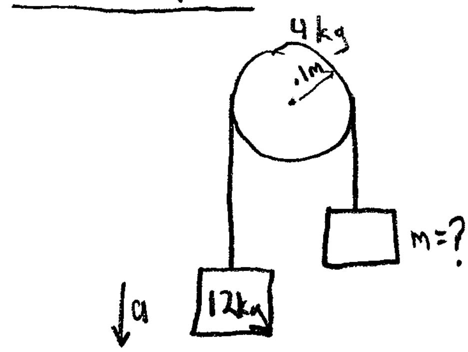
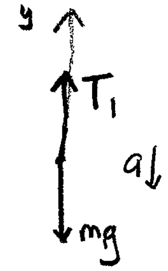
1. Torque



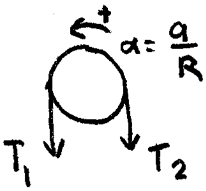
12kg block:



$$\Sigma F_y = T_1 - m_1 g = -m_1 a$$

$$\therefore T_1 = m_1 g - m_1 a$$

4kg pulley:



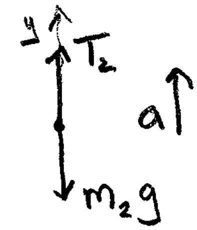
$$\Sigma \tau_z = T_1 R - T_2 R = I\alpha = \frac{1}{2} M_p R^2 \frac{a}{R}$$

$$\Rightarrow T_1 - T_2 = \frac{1}{2} M_p a$$

$$\therefore T_2 = T_1 - \frac{1}{2} M_p a$$

$$= m_1 g - m_1 a - \frac{1}{2} M_p a$$

Unknown mass:



$$\Sigma F_y = T_2 - m_2 g = m_2 a$$

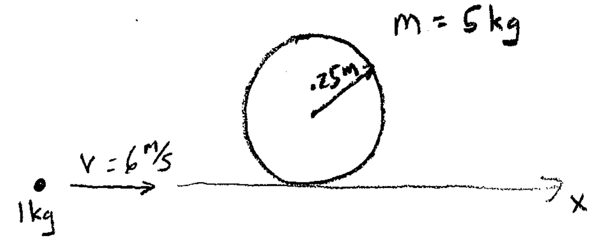
$$T_2 = m_2 (g + a)$$

$$\therefore m_2 = \frac{T_2}{g+a} = \frac{m_1 g - m_1 a - \frac{1}{2} M_p a}{g + a}$$

$$= \frac{(12 kg)(9.8 \, m/s^2 - 1.0 \, m/s^2) - \frac{1}{2}(4 kg)(1.0 \, m/s^2)}{(9.8 \, m/s^2 + 1.0 \, m/s^2)}$$

$$= 9.6 \, kg$$

2. Angular Momentum



Linear momentum is conserved:

$$m_c \vec{V}_c = (M_D + M_c)\vec{V}_{D+C}$$

$$(1\,kg)(6.0\,\tfrac{m}{s}\,\hat{i}) = (1\,kg + 5\,kg)\vec{V}_{D+C}$$

$$\boxed{\vec{V}_{D+C} = 1\,\tfrac{m}{s}\,\hat{i}}$$

Angular momentum is conserved:

The tricky part is deciding which axis of rotation to use!

If you use the center of the disk:

$$L_i = R m_c V_c = L_f = I^{CD}_{C+D}\,\omega_{C+D} + R_{cm}(m_D + m_c)V_{D+C}$$

The second term in L_f is due to the angular momentum taken out by the disk + clay center of mass motion about the axis of the disk.

with $R_{cm} = \dfrac{m_c R}{m_c + M_D}$, we have

$$R m_c V_c = I^{CD}_{C+D}\,\omega_{C+D} + \frac{m_c R}{\cancel{m_c + m_D}}\cancel{(m_c + m_D)}\,V_{D+C}$$

$$\therefore\ \omega_{D+C} = \frac{R m_c V_c - R m_c V_{D+C}}{I^{CD}_{C+D}} = \frac{R m_c V_c\left(\frac{m_D}{m_c + m_D}\right)}{I^{CD}_{C+D}}$$

with $I^{CD}_{C+D} = \frac{1}{2}M_D R^2 + m_c R^2$ then

$$\omega_{D+C} = \frac{m_c V_c\left(\frac{m_D}{m_c + m_D}\right)}{\frac{R}{2}(m_D + 2m_c)}$$

$$= \frac{(1\,kg)(6\,\tfrac{m}{s})\left(\frac{5\,kg}{6\,kg}\right)}{(.125\,m)(7\,kg)} = \boxed{5.7\ \tfrac{rad}{s} = \omega_{D+C}}$$

