$$T\left({}^{o}C\right)=\frac{5}{9}\left(T\left({}^{o}F\right)-32\right)$$
$$T\left(K\right)=T\left({}^{o}C\right)+273.15$$
$$\Delta L=\alpha L_0\Delta T$$
$$\Delta V=\beta V_0\Delta T$$
$$Q=mc\Delta T=nC\Delta T$$
$$Q_{F/V}=\pm mL_{F/V}$$
$$H=\frac{dQ}{dt}=k\frac{A}{L}\left(T_H-T_C\right)$$
$$pV=nRT$$
$$K_{tr}=\frac{3}{2}nRT$$
$C_V=\frac{3}{2}R$ ideal monatomic gas

$C_V=\frac{5}{2}R$ ideal diatomic gas w/o vibration

$$W=\int_{V_1}^{V_2}pdV$$
$$\Delta U=Q-W$$
$$e=\frac{W}{Q_H}=1-\left|\frac{Q_C}{Q_H}\right|$$
$$e_{Carnot}=1-\left|\frac{T_C}{T_H}\right|$$
$$\Delta S=\int_1^2\frac{dQ}{T}$$
$$S=k\ln w$$
$$R=8.314\,J/mol\cdot K$$
$$N_A=6.02\times10^{23}\,molecules/mole$$
1 atm = 101 325 N / ($m^2$)

$1/4\pi\varepsilon_0$ = 8.99 x $10^9$ $Nm^2/C^2$

e = -1.602 x $10^{-19}$ C

$$\vec{F}_E=q\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r}$$
$$\Phi_E=\oint\vec{E}\cdot d\vec{A}=\frac{q_{enc}}{\varepsilon_0}$$
$$V_b-V_a=-\int_a^b\vec{E}\cdot d\vec{l}$$
$$\Delta U=q\Delta V$$
$$\vec{E}=-\left(\hat{i}\frac{\partial V}{\partial x}+\hat{j}\frac{\partial V}{\partial y}+\hat{k}\frac{\partial V}{\partial z}\right)$$
$$Q=CV$$
$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots$ series

$C_{eq}=C_1+C_2+C_3+\ldots$ parallel

$$U=\frac{1}{2}CV^2$$
$$u_E=\frac{1}{2}\varepsilon_0E^2$$
$$E=\frac{E_0}{K}$$
$$I=\frac{dq}{dt}$$
$$\vec{J}=nq\vec{v}_d$$
$$\rho=\frac{E}{J}$$
$$V=IR$$
$$P=VI$$
$R_{eq}=R_1+R_2+R_3+\ldots$ series

$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots$ parallel

$q=C\mathcal{E}\left(1-e^{-t/RC}\right)$ charging

$q=Q_0e^{-t/RC}$ discharging

$$\vec{F}=q\vec{v}\times\vec{B}$$
$$\Phi_B=\int\vec{B}\cdot d\vec{A}$$
$$d\vec{F}=Id\vec{l}\times\vec{B}$$
$$\vec{\tau}=\vec{\mu}\times\vec{B},\quad\vec{\mu}=NI\vec{A}$$
$$U=-\vec{\mu}\cdot\vec{B}$$
$$\vec{B}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$
$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$
$$\oint\vec{B}\cdot d\vec{l}=\mu_0I_{enc}$$

## Calculus

***Derivatives:***

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}\ln ax = \frac{1}{x}$$

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\cos ax = -a\sin ax$$

***Integrals:***

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$$

$$\int \frac{dx}{x} = \ln x$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

# Physics 161-001 Spring 2013 Exam 3

**Name: __________________________________ Box#_____________**

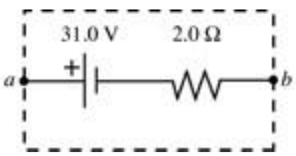
## Multiple Choice (5 points each):

**1) A tube of mercury with resistivity $7.84 \times 10^{-6}$ Ωm has an electric field inside the column of mercury of magnitude 12 V/m that is directed along the length of the tube. How much current is flowing through this tube if its radius is 2.0 mm?**

**A) 4.81 A**
**B) 12.3 A**
**C) 6.00 A**
**D) 19.2 A**
**E) 10.0 A**

$$J = \frac{E}{\rho} = \frac{12V/m}{7.84\times10^{-6}\Omega m} = 1.53\times10^{6}\ A/m^2 \text{ and}$$
$$I = J \cdot A = \left(1.53\times10^{6}\ A/m^2\right)\left(\pi\left(2\times10^{-3}m\right)^2\right) = 19.2A.$$

