#25 Angular Velocity and Acceleration Post-class

Due: 11:00am on Monday, October 22, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

A Spinning Grinding Wheel

At time $t = 0$ a grinding wheel has an angular velocity of 25.0$\mathrm{rad/s}$. It has a constant angular acceleration of 32.0$\mathrm{rad/s^2}$ until a circuit breaker trips at time t = 1.60s . From then on, the wheel turns through an angle of 438rad as it coasts to a stop at constant angular deceleration.

Part A

Through what total angle did the wheel turn between $t = 0$ and the time it stopped?

Express your answer in radians.

Hint 1. How to approach the problem

The angular motion of the grinding wheel is somewhat complicated. However, you can break it up into two simpler pieces: the period from $t = 0$ until time 1.60s and the period from time 1.60s until the grinding wheel comes to a halt. You know the angle through which the wheel turned in the second period: It is given to you in the introduction. All you need to do is find the angle through which the wheel turns in the first period. The total angle is the sum of the angles from each period.

Hint 2. Choose the most appropriate kinematic equation

Choose the most appropriate kinematic equation to use to determine the angle through which the wheel turns during the period of constant acceleration. Note that all of these equations are correct for the case of constant angular acceleration.

The variables are θ (final angle), θ_0 (initial angle), ω (final angular velocity), ω_0 (initial angular velocity), α_0 (constant angular acceleration), and t (time).

Hint 1. Factors to finding the correct equation

In order to choose the most appropriate equation you should look at the variables involved in the problem. In Part A, you are asked to find the total angle, so θ must be a part of the equation. Look at the other variables that you have been given before deciding which equation to use.

ANSWER:

- $\omega^2 = \omega_0^2 + 2\alpha_0(\theta - \theta_0)$
- $\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$
- $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_0 t^2$
- $\omega = \omega_0 + \alpha_0 t$

ANSWER:

519 rad

Correct

Part B

At what time does the wheel stop?

Express your answer in seconds.

Hint 1. What is the angular velocity when the wheel begins to slow down?

Calculate the angular velocity of the wheel when the circuit breaker trips.

Express your answer in radians per second.

Hint 1. The correct kinematic equation

To solve this problem, use the equation $\omega = \omega_0 + \alpha_0 t$, where ω is the angular velocity at time t, ω_0 is the initial angular velocity, and α_0 is the constant angular acceleration.

ANSWER:

76.2 $\mathrm{rad/s}$

Hint 2. Solving for time

After the initial time 1.60s , the wheel begins to decelerate at a constant rate, say α_d. One can solve for the additional amount of time that it takes for the wheel to slow down to zero by looking at both the equation for angular velocity $\omega = \omega_0 + \alpha_\mathrm{d} t$ and the equation $2\alpha_\mathrm{d}(\theta - \theta_0) = \omega^2 - \omega_0^2$ as a function of time. We know that during deceleration the wheel passes through the angle 438rad and that the final angular velocity is $\omega = 0$. With these two equations, we can solve for the two unknowns (time and deceleration). Remember to set the initial velocity to the velocity when the circuit breaker trips.

ANSWER:

13.1 s

Correct

Part C

What was the wheel's angular acceleration as it slowed down?

Express your answer in radians per second per second.

Hint 1. Calculating the deceleration

In solving Part B you had to use the angular velocity of the wheel when the circuit breaker tripped. To find the deceleration $\alpha_{\rm d}$ (which is a constant), just divide by the total time it took for the wheel to stop spinning. Remember that deceleration here is a negative acceleration.

ANSWER:

-6.63 $\mathrm{rad/s^2}$

Correct

Exercise 9.16

A computer disk drive is turned on starting from rest and has constant angular acceleration.

Part A

If it took 0.840s for the drive to make its *second* complete revolution, how long did it take to make the first complete revolution?

ANSWER:

t = 2.03 s

Answer Requested

Part B

What is its angular acceleration, in $\mathrm{rad/s^2}$?

ANSWER:

α = 3.06 $\mathrm{rad/s^2}$

Answer Requested

± A Spinning Electric Fan

An electric fan is turned off, and its angular velocity decreases uniformly from 540$\mathrm{rev/min}$ to 170$\mathrm{rev/min}$ in a time interval of length 3.90s .

Part A

Find the angular acceleration α in revolutions per second per second.

Hint 1. Average acceleration

Recall that if the angular velocity decreases uniformly, the angular acceleration will remain constant. Therefore, the angular acceleration is just the total change in angular velocity divided by the total change in time. Be careful of the sign of the angular acceleration.

ANSWER:

α = -1.58 $\mathrm{rev/s^2}$

Correct

Part B

Find the number of revolutions made by the fan blades during the time that they are slowing down in Part A.

Hint 1. Determine the correct kinematic equation

Which of the following kinematic equations is best suited to this problem? Here ω_0 and ω are the initial and final angular velocities, t is the elapsed time, α is the constant angular acceleration, and θ_0 and θ are the initial and final angular displacements.

Hint 1. How to chose the right equation

Notice that you were given in the problem introduction the initial and final speeds, as well as the length of time between them. In this problem, you are asked to find the number of revolutions (which here is the change in angular displacement, $\theta - \theta_0$). If you already found the angular acceleration in Part A, you could use that as well, but you would end up using a more complex equation. Also, in general, it is somewhat favorable to use given quantities instead of quantities that you have calculated.

