#33 Fluid Mechanics II Pre-class

Due: 11:00am on Friday, November 9, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Understanding Bernoulli's Equation

Bernoulli's equation is a simple relation that can give useful insight into the balance among fluid pressure, flow speed, and elevation. It applies exclusively to ideal fluids with steady flow, that is, fluids with a constant density and no internal friction forces, whose flow patterns do not change with time. Despite its limitations, however, Bernoulli's equation is an essential tool in understanding the behavior of fluids in many practical applications, from plumbing systems to the flight of airplanes.

For a fluid element of density ρ that flows along a streamline, Bernoulli's equation states that

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2,$$

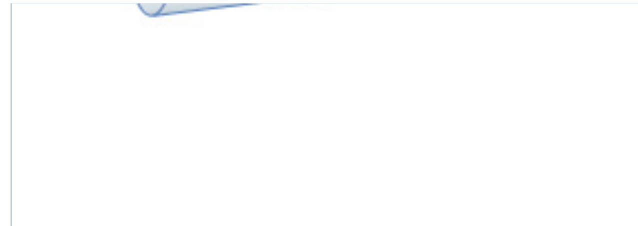
where p is the pressure, v is the flow speed, h is the height, g is the acceleration due to gravity, and subscripts 1 and 2 refer to any two points along the streamline. The physical interpretation of Bernoulli's equation becomes clearer if we rearrange the terms of the equation as follows:

$$p_1 - p_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2).$$

The term $p_1 - p_2$ on the left-hand side represents the total work done on a unit volume of fluid by the pressure forces of the surrounding fluid to move that volume of fluid from point 1 to point 2. The two terms on the right-hand side represent, respectively, the change in potential energy, $\rho g(h_2 - h_1)$, and the change in kinetic energy, $\frac{1}{2}\rho(v_2^2 - v_1^2)$, of the unit volume during its flow from point 1 to point 2. In other words, Bernoulli's equation states that the work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the change in potential and kinetic energy per unit volume that occurs during the flow. This is nothing more than the statement of conservation of mechanical energy for an ideal fluid flowing along a streamline.

Part A

Consider the portion of a flow tube shown in the figure. Point 1 and point 2 are at the same height. An ideal fluid enters the flow tube at point 1 and moves steadily toward point 2. If the cross section of the flow tube at point 1 is greater than that at point 2, what can you say about the pressure at point 2?



Hint 1. How to approach the problem

Apply Bernoulli's equation to point 1 and to point 2. Since the points are both at the same height, their elevations cancel out in the equation and you are left with a relation between pressure and flow speeds. Even though the problem does not give direct information on the flow speed along the flow tube, it does tell you that the cross section of the flow tube decreases as the fluid flows toward point 2. Apply the continuity equation to points 1 and 2 and determine whether the flow speed at point 2 is greater than or smaller than the flow speed at point 1. With that information and Bernoulli's equation, you will be able to determine the pressure at point 2 with respect to the pressure at point 1.

Hint 2. Apply Bernoulli's equation

Apply Bernoulli's equation to point 1 and to point 2 to complete the expression below. Here p and v are the pressure and flow speed, respectively, and subscripts 1 and 2 refer to point 1 and point 2. Also, use h for elevation with the appropriate subscript, and use ρ for the density of the fluid.

Express your answer in terms of some or all of the variables p_1, v_1, h_1, p_2, v_2, h_2, and ρ.

Hint 1. Flow along a horizontal streamline

Along a horizontal streamline, the change in potential energy of the flowing fluid is zero. In other words, when applying Bernoulli's equation to any two points of the streamline, $h_1 = h_2$ and they cancel out.

ANSWER:

$p_1 + \frac{1}{2}\rho v_1^2$ = $p_2 + \frac{\rho {v_2}^2}{2}$

Hint 3. Determine v_2 with respect to v_1

By applying the continuity equation, determine which of the following is true.

Hint 1. The continuity equation

The continuity equation expresses conservation of mass for incompressible fluids flowing in a tube. It says that the amount of fluid ΔV flowing through a cross section A of the tube in a time interval Δt must be the same for all cross sections, or

$$\frac{\Delta V}{\Delta t} = A_1 v_1 = A_2 v_2.$$

Therefore, the flow speed must increase when the cross section of the flow tube decreases, and vice versa.

