Lecture 11
(Electric Fields And Calculating Electric Fields)

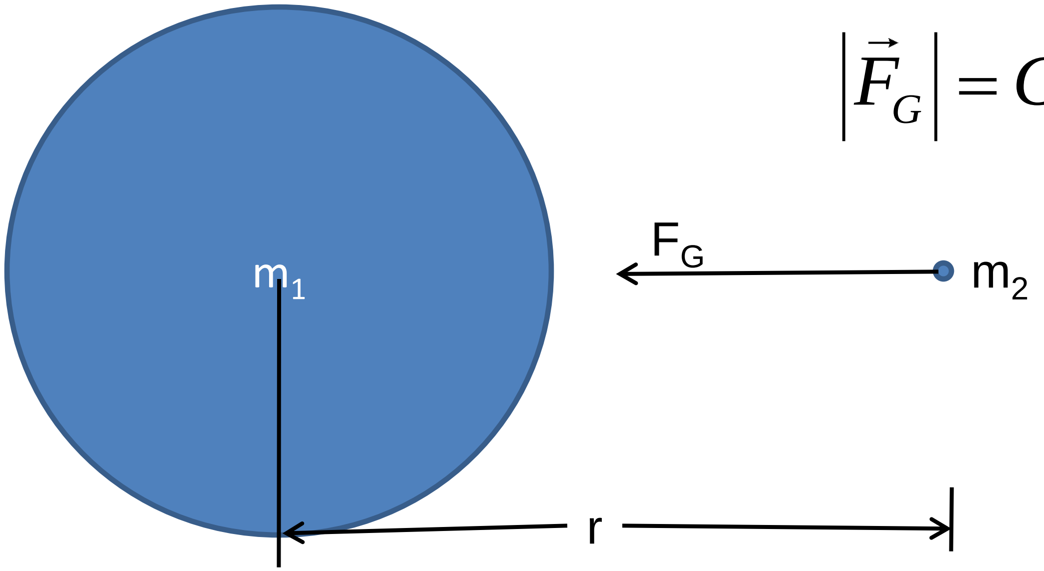
Physics 161-01 Spring 2012

Douglas Fields

Gravitational Force and Gravitational Fields

- A couple of things to remember from our study of gravity:
  - The force law only applies to point particles.
  - But, for a spherically symmetric body, we found the same force law holds.

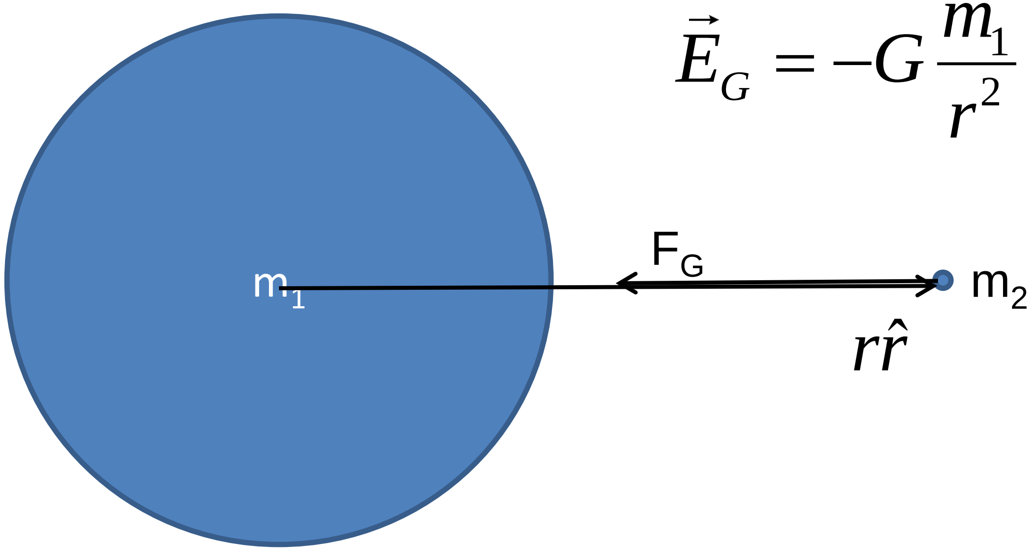
$$\left|\vec{F}_G\right| = G\frac{m_1 m_2}{r^2}$$



