Lecture 4
(Ideal Gas Equation, P-V Diagrams)

Physics 161-01 Spring 2012

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Thermodynamic State Variables

- In order to describe the thermodynamic state of an object, we will use the state variables: V (volume), n (number of moles), p (pressure), and T (temperature).
- We have already implicitly done this for a solid:

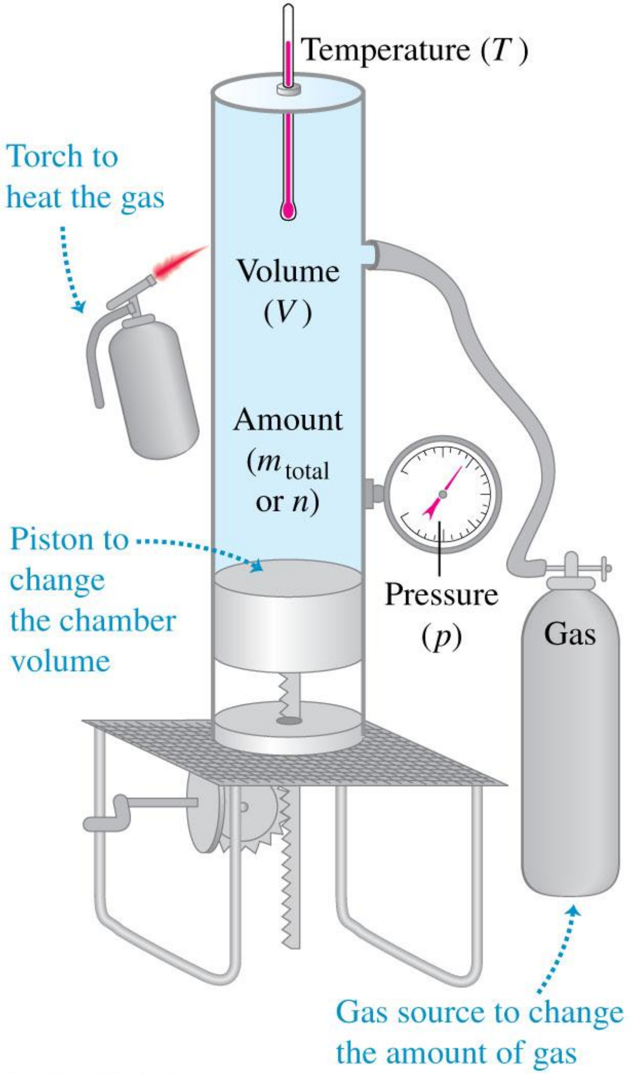
$$V = V_0\left[1 + \beta\left(T - T_0\right) - k\left(p - p_0\right)\right]$$

Moles, Molecular Mass, etc.

- One *mole* of a substance contains as many elementary entities (atoms or molecules) as there are atoms in 0.012 kg of carbon-12.
- One mole of a substance contains *Avogadro's number* N_A of molecules.
- N_A = 6.022 x 10^{23} molecules/mol
- The *molar mass* M is the mass of one mole.
- $M = N_A m$, where m is the mass of a single molecule.

Ideal Gases

- We can use a setup as in the figure to the right to determine the state of an ideal gas.
- What is ideal?
  - Non-interacting molecules
  - Point particles (volume of molecules is small compared to total volume)

