Chapter 7: Potential Energy and Energy Conservation

Due: 11:00pm on Tuesday, October 9, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Introduction to Potential Energy

Learning Goal:

Understand that conservative forces can be removed from the work integral by incorporating them into a new form of energy called potential energy that must be added to the kinetic energy to get the total mechanical energy.

The first part of this problem contains short-answer questions that review the work-energy theorem. In the second part we introduce the concept of potential energy. But for now, please answer in terms of the work-energy theorem.

Work-Energy Theorem
The work-energy theorem states

$$K_{\rm f} = K_{\rm i} + W_{\rm all},$$

where $W_{\rm all}$ is the work done by *all* forces that act on the object, and $K_{\rm i}$ and $K_{\rm f}$ are the initial and final kinetic energies, respectively.

Part A

The work-energy theorem states that a force acting on a particle as it moves over a ______ changes the ______ energy of the particle if the force has a component parallel to the motion.

Choose the best answer to fill in the blanks above:

ANSWER:

- distance / potential
- distance / kinetic (selected)
- vertical displacement / potential
- none of the above

It is important that the force have a component acting in the direction of motion. For example, if a ball is attached to a string and whirled in uniform circular motion, the string does apply a force to the ball, but since the string's force is always perpendicular to the motion it does no work and cannot change the kinetic energy of the ball.

Part B

To calculate the change in energy, you must know the force as a function of _______. The work done by the force causes the energy change.

Choose the best answer to fill in the blank above:

ANSWER:

- acceleration
- work
- distance (selected)
- potential energy

Part C

To illustrate the work-energy concept, consider the case of a stone falling from $x_{\rm i}$ to $x_{\rm f}$ under the influence of gravity.

Using the work-energy concept, we say that work is done by the gravitational _____, resulting in an increase of the ______ energy of the stone.

Choose the best answer to fill in the blanks above:

ANSWER:

- (selected) force / kinetic
- potential energy / potential
- force / potential
- potential energy / kinetic

Potential Energy You should read about potential energy in your text before answering the following questions.

Potential energy is a concept that builds on the work-energy theorem, enlarging the concept of energy in the most physically useful way. The key aspect that allows for potential energy is the existence of conservative forces, forces for which the work done on an object does not depend on the path of the object, only the initial and final positions of the object. The gravitational force is conservative; the frictional force is not.

The change in potential energy is the *negative* of the work done by conservative forces. Hence considering the initial and final potential energies is equivalent to calculating the work done by the conservative forces. When potential energy is used, it *replaces the work* done by the associated conservative force. Then only the work due to *nonconservative* forces needs to be calculated.

In summary, when using the concept of potential energy, only *nonconservative* forces contribute to the work, which now changes the total energy:

$$K_{\rm f} + U_{\rm f} = E_{\rm f} = W_{\rm nc} + E_{\rm i} = W_{\rm nc} + K_{\rm i} + U_{\rm i},$$

where $U_{\rm f}$ and $U_{\rm i}$ are the final and initial potential energies, and $W_{\rm nc}$ is the work due *only* to nonconservative forces.

Now, we will revisit the falling stone example using the concept of potential energy.

Part D

Rather than ascribing the increased kinetic energy of the stone to the work of gravity, we now (when using potential energy rather than work-energy) say that the increased kinetic energy comes from the ______ of the _______ energy.

Choose the best answer to fill in the blanks above:

ANSWER:

- work / potential
- force / kinetic
- (selected) change / potential

Part E

This process happens in such a way that *total mechanical energy*, equal to the ______ of the kinetic and potential energies, is _______.

Choose the best answer to fill in the blanks above:

ANSWER:

- (selected) sum / conserved
- sum / zero
- sum / not conserved
- difference / conserved

Potential Energy Calculations

Learning Goal:

To understand the relationship between the force and the potential energy changes associated with that force and to be able to calculate the changes in potential energy as definite integrals.

Imagine that a *conservative force field* is defined in a certain region of space. Does this sound too abstract? Well, think of a *gravitational field* (the one that makes apples fall down and keeps the planets orbiting) or an *electrostatic field* existing around any electrically charged object.

If a particle is moving in such a field, its change in potential energy does not depend on the particle's path and is determined only by the particle's initial and final positions. Recall that, in general, the component of the net force acting on a particle equals the negative derivative of the potential energy function along the corresponding axis:

$$F_x = -\frac{dU(x)}{dx}.$$

Therefore, the change in potential energy can be found as the integral

$$\Delta U = -\int_1^2 \vec{F} \cdot d\vec{s},$$

where ΔU is the change in potential energy for a particle moving from point 1 to point 2, $\vec{F}$ is the net force acting on the particle at a given point of its path, and $d\vec{s}$ is a small displacement of the particle along its path from 1 to 2.

Evaluating such an integral in a general case can be a tedious and lengthy task. However, two circumstances make it easier:

1. Because the result is *path-independent*, it is always possible to consider the most straightforward way to reach point 2 from point 1.
2. The most common real-world fields are rather simply defined.

In this problem, you will practice calculating the change in potential energy for a particle moving in three common force fields.

Note that, in the equations for the forces, $\hat{x}$ is the unit vector in the *x* direction, $\hat{y}$ is the unit vector in the *y* direction, and $\hat{r}$ is the unit vector in the radial direction in case of a spherically symmetrical force field.

Part A

Consider a *uniform gravitational field* (a fair approximation near the surface of a planet). Find

$$U(y_f) - U(y_0) = -\int_{y_0}^{y_f} \vec{F}_g \cdot d\vec{s},$$

where

$$\vec{F}_g = -mg\,\hat{y} \text{ and } d\vec{s} = dy\,\hat{y}.$$

Express your answer in terms of m, g, y_0, and y_f.

