

## Solutions

### "Back-of-the-envelope" calculations of energy levels Part 1.

The Schrödinger equation is a differential equation that must be solved to find the "stationary states" for a given potential.

However, you can often get a feeling for what the different wavefunctions look like, and what the energy levels are, by applying the de Broglie relation. Let's do this for an asymmetric "V" shaped potential. Some things to remember: the lowest energy eigenstate ( $n=1$ ) doesn't cross zero, the next higher in energy crosses zero once, the next higher crosses zero twice, etc.

On the  $V$  potential given, where are the "classical turning points" for a particle of energy 3 eV? A classical turning point is where a particle with that total energy would change direction in the given potential.

shown with big black dots.

Suppose the energy levels for the  $V$  potential are evenly spaced every 1 eV; one of the energy levels is exactly 3 eV. Is the lowest energy level 0 eV or 1 eV?

1 eV. Can't have  $x=0$   $p=0$ .

Let's make some really oversimplifying assumptions. Assume the particle cannot go into the classically forbidden region, and that its momentum is the same everywhere in the allowed region. Draw the time independent (real) wave functions for the lowest 3 levels, on the graph. Higher energy states should have more nodes.

Some of the wave does "leak" into the classically forbidden region on the right, but not on the left. Why?

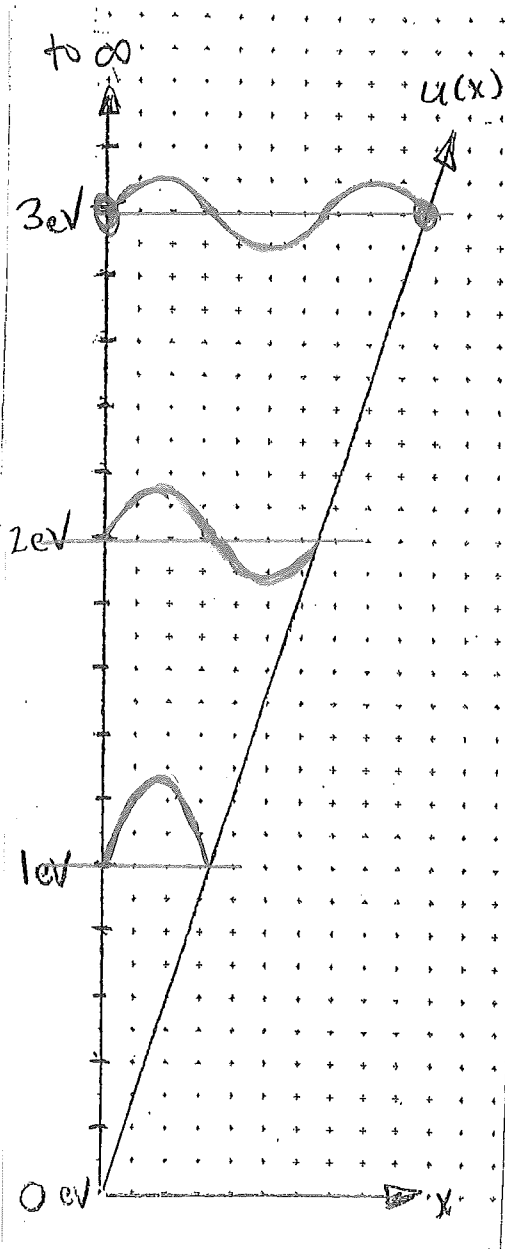
If  $U = \infty$ ,  $\psi = 0$ .

Higher energy states should have more kinetic energy, just like they do in the infinite square well. Does your  $n=2$  state have more kinetic energy than the  $n=1$  state? How can you decide, using the de Broglie idea?

