

## #37 Pendula Pre-class

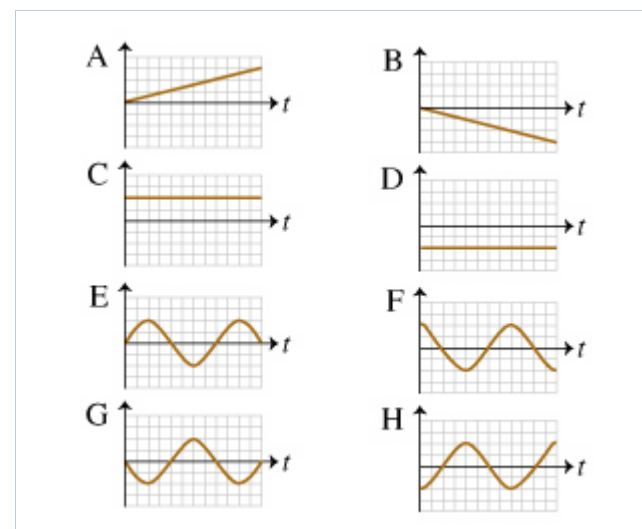
Due: 11:00am on Monday, November 19, 2012

**Note:** You will receive no credit for late submissions. To learn more, read your instructor's [Grading Policy](#)

### Simple Harmonic Motion Kinematics Graphs Conceptual Question

A simple pendulum is displaced to the left of its equilibrium position and is released from rest. Set the origin of the coordinate system at the equilibrium position of the pendulum, and let counterclockwise be the positive angular direction. Assume air resistance is so small that it can be ignored.

Refer to these graphs when answering the following questions.



#### Part A

Beginning the instant the pendulum is released, select the graph that best matches the angular position vs. time graph for the pendulum.

##### Hint 1. How to approach the problem

To find the graph of angular position vs. time, first determine the initial value of the pendulum's angular position. This will narrow down your choices of possible graphs. Then, interpret what each remaining graph says about the subsequent motion of the pendulum. You should find that only one graph describes the angular position of the pendulum correctly.

##### Hint 2. Assess the initial angular position

The origin of the coordinate system is set at the equilibrium angular position of the pendulum, with the positive direction counterclockwise. The pendulum is rotated clockwise from equilibrium and released. Therefore, is the initial angular position positive, negative, or zero?

ANSWER:

- ☐ positive
- ☒ negative
- ☐ zero

ANSWER:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ F
- ☐ G
- ☒ H

**Correct**

---

## Part B

Beginning the instant the pendulum is released, select the graph that best matches the angular velocity vs. time graph for the pendulum.

**Hint 1.** Assess the initial angular velocity

The pendulum is released from rest. Is the initial angular velocity positive, negative, or zero?

ANSWER:

- ☐ positive
- ☐ negative
- ☒ zero

**Hint 2.** Find the direction of the angular velocity a short time later

After the pendulum is released from rest, in which direction will it initially move?

ANSWER:

- ☒ counterclockwise (positive)
- ☐ clockwise (negative)

ANSWER:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☒ E
- ☐ F
- ☐ G
- ☐ H

Correct

---

### Part C

Beginning the instant the pendulum is released, select the graph that best matches the angular acceleration vs. time graph for the pendulum.

#### Hint 1. Assess the initial angular acceleration

The pendulum is released from rest, and a short time later it is moving counterclockwise. Based on this observation, is initial angular acceleration positive, negative, or zero?

ANSWER:

- ☒ positive
- ☐ negative
- ☐ zero

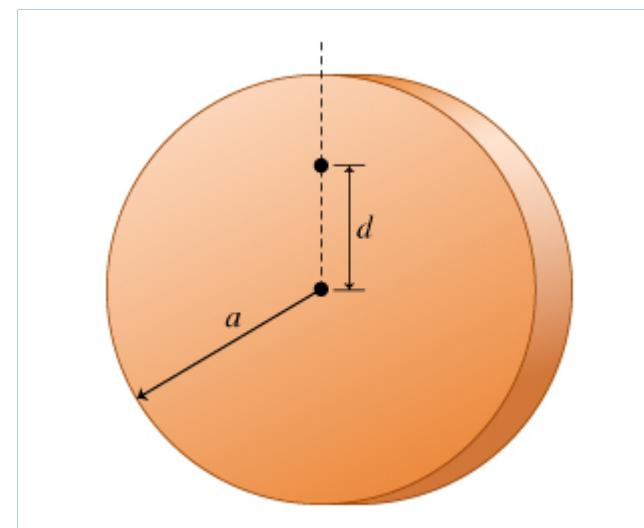
ANSWER:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☒ F
- ☐ G
- ☐ H

Correct

## Extreme Period for a Physical Pendulum

A solid, uniform disk of mass  $M$  and radius  $a$  may be rotated about any axis parallel to the disk axis, at variable distances from the center of the disk.



**Part A**

What is  $I_{\text{cm}}$ , the moment of inertia of the disk around its center of mass? You should know this formula well.

