UNM Physics 262, 25 Sep 2006
Midterm Exam 1: Optics

Name and/or CPS number: Dr. Landahl - Solution Key

Show all your work for full credit. Remember that quantities have units and vectors have components (or magnitude and direction). **ASK** if anything seems unclear.

CALCULATORS AND CELL PHONES ARE PROHIBITED.
USE OF THESE WILL RESULT IN A ZERO FOR THE EXAM.
Keep any factors of π, e, $\sqrt{2}$, etc. in your answers.

You may use a single 8.5" × 11" paper containing notes you have prepared ahead of time to assist you.

Apportion your time sensibly. Spend about 10–12 minutes per problem.

Please put a box around your final answers.

Useful constants:

$c = 3 \times 10^8$ m/s

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

1. Faraday's Law and Maxwell's equations [24 points]

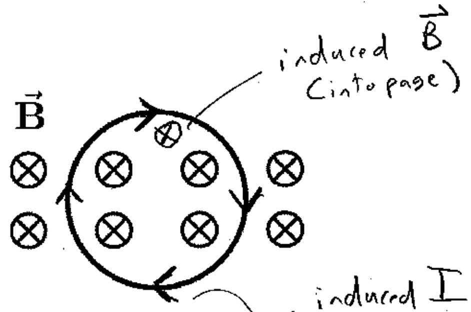


Fig. 1

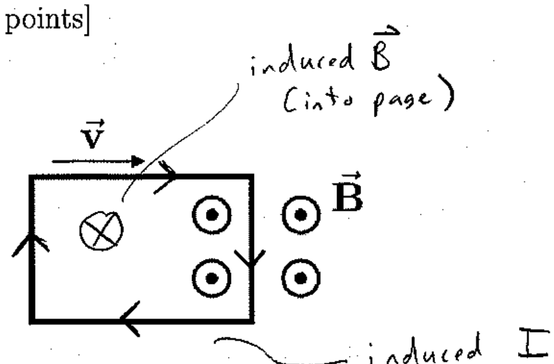


Fig. 2

(*a*) The circular wire loop in Fig. 1 lies in a magnetic field that is directed into the page and decreasing in time.

[3] *i*) Draw and label the direction of the induced magnetic field in Fig. 1. Explain your reasoning. Lenz' Law says that the induced current in the loop generates an induced $\vec{B}$ field to oppose the loss of flux into the page through the loop, so

$\boxed{\text{The induced } \vec{B} \text{ is into the page.}}$

[3] *ii*) Draw and label the direction of the current induced in the loop in Fig. 1. Explain your reasoning. Applying the right hand rule around the induced $\vec{B}$, an $\boxed{\text{induced clockwise current}}$ is the one responsible for the induced $\vec{B}$. (Thumb along $\vec{B}$, fingers curl in I direction.)

(*b*) The rectangular wire loop in Fig. 2 is moving to the right. At the instant depicted, the loop is partly in the magnetic field that is directed out of the page.

[3] *i*) Draw and label the direction of the induced magnetic field in Fig. 2. Explain your reasoning. To oppose the <u>increase</u> in flux out of the page through the loop, Lenz' Law says that the induced current in the loop generates an $\boxed{\text{induced } \vec{B} \text{ field into the page}}$ to oppose the change.

[3] *ii*) Draw and label the direction of the current induced in the loop in Fig. 2. Explain your reasoning. Applying the right hand rule around the induced $\vec{B}$, an $\boxed{\text{induced clockwise current}}$ is the one responsible for the induced $\vec{B}$.

(Thumb along $\vec{B}$, Fingers curl in I direction)

(c) The magnetic field in a cylindrical region of length L and radius R is

$$\vec{\mathbf{B}} = \frac{B_0 R}{r}\,\hat{\mathbf{z}},$$

where $r = \sqrt{x^2+y^2}$ is the distance from the cylinder's axis of symmetry and B_0 is a parameter.

[8] *i*) Write down the integral for the total magnetic energy in this cylindrical region in terms of B_0, r, L, and R. Your integral should be solely in the integration variable r.

