ECE-340, Spring 2011
Review Questions for Midterm-1

Problem 1. A boat relies on two marine engines for its operation. The probability of the first engine failing during a marine mission is 0.0001 and the probability that the second engine failing is 0.0003. It is known, however, that once the first engine fails, the second engine can fail with probability 0.1.

a) What is the probability that both engines fail during the marine mission?

Let F_1 & F_2 be the events that the 1st & second engines fail, respectively. We know that $P(F_2|F_1) = 0.1$

$P(F_1 \cap F_2) = P(F_2|F_1)P(F_1)$; $P(F_1) = 10^{-4}$

b) Suppose that the second engines fails during the mission, what is the probability that first engine also fails? We want $P(F_1|F_2)$.

$$P(F_1|F_2) = \frac{P(F_2|F_1)P(F_1)}{P(F_2)} \quad ; \quad P(F_2) = 3 \times 10^{-4}$$

c) What is the probability that at least one of the engines fails?

We want $P(F_1 \cup F_2)$.

$$P(F_1 \cup F_2) = P(F_1) + P(F_2) - P(F_1 \cap F_2)$$

d) What is the probability that the first engines fails and the second engine doesn't fail during the mission? We want $P(F_1 \cap F_2^c)$

$$= P(F_2^c|F_1)P(F_1) = (1 - P(F_2|F_1))P(F_1)$$

Problem 2.a Consider a group of 30 people. If everyone shakes hands with everyone else once, how many handshakes take place in total? $\binom{30}{2}$

Problem 2.b You are to randomly form a number consisting of numerals 2, 3, 5 and 1; what is the probability that you form an even number?

Number of ways for even number is 1x3x2x1=6
Total number of ways = 4!=24
Probability of even no. is 6/24

z y x 2

1 choice, 2 choices, 3 choices, one choice

Problem 3 In a class of 30, what is the probability that at least two students have the same birthdate? You must formulate the answer precisely but do not try to obtain a single numeric answer.

Hint: Probability that students have distinct BDs

is

1 - (365)(364)...(365-30+1) / (365)^(30)

Problem 4. An experiment consists of rolling a die twice and recording the two numbers in the order that they are generated. Assume that all outcomes are equally likely.

a) What is the sample space, Ω, for this experiment?

$\Omega = \{(1,1), \ldots, (6,6)\}$

$\Omega^{\#} = 36$

b) How many events can there be at most for this experiment?

$2^{(\Omega^{\#})}$ why?

c) Describe mathematically the event, E_2, that the maximum of the two numbers is 2 and find $\mathsf{P}(E_2)$.

$E_2 = \{(1,2), (2,1), (2,2)\}$

$P(E_2) = \frac{3}{36}$

d) Let F_2 be the event that the minimum of the two numbers is 2. Find $\mathsf{P}(F_2)$.

e) Are the events F_2 and E_2 independent? Justify your answer thoroughly.

Check $P(E_2 \cap F_2) \stackrel{?}{=} P(E_2)P(F_2)$

f) Find $\mathsf{P}(F_2|E_2)$.

Use definition of conditional probability.

Problem 5. In a binary communication system, each transmitted 1 has a probability 0.001 of being converted to 0, and each transmitted 0 is converted to a 1 with probability 0.01. It is also know that the probability of transmitting a 0 is 0.4.

a) Write an appropriate sample space for the experiment consisting of communication one symbol.

Transmitted bit : $\Omega_1 = \{0,1\}$
Received bit : $\Omega_2 = \{0,1\}$

$\Omega = \Omega_1 \times \Omega_2$

b) What is the probability that a zero is received.

Let E_{OR} be the event that "0" is received

$P(E_{OR}) = P(E_{OR}|E_{OT})P(E_{OT}) + P(E_{OR}|E_{1T})P(E_{1T})$

- why?

c) What is the probability that a 1 was transmitted given that a 1 was received?

$P(E_{1T}|E_{1R}) = P(E_{1R}|E_{1T})P(E_{1T}) / P(E_{1R})$

d) Calculate the probability that an error occurs in the reception.

Let E_e be the event that an error occurs.

Hint: $P(E_e) = P(E_e|E_{1T})P(E_{1T}) + P(E_e|E_{OT})P(E_{OT})$

What is $P(E_e|E_{1T})$?

Problem 6. Suppose that the events A and B are independent, what can you say about $\mathsf{P}(A|B)$ and $\mathsf{P}(B|A)$? Justify your answers rigorously. Be clear about any assumptions on any restrictions on the values of $\mathsf{P}(A)$ and $\mathsf{P}(B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,P(B)}{P(B)} = P(A)$$

etc.

Problem 7. A communication channel is known for converting a 0 to a 1 with probability 0.01 while converting a 1 to 0 with the same probability. To enhance robustness, each binary symbol is transmitted 7 times and a majority rule is used to decide upon the reception decision. (For example, the reception of the sequence (1,0,1,1,1,0,1) is announced as 1, etc.).

a) What is the probability of an error in reception given that 1 was sent?

$$\sum_{i=4}^{7} \binom{7}{i} (0.01)^{i} (1-0.01)^{7-i} \quad (\text{why?})$$

b) What is the probability of an error in reception? Do not attempt to reduce your answer to a final number.

