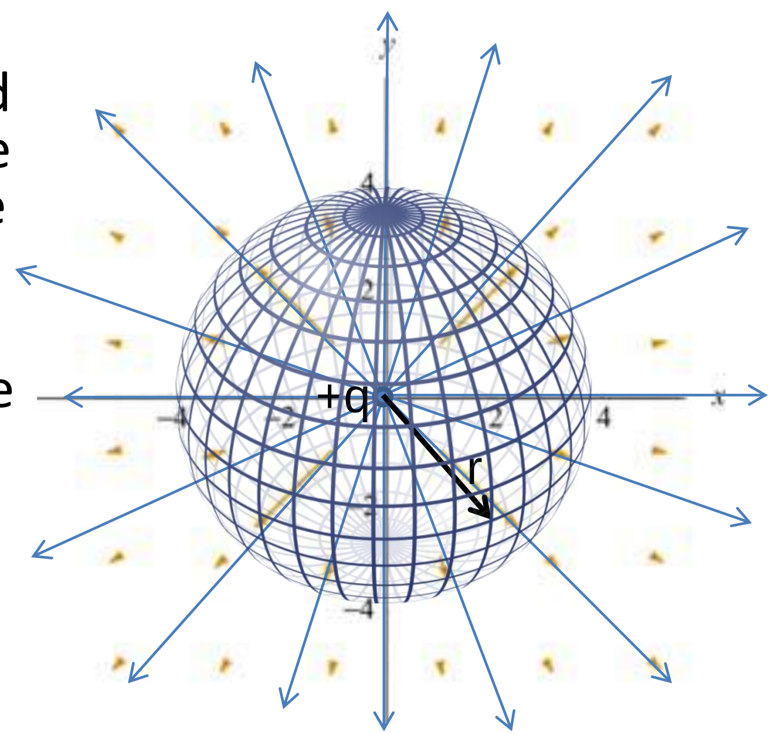
Lecture 14
(Gauss's Law)

Physics 161-01 Spring 2012
Douglas Fields

Net Flux for a Single Charge

- Last lecture we talked about flux in a very qualitative manner. But what can we say quantitatively?
- Let's examine the net flux through a sphere centered on a positive charge, q.

- First, we note that the electric field is everywhere perpendicular to the surface, so that E and dA are in the same direction everywhere on the surface.
- Since the sphere is centered on the charge, then the electric field also has the same magnitude everywhere on the surface.

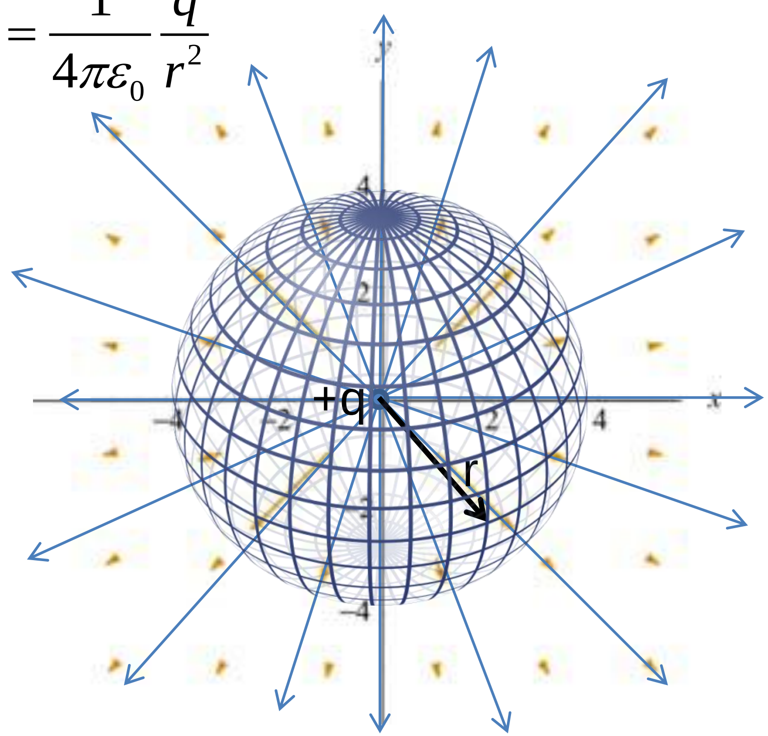


Net Flux for a Single Charge

- Since the magnitude of the field is constant, and the dot product just gives 1 everywhere, then both can be taken out of the integral over the surface for the flux.

