Chapter 10: Dynamics of Rotation Motion

Due: 11:00pm on Tuesday, November 6, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Spinning the Wheels: An Introduction to Angular Momentum

Learning Goal:

To learn the definition and applications of angular momentum including its relationship to torque.

By now, you should be familiar with the concept of *momentum*, defined as the product of an object's mass and its velocity:

$$\vec{p} = m\vec{v}.$$

You may have noticed that nearly every *translational* concept or equation seems to have an analogous *rotational* one. So, what might be the rotational analogue of momentum?

Just as the rotational analogue of force $\vec{F}$, called the torque $\vec{\tau}$, is defined by the formula

$$\vec{\tau} = \vec{r} \times \vec{F},$$

the rotational analogue of momentum $\vec{p}$, called the angular momentum $\vec{L}$, is given by the formula

$$\vec{L} = \vec{r} \times \vec{p},$$

for a single particle. For an extended body you must add up the angular momenta of all of the pieces.

There is another formula for angular momentum that makes the analogy to momentum particularly clear. For a rigid body rotating about an axis of symmetry, which will be true for all parts in this problem, the measure of inertia is given not by the mass m but by the *rotational* inertia (i.e., the moment of inertia) I. Similarly, the rate of rotation is given by the body's *angular* speed, ω. The product $I\vec{\omega}$ gives the angular momentum $\vec{L}$ of a rigid body rotating about an axis of symmetry. (Note that if the body is not rotating about an axis of symmetry, then the angular momentum and the angular velocity may not be parallel.)

Part A

Which of the following is the SI unit of angular momentum?

ANSWER:

- $\mathrm{N \cdot m/s}$
- $\mathrm{kg \cdot m/s}$
- $\mathrm{kg \cdot m^2/s^2}$
- (selected) $\mathrm{kg \cdot m^2/s}$

Part B

An object has rotational inertia I. The object, initially at rest, begins to rotate with a constant angular acceleration of magnitude α. What is the magnitude of the angular momentum L of the object after time t?

Express your answer in terms of I, α, and t.

Hint 1. How to approach the problem

Find the angular velocity first; then use the $\vec{L} = I\vec{\omega}$ definition of angular momentum.

ANSWER:

L = $I\alpha t$

Part C

A rigid, uniform bar with mass m and length b rotates about the axis passing through the midpoint of the bar perpendicular to the bar. The linear speed of the end points of the bar is v. What is the magnitude of the angular momentum L of the bar?

Express your answer in terms of m, b, v, and appropriate constants.

Hint 1. How to approach the problem

Find separately the rotational inertia and the angular velocity; then use the $\vec{L} = I\vec{\omega}$ definition of angular momentum.

Hint 2. Rotational inertia of the bar

For this axis, the rotational inertia of the bar is given by

$$I = ml^2/12,$$

where l is the length of the bar.

Hint 3. Finding the magnitude of the angular velocity

The magnitude of the angular velocity ω can be obtained from the relation

$$\omega r = v,$$

where r is the radius of rotation.

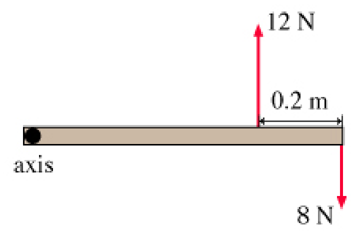
ANSWER:

L = $\frac{mbv}{6}$

Part D

The uniform bar shown in the diagram has a length of 0.80 m. The bar begins to rotate from rest in the horizontal plane about the axis passing through its left end. What will be the magnitude of the angular momentum L of the bar 6.0 s after the motion has begun? The forces acting on the bar are shown.

Express your answer in $\mathrm{kg \cdot m^2/s}$ to two significant figures.



Hint 1. How to approach the problem

First, find the net torque, and then use

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\mathrm{net}}.$$

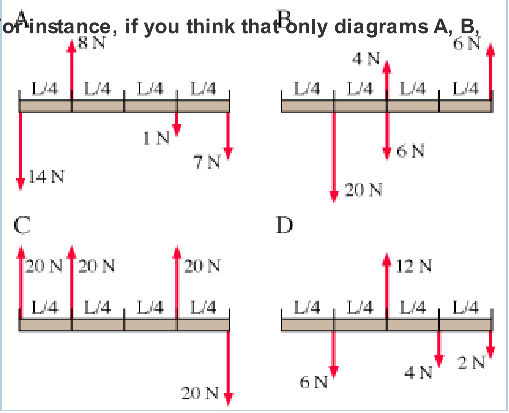
ANSWER:

L = 4.8 $\mathrm{kg \cdot m^2/s}$

Part E

Each of the four bars shown can rotate freely in the horizontal plane about its left end. For which diagrams is the net torque equal to zero?

Type in alphabetical order the letters corresponding to the correct diagrams. For instance, if you think that only diagrams A, B, and C answer the question, type ABC**.**



ANSWER:

BC

If the sum of the forces on a body is zero, then the net torque is independent of the point about which the torque is calculated. If the net force on the body is *not* zero, as is true for most of the beams in this part, then the torque will depend on the point about which you calculate the torque.

Part F

Consider the figures for Part E. For which diagrams is the angular momentum constant?

Type alphabetically the letters corresponding to the correct diagrams. For instance, if you think that only diagrams A, B, and C answer the question, type ABC**.**

Hint 1. Determining when angular momentum is constant

Recall that

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\mathrm{net}}.$$

This means that if the net torque is zero, then the angular momentum is a constant. That is, if the rate of change for $\vec{L}$ is zero, then $\vec{L}$ can't be changing.

ANSWER:

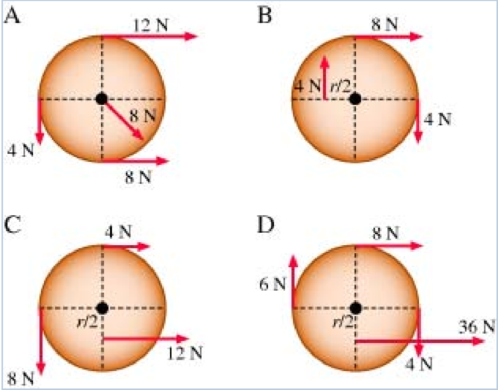
BC

Angular momentum is conserved when the net torque is zero. This is analogous to the statement from *linear* dynamics that momentum is conserved when the net force is zero.

Part G

Each of the disks in the figure has radius r. Each disk can rotate freely about the axis passing through the center of the disk perpendicular to the plane of the figure, as shown. For which diagrams is the angular momentum constant? In your calculations, use the information provided in the diagrams.

Type alphabetically the letters corresponding to the correct diagrams. For instance, if you think that only diagrams A, B, and C answer the question, type ABC.



ANSWER:

AD

Part H

Three disks are spinning independently on the same axle without friction. Their respective rotational inertias and angular speeds are I, ω (clockwise); $2I, 3\omega$ (counterclockwise); and $4I, \omega/2$ (clockwise). The disks then slide together and stick together, forming one piece with a single angular velocity. What will be the direction and the rate of rotation ω_{net} of the single piece?

Express your answer in terms of one or both of the variables I and ω and appropriate constants. Use a minus sign for clockwise rotation.

Hint 1. How to approach the problem

The angular momentum for the system of the three disks is conserved. Therefore, if you find the angular momentum of the system before the disks stick together and the moment of inertia after they have stuck, you can solve for the angular speed ω_{net} after they have stuck together.

Hint 2. Find the rotational inertia

What is the rotational inertia I_{net} of the single piece?

Express your answer in terms of I.

