#36 Simple Harmonic Motion Pre-class

Due: 11:00am on Friday, November 16, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

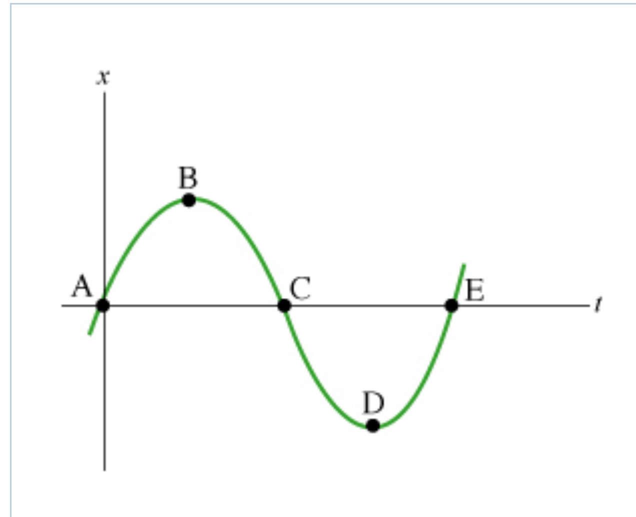
Position, Velocity, and Acceleration of an Oscillator

Learning Goal:

To learn to find kinematic variables from a graph of position vs. time.

The graph of the position of an oscillating object as a function of time is shown.

Some of the questions ask you to determine ranges on the graph over which a statement is true. When answering these questions, choose the *most complete* answer. For example, if the answer "B to D" were correct, then "B to C" would technically also be correct--but you will only recieve credit for choosing the most complete answer.



Part A

Where on the graph is $x > 0$?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part B

Where on the graph is $x < 0$?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part C

Where on the graph is $x = 0$?

ANSWER:

- A only
- C only
- E only
- A and C
- A and C and E
- B and D

Correct

Part D

Where on the graph is the velocity $v > 0$?

Hint 1. Finding instantaneous velocity

Instantaneous velocity is the derivative of the position function with respect to time,

$$v(t) = \frac{dx(t)}{dt}.$$

Thus, you can find the velocity at any time by calculating the slope of the x vs. t graph. When is the slope greater than 0 on this graph?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part E

Where on the graph is the velocity $v < 0$?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part F

Where on the graph is the velocity $v = 0$?

Hint 1. How to tell if $v = 0$

The velocity is zero when the slope of the *x* vs. *t* curve is zero: $\frac{dx(t)}{dt} = 0$.

ANSWER:

- A only
- B only
- C only
- D only
- E only
- A and C
- A and C and E
- (selected) B and D

Correct

Part G

Where on the graph is the acceleration a > 0?

Hint 1. Finding acceleration

Acceleration is the second derivative of the position function with respect to time:

$$a = \frac{d^2x(t)}{dt^2}.$$

This means that the sign of the acceleration is the same as the sign of the curvature of the *x* vs. *t* graph. The acceleration of a curve is negative for downward curvature and positive for upward curvature. Where is the curvature greater than 0?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part H

Where on the graph is the acceleration $a < 0$?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Correct

Part I

Where on the graph is the acceleration $a = 0$?

Hint 1. How to tell if $a = 0$

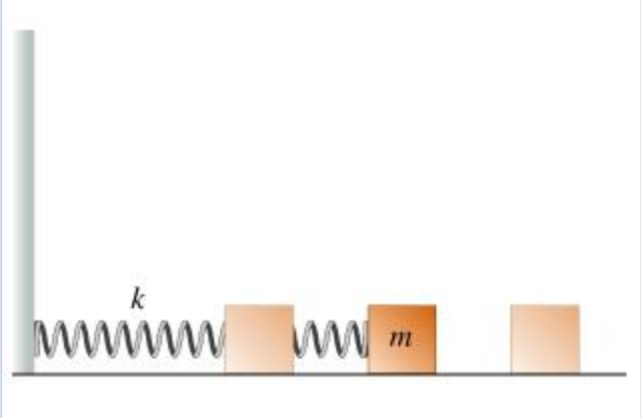
The acceleration is zero at the inflection points of the *x* vs. *t* graph. Inflection points are where the curvature of the graph changes sign.

