Lecture 30
(Magnetic Fields from Currents and Loops)

Physics 161-01 Spring 2012

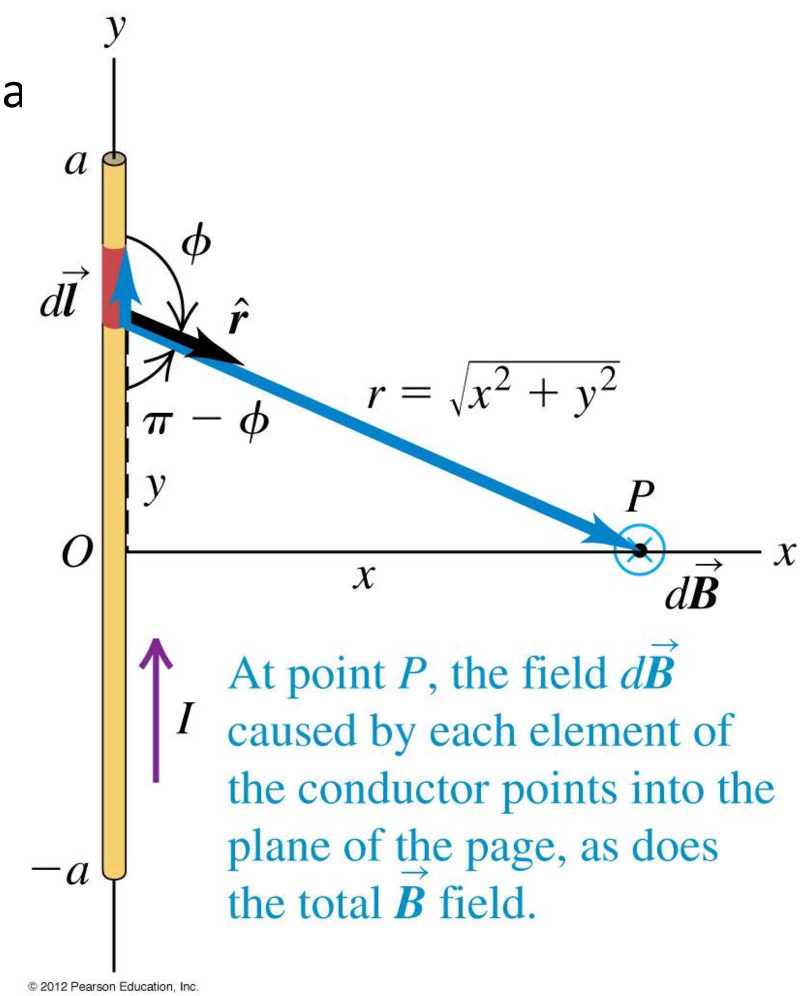
Douglas Fields

Magnetic Field from a Current Segment

- Calculus again!
- We want to find the magnetic field from a line of current 2a long at a point x away from the line along its perpendicular bisector.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

- Starting with our formulation for the magnetic field from a current element, we find a general point on the segment and put everything in the formula in terms of variables of our coordinate system.
- Each element of current is in the y-direction and has a length dy.
- The B-field from each element is in the negative z-direction, so we just have to worry about that one component.



Magnetic Field from a Current Segment

- The cross product can then be written as:

$$dB_z = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{Idy\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{Idy\sin(\pi-\phi)}{r^2}$$

