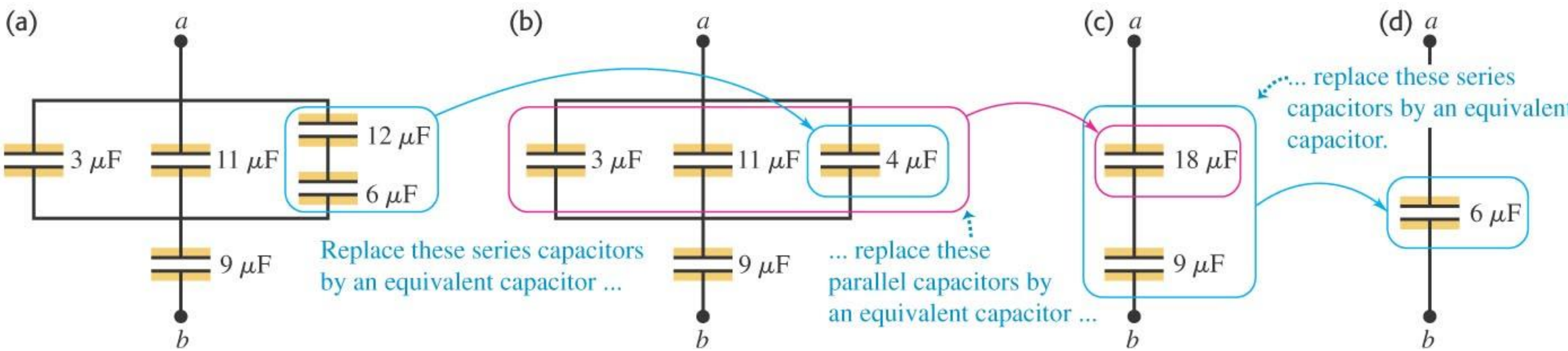
Lecture 20
(Energy in Capacitors)