**2) The emf and the internal resistance of a battery are as shown in the figure. When the terminal voltage Vab is equal to 26.2 V, what is the current through the battery?**




$$V_{ab} = 26.2 = \mathcal{E} - Ir = 31V - I(2\Omega) \Rightarrow$$
$$I = 2.4A$$

**A) 3.1 A**
**B) 15.5 A**
**C) 12.4 A**
**D) 27.9 A**
**E) 2.4 A**

**3) An electron moving in the positive z direction enters a magnetic field. If the electron experiences a magnetic deflection in the negative y direction, the direction of the magnetic field in this region points in the direction of the**

**A) -z axis.**
**B) +x axis.**
**C) +z axis.**
**D) -y axis.**
**E) -x axis.**

$F_B = q\vec{v} \times \vec{B}$. Since the electron is negatively charged, the direction is in the opposite direction given by the right hand rule.

**4) A proton is in a region where a uniform electric field of $3 \times 10^4$ V/m is perpendicular to a uniform magnetic field of 1.8 T. If its acceleration is zero, then what is its speed?**

**A) 0m/s**
**B) $6.3 \times 10^4$m/s**
**C) $2.8 \times 10^4$m/s**
**D) $4.0 \times 10^5$m/s**
**E) $1.6 \times 10^4$m/s**

$F_E = qE$ and $F_B = q\vec{v} \times \vec{B}$. For the acceleration to be zero, the net force must be zero, so $F_{Net} = qE - qvB = 0$, so

$$v = \frac{E}{B} = \frac{3 \times 10^4 \, V/m}{1.8T} = 1.6 \times 10^4 \, m/s$$

**5) A wire carries a 1.0-A current along the +x-axis through a magnetic field $\vec{B} = \left(4.0\hat{i} - 2.0\hat{j}\right)T$. If the wire experiences a force of 5.0 N in the positive z-direction as a result, how long is the wire?**

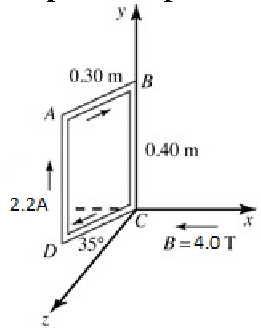
**A) 1.1 m**
**B) 1.8 m**
**C) 1.0 m**
**D) 2.0 m**
**E) 2.5 m**

$d\vec{F} = Id\vec{l} \times \vec{B}$. Only the y component of the B-field creates a force on the wire, and since the force is in the negative z direction for the whole wire, the total force is just:

$$\vec{F} = -ILB_y\hat{k} = -(1A)L(-2.0T)\hat{k} = 5.0N\hat{k} \Rightarrow$$

$$L = 2.5m$$

**6) A rigid rectangular loop, which measures 0.30 m by 0.40 m, carries a current of 2.2 A, as shown in the figure. A uniform external magnetic field of magnitude 4.0 T in the negative x-direction is present. Segment *CD* is in the *xz*-plane and forms a 35° angle with the z-axis, as shown. Find the magnitude of the external torque needed to keep the loop in static equilibrium.**




$$|\vec{\mu}| = IA = (2.2A)(0.30m \cdot 0.40m) = 0.26Am^2 \ .$$
$$\vec{\tau} = \vec{\mu} \times \vec{B} = (0.26Am^2)(4.0T)\sin(35°)\,\hat{j} = (0.6N \cdot m)\,\hat{j}$$

**A) 1.1 N · m**
**B) 1.4 N · m**
**C) 0.6 N · m**
**D) 0.3 N · m**
**E) 0.9 N · m**

**Written Problems (25 points each) SHOW ALL WORK! No credit for answers without work!**

**1) (25 points) The current in a long straight wire of radius R running along the x-axis, has a current density of $\vec{J} = \left(1\frac{A}{m^4}\right) r^2 \hat{i}$. Calculate and plot the magnetic field strength and direction as a function of the distance r from the axis of the wire, below the x-axis.**


Since it is a very long wire, we can use Ampere's Law to calculate the B-field both inside and outside the wire: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$. From cylindrical symmetry, B is uniform on a circle of some radius r centered at the axis of the wire, can come out from the integral, and the left-hand side of the equation is just $B2\pi r$ for any r. The right-hand side depends on how much of the current the loop contains –

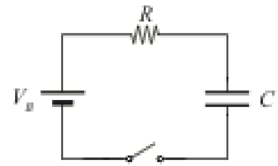
$$I_{enc} = \int_0^r \vec{J} \cdot d\vec{A} = \int_0^r r^2 (2\pi r dr) = 2\pi \frac{r^4}{4} \quad r < R$$
$$= 2\pi \frac{R^4}{4} \quad r \geq R$$

, so then the B-field under the axis points into the page and has a value:

$$B = \frac{\mu_0 r^3}{4} \quad r < R$$
$$= \frac{R^4}{4r} \quad r \geq R$$

in units of Tesla.