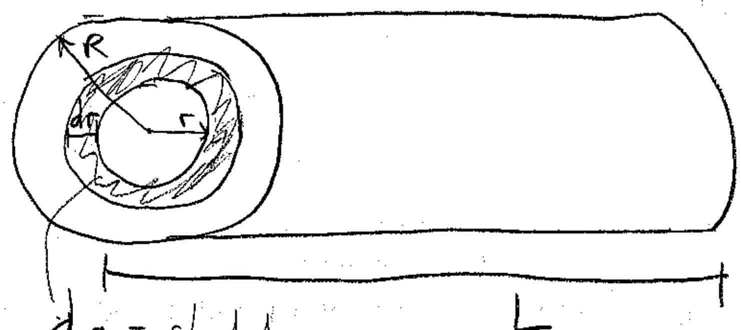
$$U = \frac{1}{2\mu_0}\int |\vec{B}|^2 d^3r \quad \leftarrow \text{[Energy of a } \vec{B} \text{ field]}$$

$$d^3 r = L\, da = L(2\pi r\, dr)$$

(uses $\vec{B}\cdot\vec{B} = |\vec{B}|^2$)

$$U = \frac{1}{2\mu_0}\int_{r=0}^{R} \frac{B_0^2 R^2}{r^2}\, \cancel{\hat{z}\cdot\hat{z}}^{\,1}\, L(2\pi r)\,dr$$

$$= \frac{2\pi L B_0^2 R^2}{2\mu_0}\int_{r=0}^{R}\frac{dr}{r}$$



da = shaded region
$= 2\pi r\, dr$

$$\boxed{U = \frac{\pi L B_0^2 R^2}{\mu_0}\int_{r=0}^{R}\frac{dr}{r}}$$

[4] *ii*) Calculate the integral to obtain an algebraic expression for the total magnetic energy in the cylinder.

Because $\displaystyle\int_{r=0}^{R}\frac{dr}{r} = \ln r\Big|_0^R = \ln R - \ln 0$

$= \ln R - (-\infty)$

$= \infty$,

$$\boxed{U = \infty}$$

2. E & M Optics [22 points]

A flashlight is turned on and has an intensity of 5 $\mathrm{W/m^2}$ at its circular opening of diameter 4 cm.

[4] *a*) What is the average Poynting vector $\langle \vec{\mathrm{S}} \rangle$ at the opening of the flashlight? Explain your reasoning.

The magnitude of the average Poynting vector is equal to the average intensity, $I_{av} = \langle |\vec{S}| \rangle$. The direction of $\vec{S}$ is the direction of energy flux, which is out of the flashlight. Using $I_{av} = 5\,\mathrm{W/m^2}$, we get $\boxed{\vec{S} = 5\,\mathrm{W/m^2} \text{ out of the flashlight}}$

[4] *b*) What is the total power of the light leaving the flashlight? Explain your reasoning.

By definition, $\langle P \rangle = \int \langle \vec{S} \rangle \cdot \hat{n}\, da$. Assuming $\langle \vec{S} \rangle$ is constant over the flashlight's aperture, this yields $|\langle P \rangle| = I \cdot A_{aperture}$.

Hence $|\langle P \rangle| = \pi r^2 I = \pi (2\times10^{-2}\,\mathrm{m})^2 \left(\frac{5\,\mathrm{W}}{\mathrm{m}^2}\right) = 20\pi \times 10^{-4}\,\mathrm{W} = \boxed{2\pi\ \mathrm{mW}}$

The flashlight leaves a circular spot of diameter 4 m on a wall 10 m away. The light reflects elastically off of the wall.

[4] *c*) What is the total power of the light hitting the wall? Explain your reasoning.

Total power is conserved in the flashlight beam, so

$$\boxed{|\langle P \rangle|_{wall} = 2\pi\ \mathrm{mW}}$$

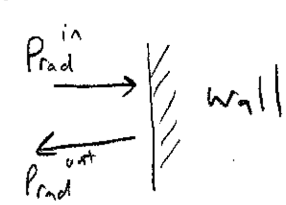
[4] *d*) What is the intensity of the light hitting the wall? Explain your reasoning.

By the same reasoning as in (b), $I_{wall} = \frac{P_{wall}}{A_{wall}} = \frac{P_{wall}}{\pi r_{spot}^2}$,

Hence $I_{wall} = \frac{2\pi\,\mathrm{mW}}{\pi (2\mathrm{m})^2} = \boxed{0.5\ \mathrm{mW/m^2}}$

[6] *e*) What is the radiation pressure exerted by the light on the wall? Explain your reasoning.

