# Physics 262 Fall 2011 Exam #1

**Name:______________________________**

**CLOSED BOOK, CALCULATORS ONLY**
**USE A RULER!   SHOW ALL WORK!   Use extra blank sheets if necessary**

$c^2 = \frac{1}{\varepsilon_O \mu_O}, \vec{\mathbf{S}} = \frac{1}{\mu_O}\vec{\mathbf{E}} \times \vec{\mathbf{B}}, I = \langle |\vec{\mathbf{S}}| \rangle,$

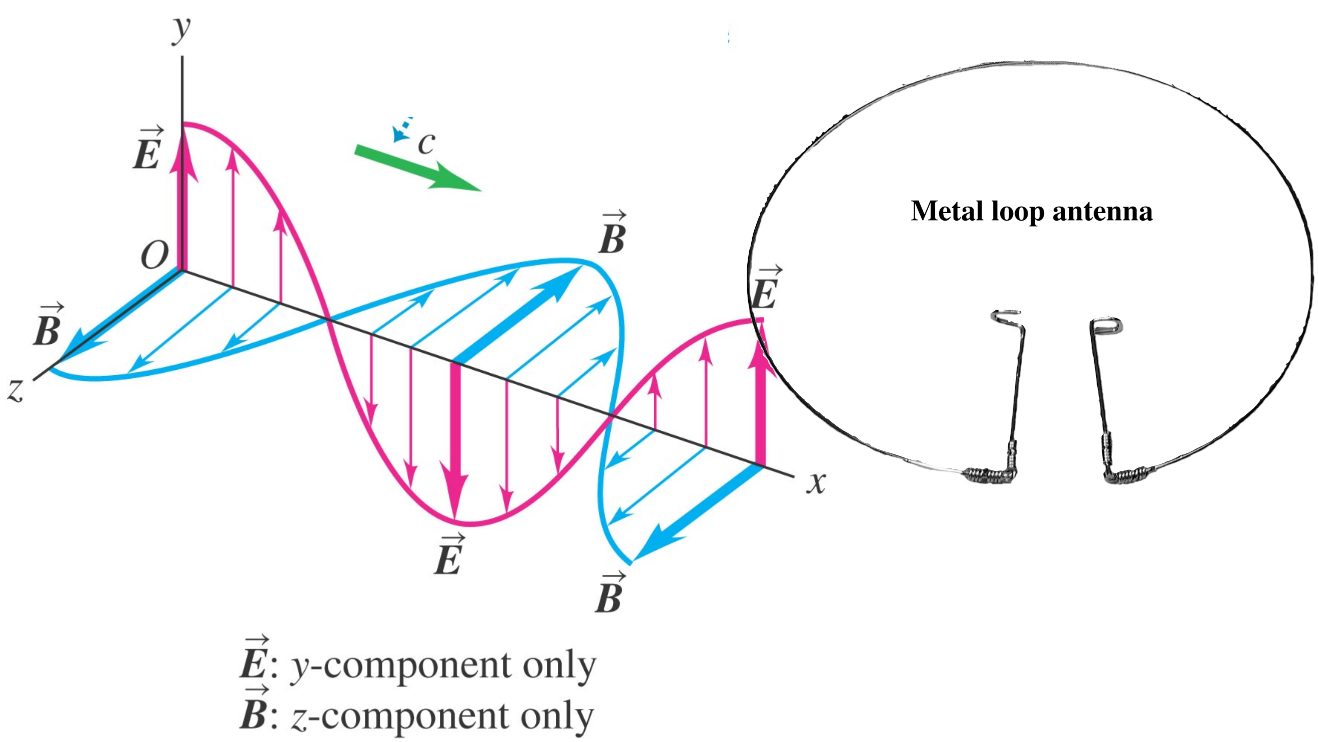
$c = 3.00 \times 10^8 \,\mathrm{m\,s^{-1}} \quad \varepsilon_O = 8.85 \times 10^{-12}\, C V^{-1} m^{-1}$

$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}, \quad \frac{1}{s} + \frac{1}{s'} = \frac{1}{f},$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

## EXTRA CREDIT (Save for last) (10 points)

5 Points: A circular metal loop antenna can be oriented so that the metal loop is in the xy, xz or yz planes. Which orientation will receive the highest signal for the electromagnetic wave with the polarization shown?






