your answer in terms of the radius of curvature r and the speed v_E of car E.

Hint 1. Uniform circular motion

The magnitude a of the acceleration of an object that moves with constant speed v along a circular path of radius r is given by

$$a = \frac{v^2}{r}.$$

ANSWER:

$a_{\rm E}$ = $\frac{{v_E}^2}{r}$

Hint 2. Find the acceleration of the car at point A

If $v_A = 2v_E$, what is the acceleration $a_{\rm A}$ of the car at point A? Let r be the radius of the two curves along which the cars at points A and E are traveling.

Express your answer in terms of the speed $v_{\rm E}$ of the car at E and the radius r.

Hint 1. Uniform circular motion

The magnitude of the acceleration of an object that moves with constant speed v along a circular path of radius r is given by

$$a = \frac{v^2}{r}.$$

ANSWER:

$a_{\rm A}$ = $\frac{4{v_E}^2}{r}$

Since the magnitude of the acceleration of an object that moves with constant speed along a circular path is proportional to the *square* of the speed, the acceleration of the car at point A is proportional to $v_A^2 = (2v_E)^2$.

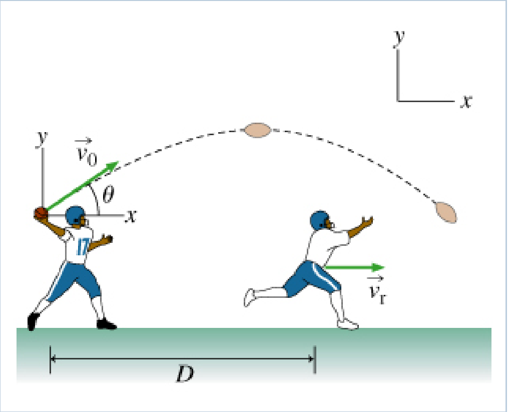
ANSWER:

- ○ The magnitude of the acceleration of the car at point A is twice that of the car at point E.
- ○ The magnitude of the acceleration of the car at point A is the same as that of the car at point E.
- ○ The magnitude of the acceleration of the car at point A is half that of the car at point E.
- ◉ The magnitude of the acceleration of the car at point A is four times that of the car at point E.

Advice for the Quarterback

A quarterback is set up to throw the football to a receiver who is running with a constant velocity $\vec{v}_{\rm r}$ directly away from the quarterback and is now a distance D away from the quarterback. The quarterback figures that the ball must be thrown at an angle θ to the horizontal and he estimates that the receiver must catch the ball a time interval $t_{\rm c}$ after it is thrown to avoid having opposition players prevent the receiver from making the catch. In the following you may assume that the ball is thrown and caught at the same height above the level playing field. Assume that the *y* coordinate of the ball at the instant it is thrown or caught is $y = 0$ and that the horizontal position of the quaterback is $x = 0$.

Use g for the magnitude of the acceleration due to gravity, and use the pictured inertial coordinate system when solving the problem.



Part A

Find v_{0y}, the vertical component of the velocity of the ball when the quarterback releases it.

Express v_{0y} in terms of t_c and g.

Hint 1. Equation of motion in y direction

What is the expression for $y(t)$, the height of the ball as a function of time?

Answer in terms of t, g, and v_{0y}.

ANSWER:

$y(t)$ = $v_{0y}t - \frac{1}{2}gt^2$

Also accepted: $y_0 + v_{0y}t - \frac{1}{2}gt^2$

Hint 2. Height at which the ball is caught, $y(t_c)$

Remember that after time t_c the ball was caught at the same height as it had been released. That is, $y(t_c) = y_0 = 0$.

ANSWER:

v_{0y} = $\frac{gt_c}{2}$

Part B

Find v_{0x}, the initial horizontal component of velocity of the ball.

Express your answer for v_{0x} in terms of D, t_c, and v_r.

Hint 1. Receiver's position

Find x_r, the receiver's position before he catches the ball.

Answer in terms of D, v_r, and t_c.