Because the reflection is <u>elastic</u>, the light hits the wall and bounces back, causing the net pressure exerted on the wall to be twice that of the light itself.



By definition, $P_{rad} = \frac{I_{wall}}{c}$, so

$P_{wall} = \frac{2 I_{wall}}{c} = \frac{2(0.5\,\mathrm{mW/m^2})}{3\times10^8\,\mathrm{m/s}} = \frac{1}{3}\times10^{-8}\,\frac{\mathrm{mW\cdot s}}{\mathrm{m^3}} = \frac{1}{3}\times10^{-11}\,\frac{\mathrm{W\cdot s}}{\mathrm{m^3}}$

$= \boxed{\frac{10}{3}\ \frac{\mathrm{pW\cdot s}}{\mathrm{m^3}}}$

3. Geometric Optics [28 points]

A mirror shows an upside-down image of an upright object. The height of the image is 3 times the object's height. The distance between the image and the vertex of the mirror is 2 m. The object is to the left of the mirror.

[6] *a*) On which side of the mirror is the image? Explain your reasoning.

Using $m = \frac{h_i}{h_p} = -3$ and $m = \frac{-i}{p}$, we have $i = 3p$.

p on incoming light side of mirror $\Rightarrow p > 0$ by sign rules.

$i = 3p \Rightarrow i > 0$ too. $i > 0 \Rightarrow i$ on outgoing light side by sign rules.

Outgoing light side = incoming light side for a mirror, $\therefore$ $\boxed{\text{Left side of mirror}}$

[6] *b*) Where is the object located? Explain your reasoning.

Obviously the object is on the left side of the mirror, as this is stated in the problem. The question is asking at what distance from the mirror surface is the object. Using $i = 3p$ from ⓐ, we have $p = i/3$. Together with $i = 2m$ ($i > 0$ from ⓐ), this gives $\boxed{p = \frac{2}{3}\text{ m to the left of the mirror}}$

[8] *c*) What is the magnitude of the radius of curvature of the mirror? Explain your reasoning.

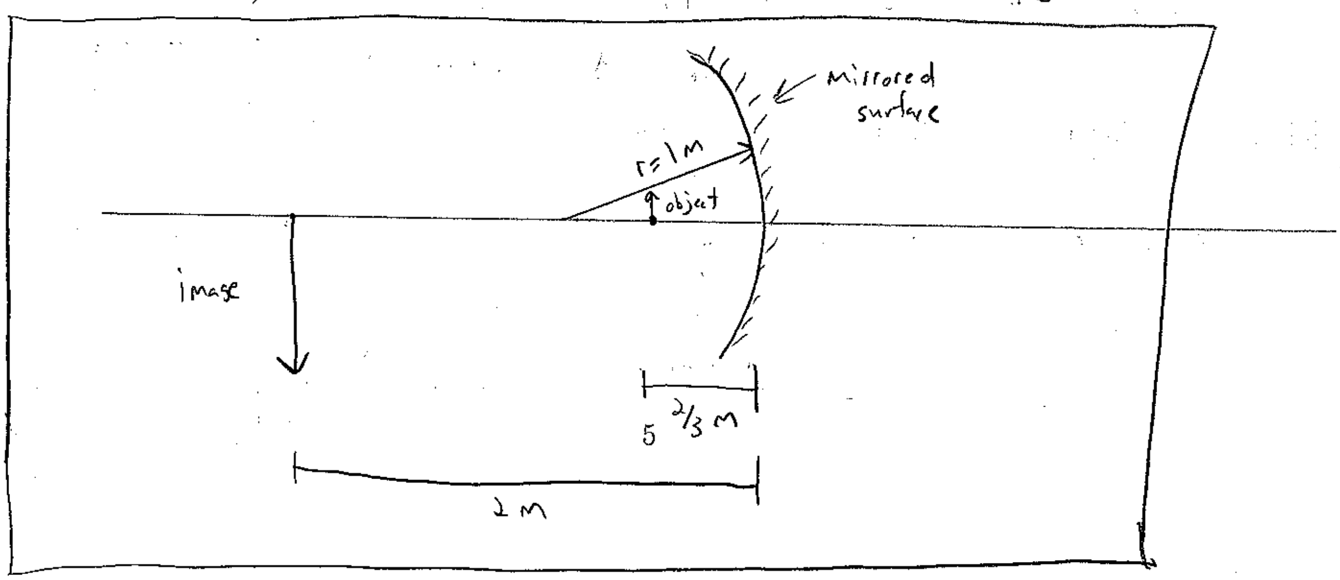
$\frac{1}{p} + \frac{1}{i} = \frac{2}{r}$ [Spherical mirror imaging formula]

