Phys. 161 HW 4 (Ch. 20)

52.

a) [12 pt.]

***a* → *b*.**

$$W_{a\to b} = \int_{V_a}^{V_b} P\,\mathrm{d}V = \int_{V_a}^{V_b} \frac{n\,R\,T_1}{V}\,\mathrm{d}V = n\,R\,T_1 \ln\frac{V_b}{V_a} = n\,R\,T_1 \ln\frac{1}{r} = -n\,R\,T_1 \ln r.$$

Since $a\to b$ is an isothermal process and $U = n\,C_V T$ for an ideal gas,

$$\Delta_{a\to b} U = U_b - U_a = 0 = Q_{a\to b} - W_{a\to b}.$$

So $Q_{a\to b} = -n\,R\,T_1 \ln r$.

***b* → *c*.** $W_{b\to c} = 0$ because the volume doesn't change.

$$\Delta_{b\to c} U = n\,C_V\,\Delta_{b\to c} T = n\,C_V\,(T_2 - T_1) = Q_{b\to c} - W_{b\to c} = Q_{b\to c}.$$

***c* → *d*.**

$$W_{c\to d} = \int_{V_c}^{V_d} P\,\mathrm{d}V = \int_{V_c}^{V_d} \frac{n\,R\,T_2}{V}\,\mathrm{d}V = n\,R\,T_2 \ln\frac{V_d}{V_c} = n\,R\,T_2 \ln r.$$

Since $c\to d$ is an isothermal process and $U = n\,C_V T$ for an ideal gas,

$$\Delta_{c\to d} U = 0 = Q_{c\to d} - W_{c\to d},$$

so $Q_{c\to d} = n\,R\,T_2 \ln r$.

***d* → *a*.** $W_{d\to a} = 0$ because the volume doesn't change.

$$\Delta_{d\to a} U = n\,C_V\,\Delta_{d\to a} T = n\,C_V\,(T_1 - T_2) = Q_{d\to a} - W_{d\to a} = Q_{d\to a}.$$

b) [6 pt.] $Q_{d\to a} = n\,C_V\,(T_1 - T_2) = -n\,C_V\,(T_2 - T_1) = -Q_{b\to c}$, so $Q_{b\to c} + Q_{d\to a} = 0$, that is, there is no net heat flow needed to make these two steps happen.

c) [7 pt.]

$$W_{\text{cycle}} = W_{a\to b} + W_{b\to c} + W_{c\to c} + W_{d\to a} = -n\,R\,T_1 \ln r + n\,R\,T_2 \ln r = n\,R\,(T_2 - T_1) \ln r.$$

$$e = \frac{W_{\text{cycle}}}{Q_{c\to d}} = \frac{n\,R\,(T_2 - T_1)\ln r}{n\,R\,T_2 \ln r} = 1 - \frac{T_1}{T_2}.$$

This is the same efficiency as a Carnot engine operating between these reservoirs. This does not violate the second law of thermodynamics *if* the internal heat transfers between the working fluid and the regenerator are done reversibly, which requires that the regenerator increases gradually in temperature while losing heat and decreases gradually in temperature while receiving heat.

If there is no regeneration, then the efficiency is

$$\frac{W_{\text{cycle}}}{Q_H}=\frac{W_{\text{cycle}}}{Q_{b\to c}+Q_{c\to d}}=\frac{n\,R\,(T_2-T_1)\ln r}{n\,C_V\,(T_2-T_1)+n\,R\,T_2\ln r}<\frac{n\,R\,(T_2-T_1)\ln r}{n\,R\,T_2\ln r}=1-\frac{T_1}{T_2}.$$

54.

a) [5 pt.] The efficiency $e={}^{W}\!/_{Q_H}=\frac{\mathrm{d}W/\mathrm{d}t}{\mathrm{d}Q_H/\mathrm{d}t}=0.4$, power output is ${}^{\mathrm{d}W}\!/_{\mathrm{d}t}=1000\,\mathrm{MW}$, so heat input rate is ${}^{\mathrm{d}Q_H}\!/_{\mathrm{d}t}=\frac{\mathrm{d}W/\mathrm{d}t}{e}={}^{1000\,\mathrm{MW}}\!/_{0.4}=2500\,\mathrm{MW}$.

b) [5 pt.]

