Lecture 29 (Magnetic Fields from Moving Charges)

Physics 161-01 Spring 2012

Douglas Fields

Source of Magnetic Fields

- Symmetry is beautiful.
- Let's re-examine the nature of electric fields and forces:
  - An electric field will cause a force $\vec{F} = q\vec{E}$ on a charge q placed at rest within the field.
  - A distribution of electric charge at rest creates an electric field in the surrounding space.
- Now, what do we know about the magnetic field and forces?
  - A magnetic field will cause a force $\vec{F} = q\vec{v} \times \vec{B}$ on a moving charge placed within the field.
  - What is the source of magnetic field?
  - A distribution of moving charges creates a magnetic field in the surrounding space!

Source of Magnetic Fields

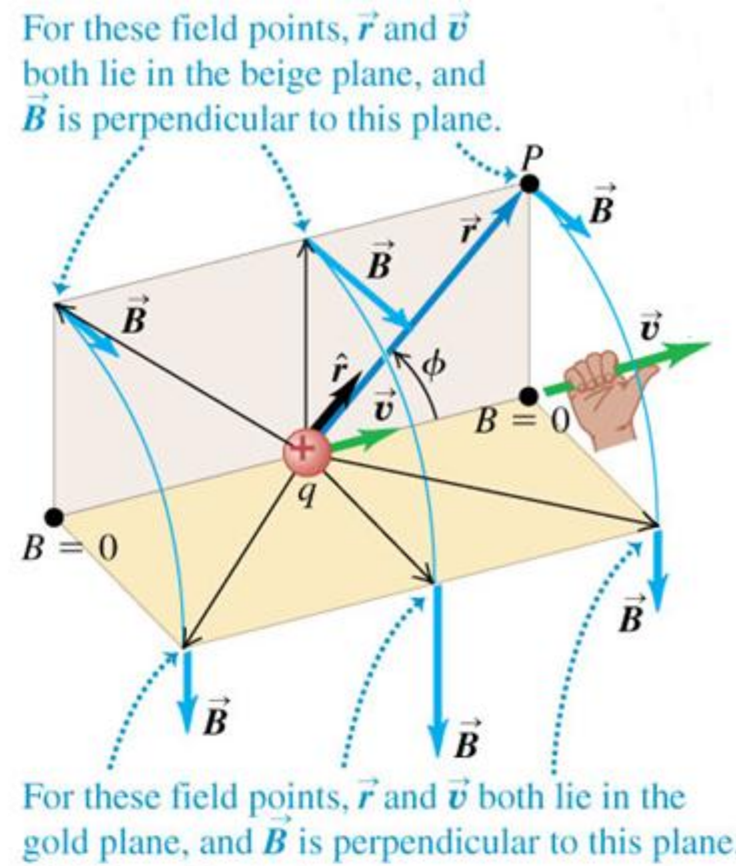
- Symmetry is beautiful.
- One of you asked me WHY??? $\vec{F} = q\vec{v} \times \vec{B}$
- Is it just from experiment?
- Is there no other more fundamental explanation?
- There is, but you would need to know Special Relativity.
- Ask yourselves: If the laws of physics mean anything, shouldn’t they work for everyone equally?
- Then ask: What if I was moving on a charged object within a magnetic field – in my perspective I’m not moving. Would I feel a force? What would be the cause of that force?
- Google **Relativistic electromagnetism.**

Magnetic Field from a Moving Charge

- For now, let's just look at the experimental revelation:

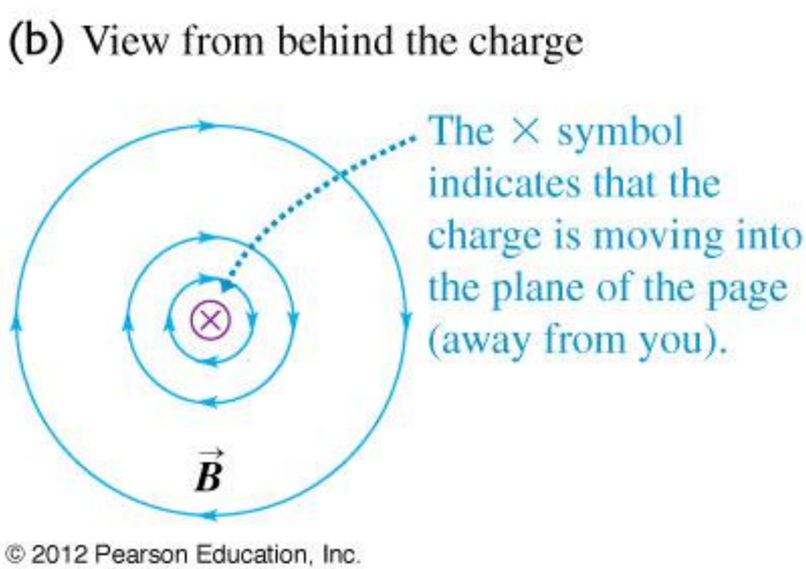
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

