#27 Energy in Rotational Motion and Moments of Inertia Post-class

Due: 11:00am on Friday, October 26, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Problem 9.93

A thin, flat, uniform disk has mass M and radius R. A circular hole of radius $R/4$, centered at a point $R/2$ from the disk's center, is then punched in the disk.

Part A

Find the moment of inertia of the disk with the hole about an axis through the original center of the disk, perpendicular to the plane of the disk. (*Hint:* Find the moment of inertia of the piece punched from the disk.)

Express your answer in terms of the given quantities.

ANSWER:

$I = \frac{247}{512}MR^2$

Correct

Part B

Find the moment of inertia of the disk with the hole about an axis through the center of the hole, perpendicular to the plane of the disk.

Express your answer in terms of the given quantities.

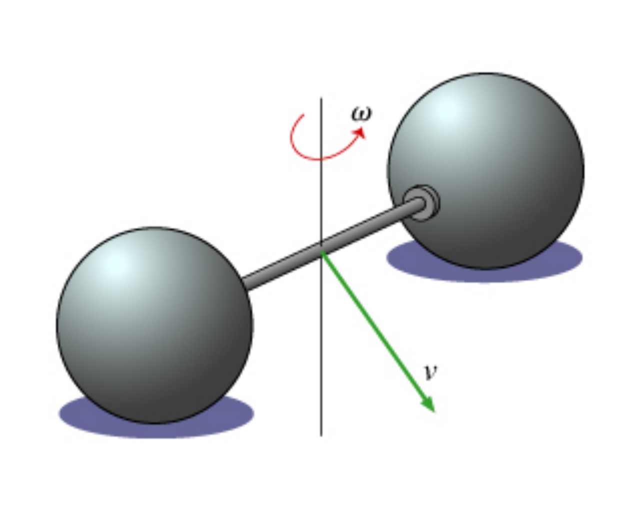
ANSWER:

$I = \frac{383}{512}MR^2$

Correct

Kinetic Energy of a Dumbbell

This problem illustrates the two contributions to the kinetic energy of an extended object: rotational kinetic energy and translational kinetic energy. You are to find the total kinetic energy $K_{\rm total}$ of a dumbbell of mass m when it is rotating with angular speed ω and its center of mass is moving translationally with speed v. Denote the dumbbell's moment of inertia about its center of mass by $I_{\rm cm}$. Note that if you approximate the spheres as point masses of mass $m/2$ each located a distance r from the center and ignore the moment of inertia of the connecting rod, then the moment of inertia of the dumbbell is given by $I_{\rm cm} = mr^2$, but this fact will not be necessary for this problem.



Part A

Find the total kinetic energy $K_{\rm tot}$ of the dumbbell.

Express your answer in terms of m, v, $I_{\rm cm}$, and ω.

Hint 1. How to approach the problem

Compute separately the rotational and translational kinetic energies of the dumbbell. Then add the two to find the total kinetic energy.

Hint 2. Find the rotational kinetic energy

What is the dumbbell's rotational kinetic energy K_r?

Give your answer in terms of some or all of the variables I_{cm}, v, and ω.

Hint 1. Formula for rotational kinetic energy

The formula for the rotational kinetic energy K_{rot} of a body with moment of inertia I about some axis, rotating with angular velocity ω about it, is

$$K_{rot} = \frac{1}{2} I \omega^2.$$

ANSWER:

K_r = $\frac{1}{2} I_{cm} \omega^2$

Hint 3. Find the translational kinetic energy

What is the dumbbell's translational kinetic energy K_t?

Give your answer in terms of some or all of the variables m, v, and ω.

ANSWER:

K_t = $\frac{1}{2} m v^2$

ANSWER:

$K_{\rm tot} = \frac{1}{2}I_{\rm cm}\omega^2 + \frac{1}{2}mv^2$

Correct

Part B

The rotational kinetic energy term is often called the kinetic energy *in* the center of mass, while the translational kinetic energy term is called the kinetic energy *of* the center of mass.

