#34 Gravitation, Potential Energy and Gauss's Law Pre-class

Due: 11:00am on Monday, November 12, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

± Escape Velocity

Learning Goal:

To introduce you to the concept of escape velocity for a rocket.

The escape velocity is defined to be the minimum speed with which an object of mass m must move to escape from the gravitational attraction of a much larger body, such as a planet of total mass M. The escape velocity is a function of the distance of the object from the center of the planet R, but unless otherwise specified this distance is taken to be the radius of the planet because it addresses the question "How fast does my rocket have to go to escape from the surface of the planet?"

Part A

The key to making a concise mathematical definition of escape velocity is to consider the energy. If an object is launched at its escape velocity, what is the total mechanical energy $E_{\rm total}$ of the object at a very large (i.e., infinite) distance from the planet? Follow the usual convention and take the gravitational potential energy to be zero at very large distances.

Hint 1. Consider various cases

If the object is given some speed *less* than its escape velocity, it will eventually return to the planet. If it is given some speed *greater* than its escape velocity, then even at very large distances, it will still have some velocity directed away from the planet. If the object is given its escape velocity *exactly*, its velocity at a very large distance from the planet will tend to zero (so, its kinetic energy will go to zero).

ANSWER:

$E_{\rm total}$ = 0

Correct

Consider the motion of an object between a point close to the planet and a point very very far from the planet. Indicate whether the following statements are true or false.

Part B

Angular momentum about the center of the planet is conserved.

ANSWER:

- (selected) true
- false

Correct

Part C

Total mechanical energy is conserved.

ANSWER:

- (selected) true
- false

Correct

Part D

Kinetic energy is conserved.

ANSWER:

- true
- (selected) false

Correct

Part E

The angular momentum about the center of the planet and the total mechanical energy will be conserved regardless of whether the object moves from small R to large R (like a rocket being launched) or from large R to small R (like a comet approaching the earth).

ANSWER:

- true
- false

Correct

What if the object is not moving directly away from or toward the planet but instead is moving at an angle θ from the normal? In this case, it will have a tangential velocity $v_{\tan} = v \sin\theta$ and angular momentum $L = mv_{\tan}R$. Since angular momentum is conserved, $v_{\tan} = L/(mR)$ for any R, so $v_{\tan}$ will go to 0 as R goes to infinity. This means that angular momentum can be conserved without adding any kinetic energy at $R = \infty$. The important aspect for determining the escape velocity will therefore be the conservation of total mechanical energy.

Part F

Find the escape velocity v_e for an object of mass m that is initially at a distance R from the center of a planet of mass M. Assume that $R \geq R_{\mathrm{planet}}$, the radius of the planet, and ignore air resistance.

Express the escape velocity in terms of R, M, m, and G, the universal gravitational constant.

Hint 1. Determine the gravitational potential energy

Find U, the gravitational potential energy of the object at a distance R from the center of the planet, with the gravitational potential energy taken to be zero when the object is infinitely far away from the planet.

Express your answer in terms of R, M, m, and G, the universal gravitational constant.

ANSWER:

U = $\frac{-GMm}{R}$

Hint 2. Determine the kinetic energy

Suppose that the object is given its escape velocity v_e. Find the initial kinetic energy K of the object when it is at its initial distance R from the center of the planet.

Express your answer in terms of v_e and other given quantities.

ANSWER:

K = $\frac{1}{2}m{v_e}^2$

Hint 3. Putting it all together

You can assume that no nonconservative forces act on the object as it moves through space. Hence, using conservation of mechanical energy, you can equate the object's initial mechanical energy (when it is at a distance R from the planet) to the object's mechanical energy at very large distances from the planet. This should allow you to solve for v_e.

ANSWER:

v_e = $\sqrt{\frac{2GM}{R}}$

Correct

Does it surprise you that the escape velocity does not depend on the mass of the object? Even more surprising is that it does not depend on the direction (as long as the trajectory misses the planet). Any angular momentum given at radius R can be conserved with a tangential velocity that vanishes as R goes to infinity, so the angle at which the object is launched does not have a significant effect on the energy at large R.