$$\left|\vec{E}(r)\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

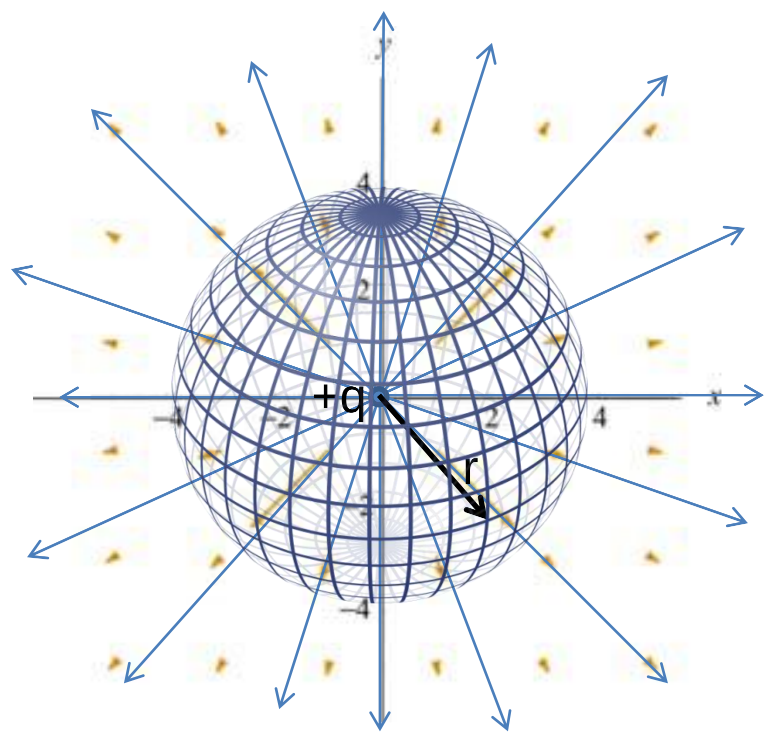
$$\begin{aligned}\Phi_{\mathrm{E,Net}} &= \oint \vec{E}\cdot d\vec{A} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\oint dA \\ &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\left(4\pi r^2\right) \\ &= \frac{q}{\varepsilon_0}\end{aligned}$$



Gauss's Law

- This relation is known as the integral form of Gauss's Law.
- In plain English, the law states that the net flux through a closed surface (Gaussian surface) is just equal to the net charge enclosed divided by the permittivity of free space.

$$\Phi_{\mathrm{E,Net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\mathrm{enc}}}{\varepsilon_0}$$



Gauss's Law

- Notice that the surface is undefined, even though we used a nice, happy sphere to show the law, the Gaussian surface can be any closed surface.

