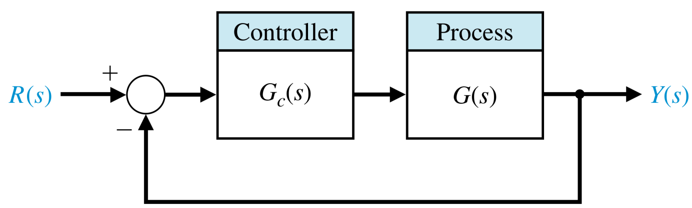
ECE 345: Introduction to Control Systems
Problem Set #6

Dr. Oishi

Due Thursday, November 29, 2012 at the start of class

This homework is open note and open book. You are welcome to discuss the problems with other students, but your solutions and Matlab code *must be written independently*. Copying will not be tolerated.

Consider the system in the figure below with $G(s) = \frac{1}{s^2+5s+6}$.



This assignment will investigate the use of two different controllers $G_c(s)$ under negative unity feedback: (1) Proportional (P) $G_c(s) = K$, (2) Proportional-integral (PI) $G_c(s) = K(1 + \frac{1}{s})$. We want to design the system such that settling time $T_s \leq 2$ seconds and overshoot M_p is less than or equal to 5% for a step input.

1. Consider the effect of the proportional controller $G_c(s) = K$.

   (a) Sketch the region in the complex plane for which the transient performance criteria is met. Use the attached complex plane plot for your sketch.

   (b) What is the type number of the closed-loop system? Calculate the steady-state error in response to a ramp input.

   (c) Sketch the root locus by hand. Locate the asymptotes, break-in / breakaway points, departure / arrival angles, as appropriate. Use the attached complex plane plot for your sketch.

   (d) For what values of K, if any, are the transient performance specifications satisfied for the closed-loop system?

2. We reconsider the same plant, but with PI control $G_c(s) = K(1 + \frac{1}{s}) = \frac{K(s+1)}{s}$.

   (a) Compare the type number of the closed-loop system under integral control to the type number of the closed-loop system under proportional control. Which controller will yield better steady-state error to a unit step input? Why?

(b) Sketch the root locus by hand. Locate the asymptotes, break-in / breakaway points, departure / arrival angles, as appropriate. Use the attached complex plane plot for your sketch.

(c) From your sketch only (without any additional calculations), determine whether there exists a value of K for which the performance criteria are satisfied. *(Neglect the effect of the pole nearest to the origin in this case.)*

(d) Is it possible to destabilize the closed-loop system by increasing the gain?

3. Now evaluate all of the above controllers for this system.

   (a) Which of the controllers yields the best steady-state response? Justify your selection.

   (b) Which of the controllers yields the best transient performance? Justify your selection. Consider the effect of the zero near the origin in the PI control, as well as the magnitude of the gain required to meet the transient performance specifications.

4. (BONUS, +5 points) We reconsider the same plant, but with integral controller $G_c(s) = \frac{K}{s}$.

   (a) Sketch the root locus by hand. Locate the asymptotes, break-in / breakaway points, departure / arrival angles, as appropriate. Use the attached complex plane plot for your sketch.

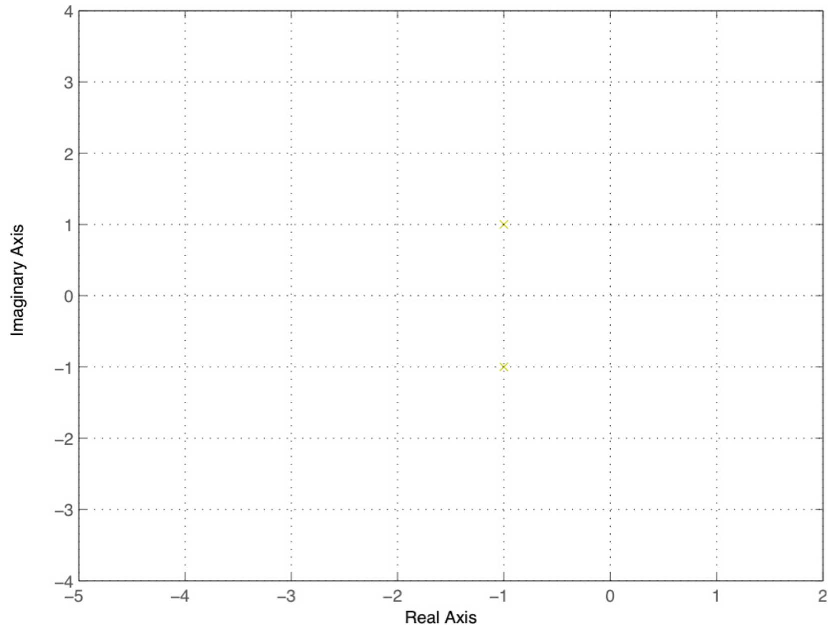
   (b) Determine for which values of K the closed-loop system is stable.

   (c) Determine the value of K for which the settling time criteria is just met. Is the overshoot criteria met for the same value of K?

