ECE 345: Introduction to Control Systems
Problem Set #2

Dr. Oishi

Due Thursday, September 13, 2012 at the start of class

This homework is open note and open book. You are welcome to discuss the problems with other students, but your solutions *must be written independently*. Copying will not be tolerated.

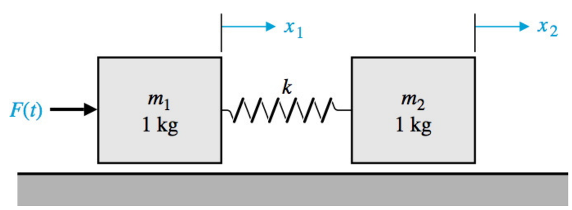
1. Consider the transfer function of a laser printer positioning system, in which the input $R(s)$ represents the desired position of the laser beam, and the output $Y(s)$ represents the actual position of the laser beam.

$$\frac{Y(s)}{R(s)} = \frac{5(s+4)}{s^2+10s+50}$$

   (a) Use the final value theorem to determine the steady-state value in response to a unit step input.

   (b) What is the impulse response of the system (in the time-domain)?

2. Consider the dynamical system shown below (from Problem Set #1),



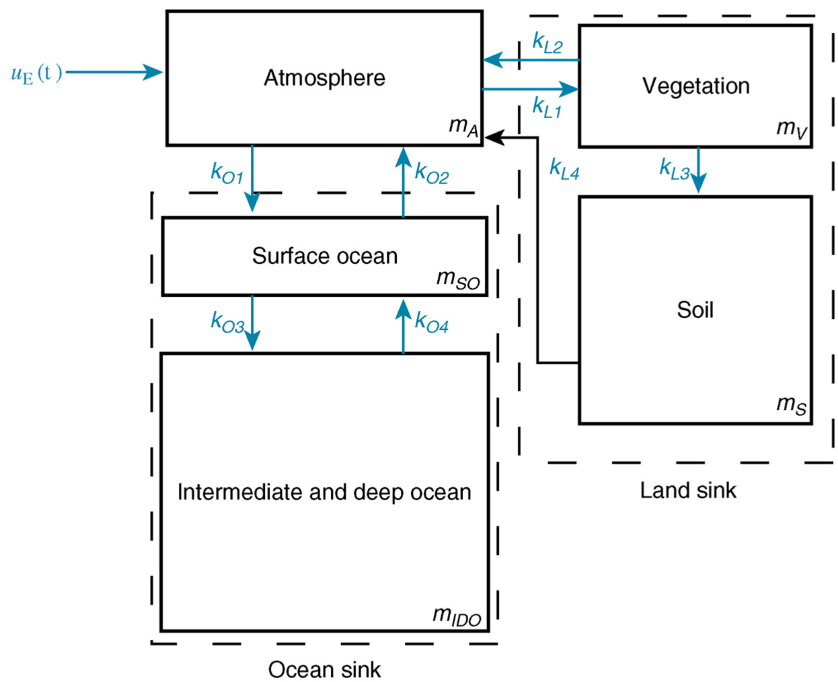
with equations of motion

$$\begin{aligned} m\ddot{x}_1 &= f(t) - k(x_1 - x_2) \\ m\ddot{x}_2 &= +k(x_1 - x_2) \end{aligned} \tag{1}$$

   (a) Reduce the equations of motion to a set of first-order differential equations.

   (b) Rewrite the system in standard state-space form, identifying the matrices A, B, C, and D with input $f(t)$, state $z(t) = [x_1(t), v_1(t), x_2(t), v_2(t)]$ and output $x_2(t)$, as above, with $v_1(t) = \dot{x}_1(t)$ and $v_2(t) = \dot{x}_2(t)$.

3. Consider the schematic description below of the global carbon cycle (Li, 2009). In the figure, $m_A(t)$ represents the amount of carbon in gigatons (GtC) present in the atmosphere of the earth, $m_V(t)$ the amount of vegetation in the soil, $m_{SO}(t)$ the amount in surface ocean, and $m_{IDO}(t)$ the amount in the intermediate and deep-ocean reservoirs. Let $u_E(t)$ represent the human generated CO_2 emissions (GtC/yr).



From the figure, atmospheric mass balance in the atmosphere can be expressed as

$$\dot{m}_A(t) = u_E(t) - (k_{O1} + k_{L1})m_A(t) + k_{L2}m_V(t) + k_{O2}m_{SO}(t) + k_{L4}m_S(t) \tag{2}$$

where the k's are constants that represent exchange coefficients (yr^{-1}).

(a) Write the remaining mass balances. That is, supply equations for $\dot{m}_{SO}(t)$, $\dot{m}_{IDO}(t)$, $\dot{m}_V(t)$, and $\dot{m}_S(t)$

(b) Express the system in state-space form with state $x(t) = \begin{bmatrix} m_A(t) \\ m_{SO}(t) \\ m_{IDO}(t) \\ m_V(t) \\ m_S(t) \end{bmatrix}$, input $u(t) = u_E(t)$, and output $y(t) = m_A(t)$. Identify the A, B, C, and D matrices.

4. State-space representations are, in general, not unique. A single system can be represented in several possible ways. Consider the following three systems:

$$\text{System A:} \quad \begin{cases} \dot{x} &= -5x + 3u \\ y &= 7x \end{cases}$$

$$\text{System B:} \quad \begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -5 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 7 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

$$\text{System C:} \quad \begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -5 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 7 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

(a) Create systems A, B, and C in Matlab via `ss`.

(b) Use Matlab's `tf` on systems A and B to show that these two systems can be represented by the *same* transfer function.

(c) Use `tfdata` to find the coefficients of the numerator and denominator of the transfer function for system C. *Hint: Use the* `'v'` *option to simplify the output since this is a single-input, single-output system.*

(d) Now construct system C as a transfer function with the numerator and denominator from the previous step, via `tf`. You should find the same transfer function as in part (b). Describe the difference in how `tf` is used in part (b) and in part (d). *Hint: What does* `tf` *take as an input?*