Capacitance: A Review

Description: Multiple-choice questions on concept of capacitance. Use after Introduction to Capacitance. Some questions are repetitions of those in other problems. This fact may get some students frustrated.

Learning Goal:

To review the meaning of capacitance and ways of changing the capacitance of a parallel-plate capacitor.

Capacitance is one of the central concepts in electrostatics. Understanding its meaning and the difference between its definition and the ways of calculating capacitance can be challenging at first. This tutorial is meant to help you become more comfortable with capacitance. Recall the fundamental formula for capacitance:

$$C = Q/V,$$

where C is the capacitance in farads, Q is the charge stored on the plates in coulombs, and V is the potential difference (or voltage) between the plates. In the following problems it may help to keep in mind that the voltage is related to the strength of the electric field E and the distance between the plates, d, by

$$V = Ed.$$

Part A

What property of objects is best measured by their capacitance?

ANSWER:

- the ability to conduct electric current
- the ability to distort an external electrostatic field
- the ability to store charge

Capacitance is a measure of the ability of a system of two conductors to store electric charge and energy. It is defined as $C = Q/V$. This ratio remains constant as long as the system retains its geometry and the amount of dielectric does not change. Capacitors are special devices designed to combine a large capacitance with a small size. However, any pair of conductors separated by a dielectric (or vacuum) has some capacitance. Even an isolated electrode has a small capacitance. That is, if a charge Q is placed on it, its potential V with respect to ground would change, and the ratio Q/V is its capacitance C.

Part B

Consider an air-filled charged capacitor. How can its capacitance be increased?

Hints (1)

Hint 1. What does capacitance depend on?

Capacitance depends on the inherent properties of the system of conductors, such as its geometry and the presence of dielectric, not on the charge placed on the conductors. Specifically, capacitance depends on the area A of the conducting plates and the distance d between the plates and is given by

$$C = \epsilon_0 \frac{A}{d},$$

where ϵ_0 is a constant called the permittivity of free space.

ANSWER:

- Increase the charge on the capacitor.
- Decrease the charge on the capacitor.
- Increase the spacing between the plates of the capacitor.
- Decrease the spacing between the plates of the capacitor.
- Increase the length of the wires leading to the capacitor plates.

Part C

Consider a charged parallel-plate capacitor. How can its capacitance be halved?

Check all that apply.

ANSWER:

- [ ] Double the charge.
- [ ] Double the plate area.
- [x] Double the plate separation.
- [ ] Halve the charge.
- [x] Halve the plate area.
- [ ] Halve the plate separation.

Part D

Consider a charged parallel-plate capacitor. Which combination of changes would quadruple its capacitance?

ANSWER:

- Double the charge and double the plate area.
- Double the charge and double the plate separation.
- Halve the charge and double the plate separation.
- Halve the charge and double the plate area.
- Halve the plate separation and double the plate area.

Exercise 24.3

Description: A parallel-plate air capacitor with a capacitance of C has a charge of magnitude q on each plate. The plates have a separation of d. (a) What is the potential difference between the plates? (b) What is the area of each plate? (c) What is the...

A parallel-plate air capacitor with a capacitance of 249pF has a charge of magnitude 0.147$\mu\mathrm{C}$ on each plate. The plates have a separation of 0.291mm .

Part A

What is the potential difference between the plates?

ANSWER:

$V = \frac{q}{C} = 590 \ \mathrm{V}$

Part B

What is the area of each plate?

Use 8.854×10^{-12}$\mathrm{F/m}$ for the permittivity of free space.

ANSWER:

Part C

What is the electric field magnitude between the plates?

ANSWER:

$E = \frac{q}{Cd} = 2.03 \times 10^{6} \ \mathrm{V/m}$

Part D

What is the surface-charge density on each plate?

ANSWER:

$\sigma = \frac{\epsilon_0 q}{Cd} = 1.80 \times 10^{-5} \ \mathrm{C/m^2}$

Exercise 24.11

Description: A capacitor is made from two hollow, coaxial, iron cylinders, one inside the other. The inner cylinder is negatively charged and the outer is positively charged; the magnitude of the charge on each is q. The inner cylinder has a radius of r_in, the...

