Lecture 30
(Fluid Mechanics)

Physics 160-01 Fall 2012

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Fluid Mechanics

- Here are the things you need to understand out of this chapter:
  - Density
  - Pressure
  - Pascal’s Law
  - Archimedes’ Principle
  - Bernoulli’s Equation

Density

- Most of you already know this…

$$\rho = \frac{M}{V} = \frac{\text{mass}}{\text{volume}}$$

- Can vary as a function of position:

$$\rho\left(x, y, z\right)$$

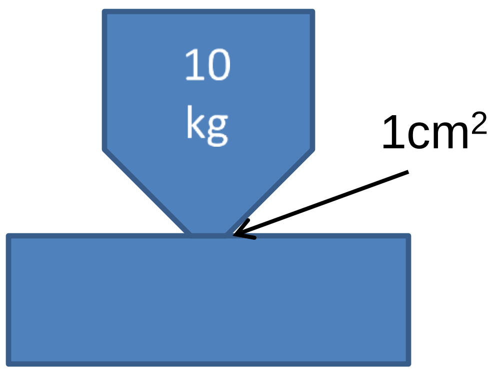
- For instance, the density of the earth's atmosphere depends upon at what height it is measured.

Pressure

- Pressure is a little more fuzzy in our minds, but we kind of know it as force distributed over an area:

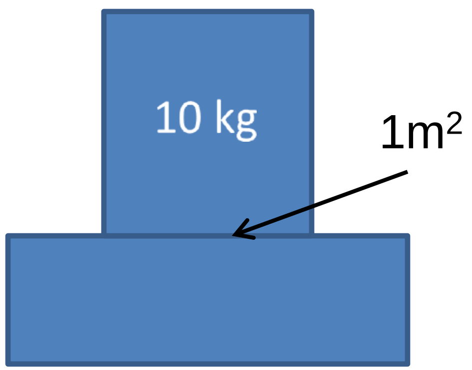
$$P = \frac{F_{\perp}}{A}$$ (Pressure is a scalar!)

- Examples:



$$P = \frac{F_{\perp}}{A} = \frac{mg}{A} = \frac{10kg \cdot 9.8\, m/s^2}{(0.01m)^2}$$

$$= 9.8 \times 10^5 \frac{N}{m^2} = 9.8 \times 10^5 \text{Pa}$$

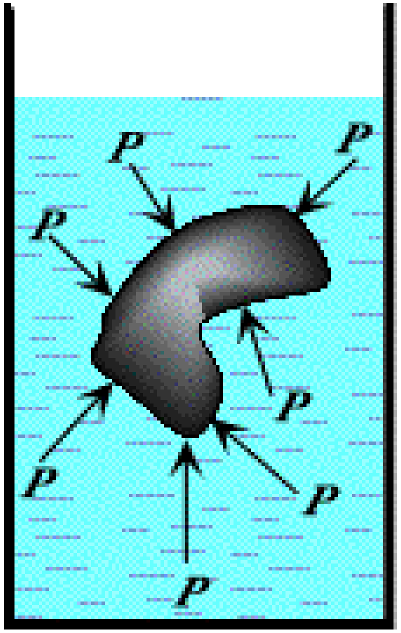


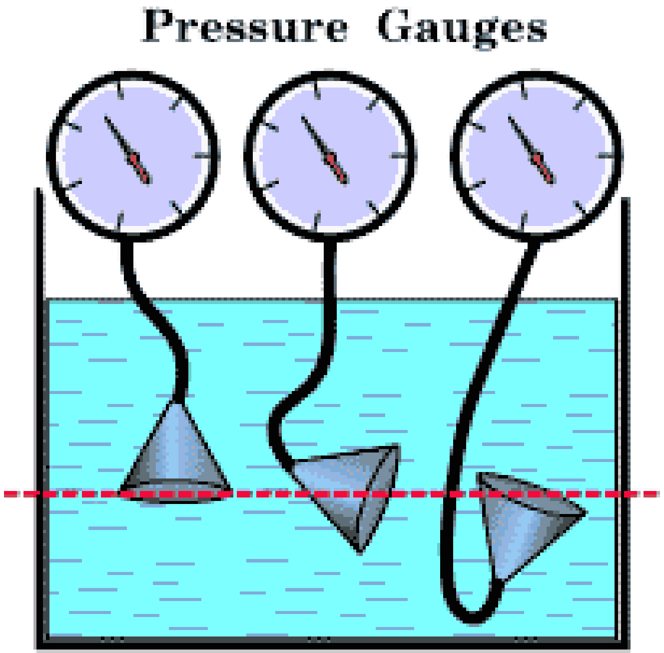
$$P = \frac{F_{\perp}}{A} = \frac{mg}{A} = \frac{10kg \cdot 9.8\, m/s^2}{(1m)^2}$$