Gravitational Force and Gravitational Fields

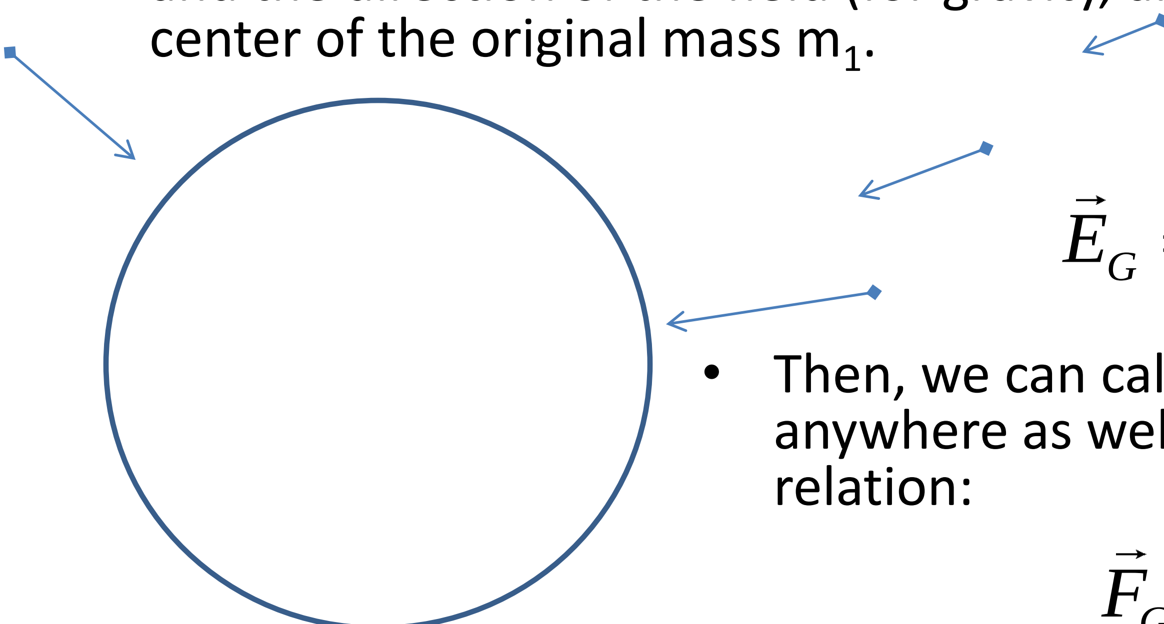
- But, if they are not touching one another, and, in fact, can be quite distant, how does m_2 know that m_1 is there?
- Let's say there is a "gravitational field" around m_1 such that when we put a mass there, there is force that only depends on the "test mass" we place there:

$$\vec{E}_G = -G\frac{m_1}{r^2}\hat{r} \Rightarrow \vec{F}_G = \vec{E}_G m_2$$



Gravitational Force and Gravitational Fields

- So, if we assume that mass m_1 doesn't move or change configuration, we can remove it and replace it by the gravitational field that it caused.
- Note that for each point in space, there is a vector associated with it that describes the strength of the field (the vector's magnitude), and the direction of the field (for gravity, always pointing at the center of the original mass m_1.



$$\vec{E}_G = -G\frac{m_1}{r^2}\hat{r}$$

- Then, we can calculate the force anywhere as well by using the relation:

$$\vec{F}_G = \vec{E}_G m_2$$

Electric Force and Electric Fields

- So, if we do the same thing with the electric force, by defining an electric field, such that:

$$\vec{F} = \vec{E}q$$

- Then, if we can know the electric field from some system of charges, we can know the force that some other charge, q, will feel.
- But how has this helped us, if we have to know the force before we can know the field?