You found that the total kinetic energy is the sum of the kinetic energy in the center of mass plus the kinetic energy of the center of mass. A similar decomposition exists for angular and linear momentum. There are also related decompositions that work for systems of masses, not just rigid bodies like a dumbbell.

It is important to understand the applicability of the formula $K_{\rm tot} = K_{\rm r} + K_{\rm t}$. Which of the following conditions are necessary for the formula to be valid?

Check all that apply.

ANSWER:

- [ ] The velocity vector $\vec{v}$ must be perpendicular to the axis of rotation.
- [ ] The velocity vector $\vec{v}$ must be perpendicular or parallel to the axis of rotation.
- [x] The moment of inertia must be taken about an axis through the center of mass.

Correct

Problem 9.98: Neutron Stars and Supernova Remnants

The Crab Nebula is a cloud of glowing gas about 10 light-years across, located about 6500 light years from the earth (the figure). It is the remnant of a star that underwent a *supernova explosion,* seen on earth in 1054 a.d. Energy is released by the Crab Nebula at a rate of about $5 \times 10^{31} \mathrm{W}$, about 10^5 times the rate at which the sun radiates energy. The Crab Nebula obtains its energy from the rotational kinetic energy of a rapidly spinning *neutron star* at its center. This object rotates once every 0.0331 s, and this period is increasing by $4.22 \times 10^{-13} \mathrm{s}$ for each second of time that elapses.

Part A

If the rate at which energy is lost by the neutron star is equal to the rate at which energy is released by the nebula, find the moment of inertia of the neutron star.

Express your answer using two significant figures.

ANSWER:

I = 1.1×10^{38} $\mathrm{kg \cdot m^2}$

Correct

Part B

Theories of supernovae predict that the neutron star in the Crab Nebula has a mass about 1.4 times that of the sun. Modeling the neutron star as a solid uniform sphere, calculate its radius in kilometers.

Express your answer using two significant figures.

ANSWER:

R = 9.9 km

Correct

Part C

What is the linear speed of a point on the equator of the neutron star?

Express your answer using two significant figures.

ANSWER:

v = 1.9×10^6 m/s

Correct

Part D

Compare to the speed of light.

Express your answer in terms of c.Express your answer using two significant figures.

ANSWER:

v = $6.3 \cdot 10^{-3}$ c

Correct

Part E

Assume that the neutron star is uniform and calculate its density.

Express your answer using two significant figures.

ANSWER:

ρ = 6.9×10^{17} $\mathrm{kg/m^3}$

Correct

Problem 9.91

We wrap a light, flexible cable around a solid cylinder with mass 9.00 kg and diameter 24.0 cm . We tie the free end of the cable to a block of mass 14.0 kg . The 14.0-kg mass is released from rest and falls, causing the cylinder to turn about a frictionless axle through its center. As the block falls, the cable unwinds without stretching or slipping, turning the cylinder.

Part A

How far will the mass have to descend to give the cylinder 280 J of kinetic energy?

ANSWER:

h = 8.39 m

Correct

Problem 9.77

It has been argued that power plants should make use of off-peak hours (such as late at night) to generate mechanical energy and store it until it is needed during peak load times, such as the middle of the day. One suggestion has been to store the energy in large flywheels spinning on nearly frictionless ball-bearings. Consider a flywheel made of iron, with a density of 7800 $\mathrm{kg/m^3}$, in the shape of a uniform disk with a thickness of 12.1 cm .

Part A

What would the diameter of such a disk need to be if it is to store an amount of kinetic energy of 10.4 MJ when spinning at an angular velocity of 87.0 rpm about an axis perpendicular to the disk at its center?

ANSWER:

d = 7.21 m

Answer Requested

Part B

What would be the centripetal acceleration of a point on its rim when spinning at this rate?

ANSWER:

a = 299 $\mathrm{m/s^2}$

Answer Requested

Score Summary:

Your score on this assignment is 76.9%.
You received 38.43 out of a possible total of 50 points.