Problem Set # 3 Solutions

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[1] $G(s) = \dfrac{5(s+4)}{s^2+10s+50}$

a) zeros: $s = -4$

poles: $s = -5 \pm 5j$ (since $s^2+10s+50 = (s+5)^2+5^2$)

b) $G(-4) = 0$

$G(-5)$ = 1/5 (and G(-5 +/- 5j) is infinite)

c) $A = \begin{bmatrix} 0 & 1 \\ -50 & -10 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 20 & 5 \end{bmatrix}$, $D = 0$

d) $C(sI-A)^{-1}B+D$

$= \begin{bmatrix} 20 & 5 \end{bmatrix} \begin{bmatrix} s & -1 \\ 50 & s+10 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0$

$= \begin{bmatrix} 20 & 5 \end{bmatrix} \dfrac{1}{s^2+10s+50} \begin{bmatrix} s+10 & 1 \\ -50 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \dfrac{1}{s^2+10s+50} \begin{bmatrix} 20 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} = \dfrac{5(s+4)}{s^2+10s+50}$ ✓

$\boxed{2}$ a) $\Delta(s) = 0 = |sI - A|$

$$= \begin{vmatrix} s-2 & 3 & 3 \\ 0 & s-5 & -3 \\ 3 & 5 & s+4 \end{vmatrix}$$

$$= (s-2)\begin{vmatrix} s-5 & -3 \\ 5 & s+4 \end{vmatrix} - 0\begin{vmatrix} 3 & 3 \\ 5 & s+4 \end{vmatrix} + 3\begin{vmatrix} 3 & 3 \\ s-5 & -3 \end{vmatrix}$$

$$= (s-2)\left[(s-5)(s+4)+15\right] + 3\left[-9 - 3(s-5)\right]$$

$$= (s-2)\left[s^2 - s - 5\right] + 3\left[-3s + 6\right]$$

$$0 = s^3 - s^2 - 5s - 2s^2 + 2s + 10 - 9s + 18$$

$$= s^3 - 3s^2 - 12s + 28$$

b) The roots of the characteristic equation (= poles of the transfer function) determine the general form of the transient response. The transient response is a linear combination of exponential decay / growth / oscillations where decay / growth / frequency of oscillations are determined by the location of the poles. For example, a pole at $s = -4$ results in e^{-4t} in transient response.

c) The transient response is a linear combination of exponential decay and ~~oscillatory~~ exponential growth. Unless C is a special value, exponential growth will dominate the transient response eventually.

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% Problem 3

A = [-1.702 50.72 263.38; 0.22 -1.418 -31.99; 0 0 -14];
B = [-272.06 0 14]';
C1 = [1 0 0 ];
C2 = [0 1 0];
D = 0;

[num1, den1]=ss2tf(A,B,C1,D)

num1 =
```

(a)

```
   1.0e+04 *

         0   -0.0272   -0.0507   -2.2888


den1 =

    1.0000   17.1200   34.9350 -122.4295

[num2, den2]=ss2tf(A,B,C2,D)

num2 =
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(b)

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         0         0 -507.7132 -788.9921


den2 =

    1.0000   17.1200   34.9350 -122.4295

eigenvalues = eig(A)
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(c)

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eigenvalues =

   -4.9034
    1.7834
  -14.0000

poles1 = roots(den1)
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(d)

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poles1 =

  -14.0000
   -4.9034
    1.7834

poles2 = roots(den2)

poles2 =

  -14.0000
   -4.9034
    1.7834
```

(f)

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zeros1 = roots(num1)

zeros1 =

  -0.9323 + 9.1246i
  -0.9323 - 9.1246i

zeros2 = roots(num2)

zeros2 =

   -1.5540
```

e) Yes. The eigenvalues of the state matrix are equal to the roots of the denominators of both $G_1(s)$ and $G_2(s)$.

f) $G_1(s)$ has 2 zeros

$G_2(s)$ has 1 zero