ANSWER:

I_{net} = $7I$

ANSWER:

$\omega_{net} = \frac{3\omega}{7}$

Record and Turntable

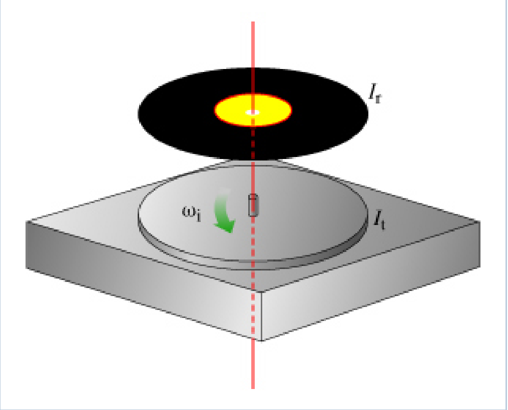
Learning Goal:

To understand how to use conservation of angular momentum to solve problems involving collisions of rotating bodies.

Consider a turntable to be a circular disk of moment of inertia I_t rotating at a constant angular velocity ω_i around an axis through the center and perpendicular to the plane of the disk (the disk's "primary axis of symmetry"). The axis of the disk is vertical and the disk is supported by frictionless bearings. The motor of the turntable is off, so there is no external torque being applied to the axis.
Another disk (a record) is dropped onto the first such that it lands coaxially (the axes coincide). The moment of inertia of the record is I_r. The initial angular velocity of the second disk is zero.

There *is* friction between the two disks.

After this "rotational collision," the disks will eventually rotate with the same angular velocity.



Part A

What is the final angular velocity, ω_f, of the two disks?

Express ω_f in terms of I_t, I_r, and ω_i.

Hint 1. How to approach the problem

Because there is friction between the record and turntable, you can't use energy conservation. However, since there are no net external torques acting on the system, angular momentum is conserved.

Hint 2. Initial angular momentum

Find the magnitude, L_i, of the (combined) initial angular momenta of the two disks.

Express L_i in terms of ω_i, I_t, and/or I_r .

ANSWER:

L_i = $I_t\omega_i$

Hint 3. Final angular momentum

Find the magnitude, L_f, of the (combined) final angular momenta of the two disks.

Express L_f in terms of ω_f, I_t, and/or I_r .

ANSWER:

L_f = $(I_t + I_r)\omega_f$

ANSWER:

ω_f = $\frac{I_t}{I_t + I_r}\omega_i$

Part B

Because of friction, rotational kinetic energy is not conserved while the disks' surfaces slip over each other. What is the final rotational kinetic energy, K_f, of the two spinning disks?

Express the final kinetic energy in terms of I_t, I_r, and the initial kinetic energy K_i of the two-disk system. No angular velocities should appear in your answer.

Hint 1. Initial rotational kinetic energy

What is the initial rotational kinetic energy of the two-disk system, K_i.

Express your answer in terms of I_t and ω_i.

Hint 1. Formula for rotational kinetic energy

The formula for the rotational kinetic energy K of a rigid body with moment of inertia I, spinning with an angular velocity ω is

$$K = \frac{1}{2} I \omega^2$$

ANSWER:

K_i = $\frac{1}{2} I_t {\omega_i}^2$

Hint 2. Final rotational kinetic energy

What is the final rotational kinetic energy of the two-disk system.

Express your answer in terms of I_t, I_r, and ω_f.

ANSWER:

K_f = $\frac{(I_t + I_r)\,{\omega_f}^2}{2}$

Hint 3. Putting it all together

Use the relationship between ω_i and ω_f (from the answer to the first part of this problem) to express K_f in terms of ω_i. Then use the equation relating K_i and ω_i to express K_f in terms of K_i.

ANSWER:

K_f = $\frac{K_i I_t}{I_t + I_r}$

Some of the energy was converted into heat and sound as the frictional force, torque acted, stopping relative motion.

Part C

Assume that the turntable deccelerated during time Δt before reaching the final angular velocity (Δt is the time interval between the moment when the top disk is dropped and the time that the disks begin to spin at the same angular velocity). What was the average torque, $\langle \tau \rangle$, acting on the bottom disk due to friction with the record?

Express the torque in terms of I_t, ω_i, ω_f, and Δt.

Hint 1. Average angular acceleration

What is the average angular acceleration, $\langle \alpha \rangle$, of the bottom disk?

Express $\langle \alpha \rangle$ in terms of ω_i, ω_f, and Δt.

Hint 1. Definition of average angular acceleration

The angular acceleration is the rate of change of angular velocity. The *average* angular acceleration is the net change in angular velocity (final angular velocity minus initial angular velocity) divided by the elapsed time.

ANSWER:

$\langle\alpha\rangle$ = $\frac{\omega_f - \omega_i}{\Delta t}$

Also accepted: $\frac{\omega_f - \omega_i}{\Delta t}$

Hint 2. Formula for torque

The torque is given by

$$\tau = I\frac{d\omega}{dt}.$$

ANSWER:

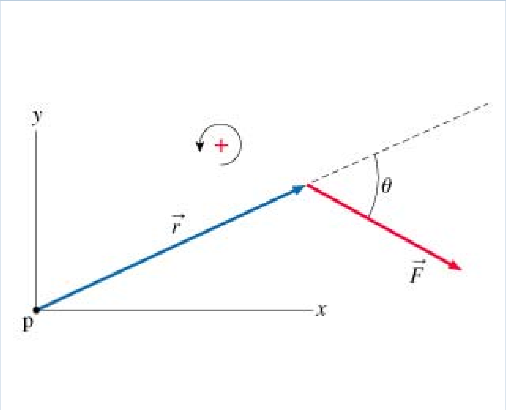
$\langle\tau\rangle$ = $I_t\frac{\omega_f - \omega_i}{\Delta t}$

Also accepted: $I_t\frac{\omega_f - \omega_i}{\Delta t}$

Torque about the z Axis

Learning Goal:

To understand two different techniques for computing the torque on an object due to an applied force.



Imagine an object with a pivot point p at the origin of the coordinate system shown . The force vector $\vec{F}$ lies in the *xy* plane, and this force of magnitude F acts on the object at a point in the *xy* plane. The vector $\vec{r}$ is the position vector relative to the pivot point p to the point where $\vec{F}$ is applied.

The torque on the object due to the force $\vec{F}$ is equal to the cross product $\vec{\tau} = \vec{r} \times \vec{F}$.

When, as in this problem, the force vector and lever arm both lie in the *xy* plane of the paper or computer screen, only the *z* component of torque is nonzero.

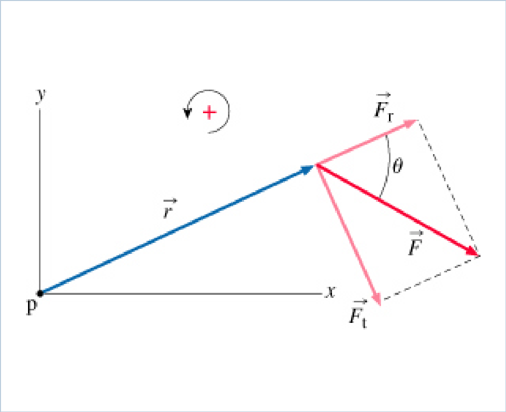
When the torque vector is parallel to the *z* axis ($\vec{\tau} = \tau\hat{z}$), it is easiest to find the magnitude and sign of the torque, τ, in terms of the angle θ between the position and force vectors using one of two simple methods: the *Tangential Component of the Force* method or the *Moment Arm of the Force* method.
Note that in this problem, the positive *z* direction is perpendicular to the computer screen and points toward you (given by the right-hand rule $\hat{x} \times \hat{y} = \hat{z}$), so a positive torque would cause counterclockwise rotation about the *z* axis.

Tangential component of the force

Part A

Decompose the force vector $\vec{F}$ into radial (i.e., parallel to $\vec{r}$) and tangential (perpendicular to $\vec{r}$) components as shown. Find the magnitude of the radial and tangential components, F_r and F_t. You may assume that θ is between zero and 90 degrees.

Enter your answer as an ordered pair. Express F_t and F_r in terms of F and θ.



Hint 1. Magnitude of $\vec{F}_r$

Use the given angle between the force vector $\vec{F}$ and its radial component $\vec{F}_r$ to compute the magnitude F_r.