$$\begin{aligned}\frac{{}^{\mathrm{d}Q_H}\!/_{\mathrm{d}t}}{2.65\times10^7\,\mathrm{J/kg}}(1\,\mathrm{day}) &= \frac{2500\,\mathrm{MW}}{2.65\times10^7\,\mathrm{J/kg}}\left(\frac{10^6\,\mathrm{W}}{1\,\mathrm{MW}}\right)(1\,\mathrm{day})\left(\frac{24\,\mathrm{hr}}{1\,\mathrm{day}}\right)\left(\frac{60\,\mathrm{min}}{1\,\mathrm{hr}}\right)\left(\frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right)\\ &= 8.15\times10^6\,\mathrm{kg}\end{aligned}$$

c) [5 pt.] $|Q_C|=Q_H-W$, ${}^{|\mathrm{d}Q_C|}\!/_{\mathrm{d}t}={}^{\mathrm{d}Q_H}\!/_{\mathrm{d}t}-{}^{\mathrm{d}W}\!/_{\mathrm{d}t}=2500\,\mathrm{MW}-1000\,\mathrm{MW}=1500\,\mathrm{MW}$.

d) [5 pt.] $|Q_C|=m_{\text{water}}\,c_{\text{water}}\,\Delta T_{\text{water}}=\rho_{\text{water}}\,V_{\text{water}}\,c_{\text{water}}\,\Delta T_{\text{water}}$, where $c_{\text{water}}=4186\,\mathrm{J/kg\cdot K}$ is the specific heat of water and $\rho_{\text{water}}=1000\,\mathrm{kg/m^3}$ is the density;

$$\frac{|\mathrm{d}Q_C|}{\mathrm{d}t}=\rho_{\text{water}}\frac{\mathrm{d}V_{\text{water}}}{\mathrm{d}t}c_{\text{water}}\,\Delta T_{\text{water}};$$

$$\begin{aligned}\frac{\mathrm{d}V_{\text{water}}}{\mathrm{d}t} &= \frac{|\mathrm{d}Q_C|/\mathrm{d}t}{c_{\text{water}}\,\rho_{\text{water}}\,\Delta T_{\text{water}}}\\ &= \frac{1500\,\mathrm{MW}}{(4186\,\mathrm{J/kg\cdot K})\,(1000\,\mathrm{kg/m^3})\,(18.5^\circ\mathrm{C}-18.0^\circ\mathrm{C})}=717.7\,\mathrm{m^3/s}.\end{aligned}$$

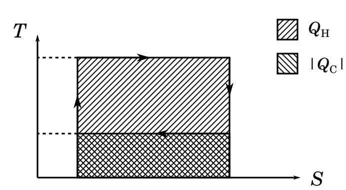
e) [5 pt.]

$$\begin{aligned}\Delta S_{\text{water}} &= \int\frac{\mathrm{d}|Q_C|}{T_{\text{water}}}=\int_{T_{\text{i,water}}}^{T_{\text{f,water}}}\frac{\rho_{\text{water}}\,V_{\text{water}}\,c_{\text{water}}}{T_{\text{water}}}\,\mathrm{d}T_{\text{water}}\\ &= \rho_{\text{water}}\,V_{\text{water}}\,c_{\text{water}}\ln\frac{T_{\text{f,water}}}{T_{\text{i,water}}}.\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}S_{\text{water}}}{\mathrm{d}t} &= \rho_{\text{water}}\frac{\mathrm{d}V_{\text{water}}}{\mathrm{d}t}c_{\text{water}}\ln\frac{T_{\text{f,water}}}{T_{\text{i,water}}}\\ &= (1000\,\mathrm{kg/m^3})(717.7\,\mathrm{m^3/s})\,(4186\,\mathrm{J/kg\cdot K})\ln\frac{291.65\,\mathrm{K}}{291.15\,\mathrm{K}}\\ &= 5.15\times10^6\,\frac{\mathrm{J/K}}{\mathrm{s}}.\end{aligned}$$

60.

