21.64.

Let $q_1 = +2.50\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge q_1, since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of q_1. Let $+q$ be a distance d to the left of q_1, so it is a distance $d + 0.600\ \text{m}$ from q_2.

$F_1 = F_2$ gives $\dfrac{kq|q_1|}{d^2} = \dfrac{kq|q_2|}{(d + 0.600\ \text{m})^2}$.

$$d = \pm\sqrt{\frac{|q_1|}{|q_2|}}(d + 0.600\ \text{m}) = \pm(0.8452)(d + 0.600\ \text{m}).$$

d must be positive, so $d = \dfrac{(0.8452)(0.600\ \text{m})}{1 - 0.8452} = 3.27\ \text{m}.$

The net force would be zero when $+q$ is at $x = -3.27\ \text{m}.$

When $+q$ is at $x = -3.27\ \text{m},$ $\vec{F}_1$ is in the $-x$ direction and $\vec{F}_2$ is in the $+x$ direction.

21.82.

The electric force on one sphere due to the other is $F_\text{C} = k\dfrac{|q^2|}{r^2}$ in the horizontal direction, the force on it due to the uniform electric field is $F_E = qE$ in the horizontal direction, the gravitational force is mg vertically downward and the force due to the string is T directed along the string. For equilibrium $\sum F_x = 0$ and $\sum F_y = 0.$

(a) The positive sphere is deflected in the same direction as the electric field, so the one that is deflected to the left is positive.

(b) The separation between the two spheres is $2(0.530\ \text{m})\sin 25^\circ = 0.4480\ \text{m}.$

$$F_\text{C} = k\frac{|q^2|}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(72.0\times10^{-9}\ \text{C})^2}{(0.4480\ \text{m})^2} = 2.322\times10^{-4}\ \text{N}.$$

$F_E = qE.$

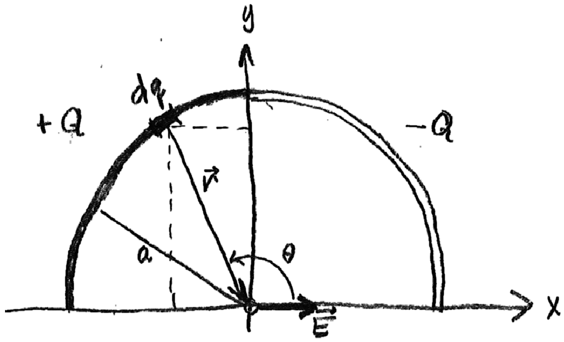
$\sum F_y = 0$ gives $T\cos 25^\circ - mg = 0$ and $T = \dfrac{mg}{\cos 25^\circ}$.

$\sum F_x = 0$ gives $T\sin 25^\circ + F_C - F_\text{E} = 0.$ $mg\tan 25^\circ + F_C = qE.$

Combining the equations and solving for E gives

$$E = \frac{mg\tan 25^\circ + F_\text{C}}{q} = \frac{(6.80\times10^{-6}\ \text{kg})(9.8\ \text{m/s}^2)\tan 25^\circ + 2.322\times10^{-4}\ \text{N}}{72.0\times10^{-9}\ \text{C}} = 3.66\times10^3\ \text{N/C}.$$

98.

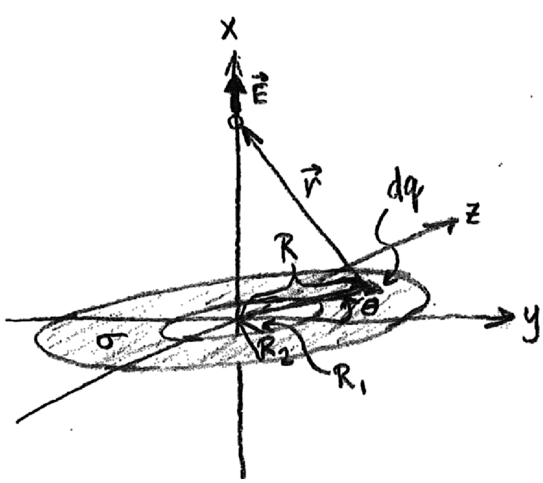


We can immediately predict that $\vec{\boldsymbol{E}}$ will be in the $+\hat{\boldsymbol{\imath}}$ direction.