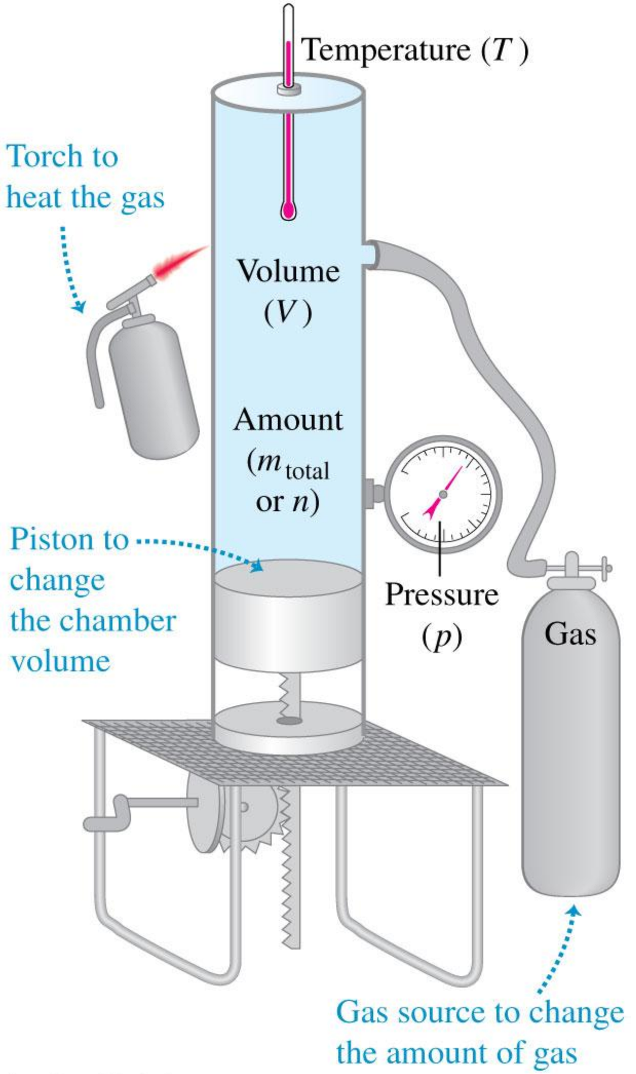


Ideal Gas Law

- Pressure times volume is constant (with temperature and amount held constant).
- Pressure is proportional to the amount (with temperature and volume held constant).
- Pressure is proportional to temperature (with amount and volume held constant).

$$pV = nRT$$

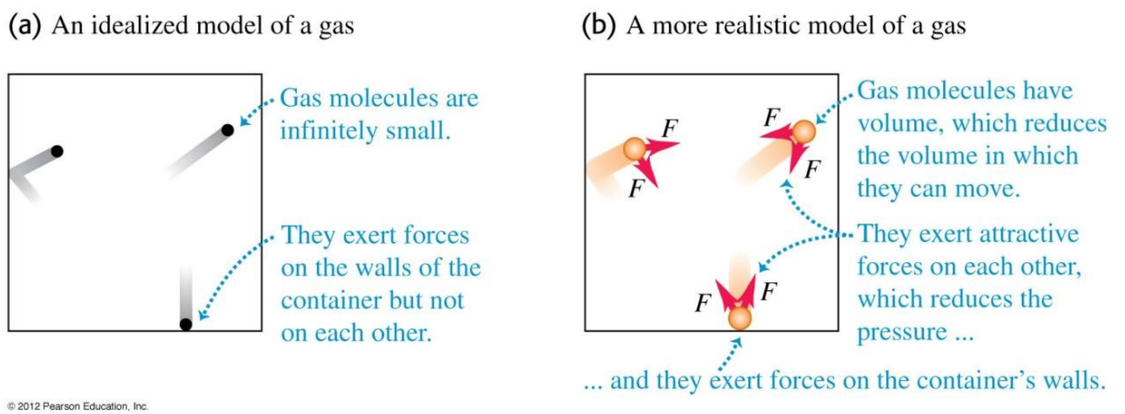
$$R = 8.314472(15)\, J/mol \cdot K$$



van der Waals Equation of State

- Including the effects of molecular size and the van der Waals forces between the molecules alters the ideal gas law, but one can ignore these effects for a dilute gas (small n relative to V).

$$\left(p+\frac{an^2}{V^2}\right)(V-nb)=nRT$$



Example 18.1 Volume of an ideal gas at STP

What is the volume of a container that holds exactly 1 mole of an ideal gas at *standard temperature and pressure* (STP), defined as $T = 0°\text{C} = 273.15\ \text{K}$ and $p = 1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}$?

SOLUTION

IDENTIFY and SET UP: This problem involves the properties of a single state of an ideal gas, so we use Eq. (18.3). We are given the pressure p, temperature T, and number of moles n; our target variable is the corresponding volume V.