ANSWER:

- ○ A only
- ○ B only
- ○ C only
- ○ D only
- ○ E only
- ○ A and C
- ◉ A and C and E
- ○ B and D

Correct

± Introduction to Simple Harmonic Motion



Consider the system shown in the figure. It consists of a block of mass m attached to a spring of negligible mass and force constant k. The block is free to move on a frictionless horizontal surface, while the left end of the spring is held fixed. When the spring is neither compressed nor stretched, the block is in equilibrium. If the spring is stretched, the block is displaced to the right and when it is released, a force acts on it to pull it back toward equilibrium. By the time the block has returned to the equilibrium position, it has picked up some kinetic energy, so it overshoots, stopping somewhere on the other side, where it is again pulled back toward equilibrium. As a result, the block moves back and forth from one side of the equilibrium position to the other, undergoing *oscillations*. Since we are ignoring friction (a good approximation to many cases), the mechanical energy of the system is conserved and the oscillations repeat themselves over and over.

The motion that we have just described is typical of most systems when they are displaced from equilibrium and experience a *restoring force* that tends to bring them back to their equilibrium position. The resulting oscillations take the name of *periodic motion*. An important example of periodic motion is *simple harmonic motion* (SHM) and we will use the mass-spring system described here to introduce some of its properties.

Part A

Which of the following statements best describes the characteristic of the restoring force in the spring-mass system described in the introduction?

Hint 1. Find which force is the restoring force

Which of the following forces plays the role of the restoring force?

ANSWER:

- gravity
- friction
- the force exerted by the spring
- the normal force

Hint 2. Hooke's law

The expression known as Hooke's law says that a spring stretched or compressed by a distance x exerts a force given by $F = -kx$, where k is a constant characteristic of the spring called the spring constant. The negative sign expresses the fact that the force exerted by the spring acts in the direction opposite the direction in which the displacement has occurred. Also note that the spring exerts a varying force that is proportional to displacement.

ANSWER:

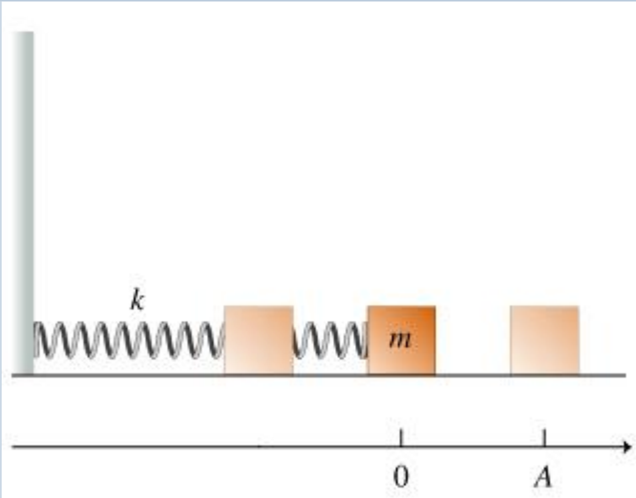
- The restoring force is constant.
- The restoring force is directly proportional to the displacement of the block.
- The restoring force is proportional to the mass of the block.
- The restoring force is maximum when the block is in the equilibrium position.

Correct

Whenever the oscillations are caused by a restoring force that is directly proportional to displacement, the resulting periodic motion is referred to as *simple harmonic motion*.

Part B

As shown in the figure, a coordinate system with the origin at the equilibrium position is chosen so that the *x* coordinate represents the displacement from the equilibrium position. (The positive direction is to the right.) What is the initial acceleration of the block, a_0, when the block is released at a distance A from its equilibrium position?



Express your answer in terms of some or all of the variables A, m, and k.

Hint 1. Find the restoring force

Find F_x, the *x* component of the net force acting on the block, when the block is at a distance A from its equilibrium position. Note that if the block is displaced a certain distance from its equilibrium position, the spring is stretched by the same distance.

Express your answer in terms of some or all of the variables A, m, and k.

Hint 1. Forces exerted on the block in the *x* direction

The *x* component of the net force acting on the block is due exclusively to the force exerted by the spring, since all the other forces (gravity and the normal force) act in the vertical direction.

ANSWER:

F_x = $-kA$

ANSWER:

a_0 = $-\frac{kA}{m}$

Correct

Part C

What is the acceleration a_1 of the block when it passes through its equilibrium position?

Express your answer in terms of some or all of the variables A, m, and k.

Hint 1. A characteristic of equilibrium

By definition, an object in equilibrium does not accelerate.

ANSWER:

a_1 = 0

Correct

Your results from Parts B and C show that the acceleration of the block is negative when the block has undergone a positive displacement. Then, the acceleration's magnitude decreases to zero as the block goes through its equilibrium position. What do you expect the block's acceleration will be when the block is to the left of its equilibrium position and has undergone a negative displacement?

Part D

Select the correct expression that gives the block's acceleration at a distance x from the equilibrium position. Note that x can be either positive or negative; that is, the block can be either to the right or left of its equilibrium position.

