ECE 340 Spring 2011

Solutions to Homework 1

P 1.2-1

a) The list of all possible outcomes is HH, HT, TH, TT, where H stands for heads and T stands for tails. From the list of the outcomes we can form the sample space as follows:

$\Omega = \{(H, H), (H, T), (T, H), (T, T)\}$.

The outcomes are equally likely only if both of the coins are fair.

b) The list of all possible outcomes is 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. From the list of the outcomes we can form the sample space as follows:

$\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 0\}$;

the outcomes are equally likely.

c) The list of all possible outcomes is 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18. From the list of the outcomes we can form the sample space as:

$\Omega = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

The outcomes are not equally likely, since the number of combinations of 0-9 digits the summation of which constitute each of the outcomes are different; for example, there more more different combinations of two digits whose summation becomes 6 than 2.

P 1.2-2

An elementary event is one of which there is only one outcome.

a) Not an elementary event. (multiple outcomes)

b) Not an elementary event. (multiple outcomes)

c) Not an elementary event. (multiple outcomes: A of spades, hearts, diamonds and clubs)

d) Yes

e) Yes

f) Yes

g) Not an elementary event. (multiple outcomes: 1,2,3,4,5.9 etc.)

h) Not an elementary event. (multiple outcomes: the sixteen letter of e can be anywhere, so more than just one possibility)

P 1.4-1

The sample space is defined as:

$S = \{1, 2, 3, 4, 5, 6\}$. Then we have:

a) $A = \{1\}$, so P(A)=1/6;

b) $B = \{4, 5, 6\}$, so P(B)=3/6=1/2;

c) $C = \{2, 4, 6\}$, so P(C)=3/6=1/2.

P 1.4-4

We have the following table for distribution of IC's:

	HI	AG	FF	DC	SR
G	180	85	41	20	20
B	20	15	9	5	5

where HI, AG, FF, DC, and SR denote Hex Inverter, And Gate, Flip Flop, Decade Counters, and Shift Registers, and G and H stand for Good and Bad, respectively. Then we have:

a) P(G $\cap$ DC)=20/400=1/20;

b) Probability of being G when it is FF equals P(G|FF)=41/50;

c) Probability of being DC when it is G equals P(DC|G)=20/(180+85+41+20+20)=20/346;

P 1.5-2

$A \cup B = \{1, 3, 5, 7, 9, 11\}$

$B \cup C = \{1, 3, 7, 9, 11\}$

$A \cup C = \{1, 3, 5, 9, 11\}$

$A \cap B = \emptyset$

$A \cap C = \{1, 3\}$

$B \cap C = \{9, 11\}$

$A \cap B \cap C = \emptyset$

$A^c = \{7, 9, 11\}$

$B^c = \{1, 3, 5\}$

$C^c = \{5, 7\}$

$A^c \cap B = \{7, 9, 11\}$

$A \cap B^c = \{1, 3, 5\}$

$(B \cap C)^c = \{1, 3, 5, 7\}$

$A - C = \{5\}$

$C - A = \{9, 11\}$

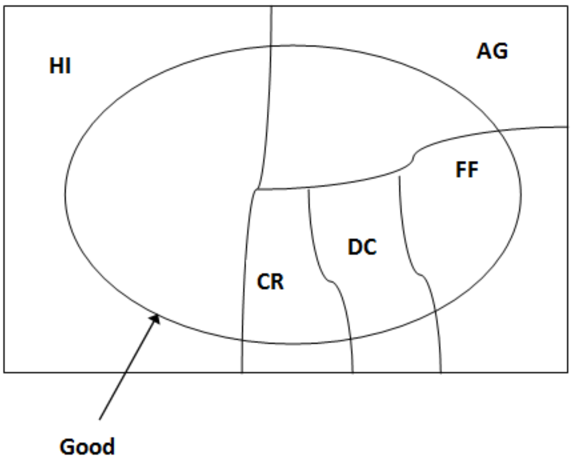
$A - B = \{1, 3, 5\}$

$(A - B) \cup B = \{1, 3, 5, 7, 9, 11\}$

$(A - B) \cup C = \{1, 3, 5, 9, 11\}$

P 1.5-3

We will have the following Venn digram for problem 1.4-4:



P 1.5-4(b)

To show $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, we first show that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$. Pick any $\omega \in A \cup (B \cap C)$. If $w \in A$, then we know $\omega \in A \cup B$ and also $\omega \in A \cup C$, so we know that $\omega \in (A \cup B) \cap (A \cup C)$. If $\omega \in (B \cap C)$ then we know that $\omega \in B$ and also $\omega \in C$, so we know that $\omega \in A \cup B$ and $\omega \in A \cup C$, which implies that $\omega \in (A \cup B) \cap (A \cup C)$. This we have established that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

Next, we show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Pick any $\omega \in (A \cup B) \cap (A \cup C)$. If $w \in A$, then $w \in A \cup (B \cap C)$; if $\omega \notin A$ then it must be true that $w \in (A^c \cap B \cap C)$, so $w \in (B \cap C)$, which implies $w \in A \cup (B \cap C)$. Therefore, we have shown that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. Hence, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

P 1.5-5(c)

To show set equality we must show $A \subset (A \cap B) \cup (A - B)$ and $(A \cap B) \cup (A - B) \subset A$. Let $\omega \in A$. Then ω either in $A \cap B$ or in $A \cap B^c = A - B$, so $\omega \in (A \cap B) \cup (A - B)$; therefore, $A \subset (A \cap B) \cup (A - B)$.

Next, let $\omega \in (A \cap B) \cup (A - B)$. Then either ω in $A \cap B$ or ω in $A \cap B^c$. But in both cases $\omega \in A$; therefore $\omega \in A$, and we have established that $(A \cap B) \cup (A - B) \subset A$.

Hence, $(A - B) \cup (A \cap B) = A$

P 1.6-1

Since each element in set S has a probability 1/6, by counting the elements in each subset and divide them by the total number of elements in the set S, we have the following probabilities:

$a)\mathsf{P}(A) = 3/6 = 0.5$

$b)\mathsf{P}(B) = 3/6 = 0.5$

$c)\mathsf{P}(C) = 4/6 = 0.667$

$d)\mathsf{P}(A \cup B) = 1$

$e)\mathsf{P}(A \cup C) = 5/6 = 0.833$

$f)\mathsf{P}((A - C) \cup B) = 4/6 = 0.667$

P 1.6-4

The DeMorgan's laws states that:

$(A \cup B)^c = A^c \cap B^c$.

$(A \cap B)^c = A^c \cup B^c$

From the DeMorgan's laws, we know that:

$A^c \cup B^c = S - (A \cap B)$

So

$\mathsf{P}(A^c \cup B^c) = 1 - \mathsf{P}(A \cap B)$