PHYSICS 161, HW#2

#1

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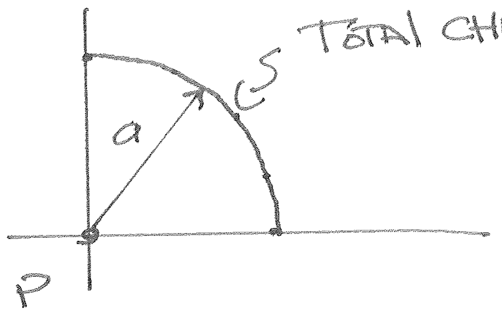
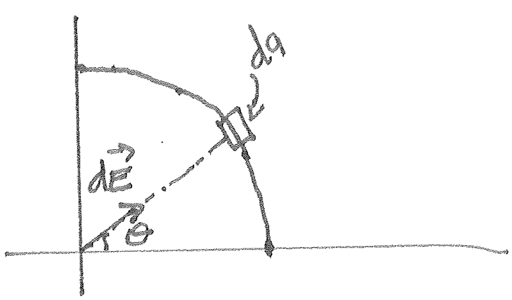


FIND E_x, E_y OF NET FIELD AT P=ORIGIN

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

For all points ON Circle $r=a$

$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{a^2}$$



$d\vec{E}$ TOWARDS dq SINCE Negatively CHARGED

FOR A CIRCLE: $\lambda = \frac{dq}{ds}$ ds = ARClength

$$\Rightarrow dq = \lambda ds$$

FROM PHYSICS I (OR GEOMETRY CLASS): $s = r\theta$ when θ IN RADIANS $\Rightarrow ds = r\,d\theta$ but $r = a$ HERE

$\Rightarrow ds = a\,d\theta$ WHERE $0 \le \theta \le \pi/2$ FOR A QUARTER CIRCLE.

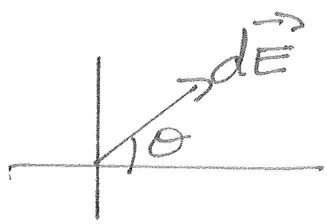
$\therefore\ dq = \lambda a\, d\theta$. WE ALREADY took CARE OF THE Negative Charge BY HAVING $d\vec{E}$ point towards dq

SO $\lambda = \frac{|-Q|}{S} = \frac{Q}{\underbrace{a(\pi/2)}} = \frac{2Q}{a\pi}$

↳ QUARTER Circle's ARCLENGTH

$dq = \left(\frac{2Q}{a\pi}\right)a\,d\theta = \frac{2Q}{\pi}\,d\theta$; $dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{a^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{1}{a^2}\right)\frac{2Q}{\pi}d\theta$

$= \frac{Q}{2\pi^2\epsilon_0 a^2}\,d\theta$



$dE_x = dE\cos\theta$

$\Rightarrow E_x = \int dE\cos\theta = \int_0^{\pi/2} \frac{Q}{2\pi^2\epsilon_0 a^2}\cos\theta\, d\theta$

$= \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\cos\theta\, d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2}\sin\theta\Big|_0^{\pi/2}$

$$\Rightarrow E_x = \frac{Q}{2\pi^2\epsilon_0 a^2}(1-0) = \frac{Q}{2\pi^2\epsilon_0 a^2}$$

$$dE_y = dE \sin\theta \Rightarrow E_y = \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2} \sin\theta \, d\theta$$

$$= \frac{Q}{2\pi^2\epsilon_0 a^2}\left[-\cos\theta\right]\Big|_0^{\pi/2} = \frac{Q}{2\pi^2\epsilon_0 a^2}\left[-(0-1)\right] =$$

$$\frac{Q}{2\pi^2\epsilon_0 a^2}\left[-(-1)\right] = \frac{Q}{2\pi^2\epsilon_0 a^2}$$

So $\boxed{E_x = E_y = \frac{Q}{2\pi^2\epsilon_0 a^2}}$ at ORIGIN

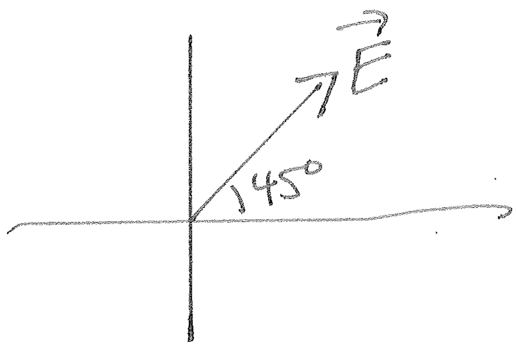
b) $\vec{E} = \frac{Q}{2\pi^2\epsilon_0 a^2}\sqrt{2}$ at 45°

Since $E = \sqrt{E_x^2 + E_y^2}$ AND $\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right)$

