Lecture 4
(Scalar and Vector Multiplication)

Physics 160-01 Fall 2012

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Multiplication of Vectors

- OK, adding and subtracting vectors seemed fairly straightforward, but how would one multiply vectors?
- There are two ways to multiply vectors and they give different answers…
  – Dot (or Scalar) Product
  – Cross (or Vector) Product
- You use the two ways for different purposes which will become clearer as you use them.

Dot (or Scalar) Product

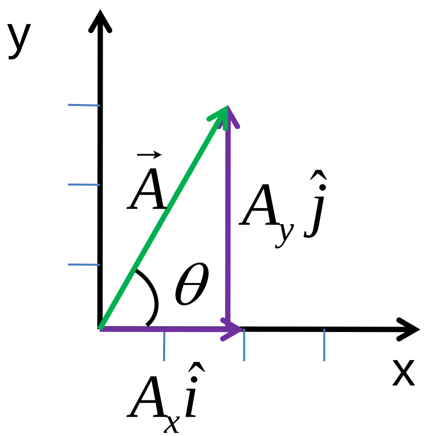
- The dot product of two vectors is written as: $\vec{A} \cdot \vec{B}$.
- The result of a dot product is a scalar (no direction).
- There are two ways to find the dot product:
  - $\vec{A} \cdot \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta_{AB} \equiv AB\cos\theta$
  - or,
  - $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
- But what IS the dot product – I mean what does it MEAN???

Dot (or Scalar) Product

- The dot product of two vectors $\vec{A}\cdot\vec{B}$ gives the length of $\vec{A}$ in the direction of $\vec{B}$ (projection of $\vec{A}$ onto $\vec{B}$) times the length of $\vec{B}$.
- Example:

$$\vec{A}\cdot\hat{i} = \left|\vec{A}\right|\left|\hat{i}\right|\cos\theta_{A\hat{i}} \equiv A\times 1\cos\theta = A\cos\theta$$

$$\vec{A}\cdot\hat{j} = \left|\vec{A}\right|\left|\hat{j}\right|\cos\theta_{A\hat{j}} \equiv A\times 1\cos\left(90-\theta\right) = A\sin\theta$$



$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$A_x = \left|\vec{A}\right|\cos\theta$$

$$A_y = \left|\vec{A}\right|\sin\theta$$

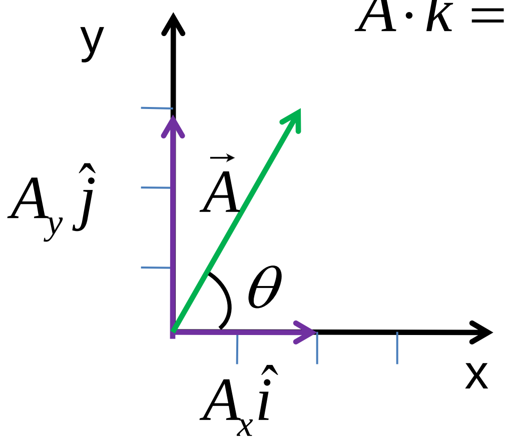
Dot (or Scalar) Product

- The dot product of two vectors $\vec{A}\cdot\vec{B}$ gives the length of $\vec{A}$ in the direction of $\vec{B}$ times the length of $\vec{B}$.
- Using the other method:

$$\vec{A}\cdot\hat{i} = A_x\cdot 1 + A_y\cdot 0 + A_z\cdot 0 = A_x$$

$$\vec{A}\cdot\hat{j} = A_x\cdot 0 + A_y\cdot 1 + A_z\cdot 0 = A_y$$

$$\vec{A}\cdot\hat{k} = A_x\cdot 0 + A_y\cdot 0 + A_z\cdot 1 = A_z$$



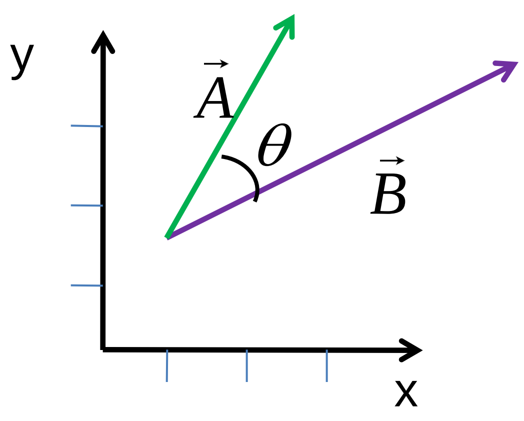
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Dot (or Scalar) Product