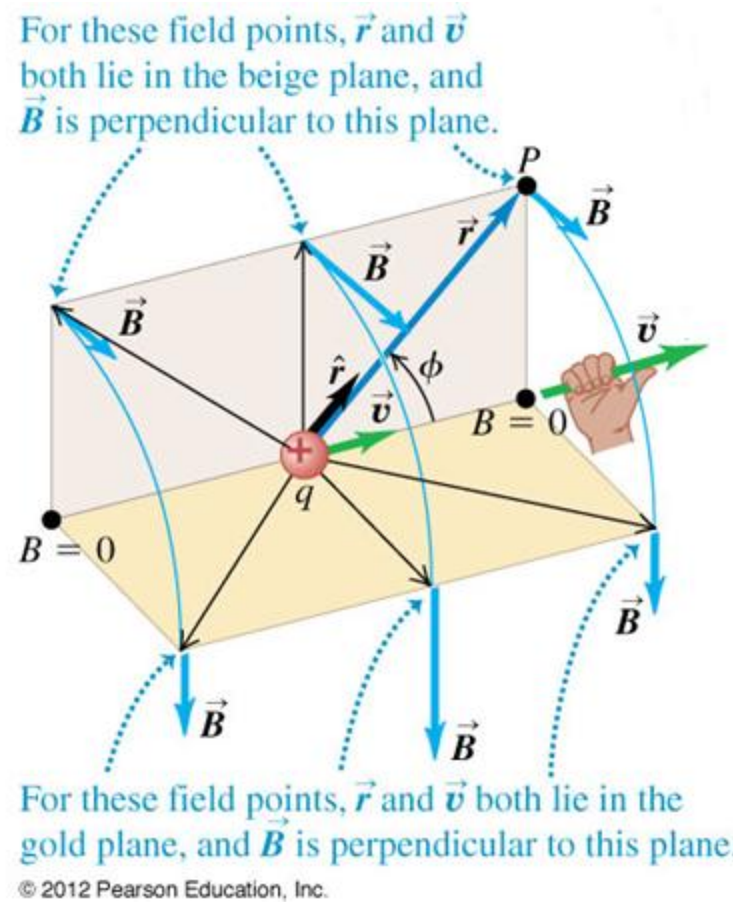
- There is a magnetic field caused by a moving charge that is perpendicular to both the direction of motion and the vector which points from the charge to the point that you want to determine the field.
- The strength of the field (its magnitude) falls as the inverse distance squared from the charge, and is proportional to the velocity.



Magnetic Field from a Moving Charge

- If you look at the situation from behind the moving charge, the field lines (remember what field lines are?) are circles around the moving charge.