ANSWER:

(F_r, F_t) = $F\cos(\theta)$, $F\sin(\theta)$

Part B

Is the following statement true or false?

The torque about point p is proportional to the length r of the position vector $\vec{r}$.

ANSWER:

- (selected) true
- false

Part C

Is the following statement true or false?

Both the radial and tangential components of $\vec{F}$ generate torque about point p.

ANSWER:

- true
- (selected) false

Part D

Is the following statement true or false?

In this problem, the tangential force vector would tend to turn an object clockwise around pivot point p.

ANSWER:

- (selected) true
- false

Part E

Find the torque τ about the pivot point p due to force $\vec{F}$. Your answer should correctly express both the magnitude and sign of τ.

Express your answer in terms of F_t and r or in terms of F, θ, and r.

ANSWER:

τ = $-rF_t$

Also accepted: $-rF\sin(\theta)$

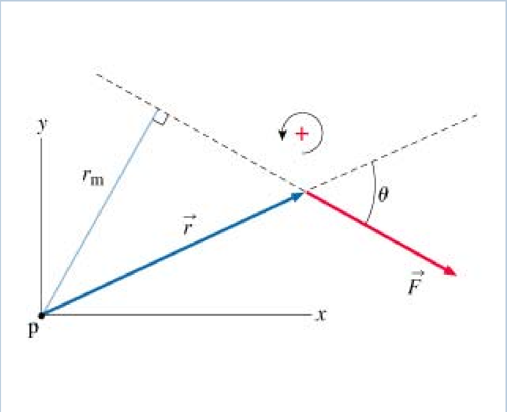
Moment arm of the force

In the figure, the dashed line extending from the force vector is called the line of action of $\vec{F}$. The perpendicular distance r_m from the pivot point p to the line of action is called the moment arm of the force.

Part F

What is the length, r_m, of the moment arm of the force $\vec{F}$ about point p?

Express your answer in terms of r and θ.



ANSWER:

r_m = $r\sin(\theta)$

Part G

Find the torque τ about p due to $\vec{F}$. Your answer should correctly express both the magnitude and sign of τ.

Express your answer in terms of r_m and F or in terms of r, θ, and F.

ANSWER:

τ = $-r_mF$

Also accepted: $-rF\sin(\theta)$

Three equivalent expressions for expressing torque about the *z* axis have been discussed in this problem:

1. Torque is defined as the cross product between the position and force vectors. When both $\vec{F}$ and $\vec{r}$ lie in the *xy* plane, only the *z* component of torque is nonzero, and the cross product simplifies to:

$$\vec{\tau} = \vec{r} \times \vec{F} = r * F * \sin(\theta)\hat{z} = \tau\hat{z}.$$

Note that a positive value for τ indicates a counterclockwise direction about the *z* axis.

2. Torque is generated by the component of $\vec{F}$ that is tangential to the position vector $\vec{r}$ (the tangential component of force):

$$\tau = r * F_t = r * F\sin(\theta).$$

3. The magnitude of torque is the product of the force and the perpendicular distance between the *z* axis and the line of action of a force, r_m, called the moment arm of the force:

$$\tau = r_m * F = r * \sin(\theta) * F.$$

Torque and Angular Acceleration

Learning Goal:

To understand and apply the formula $\tau = I\alpha$ to rigid objects rotating about a fixed axis.

To find the acceleration a of a particle of mass m, we use Newton's second law: $\vec{F}_{\rm net} = m\vec{a}$, where $\vec{F}_{\rm net}$ is the net force acting on the particle.

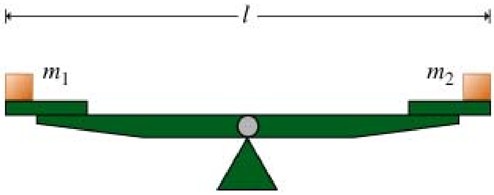
To find the angular acceleration α of a rigid object rotating about a fixed axis, we can use a similar formula: $\tau_{\rm net} = I\alpha$, where $\tau_{\rm net} = \sum \tau$ is the *net torque* acting on the object and I is its moment of inertia.

In this problem, you will practice applying this formula to several situations involving angular acceleration. In all of these situations, two objects of masses m_1 and m_2 are attached to a seesaw. The seesaw is made of a bar that has length l and is pivoted so that it is free to rotate in the vertical plane without friction.

You are to find the angular acceleration of the seesaw when it is set in motion from the horizontal position. In all cases, assume that $m_1 > m_2$.

Part A

The seesaw is pivoted in the middle, and the mass of the swing bar is negligible.



Find the angular acceleration α of the seesaw.

Express your answer in terms of some or all of the quantities m_1, m_2, l, as well as the acceleration due to gravity g.

Hint 1. Find the moment of inertia

Find the moment of inertia I of the system.

Express your answer in terms of some or all of the quantities m_1, m_2, and l.

ANSWER:

$I = (m_1 + m_2)\left(\frac{l}{2}\right)^2$

Hint 2. Find the net torque

Find the magnitude of the net torque τ acting on the system.

Express your answer in terms of some or all of the quantities m_1, m_2, l, as well as the acceleration due to gravity g.

ANSWER:

$\tau = \frac{(m_1 - m_2)(gl)}{2}$

ANSWER:

$\alpha = \frac{2g(m_1 - m_2)}{(m_1 + m_2)l}$

Part B

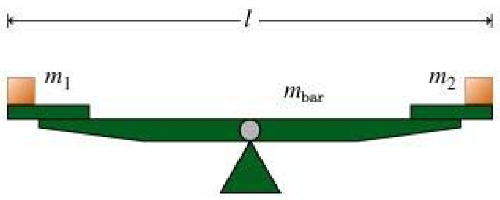
In what direction will the seesaw rotate, and what will the sign of the angular acceleration be?

ANSWER:

- The rotation is in the clockwise direction and the angular acceleration is positive.
- The rotation is in the clockwise direction and the angular acceleration is negative.
- (selected) The rotation is in the counterclockwise direction and the angular acceleration is positive.
- The rotation is in the counterclockwise direction and the angular acceleration is negative.

Part C

Now consider a similar situation, except that now the swing bar itself has mass m_{bar}.



Find the angular acceleration α of the seesaw.

Express your answer in terms of some or all of the quantities m_1, m_2, m_{bar}, l, as well as the acceleration due to gravity g.

Hint 1. What has changed?

Compared to the previous situation, what quantities have changed?

ANSWER:

- net torque only
- (selected) moment of inertia only
- both torque and moment of inertia

In calculating the torque due to gravity, the weight of the bar is applied at the center of mass of the bar. The distance between the center of mass and the pivot is used in the calculation of the torque. In this case, the center of mass of the bar coincides with the pivot point. Therefore, the corresponding torque is zero.

Hint 2. Find the moment of inertia

What is the new moment of inertia I? Recall that the moment of inertia for a uniform thin rod of mass m and length L, pivoted at its center, is given by $I = \frac{1}{12}mL^2$.

Express your answer in terms of some or all of the quantities m_1, m_2, m_{bar}, and l.

ANSWER:

$$I = \frac{m_{\text{bar}}l^2}{12} + (m_1 + m_2)\frac{l^2}{4}$$

ANSWER:

$$\alpha = \frac{6g(m_1 - m_2)}{(3(m_1 + m_2) + m_{\text{bar}})l}$$

Part D

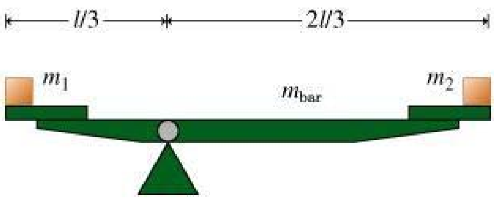
In what direction will the seesaw rotate and what will the sign of the angular acceleration be?

ANSWER:

- The rotation is in the clockwise direction and the angular acceleration is positive.
- The rotation is in the clockwise direction and the angular acceleration is negative.
- (selected) The rotation is in the counterclockwise direction and the angular acceleration is positive.
- The rotation is in the counterclockwise direction and the angular acceleration is negative.

