# #5 Speed and Velocity, Acceleration and Motion in 1D Post-class

Due: 11:00am on Friday, August 31, 2012

***Note: You will receive no credit for late submissions.*** To learn more, read your instructor's Grading Policy

## Overcoming a Head Start

Cars A and B are racing each other along the same straight road in the following manner: Car A has a head start and is a distance $D_A$ beyond the starting line at $t = 0$. The starting line is at $x = 0$. Car A travels at a constant speed $v_A$. Car B starts at the starting line but has a better engine than Car A, and thus Car B travels at a constant speed $v_B$, which is greater than $v_A$.

### Part A

How long after Car B started the race will Car B catch up with Car A?

**Express the time in terms of given quantities.**

**Hint 1.** Consider the kinematics relation

Write an expression for the displacement of Car A from the starting line at a time $t$ after Car B starts. (Note that we are taking this time to be $t = 0$.)

**Answer in terms of $v_A$, $v_B$, $D_A$, and $t$ for time, and take $x = 0$ at the starting line.**

**Hint 1.** What is the acceleration of Car A?

The acceleration of Car A is zero, so the general formula $x(t) = x_0 + v_0 t + (1/2)at^2$ has at least one term equal to zero.

ANSWER:

$x_A(t)$ = $D_A + v_A t$

**Hint 2.** What is the relation between the positions of the two cars?

The positions of the two cars are equal at time $t_{\rm catch}$.

**Hint 3.** Consider Car B's position as a function of time

Write down an expression for the position of Car B at time $t$ after starting.

**Give your answer in terms of any variables needed (use $t$ for time).**

ANSWER:

$x_B(t)$ = $v_B t$

ANSWER:

$t_{\rm catch}$ = $\dfrac{D_A}{v_B - v_A}$

**Answer Requested**

## Part B

How far from Car B's starting line will the cars be when Car B passes Car A?

**Express your answer in terms of known quantities. (You may use $t_{\rm catch}$ as well.)**

**Hint 1.** Which expression should you use?

Just use your expression for the position of either car after time $t = 0$, and substitute in the correct value for $t_{\rm catch}$ (found in the previous part).

ANSWER:

$d_{\rm pass}$ = $v_B t_{\rm catch}$

**All attempts used; correct answer displayed**

# Exercise 2.3: Trip Home

You normally drive on the freeway between San Diego and Los Angeles at an average speed of 105 $\rm km/h$, and the trip takes 2 $\rm h$ and 20 $\rm min$. On a

Friday afternoon, however, heavy traffic slows you down and you drive the same distance at an average speed of only 72.0$\mathrm{km/h}$ .

### Part A

How much longer does the trip take?

ANSWER:

$t$ = 64.2 $\mathrm{min}$

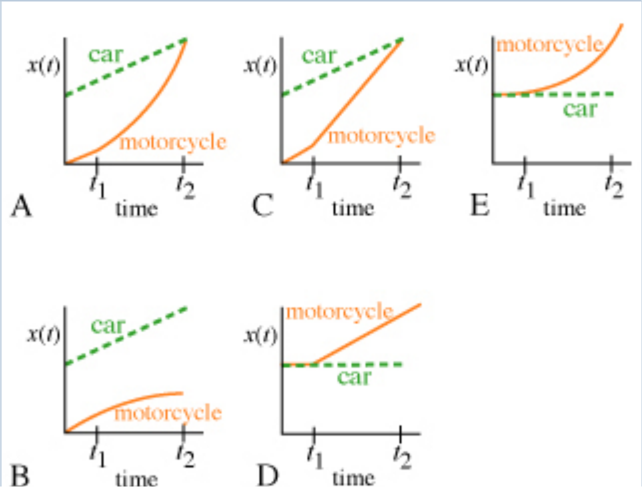
**Correct**

## ± A Motorcycle Catches a Car

A motorcycle is following a car that is traveling at constant speed on a straight highway. Initially, the car and the motorcycle are both traveling at the same speed of 18.5$\mathrm{m/s}$ , and the distance between them is 54.0$\mathrm{m}$ . After $t_1$ = 2.00$\mathrm{s}$ , the motorcycle starts to accelerate at a rate of 5.00$\mathrm{m/s^2}$ . The motorcycle catches up with the car at some time $t_2$.

