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**Homework 3 (Due Date: Thursday, Feb 7th In Class)**

# Problem 4.5

An n-channel MOSFET is biased in the saturation region at a constant  $V_{GS}$ . (a) The drain current is  $I_D = 0.250$  mA at  $V_{DS} = 1.5$  V and  $I_D = 0.258$  mA at  $V_{DS} = 3.3$  V. Determine the value of  $\lambda$  and  $r_o$ . (b) Using the results of part (a), determine  $I_D$  at  $V_{DS} = 5$  V.

$$a) \quad I_D = K_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS}) \Rightarrow K_n = \frac{I_D}{(V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})}$$

$$\frac{0.25 \text{ mA}}{(V_{GS} - V_{TN})^2 (1 + \lambda(1.5))} = \frac{0.258 \text{ mA}}{(V_{GS} - V_{TN})^2 (1 + \lambda(3.3))}$$

$$0.25 \text{ mA} (1 + \lambda(3.3)) = 0.258 \text{ mA} (1 + \lambda(1.5))$$

$$0.25 \text{ mA} + 1825 \mu = 0.258 \text{ mA} + \lambda (387 \mu) \Rightarrow 8 \mu = \lambda (438 \mu)$$

$$\lambda = \frac{8 \mu}{438 \mu} \Rightarrow \boxed{\lambda = 0.0183 \text{ V}^{-1}}$$

$$r_o = \frac{\Delta V}{\Delta I} \Rightarrow \frac{3.3 \text{ V} - 1.5 \text{ V}}{0.258 \text{ mA} - 0.25 \text{ mA}} \Rightarrow \frac{1.8}{8 \mu} \Rightarrow \boxed{r_o = 225 \text{ k}\Omega}$$

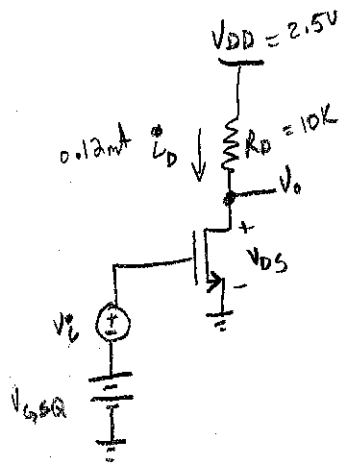
$$b) \quad 0.258 \text{ mA} = K_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS}) \Rightarrow K_n (V_{GS} - V_{TN})^2 = \frac{0.258 \text{ mA}}{1 + 3.3(0.0183)}$$

$$\Rightarrow 0.243 \text{ mA} = K_n (V_{GS} - V_{TN})^2$$

$$I_D = 0.243 \text{ mA} (1 + (0.0183)(5)) \Rightarrow \boxed{I_D = 0.2655 \text{ mA}}$$

### Problem 4.9

The circuit shown in Figure 4.1 has parameters  $V_{DD} = 2.5$  V and  $R_D = 10$  k $\Omega$ . The transistor is biased at  $I_{DQ} = 0.12$  mA. The transistor parameters are  $V_{TN} = 0.3$  V,  $k'_n = 100$   $\mu$ A/V<sup>2</sup>, and  $\lambda = 0$ . (a) Design the  $W/L$  ratio of the transistor such that the small-signal voltage gain is  $A_v = -3.8$ . (b) Repeat part (a) for  $A_v = -5.0$ .



$$a) V_0 = 2.5V - 0.12mA(10k) \Rightarrow V_0 = 1.3V$$

$$A_v = -3.8 = \frac{V_0}{V_i} \Rightarrow V_i = \frac{1.3}{-3.8} \Rightarrow V_i = -0.342V = V_{GS}$$

$$1.3 \geq -0.342V - 0.3V \quad \text{sat}$$

$$0.12mA = \frac{100\mu A}{2} \left( \frac{W}{L} \right) (-0.342 - 0.3V)^2$$

$$\frac{W}{L} = \frac{0.12mA(2)}{100\mu A(-0.642)^2} \Rightarrow \boxed{\frac{W}{L} = 5.823}$$

$$b) A_v = -5.0 = \frac{V_0}{V_i} \Rightarrow V_i = \frac{1.3V}{-5} \Rightarrow V_i = -0.26V = V_{GS}$$