Part E

This time, the swing bar of mass m_{bar} is pivoted at a different point, as shown in the figure.



Find the magnitude of the angular acceleration α of the swing bar. Be sure to use the absolute value function in your answer, since no comparison of m_1, m_2, and m_{bar} has been made.

Express your answer in terms of some or all of the quantities m_1, m_2, m_{bar}, l, as well as the acceleration due to gravity g. Enter the absolute value function as abs() **. For instance, enter abs(x*y) for $|xy|$.**

Hint 1. What has changed?

Compared to the previous situation, what quantities have changed?

ANSWER:

- net torque only
- moment of inertia only
- (selected) both torque and moment of inertia

This time, the torque due to the weight of the bar is not zero. Additionally, the torques due to the weights of the blocks are different, since their lever arms have changed.

Hint 2. Find the moment of inertia

Find the total moment of inertia I of the bar and the two masses.

Express your answer in terms of some or all of the quantities m_1, m_2, m_{bar}, and l.

Hint 1. Find the moment of inertia of the bar alone

What is the moment of inertia I_{bar} of the bar alone?

Express your answer in terms of m_{bar} and l.

Hint 1. Use the parallel-axis theorem

The moment of inertia of a uniform bar of mass m and length l about the axis passing through the midpoint (center of mass) of the bar and perpendicular to the it (as in the previous situation) is $I_{\mathrm{cm}} = \frac{1}{12} m l^2$. In the current situation, the axis passes at distance $\frac{l}{6}$ from the center of mass. The value of the moment of inertia with respect to this new axis can be determined using the parallel-axis theorem:

$$I = I_{\mathrm{CM}} + M d^2,$$

where I_{CM} is the moment of inertia of a body of mass M about an axis through its center of mass, and I is the moment of inertia of that object through an axis parallel to the original axis through its center of mass, but displaced from it by a distance d.

ANSWER:

$I_{\text{bar}} = \dfrac{m_{\text{bar}} l^2}{9}$

ANSWER:

$I = \dfrac{l^2}{9}\left(m_1 + 4m_2 + m_{\text{bar}}\right)$

Hint 3. Find the net torque

Find the magnitude of the net torque τ acting on the system. Be sure to use the absolute value function in your answer, since no comparison of m_1, m_2, and m_{bar} has been given.

Express your answer in terms of some or all of the quantities m_1, m_2, m_{bar}, l, as well as the acceleration due to gravity g. Enter the absolute value function abs() **. For instance, enter abs(x*y) for $|xy|$.**

ANSWER:

$\tau = \left|\dfrac{gl\left(m_1 - \frac{m_{\text{bar}}}{2} - 2m_2\right)}{3}\right|$

Since we cannot numerically compare the quantities m_1, m_2, and m_{bar}, we cannot determine the direction of the net torque.

ANSWER:

$\alpha = \left|\dfrac{g\left(6m_1 - 12m_2 - 3m_{\text{bar}}\right)}{\left(m_1 + 4m_2 + m_{\text{bar}}\right) \cdot 2l}\right|$

Since we cannot numerically compare the quantities m_1, m_2, and m_{bar}, we cannot determine the direction of rotation or the sign of the angular acceleration.

Part F

If m_1 is 24 kilograms, m_2 is 12 kilograms, and m_{bar} is 10 kilograms, what is the direction of rotation and the sign of the angular acceleration?

ANSWER:

- ○ The rotation is in the clockwise direction and the angular acceleration is positive.
- ◉ The rotation is in the clockwise direction and the angular acceleration is negative.
- ○ The rotation is in the counterclockwise direction and the angular acceleration is positive.
- ○ The rotation is in the counterclockwise direction and the angular acceleration is negative.

Introduction to Rotational Work and Power

Learning Goal:

To understand work and power in rotational systems and to use the work-energy theorem to determine kinematics variables.

The variables used in standard linear mechanics (the study of objects that do not rotate) all have analogues in rotational mechanics (the study of objects that rotate). Here is a summary of variables used in kinematics and dynamics.

Linear variable	Rotational variable

linear position: x	angular position: θ
linear velocity: v	angular velocity: ω
linear acceleration: a	angular acceleration: α
linear inertia (*mass*): m	moment of inertia: I
force: F	torque: τ

The kinetic energy $K_{\rm rot}$ associated with a rotating object is defined as

$$K_{\rm rot} = \frac{1}{2}I\omega^2,$$

where ω is measured in $\rm rad/s$. Applying a torque can change the angular velocity of an object, and hence its kinetic energy. The torque is said to have done work on the object, just as a force applied over a distance can change the kinetic energy of an object in linear mechanics. In terms of rotational variables, the work W done by a constant torque τ is

$$W = \tau\Delta\theta,$$

where $\Delta\theta$ describes the total angle, measured in $\rm radians$, through which the object rotates while the torque is being applied.

Consider a motor that exerts a constant torque of 25.0$\rm N \cdot m$ to a horizontal platform whose moment of inertia is 50.0$\rm kg \cdot m^2$. Assume that the platform is initially at rest and the torque is applied for 12.0$\rm rotations$. Neglect friction.

Part A

How much work W does the motor do on the platform during this process?

Enter your answer in joules to four significant figures.

Hint 1. Convert rotations to radians

How many radians are there in 12.0$\rm rotations$?

Enter your answer in radians to four significant figures.

Hint 1. The number of radians in one revolution

One rotation is the same as one revolution. For each revolution the platform makes, it travels through 2π $\rm radians$.

ANSWER:

$\Delta\theta$ = $N{\cdot}2\pi$ = 75.40 $\rm rad$

ANSWER:

W = $\tau(N{\cdot}2\pi)$ = 1885 $\rm J$

Part B

What is the rotational kinetic energy of the platform $K_{\rm rot,f}$ at the end of the process described above?

Enter your answer in joules to four significant figures.

Hint 1. How to approach the problem

We cannot use the definition $K_{\rm rot} = (1/2)I\omega^2$ since the value of ω is not known. Instead, apply the work-energy theorem. Since the platform starts at rest, its initial kinetic energy is easy to calculate.

ANSWER:

$K_{\mathrm{rot,f}}$ = $\tau(N{\cdot}2\pi)$ = 1885 J

The net work done by the motor increases the kinetic energy of the platfrom from $K_{\mathrm{rot,i}} = 0$ to $K_{\mathrm{rot,f}}$.

Now that the final kinetic energy is known, the values of other kinematic variables at this final time may also be determined.

Part C

What is the angular velocity ω_{f} of the platform at the end of this process?

Enter your answer in radians per second to three significant figures.

Hint 1. How to approach the problem

Use your knowledge of the final rotational kinetic energy of the platform to determine its final rotational speed.

ANSWER:

ω_{f} = $\sqrt{\frac{2\tau N\cdot 2\pi}{I}}$ = 8.68 rad/s

Part D

How long Δt does it take for the motor to do the work done on the platform calculated in Part A?

Enter your answer in seconds to three significant figures.

Hint 1. Determine how to approach the problem

This is a situation in which the rotational acceleration is constant, so Δt can be found using constant-acceleration kinematics relations. Which of the following lines of reasoning, all of which are correct, can be applied most directly in this situation to find Δt?

ANSWER:

- Use the equation $\omega_{\mathrm{f}} = \omega_{\mathrm{i}} + \alpha\Delta t$.
- (selected) Use the equation $\omega_{\mathrm{avg}} = \Delta\theta/\Delta t$ where $\omega_{\mathrm{avg}} = (1/2)(\omega_{\mathrm{i}} + \omega_{\mathrm{f}})$.
- Use the equation $\Delta\theta = (1/2)\alpha(\Delta t)^2$.

ANSWER:

Δt = $\frac{2N\cdot 2\pi}{\sqrt{\frac{2\tau(N\cdot 2\pi)}{I}}}$ = 17.4 s

---

Part E

What is the average power P_{avg} delivered by the motor in the situation above?