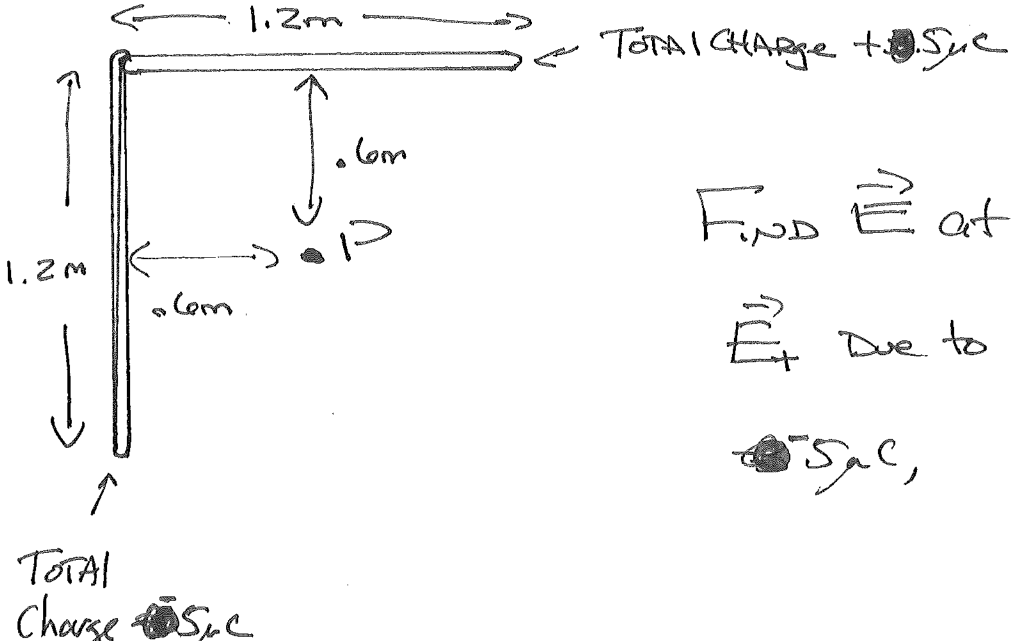
c. For $Q = 50\,\mu C$, $a = 5\,cm$

$$E = \frac{Q}{2\pi^2 \epsilon_0 a^2}\sqrt{2} = \frac{(50\times10^{-6}\,C)\sqrt{2}}{2\pi^2(8.85\times10^{-12}\,C^2/N\cdot m^2)(0.05\,m)^2}$$

$$\Rightarrow \boxed{E = 1.619\times10^{8}\,N/C = 1.6\times10^{8}\,N/C}$$

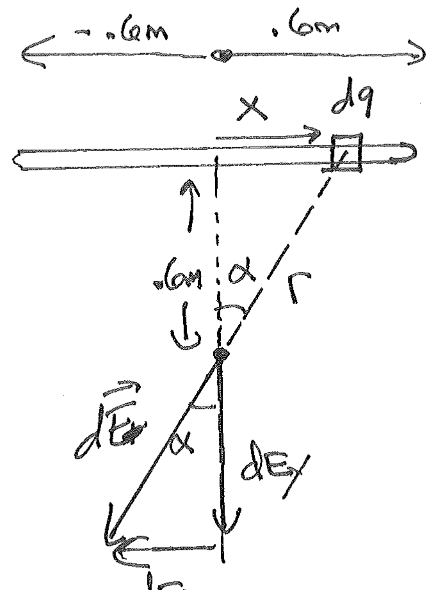


#2



Find $\vec{E}$ at P & Find $\vec{E}_+$ Due to +[redacted]5μC, $\vec{E}_-$ Due to [redacted]5μC, $\vec{E} = \vec{E}_+ + \vec{E}_-$

$\vec{E}_+$:



$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{r^2}$$

$$\lambda = \frac{[redacted]5\times10^{-6}C}{1.2m}, \quad \cos\alpha = \frac{.6m}{r}$$

substitute this later

$$\Rightarrow r = \frac{.6m}{\cos\alpha}$$

$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{\left(\frac{.6m}{\cos\alpha}\right)^2}$$

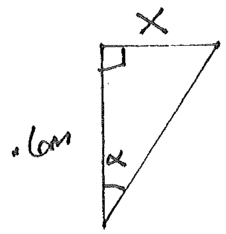
$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{(.6m)^2}\cos^2\alpha$$

Components: $dE_y = dE\cos\alpha = \left(\frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{(.6m)^2}\cos^2\alpha\right)\cos\alpha$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda\cos^3\alpha}{(.6m)^2}dx$$

$$E_y = \int_{-.6m}^{.6m} \frac{\lambda}{4\pi\epsilon_0} \frac{\cos^3\alpha}{(.6m)^2}\, dx = \frac{\lambda}{4\pi\epsilon_0} \int_{-.6m}^{.6m} \frac{\cos^3\alpha}{(.6m)^2}\, dx$$

CHANGE TO α AS INTEGRATION VARIABLE



$$\tan\alpha = \frac{x}{.6m} \Rightarrow x = .6m \tan\alpha$$

$$\Rightarrow dx = (.6m) \frac{1}{\cos^2\alpha}\, d\alpha$$

$x = .6m \Rightarrow \tan\alpha = 1 \Rightarrow \alpha = \pi/4$, $\quad x = -.6m \Rightarrow \alpha = -\pi/4$

$$\therefore E_y = \frac{\lambda}{4\pi\epsilon_0} \int_{-\pi/4}^{\pi/4} \frac{\cos^3\alpha}{(.6m)^2} \frac{(.6m)}{\cos^2\alpha}\, d\alpha = \frac{\lambda}{4\pi\epsilon_0} \int_{-\pi/4}^{\pi/4} \frac{\cos\alpha}{(.6m)}\, d\alpha$$

$$= \frac{\lambda}{4\pi\epsilon_0} \frac{1}{(.6m)} \int_{-\pi/4}^{\pi/4} \cos\alpha\, d\alpha$$

$\Leftarrow$ IN OTHER WORDS, AT CENTER OF FINITE WIRE, WE GET SAME INTEGRAL AS INFINITE WIRE BUT WITH DIFFERENT LIMITS

$$\therefore E_y = \frac{\lambda}{4\pi\epsilon_0} \left(\frac{1}{.6m}\right) \sin\alpha \Big|_{-\pi/4}^{\pi/4}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \left(\frac{1}{.6m}\right)\left(\frac{1}{\sqrt{2}} - \frac{-1}{\sqrt{2}}\right)$$

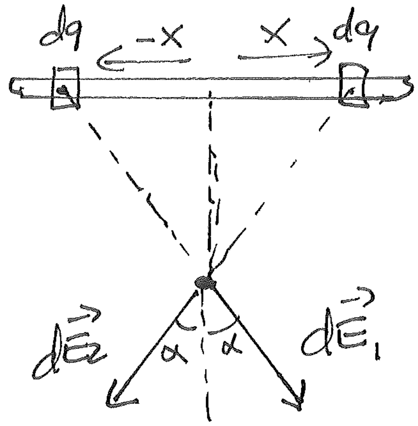
$$= \frac{\lambda}{4\pi\epsilon_0} \left(\frac{1}{.6m}\right)\sqrt{2}$$

IF YOU PREFER DECIMAL: $E_y = (9 \times 10^9\, N \cdot m^2/C^2)\left(\frac{.5 \times 10^{-6} C}{1.2m}\right)\left(\frac{1}{.6m}\right)\sqrt{2}$

$= \text{[illegible]}\ N/C$

$88388\ N/C$

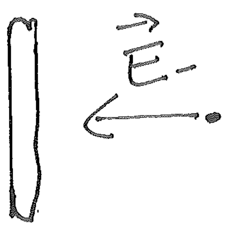
BY SYMMETRY (OR DOING THE INTEGRATION), $E_x = 0$



FOR EVERY dq at +x THERE IS ANOTHER dq at −x. THE FIELDS CREATED BY THESE PAIRS HAVE EQUAL MAGNITUDE AND ARE AT EQUAL ANGLES ⇒ X-COMPONENTS CANCEL.

