

**28.3. IDENTIFY:** A moving charge creates a magnetic field.

**SET UP:** The magnetic field due to a moving charge is  $B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2}$ .

**EXECUTE:** Substituting numbers into the above equation gives

$$(a) B = \frac{\mu_0}{4\pi} \frac{qv \sin \phi}{r^2} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} (1.6 \times 10^{-19} \text{ C})(3.0 \times 10^7 \text{ m/s}) \sin 30^\circ}{(2.00 \times 10^{-6} \text{ m})^2}.$$

$B = 6.00 \times 10^{-8} \text{ T}$ , out of the paper, and it is the same at point  $B$ .

$$(b) B = (1.00 \times 10^{-7} \text{ T} \cdot \text{m/A})(1.60 \times 10^{-19} \text{ C})(3.00 \times 10^7 \text{ m/s})/(2.00 \times 10^{-6} \text{ m})^2$$

$B = 1.20 \times 10^{-7} \text{ T}$ , out of the page.

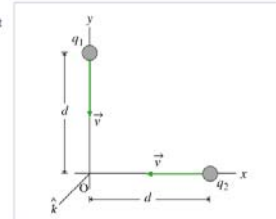
$$(c) B = 0 \text{ T since } \sin(180^\circ) = 0.$$

**EVALUATE:** Even at high speeds, these charges produce magnetic fields much less than the earth's magnetic field.

#### Force between Moving Charges

Description: compare electric and magnetic forces between moving charges. 1 part requires Biot-Savart law.

Two point charges, with charges  $q_1$  and  $q_2$ , are each moving with speed  $v$  toward the origin. At the instant shown  $q_1$  is at position  $(0, d)$  and  $q_2$  is at  $(d, 0)$ . (Note that the signs of the charges are not given because they are not needed to determine the magnitude of the forces between the charges.)



#### Part A

What is the magnitude of the electric force between the two charges?

Express  $F$  in terms of  $q_1$ ,  $q_2$ ,  $d$ , and  $\epsilon_0$ .

[Hints \(2\)](#)

**Hint 1.** Which law to use

Apply Coulomb's law:

$$F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2},$$

where  $r$  is the distance between the two charges.

**Hint 2.** Find the value of  $r^2$

What is the value of  $r^2$  for the given situation?

Express your answer in terms of  $d$ .

ANSWER:

$$r^2 = 2(d^2)$$

ANSWER:

$$F = \frac{q_1 q_2}{8\pi \epsilon_0 d^2}$$

#### Part B

What is the magnitude of the magnetic force on  $q_2$  due to the magnetic field caused by  $q_1$ ?

Express the magnitude of the magnetic force in terms of  $q_1$ ,  $q_2$ ,  $v$ ,  $d$ , and  $\mu_0$ .

**Hint 1.** How to approach the problem

First, find the magnetic field generated by charge  $q_1$  at the position of charge  $q_2$ . Then evaluate the magnetic force on  $q_2$  due to the field of  $q_1$ .

**Hint 2.** Magnitude of the magnetic field

The Biot-Savart law, which gives the magnetic field produced by a moving charge, can be written

$$\vec{B} = \frac{\mu_0 q \vec{v} \times \vec{r}}{4\pi r^3},$$

where  $\mu_0$  is the permeability of free space and  $\vec{r}$  is the vector from the charge to the point where the magnetic field is produced. Note we have  $\vec{r}$  in the numerator, not  $\hat{r}$ , necessitating an extra power of  $r$  in the denominator.

Using this equation find the expression for the magnitude of the magnetic field experienced by charge  $q_2$  due to charge  $q_1$ .

Express the magnitude of the magnetic field of  $q_1$  (at the location of  $q_2$ ) in terms of  $q_1$ ,  $v$ ,  $d$ , and  $\mu_0$ .

