#33 Fluid Mechanics I Post-class

Due: 11:00am on Friday, November 9, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's [Grading Policy]

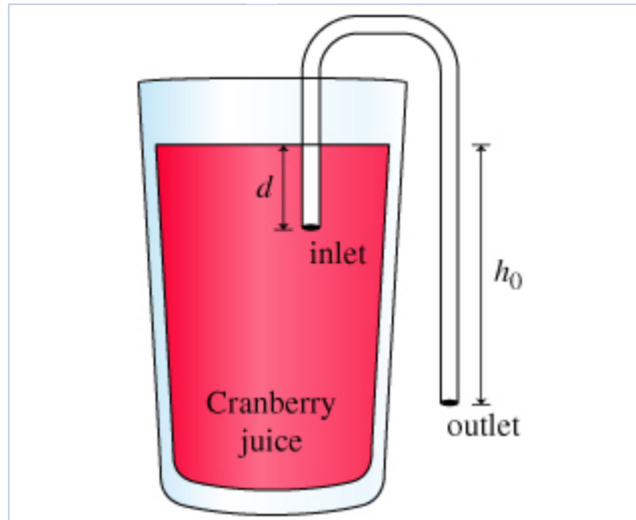
A Siphon at the Bar

Jane goes to a juice bar with her friend Neil. She is thinking of ordering her favorite drink, 7/8 orange juice and 1/8 cranberry juice, but the drink is not on the menu, so she decides to order a glass of orange juice and a glass of cranberry juice and do the mixing herself. The drinks come in two identical tall glasses; to avoid spilling while mixing the two juices, Jane shows Neil something she learned that day in class. She drinks about 1/8 of the orange juice, then takes the straw from the glass containing cranberry juice, sucks up just enough cranberry juice to fill the straw, and while covering the top of the straw with her thumb, carefully bends the straw and places the end over the orange juice glass. After she releases her thumb, the cranberry juice flows through the straw into the orange juice glass. Jane has successfully designed a siphon.
Assume that the glass containing cranberry juice has a very large diameter with respect to the diameter of the straw and that the cross-sectional area of the straw is the same at all points. Let the atmospheric pressure be p_a and assume that the cranberry juice has negligible viscosity.

Part A

Consider the end of the straw from which the cranberry juice is flowing into the glass containing orange juice, and let h_0 be the distance below the surface of cranberry juice at which that end of the straw is located: . What is the initial velocity v of the cranberry juice as it flows out of the straw? Let g denote the magnitude of the acceleration due to gravity.

Express your answer in terms of g and h_0.



Hint 1. How to approach the problem

This problem asks for the fluid speed as it flows out of the siphon tube. One of the mathematical relations involving fluid speed is Bernoulli's principle, which relates the pressure at a point at a given depth to the speed of a fluid element at that particular point. Specifically, in this problem you need to identify two distinct points along a flow tube where Bernoulli's principle applies.
Remember to define clearly your coordinate system, in particular the level at which $y = 0$.

Hint 2. Apply Bernoulli's principle

Consider the entire volume of cranberry juice as a single flow tube and complete the expression below by applying Bernoulli's principle to a point at the opening of the siphon tube and to a point at the exit of the siphon tube, that is, to a point at the end of the straw submerged in the cranberry juice and to a point at the other end of the straw.

Take the exit of the siphon tube as the origin of the *y* axis and assume that the inlet of the siphon is located at distance d from the surface of the cranberry juice. Let $\rho_{\rm j}$ be the density of the cranberry juice.

Note that the fluid speed at the inlet of the siphon tube (i.e., the straw) may be neglected as a result of the fact that the diameter of the glass containing the cranberry juice is very large with respect to the diameter of the straw.

Express your answer in terms of h_0, $p_{\rm a}$, $\rho_{\rm j}$, and g, the magnitude of the acceleration due to gravity.

Hint 1. Bernoulli's principle

For an incompressible steady flow of a fluid with no internal friction, the pressure $p_{\rm h}$ and the flow speed $v_{\rm h}$ at height h above some reference height are linked together by an important relationship, known as Bernoulli's principle. In particular, at any point at height h along a flow tube the following relation is valid:

$$p_h + \rho g h + \frac{1}{2}\rho v_h^2 = \mathrm{constant},$$

where ρ is the density of the fluid and g the magnitude of the acceleration due to gravity.

Since Bernoulli's principle is valid at any point along a flow tube, it takes the form

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

when applied to two distinct points along the flow tube. The subscripts 1 and 2 refer to such points.