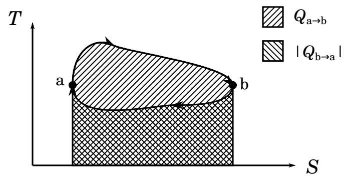
a) [7 pt.] The adiabatic steps have constant entropy, since $\mathrm{d}S={}^{\mathrm{d}Q}\!/_{T}$ for a reversible process. The isothermal steps, obviously, have constant temperature. Therefore, the cycle is a rectangle on a TS-diagram:



b) [6 pt.] For a reversible process, $\mathrm{d}Q = T\,\mathrm{d}S$, so

$$\int_a^b T\,\mathrm{d}S = Q_{a\to b}$$

$$\oint T\,\mathrm{d}S = Q_{a\to b} - |Q_{b\to a}| = Q_{\text{net}}$$



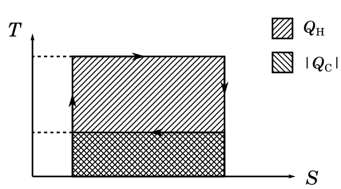
c) [6 pt.] Efficiency $e = {}^{(Q_H - |Q_C|)}\!/_{Q_H}$ for any heat engine operating between two heat reservoirs. $Q_H - |Q_C|$ is the area enclosed by the cyclic path, and Q_H is all the area under the top edge of the cyclic path. So we can visualize the efficiency as the ratio of the area enclosed by the path to the area under and inside the path. For the Carnot cycle, this is clearly

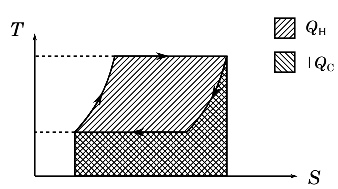
$$e = \frac{(T_H - T_C)\,(S_2 - S_1)}{T_H\,(S_2 - S_1)} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H},$$

which is known to be correct.

d) [6 pt.] For the constant-volume stages of the Stirling cycle, $\Delta S = \int \,{}^{\mathrm{d}Q}\!/_{T} = \int_{T_1}^{T} C_V\,{}^{\mathrm{d}T}\!/_{T} = C_V \ln\left({}^{T}\!/_{T_1}\right)$, creating curved paths as shown at right. It is visually clear that the ratio of the enclosed area to the total area under the path is less for the Stirling cycle than for the Carnot cycle.

If we consider a Stirling engine using regeneration, *and* the internal heat transfers are reversible, then the entropy of the whole engine is constant during the isochoric steps, and the whole engine has a single temperature at each time; therefore, the *TS* diagram for the engine would look like the one for the Carnot cycle, and it would have the same theoretical efficiency.





62. $250\,\mathrm{cm}^3$ water $= 250\,\mathrm{g} = 0.25\,\mathrm{kg}$, $20^\circ\mathrm{C} = 293.15\,\mathrm{K}$, $78^\circ\mathrm{C} = 351.15\,\mathrm{K}$, $120^\circ\mathrm{C} = 393.15\,\mathrm{K}$

a) [7 pt.] $Q = C_P\,\Delta T_{\text{water}} = m\,c\,\Delta T_{\text{water}}$, where $c = 4190\,\mathrm{J/kg\cdot K}$ is the specific heat of water. Since entropy depends only on state, the entropy change is the same as if we heated the water reversibly by gradually increasing the external temperature; therefore

$$\Delta S_{\text{water}} = \int_{T_{1,\text{water}}}^{T_{2,\text{water}}} \frac{m\,c\,\mathrm{d}T}{T} = m\,c \ln\frac{T_2}{T_1} = (0.25\,\mathrm{kg})\,(4190\,\mathrm{J/kg\cdot K}) \ln\frac{351.15\,\mathrm{K}}{293.15\,\mathrm{K}} = \boxed{189.1\,\mathrm{J/K.}}$$

b) [7 pt.]

$$\begin{aligned}\Delta S_{\text{heater}} &= \frac{-Q}{T_{\text{heater}}} = \frac{-m\,c\,\Delta T_{\text{water}}}{T_{\text{heater}}} \\ &= \frac{-(0.25\,\text{kg})\,(4190\,\text{J/kg}\cdot\text{K})\,(78°\,\text{C} - 20°\,\text{C})}{393.15\,\text{K}} = \boxed{-154.5\,\text{J/K.}}\end{aligned}$$

c) [4 pt.] $\Delta S = \Delta S_{\text{water}} + \Delta S_{\text{heater}} = 189.1\,\text{J/K} - 154.5\,\text{J/K} = \boxed{35\,\text{J/K.}}$

d) [7 pt.] Although in practice the heating element gets energy from the external world, we can imagine that it is just a large heat reservoir; therefore, we can regard the system of the water and the heating element as being isolated from the rest of the universe. Since $\Delta S > 0$ for this isolated system, the process is irreversible. This could be expected, because heat flowing across a finite temperature difference is an irreversible process.