Enter your answer in watts to three significant figures.

Hint 1. Find the average angular velocity

What is the average angular velocity of the platform? Keep in mind that the average velocity is given by the equation

$$\omega_{\mathrm{avg}} = \frac{\omega_{\mathrm{i}} + \omega_{\mathrm{f}}}{2}.$$

Enter your answer in radians per second to three significant figures.

ANSWER:

ω_{avg} = $\dfrac{\sqrt{\frac{2\tau(N\cdot 2\pi)}{I}}}{2}$ = 4.34 $\mathrm{rad/s}$

ANSWER:

P_{avg} = $\dfrac{\tau(N\cdot 2\pi)}{\frac{2N\cdot 2\pi}{\sqrt{\frac{2\tau(N\cdot 2\pi)}{I}}}}$ = 109 W

Part F

Note that the instantaneous power P delivered by the motor is directly proportional to ω, so P increases as the platform spins faster and faster. How does the instantaneous power P_f being delivered by the motor at the time t_f compare to the average power P_{avg} calculated in Part E?

Hint 1. How to approach the problem

In part E you have already calculated the average power P_{avg}. Now calculate P_f using the appropriate values of τ and ω that apply at time t_f and compare the instantaneous power to the average power. Since the platform is spinning faster and faster in this situation, you can probably guess in advance which one will be larger.

ANSWER:

- ○ $P = P_{avg}$
- ◉ $P = 2 * P_{avg}$
- ○ $P = P_{avg}/2$
- ○ none of the above

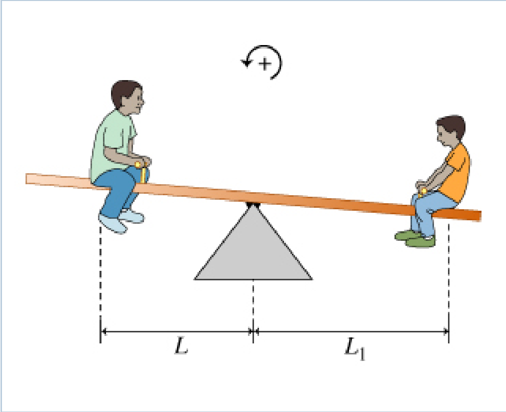
This makes sense! As another example, consider a constant-acceleration situation in which ω increases from rest at a rate of 1 $\mathrm{rad/s}$. During the first second, ω changes from 0 to 1 $\mathrm{rad/s}$. About 10 seconds later ω changes from 10 to 11 $\mathrm{rad/s}$, again in a single second. This requires about 20 times more work than the first change because kinetic energy depends on the square of ω rather than on ω alone. Since the work in both cases must be done in just one second, much more power is required to maintain the angular acceleration when the object is spinning faster.

Torques on a Seesaw: A Tutorial

Learning Goal:

To make the connection between your intuitive understanding of a seesaw and the standard formalism for torque.

This problem deals with the concept of torque, the "twist" that an off-center force applies to a body that tends to make it rotate.



Try to use your intuition to answer the following question. If your intuition fails, work the rest of the problem and return here when you feel that you are more comfortable with torques.

Part A

A mother is helping her children, of unequal weight, to balance on a seesaw so that they will be able to make it tilt back and forth without the heavier child simply sinking to the ground. Given that her heavier child of weight W is sitting a distance L to the left of the pivot, at what distance L_1 must she place her second child of weight w on the right side of the pivot to balance the seesaw?

Express your answer in terms of L, W, and w.

Hint 1. How to approach the problem

Consider whether L_1 increases or decreases as each of the variables that it depends on– W, w, and L –is made larger or smaller.

ANSWER:

$L_1 = \frac{LW}{w}$

The figure shows the seesaw slightly tilted, as will be the case when in use. This does not change the torque balance because the horizontal distances from the pivot to each child (which are called the moment arms for the vertically directed weight and must be used to calculate the torque instead of the distance along the seesaw) are reduced equally, so the sum of the torques is zero at any angle. Given that the torque is zero at all times (except when one or both children push on the ground), there will generally be no angular acceleration of the seesaw, and the seesaw will rotate at a constant velocity between pushes from the feet of the children on the ground.

Now consider this problem as a more formal introduction to torque. The torque of each child about the pivot point is the product of the child's weight and the distance of the child (strictly speaking, the child's center of mass) from the pivot. The sign of the torque is positive if it would cause a counterclockwise rotation of the seesaw. The distance is measured perpendicular to the line of force and is called the *moment arm*.

The concept of torque requires both a force and a specification of the pivot point, emphasized by the first subscript on the torque.

Part B

Find $\tau_{p,w}$, the torque about the pivot due to the weight w of the smaller child on the seesaw.

Express your answer in terms of L_1 and w.

ANSWER:

$\tau_{p,w} = -L_1 w$

The children's mother wants the seesaw to balance, which means that there can be no angular acceleration about the pivot. The balanced seesaw will then be in equilibrium since it has neither linear acceleration nor rotational acceleration.
For the linear acceleration to be zero, the vector sum of forces acting on the seesaw and children must equal zero.

For the angular acceleration to be zero, the sum of the torques about the pivot must equal zero. This can be written

$$\sum_i \tau_{p,i} = 0,$$

where $\tau_{p,i}$ is the torque about the pivot due to the i th force.

Part C

Determine $\sum_i \tau_{p,i}$, the sum of the torques on the seesaw. Consider only the torques exerted by the children.

Express your answer in terms of W, w, L, and L_1.

Hint 1. Torque from the weight of the seesaw

The seesaw is symmetric about the pivot, and so the gravitational force on the seesaw produces no net torque. More generally, when determining torques, the gravitational force on an object in a uniform gravitational field can be taken to act at the center of mass. Here the center of mass is directly above the pivot, so the weight of the seesaw has zero moment arm and produces no torque about the pivot.

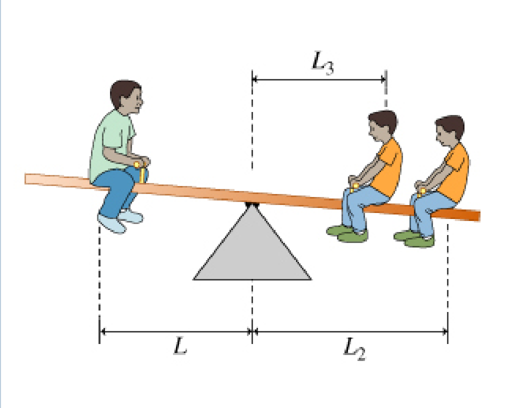
ANSWER:

$\sum_i \tau_{p,i} = 0 =$ $WL - wL_1$

Good! If you did not solve for the distance L_1 required to balance the seesaw in Part A, do so now.

The equation $\sum_i \tau_i = 0$ applies to any body that is not rotationally accelerating. Combining this equation with $\sum_i F_i = 0$ (which applies to any body that is not accelerating linearly) gives a pair of equations that are sufficient to form the basis of *statics*; these or similar, often more complicated equations govern structures that are not accelerating. The torque equation is often of more utility, however, because you can choose the pivot point arbitrarily (often so that unknown forces have no moment arm and therefore contribute no torque). The art of applying these equations to large or complicated structures constitutes a significant part of mechanical and civil engineering.

The child with weight w has an identical twin also of weight w. The two twins now sit on the same side of the seesaw, with one twin a distance L_2 from the pivot and the other a distance L_3.



Part D

Where should the mother position the child of weight W to balance the seesaw now?

Express your answer in terms of L_2, L_3, W, and w.

Hint 1. Balancing the seesaw

For the seesaw to balance, the sum of the torques from the three children $\sum_i \tau_{p,i}$ must be zero. What is the sum of the torques?

Express your answer in terms of L, L_2, L_3, W, and w.