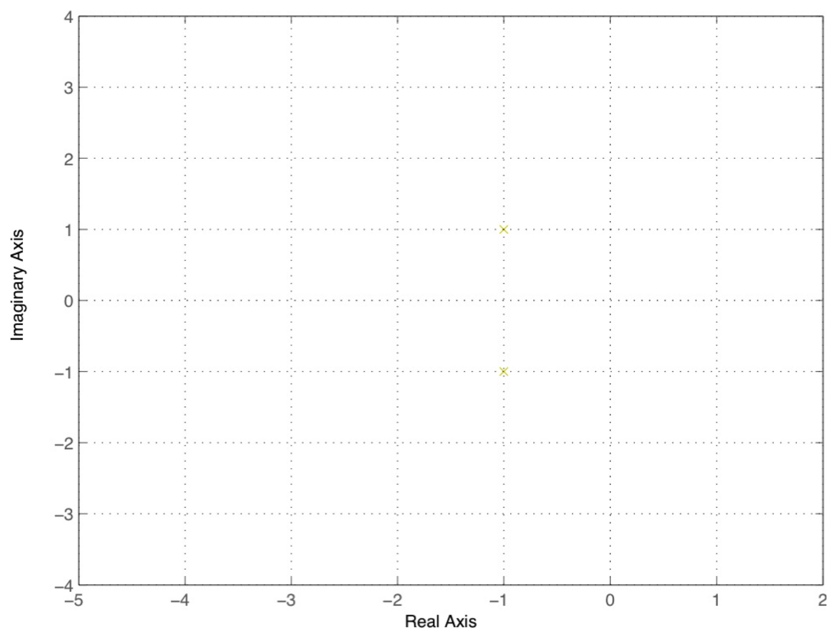
   (d) Reconsider your answer to problem 3(b): Which of the P, I, or PI controllers yields the best transient performance?

Root Locus Sketches

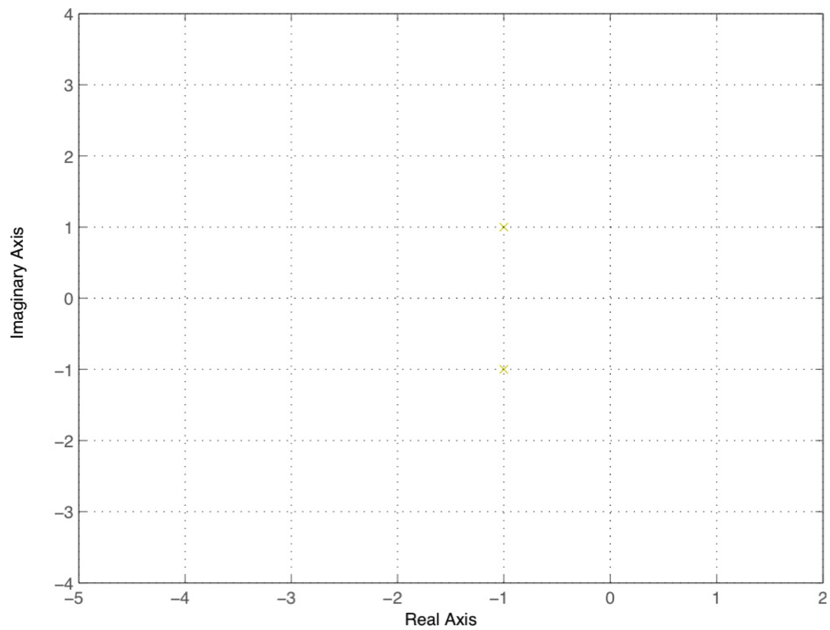
1. (a)



(c)

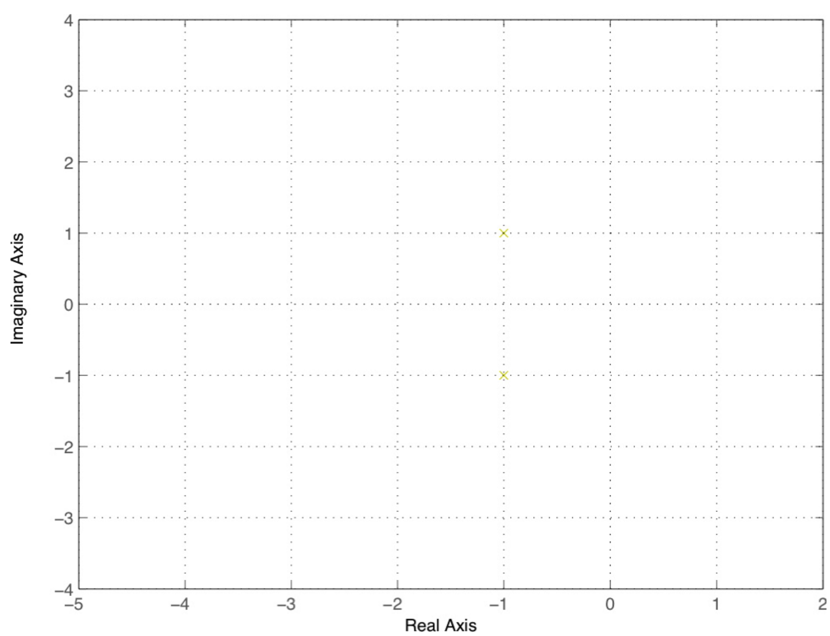


2. (b)


Imaginary Axis
Real Axis
4
3
2
1
0
−1
−2
−3
−4
−5
−4
−3
−2
−1
0
1
2

BONUS:

4. (a)


Imaginary Axis
Real Axis
4
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2