In-Class Exercise #4 Solutions

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10/30/12
ECE 345

Pre-Class Work

1. $Y(s) = G(s)\left(D(s) + k\,(R(s) - Y(s))\right)$

$= G(s)\cdot D(s) + kG(s)\cdot R(s) - kG(s)\,Y(s)$

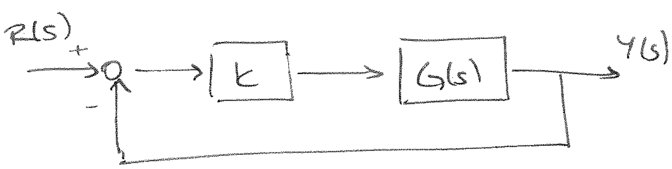
$Y(s)\left(1 + kG(s)\right) = G(s)\cdot D(s) + kG(s)\cdot R(s)$

$Y(s) = \dfrac{G(s)}{1+kG(s)} \cdot D(s) \quad \dfrac{kG(s)}{1+kG(s)} \cdot R(s)$

$= \dfrac{1}{k + s(s+k_1)} \cdot D(s) + \dfrac{k}{k + s(s+k_1)} \cdot R(s)$

$= \dfrac{1}{s^2 + k_1 s + k} \cdot D(s) + \dfrac{k}{s^2 + k_1 s + k} \cdot R(s)$

2. (a)



Since $kG(s) = \dfrac{k}{s(s+k_1)}$ has one pole at the origin,

the system is Type 1.

3. Type 1 systems can track a unit step with $e_{ss} = 0$.

4. $s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + k_1 s + k$

$\Rightarrow k_1 = 2\zeta\omega_n \qquad \Rightarrow \qquad \omega_n = \sqrt{k}$

$k = \omega_n^2 \qquad\qquad\qquad \zeta = \dfrac{k_1}{2\sqrt{k}}$

In-Class Assignment

1. Type 1 $\Rightarrow e_{ss} = 0 \quad \Rightarrow$ (b) is correct.

2. $e_{ss} = \dfrac{1}{K_v}, \quad K_v = \lim_{s \to 0} s\,K G(s)$

$$= \lim_{s \to 0} s \cdot \frac{K}{s(s+K_1)} = \frac{K}{K_1}$$

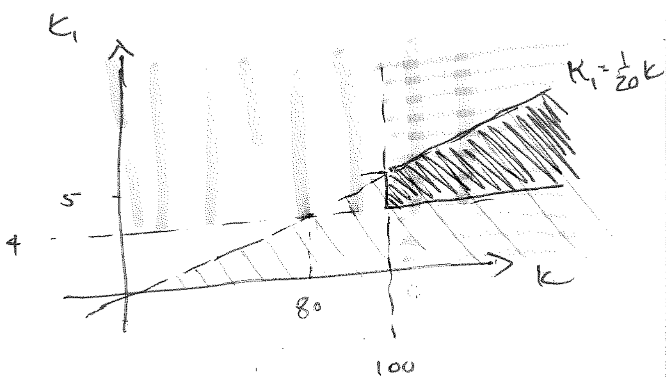
$$\therefore\ e_{ss} = \frac{K_1}{K}$$

3. (b) + (d) are correct.

4. (a) $T_s = \dfrac{4}{\zeta\omega_n} \le 2$

$$2 \le \zeta\omega_n$$

$$4 \le 2\cdot\zeta\omega_n = K_1$$



(b) $e_{ss} = \dfrac{K_1}{K} \le \dfrac{1}{20} = 0.05$

$$20\cdot K_1 \le K$$

(c) $y_{ss} = \dfrac{1}{K} \le \dfrac{1}{100}$

$$100 \le K$$

All 3 constraints can be satisfied w/ (K, K_1) taken from region.

5. y_{ss} and e_{ss} decrease as K increases. However, since $K = \sqrt{\omega_n}$ is increasing (and hence so is ω_n) and $T_s = \dfrac{4}{\zeta\omega_n}$ is constant $\Rightarrow \zeta\omega_n$ = constant, this means that ζ is decreasing as K increases. Hence the transient response will have excessive oscillating as K increases. The transient performance worsens as the steady-state perf. improves w/ increasing K.