Orbital Speed of a Satellite

Part A

Two identical satellites orbit the earth in stable orbits. One satellite orbits with a speed v at a distance r from the center of the earth. The second satellite travels at a speed that is less than v. At what distance from the center of the earth does the second satellite orbit?

Hint 1. How to approach the problem

Satellites orbiting any large object, like the earth or the moon, are in free fall. The only force that acts on a satellite that maintains a stable orbit is the gravitational attraction of the object on the satellite. This force is given by Newton's law of gravitation

$$F = G\left(\frac{Mm}{r^2}\right),$$

where G is the gravitational constant, M is the mass of the large object (e.g., in this case the earth), m is the mass of the satellite, and r is the distance from the center of the object to the satellite.

According to Newton's 2nd law the force on the satellite is also given by

$$\sum \vec{F} = m\vec{a},$$

where $a = v^2/r$ for objects undergoing uniform circular motion with speed v.

Use this information to find a relationship between a satellite's speed and its orbital distance.

Hint 2. Working with the equations

Because the gravitational attraction of the earth on the satellite is the only force acting on the satellite, you know that

$$\sum F = G\left(\frac{Mm}{r^2}\right) = ma.$$

You also know that the satellite undergoes uniform circular motion, meaning that

$$a = \frac{v^2}{r}.$$

Combining these expressions yields

$$G\left(\frac{Mm}{r^2}\right) = m\frac{v^2}{r}.$$

Simplify this expression by getting all the distance terms r on one side of the equation.

ANSWER:

- At a distance that is less than r.
- At a distance equal to r.
- ◉ At a distance greater than r.

Correct

The speed v of a satellite of mass m orbiting a distance r from the center of a larger object of mass M is given by the relationship

$$v = \sqrt{\frac{GM}{r}},$$

where G is the gravitational constant. We can't choose the radius r of the orbit and the speed v of the satellite independently; choosing a value of r automatically determines v. Also note that the satellite's motion does not depend on its mass m.

Part B

Now assume that a satellite of mass m is orbiting the earth at a distance r from the center of the earth with speed $v_{\rm e}$. An identical satellite is orbiting the moon at the same distance with a speed $v_{\rm m}$. How does the time $T_{\rm m}$ it takes the satellite circling the moon to make one revolution compare to the time $T_{\rm e}$ it takes the satellite orbiting the earth to make one revolution?

Hint 1. How to approach this problem

To compare the time it takes the satellite circling the moon to make one revolution to the time it takes the satellite orbiting the earth to make one revolution you should first compare the speeds $v_{\rm m}$ and $v_{\rm e}$ of the satellite. Then you can relate the speeds to the time it takes the satellites to make one revolution.

Hint 2. Compare the speeds of the satellites

How does the speed of the satellite orbiting the moon, $v_{\rm m}$, compare to the speed of the satellite orbiting the earth, $v_{\rm e}$?

ANSWER:

- v_m is greater than v_e.
- v_m is equal to v_e.
- v_m is less than v_e.

Hint 3. Relationship between speed and the time for one revolution

The time it takes a satellite to make one revolution is called the period T. The period depends on the speed v of the satellite and its radius r and is given by

$$v = \frac{2\pi r}{T}.$$

ANSWER:

- T_m is less than T_e.
- T_m is equal to T_e.
- T_m is greater than T_e.

Correct

The period of a satellite orbiting an object of mass M at a distance r from its center is given by

$$T = 2\pi\sqrt{\frac{r^3}{GM}}.$$

This relationship is also known as Kepler's 3rd law.