### Part A

Which of the graphs correctly displays the positions of the motorcycle and car as functions of time?



**Hint 1.** Describe the graph of the motorcycle's position

Between times $t_1$ and $t_2$, what is the shape of the graph of the motorcycle's position versus time?

**Hint 1.** What does the graph of the position as a function of time look like?

If the motorcycle's initial position is $x_0$, its initial velocity is $v_0$, and it travels at constant acceleration $a$, the position of the motorcycle, as a function of time, is given by the equation $x(t) = x_0 + v_0 t + (1/2)at^2$. This is a quadratic equation with respect to the variable $t$. What is the shape of its graph?

ANSWER:

- ○ an increasing straight line
- ◉ a parabola opening upward
- ○ a decreasing straight line
- ○ a parabola opening downward

**Hint 2.** The relative positions of the two vehicles

The motorcycle starts out behind the car, and has caught up with the car at time $t_2$. Therefore, at time $t_2$, the car and the motorcycle must have the same position.

ANSWER:

- (selected) A
- B
- C
- D
- E

**Correct**

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**Part B**

How long does it take from the moment when the motorcycle starts to accelerate until it catches up with the car? In other words, find $t_2 - t_1$.

**Express the time numerically in seconds using three significant figures.**

**Hint 1.** Using a moving reference frame

For this part, the important quantity is the *relative position*, or the *separation* of the two vehicles. You can consider the motion in a frame of reference that moves with the constant speed of the car (18.5$\mathrm{m/s}$ ). In this frame of reference, the car is standing still, both vehicles have zero initial speed, and so the calculations are simpler. With the car at zero speed and the initial speed of the motorcycle zero, the problem reduces to finding how long it takes the motorcycle to cover a distance of 54.0$\mathrm{m}$ starting at zero velocity with an acceleration of 5.00$\mathrm{m/s^2}$ .

However, if you don't feel comfortable with this approach, the rest of the hints for this part will help you with a more traditional method based on the positions of car and motorcycle with respect to the ground as functions of time.

**Hint 2.** Find the initial conditions for the position of the car

If the initial conditions are known at time $t_1$, and the motion is one of constant acceleration, the equation for the position of the car at time $t_2$ is

$$x_c(t_2) = x_{1,c} + v_{1,c}(t_2 - t_1) + \frac{1}{2}a_c(t_2 - t_1)^2,$$

where $x_c\,(t)$ is the positon of the car as a function of time, $x_{1,c}$ is its position at time $t_1$, $v_{1,c}$is the car's velocity at time $t_1$, and $a_c$ is the car's constant acceleration. (If $t_1 = 0$, the equations become more familiar.) Let us choose a frame of reference in which at time $t_1$, the motorcycle is at position $x_{1,\mathrm{m}} = 0$. What are the values of $x_{1,\mathrm{c}}$, $v_{1,\mathrm{c}}$, and $a_\mathrm{c}$ that you should use in the above equation?

**Enter your answer in the order $x_{1,\mathrm{c}}, v_{1,\mathrm{c}}, a_\mathrm{c}$, separated by commas as shown, in units of $\mathrm{meters}$, $\mathrm{m/s}$, and $\mathrm{m/s^2}$, respectively.**

ANSWER:

$x_{1,\mathrm{c}}, v_{1,\mathrm{c}}, a_\mathrm{c}$ = 54.0,18.5,0 $\mathrm{m}$, $\mathrm{m/s}$, $\mathrm{m/s^2}$

