#39 Waves Pre-class

Due: 11:00am on Monday, November 26, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Wave Notation

Learning Goal:

To understand the relationships between the parameters that characterize a wave.

It is of fundamental importance in many areas of physics to be able to deal with waves. This problem will lead you to understand the relationship of variables related to wave propagation: frequency, wavelength, velocity of propagation, and related variables. Note that these are kinematic variables that relate to the wave's propagation and do not depend on its amplitude.

Part A

Traveling waves propagate with a fixed speed usually denoted as v (but sometimes c). The waves are called _____ if their waveform repeats every time interval T.

ANSWER:

- transverse
- longitudinal
- periodic
- sinusoidal

Correct

Periodic waves are characterized by a frequency that is designated by f or ω. The quantities k and λ are related to the length of the waves.

The fundamental relationship among frequency, wavelength, and velocity is

$$v = f\lambda.$$

This relationship may be visualized as follows: In 1 s, $f \cdot (1\ \text{s})$ cycles of the wave move past an observer. In this same second the wavetrain moves a distance $v \cdot (1\ \text{s})$.

Part B

Solve this equation to find an expression for the wavelength λ.

Express your answer in terms of the velocity v and the frequency f.

ANSWER:

$\lambda = \frac{v}{f}$

Correct

Part C

If the velocity of the wave remains constant, then as the frequency of the wave is increased, the wavelength _____.

ANSWER:

- (•) decreases
- ( ) increases
- ( ) stays constant

Correct

Part D

The difference between the frequency f and the frequency ω is that f is measured in cycles per second or hertz (abbreviated Hz) whereas the units for ω are _____ per second.

Give your answer in one word. Watch your spelling to avoid a mistake!

ANSWER:

radians

Correct

Part E

Find an expression for the period of a wave T in terms of other kinematic variables.

Express your answer in terms of any of f, v, ω, and simple constants such as π.

ANSWER:

$T = \frac{1}{f}$

Correct

Part F

What is the relationship between ω and f?

ANSWER:

$\omega = 2\pi f$

Correct

Part G

What is the simplest relationship between the angular wavenumber k and just one of the other kinematic variables?

Express your answer using only one of the other kinematic variables plus constants like π.

ANSWER:

$$k = \frac{2\pi}{\lambda}$$

Correct

The units of k are radians/meter and indicate that angular wavenumber is a measure of the number of radians of phase in a unit distance, which is 2π times the number of complete wave cycles in this distance. For example, a distance of one wavelength is equivalent to one cycle, one period, and 2π radians. The angular wavenumber k is often misnamed "wavenumber," which is actually $1/\lambda$, so be careful to determine how this term is defined when reading your textbook or reviewing your lecture notes.

Surface Waves

The waves on the ocean are surface waves: They occur at the interface of water and air, extending down into the water and up into the air at the expense of becoming exponentially reduced in amplitude. They are neither transverse nor longitudinal. The water both at and below the surface travels in vertical circles, with exponentially smaller radius as a function of depth.

Both empirical measurements and calculations beyond the scope of introductory physics give the propagation speed of water waves as

$$v = \sqrt{\frac{g}{k}},$$

where $g = 9.8\ \mathrm{m/s^2}$ is the magnitude of the acceleration due to gravity and k is the wavenumber.

This relationship applies only when the following three conditions hold:

1. The water is several times deeper than the wavelength.
2. The wavelength is large enough that the surface tension of the waves can be neglected.
3. The ratio of wave height to wavelength is small.

The restoring force (analogous to the tension in a string) that restores the water surface to flatness is due to gravity, which explains why these waves are often called "gravity waves."

Part A

Find the speed v of water waves in terms of the wavelength λ.

Express the speed in terms of g, λ, and π.

Hint 1. Definition of k

Express k in terms of constants such as g and π and *one* of the following kinematic parameters used to characterize traveling wave propagation: the period T, the angular frequency ω, the frequency f, or the wavelength λ. Do **not** use v.

ANSWER:

$k = \dfrac{2\pi}{\lambda}$

ANSWER:

$v = \sqrt{\dfrac{\lambda g}{2\pi}}$

Correct

Part B

Find the speed v of a wave of wavelength $\lambda = 8.0\ \text{m}$.

Give your answer in meters per second to a precision of two significant figures.

ANSWER:

$v(\lambda = 8\ \text{m})$ = 3.5 m/s

Correct

Part C

Find the period T for a wave of wavelength λ.

Express the period in terms of π, λ, and g.

Hint 1. Formula for T

Express T in terms of constants like g and π and any *two* of the following kinematic parameters used to characterize traveling wave propagation: the angular frequency ω, the frequency f, the wavelength λ, and the velocity v.