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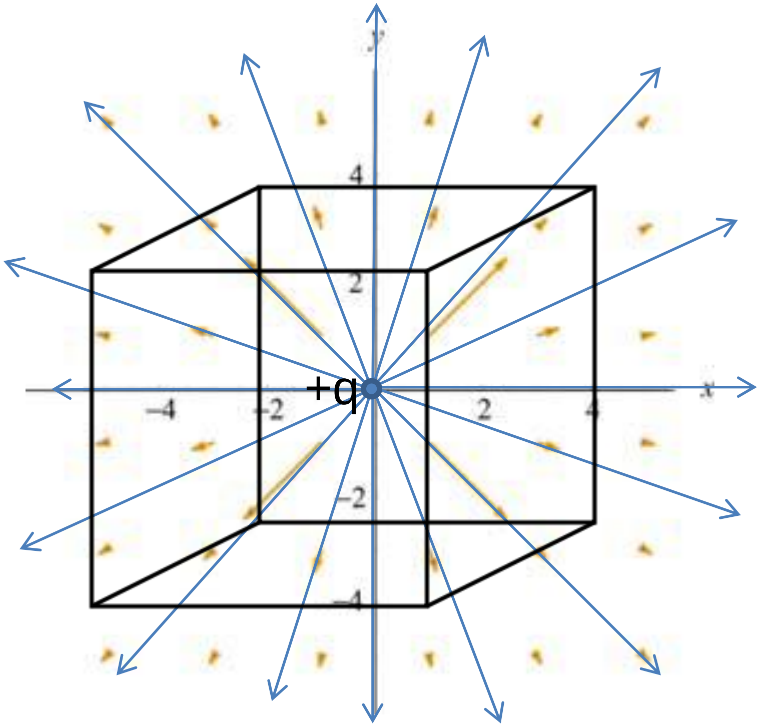
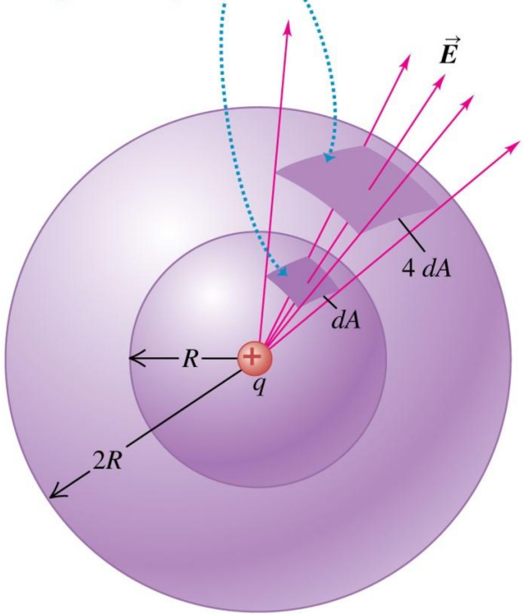


Gauss's Law

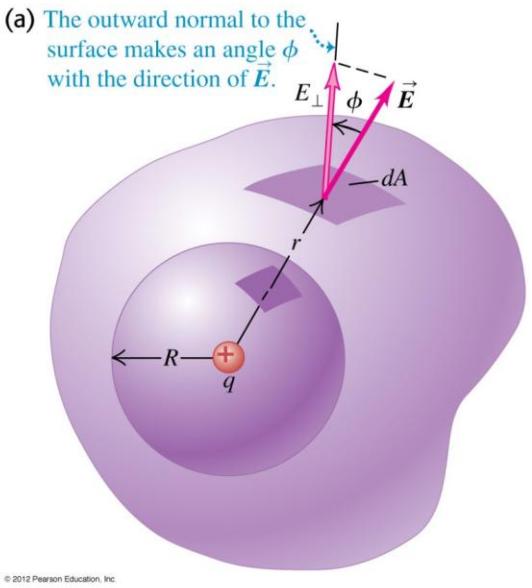
- But what if we were to double the radius of the sphere we used, wouldn't the area be bigger and so the flux be bigger?
- The area goes up by r^2, but the field goes down as $1/r^2$, so the flux remains constant.
- Again, one can see this visually by looking at the field lines. For the two surfaces, the same number of field lines passes through. That's because the field gets weaker (the field lines get farther spaced out as you go away from the charge).

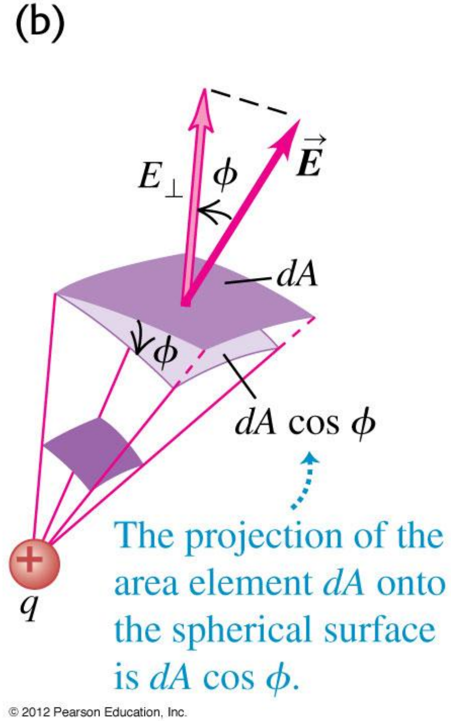
$$\Phi_{\text{E,Net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$



Gauss's Law

- You can use the last result to prove that Gauss's Law is independent of shape of the surface.