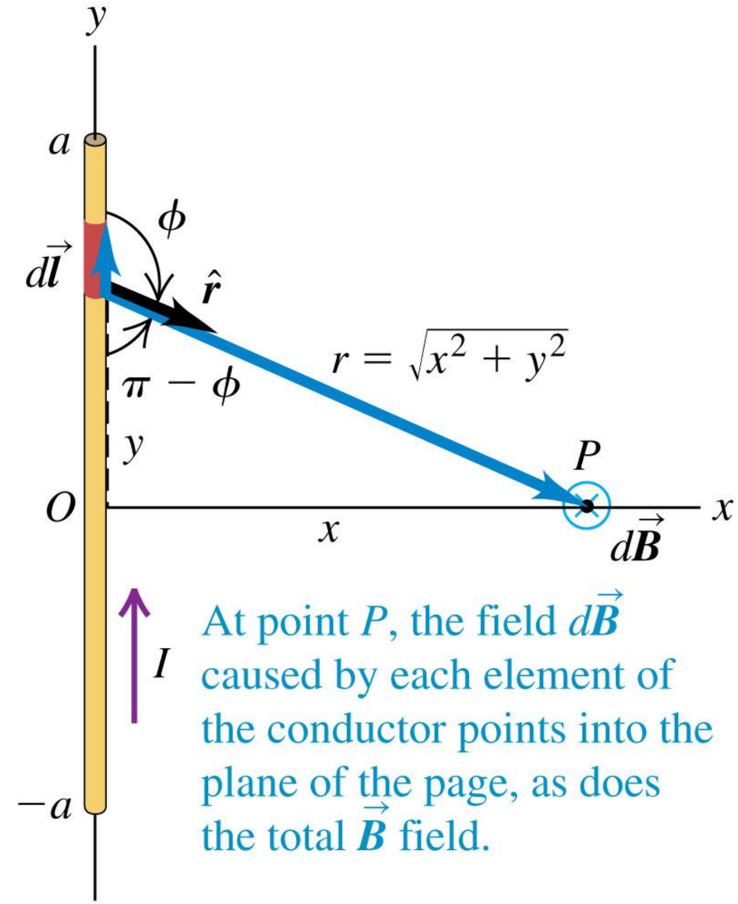
- And then we must put r and the sin function in terms of our coordinate system:

$$dB_z = \frac{\mu_0 I}{4\pi}\frac{\sin(\pi-\phi)}{r^2}dy = \frac{\mu_0 I}{4\pi}\frac{\sin(\pi-\phi)}{\left(x^2+y^2\right)^2}dy$$

$$= \frac{\mu_0 I}{4\pi}\frac{x}{\left(x^2+y^2\right)^{3/2}}dy$$

- And finally, choose our limits of integration and look up the integral:

$$B_z = \int dB_z = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x}{\left(x^2+y^2\right)^{3/2}}dy = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}$$