ANSWER:

$$T = \frac{\lambda}{v}$$

ANSWER:

$$T = \frac{\lambda}{\sqrt{\frac{\lambda g}{2\pi}}}$$

Correct

Part D

On the East Coast of the United States, the National Weather Service frequently reports waves with a period of 4.0 s. Find the wavelength λ and speed v of these waves.

Express your answers numerically as an ordered pair separated by a comma. Give an accuracy of two significant figures.

Hint 1. Relationship between wavelength and period

Find the wavelength λ of water waves in terms of their period T.

Express your answer in terms of T, g, and π.

ANSWER:

$\lambda = \frac{gT^2}{2\pi}$

ANSWER:

λ, v = 25,6.25 m, m/s

Correct

Part E

On the West Coast of the United States, the National Weather Service frequently reports waves (really swells) with a period of 15 s. Find the wavelength λ and speed v of these waves.

Express your answers numerically as an ordered pair separated by a comma. Give an accuracy of two significant figures.

ANSWER:

λ, v = 350,23 m, m/s

Correct

Standard Expression for a Traveling Wave

Learning Goal:

To understand the standard formula for a sinusoidal traveling wave.

One formula for a wave with a *y* displacement (e.g., of a string) traveling in the *x* direction is

$$y(x,t) = A\sin(kx - \omega t).$$

All the questions in this problem refer to this formula and to the wave it describes.

Part A

Which of the following are independent variables?

Hint 1. What are independent variables?

Independent variables are those that are freely varied to control the value of the function. The independent variables typically appear on the horizontal axis of a plot of the function.

ANSWER:

- x only
- t only
- A only
- k only
- ω only
- x and t
- ω and t
- A and k and ω

Correct

Part B

Which of the following are parameters that determine the characteristics of the wave?

Hint 1. What are parameters?

Parameters are constants that determine the characteristics of a particular function. For a wave these include the amplitude, frequency, wavelength, and period of the wave.

ANSWER:

- x only
- t only
- A only
- k only
- ω only
- x and t
- ω and t
- A and k and ω

Correct

Part C

What is the phase $\phi(x,t)$ of the wave?

Express the phase in terms of one or more given variables (A, k, x, t, and ω) and any needed constants like π.

Hint 1. Definition of phase

The phase is the argument of the trig function, which is expressed in radians.

ANSWER:

$\phi(x,t) = kx - \omega t$

Correct

Part D

What is the wavelength λ of the wave?

Express the wavelength in terms of one or more given variables (A, k, x, t, and ω) and any needed constants like π.

Hint 1. Finding the wavelength

Consider the form of the wave at time $t = 0$. The wave crosses the *y* axis, sloping upward at $x = 0$. The wavelength is the *x* position at which the wave next crosses the *y* axis, sloping upward (i.e., the length of one complete cycle of oscillation).

ANSWER:

$\lambda = \frac{2\pi}{k}$

Correct

Part E

What is the period T of this wave?

Express the period in terms of one or more given variables (A, k, x, t, and ω) and any needed constants like π.

ANSWER:

$T = \frac{2\pi}{\omega}$

Correct

Part F

What is the speed of propagation v of this wave?

Express the speed of propagation in terms of one or more given variables (A, k, x, t, and ω) and any needed constants like π.

Hint 1. How to find v

If you've done the previous parts of this problem, you have found the wavelength and the period of this wave. The speed of propagation is a function of these two quantities: $v = \lambda/T$.

ANSWER:

$v = \frac{\omega}{k}$

Correct

PSS 15.1 Mechanical Waves

Learning Goal:

To practice Problem-Solving Strategy 15.1 Mechanical Waves.

Waves on a string are described by the following general equation

$$y(x,t) = A\cos(kx - \omega t).$$

A transverse wave on a string is traveling in the +x direction with a wave speed of 8.75$\mathrm{m/s}$, an amplitude of 8.00×10$^{-2}\mathrm{m}$, and a wavelength of 0.520m . At time $t = 0$, the $x = 0$ end of the string has its maximum upward displacement. Find the transverse displacement y of a particle at x = 1.57m and t = 0.150s .

Problem Solving Strategy 15.1: Mechanical Waves

IDENTIFY *the relevant concepts:*
Wave problems fall into two broad categories. *Kinematics* problems are concerned with describing wave motion; they involve wave speed v, wavelength λ (or wave number k), frequency f (or angular frequency ω) and amplitude A. They may also involve the position, velocity, and acceleration of individual particles in the medium. *Dynamics* problems also use concepts from Newton's laws such as force and mass.