A capacitor is made from two hollow, coaxial, iron cylinders, one inside the other. The inner cylinder is negatively charged and the outer is positively charged; the magnitude of the charge on each is 12.5pC . The inner cylinder has a radius of 0.250mm , the outer one has a radius of 4.00mm , and the length of each cylinder is 25.0cm .

Part A

What is the capacitance?

Use 8.854×10^{-12}$\mathrm{F/m}$ for the permittivity of free space.

ANSWER:

$C = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{r_{out}}{r_{in}}\right)} = 5.02 \times 10^{-12} \ \mathrm{F}$

Part B

What applied potential difference is necessary to produce these charges on the cylinders?

ANSWER:

$V = \frac{q}{2\pi\epsilon_0 L}\ln\left(\frac{r_{out}}{r_{in}}\right) = 2.49 \ \mathrm{V}$

Exercise 24.13

Description: A spherical capacitor contains a charge of Q when connected to a potential difference of V. If its plates are separated by vacuum and the inner radius of the outer shell is r. (a) Calculate the capacitance. (b) Calculate the radius of the inner...

A spherical capacitor contains a charge of 3.50nC when connected to a potential difference of 230V . If its plates are separated by vacuum and the inner radius of the outer shell is 4.20 cm .

Part A

Calculate the capacitance.

ANSWER:

$C = \frac{Q}{V} \cdot 10^{12} = 15.2 \ \mathrm{pF}$

Part B

Calculate the radius of the inner sphere.

ANSWER:

$$r = \frac{\frac{8.99 \cdot 10^9 Q}{V} r}{r + \frac{Q}{V} \cdot 8.99 \cdot 10^9} \cdot 100 = 3.21 \ \mathrm{cm}$$

Part C

Calculate the electric field just outside the surface of the inner sphere.

ANSWER:

$$E = \frac{9.00 \cdot 10^9 Q}{\left(\frac{\frac{8.99 \cdot 10^9 Q}{V} r}{r + \frac{Q}{V} \cdot 8.99 \cdot 10^9} \right)^2} = 3.05 \times 10^4 \ \mathrm{N/C}$$

Exercise 24.15

Description: Electric eels and electric fish generate large potential differences that are used to stun enemies and prey. These potentials are produced by cells that each can generate ## V. We can plausibly model such cells as charged capacitors. (a) How should...

Electric eels and electric fish generate large potential differences that are used to stun enemies and prey. These potentials are produced by cells that each can generate $0.100 \mathrm{V}$. We can plausibly model such cells as charged capacitors.

Part A

How should these cells be connected (in series or in parallel) to produce a total potential of more than $0.100 \mathrm{V}$?

ANSWER:

- (selected) in series
- in parallel

Part B

Using the connection in part (a), how many cells must be connected together to produce the $520 \mathrm{V}$ surge of the electric eel?

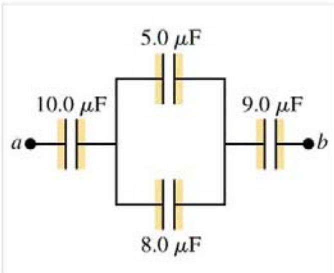
ANSWER:

$$N = \frac{V1}{V} = 5200$$

Exercise 24.22

Description: The Figure below shows a system of four capacitors, where the potential difference across ab is 50.0 V . (a) Find the equivalent capacitance of this system between a and b. (b) How much charge is stored by this combination of capacitors? (c)...

The Figure below shows a system of four capacitors, where the potential difference across ab is $50.0 \mathrm{V}$.



Part A

Find the equivalent capacitance of this system between a and b.

ANSWER:

C = 3.47 μF

Part B

How much charge is stored by this combination of capacitors?

ANSWER:

Q = 174 μC

Part C

How much charge is stored in the 10.0-μF capacitor?

ANSWER:

Q = 174 μC

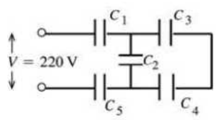
Part D

How much charge is stored in the 9.0-μF capacitor?

ANSWER:

Q = 174 μC

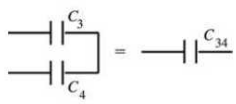
24.57. **(a) IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents.
SET UP: The network is sketched in Figure 24.57a.



