#18 Elastic Potential Energy Pre-class

Due: 11:00am on Wednesday, October 3, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Exercise 7.20

A piece of cheese with a mass of 1.13kg is placed on a vertical spring of negligible mass and a force constant k = 2500$\mathrm{N/m}$ that is compressed by a distance of 13.5cm .

Part A

When the spring is released, how high does the cheese rise from this initial position? (The cheese and the spring are *not* attached.)

Use 9.81$\mathrm{m/s^2}$ for the acceleration due to gravity. Express your answer using two significant figures.

ANSWER:

h = 2.1 m

Correct

Exercise 7.22

A glider with mass m = 0.220kg sits on a frictionless horizontal air track, connected to a spring with force constant k = 4.70$\mathrm{N/m}$. You pull on the glider, stretching the spring 0.100m , and then release it with no initial velocity. The glider begins to move back toward its equilibrium position ($x = 0$).

Part A

What is the speed of the glider when it returns to $x = 0$?

ANSWER:

v = 0.462 $\mathrm{m/s}$

Correct

Part B

What must the initial displacement of the glider be if its maximum speed in the subsequent motion is to be 2.30$\mathrm{m/s}$?

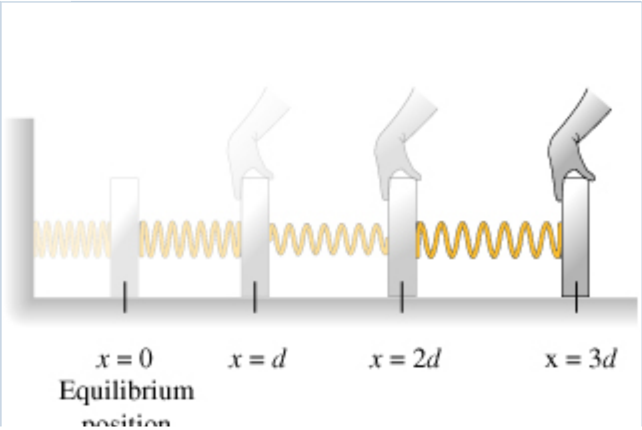
ANSWER:

x = 0.498 m

Correct

Stretching a Spring

As illustrated in the figure, a spring with spring constant k is stretched from $x = 0$ to $x = 3d$, where $x = 0$ is the equilibrium position of the spring.



position

Part A

During which interval is the largest amount of energy required to stretch the spring?

Hint 1. How to approach the problem

The force exerted *on* a spring to stretch or compress it from equilibrium is given by Hooke's law:

$$F_{\text{on spring}} = kx,$$

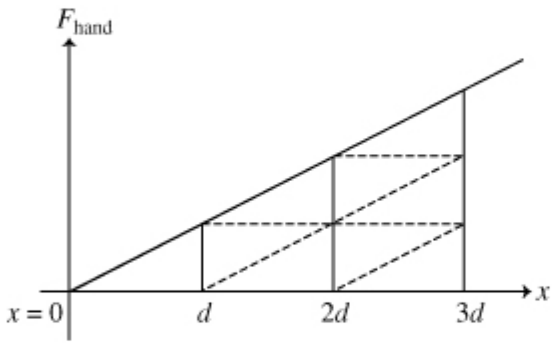
where x is the displacement of the spring from equilibrium. Notice that this force varies in magnitude: as x increases so does the magnitude of the force. On a graph of force as a function of position, the total work done by the force is represented by the area under the curve between the initial and final positions. Plot a graph of force versus displacement and compare the areas under the curve from $x = 0$ to $x = d$, $x = d$ to $x = 2d$, and $x = 2d$ to $x = 3d$.

ANSWER:

- From $x = 0$ to $x = d$
- From $x = d$ to $x = 2d$
- (selected) From $x = 2d$ to $x = 3d$
- The energy required is the same in all three intervals.

Correct

A graph of the force exerted on the spring versus the displacement of the spring is shown in the figure.



Recall that on a graph of force as a function of position, the work done by the force is represented by the area under the curve. The work done by the hand in the first segment to pull the spring from $x = 0$ to $x = d$ is represented by a single triangle. The area under the second segment from $x = d$to $x = 2d$ is three times larger than the first segment, and the area under the third segment from $x = 2d$ to $x = 3d$ is five times larger than in the first segment. So more energy is required to pull the spring through the third segment.