ANSWER:

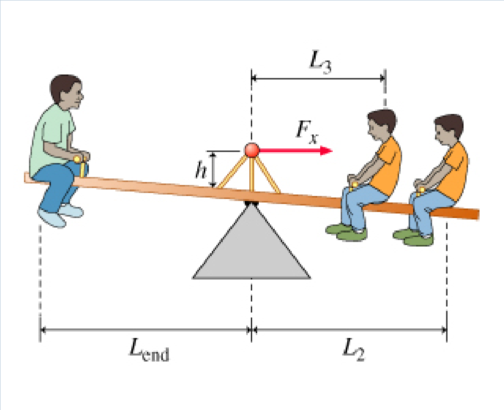
$$\sum_i \tau_{p,i} = 0 = \; LW - w(L_2 + L_3)$$

Now solve you torque equation for L.

ANSWER:

$$L = \frac{w(L_3 + L_2)}{W}$$

Bad news! When the mother finds the distance L from the previous part it turns out to be greater than $L_{\rm end}$, the distance from the pivot to the end of the seesaw. Hence, even with the child of weight W at the very end of the seesaw the twins exert more torque than the heavier child does. The mother now elects to balance the seesaw by pushing sideways on an ornament (shown in red) that is a height h above the pivot.



Part E

With what force in the rightwards direction, F_x, should the mother push? Note that if you think the force exerted by the mother should be toward the left, your final answer F_x should be negative.

Express your answer in terms of W, $L_{\rm end}$, w, L_2, L_3, and h.

Hint 1. Sign conventions

It is easy to make sign errors in torque problems, and experience shows that it is better to use standard conventions (here that F_x goes to the right, the direction of positive *x*) than to change the direction of positive torque or displacement to suit your convenience. (You are likely to forget your unconventional choice at a later point in the problem.) In this case, your intuition correctly expects that the mother must push to the left to make things balance; the equations will confirm this by giving a *negative* result for F_x. (A positive result would mean that the force would be directed to the right in the figure.)

Hint 2. Torque due to mother's push

The sum of all four torques (due to each of the three children plus the mother) must be zero.
What is the torque $\tau_{p,F}$ due to the mother's push?

Remember, a positive torque will cause counterclockwise rotation of the seesaw.

Express your answer in terms of the unknown force F_x and the height at which it is applied h.

ANSWER:

$$\tau_{p,F} = \; -F_x h$$

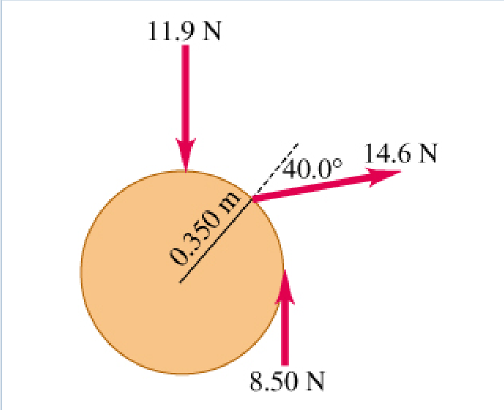
ANSWER:

$$F_x = \frac{W L_{\rm end} - w(L_2 + L_3)}{h}$$

This answer will necessarily be negative, because you were told that to balance the seesaw with the twins on the right, the child of weight W had to be "beyond the end" of the seesaw. Therefore, when $L_{\rm end}$ is the position of the child of weight W, F_x will be less than zero. Hence the mother must push to the left as you'd expect.

Exercise 10.4

Three forces are applied to a wheel of radius 0.350 $\rm m$, as shown in the figure . One force is perpendicular to the rim, one is tangent to it, and the other one makes a 40.0 $^\circ$ angle with the radius.



Part A

What is the magnitude of the net torque on the wheel due to these three forces for an axis perpendicular to the wheel and passing through its center?

ANSWER:

τ = 0.310 $\rm N \cdot m$

Part B

What is the direction of the net torque in part (A).

ANSWER:

- (●) into the page
- ( ) out of the page.

Exercise 10.11

A machine part has the shape of a solid uniform sphere of mass 230$\rm g$ and diameter 4.10$\rm cm$. It is spinning about a frictionless axle through its center, but at one point on its equator it is scraping against metal, resulting in a friction force of 0.0200 $\rm N$ at that point.

Part A

Find its angular acceleration. Let the direction the sphere is spinning be the positive sense of rotation.

ANSWER:

$\alpha = \frac{\frac{-0.0200d}{2}}{\frac{\frac{2}{5}md^2}{4}}$ = -10.6 $\rm rad/s^2$

Part B

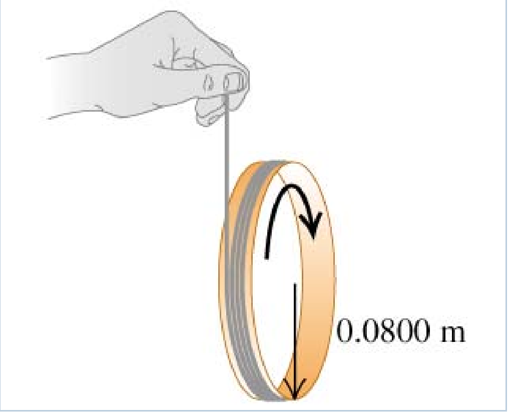
How long will it take to decrease its rotational speed by 20.0$\rm rad/s$?

ANSWER:

$t = \frac{om}{\frac{\frac{0.0200d}{2}}{\frac{\frac{2}{5}md^2}{4}}}$ = 1.89 s

Exercise 10.20

A string is wrapped several times around the rim of a small hoop with radius 8.00 cm and mass 0.180kg . The free end of the string is held in place and the hoop is released from rest (the figure). After the hoop has descended 60.0cm , calculate



Part A

the angular speed of the rotating hoop and

ANSWER:

$\omega = \frac{\sqrt{(9.8s)}}{0.08}$ = 30.3 $\mathrm{rad/s}$

Part B

the speed of its center.

ANSWER:

$v = \sqrt{(9.8s)}$ = 2.42 $\mathrm{m/s}$

Exercise 10.22

A hollow, spherical shell with mass 2.40kg rolls without slipping down a slope angled at 34.0°.

Part A

Find the acceleration.

Take the free fall acceleration to be g = 9.80$\mathrm{m/s^2}$

ANSWER:

$\frac{3}{5}g\sin(\theta)$ = 3.29 $\mathrm{m/s^2}$

Part B

Find the friction force.

Take the free fall acceleration to be g = 9.80$\mathrm{m/s^2}$

ANSWER:

$\frac{2}{5}mg\sin(\theta)$ = 5.26 N

Part C

Find the minimum coefficient of friction needed to prevent slipping.

ANSWER:

$\frac{2}{5}\tan(\theta)$ = 0.270

Exercise 10.30

The engine of an aircraft propeller delivers an amount of power 180hp to the propeller at a rotational velocity of 2300$\mathrm{rev/min}$.

Part A

How much torque does the aircraft engine provide?

ANSWER:

$\frac{P}{\omega}$ = 558 $\mathrm{N \cdot m}$

Part B

How much work does the engine do in one revolution of the propeller?

ANSWER:

$\frac{P}{\omega} \cdot 2\pi$ = 3500 J

Exercise 10.36

A woman with a mass of 51.0kg is standing on the rim of a large disk that is rotating at an angular velocity of 0.510$\mathrm{rev/s}$ about an axis through its center. The disk has a mass of 104kg and a radius of 3.70m .

Part A

Calculate the magnitude of the total angular momentum of the woman-plus-disk system. (Assume that you can treat the woman as a point.)

ANSWER:

$\left(\frac{1}{2}m_2 + m_1\right)R^2\omega$ = 4520 $\mathrm{kg \cdot m^2/s}$

Exercise 10.41

Under some circumstances, a star can collapse into an extremely dense object made mostly of neutrons and called a *neutron star.* The density of a neutron star is roughly 10^{14} times as great as that of ordinary solid matter. Suppose we represent the star as a uniform, solid, rigid sphere, both before and after the collapse. The star's initial radius was 9.0×$10^5$$\mathrm{km}$ (comparable to our sun); its final radius is 18km .

