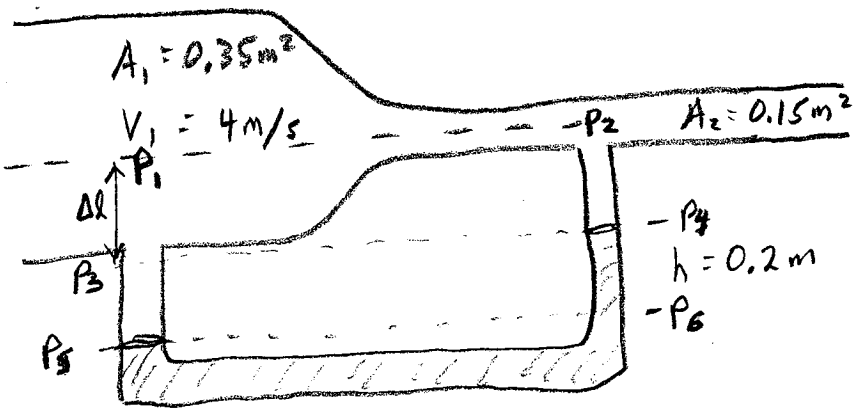
1. Fluid Mechanics



I have defined six pressures, P_1 through P_6. From Bernoulli's law:

$$P_1 + \frac{1}{2}\rho_w V_1^2 + \rho_w g h_1 = P_2 + \frac{1}{2}\rho_w V_2^2 + \rho_w g h_2$$

but $h_1 = h_2$ and we can relate the velocities by:

$$A_1 V_1 = A_2 V_2 \Rightarrow V_2 = \frac{A_1}{A_2} V_1$$

so, $$P_1 - P_2 = \frac{1}{2}\rho_w (V_2^2 - V_1^2) = \frac{1}{2}\rho_w \left[\left(\frac{A_1}{A_2}\right)^2 V_1^2 - V_1^2\right]$$

we will use this later.

Now $P_3 = P_1 + \rho_w g \Delta l$ and $P_4 = P_2 + \rho_w g \Delta l$

And $P_5 = P_3 + \rho_w g h$ and $P_6 = P_4 + \rho_\ell g h$

$= P_1 + \rho_w g \Delta l + \rho_w g h$ $= P_2 + \rho_w g \Delta l + \rho_\ell g h$

But $P_5 = P_6$ since they are in the same fluid, so:

$$P_1 + \cancel{\rho_w g \Delta l} + \rho_w g h = P_2 + \cancel{\rho_w g \Delta l} + \rho_\ell g h$$

$$P_1 - P_2 = \rho_\ell g h - \rho_w g h = \frac{1}{2}\rho_w \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] V_1^2$$

$$\rho_\ell = \frac{1}{gh}\left[\frac{1}{2}\rho_w V_1^2 \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] + \rho_w g h\right]$$

$$= \frac{1}{(9.8 \frac{m}{s^2})(.2m)}\left[\frac{1}{2}\rho_w \left(4\frac{m}{s}\right)^2 \left[\left(\frac{.35 m^2}{.15 m^2}\right)^2 - 1\right] + \rho_w \left(9.8\frac{m}{s^2}\right)(.2m)\right.$$

$$\rho_\ell = \rho_w \cdot (19.1)$$

Now, the problem said $\rho_w = 1 \frac{kg}{m^3}$, but that's wrong. $\rho_w = 1000 \frac{kg}{m^3}$

so, $\rho_\ell = 19{,}100 \frac{kg}{m^3}$ so, maybe liquid uranium...

2. Gravitation

For the sattelite to be in a circular orbit, we know:

$$\Sigma F = G \frac{m_s m_E}{(R_E + h)^2} = m_s a_s = m_s \frac{v_s^2}{(R_E + h)}$$

$$\Rightarrow v_s = \sqrt{\frac{G m_E}{(R_E + h)}}$$

$$= \sqrt{\frac{6.67 \times 10^{-11} \frac{m^3}{kg\, s^2} \cdot 5.97 \times 10^{24} kg}{(6.37 \times 10^6 m + 3.00 \times 10^6 m)}}$$

$$\boxed{v_s = 6.5 \times 10^3 \frac{m}{s}}$$