ANSWER:

x_r = $D + v_r t_c$

Hint 2. Football's position

Find x_c, the horizontal distance that the ball travels before reaching the receiver.

Answer in terms of v_{0x} and t_c.

ANSWER:

$x_c = v_{0x} t_c$

ANSWER:

$v_{0x} = \frac{D}{t_c} + v_r$

Also accepted: $\frac{D + v_r t_c}{t_c}$

Part C

Find the speed v_0 with which the quarterback must throw the ball.

Answer in terms of D, t_c, v_r, and g.

Hint 1. How to approach the problem

Remember that velocity is a vector; from solving Parts A and B you have the two components, from which you can find the magnitude of this vector.

ANSWER:

$v_0 = \sqrt{\left(\frac{D}{t_c} + v_r\right)^2 + \left(\frac{g t_c}{2}\right)^2}$

Part D

Assuming that the quarterback throws the ball with speed v_0, find the angle θ above the horizontal at which he should throw it.

Your solution should contain an inverse trig function (entered as asin, acos, **or** atan**). Give your answer in terms of already known quantities, v_{0x}, v_{0y}, and v_0.**

Hint 1. Find angle θ from v_{0x} and v_{0y}

Think of velocity as a vector with Cartesian coordinates $v_{0x}\hat{x}$ and $v_{0y}\hat{y}$. Find the angle θ that this vector would make with the *x* axis using the results of Parts A and B.

ANSWER:

$\theta = \mathrm{atan}\left(\frac{v_{0y}}{v_{0x}}\right)$

Also accepted: $\mathrm{acos}\left(\frac{v_{0x}}{v_0}\right)$, $\mathrm{asin}\left(\frac{v_{0y}}{v_0}\right)$

Battleship Shells

A battleship simultaneously fires two shells toward two identical enemy ships. One shell hits ship A, which is close by, and the other hits ship B, which is farther away. The two shells are fired at the same *speed*. Assume that air resistance is negligible and that the magnitude of the acceleration due to gravity is g.

Note that after Part B the question setup changes slightly.

Part A

What shape is the trajectory (graph of *y* vs. *x*) of the shells?

ANSWER:

- ○ straight line
- ◉ parabola
- ○ hyperbola
- ○ The shape cannot be determined.

Part B

For two shells fired at the same speed which statement about the horizontal distance traveled is correct?

Hint 1. Two things to consider

The distance traveled is the product of the *x* component of the velocity and the time in the air. How does the *y* component of the velocity affect the "air time"? What angle would give the longest range?

ANSWER:

- ○ The shell fired at a larger angle with respect to the horizontal lands farther away.
- ◉ The shell fired at an angle closest to 45 degrees lands farther away.
- ○ The shell fired at a smaller angle with respect to the horizontal lands farther away.
- ○ The lighter shell lands farther away.

Now, consider for the remaining parts of the question below that both shells are fired at an angle greater than 45 degrees with respect to the horizontal. Remember that enemy ship A is closer than enemy ship B.

Part C

Which shell is fired at the larger angle?

Hint 1. Consider the limiting case

Consider the case in which a shell is fired at 90 degrees above the horizontal (i.e., straight up). What distance x will the shell travel? Now lower the angle at which the shell is fired. What happens to the distance x?

ANSWER:

- ◉ A
- ○ B
- ○ Both shells are fired at the same angle.

Part D

Which shell is launched with a greater vertical velocity, v_y?

ANSWER:

- ◉ A
- ○ B
- ○ Both shells are launched with the same vertical velocity.

Part E

Which shell is launched with a greater horizontal velocity, v_x?

ANSWER:

- ◯ A
- ◉ B
- ◯ Both shells are launched with the same horizontal velocity.

Part F

Which shell reaches the greater maximum height?

Hint 1. What determines maximum height?

What determines the maximum height reached by the shell?

ANSWER:

- ◯ horizontal velocity
- ◉ vertical velocity
- ◯ mass of the shell

ANSWER:

- ◉ A
- ◯ B
- ◯ Both shells reach the same maximum height.

