Lecture 16
(Electric Potential Energy)

Physics 161-01 Spring 2012

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Gravitational Potential Energy

- Once again, the similarities of the gravitational force law and the electric force law bring us to review some physics from last semester.
- Remember that the gravitational potential energy was defined as:

$$\Delta U_g = -W_g = -\int_1^2 \vec{F}_g \cdot d\vec{r}$$

- If you are close to the earth's surface, you can take the force to be essentially constant (=mg) and the above just gets you

$$\Delta U_g = \int_1^2 (mg)\, dy = mg\left(y_2 - y_1\right) = mgh$$

Gravitational Potential Energy

- If we relax the condition on being close to the surface, we have to use the exact force law.

$$\Delta U_g = -W_g$$

$$U_{g2} - U_{g1} = -\int_1^2 \vec{F}_g \cdot d\vec{r}$$

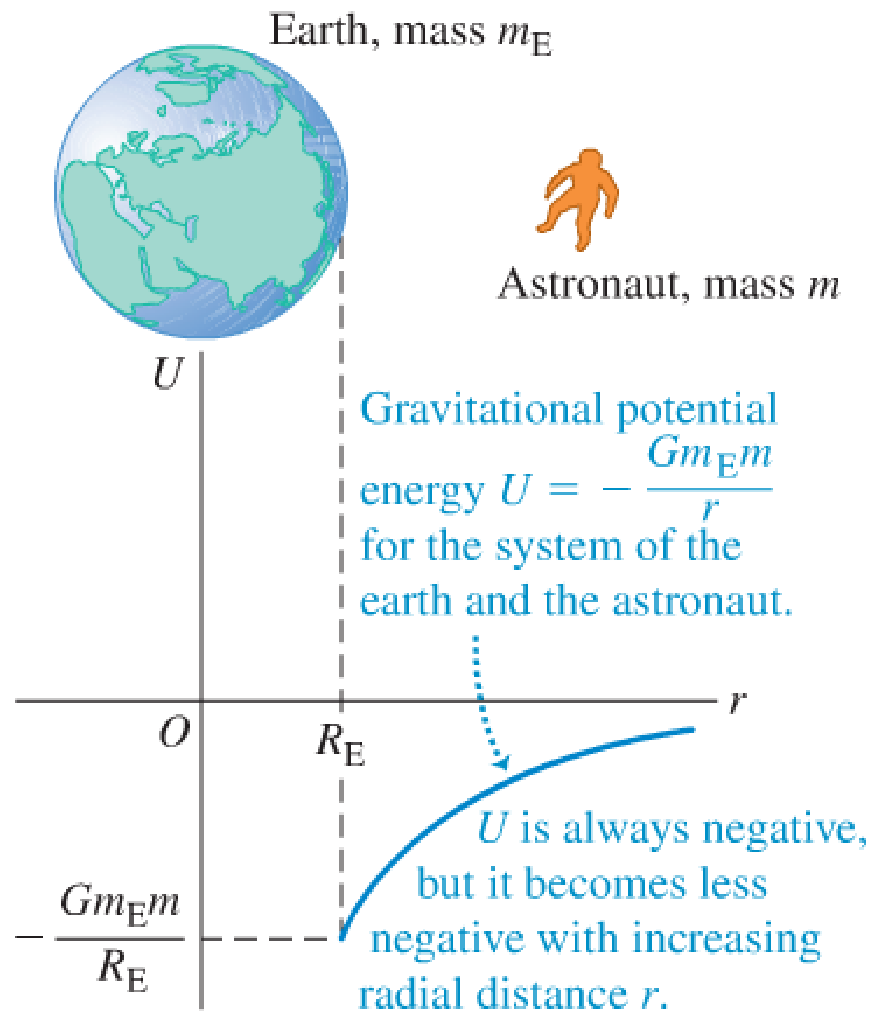
$$U_{g2} - U_{g1} = -\int_{r_1}^{r_2} \frac{-Gm_1m_2}{r^2}\hat{r} \cdot d\vec{r}$$

$$U_{g2} - U_{g1} = \frac{Gm_1m_2}{r_1} - \frac{Gm_1m_2}{r_2}$$

- Then, we can define the gravitational potential energy far from a mass (at infinity) to be zero, so: $U_g = -\frac{Gm_1m_2}{r}; \quad U_G(\infty) = 0$