5 Points: Which of Maxwell's Equations describes how this antenna works (Hint: the response of this antenna depends on the rate of change of the electromagnetic fields).

ORIENT the Antenna loop in the XY Plane so that Faraday's Law applies:

$$\oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} = -\frac{d\Phi_{\vec{\mathbf{B}}}}{dt} = -\frac{d\left(\int \vec{\mathbf{B}} \bullet d\vec{\mathbf{A}}\right)}{dt}$$

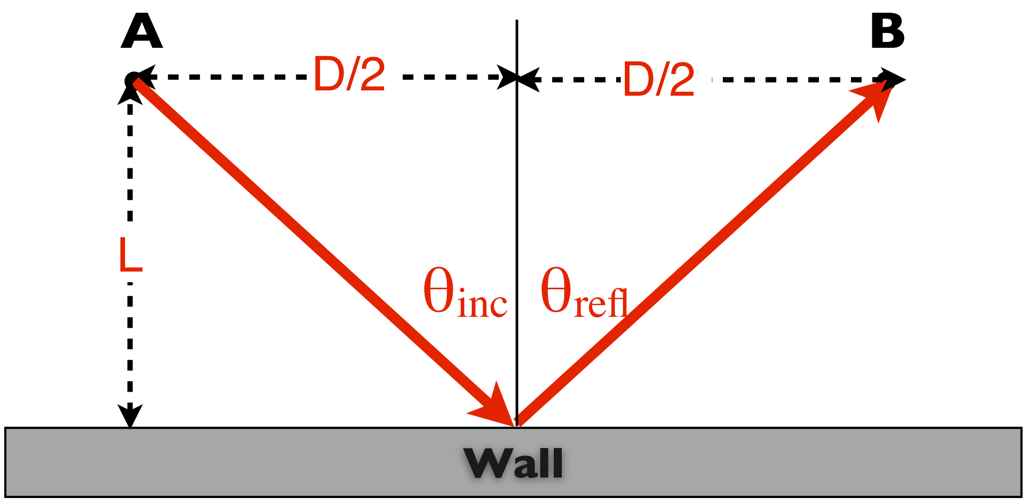
**1) (25 points)**
The laser pointer I use in class operates at a wavelength of 670 nm with a power of 0.500 mW spread uniformly over a circle 1.20 mm in diameter. It is also waterproof.

Assuming my laser pointer works in air and in water, fill in the following table. Water has an index of refraction of n=1.33

| | **Air** | **Water** |
|---|---|---|
| Wavelength, $\lambda$ | 670 nm | $\lambda$ =670 nm/n=504 nm |
| Frequency, f | $f\lambda=c$, $f=c/\lambda=4.48\times10^{14}$ Hz | $f\lambda=c$, $f=c/\lambda=4.48\times10^{14}$ Hz |
| Angular Frequency, $\omega$ | $\omega=2\pi f=2.81\times10^{15}$ $s^{-1}$ | $\omega=2\pi f=2.81\times10^{15}$ $s^{-1}$ |
| Wavenumber, k | $k=2\pi/\lambda=9.38\times10^{6}$ $m^{-1}$ | $k=2\pi n/\lambda=1.25\times10^{7}$ $m^{-1}$ |
| Period, T | $T=1/f=2.23\times10^{-15}$ s | $T=1/f=2.23\times10^{-15}$ s |

**2) 20 points**

(10 points) Sprinters must run (at a constant velocity) from point A to point B but must touch the wall at some point between starting and finishing the race. Accurately sketch the quickest path the sprinters should take under these rules.