ANSWER:

- ◉ $v_2 > v_1$
- ○ $v_2 = v_1$
- ○ $v_2 < v_1$

ANSWER:

The pressure at point 2 is
- ◉ lower than the pressure at point 1.
- ○ equal to the pressure at point 1.
- ○ higher than the pressure at point 1.

Correct

Thus, by combining the continuity equation and Bernoulli's equation, one can characterize the flow of an ideal fluid.When the cross section of the flow tube decreases, the flow speed increases, and therefore the pressure decreases. In other words, if $A_2 < A_1$, then $v_2 > v_1$ and $p_2 < p_1$.

Part B

As you found out in the previous part, Bernoulli's equation tells us that a fluid element that flows through a flow tube with decreasing cross section moves toward a region of lower pressure. Physically, the pressure drop experienced by the fluid element between points 1 and 2 acts on the fluid element as a net force that causes the fluid to ___________.

Hint 1. Effects from conservation of mass

Recall that, if the cross section A of the flow tube varies, the flow speed v *must* change to conserve mass. This means that there is a nonzero net force acting on the fluid that causes the fluid to increase or decrease speed depending on whether the fluid is flowing through a portion of the tube with a smaller or larger cross section.

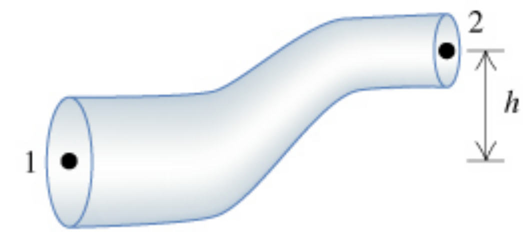
ANSWER:

- ○ decrease in speed
- ◉ increase in speed
- ○ remain in equilibrium

Correct

Part C

Now assume that point 2 is at height h with respect to point 1, as shown in the figure. The ends of the flow tube have the same areas as the ends of the horizontal flow tube shown in Part A. Since the cross section of the flow tube is decreasing, Bernoulli's equation tells us that a fluid element flowing toward point 2 from point 1 moves toward a region of lower pressure. In this case, what is the pressure drop experienced by the fluid element?



Hint 1. How to approach the problem

Apply Bernoulli's equation to point 1 and to point 2, as you did in Part A. Note that this time you must take into account the difference in elevation between points 1 and 2. Do you need to add this additional term to the other term representing the pressure drop between the two ends of the flow tube or do you subtract it?

ANSWER:

The pressure drop is

- smaller than the pressure drop occurring in a purely horizontal flow.
- equal to the pressure drop occurring in a purely horizontal flow.
- larger than the pressure drop occurring in a purely horizontal flow.

Correct

Part D

From a physical point of view, how do you explain the fact that the pressure drop at the ends of the elevated flow tube from Part C is *larger* than the pressure drop occurring in the similar but purely horizontal flow from Part A?

Hint 1. Physical meaning of the pressure drop in a tube

As explained in the introduction, the difference in pressure $p_1 - p_2$ between the ends of a flow tube represents the total work done on a unit volume of fluid by the pressure forces of the surrounding fluid to move that volume of fluid from one end to the other end of the flow tube.

ANSWER:

A greater amount of work is needed to balance the

- (selected) increase in potential energy from the elevation change.
- decrease in potential energy from the elevation change.
- larger increase in kinetic energy.
- larger decrease in kinetic energy.

Correct

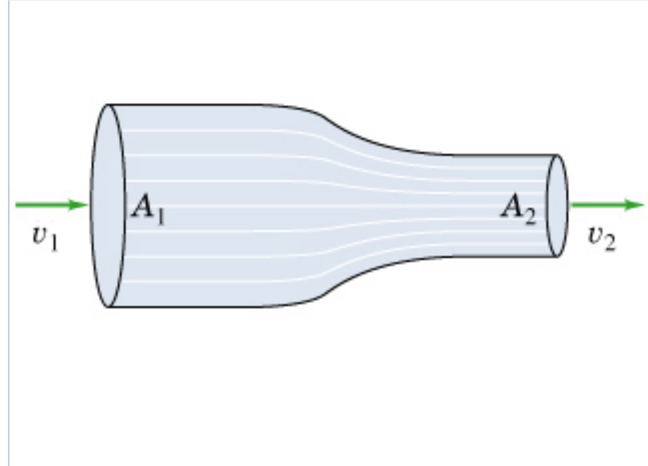
In the case of purely horizontal flow, the difference in pressure between the two ends of the flow tube had to balance only the increase in kinetic energy resulting from the acceleration of the fluid. In an elevated flow tube, the difference in pressure must also balance the increase in potential energy of the fluid; therefore a higher pressure is needed for the flow to occur.