EXECUTE: From Eq. (18.3), using R in $\text{J/mol} \cdot \text{K}$,

$$V = \frac{nRT}{p} = \frac{(1\ \text{mol})(8.314\ \text{J/mol} \cdot \text{K})(273.15\ \text{K})}{1.013 \times 10^5\ \text{Pa}}$$

$$= 0.0224\ \text{m}^3 = 22.4\ \text{L}$$

EVALUATE: At STP, 1 mole of an ideal gas occupies 22.4 L. This is the volume of a cube 0.282 m (11.1 in.) on a side, or of a sphere 0.350 m (13.8 in.) in diameter.

Example 18.3 Mass of air in a scuba tank

An "empty" aluminum scuba tank contains 11.0 L of air at 21°C and 1 atm. When the tank is filled rapidly from a compressor, the air temperature is 42°C and the gauge pressure is 2.10×10^7 Pa. What mass of air was added? (Air is about 78% nitrogen, 21% oxygen, and 1% miscellaneous; its average molar mass is 28.8 g/mol = 28.8×10^{-3} kg/mol.)

SOLUTION

IDENTIFY and SET UP: Our target variable is the difference $m_2 - m_1$ between the masses present at the end (state 2) and at the beginning (state 1). We are given the molar mass M of air, so we can use Eq. (18.2) to find the target variable if we know the number of moles present in states 1 and 2. We determine n_1 and n_2 by applying Eq. (18.3) to each state individually.

EXECUTE: We convert temperatures to the Kelvin scale by adding 273 and convert the pressure to absolute by adding 1.013×10^5 Pa. The tank's volume is hardly affected by the increased temperature and pressure, so $V_2 = V_1$. From Eq. (18.3), the numbers of moles in the empty tank (n_1) and the full tank (n_2) are

$$n_1 = \frac{p_1 V_1}{RT_1} = \frac{(1.013 \times 10^5\ \text{Pa})(11.0 \times 10^{-3}\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(294\ \text{K})} = 0.46\ \text{mol}$$

$$n_2 = \frac{p_2 V_2}{RT_2} = \frac{(2.11 \times 10^7\ \text{Pa})(11.0 \times 10^{-3}\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(315\ \text{K})} = 88.6\ \text{mol}$$

We added $n_2 - n_1 = 88.6\ \text{mol} - 0.46\ \text{mol} = 88.1\ \text{mol}$ to the tank. From Eq. (18.2), the added mass is $M(n_2 - n_1) = (28.8 \times 10^{-3}\ \text{kg/mol})(88.1\ \text{mol}) = 2.54\ \text{kg}$.

EVALUATE: The added mass is not insubstantial: You could certainly use a scale to determine whether the tank was empty or full.

Example 18.2 Compressing gas in an automobile engine

In an automobile engine, a mixture of air and vaporized gasoline is compressed in the cylinders before being ignited. A typical engine has a compression ratio of 9.00 to 1; that is, the gas in the cylinders is compressed to $\frac{1}{9.00}$ of its original volume (Fig. 18.3). The intake and exhaust valves are closed during the compression, so the quantity of gas is constant. What is the final temperature of the compressed gas if its initial temperature is 27°C and the initial and final pressures are 1.00 atm and 21.7 atm, respectively?

SOLUTION

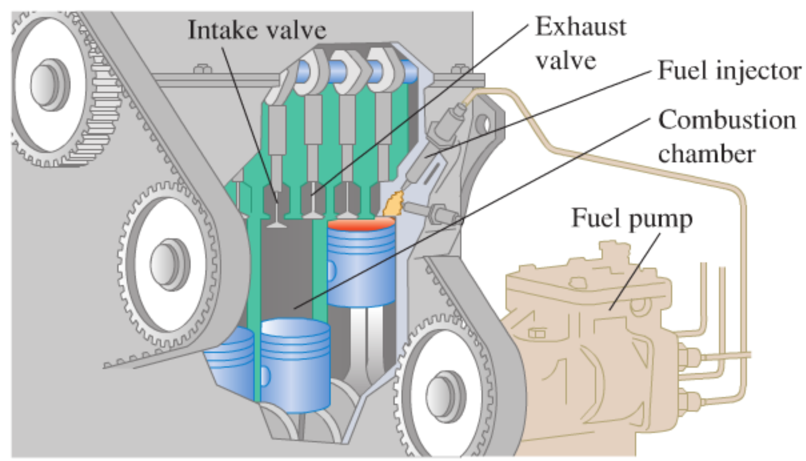
IDENTIFY and SET UP: We must compare two states of the same quantity of ideal gas, so we use Eq. (18.6). In the uncompressed state, $p_1 = 1.00$ atm and $T_1 = 27°\text{C} = 300$ K. In the compressed state 2, $p_2 = 21.7$ atm. The cylinder volumes are not given, but we have $V_1 = 9.00V_2$. The temperature T_2 of the compressed gas is the target variable.