Hint 1. Relative directions of $\vec{F}_g$ and $d\vec{s}$

Note that $\vec{F}_g$ and $d\vec{s}$ are parallel, and their dot product is simply the product of their magnitudes. That is,

$$\vec{F}_g \cdot d\vec{s} = -mg\,dy.$$

ANSWER:

$U(y_f) - U(y_0) =$ $mg\,(y_f - y_0)$

Part B

Consider the force exerted by a spring that obeys Hooke's law. Find

$$U(x_f) - U(x_0) = -\int_{x_0}^{x_f} \vec{F}_s \cdot d\vec{s},$$

where

$$\vec{F}_s = -kx\,\hat{x}, \quad d\vec{s} = dx\,\hat{x},$$

and the spring constant k is positive.

Express your answer in terms of k, x_0, and x_f.

Hint 1. Relative directions of $\vec{F}_s$ and $d\vec{s}$

Note that $\vec{F}_s$ and $d\vec{s}$ are parallel, and their dot product is simply the product of their magnitudes. That is,

$$\vec{F}_s \cdot d\vec{s} = -kx\,dx.$$

ANSWER:

$U(x_f) - U(x_0) = \frac{k}{2}\left(x_f{}^2 - x_0{}^2\right)$

Part C

Finally, consider the gravitational force generated by a spherically symmetrical massive object. The magnitude and direction of such a force are given by Newton's law of gravity:

$$\vec{F}_G = -\frac{Gm_1m_2}{r^2}\hat{r},$$

where $d\vec{s} = dr\,\hat{r}$; G, m_1, and m_2 are constants; and $r > 0$. Find

$$U(r_f) - U(r_0) = -\int_{r_0}^{r_f} \vec{F}_G \cdot d\vec{s}.$$

Express your answer in terms of G, m_1, m_2, r_0, and r_f.

Hint 1. Relative directions of $\vec{F}_G$ and $d\vec{s}$

Note that $\vec{F}_G$ and $d\vec{s}$ are parallel, and their dot product is simply the product of their magnitudes. That is,

$$\vec{F}_G \cdot d\vec{s} = -\frac{Gm_1m_2}{r^2}dr.$$

Hint 2. Integrating $1/r^2$

Recall that

$$\int \frac{dr}{r^2} = -\frac{1}{r}.$$

Carefully account for all the negative signs in your calculations.

ANSWER:

$U(r_f) - U(r_0) = (Gm_1m_2)\left(\frac{1}{r_0} - \frac{1}{r_f}\right)$

As you can see, the change in potential energy of the particle can be found by integrating the force along the particle's path. However, this method, as we mentioned before, does have an important restriction: It can only be applied to a *conservative* force field. For conservative forces such as gravity or tension *the work done on the particle does not depend on the particle's path*, and the potential energy is the function of the particle's position.

In case of a *nonconservative force*–such as a frictional or magnetic force–the potential energy can no longer be defined as a function of the particle's position, and the method that you used in this problem would not be applicable.

Potential Energy Graphs and Motion

Learning Goal:

To be able to interpret potential energy diagrams and predict the corresponding motion of a particle.

Potential energy diagrams for a particle are useful in predicting the motion of that particle. These diagrams allow one to determine the direction of the force acting on the particle at any point, the points of stable and unstable equilibrium, the particle's kinetic energy, etc.
Consider the potential energy diagram shown. The curve represents the value of potential energy U as a function of the particle's coordinate x. The horizontal line above the curve represents the constant value of the total energy of the particle E. The total energy E is the sum of kinetic (K) and potential (U) energies of the particle.

The key idea in interpreting the graph can be expressed in the equation

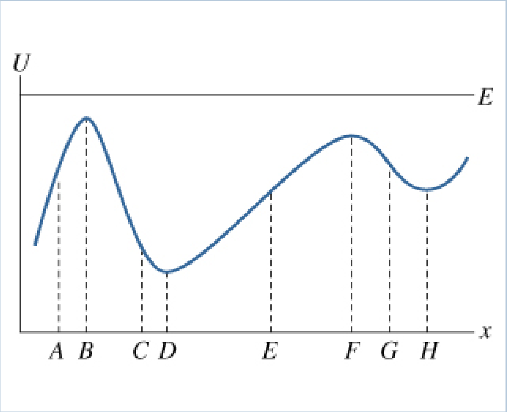
$$F_x(x) = -\frac{dU(x)}{dx},$$

where $F_x(x)$ is the *x* component of the net force as function of the particle's coordinate x. Note the negative sign: It means that the *x* component of the net force is negative when the derivative is positive and vice versa. For instance, if the particle is moving to the right, and its potential energy is increasing, the net force would be pulling the particle to the left.

If you are still having trouble visualizing this, consider the following: If a massive particle is increasing its gravitational potential energy (that is, moving upward), the force of gravity is pulling in the *opposite* direction (that is, downward).

If the *x* component of the net force is zero, the particle is said to be in *equilibrium*. There are two kinds of equilibrium:

- *Stable equilibrium* means that small deviations from the equilibrium point create a net force that accelerates the particle back toward the equilibrium point (think of a ball rolling between two hills).
- *Unstable equilibrium* means that small deviations from the equilibrium point create a net force that accelerates the particle further away from the equilibrium point (think of a ball on top of a hill).



In answering the following questions, we will assume that there is a single varying force F acting on the particle along the *x* axis. Therefore, we will use the term *force* instead of the cumbersome *x component of the net force*.