Electric Fields

- The field will turn out to obey certain rules that we can use that will simplify its determination.
- We will no longer need to know the exact distribution of charges that create the field.
- For now, we will limit ourselves to the study of ***electrostatics***, which means that the charges that create the electric fields are in a stable position.

Electric Fields in a Conductor

- An example of this simplification is the calculation of the electric field inside a conductor.
- Since electrons inside a conductor are free to move, then if there is an electric field in a conductor, they will feel a force and move until they no longer feel a force.
- Since there is a resistance to motion inside most conductors, they will eventually stop at the locations where the net electric field is zero.
- So, the electric field in a ***static situation*** inside a conductor is zero.
- We will return to this later.

Electric Field of a Point Charge

- For a point charge, q, we know that the force on a test charge q_0 is,

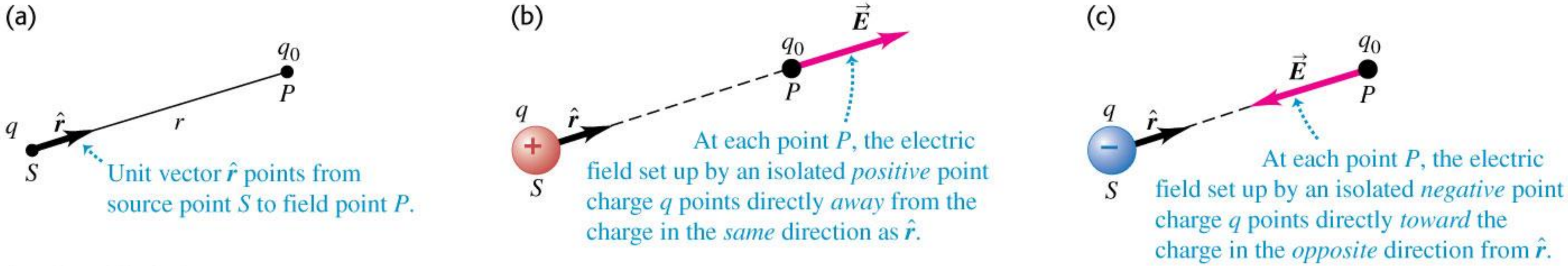
$$\vec{F}_C = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}$$

- And from the definition of the electric field,

$$\vec{F}_C = \vec{E}q_0 \Rightarrow$$

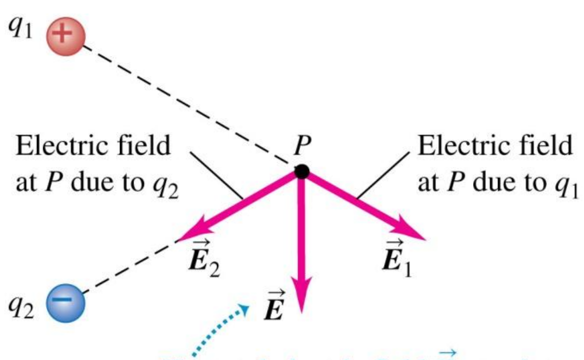
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

- Note that the electric field is in the r-hat direction for a positive charge q, i.e., it points away from positive charges.



