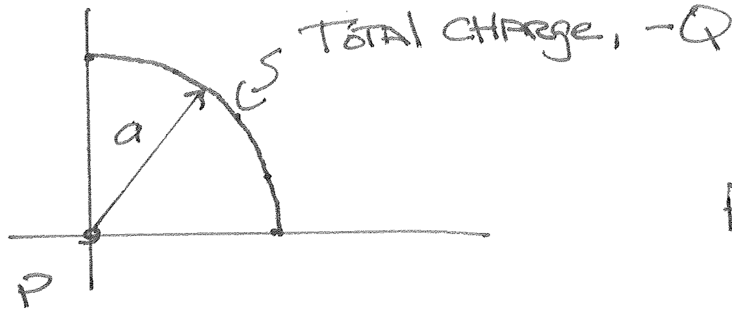
PHYSICS 161, HW #2

#1

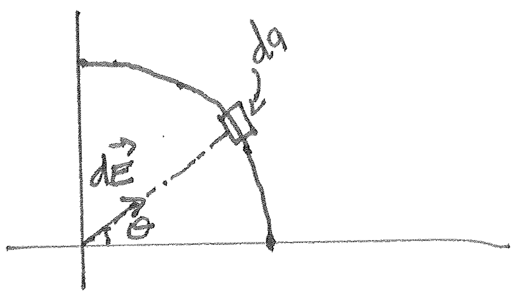


FIND E_x, E_y OF NET FIELD AT P = ORIGIN

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

FOR ALL POINTS ON CIRCLE $r = a$

$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{a^2}$$



$d\vec{E}$ TOWARDS dq SINCE NEGATIVELY CHARGED

FOR A CIRCLE: $\lambda = \frac{dq}{ds}$ ds = ARCLENGTH

$$\Rightarrow dq = \lambda ds$$

FROM PHYSICS I (OR GEOMETRY CLASS): $s = r\theta$ WHEN θ IN RADIANS $\Rightarrow ds = r\,d\theta$ BUT $r = a$ HERE

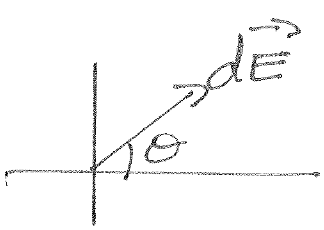
$\Rightarrow ds = a\,d\theta$ WHERE $0 \le \theta \le \pi/2$ FOR A QUARTER CIRCLE.

$\therefore dq = \lambda a\, d\theta$. WE ALREADY took CARE OF THE Negative Charge BY HAVING $d\vec{E}$ point towards dq

SO $\lambda = \frac{|-Q|}{S} = \frac{Q}{\underbrace{a(\pi/2)}_{\text{QUARTER Circle's ARClength}}} = \frac{2Q}{a\pi}$

$dq = \left(\frac{2Q}{a\pi}\right) a\, d\theta = \frac{2Q}{\pi}\, d\theta$, $dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{a^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{a^2}\right)\frac{2Q}{\pi}d\theta$

$= \frac{Q}{2\pi^2\epsilon_0 a^2}\, d\theta$



$dE_x = dE\cos\theta$

$\Rightarrow E_x = \int dE\cos\theta = \int_0^{\pi/2} \frac{Q}{2\pi^2\epsilon_0 a^2}\cos\theta\, d\theta$

$= \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\cos\theta\, d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2}\sin\theta\Big|_0^{\pi/2}$

$$\Rightarrow E_x = \frac{Q}{2\pi^2 \epsilon_0 a^2}(1-0) = \frac{Q}{2\pi^2 \epsilon_0 a^2}$$

$$dE_y = dE \sin\theta \Rightarrow E_y = \frac{Q}{2\pi^2 \epsilon_0 a^2}\int_0^{\pi/2} \sin\theta \, d\theta$$

$$= \frac{Q}{2\pi^2 \epsilon_0 a^2}\left[-\cos\theta\right]\Big|_0^{\pi/2} = \frac{Q}{2\pi^2 \epsilon_0 a^2}\left[-(0-1)\right] =$$

$$\frac{Q}{2\pi^2 \epsilon_0 a^2}\left[-(-1)\right] = \frac{Q}{2\pi^2 \epsilon_0 a^2}$$

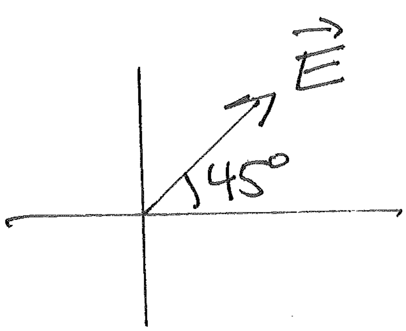
So $E_x = E_y = \frac{Q}{2\pi^2 \epsilon_0 a^2}$ at ORIGIN

$\Rightarrow \vec{E} = \frac{Q}{2\pi^2 \epsilon_0 a^2}\sqrt{2}$ at $45°$

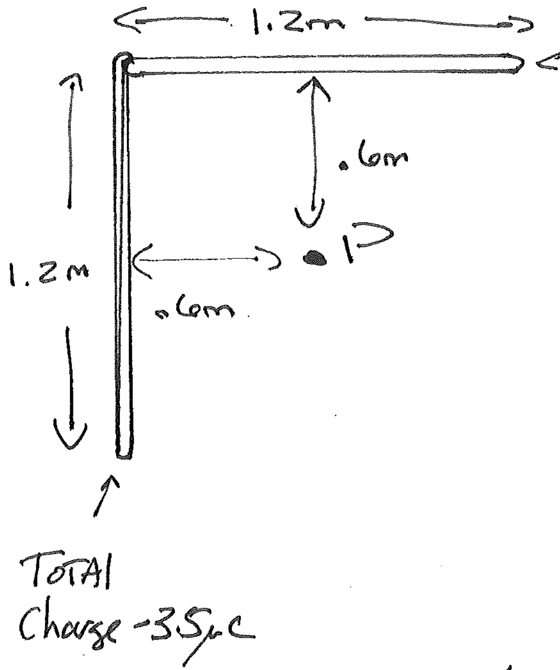
So for $Q = 20\mu C$, $a = 3cm$

$$E = \frac{(20\times10^{-6}C)}{2\pi^2(8.85\times10^{-12}C^2/Nm^2)(.03m)^2}\sqrt{2} = (1.272\times10^{8}N/C)\sqrt{2}$$

