#8 Projectile Motion Post-class

Due: 11:00am on Monday, September 10, 2012

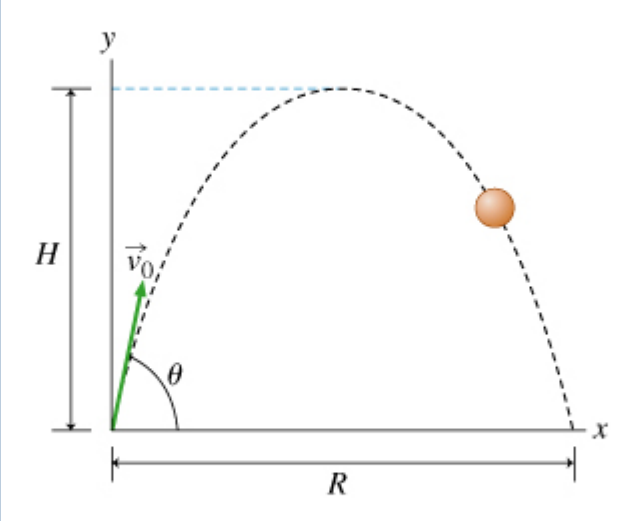
Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Projectile Motion Tutorial

Learning Goal:

Understand how to apply the equations for 1-dimensional motion to the *y* and *x* directions separately in order to derive standard formulae for the range and height of a projectile.

A projectile is fired from ground level at time $t = 0$, at an angle θ with respect to the horizontal. It has an initial speed v_0. In this problem we are assuming that the ground is level.



Part A

Find the time t_H it takes the projectile to reach its maximum height.

Express t_H in terms of v_0, θ, and g (the magnitude of the acceleration due to gravity).

Hint 1. A basic property of projectile motion

It is important to understand that the motion in the *y* direction is independent of the motion in the *x* direction because these directions are orthogonal (at right angles) to each other.

Hint 2. What condition applies at the top?

What is the value of the projectile's vertical velocity when it has reached the top of its trajectory?

ANSWER:

v_{top} = 0

Hint 3. Vertical velocity as a function of time

Find an expression for the projectile's vertical velocity as a function of time, $v_y(t)$.

Express your answer in terms of t, v_0, θ, and g, the acceleration due to gravity.

Hint 1. Initial velocity in *y* direction

What is the initial *y* component of velocity?

Provide your answer in terms of v_0 and θ.

ANSWER:

v_{y0} = $v_0 \sin(\theta)$

ANSWER:

$v_y(t)$ = $v_0 \sin(\theta) - gt$

Hint 4. Putting it all together

You now have a general expression for the y-velocity as a function of time, and you also know the the y-velocity at the top (i.e. at time t_H). This gives an equation which you can solve for t_H.

Hint 5. A list of possible answers

There are four answers in the pulldown list below, one of which is the correct answer for this question (the time to reach the top). Once you have finished this part, therefore, you will have the answer for the question. (However, you will still need to enter this answer into the answer box in the main part of the problem.)

ANSWER:

- ○ $\frac{v_0}{g\sin(\theta)}$
- ○ $\frac{v_0 g}{\sin(\theta)}$
- ◉ $\frac{v_0\sin(\theta)}{g}$
- ○ $\frac{g\sin(\theta)}{v_0}$

ANSWER:

$t_H = \frac{v_0\sin(\theta)}{g}$

Correct

Part B

Find t_R, the time at which the projectile hits the ground.

Express the time in terms of v_0, θ, and g.

Hint 1. Two possible approaches

There are two good ways to find the total flight time t_R: either by invoking the symmetry of this problem (limited to projectiles fired over level ground) or by finding a general expression for $y(t)$, the *y* position of the projectile, and setting this equal to the height at the end of the trajectory, y_R. The second method is more general (i.e., it would work if the projectile landed on a hill of height H). What is the value of y_R in this problem?

