Exam #1

1) Find the angle in degrees between the two vectors: $\vec{A} = 8\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{B} = -1\hat{i} + 3\hat{j}$. **(10pts)**

$$\vec{A} \cdot \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = A_x B_x + A_y B_y + A_z B_z \Rightarrow$$

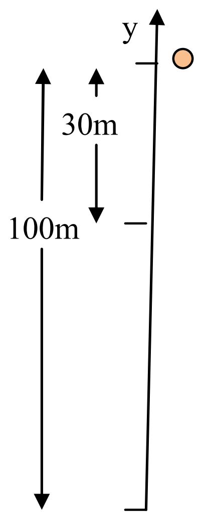
$$\theta = \cos^{-1}\left[\frac{A_x B_x + A_y B_y + A_z B_z}{\left|\vec{A}\right|\left|\vec{B}\right|}\right]$$

$$= \cos^{-1}\left[\frac{8 \cdot (-1) + 2 \cdot 3 + (-4) \cdot (0)}{\sqrt{(8)^2 + (2)^2 + (-4)^2}\sqrt{(-1)^2 + (3)^2 + (0)^2}}\right]$$

$$= \cos^{-1}\left[\frac{-2}{28.98}\right]$$

$$= 94.0^o \text{ or } 1.64 \text{ radians}$$

2) A man jumps from a building 100m high. After 30m of free fall (neglect air resistance during this stage), he deploys a parachute that stops his acceleration so that he continues to the ground at constant velocity. How much time does it take for him to reach the ground? (**20pts**)



This is a two part question. In the first part, the man free-falls starting from rest at y=100m and has a constant acceleration for 30m. He then continues down with zero acceleration until he hits the ground. So, for the first part:
y_o=100m,
y_f=70m,
v_{oy}=0m/s,
v_{fy}=?,
a_y=-9.8m/s^2,
t=?.
We can use $v_f^2=v_o^2+2a(y_f-y_o)$ to get the final velocity => v_f=-24.2m/s and also use $y_f=y_o+v_ot+1/2at^2$ to get the time at the end of the free fall => t=2.47s. Now, we're not done yet, because the man keeps falling at constant velocity! In the next part: y_o=70m,
y_f=0m,
v_{oy}=-24.2m/s,
v_{fy}=-24.2m/s,
a_y=0m/s^2,
t=?.
Now we can use $y_f=y_o+v_ot$ to get the time for this stage, t=2.89s. So, the total time is just t=2.47s+2.89s=5.36s.

3) An arrow is shot horizontally (in the positive x-direction) from the top of a building 60m high. The arrow strikes the ground at a point 100m from the base of the building. What is the initial velocity of the arrow? (**20pts**)

This is a 2-D problem and must be analyzed in each dimension.
In the x- direction,
x_o=0m,
x_f=100m,
v_{ox}=?,
$v_{fx}=v_{ox}$,
a_x=0m/s^2,
t=?.
In the y- direction,
y_o=60m,
y_f=0m,
v_{oy}=0m/s,
v_{fy}=?,
a_y=-9.8m/s^2,
t=?.
To get the initial velocity (all in the x-direction) we need to know the time (since the velocity in the x-direction is constant, and we know the distance), so look in the y-direction and use $y_f=y_o+v_{oy}t+1/2a_yt^2$ => t=3.5s.

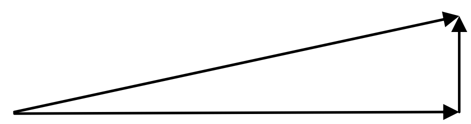
Then in the x-direction, we use $x_f=x_o+v_{ox}t+1/2a_xt^2$, with $a_x=0$ => v_{ox}=28.6m/s in the positive x direction. Remember, velocities have direction!

4) A plane is traveling at a constant speed of 1200km/hr. In order to avoid an incoming missile, the pilot banks hard in a circular path of some radius. If the pilot looses consciousness when the radial acceleration reaches 4g, what is the smallest radius of curvature that the plane can safely make? (**15pts**)

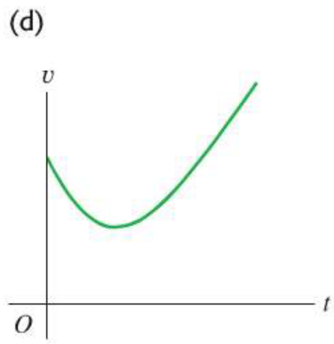
The velocity is 1,200,000m/hr*1hr/60mins*1min/60sec = 333m/s. The centripetal acceleration is given by $a = v^2/r$. When the centripetal acceleration is $4g = 4(9.8m/s^2) = 39.2m/s^2$, then the radius is determined by: $r = v^2/a = (333m/s)^2/(39.2m/s^2) = 2.8km$.

5) A plane is flying with airspeed of 500km/hr in an east direction. There is a wind blowing from the south of 100km/hr relative to the ground. What is the plane's velocity relative to the ground (magnitude and direction)? (**15pts**)

The relative velocity of the plane to the ground is the vector sum of the velocity of the plane to the wind (airspeed) and the velocity of the wind to the ground. The magnitude of these two perpendicular vectors is just the square root of the sum of the squares: sqrt(500^2+100^2) = 510km/hr. The angle it makes is given by atan(100/500) = 11.3^{o} north of east.

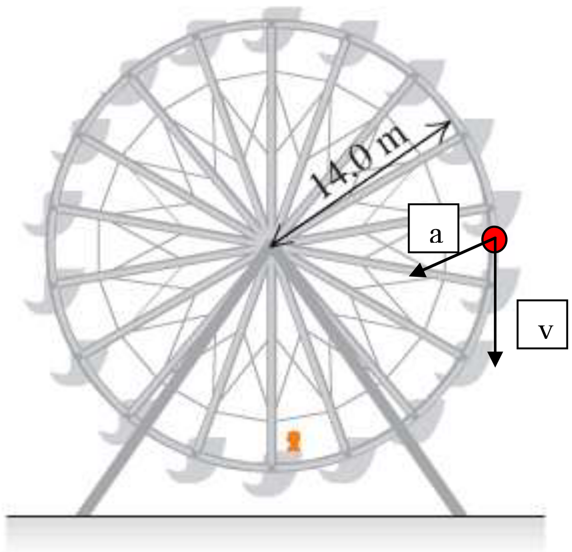


6) A stone is thrown into the air at 45° above the horizontal and feels negligible air resistance. Draw a graph below depicts the stone's ***speed*** as a function of time while it is in the air? (**10pts**)



The speed of the stone is the magnitude of the velocity. It starts with some positive value, decreases but does not go to zero (since the y-component goes to zero at the top of the trajectory, but the x-component remains the same) and then increases (since the y-component increases in magnitude as it falls).

7) A person riding on a Ferris Wheel of radius 14.0 m at some moment has a tangential speed of 1 m/s (rotating clockwise) and is speeding up at a rate of $0.05 m/s^2$. At the middle point on the right, indicated by the circle, draw arrows indicating the direction of his velocity and acceleration and determine the magnitude and exact direction of his acceleration vector. (**10pts**)



Since the wheel is speeding up, the acceleration of the rider is always given by a radial component, $a_R = v^2/r = 0.07 m/s^2$ and a tangential component, $0.05 m/s^2$. So, the magnitude of the acceleration vector is $\text{sqrt}((0.07 m/s^2)^2 + (0.05 m/s^2)^2) = 0.087 m/s^2$ and points to the center of the wheel and down at an angle of $\tan^{-1}(0.05/0.07) = 35^o$ below horizontal.