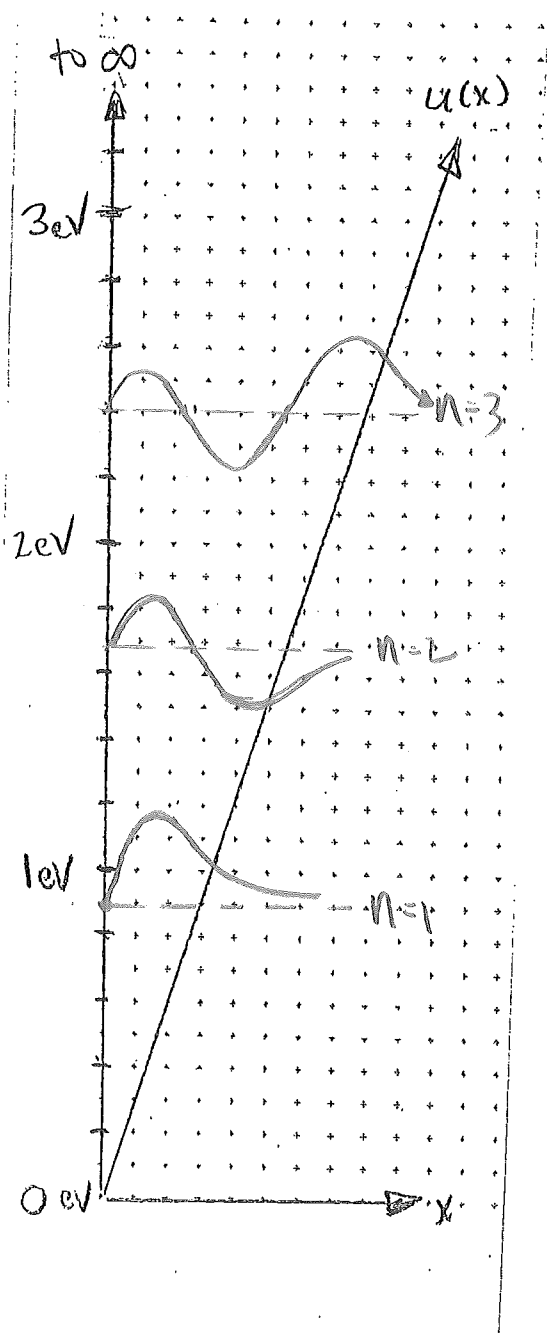
No.  $\lambda$  is the same for  $n=1,2,3$  as drawn!

Make your  $n=2$  and  $n=3$  stationary states more correct, do you need to shift their energy levels UP or DOWN? Note that the  $n=2$  state should have more kinetic energy and more potential energy than the  $n=1$  state.

The  $K$  of  $n=2$  should be bigger than for  $n=1$ . So  $\lambda_2 < \lambda_1$ . We need to move DOWN to "squeeze the wave" and make  $K \uparrow$



Draw more qualitatively correct  $n=1$ ,  $n=2$ , and  $n=3$  states on an other  $V$  potential. Effects to include: 1) place the energy levels better 2) include the fact that the particle will move slower when it has more potential energy (thus less kinetic energy) and 3) allow some of your wave to leak into the appropriate regions.



$\lambda$  is longer on right side  
because  $k$  is lower, so  $p$  is  
lower.

Energy levels are  
shifted down

And get closer together as  $n \uparrow$ .

Note that bigger  $\lambda$  also gives  
bigger amplitude on the right!

Part 2.

Let's go back to the hugely oversimplified version of the states: perfect sinusoids limited by the classical turning points. The width of the well (to the classical turning points) is simply proportional to the energy

$$d_n = \alpha E_n$$

where the subscript indicates the quantum state of interest. Question: how many wavelengths must we fit in the well width,  $d_n$ ? Answer in terms of  $n$ .

$$\frac{n}{2} \text{ wavelengths.}$$

Write down the equation expressing that this number of wavelengths fits in (is equal to)  $d_n$ .

$$\frac{n}{2} \cdot \lambda = d_n$$

Use the de Broglie idea to change this equation into a relation between  $p$  and  $E_n$ .

