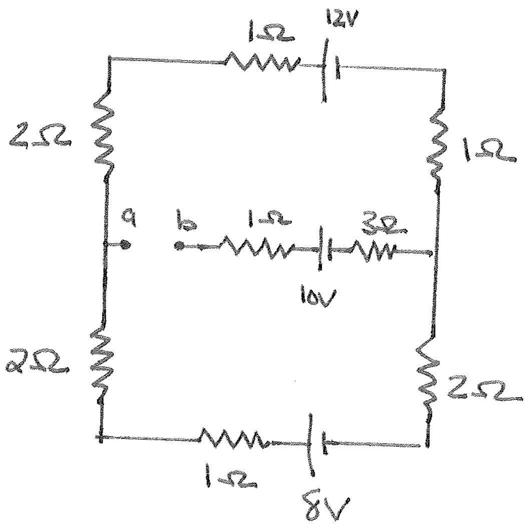
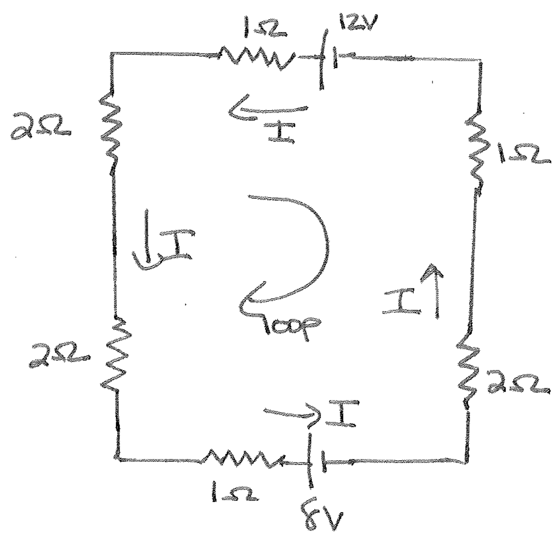
Physics 161, HW #5

#1



a) $V_{ab} = ?$ WITH GAP BETWEEN a & b NO CURRENT CAN FLOW FROM 10V battery

⇒ REMOVE THAT BRANCH WHEN CALCULATING CURRENT.



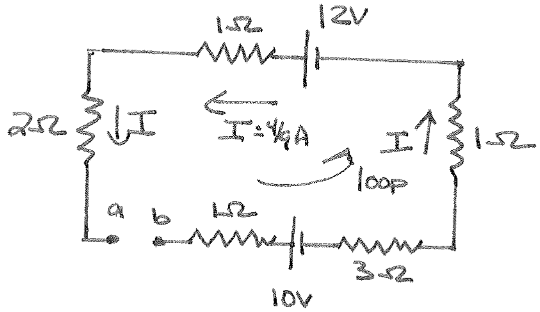
All elements IN SERIES ⇒ ONE CURRENT $\underline{I}$ AS SHOWN

Loop: $+I(1\Omega) - 12V + I(1\Omega) + I(2\Omega) + 8V + I(1\Omega) + I(2\Omega) + I(2\Omega) = 0$

$\Rightarrow I(9\Omega) - 4V = 0$

$\Rightarrow \underline{I} = 4/9\,A = .444\ldots\,A$

To Find V_{ab} do an incomplete Loop from b to a

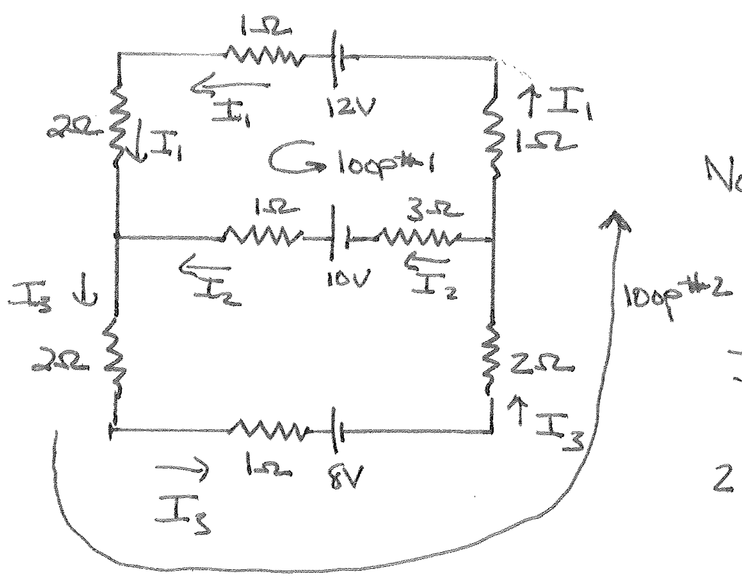


$$V_b - \emptyset A(1\Omega) - 10V - \emptyset A(3\Omega) - I(1\Omega) + 12V - I(1\Omega) - I(2\Omega) = V_a$$

$$\Rightarrow V_a - V_b = V_{ab} = 2V - I(4\Omega) = 2V - 4/9A(4\Omega)$$

$$\Rightarrow \boxed{V_{ab} = .222...V = .222V}$$

b) If a & b connected find current in 12V battery.