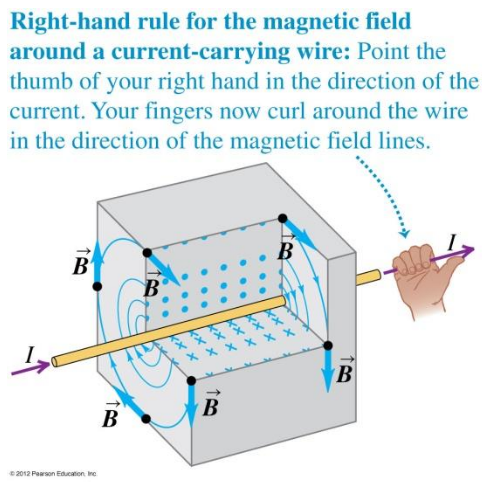


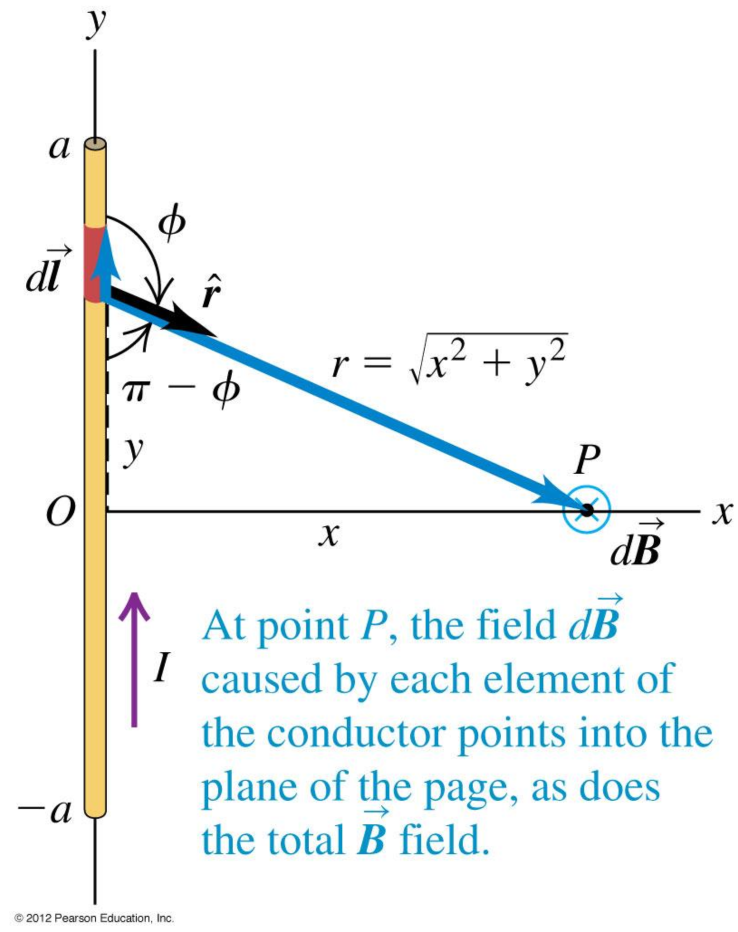
Magnetic Field from an Infinite Current

- If we take this result for a current segment, and let a go to infinity:

$$B_z = \int dB_z = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x}{\left(x^2+y^2\right)^{3/2}}dy = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}} \Rightarrow$$

$$= \lim_{a\to\infty}\frac{\mu_0 I}{4\pi}\frac{2}{x\sqrt{\frac{x^2}{a^2}+1}} = \frac{\mu_0 I}{2\pi x}$$





CPS 30-1

A long straight wire lies along the *y*-axis and carries current in the positive *y*-direction.

A positive point charge moves along the *x*-axis in the positive *x*-direction. The magnetic force that the wire exerts on the point charge is in

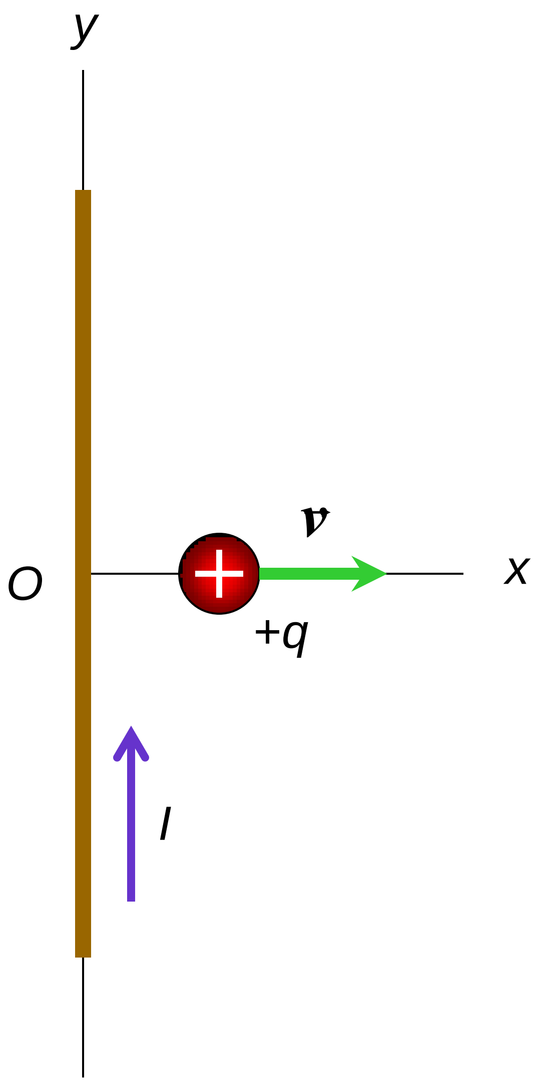
A. the positive *x*-direction.