$$\Rightarrow \boxed{E = 1.79899\times10^{8}N/C = 1.8\times10^{8}N/C}$$

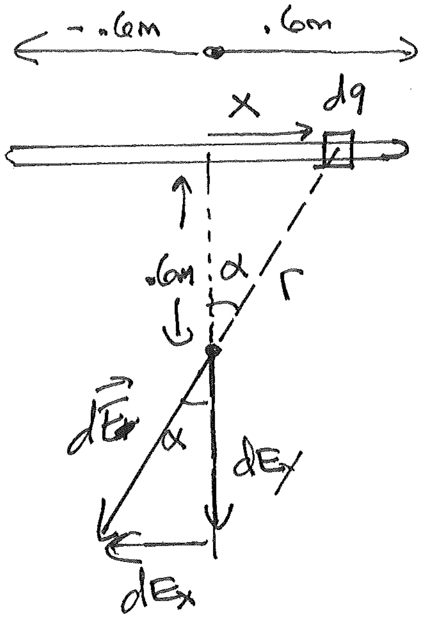


#2



Find $\vec{E}$ at P & Find $\vec{E}_+$ Due to $+3.5\mu C$, $\vec{E}_-$ Due to $-3.5\mu C$, $\vec{E} = \vec{E}_+ + \vec{E}_-$

$\vec{E}_+$:



$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{r^2}$$

$$\lambda = \frac{3.5\times10^{-6}C}{1.2m}, \quad \cos\alpha = \frac{.6m}{r}$$

substitute this later

$$\Rightarrow r = \frac{.6m}{\cos\alpha}$$

$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{\left(\frac{.6m}{\cos\alpha}\right)^2}$$

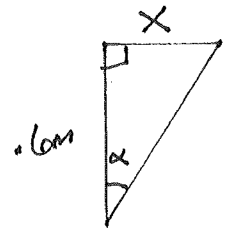
$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{(.6m)^2}\cos^2\alpha$$

Components: $dE_y = dE\cos\alpha = \left(\frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{(.6m)^2}\cos^2\alpha\right)\cos\alpha$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda\cos^3\alpha}{(.6m)^2}dx$$

$$E_y = \int_{-.6m}^{.6m} \frac{\lambda}{4\pi\epsilon_0} \frac{\cos^3\alpha}{(.6m)^2}\, dx = \frac{\lambda}{4\pi\epsilon_0} \int_{-.6m}^{.6m} \frac{\cos^3\alpha}{(.6m)^2}\, dx$$

CHANGE TO α AS INTEGRATION VARIABLE



$$\tan\alpha = \frac{x}{.6m} \Rightarrow x = .6m \tan\alpha$$

$$\Rightarrow dx = (.6m) \frac{1}{\cos^2\alpha}\, d\alpha$$

$x = .6cm \Rightarrow \tan\alpha = 1 \Rightarrow \alpha = \pi/4$, $\quad x = -.6cm \Rightarrow \alpha = -\pi/4$

$$\therefore\ E_y = \frac{\lambda}{4\pi\epsilon_0} \int_{-\pi/4}^{\pi/4} \frac{\cos^3\alpha}{(.6m)^2} \frac{(.6m)}{\cos^2\alpha}\, d\alpha = \frac{\lambda}{4\pi\epsilon_0} \int_{-\pi/4}^{\pi/4} \frac{\cos\alpha}{(.6m)}\, d\alpha$$

$$= \frac{\lambda}{4\pi\epsilon_0} \frac{1}{(.6m)} \int_{-\pi/4}^{\pi/4} \cos\alpha\, d\alpha$$

$\Leftarrow$ IN OTHER WORDS, AT CENTER OF FINITE WIRE, WE GET SAME INTEGRAL AS INFINITE WIRE BUT WITH DIFFERENT LIMITS

$$\therefore\ E_y = \frac{\lambda}{4\pi\epsilon_0} \left(\frac{1}{.6m}\right) \sin\alpha \Big|_{-\pi/4}^{\pi/4}$$

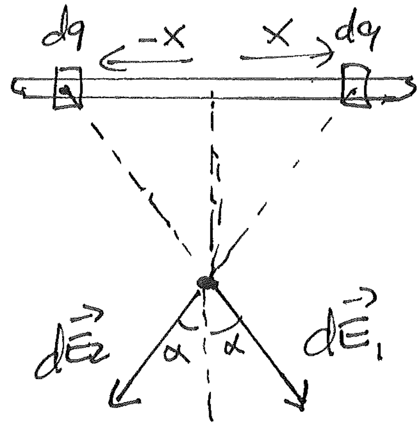
$$= \frac{\lambda}{4\pi\epsilon_0} \left(\frac{1}{.6m}\right) \left(\frac{1}{\sqrt{2}} - \frac{-1}{\sqrt{2}}\right)$$

$$= \frac{\lambda}{4\pi\epsilon_0} \left(\frac{1}{.6m}\right) \sqrt{2}$$

IF YOU PREFER DECIMAL: $E_y = (8.99 \times 10^9\, N \cdot m^2/C^2)\left(\frac{3.5 \times 10^{-6} C}{1.2m}\right)\left(\frac{1}{.6m}\right)\sqrt{2}$

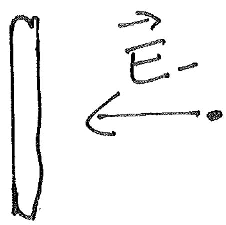
$= 61872\, N/C$

By SYMMETRY (OR DOING THE INTEGRATION), $E_x = 0$



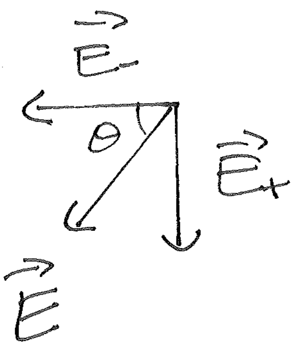
FOR EVERY dq at +x THERE IS ANOTHER dq at -x. THE FIELDS CREATED by THESE PAIRS HAVE EQUAL MAGNITUDE AND ARE AT EQUAL ANGLES ⇒ X-COMPONENTS CANCEL.