Physics 161-01 Spring 2012

Douglas Fields

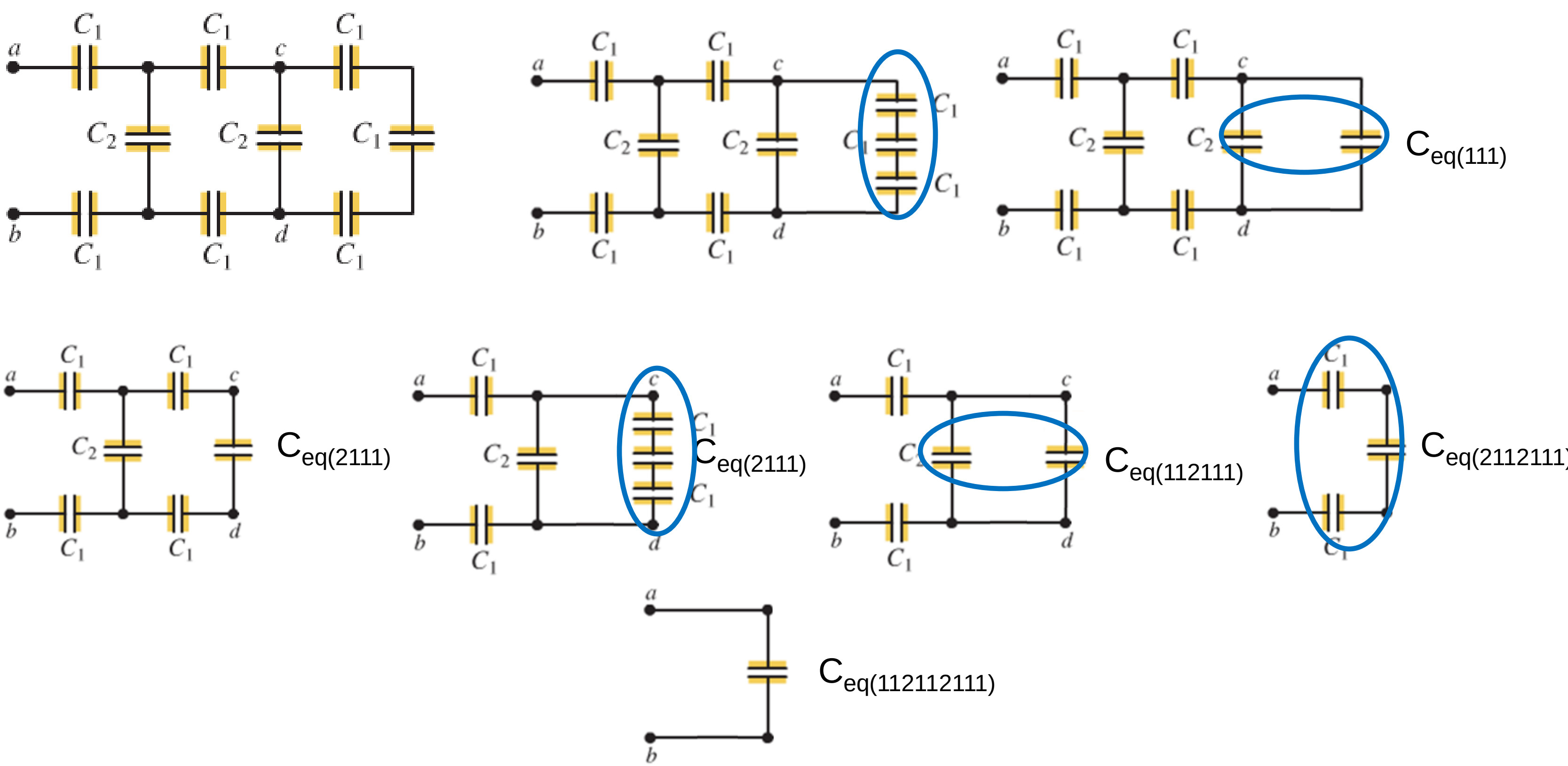
Capacitor Circuits

- When there are combinations of series and parallel capacitors, try to reduce them to more simple forms.



Capacitor Circuits

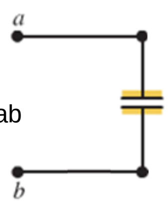
- Remember that wires can be "stretched" and "shortened", but you can't cross a junction!



Capacitor Circuits

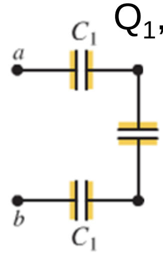
- And then, to find charges and potentials, work backwards:

V_{ab}



$Q_1 = C_{eq(112112111)} V_{ab}$

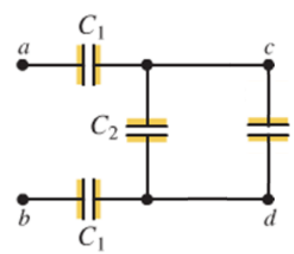
$Q_1, V = Q_1 / C_1$



$Q_1, V = Q_1 / C_{eq(2112111)}$

$C_{eq(2112111)}$

$Q_1, V = Q_1 / C_1$

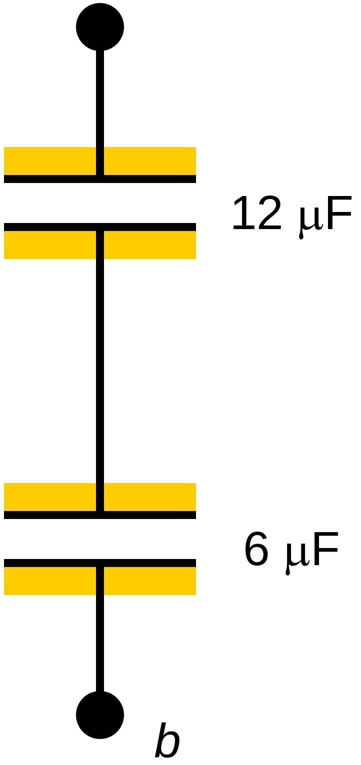


$V = Q_1 / C_{eq(2112111)}$

$C_{eq(112111)}$

CPS 20-1

A 12-μF capacitor and a 6-μF capacitor are connected together as shown. What is the equivalent capacitance of the two capacitors as a unit?



