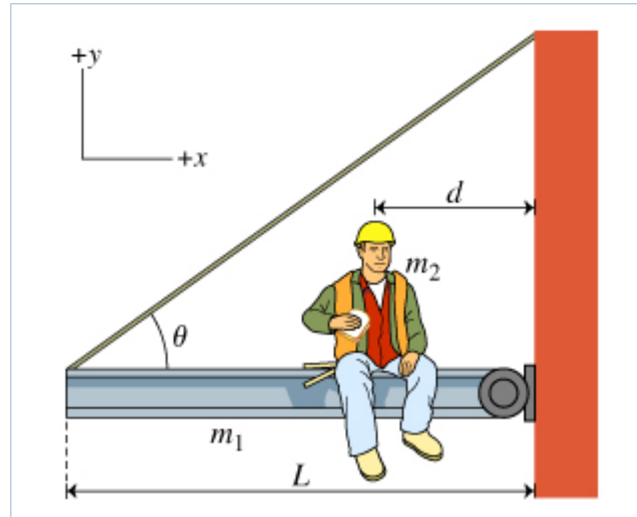
#32 Equillibrium of Rigid Bodies II Post-class

Due: 11:00am on Wednesday, November 7, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Precarious Lunch

A uniform steel beam of length L and mass m_1 is attached via a hinge to the side of a building. The beam is supported by a steel cable attached to the end of the beam at an angle θ, as shown. Through the hinge, the wall exerts an unknown force, F, on the beam. A workman of mass m_2 sits eating lunch a distance d from the building.



Part A

Find T, the tension in the cable. Remember to account for *all* the forces in the problem.

Express your answer in terms of m_1, m_2, L, d, θ, and g, the magnitude of the acceleration due to gravity.

Hint 1. Pick the best origin

This is a statics problem so the sum of torques about any axis *a* will be zero. In order to solve for T, you want to pick the axis such that T will give a torque, but as few as possible other *unknown* forces will enter the equations. So where should you place the origin for the purpose of calculating torques?

ANSWER:

- ○ At the center of the bar
- ◉ At the hinge
- ○ At the connection of the cable and the bar
- ○ Where the man is eating lunch

Hint 2. Calculate the sum torques

Now find the sum of the torques about center of the hinge. Remember that a positive torque will tend to rotate objects counterclockwise around the origin.

Answer in terms of T, L, d, m_1, m_2, θ, and g.

ANSWER:

$\Sigma\,\tau_a$ = 0 = $\left(\frac{m_1 L}{2} + m_2 d\right) g - TL\sin(\theta)$

ANSWER:

T = $\frac{g\left(m_2 d + m_1 \frac{L}{2}\right)}{L\sin\theta}$

Correct

Part B

Find F_x, the x-component of the force exerted by the wall *on* the beam (F), using the axis shown. Remember to pay attention to the direction that the wall exerts the force.

Express your answer in terms of T and other given quantities.

Hint 1. Find the sign of the force

The beam is not accelerating in the x-direction, so the sum of the forces in the x-direction is zero. Using the given coordinate system, is F_x going to have to be positive or negative?

ANSWER:

F_x = $-T\cos\theta$

Correct

Part C

Find F_y, the y-component of force that the wall exerts *on* the beam (F), using the axis shown. Remember to pay attention to the direction that the wall exerts the force.

Express your answer in terms of T, θ, m_1, m_2, and g.

ANSWER:

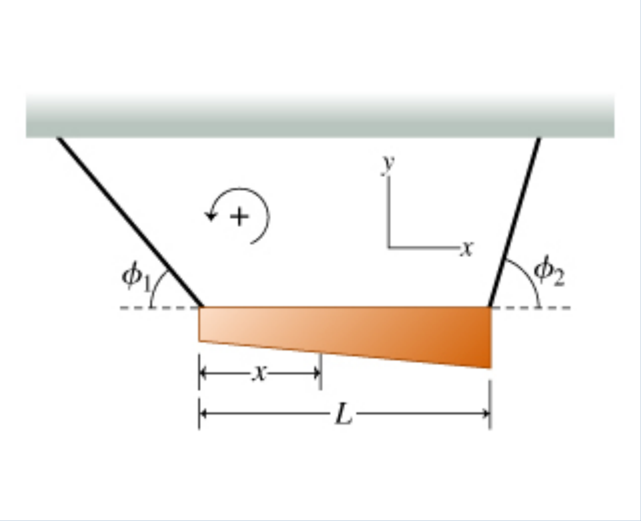
F_y = $g\left(m_1+m_2\right)-T\sin\theta$

Correct

If you use your result from part (A) in your expression for part (C), you'll notice that the result simplifies somewhat. The simplified result should show that the further the luncher moves out on the beam, the *lower* the magnitude of the upward force the wall exerts on the beam. Does this agree with your intuition?