Hint 1. How to approach the problem

Hooke's law gives you an expression for the force F exerted on the mass at a given displacement. Newton's 2nd law tells you that $a = F/m$, where a is the acceleration and m is the mass. Using this equation, you can find a formula for the acceleration of the mass attached to the spring.

ANSWER:

- $a = -kx$
- $a = kx$
- a=\frac{k}{m}x
- a=-\frac{k}{m}x

Correct

Whether the block undergoes a positive or negative displacement, its acceleration is always opposite in sign with respect to displacement. Moreover, the block's acceleration is *not* constant; instead, it is directly proportional to displacement. This is a fundamental property of simple harmonic motion.

Using the information found so far, select the correct phrases to complete the following statements.

Part E

Hint 1. How to approach the problem

In Part D, you found that a=-(k/m) x. Since the acceleration is directly proportional to displacement, it must reach its maximum value when displacement is maximum.

ANSWER:

The magnitude of the block's acceleration reaches its maximum value when the block is

- in the equilibrium position.
- (selected) at either its rightmost or leftmost position.
- between its rightmost position and the equilibrium position.
- between its leftmost position and the equilibrium position.

Correct

Part F

Hint 1. How to approach the problem

When the block is in motion, its speed can be zero only when its velocity changes sign, that is, when the direction of motion changes.

ANSWER:

The speed of the block is zero when it is

- in the equilibrium position.
- (selected) at either its rightmost or leftmost position.
- between its rightmost position and the equilibrium position.
- between its leftmost position and the equilibrium position.

Correct

Part G

Hint 1. How to approach the problem

As the block moves from its rightmost position to its leftmost position, its speed increases from zero to a certain value and then decreases back to zero. This means that as the block moves away from its rightmost position toward its leftmost position, its acceleration decreases from positive values to negative values. In particular, the location where the block's acceleration changes sign must also be the location where its speed reaches its maximum value, where it stops increasing and starts to decrease.

ANSWER:

The speed of the block reaches its maximum value when the block is

- (●) in the equilibrium position.
- ( ) at either its rightmost or leftmost position.
- ( ) between the rightmost position and the equilibrium position.
- ( ) between the leftmost position and the equilibrium position.

Correct

Part H

Because of the periodic properties of SHM, the mathematical equations that describe this motion involve sine and cosine functions. For example, if the block is released at a distance A from its equilibrium position, its displacement x varies with time t according to the equation

x= A \cos \omega t,

where ω is a constant characteristic of the system. If time is measured is seconds, ω must be expressed in radians per second so that the quantity \omega t is expressed in radians.

Use this equation and the information you now have on the acceleration and speed of the block as it moves back and forth from one side of its equilibrium position to the other to determine the correct set of equations for the block's *x* components of velocity and acceleration, v_x and a_x, respectively. In the expressions below, B and C are nonzero positive constants.

Hint 1. How to find the equation for acceleration

To determine the correct equation for the acceleration, simply substitute the equation $x= A \cos \omega t$ into the expression for a found in Part D and group all positive constants together. You can verify then whether your result is correct by calculating the acceleration at $t=0$ and comparing it with your result in Part B.

Hint 2. How to find the equation for velocity

To determine the correct equation for the velocity, recall that when $x=A$ the speed of the block is zero. Mathematically, you can calculate when $x=A$ from the given equation for displacement. When you do that, you will not find a unique value for t; rather, you will have a set of values of t at which $x=A$. At this point you simply need to determine which function among $\pm B \sin \omega t$ and $\pm B \cos \omega t$ is zero at those calculated values of t.

ANSWER:

- $v_x=-B\sin\omega t$, $a_x= C\cos\omega t$
- $v_x= B\cos\omega t$, $a_x= C\sin\omega t$
- $v_x= -B\cos\omega t$, $a_x= -C\cos\omega t$
- $v_x= -B \sin\omega t$, $a_x= -C\cos\omega t$

Correct

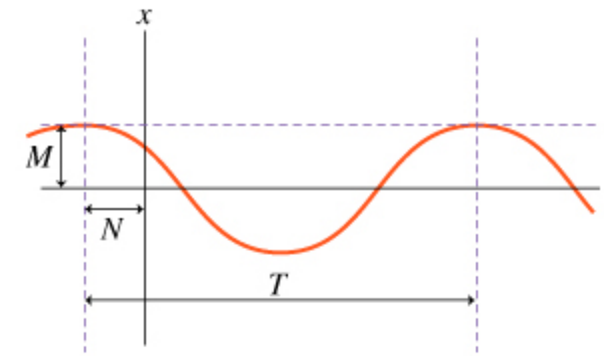
Further calculations would show that the constants B and C can be expressed in terms of A and ω.

