28.68.

(a) The magnetic field vectors are shown in Figure 28.68a.

(b) At a position on the x-axis $B_{\text{net}} = 2\dfrac{\mu_0 I}{2\pi r}\sin\theta = \dfrac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\dfrac{a}{\sqrt{x^2+a^2}} = \dfrac{\mu_0 Ia}{\pi(x^2+a^2)}$, in the positive x-direction.

(c) The graph of B versus x/a is given in Figure 28.68b.

(d) The magnetic field is a maximum at the origin, $x=0$.

(e) When $x \gg a$, $B \approx \frac{\mu_0 I a}{\pi x^2}$

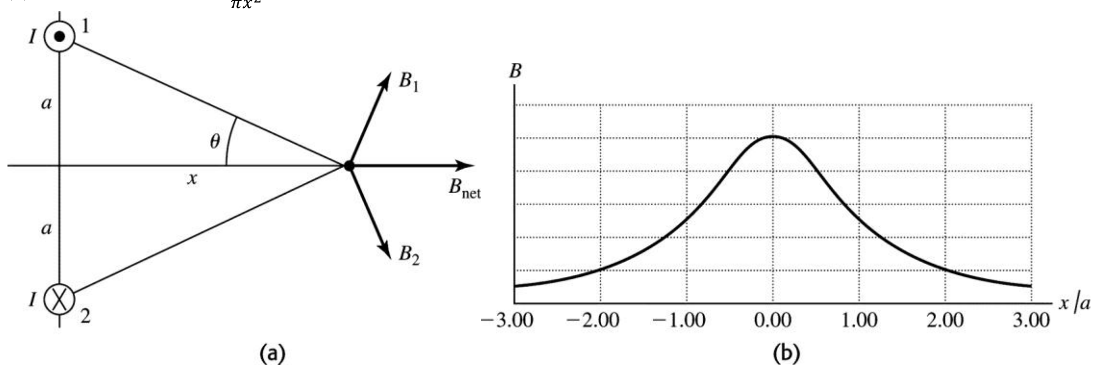


Figure 28.68

28.74 Apply $d\vec{\boldsymbol{B}} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{\boldsymbol{l}}\times\hat{\boldsymbol{r}}}{r^2}$.

The two straight segments produce zero field at P. The field at the center of a circular loop of radius R is $B=\dfrac{\mu_0 I}{2R}$, so the field at the center of curvature of a semicircular loop is $B=\dfrac{\mu_0 I}{4R}$.

The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2}\right)\left(\dfrac{1}{a}-\dfrac{1}{b}\right) = \dfrac{\mu_0 I}{4a}\left(1-\dfrac{a}{b}\right)$, out of page.

If $a=b$, $B=0$.

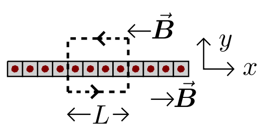
76.

a) Using the Biot-Savart law, $\vec{\boldsymbol{B}} = \frac{\mu_0}{4\,\pi} \int I\,\frac{\mathrm{d}\vec{\boldsymbol{\ell}} \times \hat{\boldsymbol{r}}}{r^2} = \frac{\mu_0\,I}{4\,\pi} \int_0^{2\pi} \frac{\left(-\frac{D}{2}\,\mathrm{d}\theta\,\hat{\boldsymbol{\theta}}\right) \times (-\hat{\boldsymbol{\rho}})}{\left(\frac{D}{2}\right)^2} = -\frac{\mu_0\,I}{4\,\pi}\,\frac{\hat{\boldsymbol{k}}}{D/2} \int_0^{2\pi} \mathrm{d}\theta = -\frac{\mu_0\,I}{D}\,\hat{\boldsymbol{k}}$ (where $\hat{\boldsymbol{\rho}}$ is the unit vector outward from point A, θ is the angle describing the location on the wire ring, and $\hat{\boldsymbol{\theta}}$ is the unit vector tangent to the ring). We can find the same result by using Equation 28.17 and the right-hand rule.

b) The wire segment has length $\pi\,D$, and is centered on point C, so it extends from $x = -\pi\,\frac{D}{2}$ to $x = \pi\,\frac{D}{2}$.