**2) (25 points) The circuit at right has a battery with voltage $V_B$, a resistor $R$, and a capacitor $C$, which is initially uncharged. At time $t = 0$, the switch is closed.**



Circuit I

i. *Before **the switch is closed, rank the voltages across the battery, resistor, capacitor, and switch. If any of the voltages are zero, state so explicitly. Explain.***

From the loop rule, $V_B - V_R - V_C - V_S = 0$. Also, since the capacitor is uncharged, $V_C = 0$, and since there is no current, $V_R = 0$, so
$V_B = V_S > V_C = V_R = 0$

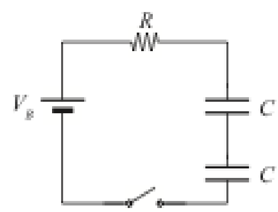
ii. ***In terms of the given quantities, what is the current through the switch and the voltage across the switch*** **just after** ***it is closed?***

At the moment the switch is closed, since there is initially no charge on the capacitor, current $I = V_B/R$ through the loop, and the voltage across the switch is zero.

iii. ***A short time after the switch is closed, is the current through*** **R greater than, less than,** ***or*** **equal to** ***the current through the switch? Explain.***

The current through the switch is ALWAYS equal to the current through the resistor.

**The capacitor is discharged, and another identical capacitor is added as shown.**



Circuit II

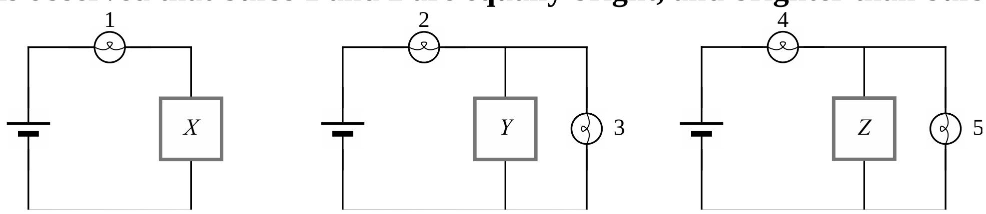
iv. ***Just after the switch is closed in circuit II, will the current through*** **R** ***be*** **greater than, less than,** ***or*** **equal to** ***the current through*** **R** ***just after the switch was closed in circuit I? Explain.***

Since both capacitors are uncharged, the potential drops across them are zero, so the current through the circuit is the same as before an equal to $V_B/R$.

v. ***How will the final charge on the first capacitor in circuit II compare to the final charge on the capacitor in circuit I? Explain.***

The potential across each capacitor (in the final state) is just one half of the battery voltage, and so the charge on each capacitor is one half of the charge in circuit 1.

**3) (20 points) In the circuits below, bulbs 1-5 are identical, and the batteries are identical and ideal. Boxes *X, Y,* and *Z* contain unknown arrangements of bulbs. It is observed that bulbs 1 and 2 are equally bright, and brighter than bulb 4.**




A) **Rank the potential difference across box *X*, box *Y*, and box *Z* according to absolute value from greatest to smallest. Explain your reasoning.**

Each bulb has identical resistance, so the brightness is an indication of the voltage across them ($P = V^2/R$). So, the potential drop across 1 and 2 are the same and more than the potential drop across 4. Then, the potential drop across X and Y must be the same, and less than that of Z.

B) **Is the brightness of bulb 3 *greater than, less than,* or *equal to* the brightness of bulb 5? Explain.**

Since the potential drop across 3 is the same as X, and the potential drop across 5 is the same as Z, then from above, bulb 5 is brighter.

C) **Suppose box *Z* is disconnected from the circuit. Determine whether the brightness of each of the bulbs below will *increase, decrease,* or *remain the same.* Explain your reasoning in each case.**

**bulb 4**

When Z is disconnected, the equivalent resistance of the circuit will go up, and so the total current through the circuit will go down (or remain the same if Z had infinite resistance originally). So, bulb 4 will get less bright.

**bulb 5**

When Z is disconnected, all the current that goes through bulb 4 will now go through bulb 5 (instead of some going through Z), and so bulb 5 will get brighter. Another way to look at it is if the current drops through bulb 4, so does the potential drop across bulb 4. Then the potential drop across bulb 5 must go up, and so its brightness.