#3 Addition and Subtraction of Vectors Post-class

Due: 11:00am on Monday, August 27, 2012

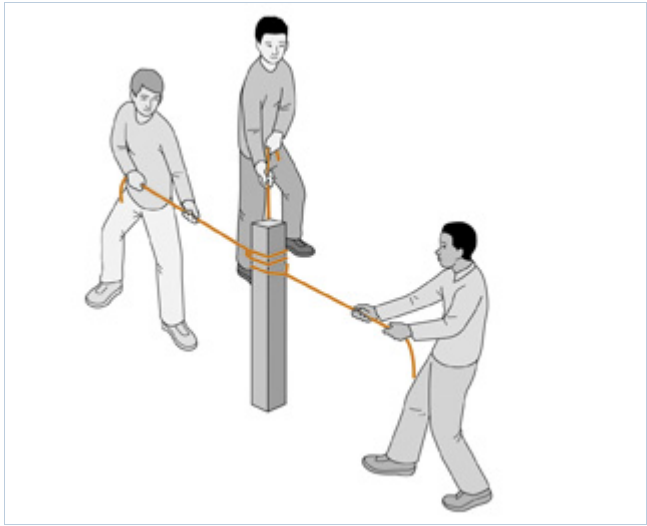
Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

PSS 1.3 Vector Addition

Learning Goal:

To practice Problem-Solving Strategy 1.3 Vector Addition.

Three people pull horizontally on ropes attached to a post, as shown in the figure. The total force that they exert on the post is zero. One of them pulls directly north with F_1 = 500N . Another pulls with F_2 = 400N in a direction 60.0° west of north. Find the magnitude and direction of the force $\vec{F}_3$ exerted by the third person so that the sum of the three forces is zero.



Problem-Solving Strategy: Vector addition

IDENTIFY *the relevant concepts:*
Decide what your target variable is. It may be the magnitude of the vector sum, the direction, or both.

SET UP *the problem using the following steps:*
Draw the individual vectors being summed and the coordinate axes being used. In your drawing, place the tail of the first vector at the origin of the coordinates; place the tail of the second vector at the head of the first vector; and so on. Draw the vector sum $\vec{R}$ from the tail of the first vector to the head of the last vector. Use your drawing to make rough estimates of the magnitude and direction of $\vec{R}$; you'll use these estimates later to check your calculations.

EXECUTE *the solution as follows:*

1. Find the *x* and *y* components of each individual vector, and record your results in a table. Be particularly careful with signs: Some components may be positive, and some may be negative.
2. Add the individual *x* components algebraically, including signs, to find R_x, the *x* component of the vector sum. Do the same for the *y* components to find R_y.
3. Then, the magnitude R and direction θ of the vector sum are given by

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \arctan\frac{R_y}{R_x}.$$

EVALUATE *your answer:*
Check your results for the magnitude and direction of the vector sum by comparing them with the rough estimates you made from your drawing. Keep in mind that the magnitude R is always positive and that θ is measured from the +*x* axis. The value of θ that you find with a calculator may be the correct one, or it may be off by 180 degrees. You can decide by examining your drawing.

IDENTIFY the relevant concepts

Force is a vector quantity, so vector addition must be used to add the three forces acting on the post. This problem requires you to find both the magnitude and direction of one of the forces, such that the total vector sum is zero.

SET UP the problem using the following steps

Part A

Add vectors to the diagram below to indicate the vector sum of the three forces acting on the post: $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. Use the conventional choice of coordinates, with the +*x* axis as east and the +*y* axis as north.

Draw vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ on the diagram below such that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. Draw $\vec{F}_1$ starting from the origin of the axes provided. Each unit on the graph is 100 N.

Hint 1. How to approach the problem

You should draw the three vectors corresponding to the forces applied to the post, adding them head to tail as outlined in the Set Up step of the strategy. You have enough information to straightforwardly draw the vectors $\vec{F}_1$ and $\vec{F}_2$. By considering that the total force exerted on the post

is zero, you can determine how to draw the unknown force $\vec{F}_3$.