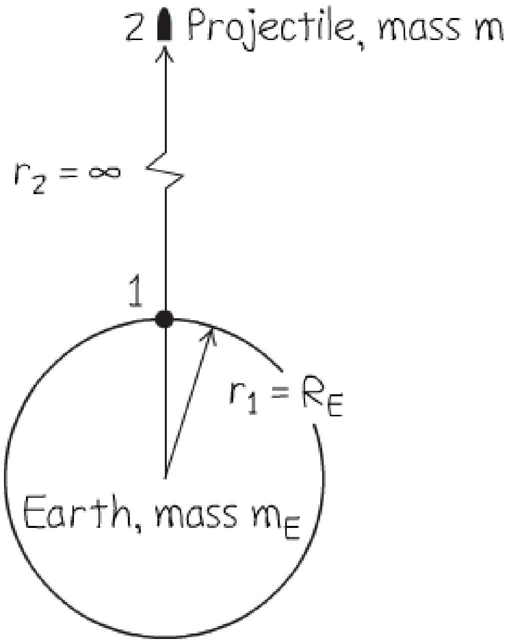
Gravitational Potential Energy

- It is important to remember exactly what we are talking about when we talk about the gravitational potential energy:
  - It is the energy stored in the configuration of the two masses m_1 and m_2.
  - In the case of a mass, m, and the earth, the potential of that two-mass system is shown in the figure to the right.



Gravitational Potential Energy

- Remember, we could use this potential energy to see how much energy we would have to add to a projectile to get it to just escape earth's gravitational attraction:



From Conservation of energy:

$$\frac{1}{2}mv_1^{\,2} + \left(-\frac{Gm_E m}{R_E}\right) = 0 + 0$$

$$\begin{aligned} v_1 &= \sqrt{\frac{2Gm_E}{R_E}} \\ &= \sqrt{\frac{2(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.97 \times 10^{24}\ \mathrm{kg})}{6.38 \times 10^6\ \mathrm{m}}} \\ &= 1.12 \times 10^4\ \mathrm{m/s}\,(= 40{,}200\ \mathrm{km/h} = 25{,}000\ \mathrm{mi/h}) \end{aligned}$$

Gravitational Potential Energy

- And, finally, if we have more than one mass involved, then the gravitational potential energy of a mass m_1 due to masses m_2, m_3, etc. is just the sum of the potential energies due to each mass.

$$U_{g1} = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_1m_4}{r_{14}} \dots$$

Electric Potential Energy

- We have already seen that Coulomb's Force Law is essentially identical to Newton's Law of Gravity, so if:

$$\Delta U_g = -W_g = \int_{r_1}^{r_2} \frac{Gm_1m_2}{r^2}\,dr = \frac{Gm_1m_2}{r_1} - \frac{Gm_1m_2}{r_2}$$

$$U_g = -\frac{Gm_1m_2}{r}; \quad U_G(\infty) = 0$$

- Then,

$$\Delta U_E = -W_E = -\int_{r_1}^{r_2} \frac{kq_1q_2}{r^2}\,dr = \frac{kq_1q_2}{r_2} - \frac{kq_1q_2}{r_1}$$

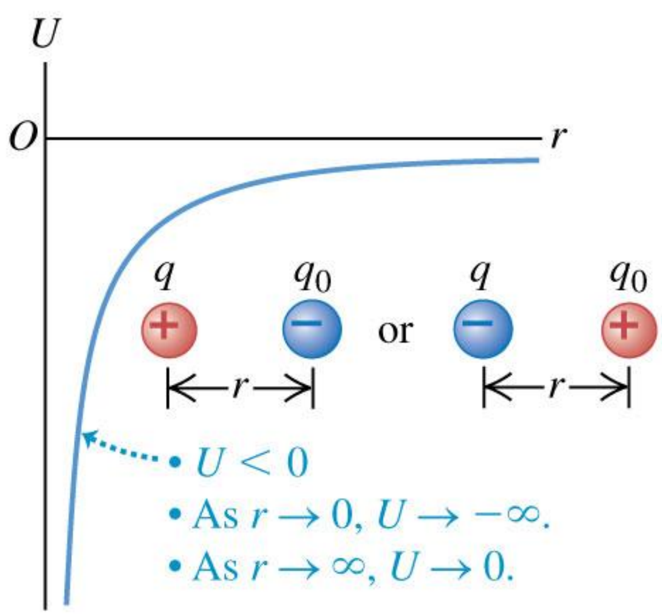
$$U_E = \frac{kq_1q_2}{r}; \quad U_E(\infty) = 0$$

Electric Potential Energy

- One difference is that charges can have different signs

$$U_E = \frac{kq_1q_2}{r}; \quad U_E(\infty) = 0$$

(b) q and q_0 have opposite signs.