[Hints \(1\)](#)

ANSWER:

$$B = \frac{\mu_0 q_1 v}{8\sqrt{2}\pi d^2}$$

**Hint 3.** Find the direction of the magnetic field

Which of the following best describes the direction of the magnetic field from  $q_1$  at  $q_2$ ? Remember, according to the Biot-Savart law, the field must be perpendicular to both  $\vec{v}$  and  $\vec{r}$ .

Ignore the effects of the sign of  $q_1$ .

ANSWER:

- ☐  $\pm \hat{i}$  (along the  $x$  axis)
- ☐  $\pm \hat{j}$  (along the  $y$  axis)
- ☒  $\pm \hat{k}$  (along the  $z$  axis into or out of the screen)

#### Hint 4. Computing the force

You can evaluate the force exerted on a moving charge by a magnetic field using the Lorentz force law:

$$\vec{F} = q\vec{v} \times \vec{B},$$

where  $\vec{F}$  is the force on the moving charge,  $\vec{B}$  is the magnetic field,  $q$  is the charge of the moving charge, and  $\vec{v}$  is the velocity of the charge. Note that, as long as  $\vec{v}$  and  $\vec{B}$  are perpendicular,  $\sin\theta = 1$ .

ANSWER:

$$F = \frac{\mu_0 q_1 q_2 v^2}{8\sqrt{2}\pi d^2}$$

#### Part C

Assuming that the charges are moving nonrelativistically ( $v \ll c$ ), what can you say about the relationship between the magnitudes of the magnetic and electrostatic forces?

**Hint 1.** How to approach the problem

Determine which force has a greater magnitude by finding the ratio of the electric force to the magnetic force and then applying the approximation. Recall that  $\epsilon_0\mu_0 = 1/c^2$ .

ANSWER:

- ☐ The magnitude of the magnetic force is greater than the magnitude of the electric force.
- ☒ The magnitude of the electric force is greater than the magnitude of the magnetic force.
- ☐ Both forces have the same magnitude.

This result holds quite generally. Magnetic forces between moving charges are much smaller than electric forces as long as the speeds of the charges are nonrelativistic.

#### Exercise 28.17

**Description:** Lightning bolts can carry currents up to approximately 20 kA. We can model such a current as the equivalent of a very long, straight wire. (a) If you were unfortunate enough to be  $d$  away from such a lightning bolt, how large a magnetic field would...

Lightning bolts can carry currents up to approximately 20 kA. We can model such a current as the equivalent of a very long, straight wire.

#### Part A

If you were unfortunate enough to be 5.2 m away from such a lightning bolt, how large a magnetic field would you experience?

Express your answer using two significant figures.

ANSWER:

$$B_1 = \frac{2 \cdot 10^{-7} \cdot 20000}{d} = 7.7 \times 10^{-4} \text{ T}$$

#### Part B

How does this field ( $B_1$ ) compare to one ( $B_2$ ) you would experience by being 5.2 cm from a long, straight household current of 10 A?

Express your answer using two significant figures.

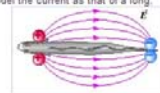
ANSWER:

$$\frac{B_1}{B_2} = 20$$

#### Exercise 28.20

**Description:** Certain fish, such as the Nile fish (Gnathonemus), concentrate charges in their head and tail, thereby producing an electric field in the water around them. This field creates a potential difference of a few volts between the head and tail, which in

certain fish, such as the Nile fish (Gnathonemus), concentrate charges in their head and tail, thereby producing an electric field in the water around them. This field creates a potential difference of a few volts between the head and tail, which in turn causes current to flow in the conducting seawater. As the fish swims, it passes near objects that have resistivities different from that of seawater, which in turn causes the current to vary. Cells in the skin of the fish are sensitive to this current and can detect changes in it. The changes in the current allow the fish to navigate. Since the electric field is weak far from the fish, we shall consider only the field running directly from the head to the tail. We can model the seawater through which that field passes as a conducting tube of area  $A$  and having a potential difference across its ends. These fish navigate by responding to changes in the current in seawater. This current is due to a potential difference of around 3.00 V generated by the fish and is about 12.0 mA within a centimeter or so from the fish. Receptor cells in the fish are sensitive to the current. Since the current is at some distance from the fish, their sensitivity suggests that these cells might be responding to the magnetic field created by the current. To get some estimate of how sensitive the cells are, we can model the current as that of a long, straight wire with the receptor cells 2.00 cm away.