$\vec{E}_-$ $\vec{E}_-$ is just $\vec{E}_+$ TURNED by $90^\circ \Rightarrow E_x \neq 0,\ E_y = 0$ but SINCE NEGATIVELY CHARGED, $\vec{E}_-$ points TOWARD LINE



ALREADY took Negative into Account

$$E_- = E_x = \frac{|\lambda|}{4\pi\epsilon_0}\left(\frac{1}{.6m}\right)\sqrt{2}$$



$\vec{E}_+, \vec{E}_-$ at $90^\circ \Rightarrow$ Components of $\vec{E}$

$$\boxed{\theta = \tan^{-1}\left(\frac{E_+}{E_-}\right) = \tan^{-1}(1) = 45^\circ}$$

$$E = \sqrt{E_+^2 + E_-^2} = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{.6m}\right)\sqrt{\sqrt{2}^2 + \sqrt{2}^2}$$

$$\sqrt{2+2} = \sqrt{4} = 2$$

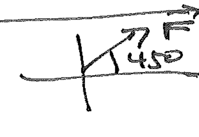
$$\Rightarrow E = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{.6m}\right)2 = (9\times10^9\ Nm^2/C^2)\left(\frac{3.5\times10^{-6}C}{1.2m}\right)\left(\frac{1}{.6m}\right)2 \Rightarrow$$

$$\boxed{E = 87500\ N/C}$$

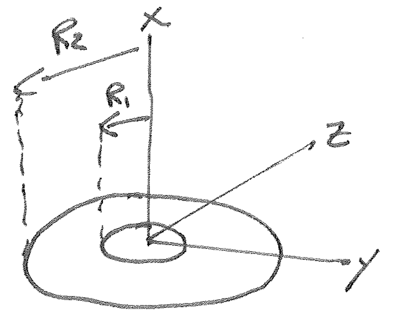
b) Electron at P : $\vec{F} = q\vec{E} \Rightarrow$

$$\boxed{F = (1.6\times10^{-19}C)(87500\ N/C) = 1.4\times10^{-14}N}$$

$\vec{F}$ opposite to $\vec{E} \Rightarrow$



#3



Annulus with charge density, σ

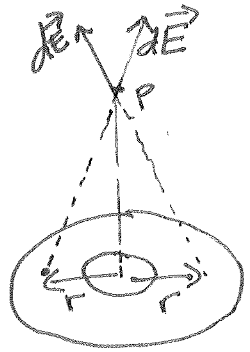
a) Determine Q_{total}.

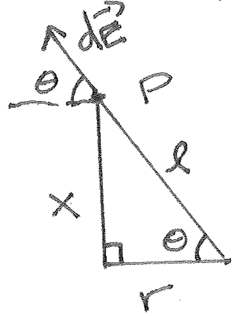
$$\text{Constant } \sigma \Rightarrow Q_T = \sigma A = \sigma(\underbrace{\pi R_2^2}_{\text{"Big" circle}} - \underbrace{\pi R_1^2}_{\text{"Little" circle}})$$

$$\Rightarrow \boxed{Q = \sigma \pi (R_2^2 - R_1^2)}$$

b) Find $\vec{E}$ on x-axis

Above annulus $(x > 0)$, by symmetry $E_y = E_z = 0$





$$dE_x = dE\sin\theta = (dE)\frac{x}{\ell} = dE\frac{x}{\sqrt{x^2+r^2}}$$

x is in the ↑ direction

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{\ell^2} = \frac{1}{4\pi\epsilon_0}\frac{\sigma\, dA}{x^2+r^2} = \frac{1}{4\pi\epsilon_0}\frac{\sigma\, r\, dr\, d\phi}{x^2+r^2}$$

$$\Rightarrow dE_x = \frac{\sigma}{4\pi\epsilon_0} \frac{r\,dr\,d\phi}{x^2+r^2} \cdot \frac{x}{\sqrt{x^2+r^2}} = \frac{\sigma x}{4\pi\epsilon_0} \frac{r\,dr\,d\phi}{(x^2+r^2)^{3/2}}$$

$$\Rightarrow E_x = \int_0^{2\pi} d\phi \int_{R_1}^{R_2} dr \, \frac{\sigma x}{4\pi\epsilon_0} \frac{r\,dr}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{4\pi\epsilon_0} (2\pi) \int_{R_1}^{R_2} \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

$$= \frac{\sigma x}{2\epsilon_0} \int_{R_1}^{R_2} \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

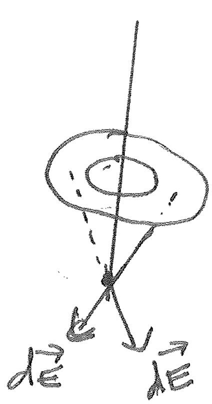
Let $u = x^2+r^2 \Rightarrow du = 2r\,dr$

$\Rightarrow \frac{1}{2} du = r\,dr$

$$\Rightarrow E_x = \frac{\sigma x}{2\epsilon_0} \int_{x^2+R_1^2}^{x^2+R_2^2} \frac{du}{2} u^{-3/2} = \frac{\sigma x}{2\epsilon_0} \cdot \frac{1}{2} \cdot -2u^{-1/2} \Big|_{x^2+R_1^2}^{x^2+R_2^2}$$

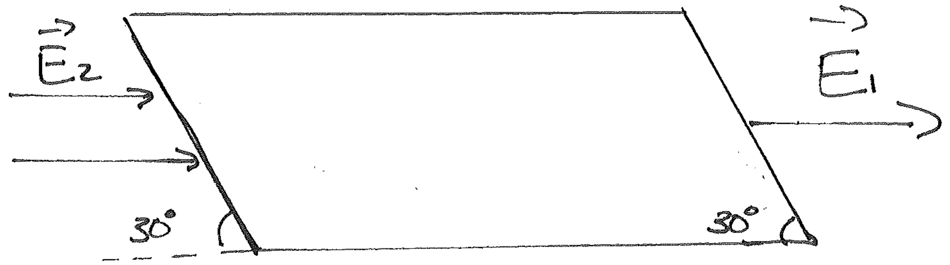
$$\Rightarrow \boxed{E_x = \frac{\sigma x}{2\epsilon_0} \left[\frac{-1}{\sqrt{x^2+R_2^2}} + \frac{1}{\sqrt{x^2+R_1^2}} \right]}$$

BELOW ANNULUS



Still only E_x, but E_x IN THE NEGATIVE X-direction. BELOW ANNULUS make $x<0 \Rightarrow$ CAN USE SAME EQUATION.