$$= 98 \frac{N}{m^2} = 98\text{Pa}$$

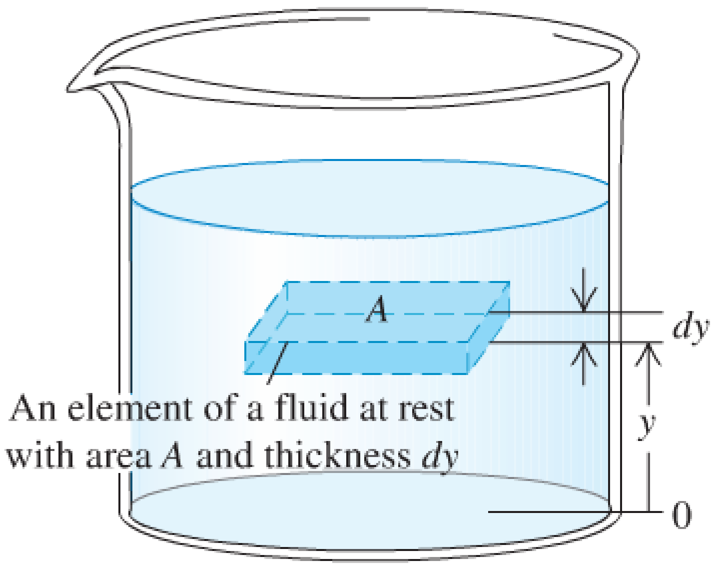
Pressure in a Fluid

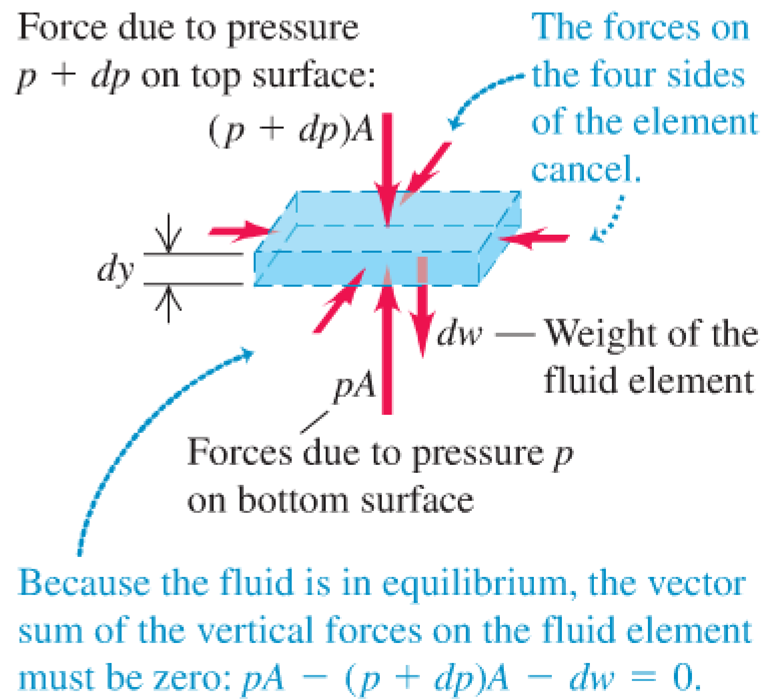
- Pressure in a fluid acts in all directions equally.





