Scalar Product:
$$\vec{A}\cdot\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\cos\theta_{AB}=A_xB_x+A_yB_y+A_zB_z$$
Vector Product:
$$\vec{A}\times\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\sin\theta_{AB}=\left(A_yB_z-A_zB_y\right)\hat{i}+\left(A_zB_x-A_xB_z\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$
Equations of motion:
$$v=v_0+at$$
$$x=x_0+v_0t+\frac{1}{2}at^2$$
$$v^2=v_0^{\,2}+2a(x-x_0)$$
Radial Acceleration:
$$a_{rad}=\frac{v^2}{r}$$
Newton's second law
$$\sum\vec{F}=m\vec{a}$$
Magnitude of kinetic friction
$$F_{f_k}=\mu_kF_N$$
Magnitude of static friction
$$F_{f_s}\le\mu_sF_N$$
Definition of work
$$W=\int\vec{F}\cdot d\vec{x}$$
Definition of kinetic energy:
$$KE=\frac{1}{2}mv^2$$
Change in gravitational potential energy:
$$\Delta U_g=mg\Delta y$$
Elastic potential energy:
$$U_{el}=\frac{1}{2}kx^2$$
Work-Energy Theorem:
$$W=\Delta U+\Delta KE$$
Center-of-mass position
$$X_{COM}=\frac{1}{M}\sum_{i=1}^{n}x_im_i$$
Definition of momentum
$$\vec{p}=m\vec{v}$$
Conservation of momentum
$$\vec{p}_i=\vec{p}_f$$
Definition of torque
$$\vec{\tau}=\vec{r}\times\vec{F}$$
CONTINUED ON BACK!!!

Newton's second law for rotation

$$\sum \vec{\tau} = I\vec{\alpha}$$

Conditions for rolling: $a_{COM} = \alpha R$ and $v_{COM} = \omega R$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$ or $\vec{L} = I\vec{\omega}$, where $I = \sum_i m_i r_i^2$

Newton's Law of Gravitation:

$F_G = \frac{Gm_1m_2}{r^2}$ and $U_G = -\frac{Gm_1m_2}{r}$ with $U_G = 0$ at infinity

Bernoulli's Equation:

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Equation for Simple Harmonic Motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Solution for above equation:

$$x(t) = A\cos(\omega t + \varphi)$$

Where,

$$\omega = 2\pi f = \frac{2\pi}{T}$$

For a spring mass oscillator,

$$\omega = \sqrt{\frac{k}{m}}$$

For a simple pendulum,

$$\omega = \sqrt{\frac{g}{L}}$$

Wave Equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v}\frac{\partial^2 y(x,t)}{\partial t^2}$$

Solution to above equation:

$$y(x,t) = A\cos(kx - \omega t)$$

Where,

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad v = \lambda f$$

Standing waves on fixed string:

$$y(x,t) = A_{SW}\sin(kx)\sin(\omega t)$$

$$f_n = n\frac{v}{2L}$$

Doppler Effect:

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

Final Exam **Name:**____________________**Box #**_________ **Physics 160-01**

1) Find a unit vector in the direction perpendicular to both of the following vectors.

$\vec{A} = 4.00\hat{i} - 3.00\hat{j} - 2.00\hat{k}$

$\vec{B} = -2.00\hat{i} - 6.00\hat{j} + 1.00\hat{k}$

A) -24***i*** + 0***j*** – 31***k***
B) -52
C) -0.45***i*** + 0***j*** – 0.89***k***
D) -0.32***i*** + 0.60***j*** – 0.61***k***
E) -0.55***i*** + 0***j*** – 0.72***k***

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & -2 \\ -2 & -6 & 1 \end{vmatrix} = (-3-12)\hat{i} + (4-4)\hat{j} + (-24-6)\hat{k}$$

$$= -15\hat{i} + 0\hat{j} - 30\hat{k}$$

$$\left|\vec{A}\times\vec{B}\right| = \sqrt{(-15)^2 + 0 + (-30)^2} = 33.54$$

$\therefore$ a unit vector is:

$$\frac{\vec{A}\times\vec{B}}{\left|\vec{A}\times\vec{B}\right|} = \frac{-25\hat{i} + 0\hat{j} - 30\hat{k}}{33.54} = -0.45\hat{i} + 0\hat{j} - 0.89\hat{k}$$

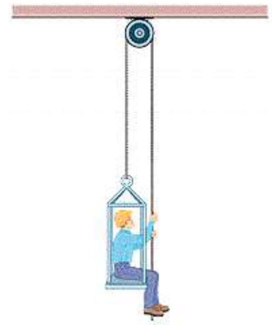
2) An arrow is shot at 45^o above the horizontal on a level field towards a target that is at the same height as the arrow's initial height and at a horizontal distance of 60m. To get the arrow to hit the target, what is the magnitude of the initial velocity of the arrow? Neglect air resistance.