± Understanding Newton's Law of Universal Gravitation

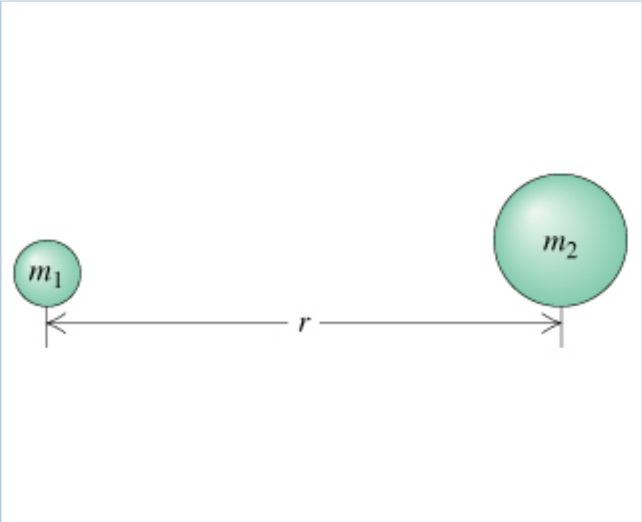
Learning Goal:

To understand Newton's law of universal gravitation and be able to apply it in two-object situations and (collinear) three-object situations; to distinguish between the use of G and g.

In the late 1600s, Isaac Newton proposed a rule to quantify the attractive force known as *gravity* between objects that have mass, such as those shown in the figure. Newton's law of universal gravitation describes the magnitude of the attractive gravitational force $\vec{F}_g$ between two objects with masses m_1 and m_2 as

$$F_g = G\left(\frac{m_1 m_2}{r^2}\right),$$

where r is the distance between the centers of the two objects and G is the gravitational constant.



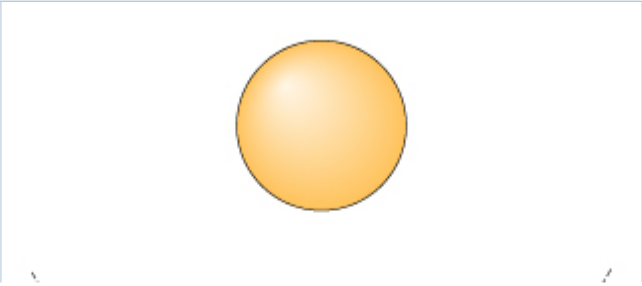
The gravitational force is attractive, so in the figure it pulls to the right on m_1 (toward m_2) and toward the left on m_2 (toward m_1). The gravitational force acting on m_1 is equal in size to, but exactly opposite in direction from, the gravitational force acting on m_2, as required by Newton's third law. The magnitude of both forces is calculated with the equation given above.

The gravitational constant G has the value

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

and should not be confused with the magnitude of the gravitational free-fall acceleration constant, denoted by g, which equals 9.80 $\mathrm{m/s^2}$ near the surface of the earth. The size of G in SI units is tiny. This means that gravitational forces are sizeable only in the vicinity of very massive objects, such as the earth. You are in fact gravitationally attracted toward all the objects around you, such as the computer you are using, but the size of that force is too small to be noticed without extremely sensitive equipment.

Consider the earth following its nearly circular orbit (dashed curve) about the sun. The earth has mass $m_{\mathrm{earth}} = 5.98 \times 10^{24}\ \mathrm{kg}$ and the sun has mass $m_{\mathrm{sun}} = 1.99 \times 10^{30}\ \mathrm{kg}$. They are separated, center to center, by $r = 93\ \text{million miles} = 150\ \text{million km}$.



Part A

What is the size of the gravitational force acting on the earth due to the sun?

Express your answer in newtons.

Hint 1. What units to select

For the force to come out in newtons, all masses and distances used must be expressed in SI units. Note that *neither* 93 million miles ($= 93 \times 10^6 \text{ miles}$) *nor* 150 million km ($= 150 \times 10^6 \text{ km}$) is in the SI unit of length, which is meters.