A. C_{eq} = 18 μF

B. C_{eq} = 9 μF

C. C_{eq} = 6 μF

D. C_{eq} = 4 μF

E. C_{eq} = 2 μF

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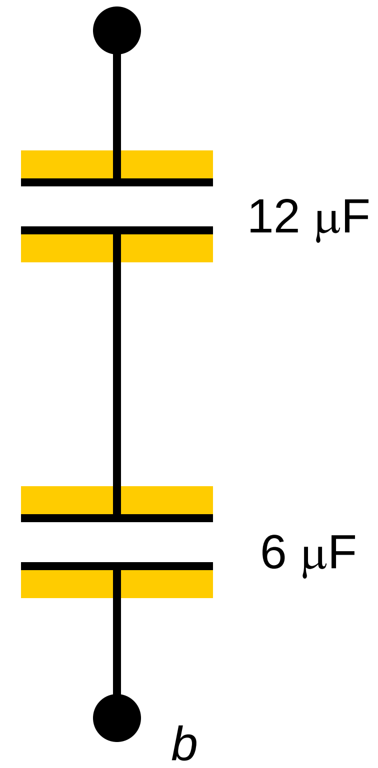

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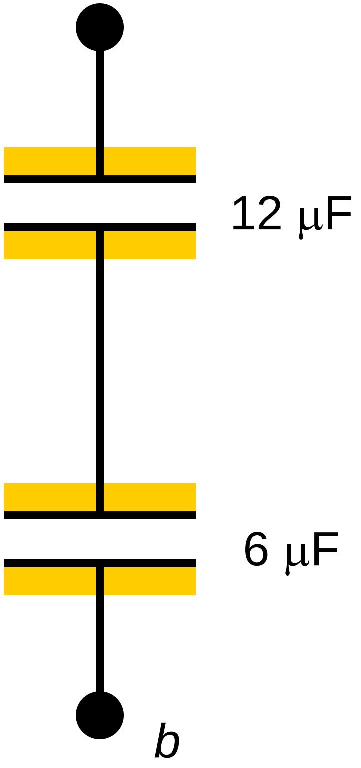
CPS 20-2

A 12-μF capacitor and a 6-μF capacitor are connected together as shown. If the charge on the 12-μF capacitor is 24 microcoulombs (24 μC), what is the charge on the 6-μF capacitor?



A. 48 μC

B. 36 μC

C. 24 μC

D. 12 μC

E. 6 μC

CPS 20-2

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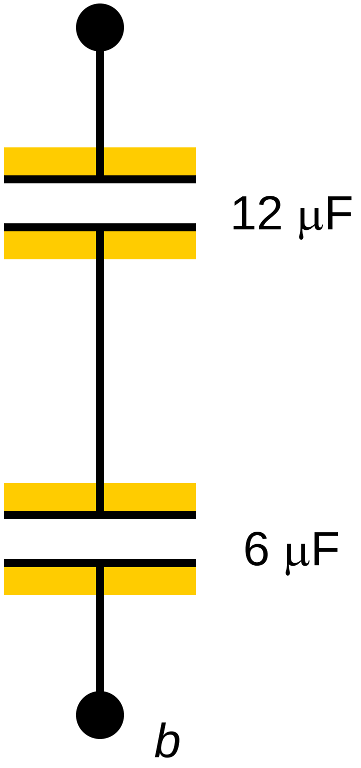