Part A

The force acting on the particle at point A is ___________.

Hint 1. Sign of the derivative

If a function increases (as x increases) in a certain region, then the derivative of the function in that region is positive.

Hint 2. Sign of the component

If x increases to the right, as in the graph shown, then a (one-dimensional) vector with a positive *x* component points to the right, and vice versa.

ANSWER:

- ○ directed to the right
- ◉ directed to the left
- ○ equal to zero

Consider the graph in the region of point A. If the particle is moving to the right, it would be "climbing the hill," and the force would "pull it down," that is, pull the particle back to the left. Another, more abstract way of thinking about this is to say that the slope of the graph at point A is *positive*; therefore, the direction of $\vec{F}$ is *negative*.

Part B

The force acting on the particle at point C is ___________.

Hint 1. Sign of the derivative

If a function increases (as x increases) in a certain region, then the derivative of the function in that region is positive, and vice versa.

Hint 2. Sign of the component

If x increases to the right, as in the graph shown, then a (one-dimensional) vector with a positive *x* component points to the right, and vice versa.

ANSWER:

- ◉ directed to the right
- ○ directed to the left
- ○ equal to zero

Part C

The force acting on the particle at point B is __________.

Hint 1. Derivative of a function at a local maximum

At a local maximum, the derivative of a function is equal to zero.

ANSWER:

- directed to the right
- directed to the left
- equal to zero (selected)

The slope of the graph is zero; therefore, the derivative $dU/dx = 0$, and $|\vec{F}| = 0$.

Part D

The acceleration of the particle at point B is __________.

Hint 1. Relation between acceleration and force

The relation between acceleration and force is given by Newton's 2nd law,

$$F = ma.$$

ANSWER:

- directed to the right
- directed to the left
- equal to zero (selected)

If the net force is zero, so is the acceleration. The particle is said to be in a state of *equilibrium*.

Part E

If the particle is located slightly to the left of point B, its acceleration is __________.

Hint 1. The force on such a particle

To the left of B, $U(x)$ is an increasing function and so its derivative is positive. This implies that the *x* component of the force on a particle at this location is negative, or that the force is directed to the left, just like at A. What can you say now about the acceleration?

ANSWER:

- directed to the right
- directed to the left (selected)
- equal to zero

Part F

If the particle is located slightly to the right of point B, its acceleration is __________.

Hint 1. The force on such a particle

To the right of B, $U(x)$ is a decreasing function and so its derivative is negative. This implies that the *x* component of the force on a particle at this location is positive, or that the force is directed to the right, just like at C. What can you now say about the acceleration?

ANSWER:

- (selected) directed to the right
- directed to the left
- equal to zero

As you can see, small deviations from equilibrium at point B cause a force that accelerates the particle further away; hence the particle is in *unstable equilibrium*.

Part G

Name all labeled points on the graph corresponding to *unstable* equilibrium.

List your choices alphabetically, with no commas or spaces; for instance, if you choose points B, D, and E, type your answer as `BDE`.

Hint 1. Definition of unstable equilibrium

Unstable equilibrium means that small deviations from the equilibrium point create a net force that accelerates the particle further away from the equilibrium point (think of a ball on top of a hill).

ANSWER:

BF

Part H

Name all labeled points on the graph corresponding to *stable* equilibrium.

List your choices alphabetically, with no commas or spaces; for instance, if you choose points B, D, and E, type your answer as `BDE`.

Hint 1. Definition of stable equilibrium

Stable equilibrium means that small deviations from the equilibrium point create a net force that accelerates the particle back toward the equilibrium point. (Think of a ball rolling between two hills.)

ANSWER:

DH

Part I

Name all labeled points on the graph where the acceleration of the particle is zero.

List your choices alphabetically, with no commas or spaces; for instance, if you choose points B, D, and E, type your answer as `BDE`.

Hint 1. Relation between acceleration and force

The relation between acceleration and force is given by Newton's 2nd law,

$$F = ma.$$

ANSWER:

BDFH

Your answer, of course, includes the locations of both stable and unstable equilibrium.

Part J

Name all labeled points such that when a particle is released from rest there, it would accelerate to the left.

List your choices alphabetically, with no commas or spaces; for instance, if you choose points B, D, and E, type your answer as `BDE`.

Hint 1. Determine the sign of the *x* component of force

If the acceleration is to the left, so is the force. This means that the *x* component of the force is __________.

ANSWER:

- ○ positive
- ◉ negative

Hint 2. What is the behavior of $U(x)$?

If the *x* component of the force at a point is negative, then the derivative of $U(x)$ at that point is positive. This means that in the region around the point $U(x)$ is __________.

ANSWER:

- ◉ increasing
- ○ decreasing

ANSWER:

AE

Part K

Consider points A, E, and G. Of these three points, which one corresponds to the greatest magnitude of acceleration of the particle?

Hint 1. Acceleration and force

The greatest acceleration corresponds to the greatest magnitude of the net force, represented on the graph by the magnitude of the slope.

ANSWER:

- ◉ A
- ○ E
- ○ G

Kinetic energy

If the total energy E of the particle is known, one can also use the graph of $U(t)$ to draw conclusions about the kinetic energy of the particle since

$$K = E - U.$$

As a reminder, on this graph, the total energy E is shown by the horizontal line.

Part L

What point on the graph corresponds to the maximum kinetic energy of the moving particle?

Hint 1. K, U, and E

Since the total energy does not change, the maximum kinetic energy corresponds to the minimum potential energy.

