27.62. **IDENTIFY:** Apply $\vec{F} = q\vec{v} \times \vec{B}$.

SET UP: $\vec{v} = -v\hat{j}$

EXECUTE: **(a)** $\vec{F} = -qv[B_x(\hat{j} \times \hat{q}) + B_y(\hat{j} \times \hat{j}) + B_z(\hat{j} \times \hat{k})] = qvB_x\hat{k} - qvB_z\hat{i}$

(b) $B_x > 0$, $B_z < 0$, sign of B_y doesn't matter.

(c) $\vec{F} = |q|vB_x - |q|vB_x\hat{k}$ and $|\vec{F}| = \sqrt{2}|q|vB_x$

EVALUATE: $\vec{F}$ is perpendicular to $\vec{v}$, so $\vec{F}$ has no y-component.

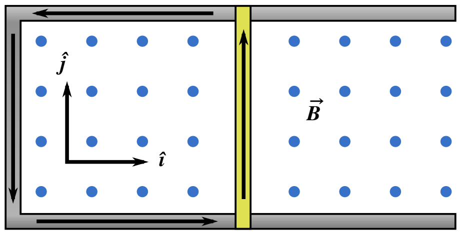
27.72. **IDENTIFY:** Turning the charged loop creates a current, and the external magnetic field exerts a torque on that current.

SET UP: The current is $I = q/T = q/(1/f) = qf = q(\omega/2\pi) = q\omega/2\pi.$ The torque is $\tau = \mu B \sin\phi.$

EXECUTE: In this case, $\phi = 90°$ and $\mu = AB$, giving $\tau = IAB.$ Combining the results for the torque and current and using $A = \pi r^2$ gives $\tau = \left(\frac{q\omega}{2\pi}\right)\pi r^2 B = \frac{1}{2} q\omega r^2 B.$

EVALUATE: Any moving charge is a current, so turning the loop creates a current causing a magnetic force.

74.



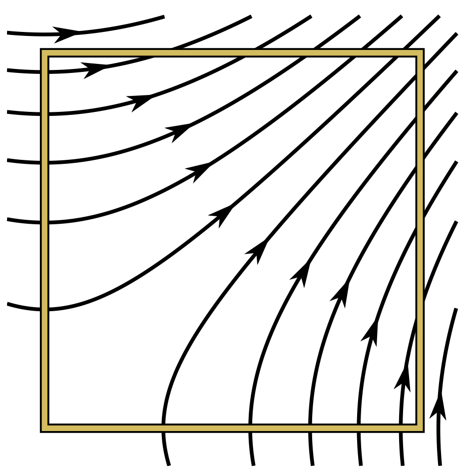
a) $\vec{\boldsymbol{\ell}} = L\,\hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{B}} = B\,\hat{\boldsymbol{k}}$, $\vec{\boldsymbol{F}} = I\,\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{B}} = \boxed{I\,L\,B\,\hat{\boldsymbol{\imath}}}$.

b) $\dfrac{\mathrm{d}\vec{\boldsymbol{v}}}{\mathrm{d}t} = \dfrac{\vec{\boldsymbol{F}}}{m} = \dfrac{I\,L\,B}{m}\,\hat{\boldsymbol{\imath}}$ and $\vec{\boldsymbol{v}}(0) = 0$, so $\vec{\boldsymbol{v}}(t) = \dfrac{I\,L\,B}{m}\,t\,\hat{\boldsymbol{\imath}} = v(t)\,\hat{\boldsymbol{\imath}}$. $d(t) = \displaystyle\int v(t)\,\mathrm{d}t = \dfrac{I\,L\,B}{2\,m}\,t^2$. Since $t = \dfrac{m}{I\,L\,B}\,v(t)$, $d = \dfrac{I\,L\,B}{2\,m}\left(\dfrac{m}{I\,L\,B}\,v\right)^2 = \boxed{\dfrac{m\,v^2}{2\,I\,L\,B}}$.

c) $d = \dfrac{(25.0\,\mathrm{kg})\,(11.2\times10^3\,\mathrm{m/s})^2}{2\,(2.0\times10^3\,\mathrm{A})\,(0.50\,\mathrm{m})\,(0.80\,\mathrm{T})} = 1.96\times10^6\,\mathrm{m} = \boxed{1960\,\mathrm{km}}$.