$$\begin{aligned}
\vec{\boldsymbol{B}} &= \frac{\mu_0}{4\,\pi} \int I\,\frac{\mathrm{d}\vec{\boldsymbol{\ell}} \times \hat{\boldsymbol{r}}}{r^2} = \frac{\mu_0}{4\,\pi} \int I\,\frac{\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{r}}}{r^3} = \frac{\mu_0\,I}{4\,\pi} \int_{-\pi D/2}^{\pi D/2} \frac{(-\mathrm{d}x\,\hat{\boldsymbol{\imath}}) \times \left(-x\,\hat{\boldsymbol{\imath}} + \frac{D}{2}\,\hat{\boldsymbol{\jmath}}\right)}{\left[x^2 + \left(\frac{D}{2}\right)^2\right]^{3/2}} \\
&= -\frac{\mu_0\,I}{4\,\pi} \int_{-\pi D/2}^{\pi D/2} \frac{\frac{D}{2}\,\mathrm{d}x\,\hat{\boldsymbol{k}}}{\left[x^2 + \left(\frac{D}{2}\right)^2\right]^{3/2}} \qquad x = \frac{D}{2}\tan\theta, \quad \mathrm{d}x = \frac{D}{2}\sec^2\theta\,\mathrm{d}\theta \\
&= -\frac{\mu_0\,I}{4\,\pi}\,\hat{\boldsymbol{k}} \int_{-\tan^{-1}\pi}^{\tan^{-1}\pi} \frac{\left(\frac{D}{2}\right)^2 \sec^2\theta\,\mathrm{d}\theta}{\left[\left(\frac{D}{2}\right)^2 \tan^2\theta + \left(\frac{D}{2}\right)^2\right]^{3/2}} \\
&= -\frac{\mu_0\,I}{4\,\pi}\,\hat{\boldsymbol{k}} \int_{-\tan^{-1}\pi}^{\tan^{-1}\pi} \frac{\left(\frac{D}{2}\right)^2 \sec^2\theta\,\mathrm{d}\theta}{\left(\frac{D}{2}\right)^3 \sec^3\theta} = -\frac{\mu_0\,I}{2\,\pi\,D}\,\hat{\boldsymbol{k}} \int_{-\tan^{-1}\pi}^{\tan^{-1}\pi} \cos\theta\,\mathrm{d}\theta \\
&= -\frac{\mu_0\,I}{2\,\pi\,D}\,\hat{\boldsymbol{k}}\,\sin\theta\Big|_{\theta=-\tan^{-1}\pi}^{\theta=\tan^{-1}\pi} = -\frac{\mu_0\,I}{2\,\pi\,D}\left[\frac{\pi}{\sqrt{1+\pi^2}} - \frac{-\pi}{\sqrt{1+\pi^2}}\right]\hat{\boldsymbol{k}} \\
&= -\frac{\mu_0\,I}{D\,\sqrt{1+\pi^2}}\,\hat{\boldsymbol{k}}.
\end{aligned}$$

c) The answer in part (b) is smaller by a factor of $\frac{1}{\sqrt{1+\pi^2}}$. This is reasonable because, in going from part (a) to part (b), most of the wire gets farther away from point A.

84. 

Consider a single conducting sheet as in Problem 83. We know that $\vec{\boldsymbol{B}}$ must be in the $\pm x$-direction by symmetry because the vertical component of the field produced by each wire is cancelled by the vertial component of the field produced by another wire. Draw a rectangular Amperian loop as shown. Ampere's law is $\oint \mathrm{d}\vec{\boldsymbol{\ell}} \cdot \vec{\boldsymbol{B}} = (-B_{x,\text{above}} + B_{x,\text{below}})\,L = \mu_0\,I_{\text{enc}} = \mu_0\,I\,n\,L$, so $B_{x,\text{below}} - B_{x,\text{above}} = \mu_0\,I\,n$, regardless of how far the top and bottom sides of the loop are from the current sheet. Therefore the magnetic field is uniform except inside the sheet. By symmetry, we should have $|\vec{\boldsymbol{B}}_{\text{above}}| = |\vec{\boldsymbol{B}}_{\text{below}}| = |B_{x,\text{above}}| = |B_{y,\text{below}}|$. The only possibility then is $\vec{\boldsymbol{B}}_{\text{below}} = \frac{1}{2}\,\mu_0\,I\,n\,\hat{\boldsymbol{\imath}}$ and $\vec{\boldsymbol{B}}_{\text{above}} = -\frac{1}{2}\,\mu_0\,I\,n\,\hat{\boldsymbol{\imath}}$.

We can now find the field in the case of two current sheets by superposition of two copies of the field due to a single sheet.

$$\vec{\boldsymbol{B}}_1 = \begin{cases} -\frac{1}{2}\,\mu_0\,I\,n\,\hat{\boldsymbol{\imath}}, & \frac{d}{2} < y \\ \frac{1}{2}\,\mu_0\,I\,n\,\hat{\boldsymbol{\imath}}, & y < \frac{d}{2}, \end{cases} \qquad \vec{\boldsymbol{B}}_2 = \begin{cases} \frac{1}{2}\,\mu_0\,I\,n\,\hat{\boldsymbol{\imath}}, & -\frac{d}{2} < y \\ -\frac{1}{2}\,\mu_0\,I\,n\,\hat{\boldsymbol{\imath}}, & y < -\frac{d}{2}, \end{cases}$$

so

$$\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}_1 + \vec{\boldsymbol{B}}_2 = \begin{cases} 0, & \frac{d}{2} < y \quad \text{(a)} \\ \mu_0\, I\, n\, \hat{\boldsymbol{\imath}}\,, & -\frac{d}{2} < y < \frac{d}{2} \quad \text{(b)} \\ 0, & y < -\frac{d}{2} \quad \text{(c)}. \end{cases}$$