Part A

If the original star rotated once in 29 days, find the angular speed of the neutron star.

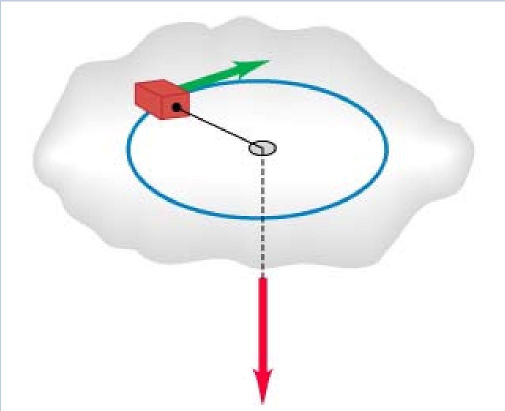
Express your answer using two significant figures.

ANSWER:

$\omega_2 = \frac{2\pi}{t \cdot 86400}\left(\frac{r_1}{r_2}\right)^2$ = 6300 $\mathrm{rad/s}$

Exercise 10.42

A small block on a frictionless, horizontal surface has a mass of $2.20 \times 10^{-2}\ \mathrm{kg}$. It is attached to a massless cord passing through a hole in the surface (the figure). The block is originally revolving at a distance of 0.300 m from the hole with an angular speed of $1.85\ \mathrm{rad/s}$. The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.150 m. Model the block as a particle.



Part A

Is angular momentum of the block conserved?

ANSWER:

- (selected) yes
- no

Part B

Why or why not?

ANSWER:

3647 Character(s) remaining

The net force is due to the tension in the rope, which always acts in the radial direction, so the angular

Part C

What is the new angular speed?

ANSWER:

ω_2 = $\omega \cdot 4$ = 7.40 $\mathrm{rad/s}$

Part D

Find the change in kinetic energy of the block.

ANSWER:

ΔK = $1.5m\,(\omega \cdot 0.3)^2$ = 1.02×10^{-2} J

Part E

How much work was done in pulling the cord?

ANSWER:

$1.5m\,(\omega \cdot 0.3)^2$ = 1.02×10^{-2} J

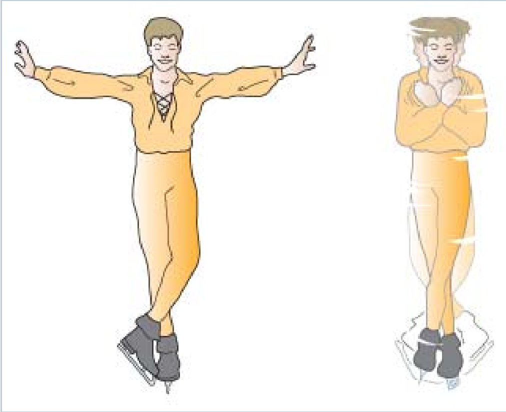
Exercise 10.43

The outstretched hands and arms of a figure skater preparing for a spin can be considered a slender rod pivoting about an axis through its center . When his hands and arms are brought in and wrapped around his body to execute the spin, the hands and arms can be considered a thin-walled hollow cylinder. His hands and arms have a combined mass $8.0\ \mathrm{kg}$. When outstretched, they span $1.6\ \mathrm{m}$; when wrapped, they form a cylinder of radius $26\ \mathrm{cm}$. The moment of inertia about the rotation axis of the remainder of his body is constant and equal to $0.40\ \mathrm{kg \cdot m^2}$.

Exercise 10.42

Part B ANSWER:

The net force is due to the tension in the rope, which always acts in the radial direction, so the angular momentum with respect to the hole is constant.



Part A

If his original angular speed is 0.40$\mathrm{rev/s}$, what is his final angular speed?

Express your answer using two significant figures.

ANSWER:

ω_2 = $\frac{\omega\left(0.4+\frac{1}{12}mr1^2\right)}{0.4+mr2^2}$ = 0.90 $\mathrm{rev/s}$

Also accepted: $\frac{\omega\left(0.4+\frac{1}{12}mr1^2\right)}{0.4+\frac{mr2^2}{2}}$ = 1.3

10.4. **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ to calculate the magnitude of each torque and use the right-hand rule to determine the direction of each torque. Add the torques to find the net torque.

SET UP: Let counterclockwise torques be positive. For the 11.9 N force (F_1), $r = 0$. For the 14.6 N force (F_2), $r = 0.350\text{ m}$ and $\phi = 40.0°$. For the 8.50 N force (F_3), $r = 0.350\text{ m}$ and $\phi = 90.0°$.

EXECUTE: $\tau_1 = 0.$ $\tau_2 = -(14.6\text{ N})(0.350\text{ m})\sin 40.0° = -3.285\text{ N}\cdot\text{m}.$

$\tau_3 = +(8.50\text{ N})(0.350\text{ m})\sin 90.0° = +2.975\text{ N}\cdot\text{m}.$

$\sum\tau = -3.285\text{ N}\cdot\text{m} + 2.975\text{ N}\cdot\text{m} = -0.31\text{ N}\cdot\text{m}.$ The net torque is $0.31\text{ N}\cdot\text{m}$ and is clockwise.

EVALUATE: If we treat the torques as vectors, $\vec{\tau}_2$ is into the page and $\vec{\tau}_3$ is out of the page.

10.11. **IDENTIFY:** Use $\sum\tau_z = I\alpha_z$ to calculate α. Use a constant angular acceleration kinematic equation to relate α_z, ω_z and t.

SET UP: For a solid uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$. Let the direction the sphere is spinning be the positive sense of rotation. The moment arm for the friction force is $l = 0.0150\text{ m}$ and the torque due to this force is negative.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{-(0.0200\text{ N})(0.0150\text{ m})}{\frac{2}{5}(0.225\text{ kg})(0.0150\text{ m})^2} = -14.8\text{ rad/s}^2$

(b) $\omega_z - \omega_{0z} = -22.5\text{ rad/s}.$ $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{-22.5\text{ rad/s}}{-14.8\text{ rad/s}^2} = 1.52\text{ s}.$

EVALUATE: The fact that α_z is negative means its direction is opposite to the direction of spin. The negative α_z causes ω_z to decrease.

10.20. **IDENTIFY:** Only gravity does work, so $W_{\text{other}} = 0$ and conservation of energy gives $K_1 + U_1 = K_2 + U_2.$ $K_2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2.$

SET UP: Let $y_2 = 0$, so $U_2 = 0$ and $y_1 = 0.750\text{ m}.$ The hoop is released from rest so $K_1 = 0.$ $v_{\text{cm}} = R\omega.$ For a hoop with an axis at its center, $I_{\text{cm}} = MR^2.$

EXECUTE: **(a)** Conservation of energy gives $U_1 = K_2.$ $K_2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2,$ so $MR^2\omega^2 = Mgy_1.$ $\omega = \dfrac{\sqrt{gy_1}}{R} = \dfrac{\sqrt{(9.80\text{ m/s}^2)(0.750\text{ m})}}{0.0800\text{ m}} = 33.9\text{ rad/s}.$

(b) $v = R\omega = (0.0800\text{ m})(33.9\text{ rad/s}) = 2.71\text{ m/s}$

EVALUATE: An object released from rest and falling in free fall for 0.750 m attains a speed of $\sqrt{2g(0.750\text{ m})} = 3.83\text{ m/s}.$ The final speed of the hoop is less than this because some of its energy is in kinetic energy of rotation. Or, equivalently, the upward tension causes the magnitude of the net force of the hoop to be less than its weight.

10.22. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the center of mass and $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass.

SET UP: Let $+x$ be down the incline and let the shell be turning in the positive direction. The free-body diagram for the shell is given in Figure 10.22. From Table 9.2, $I_{\text{cm}} = \frac{2}{3}mR^2$.