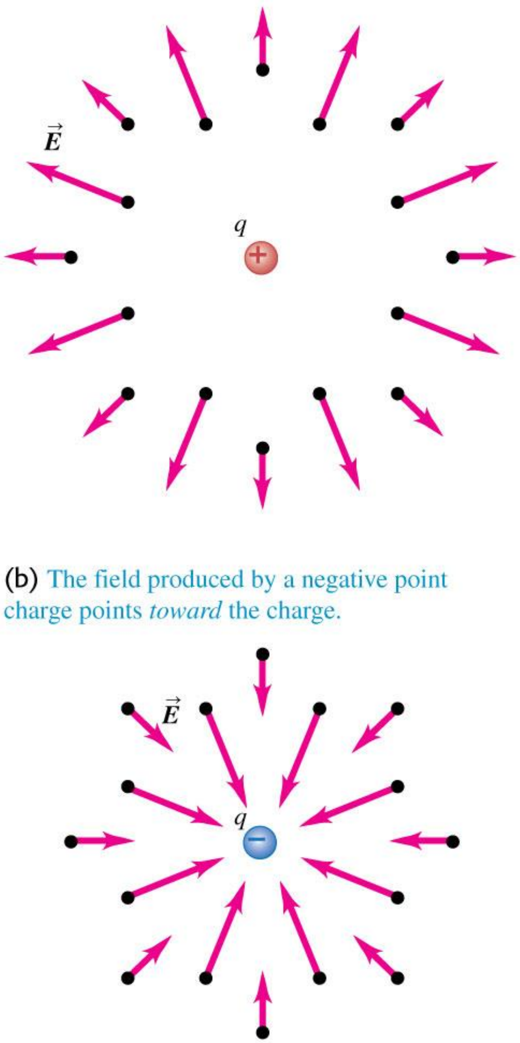
Vector Field

- The electric field is then a field of vectors.
- Each point in space has, associated with it, a vector quantity which represents both the magnitude and direction of the electric field from some charge or charges.
- Since forces add as vectors, so do electric fields:
  - The net electric field from two (or more) charges is just the vector sum of the electric fields from each charge.



The total electric field $\vec{E}$ at point P is the vector sum of $\vec{E}_1$ and $\vec{E}_2$.

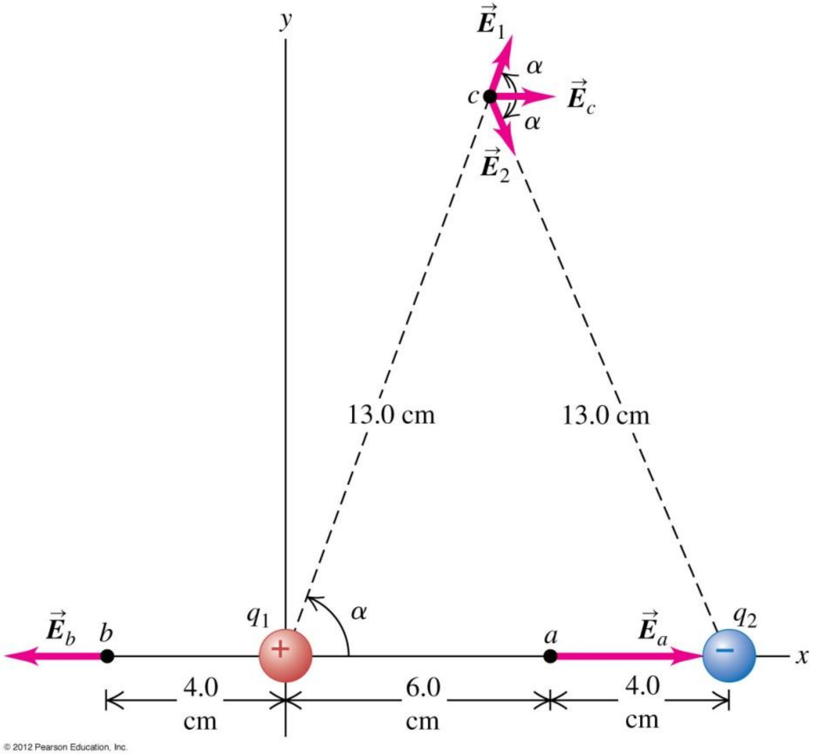
(a) The field produced by a positive point charge points *away from* the charge.



(b) The field produced by a negative point charge points *toward* the charge.