Express y_R in terms of quantities given in the introduction.

ANSWER:

y_R = 0

Hint 2. Some needed kinematics

Give an expression for the height as a function of time, $y(t)$, taking $y_0 = 0$.

Express your answer in terms of t, v_0, θ, and g.

Hint 1. Equation of motion

You should plug into the usual formula for the position with constant acceleration,

$$y(t) = y_0 + v_{0y}t + \frac{a_y t^2}{2}.$$

In this problem, what is v_{0y}, the initial y component of velocity?

Answer in terms of v_0 and θ.

ANSWER:

v_{0y} = $v_0 \sin(\theta)$

ANSWER:

$y(t)$ = $v_0 \sin(\theta)\, t - \frac{1}{2} g t^2$

Hint 3. Solving for t_R

You now know the equation for $y(t)$. By plugging in t_R you can determine $y(t_R)$. From Part B.i you know the value of y_R, which is equal to $y(t_R)$.

ANSWER:

t_R = $2\frac{v_0 \sin(\theta)}{g}$

Correct

Part C

Find H, the maximum height attained by the projectile.

Express the maximum height in terms of v_0, θ, and g.

Hint 1. Equation of motion

Keep in mind the equation of motion for $y(t)$ that you have found in Part B.ii. If you can't find the equation of motion and have not done Part B.ii, please finish this part now.

Hint 2. When is the projectile at the top of its trajectory?

At which time $t_{\rm max}$ will the projectile reach the maximum height?

Answer in terms of v_0, θ, and g.

ANSWER:

$t_{\rm max}$ = $\frac{v_0 \sin(\theta)}{g}$

Hint 3. Finding H

Remember that $H = y(t_{\rm max})$.

ANSWER:

$H(\theta)$ = $\frac{(v_0 \sin\theta)^2}{2g}$

Correct

Part D

Find the total distance R (often called the range) traveled in the *x* direction; in other words, find where the projectile lands.

Express the range in terms of v_0, θ, and g.

Hint 1. When does the projectile hit the ground?

The projectile reaches the ground at time t_R.

Hint 2. Where is the projectile as a function of time?

Give an expression for the *x* position of the particle as a function of time.

Answer in terms of t, v_0, and θ.

Hint 1. Acceleration in *x* direction

There is no acceleration in the *x* direction.

ANSWER:

$x(t)$ = $v_0 \cos(\theta)\, t$

Hint 3. Finding the range

Remember that $R = x(t_R)$.

Hint 4. A list of possible answers

Choose the correct answer for R from the following list. (You will still have to enter this answer into the main answer box to receive credit.)

ANSWER:

- $\frac{v_0^2}{g\sin(2\theta)}$
- $\frac{v_0\sin(\theta)}{g}$
- $\frac{v_0^2}{g\cos(2\theta)}$
- $\frac{v_0^2\sin(2\theta)}{g}$

ANSWER:

$R(\theta)$ = $2\frac{v_0\sin\theta}{g}v_0\cos\theta$

Correct

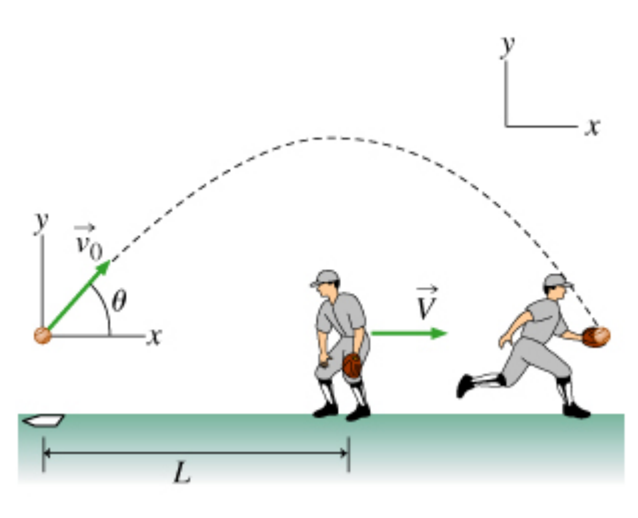
The actual formula for $R(\theta)$ is less important than how it is obtained:

1. Consider the *x* and *y* motion separately.
2. Find the time of flight from the *y*-motion
3. Find the *x*-position at the end of the flight - this is the range.