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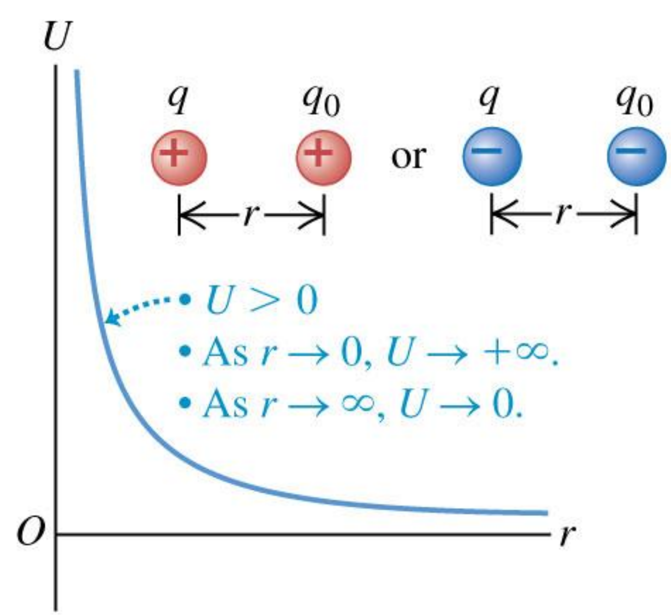
(a) q and q_0 have the same sign.



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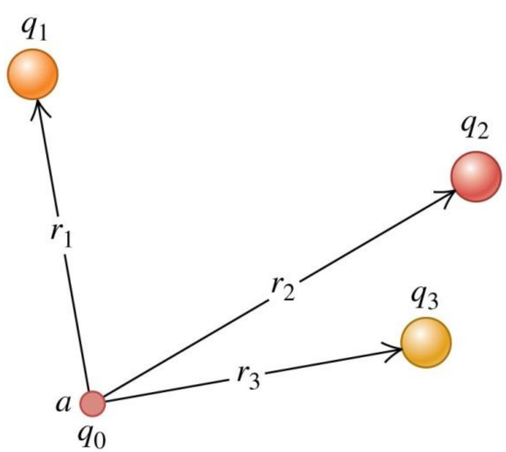
Electric Potential Energy

- And, again, if we have more than one charge involved, then the electric potential energy of a charge q_0 due to charges q_1, q_2, etc. is just the sum of the potential energies due to each charge:

$$U_{E0} = \frac{kq_0q_1}{r_1} + \frac{kq_0q_2}{r_2} + \frac{kq_0q_3}{r_3} \ldots$$

$$= \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

$$= \frac{q_0}{4\pi\varepsilon_0}\int \frac{dq}{r} \quad (\text{continuous charge distribution})$$



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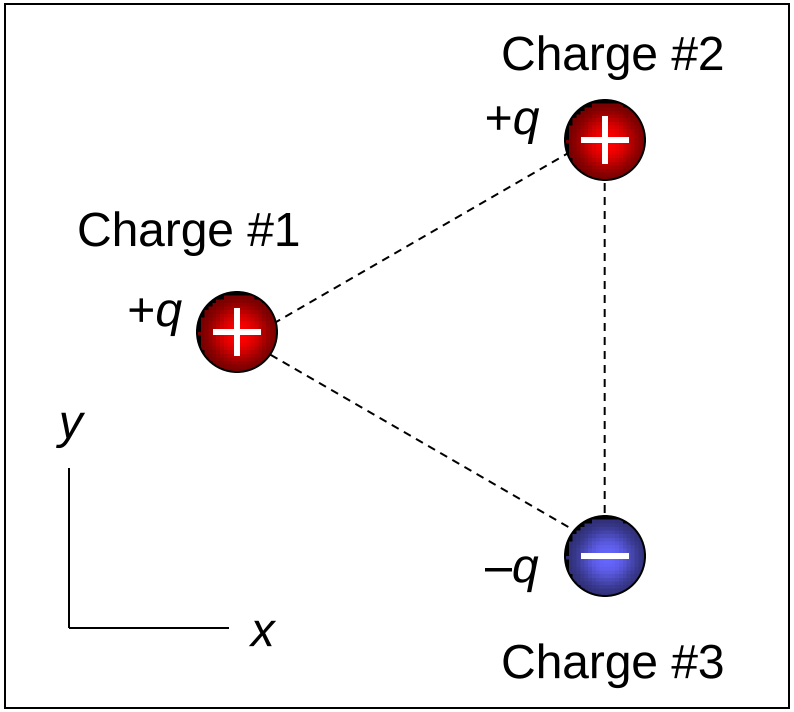
CPS 16-1