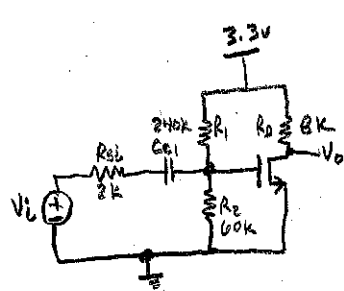
$$1.3 \geq -0.26 - 0.3 \quad \text{sat}$$

$$0.12mA = \frac{100\mu A}{2} \left( \frac{W}{L} \right) (-0.26 - 0.3)^2$$

$$\frac{W}{L} = \frac{0.12mA(2)}{100\mu A(-0.56)^2} \Rightarrow \boxed{\frac{W}{L} = 7.653}$$

# Problem 4.13

Consider the circuit in Figure 4.14 in the text. The circuit parameters are  $V_{DD} = 3.3\text{ V}$ ,  $R_D = 8\text{ k}\Omega$ ,  $R_1 = 240\text{ k}\Omega$ ,  $R_2 = 60\text{ k}\Omega$ , and  $R_{Si} = 2\text{ k}\Omega$ . The transistor parameters are  $V_{TN} = 0.4\text{ V}$ ,  $k'_n = 100\text{ }\mu\text{A/V}^2$ ,  $W/L = 80$ , and  $\lambda = 0.02\text{ V}^{-1}$ . (a) Determine the quiescent values  $I_{DQ}$  and  $V_{DSQ}$ . (b) Find the small-signal parameters  $g_m$  and  $r_o$ . (c) Determine the small-signal voltage gain.



$$V_{GS} = 3.3\text{ V} \left( \frac{60\text{ k}\Omega}{60\text{ k}\Omega + 240\text{ k}\Omega} \right) \Rightarrow V_{GS} = 0.66\text{ V}$$

Assume sat

$$I_{DQ} = \frac{100\text{ }\mu\text{A}}{2} (80) (0.66 - 0.4)^2 \Rightarrow I_{DQ} = 270.4\text{ }\mu\text{A}$$

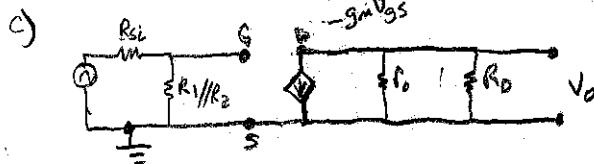
$$V_{DS} = 3.3\text{ V} - 270.4\text{ }\mu\text{A} (8\text{ k}\Omega) \Rightarrow V_{DSQ} = 1.1368\text{ V}$$

$$1.1368\text{ V} \geq 0.66\text{ V} - 0.4 \quad \text{Sat} \quad \checkmark$$

$$b) \quad g_m = 2\sqrt{k_n I_{DQ}} \quad k_n = \frac{k'_n}{2} \left( \frac{W}{L} \right) = \frac{100\text{ }\mu\text{A}}{2} (80) = k_n = 4\text{ m}$$

$$g_m = 2\sqrt{(4\text{ m})(270.4\text{ }\mu\text{A})} = g_m = 2.08\text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} \Rightarrow \frac{1}{(0.02)(270.4\text{ }\mu\text{A})} \Rightarrow r_o = 184.911\text{ k}\Omega$$



$$V_o = -g_m V_{GS} (R_o // R_D) \quad V_{GS} = V_i \left( \frac{R_1 // R_2}{R_{Si} + R_1 // R_2} \right)$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_m \left( \frac{R_1 // R_2}{R_{Si} + R_1 // R_2} \right) (R_o // R_D)}{1} \Rightarrow -g_m \left( \frac{R_1 // R_2}{R_{Si} + R_1 // R_2} \right) (R_o // R_D) = A_v$$

$$R_1 // R_2 = 48\text{ k}\Omega \quad R_o // R_D = 7.668\text{ k}\Omega$$

$$A_v = -2.08\text{ mA/V} \left( \frac{48\text{ k}\Omega}{2\text{ k}\Omega + 48\text{ k}\Omega} \right) (7.668\text{ k}\Omega) = A_v = -15.312$$

# Problem 4.15

For the NMOS common-source amplifier in Figure P4.15, the transistor parameters are:

$V_{TN} = 0.8 \text{ V}$ ,  $K_n = 1 \text{ mA/V}^2$ , and  $\lambda = 0$ . The circuit parameters are  $V_{DD} = 5 \text{ V}$ ,

$R_S = 1 \text{ k}\Omega$ ,  $R_D = 4 \text{ k}\Omega$ ,  $R_1 = 225 \text{ k}\Omega$ , and  $R_2 = 175 \text{ k}\Omega$ . (a) Calculate the quiescent values  $I_{DQ}$  and  $V_{DSQ}$ . (b) Determine the small-signal voltage gain for  $R_L = \infty$ .