$\underline{\vec{E}_-}$ $\quad \vec{E}_-$ is just $\vec{E}_+$ TURNED by $90° \Rightarrow E_x \neq 0,\ E_y = 0$

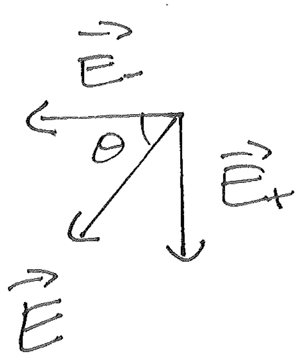
but SINCE NEGATIVELY CHARGED, $\vec{E}_-$ POINTS TOWARD LINE



ALREADY TOOK NEGATIVE INTO ACCOUNT

$$E_- = E_x = \frac{|\lambda|}{4\pi\epsilon_0}\left(\frac{1}{.6m}\right)\sqrt{2}$$

$\vec{E}_+, \vec{E}_-$ at $90° \Rightarrow$ COMPONENTS OF $\vec{E}$



$$\boxed{\theta = \tan^{-1}\left(\frac{E_+}{E_-}\right) = \tan^{-1}(1) = 45°}$$

$$E = \sqrt{E_+^2 + E_-^2} = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{.6m}\right)\underbrace{\sqrt{\sqrt{2}^2 + \sqrt{2}^2}}_{\sqrt{2+2} = \sqrt{4} = 2}$$

$$\Rightarrow E = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{.6m}\right)2 = (9 \times 10^9\ N m^2/C^2)\left(\frac{.5 \times 10^{-6} C}{1.2m}\right)\left(\frac{1}{.6m}\right)2 \Rightarrow$$

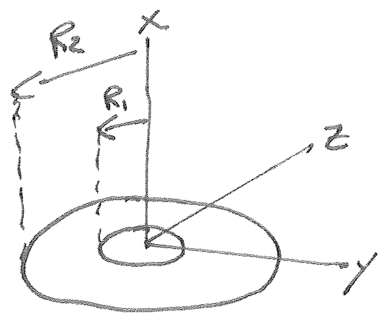
$$\boxed{E = 125000 N/C}$$

b) Electron at P : $\vec{F} = q\vec{E} \Rightarrow$

$$\boxed{F = (1.6 \times 10^{-19} C)(125000 N/C) = 2 \times 10^{-14} N}$$

$\vec{F}$ opposite to $\vec{E} \Rightarrow$ $\vec{F}$ $45°$

#3



Annulus with charge density, σ

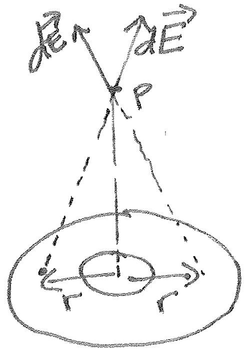
a) Determine Q_{TOTAL}.

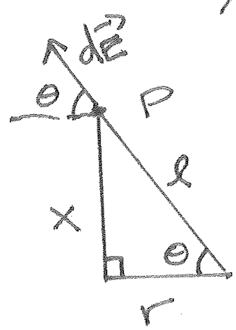
Constant $\sigma \Rightarrow Q_T = \sigma A = \sigma(\underbrace{\pi R_2^2}_{\text{"Big" circle}} - \underbrace{\pi R_1^2}_{\text{"Little" Circle}})$

$$\Rightarrow \boxed{Q = \sigma\pi(R_2^2 - R_1^2)}$$

b) Find $\vec{E}$ on X-axis

Above Annulus $(x > 0)$, By Symmetry $E_y = E_z = 0$





$$dE_x = \underbrace{dE\sin\theta}_{x \text{ is in the } \uparrow \text{ direction}} = (dE)\frac{x}{\ell} = dE\frac{x}{\sqrt{x^2+r^2}}$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{\ell^2} = \frac{1}{4\pi\epsilon_0}\frac{\sigma\, dA}{x^2+r^2} = \frac{1}{4\pi\epsilon_0}\frac{\sigma\, r\,dr\,d\phi}{x^2+r^2}$$

$$\Rightarrow dE_x = \frac{\sigma}{4\pi\epsilon_0} \frac{r\,dr\,d\phi}{x^2+r^2} \cdot \frac{x}{\sqrt{x^2+r^2}} = \frac{\sigma x}{4\pi\epsilon_0} \frac{r\,dr\,d\phi}{(x^2+r^2)^{3/2}}$$

$$\Rightarrow E_x = \int_0^{2\pi} d\phi \int_{R_1}^{R_2} dr \frac{\sigma x}{4\pi\epsilon_0} \frac{r\,dr}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{4\pi\epsilon_0} (2\pi) \int_{R_1}^{R_2} \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

$$= \frac{\sigma x}{2\epsilon_0} \int_{R_1}^{R_2} \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

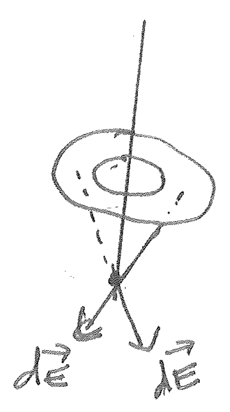
Let $u = x^2 + r^2 \Rightarrow du = 2r\,dr$

$\Rightarrow \frac{1}{2} du = r\,dr$

$$\Rightarrow E_x = \frac{\sigma x}{2\epsilon_0} \int_{x^2+R_1^2}^{x^2+R_2^2} \frac{du}{2} u^{-3/2} = \frac{\sigma x}{2\epsilon_0} \cdot \frac{1}{2} \cdot -2u^{-1/2} \Big|_{x^2+R_1^2}^{x^2+R_2^2}$$

