Chapter 12: Fluid Mechanics

Due: 11:00pm on Tuesday, November 13, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Archimedes' Principle

Learning Goal:

To understand the applications of Archimedes' principle.

Archimedes' principle is a powerful tool for solving many problems involving equilibrium in fluids. It states the following:
When a body is partially or completely submerged in a fluid (either a liquid or a gas), the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.
As a result of the upward Archimedes force (often called the *buoyant force*), some objects may float in a fluid, and all of them appear to weigh less. This is the familiar phenomenon of *buoyancy*.

Quantitatively, the buoyant force can be found as

$$F_{\rm buoyant} = \rho_{\rm fluid} g V,$$

where $F_{\rm buoyant}$ is the force, $\rho_{\rm fluid}$ is the density of the fluid, g is the magnitude of the acceleration due to gravity, and V is the volume of the displaced fluid.

In this problem, you will be asked several qualitative questions that should help you develop a feel for Archimedes' principle.

An object is placed in a fluid and then released. Assume that the object either floats to the surface (settling so that the object is partly above and partly below the fluid surface) or sinks to the bottom. (Note that for Parts A through D, you should assume that the object has settled in equilibrium.)

Part A

Consider the following statement:
The magnitude of the buoyant force is equal to the weight of fluid displaced by the object.
Under what circumstances is this statement true?

Hint 1. Archimedes' principle

The statement of Part A is one way of expressing Archimedes' principle.

ANSWER:

- (selected) for every object submerged partially or completely in a fluid
- only for an object that floats
- only for an object that sinks
- for no object submerged in a fluid

Use Archimedes' principle to answer the rest of the questions in this problem.

Part B

Consider the following statement:
The magnitude of the buoyant force is equal to the weight of the amount of fluid that has the same total volume as the object.
Under what circumstances is this statement true?

Hint 1. Consider Archimedes' principle

Archimedes' principle deals with the displaced volume. When is the displaced volume equal to the total volume of the object?

ANSWER:

- ○ for an object that is partially submerged in a fluid
- ○ only for an object that floats
- ◉ for an object completely submerged in a fluid
- ○ for no object partially or completely submerged in a fluid

Part C

Consider the following statement:
The magnitude of the buoyant force equals the weight of the object.
Under what circumstances is this statement true?

Hint 1. Forces and equilibrium

If the buoyant force and the weight of the object are equal, the object can be in equilibrium without any other forces acting on it.

ANSWER:

- ○ for every object submerged partially or completely in a fluid
- ◉ for an object that floats
- ○ only for an object that sinks
- ○ for no object submerged in a fluid

Part D

Consider the following statement:
The magnitude of the buoyant force is less than the weight of the object.
Under what circumstances is this statement true?

ANSWER:

- ○ for every object submerged partially or completely in a fluid
- ○ for an object that floats
- ◉ for an object that sinks
- ○ for no object submerged in a fluid

Now apply what you know to some more complicated situations.

Part E

An object is floating in equilibrium on the surface of a liquid. The object is then removed and placed in another container, filled with a denser liquid. What would you observe?

Hint 1. Density and equilibrium

If the second liquid is denser than the first one, the fluid would exert a greater upward force on the object if it were held submerged as deeply as in the first case.

ANSWER:

- ○ The object would sink all the way to the bottom.
- ○ The object would float submerged more deeply than in the first container.
- ◉ The object would float submerged less deeply than in the first container.
- ○ More than one of these outcomes is possible.

Part F

An object is floating in equilibrium on the surface of a liquid. The object is then removed and placed in another container, filled with a less dense liquid. What would you observe?

Hint 1. Density and equilibrium

If the second liquid is less dense than the first one, the liquid would exert a smaller upward force on the object if it were held submerged as deeply as in the first case.

ANSWER:

- ○ The object would sink all the way to the bottom.
- ○ The object would float submerged more deeply than in the first container.
- ○ The object would float submerged less deeply than in the first container.
- ◉ More than one of these outcomes is possible.