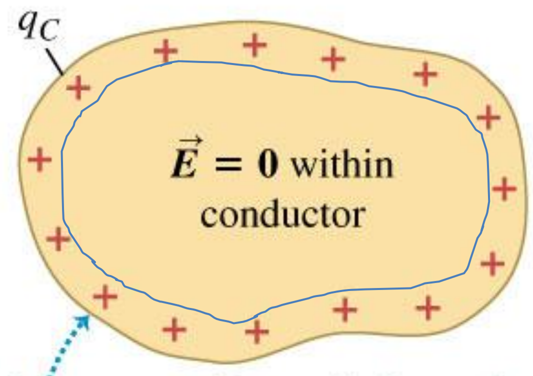


Conductors

- Let's use what we know about conductors in a electrostatics together with Gauss's Law to expand our knowledge…
  - E = 0 inside a conductor

$$\Phi_{\text{E,Net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

(a) Solid conductor with charge q_C



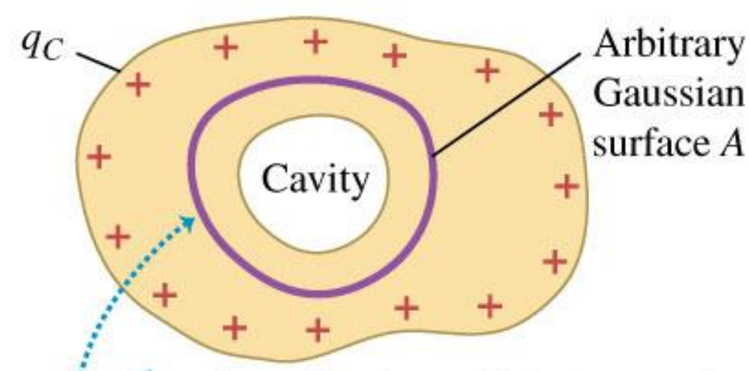
The charge q_C resides entirely on the surface of the conductor. The situation is electrostatic, so $\vec{E} = \mathbf{0}$ within the conductor.

Conductors

- Let's use what we know about conductors in a electrostatics together with Gauss's Law to expand our knowledge…
  - E = 0 inside a conductor
  - All (free) charges on a conductor lie on it's surfaces.

$$\Phi_{\text{E,Net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

(b) The same conductor with an internal cavity



Because $\vec{E} = \mathbf{0}$ at all points within the conductor, the electric field at all points on the Gaussian surface must be zero.

Conductors

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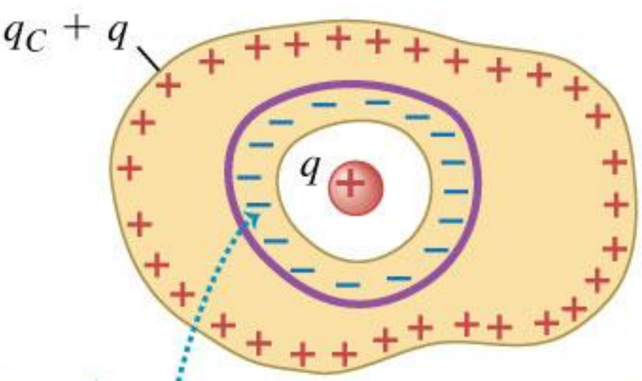
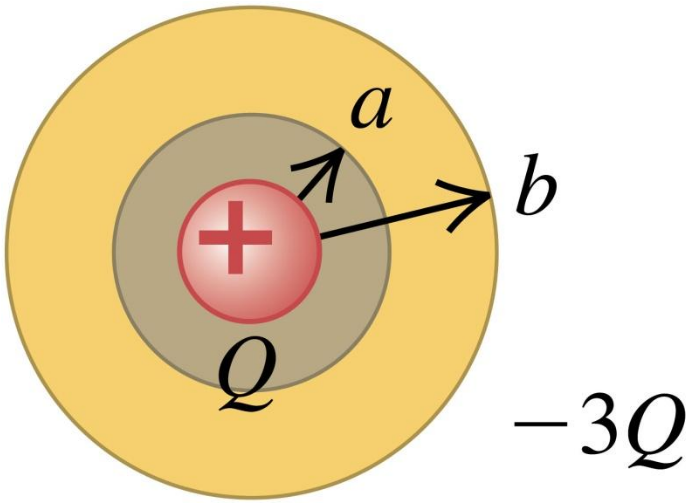
(c) An isolated charge q placed in the cavity