ANSWER:

D

It makes sense that the kinetic energy of the particle is maximum at one of the (force) equilibrium points. For example, think of a pendulum (which has only one force equilibrium point--at the very bottom).

Part M

At what point on the graph does the particle have the lowest speed?

ANSWER:

B

As you can see, many different conclusions can be made about the particle's motion merely by looking at the graph. It is helpful to understand the character of motion qualitatively before you attempt quantitative problems. This problem should prove useful in improving such an understanding.

Ups and Downs

Learning Goal:

To apply the law of conservation of energy to an object launched upward in the gravitational field of the earth.

In the absence of nonconservative forces such as friction and air resistance, the total mechanical energy in a closed system is conserved. This is one particular case of the *law of conservation of energy*.

In this problem, you will apply the law of conservation of energy to different objects launched from the earth. The energy transformations that take place involve the object's kinetic energy $K = (1/2)mv^2$ and its gravitational potential energy $U = mgh$. The law of conservation of energy for such cases implies that the sum of the object's kinetic energy and potential energy does not change with time. This idea can be expressed by the equation

$$K_i + U_i = K_f + U_f,$$

where "i" denotes the "initial" moment and "f" denotes the "final" moment. Since any two moments will work, the choice of the moments to consider is, technically, up to you. That choice, though, is usually suggested by the question posed in the problem.

First, let us consider an object launched vertically upward with an initial speed v. Neglect air resistance.

Part A

As the projectile goes upward, what energy changes take place?

ANSWER:

- Both kinetic and potential energy decrease.
- Both kinetic and potential energy increase.
- Kinetic energy decreases; potential energy increases.
- Kinetic energy increases; potential energy decreases.

Part B

At the top point of the flight, what can be said about the projectile's kinetic and potential energy?

ANSWER:

- Both kinetic and potential energy are at their maximum values.
- Both kinetic and potential energy are at their minimum values.
- Kinetic energy is at a maximum; potential energy is at a minimum.
- Kinetic energy is at a minimum; potential energy is at a maximum.

Strictly speaking, it is not the ball that possesses potential energy; rather, it is the system "Earth-ball." Although we will often talk about "the gravitational potential energy of an elevated object," it is useful to keep in mind that the energy, in fact, is associated with the interactions between the earth and the elevated object.

Part C

The potential energy of the object at the moment of launch ___________.

ANSWER:

- is negative
- is positive
- is zero
- depends on the choice of the "zero level" of potential energy

Usually, the zero level is chosen so as to make the relevant calculations simpler. In this case, it makes good sense to assume that $U = 0$ at the ground level--but this is not, by any means, the only choice!

Part D

Using conservation of energy, find the maximum height $h_{\rm max}$ to which the object will rise.

Express your answer in terms of v and the magnitude of the acceleration of gravity g.

ANSWER:

$h_{\rm max} = \frac{v^2}{2g}$

You may remember this result from kinematics. It is comforting to know that our new approach yields the same answer.

Part E

At what height h above the ground does the projectile have a speed of $0.5v$?

Express your answer in terms of v and the magnitude of the acceleration of gravity g.

ANSWER:

$h = \frac{3v^2}{8g}$

Part F

What is the speed u of the object at the height of $(1/2)h_{\rm max}$?

Express your answer in terms of v and g. Use three significant figures in the numeric coefficient.

Hint 1. How to approach the problem

You are being asked for the speed at half of the maximum height. You know that at the initial height ($h=0$), the speed is v. All of the energy is kinetic energy, and so, the total energy is $(1/2)mv^2$. At the maximum height, all of the energy is potential energy. Since the gravitational potential energy is proportional to h, half of the initial kinetic energy must have been converted to potential energy when the projectile is at $(1/2)h_{\rm max}$. Thus, the kinetic energy must be half of its original value (i.e., $(1/4)mv^2$ when $h=(1/2)h_{\rm max}$). You need to determine the speed, as a multiple of v, that corresponds to such a kinetic energy.

ANSWER:

u = $0.707v$

Let us now consider objects launched at an angle. For such situations, using conservation of energy leads to a quicker solution than can be produced by kinematics.

Part G

A ball is launched as a projectile with initial speed v at an angle θ above the horizontal. Using conservation of energy, find the maximum height $h_{\rm max}$ of the ball's flight.

Express your answer in terms of v, g, and θ.

Hint 1. Find the final kinetic energy

Find the final kinetic energy $K_{\rm f}$ of the ball. Here, the best choice of "final" moment is the point at which the ball reaches its maximum height, since this is the point we are interested in.

Express your answer in terms of v, m, and θ.

Hint 1. Find the speed at the maximum height

The speed of the ball at the maximum height is __________.

ANSWER:

- ○ 0
- ○ v
- ◉ $v\cos\theta$
- ○ $v\sin\theta$
- ○ $v\tan\theta$

ANSWER:

$K_{\rm f}$ = $0.5m\left(v\cos\left(\theta\right)\right)^2$

ANSWER:

$h_{\rm max}$ = $\frac{\left(v\sin\left(\theta\right)\right)^2}{2g}$

Part H

A ball is launched with initial speed v from ground level up a frictionless slope. The slope makes an angle θ with the horizontal. Using conservation of energy, find the maximum *vertical* height $h_{\rm max}$ to which the ball will climb.

Express your answer in terms of v, g, and θ. You may or may not use all of these quantities.

ANSWER:

$h_{\rm max}$ = $\frac{v^2}{2g}$

Interestingly, the answer does *not* depend on θ. The difference between this situation and the projectile case is that the ball moving up a slope has no kinetic energy at the top of its trajectory whereas the projectile launched at an angle does.