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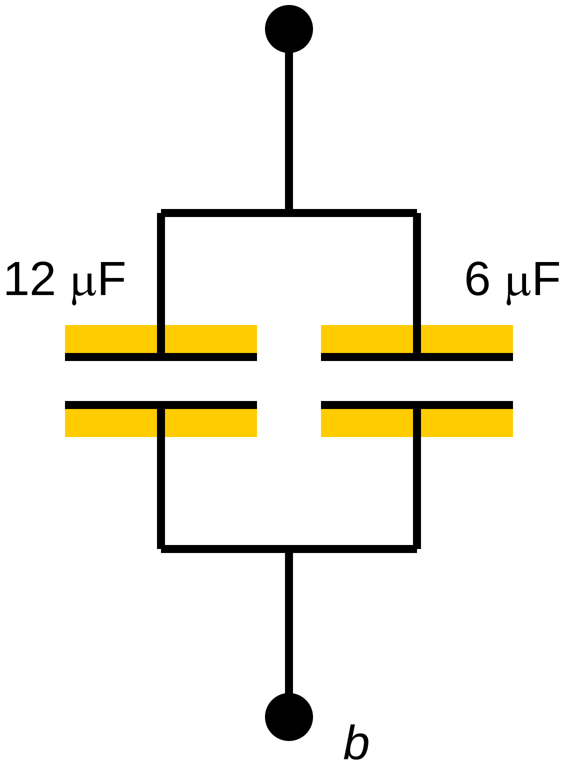
CPS 20-3

A 12-μF capacitor and a 6-μF capacitor are connected together as shown. If the charge on the 12-μF capacitor is 24 microcoulombs (24 μC), what is the charge on the 6-μF capacitor?



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CPS 20-3

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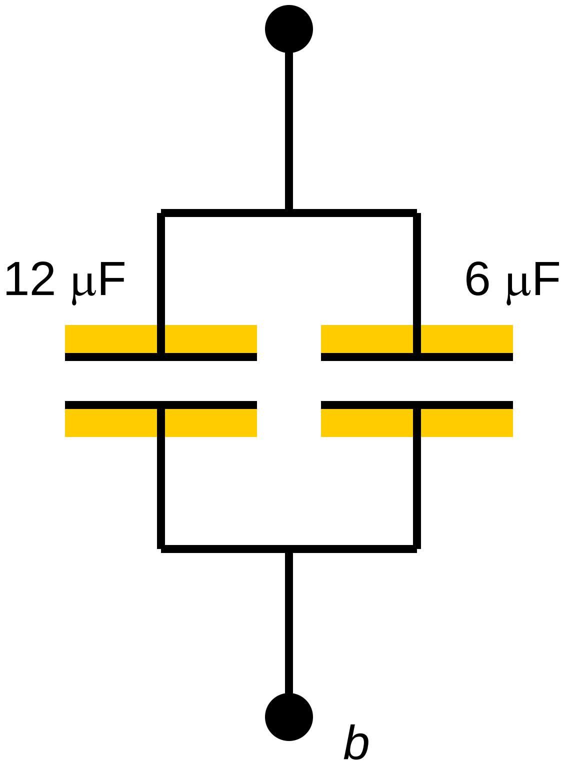

