Lecture 18 (Equipotential Surfaces and Gradients)

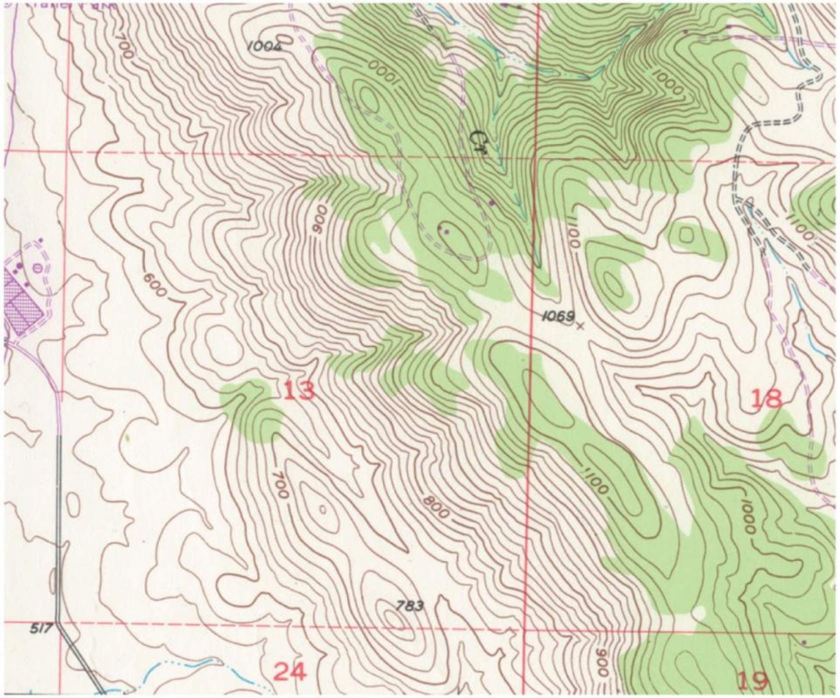
Physics 161-01 Spring 2012

Douglas Fields

Equipotential Surfaces

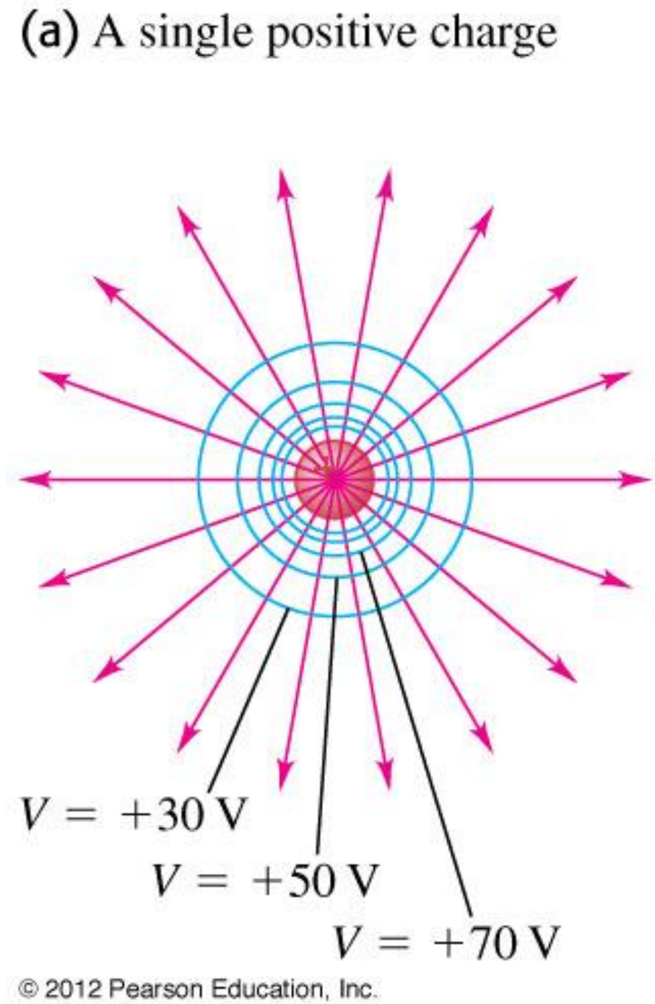
- Lines or surfaces of constant potential are called equipotential lines or surfaces.
- Since a charge moving along an equipotential surface will always have the same potential energy, then the electric field does no work.
- Hence the equipotential surface must be perpendicular to the field lines everywhere.

- One can think of equipotential surfaces in the same way as contour lines on a topo map.
- Notice that the contour lines are everywhere perpendicular to the "line of fall" that a stone would take rolling down a hill.