Pressure and Depth





$$\sum F_y = 0$$

$$pA - (p + dp)A - mg = 0$$

$$pA - (p + dp)A - (\rho V)g = 0$$

$$pA - (p + dp)A - (\rho A dy)g = 0$$

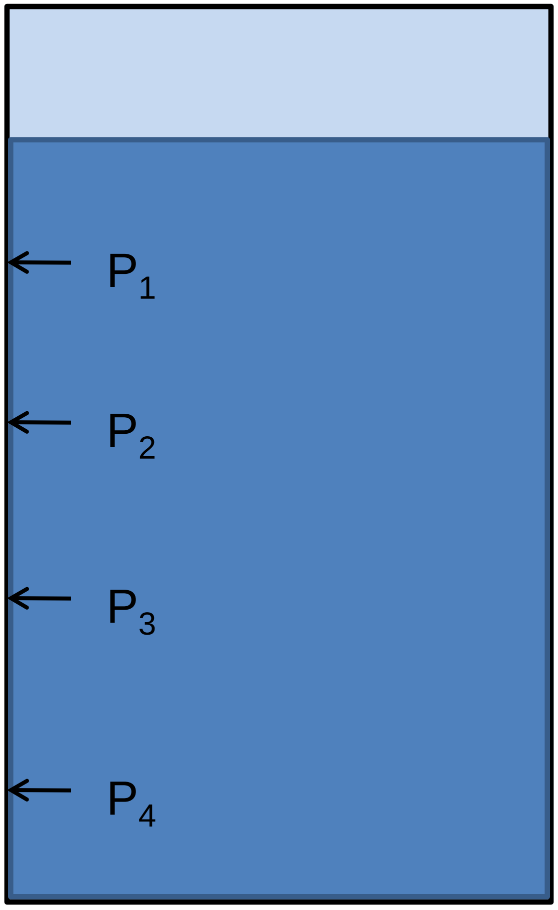
$$p - (p + dp) - (\rho dy)g = 0$$

$$dp = -\rho g dy$$

$$\int_{p_0}^{p} dp = -\rho g \int_{y_0}^{y} dy \Rightarrow$$

$$p - p_0 = -\rho g (y - y_0)$$

Demo



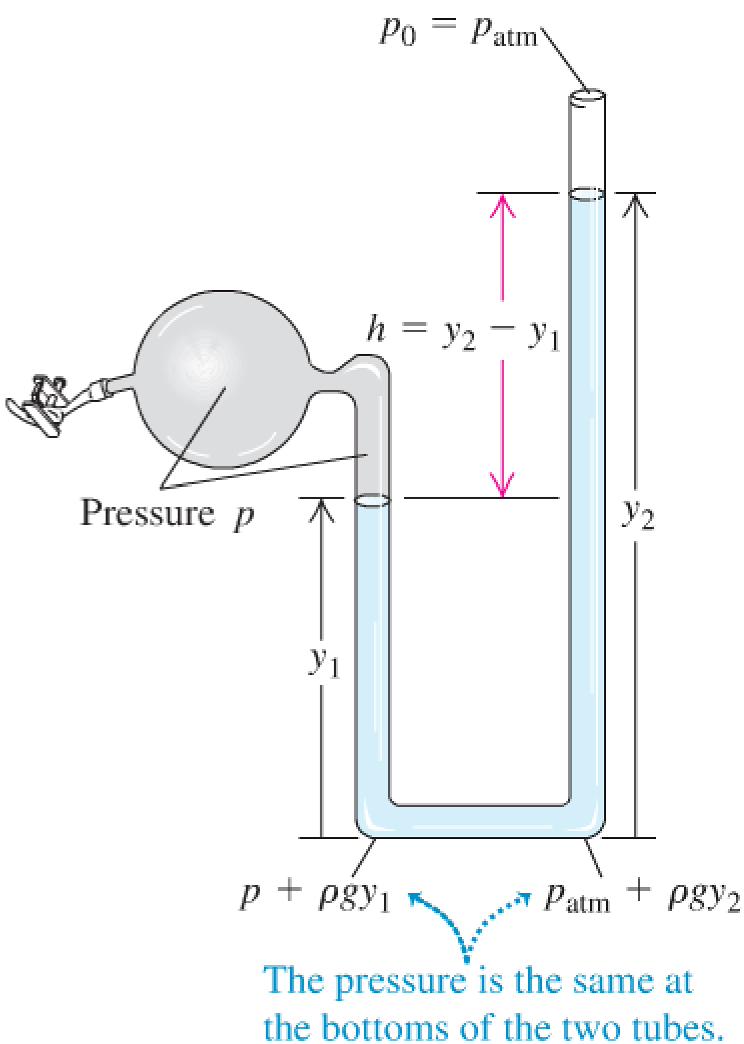
Even Train Modelers Understand

Atmospheric Pressure

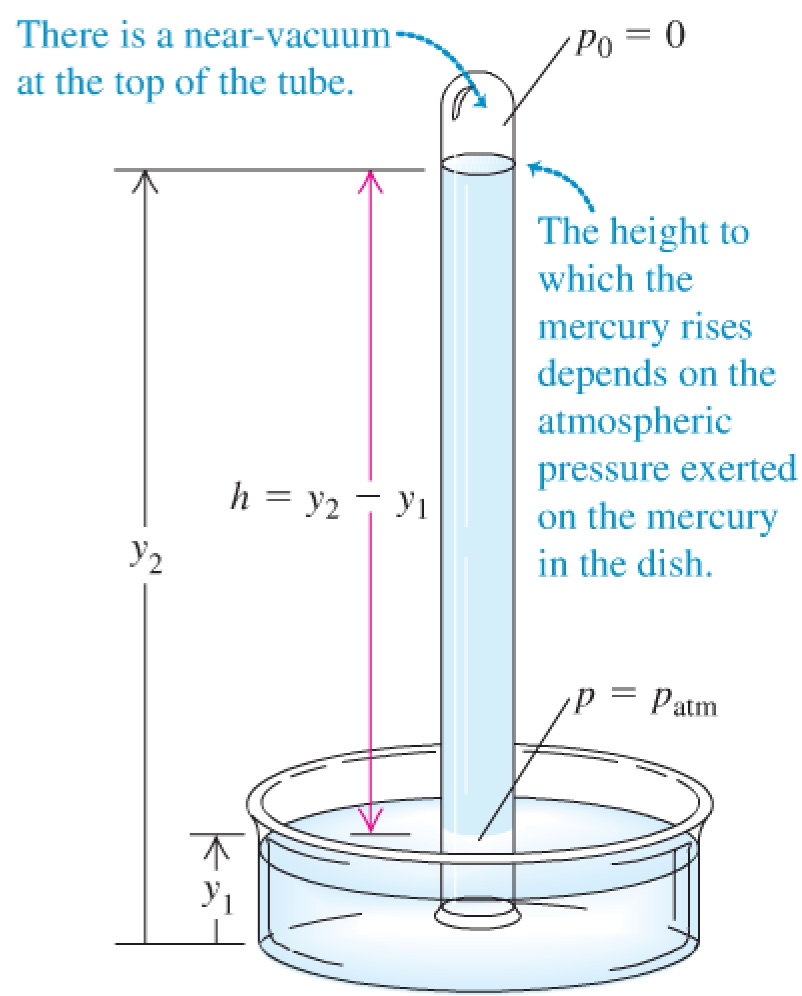
- The Pressure at the surface of our water tank isn't zero, but is the result of all the air above it.
- The only difference with the liquid pressure formulation we just did is that the density of air can change.
  - Air is a gas, and so it is compressible.
  - Water is a liquid and is not compressible.

