Chapter 13: Gravitation

Due: 11:00pm on Tuesday, November 20, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

A Matter of Some Gravity

Learning Goal:

To understand Newton's law of gravitation and the distinction between inertial and gravitational masses.

In this problem, you will practice using Newton's law of gravitation. According to that law, the magnitude of the gravitational force F_g between two small particles of masses m_1 and m_2, separated by a distance r, is given by

$$F_g = G\frac{m_1 m_2}{r^2},$$

where G is the universal gravitational constant, whose numerical value (in SI units) is $6.67 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$.

This formula applies not only to small particles, but also to spherical objects. In fact, the gravitational force between two uniform spheres is the same as if we concentrated all the mass of each sphere at its center. Thus, by modeling the Earth and the Moon as uniform spheres, you can use the particle approximation when calculating the force of gravity between them.

Be careful in using Newton's law to choose the correct value for r. To calculate the force of gravitational attraction between two uniform spheres, the distance r in the equation for Newton's law of gravitation is the distance between the centers of the spheres. For instance, if a small object such as an elephant is located on the surface of the Earth, the radius of the Earth $r_{\rm Earth}$ would be used in the equation. Note that the force of gravity acting on an object located near the surface of a planet is often called *weight*.

Also note that in situations involving satellites, you are often given the altitude of the satellite, that is, the distance from the satellite to the surface of the planet; this is not the distance to be used in the formula for the law of gravitation.

There is a potentially confusing issue involving mass. Mass is defined as a measure of an object's inertia, that is, its ability to resist acceleration. Newton's second law demonstrates the relationship between mass, acceleration, and the net force acting on an object: $\vec{F}_{\rm net} = m\vec{a}$. We can now refer to this measure of inertia more precisely as the *inertial mass*.

On the other hand, the masses of the particles that appear in the expression for the law of gravity seem to have nothing to do with inertia: Rather, they serve as a measure of the strength of gravitational interactions. It would be reasonable to call such a property *gravitational mass*.

Does this mean that every object has two different masses? Generally speaking, yes. However, the good news is that according to the latest, highly precise, measurements, the inertial and the gravitational mass of an object are, in fact, equal to each other; it is an established consensus among physicists that there is only one mass after all, which is a measure of both the object's inertia and its ability to engage in gravitational interactions. Note that this consensus, like everything else in science, is open to possible amendments in the future.
In this problem, you will answer several questions that require the use of Newton's law of gravitation.

Part A

Two particles are separated by a certain distance. The force of gravitational interaction between them is F_0. Now the separation between the particles is tripled. Find the new force of gravitational interaction F_1.

Express your answer in terms of F_0.

ANSWER:

$F_1 = \frac{F_0}{9}$

Note that the gravitational force between two objects is inversely proportional to the square of the distance between them. If the distance is tripled, the force of gravitational attraction is nine times weaker.

Part B

A satellite revolves around a planet at an altitude equal to the radius of the planet. The force of gravitational interaction between the satellite and the planet is F_0. Then the satellite moves to a different orbit, so that its altitude is tripled. Find the new force of gravitational interaction F_2.

Express your answer in terms of F_0.

Hint 1. Altitude versus distance

If the altitude above the surface of the planet increases by a factor of 3, by what factor does the distance from the center of the planet increase?

Your answer should be an integer.

ANSWER:

2

ANSWER:

$F_2 = \frac{F_0}{4}$

Part C

A satellite revolves around a planet at an altitude equal to the radius of the planet. The force of gravitational interaction between the satellite and the planet is F_0. Then the satellite is brought back to the surface of the planet. Find the new force of gravitational interaction F_4.

Express your answer in terms of F_0.

ANSWER:

$F_4 = 4F_0$

Part D

Two satellites revolve around the Earth. Satellite A has mass m and has an orbit of radius r. Satellite B has mass $6m$ and an orbit of unknown radius r_b. The forces of gravitational attraction between each satellite and the Earth is the same. Find r_b.

Express your answer in terms of r.

ANSWER:

$r_b = \sqrt{6}r$

Part E

An adult elephant has a mass of about 5.0 tons. An adult elephant shrew has a mass of about 50 grams. How far r from the center of the Earth should an elephant be placed so that its weight equals that of the elephant shrew on the surface of the Earth? The radius of the Earth is 6400 km. ($1\ \mathrm{ton} = 10^3\ \mathrm{kg}$.)

Express your answer in kilometers.

ANSWER:

$r = 2.00\times10^6\ \mathrm{km}$

The table below gives the masses of the Earth, the Moon, and the Sun.

