43.2. **IDENTIFY:** Calculate the spin magnetic energy shift for each spin component. Calculate the energy splitting between these states and relate this to the frequency of the photons.

(a) SET UP: From Example 43.2, when the z-component of $\vec{S}$ (and $\vec{\mu}$) is parallel to $\vec{B}$, $U = -|\mu_z|B = -2.7928\mu_n B$. When the z-component of $\vec{S}$ (and $\vec{\mu}$) is antiparallel to $\vec{B}$, $U = +|\mu_z|B = +2.7928\mu_n B$. The state with the proton spin component parallel to the field lies lower in energy. The energy difference between these two states is $\Delta E = 2(2.7928\mu_n B)$.

EXECUTE: $\Delta E = hf$ so $f = \dfrac{\Delta E}{h} = \dfrac{2(2.7928\mu_n)B}{h} = \dfrac{2(2.7928)(5.051\times10^{-27}\text{ J/T})(1.65\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}}$

$f = 7.03\times10^{7}\text{ Hz} = 70.3\text{ MHz}$

And then $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\text{ m/s}}{7.03\times10^{7}\text{ Hz}} = 4.26\text{ m}$

EVALUATE: From Figure 32.4 in the textbook, these are radio waves.

(b) SET UP: From Eqs. (27.27) and (41.40) and Figure 41.18 in the textbook, the state with the z-component of $\vec{\mu}$ parallel to $\vec{B}$ has lower energy. But, since the charge of the electron is negative, this is the state with the electron spin component antiparallel to $\vec{B}$. That is, for $m_s = -\frac{1}{2}$, the state lies lower in energy.

EXECUTE: For the $m_s = +\frac{1}{2}$ state,

$$U = +(2.00232)\left(\frac{e}{2m}\right)\left(+\frac{\hbar}{2}\right)B = +\tfrac{1}{2}(2.00232)\left(\frac{e\hbar}{2m}\right)B = +\tfrac{1}{2}(2.00232)\mu_B B.$$

For the $m_s = -\frac{1}{2}$ state, $U = -\frac{1}{2}(2.00232)\mu_B B$. The energy difference between these two states is $\Delta E = (2.00232)\mu_B B$.

$$\Delta E = hf \text{ so } f = \frac{\Delta E}{h} = \frac{2.00232\mu_B B}{h} = \frac{(2.00232)(9.274\times10^{-24}\text{ J/T})(1.65\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 4.62\times10^{10}\text{ Hz} = 46.2\text{ GHz}.$$

And $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\text{ m/s}}{4.62\times10^{10}\text{ Hz}} = 6.49\times10^{-3}\text{ m} = 6.49\text{ mm}.$

EVALUATE: From Figure 32.4 in the textbook, these are microwaves. The interaction energy with the magnetic field is inversely proportional to the mass of the particle, so it is less for the proton than for the electron. The smaller transition energy for the proton produces a larger wavelength.

43.74. **IDENTIFY:** The tritium (H-3) decays to He-3. The ratio of the number of He-3 atoms to H-3 atoms allows us to calculate the time since the decay began, which is when the H-3 was formed by the nuclear explosion. The H-3 decay is exponential.

SET UP: The number of tritium (H-3) nuclei decreases exponentially as $N_H = N_{0,H}e^{-\lambda t}$, with a half-life of 12.3 years. The amount of He-3 present after a time t is equal to the original amount of tritium minus the number of tritium nuclei that are still undecayed after time t.

EXECUTE: The number of He-3 nuclei after time t is

$$N_{He} = N_{0,H} - N_H = N_{0,H} - N_{0,H}e^{-\lambda t} = N_{0,H}(1 - e^{-\lambda t}).$$

Taking the ratio of the number of He-3 atoms to the number of tritium (H-3) atoms gives

$$\frac{N_{He}}{N_H} = \frac{N_{0,H}(1-e^{-\lambda t})}{N_{0,H}e^{-\lambda t}} = \frac{1-e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1.$$

Solving for t gives $t = \dfrac{\ln(1 + N_{He}/N_H)}{\lambda}$. Using the given numbers and $T_{1/2} = \dfrac{\ln 2}{\lambda}$, we have

$\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{12.3\text{ y}} = 0.0563/\text{y}$ and $t = \dfrac{\ln(1+4.3)}{0.0563/\text{y}} = 30$ years.

EVALUATE: One limitation on this method would be that after many years the ratio of H to He would be too small to measure accurately.