- The dot product of two vectors $\vec{A} \cdot \vec{B}$ gives the length of $\vec{A}$ in the direction of $\vec{B}$ times the length of $\vec{B}$.
- Another Example:

$$\vec{A} \cdot \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| \cos\theta_{AB} \equiv \left(A\cos\theta\right)B = A_B B \Rightarrow$$

$$A_B = \frac{\vec{A} \cdot \vec{B}}{\left|\vec{B}\right|}; \quad B_A = \frac{\vec{A} \cdot \vec{B}}{\left|\vec{A}\right|}$$



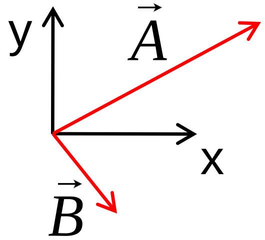
Dot (or Scalar) Product

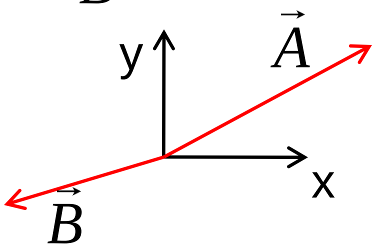
- Physics Example:
- Work – force acting over a distance.

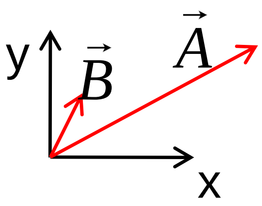
$$W = \vec{F} \cdot \vec{D}$$

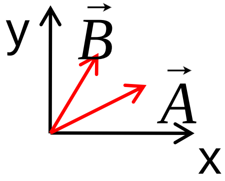
CPS Question 3-1

- Which of the dot products $\vec{A} \cdot \vec{B}$ has the greatest *absolute* magnitude?

A. 

B. 

C. 

D. 

Dot (or Scalar) Product

- Commutative and Distributive Laws are obeyed by the dot product:

$$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$$

$$\vec{A}\cdot\left(\vec{B}+\vec{C}\right)=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

Dot (or Scalar) Product

- This explains the second method then:

$$
\begin{aligned}
\vec{A}\cdot\vec{B} &= \left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\cdot\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)\\
&= A_x\hat{i}\cdot B_x\hat{i}+A_x\hat{i}\cdot B_y\hat{j}+A_x\hat{i}\cdot B_z\hat{k}\\
&\quad +A_y\hat{j}\cdot B_x\hat{i}+A_y\hat{j}\cdot B_y\hat{j}+A_y\hat{j}\cdot B_z\hat{k}\\
&\quad +A_z\hat{k}\cdot B_x\hat{i}+A_z\hat{k}\cdot B_y\hat{j}+A_z\hat{k}\cdot B_z\hat{k}\\
&= A_xB_x+A_yB_y+A_zB_z
\end{aligned}
$$

Dot (or Scalar) Product

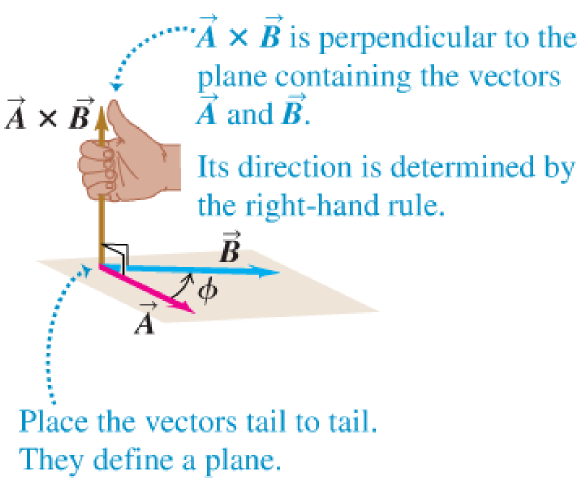
- Usefulness of combining the two methods:

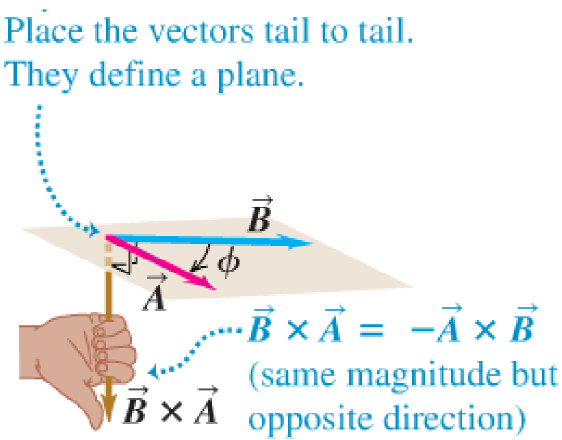
$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z=\left|\vec{A}\right|\left|\vec{B}\right|\cos\theta_{AB}\Rightarrow$$

$$\cos\theta_{AB}=\frac{A_xB_x+A_yB_y+A_zB_z}{\left|\vec{A}\right|\left|\vec{B}\right|}$$

Cross (or Vector) Product

- The cross product of two vectors is written as: $\vec{A}\times\vec{B}$.
- The result of a vector product is a vector (has direction).
- To find the magnitude of a cross product:
  - $\left|\vec{A}\times\vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta_{AB} \equiv AB\sin\theta$
  - Its direction is perpendicular to both $\vec{A}$ and $\vec{B}$, and given by the Right-Hand-Rule:





Cross (or Vector) Product

- Another way of finding $\vec{A}\times\vec{B}$:
  - $\vec{A}\times\vec{B}=\left(A_yB_z-A_zB_y\right)\hat{i}+\left(A_zB_x-A_xB_z\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$
  - Good way to remember using determinant:

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

Cross (or Vector) Product

- Commutative law is NOT obeyed by the cross product:
  - $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- Distributive law is obeyed:
  - $\vec{A}\times\left(\vec{B}+\vec{C}\right) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

Cross (or Vector) Product

- Let's use this to get the second method:
  - $$\vec{A}\times\vec{B}=\left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\times\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$
    $$=A_x\hat{i}\times B_x\hat{i}+A_x\hat{i}\times B_y\hat{j}+A_x\hat{i}\times B_z\hat{k}$$
    $$+A_y\hat{j}\times B_x\hat{i}+A_y\hat{j}\times B_y\hat{j}+A_y\hat{j}\times B_z\hat{k}$$
    $$+A_z\hat{k}\times B_x\hat{i}+A_z\hat{k}\times B_y\hat{j}+A_z\hat{k}\times B_z\hat{k}$$
  - with $\hat{i}\times\hat{j}=\hat{k};\quad \hat{j}\times\hat{k}=\hat{i};\quad \hat{k}\times\hat{i}=\hat{j}$
  - then

$$\vec{A}\times\vec{B}=\left(A_yB_z-A_zB_y\right)\hat{i}+\left(A_zB_x-A_xB_z\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$

Cross (or Vector) Product

- But what IS the vector product – I mean what does it MEAN???
- It gives a sense of the perpendicularity and length of two vectors.

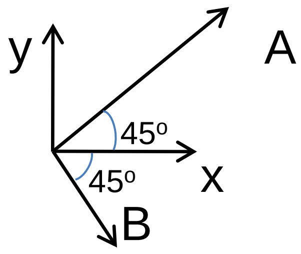
Cross (or Vector) Product

- Physics Example:
- Torque – what does it take to turn a sticky bolt?

$$\vec{\tau} = \vec{r} \times \vec{F}$$

CPS Question 4-1

- $\vec{A}\times\vec{B}=?,\ \vec{A},\vec{B}$ in the x-y plane



A. 0

B. $\left|\vec{A}\right|\left|\vec{B}\right|\sin 45^{o}$

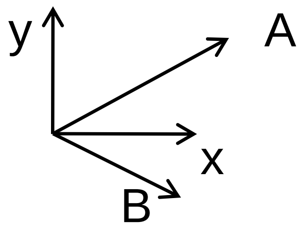
C. $\left|\vec{A}\right|\left|\vec{B}\right|$ in the positive z-direction

D. $\left|\vec{A}\right|\left|\vec{B}\right|$ in the negative z-direction

E. $\left|\vec{A}\right|\left|\vec{B}\right|\sin 45^{o}$ in the negative z-direction

CPS Question 4-2

- What is $\vec{A}\cdot\left(\vec{A}\times\vec{B}\right)=?$, $\vec{A},\vec{B}$ in the x-y plane



A. 0

B. $\left|\vec{A}\right|^2\left|\vec{B}\right|\sin 45^o$

C. $\left|\vec{A}\right|^2\left|\vec{B}\right|$ in the positive z-direction

D. $\left|\vec{A}\right|^2\left|\vec{B}\right|$ in the negative z-direction

E. not enough information