Name	Mass (kg)
Earth	5.97×10^{24}

Moon 7.35×10^{22}

Sun 1.99×10^{30}

The average distance between the Earth and the Moon is $3.84 \times 10^{8}\ \mathrm{m}$. The average distance between the Earth and the Sun is $1.50 \times 10^{11}\ \mathrm{m}$. Use this information to answer the following questions.

Part F

Find the net gravitational force $F_{\rm net}$ acting on the Earth in the Sun-Earth-Moon system during the new moon (when the moon is located directly between the Earth and the Sun).

Express your answer in newtons to three significant figures.

Hint 1. How to apprach the problem

Sketch a diagram, drawing the forces acting on the Earth and adding them as vectors.

Hint 2. Find the force of attraction between the Earth and the Moon

Find the force of gravitational attraction $F_{\rm Earth/Moon}$ between the Earth and the Moon.

Express your answer in newtons to three significant figures.

ANSWER:

$F_{\rm Earth/Moon}$ = 1.98×10^{20} N

Hint 3. Find the force of attraction between the Earth and the Sun

Find the force of gravitational attraction $F_{\rm Earth/Sun}$ between the Earth and the Sun.

Express your answer in newtons to four significant figures.

ANSWER:

$F_{\rm Earth/Sun}$ = 3.520×10^{22} N

ANSWER:

$F_{\rm net}$ = 3.54×10^{22} N

Part G

Find the net gravitational force $F_{\rm net}$ acting on the Earth in the Sun-Earth-Moon system during the full moon (when the Earth is located directly between the moon and the sun).

Express your answer in newtons to three significant figures.

ANSWER:

$F_{\rm net}$ = 3.50×10^{22} N

Understanding Mass and Weight

Learning Goal:

To understand the distinction between mass and weight and to be able to calculate the weight of an object from its mass and Newton's law of gravitation.

The concepts of *mass* and *weight* are often confused. In fact, in everyday conversations, the word "weight" often replaces "mass," as in "My weight is seventy-five kilograms" or "I need to lose some weight." Of course, mass and weight *are* related; however, they are also very different.

Mass, as you recall, is a measure of an object's *inertia* (ability to resist acceleration). Newton's 2nd law demonstrates the relationship among an object's mass, its acceleration, and the net force acting on it: $\vec{F}_{\rm net} = m\vec{a}$. Mass is an intrinsic property of an object and is independent of the object's location.

Weight, in contrast, is defined as the *force due to gravity* acting on the object. That force depends on the strength of the gravitational field of the planet: $\vec{W} = m\vec{g}$, where $\vec{W}$ is the weight of an object, m is the mass of that object, and $\vec{g}$ is the local acceleration due to gravity (in other words, the strength of the gravitational field at the location of the object). Weight, unlike mass, is *not* an intrinsic property of the object; it is determined by both the object and its location.

Part A

Which of the following quantities represent mass?

Check all that apply.

ANSWER:

- [ ] 12.0 lbs
- [x] 0.34 g
- [x] 120 kg
- [ ] 1600 kN
- [ ] 0.34 m
- [ ] 411 cm
- [ ] 899 MN

Part B

Which of the following quantities would be acceptable representations of weight?

Check all that apply.

ANSWER:

- [x] 12.0 lbs
- [ ] 0.34 g
- [ ] 120 kg
- [x] 1600 kN
- [ ] 0.34 m
- [ ] 411 cm
- [x] 899 MN

Weight is a *force* and is measured in *newtons* (or kilonewtons, meganewtons, etc.) or in *pounds* (or tons, megatons, etc.).

Using the universal law of gravity, we can find the weight of an object feeling the gravitational pull of a nearby planet. We can write an expression $W = GmM/r^2$, where W is the weight of the object, G is the gravitational constant, m is the mass of that object, M is mass of the planet, and r is the distance from the center of the planet to the object. If the object is on the surface of the planet, r is simply the radius of the planet.

Part C

The gravitational field on the surface of the earth is stronger than that on the surface of the moon. If a rock is transported from the moon to the earth, which properties of the rock change?

ANSWER:

- ○ mass only
- ◉ weight only
- ○ both mass and weight
- ○ neither mass nor weight

Part D

An object is lifted from the surface of a spherical planet to an altitude equal to the radius of the planet. As a result, which of the following changes in the properties of the object take place?