Part I

A ball is launched with initial speed v from the ground level up a frictionless hill. The hill becomes steeper as the ball slides up; however, the ball remains in contact with the hill at all times. Using conservation of energy, find the maximum *vertical* height $h_{\rm max}$ to which the ball will climb.

Express your answer in terms of v and g.

ANSWER:

$h_{\rm max}$ = $\frac{v^2}{2g}$

The profile of the hill does not matter; the equation

$$K_i + U_i = K_f + U_f$$

would have the same terms regardless of the steepness of the hill.

Exercise 7.5

A baseball is thrown from the roof of 20.6$\rm m$ -tall building with an initial velocity of magnitude 12.7$\rm m/s$ and directed at an angle of 51.5° above the horizontal.

Part A

What is the speed of the ball just before it strikes the ground? Use energy methods and ignore air resistance.

ANSWER:

v_2 = $\sqrt{(v_1)^2 + 2gh}$ = 23.8 $\rm m/s$

Part B

What is the answer for part (A) if the initial velocity is at an angle of 51.5° *below* the horizontal?

ANSWER:

v_2 = $\sqrt{(v_1)^2 + 2gh}$ = 23.8 $\rm m/s$

Part C

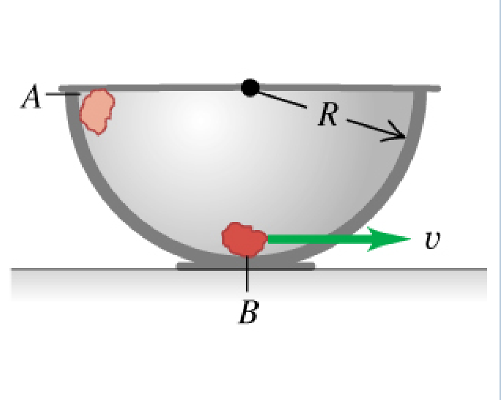
If the effects of air resistance are included, will part (A) or (B) give the higher speed?

ANSWER:

- The part (A) will give the higher speed.
- The part (B) will give the higher speed.

Exercise 7.9

A small rock with mass 0.20kg is released from rest at point A, which is at the top edge of a large, hemispherical bowl with radius R = 0.50m (the figure). Assume that the size of the rock is small compared to R, so that the rock can be treated as a particle, and assume that the rock slides rather than rolls. The work done by friction on the rock when it moves from point A to point B at the bottom of the bowl has magnitude 0.22 J.



Part A

Between points A and B, how much work is done on the rock by the normal force?

Express your answer using two significant figures.

ANSWER:

W = 0 J

Part B

Between points A and B, how much work is done on the rock by gravity?

Express your answer using two significant figures.

ANSWER:

W = $9.8mR$ = 0.98 J

Part C

What is the speed of the rock as it reaches point B?

Express your answer using two significant figures.

ANSWER:

$v = \sqrt{\frac{2\left(9.8mR - .22\right)}{m}}$ = 2.8 $\mathrm{m/s}$

Part D

Of the three forces acting on the rock as it slides down the bowl, which (if any) are constant and which are not? Explain.

ANSWER:

3663 Character(s) remaining

Gravity is constant and equal to mg. n is not constant; it is zero at A and not zero at B. Therefore, f_k = mu_k*n is

Part E

Just as the rock reaches point B, what is the normal force on it due to the bottom of the bowl?

Exercise 7.9

Part D ANSWER:

Gravity is constant and equal to mg. n is not constant; it is zero at A and not zero at B. Therefore, $f_k = \mu_k n$ is also not constant.

Express your answer using two significant figures.

ANSWER:

$n = m\left(9.8 + \frac{\frac{2(9.8mR-.22)}{m}}{R}\right)$ = 5.0 N

Exercise 7.15

A force of 600 N stretches a certain spring a distance of 0.400 m .

Part A

What is the potential energy of the spring when it is stretched a distance of 0.400 m ?

ANSWER:

$U_1 = \frac{Fl}{2}$ = 120 J

Part B

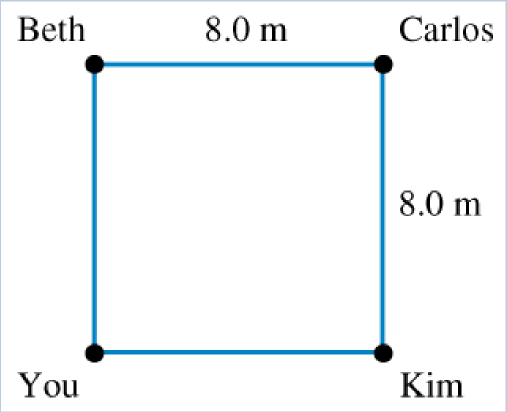
What is its potential energy when it is compressed a distance of 6.00 cm ?

ANSWER:

$U_2 = \frac{\frac{Fyy}{2}}{l}$ = 2.70 J

Exercise 7.31

You and three friends stand at the corners of a square whose sides are 8.0 m long in the middle of the gym floor, as shown in the figure . You take your physics book and push it from one person to the other. The book has a mass of 1.3 kg , and the coefficient of kinetic friction between the book and the floor is $\mu_k =$ 0.26.



Part A

The book slides from you to Beth and then from Beth to Carlos, along the lines connecting these people. What is the work done by friction during this displacement?

Express your answer using two significant figures.

ANSWER:

$-2m \cdot 9.8f \cdot 8$ = -53 J

Part B

You slide the book from you to Carlos along the diagonal of the square. What is the work done by friction during this displacement?

Express your answer using two significant figures.