EXECUTE: We solve Eq. (18.6) for T_2:

$$T_2 = T_1 \frac{p_2 V_2}{p_1 V_1} = (300\ \text{K}) \frac{(21.7\ \text{atm})V_2}{(1.00\ \text{atm})(9.00V_2)} = 723\ \text{K} = 450°\text{C}$$

EVALUATE: This is the temperature of the air–gasoline mixture *before* the mixture is ignited; when burning starts, the temperature becomes higher still.

18.3 Cutaway of an automobile engine. While the air–gasoline mixture is being compressed prior to ignition, the intake and exhaust valves are both in the closed (up) position.



CPS 4-1

A quantity of an ideal gas is contained in a balloon. Initially the gas temperature is 27°C.

You double the pressure on the balloon and change the temperature so that the balloon shrinks to one-quarter of its original volume. What is the new temperature of the gas?

A. 54°C

B. 27°C

C. 13.5°C

D. –123°C

E. –198°C

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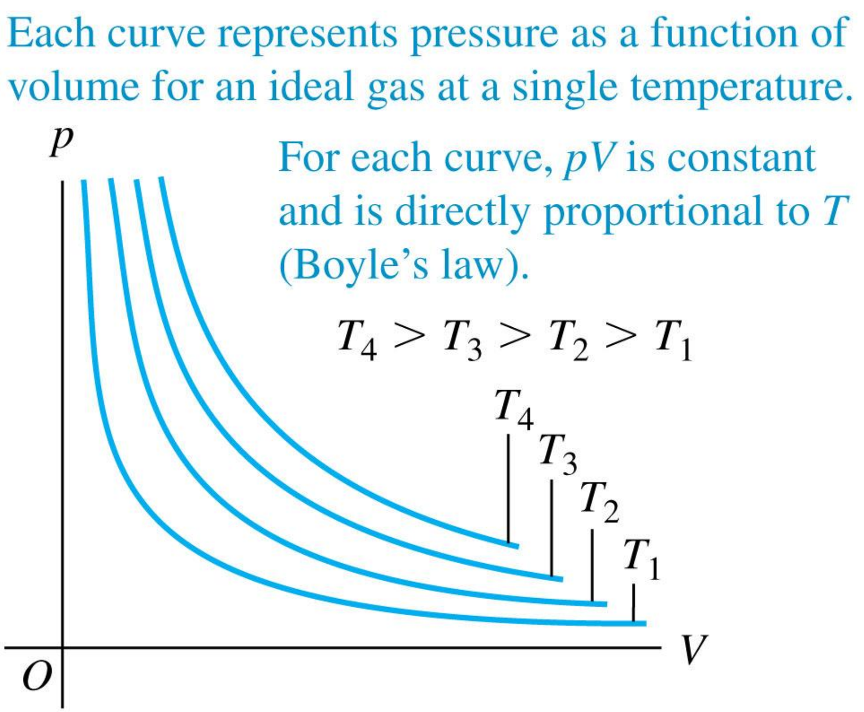
P-V Diagrams

