# Lecture 36 (Magnetic Field Energy & R-L Circuits)

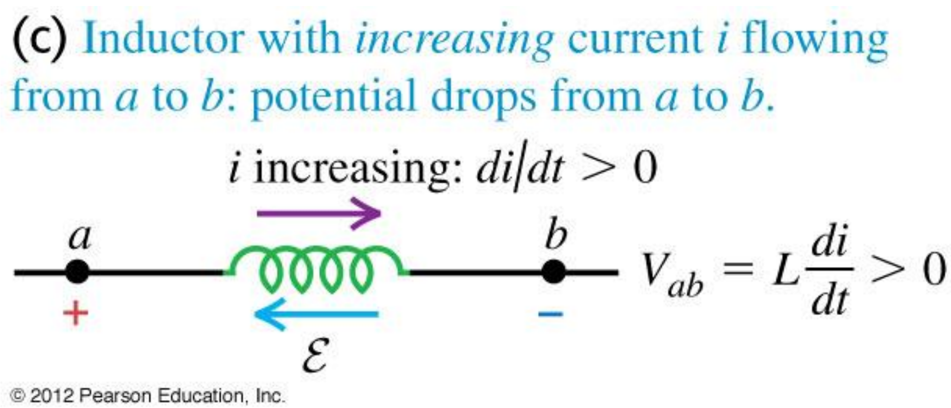
Physics 161-01 Spring 2012

Douglas Fields

# Energy Stored in Inductors

- So, we can now recognize that there is a potential across an inductor with an increasing current that opposes that increase.
- What is the energy needed to move a charge across that potential difference?

$$U = qV_{ab} = qL\frac{di}{dt}$$



# Energy Stored in Inductors

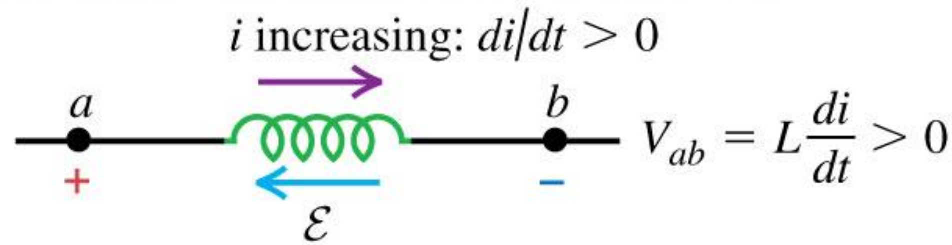
- Now, what is the instantaneous power needed to put a current through the inductor?

$$U_q = qL\frac{di}{dt} \Rightarrow$$

$$P_i = \frac{dU}{dt} = \frac{dq}{dt}L\frac{di}{dt} + qL\cancel{\frac{d^2 i}{dt^2}} \Rightarrow$$

$$P = iL\frac{di}{dt}$$

**(c)** Inductor with *increasing* current $i$ flowing from $a$ to $b$: potential drops from $a$ to $b$.



# Energy Stored in Inductors

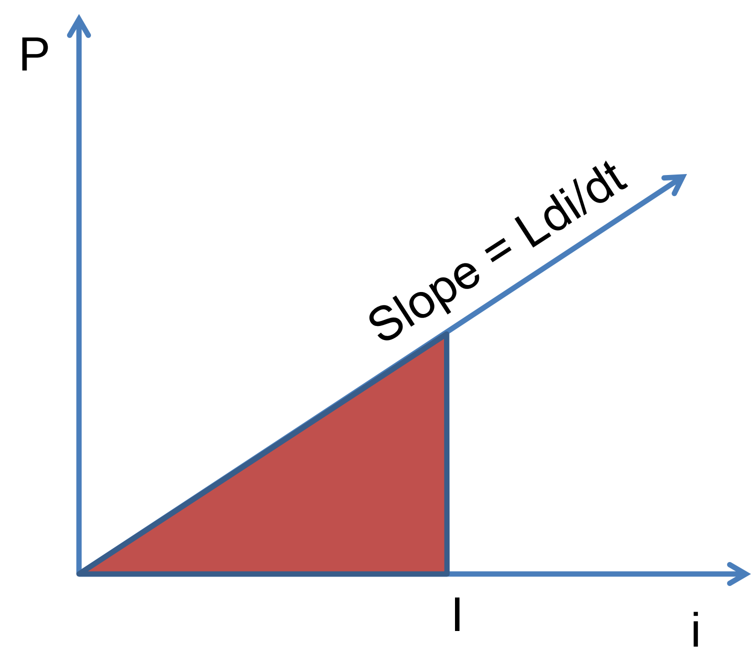
- Now, we are ready to ask, how much energy total does it take to get a current I going through an inductor starting from no current?