Part G

Which shell has the longest travel time (time elapsed between being fired and hitting the enemy ship)?

Hint 1. Consider the limiting case

If a shell is fired exactly horizontally (0 degrees) the shell hits the ground right away. As the angle above the horizontal increases, what happens to the time of travel? Does this change as the angle becomes greater than 45 degrees?

ANSWER:

- ◉ A
- ◯ B
- ◯ Both shells have the same travel time.

Exercise 3.17

A major leaguer hits a baseball so that it leaves the bat at a speed of 28.5$\mathrm{m/s}$ and at an angle of 36.7° above the horizontal. You can ignore air resistance.

Part A

At what *two* times is the baseball at a height of 10.0m above the point at which it left the bat?

Give your answers in ascending order separated with comma.

ANSWER:

$t_1, t_2 = \frac{v_0 \sin(\alpha)}{9.80} - \sqrt{\left(\frac{v_0 \sin(\alpha)}{9.80}\right)^2 - \frac{2h}{9.80}}, \frac{v_0 \sin(\alpha)}{9.80} + \sqrt{\left(\frac{v_0 \sin(\alpha)}{9.80}\right)^2 - \frac{2h}{9.80}}$ = 0.748, 2.73 s

Part B

Calculate the horizontal component of the baseball's velocity at an earlier time calculated in part (a).

ANSWER:

v_x = $v_0\cos(\alpha)$ = 22.9 $\mathrm{m/s}$

Part C

Calculate the vertical component of the baseball's velocity at an earlier time calculated in part (a).

ANSWER:

v_y = $9.80\sqrt{\left(\frac{v_0\sin(\alpha)}{9.80}\right)^2-\frac{2h}{9.80}}$ = 9.70 $\mathrm{m/s}$

Part D

Calculate the horizontal component of the baseball's velocity at a later time calculated in part (a).

ANSWER:

v_x = $v_0\cos(\alpha)$ = 22.9 $\mathrm{m/s}$

Part E

Calculate the vertical component of the baseball's velocity at a later time calculated in part (a).

ANSWER:

v_y = $-9.80\sqrt{\left(\frac{v_0\sin(\alpha)}{9.80}\right)^2-\frac{2h}{9.80}}$ = -9.70 $\mathrm{m/s}$

Part F

What is the magnitude of the baseball's velocity when it returns to the level at which it left the bat?

ANSWER:

v = v_0 = 28.5 $\mathrm{m/s}$

Part G

What is the direction of the baseball's velocity when it returns to the level at which it left the bat?

ANSWER:

θ = $\frac{\alpha\cdot 180}{\pi}$ = 36.7 ° below the horizontal

Exercise 3.28

The radius of the earth's orbit around the sun (assumed to be circular) is $1.50\times10^{8}\ \mathrm{km}$, and the earth travels around this orbit in 365 days.

Part A

What is the magnitude of the orbital velocity of the earth in $\mathrm{m/s}$?

ANSWER:

2.97×10^{4} $\mathrm{m/s}$

Part B

What is the radial acceleration of the earth toward the sun?

ANSWER:

5.91×10^{-3} $\mathrm{m/s^2}$

Part C

What is the magnitude of the orbital velocity of the planet Mercury (orbit radius $= 5.79 \times 10^{7}\ \mathrm{km}$, orbital period = 88.0 days)?

ANSWER:

4.78×10^{4} $\mathrm{m/s}$

Part D

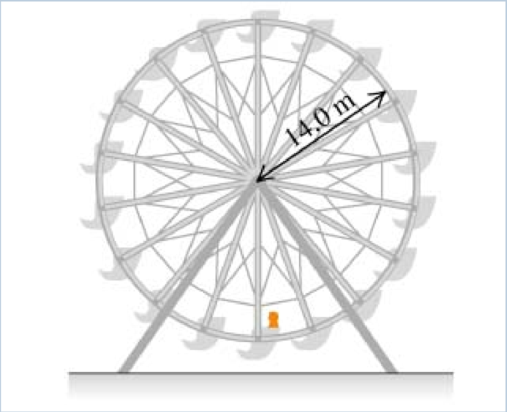
What is the radial acceleration of the Mercury?