Cosine Wave

The graph shows the position x of an oscillating object as a function of time t. The equation of the graph is

x(t) = A\cos{(\omega t + \phi)},

where A is the amplitude, ω is the angular frequency, and ϕ is a phase constant. The quantities M, N, and T are measurements to be used in your answers.



Part A

What is A in the equation?

Hint 1. Maximum of $x(t)$

What is the maximum value of x on the graph and what is the maximum of $x(t)$ as described by the equation? The equation is just a constant multiplied by a cosine function. Cosine can only range from -1 to 1.

ANSWER:

- ○ T
- ◉ M
- ○ $2M$
- ○ M/T
- ○ T/2

Correct

Part B

What is ω in the equation?

Hint 1. Period

Think of the simpler equation x = \cos{(\omega t)}. The period T is the same as before. What does x equal when t = T? Use the result to solve for ω.

ANSWER:

- T
- M
- 2\pi T
- 2\pi/T
- 2/T
- 1/T

Correct

Part C

What is ϕ in the equation?

Hint 1. Using the graph and trigonometry

What is x equal to when t = -N? Use your result for ω to solve for ϕ in terms of T, M, and N.

Hint 2. Using the graph and Part B

You might be able to find ϕ in terms of ω and then use your result from Part B.

ANSWER:

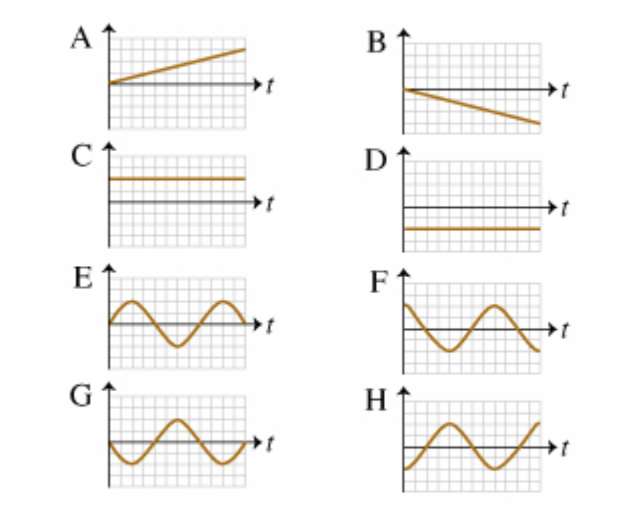
- N
- T-N
- 2\pi N/T
- -2\pi N/T
- \arccos(2\pi N/T)

Correct

Simple Harmonic Motion Conceptual Question

An object of mass m is attached to a vertically oriented spring. The object is pulled a short distance below its equilibrium position and released from rest. Set the origin of the coordinate system at the equilibrium position of the object and choose upward as the positive direction. Assume air resistance is so small that it can be ignored.

Refer to these graphs when answering the following questions.



Part A

Beginning the instant the object is released, select the graph that best matches the position vs. time graph for the object.

Hint 1. How to approach the problem

To find the graph that best matches the object's position vs. time, first determine the initial value of the position. This will narrow down your choices of possible graphs. Then, interpret what the remaining graphs say about the subsequent motion of the object. You should find that only one graph describes the position of the object correctly.

Hint 2. Find the initial position

The origin of the coordinate system is set at the equilibrium position of the object, with the positive direction upward. The object is pulled below equilibrium and released. Therefore, is the initial position positive, negative, or zero?

ANSWER:

- ○ positive
- ◉ negative
- ○ zero

ANSWER:

- ○ A
- ○ B
- ○ C
- ○ D
- ○ E
- ○ F
- ○ G
- ◉ H

Correct

Part B

Beginning the instant the object is released, select the graph that best matches the velocity vs. time graph for the object.

Hint 1. Find the initial velocity

The object is released from rest. Is the initial velocity positive, negative, or zero?

ANSWER:

- ○ positive
- ○ negative
- ◉ zero

Hint 2. Find the velocity a short time later

After the object is released from rest, in which direction will it initially move?

ANSWER:

- ◉ upward (positive)
- ○ downward (negative)

ANSWER:

- ○ A
- ○ B
- ○ C
- ○ D
- ◉ E
- ○ F
- ○ G
- ○ H

Correct

Part C

Beginning the instant the object is released, select the graph that best matches the acceleration vs. time graph for the object.

Hint 1. Find the initial acceleration

The object is released from rest, and a short time later it is moving upward. Based on this observation, what is the direction of the initial acceleration?

ANSWER:

- ◉ positive
- ○ negative
- ○ neither positive nor negative (i.e., there is no acceleration)

ANSWER:

- A
- B
- C
- D
- E
- F
- G
- H

Correct

Score Summary:

Your score on this assignment is 91.3%.
You received 18.26 out of a possible total of 20 points.