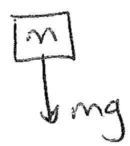
Bungee-cord jumper Sim | Exercise #1

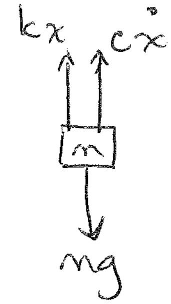
M. Oishi 8/28/12

Pre-Lecture

1. Free fall:



2. Active cord:



3. $\Sigma F = ma$, $a = \ddot{x}$ "+" is ↓

Free fall: $mg = m\ddot{x}$

Active cord: $mg - kx - c\dot{x} = m\ddot{x}$

4. i) Parabolic motion.

$$v(t) = \int_0^t \ddot{x}(\tau)\,d\tau = \int_0^t g \cdot d\tau$$
$$= gt$$

$$x(t) = \int_0^t \dot{x}(\tau)\,d\tau = \int_0^t g\tau \cdot d\tau$$
$$= \frac{1}{2}gt^2$$

For $x(t) = x_0 = 50\,m$

ii) $$t_0 = \sqrt{\frac{2x_0}{g}} \simeq 3.2 \text{ sec.}$$

so $v_0 = g t_0 \simeq 31.3$ m/s

In-class

1. $\mathcal{L}\{m\ddot{x}\} = \mathcal{L}\{mg - kx - c\dot{x}\}$

with $x(0) = x_0 = 0$
$\dot{x}(0) = v_0$

$$m\left[s^2 X(s) - s\cdot x_0 - v_0\right] = mg\cdot\frac{1}{s} - k\left[X(s)\right] - c\left[s X(s) - x_0\right]$$

$$X(s)\left[ms^2 + cs + k\right] = \frac{mg}{s} + mv_0 \qquad (a)$$

2. $$\lim_{t\to\infty} x(t) = \lim_{s\to 0} s\,X(s)$$

$$= \lim_{s\to 0} s\,\frac{(mg/s + mv_0)}{ms^2 + cs + k}$$

$$x_{ss} \triangleq mg/k$$

3. For $x_{ss} \leq 50$ (since $L_0 = 50$m & the platform height is 102m for a 2m ball jump)

$$\frac{mg}{k} \leq 50$$

$$\Rightarrow \boxed{\frac{mg}{50} \leq k}$$ no restrictions on c to meet height requirement.

4. $\Delta x = x - x_{ss} \Rightarrow \Delta v = \Delta\dot{x} = \dot{x}$

$$m\left[s^2\Delta X(s) - s\Delta x_0 - v_0\right] = -k\cdot\Delta X(s) - c\left[s\Delta X(s) - \Delta x_0\right]$$

$$\Delta X(s)\left[ms^2 + cs + k\right] = mv_0 + (ms + c)\Delta x_0$$

$$\Delta X(s) = \frac{mv_0 - cx_{ss} - mx_{ss}\cdot s}{ms^2 + cs + k}$$

$$= \frac{v_0 - c/m\, x_{ss} - x_{ss}\cdot s}{s^2 + c/m\cdot s + k/m}$$

Note that for $(s+a)^2 + b^2 = s^2 + 2as + a^2 + b^2 = s^2 + c/m\, s + k/m$

$\Rightarrow 2a = c/m$, $\quad a^2 + b^2 = k/m$

$a = \frac{c}{2m}$ $\qquad b = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \frac{\sqrt{4mk - c^2}}{2m}$

So $$\Delta X(s) = \frac{A(s+a)}{(s+a)^2+b^2} + \frac{B\cdot(b)}{(s+a)^2+b^2}$$

$$\therefore\ A(s+a) + Bb = -m x_{ss}\cdot s + m v_0 - c x_{ss}$$

$$A\cdot s + A\cdot\frac{c}{2m} + B\frac{\sqrt{4mk-c^2}}{2m} = -m x_{ss}\cdot s + m v_0 - c x_{ss}$$

Matching coefficients,

$$A = -m x_{ss}, \qquad A\cdot\frac{c}{2m} + B\frac{\sqrt{4mk-c^2}}{2m} = m v_0 - c x_{ss}$$

$$B\frac{\sqrt{4mk-c^2}}{2m} = m v_0 - c x_{ss} + m x_{ss}\cdot\frac{c}{2m}$$

$$B = \frac{2m(m v_0 - c x_{ss}/2)}{\sqrt{4mk-c^2}}$$

$$\therefore\ \Delta x(t) = \mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+b^2}\right\}\cdot A + B\cdot\mathcal{L}^{-1}\left\{\frac{b}{(s+a)^2+b^2}\right\}$$

$$= Ae^{-at}\cos(bt)\cdot u(t) + Be^{-at}\sin(bt)\,u(t)$$

$$= e^{-at}u(t)\left[A\cos(bt) + B\sin(bt)\right]$$

w/ typo in MC options, closest answer is (a)