Hint 2. How to draw the vector of the force exerted by the third person

The total force on the post is zero: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. When a set of vectors sum to zero, the head of the last vector must be located at the tail of the first vector. $\vec{F}_3$ is therefore drawn from the head of $\vec{F}_2$ to the tail of $\vec{F}_1$.

ANSWER:

Correct

EXECUTE the solution as follows

Part B

Calculate the magnitude of the force exerted by the third person.

Express the magnitude of $\vec{F}_3$ in newtons to three significant figures.

Hint 1. How to approach the problem

The total force on the post is zero: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$.

This can be written as $\vec{F}_3 = -\left(\vec{F}_1 + \vec{F}_2\right)$.

Add the *x* components of $\vec{F}_1$ and $\vec{F}_2$ to find the negative of the *x* component of $\vec{F}_3$. Similarly, add the *y* components of $\vec{F}_1$ and $\vec{F}_2$ to find the negative of the *y* component of $\vec{F}_3$. Once you know $\vec{F}_3$, use the formula

$$R = \sqrt{R_x^2 + R_y^2}$$

from the introduction.

Hint 2. Determine the vector components of the force exerted by the second person

It is important to use the correct angle and trigonometric function when calculating the component of a vector. If the direction of the vector (with magnitude A) is described by the angle θ that it makes with the +*x* axis, then its components are

$$A_x = A\cos\theta \qquad A_y = A\sin\theta.$$

However, the angle given in this problem is *not* measured from the +*x* axis. You should use geometry to calculate an angle from the +*x* axis before using the above formulas.

Calculations using the angle measured from the +*x* axis will also give the correct sign (positive or negative) for the components.

Consider the force vector $\vec{F}_2$ = 400 N , 60.0° west of north. Calculate the *x* and *y* components of the force, F_{2x} and F_{2y}, using the conventional choice of coordinates, with the +*x* axis as east and the +*y* axis as north.

Express the *x* and *y* components of $\vec{F}_2$ in newtons, separated by a comma, to three significant figures.

ANSWER:

F_{2x}, F_{2y} = -346,200 N

Answer Requested

ANSWER:

F_3 = 781 N

Correct

Part C

Determine the direction in which the third person is pulling.

Express the angle in degrees $(0 \leq \theta < 360^\circ)$ that $\vec{F}_3$ makes, counterclockwise, with the +*x* axis. Express your answer to three significant figures.

ANSWER:

θ = 296 °

Correct

EVALUATE your answer

Part D

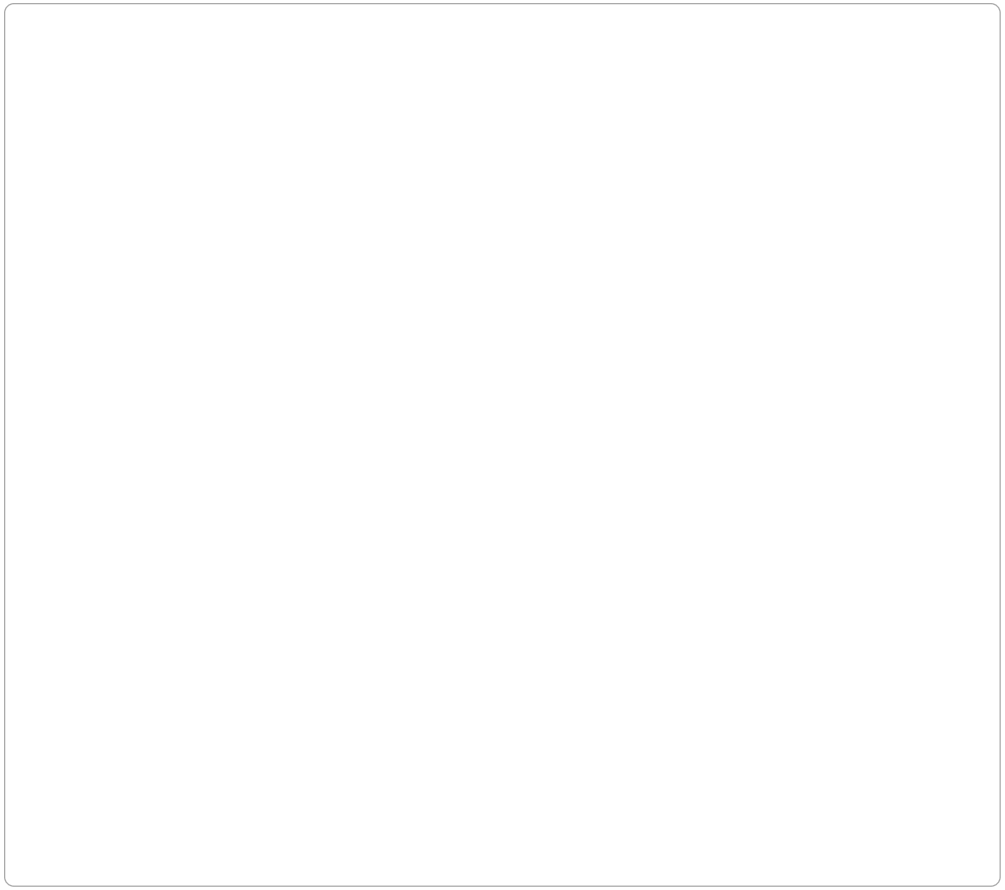
By adding vectors using components, you have been able to determine an unknown force, both magnitude and direction, to three significant figures: $\vec{F}_3$ = 781 N, 63.7° south of east.

You can check your answer by comparing it to the diagram in Part A above.

Some other students calculated $\vec{F}_3$, not as correctly as you, and came up with a set of different values. Pretend that you do not know the actual value of $\vec{F}_3$, and by considering the vector addition diagram in Part A, decide which of the following values could possibly be correct, and which are definitely incorrect.

Drag the appropriate values to their respective bins.

ANSWER:



Correct

Adding Scalar Multiples of Vectors Graphically

Draw the vectors indicated. You may use any extra (unlabeled) vectors that are helpful; but, keep in mind that the unlabeled vectors should not be part of your submission.

Part A

Draw the vector $\vec{C} = \vec{A} + 2\vec{B}$.

The length and orientation of the vector will be graded. The location of the vector is not important.

Hint 1. How to approach the problem

You can add the vectors graphically or using components, but a graphical approach will be the simplest. It may help to draw the vector $2\vec{B}$ first.

Hint 2. Draw $2\vec{B}$

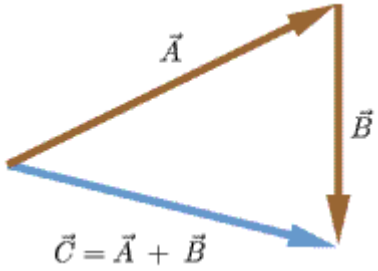
Draw the vector $\vec{B}_{\text{scaled}} = 2\vec{B}$.

The length and orientation of the vector will be graded. The location of the vector is not important.