For $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$.

CPS 14-1

A conducting spherical shell with inner radius *a* and outer radius *b* has a positive point charge *Q* located at its center. The total charge on the shell is –3*Q*, and it is insulated from its surroundings. In the region *a* < *r* < *b*,



A. the electric field points radially outward.

B. the electric field points radially inward.

C. the electric field is zero.

D. not enough information given to decide

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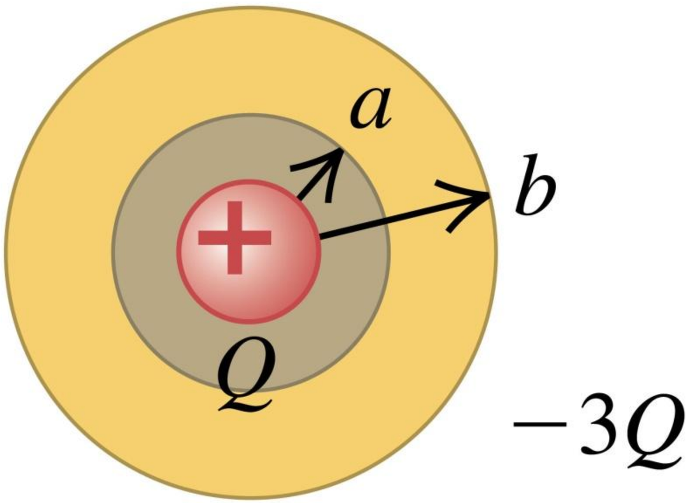

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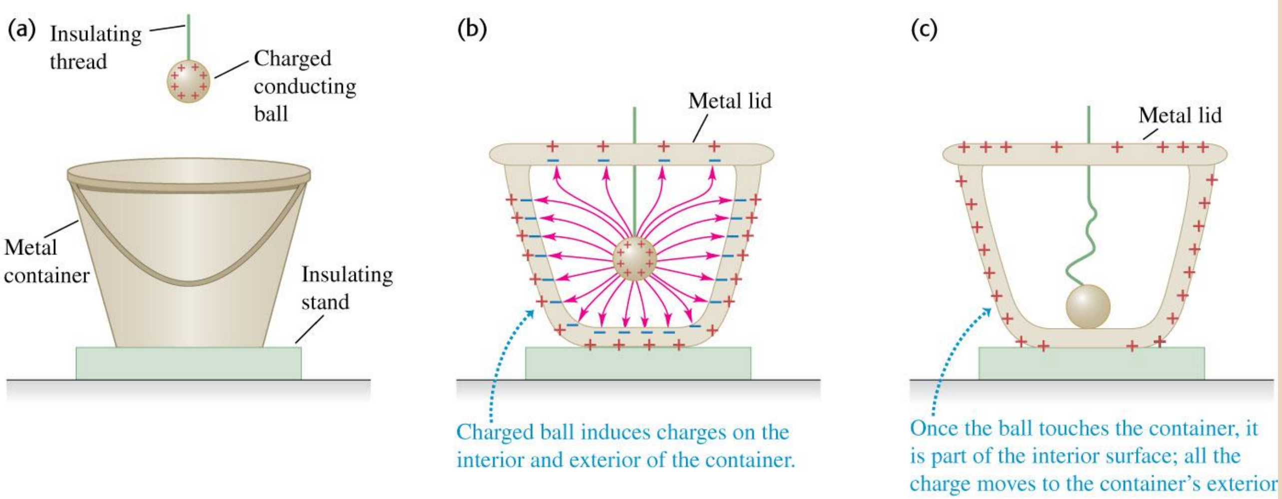
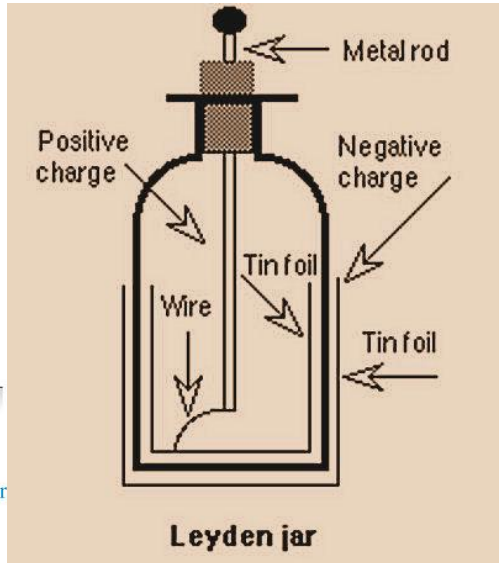
Conductors