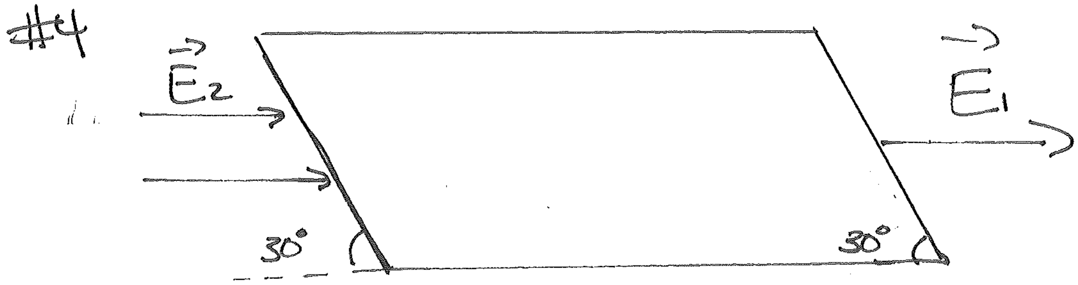
$$\Rightarrow \boxed{E_x = \frac{\sigma x}{2\epsilon_0} \left[\frac{-1}{\sqrt{x^2+R_2^2}} + \frac{1}{\sqrt{x^2+R_1^2}} \right]}$$

BELOW ANNULUS



STILL ONLY E_x, but E_x IN THE NEGATIVE X-direction. ~~but~~ BELOW ANNULUS make $x < 0$ $\Rightarrow$ CAN USE SAME EQUATION.

#4



$$E_2 = 7.5\times10^4\,N/C$$

$$E_1 = 3.5\times10^4\,N/C$$

a) FIND NET CHARGE :

FIELDS PASS THROUGH TWO FACES OF PARALLELPIPED :

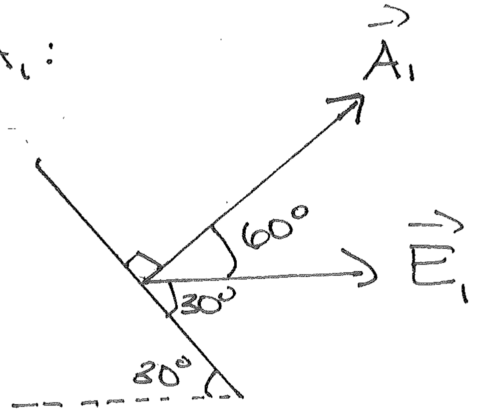
"FRONT" $= A_1 = \cancel{\vec{A}_1}$ AREA THAT $\vec{E}_1$ IS COMING OUT OF

"BACK" $= A_2 =$ AREA THAT $\vec{E}_2$ IS GOING INTO

$$A_1 = A_2 = (.05m)(.06m) = .003m^2$$

FOR BOTH AREAS, FIELD IS UNIFORM $\Rightarrow \Phi = \vec{E}\cdot\vec{A}$

FOR A_1 :



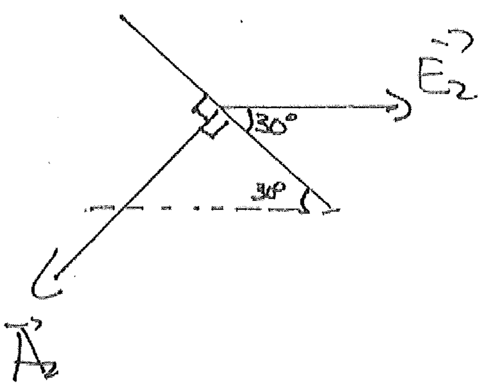
$\vec{A}_1$ IS 90° TO SURFACE

$\vec{E}_1$ IS HORIZONTAL $\Rightarrow$ 30° TO SURFACE

$$\Rightarrow \Phi_1 = \vec{E}_1\cdot\vec{A}_1 = E_1 A_1 \cos 60°$$

$$\Rightarrow \Phi_1 = E_1 A_1 \left(\tfrac{1}{2}\right) = \tfrac{1}{2}E_1 A_1$$

For A_2:



$\vec{A}_2$ points OUTWARDS and 90° to SURFACE

$$\therefore \quad \Phi_2 = \vec{E}_2 \cdot \vec{A}_2 = E_2 A_2 \cos 120^\circ$$

$$= E_2 A_2 \left(-\tfrac{1}{2}\right) = -\tfrac{1}{2} E_2 A_2$$

$$\Phi_{TOTAL} = \Phi_1 + \Phi_2 = \tfrac{1}{2} E_1 A_1 - \tfrac{1}{2} E_2 A_2$$

$$A_1 = A_2 \Rightarrow \Phi_{TOTAL} = \tfrac{1}{2}(E_1 - E_2) A = \tfrac{1}{2}\left(3.5\times10^4 \, N/C - 7.5\times10^4 \, N/C\right)(0.003 \, m^2)$$

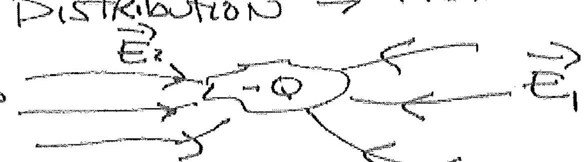
$$\Rightarrow \Phi_{TOTAL} = \tfrac{1}{2}\left(-4\times10^4 \, N/C\right)(0.003 \, m^2) \Rightarrow \boxed{\Phi_{TOTAL} = -60 \, N\cdot m^2/C}$$

Gauss's Law: $\Phi_{TOTAL} = \frac{Q_{encl}}{\epsilon_0} \Rightarrow Q_{encl} = \Phi_{total}\,\epsilon_0 = \left(-60 \, N\cdot m^2/C\right)\left(8.85\times10^{-12} \, \frac{C^2}{N\cdot m^2}\right)$

$$\Rightarrow \boxed{Q_{encl} = -5.31\times10^{-10} \, C = -0.531 \, nC}$$

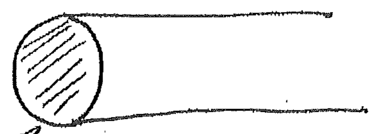
b) Is $\vec{E}_1, \vec{E}_2$ produced only by charge inside? → NO

If only a negative charge distribution ⇒ field lines would only go towards parallelpiped



but $\vec{E}_1$ is coming out ⇒ there must be negative charge outside as well.

