Physics 161-001 Spring 2012 Exam 2

Name: Solutions **Box#**__________

Multiple Choice (5 points each):

1) A point charge is placed at the center of a spherical Gaussian surface. The net electric flux through the surface is *changed* if

A) the sphere is replaced by a larger sphere with the point charge still centered
B) the point charge is moved off center (but still inside the original sphere)
C) a second point charge is moved to just outside the sphere (the original charge is still centered)
D) more than one of these
E) none of these

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

net electric flux

only depends on charge enclosed

2) The electric field at a distance of 15cm from an isolated point charge of 3 x 10^{-7} C is:

A) 2,700 N/C
B) 1,800 N/C
C) 1.2 x 10^5 N/C
D) 18,000 N/C
E) none of the above

$$\vec{E} = k\frac{q}{r^2} = 9.0\times10^9\,\frac{Nm^2}{C^2}\cdot\frac{3\times10^{-7}\,C}{(0.15\,m)^2}$$

$$= 1.2\times10^5\,\frac{N}{C}$$

3) A solid spherical conductor of radius 5m with -5C of charge on its outer surface has a potential V = 0 at its center. What is the potential at the surface?

A) 5V
B) 1V
C) 0V
D) –1V
E) –5V

Since $\vec{E}=0 \Rightarrow V = \text{constant}$ inside a conductor,

4) **If the potential in a region is given by $V(x,y,z) = 4xz-3x^2y$ (with appropriate units), then the x-component of the electric field at the point (x=2m, y=1m, z=0m) is:**

A) -12 V/m
B) -8 V/m
C) 0 V/m
D) +4 V/m
E) +12 V/m

$$E_x = -\frac{\partial V}{\partial x} = -4z + 6x \cdot y$$

so $E_x(2,1,0) = -4(0m) + 6(2m)(1m)$

$= 12 \frac{V}{m}$

5) **A 5.0 Coulomb charge is 10m from a -2.0 Coulomb charge. The electrostatic force on the positive charge is:**

A) 9.0×10^8 N toward the negative charge
B) 9.0×10^8 N away from the negative charge
C) 9.0×10^9 N toward the negative charge
D) 9.0×10^9 N away from the negative charge
E) 9.0×10^7 N toward the negative charge

$$\vec{F} = k\frac{q_1 q_2}{r^2} = 9\times10^9 \frac{Nm^2}{C^2} \cdot \frac{5C \cdot 2C}{(10m)^2}$$

$= 9\times10^8$ N

Since they are oppositely charged, the force is attractive.

6) **A 2μC charge is placed 2m from the origin on the x-axis, and a -2μC charge is placed 2m from the origin on the y-axis. The potential at the origin is:**

A) 1.8×10^4 V
B) 1.3×10^4 V
C) 9.0×10^3 V
D) $(9.0 \times 10^3$ V)i - $(9.0 \times 10^3$ V)j
E) 0V

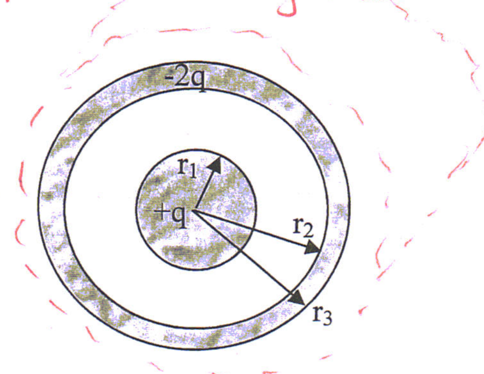
Since potentials just add, and the charges are equal and opposite sign, and the same distance from the origin, then $V=0$

Written Problems. SHOW ALL WORK! No credit for answers without work!

1) (25 points) A very long conducting cylindrical rod of length L and radius r_1 with a total charge +q is surrounded by a conducting cylindrical shell (also of length L, inner radius r_2, outer radius r_3) with a total charge –2q.