$$\frac{n}{2} \cdot \frac{h}{p} = \alpha E_n \quad (\text{substituting for } d_n)$$

Ignore potential energy and write a different relationship between  $p$  and  $E_n$ .

$$\frac{p^2}{2m} = E_n$$

Eliminate  $p$  between your two equations and find a relationship between  $E_n$  and  $n$ .

$$p = (2mE_n)^{1/2} \quad \text{so} \quad \frac{nh}{\sqrt{2m}} = \alpha E_n^{3/2}$$

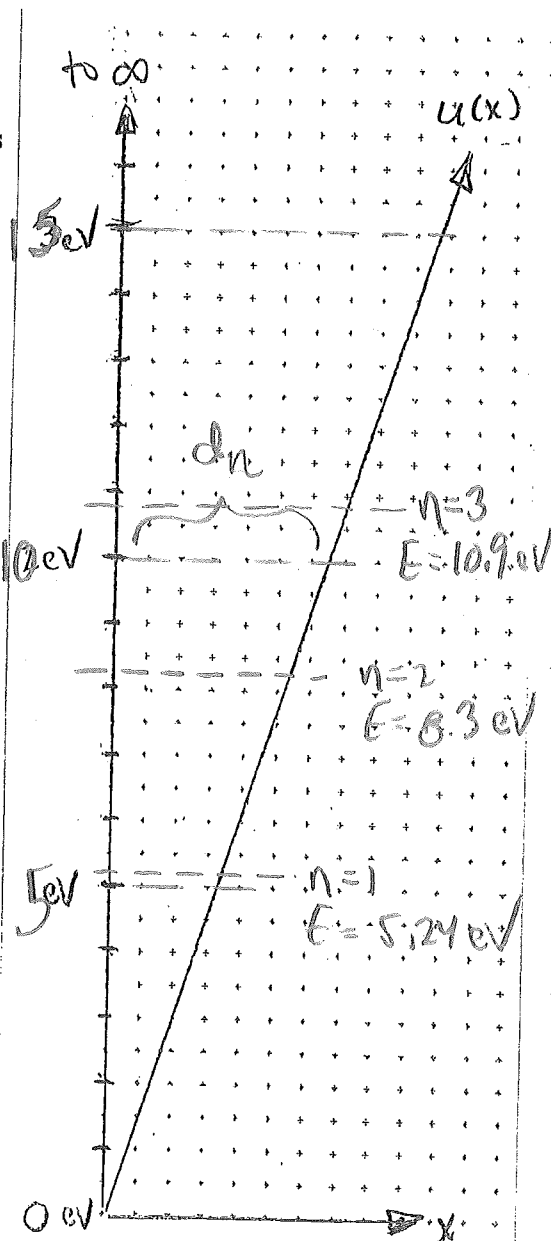
$$\boxed{E_n \propto n^{2/3}}$$

You might be able to make such a  $V$  potential for an electron using nanotechnology. Suppose  $\alpha = 0.1 \text{ nm/eV}$ . Draw these de Broglie waves at their appropriate energies on the  $V$  potential. What is the longest wavelength of light your nanowell can absorb in the ground state?

$$E_n = \left( \frac{n^2 h^2}{2m \alpha^2} \right)^{1/3}$$

$$h = 4 \times 10^{-15} \text{ eV}\cdot\text{s}$$

$$= n^{2/3} \cdot \left( \frac{(4 \times 10^{-15} \text{ eV}\cdot\text{s})^2}{2 \cdot 0.5 \times 10^{-6} \frac{\text{eV}}{\text{c}} \cdot (0.1 \frac{\text{nm}}{\text{eV}})^2} \right)^{1/3} = n^{2/3} \cdot 5.24 \text{ eV}$$



$$n=1 \quad E_1 = 5.24 \text{ eV}$$

$$n=2 \quad E_2 = 8.32 \text{ eV}$$

$$\Delta E = 3.08 \text{ eV} \quad \lambda = 403 \text{ nm}$$