22.36. The electric field is perpendicular to the square but varies in magnitude over the surface of the square, so we will need to integrate to find the flux.

$\vec{E} = (964\ \text{N/(C}\cdot\text{m))}x\hat{k}$. Consider a thin rectangular slice parallel to the y-axis and at coordinate x with width dx. $d\vec{A} = (Ldx)\hat{k}$. $d\Phi_E = \vec{E}\cdot d\vec{A} = (964\ \text{N/(C}\cdot\text{m))}Lxdx$.

$$\Phi_E = \int_0^L d\Phi_E = (964\ \text{N/(C}\cdot\text{m))}L\int_0^L xdx = (964\ \text{N/(C}\cdot\text{m))}L\left(\frac{L^2}{2}\right).$$

$$\Phi_E = \frac{1}{2}(964\ \text{N/(C}\cdot\text{m))}(0.350\ \text{m})^3 = 20.7\ \text{N}\cdot\text{m}^2/\text{C}.$$

22.40.

Use a Gaussian surface that is a cylinder of radius r, length l and that has the line of charge along its axis. The charge on a length l of the line of charge or of the tube is $q = \alpha l$.

(a) (i) For $r < a$, Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{e_0} = \dfrac{\alpha l}{e_0}$ and $E = \dfrac{\alpha}{2\pi e_0 r}$.

(ii) The electric field is zero because these points are within the conducting material.

(iii) For $r > b$, Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{e_0} = \dfrac{2\alpha l}{e_0}$ and $E = \dfrac{\alpha}{\pi e_0 r}$.

The graph of E versus r is sketched in Figure 22.40.

(b) (i) The Gaussian cylinder with radius r, for $a < r < b$, must enclose zero net charge, so the charge per unit length on the inner surface is $-\alpha$. (ii) Since the net charge per length for the tube is $+\alpha$ and there is $-\alpha$ on the inner surface, the charge per unit length on the outer surface must be $+2\alpha$.

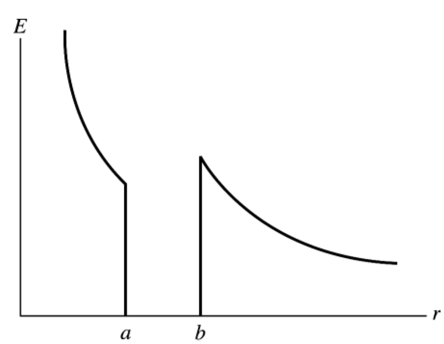


Figure 22.40

46.

a) By symmetry, $\vec{\boldsymbol{E}}$ must point toward or away from the center at every point. If we integrate over a spherical surface of radius r centered on the origin, then

$$\oint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = \oint E_r\, \hat{\boldsymbol{r}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = E_r\, A = 4\,\pi\, r^2\, E_r = \frac{Q_{\text{enc}}}{\epsilon_0},$$

so $E_r = \frac{1}{4\,\pi\,\epsilon_0} \frac{Q_{\text{enc}}}{r^2}$.

$\boldsymbol{r < a}$**.** $Q_{\text{enc}} = Q$, $E_r = \frac{1}{4\,\pi\,\epsilon_0} \frac{Q}{r^2}$. The electric field magnitude is $E = |E_r| = \frac{1}{r\,\pi\,\epsilon_0} \frac{Q}{r^2}$.

$\boldsymbol{a < r < b}$**.** $E_r = \frac{1}{4\,\pi\,\epsilon_0} \frac{Q_{\text{enc}}}{r^2}$, but E_r must be zero because we are inside the conductor. Therefore $Q_{\text{enc}} = 0 = Q + Q_{\text{inner surface}}$, so the charge on the inner surface of the conductor must be $-Q$. This charge has to be exactly on the surface because $Q_{\text{enc}} = 0$ for any $r > a$, no matter how close r is to a.

The electric field magnitude is $E = |E_r| = 0$.