Now have 3 currents, I_1, I_2, I_3 as shown

Junction Rule: $I_1 + I_2 = I_3$

2 Loops: "upper" Loop and Outer Loop

From loop #1: $-I_1(2\Omega) + I_2(1\Omega) - 10V + I_2(3\Omega) - I_1(1\Omega) + 12V - I_1(1\Omega) = 0$

$$\Rightarrow -I_1(4\Omega) + I_2(4\Omega) + 2V = 0 \Rightarrow I_2 = \frac{I_1(4\Omega) - 2V}{4\Omega}$$

$$\Rightarrow I_2 = I_1 - .5A$$

Loop #2: $-I_1(2\Omega) - I_3(2\Omega) - I_3(1\Omega) - 8V - I_3(2\Omega) - I_1(1\Omega) + 12V - I_1(1\Omega) = 0$

$\Rightarrow -I_1(4\Omega) - I_3(5\Omega) + 4V = 0$

$\Rightarrow I_3 = \dfrac{4V - I_1(4\Omega)}{5\Omega} = .8A - .8I_1$

$I_1 + I_2 = I_3 \Rightarrow I_1 + (I_1 - .5A) = (.8A - .8I_1)$

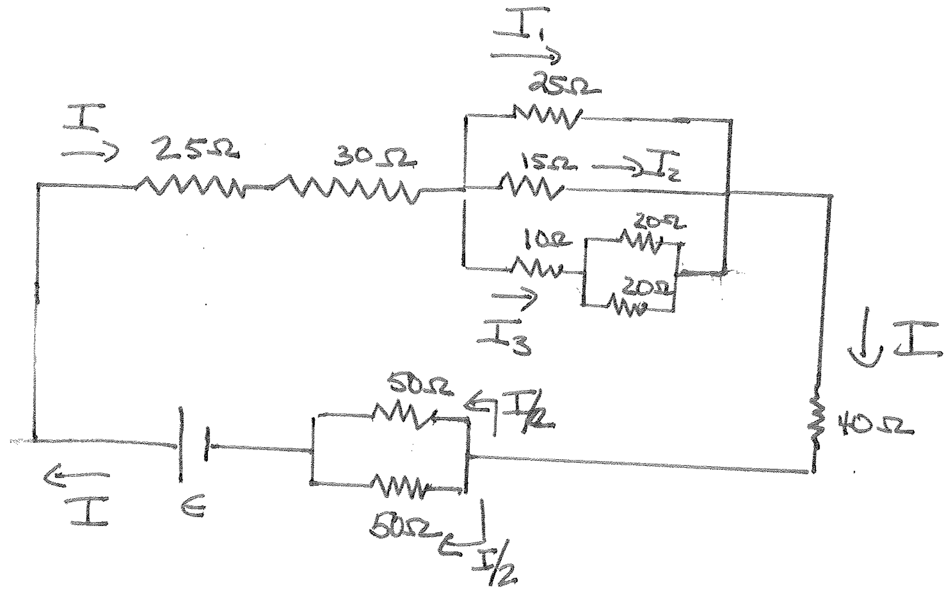
$\Rightarrow 2I_1 - .5A = .8A - .8I_1 \Rightarrow 2.8I_1 = 1.3A$

$\Rightarrow \boxed{I_1 = .46428A = .464A \rightarrow \left(\frac{13}{28}A\right)}$

For Completeness: $I_2 = -.0357A \rightarrow$ labeled wrong!

$I_3 = .4286A$

#2



MAXIMUM POWER FOR All Resistors is ~~2~~ 1.75 Watt. FIND MAX EMF

$P = I^2 R$ ⇒ Resistor WHICH gets MAXIMUM Current uses Most Power

Let I be current FROM Battery

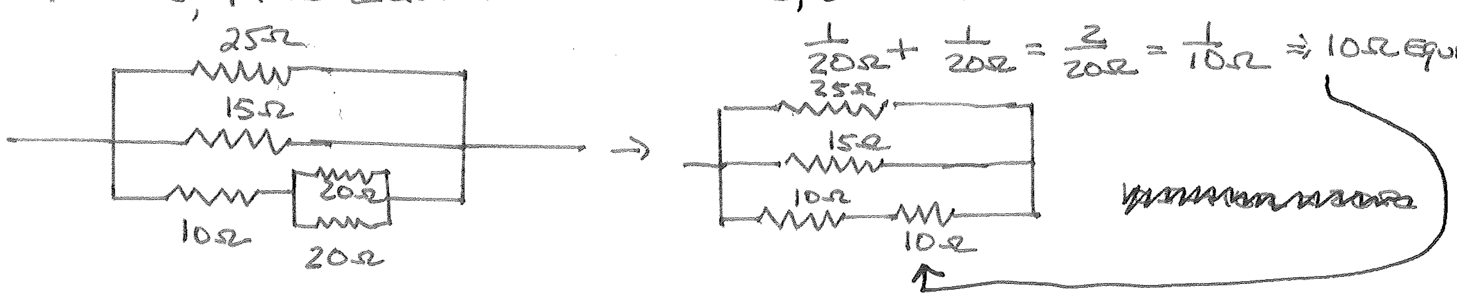
As labeled ABOVE $I_1 + I_2 + I_3 = I$ ⇒ EACH SMALLER THAN I