Streamlines and Fluid Flow

Learning Goal:

To understand the continuity equation.

Streamlines represent the path of the flow of a fluid. You can imagine that they represent a time-exposure photograph that shows the paths of small particles carried by the flowing fluid. The figure shows streamlines for the flow of an incompressible fluid in a tapered pipe of circular cross section. The speed of the fluid as it enters the pipe on the left is v_1. Assume that the cross-sectional areas of the pipe are A_1 at its entrance on the left and A_2 at its exit on the right.



Part A

Find F_1, the volume of fluid flowing into the pipe per unit of time. This quantity is also known as the *volumetric flow rate*.

Express the volumetric flow rate in terms of any of the quantities given in the problem introduction.

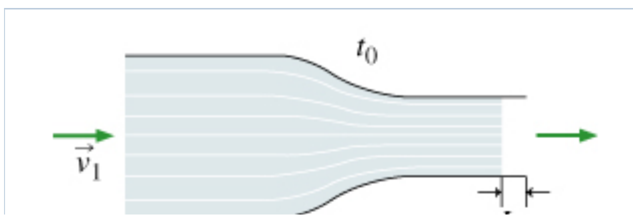
Hint 1. Find the volume of fluid entering the pipe

The volumetric flow rate has units of volume per unit time (cubic meters per second). What is the volume ΔV of fluid entering the pipe in time Δt?

Express your answer in terms of any or all of the following quantities: v_1, A_1, and Δt.

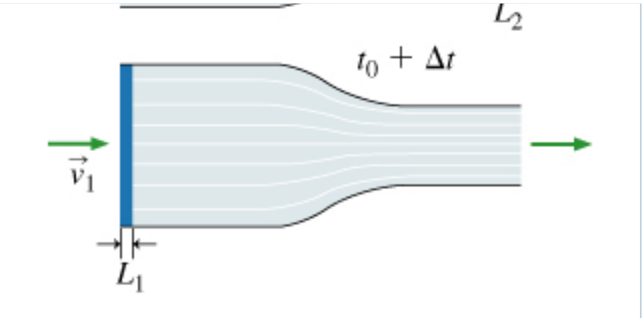
Hint 1. How far does the fluid move in time Δt?

The figure shows a snapshot of the pipe at two instants in time: t_0 and $t_0 + \Delta t$. In the time interval Δt, some additional fluid (shown in blue) has entered the pipe from the left. The speed of the left-hand boundary of the blue-colored fluid is v_1. What is the length L_1 of the region of additional fluid that has entered the



pipe in the time interval Δt?

Express your answer in terms of v_1 and Δt.



ANSWER:

L_1 = $v_1 \Delta t$

ANSWER:

ΔV = $A_1 v_1 \Delta t$

ANSWER:

F_1 = $v_1 A_1$

Correct

Part B

Because the fluid is assumed to be incompressible and mass is conserved, at a particular moment in time, the amount of fluid that flows into the pipe must equal the amount of fluid that flows out. This fact is embodied in the *continuity equation*. Using the continuity equation, find the velocity v_2 of the fluid flowing out of the right end of the pipe.

Express your answer in terms of any of the quantities given in the problem introduction.

Hint 1. Find the volumetric flow rate out of the pipe

Find F_2, the volume of fluid flowing out of the pipe per unit of time.

Express your answer in terms of v_2 and A_2.

ANSWER:

F_2 = v_2A_2

Hint 2. Apply the continuity equation

The continuity equation states that the volumetric flows into and out of the pipe must be the same. Fill in the right-hand side of the continuity equation for this problem.

Express your answer in terms of v_2 and any of the quantities given in the problem introduction.

ANSWER:

v_1A_1 = v_2A_2

ANSWER:

v_2 = $v_1\frac{A_1}{A_2}$

Correct

Part C

If you are shown a picture of streamlines in a flowing fluid, you can conclude that the __________ of the fluid is greater where the streamlines are closer together.

Enter a one-word answer.

ANSWER:

speed

Correct

Thus the velocity of the flow increases with increasing density (number per unit area) of streamlines.