ANSWER:

- mass increases; weight decreases
- mass decreases; weight decreases
- mass increases; weight increases
- mass increases; weight remains the same
- mass remains the same; weight decreases
- mass remains the same; weight increases
- mass remains the same; weight remains the same

Part E

If acceleration due to gravity on the earth is g, which formula gives the acceleration due to gravity on Loput?

Hint 1. What equations to use

Combine $W = mg$ and $W = GmM/r^2$.

ANSWER:

- $g\frac{1.7}{5.6}$
- $g\frac{1.7^2}{5.6}$
- $g\frac{1.7^2}{5.6^2}$
- $g\frac{5.6}{1.7}$
- $g\frac{5.6^2}{1.7^2}$
- $g\frac{5.6}{1.7^2}$

Part F

If the acceleration due to gravity on the earth is 9.8 $\mathrm{m/s^2}$, what is the acceleration due to gravity on Rams?

Express your answer in meters per second squared and use two significant figures.

ANSWER:

5.7 $\mathrm{m/s^2}$

Part G

Which planet should Punch travel to if his goal is to weigh in at 118 lb? Refer to the table of planetary masses and radii given to determine your answer.

Hint 1. Determine the percentage difference in weight

To make the scale read 118 lb, the 236-lb Punch has to travel to a planet where the gravitational field is what percentage of that on the earth?

ANSWER:

- 25%
- 50%
- 200%
- 400%

ANSWER:

- Tehar
- Loput
- Cremury
- Suven
- Pentune (selected)
- Rams

Part H

As Punch Taut travels to Pentune, what *actually* happens to his mass and his weight?

ANSWER:

- mass increases; weight decreases
- mass decreases; weight decreases
- mass increases; weight increases
- mass increases; weight remains the same
- mass remains the same; weight decreases (selected)
- mass remains the same; weight increases
- mass remains the same; weight remains the same

Of course, the "weight classes" in boxing are really "mass classes": It is the relative mass of the boxers that matters. The masses and the weights of the athletes are directly proportional–as long as everyone is on the same planet!

± Escape Velocity

Learning Goal:

To introduce you to the concept of escape velocity for a rocket.

The escape velocity is defined to be the minimum speed with which an object of mass m must move to escape from the gravitational attraction of a much larger body, such as a planet of total mass M. The escape velocity is a function of the distance of the object from the center of the planet R, but unless otherwise specified this distance is taken to be the radius of the planet because it addresses the question "How fast does my rocket have to go to escape from the surface of the planet?"

Part A

The key to making a concise mathematical definition of escape velocity is to consider the energy. If an object is launched at its escape velocity, what is the total mechanical energy $E_{\rm total}$ of the object at a very large (i.e., infinite) distance from the planet? Follow the usual convention and take the gravitational potential energy to be zero at very large distances.

Hint 1. Consider various cases

If the object is given some speed *less* than its escape velocity, it will eventually return to the planet. If it is given some speed *greater* than its escape velocity, then even at very large distances, it will still have some velocity directed away from the planet. If the object is given its escape velocity *exactly*, its velocity at a very large distance from the planet will tend to zero (so, its kinetic energy will go to zero).

ANSWER:

$E_{\rm total}$ = 0

Consider the motion of an object between a point close to the planet and a point very very far from the planet. Indicate whether the following statements are true or false.

Part B

Angular momentum about the center of the planet is conserved.

ANSWER:

- (selected) true
- false

Part C

Total mechanical energy is conserved.

ANSWER:

- (selected) true
- false

Part D

Kinetic energy is conserved.

ANSWER:

- true
- (selected) false

Part E

The angular momentum about the center of the planet and the total mechanical energy will be conserved regardless of whether the object moves from small R to large R (like a rocket being launched) or from large R to small R (like a comet approaching the earth).

ANSWER:

- (selected) true
- false

What if the object is not moving directly away from or toward the planet but instead is moving at an angle θ from the normal? In this case, it will have a tangential velocity $v_{\rm tan} = v\sin\theta$ and angular momentum $L = mv_{\rm tan}R$. Since angular momentum is conserved, $v_{\rm tan} = L/(mR)$ for any R, so $v_{\rm tan}$ will go to 0 as R goes to infinity. This means that angular momentum can be conserved without adding any kinetic energy at $R = \infty$. The important aspect for determining the escape velocity will therefore be the conservation of total mechanical energy.

Part F

Find the escape velocity $v_{\rm e}$ for an object of mass m that is initially at a distance R from the center of a planet of mass M. Assume that $R \geq R_{\rm planet}$, the radius of the planet, and ignore air resistance.

Express the escape velocity in terms of R, M, m, and G, the universal gravitational constant.