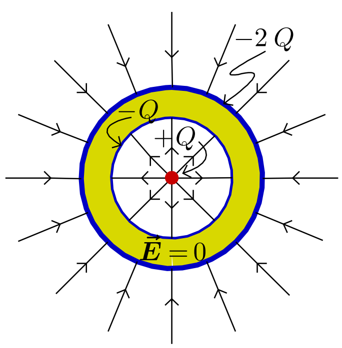
$\boldsymbol{r > b}$**.** $Q_{\text{enc}} = Q + (-3\,Q) = -2\,Q$, $E_r = \frac{1}{4\,\pi\,\epsilon_0} \frac{-2\,Q}{r^2} = -\frac{1}{2\,\pi\,\epsilon_0} \frac{Q}{r^2}$. The final $-2\,Q$ of charge must be exactly on the outer surface of the conductor, because $Q_{\text{enc}} = 0$ for any r between a and b, and Q_{enc} must be $-2\,Q$ as soon as $r > b$.

The electric field magnitude is $E = |E_r| = \frac{1}{2\,\pi\,\epsilon_0} \frac{Q}{r^2}$.

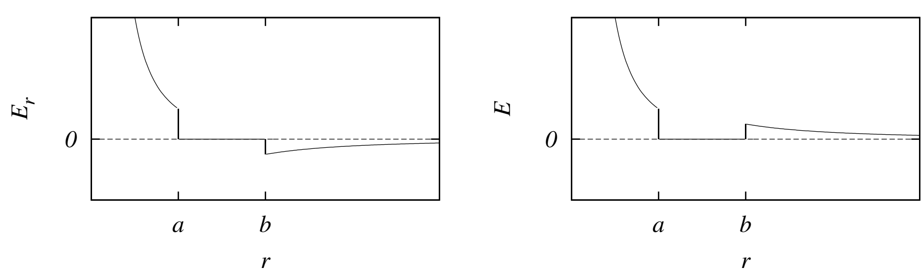
b) $\sigma_{\text{inner}} = \frac{-Q}{A_{\text{inner}}} = \frac{-Q}{4\,\pi\,a^2}$.

c) $\sigma_{\text{outer}} = \frac{-2\,Q}{A_{\text{outer}}} = \frac{-2\,Q}{4\,\pi\,b^2} = \frac{-Q}{2\,\pi\,b^2}$.

d)



e)



62. For a section of length ℓ of a *solid* cylinder with charge density ρ, $\vec{\boldsymbol{E}}$ is in the $\hat{\boldsymbol{r}}$ direction (I'm defining $\hat{\boldsymbol{r}}$ to be the unit vector pointing away from the axis), and inside the cylinder, $\oint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = 2\,\pi\,r\,\ell\,E_r = \frac{Q_{\mathrm{enc}}}{\epsilon_0} = \frac{\pi\,r^2\,\ell\,\rho}{\epsilon_0}$, so $\vec{\boldsymbol{E}} = \frac{\rho\,r}{2\,\epsilon_0}\,\hat{\boldsymbol{r}} = \frac{\rho}{2\,\epsilon_0}\,\vec{\boldsymbol{r}}$. The cylinder with a hole can be viewed as a solid cylinder with a smaller cylinder with equal and opposite charge density superimposed on it. The field due to the solid cylinder is $\vec{\boldsymbol{E}}_1 = \frac{\rho}{2\,\epsilon_0}\,\vec{\boldsymbol{r}}$ (when inside this cylinder). The vector from the axis of the smaller cylinder is not $\vec{\boldsymbol{r}}$ but $\vec{\boldsymbol{r}} - b\,\hat{\boldsymbol{\imath}}$, since its axis is offset by a distance b in the $\hat{\boldsymbol{\imath}}$ direction. Therefore the field due to the smaller cylinder is $\vec{\boldsymbol{E}}_2 = \frac{-\rho}{2\,\epsilon_0}\,(\vec{\boldsymbol{r}} - b\,\hat{\boldsymbol{\imath}})$ (when inside the smaller cylinder, i.e., inside the hole). Then the total field (inside the hole) is $\vec{\boldsymbol{E}} = \vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 = \frac{\rho\,b}{2\,\epsilon_0}\,\hat{\boldsymbol{\imath}}$ which is uniform and points away from the axis of the larger cylinder.