(a) Use Gauss's law to find the electric field at points outside the conducting shell ($r > r_3$).

Use a cylindrical Gaussian surface of radius r and length L:



$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{cap\,1} \vec{E}\cdot d\vec{A}}_{\emptyset} + \underbrace{\int_{cap\,2} \vec{E}\cdot d\vec{A}}_{\emptyset} + \int_{cyl.} \vec{E}\cdot d\vec{A}$$

$$= E\,2\pi r L = \frac{q_{enc}}{\epsilon_0} = \frac{+q-2q}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{-\frac{q}{L}}{2\pi r \epsilon_0}}$$

(b) Use Gauss's law to find the electric field in all of the regions between the rod and the shell ($r_1 > r > r_2$).

Again, use a cylindrical Gaussian surface:

$$\oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{cap\,1} \vec{E}\cdot d\vec{A}}_{\emptyset} + \underbrace{\int_{cap\,2} \vec{E}\cdot d\vec{A}}_{0} + \int_{cyl} \vec{E}\cdot d\vec{A}$$

$$= E\,2\pi r L = \frac{q_{enc}}{\epsilon_0} = \frac{+q}{2\pi r \epsilon_0} \Rightarrow$$

$$\boxed{E = \frac{q}{2\pi r L \epsilon_0}}$$

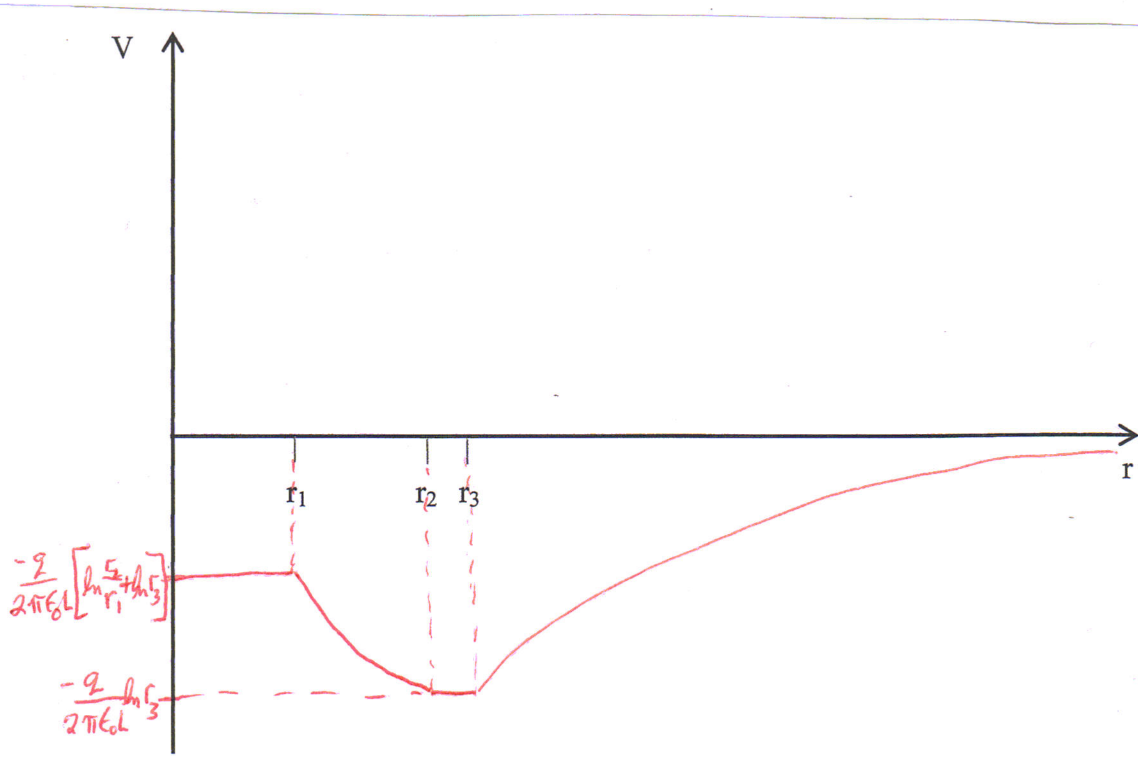
(c) What is the potential difference between the rod and the shell?