$\vec{\boldsymbol{r}}$ points from the point on the semicircle, *to* the origin, so $\vec{\boldsymbol{r}} = -a\cos\theta\,\hat{\boldsymbol{\imath}} - a\sin\theta\,\hat{\boldsymbol{\jmath}}$, $r = a$, and $\hat{\boldsymbol{r}} = -\cos\theta\,\hat{\boldsymbol{\imath}} - \sin\theta\,\hat{\boldsymbol{\jmath}}$.

$$\begin{aligned}
\vec{\boldsymbol{E}} &= \int \frac{1}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{r}}}{r^2}\,\mathrm{d}q = \int_0^\pi \frac{-\cos\theta\,\hat{\boldsymbol{\imath}} - \sin\theta\,\hat{\boldsymbol{\jmath}}}{4\pi\epsilon_0 a^2}\lambda\,a\,\mathrm{d}\theta \\
&= -\frac{1}{4\pi\epsilon_0 a}\int_0^\pi (\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}})\,\lambda\,\mathrm{d}\theta \\
&= -\frac{1}{4\pi\epsilon_0 a}\int_0^{\pi/2} (\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}})\left(\frac{-Q}{\pi a/2}\right)\mathrm{d}\theta - \frac{1}{4\pi\epsilon_0 a}\int_{\pi/2}^{\pi} (\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}})\left(\frac{Q}{\pi a/2}\right)\mathrm{d}\theta \\
&= \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2} (\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}})\,\mathrm{d}\theta - \frac{Q}{2\pi^2\epsilon_0 a^2}\int_{\pi/2}^{\pi} (\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}})\,\mathrm{d}\theta \\
&= \left[\frac{Q}{2\pi^2\epsilon_0 a^2}(\sin\theta\,\hat{\boldsymbol{\imath}} - \cos\theta\,\hat{\boldsymbol{\jmath}})\right]_{\theta=0}^{\pi/2} - \left[\frac{Q}{2\pi^2\epsilon_0 a^2}(\sin\theta\,\hat{\boldsymbol{\imath}} - \cos\theta\,\hat{\boldsymbol{\jmath}})\right]_{\theta=\pi/2}^{\pi} \\
&= \frac{Q}{2\pi^2\epsilon_0 a^2}[(\hat{\boldsymbol{\imath}} - 0\,\hat{\boldsymbol{\jmath}}) - (0\,\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}})] - \frac{Q}{2\pi^2\epsilon_0 a^2}[(0\,\hat{\boldsymbol{\imath}} - (-1)\,\hat{\boldsymbol{\jmath}}) - (\hat{\boldsymbol{\imath}} - 0\,\hat{\boldsymbol{\jmath}})] \\
&= \frac{Q}{2\pi^2\epsilon_0 a^2}(\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}) - \frac{Q}{2\pi^2\epsilon_0 a^2}(-\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}) = \boxed{\frac{Q}{\pi^2\epsilon_0 a^2}\,\hat{\boldsymbol{\imath}}}.
\end{aligned}$$

104.



a) $Q = \sigma A = \sigma(\pi R_2^2 - \pi R_1^2) = \boxed{\pi\sigma(R_2^2 - R_1^2)}$

b) Using symmetry, we predict that, on the x-axis, $\vec{\boldsymbol{E}}$ will point in the $\pm\hat{\boldsymbol{\imath}}$ direction. Let R be the distance of a point from the x-axis. Then $\mathrm{d}q = \sigma\,\mathrm{d}A = \sigma\,R\,\mathrm{d}\theta\,\mathrm{d}R$. $\vec{\boldsymbol{r}} = x\,\hat{\boldsymbol{\imath}} - R\cos\theta\,\hat{\boldsymbol{\jmath}} - R\sin\theta\,\hat{\boldsymbol{k}}$, $r = |\vec{\boldsymbol{r}}\,| = \sqrt{x^2 + R^2\cos^2\theta + R^2\sin^2\theta} = \sqrt{x^2+R^2}$, $\hat{\boldsymbol{r}} = \vec{\boldsymbol{r}}\,/r$.