If the fluid in the second container is less dense than the object, then the object will sink all the way to the bottom. If the fluid in the second container is denser than the object (though less dense than the fluid in the original container), the object will still float, but its depth will be greater than it was in the original container.

Part G

Two objects, T and B, have identical size and shape and have uniform density. They are carefully placed in a container filled with a liquid. Both objects float in equilibrium. Less of object T is submerged than of object B, which floats, fully submerged, closer to the bottom of the container. Which of the following statements is true?

ANSWER:

- ○ Object T has a greater density than object B.
- ◉ Object B has a greater density than object T.
- ○ Both objects have the same density.

Since both objects float, the buoyant force in each case is equal to the object's weight. Block B displaces more fluid, so it must be heavier than block T. Given that the two objects have the same volume, block B must also be denser. In fact, since the weight equals the buoyant force, and B is fully submerged, $\rho_B V g = \rho_{\rm liquid} V g$, where all the symbols have their usual meaning. From this equation, one can see that the density of B must equal the density of the fluid.

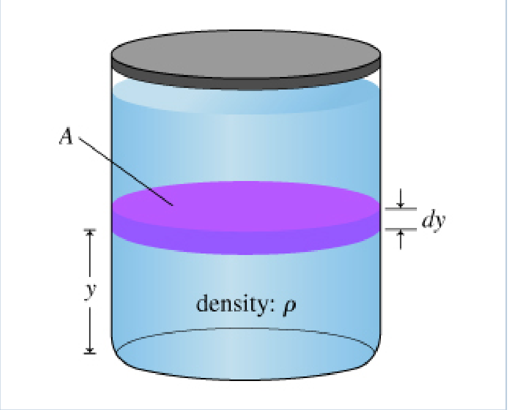
Relating Pressure and Height in a Container

Learning Goal:

To understand the derivation of the law relating height and pressure in a container.

In this problem, you will derive the law relating pressure to height in a container by analyzing a particular system.

A container of uniform cross-sectional area A is filled with liquid of uniform density ρ. Consider a thin horizontal layer of liquid (thickness dy) at a height y as measured from the bottom of the container. Let the pressure exerted upward on the bottom of the layer be p and the pressure exerted downward on the top be $p + dp$. Assume throughout the problem that the system is in equilibrium (the container has not been recently shaken or moved, etc.).



Part A

What is F_{up}, the magnitude of the force exerted upward on the bottom of the liquid?

Hint 1. Formula for the force

Force is equal to pressure times area.

ANSWER:

$F_{\mathrm{up}} = pA$

Part B

What is F_{down}, the magnitude of the force exerted downward on the top of the liquid?

Hint 1. Formula for the force

Force is equal to pressure times area.

ANSWER:

$F_{\mathrm{down}} = (p + dp)\,A$

Part C

What is the weight w_{layer} of the thin layer of liquid?

Express your answer in terms of quantities given in the problem introduction and g, the magnitude of the acceleration due to gravity.

Hint 1. How to approach the problem

The weight of the layer is given by the formula $w = mg$, where m is the mass of the layer and g is the magnitude of the acceleration due to gravity.

Hint 2. Mass of the layer

Use the definition of density to write the mass m of the layer of liquid in terms of its density and its volume (express volume in terms of physical dimensions given in the problem introduction).

Express your answer in terms of quantities given in the problem introduction.

Hint 1. Definition of density

The density of an object is equal to its mass divided by its volume.

Hint 2. Volume of the layer

What is the volume dV of the thin layer of liquid?

Express your answer in terms of quantities given in the problem introduction.

ANSWER:

$dV = Ady$

ANSWER:

$m = \rho A dy$

ANSWER:

$w_{\text{layer}} = \rho A dy g$

Part D

Since the liquid is in equilibrium, the net force on the thin layer of liquid is zero. Complete the force equation for the sum of the vertical forces acting on the liquid layer described in the problem introduction.

Express your answer in terms of quantities given in the problem introduction and taking upward forces to be positive.

