Faraday's Law and Induced Emf

Description: Discusses Faraday's law, presents a sequence of questions related to finding the induced emf under different circumstances.

Learning Goal:

To understand the terms in Faraday's law and to be able to identify the magnitude and direction of induced emf.

Faraday's law states that induced emf is directly proportional to the time rate of change of magnetic flux. Mathematically, it can be written as

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t},$$

where $\mathcal{E}$ is the emf induced in a closed loop, and

$$\frac{\Delta\Phi_B}{\Delta t}$$

is the rate of change of the magnetic flux through a surface bounded by the loop. For uniform magnetic fields the magnetic flux is given by $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos(\theta)$, where θ is the angle between the magnetic field $\vec{B}$ and the normal to the surface of area A.

To find the *direction* of the induced emf, one can use Lenz's law:

The induced current's magnetic field opposes the change in the magnetic flux that induced the current.

For example, if the magnetic flux through a loop *increases*, the induced magnetic field is directed *opposite* to the "parent" magnetic field, thus countering the increase in flux. If the flux *decreases*, the induced current's magnetic field has the same direction as the parent magnetic field, thus countering the decrease in flux.

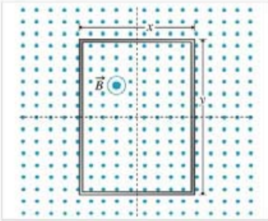
Recall that to relate the direction of the electric current and its magnetic field, you can use the *right-hand rule*. When the fingers on your right hand are curled in the direction of the current in a loop, your thumb gives the direction of the magnetic field generated by this current.

In this problem, we will consider a rectangular loop of wire with sides x and y placed in a region where a uniform magnetic field $\vec{B}$ exists (see the diagram). The resistance of the loop is R.

Item Type: Tutorial | **Difficulty:** 1 | **Time:** 10m | Learning Outcomes ▾ | Contact the Publisher **Manage this Item:** Solution View

In this problem, we will consider a rectangular loop of wire with sides x and y placed in a region where a uniform magnetic field $\vec{B}$ exists (see the diagram). The resistance of the loop is R.

Initially, the field is perpendicular to the plane of the loop and is directed out of the page. The loop can rotate about either the vertical or horizontal axis, passing through the midpoints of the opposite sides, as shown.



Part A

Which of the following changes would induce an electromotive force (emf) in the loop? When you consider each option, assume that no other changes occur.

Check all that apply.

ANSWER:

- [x] The magnitude of $\vec{B}$ increases.
- [x] The magnitude of $\vec{B}$ decreases.
- [x] The loop rotates about the vertical axis (vertical dotted line) shown in the diagram.
- [x] The loop rotates about the horizontal axis (horizontal dotted line) shown in the diagram.
- [ ] The loop moves to the right while remaining in the plane of the page.
- [ ] The loop moves toward you, out of the page, while remaining parallel to itself.

Part B

Find the flux Φ_B through the loop.

Express your answer in terms of x, y, and B.

ANSWER:

Φ_B = xyB

Part C

If the magnetic field steadily decreases from B to zero during a time interval t, what is the magnitude $\mathcal{E}$ of the induced emf?

Express your answer in terms of x, y, B, and t.

+ Hints (1)

ANSWER:

$\mathcal{E}$ = $\frac{xyB}{t}$

Part D

If the magnetic field steadily decreases from B to zero during a time interval t, what is the magnitude I of the induced current?

Express your answer in terms of x, y, B, t, and the resistance R of the wire.

ANSWER:

I = $\frac{xyB}{Rt}$

Part E

If the magnetic field steadily decreases from B to zero during a time interval t, what is the direction of the induced current?

ANSWER:

- ○ clockwise
- ● counterclockwise

The flux decreases, so the induced magnetic field must be in the same direction as the original (parent) magnetic field. Therefore, the induced magnetic field is out of the page. Using the right-hand rule, we deduce that the direction of the current is counterclockwise.

Part F

Which of the following changes would result in a clockwise emf in the loop? When you consider each option, assume that no other changes occur.

Check all that apply.