ANSWER:

$-\sqrt{2} \cdot 8.0m \cdot 9.8f$ = -37 J

Exercise 7.31

Part E ANSWER:

The work required to go from one point to another is not path independent, and the work required for a round trip is not zero, so friction is not a conservative force.

Part C

You slide the book to Kim who then slides it back to you. What is the total work done by friction during this motion of the book?

Express your answer using two significant figures.

ANSWER:

$-2m\cdot 9.8f\cdot 8.0$ = -53 J

Part D

Is the friction force on the book conservative or nonconservative?

ANSWER:

- conservative
- nonconservative

Part E

Explain.

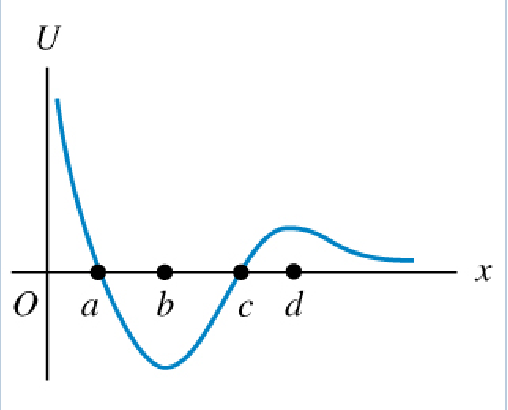
ANSWER:

3633 Character(s) remaining

The work required to go from one point to another is not path independent, and the work required for a round trip is

Exercise 7.38

A marble moves along the x-axis. The potential-energy function is shown in the figure .



Part A

At which of the labeled x-coordinates is the force on the marble zero?

ANSWER:

- a and c
- b and d

Part B

Which of the labeled x-coordinates is a position of stable equilibrium?

ANSWER:

- a
- b (selected)
- c
- d

Part C

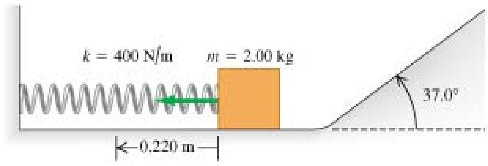
Which of the labeled x-coordinates is a position of unstable equilibrium?

ANSWER:

- a
- b
- c
- d (selected)

Problem 7.42

A 2.00-kg block is pushed against a spring with negligible mass and force constant k = 400 $\mathrm{N/m}$, compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37.0° .



Part A

What is the speed of the block as it slides along the horizontal surface after having left the spring?

ANSWER:

v = 3.11 $\mathrm{m/s}$

Part B

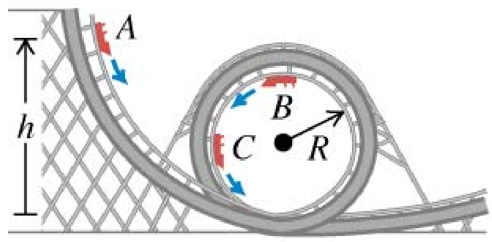
How far does the block travel up the incline before starting to slide back down?

ANSWER:

L = 0.821 m

Problem 7.46

A car in an amusement park ride rolls without friction around the track shown in the figure . It starts from rest at point A at a height h above the bottom of the loop. Treat the car as a particle.



Part A

What is the minimum value of h (in terms of R) such that the car moves around the loop without falling off at the top (point B)?

ANSWER:

h_{min} = $\frac{5R}{2}$ = 5.46

Also accepted: $2.5R$ = 7.67

Part B

If the car starts at height $h = 5.00$ R and the radius is R = 23.0m , compute the speed of the passengers when the car is at point C, which is at the end of a horizontal diameter.

ANSWER:

v_C = $\sqrt{2\,(kR_1 - R_1)\,g}$ = 42.5 $\mathrm{m/s}$

Part C

If the car starts at height $h = 5.00$ R and the radius is R = 23.0m , compute the radial acceleration of the passengers when the car is at point C, which is at the end of a horizontal diameter.

ANSWER:

a_{rad} = $2\,(k-1)\,g$ = 78.4 $\mathrm{m/s^2}$

Part D

If the car starts at height $h = 5.00$ R and the radius is R = 23.0m , compute the tangential acceleration of the passengers when the car is at point C, which is at the end of a horizontal diameter.

ANSWER:

$|a_{\mathrm{tan}}|$ = 9.80 $\mathrm{m/s^2}$

Problem 7.51

A bungee cord is 30.0 m long and, when stretched a distance x, it exerts a restoring force of magnitude kx. Your father-in-law (mass 98.0kg) stands on a platform 45.0 m above the ground, and one end of the cord is tied securely to his ankle and the other end to the platform. You have promised him that when he steps off the platform he will fall a maximum distance of only 41.0 m before the cord stops him. You had several bungee cords to select from, and you tested them by stretching them out, tying one end to a tree, and pulling on the other end with a force of 420 N .

Part A

When you do this, what distance will the bungee cord that you should select have stretched?

ANSWER:

$x = \frac{F}{\frac{2m \cdot 9.8 \cdot 41}{11^2}} = 0.645 \ \mathrm{m}$

7.5. IDENTIFY and **SET UP:** Use energy methods. Points 1 and 2 are shown in Figure 7.5.

(a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Solve for K_2 and then use $K_2 = \frac{1}{2}mv_2^2$ to obtain v_2.

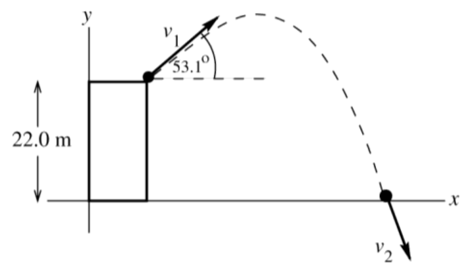


Figure 7.5

$W_{\text{other}} = 0$ (The only force on the ball while it is in the air is gravity.)