ANSWER:

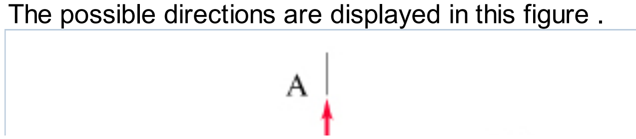
3.53×10^{22} N

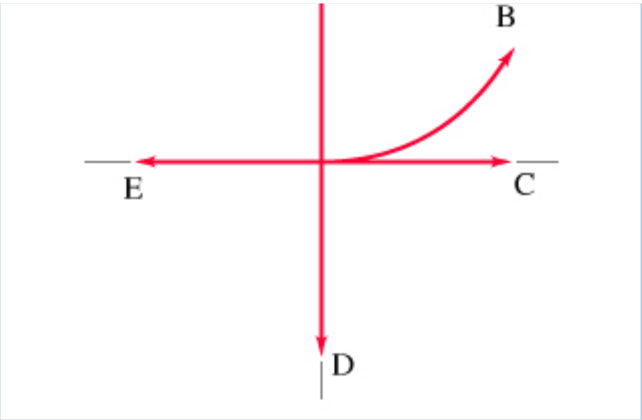
Correct

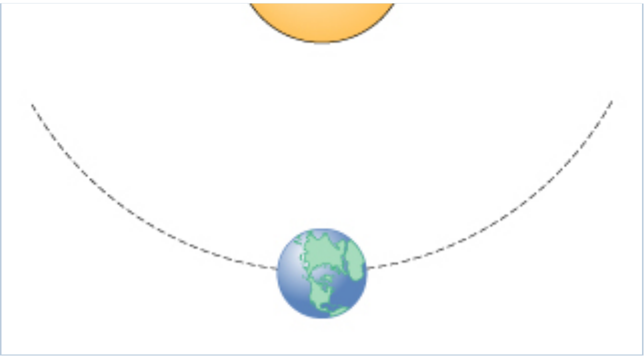
This force causes the earth to orbit the sun.

Part B

At the moment shown in the figure of the earth and sun , what is the direction of the gravitational force acting on the earth? The possible directions are displayed in this figure .







ANSWER:

- A
- B
- C
- D
- E

Correct

As the earth proceeds around its orbit, the direction of the gravitational force acting on it changes so that the force always points directly toward the sun.

Part C

What is the size of the gravitational force acting on the sun due to the earth?

Hint 1. Newton's third law

Newton's third law states that if object A exerts a force on object B, then object B must exert a force of the same type on object A. These two forces are equal in size and exactly opposite in direction. Since they act on different objects, these two forces never cancel one another out.

ANSWER:

- The earth does not exert any gravitational force on the sun.
- The earth exerts some force on the sun, but less than $3.53 \times 10^{22}\ \mathrm{N}$ because the earth, which is exerting the force, is so much less massive than the sun.
- The earth exerts $3.53 \times 10^{22}\ \mathrm{N}$ of force on the sun, exactly the same amount of force the sun exerts on the earth found in Part A.
- The earth exerts more than $3.53 \times 10^{22}\ \mathrm{N}$ of force on the sun because the sun, which is experiencing the force, is so much more massive than the earth.

Correct

Also note that the force exerted on the sun by the earth is directed from the sun toward the earth, downward at the moment shown in the figure. This is exactly the opposite direction of the force exerted on the earth by the sun found in Part B, in accordance with Newton's third law.
Recall that Newton's second law states that $\Sigma\vec{F} = m\vec{a}$. The sun does not accelerate as much as the earth due to this gravitational force since the sun is so much more massive. This force causes the earth to follow a nearly circular path as it orbits the sun, but the same amount of force only causes the sun to wobble back and forth very slightly.

Part D

Which of the following changes to the earth-sun system would reduce the magnitude of the force between them to one-fourth the value found in Part A?

Check all that apply.

ANSWER:

- [x] Reduce the mass of the earth to one-fourth its normal value.
- [x] Reduce the mass of the sun to one-fourth its normal value.
- [x] Reduce the mass of the earth to one-half its normal value and the mass of the sun to one-half its normal value.
- [ ] Increase the separation between the earth and the sun to four times its normal value.