(c) Determine the value of  $R_L$  that will reduce the small-signal voltage gain to 75 percent of the value found in part (b).

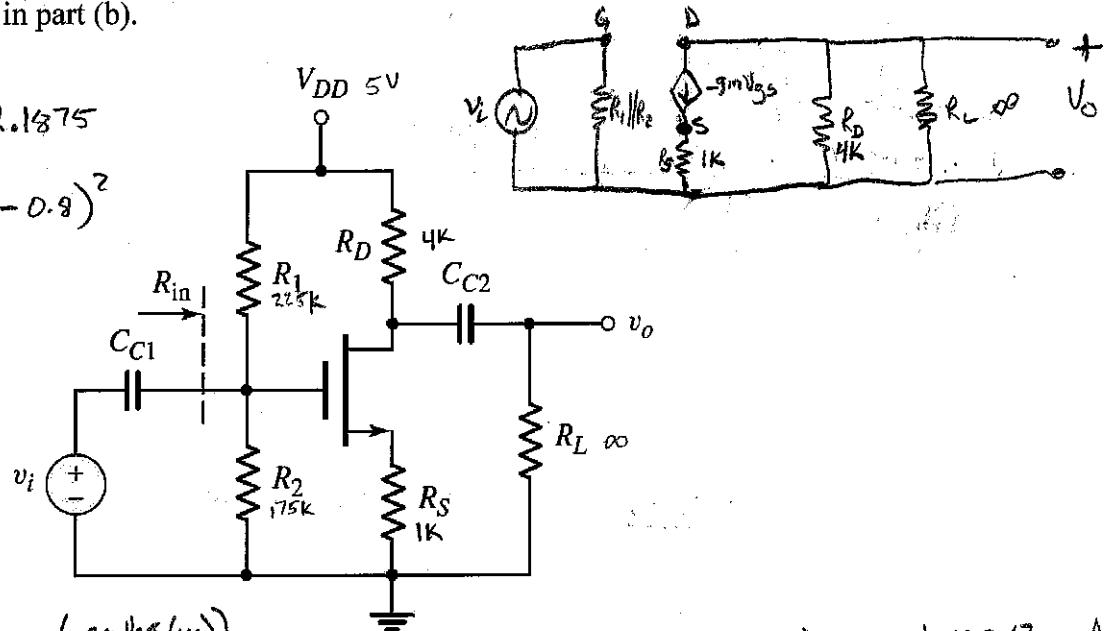
$$a) V_G = 5 \left( \frac{175 \text{ k}}{175 \text{ k} + 225 \text{ k}} \right) \Rightarrow V_G = 2.1875$$

$$I_{DQ} = 1 \text{ mA} \left( 2.1875 - I_{DQ}(1 \text{ k}) - 0.8 \right)^2$$

$$I_{DQ} = 607.852 \mu\text{A}$$

$$V_{DS} = 5 - (607.852 \mu\text{A})(4 \text{ k}) - (607.852 \mu\text{A})(1 \text{ k})$$

$$V_{DSQ} = 1.9607 \text{ V}$$



$$b) V_{GS} = V_G - V_S \Rightarrow V_G - V_S = V_{GS}$$

$$V_G = V_{GS} - V_S \Rightarrow V_G = V_{GS} - (-g_m V_{GS}(1 \text{ k}))$$

$$V_G = V_{GS}(1 + g_m(1 \text{ k})) \quad \text{Figure P4.15}$$

$$V_o = -g_m V_{GS}(R_D || R_L)$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_m(R_D || R_L)}{1 + g_m(1 \text{ k})}$$

$$g_m = 2\sqrt{K_n I_{DQ}} \Rightarrow g_m = 2\sqrt{(1 \text{ mA/V}^2)(607.852 \mu\text{A})}$$

$$g_m = 1.559 \text{ mS}$$

$$A_v = \frac{-1.559(4 \text{ k} || \infty)}{1 + (1.559 \text{ m})(1 \text{ k})} \Rightarrow A_v = -2.437$$

$$c) 0.75(-2.437) = -1.828 = A_v$$

$$-1.828 = \frac{-g_m(R_D || R_L)}{1 + g_m(1 \text{ k})}$$

$$R_D || R_L = \frac{1.828(1 + g_m(1 \text{ k}))}{-g_m}$$

$$\frac{1}{R_D} + \frac{1}{R_L} = \frac{1.828(1 + g_m(1 \text{ k}))}{g_m}$$

$$\frac{g_m}{1.828(1 + g_m(1 \text{ k}))} - \frac{1}{R_D} = \frac{1}{R_L}$$

$$R_L = \frac{1}{\frac{1.559 \text{ m}}{1.828(1 + 1.559 \text{ m}(1 \text{ k}))} - \frac{1}{4 \text{ k}}}$$