$$P = iL\frac{di}{dt} \Rightarrow$$

$$Pdt = iLdi \Rightarrow$$

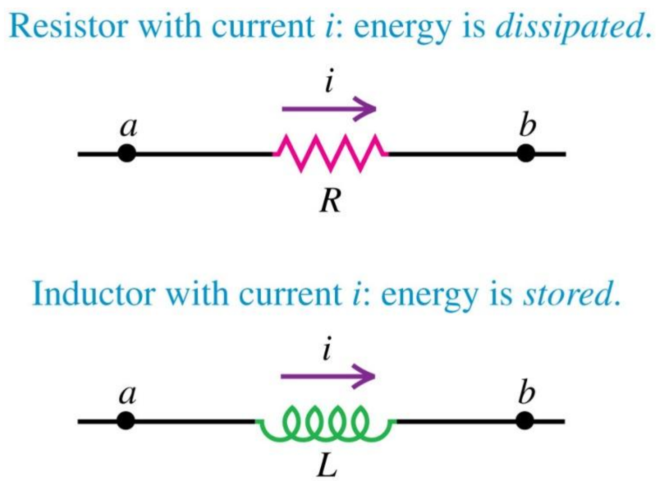
$$\int Pdt = U_{Total} = L\int_0^I idi = \frac{1}{2}LI^2$$

$$U_{Total} = \frac{1}{2}LI^2$$



# Energy Stored in Inductors

- Now, it is important to understand this energy, and especially "where it goes".
- Remember that the power, P=IV that it takes to put a current through a resistor is lost to thermal energy.
- But also remember that the energy that it takes to charge a capacitor is NOT lost, it is stored in the electric field between the capacitor plates and can be recovered (by discharging the capacitor).
- In this sense, an inductor is like a capacitor, the energy is not lost, but stored.
- Where is it stored?
- In the magnetic field within the inductor!



# CPS 36-1

A steady current flows through an inductor. If the current is doubled while the inductance remains constant, the amount of energy stored in the inductor

A. increases by a factor of $\sqrt{2}$ .

B. increases by a factor of 2.

C. increases by a factor of 4.

D. increases by a factor that depends on the geometry of the inductor.

E. none of the above

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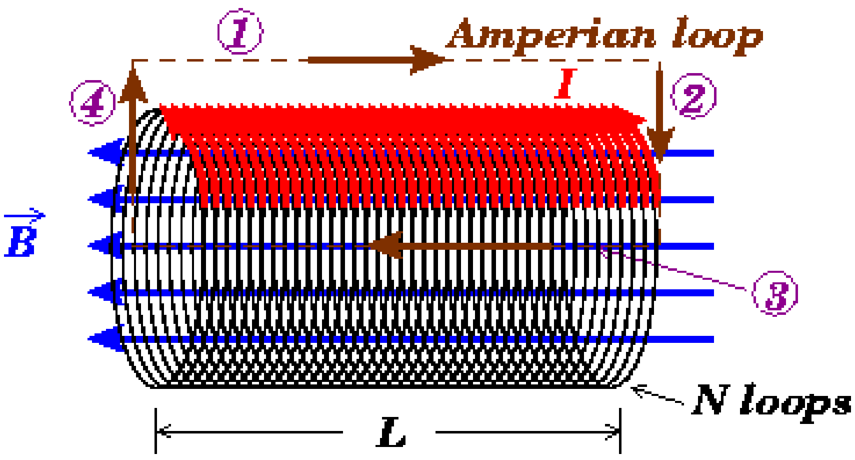
# Energy Stored in Inductors