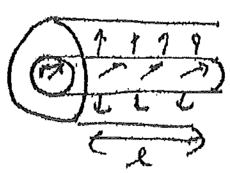
\#5



Solid cylinder, charge density ρ, RADIUS R.

a) find E for $r<R$

by symmetry E is outwards AND constant at constant r $\Rightarrow$ USE GAUSSIAN Cylinder



Flux through Top & Bottom $=0$

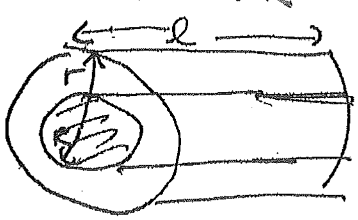
Area of sides

$$\oint \vec{E}\cdot d\vec{A} = EA = E(2\pi r \ell)$$

$$\frac{Q_{encl}}{\epsilon_0} = \rho\frac{(\pi r^2 \ell)}{\epsilon_0}$$

$$\Rightarrow E(2\pi r \ell) = \rho\frac{(\pi r^2 \ell)}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho r}{2\epsilon_0}}$$

b) find E for $r>R$



$$\oint \vec{E}\cdot d\vec{A} = E(2\pi r \ell), \quad \frac{Q_{enc}}{\epsilon_0} = \rho\frac{(\pi R^2 \ell)}{\epsilon_0}$$

only CHARGE up to R, so Q_{encl} the same for any value of r

$$\Rightarrow E(2\pi r \ell) = \rho\frac{(\pi R^2 \ell)}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho R^2}{2\epsilon_0 r}}$$

$$\rho = \frac{\text{Charge}}{\text{Volume}} = \frac{\text{charge}}{\text{Area} \times \text{length}} \Rightarrow \rho = \frac{1}{\text{Area}} \times \left(\frac{\text{charge}}{\text{length}}\right) = \frac{1}{\pi R^2} \lambda$$

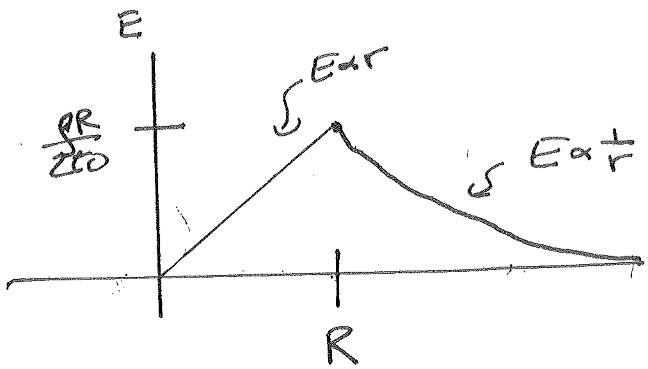
$$\Rightarrow \rho R^2 = \frac{\lambda}{\pi} \Rightarrow \boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

← SAME AS INFINITE WIRE (which is what A cylinder is outside)

c) $E|_{r=R}$ for both

part a: $E = \frac{\rho R}{2\epsilon_0}$ part b: $E = \frac{\rho R^2}{2\epsilon_0 R} = \frac{\rho R}{2\epsilon_0} \Rightarrow$ SAME Result

d) GRAPH E



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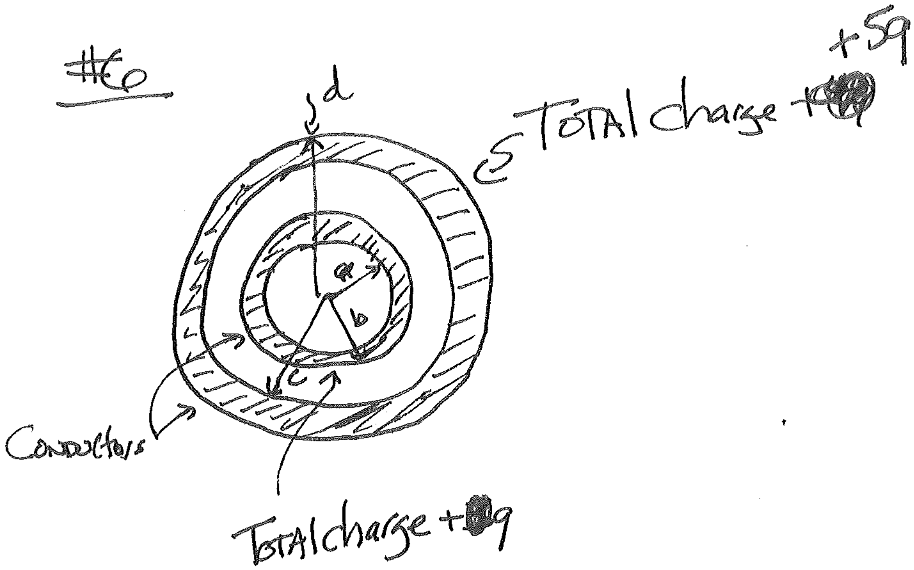


a) WHAT IS TOTAL CHARGE ON INNER AND OUTER SURFACE OF CONDUCTORS.

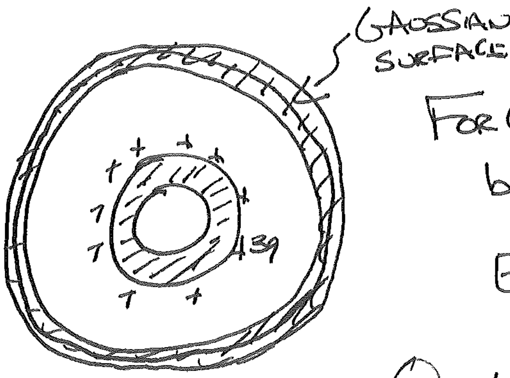
CONDUCTORS : HAVE $E = 0$ INSIDE AND ONLY HAVE CHARGE ON SURFACE.

~~By~~ GAUSS'S LAW ⇒ FOR ANY SURFACE WITH ITS BOUNDARY IN THE CONDUCTOR $\Phi_{TOTAL} = 0$ SINCE $E = 0$ ⇒ $Q_{encl} = 0$

NO CHARGE FOR $r < a$ ⇒ THERE CAN BE NO CHARGE ON INNER SURFACE OF SMALL shell, i.e, $r = a$

OTHERWISE, GAUSSIAN SURFACE at ~~r~~ $a < r < b$ would have $Q_{encl} \neq 0$ but $E = 0$ for $a < r < b$.