B. the negative *x*-direction.

C. the positive *y*-direction.

D. the negative *y*-direction.

E. none of the above



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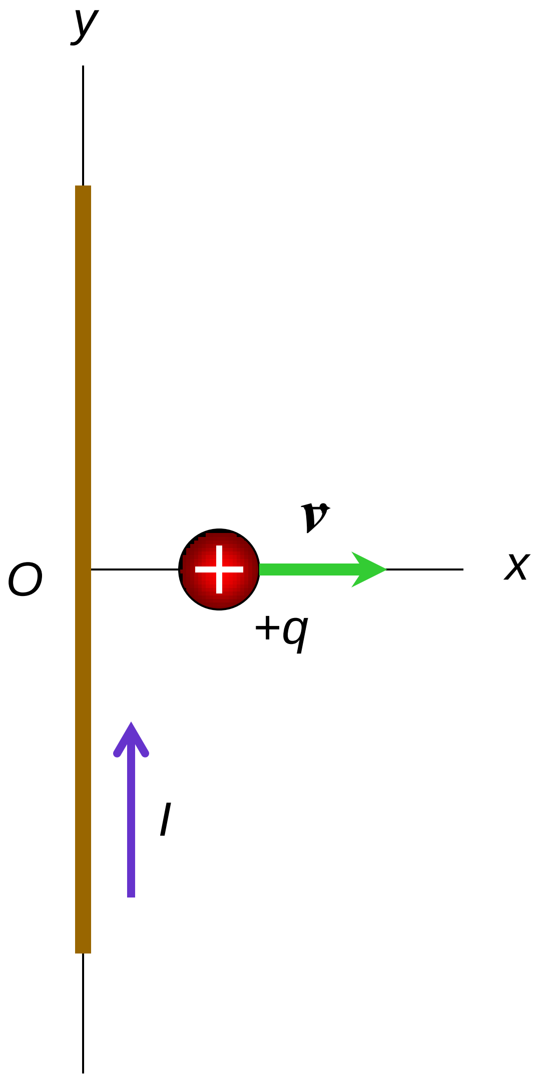
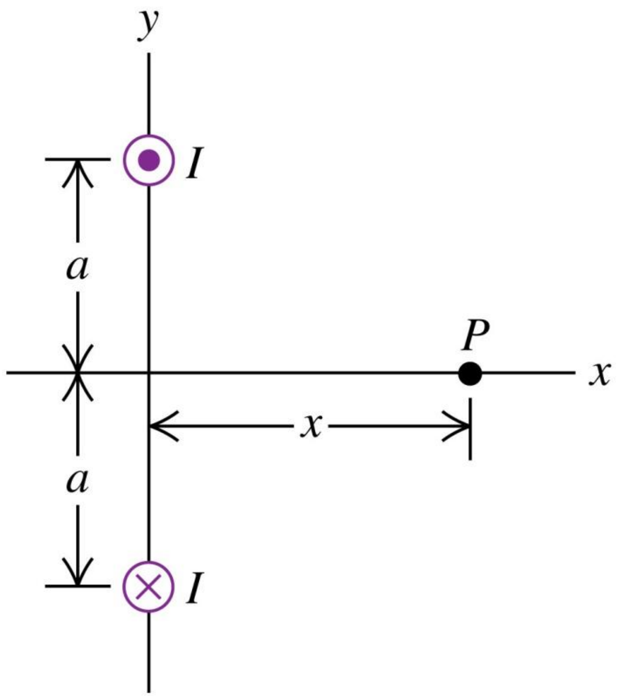


CPS 30-2

Two long, straight wires are oriented perpendicular to the *xy*-plane. They carry currents of equal magnitude *I* in opposite directions as shown. At point *P*, the magnetic field due to these currents is in