Hint 1. How to approach the problem

If you have completed the previous parts, you have already done most of the work needed to answer this part. Just add together the forces that you found in the previous three parts. All three of the forces act along the *y* axis; some are directed upward, and others are directed downward. Those that act downward should appear with a negative sign.

ANSWER:

$\sum_i F_{y,i} = = pA - (p + dp)A - \rho A dy g$

Part E

Solve the sum-of-forces equation just derived,

$$0 = \sum_i F_{y,i} = pA - (p + dp)A - \rho A g\, dy,$$

to obtain an expression for dp and thus a differential equation for pressure.

ANSWER:

$dp = -\rho g dy$

Part F

Integrate both sides of the differential equation you found for dp to obtain an equation for p. Your equation should then include a constant that depends on initial conditions. Determine the value of this constant by assuming that the pressure at some reference height y_0 is p_0.

Express your answer in terms of quantities given in the problem introduction along with y_0 and p_0.

Hint 1. Integrate dp

What is the expression obtained by integrating the left-hand side of the equation $dp = -\rho g\, dy$? Although the indefinite integral of the left-hand side of the equation should include a constant determined by initial conditions, you can combine it with the constant on the right-hand side. Leave it out of your answer here.

ANSWER:

$\int dp =$ p

Hint 2. Integrate $\rho g\, dy$

What is the expression obtained by integrating the right-hand side of the equation $dp = -\rho g\, dy$? Here you will need to include the constant determined by initial conditions; call it C.

ANSWER:

$-\int \rho g\, dy =$ $-\rho g y + C$

Hint 3. Determine the constant of integration

According to the statement of the initial conditions, $p(y_0) = p_0$. Using this fact, find the value of the constant C.

Express your answer in terms of p_0, y_0, and other given quantities.

ANSWER:

$C =$ $p_0 + \rho g y_0$

ANSWER:

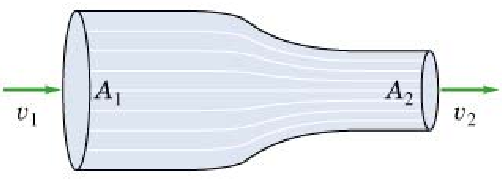
$p =$ $p_0 + \rho g\,(y_0 - y)$

Streamlines and Fluid Flow

Learning Goal:

To understand the continuity equation.

Streamlines represent the path of the flow of a fluid. You can imagine that they represent a time-exposure photograph that shows the paths of small particles carried by the flowing fluid. The figure shows streamlines for the flow of an incompressible fluid in a tapered pipe of circular cross section. The speed of the fluid as it enters the pipe on the left is v_1. Assume that the cross-sectional areas of the pipe are A_1 at its entrance on the left and A_2 at its exit on the right.



Part A

Find F_1, the volume of fluid flowing into the pipe per unit of time. This quantity is also known as the *volumetric flow rate*.

Express the volumetric flow rate in terms of any of the quantities given in the problem introduction.

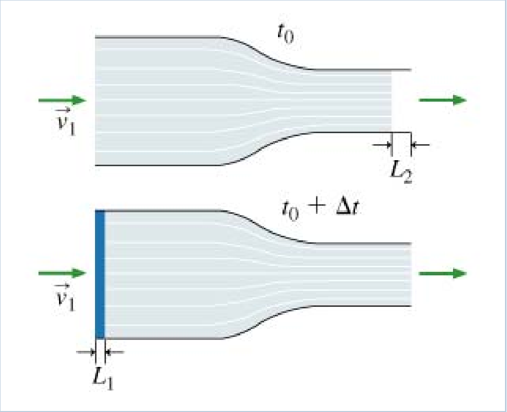
Hint 1. Find the volume of fluid entering the pipe

The volumetric flow rate has units of volume per unit time (cubic meters per second). What is the volume ΔV of fluid entering the pipe in time Δt?

Express your answer in terms of any or all of the following quantities: v_1, A_1, and Δt.

Hint 1. How far does the fluid move in time Δt?