#4



$E_2 = 7.5 \times 10^4\, N/C$

$E_1 = 3 \times 10^4\, N/C$

a) FIND NET CHARGE :

FIELDS PASS THROUGH TWO FACES OF PARALLELPIPED :

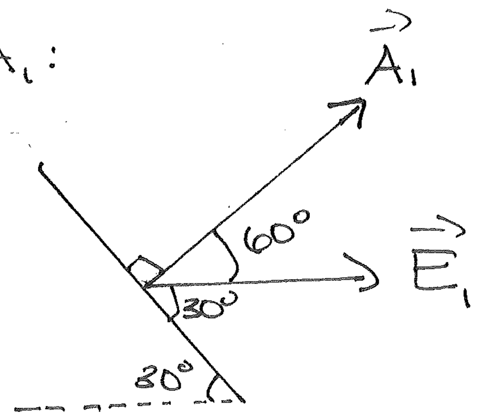
"FRONT" $= A_1 = \vec{A}_1$ AREA THAT $\vec{E}_1$ IS coming out of

"BACK" $= A_2 =$ AREA THAT $\vec{E}_2$ is going INTO

$A_1 = A_2 = (.05m)(.06m) = .003m^2$

FOR BOTH AREAS, FIELD IS UNIFORM $\Rightarrow \Phi = \vec{E} \cdot \vec{A}$

FOR A_1 :



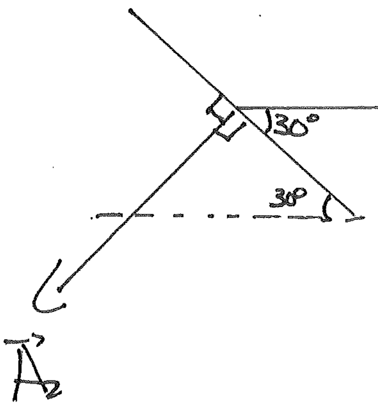
$\vec{A}_1$ IS 90° to SURFACE

$\vec{E}_1$ IS HORIZONTAL $\Rightarrow$ 30° to SURFACE

$\Rightarrow \Phi_1 = \vec{E}_1 \cdot \vec{A}_1 = E_1 A_1 \cos 60°$

$\Rightarrow \Phi_1 = E_1 A_1 \left(\frac{1}{2}\right) = \frac{1}{2} E_1 A_1$

FOR A_2:



$\vec{A}_2$ POINTS <u>OUTWARDS</u> AND 90° to SURFACE

$$\therefore \Phi_2 = \vec{E}_2 \cdot \vec{A}_2 = E_2 A_2 \cos 120^\circ$$
$$= E_2 A_2 \left(-\tfrac{1}{2}\right) = -\tfrac{1}{2} E_2 A_2$$

$$\Phi_{TOTAL} = \Phi_1 + \Phi_2 = \tfrac{1}{2} E_1 A_1 - \tfrac{1}{2} E_2 A_2$$

$$A_1 = A_2 \Rightarrow \Phi_{TOTAL} = \tfrac{1}{2}(E_1 - E_2) A = \tfrac{1}{2}(3\times10^4 \, N/C - 7.5\times10^4 \, N/C)(.003 m^2)$$

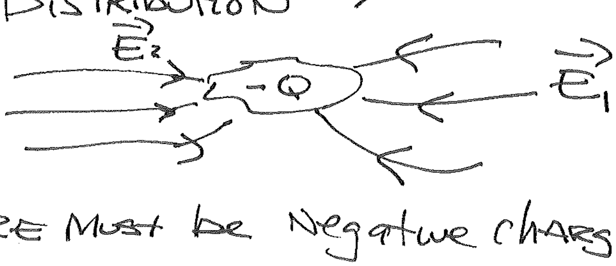
$$\Rightarrow \Phi_{TOTAL} = -67.5 \, N \cdot m^2/C$$

GAUSS'S LAW: $\Phi_{TOTAL} = \frac{Q_{encl}}{\epsilon_0} \Rightarrow Q_{encl} = \Phi_{TOTAL}\, \epsilon_0 = (-67.5 \, N m^2/C)(8.85\times10^{-12} \, C^2/N m^2)$

$$\Rightarrow \boxed{Q_{encl} = -5.97\times10^{-10} C}$$

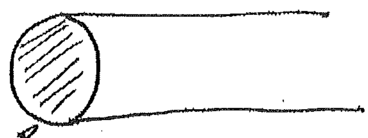
b) Is $\vec{E}_1, \vec{E}_2$ PRODUCED ONLY by CHARGE INSIDE? → NO

IF only A NEGATIVE CHARGE DISTRIBUTION ⇒ FIELD LINES would only go TOWARDS parallelpiped



but $\vec{E}_1$ IS COMING OUT ⇒ THERE MUST be NEGATIVE CHARGE OUTSIDE AS well.