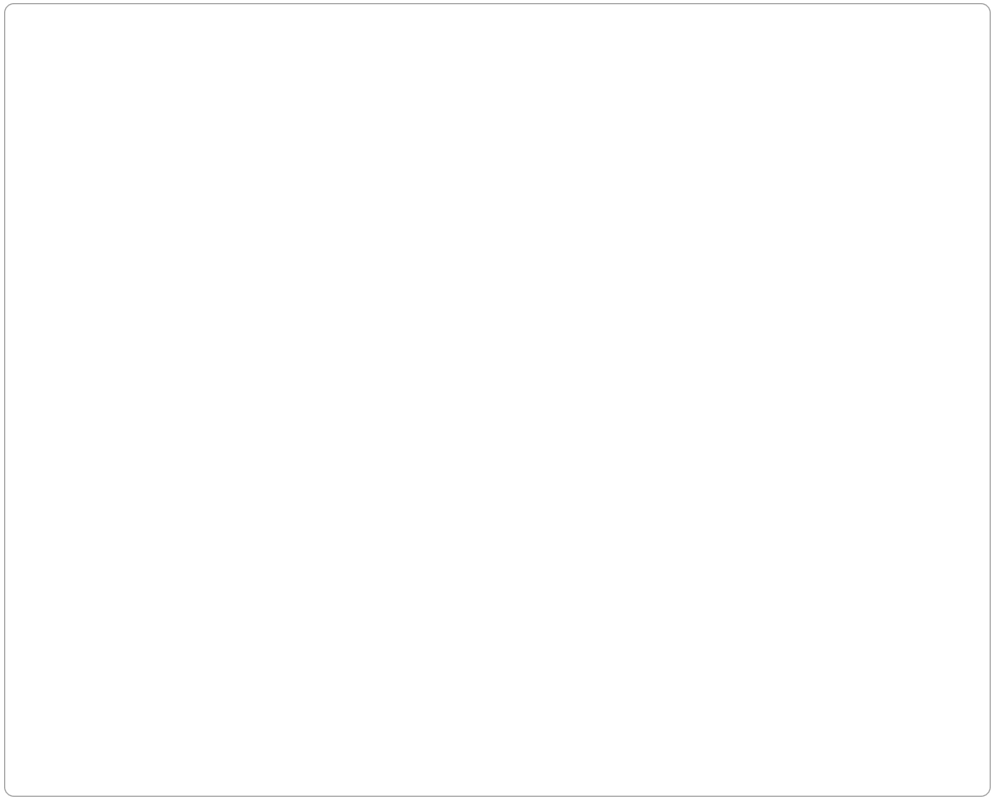
ANSWER:

Hint 3. Adding vectors graphically

To add two vectors, slide one vector (without rotating it) until its tip coincides with the tail of the second vector. The sum of the two vectors is the vector that goes from the tail of the first vector to the tip of the second:



ANSWER:



Correct

Now use the same technique to answer the next two parts.

Part B

Draw the vector $\vec{C} = 1.5\vec{A} - 3\vec{B}$.

The length and orientation of the vector will be graded. The location of the vector is not important.

Hint 1. Find $1.5\vec{A}$ and $-3\vec{B}$

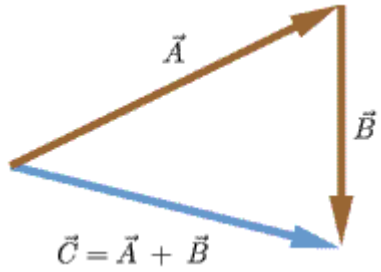
Draw the vectors $\vec{A}_{\text{scaled}} = 1.5\vec{A}$ and $\vec{B}_{\text{scaled}} = -3\vec{B}$. Recall that multiplying a vector by a negative number reverses its direction.

The length and orientation of the vectors will be graded. The locations of the vectors are not important.

ANSWER:

Hint 2. Adding vectors graphically

To add two vectors, slide one vector (without rotating it) until its tip coincides with the tail of the second vector. The sum of the two vectors is the vector that goes from the tail of the first vector to the tip of the second:



ANSWER:

Correct

Part C

Draw the vector $\vec{C} = 0.5\vec{A} + 2\vec{B}$.

The length and orientation of the vector will be graded. The location of the vector is not important.

Hint 1. Find $0.5\vec{A}$ and $2\vec{B}$

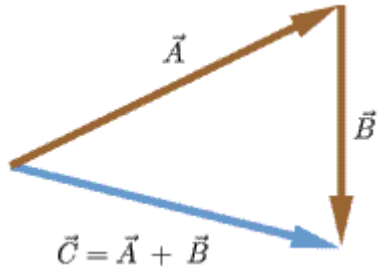
Draw the vectors $\vec{A}_{\text{scaled}} = 0.5\vec{A}$ and $\vec{B}_{\text{scaled}} = 2\vec{B}$.