The electric potential energy of two point charges approaches zero as the two point charges move farther away from each other.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential energy of charge 1 is



A. positive.

B. negative.

C. zero.

D. not enough information given to decide

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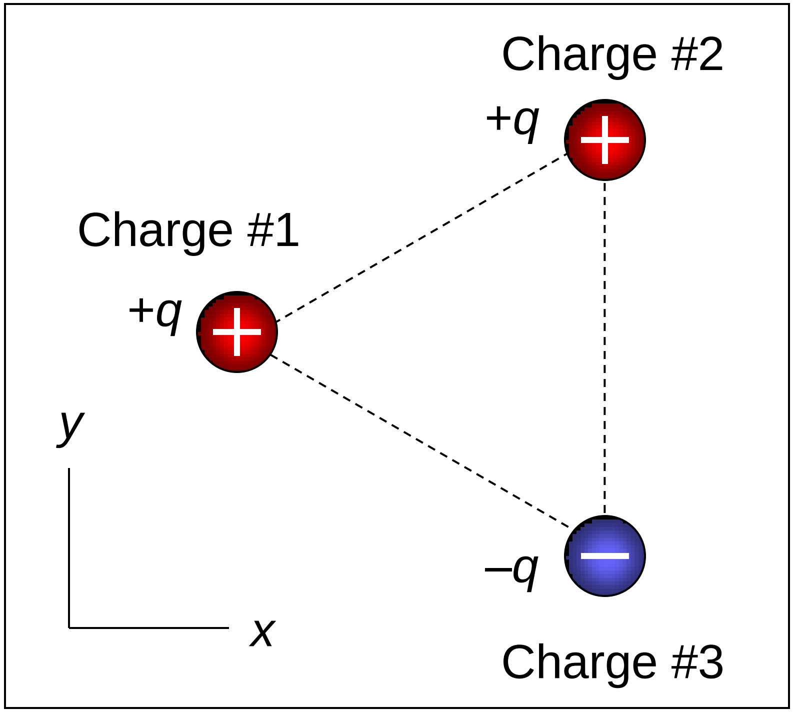

A. positive.

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C. zero.

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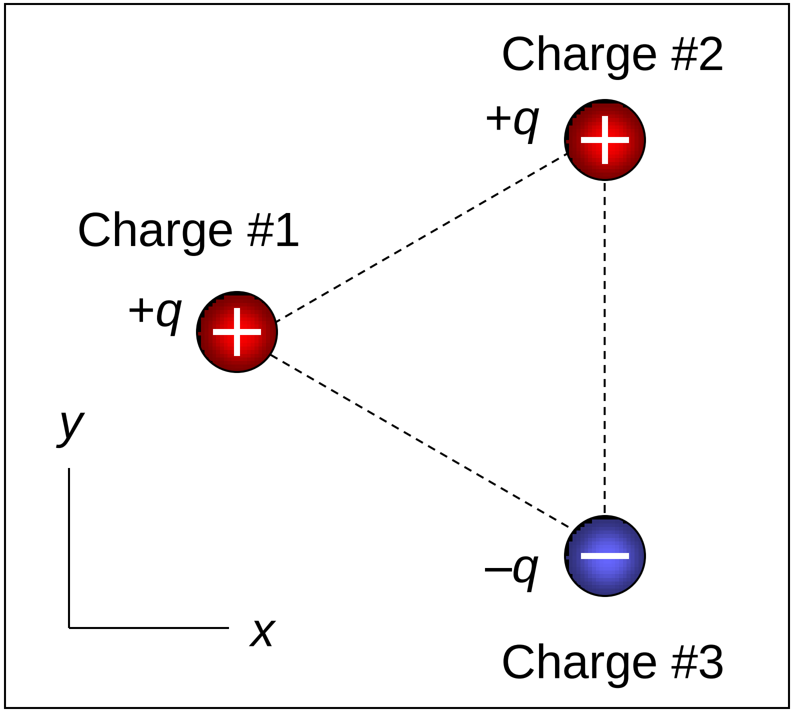
CPS 16-2