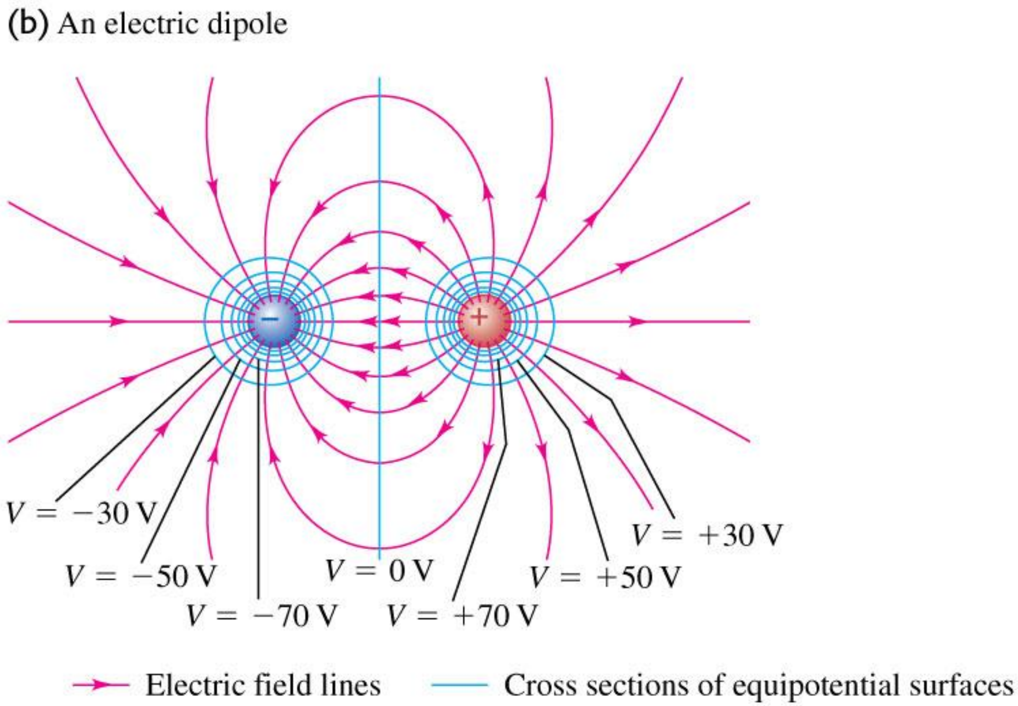
Equipotential Surfaces

- For a point charge, the equipotential surfaces are just spheres.
- Notice that the equipotential spheres get closer together as you approach the charge.
- Notice that the field and the equipotential surface are perpendicular.



Equipotential Surfaces

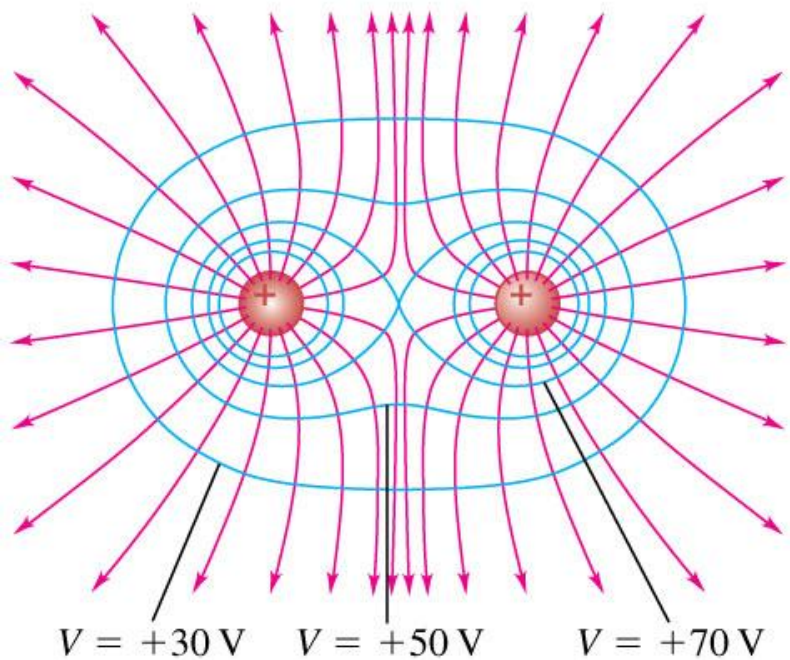
- For a dipole, there is a plane that divides the dipole that sits at V = 0. (Notice that the plane extends to infinity, so V = 0 at infinity as well.)