As always, make sure that you identify the target variable(s) for the problem.

SET UP *the problem* using the following steps:

1. Make a list of the quantities whose values are given. To help you visualize the situation, you will find it useful to sketch graphs of y position versus x position and of y position versus time t.
2. Decide which equations you will need to use. If any two of v, f, and λ are given, use $v = \lambda f$ to find the third quantity. If the problem involves the angular frequency ω and/or the wave number k, use the definitions of those quantities and $\omega = vk$. You may also need to use the various forms in which the wave function can be expressed.
3. If the wave speed isn't given and you don't have enough information to determine it using $v = \lambda f$, you may be able to use the relationship between v and the mechanical properties of the system.

EXECUTE *the solution* as follows:
Solve for the unknown quantities using the equations you've selected.

EVALUATE *your answer:*
Look at your results with a critical eye. Check to see whether the values of v, f, and λ (or v, ω, and k) agree with the relationships. If you've calculated the wave function, check one or more special cases for which you can guess what the results ought to be.

IDENTIFY the relevant concepts

The target variable is the transverse displacement y at a specified location x and time t. Because this problem is concerned with the motion of the wave, it is a kinematics problem.

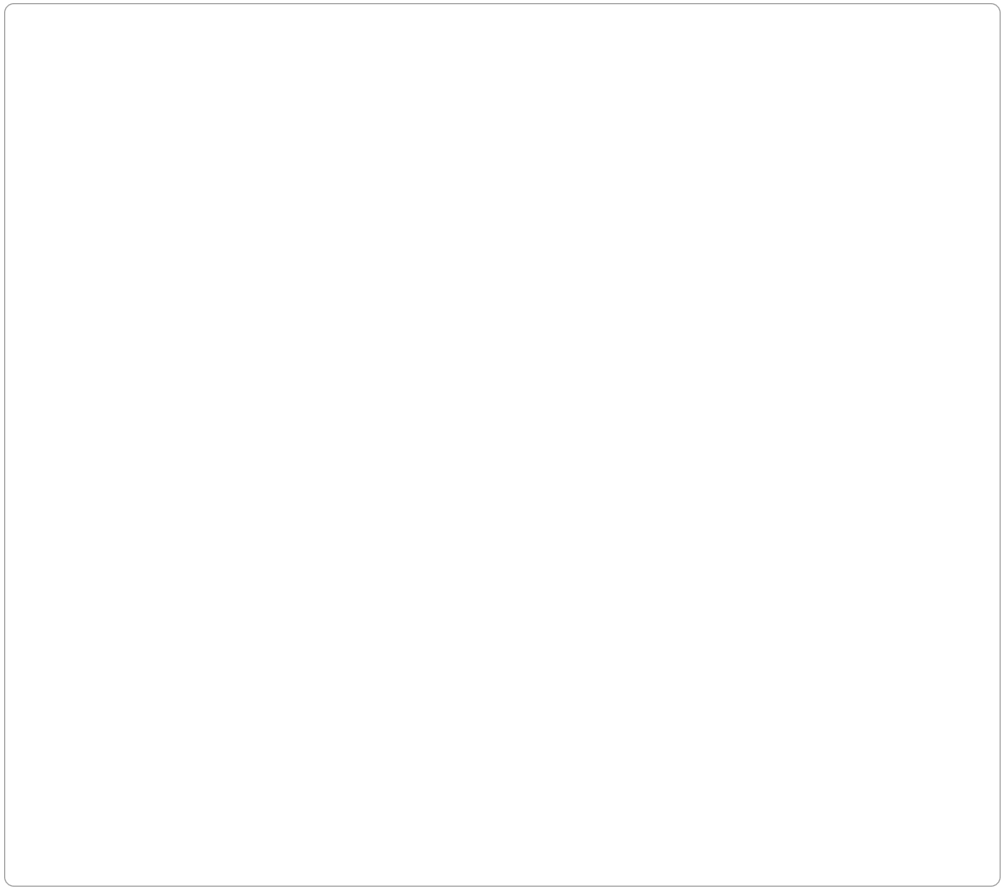
SET UP the problem using the following steps

Part A

Classify each of these variables as known or unknown.

Drag the appropriate variables to their respective bins.

ANSWER:



Correct

Part B

Calculate the frequency f of the wave on the string.