### Hint 3. Find the initial conditions for the position of the motorcycle

If initial conditions are known at time $t_1$, and the motion is one of constant acceleration, the equation for the position of the motorcycle at time $t_2$ is

$$x_\mathrm{m}(t_2) = v_{1,\mathrm{m}}(t_2 - t_1) + \frac{1}{2}a_\mathrm{m}(t_2 - t_1)^2,$$

where the meaning of the symbols is analogous to that of Part B.2. Observe that there is no term involving the initial position, because here we have assumed that at time $t_1$, the motorcycle is at position $x_{1,\mathrm{m}} = 0$. What are the values of $v_{1,\mathrm{m}}$ and $a_\mathrm{m}$ that you should use in the above equation?

**Enter your answer in the order $v_{1,m}, a_\mathrm{m}$, separated by commas as shown, in units of $\mathrm{m/s}$ and $\mathrm{m/s^2}$ respectively.**

ANSWER:

$v_{1,m}, a_\mathrm{m}$ = 18.5,5.00 $\mathrm{m/s}$, $\mathrm{m/s^2}$

### Hint 4. Solving for the time

At time $t_2$, the car and motorcycle must be at the same position, since they are side by side. This means that you can set $x_\mathrm{c}(t_2)$ and $x_\mathrm{m}(t_2)$, the positions of the car and motorcycle at time $t_2$, equal to each other, and then solve for the quantity $t_2 - t_1$. You should find that some terms cancel out on either side of the equation, which will make your calculations simpler.

ANSWER:

$t_2 - t_1$ = 4.65 s

**Answer Requested**

## Part C

How far does the motorcycle travel from the moment it starts to accelerate (at time $t_1$) until it catches up with the car (at time $t_2$)? Should you need to use an answer from a previous part, make sure you use the unrounded value.

**Answer numerically in meters using three significant figures.**

**Hint 1.** Find the initial conditions for the position of the motorcycle

If the initial conditions are known at time $t_1$, and the motion has constant acceleration, the equation for the position of the motorcycle at time $t_2$ is

$$x_{\rm m}(t_2) = v_{1,\rm m}(t_2 - t_1) + \frac{1}{2}a_{\rm m}(t_2 - t_1)^2,$$

as discussed in Part B.3. Here we have again assumed that at time $t_1$, the motorcycle is at position $x_{1,\rm m} = 0$. What are the values of $v_{1,\rm m}$ and $a_{\rm m}$ that you should use in the above equation?

**Enter your answer in the order $v_{1,\rm m}, a_{\rm m}$, separated by commas as shown, in units of $\rm m/s$ and $\rm m/s^2$ respectively.**

ANSWER:

$v_{1,\rm m}, a_{\rm m}$ = 18.5,5.00 $\rm m/s$ and $\rm m/s^2$

ANSWER:

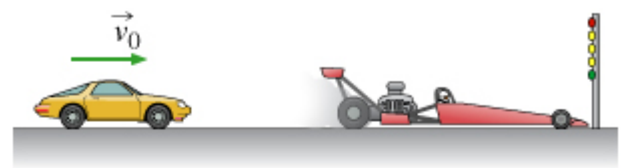
$x_{\rm m}(t_2)$ = 140 m

**All attempts used; correct answer displayed**

## Rearending Drag Racer

To demonstrate the tremendous acceleration of a top fuel drag racer, you attempt to run your car into the back of a dragster that is "burning out" at the red light before the start of a race. (Burning out means spinning the tires at high speed to heat the tread and make the rubber sticky.)

You drive at a constant speed of $v_0$ toward the stopped dragster, not slowing down in the face of the imminent collision. The dragster driver sees you coming but waits until the last instant to put down the hammer, accelerating from the starting line at constant acceleration, $a$. Let the time at which the dragster starts to accelerate be $t = 0$.




### Part A

What is $t_{\rm max}$, the longest time after the dragster begins to accelerate that you can possibly run into the back of the dragster if you continue at your initial velocity?

**Hint 1.** Calculate the velocity

At $t_{\rm max}$, what will the velocity of the drag car be?