$$\Delta V = -\int_{r_1}^{r_2} \vec{E}\cdot d\vec{r} = \frac{-q}{2\pi\epsilon_0 L}\int_{r_1}^{r_2}\frac{1}{r}\,dr = \boxed{\frac{-q}{2\pi\epsilon_0 L}\ln\left(\frac{r_2}{r_1}\right)}$$

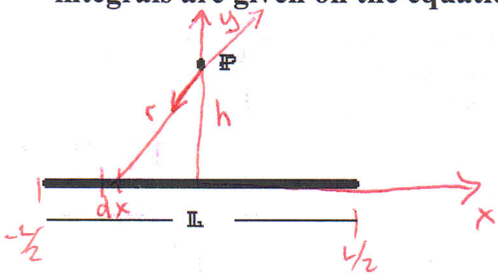
(d) What is the capacitance of this object?

$$C = \frac{Q}{V} = 2\pi\epsilon_0 L\left[\ln\left(\frac{r_1}{r_2}\right)\right]^{-1}$$

(e) Draw a graph of the potential vs. distance from the axis of the rod, assuming V = 0 at infinity. Shape and scale are important.



2) **(20 points) A thin non-conducting rod of length L has charge -q uniformly distributed along its length. Find the magnitude and direction of the electric field at point P a distance h above and on the perpendicular bisector of the rod. Show your choice of coordinate system and all work. If you need them, indefinite integrals are given on the equation sheet.**



$$dE = k\frac{dq}{r^2}$$

$$\lambda = \frac{-q}{L}$$

$$dq = \lambda\, dx$$

$$r = \sqrt{x^2+h^2}$$

$$\therefore\ dE = k\frac{\lambda\, dx}{(x^2+h^2)}$$

$$dE_x = dE\cdot\cos\theta = dE\cdot\frac{h}{r} \Rightarrow$$

$$dE_y = k\frac{\lambda h\, dx}{(x^2+h^2)^{3/2}}$$

$$\therefore\ E_y = \int_{-L/2}^{L/2}\frac{k\lambda h}{(x^2+h^2)^{3/2}}\,dx$$

$$= 2\int_0^{L/2}\frac{k\lambda h}{(x^2+h^2)^{3/2}}\,dx$$

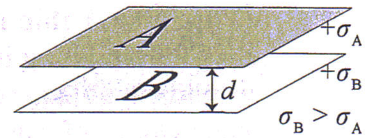
$$= -\frac{2qkh}{L}\int_0^{L/2}\frac{dx}{(x^2+h^2)^{3/2}}$$

$$= -\frac{2kq\cancel{h}}{L}\,\frac{1}{h^{\cancel{2}}}\,\frac{x}{(x^2+h^2)^{1/2}}\Bigg|_0^{L/2}$$

$$= \frac{kq}{h}\,\frac{1}{\left(\frac{L^2}{4}+h^2\right)^{1/2}}$$

3) **(20 points) The figure at right shows a small portion of two parallel, infinite sheets of positive surface charge densities $+\sigma_A$ and $+\sigma_B$. The sheets are a distance *d* apart. The surface charge density of sheet B is greater than that of sheet A (*i.e.*, $\sigma_B > \sigma_A$).**

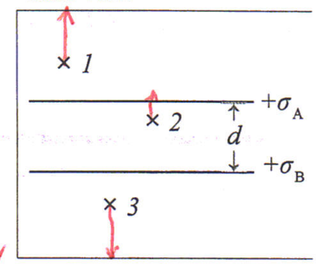
Perspective view



a) **On the side-view diagram at right, draw *vectors* with their tails at each "x" to represent the net electric field at points *1, 2,* and *3.* Your drawing should be qualitatively correct in both magnitude and direction. Explain.**

The E-field from each sheet points away from the sheet. They add above sheet A and below sheet B and partially cancel in between since $\sigma_B > \sigma_A$.