Equipotential Surfaces

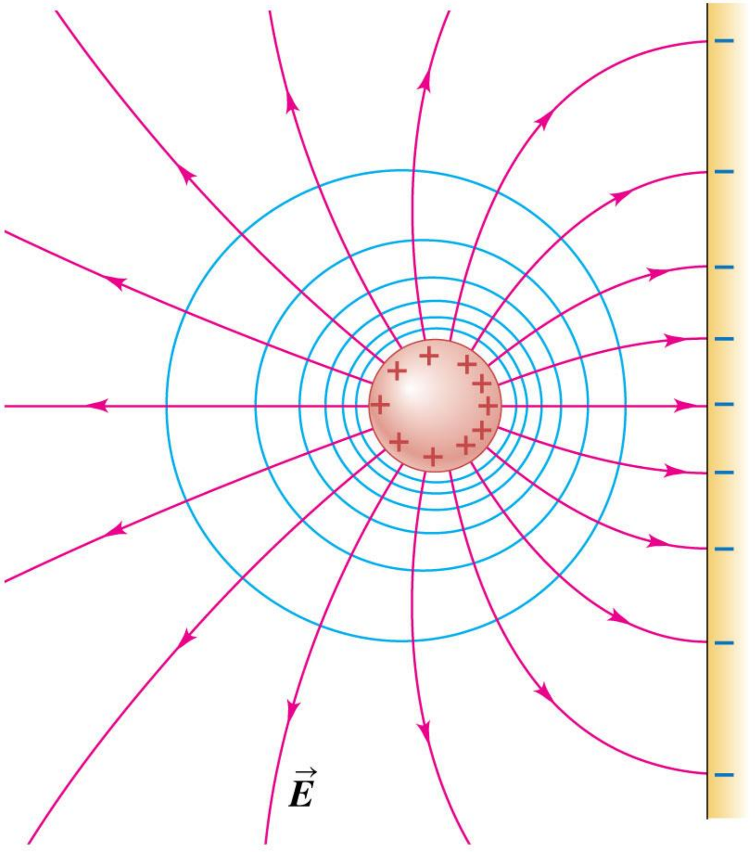
- For two positive charges.