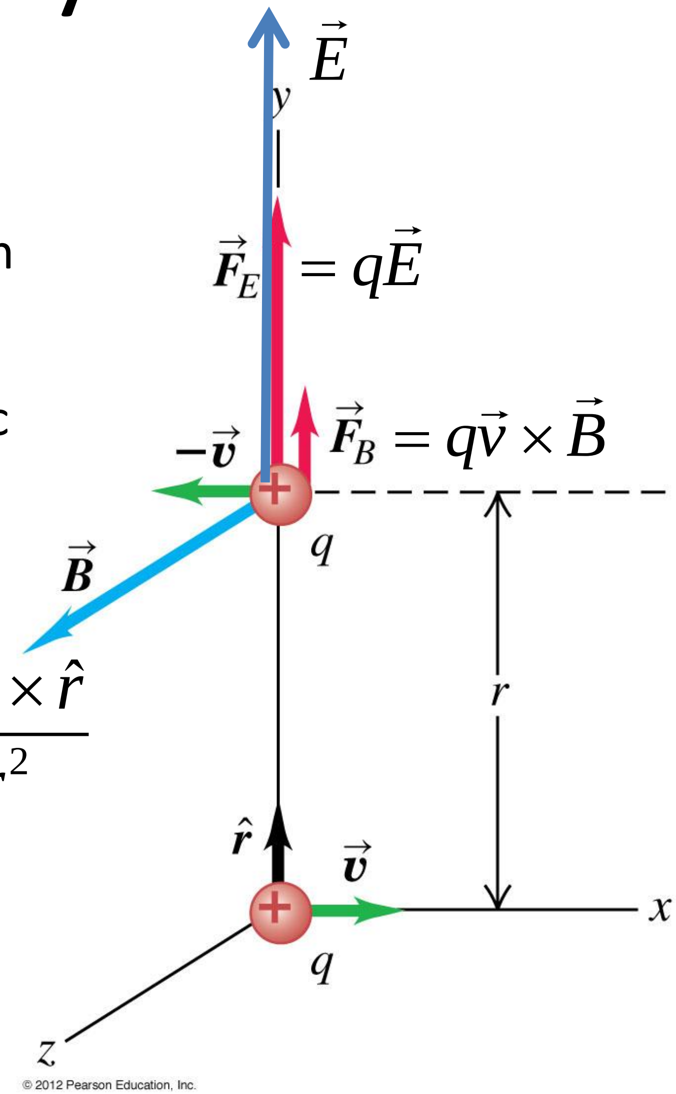


$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Symmetry

- Just as a charged particle will create an electric field, which can affect another charged particle, a moving charged particle will create a magnetic field which can affect another moving charged particle.
- But even moving charges have an electric field too!

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$



Magnetic Field from a Current Element

- Let's now look at a segment of a thin wire, of length *dl*, and cross-sectional area *A* with a current through it.
- For each moving charge:

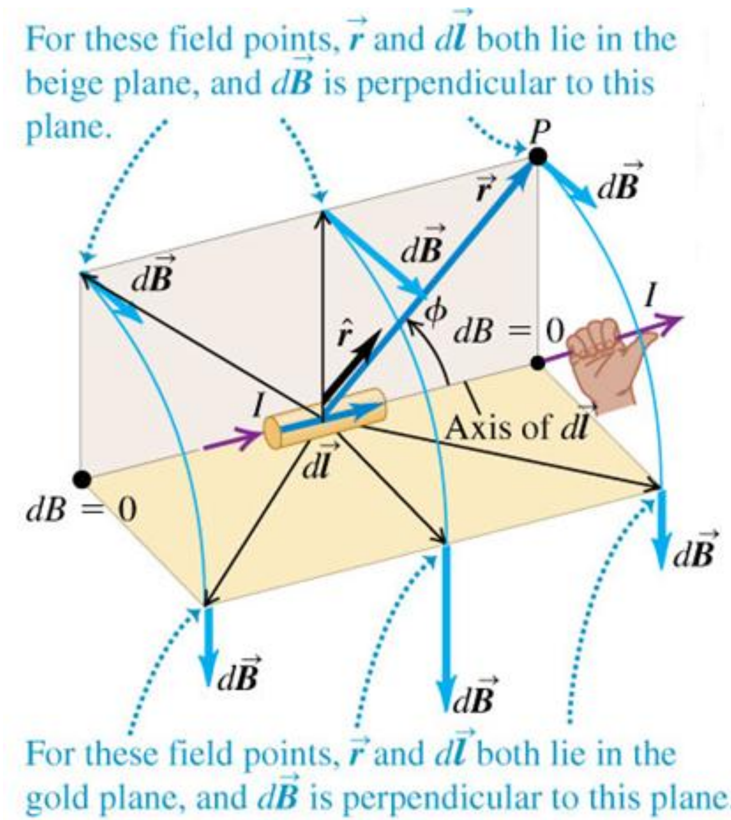
$$\vec{B}_i = \frac{\mu_0}{4\pi}\frac{q\vec{v}_d \times \hat{r}}{r^2}$$