**Your answer should not contain tmax, as that time is not yet known.**

**Hint 1.** Consider the speed of both cars

No collision can occur if the dragster has greater speed than the speed of the car behind it.

ANSWER:

$v_{\rm d}\left(t_{\rm max}\right)$ = $v_0$

ANSWER:

$t_{\rm max}$ = $\frac{v_0}{a}$

**All attempts used; correct answer displayed**

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### Part B

Assuming that the dragster has started at the last instant possible (so your front bumper *almost* hits the rear of the dragster at $t = t_{\rm max}$), find your distance from the dragster when he started. If you calculate positions on the way to this solution, choose coordinates so that the position of the drag car is 0 at $t = 0$. Remember that you are solving for a distance (which is a magnitude, and can never be negative), not a position (which *can* be negative).

**Hint 1.** Drag car position at time $t_{\rm max}$

Taking $x = 0$ at the position of the dragster at $t = 0$, find $x_{\rm d}\left(t_{\rm max}\right)$, the position of the dragster at $t_{\rm max}$.

**Express your answer in terms of $t_{\rm max}$ and given quantities.**

ANSWER:

$x_d(t_{max})$ = $\frac{1}{2}a{t_{max}}^2$ .

**Hint 2.** Distance car travels until tmax

Find $D_{car}$, the distance you travel from $t = 0$ to $t_{max}$.

ANSWER:

$D_{car}$ = $v_0 t_{max}$

**Hint 3.** Starting position of car

Express $D_{car}$, the distance the car travels in terms of the starting distance of the car from the starting line at time $t = 0$, $D_{start}$, and the position of the drag car at time $t_{max}$, $x_d(t_{max})$. Note that $D_{start}$ is a distance and can't be negative. This should affect your use of signs.

**Express your answer in terms of $t_{max}$, $x_d(t_{max})$, and $D_{start}$.**

ANSWER:

$D_{car}$ = $x_d(t_{max}) + D_{start}$

**Hint 4.** Obtaining the Solution

Equate your two expressions for the distance traveled by the car up to $t_{max}$, substitute for $t_{max}$ in terms of $v_0$ and $a$, and solve for $D_{start}$, the initial distance of the car from the starting line. Your answer should be in terms of $v_0$ and $a$.

ANSWER:

$D_{start}$ = $\frac{1}{2}a\left(\frac{v_0}{a}\right)^2$

All attempts used; correct answer displayed

### Part C

Find numerical values for $t_{max}$ and $D_{start}$ in seconds and meters for the (reasonable) values $v_0 = 60 \text{ mph}$ (26.8 m/s) and $a = 50 \text{ m/s}^2$.

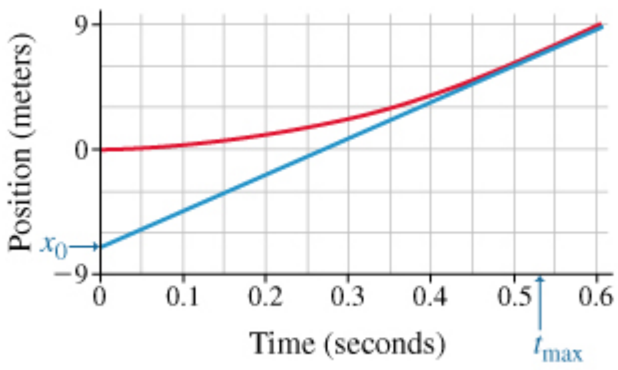
**Separate your two numerical answers by commas, and give your answer to two significant figures.**

ANSWER:

$t_{max}$, $D_{start}$ = 0.54,7.2 s, m

All attempts used; correct answer displayed

The blue curve shows how the car, initially at $x_0$, continues at constant velocity (blue) and just barely touches the accelerating drag car (red) at $t_{max}$.



## Score Summary:

Your score on this assignment is 24.3%.
You received 9.7 out of a possible total of 40 points.