If you remember these steps, you can deal with many variants of the basic problem, such as: a cannon on a hill that fires horizontally (i.e. the second half of the trajectory), a projectile that lands on a hill, or a projectile that must hit a moving target.

± Speed of a Softball

A softball is hit over a third baseman's head with speed v_0 and at an angle θ from the horizontal. Immediately after the ball is hit, the third baseman turns around and runs straight back at a constant velocity $V = 7.00 \ \mathrm{m/s}$, for a time $t = 2.00 \ \mathrm{s}$. He then catches the ball at the same height at which it left the bat. The third baseman was initially $L = 18.0 \ \mathrm{m}$ from the location where the ball was hit at home plate.



Part A

Find v_0, the initial speed. Use $g = 9.80 \ \mathrm{m/s^2}$ for the magnitude of the acceleration due to gravity.

Express your answer numerically in meters per second to three significant figures.

Hint 1. Find the initial velocity in the *x* direction

Consider v_{0x} and v_{0y}, the respective horizontal and vertical components of the softball's initial velocity. Since we assume there is no air resistance, the horizontal component of the ball's velocity cannot change. What is v_{0x}?

Express your answer in terms of L, V, and t.

Hint 1. Distance traveled in the *x* direction

What is the distance x the ball travels horizontally before it is caught?

Express your answer in terms of L, V, and t.

ANSWER:

x = $L+Vt$

Try Again; 3 attempts remaining

ANSWER:

v_{0x} = $\frac{L+Vt}{t}$

Answer Requested

Hint 2. Find the initial velocity in the *y* direction

Using the equation of motion in the y direction, $y(t) = y_0 + v_{y0}t - \frac{gt^2}{2}$, find an expression for v_{0y}. Remember, the final height of the ball equals its initial height.

Give your answer in terms of g, the magnitude of the acceleration due to gravity, and t.

ANSWER:

v_{0y} = $\frac{1}{2}gt$

Answer Requested

Hint 3. Find the total initial velocity

Once you have v_{0x} and v_{0y}, you can find v_0 by using

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2}.$$

ANSWER:

v_0 = 18.8 $\mathrm{m/s}$

Correct

Part B

Find the angle θ in degrees.

Express your answer numerically in degrees to three significant figures.

ANSWER:

θ = 31.5 °

Correct

Part C

Find a vector expression for the velocity $\vec{v}$ of the softball 0.100 s before the ball is caught.

Use the notation v_x, v_y, an ordered pair of values separated by a commas Express your answer numerically in meters per second to three significant figures.

Hint 1. $v_x(t)$ vs $v_y(t)$

v_x is constant during the softball's motion, but v_y is a function of time.

Hint 2. Find v_y as a function of time

What is the equation for $v_y(t)$?

Give your answer in terms of v_{0y}, g and time t.

ANSWER:

$v_y(t)$ = $v_{0y} - gt$

Hint 3. Unit vectors

Remember that v_x is a projection of the velocty $\vec{v}$ onto the $\hat{i}$ vector, and v_y is a projection of the velocty $\vec{v}$ onto the $\hat{j}$ vector.

ANSWER:

$\vec{v}$ = 16.0,-8.82 $\mathrm{m/s}$

Correct

Part D

Find a vector expression for the position $\vec{r}$ of the softball 0.100 s before the ball is caught.

Use the notation x, y, an ordered pair of values separated by a comma, where x and y are expressed numerically in meters, as measured from the point where the softball initially left the bat. Express your answer to three significant figures.