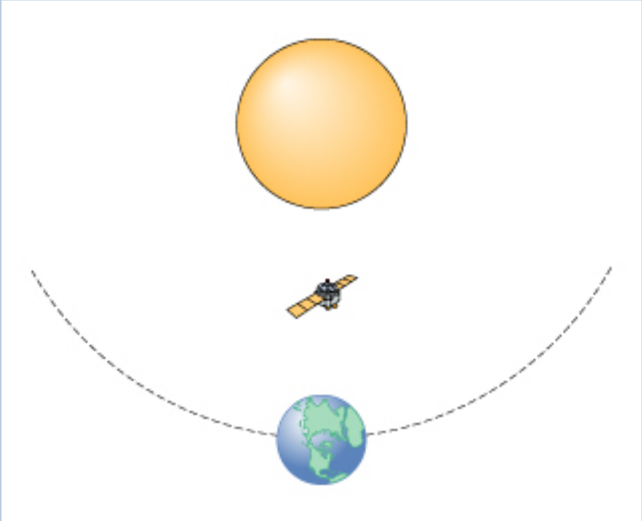
Correct

The actions in choices A, B, and C would reduce the force to one-fourth its original value by modifying the product of the masses. Increasing the separation distance by a factor of 4 would reduce the force by a factor of $4^2 = 16$. To reduce the force by only a factor of 4, increase the separation distance by a factor of 2 instead because $2^2 = 4$.

Part E

With the sun and the earth back in their regular positions, consider a space probe with mass $m_p = 125\ \mathrm{kg}$ launched from the earth toward the sun.

When the probe is exactly halfway between the earth and the sun along the line connecting them, what is the direction of the net gravitational force acting on the probe? Ignore the effects of other massive objects in the solar system, such as the moon and other planets.

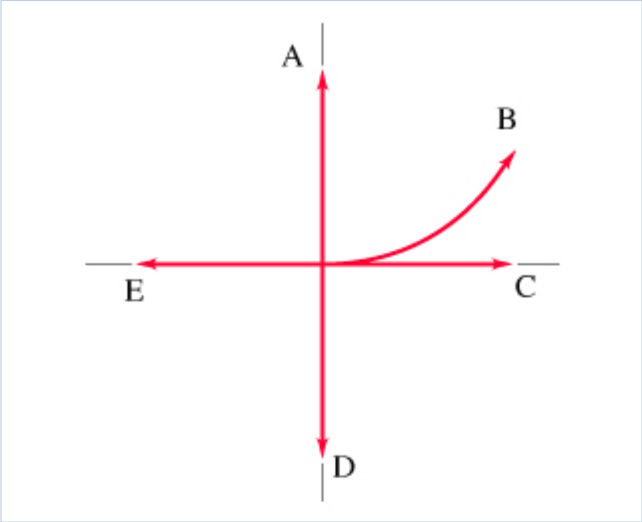


Hint 1. How to approach the problem

The sun and the earth each exert a gravitational force on the probe. Use Newton's law of universal gravitation to find the magnitude and direction of each of these forces. Then sum them (as vectors!) to find the net force acting on the probe.

Hint 2. Direction of the gravitational force due to the earth

What is the direction of the gravitational force acting on the probe due to the earth (refer to the figure)?

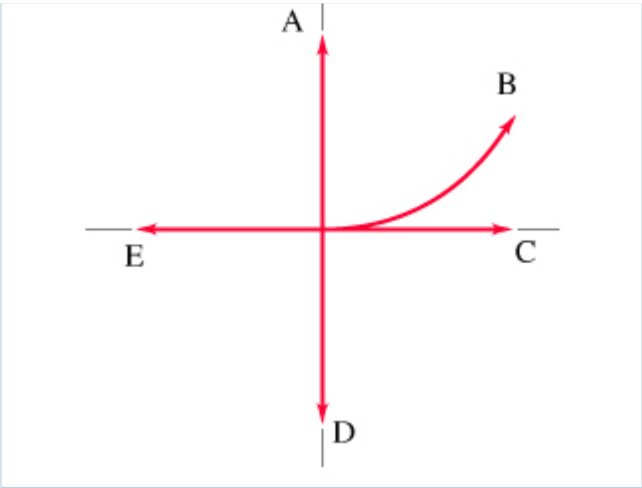


ANSWER:

- A
- B
- C
- D
- E

Hint 3. Direction of the gravitational force due to the sun

What is the direction of the gravitational force acting on the probe due to the sun (refer to the figure)?



