ECE 345: Introduction to Control Systems
Problem Set #3

Dr. Oishi

Due Thursday, September 20, 2012 at the start of class

This homework is open note and open book. You are welcome to discuss the problems with other students, but your solutions and Matlab code *must be written independently.* Copying will not be tolerated.

1. Consider the laser printer positioning system described in Problem Set #2 with transfer function

$$G(s) = \frac{5(s+4)}{s^2+10s+50}$$

   (a) Compute the poles and zeros of this transfer function.

   (b) What magnitude will $G(s)$ have when $s=-4$? When $s=-5$?

   (c) Find the phase variable representation of this system. That is, identify state-space matrices A, B, C, D corresponding to the above transfer function.

   (d) Show that the transfer function obtained from the above state-space matrices via $C(sI_A)^{-1}B+D$ is the same as the one initially given in the problem statement.

2. Consider the state-space system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 2 & -3 & -3 \\ 0 & 5 & 3 \\ -3 & -5 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 3 & 6 \end{bmatrix} x \end{aligned}$$

   (a) What is the characteristic equation of the system?

   (b) Describe briefly (e.g., one or two sentences) how the roots of the characteristic equation affect the transient response of generic system.

   (c) This system has one pole on the negative real line, and a complex conjugate pair of poles in the right-half of the complex plane. Based on just this information, would you expect to see exponential growth or decay predominantly in the transient response after t is large enough?

3. The F4-E military aircraft uses elevator deflection δ_e of the horizontal stabilizers and canard deflection δ_c to control the aircraft's normal acceleration a_n and pitch rate q. A commanded deflection δ_{com} is used to effect a change in both δ_e and δ_c. The aircraft longitudinal dynamics can be described by the state-space model with state $x = \begin{bmatrix} a_n \\ q \\ \delta_e \end{bmatrix}$ and input δ_{com}.

$$\dot{x} = \begin{bmatrix} -1.702 & 50.72 & 263.38 \\ 0.22 & -1.418 & -31.99 \\ 0 & 0 & -14 \end{bmatrix} x + \begin{bmatrix} -272.06 \\ 0 \\ 14 \end{bmatrix} \delta_{\text{com}}$$

Use the `diary` command to record your Matlab session and hand in your record of commands and results generated. Use notation consistent with what was covered in class.

(a) Using Matlab, use `ss2tf` to find the transfer function $G_1(s) = \frac{A_n(s)}{\delta_{\text{com}}(s)}$. Call the numerator coefficients `num1` and the denominator coefficients `den1`. What is the output matrix C_1 in this case?

(b) Similarly, find the transfer function $G_2(s) = \frac{Q(s)}{\delta_{\text{com}}(s)}$. What is the output matrix C_2?

(c) Use `eig` to find the eigenvalues of the state matrix.

(d) Use `roots` to find the poles of $G_1(s)$ and $G_2(s)$.

(e) Are the denominators of the two transfer functions the same? In one sentence, describe why or why not. How do they relate to your answer for part (c)?

(f) Which of the two transfer functions has more zeros? Use `roots` to determine where the zeros of that transfer function are located.