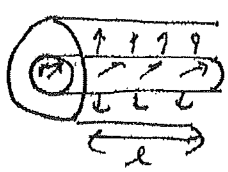
#5



Solid cylinder, charge density ρ, RADIUS R.

a) find E for $r<R$

by symmetry E is outwards AND constant at constant r $\Rightarrow$ USE GAUSSIAN Cylinder



Flux through Top & Bottom $= 0$

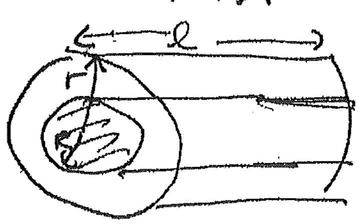
Area of sides

$$\oint \vec{E}\cdot d\vec{A} = EA = E(2\pi r \ell)$$

$$\frac{Q_{encl}}{\epsilon_0} = \rho\frac{(\pi r^2 \ell)}{\epsilon_0}$$

$$\Rightarrow E(2\pi r \ell) = \rho\frac{(\pi r^2 \ell)}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho r}{2\epsilon_0}}$$

b) find E for $r>R$



$$\oint \vec{E}\cdot d\vec{A} = E(2\pi r \ell), \quad \frac{Q_{enc}}{\epsilon_0} = \rho\frac{(\pi R^2 \ell)}{\epsilon_0}$$

$$\Rightarrow E(2\pi r \ell) = \rho\frac{(\pi R^2 \ell)}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho R^2}{2\epsilon_0 r}}$$

$$\rho = \frac{\text{Charge}}{\text{Volume}} = \frac{\text{charge}}{\text{Area} \times \text{length}} \Rightarrow \rho = \frac{1}{\text{Area}} \times \left(\frac{\text{charge}}{\text{length}}\right) = \frac{1}{\pi R^2}\lambda$$

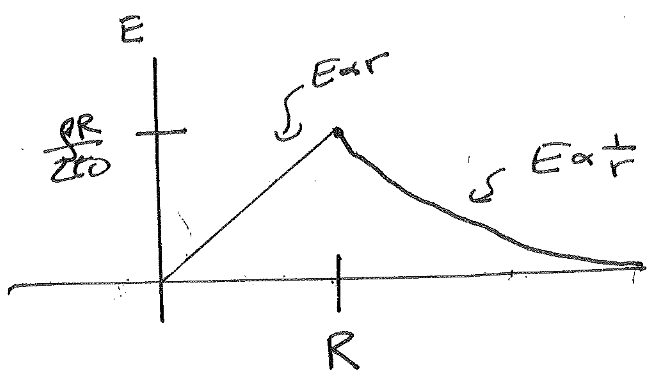
$$\Rightarrow \rho R^2 = \frac{\lambda}{\pi} \Rightarrow \boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

← SAME AS INFINITE WIRE (which is what A cylinder is outside)

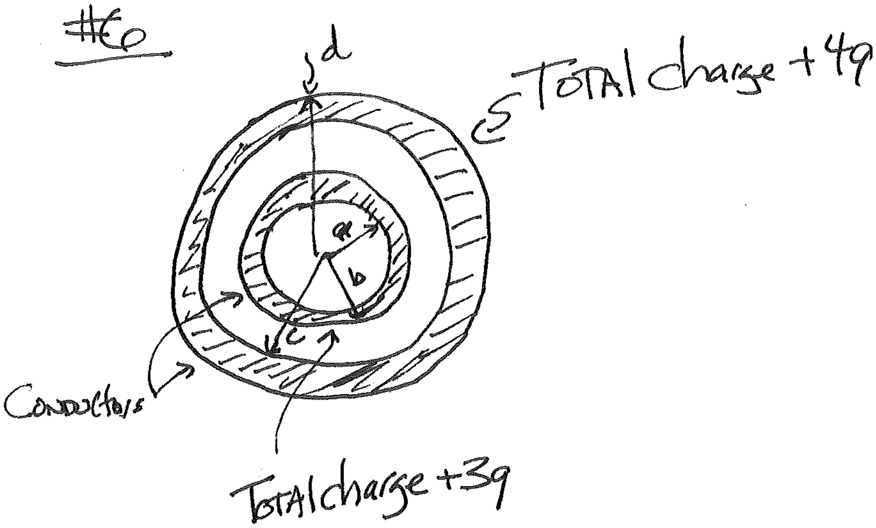
c) $E|_{r=R}$ for both

part a: $E = \frac{\rho R}{2\epsilon_0}$ part b: $E = \frac{\rho R^2}{2\epsilon_0 R} = \frac{\rho R}{2\epsilon_0}$ ⇒ SAME Result

d) GRAPH E



#6



a) WHAT IS TOTAL CHARGE ON INNER AND OUTER SURFACE OF CONDUCTORS.

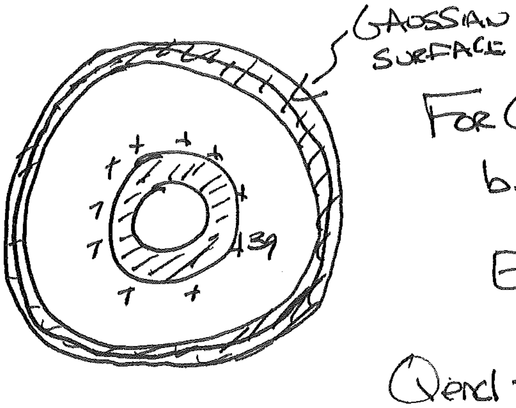
CONDUCTORS : HAVE $E=0$ INSIDE AND ONLY HAVE CHARGE ON SURFACE.

~~By~~ GAUSS'S LAW ⇒ FOR ANY SURFACE WITH ITS BOUNDARY IN THE CONDUCTOR $\Phi_{TOTAL}=0$ SINCE $E=0$ ⇒ $Q_{encl}=0$

NO CHARGE FOR $r<a$ ⇒ THERE CAN BE NO CHARGE ON INNER SURFACE OF SMALL shell, i.e, $r=a$

OTHERWISE, GAUSSIAN SURFACE at ~~r~~ $a<r<b$ would have $Q_{encl}\neq 0$ but $E=0$ for $a<r<b$.