$C_1 = C_5 = 8.4\ \mu F$
$C_2 = C_3 = C_4 = 4.2\ \mu F$

Figure 24.57a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.57b):

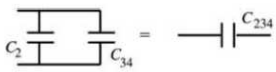


$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}$$
$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3 C_4}$$

Figure 24.57b

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4.2\ \mu F)(4.2\ \mu F)}{4.2\ \mu F + 4.2\ \mu F} = 2.1\ \mu F$$

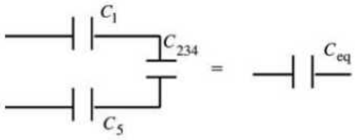
C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.57c):



$C_{234} = C_2 + C_{34}$
$C_{234} = 4.2\ \mu F + 2.1\ \mu F$
$C_{234} = 6.3\ \mu F$

Figure 24.57c

C_1, C_5 and C_{234} are in series and can be replaced by C_{eq} (Figure 24.57d):



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}$$

$$\frac{1}{C_{eq}} = \frac{2}{8.4\ \mu\text{F}} + \frac{1}{6.3\ \mu\text{F}}$$

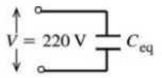
$$C_{eq} = 2.5\ \mu\text{F}$$

Figure 24.57d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.

(b) IDENTIFY and SET UP: In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

EXECUTE: The equivalent circuit is drawn in Figure 24.57e.



$$Q_{eq} = C_{eq}V$$

$$Q_{eq} = (2.5\ \mu\text{F})(220\ \text{V}) = 550\ \mu\text{C}$$

Figure 24.57e

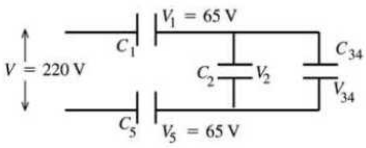
$Q_1 = Q_5 = Q_{234} = 550\ \mu\text{C}$ (capacitors in series have same charge)

$$V_1 = \frac{Q_1}{C_1} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550\ \mu\text{C}}{6.3\ \mu\text{F}} = 87\ \text{V}$$

Now draw the network as in Figure 24.57f.



$V_2 = V_{34} = V_{234} = 87\ \text{V}$

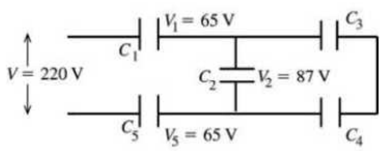
capacitors in parallel have the same potential

Figure 24.57f

$Q_2 = C_2V_2 = (4.2\ \mu\text{F})(87\ \text{V}) = 370\ \mu\text{C}$

$Q_{34} = C_{34}V_{34} = (2.1\ \mu\text{F})(87\ \text{V}) = 180\ \mu\text{C}$

Finally, consider the original circuit (Figure 24.57g).



$Q_3 = Q_4 = Q_{34} = 180\ \mu\text{C}$

capacitors in series have the same charge

$$V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}$$

$$V_4 = \frac{Q_4}{C_4} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}$$

Summary: $Q_1 = 550\ \mu\text{C}$, $V_1 = 65\ \text{V}$

$Q_2 = 370\ \mu\text{C}$, $V_2 = 87\ \text{V}$

$Q_3 = 180\ \mu\text{C}$, $V_3 = 43\ \text{V}$

$Q_4 = 180\ \mu\text{C}$, $V_4 = 43\ \text{V}$

$Q_5 = 550\ \mu\text{C}$, $V_5 = 65\ \text{V}$

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220\ \text{V}$ (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4$

24.65. **(a) IDENTIFY** and **SET UP:** Q is constant. $C = KC_0$; use Eq. (24.1) to relate the dielectric constant K to the ratio of the voltages without and with the dielectric.

EXECUTE: With the dielectric: $V = Q/C = Q/(KC_0)$

without the dielectric: $V_0 = Q/C_0$

$V_0/V = K$, so $K = (45.0\text{ V})/(11.5\text{ V}) = 3.91$

EVALUATE: Our analysis agrees with Eq. (24.13).