ANSWER:

- A
- B
- C
- D
- E

Hint 4. Comparing the gravitational forces acting on the probe

Which exerts the stronger gravitational force on the probe?

ANSWER:

- The sun exerts a stronger force.
- The earth exerts a stronger force
- the forces exerted on the probe by the sun and by the earth are the same size

ANSWER:

- (selected) The force is toward the sun.
- The force is toward the earth.
- There is no net force because neither the sun nor the earth attracts the probe gravitationally at the midpoint.
- There is no net force because the gravitational attractions on the probe due to the sun and the earth are equal in size but point in opposite directions, so they cancel each other out.

Correct

In calculating these forces, the mass of the probe is the same, as is the distance involved. However, the sun is over 300,000 times as massive as the earth, so at this location the probe is attracted much more strongly toward the sun than toward the earth.

Perhaps the most-common gravitational calculation is to determine the "weight" of an object of mass m_o sitting on the surface of the earth (i.e., the size of the gravitational attraction between the earth and the object). According to Newton's law of universal gravitation, the gravitational force on the object is

$$F_g = G\left(\frac{m_e m_o}{r_e^2}\right) = \left(\frac{G m_e}{r_e^2}\right) m_o,$$

where m_e is the mass of the earth and r_e is the distance between the object and the center of the earth (i.e., r_e = radius of the earth = $6.38 \times 10^3\ \mathrm{km}$).

Part F

What is the value of the composite constant $\left(\frac{G m_e}{r_e^2}\right)$, to be multiplied by the mass of the object m_o in the equation above?

Express your answer numerically in meters per second per second.

ANSWER:

9.80 $\mathrm{m/s^2}$

Correct

This particular combination of constants occurs so often that it is given its own name, g, and called the "gravitational free-fall acceleration near the surface of the earth." Using g leads to the more familiar formula for calculating the gravitational weight F_g of an object of mass m, namely

$$F_g = mg.$$

This equation is limited to calculating the weight of objects near the surface of the earth. When calculating gravitational forces between objects under other conditions, such as in the earth/probe calculation done earlier, use Newton's law of universal gravitation instead.

Understanding Mass and Weight

Learning Goal:

To understand the distinction between mass and weight and to be able to calculate the weight of an object from its mass and Newton's law of gravitation.

The concepts of *mass* and *weight* are often confused. In fact, in everyday conversations, the word "weight" often replaces "mass," as in "My weight is seventy-five kilograms" or "I need to lose some weight." Of course, mass and weight *are* related; however, they are also very different.

Mass, as you recall, is a measure of an object's *inertia* (ability to resist acceleration). Newton's 2nd law demonstrates the relationship among an object's mass, its acceleration, and the net force acting on it: $\vec{F}_{\rm net} = m\vec{a}$. Mass is an intrinsic property of an object and is independent of the object's location.

Weight, in contrast, is defined as the *force due to gravity* acting on the object. That force depends on the strength of the gravitational field of the planet: $\vec{W} = m\vec{g}$, where $\vec{W}$ is the weight of an object, m is the mass of that object, and $\vec{g}$ is the local acceleration due to gravity (in other words, the strength of the gravitational field at the location of the object). Weight, unlike mass, is *not* an intrinsic property of the object; it is determined by both the object and its location.

Part A

Which of the following quantities represent mass?

Check all that apply.

ANSWER:

- [ ] 12.0 lbs
- [x] 0.34 g
- [x] 120 kg
- [ ] 1600 kN
- [ ] 0.34 m
- [ ] 411 cm
- [ ] 899 MN

Correct

Part B

Which of the following quantities would be acceptable representations of weight?

Check all that apply.

ANSWER:

- [x] 12.0 lbs
- [ ] 0.34 g
- [ ] 120 kg
- [x] 1600 kN
- [ ] 0.34 m
- [ ] 411 cm
- [x] 899 MN

Correct

Weight is a *force* and is measured in *newtons* (or kilonewtons, meganewtons, etc.) or in *pounds* (or tons, megatons, etc.).