EXECUTE: **(a)** $\sum F_x = ma_x$ gives $mg\sin\beta - f = ma_{\text{cm}}$. $\sum \tau_z = I\alpha_z$ gives $fR = \left(\frac{2}{3}mR^2\right)\alpha$.

With $\alpha = a_{\text{cm}}/R$ this becomes $f = \frac{2}{3}ma_{\text{cm}}$. Combining the equations gives

$mg\sin\beta - \frac{2}{3}ma_{\text{cm}} = ma_{\text{cm}}$ and $a_{\text{cm}} = \dfrac{3g\sin\beta}{5} = \dfrac{3(9.80\ \text{m/s}^2)(\sin 38.0°)}{5} = 3.62\ \text{m/s}^2$.

$f = \frac{2}{3}ma_{\text{cm}} = \frac{2}{3}(2.00\ \text{kg})(3.62\ \text{m/s}^2) = 4.83\ \text{N}$. The friction is static since there is no slipping at the point of contact. $n = mg\cos\beta = 15.45\ \text{N}$. $\mu_s = \dfrac{f}{n} = \dfrac{4.83\ \text{N}}{15.45\ \text{N}} = 0.313$.

(b) The acceleration is independent of *m* and doesn't change. The friction force is proportional to *m* so will double; $f = 9.66\ \text{N}$. The normal force will also double, so the minimum μ_s required for no slipping wouldn't change.

EVALUATE: If there is no friction and the object slides without rolling, the acceleration is $g\sin\beta$. Friction and rolling without slipping reduce *a* to 0.60 times this value.

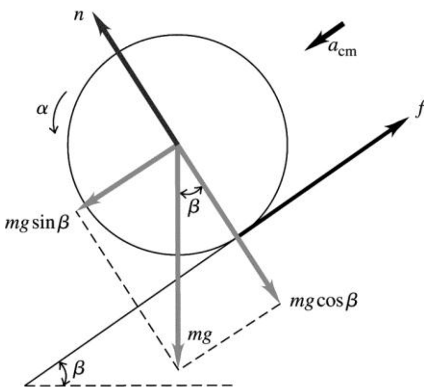


Figure 10.22

10.30. IDENTIFY: Apply $P = \tau\omega$ and $W = \tau\Delta\theta$.

SET UP: *P* must be in watts, $\Delta\theta$ must be in radians, and ω must be in rad/s. $1\ \text{rev} = 2\pi\ \text{rad}$. $1\ \text{hp} = 746\ \text{W}$. $\pi\ \text{rad/s} = 30\ \text{rev/min}$.

EXECUTE: **(a)** $\tau = \dfrac{P}{\omega} = \dfrac{(175\ \text{hp})(746\ \text{W/hp})}{(2400\ \text{rev/min})\left(\dfrac{\pi\ \ \text{rad/s}}{30\ \text{rev/min}}\right)} = 519\ \text{N}\cdot\text{m}$.

(b) $W = \tau\Delta\theta = (519\ \text{N}\cdot\text{m})(2\pi\ \text{rad}) = 3260\ \text{J}$

EVALUATE: $\omega = 40\text{ rev/s},$ so the time for one revolution is $0.025\text{ s}.$ $P = 1.306\times 10^5\text{ W},$ so in one revolution, $W = Pt = 3260\text{ J},$ which agrees with our result.

10.36.IDENTIFY: $L = I\omega$ and $I = I_{\text{disk}} + I_{\text{woman}}.$

SET UP: $\omega = 0.50\text{ rev/s} = 3.14\text{ rad/s}.$ $I_{\text{disk}} = \frac{1}{2}m_{\text{disk}}R^2$ and $I_{\text{woman}} = m_{\text{woman}}R^2.$

EXECUTE: $I = (55\text{ kg} + 50.0\text{ kg})(4.0\text{ m})^2 = 1680\text{ kg}\cdot\text{m}^2.$

$L = (1680\text{ kg}\cdot\text{m}^2)(3.14\text{ rad/s}) = 5.28\times 10^3\text{ kg}\cdot\text{m}^2/\text{s}$

EVALUATE: The disk and the woman have similar values of *I*, even though the disk has twice the mass.

10.41.IDENTIFY: Apply conservation of angular momentum.

SET UP: For a uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2.$

EXECUTE: The moment of inertia is proportional to the square of the radius, and so the angular velocity will be proportional to the inverse of the square of the radius, and the final angular velocity is

$$\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2\pi\text{ rad}}{(30\text{ d})(86{,}400\text{ s/d})}\right)\left(\frac{7.0\times 10^5\text{ km}}{16\text{ km}}\right)^2 = 4.6\times 10^3\text{ rad/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega.$ L is constant and ω increases by a large factor, so there is a large increase in the rotational kinetic energy of the star. This energy comes from potential energy associated with the gravity force within the star.

10.42. **IDENTIFY** and **SET UP:** $\vec{L}$ is conserved if there is no net external torque.

Use conservation of angular momentum to find ω at the new radius and use $K = \frac{1}{2}I\omega^2$ to find the change in kinetic energy, which is equal to the work done on the block.

EXECUTE: **(a)** Yes, angular momentum is conserved. The moment arm for the tension in the cord is zero so this force exerts no torque and there is no net torque on the block.

(b) $L_1 = L_2$ so $I_1\omega_1 = I_2\omega_2.$ Block treated as a point mass, so $I = mr^2,$ where *r* is the distance of the block from the hole.

$mr_1^2\omega_1 = mr_2^2\omega_2$

$$\omega_2 = \left(\frac{r_1}{r_2}\right)^2 \omega_1 = \left(\frac{0.300\text{ m}}{0.150\text{ m}}\right)^2 (1.75\text{ rad/s}) = 7.00\text{ rad/s}$$

(c) $K_1 = \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} m r_1^2 \omega_1^2 = \frac{1}{2} m v_1^2$

$v_1 = r_1 \omega_1 = (0.300\text{ m})(1.75\text{ rad/s}) = 0.525\text{ m/s}$

$K_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(0.0250\text{ kg})(0.525\text{ m/s})^2 = 0.00345\text{ J}$

$K_2 = \frac{1}{2} m v_2^2$

$v_2 = r_2 \omega_2 = (0.150\text{ m})(7.00\text{ rad/s}) = 1.05\text{ m/s}$

$K_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}(0.0250\text{ kg})(1.05\text{ m/s})^2 = 0.01378\text{ J}$

$\Delta K = K_2 - K_1 = 0.01378\text{ J} - 0.00345\text{ J} = 0.0103\text{ J}$

(d) $W_{\text{tot}} = \Delta K$

But $W_{\text{tot}} = W$, the work done by the tension in the cord, so $W = 0.0103\text{ J}$.

EVALUATE: Smaller *r* means smaller *I*. $L = I\omega$ is constant so ω increases and *K* increases. The work done by the tension is positive since it is directed inward and the block moves inward, toward the hole.

10.43. **IDENTIFY:** Apply conservation of angular momentum to the motion of the skater.

SET UP: For a thin-walled hollow cylinder $I = mR^2$. For a slender rod rotating about an axis through its center, $I = \frac{1}{12} m l^2$.

EXECUTE: $L_i = L_f$ so $I_i \omega_i = I_f \omega_f$.

$I_i = 0.40\text{ kg}\cdot\text{m}^2 + \frac{1}{12}(8.0\text{ kg})(1.8\text{ m})^2 = 2.56\text{ kg}\cdot\text{m}^2$.

$I_f = 0.40\text{ kg}\cdot\text{m}^2 + (8.0\text{ kg})(0.25\text{ m})^2 = 0.90\text{ kg}\cdot\text{m}^2$.

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{2.56\text{ kg}\cdot\text{m}^2}{0.90\text{ kg}\cdot\text{m}^2}\right)(0.40\text{ rev/s}) = 1.14\text{ rev/s}.$$

EVALUATE: $K = \frac{1}{2} I\omega^2 = \frac{1}{2} L\omega$. ω increases and *L* is constant, so *K* increases. The increase in kinetic energy comes from the work done by the skater when he pulls in his hands.