All CHARGE ON SURFACE ⇒ All OF ~~q~~ $+q$ CHARGE MUST BE ON OUTER SURFACE OF SMALL Shell, i.e., $r = b$



FOR GAUSSIAN SURFACE WITH BOUNDARY between c & d

$E = 0$ for $c < r < d \Rightarrow Q_{encl} = 0$

$Q_{encl} = Q_{IS} + Q_{OS} + Q_{IL}$

Q_{IS}: charge ON INNER SURFACE of Small shell

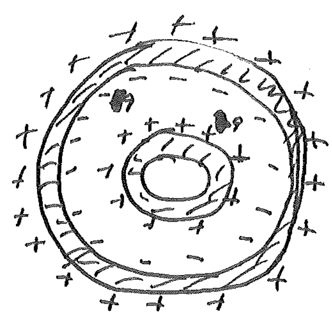
Q_{OS}: charge ON outer SURFACE OF SMALL shell

Q_{IL}: charge ON INNER SURFACE OF LARGE shell

$Q_{IS} = 0, \quad Q_{OS} = q, \quad Q_{IL} = ?$

$0 = 0 + q + Q_{IL} \Rightarrow \boxed{Q_{IL} = -q}$

so:



Outer shell has total charge $+5q$

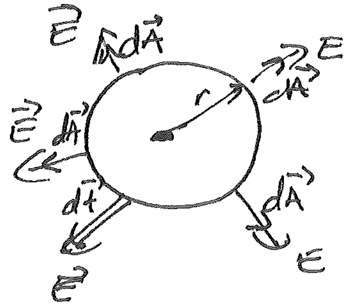
THERE IS $-q$ ON INNER SURFACE

$\Rightarrow \boxed{Q_{OL} = +6q}$ to make $Q_{IL} + Q_{OL} = +5q$

Q_{OL}: outer SURFACE, LARGE Shell

Notice: So for distances LARGER than $r = d$, the whole ARRANGEMENT MAKES the SAME Electric field AS A point Charge with VALUE $+6q$ = TOTAL CHARGE OF the two Distributions. This is why we CAN TREAT SO MANY objects AS point Charges. If they're "SMALL", their Fields look like point charges.

b) FIND E: FROM textbook AND Symmetry, we KNOW THAT SPHERICAL CONDUCTORS Create RADIAL FIELDS $\Rightarrow$ INWARD OR OUTWARD WITH Constant magnitude at A FIXED RADIUS $\Rightarrow$ FOR A GAUSSIAN sphere Concentric WITH SHELLS

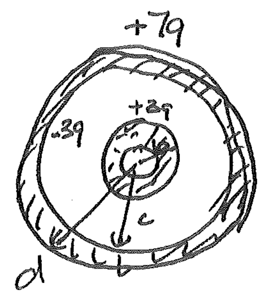


$$\oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA = EA = \Phi_{TOTAL}$$

FOR A SPHERE $A = 4\pi r^2$

GAUSS'S LAW: $\Phi_{TOTAL} = \dfrac{Q_{encl}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \dfrac{Q_{encl}}{\epsilon_0}$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q_{encl}}{r^2}$$



For $0<r<a$, $Q_{encl} = 0 \Rightarrow E = 0$

For $a<r<b$, $E=0$ since $Q_{encl} = 0$ (AND INSIDE CONDUCTOR)

For $b<r<c$, $Q_{encl} = +q \Rightarrow \boxed{E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}}$

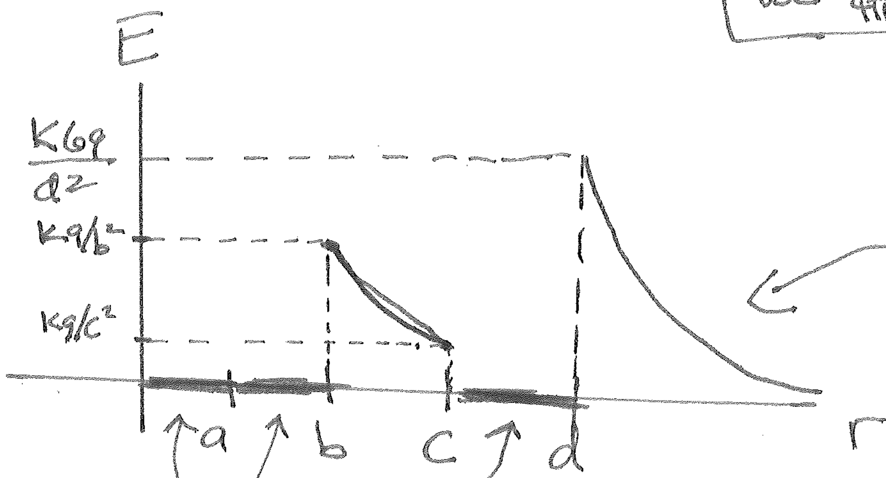
positive $q \Rightarrow$ OUTWARD field

For $c<r<d$, $E=0$ since $Q_{encl}=0$ (INSIDE CONDUCTOR)

For $r>d$, $Q_{encl} = +6q \Rightarrow \boxed{E = \dfrac{1}{4\pi\epsilon_0}\dfrac{6q}{r^2}}$ OUTWARD

In graph Form

$\boxed{\text{Use } \frac{1}{4\pi\epsilon_0} = K}$

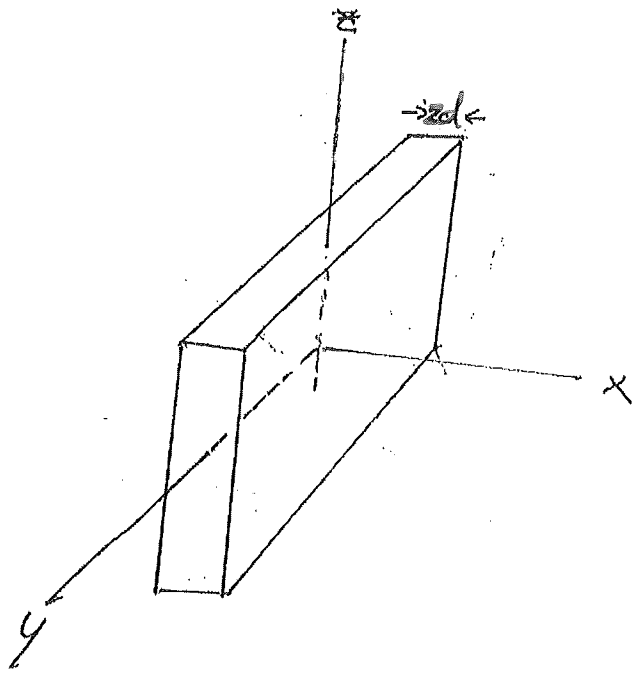


$\underbrace{E = 0}$

Inside Distribution, Electric field is "Crazy" = different values at different places