Pressure Gauges

(a) Open-tube manometer

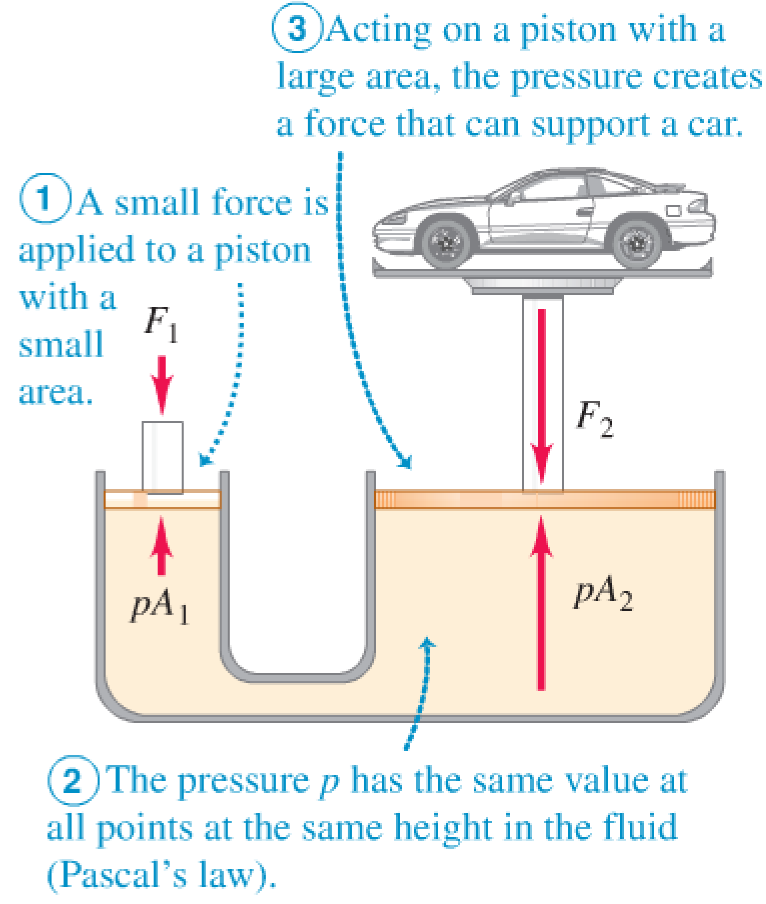


(b) Mercury barometer



Pascal's Law

- Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the container.