$$\mathrm{d}\vec{\boldsymbol{E}} = \frac{1}{4\,\pi\,\epsilon_0}\frac{\hat{\boldsymbol{r}}}{r^2}\,\mathrm{d}q = \frac{1}{4\,\pi\,\epsilon_0}\frac{\vec{\boldsymbol{r}}}{r^3}\,\mathrm{d}q = \frac{\sigma}{4\,\pi\,\epsilon_0}\frac{R\left(x\,\hat{\boldsymbol{\imath}} - R\cos\theta\,\hat{\boldsymbol{\jmath}} - R\sin\theta\,\hat{\boldsymbol{k}}\right)}{(x^2+R^2)^{3/2}}\,\mathrm{d}\theta\,\mathrm{d}R$$

$$\begin{aligned}
\vec{\boldsymbol{E}} &= \frac{\sigma}{4\,\pi\,\epsilon_0}\int_{R_1}^{R_2}\int_0^{2\pi}\frac{R\left(x\,\hat{\boldsymbol{\imath}} - R\cos\theta\,\hat{\boldsymbol{\jmath}} - R\sin\theta\,\hat{\boldsymbol{k}}\right)}{(x^2+R^2)^{3/2}}\,\mathrm{d}\theta\,\mathrm{d}R \\
&= \frac{\sigma}{4\,\pi\,\epsilon_0}\int_{R_1}^{R_2}\left.\frac{R\left(x\,\theta\,\hat{\boldsymbol{\imath}} - R\sin\theta\,\hat{\boldsymbol{\jmath}} + R\cos\theta\,\hat{\boldsymbol{k}}\right)}{(x^2+R^2)^{3/2}}\right|_{\theta=0}^{2\pi}\mathrm{d}R \\
&= \frac{\sigma}{4\,\pi\,\epsilon_0}\int_{R_1}^{R_2}\frac{2\,\pi\,x\,R\,\hat{\boldsymbol{\imath}}}{(x^2+R^2)^{3/2}}\,\mathrm{d}R = \frac{\sigma\,x}{4\,\epsilon_0}\int_{R_1}^{R_2}\frac{2\,R\,\mathrm{d}R}{(x^2+R^2)^{3/2}}\,\hat{\boldsymbol{\imath}} \\
&= \frac{\sigma\,x}{4\,\epsilon_0}\frac{1}{(-1/2)}\left.\frac{\hat{\boldsymbol{\imath}}}{\sqrt{x^2+R^2}}\right|_{R=R_1}^{R_2} = \boxed{\frac{\sigma\,x}{2\,\epsilon_0}\left(\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}}\right)\hat{\boldsymbol{\imath}}}.
\end{aligned}$$

This is in the $+\hat{\boldsymbol{\imath}}$ direction when $x > 0$ and the $-\hat{\boldsymbol{\imath}}$ direction when $x < 0$, as one would expect.

c) If $x \ll R_1$ (and therefore $x \ll R_2$), $\sqrt{x^2+R_1^2} \approx R_1$ and $\sqrt{x^2+R_2^2} \approx R_2$, so

$$\vec{\boldsymbol{E}} \approx \frac{\sigma}{2\,\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)x\,\hat{\boldsymbol{\imath}}.$$

d) The x component of the force on the charge is $F_x = -q\,E_x \approx -q\,\frac{\sigma}{2\,\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)x$. This is the same equation as the force of a spring, with spring constant $k = q\,\frac{\sigma}{2\,\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. We know that this produces oscillations at an angular frequency $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{q\,\sigma}{2\,\epsilon_0\,m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$, or linear frequency $f = \frac{1}{2\,\pi}\sqrt{\frac{q\,\sigma}{2\,\epsilon_0\,m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$.