Part B

A spring is stretched from $x = 0$ to $x = d$, where $x = 0$ is the equilibrium position of the spring. It is then compressed from $x = 0$ to $x = -d$. What can be said about the energy required to stretch or compress the spring?

Hint 1. How to approach the problem

Recall that on a graph of force as a function of position, the work done by the force is represented by the area "under" the curve, or more accurately, the area between the curve and the horizontal axis. Plot a graph of force versus displacement and compare the areas "under" the curve from $x = 0$ to $x = d$ and $x = 0$ to $x = -d$.

ANSWER:

- More energy is required to stretch the spring than to compress it.
- The same amount of energy is required to either stretch or compress the spring.
- Less energy is required to stretch the spring than to compress it.

Correct

The work done to stretch or compress a spring from equilibrium is given by

$$W_{\text{on spring}} = \frac{1}{2}kx^2,$$

where x is the distance away from equilibrium that the spring moves. Since x is squared in the equation for work, stretching ($x > 0$) or compressing ($x < 0$) a spring by the same distance requires the same positive amount of work.

Part C

Now consider two springs A and B that are attached to a wall. Spring A has a spring constant that is four times that of the spring constant of spring B. If the same amount of energy is required to stretch both springs, what can be said about the distance each spring is stretched?

Hint 1. How to approach this problem

The work done to stretch or compress a spring is given by

$$W_{\text{on spring}} = \frac{1}{2}kx^2,$$

where x is the distance away from equilibrium that the spring is displaced.

Use this expression to relate the information provided about the work done on each spring and the spring constants to the distance each spring stretches.

Hint 2. Use proportional reasoning to find a relationship between the springs

From the problem statement you know that

$$W_A = W_B,$$

where

$$W = \frac{1}{2}kx^2$$

and

$$k_A = 4k_B.$$

Use this information to find an expression for $\frac{(x_A)^2}{(x_B)^2}$.

ANSWER:

$\frac{(x_A)^2}{(x_B)^2}$ = 0.25

ANSWER:

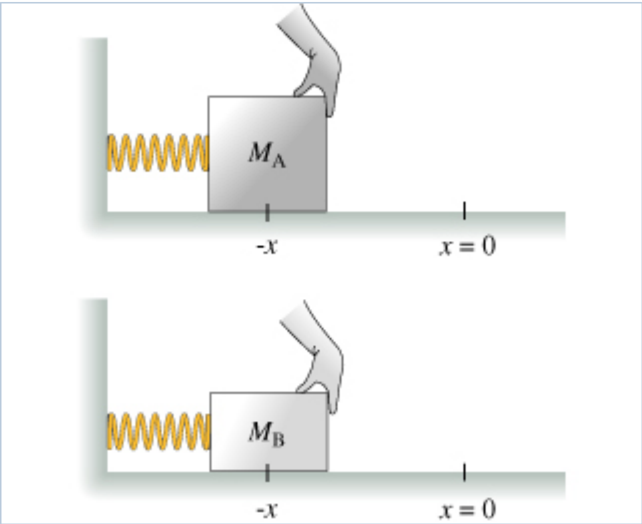
- Spring A must stretch 4 times as far as spring B
- Spring A must stretch 2 times as far as spring B.
- Spring A must stretch the same distance as spring B.
- Spring A must stretch half the distance spring B stretches.
- Spring A must stretch one-quarter of the distance spring B stretches.

Correct

The energy required to stretch a spring is proportional to k and to x^2. If k_A is four times k_B, x_A must be half that of x_B, so the energy required is the same for both springs.

Part D

Two identical springs are attached to two different masses, M_A and M_B, where M_A is greater than M_B. The masses lie on a frictionless surface. Both springs are compressed the same distance, d, as shown in the figure. Which of the following statements descibes the energy required to compress spring A and spring B?



ANSWER:

- Spring A requires more energy than spring B.
- Spring A requires the same amount of energy as spring B.
- Spring A requires less energy than spring B.
- Not enough information is provided to answer the question.

Correct

Good job; you have realized an important fact. The work done on a spring to compress it a distance d is given by $\frac{1}{2}kd^2$. The amount of mass attached to the spring does not affect the work required to stretch or compress the spring.

Score Summary:

Your score on this assignment is 100.1%.
You received 15.02 out of a possible total of 15 points.