The length and orientation of the vectors will be graded. The locations of the vectors are not important.

ANSWER:

Hint 2. Adding vectors graphically

To add two vectors, slide one vector (without rotating it) until its tip coincides with the tail of the second vector. The sum of the two vectors is the vector that goes from the tail of the first vector to the tip of the second:

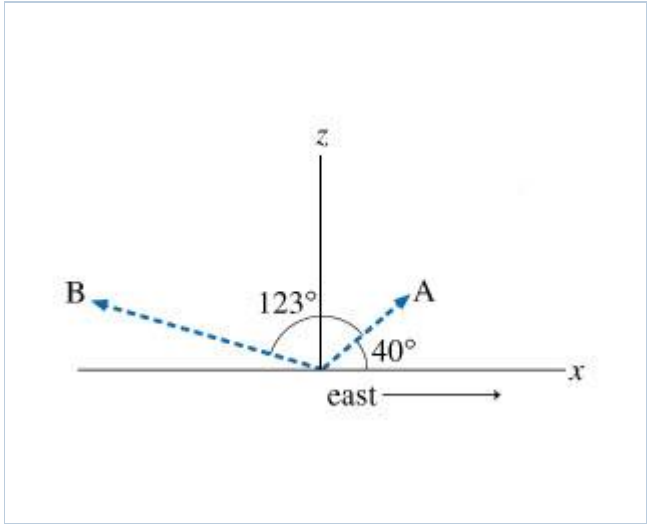


ANSWER:

Correct

Tracking a Plane

A radar station, located at the origin of *xz* plane, as shown in the figure , detects an airplane coming straight at the station from the east. At first observation (point A), the position of the airplane relative to the origin is $\vec{R}_A$. The position vector $\vec{R}_A$ has a magnitude of 360 m and is located at exactly 40 $\mathrm{degrees}$ above the horizon. The airplane is tracked for another 123 $\mathrm{degrees}$ in the vertical east-west plane for 5.0 s, until it has passed directly over the station and reached point B. The position of point B relative to the origin is $\vec{R}_B$ (the magnitude of $\vec{R}_B$ is 880 m). The contact points are shown in the diagram, where the *x* axis represents the ground and the positive *z* direction is upward.



Part A

Define the displacement of the airplane while the radar was tracking it: $\vec{R}_{BA} = \vec{R}_B - \vec{R}_A$. What are the components of $\vec{R}_{\mathrm{BA}}$?

Express $\vec{R}_{\mathrm{BA}}$ in meters as an ordered pair, separating the *x* and *z* components with a comma, to two significant figures.

Hint 1. How to approach the problem

Keep in mind that $\vec{R}_{BA} = \vec{R}_B - \vec{R}_A$.According to the rules of vector addition and subtraction, the *x* component of $\vec{R}_{\mathrm{BA}}$ is $(\vec{R}_{BA})_x = (\vec{R}_B)_x - (\vec{R}_A)_x$.

Hint 2. Finding the components of $\vec{R}_A$

What are the components of $\vec{R}_A$ in the $\hat{x}$ and $\hat{z}$ directions?

Express your answer in meters as an ordered pair, separating the *x* and *z* values with commas, to three significant figures.

ANSWER:

R_{A_x}, R_{A_z} = 276,231 m

Correct

Hint 3. Finding the components of $\vec{R}_B$

What are the components of $\vec{R}_B$ in the $\hat{x}$ and $\hat{z}$ directions?

Express your answer in meters as an ordered pair, separating the *x* and *z* components with a comma, to three significant figures.

ANSWER:

R_{B_x}, R_{B_z} = -842,257 m

Answer Requested

ANSWER:

$\vec{R}_{BA}$ = -1100,26 m

Correct

Score Summary:

Your score on this assignment is 84.1%.
You received 25.24 out of a possible total of 30 points.