Hint 2. Find the pressure at the inlet of the siphon tube

In a fluid of uniform density ρ, the pressure p at depth d is given by the pressure $p_{\rm surf}$ at the surface plus a term due to the weight of the fluid above that point. In other words,

$$p = p_{\rm surf} + \rho g d,$$

where g is the acceleration due to gravity.

Observe that this is a special case of Bernoulli's law for a fluid that has no velocity and with the reference height taken to be the surface of the fluid.

From this information, find an expression for the pressure $p_{\rm inlet}$ at the inlet of the siphon, i.e., at the end of the straw submerged into cranberry juice. Assume that the inlet of the siphon is located at distance d from the surface of the cranberry juice and let $\rho_{\rm j}$ be the density of cranberry juice.

Express your answer as a function of $p_{\rm a}$, $\rho_{\rm j}$, g, and d.

ANSWER:

$p_{\rm inlet}$ = $p_a + \rho_j g d$

ANSWER:

$p_a + \frac{1}{2}\rho_j v^2$ = $p_a + \rho_j g h_0$

ANSWER:

v = $\sqrt{2gh_0}$

Correct

The speed of fluid flowing from the outlet of a siphon tube is the same as the speed that a body would acquire in falling from rest through a distance h_0. This result is valid also for fluid flowing from an opening in a container at distance h_0 below the surface of the fluid.

Part B

Given the information found in Part A, find the time t it takes to Jane to transfer enough cranberry juice into the orange juice glass to make her favorite drink if $h_0 = 10.0$ centimeters. Assume that the flow rate of the liquid is constant, and that the glasses are cylindrical with a diameter of 7.0 centimeters and are filled to height 14.0 centimeters. Take the diameter of the straw to be 0.4 centimeters.

Express your answer numerically in seconds to two significant figures.

Hint 1. How to approach the problem

To make her favorite drink, Jane has to transfer 1/8 of the cranberry juice into the orange juice glass. To calculate the time t needed to remove 1/8 of the cranberry juice, you need to know at what volume flow rate $\frac{dV}{dt}$ the cranberry juice flows into the glass containing orange juice. By simply dividing the volume you need to remove by the volume flow rate, you will find the time needed to complete the fluid transfer.

Hint 2. Find the volume flow rate

Consider an incompressible fluid of density ρ that is steadily moving, and let A be the cross-sectional area of a flow tube. The volume ΔV of fluid flowing across the cross section at speed v during a small interval of time Δt is given by $Av\Delta t$. Then, the rate at which fluid volumes cross a portion of the flow tube is

$$\frac{dV}{dt} = Av.$$

From this expression and the information found in Part A, find the volume flow rate of cranberry juice as it flows out of the straw into the orange juice glass.

Express your answer numerically in centimeters per cubic second to three significant figures. Use 9.81 $\mathrm{m/s^2}$ as the magnitude of the acceleration due to gravity.

ANSWER:

$\frac{dV}{dt}$ = 17.6 $\mathrm{cm^3/s}$

ANSWER:

t = 3.8 s

Correct

A Water Tank on Mars

You are assigned the design of a cylindrical, pressurized water tank for a future colony on Mars, where the acceleration due to gravity is 3.71 meters per second per second. The pressure at the surface of the water will be 150kPa , and the depth of the water will be 14.2m . The pressure of the air in the building outside the tank will be 90.0kPa .

Part A

Find the net downward force on the tank's flat bottom, of area 2.20$\mathrm{m^2}$, exerted by the water and air inside the tank and the air outside the tank.

Express your answer numerically in Newtons, to three significant figures.

Hint 1. The net force

The net force on the tank's flat bottom is the sum of the (downward) force exerted on the bottom of the tank by the water and the (upward) force exerted on the bottom of the tank by the air outside the tank.

Hint 2. What is Pa (a Pascal)?

The SI unit of pressure, a Pascal, is a force per unit area: 1 Pa = 1 N/m^2.

Hint 3. Density of water

The density of water is 1 g/cm^3.

Hint 4. Find the force exerted on the tank's bottom by the air outside the tank

Write an expression for the force exerted on the tank's bottom by the air outside the tank.

Express your answer numerically to three significant figures.

Hint 1. Pressure and force

The pressure p in a gas is defined as the normal force F exerted by the gas on a surface in contact with it. If the force is the same at all points of a finite plane surface with area A, then the pressure is uniform and given by

$$p = \frac{F}{A}.$$

ANSWER:

1.98×10^5 N

Hint 5. Find the force exerted on the tank's bottom by the water

Write an expression for the force exerted on the tank's bottom by the water in the tank. Keep in mind that the water tank is located on Mars, so weights depend on the acceleration due to gravity on that particular planet.