Flow Velocity of Blood Conceptual Question

Arteriosclerotic plaques forming on the inner walls of arteries can decrease the effective cross-sectional area of an artery. Even small changes in the effective area of an artery can lead to very large changes in the blood pressure in the artery and possibly to the collapse of the blood vessel.
Imagine a healthy artery, with blood flow velocity of $v_0 = 0.14 \ \mathrm{m/s}$ and mass per unit volume of $\rho = 1050 \ \mathrm{kg/m^3}$. The kinetic energy per unit volume of blood is given by

$$K_0 = \frac{1}{2}\rho v_0^2.$$

Imagine that plaque has narrowed an artery to one-fifth of its normal cross-sectional area (an 80% blockage).

Part A

Compared to normal blood flow velocity, v_0, what is the velocity of blood as it passes through this blockage?

Hint 1. Continuitity equation and reduced cross-sectional area

By the equation of continuity, as the cross-sectional area of an artery decreases because of plaque formation, the velocity of blood through that region of the artery will increase. The new flow speed can be calculated by rearranging the equation of continuity,

$$A_1 v_1 = A_2 v_2;$$

so

v_2 = {A_1 \over A_2}v_1

where A_1 and A_2 are the initial and final cross-sectional areas, and v_1 and v_2 are the initial and final velocities of the blood, respectively.

ANSWER:

- $80v_0$
- $20v_0$
- $5v_0$
- $v_0/5$

Correct

Part B

By what factor does the kinetic energy per unit of blood volume change as the blood passes through this blockage?

ANSWER:

- 25
- 5
- 1

Correct

Part C

As the blood passes through this blockage, what happens to the blood pressure?

Hint 1. Blood pressure and blood velocity

Bernoulli's equation states that the sum of the pressure, the kinetic energy per volume, and the gravitational energy per volume of a fluid is constant. For initial and final pressures p_1 and p_2, initial and final velocities v_1 and v_2, and mass per unit volume of blood, ρ, ignoring the effects of changes in gravitational energy leads to

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2.$$

Basically, the sum of kinetic energy and pressure must remain constant in an artery. This leads to a very serious health risk. As blood velocity increases, blood pressure in a section of artery can drop to a dangerously low level, and the blood vessel can collapse, completely cutting off blood flow, owing to lack of sufficient internal pressure.

Hint 2. Calculating the change in blood pressure

From Bernoulli's equation, the change in pressure is the negative of the change in kinetic energy per unit volume. For initial and final kinetic energies per unit volume of the blood, K_1 and K_2, respectively,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2,$$

or

p_1 + K_1 = p_2 + K_2,

where $K = (1/2)\rho v^2$. Rearranging this equation yields

$$p_2 - p_1 = -(K_2 - K_1)$$

or

$$\Delta p = -\Delta K.$$

ANSWER:

- It increases by about 250 Pa.
- It increases by about 41 Pa.
- It stays the same.
- It decreases by about 41 Pa.
- It decreases by about 250 Pa.

Correct

Since the kinetic energy increases by a factor of 25,

\Delta K = 25 \times K_0\; - K_0 \; = 24 \times K_0 \; = 247 \; {\rm Pa}\; \approx\; 250\; {\rm Pa}·

Bernoulli's equation tells you that \Delta K = - \Delta p·

As the blood velocity increases through a blockage, the blood pressure in that section of the artery can drop to a dangerously low level. In extreme cases, the blood vessel can collapse, completely cutting off blood flow, owing to lack of sufficient internal pressure. In the next three parts, you will see how a small increase in blockage can cause a much larger pressure change.

For parts D - F imagine that plaque has grown to a 90% blockage.

Part D

Relative to its initial, healthy state, by what factor does the velocity of blood increase as the blood passes through this blockage?

Express your answer numerically.

ANSWER:

10

Correct

Part E

By what factor does the kinetic energy per unit of blood volume increase as the blood passes through this blockage?

Express your answer numerically.

ANSWER:

100

Correct

Part F

What is the magnitude of the drop in blood pressure, Delta p, as the blood passes through this blockage? Use K_0 as the normal (i.e., unblocked) kinetic energy per unit volume of the blood.

Express your answer in pascals using two significant figures.

ANSWER:

Delta p = 1000 Pa

Correct

Score Summary:

Your score on this assignment is 102.2%.

You received 15.33 out of a possible total of 15 points.