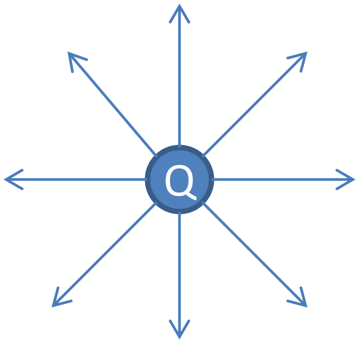
- This is the basic idea behind the very first method to store electricity, the Leyden Jar:

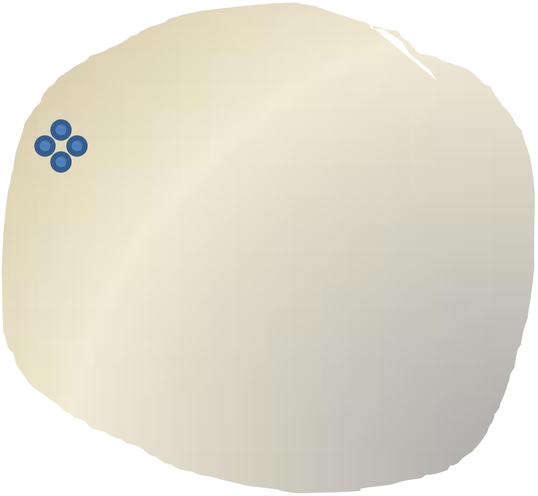




Conductors

- Charge can sit on the surface of a conductor.
- For a static situation, the electric field must be ***perpendicular*** to the surface.
- Why? Because if it wasn't, the charge would move along the surface.
  - E = 0 inside a conductor.
  - E is perpendicular to the surface immediately outside a conductor.
  - All (free) charges on a conductor lie on it's surfaces.





CPS 14-2

There is a negative surface charge density in a certain region on the surface of a solid conductor.

Just beneath the surface of this region, the electric field

A. points outward, toward the surface of the conductor.

B. points inward, away from the surface of the conductor.

C. points parallel to the surface.

D. is zero.

E. not enough information given to decide

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D. is zero.

E. not enough information given to decide

Conductors

- If we stay very close to the conductor and put a very small “pill box” (just a cylinder) through the surface of the conductor, then using what we’ve just discovered:
  - E = 0 inside a conductor.
  - E is perpendicular to the surface immediately outside a conductor.
  - All (free) charges on a conductor lie on it’s surfaces.

$$\Phi_{\text{E,Net}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \Rightarrow$$

$$EA_{\text{top cap}} = \frac{\sigma A_{\text{top cap}}}{\varepsilon_0} \Rightarrow$$

$$E = \frac{\sigma(\text{local})}{\varepsilon_0} \quad (\text{Near a conductor})$$