OTHER parallel even easier, TWO EQUAL RESISTORS ⇒ EACH get $I/2$ Current

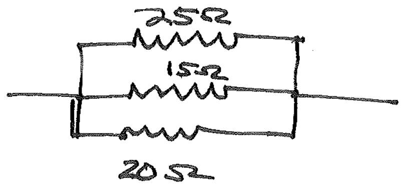
⇒ THE 25Ω, 30Ω, 40Ω All get MAX Current, so 40Ω being largest ⇒ ~~1~~ 1.75 Watt $= I^2(40\,\Omega)$ ⇒ $I =$ 0.209165 A

To FIND ε, FIND EQUIVALENT resistance, Let's Look at Circuit in Pieces

$\frac{1}{20\Omega} + \frac{1}{20\Omega} = \frac{2}{20\Omega} = \frac{1}{10\Omega}$ ⇒ 10Ω Equivalent



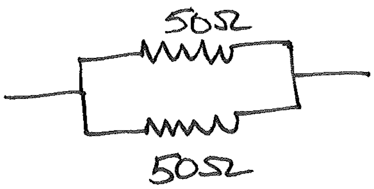
$10\Omega + 10\Omega = 20\Omega \Rightarrow$



$=$

$$\frac{1}{R''} = \frac{1}{25\Omega} + \frac{1}{15\Omega} + \frac{1}{20\Omega} = .1566\ldots\,\Omega^{-1} \Rightarrow R'' = \frac{1}{.1566\,\Omega^{-1}} = 6.383\Omega$$

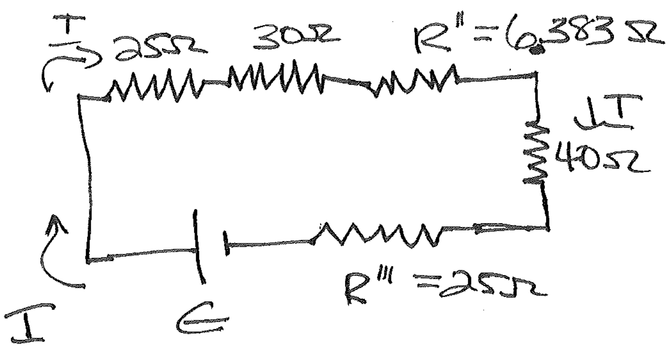
Next Piece:



$\rightarrow$

$$R''' = \frac{(50\Omega)(50\Omega)}{(50\Omega + 50\Omega)} = 25\Omega$$

$\therefore$



All in Series $\Rightarrow$

$$R_{EQ} = 25\Omega + 30\Omega + 6.383\Omega + 25\Omega$$

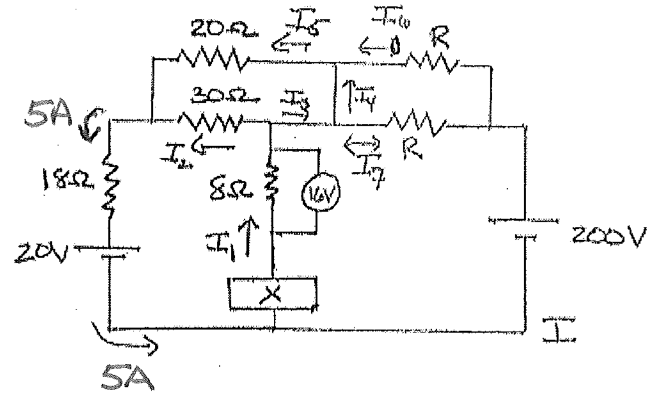
$$\Rightarrow R_{EQ} = 126.383\Omega$$

$$\varepsilon = I R_{EQ} \Rightarrow \varepsilon = (0.209165\text{A})(126.383\Omega) = 26.4349\text{V}$$

$$\Rightarrow \boxed{\varepsilon = 26.4\text{V}}$$

#3

[redacted]



Lower end of 8Ω resistor at higher potential.

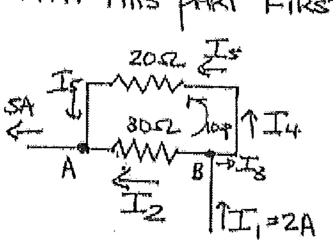
Current flows from higher to low potential ⇒ current through 8Ω is ↑. Let current through 8Ω be I_1.

a) Find EMF of X.