#### Part A

What is the strength of the magnetic field at the receptor cells?

ANSWER:

$$B = 0.12 \text{ } \mu\text{T}$$

### Magnetic Field from Two Wires

**Description:** Calculate, at several different locations, the net magnetic field due to two straight infinite wires carrying anti-parallel currents. Students are asked to look for a pattern in the results as the points of interest become more and more remote from the wires.

#### Learning Goal:

To understand how to use the principle of superposition in conjunction with the Biot-Savart (or Ampere's) law.

From the Biot-Savart law, it can be calculated that the magnitude of the magnetic field due to a long straight wire is given by

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi d},$$

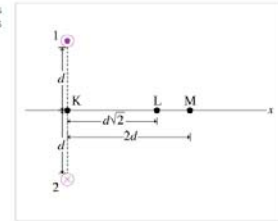
where  $\mu_0 (= 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})$  is the permeability constant,  $I$  is the current in the wire, and  $d$  is the distance from the wire to the location at which the magnitude of the magnetic field is being calculated.

The same result can be obtained from Ampere's law as well.

The direction of vector  $\vec{B}$  can be found using the right-hand rule. (Take care in applying the right-hand rule. Many students mistakenly use their left hand while applying the right-hand rule since those who use their right hand for writing sometimes automatically use their "pencil-free hand" to determine the direction of  $\vec{B}$ .)

In this problem, you will be asked to calculate the magnetic field due to a set of two wires with antiparallel currents as shown in the diagram. Each of the wires carries a current of magnitude  $I$ . The current in wire 1 is directed out of the page and that in wire 2 is directed into the page. The distance between the wires is  $2d$ . The  $x$  axis is perpendicular to the line connecting the wires and is equidistant from the wires.

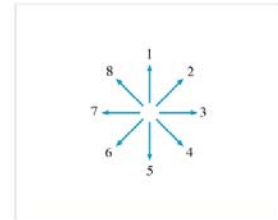
As you answer the questions posed here, try to look for a pattern in your answers.



#### Part A

Which of the vectors best represents the direction of the magnetic field created at point K (see the diagram in the problem introduction) by wire 1 alone?

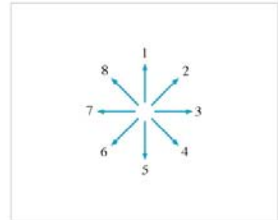
Enter the number of the vector with the appropriate direction.



#### Part B

Which of the vectors best represents the direction of the magnetic field created at point K by wire 2 alone?

Enter the number of the vector with the appropriate direction.

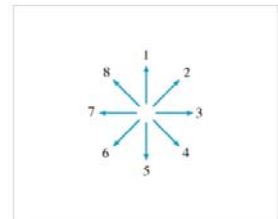


ANSWER:

#### Part C

Which of these vectors best represents the direction of the net magnetic field created at point K by both wires?

Enter the number of the vector with the appropriate direction.



ANSWER:

#### Part D

Find the magnitude of the magnetic field  $B_{1K}$  created at point K by wire 1.

Express your answer in terms of  $I$ ,  $d$ , and appropriate constants.

ANSWER:

This result is fairly obvious because of the symmetry of the problem. At point K, the two wires each contribute equally to the magnetic field. At points L and M you should also consider the symmetry of the problem. However, be careful! The vectors will add up in a more complex way.

#### Part F

Point L is located a distance  $d\sqrt{2}$  from the midpoint between the two wires. Find the magnitude of the magnetic field  $B_{1L}$  created at point L by wire 1.