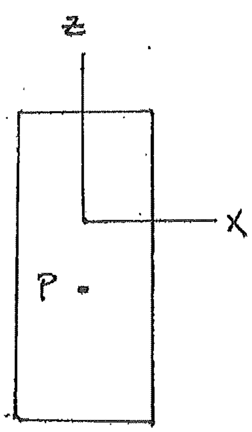
Outside distribution, Electric field looks just like A point charge, so we don't have to worry about all the complications!

#7



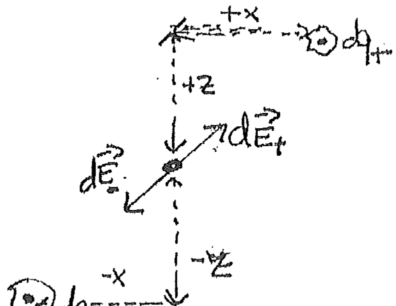
LARGE INSULATING SLAB, WITH DENSITY ρ

a) Explain why $E=0$ at $x=0$



By symmetry at any Point P along the $x=0$ line:

- Any $d\vec{E}_+$ due to charge to the right and up of P is canceled by a $d\vec{E}_-$ due to charge to left and down of P. → To include y, think into page and out of page too



$$d\vec{E}_+ + d\vec{E}_- = 0$$

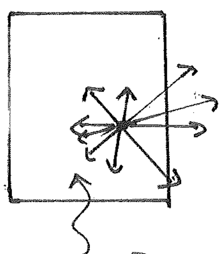
Of course only true because we assume z and y are infinite

b) Find $\vec{E}$ at all points

AT ANY POINT OFF $X=0$ THERE IS "MORE" CHARGE ON ONE SIDE THAN THE OTHER IN X. But still "EQUAL" CHARGE Along Y AND Z

$\Rightarrow$ $\vec{E}$ IN the x-direction

For $X>0$



$\vec{dE}_{+}$'s greater $+x$ components since more charge to left of P

$\vec{dE}_{-}$'s SAME y's as $\vec{E}_{+}$'s but smaller x-components

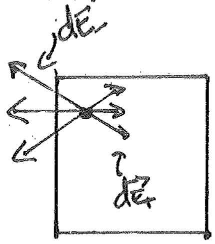
$\Rightarrow$ $\overset{P}{\bullet}\!\longrightarrow \vec{E}$

AGAIN BECAUSE $-\infty<y<\infty$, $-\infty<z<\infty$

true FOR ANY point on the $x=x_0$ LINE

$\Rightarrow$ E constant for $X=$ constant

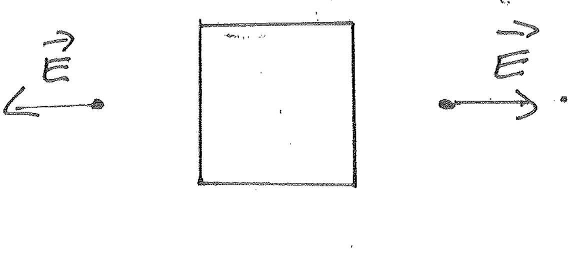
For $X<0$



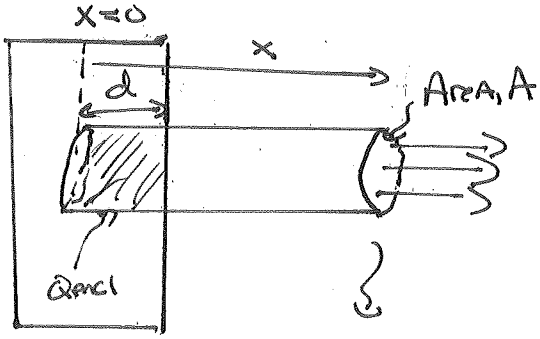
$\Rightarrow$ $\vec{E} \longleftarrow\!\bullet$

$\vec{E}$ IN negative x-component

SAME RESULT FOR points OUTSIDE OF SLAB



$\vec{E}$ IN x-direction $\Rightarrow$ USE GAUSSIAN CYLINDER WITH TOP AND BOTTOM parallel to x. Flux Through SIDES will BE ZERO.



$x > d$

$\Phi_{TOP} = EA$
Since E constant

Put other end of cylinder at $x=0$ so $E=0$

$\Rightarrow \Phi_{BOTTOM} = 0$

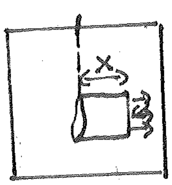
$$\Phi_{TOTAL} = \Phi_{Top} + \Phi_{Bottom} + \Phi_{sides} = EA + 0 + 0 = EA$$

$Q_{encl} = \rho\,\text{Volume} = \rho A d$

Gauss's Law $\Rightarrow EA = \rho \frac{Ad}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho d}{\epsilon_0}}$

Same as infinite sheet of charge

$0 < x < d$



$\Phi_{TOTAL} = EA$ still, $Q_{encl} = \rho A x$

$\Rightarrow EA = \frac{\rho A x}{\epsilon_0} \Rightarrow \boxed{E = \frac{\rho x}{\epsilon_0}}$

For $-d < x < 0$ and $x \leq -d$ same results $E = \frac{\rho x}{\epsilon_0}$ ($x<0$ gives $E = \leftarrow$ as it should)

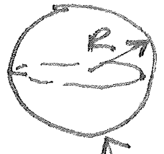
$x \leq -d$ $E = \frac{\rho d}{\epsilon_0}$ with $\leftarrow$ direction

So maybe be $\vec{E} = -\frac{\rho d}{\epsilon_0}\hat{\imath}$ to be careful

#8

$$\rho(r) = \begin{cases} \rho_0 \left(1 - r^2/R^2\right) & r \leq R \\ 0 & r > R \end{cases}$$

Where $\rho_0 = \dfrac{15Q}{8\pi R^3}$



Non-Uniform Distribution

a) Show total charge is Q.