For 8Ω: $V_{ab} = I_1 R \Rightarrow 16V = I_1(8\Omega) \Rightarrow I_1 = 2A$

Label remaining currents, $I_2, I_3, I_4, I_5, I_6, I_7$ as shown

Look at this part first:



From loop rule: $-I_5(20\Omega) + I_2(30\Omega) = 0$

$\Rightarrow I_5 = \frac{3}{2} I_2 = 1.5 I_2$

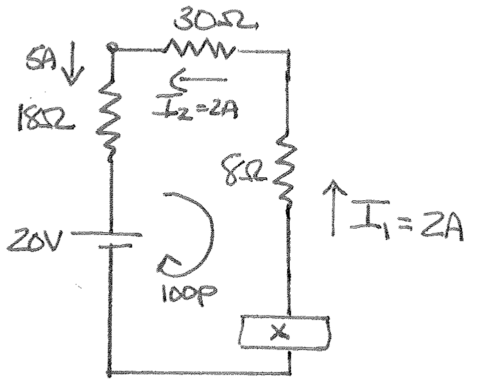
at A: Junction rule: $I_5 + I_2 = 5A$

$\Rightarrow \frac{3}{2} I_2 + I_2 = 5A \Rightarrow \frac{5}{2} I_2 = 5A$

$\Rightarrow I_2 = 2A \Rightarrow I_5 = 3A$

At B: Junction $\Rightarrow I_1 = I_2 + I_3 \Rightarrow 2A = 2A + I_3 \Rightarrow I_3 = 0$

NOW DO THE FOLLOWING LOOP:



$$+2A(30\Omega) + 2A(8\Omega) + X + 20V + 5A(18\Omega) = 0$$

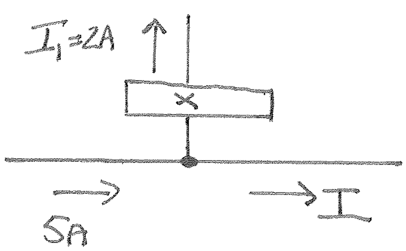
Assume for Battery X

$$\Rightarrow 186V + X = 0 \Rightarrow X = -186V$$

$$\Rightarrow X = 186V$$

b) FIND I (WITH DIRECTION)

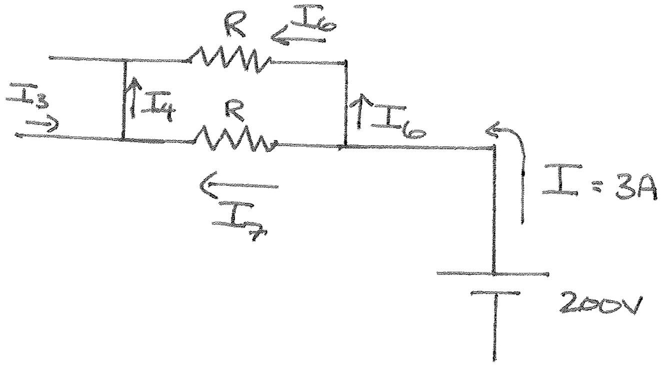
LOOK AT ~~LOWER~~ JUNCTION BELOW Battery X



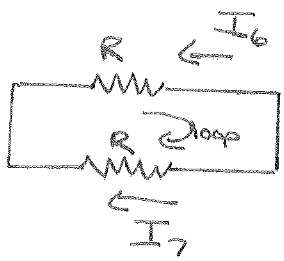
JUNCTION RULE gives DIRECTION, I MUST BE AWAY FROM JUNCTION SO THAT

$$\boxed{5A = 2A + I \Rightarrow I = 3A}$$

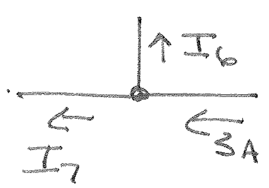
c) Find R:



Earlier found $I_3 = 0 \Rightarrow I_3 + I_7 = I_4 \Rightarrow I_4 = I_7$ (Actually don't need)



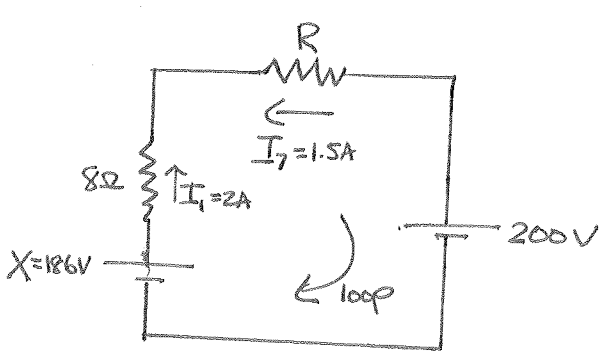
Loop Rule: $+I_6R - I_7R = 0 \Rightarrow I_6 = I_7$