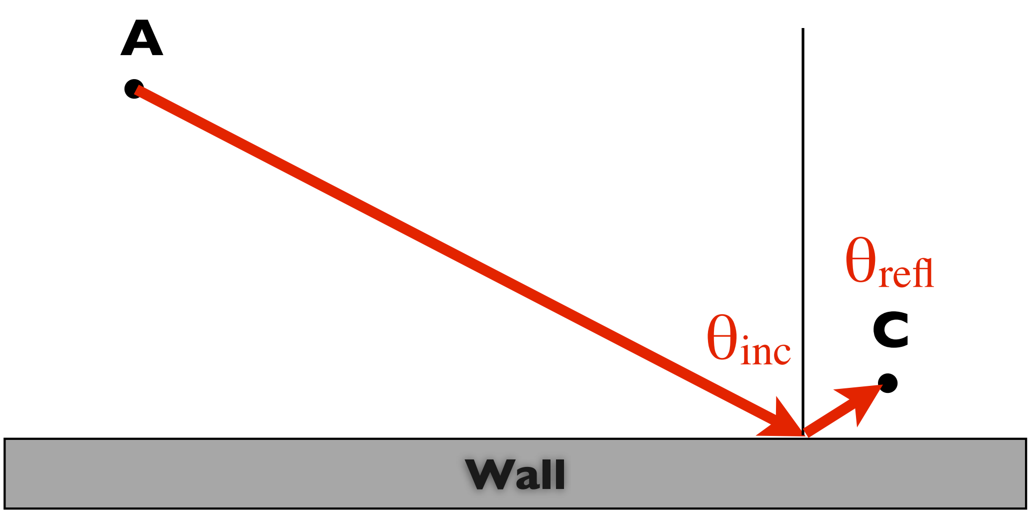



(5 points) In a sentence or two, describe how you could you prove (mathematically) that your path takes less time than any other path. What does this have to do with geometric optics?

Could calculate the path length in terms of L, D, $\theta_{inc}$ and $\theta r_{efl}$, then minimize the time it takes to get from A to B with respect to time by taking the derivative with respect to time.

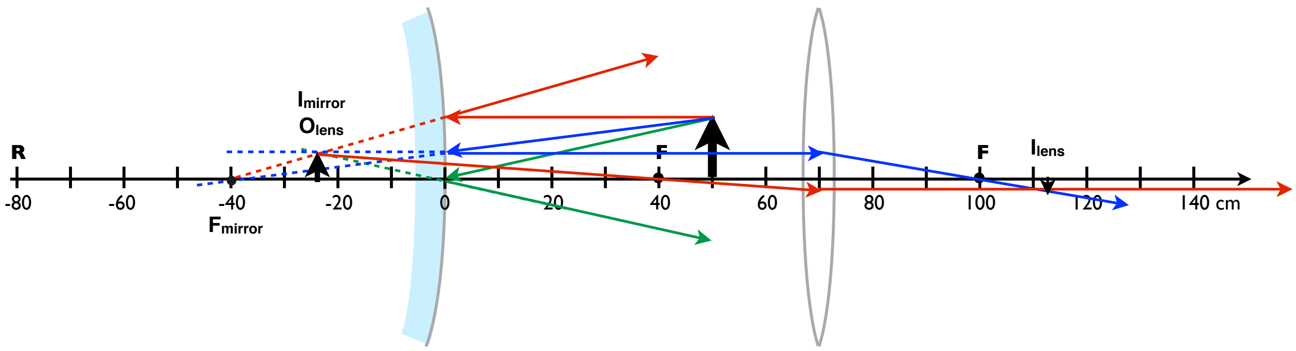
This is Fermat's Principal of Least Action and the derivation of the law of reflection: angle of incidence = angle of reflection. See problem 33.56 in your book

(5 points) Suppose the sprinters must run from Point A to Point C, again by touching the wall at some point between starting and finishing. Again, accurately sketch the path taking the shortest time between A and C.



## 3) (30 points)

An optical system comprises in turn, from left to right: a convex mirror of radius 80 cm, an erect object 20 mm high, a lens of focal length +30 cm, and an observer. The object is between the lens and the mirror, 20 cm from the lens and 50 cm from the mirror. The observer views the image that is formed first by reflection and then by refraction.




## 3) A 15 points

On the diagram above or on the next page, draw as many principal rays as you need to find the position of the image in the convex mirror and the final image as viewed from the right.

## 3) B 15 points

Use the lens and mirror equations and the magnification equations and find the exact position of the image and the exact lateral magnification for the approximate findings above.