Hint 1. Determine the gravitational potential energy

Find U, the gravitational potential energy of the object at a distance R from the center of the planet, with the gravitational potential energy taken to be zero when the object is infinitely far away from the planet.

Express your answer in terms of R, M, m, and G, the universal gravitational constant.

ANSWER:

U = $\frac{-GMm}{R}$

Hint 2. Determine the kinetic energy

Suppose that the object is given its escape velocity v_e. Find the initial kinetic energy K of the object when it is at its initial distance R from the center of the planet.

Express your answer in terms of v_e and other given quantities.

ANSWER:

$K = \frac{1}{2}m{v_e}^2$

Hint 3. Putting it all together

You can assume that no nonconservative forces act on the object as it moves through space. Hence, using conservation of mechanical energy, you can equate the object's initial mechanical energy (when it is at a distance R from the planet) to the object's mechanical energy at very large distances from the planet. This should allow you to solve for v_e.

ANSWER:

$v_e = \sqrt{\frac{2GM}{R}}$

Does it surprise you that the escape velocity does not depend on the mass of the object? Even more surprising is that it does not depend on the direction (as long as the trajectory misses the planet). Any angular momentum given at radius R can be conserved with a tangential velocity that vanishes as R goes to infinity, so the angle at which the object is launched does not have a significant effect on the energy at large R.

Properties of Circular Orbits

Learning Goal:

To teach you how to find the parameters characterizing an object in a circular orbit around a much heavier body like the earth.

The motivation for Isaac Newton to discover his laws of motion was to explain the properties of planetary orbits that were observed by Tycho Brahe and analyzed by Johannes Kepler. A good starting point for understanding this (as well as the speed of the space shuttle and the height of geostationary satellites) is the simplest orbit--a circular one. This problem concerns the properties of circular orbits for a satellite orbiting a planet of mass M.

For all parts of this problem, where appropriate, use G for the universal gravitational constant.

Part A

Find the orbital speed v for a satellite in a circular orbit of radius R.

Express the orbital speed in terms of G, M, and R.

Hint 1. Find the force

Find the radial force F on the satellite of mass m. (Note that m will cancel out of your final answer for v.)

Express your answer in terms of m, M, G, and R. Indicate outward radial direction with a positive sign and inward radial direction with a negative sign.

ANSWER:

$F = \frac{-GMm}{R^2}$

Hint 2. A basic kinematic relation

Find an expression for the radial acceleration a_r for the satellite in its circular orbit.

Express your answer in terms of v and R. Indicate outward radial direction with a positive sign and inward radial direction with a negative sign.

ANSWER:

$a_r = \frac{-v^2}{R}$

Hint 3. Newton's 2nd law

Apply $\vec{F} = m\vec{a}$ to the radial coordinate.

ANSWER:

$v = \sqrt{\frac{GM}{R}}$

Part B

Find the kinetic energy K of a satellite with mass m in a circular orbit with radius R.

Express your answer in terms of m, M, G, and R.

ANSWER:

$K = \frac{GMm}{2R}$

Part C

Express the kinetic energy K in terms of the potential energy U.

Hint 1. Potential energy

What is the potential energy U of the satellite in this orbit?

Express your answer in terms of m, M, G, and R.

ANSWER:

$U = \frac{-GMm}{R}$

ANSWER:

$K = \frac{-1}{2}U$

This is an example of a powerful theorem, known as the *Virial Theorem*. For any system whose motion is periodic or remains forever bounded, and whose potential behaves as

$$U \propto R^n,$$

Rudolf Clausius proved that

$$\langle K \rangle = \frac{n}{2}\langle U \rangle,$$

where the brackets denote the temporal (time) average.

Part D

Find the orbital period T.

Express your answer in terms of G, M, R, and π.

Hint 1. How to get started

Use the fact that the period is the time to make one orbit. Then $\mathrm{time} = \mathrm{distance}/\mathrm{velocity}$.

ANSWER:

T = $2\pi R^{\frac{3}{2}}(GM)^{\frac{-1}{2}}$

Part E

Find an expression for the square of the orbital period.

Express your answer in terms of G, M, R, and π.

ANSWER:

T^2 = $\frac{(2\pi)^2}{GM}R^3$

This shows that *the square of the period is proportional to the cube of the semi-major axis*. This is Kepler's Third Law, in the case of a circular orbit where the semi-major axis is equal to the radius, R.

Part F

Find L, the magnitude of the angular momentum of the satellite with respect to the center of the planet.

Express your answer in terms of m, M, G, and R.

