ICE #2 Solutions

M. [illegible]
ECE 345
9/27/12

Pre-class work

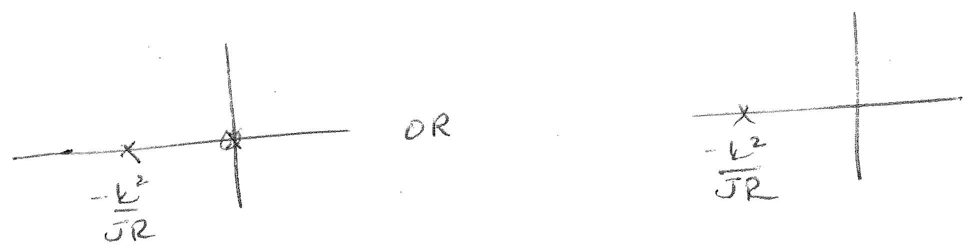
1. $G(s) = C(sI - A)^{-1}B + D$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & -1 \\ 0 & s+\frac{k^2}{JR} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{k}{JR} \end{bmatrix} + 0$$

$$= \frac{1}{s(s+\frac{k^2}{JR})} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s+\frac{k^2}{JR} & +1 \\ 0 & s \end{bmatrix} \begin{bmatrix} 0 \\ \frac{k}{JR} \end{bmatrix}$$

$$= \frac{\cancel{s} \cdot \frac{k}{JR}}{\cancel{s}(s + k^2/JR)} = \frac{k/JR}{s + k^2/JR}$$

2.



would both be considered correct.

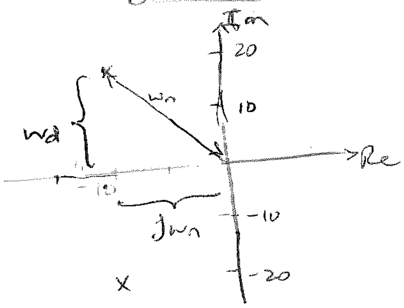
3. Time constant is $\tau = \frac{1}{(k^2/JR)} = JR/k^2$

4. $\tau = \frac{1}{100} = 100 \cdot \frac{R}{k^2}$

$$\frac{k^2}{R} = 10000 = 1e4$$

In-class assignment

1.



$$G(s) = \frac{125}{s^2 + 20s + 500}$$

$= 10^2 + 20^2$

$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 20s + 500$

$\Rightarrow \zeta\omega_n = 10, \quad \omega_n^2 = 500$

· natural frequency $\omega_n = \sqrt{500} = 10\sqrt{5}$

· damped frequency $\omega_d = \omega_n\sqrt{1-\zeta^2} = 20$

· decay rate $\zeta\omega_n = 10$

$\Rightarrow$ (c) is correct.

2. Since ω_n is the hypotenuse, $\omega_n > \omega_d$.

3. $T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{10} = 0.4 < \frac{1}{2}$ second.

overshoot $= e^{-\zeta\pi/\sqrt{1-\zeta^2}}$, $\quad \zeta = \dfrac{10}{\omega_n} = \dfrac{10}{10\sqrt{5}} = \dfrac{1}{\sqrt{5}}$

$= .2079 \Rightarrow 20\%$

$\Rightarrow$ (b) is correct.

4. Since for $s^2 + 20s + K$, $\zeta\omega_n = 10$, $\omega_n^2 = K$

$\zeta = \dfrac{10}{\sqrt{K}}$

$\therefore$ As K increases, ζ decreases $\Rightarrow$ more overshoot.

For $K = 200$, $\zeta = 1/\sqrt{2} \Rightarrow 4.3\%$ overshoot

Lower values of K will also satisfy overshoot requirement

$\Rightarrow$ (c), (d) are correct

5. $0 = s^2 + 20s + K$ is:

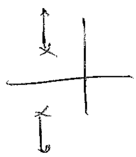
a) overdamped for $K < 100$.

Note that for $K = 100$, poles are co-located at $s = -10$

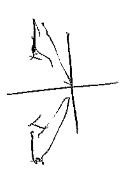
For $K > 100$, we know poles are complex conjugate pair at

$$s = -10 \pm j \cdot \omega_d$$
$$= -10 \pm j\sqrt{K} \cdot \sqrt{1-\zeta^2}$$
$$= -10 \pm j\sqrt{K} \cdot \sqrt{1 - 1/K}$$
$$= -10 \pm j\sqrt{K-1}$$

$\therefore$ For $K<100$, $\zeta > 1 \Rightarrow$ poles are on the real line. $\Rightarrow$ (a) is true.

b)  As K increases beyond $K=100$, settling time is <u>unchanged</u>. $\Rightarrow$ (b) is false.

(c) As K increases, $\omega_d = \omega_n\sqrt{1-\zeta^2}$ increases, so $T_p = \frac{\pi}{\omega_d}$ decreases. $\Rightarrow$ (c) is true.

(d)  As K increases, ζ decreases since $\zeta = \frac{10}{\omega_n} = \frac{10}{\sqrt{K}}$.
$\therefore$ Overshoot increases. $\Rightarrow$ (d) is true.

(e) See above. (e) is true.

$\Rightarrow$ (a), (c), (d), (e)

6. (a) The time constant for the extender system is $\tau = \frac{1}{\zeta\omega_n} = \frac{1}{10}$ which is <u>slower</u> than the DC motor system.

(b) Motor dynamics which are comparable in speed, or slower in speed, to the extender system will interfere, possibly causing delays that could destabilize the system. A extender command would be implemented excessively slowly, if the time constants of the systems are comparable.