**Express your answer in terms of given variables.**

ANSWER:

$$I_{\text{cm}} = \frac{1}{2}Ma^2$$

**Correct**

**Part B**

If you use this disk as a pendulum bob, what is  $T(d)$ , the period of the pendulum, if the axis is a distance  $d$  from the center of mass of the disk?

**Express the period of the pendulum in terms of given variables.**

**Hint 1. Formula for  $T$** 

The formula for the period of a physical pendulum is

$$T = 2\pi \sqrt{\frac{I(d)}{mgd}}$$

where  $T$  is the period,  $I(d)$  is the moment of inertia of the pendulum about the rotation axis,  $m$  is the mass of the pendulum,  $d$  is the distance between the center of mass of the pendulum and its axis of rotation, and  $g$  is the magnitude of the acceleration due to gravity.

**Hint 2. Moment of inertia**

Find  $I_d$ , the moment of inertia of the disk about an axis a distance  $d$  from the disk's center of mass.

**Express the moment of inertia of the disk in terms of  $M$ ,  $a$ , and  $d$ .**

**Hint 1. Parallel Axis Theorem**

The Parallel Axis Theorem states that

$$I(r) = I_{\text{cm}} + Mr^2,$$

where  $I(r)$  is the moment of inertia of an object of mass  $M$  about an axis a distance  $r$  from the object's center of mass and  $I_{\text{cm}}$  is the moment of inertia about the object's center of mass.

ANSWER:

$$I_d = M(a^2/2 + d^2)$$

ANSWER:

$$T(d) = 2\pi \sqrt{\frac{a^2/2 + d^2}{gd}}$$

**Correct**

**Part C**

The period of the pendulum has an extremum (a local maximum or a local minimum) for some value of  $d$  between zero and infinity. Is it a local maximum or a local minimum?

**Hint 1. Physical reasoning**

Reason physically. Find the period if  $d$  is very small or very large. If the period is zero (the lowest possible value) in both limits, then the extremum must be a local maximum. If the period is infinite in both limits, the extremum must be a local minimum.

**Hint 2.** As  $d$  approaches zero

What value does  $T$  approach when  $d \rightarrow 0$ ?

ANSWER:

- ☐ zero
- ☒ infinity

**Hint 3.** As  $d$  approaches infinity

What value does  $T$  approach as  $d \rightarrow \infty$ ?

ANSWER:

- ☐ zero
- ☒ infinity

ANSWER:

- ☐ maximum
- ☒ minimum

**Correct**



**Part D**

What is  $T_{\min}$ , the minimum period of the pendulum?

Your answer for the minimum period should include given variables.

**Hint 1.** How to minimize the period

What value of  $d$  minimizes the period of the pendulum?

**Hint 1.** Some calculus

When the derivative of  $T$  with respect to  $d$  is zero,  $T$  is either at a maximum or a minimum. By plugging in a few values you can verify that, in this case, it is a minimum.

ANSWER:

$$d = a/\sqrt{2}$$

ANSWER:

$$T_{\min} = 2\pi \sqrt{\frac{a^2}{\frac{ga}{\sqrt{2}}}}$$

**Correct**

## Changing the Period of a Pendulum

A simple pendulum consisting of a bob of mass  $m$  attached to a string of length  $L$  swings with a period  $T$ .

### Part A

If the bob's mass is doubled, approximately what will the pendulum's new period be?

#### Hint 1. Period of a simple pendulum

The period  $T$  of a simple pendulum of length  $L$  is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where  $g$  is the acceleration due to gravity.

ANSWER:

- ☐  $T/2$
- ☒  $T$
- ☐  $\sqrt{2}T$
- ☐  $2T$

**Correct**

### Part B

If the pendulum is brought on the moon where the gravitational acceleration is about  $g/6$ , approximately what will its period now be?

**Hint 1.** How to approach the problem

Recall the formula of the period of a simple pendulum. Since the gravitational acceleration appears in the denominator, the period must increase when the gravitational acceleration decreases.

ANSWER:

- ☐  $T/6$
- ☐  $T\sqrt{6}$
- ☒  $\sqrt{6}T$
- ☐  $6T$

**Correct**

---

**Part C**

If the pendulum is taken into the orbiting space station what will happen to the bob?

**Hint 1.** How to approach the problem

Recall that the oscillations of a simple pendulum occur when a pendulum bob is raised above its equilibrium position and let go, causing the pendulum bob to fall. The gravitational force acts to bring the bob back to its equilibrium position. In the space station, the earth's gravity acts on both the station and everything inside it, giving them the same acceleration. These objects are said to be in free fall.

ANSWER:

- ☐ It will continue to oscillate in a vertical plane with the same period.
- ☐ It will no longer oscillate because there is no gravity in space.
- ☒ It will no longer oscillate because both the pendulum and the point to which it is attached are in free fall.
- ☐ It will oscillate much faster with a period that approaches zero.

**Correct**

In the space station, where all objects undergo the same acceleration due to the earth's gravity, the tension in the string is zero and the bob does not fall relative to the point to which the string is attached.

**Score Summary:**

Your score on this assignment is 102.9%.

You received 15.44 out of a possible total of 15 points.