The figure shows a snapshot of the pipe at two instants in time: t_0 and $t_0 + \Delta t$. In the time interval Δt, some additional fluid (shown in blue) has entered the pipe from the left. The speed of the left-hand boundary of the blue-colored fluid is v_1. What is the length L_1 of the region of additional fluid that has entered the pipe in the time interval Δt?



Express your answer in terms of v_1 and Δt.

ANSWER:

L_1 = $v_1 \Delta t$

The volume of (blue) fluid that has entered the pipe in time Δt is equal to the length you just found times the cross-sectional area of the left-hand opening of the pipe.

ANSWER:

ΔV = $A_1 v_1 \Delta t$

The volume of fluid entering the pipe per unit time is $\Delta V / \Delta t$.

ANSWER:

F_1 = $v_1 A_1$

Part B

Because the fluid is assumed to be incompressible and mass is conserved, at a particular moment in time, the amount of fluid that flows into the pipe must equal the amount of fluid that flows out. This fact is embodied in the *continuity equation*. Using the continuity equation, find the velocity v_2 of the fluid flowing out of the right end of the pipe.

Express your answer in terms of any of the quantities given in the problem introduction.

Hint 1. Find the volumetric flow rate out of the pipe

Find F_2, the volume of fluid flowing out of the pipe per unit of time.

Express your answer in terms of v_2 and A_2.

ANSWER:

F_2 = $v_2 A_2$

Hint 2. Apply the continuity equation

The continuity equation states that the volumetric flows into and out of the pipe must be the same. Fill in the right-hand side of the continuity equation for this problem.

Express your answer in terms of v_2 and any of the quantities given in the problem introduction.

ANSWER:

$v_1 A_1 = v_2 A_2$

ANSWER:

$v_2 = \frac{v_1 A_1}{A_2}$

Part C

If you are shown a picture of streamlines in a flowing fluid, you can conclude that the __________ of the fluid is greater where the streamlines are closer together.

Enter a one-word answer.

ANSWER:

velocity

Also accepted: speed

Thus the velocity of the flow increases with increasing density (number per unit area) of streamlines.

Understanding Bernoulli's Equation

Bernoulli's equation is a simple relation that can give useful insight into the balance among fluid pressure, flow speed, and elevation. It applies exclusively to ideal fluids with steady flow, that is, fluids with a constant density and no internal friction forces, whose flow patterns do not change with time. Despite its limitations, however, Bernoulli's equation is an essential tool in understanding the behavior of fluids in many practical applications, from plumbing systems to the flight of airplanes.

For a fluid element of density ρ that flows along a streamline, Bernoulli's equation states that

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2,$$

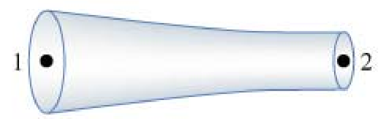
where p is the pressure, v is the flow speed, h is the height, g is the acceleration due to gravity, and subscripts 1 and 2 refer to any two points along the streamline. The physical interpretation of Bernoulli's equation becomes clearer if we rearrange the terms of the equation as follows:

$$p_1 - p_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2).$$

The term $p_1 - p_2$ on the left-hand side represents the total work done on a unit volume of fluid by the pressure forces of the surrounding fluid to move that volume of fluid from point 1 to point 2. The two terms on the right-hand side represent, respectively, the change in potential energy, $\rho g(h_2 - h_1)$, and the change in kinetic energy, $\frac{1}{2}\rho(v_2^2 - v_1^2)$, of the unit volume during its flow from point 1 to point 2. In other words, Bernoulli's equation states that the work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the change in potential and kinetic energy per unit volume that occurs during the flow. This is nothing more than the statement of conservation of mechanical energy for an ideal fluid flowing along a streamline.

Part A

Consider the portion of a flow tube shown in the figure. Point 1 and point 2 are at the same height. An ideal fluid enters the flow tube at point 1 and moves steadily toward point 2. If the cross section of the flow tube at point 1 is greater than that at point 2, what can you say about the pressure at point 2?