Express your answer in terms of  $I$ ,  $d$ , and appropriate constants.

[Hints \(1\)](#)

**Hint 1.** How to approach the problem

Use the distances provided and the Pythagorean Theorem to find the distance between wire 1 and point L.

ANSWER:

$$B_{1L} = \frac{\mu_0 I}{2\pi d\sqrt{3}}$$

#### Part G

Point L is located a distance  $d\sqrt{2}$  from the midpoint between the two wires. Find the magnitude of the *net* magnetic field  $B_L$  created at point L by both wires.

Express your answer in terms of  $I$ ,  $d$ , and appropriate constants.

[Hints \(7\)](#)

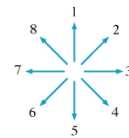
**Hint 1.** How to approach the problem

Sketch a detailed diagram with all angles marked; draw vectors  $\vec{B}_{1L}$  and  $\vec{B}_{2L}$ ; then add them using the parallelogram rule.

**Hint 2.** Find the direction of the magnetic field due to wire 1

Which of the vectors best represents the direction of the magnetic field created at point L (see the diagram in the problem introduction) by wire 1 alone?

Enter the number of the vector with the appropriate direction.



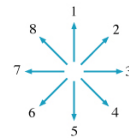
ANSWER:

2

**Hint 3.** Find the direction of the magnetic field due to wire 2

Which of the vectors best represents the direction of the magnetic field created at point L by wire 2 alone?

Enter the number of the vector with the appropriate direction.



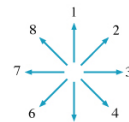
ANSWER:

4

**Hint 4.** Find the direction of the net magnetic field

Which of the vectors best represents the direction of the *net* magnetic field created at point L by *both* wires?

Enter the number of the vector with the appropriate direction.



ANSWER:

3

Note that the directions of the magnetic fields created by individual wires at point L are different from each other and from those at point K, however, the direction of the *net* magnetic field at points K and L is the same.

**Hint 5.** Angle between magnetic field due to wire 1 and the  $x$  axis

Use the distances provided and your knowledge of right angle triangle trigonometry to find the angle between the magnetic field due to wire 1 at point L and the  $x$  axis.

**Hint 6.** Find the angle between magnetic field due to wire 1 and the  $x$  axis

Use the distances provided and your knowledge of right angle triangle trigonometry to find the angle  $\theta_L$  between the magnetic field due to wire 1 at point L and the  $x$  axis.

Express your answer numerically, in degrees.

ANSWER:

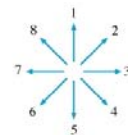
$\theta_L = 54.7^\circ$

**Hint 7.** Net magnetic field

Consider the symmetry of the magnetic field at point L due to wire 1 and the magnetic field due to wire 2. You should note that the  $y$  components of these two vectors are of equal magnitude but are opposite in direction. Therefore they will cancel when added together, leaving you only to worry about the  $x$  components. Find the  $x$  component of the magnetic field at point L due to wire 1 by using the magnitude of the vector (found in Part F) and the angle between the  $x$  axis and the magnetic field vector (found in the previous hint). Because of symmetry, the  $x$  component of the magnetic field at point L due to wire 2 is the same size. To find the net magnetic field at point L, you need to add together the  $x$  components of the magnetic field at point L due to wire 1 and of the magnetic field due to wire 2.

ANSWER:

$$B_L = \frac{\mu_0 I}{3\pi d}$$



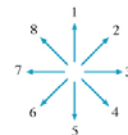
ANSWER:

1

#### Part K

Which of the vectors best represents the direction of the magnetic field created at point X by wire 2 alone?

Enter the number of the vector with the appropriate direction.



ANSWER:

5

As you can see, at a very large distance, the individual magnetic fields (and the corresponding magnetic field lines) created by the wires are directed nearly opposite to each other, thus ensuring that the net magnetic field is very, very small even compared to the magnitudes of the individual magnetic fields, which are also relatively small at a large distance from the wires. Thus, at a large distance, the magnetic fields due to the two wires almost cancel each other out! (That is, if point X is very far from each wire, then the ratio  $B_X/D_{1X}$  is very close to zero.)