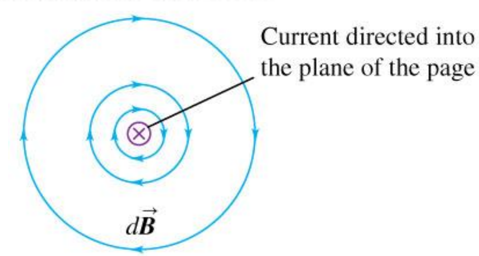
- If we then add the magnetic field from all the charges (assuming the area and length are small relative to the distance r), then:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{n\left(Adl\right)q\vec{v}_d \times \hat{r}}{r^2} \Rightarrow$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

- Where, again, *dl* is taken in the direction of the current.





CPS 29-1

A positive point charge is moving directly toward point *P*. The magnetic field that the point charge produces at point *P*

A. points from the charge toward point *P.*

B. points from point *P* toward the charge.

C. is perpendicular to the line from the point charge to point *P.*

D. is zero.

E. The answer depends on the speed of the point charge.

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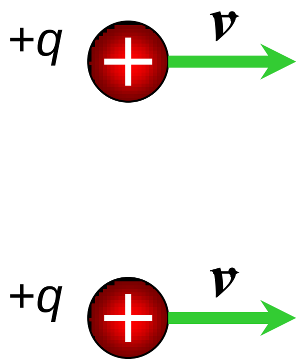
CPS 29-2

Two positive point charges move side by side in the same direction with the same velocity.

What is the direction of the magnetic force that the upper point charge exerts on the lower one?



A. toward the upper point charge (the force is attractive)

B. away from the upper point charge (the force is repulsive)

C. in the direction of the velocity

D. opposite to the direction of the velocity

E. none of the above

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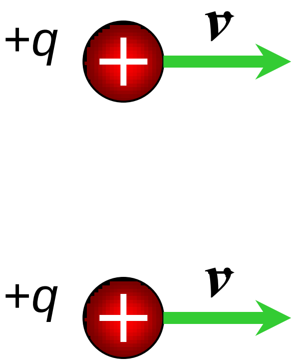

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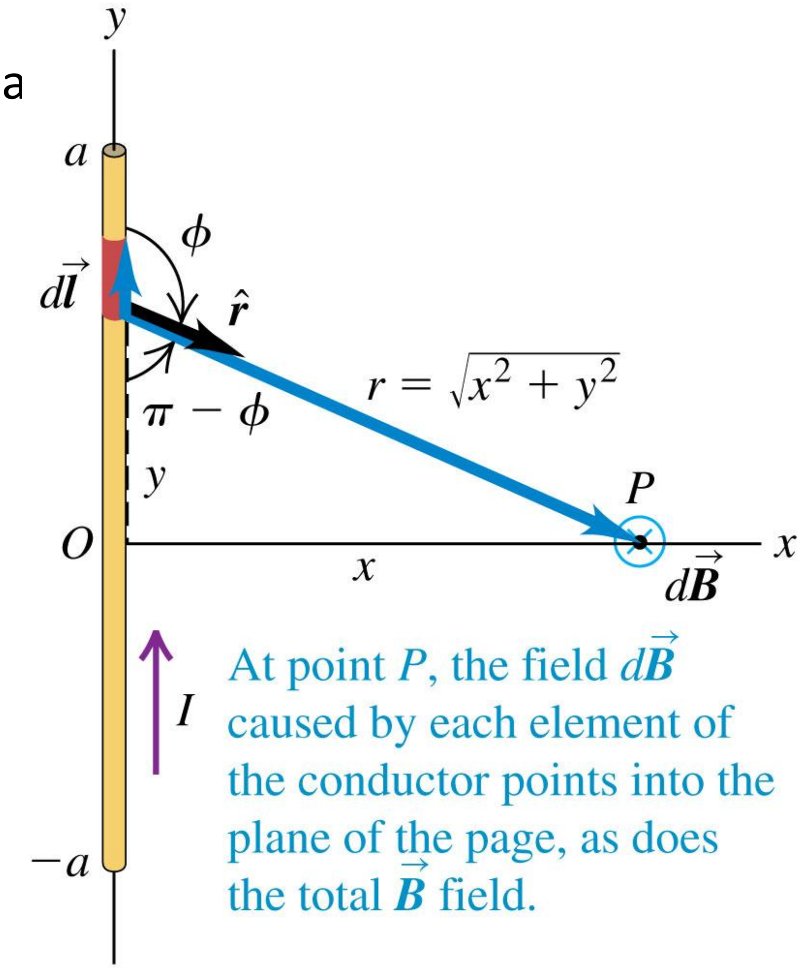
Magnetic Field from a Current Segment