Junction: $3A = I_7 + I_6 = I_7 + I_7$

$\Rightarrow 3A = 2I_7 \Rightarrow I_7 = 1.5A$

Final Loop:



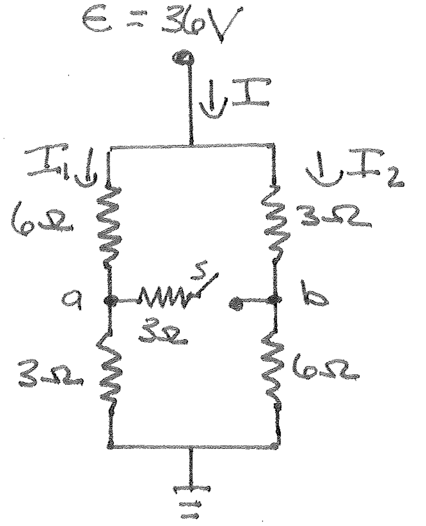
$+1.5A(R) - 200V + 186V - 2A(8\Omega) = 0$

$\Rightarrow 1.5A(R) - 30V = 0$

$\Rightarrow \boxed{R = 20\Omega}$

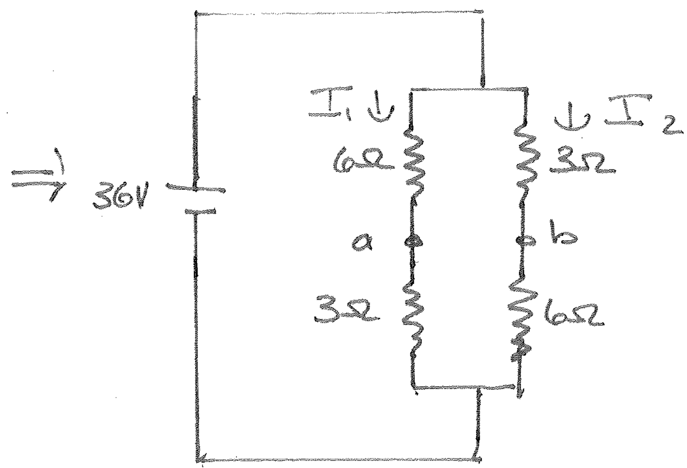
#4

[redacted]

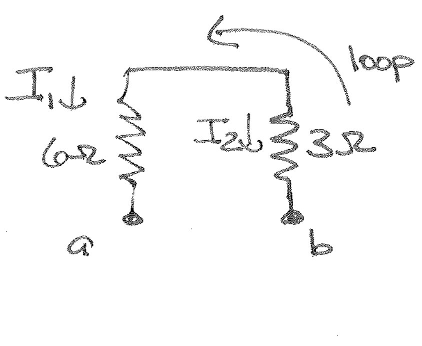


WITH SWITCH OPEN

$\Rightarrow$



a) WHAT IS V_{ab} WITH SWITCH OPEN: DO AN INCOMPLETE loop FROM b to a:

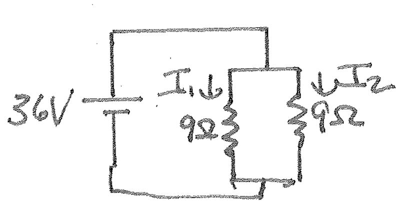


$$V_b + I_2(3\Omega) - I_1(6\Omega) = V_a$$

$$\Rightarrow V_a - V_b = V_{ab} = I_2(3\Omega) - I_1(6\Omega)$$

TO FIND I_1 AND I_2, WE CAN FIND R_{EQ}.

BOTH PAIRS OF 6Ω AND 3Ω IN SERIES $\Rightarrow 9\Omega$



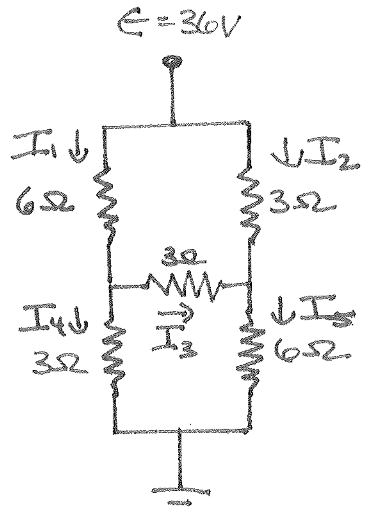
BOTH 9Ω IN parallel to BATTERY $\Rightarrow$

$$I_1 = I_2 = \frac{36V}{9\Omega} = 4A$$

$$\therefore \; V_{ab} = I_2(3\Omega) - I_1(6\Omega) = 4A(3\Omega) - 4A(6\Omega) = 4A(-3\Omega)$$