- Let's use a simple inductor (solenoid) to relate the energy stored within it to the magnetic field strength within it.
- (Remember that we did a similar thing with a parallel plate capacitor to relate the energy stored within it to the electric field strength between the plates.)

$$L = N\frac{\Phi_B}{i} = N\frac{BA}{i},$$

$$B = \mu_0 \frac{N}{L} i \Rightarrow$$

$$L = N\frac{\mu_0 \frac{N}{L} iA}{i} = \frac{\mu_0 N^2 A}{L}$$



# Energy Stored in Inductors

- Then,

$$L = N\frac{\Phi_B}{i} = N\frac{BA}{i}, \quad B = \mu_0\frac{N}{L}i \Rightarrow$$

$$L = \frac{\mu_0 N^2 A}{L}$$

$$U_{Total} = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{L}I^2$$

- Now, as before, we can ask what is the energy per volume, or energy density?

$$u_B = \frac{U_{Total}}{V} = \frac{1}{2}\frac{\mu_0 N^2 A}{L^2 A}I^2 = \frac{1}{2}\frac{\mu_0 N^2}{L^2}I^2 = \frac{1}{2\mu_0}B^2$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

# Energy Stored in Inductors

- Again, as before, the energy per unit volume (energy density) that we calculate here is a general result.

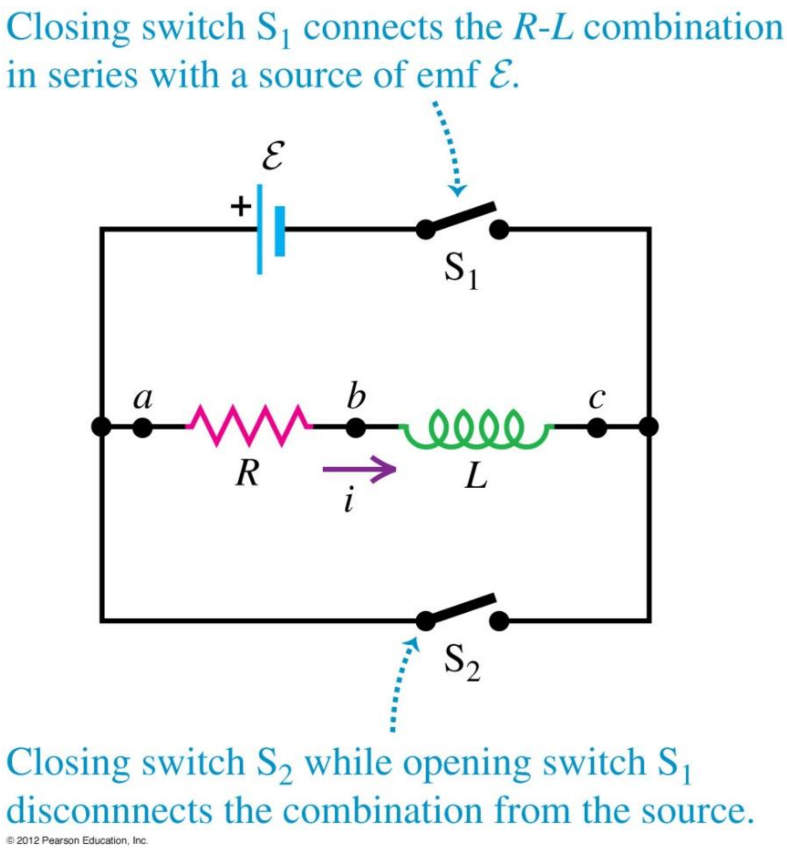
$$u_B = \frac{1}{2\mu_0} B^2 \qquad\qquad u_E = \frac{1}{2}\varepsilon_0 E^2$$

- These really represent the energy stored in an electric or magnetic field in vacuum.
- In a dielectric or magnetic medium, these are altered by:

$$u_B = \frac{1}{2K_M \mu_0} B^2 = \frac{1}{2\mu} B^2 \qquad u_E = \frac{1}{2} K\varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2$$

# RL Circuits

- Now, let’s make the first step into understanding inductors in circuits.
- In the book, you will see the figure to the right.
- With switch S2 open and switch S1 closed, we have a resistor, inductor and an EMF source in series.
- In a bit, we will open switch S1 and close switch S2, to see what happens…



# RL Circuits

- It would be good to go through this in the book

**Problem-Solving Strategy 30.1** **Inductors in Circuits**

**IDENTIFY** *the relevant concepts:* An inductor is just another circuit element, like a source of emf, a resistor, or a capacitor. One key difference is that when an inductor is included in a circuit, all the voltages, currents, and capacitor charges are in general functions of time, not constants as they have been in most of our previous circuit analysis. But Kirchhoff's rules (see Section 26.2) are still valid. When the voltages and currents vary with time, Kirchhoff's rules hold at each instant of time.