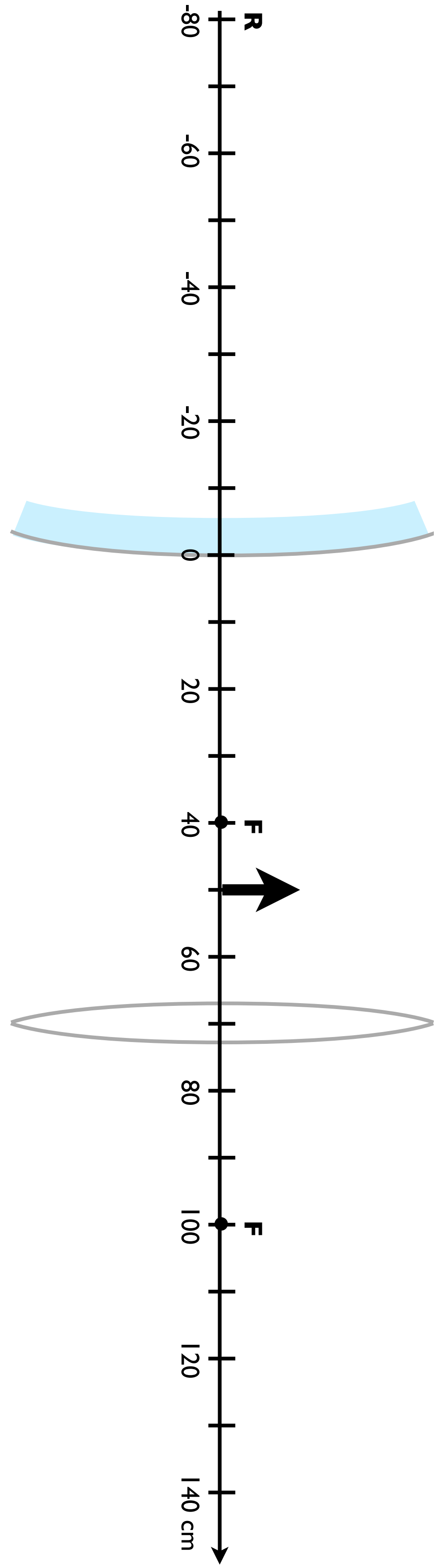
Mirror:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}:\ \frac{1}{0.5\text{m}}+\frac{1}{s'}=-\frac{1}{0.4\text{m}}, s'=-0.22\text{m},\ m_{mirror}=-\frac{s'}{s}=-\frac{-0.22\text{m}}{0.5\text{m}}=+.44$$

Lens:

$$\frac{1}{(0.22\text{m}+0.70\text{m})}+\frac{1}{s'}=\frac{1}{0.3\text{m}}, s'=+0.42\text{m (this is 0.70+0.42 =1.12m from mirror)},\ m_{lens}=-\frac{s'}{s}=-\frac{0.42\text{m}}{0.92\text{m}}=-.45, \text{ overall } m=.20$$

**Problem 3 extra diagram**



**4) (5 points)**

At a given time, an electromagnetic plane wave propagating in the negative z-direction is polarized in the positive y direction. What is the direction of the magnetic field? Assume a right-handed coordinate system.

The B Field is in the Positive-x direction so that **E**x**B** will be in the negative-z direction

**5) 10 points**

Total internal reflection occurs for light traveling from a higher index material to a lower index material when the angle of incidence exceeds a critical value.

The Brewster or polarizing angle is the one angle where the angle between the reflected ray and the refracted (transmitted) ray is 90°.

A) (5 points) Is the Brewster (or polarizing) angle of incidence greater than or less than the critical angle of incidence for total internal reflection? Can the Brewster angle occur simultaneously with total internal reflection?

*DEFINITIONS* :

$$\tan(\theta_P) = \frac{n_2}{n_1} = \sin(\theta_C)$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} > \sin(\theta),\ \arctan\left(\frac{n_2}{n_1}\right) < \arcsin\left(\frac{n_2}{n_1}\right)$$

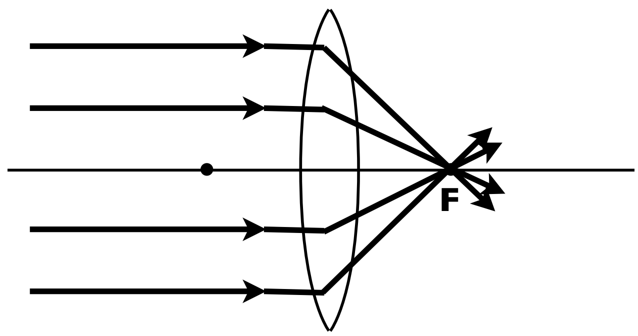
The Brewster Angle polarization condition always occurs for a smaller angle of incidence than the critical angle and therefore, Brewster polarization cannot occur simultaneously with total internal reflection

For normal materials, the maximum index of refraction is $n \approx 4.0$ However, nothing fundamental prohibits much higher indices and indeed, an area of research called "metamaterials" includes building materials with unusually high indices of refraction.