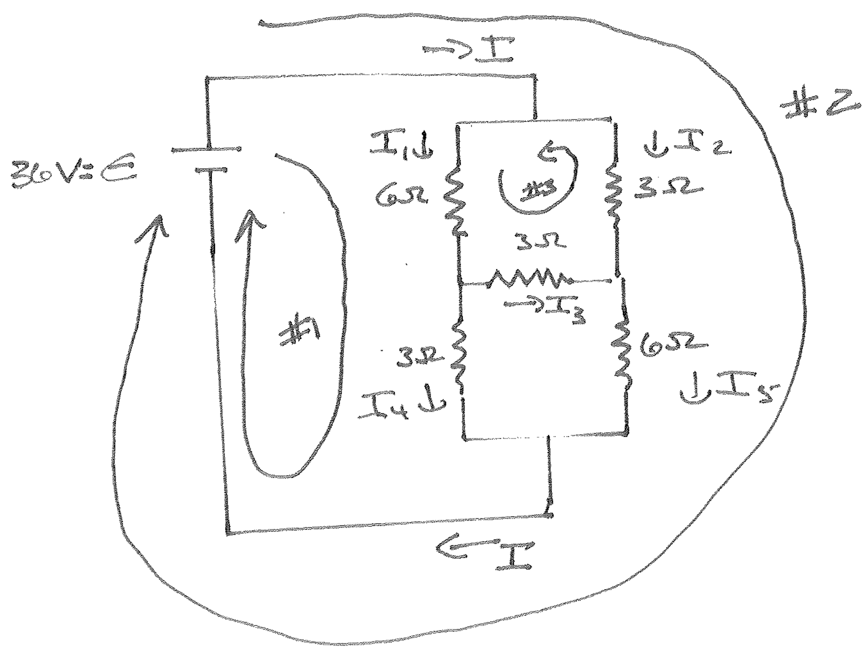
$$\Rightarrow \boxed{V_{ab} = -12V}$$

b) WHEN SWITCH CLOSED, WHAT CURRENT THROUGH SWITCH?



$I_3 = ?$

NO OBVIOUS PARALLEL & SERIES $\Rightarrow$ KIRCHOFF'S



JUNCTION: $I_1 + I_2 = I$

$I_1 = I_3 + I_4 \Rightarrow \boxed{I_4 = I_1 - I_3}$

$I_2 + I_3 = I_5 \Rightarrow \boxed{I_5 = I_2 + I_3}$

get RID OF I_4 AND I_5

Loop #3: $-I_1(6\Omega) - I_3(3\Omega) + I_2(3\Omega) = 0$

$$\Rightarrow \frac{-I_1(6\Omega) - I_3(3\Omega) + I_2(3\Omega)}{3\Omega} = 0$$

$$\Rightarrow -2I_1 - I_3 + I_2 = 0$$

$I_1 = \frac{1}{3}I_3 + 4A$, $I_2 = -\frac{2}{3}I_3 + 4A$

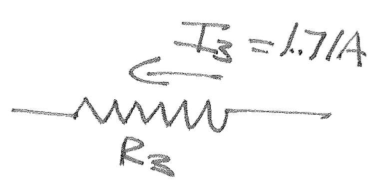
$$\Rightarrow -2\left(\frac{1}{3}I_3 + 4A\right) - I_3 + \left(-\frac{2}{3}I_3 + 4A\right) = 0$$

$$\Rightarrow -\frac{2}{3}I_3 - 8A - I_3 - \frac{2}{3}I_3 + 4A = 0$$

$$\Rightarrow \left(-\frac{2}{3} - 1 - \frac{2}{3}\right)I_3 - 4A = 0 \Rightarrow -\frac{7}{3}I_3 = 4A$$

$$\Rightarrow \boxed{I_3 = -\frac{12}{7}A = -1.71A}$$

LABELED I_3 IN WRONG DIRECTION



Loop #1: $-I_1(6\Omega) - I_4(3\Omega) + 36V = 0$

$$\Rightarrow \frac{+I_1(6\Omega) + I_4(3\Omega)}{3\Omega} = \frac{36V}{3\Omega}$$

$$\Rightarrow 2I_1 + I_4 = 12A$$

$$I_4 = I_1 - I_3 \Rightarrow 2I_1 + (I_1 - I_3) = 12A$$

$$\Rightarrow 3I_1 - I_3 = 12A \Rightarrow \boxed{I_1 = \tfrac{1}{3}I_3 + 4A}$$

Loop #2: $-I_2(3\Omega) - I_5(6\Omega) + 36V = 0$

$$\Rightarrow \frac{I_2(3\Omega) + I_5(6\Omega)}{3\Omega} = \frac{36V}{3\Omega}$$

$$\Rightarrow I_2 + 2I_5 = 12A$$

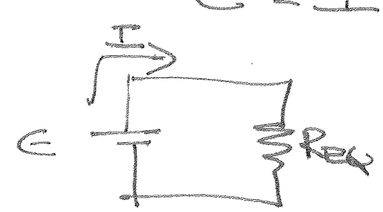
$$I_5 = I_2 + I_3 \Rightarrow I_2 + 2(I_2 + I_3) = 12A$$

$$\Rightarrow 3I_2 + 2I_3 = 12A \Rightarrow \boxed{I_2 = -\tfrac{2}{3}I_3 + \overset{4}{\cancel{12}}A}$$

c). WHAT IS R_{EQ}?

