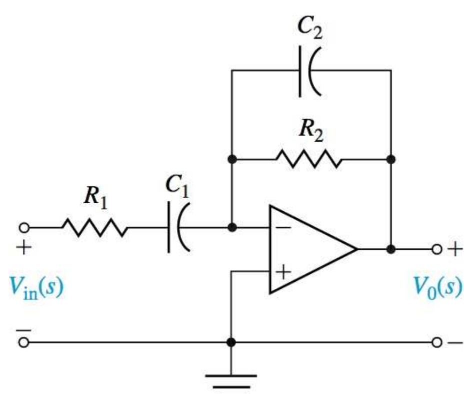
ECE 345: Introduction to Control Systems
Problem Set #1

Dr. Oishi

Due Thursday, September 6, 2012 at the start of class

This homework is open note and open book. You are welcome to discuss the problems with other students, but your solutions *must be written independently*. Copying will not be tolerated.

1. Consider the linear op-amp circuit shown below.



(a) Assuming all initial conditions are zero, find the transfer function $G(s) = V_0(s)/V_{in}(s)$.

Now assume the initial conditions are equal to zero, and that $C_1 = C_2 = 1$, $R_1 = 2$, $R_2 = 1$.

(b) What is the system response $y(t)$ to a step input? *Hint: Partial fraction expansion may make use of known Laplace transform pairs easier.*

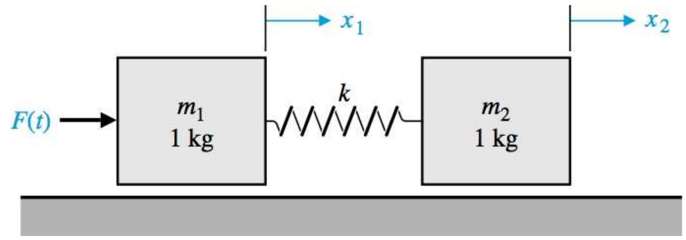
2. Consider a black-box system which we assume to be LTI. When a ramp input $r(t) = t, t \geq 0$ is applied to the system (with zero initial conditions), the resulting output response is

$$y(t) = e^{-t} - \frac{1}{4}e^{-2t} - \frac{3}{4} + \frac{1}{2}t, \quad t \geq 0 \tag{1}$$

(a) Find the transfer function $G(s)$ for this system.

(b) What is the system response $y(t)$ to an impulse input?

3. Consider the dynamical system shown below. Using the constants m_1, m_2, k as described in the text, complete the following:



(a) Draw a free-body diagram for each mass. Write the equations of motion in terms of $x_1(t)$ and $x_2(t)$ and their derivatives.

(b) What is the transfer function with input $F(s)$ and output $X_2(s)$?

(c) Use the Final Value Theorem to determine the steady-state response of the system to a step input.

4. Consider an amplifier with a deadband region, modeled as $v_0(v_{in}) = 3.5v_{in}^3$, where v_{in} is the input voltage to the amplifier, and v_0 is the resulting output voltage.

(a) Create a linear approximation for this amplifier around $v_{in} = 0$.

(b) Create a linear approximation for this amplifier around $v_{in} = 0.6$ volt.

(c) Sketch the nonlinear amplifier v_0 as a function of v_{in}. On the same diagram, sketch the two linear approximations in the neighborhood of each linearization point, respectively.

5. For this exercise, you will hand in a history of Matlab command-line inputs and outputs (hence you should not silence the output with '`;`'). Use `diary` to record your session. The commands `conv`, `roots`, `residual`, `poly` may be helpful. In addition, it is almost always helpful to refer to Matlab's `help` command or to the online documentation.

Consider the following transfer functions:

$$G_1(s) = \frac{20(s+2)(s+3)(s+6)(s+8)}{s(s+7)(s+9)(s+10)(s+15)},\ G_2(s) = \frac{5(s+2)}{s^2+2s+10},\ G_3(s) = \frac{5(s+2)}{s^2+6s+10} \tag{2}$$

(a) Find $G_1(s)$ expressed as one polynomial divided by another polynomial.

(b) Find the roots of the denominator of $G_2(s)$ and of $G_3(s)$ using the `roots` command.

(c) Construct $G_2(s)$ and $G_3(s)$ as transfer functions via the `tf` command.

(d) Calculate the step response of each of $G_2(s), G_3(s)$ via the `step` command. Comment in one or two sentences about the significant similarities and differences between the two responses. (You do not need to hand in the plots, but may do so if it aids in your discussion.)