ANSWER:

- $\omega = \omega_0 + \alpha t$
- $\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2$
- $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
- $\theta - \theta_0 = \frac{1}{2}(\omega + \omega_0)t$

ANSWER:

23.1 rev

Correct

Part C

How many more seconds are required for the fan to come to rest if the angular acceleration remains constant at the value calculated in Part A?

Hint 1. Finding the total time for spin down

To find the total time for spin down, just calculate when the velocity will equal zero. This is accomplished by setting the initial velocity plus the acceleration multipled by the time equal to zero and then solving for the time. One can then just subtract the time it took to reach 170$\mathrm{rev/min}$ from the total time. Be careful of your signs when you set up the equation.

ANSWER:

1.79 s

Correct

± An Electric Ceiling Fan

An electric ceiling fan is rotating about a fixed axis with an initial angular velocity magnitude of 0.250$\mathrm{rev/s}$. The magnitude of the angular acceleration is 0.889$\mathrm{rev/s^2}$. Both the the angular velocity and angular accleration are directed clockwise. The electric ceiling fan blades form a circle of diameter 0.710m .

Part A

Compute the fan's angular velocity magnitude after time 0.205s has passed.

Express your answer numerically in revolutions per second.

Hint 1. Angular velocity and acceleration

Recall that angular velocity ω and angular acceleration α are related by a kinematic formula similar to that for linear velocity and linear acceleration: $\omega(t) = \omega(0) + \alpha t$. You may use either revolutions per second and $\mathrm{rev/s^2}$ as your units, or radians per second and $\mathrm{rad/s^2}$.

ANSWER:

0.432 $\mathrm{rev/s}$

Correct

Part B

Through how many revolutions has the blade turned in the time interval 0.205s from Part A?

Express the number of revolutions numerically.

Hint 1. Angle and angular velocity

Recall that the angle θ, angular velocity ω, and angular acceleration α are related by a kinematic formula similar to that for linear displacement, velocity, and acceleration: $\theta(t) = \theta(0) + \omega(0)t + \frac{1}{2}\alpha t^2$.

You may use either rev, $\mathrm{rev/s}$, and $\mathrm{rev/s^2}$ as your units, or rad, $\mathrm{rad/s}$, and $\mathrm{rad/s^2}$.

ANSWER:

6.99×10^{-2} rev

Correct

Part C

What is the tangential speed $v_{\mathrm{tan}}(t)$ of a point on the tip of the blade at time t = 0.205s ?

Express your answer numerically in meters per second.

Hint 1. Relating angular and linear speed

A particle moving with angular velocity ω in a circle of radius r has *linear* speed $v = \omega r$.

To use this equation with angular velocity, you must express the angular velocity in $\mathrm{rad/s}$.

Hint 2. Converting revolutions to radians

One revolution is equal to 2π radians.

ANSWER:

$v_{\mathrm{tan}}(t)$ = 0.964 $\mathrm{m/s}$

Answer Requested

Part D

What is the magnitude a of the resultant acceleration of a point on the tip of the blade at time t = 0.205s ?

Express the acceleration numerically in meters per second squared.

Hint 1. How to approach the problem

Since the fan blade is both moving in a circle and speeding up, the tip of the blade must have both tangential and radial acceleration. Add them to find the total acceleration.
Keep in mind that acceleration is a vector, and in order to find the total acceleration, one must use vector addtion (that is, one may not simply add the magnitudes).

Hint 2. Find the centripetal acceleration

Calculate the magnitude $a_{\rm cent}$ of the instantaneous centripetal acceleration of the point at the end of the fan blade. This is the acceleration perpendicular to the direction of motion.

Express your answer numerically in meters per second squared.

Hint 1. Definition of centripetal acceleration

Centripetal acceleration $a_{\rm cent}$ for an object moving with tangential velocity $v_{\rm tan}$ in a circular path of radius r is given by

$$a_{\rm cent} = \frac{v_{\rm tan}^2}{r}.$$

ANSWER:

$a_{\rm cent}$ = 2.62 $\rm m/s^2$

Hint 3. Find the tangential acceleration

Calculate the magnitude $a_{\rm tan}$ of the instantaneous tangential acceleration (along the direction of motion) of a point on the tip of the blade at time 0.205$\rm s$.

Express your answer numerically in meters per second squared.

Hint 1. Definition of tangential acceleration

The tangential acceleration $a_{\rm tan}$ is the rate at which a point increases velocity along its line of motion. If the point is moving with angular acceleration α on a circular path of radius r, then

$$a_{\rm tan} = r\alpha.$$

To use this equation, however, you must express the angular acceleration in radians per second squared.

Hint 2. Converting revolutions to radians

One revolution is equal to 2π radians.

ANSWER:

$a_{\rm tan}$ = 1.98 $\rm m/s^2$

Hint 4. Calculating the vector sum

Notice that the centripetal and tangential accelerations are perpendicular. Thus, you can think of them as the two components of the total acceleration a. This makes the magnitude of the total acceleration $a = \sqrt{a_{\rm tan}^2 + a_{\rm cent}^2}$, where $a_{\rm tan}$ is the magnitude of the tangential acceleration and $a_{\rm cent}$ is the magnitude of the centripetal acceleration.

ANSWER:

a = 3.28 $\rm m/s^2$

Answer Requested

Score Summary:

Your score on this assignment is 62.3%.
You received 24.93 out of a possible total of 40 points.