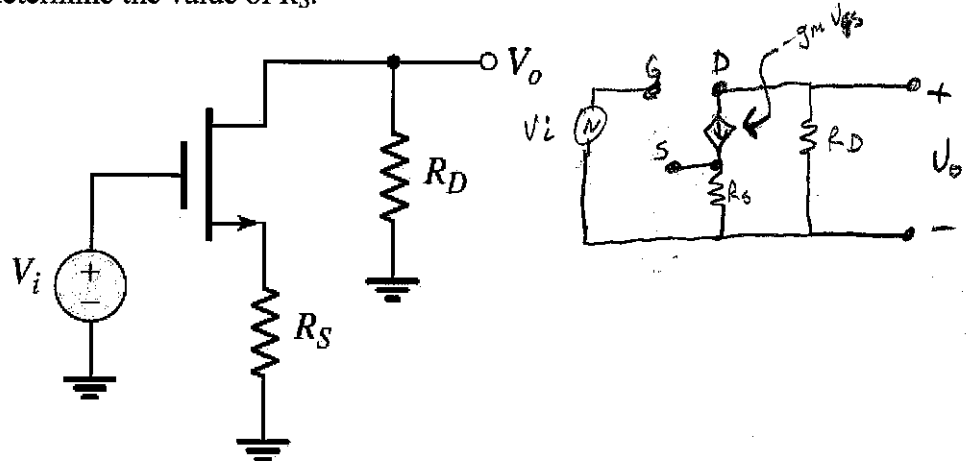
$$R_L = 12 \text{ k}\Omega$$

# Problem 4.19

Consider the ac equivalent circuit shown in Figure P4.18. Assume  $r_o = \infty$  for the transistor. The small-signal voltage gain is  $A_v = -8$  for the case when  $R_S = 1 \text{ k}\Omega$ .

(a) When  $R_S$  is shorted ( $R_S = 0$ ), the magnitude of the voltage gain doubles. Assuming the small-signal transistor parameters do not change, what are the values of  $g_m$  and  $R_D$ ? (b) A new value of  $R_S$  is inserted into the circuit and the voltage gain becomes  $A_v = -10$ . Using the results of part (a), determine the value of  $R_S$ .

$$\begin{aligned} a) \quad A_v &= \frac{V_o}{V_i} \\ V_{GS} &= V_G - V_S \\ V_{GS} &= V_i - (g_m V_{GS})(R_S) \\ V_i &= V_{GS} + g_m V_{GS} R_S \\ V_i &= V_{GS}(1 + g_m R_S) \\ V_o &= -g_m V_{GS} R_D \\ A_{v1} &= \frac{-g_m R_D}{1 + g_m R_S} \end{aligned}$$



$$2A_{v2} = \frac{-g_m R_D}{1 + g_m R_S} \Rightarrow A_{v2} = \frac{-g_m R_D}{2} \quad \text{Figure P4.18}$$

$$A_{v1} = A_{v2}$$

$$\frac{-g_m R_D}{1 + g_m R_S} = \frac{-g_m R_D}{2} \Rightarrow \frac{-2g_m R_D}{-g_m R_D} = 1 + g_m R_S$$

$$2 = 1 + g_m R_S \Rightarrow 1 = g_m R_S \Rightarrow g_m = \frac{1}{1\text{k}}$$

$$\boxed{g_m = 1\text{mA/V}}$$

$$2(-8) = -(1\text{mA})(R_D) \Rightarrow \frac{-16}{-1\text{mA}} = \boxed{R_D = 16\text{k}\Omega}$$

b)

$$-10 = \frac{-1\text{mA}(16\text{k})}{1 + (1\text{mA})(R_S)} \Rightarrow 1 + (1\text{mA})(R_S) = \frac{-1\text{mA}(16\text{k})}{-10}$$

$$R_S = \frac{1.6 - 1}{1\text{mA}} \Rightarrow \boxed{R_S = 600\Omega}$$