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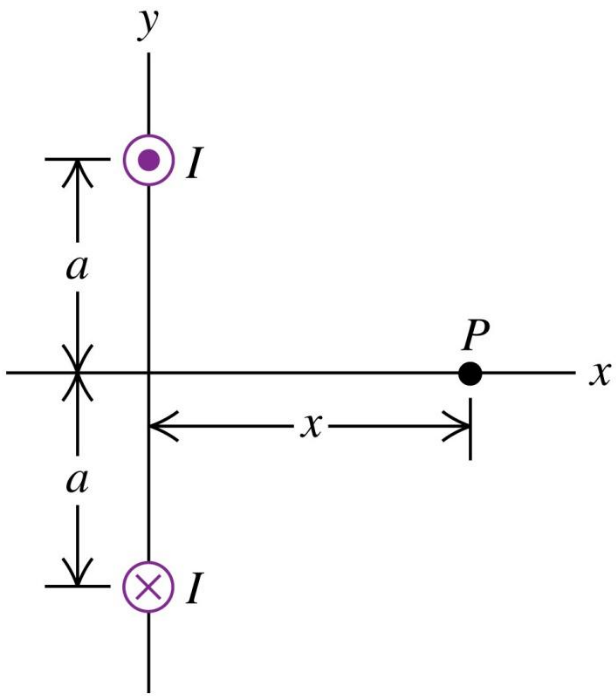

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Magnetic Field from a Current Loop

- Calculus once again…

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2}$$

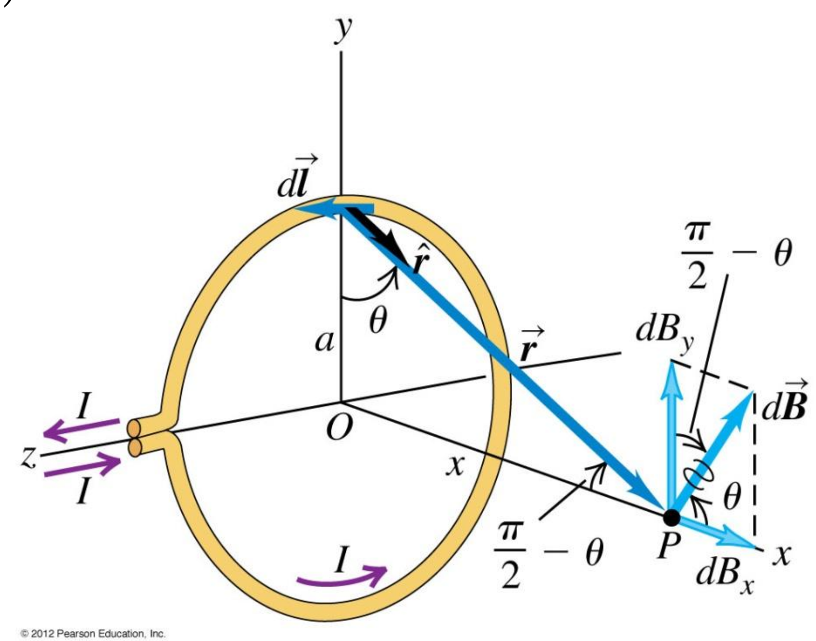
$$dB_x = \frac{\mu_0}{4\pi}\frac{Idl}{\left(x^2 + a^2\right)}\cos\theta = \frac{\mu_0}{4\pi}\frac{Idl}{\left(x^2 + a^2\right)}\frac{a}{\sqrt{x^2 + a^2}}$$

$$= \frac{\mu_0}{4\pi}\frac{Ia}{\left(x^2 + a^2\right)^{3/2}}dl$$

- But simple:

$$B_x = \frac{\mu_0}{4\pi}\frac{Ia}{\left(x^2 + a^2\right)^{3/2}}\int_{\text{Around circle}} dl$$

$$= \frac{\mu_0}{4\pi}\frac{Ia}{\left(x^2 + a^2\right)^{3/2}}2\pi a = \frac{\mu_0 I a^2}{2\left(x^2 + a^2\right)^{3/2}}$$