All CHARGE ON SURFACE ⇒ All OF ~~q~~ $+3q$ CHARGE MUST BE ON OUTER SURFACE OF SMALL Shell, i.e., $r=b$



FOR GAUSSIAN SURFACE WITH BOUNDARY between c & d

$E = 0$ for $c < r < d \Rightarrow Q_{encl} = 0$

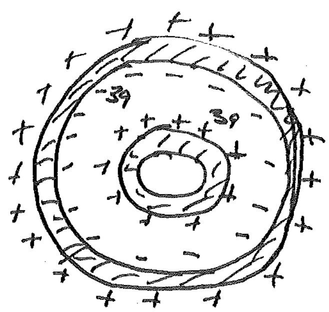
$$Q_{encl} = Q_{IS} + Q_{OS} + Q_{IL}$$

charge ON INNER SURFACE of small shell — charge ON outer SURFACE of SMALL shell — charge ON INNER SURFACE OF LARGE shell

$Q_{IS} = 0$, $Q_{OS} = 3q$, $Q_{IL} = ?$

$$0 = 0 + 3q + Q_{IL} \Rightarrow \boxed{Q_{IL} = -3q}$$

SO:



Outer shell has total charge $+4q$

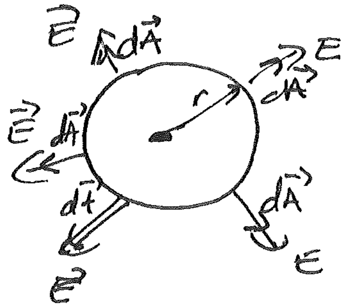
THERE IS $-3q$ ~~ON~~ ON INNER SURFACE

$\Rightarrow \boxed{Q_{OL} = +7q}$ to make $Q_{IL} + Q_{OL} = +4q$

OUTER SURFACE, LARGE Shell

b) FIND E: From textbook AND Symmetry, we KNOW THAT SPHERICAL CONDUCTORS Create RADIAL FIELDS

⇒ INWARD OR OUTWARD WITH Constant magnitude at A FIXED RADIUS ⇒ FOR A GAUSSIAN sphere Concentric WITH SHELLS

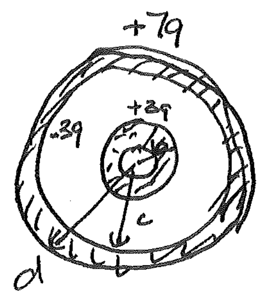


$$\oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA = EA = \Phi_{TOTAL}$$

FOR A SPHERE $A = 4\pi r^2$

GAUSS'S LAW: $\Phi_{TOTAL} = \frac{Q_{encl}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0}$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q_{encl}}{r^2}$$



For $0 < r < a$, $Q_{encl} = 0 \Rightarrow E = 0$

For $a < r < b$, $E = 0$ Since $Q_{encl} = 0$ (AND INSIDE CONDUCTOR)

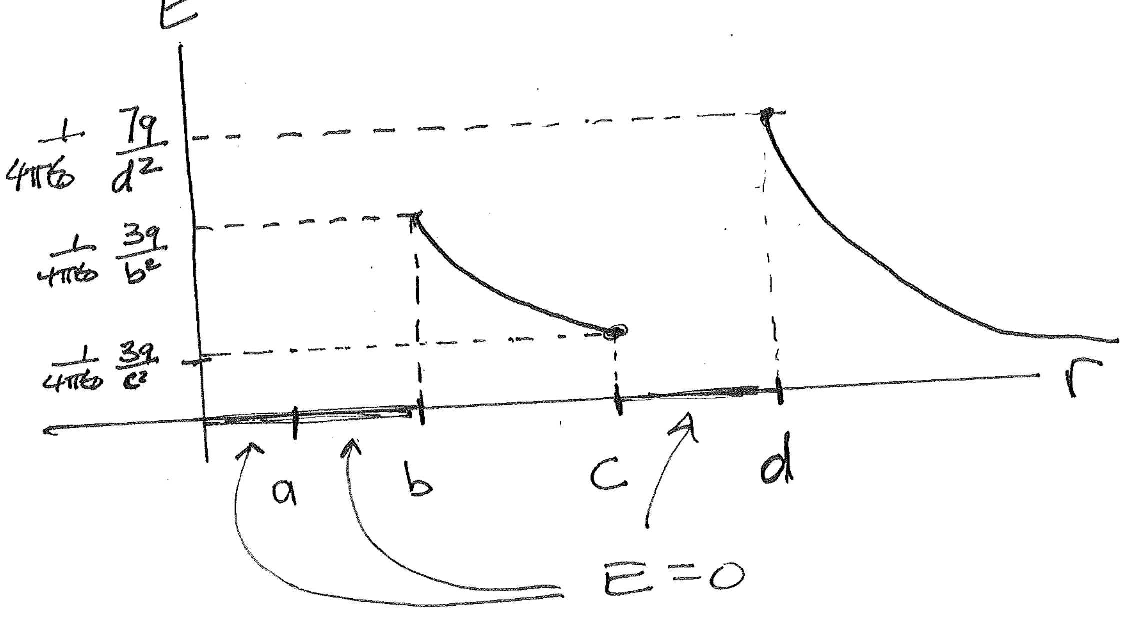
For $b < r < c$, $Q_{encl} = +3q \Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0}\frac{3q}{r^2}}$

Positive q ⇒ OUTWARD Field

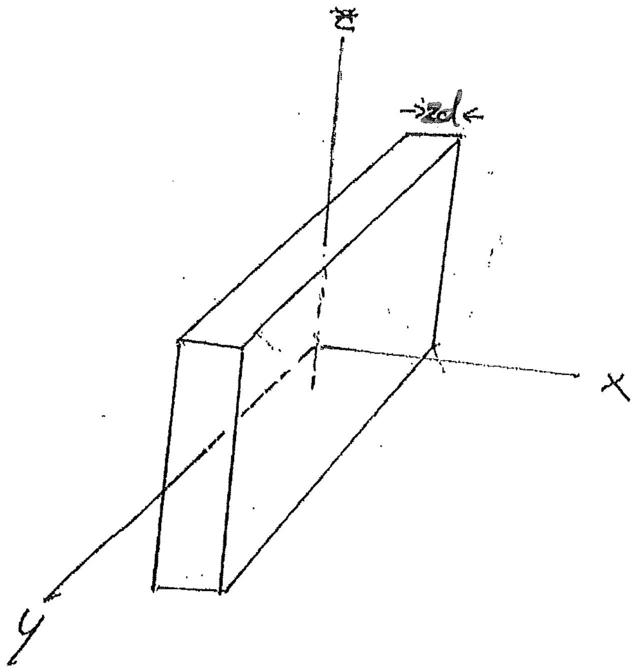
For $c < r < d$, $E = 0$ since $Q_{encl} = 0$ (INSIDE CONDUCTOR)