Another way to think of this is as follows: If you are really far from the wires, then it's hard to tell them apart. It would seem as if the current were traveling up and down, almost along the same line, thereby appearing much the same as a single wire with almost no net current (because the up and down currents almost cancel each other), and therefore almost no magnetic field. Note that this only works for points very far from the wires; otherwise it's easy to tell that the wires are separated and the currents don't cancel, since they are going up and down at different locations.

It comes as no surprise then that one way to eliminate unnecessary magnetic fields in electric circuits is to twist together the wires carrying equal currents in opposite directions.

#### Ampère's Law Explained

**Description:** Discusses terms in and use of Ampère's law, and whether it can be used in a number of specific examples.

#### Learning Goal:

To understand Ampère's law and its application.

Ampère's law is often written  $\oint \vec{B}(\vec{r}) \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$ .

#### Part A

The integral on the left is

ANSWER:

- ☐ the integral throughout the chosen volume.
- ☐ the surface integral over the open surface.
- ☐ the surface integral over the closed surface bounded by the loop.
- ☒ the line integral along the closed loop.
- ☐ the line integral from start to finish.

**Part B**

What physical property does the symbol  $I_{\text{enc}}$  represent?

ANSWER:

- ☐ The current along the path in the same direction as the magnetic field
- ☐ The current in the path in the opposite direction from the magnetic field
- ☐ The total current passing through the loop in either direction
- ☒ The net current through the loop

The positive direction of the line integral and the positive direction for the current are related by the right-hand rule: Wrap your right-hand fingers around the closed path, then the direction of your fingers is the positive direction for  $d\vec{l}$  and the direction of your thumb is the positive direction for the net current.

Note also that the angle the current-carrying wire makes with the surface enclosed by the loop doesn't matter. (If the wire is at an angle, the normal component of the current is decreased, but the area of intersection of the wire and the surface is correspondingly increased.)

**Part C**

The circle on the integral means that  $\vec{B}(\vec{r})$  must be integrated

ANSWER:

- ☐ over a circle or a sphere.
- ☒ along any closed path that you choose.
- ☐ along the path of a closed physical conductor.
- ☐ over the surface bounded by the current-carrying wire.

**Part D**

Which of the following choices of path allow you to use Ampère's law to find  $\vec{B}(\vec{r})$ ?

- a. The path must pass through the point  $\vec{r}$ .
- b. The path must have enough symmetry so that  $\vec{B}(\vec{r}) \cdot d\vec{l}$  is constant along large parts of it.
- c. The path must be a circle.

ANSWER:

- ☐ a only
- ☒ a and b
- ☐ a and c
- ☐ b and c

**Part E**

Ampère's law can be used to find the magnetic field around a straight current-carrying wire.

Is this statement true or false?

ANSWER:

- ☒ true
- ☐ false

In fact Ampère's law can be used to find the magnetic field inside a cylindrical conductor (i.e., at a radius  $r$  less than the radius of the wire,  $R$ ). In this case  $I_{\text{enc}}$  is just that current inside  $r$ , not the current inside  $R$  (which is the total current in the wire).

**Part F**

Ampère's law can be used to find the magnetic field at the center of a square loop carrying a constant current.

Is this statement true or false?

ANSWER:

- ☐ true
- ☒ false

The key point is that to be able to use Ampère's law, the path along which you take the line integral of  $\vec{B}$  must have sufficient symmetry to allow you to pull the magnitude of  $B$  outside the integral. Whether the current distribution has symmetry is incidental.

**Part G**

Ampère's law can be used to find the magnetic field at the center of a circle formed by a current-carrying conductor.

Is this statement true or false?