Hint 1. How to approach the problem

Apply Bernoulli's equation to point 1 and to point 2. Since the points are both at the same height, their elevations cancel out in the equation and you are left with a relation between pressure and flow speeds. Even though the problem does not give direct information on the flow speed along the flow tube, it does tell you that the cross section of the flow tube decreases as the fluid flows toward point 2. Apply the continuity equation to points 1 and 2 and determine whether the flow speed at point 2 is greater than or smaller than the flow speed at point 1. With that information and Bernoulli's equation, you will be able to determine the pressure at point 2 with respect to the pressure at point 1.

Hint 2. Apply Bernoulli's equation

Apply Bernoulli's equation to point 1 and to point 2 to complete the expression below. Here p and v are the pressure and flow speed, respectively, and subscripts 1 and 2 refer to point 1 and point 2. Also, use h for elevation with the appropriate subscript, and use ρ for the density of the fluid.

Express your answer in terms of some or all of the variables p_1, v_1, h_1, p_2, v_2, h_2, and ρ.

Hint 1. Flow along a horizontal streamline

Along a horizontal streamline, the change in potential energy of the flowing fluid is zero. In other words, when applying Bernoulli's equation to any two points of the streamline, $h_1 = h_2$ and they cancel out.

ANSWER:

$p_1 + \frac{1}{2}\rho v_1^2 = \; p_2 + \frac{\rho {v_2}^2}{2}$

Along a horizontal streamline, Bernoulli's equation reduces to

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2,$$

or

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2).$$

Since the density of the fluid is always a positive quantity, the sign of $v_2^2 - v_1^2$ gives you the sign of $p_1 - p_2$, allowing you to determine whether p_2 is greater than or smaller than p_1.

Hint 3. Determine v_2 with respect to v_1

By applying the continuity equation, determine which of the following is true.

Hint 1. The continuity equation

The continuity equation expresses conservation of mass for incompressible fluids flowing in a tube. It says that the amount of fluid ΔV flowing through a cross section A of the tube in a time interval Δt must be the same for all cross sections, or

$$\frac{\Delta V}{\Delta t} = A_1 v_1 = A_2 v_2.$$

Therefore, the flow speed must increase when the cross section of the flow tube decreases, and vice versa.

ANSWER:

- (selected) $v_2 > v_1$
- $v_2 = v_1$
- $v_2 < v_1$

ANSWER:

The pressure at point 2 is
- (selected) lower than the pressure at point 1.
- equal to the pressure at point 1.
- higher than the pressure at point 1.

Thus, by combining the continuity equation and Bernoulli's equation, one can characterize the flow of an ideal fluid.When the cross section of the flow tube decreases, the flow speed increases, and therefore the pressure decreases. In other words, if $A_2 < A_1$, then $v_2 > v_1$ and $p_2 < p_1$.

Part B

As you found out in the previous part, Bernoulli's equation tells us that a fluid element that flows through a flow tube with decreasing cross section moves toward a region of lower pressure. Physically, the pressure drop experienced by the fluid element between points 1 and 2 acts on the fluid element as a net force that causes the fluid to __________.

Hint 1. Effects from conservation of mass

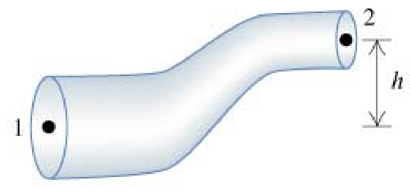
Recall that, if the cross section A of the flow tube varies, the flow speed v *must* change to conserve mass. This means that there is a nonzero net force acting on the fluid that causes the fluid to increase or decrease speed depending on whether the fluid is flowing through a portion of the tube with a smaller or larger cross section.

ANSWER:

- decrease in speed
- (selected) increase in speed
- remain in equilibrium

Part C

Now assume that point 2 is at height h with respect to point 1, as shown in the figure. The ends of the flow tube have the same areas as the ends of the horizontal flow tube shown in Part A. Since the cross section of the flow tube is decreasing, Bernoulli's equation tells us that a fluid element flowing toward point 2 from point 1 moves toward a region of lower pressure. In this case, what is the pressure drop experienced by the fluid element?