**SET UP** *the problem* using the following steps:

1. Follow the procedure described in Problem-Solving Strategy 26.2 (Section 26.2). Draw a circuit diagram and label all quantities, known and unknown. Apply the junction rule immediately so as to express the currents in terms of as few quantities as possible.
2. Determine which quantities are the target variables.

**EXECUTE** *the solution* as follows:

1. As in Problem-Solving Strategy 26.2, apply Kirchhoff's loop rule to each loop in the circuit.
2. Review the sign rules given in Problem-Solving Strategy 26.2. To get the correct sign for the potential difference between the terminals of an inductor, apply Lenz's law and the sign rule described in Section 30.2 in connection with Eq. (30.7) and Fig. 30.6. In Kirchhoff's loop rule, when we go through an inductor in the *same* direction as the assumed current, we encounter a voltage *drop* equal to $L\,di/dt$, so the corresponding term in the loop equation is $-L\,di/dt$. When we go through an inductor in the *opposite* direction from the assumed current, the potential difference is reversed and the term to use in the loop equation is $+L\,di/dt$.
3. Solve for the target variables.

**EVALUATE** *your answer:* Check whether your answer is consistent with the behavior of inductors. By Lenz's law, if the current through an inductor is changing, your result should indicate that the potential difference across the inductor opposes the change.

# RL Circuits

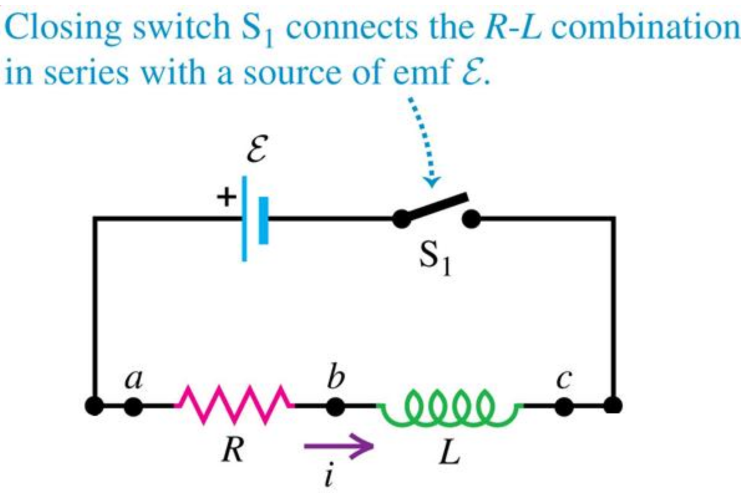
- Even though when we close switch S1, the current through the circuit will vary with time, Kirchhoff's loop rule still applies at any instant:

$$\sum \Delta V = \mathcal{E} - iR - L\frac{di}{dt} = 0$$

- Now, we can solve this equation for di/dt:

$$\mathcal{E} - iR - L\frac{di}{dt} = 0 \Rightarrow$$

$$\frac{di}{dt} = \frac{\mathcal{E} - iR}{L}$$



# RL Circuits

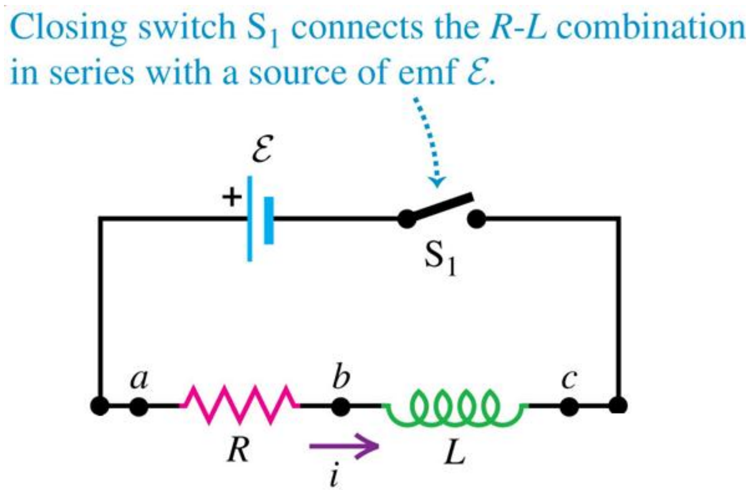
- And now, solve through integration:

$$\frac{di}{dt} = \frac{\mathcal{E} - iR}{L} \Rightarrow$$

$$\frac{di}{\mathcal{E} - iR} = \frac{1}{L} dt \Rightarrow$$

$$\frac{di}{i - \frac{\mathcal{E}}{R}} = -\frac{R}{L} dt \Rightarrow$$

$$\int_0^i \frac{di'}{\left(i' - \frac{\mathcal{E}}{R}\right)} = -\int_0^t \frac{R}{L} dt' \Rightarrow$$




$$\ln\left(\frac{i - \frac{\mathcal{E}}{R}}{-\frac{\mathcal{E}}{R}}\right) = -\frac{R}{L} t \Rightarrow$$

$$i - \frac{\mathcal{E}}{R} = -\frac{\mathcal{E}}{R} e^{-\frac{R}{L}t} \Rightarrow \quad i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

# RL Circuits

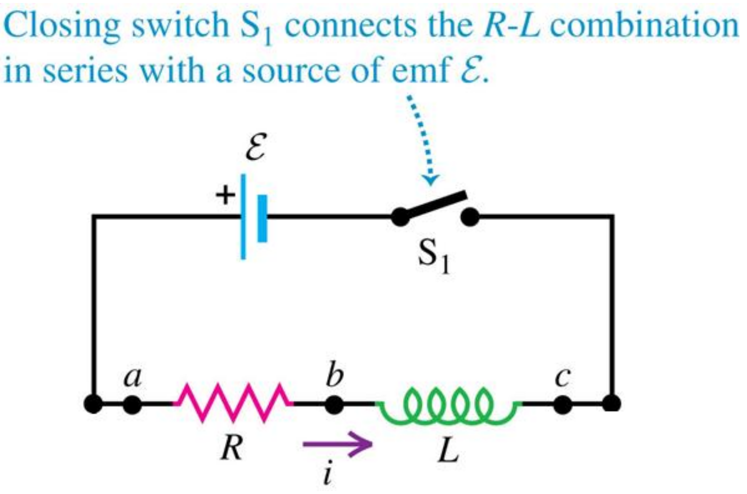
- And now, let's look at what happens as a function of time:

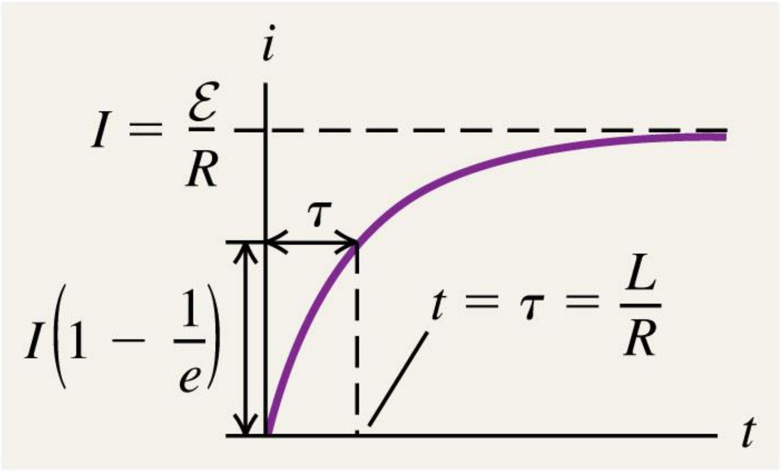
$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\tau = \frac{L}{R}$$

$$i(0) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{0}{\tau}}\right) = 0$$

