2.64 Let $+x$ be the direction that the train is traveling

$t = 0\,s$ to $t = 14.0\,s$: $x_1 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(1.60\,m/s^2)(14.0\,s)^2 = 157\,m$

at $t = 14.0\,s$: $v_x = v_{0x} + a_x t = (1.60\,m/s^2)(14.0\,s) = 22.4\,m/s$

In the next $70.0\,s$: $a_x = 0$ $x_2 = v_{0x}\cdot t = (22.4\,m/s)(70.0\,s) = 1568\,m$

During the train is slowing down: $v_{0x} = 22.4\,m/s$ $a_x = -3.50\,m/s$ $v_x = 0$

$$v_x^2 = v_{0x}^2 + 2a_x x_3$$

Solve for x_3, $x_3 = 72\,m$

Total distance: $x_1 + x_2 + x_3 = 157\,m + 1568\,m + 72\,m = 1800\,m$

2.74

Let $+x$ be to the right. Let $x = 0$ at the initial location of car 1, so $x_{01} = 0$ and $x_{02} = D$. The cars collide when $x_1 = x_2$. $v_{0x1} = 0$, $a_{x1} = a_x$, $v_{0x2} = 2v_0$ and $a_{x2} = 0$.

(a) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_1 = \frac{1}{2}a_x t^2$ and $x_2 = D - v_0 t$. $x_1 = x_2$ gives $\frac{1}{2}a_x t^2 = D - v_0 t$.

$\frac{1}{2}a_x t^2 + v_0 t - D = 0$. The quadratic formula gives $t = \frac{1}{a_x}\left(-v_0 \pm \sqrt{v_0^2 + 2a_x D}\right)$. Only the positive root is physical, so $t = \frac{1}{a_x}\left(-v_0 + \sqrt{v_0^2 + 2a_x D}\right)$.

(b) $v_1 = a_x t = \sqrt{v_0^2 + 2a_x D} - v_0$

(c) The x-t and v_x-t graphs for the two cars are sketched in Figure 2.74.

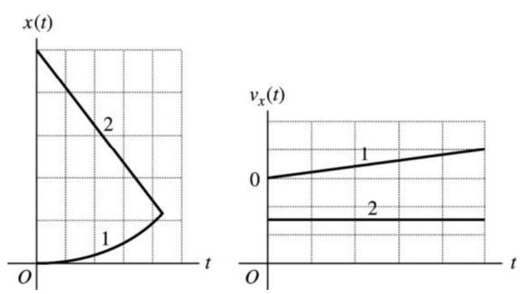

Figure 2.74

Homework 2 solution (2.86, 2.94)

2.86 (a) In the first stage, it is given that the acceleration is constant, so we can use the formula $x = x_0 + v_0 t + \frac{1}{2}at^2$ with $x_0 = 0$ (setting the initial position to $x = 0$) and $v_0 = 0$ (the rocket is at rest at the beginning).

$$x_{\text{1st stage}} = \frac{1}{2}(3.50\text{m/s}^2)(25.0\text{s})^2 = 1093.75\text{m}$$

The final velocity of the first stage is $v = at = (3.50\text{m/s}^2)(25.0\text{s}) = 87.5\text{m/s}$. The second stage lasts for 10 seconds with the rocket's final velocity 132.5m/s. So the difference in the final and the initial velocity is $132.5 - 87.5 = 45.0\text{m/s}$. Assuming that the acceleration is constant, this gives $a = \frac{\Delta v}{\Delta t} = \frac{45.0\text{m/s}}{10\text{s}} = 4.5\text{m/s}^2$. (For those of you who are put in the acceleration of -9.8m/s^2 here, that makes the rocket slows down, not speed up, contrary to what happened in the problem.) Then we may use the same formula to find out that

$$x_{\text{2nd stage}} = (87.5\text{m/s})(10.0\text{s}) + \frac{1}{2}(4.50\text{m/s}^2)(10.0\text{s})^2 = 1100\text{m}$$

(A fine point here: why do we want to assume that the acceleration is constant in the second stage? The problem only says that the second stage boosts the rocket's velocity up to some final velocity. If we only assume that the acceleration is a nondecreasing function of time, it implies that the second derivative of the velocity function is nonnegative. That is, the graph of the velocity as a function of time curves upwards. But it does not tell us how fast the graph curves upwards. So the graph is not unique, and the area under the graph, which is the total distance covered, is not unique too. That is, there are many answers to this problem if the acceleration is an unknown function of time.)

After the fuel runs out, the rocket slows down until it stops and reaches the maximum height. From now on, the acceleration is -9.8m/s^2.

How long does it take for the rocket to stop? $v = v_0 + at \implies t = \frac{v - v_0}{a} = \frac{0\text{m/s} - 132.5\text{m/s}}{-9.8\text{m/s}^2} \approx 13.52\text{s}$. Thus,

$$x_{\text{no fuel}} = (132.5\text{m/s})(13.52\text{s}) - \frac{1}{2}(9.8\text{m/s}^2)(13.52\text{s})^2 \approx 895.7\text{m}$$

Therefore,

$$x_{\text{total}} = 1093.75\text{m} + 1100\text{m} + 895.7\text{m} = \boxed{3089.45\text{m}}$$

(b) How long does it take for the rocket to fall back to the launch pad? $x = \frac{1}{2}at^2 \implies t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2\cdot(3089.45\text{m})}{-9.8\text{m/s}^2}} \approx 25.11\text{s}$. So the total time is about $13.52\text{s} + 25.11\text{s} \approx \boxed{38.6\text{s}}$.

(c) $v^2 = v_0^2 + 2a(x - x_0) \implies v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{0 + 2(-9.8\text{m/s}^2)(-3089.45\text{m})} \approx \boxed{246\text{m/s}}$

2.94 (a) Basically, we want to find out when the distance travelled by the two balls add up to H.

$$\begin{aligned} H &= x_{\text{1st ball}} + x_{\text{2nd ball}} \\ &= [v_0 t - \frac{1}{2}gt^2] + [\frac{1}{2}gt^2] = v_0 t \\ \therefore t &= \boxed{\frac{H}{v_0}} \end{aligned}$$

(b) We want to find H such that $t = \frac{H}{v_0}$ from the previous part is the time when the first ball stops. $v = v_0 + at \implies t = \frac{v - v_0}{a} = \frac{0 - v_0}{-g} = \frac{v_0}{g}$. Thus,

$$\begin{aligned} \frac{v_0}{g} &= \frac{H}{v_0} \\ \therefore H &= \boxed{\frac{v_0^2}{g}} \end{aligned}$$