(c) Two equal positive charges



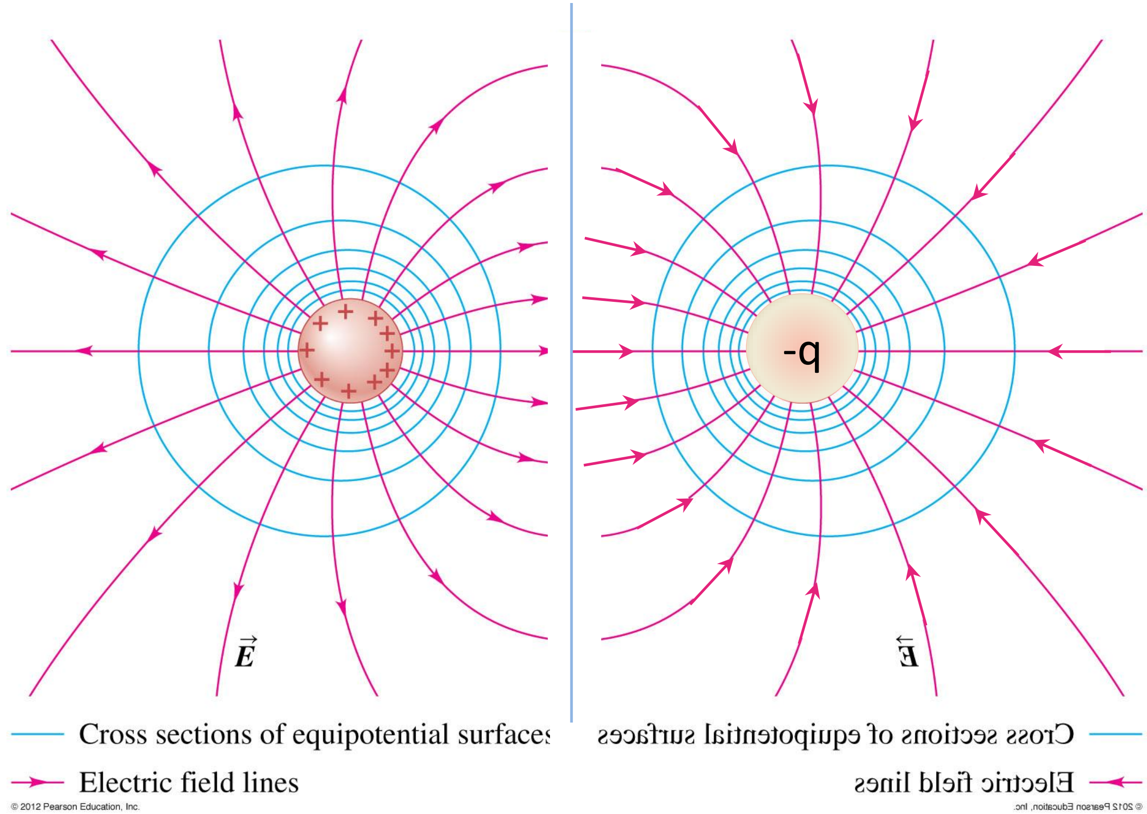
Equipotential Surfaces

- A charge near a conducting plane.
- Note: the conductor is all at one potential.
- Notice how the electric field is strongest closest to the charge.
- The charge distribution (called an image charge) on the conductor is distributed so that the electric field is perpendicular to the conductor, or equivalently, so that the potential everywhere on the conductor is constant.



Equipotential Surfaces

- Since we have a charge, with an equipotential plane (it doesn’t matter that that is because of the charge distribution on the conductor), we see the similarity with a previously seen distribution…



CPS 18-1

Where an electric field line crosses an equipotential surface, the angle between the field line and the equipotential is

A. zero.

B. between zero and 90°.

C. 90°.

D. not enough information given to decide

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CPS 18-2

Consider a point *P* in space where the electric potential is zero. Which statement is correct?

A. A point charge placed at *P* would feel no electric force.

B. The electric field at points around *P* is directed toward *P.*

C. The electric field at points around *P* is directed away from *P.*

D. none of the above

E. not enough information given to decide

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Electric Field from the Potential

- Let's say that somehow we have determined the electric potential everywhere in space from a charge distribution.

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{r} \Rightarrow$$

$$\int_a^b dV = -\int_a^b \vec{E} \cdot d\vec{r} \Rightarrow$$

$$dV = -\vec{E} \cdot d\vec{r}$$

$$dV = -\left(E_x \hat{i} + E_y \hat{j} + E_z \hat{k}\right) \cdot \left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$dV = -E_x dx - E_y dy - E_z dz$$

Electric Field from the Potential

- If we now hold y and z constant (so that dy and dz are zero) then,

$$dV = -E_x dx - \cancel{E_y dy} - \cancel{E_z dz} \Rightarrow$$

$$dV = -E_x dx \Rightarrow$$

$$E_x = -\left.\frac{dV}{dx}\right|_{\text{y and z constant}} \equiv -\frac{\partial V}{\partial x}$$

- Likewise then, for the other components of the electric field,

$$E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

Electric Field from the Potential

- We can write this all together:

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$\vec{E} = \frac{-\partial V}{\partial x} \hat{i} + \frac{-\partial V}{\partial y} \hat{j} + \frac{-\partial V}{\partial z} \hat{k}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \equiv \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

CPS 18-3

The direction of the electric potential gradient at a certain point

A. is the same as the direction of the electric field at that point.

B. is opposite to the direction of the electric field at that point.

C. is perpendicular to the direction of the electric field at that point.

D. not enough information given to decide

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Electric Field from the Potential

- Let’s do an example.

$$V\left(x,y,z\right)=\left(5\frac{V}{m}\right)x+4V$$

$$\vec{E}=-\vec{\nabla}V=\frac{\partial V}{\partial x}\hat{i}+\frac{\partial V}{\partial y}\hat{j}+\frac{\partial V}{\partial z}\hat{k}$$

$$=\frac{\partial\left(5x+3\right)}{\partial x}\hat{i}+\frac{\partial\left(5x+3\right)}{\partial y}\hat{j}+\frac{\partial\left(5x+3\right)}{\partial z}\hat{k}$$

$$=5\frac{V}{m}\hat{i}$$

Electric Field from the Potential

- What about if we are given a potential that has some spherical or cylindrical symmetry?
- We can use the same relationship, but we have to use the del-operator derived in that coordinate system:
  - Relationship between different coordinate systems

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \equiv \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{\phi}$$

$$\vec{E} = \frac{-\partial\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right)}{\partial r}\hat{r} - \frac{1}{r}\cancel{\frac{\partial\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right)}{\partial \theta}}\hat{\theta} - \frac{1}{r\sin\theta}\cancel{\frac{\partial\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right)}{\partial \phi}}\hat{\phi}$$

$$\vec{E} = \frac{-q}{4\pi\varepsilon_0}\frac{\partial\left(\frac{1}{r}\right)}{\partial r}\hat{r}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}$$