ANSWER:

- [x] The magnitude of $\vec{B}$ increases.
- [ ] The magnitude of $\vec{B}$ decreases.
- [ ] The loop rotates through 45 degrees about the vertical axis (vertical dotted line) shown in the diagram.
- [ ] The loop rotates through 45 degrees about the horizontal axis (horizontal dotted line) shown in the diagram.
- [ ] The loop moves to the right while remaining in the plane of the page.
- [ ] The loop moves toward you, out of the page, while remaining parallel to itself.

Clockwise emf implies that the induced magnetic field is directed into the page. Therefore, the magnetic flux of the original field must be increasing. Only the first option corresponds to increasing flux.

Exercise 29.4

Description: A closely wound search coil has an area of A, N turns, and a resistance of R_1. It is connected to a charge-measuring instrument whose resistance is R_2. When the coil is rotated quickly from a position parallel to a uniform magnetic field to one...

A closely wound search coil has an area of 3.19 cm^2, 145 turns, and a resistance of 60.1 Ω. It is connected to a charge-measuring instrument whose resistance is 46.9 Ω. When the coil is rotated quickly from a position parallel to a uniform magnetic field to one perpendicular to the field, the instrument indicates a charge of 3.42×10^{-5} C.

Part A

What is the magnitude of the field?

ANSWER:

$B = \dfrac{Q(R_1 + R_2)}{NA} = 7.91 \times 10^{-2}$ T

Exercise 29.9

Description: A circular loop of flexible iron wire has an initial circumference of 165.0 (cm), but its circumference is decreasing at a constant rate of 12.0 (cm/s) due to a tangential pull on the wire. The loop is in a constant, uniform magnetic field oriented...

A circular loop of flexible iron wire has an initial circumference of 165.0 cm, but its circumference is decreasing at a constant rate of 12.0 cm/s due to a tangential pull on the wire. The loop is in a constant, uniform magnetic field oriented perpendicular to the plane of the loop of magnitude 0.500 T.

Part A

Find the emf induced in the loop, at the instant when 9.0 s have passed.

Express your answer using two significant figures.

ANSWER:

$\mathcal{E} = 5.5 \times 10^{-3}$ V

Part B

Find the direction of the induced current in the loop, as viewed looking along the direction of the magnetic field inside the loop.

ANSWER:

- (•) clockwise
- ( ) counterclockwise

29.14. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi_B = NBA\cos\phi$ and the induced emf is $\mathcal{E} = -d\Phi_B/dt$.

EXECUTE: The flux is constant in each case, so the induced emf is zero in all cases.

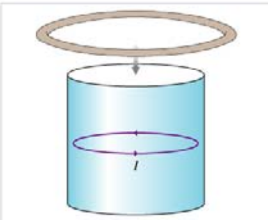
EVALUATE: Even though the coil is moving within the magnetic field and has flux through it, this flux is not *changing*, so no emf is induced in the coil.

Conceptual Induction

Description: Determine the induced current in a loop passing over a solenoid.

A loop of wire is initially held above a short solenoid. A constant counterclockwise (as viewed from above) current I passes through the turns of the solenoid. The loop of wire is steadily lowered, eventually "encircling" the solenoid.

Throughout this problem, when you answer questions about the direction of current, assume that you are viewing the wire loop from above, looking downward.



Part A

What is the direction of the induced current in the loop when the loop is above the solenoid, moving downward?

[+] Hints (2)

ANSWER:

ANSWER:

- (●) clockwise
- ( ) counterclockwise
- ( ) no current

Part B

What is the direction of the induced current at the instant that the loop is at the midpoint of the solenoid and still moving downward?

[+] Hints (1)

ANSWER:

- ( ) clockwise
- ( ) counterclockwise
- (●) no current

Part C

What is the direction of the induced current when the loop is below the solenoid and moving downward?

ANSWER:

- ( ) clockwise
- (●) counterclockwise
- ( ) no current

29.18. **IDENTIFY:** Apply Lenz's law.

SET UP: The field of the induced current is directed to oppose the change in flux in the primary circuit.

EXECUTE: **(a)** The magnetic field in A is to the left and is increasing. The flux is increasing so the field due to the induced current in B is to the right. To produce magnetic field to the right, the induced current flows through R from right to left.