A. 48 μC

B. 36 μC

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D. 12 μC

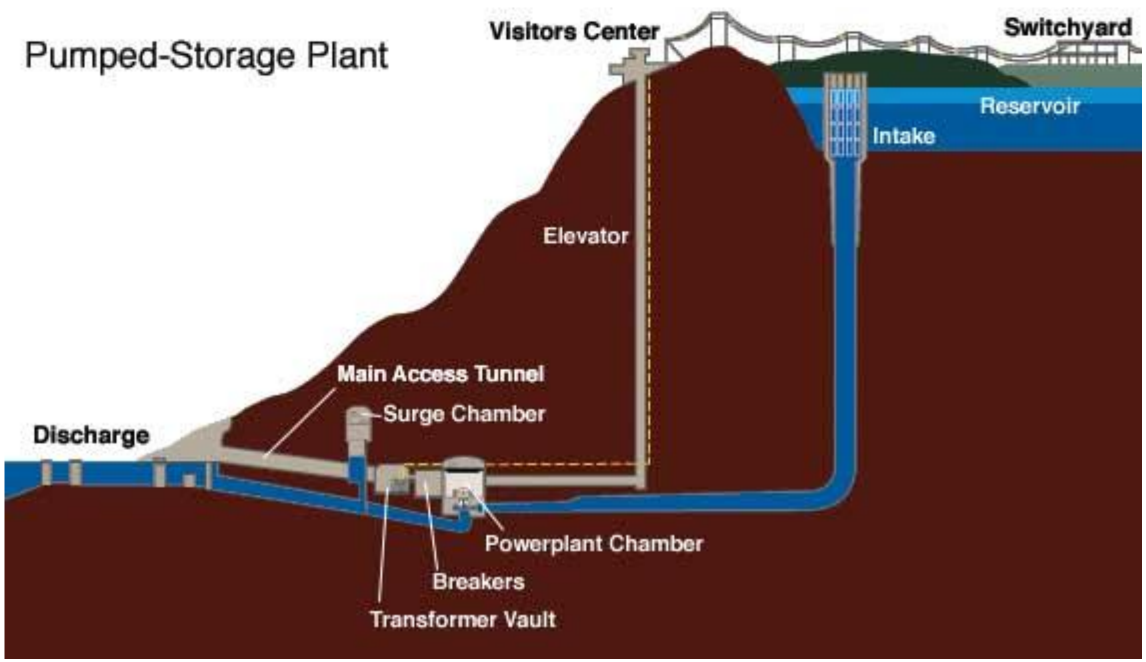
E. 6 μC

Storing Energy

- Let me ask you a question.
- Let’s say that today, you have a lot of spare energy, but that there is a high likelihood that tomorrow you would need more than you will have.
- What would you do to prepare for tomorrow?