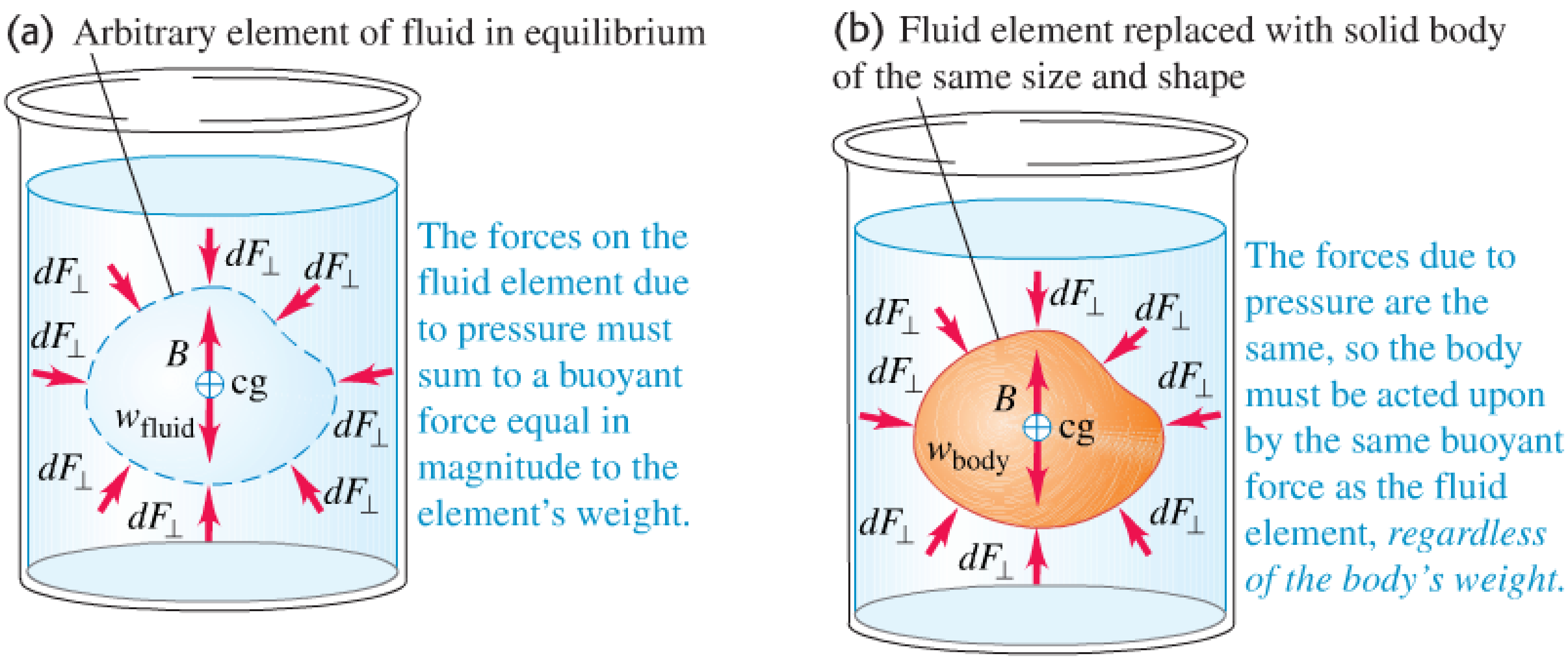


Buoyancy

- When a body is immersed in a fluid, there is a buoyant force exerted on that body.
- What causes that force?

Archimedes’ Principle

- The buoyant force acting on a body wholly or partially immersed in a fluid is equal to the weight of the fluid displaced by that body.

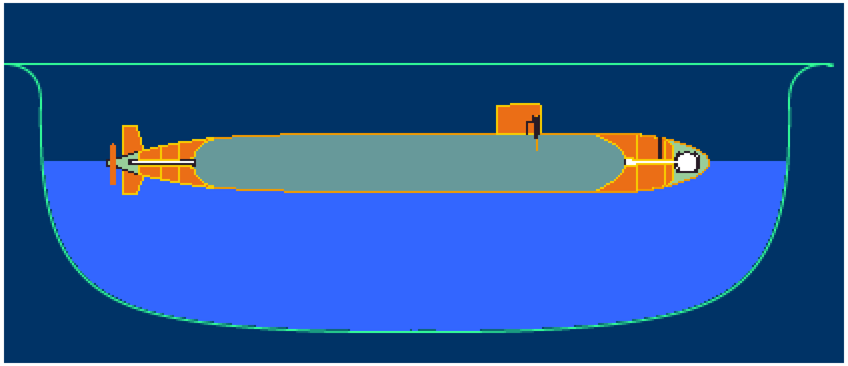


Floating and Sinking

- An object floats on the surface when it displaces more weight of the fluid than it weighs.
- An object will float in mid-fluid when it displaces exactly the same weight of fluid as it weighs itself.
- An object will sink when it displaces less weight of fluid than it weighs.

CPS 35-1

- Here we have a toy submarine floating in a bathtub. It's a really fancy sub, made out of steel. The sub weighs one pound. When completely submerged, it displaces two pounds of water. What could you do to cause the sub to sink to the bottom of the tub?



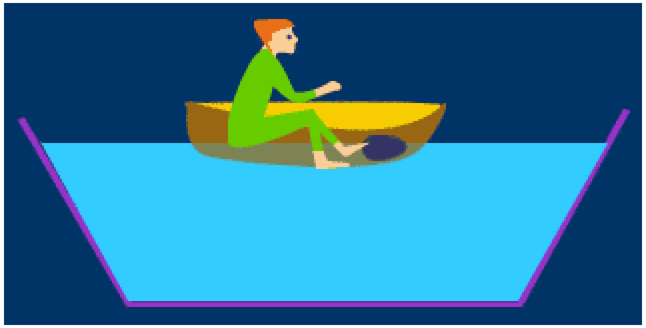
A. Add one pound of sand to the sub's interior.