A Bar Suspended by Two Wires

A nonuniform, horizontal bar of mass m is supported by two massless wires against gravity. The left wire makes an angle ϕ_1 with the horizontal, and the right wire makes an angle ϕ_2. The bar has length L.



Part A

Find the position of the center of mass of the bar, x, measured from the bar's left end.

Express the center of mass in terms of L, ϕ_1, and ϕ_2.

Hint 1. Nature of the problem

This is a statics problem. There is no net force or torque acting on the bar.

Hint 2. Torques about left end of bar

The net torque is zero about any point you select. Here we ask you to find the net torque of the system about the left end of the bar. Label the tension in the left wire T_1, and label the other wire's tension T_2. The weight of the bar is $W = mg$. Note that the vector sum of T_1, T_2, and W is zero. Using the sign convention shown in the picture, express the sum of the torques about the left end of the bar.

Answer in terms of L, x, W, T_2, T_1, ϕ_2, and/or ϕ_1. Note that not all of these quantities will appear in your answer.

ANSWER:

$\sum \tau_{\rm left} = 0 =$ $LT_2\sin(\phi_2) - Wx$

Hint 3. Forces: *x* components

Assume that the tensions in the left and right wires are T_1 and T_2, respectively. What is the sum of the *x* components of the forces ΣF_x? Because this is a statics problem, these forces will sum to zero.

Use the sign convention indicated in the figure, and express your answer in terms of L, x, W, T_2, T_1, ϕ_2, and/or ϕ_1. Note that not all of these quantities will appear in your answer.

ANSWER:

$\sum F_x = 0 =$ $-T_1\cos(\phi_1) + T_2\cos(\phi_2)$

Hint 4. Forces: *y* components

Assuming that the tensions in the left and right wires are T_1 and T_2, respectively, what is the sum of the *y* components of the forces ΣF_y? Because this is a statics problem, these forces will sum to zero.

Use the sign convention indicated in the figure, and express your answer in terms of L, x, W, T_2, T_1, ϕ_2, and/or ϕ_1. Note that not all of these quantities will appear in your answer.

ANSWER:

$\sum F_y = 0 =$ $T_1\sin(\phi_1) + T_2\sin(\phi_2) - W$

Hint 5. Eliminate weight from your equations

You should have found three equations by now. It is possible to eliminate two variables and solve for x in terms of the others. As an intermediate step, solve your torque equation for x in terms of W, T_2, L, etc. and then solve your *y*-component force equation for W and substitute back into your expression for x. In other words, find an expression for x.

Answer in terms of T_1, T_2, ϕ_1, ϕ_2, and L.

ANSWER:

$$x = \frac{T_2 \sin(\phi_2) L}{T_1 \sin(\phi_1) + T_2 \sin(\phi_2)}$$

Hint 6. A useful trig identity

The dimensions for the expression you just found for x are correct, since the units of the tensions cancel out, leaving the units of length in the numerator. If you now solve the *x*-component force equation for T_1 in terms of T_2, and substitute into your equation for x, you should find the following trig identity useful:

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b).$$

Alternatively, you could express your answer in terms of $\tan(\phi_1)$ and $\tan(\phi_2)$.

ANSWER:

$$x = \frac{L}{\left(\frac{\tan(\phi_1)}{\tan(\phi_2)}\right) + 1}$$

Correct

PSS 11.1 Equilibrium of a Rigid Body

Learning Goal:

To practice Problem-Solving Strategy 11.1 Equilibrium of a Rigid Body.

A horizontal uniform bar of mass 3kg and length 3.0 m is hung horizontally on two vertical strings. String 1 is attached to the end of the bar, and string 2 is attached a distance 0.7m from the other end. A monkey of mass 1.5kg walks from one end of the bar to the other. Find the tension T_1 in string 1 at the moment that the monkey is halfway between the ends of the bar.

Problem-Solving Strategy 11.1 Equilibrium of a Rigid Body

IDENTIFY *the relevant concepts:*
The first and second conditions for equilibrium are useful whenever there is a rigid body that is not rotating and not accelerating in space.