- Calculus again!
- We want to find the magnetic field from a line of current 2a long at a point x away from the line along its perpendicular bisector.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

- Starting with our formulation for the magnetic field from a current element, we find a general point on the segment and put everything in the formula in terms of variables of our coordinate system.
- Each element of current is in the y-direction and has a length dy.
- The B-field from each element is in the negative z-direction, so we just have to worry about that one component.



Magnetic Field from a Current Segment

- The cross product can then be written as:

$$dB_z = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{Idy\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{Idy\sin(\pi-\phi)}{r^2}$$

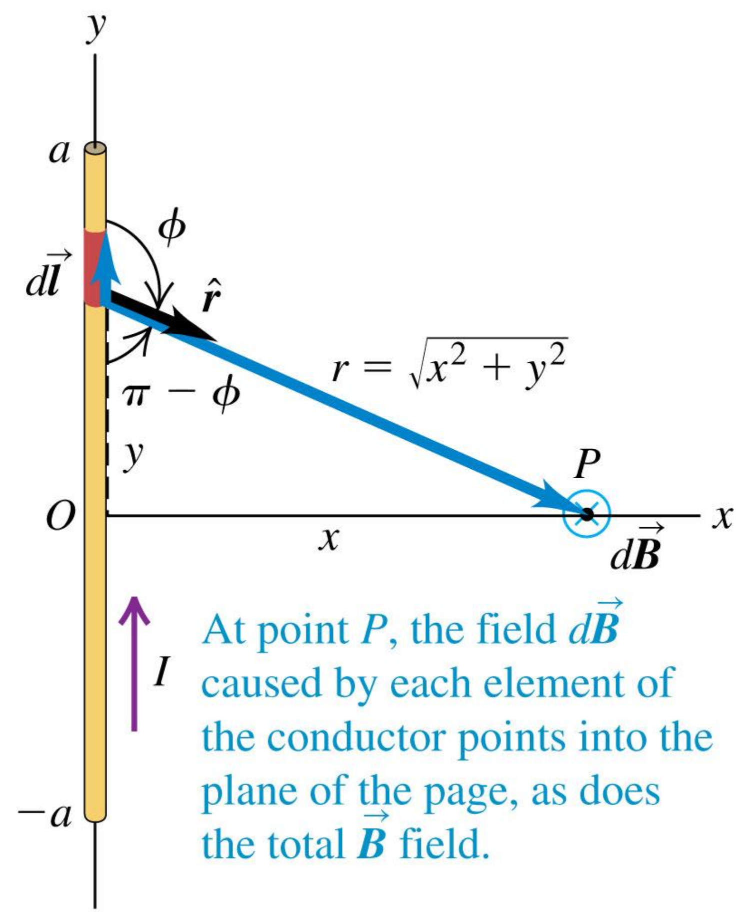
- And then we must put r and the sin function in terms of our coordinate system:

$$dB_z = \frac{\mu_0 I}{4\pi}\frac{\sin(\pi-\phi)}{r^2}dy = \frac{\mu_0 I}{4\pi}\frac{\sin(\pi-\phi)}{\left(x^2+y^2\right)^2}dy$$

$$= \frac{\mu_0 I}{4\pi}\frac{x}{\left(x^2+y^2\right)^{3/2}}dy$$

- And finally, choose our limits of integration and look up the integral:

$$B_z = \int dB_z = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x}{\left(x^2+y^2\right)^{3/2}}dy = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}$$



Magnetic Field from an Infinite Current

- If we take this result for a current segment, and let a go to infinity:

$$B_z = \int dB_z = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x}{\left(x^2+y^2\right)^{3/2}}\,dy = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}} \Rightarrow$$

$$= \lim_{a\to\infty}\frac{\mu_0 I}{4\pi}\frac{2}{x\sqrt{\frac{x^2}{a^2}+1}} = \frac{\mu_0 I}{2\pi x}$$