This last solution is a singular solution since, if $f''(t) \neq 0$, it cannot be obtained from the family of solutions $y = cx + f(c)$.

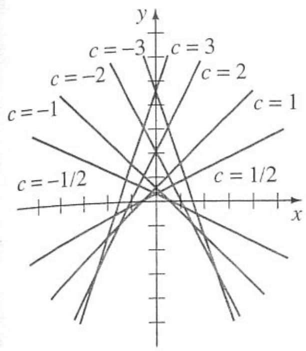

Figure 2.13

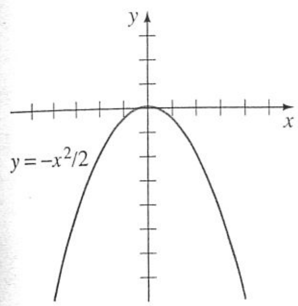

Figure 2.14

EXAMPLE 3 Solving a Clairaut DE

Solve $y = xy' + \dfrac{1}{2}(y')^2$.

Solution We first make the identification $f(y') = \frac{1}{2}(y')^2$ so that $f(t) = \frac{1}{2}t^2$. It follows from the preceding discussion that a family of solutions is

$$y = cx + \frac{1}{2}c^2. \tag{9}$$

The graph of this family is given in Figure 2.13. Since $f'(t) = t$, a singular solution is obtained from (8):

$$x = -t, \quad y = \frac{1}{2}t^2 - t \cdot t = -\frac{1}{2}t^2.$$

After eliminating the parameter, we find this latter solution is the same as $y = -\frac{1}{2}x^2$. One can readily see that this function is not part of the family (9). See Figure 2.14. ■

EXERCISES 2.6

Answers to odd-numbered problems begin on page A-3.

In Problems 1–6 solve the given Bernoulli equation.

1. $x\dfrac{dy}{dx} + y = \dfrac{1}{y^2}$

2. $\dfrac{dy}{dx} - y = e^x y^2$

3. $\dfrac{dy}{dx} = y(xy^3 - 1)$

4. $x\dfrac{dy}{dx} - (1 + x)y = xy^2$

5. $x^2\dfrac{dy}{dx} + y^2 = xy$

6. $3(1 + x^2)\dfrac{dy}{dx} = 2xy(y^3 - 1)$

In Problems 7–10 solve the given differential equation subject to the indicated initial condition.

7. $x^2\dfrac{dy}{dx} - 2xy = 3y^4, \quad y(1) = \dfrac{1}{2}$

8. $y^{1/2}\dfrac{dy}{dx} + y^{3/2} = 1, \quad y(0) = 4$

9. $xy(1 + xy^2)\dfrac{dy}{dx} = 1, \quad y(1) = 0$

10. $2\dfrac{dy}{dx} = \dfrac{y}{x} - \dfrac{x}{y^2}, \quad y(1) = 1$

In Problems 11–16 solve the given Ricatti equation; y_1 is a known solution of the equation.

11. $\dfrac{dy}{dx} = -2 - y + y^2, \quad y_1 = 2$