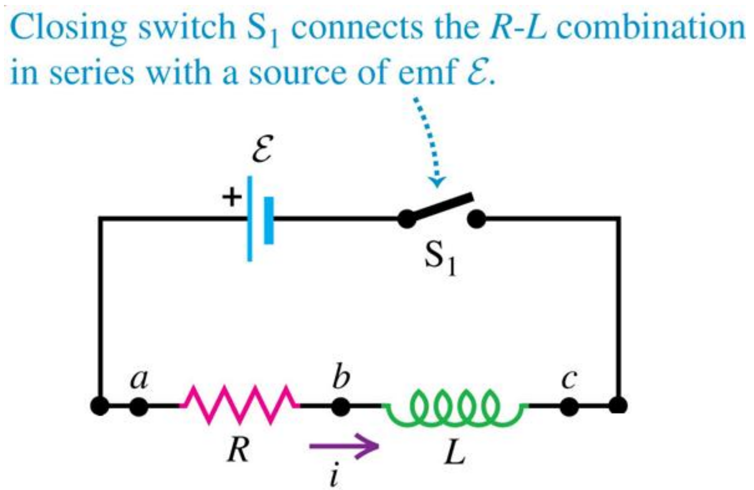
$$i(\infty) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{\infty}{\tau}}\right) = \frac{\mathcal{E}}{R}$$



# CPS 36-2

An inductance *L* and a resistance *R* are connected to a source of emf as shown. When switch $S_1$ is closed, a current begins to flow. The *final* value of the current is




A. directly proportional to *RL.*

B. directly proportional to *R*/*L.*

C. directly proportional to *L*/*R.*

D. directly proportional to 1/(*RL*).

E. independent of *L.*

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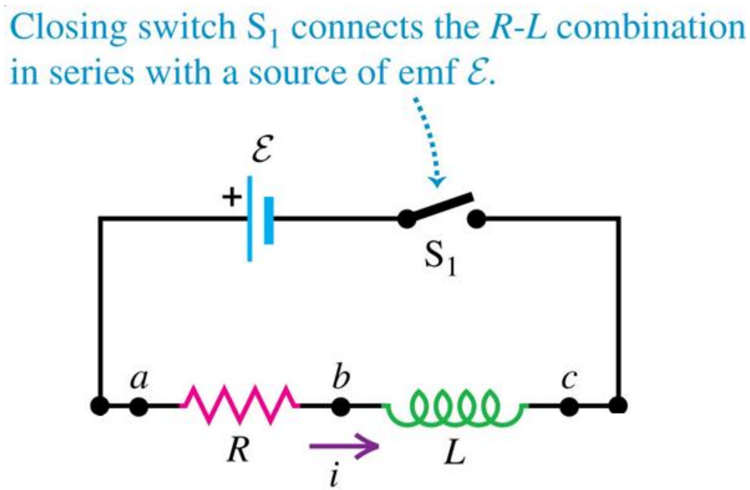


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# Power in RL Circuits

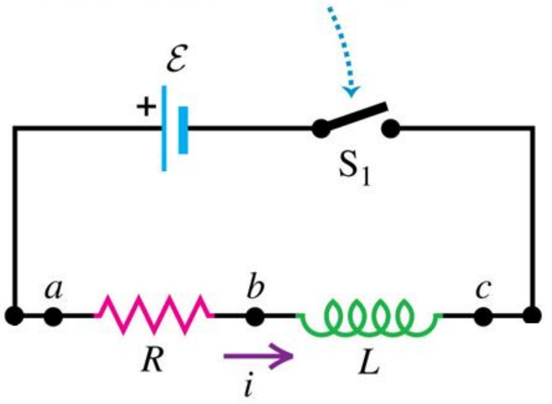
- Let's take our starting equation from Kirchhoff's loop rule and multiply by the current:

$$\mathcal{E} - iR - L\frac{di}{dt} = 0 \Rightarrow$$

$$\mathcal{E}i - i^2R - Li\frac{di}{dt} = 0$$

- So, we can see that the power supplied by the battery is split between the thermal energy dissipated by the resistor, and the power going into stored energy in the inductor.