For $r > d$, $Q_{encl} = +7q \Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0}\frac{7q}{r^2}}$ OUTWARD

In graph Form

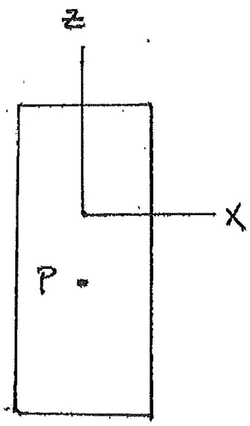


#7



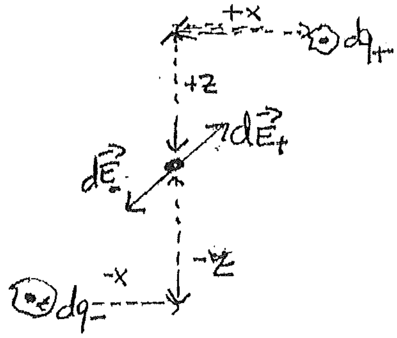
INSULATING LARGE SLAB, WITH DENSITY ρ

a) Explain why $E=0$ at $x=0$



By symmetry at any Point P along the $x=0$ line:

· Any $d\vec{E}_+$ due to charge to the right and up of P is canceled by a $d\vec{E}_-$ due to charge to left and down of P. → To include y, think into page and out of page too



$$d\vec{E}_+ + d\vec{E}_- = 0$$

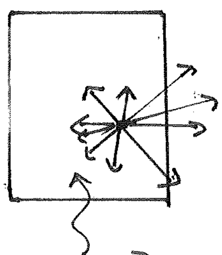
Of course only true because we assume z and y are infinite

b) Find $\vec{E}$ at all points

At any point off $x=0$ there's "more" charge on one side than the other in x. But still "equal" charge along y and z

$\Rightarrow$ $\vec{E}$ in the x-direction

For $x>0$



$d\vec{E}_+$'s greater $+x$ components since more charge to left of P

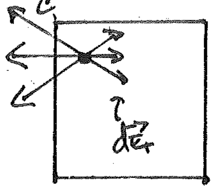
$d\vec{E}_-$'s same y's as $\vec{E}_+$'s but smaller x-components

$\Rightarrow$ P $\longrightarrow \vec{E}$

Again because $-\infty<y<\infty$, $-\infty<z<\infty$ true for any point on the $x=x_0$ line

$\Rightarrow$ E constant for $x=$ constant

For $x<0$



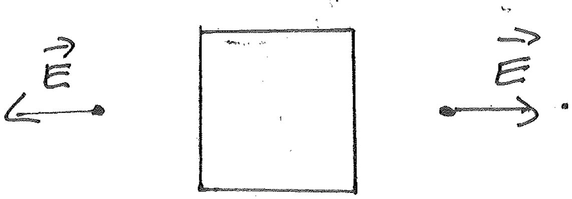
$d\vec{E}_-$

$d\vec{E}_+$

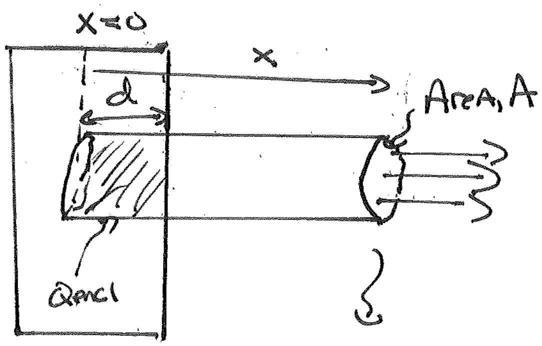
$\Rightarrow$ $\vec{E} \longleftarrow$

$\vec{E}$ in negative x-component

Same result for points outside of slab



$\vec{E}$ in x-direction $\Rightarrow$ use Gaussian cylinder with top and bottom parallel to x. Flux through sides will be zero.



$x > d$

Put other end of cylinder at $x=0$ so $E=0$

$\Rightarrow \Phi_{BOTTOM} = 0$

$\Phi_{TOP} = EA$

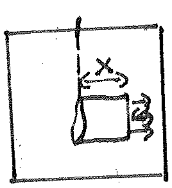
since E constant

$$\Phi_{TOTAL} = \Phi_{top} + \Phi_{bottom} + \Phi_{sides} = EA + 0 + 0 = EA$$

$$Q_{encl} = \rho\,\text{Volume} = \rho A d$$

Gauss's Law $\Rightarrow EA = \rho \frac{Ad}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho d}{\epsilon_0}}$

$0 < x < d$



$\Phi_{TOTAL} = EA$ still, $Q_{encl} = \rho A x$

$$\Rightarrow EA = \frac{\rho A x}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho x}{\epsilon_0}}$$

For $-d < x < 0$ AND $x \leq -d$ same results $E = \frac{\rho x}{\epsilon_0}$ ($x<0$ gives $E = \leftarrow$ as it should)

$x \leq -d$ $\quad E = \frac{\rho d}{\epsilon_0}$ with $\leftarrow$ direction

So maybe be $\vec{E} = -\frac{\rho d}{\epsilon_0}\hat{i}$ to be careful

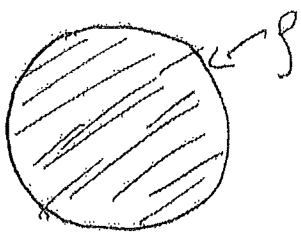
#8

[redacted]

$$\rho(r) = \rho_0 (1 - r/R) \qquad r \leq R$$

$$\rho(r) = 0 \qquad r > R$$

WHERE $\rho_0 = \frac{3Q}{\pi R^3}$ ← Constant



spherically symmetric ⇒ OUTWARDS electric field. → ρ_0 is positive

a) SHOW TOTAL charge contained is Q

$$\rho = \frac{dQ}{dV} \Rightarrow dQ = \rho\, dV \Rightarrow \int dQ = \int \rho\, dV \Rightarrow Q_{encl} = \int_0^r \rho\, dV$$