The electric potential energy of two point charges approaches zero as the two point charges move farther away from each other.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential energy of charge 3 is



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CPS 16-2

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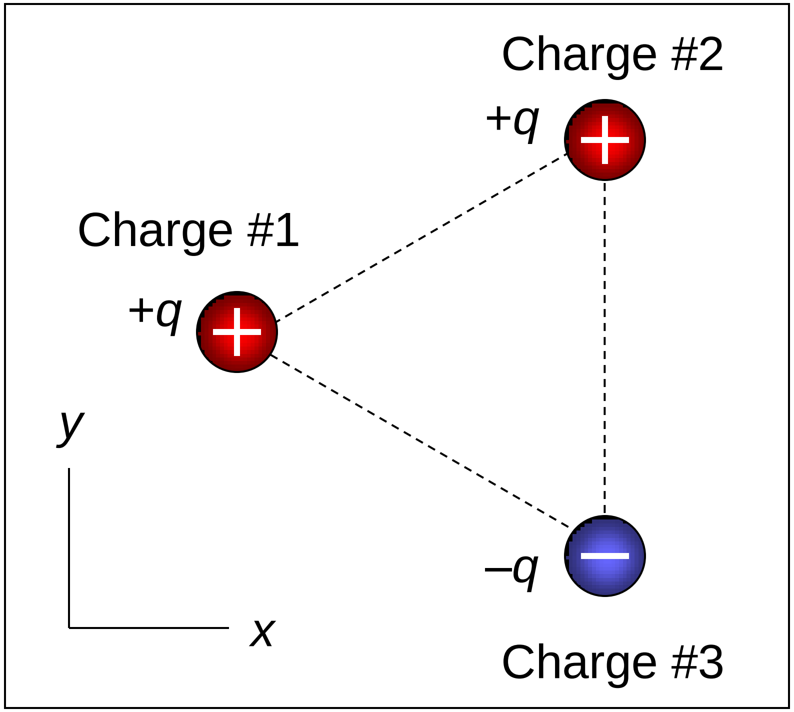

A. positive.

B. negative.

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Total Electric Potential Energy

- We can also ask "How much total potential energy is stored in the entire charge configuration?"

$$U_E = \frac{kq_0q_1}{r_{01}} + \frac{kq_0q_2}{r_{02}} + \frac{kq_0q_3}{r_{03}} \ldots + \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} \ldots + \frac{kq_2q_3}{r_{23}} \ldots$$

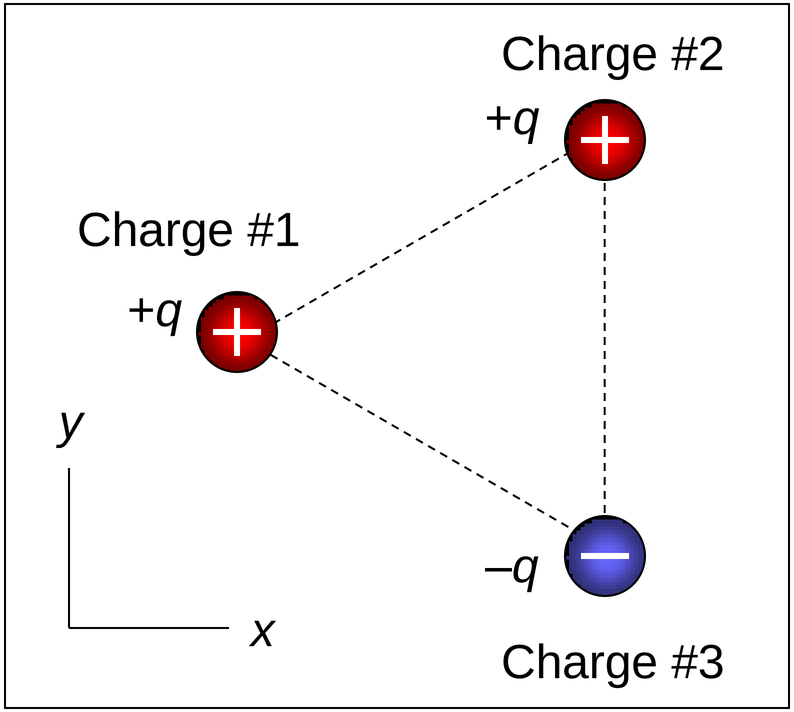
$$= \frac{1}{4\pi\varepsilon_0} \sum_{i<j} \frac{kq_iq_j}{r_{ij}}$$