But, that is not the correct way (although that way will be accepted for full credit). The reason is this: the axis of rotation after the collision is the center of mass! Let's do it that way:

$$L_i = (R - R_{cm})\, m_c V_c$$

$$L_f = I^{cm}_{D+c}\, \omega_{D+c}$$

$$\therefore \quad R\left(\frac{m_D}{m_c+m_D}\right) m_c V_c = I^{cm}_{D+c}\, \omega_{D+c}$$

Now,
$$I^{cm}_{D+c} = \tfrac{1}{2} m_D R^2 + m_D R_{cm}^2 + m_c (R - R_{cm})^2$$

$$= \frac{1}{2} m_D R^2 + \frac{m_D m_c^2}{(m_c+m_D)^2} R^2 + \frac{m_c m_D^2}{(m_c+m_D)^2} R^2$$

$$= \frac{m_D R^2}{2(m_c+m_D)^2}\left((m_c+m_D)^2 + 2m_c^2 + 2m_c m_D\right)$$

$$= \frac{m_D R^2}{2(m_c+m_D)^2}\left(3m_c^2 + 4m_c m_D + m_D^2\right)$$

$$= \frac{m_D R^2}{2(m_c+m_D)^2}\left[(m_c+m_D)^2 + 2m_c(m_c+m_D)\right]$$

$$= \frac{R^2}{2}\left[m_D + 2m_c\left(\frac{m_D}{m_c+m_D}\right)\right]$$

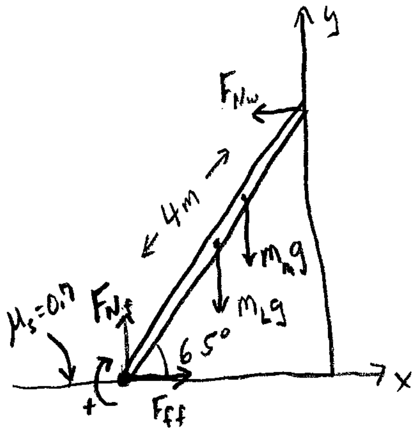
Then,
$$\omega_{D+c} = \frac{R\left(\frac{m_D}{m_c+m_D}\right) m_c V_c}{\frac{R^2}{2}\left[m_D + 2m_c\left(\frac{m_D}{m_c+m_D}\right)\right]}$$

$$= \frac{m_c V_c \left(\frac{m_D}{m_c+m_D}\right)}{\frac{R}{2}\left(m_D + 2m_c\left(\frac{m_D}{m_c+m_D}\right)\right)}$$

$$\omega_{D+c} = \frac{(1\,kg)(6\,\frac{m}{s})\left(\frac{5\,kg}{6\,kg}\right)}{(.125\,m)\left(5\,kg + 2\,kg\left(\frac{5\,kg}{6\,kg}\right)\right)}$$

$$\boxed{\omega_{D+c} = 6\,\frac{rad}{s}}$$

3. Equillibrium



$$\Sigma F_x = F_{ff} - F_{Nw} = 0 \Rightarrow F_{ff} = F_{Nw}$$

$$\Sigma \tau = (m_L g)(2m)\cos 65° + (m_m g)(3m)\cos 65° - F_{Nw}(4m)\sin 65° = 0$$

at maximum force of friction:

$$F_{ff} = F_{Nf} \cdot \mu_s$$

and

$$\Sigma F_y = F_{Nf} - m_L g - m_m g = 0 \Rightarrow$$

$$F_{Nf} = (m_L + m_m) g$$

$$\therefore \quad F_{ff} = (m_L + m_m) g \cdot \mu_s = F_{Nw}$$

$$\therefore \quad (m_L g)(2m)\cos 65° + (m_m g)(3m)\cos 65° - (m_L + m_m) g \mu_s (4m)\sin 65° = 0$$

$$m_m\left[(g)(3m)\cos 65° - (g)\mu_s(4m)\sin 65°\right] = m_L g \mu_s (4m)\sin 65° - m_L g(2m)\cdot\cos 65°$$

$$m_m = \frac{m_L \cancel{g}\left[\mu_s(4m)\sin 65° - (2m)\cos 65°\right]}{\cancel{g}(3m)\cos 65° - \cancel{g}\mu_s(4m)\sin 65°}$$

$$= \frac{20\,kg\left[(0.7)(4m)\sin 65° - (2m)\cos 65°\right]}{(3m)\cos 65° - (0.7)(4m)\sin 65°}$$

$$= -26.6\,kg \;!$$

So, the man never makes it 3m up the ladder...