NOT BEING COMBINATIONS OF SERIES AND parallel this CIRCUIT DOESN'T HAVE A "REAL" R_{EQ}.

BUT WE CAN DEFINE AN EFFECTIVE R_{EQ} FROM THE CONDITION THAT $\mathcal{E} = I R_{EQ}$ WHERE I = CURRENT FROM Battery.



FROM JUNCTION RULE: $I = I_1 + I_2$

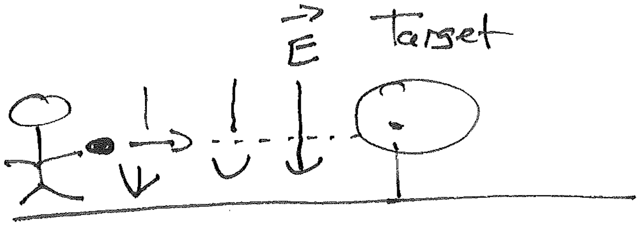
$$I_1 = \frac{1}{3} I_3 + 4A = \frac{1}{3}\left(-\frac{12}{7}A\right) + 4A = -\frac{4}{7}A + 4A = \frac{24}{7}A = 3.4285A$$

$$I_2 = -\frac{2}{3} I_3 + 4A = -\frac{2}{3}\left(-\frac{12}{7}A\right) + 4A = +\frac{8}{7}A + 4A = \frac{36}{7}A = 5.143A$$

$$I = I_1 + I_2 = \frac{24}{7}A + \frac{36}{7}A = \frac{60}{7}A = 8.5711A$$

$$\boxed{R_{EQ} = \frac{\mathcal{E}}{I} = \frac{36V}{60/7\,A} = \frac{252}{60}\,\Omega = 4.2\Omega}$$

#5



Coin:

$M = 2.5g = 2.5\times10^{-3}kg$

$Q = 4500\mu C = 4500\times10^{-6}C$
$= 4.5\times10^{-3}C$

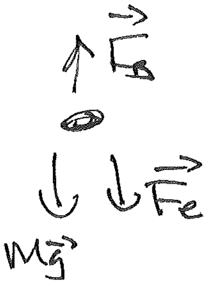
$V_0 = 15.2m/s$, Horizontal

$\vec{E} = 3.75N/C$, Down

WHAT UNIFORM B to hit target?

Forces on Coin: Gravity, Electric Force, $\vec{F_e}$. Positive charge, Downward $\vec{E} \Rightarrow$ Downward force

Target Directly AHEAD of Coin $\Rightarrow$ Coin Needs to go Horizontally. Initial velocity is Horizontal $\Rightarrow$ $\boxed{\text{ZERO Net FORCE}}$



$\sum\vec{F} = 0 \Rightarrow F_B - Mg - F_e = 0 \Rightarrow F_B = Mg + F_e$

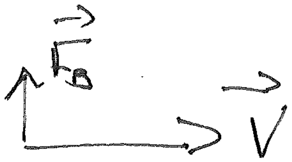
$F_B = Mg + qE$

$F_B = (2.5\times10^{-3}kg)(9.8m/s^2) + (4.5\times10^{-3}C)(3.75N/C)$

$= .0245N + .016875N = .041375N$

$\Rightarrow \vec{F_B} = .041375N$, up

$\vec{F}_B = q\vec{v} \times \vec{B}$, $\vec{v} = \vec{v}_0 = 15.2 m/s$, to Right



RHR $\Rightarrow$ $\vec{B}$ must be into page to make $\vec{F}_B$ up $\Rightarrow \vec{B} = \otimes$

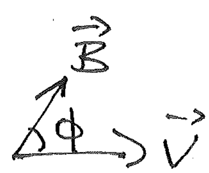
$$\Rightarrow F_B = qvB\sin 90° = qvB \Rightarrow B = \frac{F_B}{qv} = \frac{.041375 N}{(4.5\times10^{-3}C)(15.2 m/s)}$$

$$\Rightarrow \boxed{B = 0.605 T}$$ $\Leftarrow$ Very LARGE, but not impossibly so.

OF COURSE THIS IS THE SMALLest MAGnetic field NEEDED.

$\vec{B}$ doesn't have to be straight INTO THE PAGE to make $\vec{F}_B$ point upwards. As long as $\vec{B}$ has A Component INto the page $\vec{F}_B$ will be upward. THE OTHER Component of $\vec{B}$, parallel to $\vec{v}$, Causes No Force.