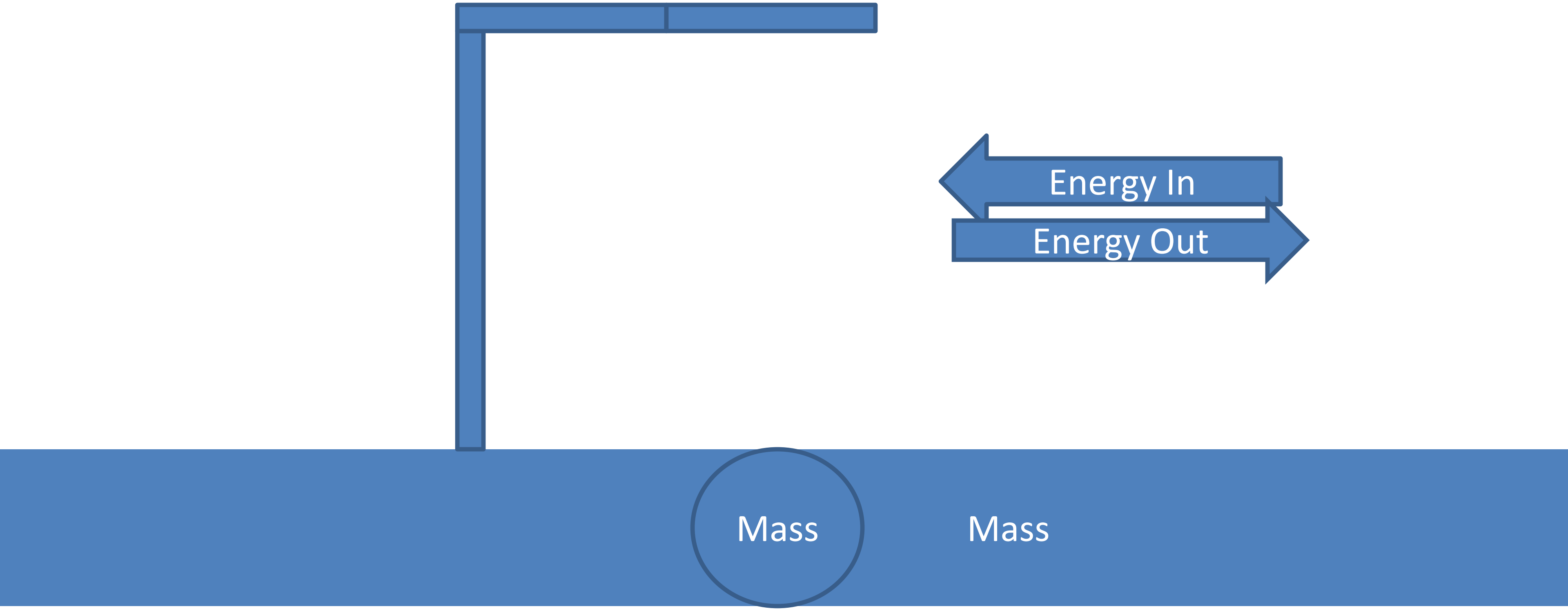
Storing Energy

- One possibility is to take something at ground level, use the spare energy you have today to do work on that something and raise it to a higher level.
- Then, tomorrow, you could use that gravitational potential energy to do work…



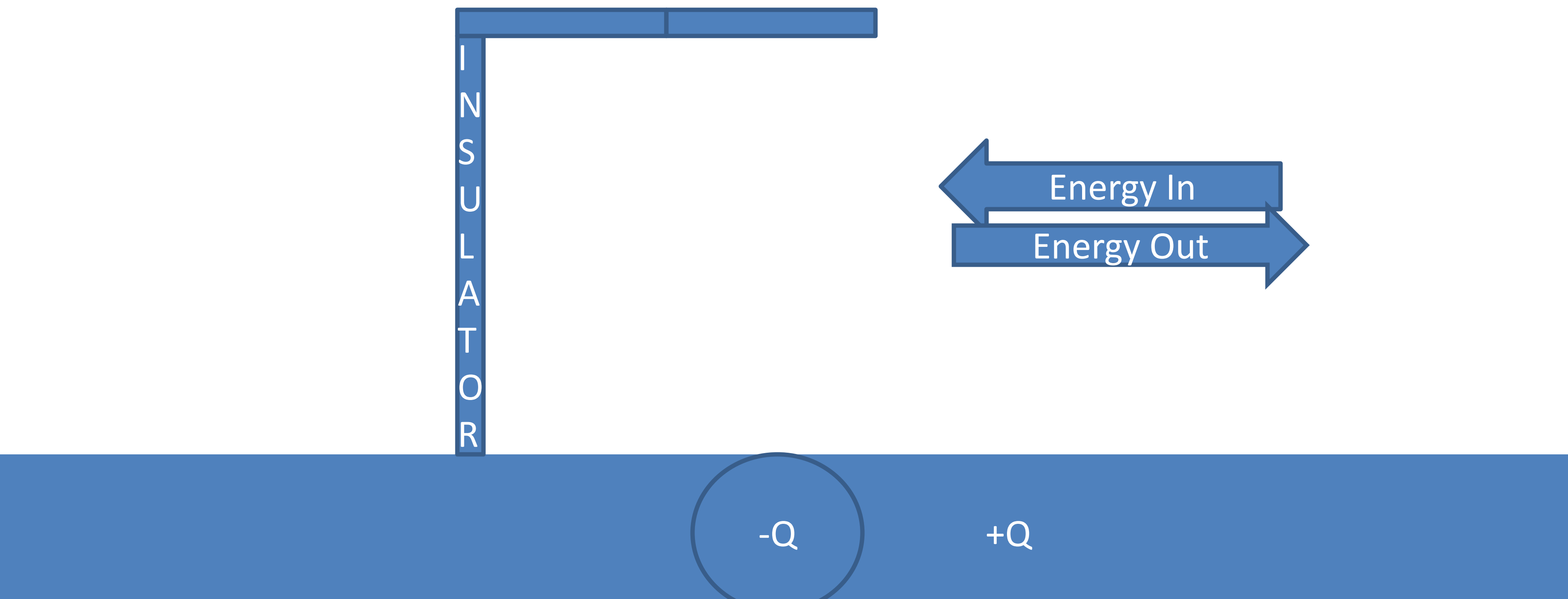
Storing Energy in Gravitation

- The basic idea of this is demonstrated below:



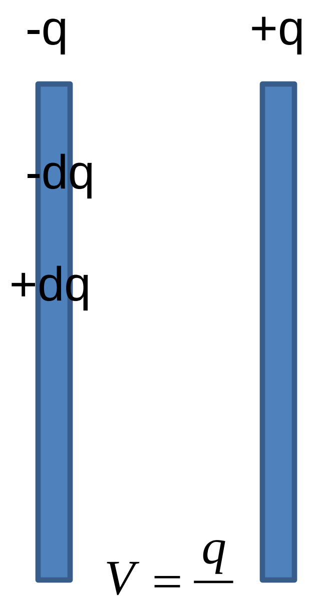
Storing Energy with Charge

- The basic idea of this is demonstrated below:



Energy Stored in Capacitors

- Let’s look at a parallel plate capacitor with some initial charge on it.
- How much work does it take to put more charge on it?

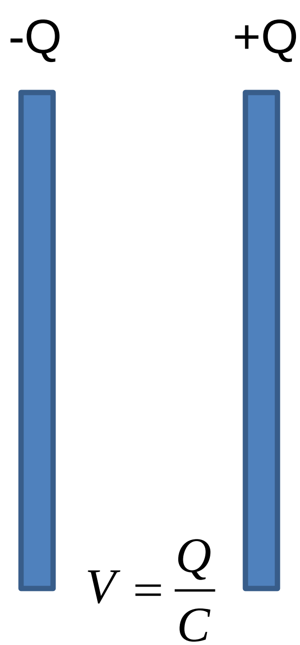


$$V = \frac{q}{C}$$

$$dW = Vdq = \frac{q}{C}dq$$

Energy Stored in Capacitors

- So, how much work does it take to charge the capacitor from zero to Q?
- Then this is also the potential energy stored in the capacitor.



$$dW = V dq = \frac{q}{C} dq \Rightarrow$$

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = U$$

Energy Stored in Capacitors

- We can think of the capacitor as an electric analog of a stretched spring:

$$U = \frac{Q^2}{2C} = \frac{1}{2}\left(\frac{1}{C}\right)Q^2$$

- With 1/C analogous to the spring constant, and Q, the charge analogous to the extent the spring is compressed or stretched.
- Note that the smaller the capacitance, the larger the "spring constant", and the harder it is to put charge on it.

CPS 20-5

You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates.

If the charges $+Q$ and $–Q$ on the two plates are kept constant in this process, the energy stored in the capacitor

A. becomes 4 times greater.

B. becomes twice as great.

C. remains the same.

D. becomes 1/2 as great.

E. becomes 1/4 as great.

CPS 20-5

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Energy Stored in Electric Fields

- Let me play around a little with this result.

$$U = \frac{Q^2}{2C} = \frac{1}{2}\frac{\left(CV\right)^2}{C} = \frac{1}{2}CV^2$$

- And now, let me look at the potential energy in the capacitor ***per volume of the capacitor.***

$$\frac{U}{\text{Volume}} \equiv u = \frac{1}{2}\frac{CV^2}{Ad} = \frac{1}{2}\frac{\frac{\varepsilon_0 A}{d}V^2}{Ad} = \frac{1}{2}\frac{\varepsilon_0 V^2}{d^2} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

Energy Stored in Electric Fields

- This result turns out to be independent of the geometry of the capacitor, and is completely general:
- Energy is stored in all electric fields, and has a density:

$$u = \frac{1}{2}\varepsilon_0 E^2$$