ANSWER:

- ☐ true
- ☒ false

**Part H**

Ampère's law can be used to find the magnetic field inside a toroid. (A toroid is a doughnut shape wound uniformly with many turns of wire.)

Is this statement true or false?

ANSWER:

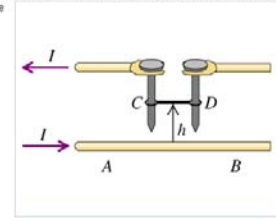
- ☒ true
- ☐ false

Therefore, though Ampère's law holds quite generally, it is useful in finding the magnetic field only in some cases, when a suitable path through the point of interest exists, typically such that all other points on the path have the same magnetic field through them.

### Exercise 28.34

**Description:** A long horizontal wire  $AB$  rests on the surface of a table and carries a current  $I$ . Horizontal wire  $CD$  is vertically above wire  $AB$  and is free to slide up and down on the two vertical metal guides  $C$  and  $D$  (the figure). Wire  $CD$  is connected through the sliding contacts to another wire that also carries a current  $I$ , opposite in direction to the current in wire  $AB$ . The mass per unit length of the wire  $CD$  is  $\lambda$ .

A long horizontal wire  $AB$  rests on the surface of a table and carries a current  $I$ . Horizontal wire  $CD$  is vertically above wire  $AB$  and is free to slide up and down on the two vertical metal guides  $C$  and  $D$  (the figure). Wire  $CD$  is connected through the sliding contacts to another wire that also carries a current  $I$ , opposite in direction to the current in wire  $AB$ . The mass per unit length of the wire  $CD$  is  $\lambda$ .



#### Part A

To what equilibrium height  $h$  will the wire  $CD$  rise, assuming that the magnetic force on it is due entirely to the current in the wire  $AB$ ?

Express your answer in terms of the variables  $I$ ,  $\lambda$ , and appropriate constants ( $\mu_0$ ,  $\pi$ ,  $g$ ).

ANSWER:

$$h = \frac{\mu_0 I^2}{2\pi g \lambda}$$

#### 28.42. IDENTIFY: Apply Ampere's law.

**SET UP:** From the right-hand rule, when going around the path in a counterclockwise direction currents out of the page are positive and currents into the page are negative.

**EXECUTE:** Path  $a$ :  $I_{\text{encl}} = 0 \Rightarrow \oint \vec{B} \cdot d\vec{l} = 0$ .

Path  $b$ :  $I_{\text{encl}} = -I_1 = -4.0 \text{ A} \Rightarrow \oint \vec{B} \cdot d\vec{l} = -\mu_0(4.0 \text{ A}) = -5.03 \times 10^{-6} \text{ T} \cdot \text{m}$ .

Path  $c$ :  $I_{\text{encl}} = -I_1 + I_2 = -4.0 \text{ A} + 6.0 \text{ A} = 2.0 \text{ A} \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0(2.0 \text{ A}) = 2.51 \times 10^{-6} \text{ T} \cdot \text{m}$ .

Path  $d$ :  $I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0 \text{ A} \Rightarrow \oint \vec{B} \cdot d\vec{l} = +\mu_0(4.0 \text{ A}) = 5.03 \times 10^{-6} \text{ T} \cdot \text{m}$ .

**EVALUATE:** If we instead went around each path in the clockwise direction, the sign of the line integral would be reversed.

#### 28.51. IDENTIFY and SET UP: Use the appropriate expression for the magnetic field produced by each current configuration.

**EXECUTE:** (a)  $B = \frac{\mu_0 I}{2\pi r}$  so  $I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(2.00 \times 10^{-2} \text{ m})(37.2 \text{ T})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} = 3.72 \times 10^6 \text{ A}$ .

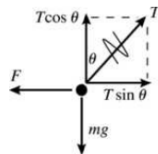
(b)  $B = \frac{N\mu_0 I}{2R}$  so  $I = \frac{2RB}{N\mu_0} = \frac{2(0.210 \text{ m})(37.2 \text{ T})}{(100)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})} = 1.24 \times 10^5 \text{ A}$ .