Express your answer numerically in Newtons, to three significant figures.

Hint 1. Find the pressure exerted by the water on the bottom of the tank

In a fluid of uniform density ρ, the pressure p at a depth H is given by the pressure p_{surf} at the surface plus a term due to the weight of the fluid above that point. In other words,

$$p = p_{surf} + \rho g H,$$

where g is the magnitude of the acceleration due to gravity.

Given this information, find the pressure exerted by the water on the bottom of the tank.

Express your answer numerically in pascals, to three significant figures.

ANSWER:

2.03×10^5 Pa

Hint 2. Pressure and force

The pressure p in a gas is defined as the normal force F exerted by the gas on a surface in contact with it. If the force is the same at all points of a finite plane surface with area A, then the pressure is uniform and given by

$$p = \frac{F}{A}.$$

ANSWER:

4.46×10^5 N

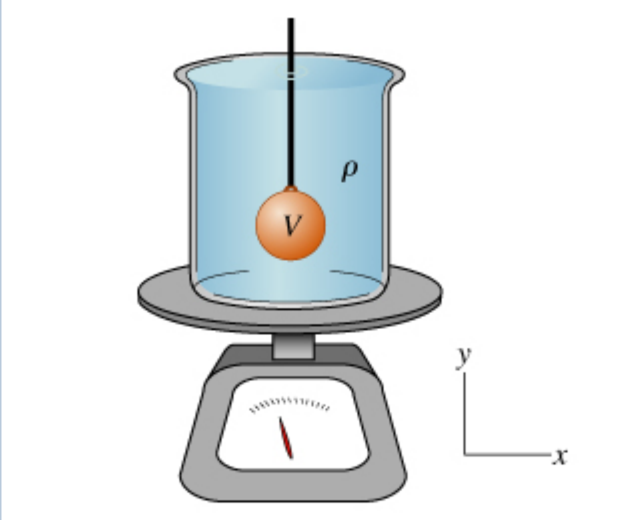
ANSWER:

2.48×10^5 N

All attempts used; correct answer displayed

Submerged Sphere in a Beaker

A cylindrical beaker of height 0.100m and negligible weight is filled to the brim with a fluid of density ρ = 890$\mathrm{kg/m^3}$. When the beaker is placed on a scale, its weight is measured to be 1.00N .



A ball of density ρ_b = 5000$\mathrm{kg/m^3}$ and volume V = 60.0$\mathrm{cm^3}$ is then submerged in the fluid, so that some of the fluid spills over the side of the beaker. The ball is held in place by a stiff rod of negligible volume and weight. Throughout the problem, assume the acceleration due to gravity is g = 9.81$\mathrm{m/s^2}$.

Part A

What is the weight W_b of the ball?

Express your answer numerically in newtons.

Hint 1. Find the mass of the ball

Find the mass m_b of the ball.

Express your answer numerically in kilograms.

ANSWER:

m_b = 0.300 kg

Hint 2. Converting between cubic meters and cubic centimeters

Since $1\ \mathrm{m} = 100\ \mathrm{cm}$, then $1\ \mathrm{m}^3 = 10^6\ \mathrm{cm}^3$ and $1\ \mathrm{cm}^3 = 10^{-6}\ \mathrm{m}^3$.

ANSWER:

W_b = 2.94 N

Correct

Part B

What is the reading W_2 of the scale when the ball is held in this submerged position? Assume that none of the water that spills over stays on the scale.

Calculate your answer from the quantities given in the problem and express it numerically in newtons.

Hint 1. How to approach the problem

One way to answer this question is to compute the pressure at the bottom of the beaker. Since the walls of the beaker are vertical, the total vertical force on the beaker due to the fluid is a result of this pressure. Alternatively, you can compute the weight of the water in the beaker (recall that some water is lost over the edge when the ball is submerged) and add the force exerted by the ball on the water (which is transmitted through the water to the beaker).

Hint 2. Find the pressure at the bottom of the beaker

What is the pressure p_2 of the fluid at the bottom of the beaker?

Express your answer numerically in pascals

ANSWER:

p_2 = 873 Pa

Hint 3. Find the pressure before the ball is submerged

What was the pressure p_1 of the fluid at the bottom of the beaker before the ball was submerged?

Express your answer numerically in pascals.