Hint 1. How to approach the problem

Apply Bernoulli's equation to point 1 and to point 2, as you did in Part A. Note that this time you must take into account the difference in elevation between points 1 and 2. Do you need to add this additional term to the other term representing the pressure drop between the two ends of the flow tube or do you subtract it?

ANSWER:

The pressure drop is
- smaller than the pressure drop occurring in a purely horizontal flow.
- equal to the pressure drop occurring in a purely horizontal flow.
- (selected) larger than the pressure drop occurring in a purely horizontal flow.

Part D

From a physical point of view, how do you explain the fact that the pressure drop at the ends of the elevated flow tube from Part C is *larger* than the pressure drop occurring in the similar but purely horizontal flow from Part A?

Hint 1. Physical meaning of the pressure drop in a tube

As explained in the introduction, the difference in pressure $p_1 - p_2$ between the ends of a flow tube represents the total work done on a unit volume of fluid by the pressure forces of the surrounding fluid to move that volume of fluid from one end to the other end of the flow tube.

ANSWER:

A greater amount of work is needed to balance the
- (selected) increase in potential energy from the elevation change.
- decrease in potential energy from the elevation change.
- larger increase in kinetic energy.
- larger decrease in kinetic energy.

In the case of purely horizontal flow, the difference in pressure between the two ends of the flow tube had to balance only the increase in kinetic energy resulting from the acceleration of the fluid. In an elevated flow tube, the difference in pressure must also balance the increase in potential energy of the fluid; therefore a higher pressure is needed for the flow to occur.

Exercise 12.4

You win the lottery and decide to impress your friends by exhibiting a million-dollar cube of gold. At the time, gold is selling for $\$$ 426.60 per troy ounce, and 1.0000 troy ounce equals 31.1035 g.

Part A

How tall would your million-dollar cube be?

ANSWER:

d = 15.6 cm

Exercise 12.10

Part A

Calculate the difference in blood pressure between the feet and top of the head for a person who is 1.60m tall.

ANSWER:

P = $1060 \cdot 9.80 h$ = 1.66×10^4 Pa

Part B

Consider a cylindrical segment of a blood vessel 3.50cm long and 2.20mm in diameter. What *additional* outward force would such a vessel need to withstand in the person's feet compared to a similar vessel in her head?

ANSWER:

F = $1060 \cdot 9.80 \pi h d l$ = 4.02 N

Exercise 12.23

Part A

For the hydraulic lift shown in Figure 12.7 in the textbook, what must be the ratio of the diameter of the vessel at the car to the diameter of the vessel where the force F_1 is applied so that a 1530 kg car can be lifted with a force F_1 of just 110 N ?

ANSWER:

$\frac{D_{car}}{D_{F_1}} = \sqrt{\frac{9.8M}{F_1}}$ = 11.7

Exercise 12.30

A hollow, plastic sphere is held below the surface of a freshwater lake by a cord anchored to the bottom of the lake. The sphere has a volume of 0.690 $\mathrm{m^3}$ and the tension in the cord is 800 N .

Part A

Calculate the buoyant force exerted by the water on the sphere.

ANSWER:

$F = 1000{\cdot}9.8V$ = 6760 N

Part B

What is the mass of the sphere?

ANSWER:

$m = \frac{1000{\cdot}9.8V - T}{9.8}$ = 608 kg

Part C

The cord breaks and the sphere rises to the surface. When the sphere comes to rest, what fraction of its volume will be submerged?

ANSWER:

$\frac{9800V - T}{9800V}{\cdot}100$ = 88.2 %

Exercise 12.36

Water is flowing in a pipe with a varying cross-sectional area, and at all points the water completely fills the pipe. At point 1 the cross-sectional area of the pipe is $0.070\,\mathrm{m^2}$, and the magnitude of the fluid velocity is $3.50\ \mathrm{m/s}$.

Part A

What is the fluid speed at point in the pipe where the cross-sectional area is $0.105\,\mathrm{m^2}$?

Express your answer using two significant figures.