SET UP *the problem using the following steps:*

1. Draw a sketch of the physical situation, including dimensions, and select the body in equilibrium to be analyzed.
2. Draw a free-body diagram showing the forces acting on the selected body and no others. Do not include forces exerted by this body on other bodies. Be careful to show correctly the point at which each force acts.
3. Choose coordinate axes, and specify a positive direction of rotation for torques. Represent forces in terms of their components with respect to the axes you have chosen.
4. In choosing a point about which to compute torques, note that if a force has a line of action that goes through a particular point, the torque of the force with respect to that point is zero. You can often eliminate unknown forces or components from the torque equation by a clever choice of point for your calculation. The body doesn't actually have to be pivoted about an axis through the chosen point.

EXECUTE *the solution as follows:*

1. Write equations expressing the equilibrium conditions. Keep in mind that $\sum F_x = 0$, $\sum F_y = 0$, and $\sum \tau_z = 0$ are always separate equations; never add *x* and *y* components in a single equation. Also, recall that when a force is represented in terms of its components, you can compute the torque of that force by finding the torque of each component separately, each with its appropriate lever arm and sign, and adding the results.
2. You always need as many equations as you have unknowns. Depending on the number of unknowns, you may need to compute torques with respect to two or more axes to obtain enough equations. Often, there are several equally good sets of force and torque equations for a particular problem.

EVALUATE *your answer:*
A useful way to check your results is to rewrite the second condition for equilibrium, $\sum \tau_z = 0$, using a different choice of origin. If you've done everything correctly, you'll get the same answers using this new choice of origin as you did with your original choice.

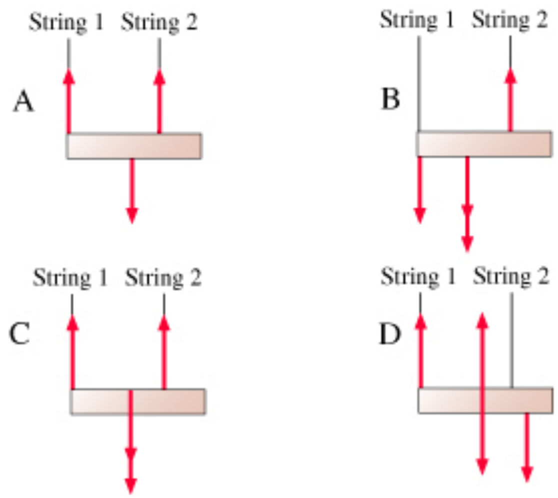
IDENTIFY the relevant concepts

The rigid body under consideration is the bar. Because there is no indication to suggest otherwise, it is reasonable to assume that the bar remains at rest as the monkey walks from one end of the bar to the other. This means that the conditions for equilibrium hold at any time, including the instant at which the monkey is halfway between its ends. Thus, the above strategy can be applied.

SET UP the problem using the following steps

Part A

Which of the following diagrams correctly represents the forces acting on the bar at the moment described in the problem introduction? (Note that the forces are not necessarily drawn to scale.)



ANSWER:

- diagram A
- diagram B
- diagram C
- diagram D

Correct

Four vertical forces act upon the bar: the upward tension in both strings, the weight of the bar, and the downward force exerted by the monkey. Note that whereas the weight of the bar is represented as a force acting at the center of gravity of the bar (which coincides with the center of mass the bar), the force exerted by the monkey acts at the point of contact between the monkey and the bar, which is a point on the top surface of the bar halfway between its ends. This may be clearer if you draw your own free-body diagram. Be sure to label all the forces in your diagram.

Part B

One of the equilibrium conditions that should be applied in this problem requires that you write a torque equation for the bar. Which of the following choices of reference point for calculating torques would lead to a torque equation in which the only unknown quantity is T_1?

Hint 1. The point of reference for calculating torques

In equilibrium problems, the choice of reference point for calculating torques is completely arbitrary. In general, it helps to pick the point so as to simplify the calculations as much as possible. For example, if a force has a line of action that goes through a particular point, the torque of the force with respect to that point is zero and its contribution would not appear in the torque equation $\sum \tau_z = 0$.

ANSWER:

- (selected) the point of attachment of string 2
- the center of mass of the bar
- the end of the bar closest to string 2
- the point of attachment of string 1

Correct

If you choose the point of attachment of string 2 as the reference point for calculating torques, the torque equation will not depend on T_2, the (unknown) tension in string 2, because the line of action of T_2 goes through the point of attachment of string 2, and the lever arm of the force in this case would be zero. Eliminating terms from the torque equation, particularly those involving unknown quantities, generally makes it easier to solve. Other choices of reference point would also work, but they might lead to more complicated mathematics. Keep in mind that once you make your choice, you must use the same point to calculate all the torques on a body.