Hint 1. Definition of angular momentum

Recall that $\vec{L} = \vec{R} \times \vec{p}$, where $\vec{p}$ is the momentum of the object and $\vec{R}$ is the vector from the pivot point. Here the pivot point is the center of the planet, and since the object is moving in a circular orbit, $\vec{p}$ is perpendicular to $\vec{R}$.

ANSWER:

L = $m\sqrt{(GMR)}$

Part G

The quantities v, K, U, and L all represent physical quantities characterizing the orbit that depend on radius R. Indicate the exponent (power) of the radial dependence of the absolute value of each.

Express your answer as a comma-separated list of exponents corresponding to v, K, U, and L, in that order. For example, -1,-1/2,-0.5,-3/2 would mean $v \propto R^{-1}$, $K \propto R^{-1/2}$, and so forth.

Hint 1. Example of a power law

The potential energy behaves as $U = GMm/R$, so U depends inversely on R. Therefore, the appropriate power for this is -1 (i.e., $U \propto R^{-1}$).

ANSWER:

-0.500, -1, -1, 0.500

Exercise 13.4

Two uniform spheres, each with mass M and radius R, touch one another.

Part A

What is the magnitude of their gravitational force of attraction?

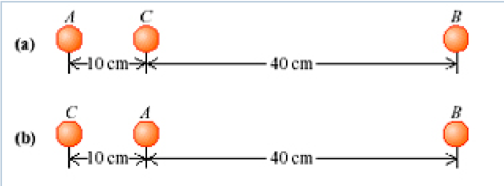
Express your answer in terms of the variables M, R, and appropriate constants.

ANSWER:

$$\frac{GM^2}{4R^2}$$

Exercise 13.6

Each mass is 4.00kg .



Part A

Find the magnitude of the net gravitational force on mass A due to masses B and C in the figure (a).

ANSWER:

F = $m^2 \cdot 6.67 \cdot 10^{-11}\left(\frac{1}{0.01}+\frac{1}{0.25}\right)$ = 1.11×10^{-7} N

Part B

Find the direction of the net gravitational force on mass A due to masses B and C in the figure (a).

ANSWER:

- ○ To the left
- ◉ To the right

Part C

Find the magnitude of the net gravitational force on mass A due to masses B and C in the figure (b).

ANSWER:

F = $m^2 \cdot 6.67 \cdot 10^{-11}\left(\frac{1}{0.01}-\frac{1}{0.16}\right)$ = 1.00×10^{-7} N

Part D

Find the direction of the net gravitational force on mass A due to masses B and C in the figure (b).

ANSWER:

- ◉ To the left
- ○ To the right

Exercise 13.9

A particle of mass 3 m is located 1.70m from a particle of mass m.

Part A

Where should you put a third mass M so that the net gravitational force on M due to the two masses is exactly zero?

ANSWER:

$x = \frac{L}{1+\frac{1}{\sqrt{3}}}$ = 1.08 m from the mass 3m

Part B

Is the equilibrium of M at this point stable or unstable for points along the line connecting m and 3m?

ANSWER:

- ○ equilibrium is stable
- ◉ equilibrium is unstable

Part C

Is the equilibrium of M at this point stable or unstable for points along the line passing through M and perpendicular to the line connecting m and 3 m?

ANSWER:

- ◉ equilibrium is stable
- ○ equilibrium is unstable

Exercise 13.11

Part A

At what distance above the surface of the earth is the acceleration due to the earth's gravity 0.755$\mathrm{m/s^2}$ if the acceleration due to gravity at the surface has magnitude $9.80\ \mathrm{m/s^2}$?

ANSWER:

$\left(\sqrt{\frac{9.8}{g_1}}-1\right)\cdot 6.38\left(10^6\right)$ = 1.66×10^{7} m

Exercise 13.18

Ten days after it was launched toward Mars in December 1998, the *Mars Climate Orbiter* spacecraft (mass 629 kg) was $2.87\times10^{6}\ \mathrm{km}$ from the earth and traveling at $1.20\times10^{4}\ \mathrm{km/h}$ relative to the earth.

Part A

At this time, what was the spacecraft's kinetic energy relative to the earth?

ANSWER:

K = 3.49×10^{9} J

Part B

What was the potential energy of the earth-spacecraft system?

ANSWER:

U = −8.73×10^{7} J

Exercise 13.20

On July 15, 2004, NASA launched the Aura spacecraft to study the earth's climate and atmosphere. This satellite was injected into an orbit 705 km above the earth's surface, and we shall assume a circular orbit.