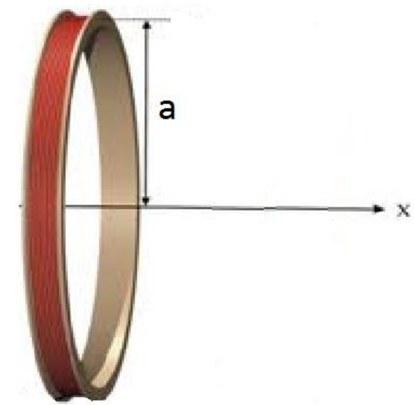
Magnetic Field from a Coil

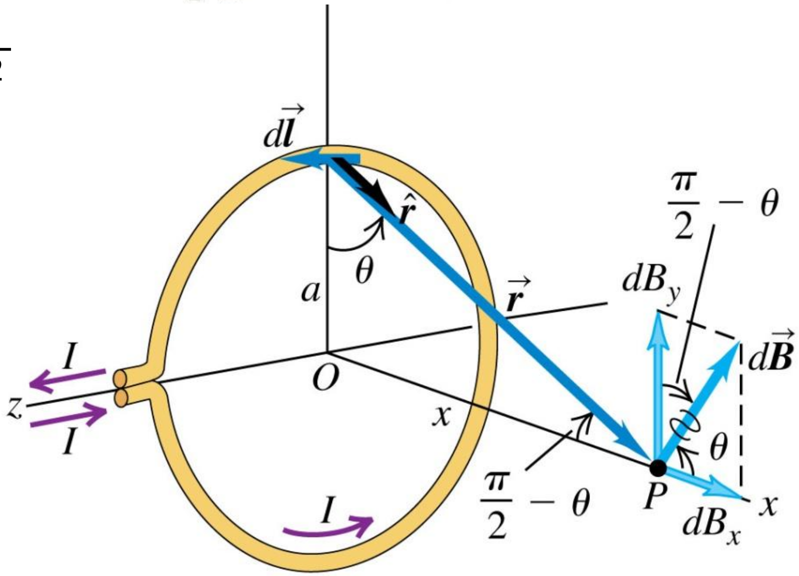
- If, instead of one turn, there were N turns (closely spaced so that they all were distance x from the point that we want the field), then:

$$B_x = \frac{\mu_0}{4\pi}\frac{Ia}{\left(x^2+a^2\right)^{3/2}}\int_{\text{Entire Length of wire}} dl$$

$$= \frac{\mu_0}{4\pi}\frac{Ia}{\left(x^2+a^2\right)^{3/2}}\left(N2\pi a\right) = \frac{\mu_0 INa^2}{2\left(x^2+a^2\right)^{3/2}}$$

- Or, just N times the field of one winding, as expected.

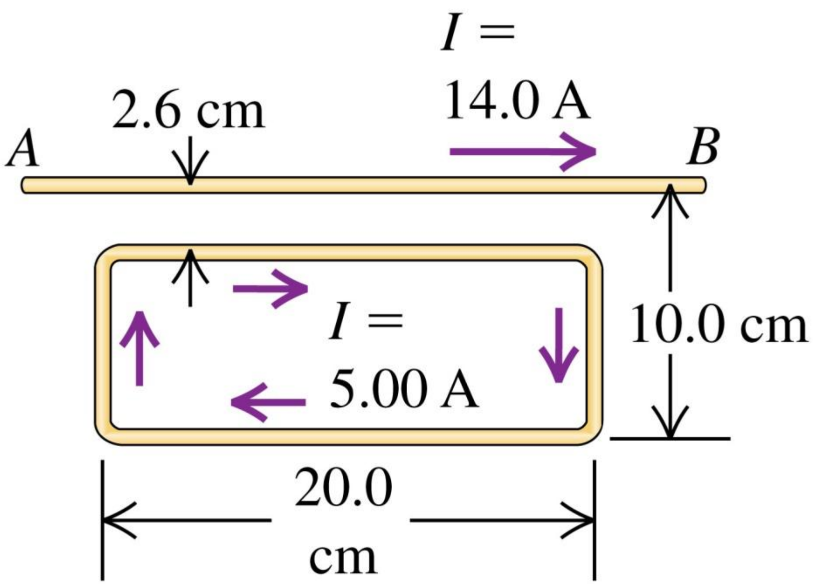




CPS 30-3

The long, straight wire *AB* carries a 14.0-A current as shown. The rectangular loop has long edges parallel to *AB* and carries a clockwise 5.00-A current.