A) 22 m/s
B) 24 m/s
C) 26 m/s
D) 28 m/s
E) 30 m/s

This is a 2-D problem and must be analyzed in each dimension.
In the x- direction,
x_o=0m,
x_f=60m,
$v_{ox}=v_o\cos 45$,
$v_{fx}=v_{ox}$,
$a_x=0m/s^2$,
t=?.
In the y- direction,
y_o=0m,
y_f=0m,
$v_{oy}= v_o\sin 45$,
$v_{fy}= -v_o\sin 45$,
$a_y=-9.8m/s^2$,
t=?.
To get the initial velocity we need to know the amount of time it takes the arrow to go up and then come back down, so look in the y-direction and use $y_f=y_o+v_{oy}t+1/2a_yt^2$ => $t=-2v_{oy}/a_y$.

Then in the x-direction, we use $x_f=x_o+v_{ox}t+1/2a_xt^2$, with $a_x=0$
=> $60m = v_{ox}(-2v_{oy}/a_y) = v_o^2\cos 45*\sin 45*(2/9.8m/s^2)$ =>
$v_o = sqrt[60m/(\sin 45*\cos 45)*(9.8m/s^2/2)] = 24m/s$

3) The figure below shows a man sitting in a bosun's chair that dangles from a massless rope, which runs over a massless, frictionless pulley and back down to the man's hand. The combined mass of the man and chair is 76.0 kg. With what force must the man pull on the rope for him to rise with an upward acceleration of 1.60 m/s^2?



A) 977 N
B) 744 N
C) 121 N
D) 433 N
E) 862 N

The man and chair has two vertical forces acting on him: the tension in the rope at the top of the chair, and the tension in the rope at his hands.
Newton's second law gives us:

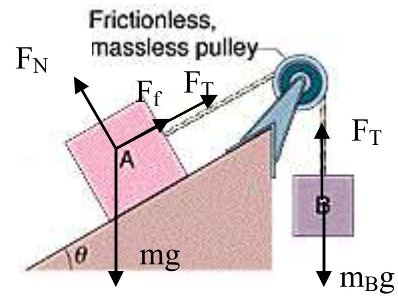
$$\sum F_y = 2T - mg = ma \Rightarrow$$

$$T = \frac{1}{2}(ma + mg) = \frac{1}{2}\left(76kg \cdot 1.6\, m/s^2 + 76kg \cdot 9.8\, m/s^2\right) = 433N$$

This then is also the force he pulls on the rope.

4) In the figure below, two blocks are connected over a pulley. The mass of block *A* is 15 kg and the coefficient of kinetic friction between *A* and the incline is 0.30. Angle θ is 25°. Block *A* slides down the incline at constant speed. What is the mass of block *B*?

A) 2.26 kg
B) 4.24 kg
C) 6.34 kg
D) 2.97 kg
E) 3.31 kg



For block A:

$\sum F_y = F_N - m_A g\cos(25) = 0$

$\sum F_x = m_A g\sin(25) - F_f - F_T = 0$

For block B:

$\sum F_y = F_T - m_B g = 0$

So,

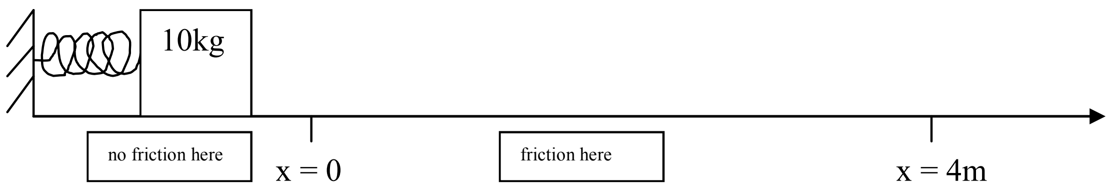
$m_B g = F_T = -F_f + m_A g\sin(25)$

$m_B g = -\mu_k F_N + m_A g\sin(25)$

$m_B = -(0.30)m_A\cos(25) + m_A\sin(25)$

$m_B = 2.26\text{kg}$

As the figure below shows, a 10kg block is accelerated by a compressed spring whose spring constant is 800N/m. After leaving the spring at the spring's relaxed length, the block travels over a horizontal surface with a coefficient of kinetic friction of 0.25, for a distance of 4m before stopping.



