

30.48.

The energy stored is $U = \frac{1}{2} Li^2$.

$$(a) \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B 2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}.$$

$$(b) d\Phi_B = B dA = \frac{\mu_0 i}{2\pi r} l dr.$$

$$(c) \Phi_B = \int_a^b d\Phi_B = \frac{\mu_0 i l}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 i l}{2\pi} \ln(b/a).$$

$$(d) L = \frac{N\Phi_B}{i} = l \frac{\mu_0}{2\pi} \ln(b/a).$$

$$(e) U = \frac{1}{2} Li^2 = \frac{1}{2} l \frac{\mu_0}{2\pi} \ln(b/a) i^2 = \frac{\mu_0 l i^2}{4\pi} \ln(b/a).$$

30.64.

At $t = 0$, $i = 0$ through each inductor. At $t \rightarrow \infty$, the voltage is zero across each inductor.

In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \rightarrow \infty$ replace each inductor by a wire.

(a) Initially the inductor blocks current through it, so the simplified equivalent circuit is shown in

Figure 30.64a. $i = \frac{\mathcal{E}}{R} = \frac{50 \text{ V}}{150 \Omega} = 0.333 \text{ A}$. $V_1 = (100 \Omega)(0.333 \text{ A}) = 33.3 \text{ V}$.

$V_4 = (50 \Omega)(0.333 \text{ A}) = 16.7 \text{ V}$. $V_3 = 0$ since no current flows through it. $V_2 = V_4 = 16.7 \text{ V}$, since the inductor is in parallel with the $50\text{-}\Omega$ resistor. $A_1 = A_3 = 0.333 \text{ A}$, $A_2 = 0$.

(b) Long after S is closed, steady state is reached, so the inductor has no potential drop across it. The simplified circuit is sketched in Figure 30.64b. $i = \mathcal{E}/R = \frac{50 \text{ V}}{130 \Omega} = 0.385 \text{ A}$.

$V_1 = (100 \Omega)(0.385 \text{ A}) = 38.5 \text{ V}$; $V_2 = 0$; $V_3 = V_4 = 50 \text{ V} - 38.5 \text{ V} = 11.5 \text{ V}$.

$i_1 = 0.385 \text{ A}$; $i_2 = \frac{11.5 \text{ V}}{75 \Omega} = 0.153 \text{ A}$; $i_3 = \frac{11.5 \text{ V}}{50 \Omega} = 0.230 \text{ A}$.

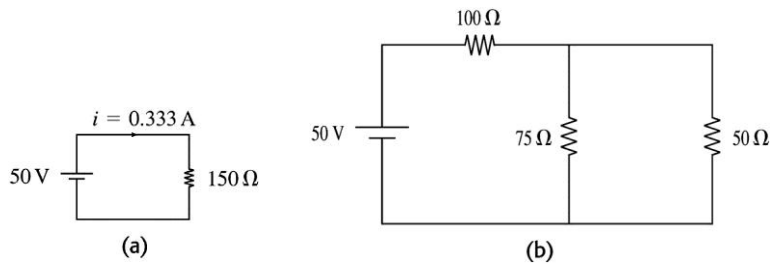


Figure 30.64

68.

- a) When S_2 is first closed, the energy in the inductor is $U_L = \frac{1}{2} L I_0^2$, where $I_0 = 3.50 \text{ A}$ is the current that was flowing at that moment. At this point, there is no energy in the capacitor because it is uncharged. At some later time, all the energy in the circuit will be in the capacitor, at which point its energy will be $U_C = \frac{q_{\text{max}}^2}{2C}$. Since the energy is not being dissipated (e.g. by a resistor), this is the same as the initial amount of energy: $\frac{q_{\text{max}}^2}{2C} = \frac{1}{2} L I_0^2$, so

$$q_{\text{max}} = \sqrt{LC} I_0 = \sqrt{(2.0 \text{ mH})(5.0 \text{ }\mu\text{F})} (3.50 \text{ A}) = \boxed{3.5 \times 10^{-4} \text{ C}} = 350 \text{ }\mu\text{C}.$$

- b) Since all of the energy is in the capacitor, the current in the inductor is zero.

70.

- a) R_1 is connected directly to the battery, which we assume is ideal and therefore has emf $\mathcal{E} = 60.0 \text{ V}$ regardless of what else is happening. So the current through R_1 is $i_1 = \frac{\mathcal{E}}{R_1} = \frac{60.0 \text{ V}}{40.0 \text{ }\Omega} = \boxed{1.50 \text{ A}}$ from a to b . We know the rate of change of current $\left(\frac{di}{dt}\right)$ through the inductor, so we know the potential across it is $v_{cd} = L \frac{di}{dt}$. Then the potential across R_2 is $v_{ac} = \mathcal{E} - v_{cd} = \mathcal{E} - L \frac{di}{dt}$, which means the current through R_2 is

$$i_2 = \frac{v_{ac}}{R_2} = \frac{1}{R_2} \left(\mathcal{E} - L \frac{di}{dt} \right) = \frac{1}{25.0 \text{ }\Omega} [60.0 \text{ V} - (0.300 \text{ H})(50.0 \text{ A/s})] = \boxed{1.80 \text{ A}}$$

from a to c .

- b) After the switch has been closed a long time, the current through R_2 is steady, so $\frac{di_2}{dt} = 0$, so $v_{cd} = L \frac{di_2}{dt} = 0$. Then $v_{ac} = \mathcal{E}$ and $i_2 = \frac{v_{ac}}{R_2} = \frac{\mathcal{E}}{R_2}$; this is the current that the inductor will try to maintain as the switch is opened. So at the instant the switch is opened, a current $\frac{\mathcal{E}}{R_2} = \frac{60.0 \text{ V}}{25.0 \text{ }\Omega} = \boxed{2.40 \text{ A}}$ flows from d to a through R_1 and from a to c through R_2 , powered by the inductor.