Using the universal law of gravity, we can find the weight of an object feeling the gravitational pull of a nearby planet. We can write an expression W=GmM/r^2, where W is the weight of the object, G is the gravitational constant, m is the mass of that object, M is mass of the planet, and r is the distance from the center of the planet to the object. If the object is on the surface of the planet, r is simply the radius of the planet.

Part C

The gravitational field on the surface of the earth is stronger than that on the surface of the moon. If a rock is transported from the moon to the earth, which properties of the rock change?

ANSWER:

- ○ mass only
- ◉ weight only
- ○ both mass and weight
- ○ neither mass nor weight

Correct

Part D

An object is lifted from the surface of a spherical planet to an altitude equal to the radius of the planet. As a result, which of the following changes in the properties of the object take place?

ANSWER:

- mass increases; weight decreases
- mass decreases; weight decreases
- mass increases; weight increases
- mass increases; weight remains the same
- mass remains the same; weight decreases
- mass remains the same; weight increases
- mass remains the same; weight remains the same

Correct

Punch Taut is a down-on-his-luck heavyweight boxer. One day, he steps on the bathroom scale and "weighs in" at 236 lb. Unhappy with his recent bouts, Punch decides to go to a different planet where he would weigh in at 118 lb so that he can compete with the bantamweights who are not allowed to exceed 118 lb. His plan is to travel to Xobing, a newly discovered star with a planetary system. Here is a table listing the planets in that system:

Name	Mass (M_earth)	Radius (R_earth)
Tehar	2.1	0.80
Loput	5.6	1.7
Cremury	0.36	0.30
Suven	12	2.8
Pentune	8.3	4.1
Rams	9.3	4.0

In this table, the mass and the radius of each planet are given in terms of the corresponding properties of the earth. For instance, Tehar has a mass equal to 2.1 earth masses and a radius equal to 0.80 earth radii.

Part E

If acceleration due to gravity on the earth is g, which formula gives the acceleration due to gravity on Loput?

Hint 1. What equations to use

Combine W=mg and W=GmM/r^2.

ANSWER:

- g\frac{1.7}{5.6}\;
- g\frac{1.7^2}{5.6}
- g\frac{1.7^2}{5.6^2}
- g\frac{5.6}{1.7}\;
- g\frac{5.6^2}{1.7^2}
- g\frac{5.6}{1.7^2}

Correct

Part F

If the acceleration due to gravity on the earth is 9.8 $\mathrm{m/s^2}$, what is the acceleration due to gravity on Rams?

Express your answer in meters per second squared and use two significant figures.

ANSWER:

5.7 $\mathrm{m/s^2}$

Correct

Part G

Which planet should Punch travel to if his goal is to weigh in at 118 lb? Refer to the table of planetary masses and radii given to determine your answer.

Hint 1. Determine the percentage difference in weight

To make the scale read 118 lb, the 236-lb Punch has to travel to a planet where the gravitational field is what percentage of that on the earth?

ANSWER:

- 25%
- 50%
- 200%
- 400%

ANSWER:

- Tehar
- Loput
- Cremury
- Suven
- Pentune
- Rams

Correct

Part H

As Punch Taut travels to Pentune, what *actually* happens to his mass and his weight?

ANSWER:

- mass increases; weight decreases
- mass decreases; weight decreases
- mass increases; weight increases
- mass increases; weight remains the same
- mass remains the same; weight decreases
- mass remains the same; weight increases
- mass remains the same; weight remains the same

Correct

Of course, the "weight classes" in boxing are really "mass classes": It is the relative mass of the boxers that matters. The masses and the weights of the athletes are directly proportional--as long as everyone is on the same planet!

Properties of Circular Orbits

Learning Goal:

To teach you how to find the parameters characterizing an object in a circular orbit around a much heavier body like the earth.