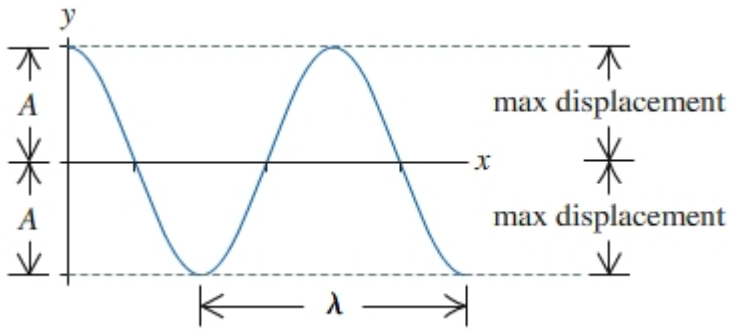
Express your answer in hertz.

ANSWER:

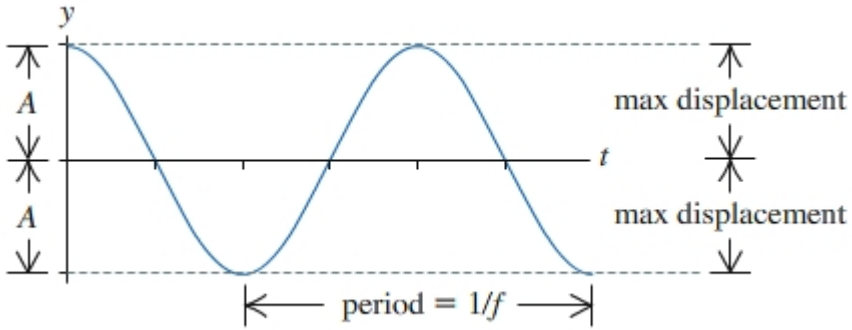
f = 16.8 Hz

Correct

Based on the general wave equation provided, the wave number and angular frequency will be used to describe the specific wave equation. The wave equation can be graphed in two basic ways. First, the transverse displacement y can be graphed as a function of the position x with time held constant, showing a snapshot of the wave's appearance at an instant in time.



Second, the transverse displacement y can be graphed as a function of time t with the position held constant, showing how the wave changes in time at a particular location x.



EXECUTE the solution as follows

Part C

Find the transverse displacement y of a particle at x = 1.57 m at time t = 0.150 s .

Express your answer in meters.

Hint 1. How to approach the problem

Use the known values λ and f to find k and ω. Then using the general form of the wave equation given in the problem to write the specific wave equation for the string. Once you have the wave equation, you can solve for the transverse displacement y at any position x and time t.

Hint 2. Calculate wave number and angular frequency

Calculate the values for ω and k, specific for the wave on the string.

Enter the values for ω in radians per second and k in radians per meter separated by commas.

Hint 1. Converting to angular wave parameters

The conversions from frequency f to angular frequency ω and wavelength λ to wave number k are given by

$$\omega = 2\pi f$$
$$k = \frac{2\pi}{\lambda}.$$

Additionally, ω and k can be related through the wave speed v by $v = \omega k$.

ANSWER:

ω, k = 106,12.1 rad/s, rad/m

Hint 3. Determine the correct wave equation

Choose the correct wave equation for the string.

ANSWER:

- $y(x,t) = (8.00 \times 10^{-2}\text{m})\cos[(106\text{rad/s})x - (12.1\text{rad/m})t]$
- $y(x,t) = (12.1\text{rad/m})\cos[(8.00 \times 10^{-2}\text{m})x - (106\text{rad/s})t]$
- $y(x,t) = (8.00 \times 10^{-2}\text{m})\cos[(12.1\text{rad/m})x - (106\text{rad/s})t]$
- $y(x,t) = (12.1\text{rad/m})\cos[(106\text{rad/s})x - (8.00 \times 10^{-2}\text{m})t]$

ANSWER:

y = −8.00×10^{-2} m

Correct

Note that if $y = A$ at a given x and t, then the position x represents a point of maximum displacement at the time specified.

EVALUATE your answer

Part D

In general, the cosine function has maximum displacements, either positive or negative, when its argument is equal to an integer multiple of π. When t = 0.150s , k = 12.1rad/m , and ω = 106rad/s use the wave equation to select all of the x positions that correspond to points of maximum displacement.

Check all that apply.

ANSWER:

- ☐ 0.786 m
- ☑ 1.57 m
- ☐ 1.70 m
- ☑ 1.83 m
- ☐ 1.96 m
- ☑ 2.09 m

Correct

Using the relationship between v, λ, and f (or v, k, and ω), the kinematics quantities of a wave can be determined. Once the kinematics variables are known, the wave equation can be specified. The wave equation provides information about the change in transverse displacement of a wave as both time and position vary. Additionally, knowing the wave equation allows determination of velocities and accelerations for any particle along the string.

Score Summary:

Your score on this assignment is 100.2%.
You received 20.03 out of a possible total of 20 points.