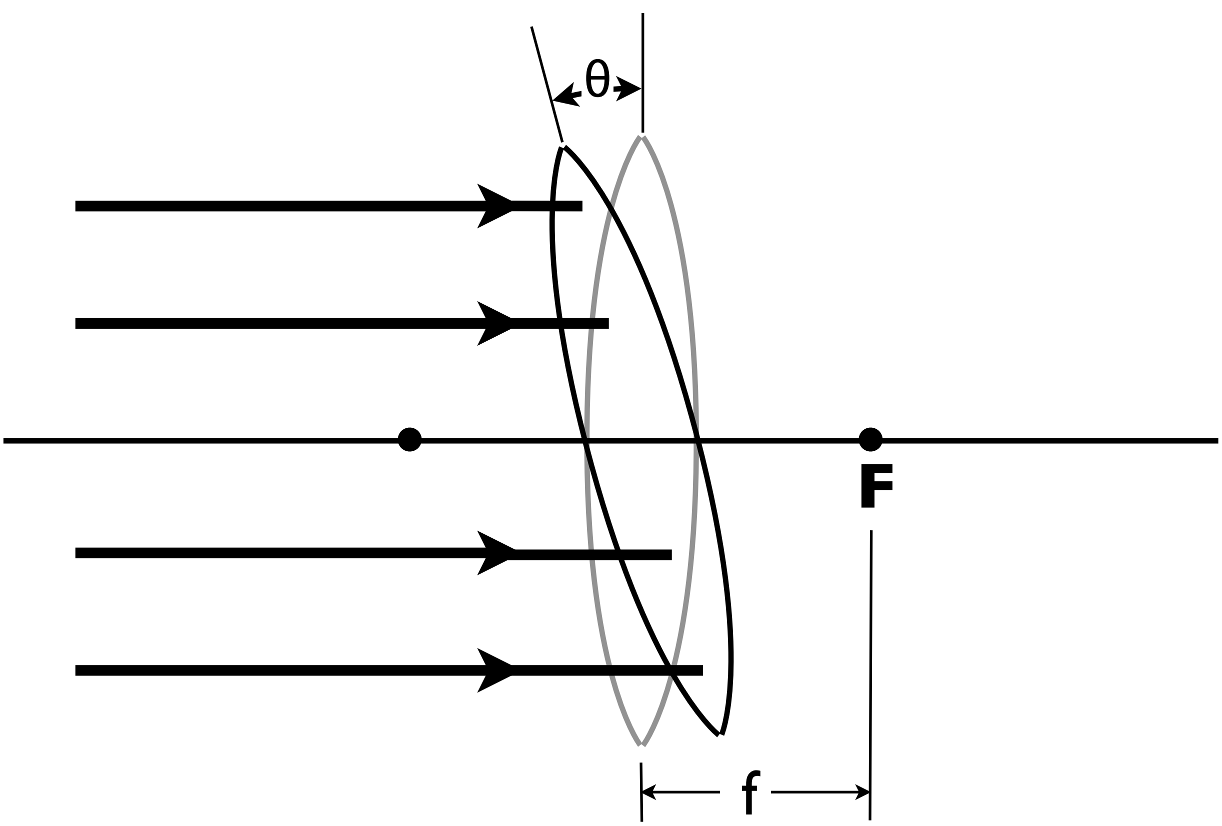
B) (5 points) Suppose light is incident from a material with an index of $n_1=100$ onto air $n_2=1.0$. What is the difference between the critical angle for total internal reflection and the Brewster (or polarizing) angle?

The Brewster Angle polarization condition occurs for arctan (0.01)=0.57294°=0.0099997 radians while the critical angle occurs for arcsin(0.01)=0.57297°=.0100002 radians. This might be an interesting effect to try to use as a switch or other device if such a metamaterial could be made.

**6) 10 points**
Consider a converging lens with focal length f. Rays from a object at infinity converge at the focal point as shown.




Where does the point of convergence move to if the lens is rotated a small angle θ? Specify the direction and the distance away from point "F." (Hint: Consider the principal ray through the center of the lens).
Strictly speaking based on the central Principal Ray and objects at infinity, point F does not move. You could research the "Scheimpflug principle" on Wikipedia to understand the effect for objects closer than that.



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