$K_1 = \frac{1}{2}mv_1^2$; $K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1$, $y_1 = 22.0\text{ m}$

$U_2 = mgy_2 = 0$, since $y_2 = 0$ for our choice of coordinates.

EXECUTE: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(12.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(22.0\text{ m})} = 24.0\text{ m/s}$$

EVALUATE: The projection angle of $53.1°$ doesn't enter into the calculation. The kinetic energy depends only on the magnitude of the velocity; it is independent of the direction of the velocity.

(b) Nothing changes in the calculation. The expression derived in part (a) for v_2 is independent of the angle, so $v_2 = 24.0\text{ m/s}$, the same as in part (a).

(c) The ball travels a shorter distance in part (b), so in that case air resistance will have less effect.

7.9. IDENTIFY: $W_{\text{tot}} = K_B - K_A$. The forces on the rock are gravity, the normal force and friction.

SET UP: Let $y = 0$ at point B and let $+y$ be upward. $y_A = R = 0.50\text{ m}$. The work done by friction is negative; $W_f = -0.22\text{ J}$. $K_A = 0$. The free-body diagram for the rock at point B is given in Figure 7.9. The acceleration of the rock at this point is $a_{\text{rad}} = v^2/R$, upward.

EXECUTE: **(a)** (i) The normal force is perpendicular to the displacement and does zero work.

(ii) $W_{\text{grav}} = U_{\text{grav},A} - U_{\text{grav},B} = mgy_A = (0.20\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m}) = 0.98\text{ J}$.

(b) $W_{\text{tot}} = W_n + W_f + W_{\text{grav}} = 0 + (-0.22\text{ J}) + 0.98\text{ J} = 0.76\text{ J}$. $W_{\text{tot}} = K_B - K_A$ gives

$\frac{1}{2}mv_B^2 = W_{\text{tot}}$. $v_B = \sqrt{\dfrac{2W_{\text{tot}}}{m}} = \sqrt{\dfrac{2(0.76\text{ J})}{0.20\text{ kg}}} = 2.8\text{ m/s}$.

(c) Gravity is constant and equal to mg. n is not constant; it is zero at A and not zero at B. Therefore, $f_k = \mu_k n$ is also not constant.

(d) $\Sigma F_y = ma_y$ applied to Figure 7.9 gives $n - mg = ma_{\text{rad}}$.

$$n = m\left(g + \frac{v^2}{R}\right) = (0.20\ \text{kg})\left(9.80\ \text{m/s}^2 + \frac{[2.8\ \text{m/s}]^2}{0.50\ \text{m}}\right) = 5.1\ \text{N}.$$

EVALUATE: In the absence of friction, the speed of the rock at point *B* would be

$\sqrt{2gR} = 3.1\ \text{m/s}.$ As the rock slides through point *B*, the normal force is greater than the weight

$mg = 2.0\ \text{N}$ of the rock.

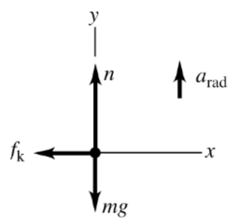


Figure 7.9

7.15. IDENTIFY: Apply $U_{\text{el}} = \frac{1}{2}kx^2.$

SET UP: $kx = F,$ so $U = \frac{1}{2}Fx,$ where *F* is the magnitude of force required to stretch or

compress the spring a distance *x*.

EXECUTE: **(a)** $(1/2)(800\ \text{N})(0.200\ \text{m}) = 80.0\ \text{J}.$

(b) The potential energy is proportional to the square of the compression or extension;

$(80.0\ \text{J})\ (0.050\ \text{m}/0.200\ \text{m})^2 = 5.0\ \text{J}.$

EVALUATE: We could have calculated $k = \frac{F}{x} = \frac{800\ \text{N}}{0.200\ \text{m}} = 4000\ \text{N/m}$ and then used

$U_{\text{el}} = \frac{1}{2}kx^2$ directly.

7.31. IDENTIFY and **SET UP:** The friction force is constant during each displacement and Eq. (6.2) can be used to calculate work, but the direction of the friction force can be different for different displacements.

$f = \mu_{\text{k}}mg = (0.25)(1.5\ \text{kg})(9.80\ \text{m/s}^2) = 3.675\ \text{N};$ direction of $\vec{f}$ is opposite to the motion.

EXECUTE: **(a)** The path of the book is sketched in Figure 7.31a.

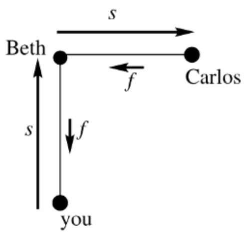


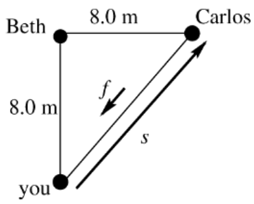
Figure 7.31a

For the motion from you to Beth the friction force is directed opposite to the displacement $\vec{s}$ and $W_1 = -fs = -(3.675\text{ N})(8.0\text{ m}) = -29.4\text{ J}$.

For the motion from Beth to Carlos the friction force is again directed opposite to the displacement and $W_2 = -29.4\text{ J}$.

$$W_{\text{tot}} = W_1 + W_2 = -29.4\text{ J} - 29.4\text{ J} = -59\text{ J}$$

(b) The path of the book is sketched in Figure 7.31b.