- We will find that representing the state variables pressure as a function of volume for different temperatures will allow us to determine the much about how a thermodynamic system is behaving.
- In particular, we will see that we can determine the amount of work that the system does by examining these graphs.
- Note, that for a constant number of moles of an ideal gas, each isotherm (curve of constant temperature) is just a hyperbolic curve:

$$pV = nRT \Rightarrow$$

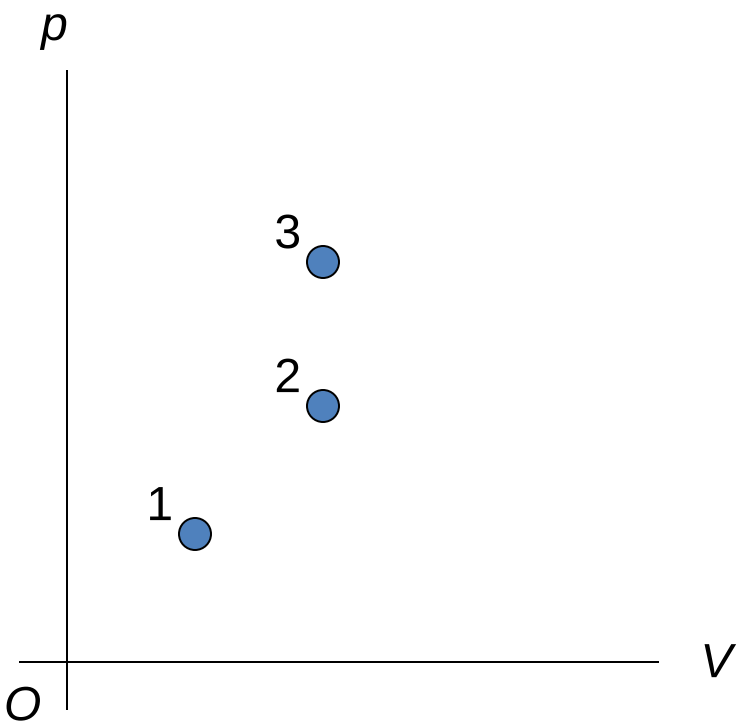
$$p = \frac{nRT}{V}$$



CPS 4-2

This *p-V* diagram shows three possible states of a certain amount of an ideal gas.

Which state is at the *highest* temperature?



A. state #1

B. state #2

C. state #3

D. Two of these are tied for highest temperature.

E. All three of these are at the same temperature.

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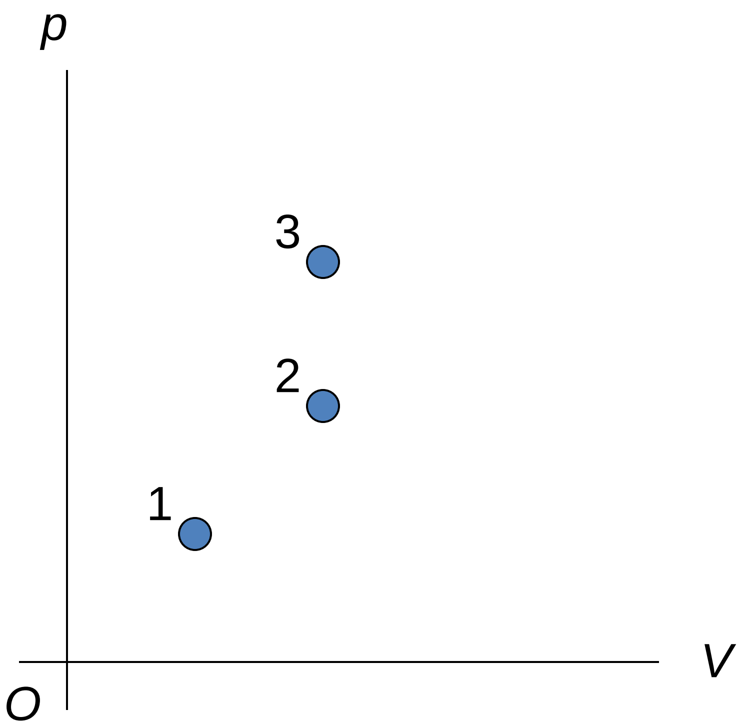

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P-V Diagrams

- For a substance that undergoes a phase transition, a p-V diagram can give a lot of information about the state of the material at different pressures, temperatures and volumes.