5) What is the increase in the thermal energy of the block-floor system?

A) 9.8 J
B) 52 J
C) 98 J
D) 49 J
E) 106 J

You know that the block travels a distance of 4m with a frictional force acting on it converting kinetic energy to thermal energy. The frictional force is $\mu_k mg$ since, in this case, the normal force is just equal to the weight. So, the work done by the frictional force is $\mu_k mgd$ and is equal to the energy transferred to thermal energy. $W = \mu_k mgd = 0.25(10\text{kg})(9.8\text{m/s}^2)(4\text{m}) = 98\text{J}$

6) What is the maximum speed of the block?

A) 3.1 m/s
B) 2.2 m/s
C) 9.8 m/s
D) 5.9 m/s
E) 4.4 m/s

All of the thermal energy was originally in kinetic energy at the point where the block left the spring. So, $98\text{J} = 1/2mv^2 \Rightarrow v = 4.4\text{m/s}$.

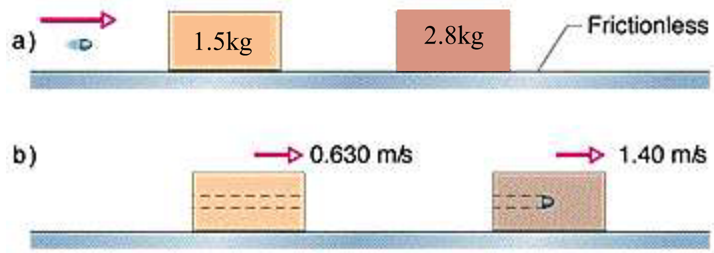
7) Through what distance is the spring compressed before the block begins to move?

A) 0.63 m
B) 0.44 m
C) 1.00 m
D) 0.49 m
E) 0.18 m

The kinetic energy (and also the thermal energy) was originally in the form of elastic potential energy of the spring. So, $98\text{J} = 1/2kx^2 \Rightarrow x = 0.49\text{m}$.

8) In the figure below, a 10.0 g bullet is fired horizontally at two blocks at rest on a frictionless tabletop. The bullet passes through the first block, with mass 1.50 kg, and embeds itself in the second, with mass 2.80 kg. Speeds of 0.630 m/s and 1.40 m/s, respectively, are thereby given to the blocks. Neglect the mass removed from the first block by the bullet. Find the bullet's original speed.

A) 937 m/s
B) 721 m/s
C) 854 m/s
D) 488 m/s
E) 518 m/s

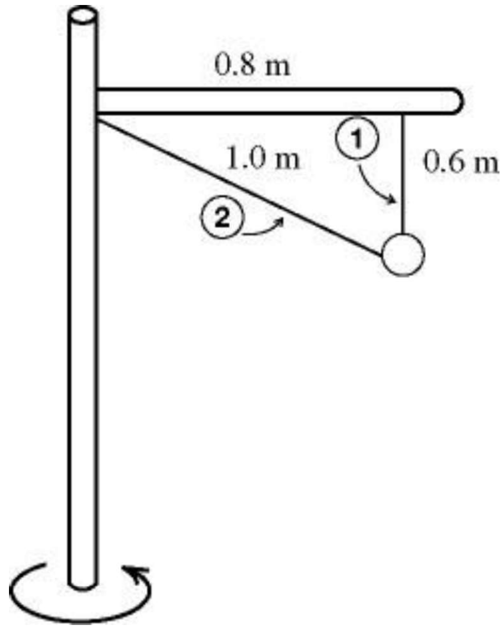


$p_i = m_b v_b$ $\qquad$ $p_f = m_1 v_1 + (m_2 + m_b) v_2$

$(.01\text{kg})v_b + (1.5\text{kg})(0.63\text{m/s}) + (2.81\text{kg})(1.4\text{m/s})$
$v_b = 488\text{m/s}$

9) A ball of unknown mass is suspended by two wires from a horizontal arm, which is attached to a thin, vertical shaft, as shown in the figure below. The shaft rotates one complete turn in 5 seconds. The tension in wire 2 is determined to be 50N. What is the mass of the ball?