Part A

How many hours does it take this satellite to make one orbit?

ANSWER:

T = 1.65 hours

Part B

How fast (in $\mathrm{km/s}$)is the Aura spacecraft moving?

ANSWER:

v = 7.49 $\mathrm{km/s}$

Exercise 13.26

In March 2006, two small satellites were discovered orbiting Pluto, one at a distance of 48,000 km and the other at 64,000 km. Pluto already was known to have a large satellite Charon, orbiting at 19,600 km with an orbital period of 6.39 days.

Part A

Assuming that the satellites do not affect each other, find the orbital periods of the two small satellites *without* using the mass of Pluto.

Enter your answers numerically separated by a comma.

ANSWER:

T_1, T_2 = 24.5, 37.7 days

Also accepted: 37.7, 24.5

Problem 13.58

Your starship, the *Aimless Wanderer,* lands on the mysterious planet Mongo. As chief scientist-engineer, you make the following measurements: a 2.50-kg stone thrown upward from the ground at 11.0$\mathrm{m/s}$ returns to the ground in 9.00s ; the circumference of Mongo at the equator is 3.00×10^5 km ; and there is no appreciable atmosphere on Mongo.

Part A

The starship commander, Captain Confusion, asks for the following information: what is the mass of Mongo?

Express your answer to three significant figures.

ANSWER:

$m = \frac{\frac{\frac{\frac{vL^2}{2}}{\pi^2}}{6.673\cdot 10^{-11}}}{t}$ = 8.35×10^{25} kg

Part B

If the *Aimless Wanderer* goes into a circular orbit 1.00×10^4km above the surface of Mongo, how many hours will it take the ship to complete one orbit?

Express your answer to three significant figures.

ANSWER:

$$t = \frac{\frac{4\pi^2}{L}\sqrt{\frac{\frac{t\left(\frac{\frac{L}{2}}{\pi}+s\right)^3}{v}}{2}}}{3600} = 10.3 \quad \mathrm{h}$$

13.4. IDENTIFY: Apply Eq. (13.2), generalized to any pair of spherically symmetric objects.

SET UP: The separation of the centers of the spheres is $2R$.

EXECUTE: The magnitude of the gravitational attraction is $GM^2/(2R)^2 = GM^2/4R^2$.

EVALUATE: Eq. (13.2) applies to any pair of spherically symmetric objects; one of the objects doesn't have to be the earth.

13.6. IDENTIFY: The net force on A is the vector sum of the force due to B and the force due to C. In part (a), the two forces are in the same direction, but in (b) they are in opposite directions.

SET UP: Use coordinates where $+x$ is to the right. Each gravitational force is attractive, so is toward the mass exerting it. Treat the masses as uniform spheres, so the gravitational force is the same as for point masses with the same center-to-center distances. The free-body diagrams for (a) and (b) are given in Figures 13.6a and 13.6b. The gravitational force is $F_{\text{grav}} = Gm_1m_2/r^2$.

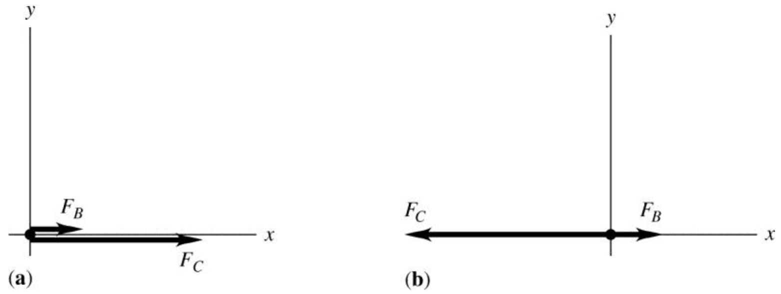


Figure 13.6

EXECUTE: **(a)** Calling F_B the force due to mass B and likewise for C, we have

$$F_B = G\frac{m_A m_B}{r_{AB}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\ \text{kg})^2}{(0.50\ \text{m})^2} = 1.069\times10^{-9}\ \text{N}$$ and

$$F_C = G\frac{m_A m_C}{r_{AC}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\ \text{kg})^2}{(0.10\ \text{m})^2} = 2.669\times10^{-8}\ \text{N}.$$ The net force is

$$F_{\text{net},\,x} = F_{Bx} + F_{Cx} = 1.069\times10^{-9}\ \text{N} + 2.669\times10^{-8}\ \text{N} = 2.8\times10^{-8}\ \text{N}$$ to the right.