Hint 1. Equations of motion

Remember the equations of motion for *x* and y positions:

$$x(t) = x_0 + v_{0x}t$$

and

$$y(t) = y_0 + v_{0y}t - \frac{gt^2}{2}.$$

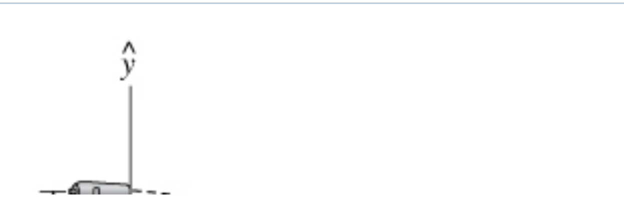
ANSWER:

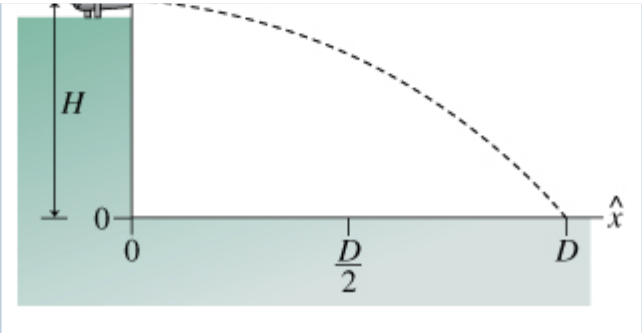
$\vec{r}$ = 30.4,0.932 m

Correct

± Horizontal Cannon on a Cliff

A cannonball is fired horizontally from the top of a cliff. The cannon is at height H = 70.0m above ground level, and the ball is fired with initial horizontal speed v_0. Assume acceleration due to gravity to be g = 9.80$\mathrm{m/s^2}$.





Part A

Assume that the cannon is fired at time $t = 0$ and that the cannonball hits the ground at time t_g. What is the *y* position of the cannonball at the time $t_g/2$?

Answer numerically in units of meters.

Hint 1. How to approach the problem

In this problem, you are asked to find the height y at a certain time. Nothing is asked or given about the distance coordinate x. Therefore, you only need to consider the *y* equations of motion and variables. Write down the known and unknown *y* variables. Then find the appropriate equation(s) and substitute for the values.

Hint 2. Identify the knowns and unknowns

The information given in the introduction can be used to determine the knowns and unknowns in the problem. For this part, you need to consider only the *y* variables. In terms of the given coordinate system, the initial height y_0 can be chosen to be 70.0m . Of course, the acceleration in the *y* direction is exclusively the acceleration due to gravity: $a_y = -g$.

Which of the following quantities is/are also known?

Check all that apply.

ANSWER:

- [x] y at time t_g
- [x] y_0
- [ ] v_y at time t_g
- [x] v_{0y}

Hint 3. Determine which equation to use to find the height at the requested time

Three equations that describe motion in the *y* direction are given below. Which would you use to determine the height y of the cannonball at time $t_g/2$?

ANSWER:

- ○ $v_y = v_{0y} - gt$
- ◉ $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$
- ○ $v_y^2 = v_{0y}^2 - 2g(y - y_0)$

Hint 4. Find t_g

What is the value of t_g?

Express your answer numerically in seconds.

Hint 1. Identify which equation to use to find t_g

Which of the equations below could you use to find t_g?

ANSWER:

- $x = x_0 + v_{0x}t$
- $v_y = v_{0y} - gt$
- (selected) $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

ANSWER:

t_g = 3.78 s

ANSWER:

$y(t_g/2)$ = 52.5 m

Correct

The same answer can be obtained more easily (perhaps you did it this way) if you notice that $v_{0y} = 0$. This means that the vertical displacement is given by $\Delta y = -\frac{1}{2}gt^2$ and therefore $\Delta y(t_g/2)$ is one-quarter of H; then $y(t_g/2) = \frac{3}{4}H$.