A) 24.3 kg
B) 15.2 kg
C) 45.1 kg
D) 28.5 kg
E) 31.7 kg



The ball is in uniform circular motion with a radius of 0.8 m, and it travels one circumference per second, so its linear velocity is: $v = \frac{2\pi r}{T} = 1\, m/s$

The radial acceleration of the ball is given by

$$a_{rad} = \frac{v^2}{r} = \frac{1 m^2/s^2}{0.8m} = 1.26\, m/s^2$$

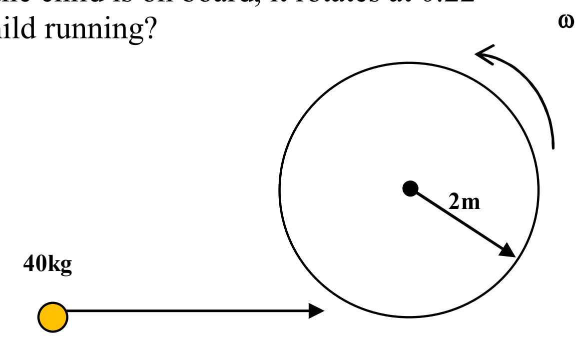
Newton's second law gives us:

$$\sum F_x = T_{2x} = 50N\frac{0.8}{1} = ma_{rad} = m \cdot 1.26\, m/s^2 \Rightarrow$$

$$m = 31.7kg$$

10) A merry-go-round with a radius of 2m and a moment of inertia $100 kgm^2$ is rotating at 0.20 revolutions per second. A child of mass 40kg runs at the merry-go-round in a line tangent to its edge, and grabs onto it. After the child is on board, it rotates at 0.22 revolutions per second. How fast was the child running?

A) 2.9 m/s
B) 4.7 m/s
C) 3.1 m/s
D) 4.2 m/s
E) 3.3 m/s



This involves conservation of angular momentum, $L_i = L_f$.

$$L_i = (I\omega_i)_{merry-go-round} + (\vec{r} \times \vec{p})_{child}$$

$$= (100 kgm^2)\left(0.2\frac{rev}{s}\right)\left(\frac{2\pi rad}{rev}\right) + (2m)(40kg)v$$

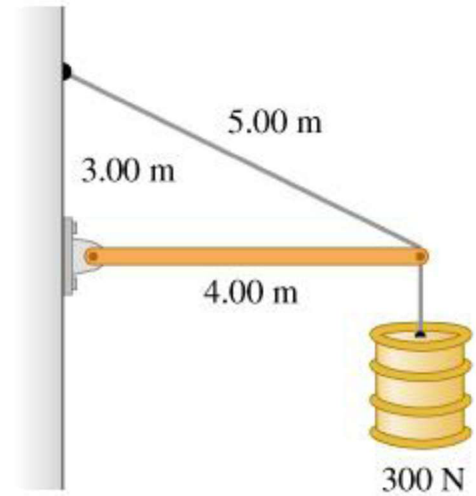
$$L_f = \left[I_{merry-go-round} + I_{child}\right]\omega_f$$

$$= \left[(100 kgm^2) + (40kg)(2m)^2\right]\left(0.22\frac{rev}{s}\right)\left(\frac{2\pi rad}{rev}\right)$$

Solving for v, we get v=2.9m/s

11) The horizontal beam in the figure weighs 250 N, and its center of gravity is at its center. Find the horizontal component of the force exerted on the beam at the wall.

A) 300 N
B) 500 N
C) 433 N
D) 566 N
E) 320 N



This is an equilibrium problem. First, examine the barrel:

$$\sum F_y = T_2 - 300N = 0 \Rightarrow$$

$$T_2 = 300N$$

Then examine the beam, noting that the angle with the top cable and the beam is given by cosθ = 4/5 so θ = 36.9°:

$$\sum F_y = H_y - T_2 - 250N + T_1 \sin(36.9) = 0$$

and

$$\sum F_x = H_x - T_1 \cos(36.9) = 0$$

and the torques around the hinge are

$$\sum \tau_z = -T_2(4m) - 250N(2m) + T_1(4m)\sin(180 - 36.9) =$$

$$300N(4m) + 250N(2m) = T_1(4m)\sin(143.1) \Rightarrow$$

$$T_1 = 708N$$

so,

$$H_x = 566N$$