(c)  $B = \mu_0 \frac{N}{L} I$  so  $I = \frac{BL}{\mu_0 N} = \frac{(37.2 \text{ T})(0.320 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(40,000)} = 237 \text{ A}$ .

**EVALUATE:** Much less current is needed for the solenoid, because of its large number of turns per unit length.

#### 28.71. IDENTIFY: Apply $\sum \vec{F} = 0$ to one of the wires. The force one wire exerts on the other depends on $I$ so $\sum \vec{F} = 0$ gives two equations for the two unknowns $T$ and $I$ .

**SET UP:** The force diagram for one of the wires is given in Figure 28.71.



The force one wire exerts on the other is  $F = \left( \frac{\mu_0 I^2}{2\pi r} \right) L$ , where

$r = 2(0.040 \text{ m}) \sin \theta = 8.362 \times 10^{-3} \text{ m}$  is the distance between the two wires.

Figure 28.71

**EXECUTE:**  $\sum F_y = 0$  gives  $T \cos \theta = mg$  and  $T = mg / \cos \theta$

$\sum F_x = 0$  gives  $F = T \sin \theta = (mg / \cos \theta) \sin \theta = mg \tan \theta$

And  $m = \lambda L$ , so  $F = \lambda L g \tan \theta$

$$\left( \frac{\mu_0 I^2}{2\pi r} \right) L = \lambda L g \tan \theta$$

$$I = \sqrt{\frac{\lambda g r \tan \theta}{(\mu_0 / 2\pi)}}$$

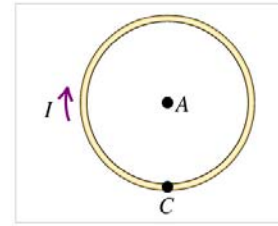
$$I = \sqrt{\frac{(0.0125 \text{ kg/m})(9.80 \text{ m/s}^2)(\tan 6.00^\circ)(8.362 \times 10^{-3} \text{ m})}{2 \times 10^{-7} \text{ T} \cdot \text{m/A}}} = 23.2 \text{ A}$$



### Problem 28.76

**Description:** A circular wire of diameter  $D$  lies on a horizontal table and carries a current  $I$ . In the figure point  $A$  marks the center of the circle and point  $C$  is on its rim. (a) Find the magnitude of the magnetic field at point  $A$ . (b) Find the direction of...

A circular wire of diameter  $D$  lies on a horizontal table and carries a current  $I$ . In the figure point  $A$  marks the center of the circle and point  $C$  is on its rim.



#### Part A

Find the magnitude of the magnetic field at point  $A$ .

Express your answer in terms of the variables  $I$ ,  $D$ , and appropriate constants ( $\mu_0$  and  $\pi$ ).

ANSWER:

$$B_1 = \frac{\mu_0 I}{D}$$

#### Part B

Find the direction of the magnetic field at point  $A$ .

ANSWER:

- ☐ out of the page  
☒ into the page

#### Part C

The wire is now unwrapped so it is straight, centered on point  $C$ , and perpendicular to the line  $AC$ , but the same current is maintained in it. Now find the magnetic field at point  $A$ .

Express your answer in terms of the variables  $I$ ,  $D$ , and appropriate constants ( $\mu_0$  and  $\pi$ ).

ANSWER:

$$B_2 = \frac{\mu_0 I}{D\sqrt{1+\pi^2}}$$

#### Part D

Which field is greater: the one in part (a) or in part (c)?

ANSWER:

- ☒ the one in part (a)  
☐ the one in part (c)

#### Part E

By what factor?

ANSWER:

$$\frac{B_1}{B_2} = 3.30$$

Also accepted: 3.30

#### Part F

Why is this result physically reasonable?

ANSWER:

**Answer Key:**

It is reasonable that the field due to the circular wire is greater than the field due to the straight wire because more of the current is close to point  $A$  for the circular wire than it is for the straight wire.