See hint in Problem 5(d)

Problem 8. Circle the most correct answer:

(i) Suppose that F_1 and F_2 are events, then

a) $\mathsf{P}(F_1 \cup F_2) = \mathsf{P}(F_1) + \mathsf{P}(F_2)$.

b) $\mathsf{P}(F_1 \cup F_2) = \mathsf{P}(F_1) + \mathsf{P}(F_2)$ only if $F_1 \cap F_2 = \emptyset$.

c) $\mathsf{P}(F_1 \cup F_2) = \mathsf{P}(F_1) + \mathsf{P}(F_2)$ only if F_1 and F_2 are independent.

d) $\mathsf{P}(F_1 \cup F_2) = \mathsf{P}(F_1) + \mathsf{P}(F_2)$ only if F_1 and F_2 are independent and $F_1 \cap F_2 = \emptyset$.

e) None of the above.

(ii) Suppose that F_1 and F_2 are events, then

a) $\mathsf{P}(F_1 \cap F_2) = \mathsf{P}(F_1)\mathsf{P}(F_2)$.

b) $\mathsf{P}(F_1 \cap F_2) = \mathsf{P}(F_1)\mathsf{P}(F_2)$ only if $F_1 \cap F_2 = \emptyset$.

c) $\mathsf{P}(F_1 \cap F_2) = \mathsf{P}(F_1)\mathsf{P}(F_2)$ only if F_1 and F_2 are independent.
d) $\mathsf{P}(F_1 \cap F_2) = \mathsf{P}(F_1)\mathsf{P}(F_2)$ only if F_1 and F_2 are independent and $F_1 \cap F_2 = \emptyset$.
e) $\mathsf{P}(F_1 \cap F_2) = \mathsf{P}(F_1) + \mathsf{P}(F_2) - \mathsf{P}(F_1 \cup F_2)$.
f) c and e.

(iii) Suppose that F and G are subsets of some sample space Ω, and suppose that $F \cap G = G$. Then it must necessarily be true that
a) G and F are disjoint.
b) $\mathsf{P}(G) \leq \mathsf{P}(F)$.
c) $\mathsf{P}(G) < \mathsf{P}(F)$.
d) $\mathsf{P}(G) \geq \mathsf{P}(F)$.
e) None of the above.

Problem 9. Suppose that A and B are events with the property that either $\mathsf{P}(A) = 0$ or $\mathsf{P}(B) = 0$. Show that A and B must be independent.

$P(A \cap B) \leq P(A)$ (why?)
$P(A \cap B) \leq P(B)$ (why?)

$\therefore$ $P(A \cap B) = 0$ (why?). But $P(A)\,P(B) = 0$

Problem 10. Think of a student who is taking three courses in a specific semester.
a) If we define an outcome to be the student's final grades in the three courses at the end of the term, what would be an appropriate sample space, Ω, for this experiment? You may assume that the only letter grades offered are A, B, C, D and F.

$\Omega = \{A, B, C, D, F\}^3$

b) How many elements are in Ω?

c) How many events can there be in total for this experiment?

d) Describe mathematically the event, E_1, that the student receives at least two A's and no grade lower than C.

$\{(A,A,A), (A,A,B), (A,A,C), \ldots\}$
(Complete it)

e) Describe mathematically the event, E_2, that the student receives at least one A and no grade lower than B.

$\{(A,A,A), (A,A,B), \ldots, (A,B,B), \ldots\}$
complete this

f) If $\mathsf{P}(E_1) = 0.7$ and $\mathsf{P}(E_1 \cap E_2) = 0.2$, calculate $\mathsf{P}(E_1 \setminus E_2)$.

minus

$P(E_1 \setminus E_2) = P(E_1 \setminus E_1 \cap E_2)$
$= P(E_1) - P(E_1 \cap E_2)$

Problem 11. A basketball player has a career success rate of 92% in her free throws. During a game, she attempts 6 free throws. Assume that different throws are independent.

a) What is the probability that she has at most one miss.

$\binom{6}{0}(1-0.92)^0 (0.92)^6 + \binom{6}{1}(1-0.92)^1(0.92)^5$

b) What is the probability that she gets exactly four scores.

$\binom{6}{4}(.92)^4(1-0.92)^2$

Problem 12. Two roommates, Carl and Michael, share a single phone line. Their phone-bill history indicates that 12% of Carl's calls are long distance while 24% of Michael's calls are long distance. It is also known that Michael makes 44% of the total calls. If a long distance call is made, what is the probability that it is made by Michael?

E_{LD} = event that long distance call is made
E_C : event that Carl makes a call
E_M : " " Michael " "

We want $P(E_M | E_{LD})$

$$P(E_M | E_{LD}) = \frac{P(E_{LD} | E_M) P(E_M)}{P(E_{LD})}$$

We know $P(E_{LD} | E_M) = 0.24$; $P(E_{LD}) = P(E_{LD} | E_M)P(E_M) + P(E_{LD} | E_C)P(E_C)$