Dipole

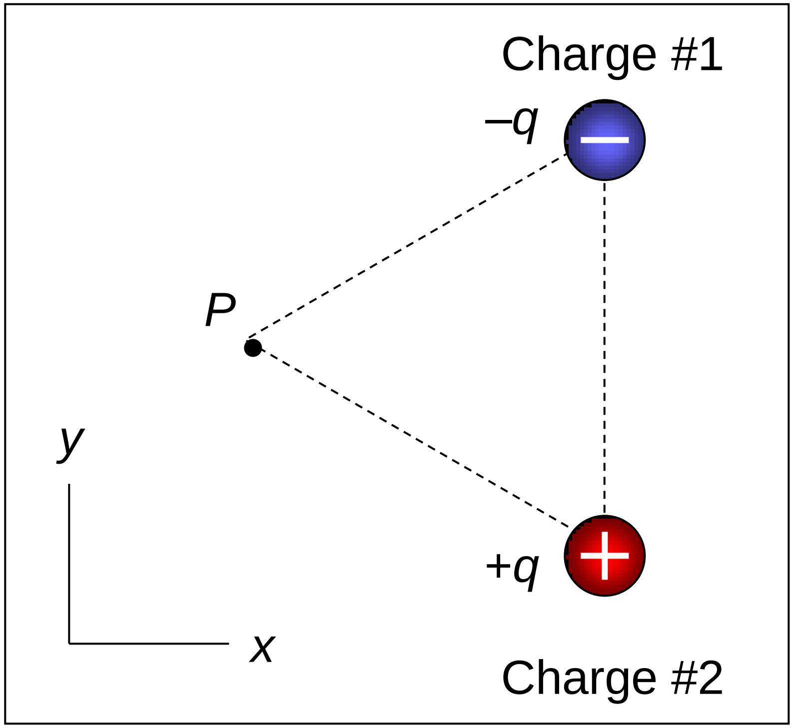
- As a first step in complexity, let's look at the electric field of two, opposite charges separated by some small distance.
- This is called an electric dipole.



CPS 11-1

Two point charges and a point *P* lie at the vertices of an equilateral triangle as shown. Both point charges have the same magnitude *q* but opposite signs. There is nothing at point *P*.

The net electric field that charges #1 and #2 produce at point *P* is in



A. the +*x*-direction.

B. the –*x*-direction.

C. the +*y*-direction.

D. the –*y*-direction.

E. none of the above

CPS 11-1

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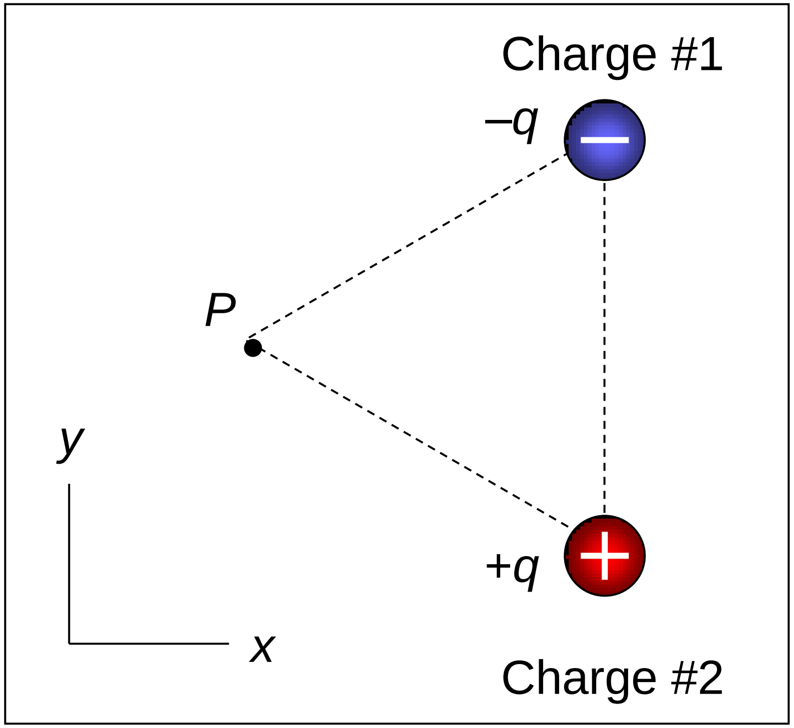