B. Add one pound of sand to the sub's interior, plus a little more.

C. Nothing. Since the boat displaces more water than it weighs, it's already on its way down.

CPS 35-2

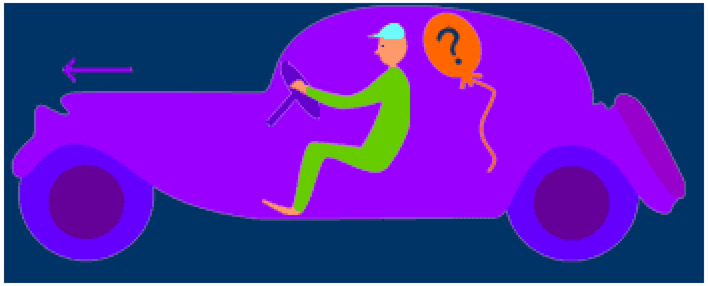
- Here we have a boat in a swimming pool. In the boat is an inquisitive experimenter. Also in the boat is a rock. Our experimenter picks up the rock and tosses it into the pool. The rock sinks to the bottom. No water leaves the pool from the splash made by the rock. Now for the question: Does the pool's water level rise, lower, or stay the same?



A. The water level rises.
B. The water level lowers.
C. The water level stays the same.

CPS 35-3

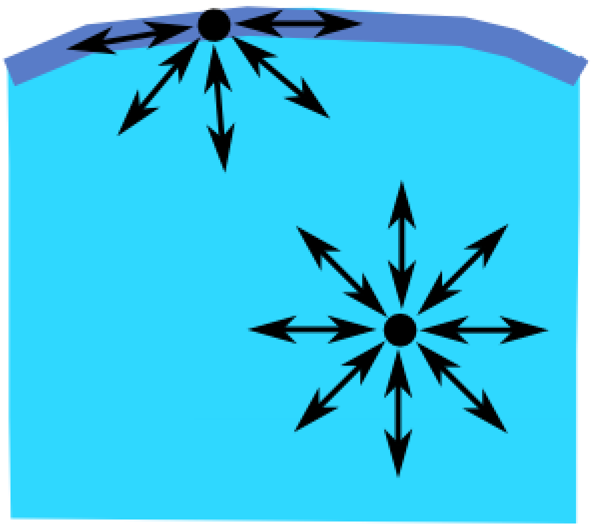
- You're sitting in a car that's not moving. Also in the car is a helium-filled balloon, which is resting up against the car's ceiling somewhere near its middle. The driver hits the gas and the car accelerates forward. You're thrown back into your seat. What happens to the balloon? (Before you answer, think about what will make the balloon act the way you think it will.)

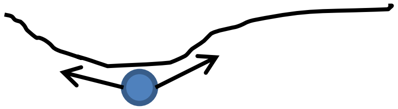


A. It floats toward the back of the car.
B. It floats toward the front of the car.
C. It stays put.

Surface Tension

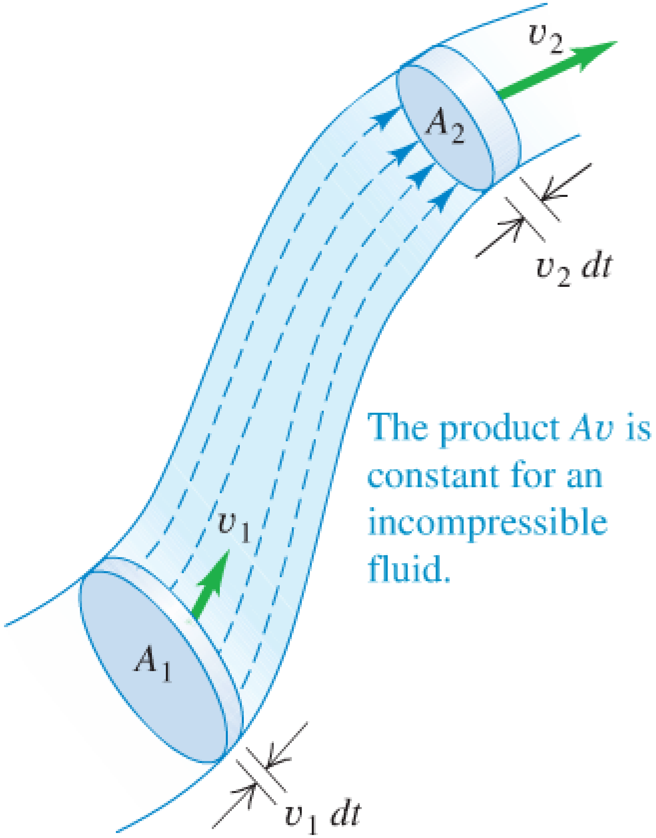
Surface Tension





Fluid Flow

- Flow lines (or streamlines) show the motion of parts of the fluid.
- If the streamlines get closer together, the velocity must get higher.