(b) The magnetic field in A is to the right and is decreasing. The flux is decreasing so the field due to the induced current in B is to the right. To produce magnetic field to the right the induced current flows through R from right to left.

(c) The magnetic field in A is to the right and is increasing. The flux is increasing so the field due to the induced current in B is to the left. To produce magnetic field to the left the induced current flows through R from left to right.

EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.20. **IDENTIFY:** The changing flux through the loop due to the changing magnetic field induces a current in the wire. Energy is dissipated by the resistance of the wire due to the induced current in it.

SET UP: The magnitude of the induced emf is $|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \pi r^2\left|\frac{dB}{dt}\right|$, $P = I^2R$, $I = \varepsilon/R$.

EXECUTE: **(a)** $\vec{B}$ is out of page and Φ_B is decreasing, so the field of the induced current is directed out of the page inside the loop and the induced current is counterclockwise.

(b) $|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \pi r^2\left|\frac{dB}{dt}\right|$. The current due to the emf is

$I = \frac{|\varepsilon|}{R} = \frac{\pi r^2}{R}\left|\frac{dB}{dt}\right| = \frac{\pi(0.0480\text{ m})^2}{0.160\ \Omega}(0.680\text{ T/s}) = 0.03076\text{ A}$. The rate of energy dissipation is

$P = I^2R = (0.03076\text{ A})^2(0.160\ \Omega) = 1.51\times10^{-4}\text{ W}$.

EVALUATE: Both the current and resistance are small, so the power is also small.

29.24. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi_B = NBA\cos\phi$ and the induced emf is $\varepsilon = -d\Phi_B/dt$.

EXECUTE: **(a)** and **(c)** The magnetic flux is constant, so the induced emf is zero.

(b) The area inside the field is changing. If we let x be the length (along the 30.0-cm side) in the field, then $A = (0.400\text{ m})x$. $\Phi_B = BA = (0.400\text{ m})x$

$$|\varepsilon| = |d\Phi_B/dt| = B\,d[(0.400\text{ m})x]/dt = B(0.400\text{ m})dx/dt = B(0.400\text{ m})v$$

$$\varepsilon = (1.25\text{ T})(0.400\text{ m})(0.0200\text{ m/s}) = 0.0100\text{ V}$$

EVALUATE: It is not a large *flux* that induces an emf, but rather a large *rate of change* of the flux. The induced emf in part (b) is small enough to be ignored in many instances.

Exercise 29.26

Description: Airplanes and trains move through the earth's magnetic field at rather high speeds, so it is reasonable to wonder whether this field can have a substantial effect on them. We shall use a typical value of ## G for the earth's field. (a) The French...

Airplanes and trains move through the earth's magnetic field at rather high speeds, so it is reasonable to wonder whether this field can have a substantial effect on them. We shall use a typical value of 0.50G for the earth's field.

Part A

The French TGV train and Japanese "bullet train" reach speeds of up to 180mph with wheels moving on tracks about 1.5m apart. At top speed moving perpendicular to the earth's magnetic field, what potential difference is induced across the tracks as the wheels roll?

ANSWER:

$\mathcal{E}$ = 6.03 mV

Also accepted: 6.00

Part B

Does this seem large enough to produce noticeable effects?

ANSWER:

- ○ Yes, this potential difference seems large enough to cause noticeable effects.
- ● No, this potential difference is much too small to be noticeable.

Part C

The Boeing 747-400 series of aircraft has a wingspan of 64.4m and a cruising speed of 565mph . If there is no wind blowing (so that this is also their speed relative to the ground),what is the maximum potential difference that could be induced between the opposite tips of the wings?

ANSWER:

$\mathcal{E}$ = 0.813 V

Part D

Does this seem large enough to cause problems with the plane?

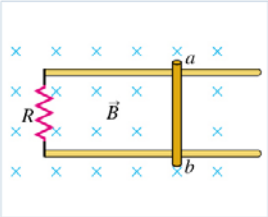
ANSWER:

- ○ Yes, this potential difference seems large enough to cause noticeable effects.
- ● No, this potential difference is too small to be noticeable.