ANSWER:

p_1 = 873 Pa

Hint 4. The force exerted by the ball on the water

According to Newton's 3rd law, the force exerted by the ball on the water is equal and opposite to the buoyant force exerted by the water on the ball.

ANSWER:

W_2 = 1.00 N

Correct

The "extra force" that the ball exerts on the water is equal to the force that the water exerts on the ball, that is, the weight of the displaced water. Therefore, the reading does not change.

Part C

What is the force F_r applied to the ball by the rod? Take upward forces to be positive (e.g., if the force on the ball is downward, your answer should be negative).

Express your answer numerically in newtons.

Hint 1. How to approach the problem

The ball is stationary. Therefore, the total force acting on it must be zero. The total force includes the force of the rod, the gravitational force, and the buoyant force.

Hint 2. Find the buoyant force

Using Archimedes' principle, find F_b, the buoyant force acting on the submerged ball.

Express your answer numerically in newtons.

ANSWER:

F_b = 0.524 N

Hint 3. Find the expression for the net force

Taking the positive *y* direction to be vertically upward, choose an equation correctly expressing the net force acting on the ball in terms of the existing forces. Let F_r, the force exerted by the rod, be directed upward.

Hint 1. How to approach this problem

There are three forces acting on the ball: the weight acting downward, the buoyant force acting upward, and the force from the rod, assumed to be acting upward.

ANSWER:

- ○ $(\rho - \rho_b)Vg - F_r$
- ◉ $(\rho - \rho_b)Vg + F_r$
- ○ $\rho Vg + F_r$
- ○ $-\rho_b Vg + F_r$

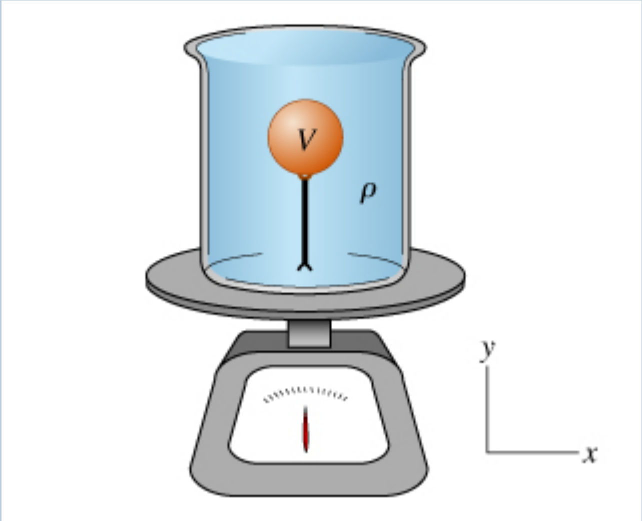
ANSWER:

F_r = 2.42 N

Correct

The force *does* act upward as one would expect because the ball is denser than the fluid and would have sunk if it weren't for the rod.

The rod is now shortened and attached to the bottom of the beaker. The beaker is again filled with fluid, the ball is submerged and attached to the rod, and the beaker with fluid and submerged ball is placed on the scale.



Part D

What weight W_3 does the scale now show?

Express your answer numerically in newtons.

Hint 1. How to approach the problem

It makes no difference how the ball and fluid are arranged inside the beaker. If there are no external forces, the weight reading would be equal to the total weight of the beaker and its contents.

Hint 2. Find the weight of the fluid in the beaker

What is the weight W_f of the fluid that overflowed the beaker when the ball was originally submerged?

Express your answer numerically in newtons.

ANSWER:

W_f = 0.524 N

ANSWER:

W_3 = 3.42 N

Correct

Note that in this case it is not necessary to know the buoyant force acting on the ball in to answer the question.

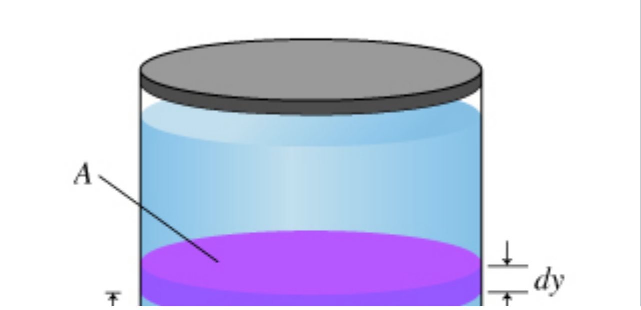
Relating Pressure and Height in a Container

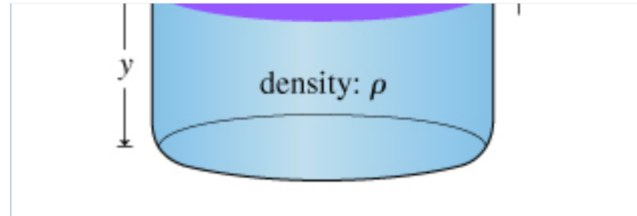
Learning Goal:

To understand the derivation of the law relating height and pressure in a container.