The motivation for Isaac Newton to discover his laws of motion was to explain the properties of planetary orbits that were observed by Tycho Brahe and analyzed by Johannes Kepler. A good starting point for understanding this (as well as the speed of the space shuttle and the height of geostationary satellites) is the simplest orbit--a circular one. This problem concerns the properties of circular orbits for a satellite orbiting a planet of mass M.

For all parts of this problem, where appropriate, use G for the universal gravitational constant.

Part A

Find the orbital speed v for a satellite in a circular orbit of radius R.

Express the orbital speed in terms of G, M, and R.

Hint 1. Find the force

Find the radial force F on the satellite of mass m. (Note that m will cancel out of your final answer for v.)

Express your answer in terms of m, M, G, and R. Indicate outward radial direction with a positive sign and inward radial direction with a negative sign.

ANSWER:

F = -G*M*m/R^2

Hint 2. A basic kinematic relation

Find an expression for the radial acceleration a_r for the satellite in its circular orbit.

Express your answer in terms of v and R. Indicate outward radial direction with a positive sign and inward radial direction with a negative sign.

ANSWER:

a_r = -v^2/R

Hint 3. Newton's 2nd law

Apply $\vec{F} = m\vec{a}$ to the radial coordinate.

ANSWER:

v = \sqrt{\frac{MG}{R}}

Correct

Part B

Find the kinetic energy K of a satellite with mass m in a circular orbit with radius R.

Express your answer in terms of m, M, G, and R.

ANSWER:

K = \frac{mMG}{2R}

Correct

Part C

Express the kinetic energy K in terms of the potential energy U.

Hint 1. Potential energy

What is the potential energy U of the satellite in this orbit?

Express your answer in terms of m, M, G, and R.

ANSWER:

U = $\frac{-GMm}{R}$

ANSWER:

K = -\frac{U}{2}

Correct

This is an example of a powerful theorem, known as the *Virial Theorem*. For any system whose motion is periodic or remains forever bounded, and whose potential behaves as

U \propto R^n,

Rudolf Clausius proved that

\avg{K}=\frac{n}{2}\avg{U},

where the brackets denote the temporal (time) average.

Part D

Find the orbital period T.

Express your answer in terms of G, M, R, and π.

Hint 1. How to get started

Use the fact that the period is the time to make one orbit. Then {\rm time=distance/velocity}.

ANSWER:

T = \sqrt{\frac{4{\pi}^{2}}{GM}R^{3}}

Correct

Part E

Find an expression for the square of the orbital period.

Express your answer in terms of G, M, R, and π.

ANSWER:

T^2 = \frac{4{\pi}^{2}}{GM}R^{3}

Correct

This shows that *the square of the period is proportional to the cube of the semi-major axis*. This is Kepler's Third Law, in the case of a circular orbit where the semi-major axis is equal to the radius, R.

Part F

Find L, the magnitude of the angular momentum of the satellite with respect to the center of the planet.

Express your answer in terms of m, M, G, and R.

Hint 1. Definition of angular momentum

Recall that \vec{L} = \vec{R} \times \vec{p}, where $\vec{p}$ is the momentum of the object and $\vec{R}$ is the vector from the pivot point. Here the pivot point is the center of the planet, and since the object is moving in a circular orbit, $\vec{p}$ is perpendicular to $\vec{R}$.

ANSWER:

L = m\sqrt{MGR}

Correct

Part G

The quantities v, K, U, and L all represent physical quantities characterizing the orbit that depend on radius R. Indicate the exponent (power) of the radial dependence of the absolute value of each.

Express your answer as a comma-separated list of exponents corresponding to v, K, U, and L, in that order. For example, -1,-1/2,-0.5,-3/2 would mean v \propto R^{-1}, K \propto R^{-1/2}, and so forth.

Hint 1. Example of a power law

The potential energy behaves as U=GMm/R, so U depends inversely on R. Therefore, the appropriate power for this is {-1} (i.e., U \propto R^{-1}).

ANSWER:

-0.500,-1,-1,0.500

Correct

Score Summary:

Your score on this assignment is 102.1%.
You received 25.52 out of a possible total of 25 points.