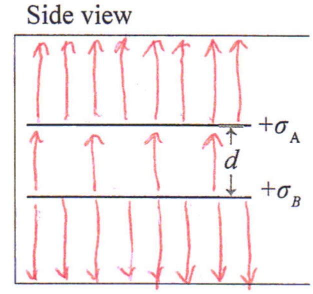
Side view



b) **On the side-view diagram at right, draw *electric field lines* to represent the net electric field above, between, and below the sheets. Your drawing should be qualitatively correct in both magnitude and direction. Explain.**

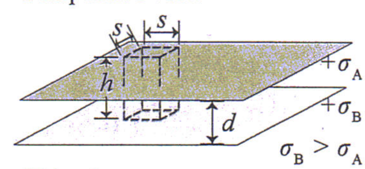
The electric field lines are spaced the same above A and below B since the field strengths there are the same. The field is weaker in-between.

Side view



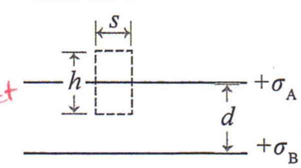
An imaginary closed surface with sides *h, s,* and *s* is shown at right.

Perspective view



c) **Evaluate the quantity $\oint \vec{E} \bullet d\vec{A}$ over the surface in terms of the magnitude of the electric field at points 1-3 from part A ($|\vec{E}_1|$, $|\vec{E}_2|$, and $|\vec{E}_3|$) and/or other relevant quantities. (Your expression should not contain any charge densities.) Explain and show your work.**

Side view



$$\oint \vec{E} \cdot d\vec{A} = s^2 E_1 + 4 s \cdot h \cdot \emptyset - s^2 E_2$$

dot product

$$= s^2 (E_1 - E_2)$$

d) **Use the given imaginary surface and Gauss' law to find an expression for σ_A in terms of the magnitudes $|\vec{E}_1|$, $|\vec{E}_2|$, $|\vec{E}_3|$, and/or other relevant quantities. Explain and show your work.**

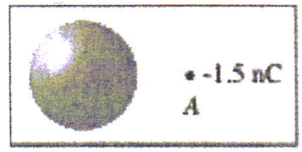
$$\cancel{s^2}(E_1 - E_2) = \frac{\sigma_A \cdot \cancel{s^2}}{\epsilon_0}$$

q_{enc}

net flux

$$\sigma_A = \epsilon_0 (E_1 - E_2)$$

4) (10 points) A test charge *A* (-1.5 nC) is held at rest 10 cm from the surface of a positively charged sphere that is fixed in place. Charge *A* is released from rest, and reaches the surface of the sphere with 14 J of kinetic energy.

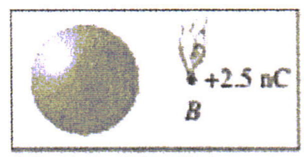


a) Find the absolute value of the electric potential difference ΔV_A between charge *A*'s starting point and the surface of the sphere. Show your work.

$$KE_f = 14\,J = \Delta U_{E_i} = q(V_f - V_i)$$

$$\therefore \Delta V = \frac{14\,J}{-1.5\,nC} = -9.3 \times 10^9\,V$$

Test charge *B* (+2.5 nC) is also initially held at rest 10 cm from the surface of a positively charged sphere identical to the one above. Charge *B* is moved by a hand at constant velocity to the sphere's surface.



b) Is the absolute value of the electric potential difference between charge *B*'s starting point and the surface of the sphere, ΔV_B, *greater than, less than,* or *equal to* the absolute value of ΔV_A? Explain.

Equal to! The potential is not dependent on the test charge (or anything else besides the charged sphere).