86.

a)



b) Bottom side: $\vec{\boldsymbol{F}}_b = \displaystyle\int I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{B}} = \int_0^L I\,(-\hat{\boldsymbol{\imath}}\,\mathrm{d}x) \times B_0\,\frac{x}{L}\,\hat{\boldsymbol{\jmath}} = -\frac{I\,B_0}{L}\,\hat{\boldsymbol{k}} \int_0^L x\,\mathrm{d}x = -\frac{1}{2}\,I\,B_0\,L\,\hat{\boldsymbol{k}}$

Left side: $\vec{\boldsymbol{F}}_\ell = \displaystyle\int I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{B}} = \int_0^L I\,(\hat{\boldsymbol{\jmath}}\,\mathrm{d}y) \times B_0\,\frac{y}{L}\,\hat{\boldsymbol{\imath}} = \frac{I\,B_0}{L}\,(-\hat{\boldsymbol{k}}) \int_0^L y\,\mathrm{d}y = -\frac{1}{2}\,I\,B_0\,L\,\hat{\boldsymbol{k}}$

Top side: $\vec{\boldsymbol{F}}_t = \displaystyle\int I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{B}} = \int_0^L I\,(\hat{\boldsymbol{\imath}}\,\mathrm{d}x) \times \left(B_0\,\hat{\boldsymbol{\imath}} + B_0\,\frac{x}{L}\,\hat{\boldsymbol{\jmath}}\right) = \frac{I\,B_0}{L}\,\hat{\boldsymbol{k}} \int_0^L x\,\mathrm{d}x = \frac{1}{2}\,I\,B_0\,L\,\hat{\boldsymbol{k}}$

Right side: $\vec{\boldsymbol{F}}_r = \displaystyle\int I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{B}} = \int_0^L I\,(-\hat{\boldsymbol{\jmath}}\,\mathrm{d}y) \times \left(B_0\,\frac{y}{L}\,\hat{\boldsymbol{\imath}} + B_0\,\hat{\boldsymbol{\jmath}}\right) = \frac{I\,B_0}{L}\,\hat{\boldsymbol{k}} \int_0^L y\,\mathrm{d}y = \frac{1}{2}\,I\,B_0\,L\,\hat{\boldsymbol{k}}$

c) Since the forces are perpendicular to the plane of the loop, the lever arms for the top and bottom sides have length $\frac{L}{2}$ (the distance from the center to the edge of the loop). Since the force on the top side is coming out of the page and the force on the bottom side is going into the page, both sides contribute a torque in the $+x$ direction, which has magnitude $\frac{L}{2}\frac{1}{2}IB_0L = \frac{1}{4}IB_0L^2$ (note that $\vec{\boldsymbol{F}}_b$ and $\vec{\boldsymbol{F}}_t$ have the same magnitude). The torque about the x axis is then $\tau_x = \frac{1}{2}IB_0L^2$.

Since the force on the left and right sides is not uniform, each one also contributes a torque about the x axis. However, the infinitesimal force on each infinitesimal piece of the left side is opposite the force on the piece of the right side directly across from it; therefore the torques on the left and right sides cancel each other.

d) Now we only have to consider the left and right sides, which feel forces of equal magnitude and opposite direction, similarly to part (c). Since the force on the right side is out of the page and the force on the left side is into the page, both sides contribute a torque in the $-y$ direction, with magnitude $\frac{L}{2}\frac{1}{2}IB_0L = \frac{1}{4}IB_0L^2$. Therefore the torque about the y axis is $\tau_y = -\frac{1}{2}IB_0L^2$.

e) It's not really appropriate to say $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$ when there is a nonuniform magnetic field, because $\vec{\boldsymbol{\mu}}$ is a property of the whole loop, so we would need a single $\vec{\boldsymbol{B}}$ value to cross it with, and it isn't clear where in the loop we should calculate $\vec{\boldsymbol{B}}$ to use in this formula. (In this case it might work if you take $\vec{\boldsymbol{B}}$ at the center of the loop, but with a less symmetric loop or a less symmetric $\vec{\boldsymbol{B}}$ field this would not work.)