Part B

Given that the projectile lands at a distance D = 170m from the cliff, as shown in the figure, find the initial speed of the projectile, v_0.

Express the initial speed numerically in meters per second.

Hint 1. How to approach the problem

The initial speed v_0 can be determined if you know either v_{0x} or v_{0y} and the angle of elevation of the cannon θ. Then you could use either

$$v_{0x} = v_0 \cos\theta.$$

or

$$v_{0y} = v_0 \sin\theta.$$

In this case, $\theta = 0$, so there is no component of velocity in the vertical direction, and the second equation is not useful. You need to determine v_{0x} from the given information. In this case, $v_0 = v_{0x}$ because the cannonball is launched with only an initial horizontal velocity. You are given the horizontal distance traveled by the cannonball and need to find its horizontal velocity (which is constant because the only force acting on the cannonball, gravity, acts exclusively in the vertical direction). You are not asked for or given any information about the *y* variables. Therefore, you need to consider only the *x* variables and equations.

Hint 2. Knowns and unknowns

Since you are asked to find v_{0x}, you need to determine the knowns and unknowns only for the *x* variables. In the coordinate system shown in the figure in the problem introduction, the known/given *x* variables are

$$x_0 = 0 \text{ and}$$

$$x(t_g) = 170 \, \mathrm{m} \; .$$

Hint 3. The equation to use

The equation that describes the motion in the *x* direction is

$$x(t) = x_0 + v_x t.$$

Substitute for the known quantities in this equation and solve for

$$v_x = v_0.$$

Keep in mind that $v_0 = v_{0x} = v_x$ because the cannonball is launched with an initial horizontal velocity and no initial vertical velocity, and the horizontal component of the cannonball's velocity is constant.

ANSWER:

v_0 = 45.0 $\mathrm{m/s}$

Correct

Part C

What is the *y* position of the cannonball when it is at distance $D/2$ from the hill? *If you need to*, you can use the trajectory equation for this projectile, which gives y in terms of x directly:

$$y = H - \frac{gx^2}{2v_{0x}^2}.$$

You should already know v_{0x} from the previous part.

Express the position of the cannonball numerically in meters.

ANSWER:

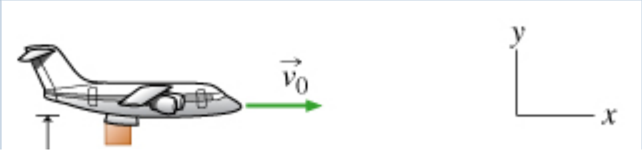
$y_{D/2}$ = 52.5 m

Correct

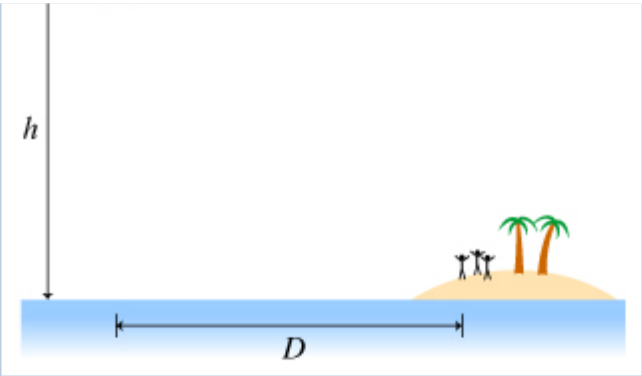
Not surprisingly, the answer to this part is the same as that in Part A because a projectile travels equal horizontal distances in equal amounts of time.

± Delivering a Package by Air

A relief airplane is delivering a food package to a group of people stranded on a very small island. The island is too small for the plane to land on, and the only way to deliver the package is by dropping it. The airplane flies horizontally with constant speed of 200mph at an altitude of 600m .