ANSWER:

v = 2.3 $\mathrm{m/s}$

Part B

What is the fluid speed at point in the pipe where the cross-sectional area is $0.047\,\mathrm{m^2}$?

Express your answer using two significant figures.

ANSWER:

v = 5.2 m/s

Part C

Calculate the volume of water discharged from the open end of the pipe in $1.00 \mathrm{hour}$.

Express your answer using two significant figures.

ANSWER:

V = 880 $\mathrm{m^3}$

Problem 12.56

It has been proposed that we could explore Mars using inflated balloons to hover just above the surface. The buoyancy of the atmosphere would keep the balloon aloft. The density of the Martian atmosphere is 0.0154 $\mathrm{kg/m^3}$ (although this varies with temperature). Suppose we construct these balloons of a thin but tough plastic having a density such that each square meter has a mass of 4.90g . We inflate them with a very light gas whose mass we can neglect.

Part A

What should be the radius of these balloons so they just hover above the surface of Mars?

ANSWER:

$R = \frac{3}{0.0154}\rho$ = 0.955 m

Part B

What should be the mass of these balloons so they just hover above the surface of Mars?

ANSWER:

$m = \frac{36 \cdot 3.1415926}{0.0154^2}\rho^3$ = 5.61×10^{-2} kg

Part C

If we released one of the balloons from part A on earth, where the atmospheric density is 1.20 $\mathrm{kg/m^3}$, what would be its initial acceleration assuming it was the same size as on Mars?

ANSWER:

a = 754 $\mathrm{m/s^2}$

Part D

Would it go up or down?

ANSWER:

- (selected) Up
- Down

Part E

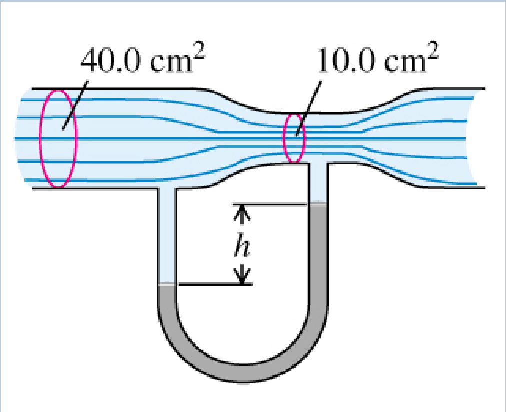
If on Mars these balloons have five times the radius found in part A, how heavy an instrument package could they carry?

ANSWER:

$m_{\mathrm{tools}} = \frac{\frac{4}{3} \cdot 3.1415926 \cdot 125 \cdot 27\rho^3}{0.0154^2} - \frac{4 \cdot 3.1415926 \cdot 25 \cdot 9\rho^3}{0.0154^2}$ = 5.61 kg

Problem 12.94

The horizontal pipe, shown in the figure , has a cross-sectional area of $40.0\ \mathrm{cm}^2$ at the wider portions and $10.0\ \mathrm{cm}^2$ at the constriction. Water is flowing in the pipe, and the discharge from the pipe is $6.00 \times 10^{-3}\ \mathrm{m}^3/\mathrm{s}\ (6.00\ \mathrm{L/s})$. The density of mercury is $\rho_{\mathrm{Hg}} = 13.6 \times 10^3\ \mathrm{kg/m}^3$ and the density of water is $\rho_{\mathrm{w}} = 1.00 \times 10^3\ \mathrm{kg/m}^3$.



Part A

Find the flow speed at the wide portion.

ANSWER:

v = 1.50 $\mathrm{m/s}$

Part B

Find the flow speed at the narrow portion.

ANSWER:

v = 6.00 $\mathrm{m/s}$

Part C

What is the pressure difference between these portions?