A. the +*x*-direction.

B. the –*x*-direction.

C. the +*y*-direction.

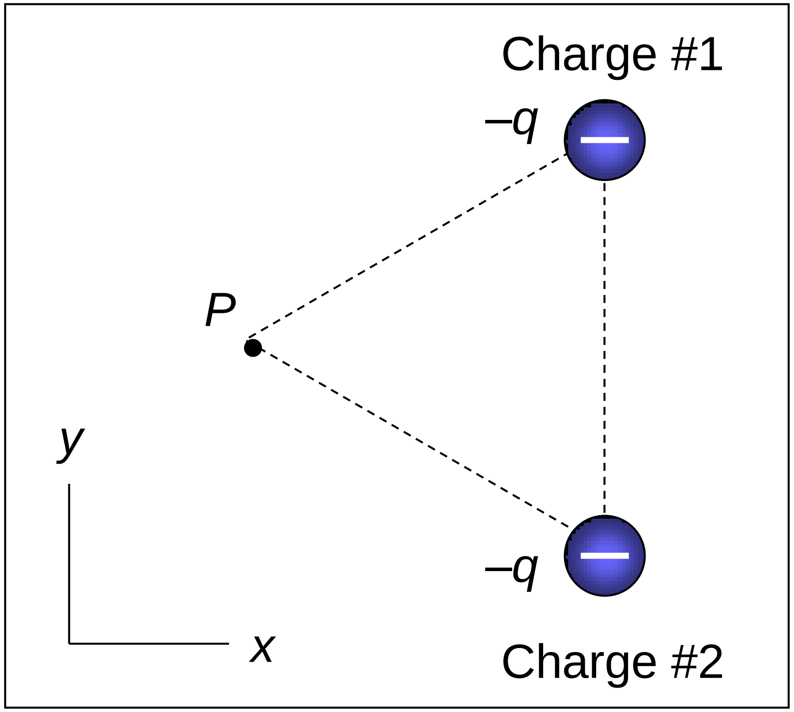
D. the –*y*-direction.

E. none of the above

CPS 11-2

Two point charges and a point *P* lie at the vertices of an equilateral triangle as shown. Both point charges have the same negative charge ($-q$). There is nothing at point *P*.

The net electric field that charges #1 and #2 produce at point *P* is in



A. the $+x$-direction.

B. the $-x$-direction.

C. the $+y$-direction.

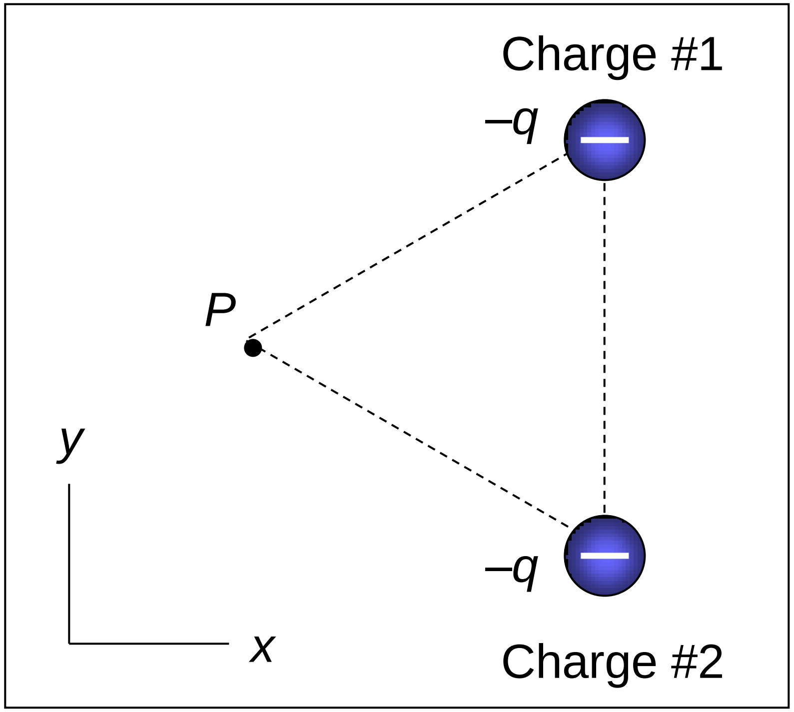
D. the $-y$-direction.