Bernoulli's Equation

- Examine the movement of a bit of a fluid (from point a to point c) over a time Δt. Fluid at point a will move to point b during that time. Fluid starting at point c will move to point d during the same time.
- Now, let's use the work energy theorem: $dW = dK + dU$
- The work done by the external fluid from point a to b minus the work done by the external fluid from point c to d is:

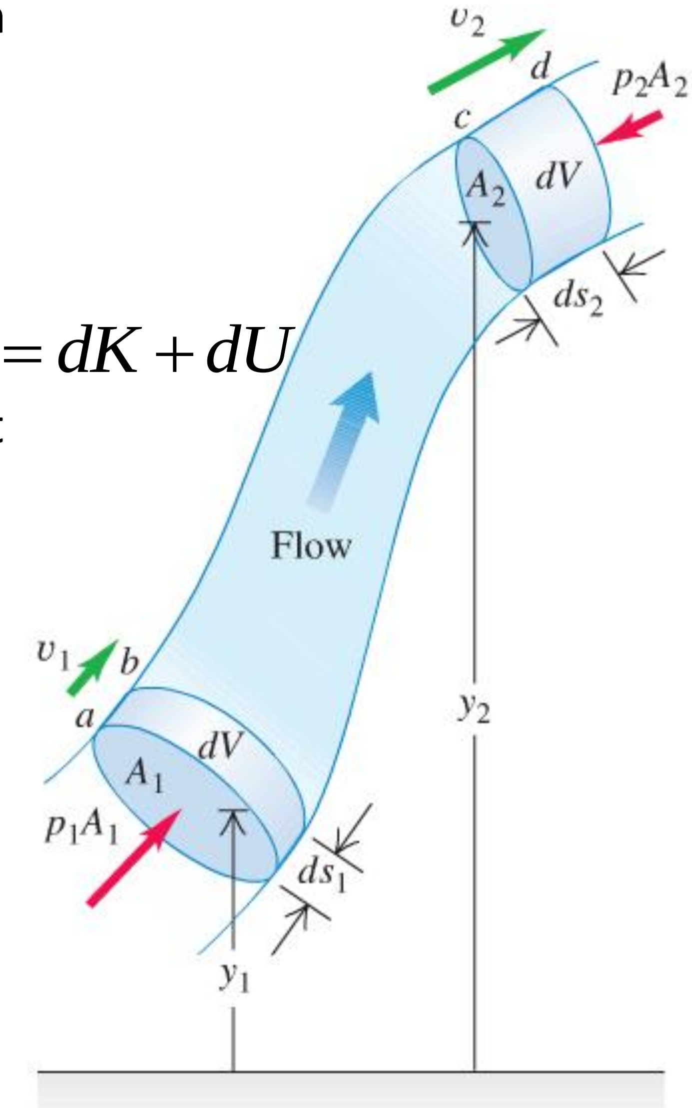
$$dW = p_1 A_1 ds_1 - p_2 A_2 ds_2 = \left(p_1 - p_2\right) dV$$

- The change in the kinetic energy of the fluid is just:

$$dK = \frac{1}{2}\rho dV\left(v_2^2 - v_1^2\right)$$

- The change in the gravitational potential energy is:

$$dU = \rho dV\left(y_2 - y_1\right)$$



Bernoulli's Equation

- Putting this all together, we have:

