Scalar Product:

$$\vec{A}\cdot\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\cos\theta_{AB}=A_xB_x+A_yB_y+A_zB_z$$

Vector Product:

$$\vec{A}\times\vec{B}=\left(A_yB_z-A_zB_y\right)\hat{i}+\left(A_zB_x-A_xB_z\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$

$$\left|\vec{A}\times\vec{B}\right|=\left|\vec{A}\right|\left|\vec{B}\right|\sin\theta_{AB}$$

Equations of motion:

$$v_y=v_{0y}+a_yt$$

$$x=x_0+v_{0x}t+\frac{1}{2}a_xt^2$$

$$v_x{}^2=v_{0x}{}^2+2a_x(x-x_0)$$

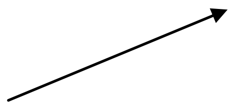
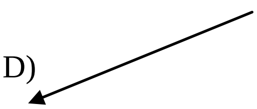
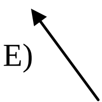
Radial Acceleration:

$$a_{rad}=\frac{v^2}{r}$$

Exam #1 Physics 160-01

Name:______________________________ Box #________________

1) Given the two vectors drawn below, which answer best represents $3\vec{X} - \vec{Y}$?

A)  B)  C)  D)  E) 

2) Find the angle in degrees between the two vectors: $\vec{A} = 3\hat{i} - 4\hat{j} - 6\hat{k}$ and $\vec{B} = 2\hat{i} - 8\hat{j}$.

A) 66.2^o
B) 108^o
C) 53.8^o
D) 1.98^o
E) 114^o

$$\vec{A}\cdot\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = A_xB_x + A_yB_y + A_zB_z \Rightarrow$$

$$\theta = \cos^{-1}\left[\frac{A_xB_x + A_yB_y + A_zB_z}{\left|\vec{A}\right|\left|\vec{B}\right|}\right]$$

$$= \cos^{-1}\left[\frac{3\cdot(2) + (-4)\cdot(-8) + (-6)\cdot(0)}{\sqrt{(3)^2 + (-4)^2 + (-6)^2}\sqrt{(2)^2 + (-8)^2 + (0)^2}}\right]$$

$$= \cos^{-1}\left[\frac{38}{64.4}\right]$$

$$= 53.8^o$$

3) An arrow is shot horizontally (in the positive x-direction) from the top of a building at a speed of 39.6 m/s. The arrow strikes the ground at a point 80m horizontally from the base of the building. What is the height of the building?

A) 20 m
B) 30 m
C) 40 m
D) 10 m
E) 60 m

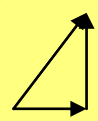
This is a 2-D problem and must be analyzed in each dimension.
In the x- direction,
x_o=0m,
x_f=80m,
v_{ox}=?,
$v_{fx}=v_{ox}$=39.6m/s,
a_x=0m/s^2,
t=?.
In the y- direction,
y_o=?,
y_f=0m,
v_{oy}=0m/s,
v_{fy}=?,
a_y=-9.8m/s^2,
t=?.
To get the initial height we need to know the time (since the velocity in the x-direction is constant, and we know the distance), so look in the x-direction, we use $x_f=x_o+v_{ox}t+1/2a_xt^2$, with $a_x=0$ => t=2.02s.

Then in the y-direction and use $y_f=y_o+v_{oy}t+1/2a_yt^2$ => y_0=20m.

4) A person is swimming across a river that is 200 m wide. They swim at a constant speed relative to the water of 0.2 m/s and in a direction straight across the river (perpendicular to the flow of water). When they reach the opposite shore, they notice that they have drifted 300 m downstream. What was the speed of the swimmer relative to the ground?

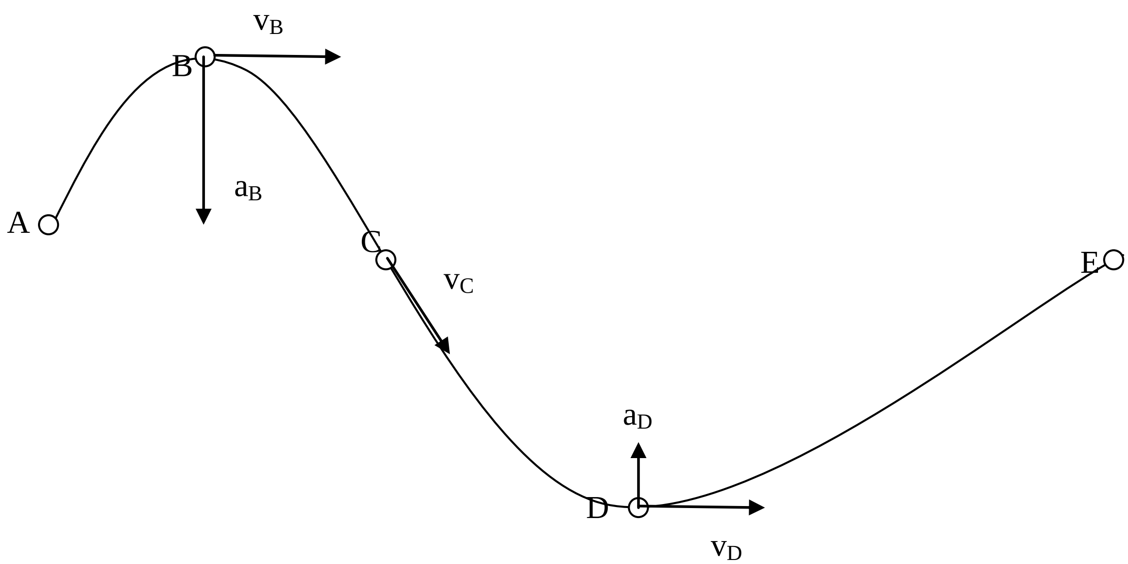
A) 0.20 m/s
B) 0.50 m/s
C) 0.36 m/s
D) 0.18 m/s
E) 0.26 m/s