In this problem, you will derive the law relating pressure to height in a container by analyzing a particular system.

A container of uniform cross-sectional area A is filled with liquid of uniform density ρ. Consider a thin horizontal layer of liquid (thickness dy) at a height y as measured from the bottom of the container. Let the pressure exerted upward on the bottom of the layer be p and the pressure exerted downward on the top be $p + dp$. Assume throughout the problem that the system is in equilibrium (the container has not been recently shaken or moved, etc.).





Part A

What is F_{up}, the magnitude of the force exerted upward on the bottom of the liquid?

Hint 1. Formula for the force

Force is equal to pressure times area.

ANSWER:

F_{up} = pA

Correct

Part B

What is F_{down}, the magnitude of the force exerted downward on the top of the liquid?

Hint 1. Formula for the force

Force is equal to pressure times area.

ANSWER:

F_{down} = $(p+dp)\,A$

Correct

Part C

What is the weight w_{layer} of the thin layer of liquid?

Express your answer in terms of quantities given in the problem introduction and g, the magnitude of the acceleration due to gravity.

Hint 1. How to approach the problem

The weight of the layer is given by the formula $w = mg$, where m is the mass of the layer and g is the magnitude of the acceleration due to gravity.

Hint 2. Mass of the layer

Use the definition of density to write the mass m of the layer of liquid in terms of its density and its volume (express volume in terms of physical dimensions given in the problem introduction).

Express your answer in terms of quantities given in the problem introduction.

Hint 1. Definition of density

The density of an object is equal to its mass divided by its volume.

Hint 2. Volume of the layer

What is the volume dV of the thin layer of liquid?

Express your answer in terms of quantities given in the problem introduction.

ANSWER:

dV = Ady

ANSWER:

m = ρAdy

ANSWER:

w_{layer} = $\rho Agdy$

Correct

Part D

Since the liquid is in equilibrium, the net force on the thin layer of liquid is zero. Complete the force equation for the sum of the vertical forces acting on the liquid layer described in the problem introduction.

Express your answer in terms of quantities given in the problem introduction and taking upward forces to be positive.

Hint 1. How to approach the problem

If you have completed the previous parts, you have already done most of the work needed to answer this part. Just add together the forces that you found in the previous three parts. All three of the forces act along the *y* axis; some are directed upward, and others are directed downward. Those that act downward should appear with a negative sign.

ANSWER:

$\sum_i F_{y,i} =$ = $pA-(p+dp)\,A-\rho Adyg$

Correct

Part E

Solve the sum-of-forces equation just derived,

$$0=\sum_i F_{y,i}=pA-(p+dp)A-\rho Ag\,dy,$$

to obtain an expression for dp and thus a differential equation for pressure.

ANSWER:

dp = $-\rho gdy$

Correct

Part F

Integrate both sides of the differential equation you found for dp to obtain an equation for p. Your equation should then include a constant that depends on initial conditions. Determine the value of this constant by assuming that the pressure at some reference height y_0 is p_0.

Express your answer in terms of quantities given in the problem introduction along with y_0 and p_0.

Hint 1. Integrate dp

What is the expression obtained by integrating the left-hand side of the equation $dp = -\rho g\, dy$? Although the indefinite integral of the left-hand side of the equation should include a constant determined by initial conditions, you can combine it with the constant on the right-hand side. Leave it out of your answer here.

ANSWER:

$\int dp = p$

Hint 2. Integrate $\rho g\, dy$

What is the expression obtained by integrating the right-hand side of the equation $dp = -\rho g\, dy$? Here you will need to include the constant determined by initial conditions; call it C.

ANSWER:

$-\int \rho g\, dy = -\rho g y + C$

Hint 3. Determine the constant of integration

According to the statement of the initial conditions, $p(y_0) = p_0$. Using this fact, find the value of the constant C.

Express your answer in terms of p_0, y_0, and other given quantities.

ANSWER:

C = $p_0 + \rho g y_0$

ANSWER:

p = $p_0 + \rho g \left(y_0 - y \right)$

Correct

Score Summary:

Your score on this assignment is 78.2%.
You received 31.28 out of a possible total of 40 points.