E. none of the above

CPS 11-2

Two point charges and a point P lie at the vertices of an equilateral triangle as shown. Both point charges have the same negative charge ($-q$). There is nothing at point P.

The net electric field that charges #1 and #2 produce at point P is in



A. the +x-direction.

B. the –x-direction.

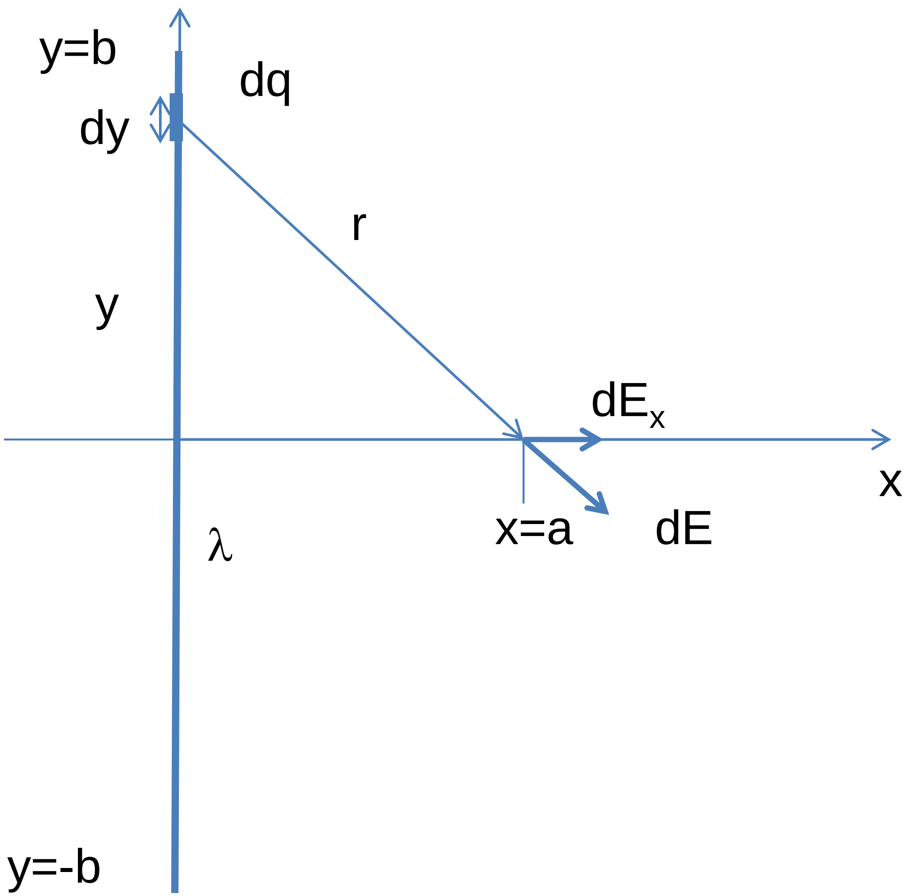
C. the +y-direction.

D. the –y-direction.

E. none of the above

Electric Fields of Extended Objects

- Finite line charge



$$\left|d\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$\left|d\vec{E}\right|_x = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\left(\frac{a}{r}\right) = \frac{a}{4\pi\varepsilon_0}\frac{dq}{r^3}$$

$$\left|d\vec{E}\right|_x = \frac{a\lambda}{4\pi\varepsilon_0}\frac{dy}{\left(a^2+y^2\right)^{3/2}}$$

$$\text{Now,}\quad \int\frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}\quad \text{so,}$$

$$\left|\vec{E}\right|_x = \frac{a\lambda}{4\pi\varepsilon_0}\int_{-b}^{b}\frac{dy}{\left(a^2+y^2\right)^{3/2}} = \frac{a\lambda}{4\pi\varepsilon_0}\left[\frac{y}{a^2\sqrt{a^2+y^2}}\right]_{-b}^{b}$$

$$\left|\vec{E}\right|_x = \frac{\lambda}{4\pi\varepsilon_0 a}\frac{2b}{\sqrt{a^2+b^2}} = \frac{\lambda}{2\pi\varepsilon_0 a\sqrt{\frac{a}{b}^2+1}}$$

if we let $b \to \infty$

$$\left|\vec{E}\right|_x = \frac{\lambda}{2\pi\varepsilon_0 a\sqrt{\frac{a}{b}^2+1}} \to \frac{\lambda}{2\pi\varepsilon_0 a}$$