The swimmer covers the 200 m in 1000 s. In that same time, the river brings them downstream 300 m, so the river is flowing at 0.3 m/s. The speed of the swimmer relative to the ground is just the vector sum of the velocities, so the magnitude of that is just sqrt($(0.2\text{ m/s})^2+(0.3\text{ m/s})^2$) = 0.36 m/s.

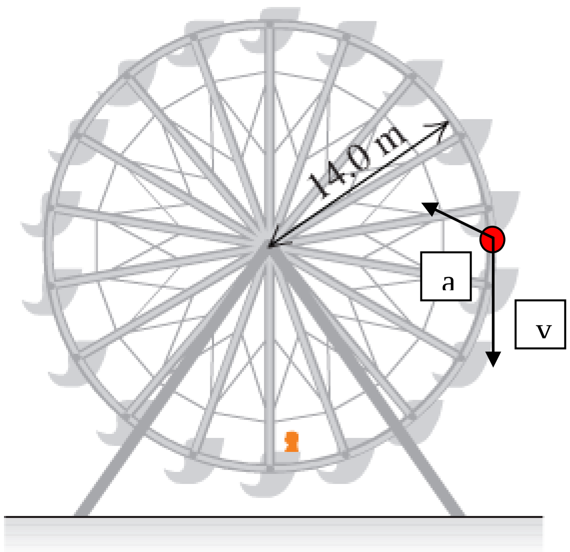


An object moves along the track shown in the top-view diagram below. The object moves from point A to point E with constant speed.

5) Draw arrows **on the diagram** to represent the direction *and* relative magnitude of the instantaneous velocity *and* acceleration of the object at points B, C, and D.

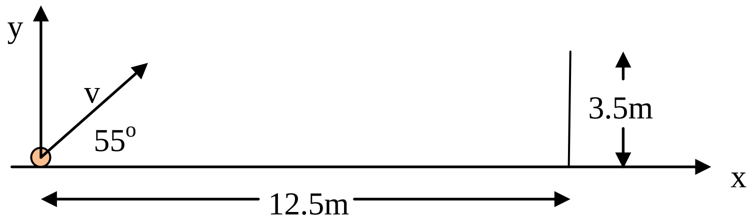


6) A person riding on a Ferris Wheel of radius 14.0 m at some moment has a tangential speed of 2 m/s (rotating clockwise) and is slowing down at a rate of 0.1 m/s^2. At the middle point on the right, indicated by the circle, draw arrows indicating the direction of his velocity and acceleration and determine the magnitude and exact direction of his acceleration vector.



Since the wheel is speeding up, the acceleration of the rider is always given by a radial component, $a_R = v^2/r = 0.28 m/s^2$ and a tangential component, $0.1 m/s^2$. So, the magnitude of the acceleration vector is sqrt($(0.28\ m/s^2)^2 + (0.1\ m/s^2)^2$) = $0.30 m/s^2$ and points to the center of the wheel and up at an angle of $\tan^{-1}(0.1/0.28) = 20^o$ above the horizontal.

7) A child wants to kick a ball a horizontal distance of 12.5 m over a fence 3.5 m high. They kick the ball at an angle of 55° above the horizontal. At what speed should they kick the ball so that it ***just*** passes over the fence?



y_o=0m,
y_f=3.5m,
v_{oy}=vsin55 m/s,
v_{fy}=?,
a_y=-9.8m/s^2,
t=?.

and

x_o=0m,
x_f=12.5m,
v_{ox}=vcos55 m/s,
v_{fx}= " ,
a_x=0 m/s^2,
t=?.

From the x-data, we can get that 12.5 m = v cos(55) t and then solve for t and substitute back into the equation of motion in the y-direction:

$$t = \frac{12.5m}{v\cos(55)}$$

$$y_f = y_0 + v\sin(55)t + \frac{1}{2}\left(-9.8\frac{m}{s^2}\right)t^2 \Rightarrow$$

$$3.5m = 0m + v\sin(55)\left(\frac{12.5m}{v\cos(55)}\right) - 4.9\frac{m}{s^2}\left(\frac{12.5m}{v\cos(55)}\right)^2 \Rightarrow$$

$$3.5m - 12.5m\tan(55) = -4.9\frac{m}{s^2}\left(\frac{12.5m}{v\cos(55)}\right)^2 \Rightarrow$$

$$14.4m = \frac{2327\frac{m^3}{s^2}}{v^2} \Rightarrow v = 12.7\frac{m}{s}$$