$p > 0$, $i > 0$, so this form of the formula is good. Solving for r:

$\frac{3}{2m} + \frac{1}{2m} = \frac{2}{r} \Rightarrow \frac{4}{2m} = \frac{2}{r} \Rightarrow r = 1m \Rightarrow \boxed{|r| = 1\text{ m}}$

[8] *d*) Draw a picture of the situation, indicating where the object and image are located in relation to the mirror, and which way the mirror is curved.

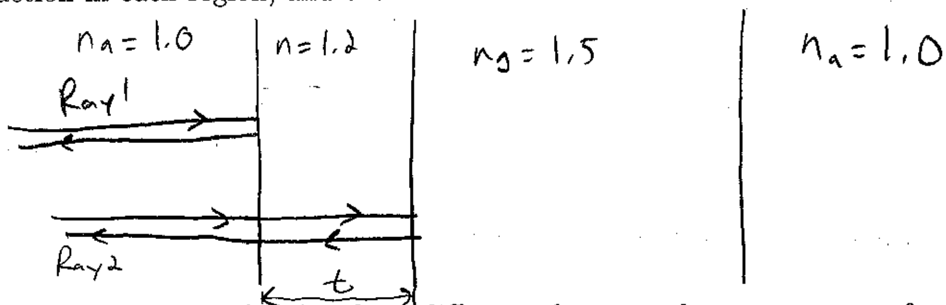
Because $r > 0$, the center of curvature is on the outgoing light side (the left).



4. Wave Optics [26 points]

A planar glass lens (refractive index $n_g = 1.5$) is coated on one side with a thin film of refractive index $n = 1.2$. The thickness of the film is t. This thickness is the minimum possible that will completely reflect UV light of wavelength $\lambda = 240$ nm that strikes it from the air (refractive index $n_a = 1.0$).

[6] *a*) Sketch the situation, labeling rays for the two waves that interfere after reflection, the indices of refraction in each region, and the film's thickness.



[8] *b*) Write down an expression for the phase difference between these two waves after they have reflected and recombine. Express your answer in terms of t, n, and λ. Explain your reasoning.

Ray 1 undergoes a π phase shift upon reflection ($n_a < n$) as does Ray 2 ($n < n_g$). The remaining phase difference is due to Ray 2's extra travel time in the thin film:

$$\Delta\phi = \phi_2 - \phi_1 = \pi + k_n(2t) - \pi = 2nt\left(\frac{2\pi}{\lambda}\right) = \boxed{\frac{4\pi n t}{\lambda}}$$

[6] *c*) What is the thickness of the film? Explain your reasoning.

The film completely reflects UV light, so Ray 1 and Ray 2 interfere constructively. Hence $\Delta\phi = 0 + 2\pi m$, i.e. $\frac{4\pi n t}{\lambda} = 2\pi m$.

Solving for t: $t = \frac{m\lambda}{2n}$. The minimum (nonzero) thickness is at $m=1$:

$$t = \frac{\lambda}{2n} = \frac{(240 \text{ nm})}{2(1.2)} = \boxed{100 \text{ nm}}$$

[6] *d*) What is the amplitude reflection coefficient of light (of any wavelength) entering glass from the film at normal incidence? Explain your reasoning.

By definition, $r = \frac{n_i - n_t}{n_i + n_t}$. For film $\to$ glass, $n_i = n$, $n_t = n_g$.

So $$r = \frac{n - n_g}{n + n_g} = \frac{1.2 - 1.5}{1.2 + 1.5} = \frac{-0.3}{2.7} = \boxed{\frac{-1}{9}}$$

(The minus sign indicates a π phase shift.)