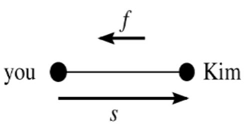


$$s = \sqrt{2(8.0\text{ m})^2} = 11.3\text{ m}$$

Figure 7.31b

$\vec{f}$ is opposite to $\vec{s}$, so $W = -fs = -(3.675\text{ N})(11.3\text{ m}) = -42\text{ J}$

(c)

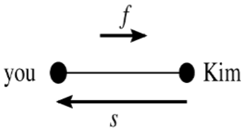


For the motion from you to Kim (Figure 7.31c)

$W = -fs$

$W = -(3.675\text{ N})(8.0\text{ m}) = -29.4\text{ J}$

Figure 7.31c



For the motion from Kim to you (Figure 7.31d)

$W = -fs = -29.4\text{ J}$

Figure 7.31d

The total work for the round trip is $-29.4\text{ J} - 29.4\text{ J} = -59\text{ J}$.

(d) EVALUATE: Parts (a) and (b) show that for two different paths between you and Carlos, the work done by friction is different. Part (c) shows that when the starting and ending points are the same, the total work is not zero. Both these results show that the friction force is nonconservative.

7.38. IDENTIFY: Apply Eq. (7.16).

SET UP: $\dfrac{dU}{dx}$ is the slope of the *U* versus *x* graph.

EXECUTE: **(a)** Considering only forces in the *x*-direction, $F_x = -\dfrac{dU}{dx}$ and so the force is zero when the slope of the *U vs x* graph is zero, at points *b* and *d*.

(b) Point *b* is at a potential minimum; to move it away from *b* would require an input of energy, so this point is stable.

(c) Moving away from point *d* involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so *d* is an unstable point.

EVALUATE: At point *b*, F_x is negative when the marble is displaced slightly to the right and F_x is positive when the marble is displaced slightly to the left, the force is a restoring force, and the equilibrium is stable. At point *d*, a small displacement in either direction produces a force directed away from *d* and the equilibrium is unstable.

7.42. IDENTIFY: Apply Eq. (7.14).

SET UP: Only the spring force and gravity do work, so $W_{\text{other}} = 0.$ Let $y = 0$ at the horizontal surface.

EXECUTE: **(a)** Equating the potential energy stored in the spring to the block's kinetic energy,

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2, \quad \text{or} \quad v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400\ \text{N/m}}{2.00\ \text{kg}}}(0.220\ \text{m}) = 3.11\ \text{m/s}.$$

(b) Using energy methods directly, the initial potential energy of the spring equals the final gravitational potential energy, $\tfrac{1}{2}kx^2 = mgL\sin\theta,$ or

$$L = \frac{\tfrac{1}{2}kx^2}{mg\sin\theta} = \frac{\tfrac{1}{2}(400\ \text{N/m})(0.220\ \text{m})^2}{(2.00\ \text{kg})(9.80\ \text{m/s}^2)\sin 37.0^\circ} = 0.821\ \text{m}.$$

EVALUATE: The total energy of the system is constant. Initially it is all elastic potential energy stored in the spring, then it is all kinetic energy and finally it is all gravitational potential energy.

7.46. IDENTIFY: Apply Eq. (7.14) to relate *h* and v_B. Apply $\Sigma\vec{F} = m\vec{a}$ at point *B* to find the minimum speed required at *B* for the car not to fall off the track.

SET UP: At *B*, $a = v_B^2/R,$ downward. The minimum speed is when $n \to 0$ and $mg = mv_B^2/R.$

The minimum speed required is $v_B = \sqrt{gR}.$ $K_1 = 0$ and $W_{\text{other}} = 0.$

EXECUTE: **(a)** Eq. (7.14) applied to points A and B gives $U_A - U_B = \frac{1}{2}mv_B^2$. The speed at the top must be at least $\sqrt{gR}$. Thus, $mg(h-2R) > \frac{1}{2}mgR$, or $h > \frac{5}{2}R$.

(b) Apply Eq. (7.14) to points A and C. $U_A - U_C = (2.50)Rmg = K_C$, so

$$v_C = \sqrt{(5.00)gR} = \sqrt{(5.00)(9.80\text{ m/s}^2)(20.0\text{ m})} = 31.3\text{ m/s}.$$

The radial acceleration is $a_{\text{rad}} = \dfrac{v_C^2}{R} = 49.0\text{ m/s}^2$. The tangential direction is down, the normal force at point C is horizontal, there is no friction, so the only downward force is gravity, and $a_{\tan} = g = 9.80\text{ m/s}^2$.

EVALUATE: If $h > \frac{5}{2}R$, then the downward acceleration at B due to the circular motion is greater than g and the track must exert a downward normal force n. n increases as h increases and hence v_B increases.

7.51.IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the person.

SET UP: Point 1 is where he steps off the platform and point 2 is where he is stopped by the cord. Let $y = 0$ at point 2. $y_1 = 41.0\text{ m}$. $W_{\text{other}} = -\frac{1}{2}kx^2$, where $x = 11.0\text{ m}$ is the amount the cord is stretched at point 2. The cord does negative work.

EXECUTE: $K_1 = K_2 = U_2 = 0$, so $mgy_1 - \frac{1}{2}kx^2 = 0$ and $k = 631\text{ N/m}$.

Now apply $F = kx$ to the test pulls:

$F = kx$ so $x = F/k = 0.602\text{ m}$.

EVALUATE: All his initial gravitational potential energy is taken away by the negative work done by the force exerted by the cord, and this amount of energy is stored as elastic potential energy in the stretched cord.