Exercise 29.28

Description: A L-m-long metal bar is pulled to the right at a steady v perpendicular to a uniform, B-T magnetic field. The bar rides on parallel metal rails connected through R-Omega, as shown in the figure , so the apparatus makes a complete circuit. You can...

A 1.20-m-long metal bar is pulled to the right at a steady 5.3$\mathrm{m/s}$ perpendicular to a uniform, 0.800-T magnetic field. The bar rides on parallel metal rails connected through R = 25.3-Ω, as shown in the figure , so the apparatus makes a complete circuit. You can ignore the resistance of the bar and the rails.



Part A

Calculate the magnitude of the emf induced in the circuit.

Express your answer using two significant figures.

ANSWER:

$\mathcal{E}$ = BLv = 5.1 V

Part B

Find the direction of the current induced in the circuit.

ANSWER:

- ○ clockwise
- ● counterclockwise

Part C

Calculate the current through the resistor.

Express your answer using two significant figures.

ANSWER:

$I = \frac{BLv}{R}$ = 0.20 A

29.37. IDENTIFY: Apply Eq. (29.11) with $\Phi_B = \mu_0 niA$.

SET UP: $A = \pi r^2$, where $r = 0.0110$ m. In Eq. (29.11), $r = 0.0350$ m.

EXECUTE: $|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}(BA)\right| = \left|\frac{d}{dt}(\mu_0 niA)\right| = \mu_0 nA\left|\frac{di}{dt}\right|$ and $|\varepsilon| = E(2\pi r)$. Therefore, $\left|\frac{di}{dt}\right| = \frac{E2\pi r}{\mu_0 nA}$.

$$\left|\frac{di}{dt}\right| = \frac{(8.00\times10^{-6}\text{ V/m})2\pi(0.0350\text{ m})}{\mu_0(400\text{ m}^{-1})\pi(0.0110\text{ m})^2} = 9.21\text{ A/s}.$$

29.39. IDENTIFY: Apply Faraday's law in the form $|\varepsilon_{\text{av}}| = N\left|\frac{\Delta\Phi_B}{\Delta t}\right|$.

SET UP: The magnetic field of a large straight solenoid is $B = \mu_0 nI$ inside the solenoid and zero outside. $\Phi_B = BA$, where A is 8.00 cm^2, the cross-sectional area of the long straight solenoid.

EXECUTE: $|\varepsilon_{\text{av}}| = N\left|\frac{\Delta\Phi_B}{\Delta t}\right| = \left|\frac{NA(B_f - B_i)}{\Delta t}\right| = \frac{NA\mu_0 nI}{\Delta t}$.

$$\varepsilon_{\text{av}} = \frac{\mu_0(12)(8.00\times10^{-4}\text{ m}^2)(9000\text{ m}^{-1})(0.350\text{ A})}{0.0400\text{ s}} = 9.50\times10^{-4}\text{ V}.$$

EVALUATE: An emf is induced in the second winding even though the magnetic field of the solenoid is zero at the location of the second winding. The changing magnetic field induces an electric field outside the solenoid and that induced electric field produces the emf.

29.43. IDENTIFY: $q = CV$. For a parallel-plate capacitor, $C = \frac{\epsilon A}{d}$, where $\epsilon = K\epsilon_0$. $i_C = dq/dt$. $j_D = \epsilon\frac{E}{dt}$.

SET UP: $E = q/\epsilon A$ so $dE/dt = i_C/\epsilon A$.

EXECUTE: **(a)** $q = CV = \left(\frac{\epsilon A}{d}\right)V = \frac{(4.70)\epsilon_0(3.00\times10^{-4}\text{ m}^2)(120\text{ V})}{2.50\times10^{-3}\text{ m}} = 5.99\times10^{-10}$ C.

(b) $\frac{dq}{dt} = i_C = 6.00\times10^{-3}$ A.

(c) $j_D = \epsilon\frac{dE}{dt} = K\epsilon_0\frac{i_C}{K\epsilon_0 A} = \frac{i_C}{A} = j_C$, so $i_D = i_C = 6.00\times10^{-3}$ A.

EVALUATE: $i_D = i_C$, so Kirchhoff's junction rule is satisfied where the wire connects to each capacitor plate.

