40.72. **IDENTIFY and SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** $E = K + U(x) = \dfrac{p^2}{2m} + U(x) \Rightarrow p = \sqrt{2m(E - U(x))}$. $\lambda = \dfrac{h}{p} \Rightarrow \lambda(x) = \dfrac{h}{\sqrt{2m(E - U(x))}}$.

(b) As $U(x)$ gets larger (i.e., $U(x)$ approaches E from below—recall $k \geq 0$), $E - U(x)$ gets smaller, so $\lambda(x)$ gets larger.

(c) When $E = U(x)$, $E - U(x) = 0$, so $\lambda(x) \to \infty$.

(d) $\displaystyle\int_a^b \frac{dx}{\lambda(x)} = \int_a^b \frac{dx}{h/\sqrt{2m(E - U(x))}} = \frac{1}{h}\int_a^b \sqrt{2m(E - U(x))}\, dx = \frac{n}{2} \Rightarrow \int_a^b \sqrt{2m(E - U(x))}\, dx = \frac{hn}{2}$.

(e) $U(x) = 0$ for $0 < x < L$ with classical turning points at $x = 0$ and $x = L$. So,

$\displaystyle\int_a^b \sqrt{2m(E - U(x))}\, dx = \int_0^L \sqrt{2mE}\, dx = \sqrt{2mE}\int_0^L dx = \sqrt{2mE}\, L$. So, from part (d),

$\displaystyle\sqrt{2mE}\, L = \frac{hn}{2} \Rightarrow E = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2 n^2}{8mL^2}$.

EVALUATE: **(f)** Since $U(x) = 0$ in the region between the turning points at $x = 0$ and $x = L$, the result is the *same* as part (e). The height U_0 never enters the calculation. WKB is best used with *smoothly* varying potentials $U(x)$.