Electric Fields of Extended Objects

- Finite disk.

$$\left|d\vec{E}\right|_z = \frac{z}{4\pi\varepsilon_0}\frac{\sigma\rho\, d\theta\, d\rho}{\left(\rho^2+z^2\right)^{3/2}}$$

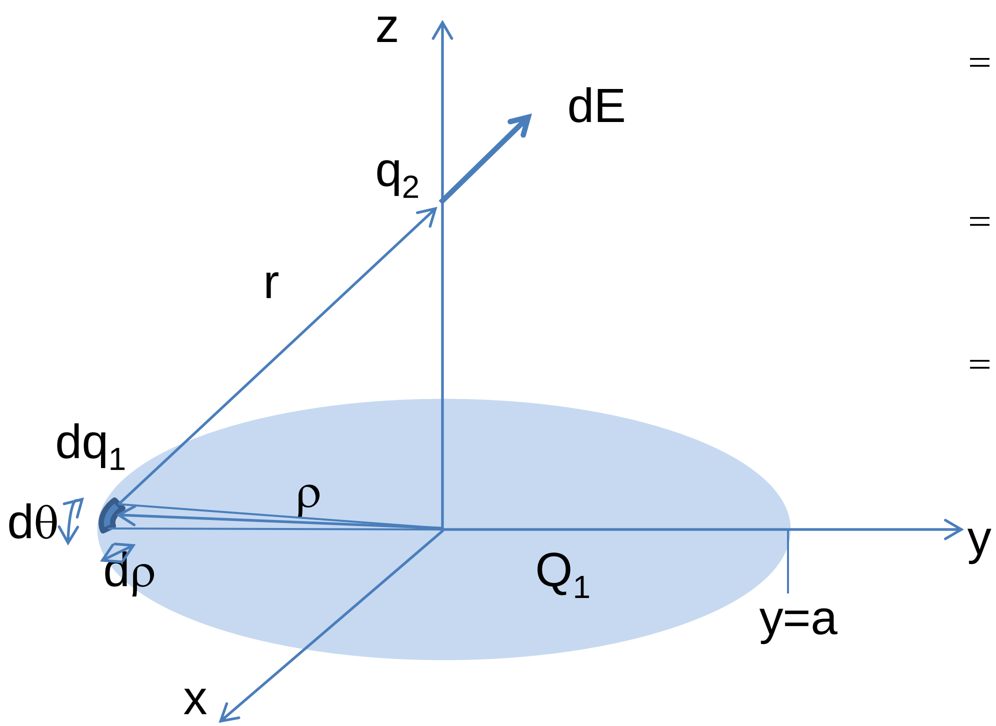
$$E_z = \frac{z\sigma}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{a}\frac{\rho}{\left(\rho^2+z^2\right)^{3/2}}\,d\rho\, d\theta$$

$$= \frac{z\sigma}{4\pi\varepsilon_0}\int_0^{2\pi} d\theta \int_0^{a}\frac{\rho}{\left(\rho^2+z^2\right)^{3/2}}\,d\rho$$

$$= \frac{z\sigma}{4\pi\varepsilon_0}2\pi\frac{1}{2}\int_0^{a}\frac{2\rho}{\left(\rho^2+z^2\right)^{3/2}}\,d\rho$$

$$= \frac{z\sigma}{2\varepsilon_0}\left[\frac{1}{\sqrt{\rho^2+z^2}}\right]_0^a$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{1}{\sqrt{a^2/z^2+1}}\right]$$



Letting a $\rightarrow \infty$, we get for an infinite plane:

$$E_z = \frac{\sigma}{2\varepsilon_0}$$