- Then,

CPS 16-3

The electric potential energy of two point charges approaches zero as the two point charges move farther away from each other.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential energy of the system of three charges is



A. positive.

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CPS 16-3

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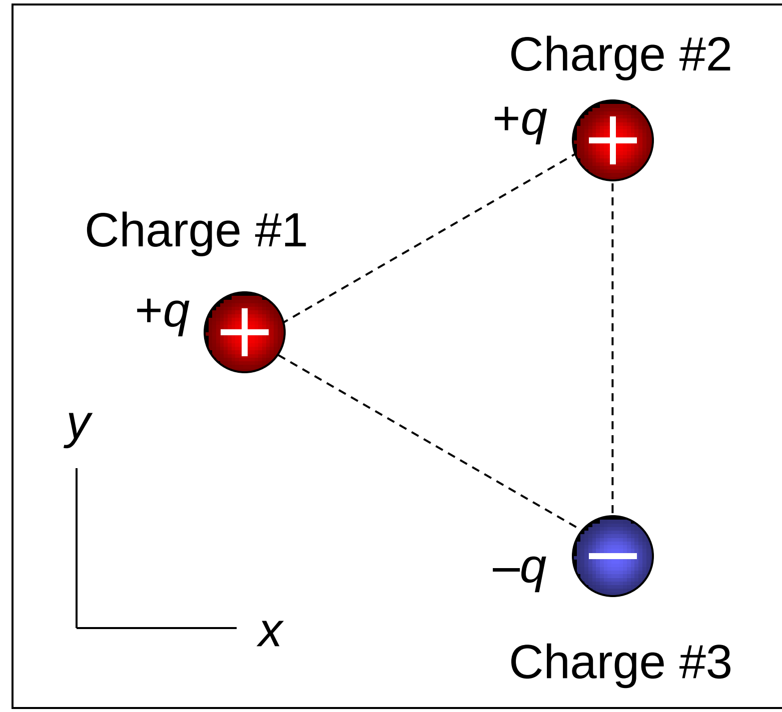


A. positive.

B. negative.

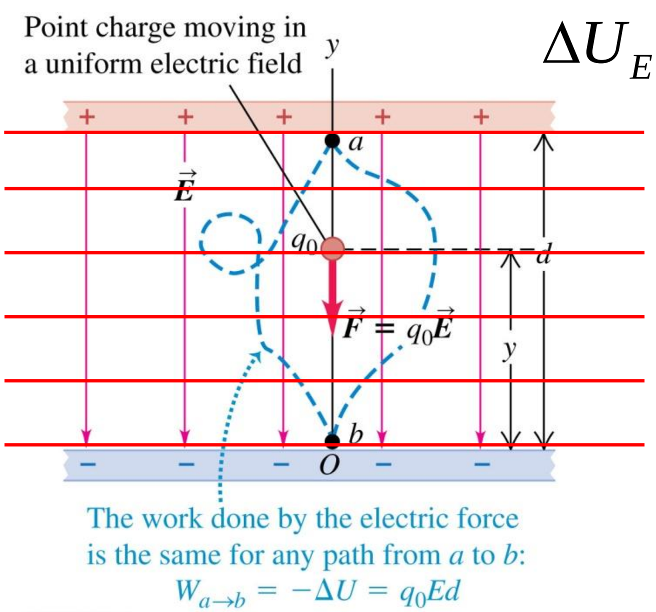
C. zero.

D. not enough information given to decide

Electric Potential Energy and the Electric Field

- If we just want to talk about the potential energy of a charge in relation to some charge distribution, you could just use the net electric field of the charge distribution:

$$\Delta U_E = -W_E = -\int_{r_1}^{r_2} q\vec{E}\cdot d\vec{r}$$



U_E = qEd

U_E = 0

U_E = 0