29.60. IDENTIFY: Apply Newton's second law to the bar. The bar will experience a magnetic force due to the induced current in the loop. Use $a = dv/dt$ to solve for v. At the terminal speed, $a = 0$.

SET UP: The induced emf in the loop has a magnitude BLv. The induced emf is counterclockwise, so it opposes the voltage of the battery, ε.

EXECUTE: **(a)** The net current in the loop is $I = \frac{\varepsilon - BLv}{R}$. The acceleration of the bar is $a = \frac{F}{m} = \frac{ILB\sin(90°)}{m} = \frac{(\varepsilon - BLv)LB}{mR}$. To find $v(t)$, set $\frac{dv}{dt} = a = \frac{(\varepsilon - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

$\int_0^v \frac{dv}{(\varepsilon - BLv)} = \int_0^t \frac{LB}{mR}dt \to v = \frac{\varepsilon}{BL}(1 - e^{-B^2L^2t/mR}) = (22\text{ m/s})(1 - e^{-t/15\text{ s}})$. The graph of v versus t is sketched in Figure 29.60. Note that the graph of this function is similar in appearance to that of a charging capacitor.

(b) Just after the switch is closed, $v = 0$ and $I = \varepsilon/R = 2.4$ A, $F = ILB = 1.296$ N, and $a = F/m = 1.4\text{ m/s}^2$.

(c) When $v = 2.0$ m/s, $a = \frac{[12\text{ V} - (1.5\text{ T})(0.36\text{ m})(2.0\text{ m/s})](0.36\text{ m})(1.5\text{ T})}{(0.90\text{ kg})(5.0\ \Omega)} = 1.3\text{ m/s}^2$.

(d) Note that as the speed increases, the acceleration decreases. The speed will asymptotically approach the terminal speed $\frac{\varepsilon}{BL} = \frac{12\text{ V}}{(1.5\text{ T})(0.36\text{ m})} = 22$ m/s, which makes the acceleration zero.

EVALUATE: The current in the circuit is counterclockwise and the magnetic force on the bar is to the right. The energy that appears as kinetic energy of the moving bar is supplied by the battery.

29.76. **IDENTIFY:** A current is induced in the loop because of its motion and because of this current the magnetic field exerts a torque on the loop.

SET UP: Each side of the loop has mass $m/4$ and the center of mass of each side is at the center of each side. The flux through the loop is $\Phi_B = BA\cos\phi$.

EXECUTE: **(a)** $\vec{\boldsymbol{\tau}}_{\text{g}} = \Sigma\vec{\boldsymbol{r}}_{\text{cm}} \times m\vec{\boldsymbol{g}}$ summed over each leg.

$$\tau_{\text{g}} = \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + (L)\left(\frac{m}{4}\right)g\sin(90° - \phi)$$

$\tau_{\text{g}} = \dfrac{mgL}{2}\cos\phi$ (clockwise).

$\tau_B = \left|\vec{\boldsymbol{\tau}} \times \vec{\boldsymbol{B}}\right| = IAB\sin\phi$ (counterclockwise).

$I = \dfrac{\varepsilon}{R} = -\dfrac{BA}{R}\dfrac{d}{dt}\cos\phi = \dfrac{BA}{R}\dfrac{d\phi}{dt}\sin\phi = \dfrac{BA\omega}{R}\sin\phi$. The current is going counterclockwise looking to the $-\hat{\boldsymbol{k}}$ direction. Therefore, $\tau_B = \dfrac{B^2A^2\omega}{R}\sin^2\phi = \dfrac{B^2L^4\omega}{R}\sin^2\phi$. The net torque is $\tau = \dfrac{mgL}{2}\cos\phi - \dfrac{B^2L^4\omega}{R}\sin^2\phi$, opposite to the direction of the rotation.

(b) $\tau = I\alpha$ (I being the moment of inertia). About this axis $I = \dfrac{5}{12}mL^2$. Therefore,

$$\alpha = \frac{12}{5}\frac{1}{mL^2}\left[\frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi\right] = \frac{6g}{5L}\cos\phi - \frac{12B^2L^2\omega}{5mR}\sin^2\phi.$$