Spherical ⇒ $dV = 4\pi r^2 dr$ such that $\int dV = \int 4\pi r^2 dr = 4\pi \int r^2 dr = \frac{4}{3}\pi r^3$

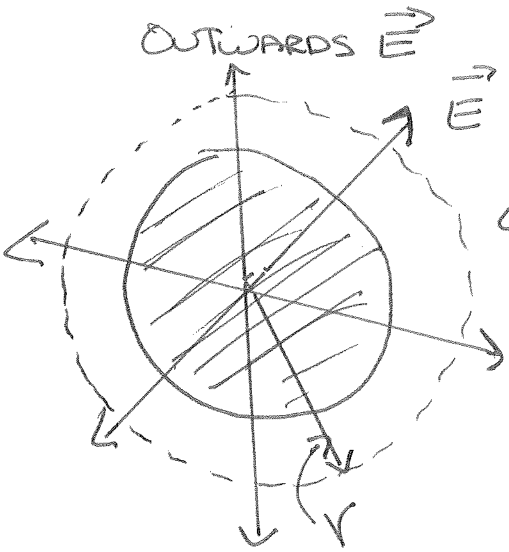
↑ Volume of sphere!

$$\text{Total charge} \Rightarrow Q_{TOTAL} = \int_0^R \rho\, 4\pi r^2 dr = \int_0^R \rho_0 (1 - r/R)\, 4\pi r^2 dr = \rho_0 4\pi \int_0^R (r^2 - r^3/R)\, dr$$

$$= 4\pi\rho_0 \left[\frac{1}{3} r^3 - \frac{1}{4R} r^4\right]\Big|_0^R = 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{R^4}{4R}\right] = 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{R^3}{4}\right]$$

$$= 4\pi\rho_0 R^3 \left(\frac{1}{3} - \frac{1}{4}\right) = 4\pi\rho_0 R^3 \left(\frac{1}{12}\right) = \frac{1}{3}\pi\rho_0 R^3 = \frac{\pi}{3}\left(\frac{3Q}{\pi R^3}\right) R^3 = Q.$$

b) SHOW For $r \geq R$ that E = point charge $= \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$



GAUSSIAN sphere Because $\vec{E}$ & $d\vec{A}$ ~~are~~ parallel

By symmetry E has SAME MAGNITUDE for constant r.

$\Rightarrow \Phi_E = EA = E(4\pi r^2)$

For $r > R$, $Q_{encl} = Q$ (AS WE just showED)

GAUSS'S LAW, $\Phi_E = Q_{encl}/\epsilon_0$

$$\Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}}$$

c) $r \leq R$ SAME GAUSSIAN sphere AND $\Phi_E = E(4\pi r^2)$ [FOR THE SAME REASONS]

$r < R$, less than total Q enclosed

$$Q_{encl} = \int_0^r \rho\, 4\pi r^2 dr = \int_0^r \rho_0 (1 - r/R) 4\pi r^2 dr$$

$$= 4\pi\rho_0 \int_0^r (r^2 - r^3/R)\, dr$$

$$= 4\pi\rho_0 \left[\frac{1}{3}r^3 - \frac{1}{4R}r^4\right]\Big|_0^r = 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

$$Q_{encl} = 4\pi\left(\frac{3Q}{\pi R^3}\right)\left(\frac{r^3}{3} - \frac{r^4}{4R}\right) = \frac{Q}{R^3}\left(4r^3 - \frac{3r^4}{R}\right)$$

$$= Q\left(\frac{4r^3}{R^3} - \frac{3r^4}{R^4}\right)$$

GAUSS'S LAW: $\Phi_E = \frac{Q_{encl}}{\epsilon_0} \Rightarrow E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}\left(\frac{4r^3}{R^3} - \frac{3r^4}{R^4}\right)$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}\left(\frac{4r^3}{R^3} - \frac{3r^4}{R^4}\right) \Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0}\left(\frac{4r}{R^3} - \frac{3r^2}{R^4}\right)}$$

OUTWARDS IN DIRECTION

I'll FIND MAX FIRST to help with plotting

$E_{max} \Rightarrow \frac{dE}{dr} = 0 \Rightarrow \frac{Q}{4\pi\epsilon_0}\left(\frac{4}{R^3} - \frac{6r}{R^4}\right) = 0$

$$\Rightarrow \frac{4}{R^3} = \frac{6r}{R^4} \Rightarrow \frac{4R^4}{6R^3} = r \Rightarrow \boxed{r = \frac{2}{3}R}$$

INSIDE DISTRIBUTION

at $r = \frac{2}{3}R$, $E = \frac{Q}{4\pi\epsilon_0}\left[\frac{4}{R^3}\left(\frac{2}{3}R\right) - \frac{3}{R^4}\left(\frac{2}{3}R\right)^2\right]$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{8}{3}\cdot\frac{1}{R^2} - \frac{4}{3}\cdot\frac{1}{R^2}\right] = \frac{Q}{4\pi\epsilon_0}\frac{4}{3}\cdot\frac{1}{R^2}$$

$$\Rightarrow \boxed{E_{max} = \frac{Q}{3\pi\epsilon_0 R^2}}$$

$$E = \frac{Q}{4\pi\epsilon_0}\left(\frac{4r}{R^3} - \frac{3r^2}{R^4}\right) \Rightarrow E = 0 \text{ at } r = 0$$

$$E = \frac{Q}{4\pi\epsilon_0}\left(\frac{4}{R^2} - \frac{3}{R^2}\right) = \frac{Q}{4\pi\epsilon_0}\frac{1}{R^2} \text{ at } r = R$$

and behaves as $\frac{1}{r^2}$ for $r > R$