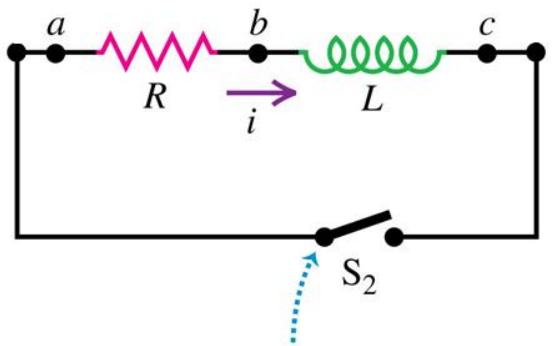
Closing switch $S_1$ connects the $R$-$L$ combination in series with a source of emf $\mathcal{E}$.




$$P = iL\frac{di}{dt}$$

# RL Circuits

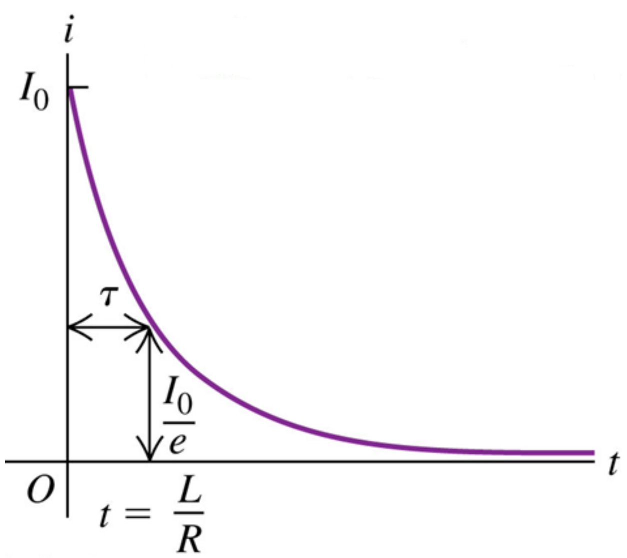
- What about an inductor with stored energy connected in series with a resistor?




Closing switch $S_2$ while opening switch $S_1$ disconnects the combination from the source.

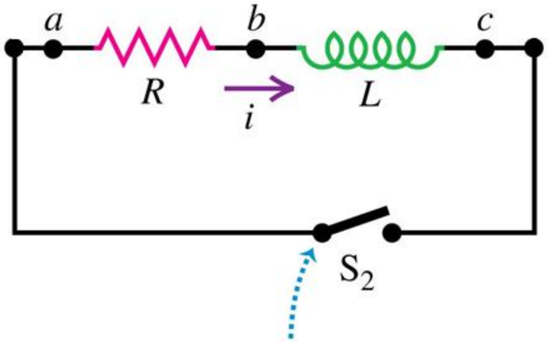
$$i = I_0\left(e^{-\frac{R}{L}t}\right) = I_0\left(e^{-\frac{t}{\tau}}\right)$$

$$\tau = \frac{L}{R}$$



# CPS 36-3

An inductance *L* and a resistance *R* are connected to a source of emf as shown. Initially switch $S_1$ is closed, switch $S_2$ is open, and current flows through *L* and *R.* When $S_2$ is closed, the *rate* at which this current decreases




Closing switch $S_2$ while opening switch $S_1$ disconnnects the combination from the source.

A. remains constant.

B. increases with time.

C. decreases with time.

D. not enough information given to decide

# CPS 36-3

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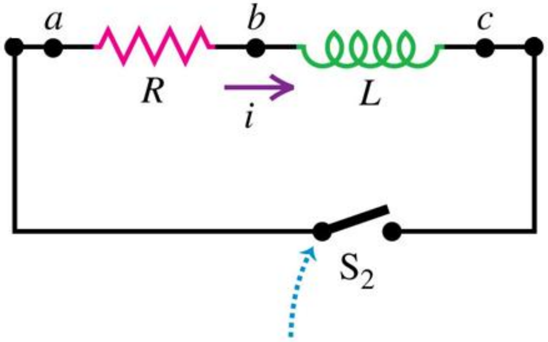

Closing switch $S_2$ while opening switch $S_1$ disconnnects the combination from the source.

A. remains constant.

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D. not enough information given to decide