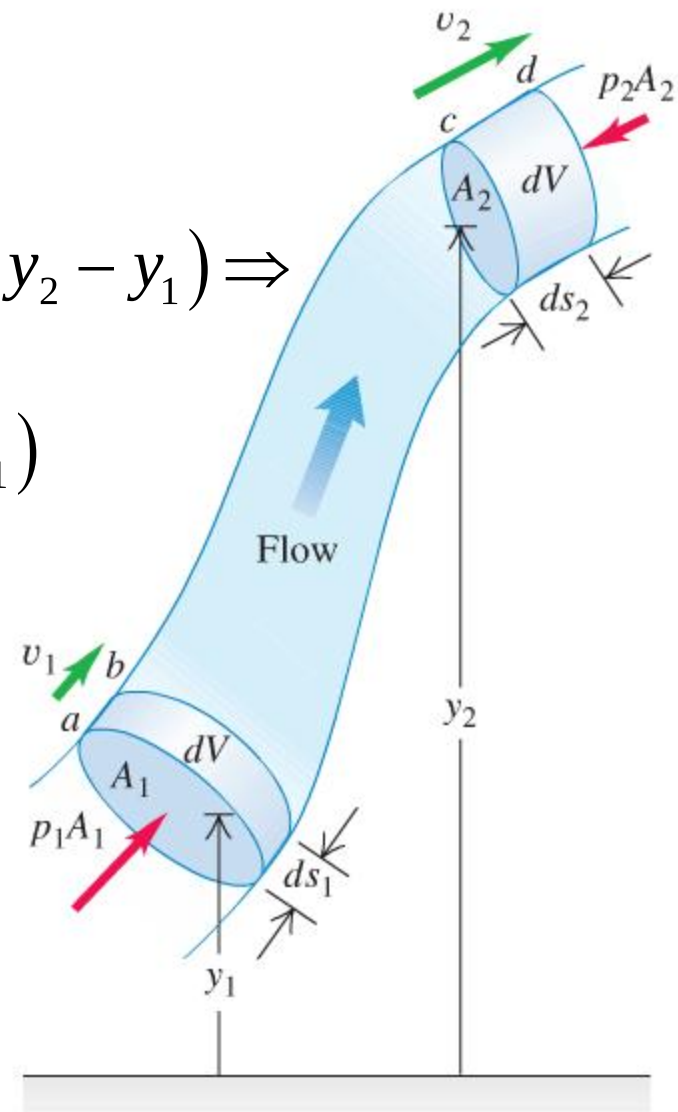
$$dW = dK + dU \Rightarrow$$

$$\left(p_1 - p_2\right)dV = \frac{1}{2}\rho dV\left(v_2^2 - v_1^2\right) + \rho dV\left(y_2 - y_1\right) \Rightarrow$$

$$\left(p_1 - p_2\right) = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho\left(y_2 - y_1\right)$$

or

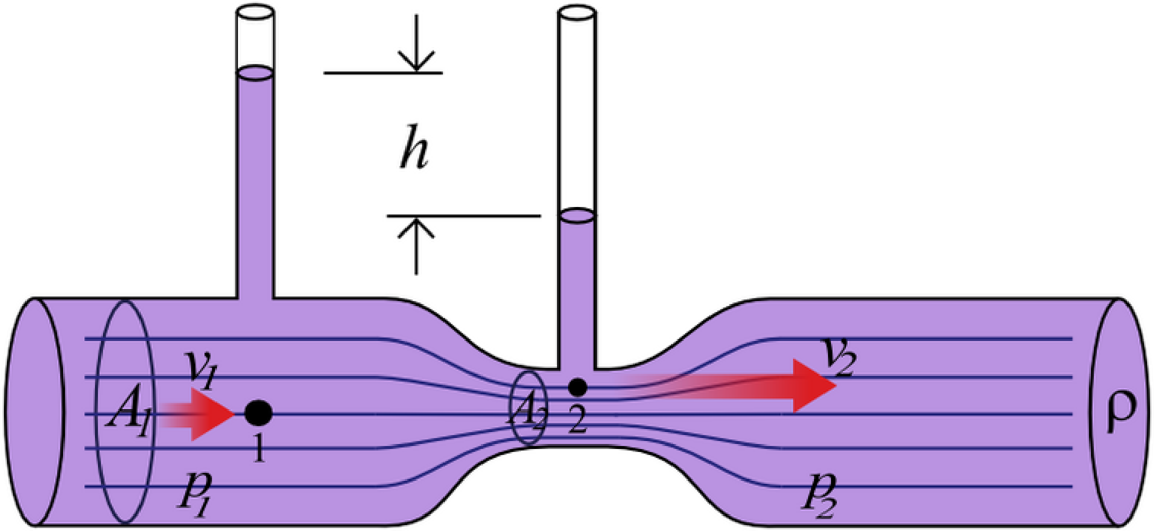
$$p_1 + \frac{1}{2}\rho v_1^2 + \rho y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho y_2$$



Bernoulli's Equation

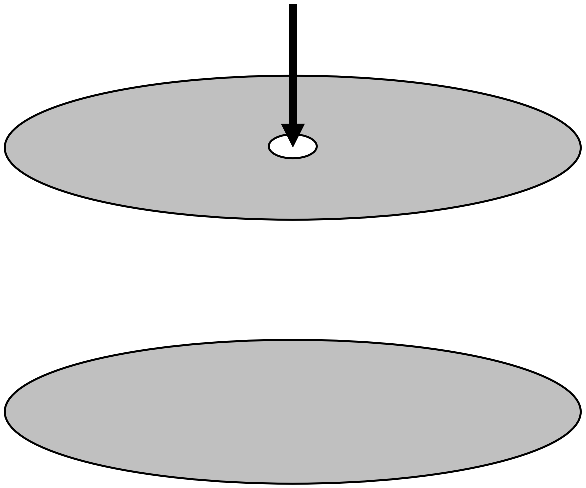
$$p_1 + \frac{1}{2}\rho v_1^2 + \rho y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho y_2$$

- Consequences:
  - When the velocities are zero (static) we get back our pressure relationship with height.
  - When the heights are the same, note that higher velocities give lower pressures.



CPS 35-4

- One of the two plates of the demonstration has air flowing out of the center. What will happen when the two plates approach each other?



A) The air will blow them apart.

B) The air will suck them together.

C) Nothing, these demonstrations never work.

D) Not enough information to solve.

Demos