(b) Following the same procedure as in (a), we have

$$F_B = G\frac{m_A m_B}{r_{AB}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\ \text{kg})^2}{(0.40\ \text{m})^2} = 1.668\times10^{-9}\ \text{N}$$

$$F_C = G\frac{m_A m_C}{r_{AC}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(2.00\ \text{kg})^2}{(0.10\ \text{m})^2} = 2.669\times10^{-8}\ \text{N}$$

$$F_{\text{net},\,x} = F_{Bx} + F_{Cx} = 1.668\times10^{-9}\ \text{N} - 2.669\times10^{-8}\ \text{N} = -2.5\times10^{-8}\ \text{N}$$

The net force on A is $2.5\times10^{-8}\ \text{N}$, to the left.

EVALUATE: As with any force, the gravitational force is a vector and must be treated like all other vectors. The formula $F_{grav} = Gm_1m_2/r^2$ only gives the magnitude of this force.

13.9.IDENTIFY: Use Eq. (13.2) to calculate the gravitational force each particle exerts on the third mass. The equilibrium is stable when for a displacement from equilibrium the net force is directed toward the equilibrium position and it is unstable when the net force is directed away from the equilibrium position.

SET UP: For the net force to be zero, the two forces on M must be in opposite directions. This is the case only when M is on the line connecting the two particles and between them. The free-body diagram for M

is given in Figure 13.9. $m_1 = 3m$ and $m_2 = m.$ If M is a distance x from $m_1,$ it is a distance $1.00\text{ m} - x$ from $m_2.$

EXECUTE: **(a)** $F_x = F_{1x} + F_{2x} = -G\dfrac{3mM}{x^2} + G\dfrac{mM}{(1.00\text{ m} - x)^2} = 0.\ 3\ (1.00\text{ m} - x)^2 = x^2.$

$1.00\text{ m} - x = \pm x/\sqrt{3}.$ Since M is between the two particles, x must be less than 1.00 m and $x = \dfrac{1.00\text{ m}}{1 + 1/\sqrt{3}} = 0.634\text{ m}.$ M must be placed at a point that is 0.634 m from the particle of mass $3m$ and 0.366 m from the particle of mass m.

(b) (i) If M is displaced slightly to the right in Figure 13.9, the attractive force from m is larger than the force from $3m$ and the net force is to the right. If M is displaced slightly to the left in Figure 13.9, the attractive force from $3m$ is larger than the force from m and the net force is to the left. In each case the net force is away from equilibrium and the equilibrium is unstable.

(ii) If M is displaced a very small distance along the y axis in Figure 13.9, the net force is directed opposite to the direction of the displacement and therefore the equilibrium is stable.

EVALUATE: The point where the net force on M is zero is closer to the smaller mass.

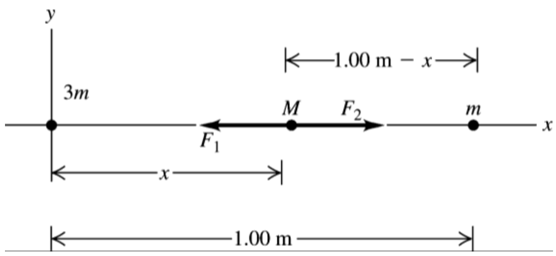


Figure 13.9

13.11.IDENTIFY: $F_g = G\dfrac{mm_E}{r^2},$ so $a_g = G\dfrac{m_E}{r^2},$ where r is the distance of the object from the center of the earth.

SET UP: $r = h + R_E,$ where h is the distance of the object above the surface of the earth and $R_E = 6.38 \times 10^6\text{ m}$ is the radius of the earth.

EXECUTE: To decrease the acceleration due to gravity by one-tenth, the distance from the center of the earth must be increased by a factor of $\sqrt{10}$, and so the distance above the surface of the earth is $(\sqrt{10}-1)R_E = 1.38\times 10^7\text{ m}$.

EVALUATE: This height is about twice the radius of the earth.

13.18. IDENTIFY: The kinetic energy is $K = \frac{1}{2}mv^2$ and the potential energy is $U = -\dfrac{GMm}{r}$.

SET UP: The mass of the earth is $M_E = 5.97\times 10^{24}\text{ kg}$.

EXECUTE: **(a)** $K = \frac{1}{2}(629\text{ kg})(3.33\times 10^3\text{ m/s})^2 = 3.49\times 10^9\text{ J}$

(b) $U = -\dfrac{GM_E m}{r} = -\dfrac{(6.673\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times 10^{24}\text{ kg})(629\text{ kg})}{2.87\times 10^9\text{ m}} = -8.73\times 10^7\text{ J}.$

EVALUATE: The total energy $K+U$ is positive.

