HW 2 Solutions

Spring 2013 Physics 262

33.57. a. The distance traveled at speed v_1 is $d_1 = \sqrt{h_1^2 + x^2}$. The distance traveled at speed v_2 is $d_2 = \sqrt{h_2^2 + (l-x)^2}$. Therefore the time taken is

$$t = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l-x)^2}}{v_2}.$$

b. $$\frac{\partial t}{\partial x} = \frac{1}{v_1}\frac{1}{2}\frac{2\,x}{\sqrt{h_1^2 + x^2}} + \frac{1}{v_2}\frac{1}{2}\frac{2\,(l-x)\,(-1)}{\sqrt{h_2^2 + (l-x)^2}} = \frac{x}{v_1\sqrt{h_1^2 + x^2}} - \frac{l-x}{v_2\sqrt{h_2^2 + (l-x)^2}},$$

so when $\frac{\partial t}{\partial x} = 0$,

$$\frac{x}{v_1\sqrt{h_1^2 + x^2}} = \frac{l-x}{v_2\sqrt{h_2^2 + (l-x)^2}}.$$

(If you want to be careful, you can check

$$\begin{aligned}
\frac{\partial^2 t}{\partial x^2} &= \frac{1}{v_1\sqrt{h_1^2 + x^2}} + \frac{1}{v_1}\left(-\frac{1}{2}\right)\frac{x\,(2\,x)}{(h_1^2 + x^2)^{3/2}} + \\
&\quad \frac{1}{v_2\sqrt{h_2^2 + (l-x)^2}} - \frac{1}{v_2}\left(-\frac{1}{2}\right)\frac{(l-x)\,2\,(l-x)\,(-1)}{[h_2^2 + (l-x)^2]^{3/2}} \\
&= \frac{h_1^2 + x^2 - x^2}{v_1\,(h_1^2 + x^2)^{3/2}} + \frac{h_2^2 + (l-x)^2 - (l-x)^2}{[h_2^2 + (l-x)^2]^{3/2}} \\
&= \frac{h_1^2}{v_1\,(h_1^2 + x^2)^{3/2}} + \frac{h_2^2}{[h_2^2 + (l-x)^2]^{3/2}} > 0,
\end{aligned}$$

so this is a minimum and not some other stationary point.)

But

$$\sin\theta_1 = \frac{x}{d_1} = \frac{x}{\sqrt{h_1^2 + x^2}}, \qquad \sin\theta_2 = \frac{l-x}{d_2} = \frac{l-x}{\sqrt{h_2^2 + (l-x)^2}},$$

so we have

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

$$n_1\sin\theta_1 = n_2\sin\theta_2.$$

33.61. a. The maximum transmitted intensity occurs when $\theta = \phi$. The largest intensity in the table is $24.8\,\mathrm{W/m^2}$, occurring at $\phi = 30°$ and $\phi = 40°$, so the actual maximum is probably at $\phi = 35°$. Therefore $\theta = 35°$.

b. When $|\phi - \theta| = 90°$, meaning $\phi = 125°$, all the polarized light is blocked and half the unpolarized light is blocked, so $\frac{1}{2}\, I_0 = 5.2\,\mathrm{W/m^2}$, $I_0 = 10.4\,\mathrm{W/m^2}$. When $\phi = \theta$, $I = \frac{1}{2}\, I_0 + I_{\mathrm{p}}$, so $I_{\mathrm{p}} = 24.8\,\mathrm{W/m^2} - 5.2\,\mathrm{W/m^2} = 19.6\,\mathrm{W/m^2}$.