Now, choose a set of coordinate axes, and specify a positive direction of rotation for torques.

EXECUTE the solution as follows

Part C

Find T_1, the magnitude of the force of tension in string 1, at the moment that the monkey is halfway between the ends of the bar.

Express your answer in newtons using three significant figures.

Hint 1. How to approach the problem

There are two unknowns in this problem: the tensions in each string. Therefore, you will in general need to use two equations, as explained in the Problem-Solving Strategy above. However, as highlighted in Part B, by a clever choice of reference point for your calculation, you can eliminate one of the unknown forces so that the torque equation involves only the unknown tension T_1 and is independent of the tension T_2 in string 2. Solving this equation for T_1 will give you the tension in string 1 at the moment that the monkey is halfway between the ends of the bar.

Hint 2. Find the torque exerted by the weight of the bar

Let the length of the bar be l, and let d be the distance from the point of attachment of string 2 to the closest end of the bar. If the mass of the bar is m, what is the torque τ_{bar} exerted by the weight of the bar about the point of attachment of string 2? Take the counterclockwise direction as the positive direction of rotation.

Express your answer in terms of some or all of the variables l, d, and m. If necessary, use g for the acceleration due to gravity.

ANSWER:

$\tau_{\rm bar}$ = $mg\left(\frac{l}{2}-d\right)$

Hint 3. Find the torque exerted by the monkey

Let the length of the bar be l, and let d be the distance from the point of attachment of string 2 to the closest end of the bar. If the mass of the monkey is $m_{\rm m}$, what is the torque $\tau_{\rm mon}$ exerted by the monkey about the point of attachment of string 2? Take the counterclockwise direction as the positive direction of rotation.

Express your answer in terms of some or all of the variables l, d, and $m_{\rm m}$. If necessary, use g for the acceleration due to gravity.

Hint 1. The force exerted by the monkey on the bar

Because the monkey moves in the horizontal direction, in the vertical direction it must be in equilibrium at any time. Therefore, at the moment that the monkey is halfway between the ends of the bar, the magnitude of the normal force exerted by the bar on the monkey must equal the weight of the monkey. However, the magnitude of the normal force acting on the monkey equals, according to Newton's 3rd law, the magnitude of the force applied by the monkey to the bar; therefore, the force applied by the monkey to the bar simply equals its weight.

ANSWER:

$\tau_{\rm mon}$ = $m_m g\left(\frac{l}{2}-d\right)$

Hint 4. Find the torque exerted by the tension in string 1

Let the length of the bar be l, and let d be the distance from the point of attachment of string 2 to the closest end of the bar. What is the torque τ_1 exerted by the tension T_1 in string 1 about the point of attachment of string 2? Take the counterclockwise direction as the positive direction of rotation.

Express your answer in terms of some or all of the variables l, d, and T_1. If necessary, use g for the acceleration due to gravity.

ANSWER:

τ_1 = $-T_1(l-d)$

ANSWER:

T_1 = 15.4 N

Correct

EVALUATE your answer

Part D

For the bar to experience no net force, what must be the tension T_2 in string 2 when the monkey is halfway between the ends of the bar?

Express your answer in newtons using three significant figures.

Hint 1. How to calculate the tension in string 2

Since all the forces acting on the bar are vertical, the bar will experience no net force if $\sum F_y = 0$. By using the value for T_1 calculated in Part C, this equilibrium condition will allow you to write an equation that can be solved for T_2 .

Alternatively, you can calculate the tension in string 2 by applying again the equilibrium condition $\sum \tau_z = 0$, but this time choosing a different point about which to compute torques.

ANSWER:

T_2 = 28.8 N

Correct

To evaluate your results from Parts C and D, compare the forces of tension in the strings by considering the tendency of the bar to rotate about its center of mass. The torques exerted by the monkey and the weight of the bar about that point are both zero; therefore, the only contribution to rotation would come from the strings. Since the bar is in equilibrium, the torques due to the forces of tension must balance each other out. Because the lever arm for T_1 is greater than that for T_2, it follows that $T_1 < T_2$. Your results do make sense!

Score Summary:

Your score on this assignment is 103.9%.
You received 31.17 out of a possible total of 30 points.