SOLN'S
In-class exercise #3
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Pre-lecture work.

$\boxed{1.}$

$$ms^2 Z(s) = -\theta_1 s Z(s) + \theta_2 T(s)$$
$$sT(s) = -\theta_3 T(s) + \theta_4 Q(s)$$

$$\Rightarrow T(s) = \frac{\theta_4}{s+\theta_3} \cdot Q(s)$$

$$\Rightarrow Z(s)(ms^2 + \theta_1 s) = \theta_2 \cdot \frac{\theta_4}{s+\theta_3} \cdot Q(s)$$

$$\frac{Z(s)}{Q(s)} = \frac{\theta_2/m \cdot \theta_4}{s(s+\theta_1/m)(s+\theta_3)}$$

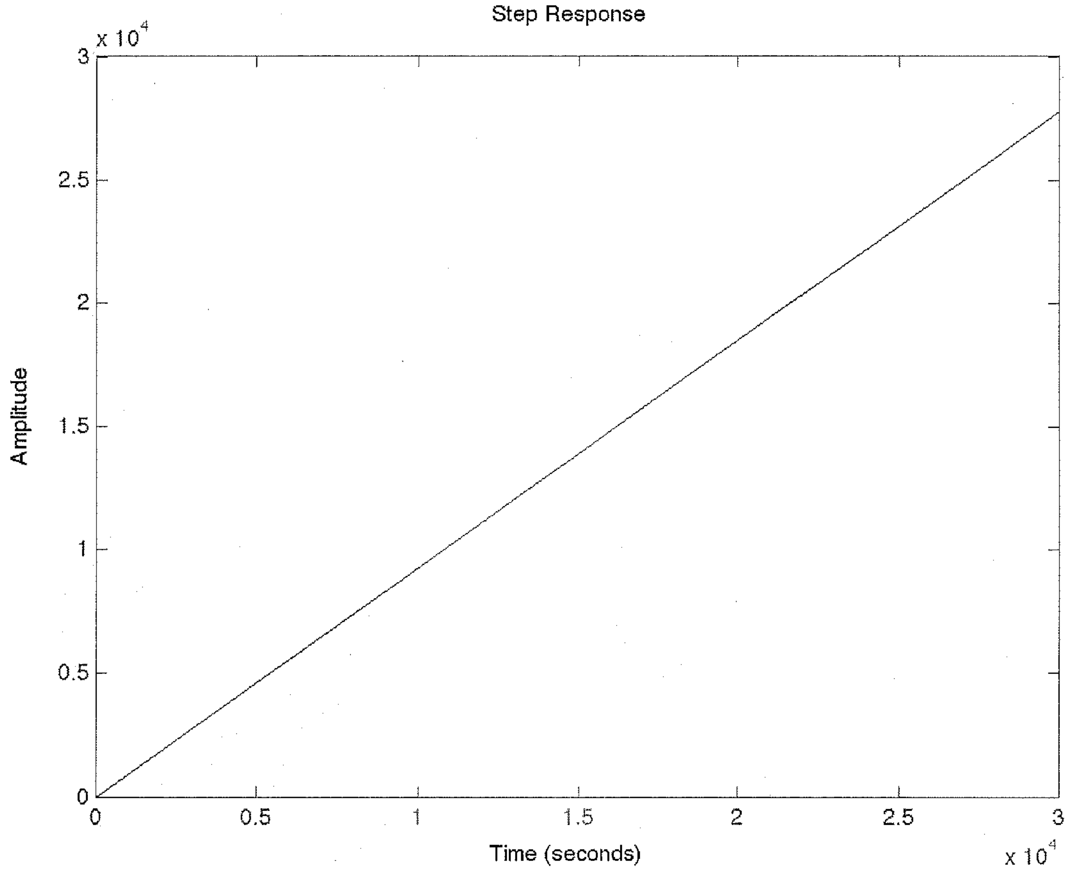
2. Poles at $s = 0, -\theta_1/m, -\theta_3$

$\Rightarrow$ marginal stability

3. See next page for step response plot

4. The system is BIBO unstable.

A bounded input (a step) results in an unbounded output (a ramp).


Step Response
x 10^4
3
2.5
2
1.5
1
0.5
0
Amplitude
0
0.5
1
1.5
2
2.5
3
Time (seconds)
x 10^4

In-class work

1. $\Delta(s) = D(s) + kN(s)$

$$= s(s+\theta_1/m)(s+\theta_3) + k\cdot\theta_2/m\cdot\theta_4$$

$$= s^3 + s^2\left(\frac{\theta_1}{m}+\theta_3\right) + s\cdot\frac{\theta_1}{m}\cdot\theta_3 + k\cdot\frac{\theta_2}{m}\cdot\theta_4$$

$$= s^3 + a_2\cdot s^2 + a_1\cdot s + a_0\cdot K$$

2. $(s+\alpha)(s^2+2\zeta\omega_n s+\omega_n^2)$

$$= s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s + \alpha s^2 + 2\zeta\omega_n\alpha\cdot s + \omega_n^2\alpha$$

$$= s^3 + (2\zeta\omega_n+\alpha)s^2 + (\omega_n^2+2\zeta\omega_n\alpha)s + \omega_n^2\alpha$$

$$= s^3 + (\omega_n+\alpha)s^2 + (\omega_n^2+\omega_n\alpha)s + \omega_n^2\alpha$$

$$\Rightarrow \begin{cases} a_2 = \omega_n+\alpha \\ a_1 = \omega_n^2+\omega_n\alpha = \omega_n(\omega_n+\alpha) \\ Ka_0 = \omega_n^2\cdot\alpha \end{cases} \Rightarrow a_1 = \omega_n\cdot a_2, \quad \frac{a_1}{a_2} = \omega_n \quad *$$

and $\alpha = a_2 - \omega_n = a_2 - \frac{a_1}{a_2}$

So $K = \dfrac{\omega_n^2\alpha}{a_0} = \dfrac{\left(\frac{a_1}{a_2}\right)^2\left(a_2-\frac{a_1}{a_2}\right)}{a_0}$

$\Rightarrow$ (b) is correct

4. The open-loop system is marginally stable; closed-loop is asymptotically stable. The open-loop pole at 0 means that in response to an input $\frac{1}{s}$, the output will have a $\frac{1}{s^2}$ term ($= t\cdot c$ term in the time-domain), hence the ramp output. The closed loop has poles in the open LHP which implies convergence in $y(t)$ to 0 as $t\to\infty$.

3. $\alpha > 0$, and $\zeta\omega_n > 0$ $\Rightarrow$ all poles of closed-loop system in open LHP

$\therefore$ Asymptotically stable, $\Rightarrow$ BIBO stable.

(a) & (d) is correct.

5. (c) or (d) indicate an unbounded response to a step input, not possible in a BIBO system. With positive coefficients in numerator of KG(s), (a) is most reasonable.

6. Stability is improved through feedback. Asymptotic stability is stronger than marginal stability. BIBO stability is gained only through feedback.