13.20. IDENTIFY: The time to complete one orbit is the period *T*, given by Eq. (13.12). The speed *v* of the satellite is given by $v = \dfrac{2\pi r}{T}$.

SET UP: If *h* is the height of the orbit above the earth's surface, the radius of the orbit is $r = h + R_E$. $R_E = 6.38\times 10^6\text{ m}$ and $m_E = 5.97\times 10^{24}\text{ kg}$.

EXECUTE: **(a)**

$$T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}} = \frac{2\pi(7.05\times 10^5\text{ m} + 6.38\times 10^6\text{ m})^{3/2}}{\sqrt{(6.67\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times 10^{24}\text{ kg})}} = 5.94\times 10^3\text{ s} = 99.0\text{ min}$$

(b) $v = \dfrac{2\pi(7.05\times 10^5\text{ m} + 6.38\times 10^6\text{ m})}{5.94\times 10^3\text{ s}} = 7.49\times 10^3\text{ m/s} = 7.49\text{ km/s}$

EVALUATE: The satellite in Example 13.6 is at a lower altitude and therefore has a smaller orbit radius than the satellite in this problem. Therefore, the satellite in this problem has a larger period and a smaller orbital speed. But a large percentage change in *h* corresponds to a small percentage change in *r* and the values of *T* and *v* for the two satellites do not differ very much.

13.26. IDENTIFY: The period of each satellite is given by Eq. (13.12). Set up a ratio involving *T* and *r*.

SET UP: $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_p}}$ gives $\dfrac{T}{r^{3/2}} = \dfrac{2\pi}{\sqrt{Gm_p}} = \text{constant}$, so $\dfrac{T_1}{r_1^{3/2}} = \dfrac{T_2}{r_2^{3/2}}$.

EXECUTE: $T_2 = T_1\left(\dfrac{r_2}{r_1}\right)^{3/2} = (6.39\text{ days})\left(\dfrac{48{,}000\text{ km}}{19{,}600\text{ km}}\right)^{3/2} = 24.5\text{ days}$. For the other satellite,

$$T_2 = (6.39\text{ days})\left(\frac{64{,}000\text{ km}}{19{,}600\text{ km}}\right)^{3/2} = 37.7\text{ days}.$$

EVALUATE: T increases when r increases.

13.58.IDENTIFY: Use the measurements of the motion of the rock to calculate g_M, the value of g on Mongo. Then use this to calculate the mass of Mongo. For the ship, $F_g = ma_{rad}$ and $T = \dfrac{2\pi r}{v}$.

SET UP: Take $+y$ upward. When the stone returns to the ground its velocity is 12.0 m/s, downward. $g_M = G\dfrac{m_M}{R_M^2}$. The radius of Mongo is $R_M = \dfrac{c}{2\pi} = \dfrac{2.00\times10^8\text{ m}}{2\pi} = 3.18\times10^7\text{ m}$. The ship moves in an orbit of radius $r = 3.18\times10^7\text{ m} + 3.00\times10^7\text{ m} = 6.18\times10^7\text{ m}$.

EXECUTE: **(a)** $v_{0y} = +12.0\text{ m/s}$, $v_y = -12.0\text{ m/s}$, $a_y = -g_M$ and $t = 6.00\text{ s}$. $v_y = v_{0y} + a_y t$ gives $-g_M = \dfrac{v_y - v_{0y}}{t} = \dfrac{-12.0\text{ m/s} - 12.0\text{ m/s}}{6.00\text{ s}}$ and $g_M = 4.00\text{ m/s}^2$.

$$m_M = \frac{g_M R_M^2}{G} = \frac{(4.00\text{ m/s}^2)(3.18\times10^7\text{ m})^2}{6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = 6.06\times10^{25}\text{ kg}$$

(b) $F_g = ma_{rad}$ gives $G\dfrac{m_M m}{r^2} = m\dfrac{v^2}{r}$ and $v^2 = \dfrac{Gm_M}{r}$.

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_M}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_M}} = \frac{2\pi(6.18\times10^7\text{ m})^{3/2}}{\sqrt{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(6.06\times10^{25}\text{ kg})}}$$

$$T = 4.80\times10^4\text{ s} = 13.3\text{ h}$$

EVALUATE: $R_M = 5.0R_E$ and $m_M = 10.2m_E$, so $g_M = \dfrac{10.2}{(5.0)^2}g_E = 0.408g_E$, which agrees with the value calculated in part (a).