What is the direction of the net magnetic force that the straight wire *AB* exerts on the loop?



A. to the right

B. to the left

C. upward (toward *AB*)

D. downward (away from *AB*)

E. misleading question—the net magnetic force is zero

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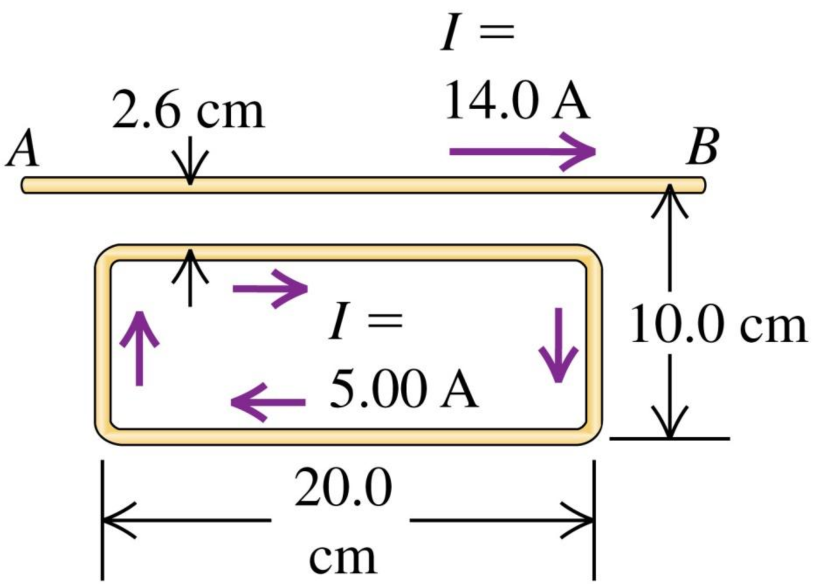

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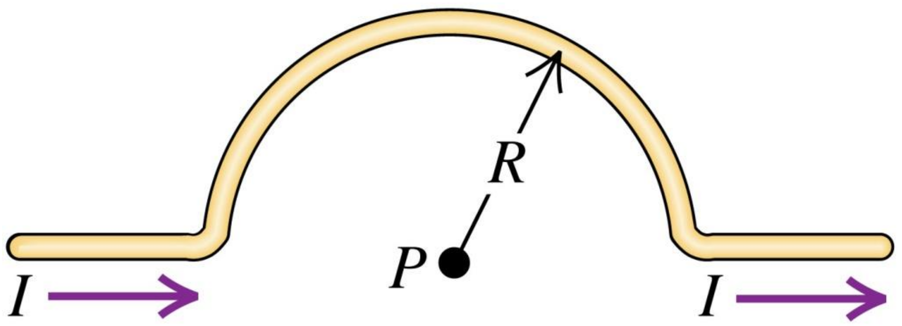
CPS 30-3

A wire consists of two straight sections with a semicircular section between them. If current flows in the wire as shown, what is the direction of the magnetic field at *P* due to the current?



A. to the right

B. to the left

C. out of the plane of the figure

D. into the plane of the figure

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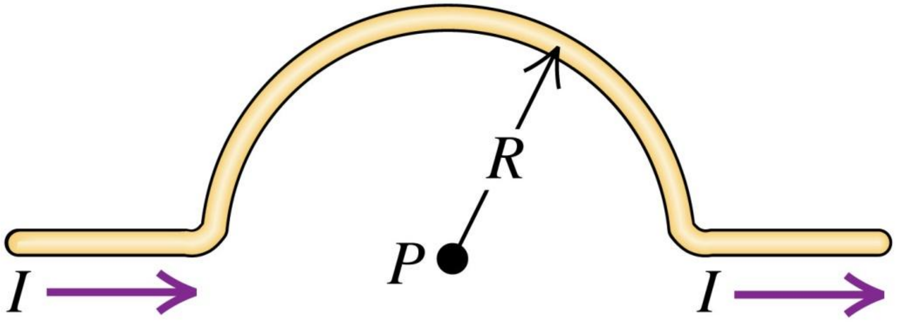

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Properties of the Magnetic Field