**28.84. IDENTIFY:** Find the vector sum of the fields due to each sheet.

**SET UP:** Problem 28.83 shows that for an infinite sheet  $B = \frac{1}{2}\mu_0 In$ . If  $I$  is out of the page,  $\vec{B}$  is to the left above the sheet and to the right below the sheet. If  $I$  is into the page,  $\vec{B}$  is to the right above the sheet and to the left below the sheet.  $B$  is independent of the distance from the sheet. The directions of the two fields at points  $P$ ,  $R$  and  $S$  are shown in Figure 28.84.

**EXECUTE: (a)** Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

**(b)** In between the sheets the two fields add up to yield  $B = \mu_0 nI$ , to the right.

**(c)** Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

**EVALUATE:** The two sheets with currents in opposite directions produce a uniform field between the sheets and zero field outside the two sheets. This is analogous to the electric field produced by large parallel sheets of charge of opposite sign.



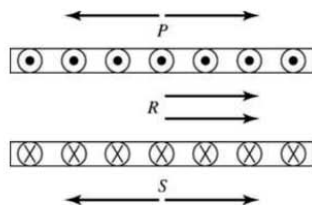


Figure 28.84

**28.86. IDENTIFY:** Approximate the moving belt as an infinite current sheet.

**SET UP:** Problem 28.83 shows that  $B = \frac{1}{2}\mu_0 I n$  for an infinite current sheet. Let  $L$  be the width of the sheet, so  $n = 1/L$ .

**EXECUTE:** The amount of charge on a length  $\Delta x$  of the belt is  $\Delta Q = L\Delta x\sigma$ , so  $I = \frac{\Delta Q}{\Delta t} = L \frac{\Delta x}{\Delta t} \sigma = Lv\sigma$ .

Approximating the belt as an infinite sheet  $B = \frac{\mu_0 I}{2L} = \frac{\mu_0 v \sigma}{2}$ .  $\vec{B}$  is directed out of the page, as shown in

Figure 28.86.

**EVALUATE:** The field is uniform above the sheet, for points close enough to the sheet for it to be considered infinite.



Figure 28.86

**28.87. IDENTIFY:** The current-carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role.

**SET UP:** The magnetic force per unit length is  $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi d}$ , and the acceleration obeys the equation

$F/L = m/L a$ . The rms current over a short discharge time is  $I_0/\sqrt{2}$ .

**EXECUTE: (a)** First get the force per unit length:

$$\frac{F}{L} = \frac{\mu_0 I^2}{2\pi d} = \frac{\mu_0 \left(\frac{I_0}{\sqrt{2}}\right)^2}{2\pi d} = \frac{\mu_0 \left(\frac{V}{R}\right)^2}{4\pi d} = \frac{\mu_0 \left(\frac{Q_0}{RC}\right)^2}{4\pi d}$$

Now apply Newton's second law using the result above:  $\frac{F}{L} = \frac{m}{L} a = \lambda a = \frac{\mu_0 \left(\frac{Q_0}{RC}\right)^2}{4\pi d}$ . Solving for  $a$  gives

$$a = \frac{\mu_0 Q_0^2}{4\pi \lambda R^2 C^2 d}. \text{ From the kinematics equation } v_x = v_{0x} + a_x t, \text{ we have } v_0 = at = aRC = \frac{\mu_0 Q_0^2}{4\pi \lambda R C d}.$$

$$\text{(b) Conservation of energy gives } \frac{1}{2}mv_0^2 = mgh \text{ and } h = \frac{v_0^2}{2g} = \frac{\left(\frac{\mu_0 Q_0^2}{4\pi \lambda R C d}\right)^2}{2g} = \frac{1}{2g} \left(\frac{\mu_0 Q_0^2}{4\pi \lambda R C d}\right)^2.$$

**EVALUATE:** Once the wires have swung apart, we would have to consider gravity in applying Newton's second law.