The positive *x* and *y* directions are defined in the figure. For all parts, assume that the "island" refers to the point at a distance D from the point at which the package is released, as shown in the figure. Ignore the height of this point above sea level. Assume that the acceleration due to gravity is g = 9.80$\mathrm{m/s^2}$.



Part A

After a package is ejected from the plane, how long will it take for it to reach sea level from the time it is ejected? Assume that the package, like the plane, has an initial velocity of 200mph in the horizontal direction.

Express your answer numerically in seconds. Neglect air resistance.

Hint 1. Knowns and unknowns: what are the initial conditions?

Take the origin of the coordinate system to be at the point on the surface of the water directly below the point at which the package is released. The directions of the axes are shown in the figure in the problem introduction. In this coordinate system, what are the values of x_0, y_0, v_{0x}, v_{0y} of the package?

Express your answers numerically and enter them, separated by commas, in the order x_0, y_0, v_{0x}, v_{0y}. Use units of meters and mph for distances and speeds, respectively.

Hint 1. Initial velocity in the *y* direction

Because the package is ejected horizontally, the vertical component of its initial velocity is zero.

ANSWER:

x_0, y_0, v_{0x}, v_{0y} = 0,600,200,0 $\mathrm{m, m, mph, mph}$

Hint 2. What are the knowns and unknowns when the package hits the ground?

Take the origin of the coordinate system to be at the point on the surface of the water directly below the point at which the package is released. The directions of the axes are shown in the figure in the problem introduction. Let t_g be the time when the package hits the ground. In this coordinate system, $v_x(t_g) = 200\mathrm{mph}$ since there is no acceleration in the *x* direction. Which of the following values is/are known?

Check all that apply.

ANSWER:

- [ ] $x(t_g)$
- [x] $y(t_g)$
- [ ] $v_y(t_g)$

Hint 3. Find the best equation to use

Which of the equations below could you use to find the time t_g when the packet hits the ground?

Hint 1. How to determine which equation to use

Only one of the quantities $x(t_g)$, $y(t_g)$, $v_y(t_g)$ is known. Which one? You have to use the equation that contains this variable.

ANSWER:

- $x = x_0 + v_{0x}t$
- $v_y = v_{0y} - gt$
- $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

ANSWER:

t = 11.1 s

Correct

Part B

If the package is to land right on the island, at what horizontal distance D from the plane to the island should the package be released?

Express the distance numerically in meters.

Hint 1. How to approach the problem

You are asked to find D, which is also the change in the *x* coordinate of the package over the time spent in the air. You should have calculated this time interval in Part A. Use it to find D.

Hint 2. The equation for $x\,(t)$

Since there is no acceleration in the horizontal direction, the equation for $x\,(t)$ is

$$x(t) = x_0 + v_{0x}t.$$

ANSWER:

D = 989 m

Correct

Part C

What is the *speed* v_f of the package when it hits the ground?

Express your answer numerically in miles per hour.

Hint 1. How to approach the problem

The speed is the magnitude of the velocity, i.e., $\sqrt{v_x^2 + v_y^2}$. You already know that $v_x = 200\mathrm{mph}$, so you need to find v_y.

Hint 2. The equation for the velocity in the *y* direction

The equation for the velocity in the *y* direction $v_y\,(t)$ is

$$v_y(t) = v_{0y} + a_y t.$$

If you have completed the earlier parts, you should know all the quantities on the right side.

ANSWER:

v_f = 314 mph

Correct

Part D

The speed at which the package hits the ground is really fast! If a package hits the ground at such a speed, it can be crushed and also cause some serious damage on the ground. Which of the following would help decrease the speed with which the package hits the ground?

ANSWER:

- Increase the plane's speed and height
- (selected) Decrease the plane's speed and height

Correct

This is why it would be nice for rescue teams to have hybrid airplane-helicopters. Of course, then they can just airlift the stranded group.

Score Summary:

Your score on this assignment is 98%.
You received 39.18 out of a possible total of 40 points.