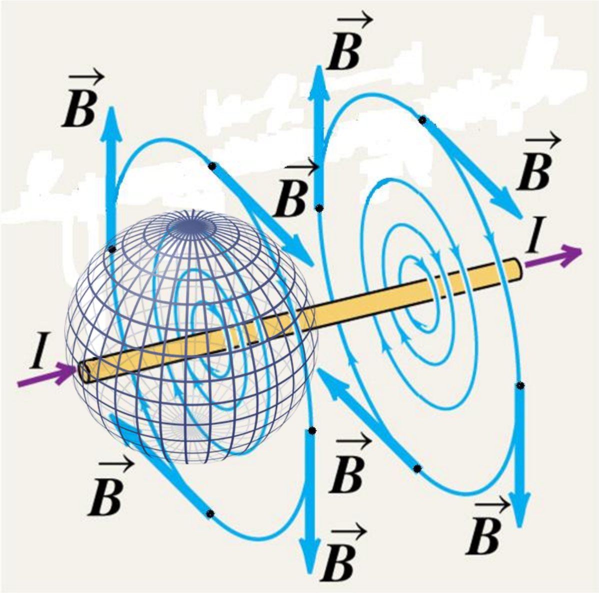
- Remember that we discovered that the formulation of Gauss's Law for magnetic fields is just:

$$\oint \vec{B} \cdot d\vec{A} = 0$$

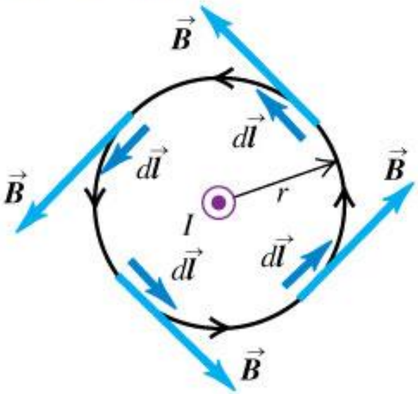
- Another important property of the magnetic field can be seen by noticing that the magnetic field lines always form loops around currents:

$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow$$

$$\int_{\text{Closed Path}} \vec{B} \cdot d\vec{l} = \int_0^{2\pi} \frac{\mu_0 I}{2\pi r} r\, d\theta = \mu_0 I$$



Result: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

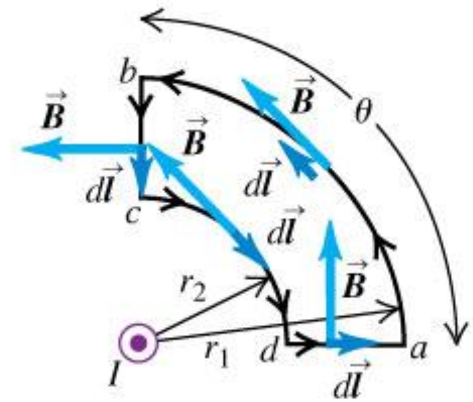


Properties of the Magnetic Field

- What if the path doesn’t enclose the current?

(c) An integration path that does not enclose the conductor

Result: $\oint \vec{B} \cdot d\vec{l} = 0$



$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow$$

$$\int\limits_{\text{Closed Path}} \vec{B} \cdot d\vec{l} = \int\limits_{a\to b} \vec{B} \cdot d\vec{l} + \cancel{\int\limits_{b\to c} \vec{B} \cdot d\vec{l}} + \int\limits_{c\to d} \vec{B} \cdot d\vec{l} + \cancel{\int\limits_{d\to a} \vec{B} \cdot d\vec{l}}$$

$$\int\limits_{\text{Closed Path}} \vec{B} \cdot d\vec{l} = \int_0^{\pi/2} \frac{\mu_0 I}{2\pi r} r d\theta + \int_{\pi/2}^{0} \frac{-\mu_0 I}{2\pi r} r(-d\theta)$$

$$\int\limits_{\text{Closed Path}} \vec{B} \cdot d\vec{l} = \frac{\mu_0 I}{4} - \frac{\mu_0 I}{4} = 0$$