5. $$d(t) = d_0 e^{-\frac{c}{2m}t} = 0.02\, d_0$$

$$-\frac{c}{2m}\cdot t = \ln(1/50)$$

$$t = \frac{2m}{c}\ln(50)$$

Subtract out time for free fall, then for hang time:

$$25\text{ sec} \le \frac{m}{c}\cdot 2\ln 50 + t_0 \le 50\text{ sec}.$$

$$\underbrace{\frac{2\ln 50}{25 - t_0}}_{\in[17.9,\ 35.9]\ \frac{kg}{s}} \ge c \ge \underbrace{\frac{2\ln 50}{50 - t_0}\cdot m}_{\in[8.4,\ 16.7]\ \frac{kg}{s}}$$

$\to$ So $c \in [16.7, 17.9]$ kg/s will work.

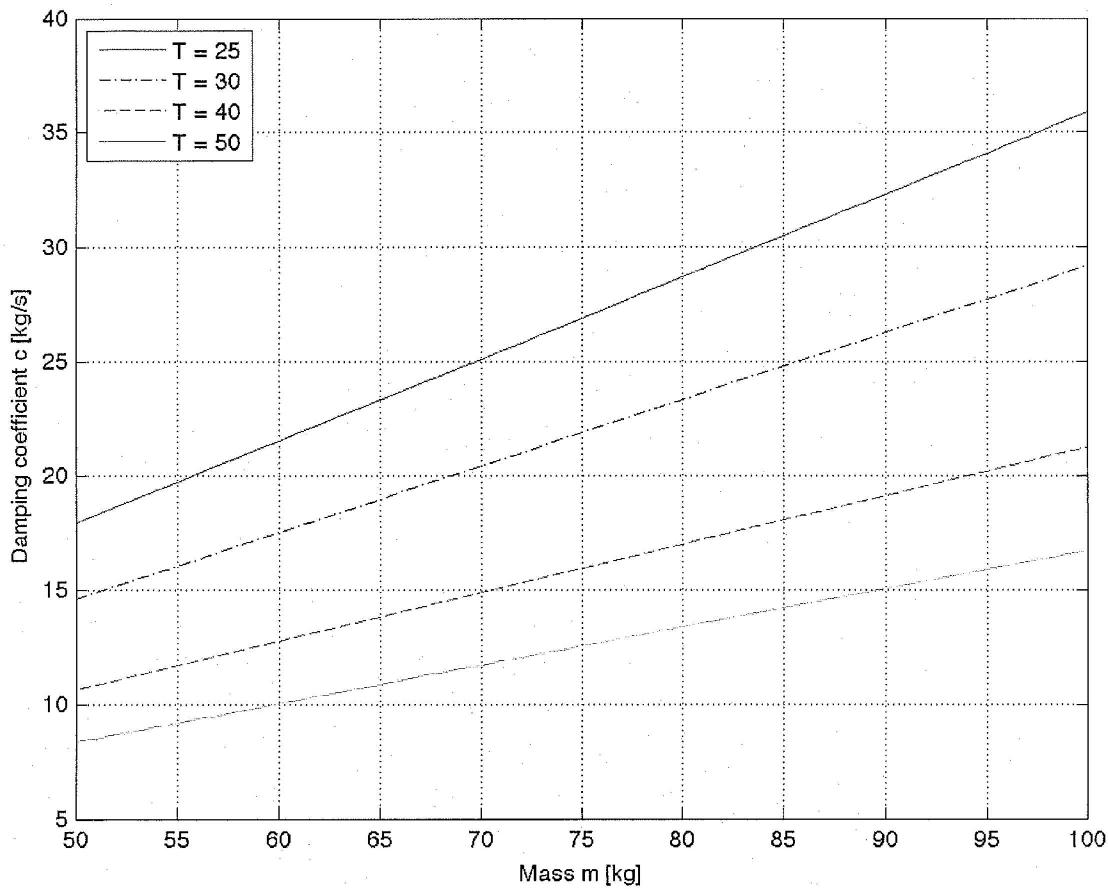
6. (b), (c), (d) are true.

(b): see answer to #5, & plot next page

(c): As $m\downarrow$, $b\uparrow$, hence frequency of oscillations increases (see #4)

(d): Jumping may change v_0, but x_{ss} (see #3) is <u>not</u> dependent on v_0.

(e): As $k\uparrow$, $x_{ss}\downarrow$, meaning jumper comes to rest <u>higher</u> (= further from the ground)


40
35
30
25
20
15
10
5
Damping coefficient c [kg/s]
T = 25
T = 30
T = 40
T = 50
50
55
60
65
70
75
80
85
90
95
100
Mass m [kg]