ANSWER:

3.95×10^{-2} $\mathrm{m/s^2}$

Exercise 3.29

A Ferris wheel with radius 14.0 m is turning about a horizontal axis through its center (the figure). The linear speed of a passenger on the rim is constant and equal to 7.60$\mathrm{m/s}$.



Part A

What is the magnitude of the passenger's acceleration as she passes through the lowest point in her circular motion?

ANSWER:

$a = \frac{vv}{14} = 4.13$ $\mathrm{m/s^2}$

Part B

What is the direction of the passenger's acceleration as she passes through the lowest point in her circular motion?

ANSWER:

- (●) towards the center
- ( ) outwards the center

Part C

What is the magnitude of the passenger's acceleration as she passes through the highest point in her circular motion?

ANSWER:

$a = \frac{vv}{14} = 4.13$ $\mathrm{m/s^2}$

Part D

What is the direction of the passenger's acceleration as she passes through the highest point in her circular motion?

ANSWER:

- (selected) towards the center
- outwards the center

Part E

How much time does it take the Ferris wheel to make one revolution?

ANSWER:

T = $\frac{2\pi \cdot 14}{v}$ = 11.6 s

Exercise 3.38

An airplane pilot wishes to fly due west. A wind of 84.0$\mathrm{km/h}$ is blowing toward the south.

Part A

If the airspeed of the plane (its speed in still air) is 425.0$\mathrm{km/h}$, in which direction should the pilot head?

Express your answer as an angle measured north of west.

ANSWER:

θ = $\frac{\mathrm{asin}\left(\frac{v_w}{v_a}\right) \cdot 180}{\pi}$ = 11.4 $^\circ$ north of west

Part B

What is the speed of the plane over the ground?

ANSWER:

v = $\sqrt{(v_a{}^2 - v_w{}^2)}$ = 417 $\mathrm{km/h}$

Problem 3.53

In fighting forest fires, airplanes work in support of ground crews by dropping water on the fires. A pilot is practicing by dropping a canister of red dye, hoping to hit a target on the ground below.

Part A

If the plane is flying in a horizontal path at an altitude of 97.0m above the ground and with a speed of 62.0$\mathrm{m/s}$, at what horizontal distance from the target should the pilot release the canister? Ignore air resistance.

Take free fall acceleration to be g.

ANSWER:

x = $v\sqrt{\frac{2h}{g}}$ = 276 m

Problem 3.74

A student sits atop a platform a distance h above the ground. He throws a large firecracker horizontally with a speed v. However, a wind blowing parallel to the ground gives the firecracker a constant horizontal acceleration with magnitude a. This results in the firecracker reaching the ground directly under the student.

Part A

Determine the height h in terms of v, a, and g. You can ignore the effect of air resistance on the vertical motion.

ANSWER:

$h = \frac{2gv^2}{a^2}$

Problem 3.82

When a train's velocity is 12.0 $\mathrm{m/s}$ eastward, raindrops that are falling vertically with respect to the earth make traces that are inclined 30.0 $^\circ$ to the vertical on the windows of the train.

Part A

What is the horizontal component of a drop's velocity with respect to the earth?

ANSWER:

v_x = 0 $\mathrm{m/s}$

Part B

What is the horizontal component of a drop's velocity with respect to the train?

ANSWER:

v_x = -12.0 $\mathrm{m/s}$

Part C

What is the magnitude of the velocity of the raindrop with respect to the earth?

ANSWER:

v = 20.8 $\mathrm{m/s}$

Part D

What is the magnitude of the velocity of the raindrop with respect to the train?

ANSWER:

v = 24.0 $\mathrm{m/s}$