

9. (a) The Earth time required for the trip is d/v . The proper time is $(d/v)\sqrt{1 - v^2/c^2}$, and this should equal 10 years,

$$\frac{25,000 \text{ years} \times c}{v} \sqrt{1 - v^2/c^2} = 10 \text{ years}$$

We can approximate $v \simeq c$ in the denominator, and $1 - v^2/c^2 \simeq 2(1 - v/c)$ under the square root. Then

$$(25,000)^2 \times 2 \times (1 - v/c) \simeq 10^2$$

from which

$$v/c \simeq 1 - 8.0 \times 10^{-8}$$

- (b) The Earth time is $t = d/v \simeq 25,000 \text{ light-years}/c = 25,000 \text{ years}$

11. The worldlines intersect at $vt = \frac{1}{2}at^2$, that is, $t = 2v/a$. For the worldline of constant velocity,

$$\int_0^{2v/a} \sqrt{1 - v^2/c^2} dt = \frac{2v}{a} \sqrt{1 - v^2/c^2}$$

and for the curved worldline,

$$\int_0^{2v/a} \sqrt{1 - a^2 t^2/c^2} dt = \frac{c}{2a} \left[\frac{2v}{c} \sqrt{1 - 4v^2/c^2} + \sin^{-1} \frac{2v}{c} \right]$$

The first of these is larger, and gives the longer proper time.

16. In the spaceship frame, the time for the forward trip and the time for the backward trip are the same, $\Delta t' = L/c$

In the laboratory frame, the distance between the mirrors is contracted to $L\sqrt{1 - V^2/c^2}$, and the difference between the speeds of the light signal and the mirrors is $c - V$ on the forward trip, and $c + V$ for the backward trip. Hence

$$\Delta t = \frac{L\sqrt{1 - V^2/c^2}}{c - V} \quad \text{for the forward trip and}$$

$$\Delta t = \frac{L\sqrt{1 - V^2/c^2}}{c + V} \quad \text{for the backward trip.}$$

The roundtrip time in the spaceship frame is $2L/c$, and the roundtrip time in the laboratory frame is

$$\frac{L\sqrt{1 - V^2/c^2}}{c - V} + \frac{L\sqrt{1 - V^2/c^2}}{c + V} = \frac{2L}{c} \frac{1}{\sqrt{1 - V^2/c^2}}$$

This is in agreement with the expected time dilation.

19. As received by the mirror, the frequency of the light is Doppler-shifted according to Eq. (19),

$$\nu' = \sqrt{\frac{1 + v/c}{1 - v/c}} \nu$$

The mirror acts as a source of frequency ν' . When you receive this light, it is again

Doppler-shifted according to Eq. (19), so the frequency you receive is

$$\nu'' = \sqrt{\frac{1 + v/c}{1 - v/c}} \nu' = \frac{1 + v/c}{1 - v/c} \nu$$

21. When approaching the Sun,

$$\lambda = \sqrt{\frac{1 - v/c}{1 + v/c}} \times 550 \text{ nm} = \sqrt{\frac{1 - 0.8}{1 + 0.8}} \times 550 \text{ nm} = 183 \text{ nm}$$

When abeam,

$$\lambda = \frac{1}{\sqrt{1 - v^2/c^2}} \times 550 \text{ nm} = \frac{1}{\sqrt{1 - 0.8^2}} \times 550 \text{ nm} = 917 \text{ nm}$$

When receding from the Sun,

$$\lambda = \sqrt{\frac{1 + v/c}{1 - v/c}} \times 550 \text{ nm} = \sqrt{\frac{1 + 0.8}{1 - 0.8}} \times 550 \text{ nm} = 1650 \text{ nm}$$

36. In the reference frame of the laboratory, both ends of the meter rod reach equal

heights at the same time (the rod descends while remaining parallel to the street). However, in the reference frame of the spaceship, the clocks at the "leading" end of the laboratory reference frame (the left end) are late, and consequently the rod's left end reaches a given height later than the right end. This means the rod is tilted, with its left end higher than its right. The rod slides through the manhole at an angle (see Figure 3.24b).

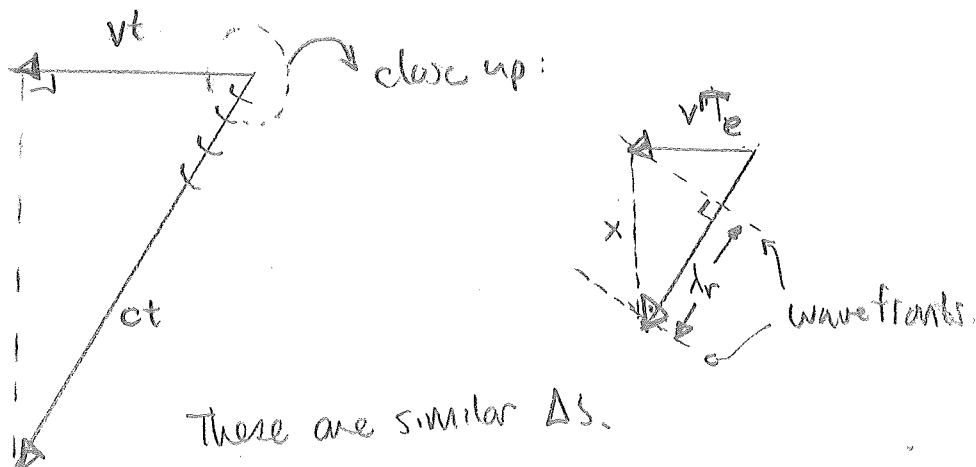
38. An instant of spaceship time, say, $t' = 0$, corresponds to $t = 0$ at $x = 0$ in the laboratory frame, and it corresponds to $t = xV/c^2$ at $x > 0$. Suppose that this instant of spaceship time corresponds to the instant when the rear end of the rod has just barely reached the gap and begun to fall. At this instant, a point of the rod with some given value $x > 0$ will already have fallen a distance $y' = y = -\frac{1}{2}gt^2 = -\frac{1}{2}g(xV/c^2)^2$. Hence

the instantaneous configuration of the rod is bent along a downward parabolic arc, as illustrated in Fig. 3.25b, and the rod "wriggles" out of the gap.

"Transverse" could be where the emitter is when emitting, or where the receiver is when detecting. It cannot be both.

Both observers will agree on what event (emission or detection) is transverse.

The book formula is for a transverse emitter. For a transverse detector, the emitter must have been moving slightly toward the detector when the signal was emitted, as shown below.



These are similar Δ s.

$$\frac{\lambda_r}{x} = \frac{x}{cT_e} \quad \text{Time between emissions, mod. by receiver}$$

$$\lambda_r = \frac{x^2}{cT_e} = \frac{c^2T_e^2 - v^2T_e^2}{cT_e} = \frac{1}{\gamma^2} cT_e$$

Because moving clocks run slow, $T_e = \gamma T_e'$

$$\text{So } \lambda_r = \frac{1}{\gamma^2} cT_e' = \frac{1}{\gamma} \frac{cT_e'}{\gamma} = \frac{\lambda_e'}{\gamma}$$

$$\text{And } f_r = \gamma f_e'$$

This is the same result you get (more easily) from the emitter frame: the detector clock is slow!