(b) IDENTIFY: The capacitor can be treated as equivalent to two capacitors C_1 and C_2 in parallel, one with area $2A/3$ and air between the plates and one with area $A/3$ and dielectric between the plates.

SET UP: The equivalent network is shown in Figure 24.65.

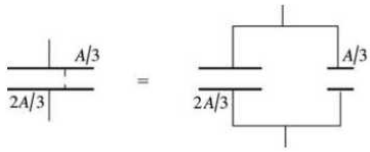


Figure 24.65

EXECUTE: Let $C_0 = \epsilon_0 A/d$ be the capacitance with only air between the plates. $C_1 = KC_0/3$, $C_2 = 2C_0/3$;

$C_{\text{eq}} = C_1 + C_2 = (C_0/3)(K+2)$

$$V = \frac{Q}{C_{\text{eq}}} = \frac{Q}{C_0}\left(\frac{3}{K+2}\right) = V_0\left(\frac{3}{K+2}\right) = (45.0\text{ V})\left(\frac{3}{5.91}\right) = 22.8\text{ V}$$

EVALUATE: The voltage is reduced by the dielectric. The voltage reduction is less when the dielectric doesn't completely fill the volume between the plates.

24.72. **IDENTIFY:** The capacitor is equivalent to two capacitors in parallel, as shown in Figure 24.72.

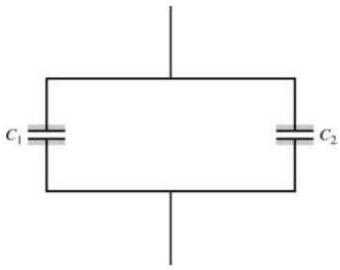


Figure 24.72

SET UP: Each of these two capacitors have plates that are 12.0 cm by 6.0 cm. For a parallel-plate capacitor with dielectric filling the volume between the plates, $C = K\epsilon_0 \dfrac{A}{d}$. For two capacitors in parallel, $C = C_1 + C_2$. The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: **(a)** $C = C_1 + C_2$.

$$C_2 = \epsilon_0 \frac{A}{d} = \frac{(8.854\times10^{-12}\text{ F/m})(0.120\text{ m})(0.060\text{ m})}{4.50\times10^{-3}\text{ m}} = 1.42\times10^{-11}\text{ F}.$$

$C_1 = KC_2 = (3.40)(1.42\times10^{-11}\text{ F}) = 4.83\times10^{-11}\text{ F}$. $C = C_1 + C_2 = 6.25\times10^{-11}\text{ F} = 62.5\text{ pF}$.

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.25\times10^{-11}\text{ F})(18.0\text{ V})^2 = 1.01\times10^{-8}\text{ J}$.

(c) Now $C_1 = C_2$ and $C = 2(1.42\times10^{-11}\text{ F}) = 2.84\times10^{-11}\text{ F}$.

$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.84\times10^{-11}\text{ F})(18.0\text{ V})^2 = 4.60\times10^{-9}\text{ J}$.

EVALUATE: The plexiglass increases the capacitance and that increases the energy stored for the same voltage across the capacitor.

24.75. **IDENTIFY:** The object is equivalent to two identical capacitors in parallel, where each has the same area A, plate separation d and dielectric with dielectric constant K.

SET UP: For each capacitor in the parallel combination, $C = \frac{\epsilon_0 A}{d}$.

EXECUTE: **(a)** The charge distribution on the plates is shown in Figure 24.75.

(b) $C = 2\left(\frac{\epsilon_0 A}{d}\right) = \frac{2(4.2)\epsilon_0(0.120 \text{ m})^2}{4.5\times10^{-4} \text{ m}} = 2.38\times10^{-9} \text{ F}.$

EVALUATE: If two of the plates are separated by both sheets of paper to form a capacitor,

$C = \frac{\epsilon_0 A}{2d} = \frac{2.38\times10^{-9} \text{ F}}{4}$, smaller by a factor of 4 compared to the capacitor in the problem.

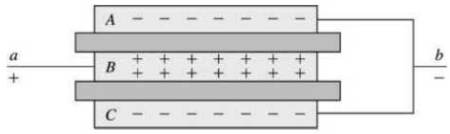


Figure 24.75