Looking From "Above":

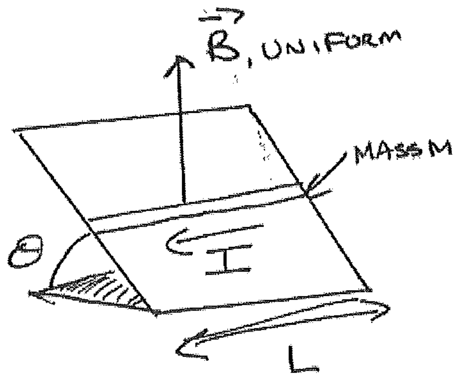


(Now $\vec{F}_B = \odot$, $\phi = 90°$ gives previous Result)

$$F_B = qvB\sin\phi \Rightarrow B = \frac{F_B}{qv\sin\phi} = \frac{0.605T}{\sin\phi}$$ $0 < \phi \leq 90°$

$\Rightarrow B \geq 0.605T$

#6



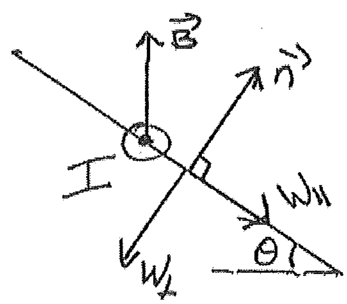
WHAT Current Keeps WIRE FROM SLIDING DOWN

Forces on Wire : gravity WHICH ON AN INCLINE splits itself into parallel AND perpendicular Components $W_{\parallel} = Mg\sin\theta$, $W_{\perp} = Mg\cos\theta$,

A NORMAL Force, $\vec{n}$ WHICH IS PERPENDICULAR

AND MAGNETIC Force, $\vec{F}_M$

IF WE DRAW THE INCLINE FROM THE SIDE, SUCH THAT $\vec{B}$ is straight up, the incline is at angle θ, the current will be into or out of page.



WE SEE THAT F_M MUST CANCEL $W_{\parallel}$.

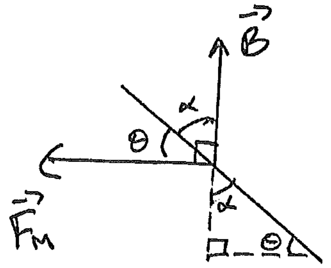
FROM RIGHT HAND RULE, current MUST BE Flowing out OF PAGE

$I = \odot$

So ON ORIGINAL DRAWING Current Flowing RIGHT TO LEFT (AS SHOWN).

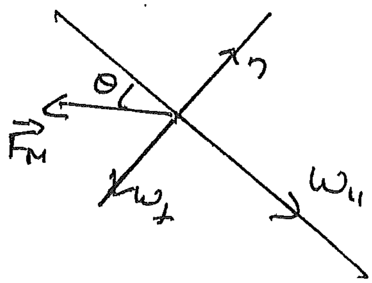
$I = \odot$, $\vec{B} = \uparrow$ $\Rightarrow$ $F_M = ILB\sin 90° = ILB$

but $\vec{F}_M$ IS <u>NOT</u> opposite to $W_{\parallel}$. IT IS 90° to $\vec{B}$.

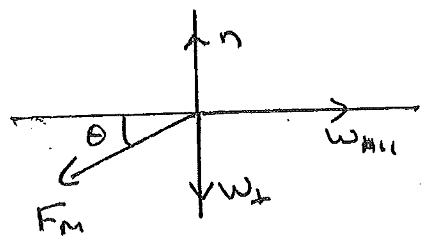


$\alpha = 90° - \theta$

SO THE FREE BODY DIAGRAM LOOKS LIKE:



WHICH IS EASIER TO DRAW AS:



$\sum F_{\parallel} = W_{\parallel} - F_M \cos\theta$. NOT MOVING $\Rightarrow \sum F_{\parallel} = 0$

$$\Rightarrow W_{\parallel} = F_M \cos\theta \Rightarrow Mg\sin\theta = ILB\cos\theta \Rightarrow I = \frac{Mg\sin\theta}{LB\cos\theta} = \frac{Mg}{LB}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$\Rightarrow \boxed{I = \frac{Mg\tan\theta}{LB}}$$

For Completeness: $\sum F_n = n - W_\perp - F_M \sin\theta = \overset{\text{NOT MOVING}}{0}$

$$\Rightarrow n = W_\perp + F_M\sin\theta = Mg\cos\theta + ILB\sin\theta = Mg\cos\theta + \frac{Mg}{LB}\left(\frac{\sin\theta}{\cos\theta}\right)LB\sin\theta$$

$$\Rightarrow n = Mg\cos\theta + Mg\frac{\sin^2\theta}{\cos\theta} = Mg\frac{\cos^2\theta}{\cos\theta} + Mg\frac{\sin^2\theta}{\cos\theta} = \frac{Mg}{\cos\theta}(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow n = \frac{Mg}{\cos\theta} = Mg\sec\theta$$