$\rho = \dfrac{dQ}{dV} \Rightarrow dq = \rho \, dV$

$\Rightarrow \int dq = \int \rho \, dV \Rightarrow \boxed{Q_{encl} = \int \rho \, dV}$

For spherically symmetric Distribution $dV = 4\pi r^2 dr$

Such That $\int dV = \int 4\pi r^2 dr = 4\pi \int r^2 dr = 4\pi \left(\frac{r^3}{3}\right) = \frac{4}{3}\pi r^3 =$ Volume!

∴ Since there's only CHARGE for $0 < r < R$, $Q_{TOTAL} = \int_0^R \rho \, 4\pi r^2 dr$

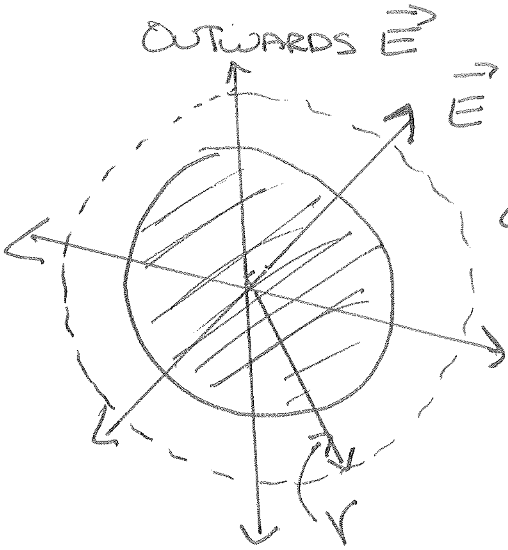
$$\Rightarrow Q_{TOTAL} = \int_0^R \rho_0 \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr = 4\pi \rho_0 \int_0^R \left(r^2 - \frac{r^4}{R^2}\right) dr$$

$$= 4\pi \rho_0 \left[\frac{r^3}{3} - \frac{r^5}{5R^2}\right]\Bigg|_0^R = 4\pi \rho_0 \left[\frac{R^3}{3} - \frac{R^5}{5R^2}\right] = 4\pi \rho_0 \left[\frac{R^3}{3} - \frac{R^3}{5}\right]$$

$$= 4\pi \rho_0 R^3 \left(\frac{1}{3} - \frac{1}{5}\right) = 4\pi \rho_0 R^3 \left(\frac{2}{15}\right) = \frac{8\pi}{15} R^3 \rho_0 = \frac{8\pi}{15} R^3 \left(\frac{15Q}{8\pi R^3}\right)$$

$$= Q.$$

b) SHOW FOR $r \geq R$ that E = point charge $= \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$



GAUSSIAN sphere BECAUSE $\vec{E}$ & $d\vec{A}$ parallel

By symmetry E has SAME MAGNITUDE for constant r.

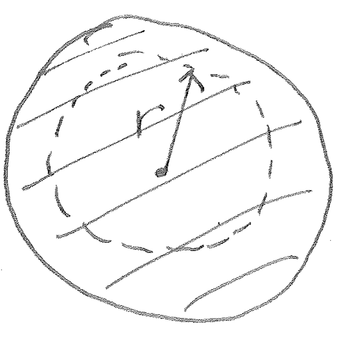
$\Rightarrow \Phi_E = EA = E(4\pi r^2)$

For $r > R$, $Q_{encl} = Q$ (AS WE just SHOWED)

GAUSS'S LAW, $\Phi_E = Q_{encl}/\epsilon_0$

$\Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}}$

c) $r \leq R$ SAME GAUSSIAN sphere AND $\Phi_E = E(4\pi r^2)$ [FOR THE SAME REASONS]



$r < R$, less than total Q enclosed

$Q_{encl} = \int_0^r \rho \, 4\pi r^2 dr = \int_0^r \rho_0 (1 - r^2/R^2) 4\pi r^2 dr$

$= 4\pi\rho_0 \int_0^r (r^2 - r^4/R^2) dr$

$= 4\pi\rho_0 \left[\frac{1}{3}r^3 - \frac{1}{5R^2}r^5\right]\Big|_0^r = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$

$$Q_{encl} = 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right) = 4\pi\left(\frac{15Q}{8\pi R^3}\right)\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

$$= \cancel{4\pi}\left(\frac{15Q}{\cancel{8\pi}_{\,2} R^3}\right)\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right) = \frac{15Q}{2R^3}\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

$$= Q\left(\frac{15r^3}{6R^3} - \frac{15r^5}{10R^5}\right) = Q\left(\frac{5r^3}{2R^3} - \frac{3r^5}{2R^5}\right)$$

$$= \frac{Q}{2}\left(\frac{5r^3}{R^3} - \frac{3r^5}{R^5}\right)$$

GAUSS'S LAW: $\Phi_E = \dfrac{Q_{encl}}{\varepsilon_0} \Rightarrow E(4\pi r^2) = \dfrac{Q}{2\varepsilon_0}\left(\dfrac{5r^3}{R^3} - \dfrac{3r^5}{R^5}\right)$

$$\Rightarrow E = \frac{Q}{8\pi\varepsilon_0 r^2}\left(\frac{5r^3}{R^3} - \frac{3r^5}{R^5}\right)$$

$$\Rightarrow \boxed{E = \frac{Q}{8\pi\varepsilon_0}\left(\frac{5r}{R^3} - \frac{3r^3}{R^5}\right)}$$

($\vec{E}$ is outwards.)

at $r = R$

$$E = \frac{Q}{8\pi\varepsilon_0}\left(\frac{5R}{R^3} - \frac{3R^3}{R^5}\right) = \frac{Q}{8\pi\varepsilon_0}\left(\frac{5}{R^2} - \frac{3}{R^2}\right)$$

$$= \frac{Q}{8\pi\varepsilon_0}\left(\frac{2}{R^2}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{R^2}$$

OUTSIDE $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ So at $r = R$ $E = \dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{R^2}$ SAME

So, even for non-uniform charge distributions

if they are spherically symmetric, they will look like a point charge outside. This is why we can treat ~~Atoms~~ ions as point charges.

We know they really aren't uniform or even a charged "point", but Gauss's Law tells us that their field looks like they are!