ANSWER:

ΔP = 1.69×10^4 Pa

Part D

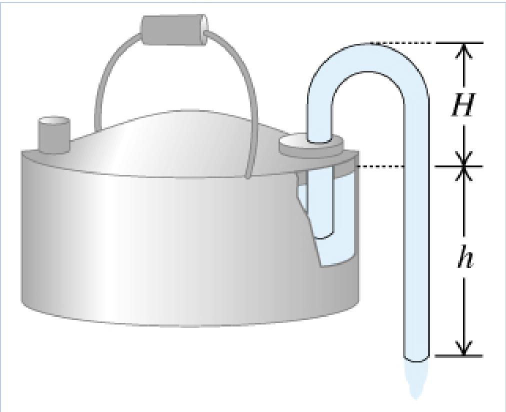
What is the difference in height between the mercury columns in the U-shaped tube?

ANSWER:

Δh = 13.7 cm

Problem 12.98

A *siphon,* as shown in the figure , is a convenient device for removing liquids from containers. To establish the flow, the tube must be initially filled with fluid. Let the fluid have density ρ, and let the atmospheric pressure be p_{a}. Assume that the cross-sectional area of the tube is the same at all points along it.



Part A

If the lower end of the siphon is at a distance h below the surface of the liquid in the container, what is the speed of the fluid as it flows out the lower end of the siphon? (Assume that the container has a very large diameter, and ignore any effects of viscosity.)

Express your answer in terms of the given quantities and appropriate constants.

ANSWER:

v = $\sqrt{2gh}$

Part B

A curious feature of a siphon is that the fluid initially flows "uphill." What is the greatest height H that the high point of the tube can have if flow is still to occur?

Express your answer in terms of the given quantities and appropriate constants.

ANSWER:

H = $\frac{p_a}{\rho g} - h$

12.4. IDENTIFY: Find the mass of gold that has a value of $\$1.00\times10^6$. Then use the density of gold to find the volume of this mass of gold.

SET UP: For gold, $\rho = 19.3\times10^3\ \text{kg/m}^3$. The volume V of a cube is related to the length L of one side by $V = L^3$.

EXECUTE: $m = (\$1.00\times10^6)\left(\dfrac{1\ \text{troy ounce}}{\$426.60}\right)\left(\dfrac{31.1035\times10^{-3}\ \text{kg}}{1\ \text{troy ounce}}\right) = 72.9\ \text{kg}$. $\rho = \dfrac{m}{V}$ so

$$V = \frac{m}{\rho} = \frac{72.9\ \text{kg}}{19.3\times10^3\ \text{kg/m}^3} = 3.78\times10^{-3}\ \text{m}^3. \quad L = V^{1/3} = 0.156\ \text{m} = 15.6\ \text{cm}.$$

EVALUATE: The cube of gold would weigh about 160 lbs.

12.10. IDENTIFY: The difference in pressure at points with heights y_1 and y_2 is $p - p_0 = \rho g(y_1 - y_2)$. The outward force $F_\perp$ is related to the surface area A by $F_\perp = pA$.

SET UP: For blood, $\rho = 1.06\times10^3\ \text{kg/m}^3$. $y_1 - y_2 = 1.65\ \text{m}$. The surface area of the segment is πDL, where $D = 1.50\times10^{-3}\ \text{m}$ and $L = 2.00\times10^{-2}\ \text{m}$.

EXECUTE: **(a)** $p_1 - p_2 = (1.06\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.65\ \text{m}) = 1.71\times10^4\ \text{Pa}$.

(b) The additional force due to this pressure difference is $\Delta F_\perp = (p_1 - p_2)A$.

$$A = \pi DL = \pi(1.50\times10^{-3}\ \text{m})(2.00\times10^{-2}\ \text{m}) = 9.42\times10^{-5}\ \text{m}^2.$$

$$\Delta F_\perp = (1.71\times10^4\ \text{Pa})(9.42\times10^{-5}\ \text{m}^2) = 1.61\ \text{N}.$$

EVALUATE: The pressure difference is about $\frac{1}{6}$ atm.

12.23. IDENTIFY: $F_2 = \dfrac{A_2}{A_1}F_1$. F_2 must equal the weight $w = mg$ of the car.

SET UP: $A = \pi D^2/4$. D_1 is the diameter of the vessel at the piston where F_1 is applied and D_2 of the diameter at the car.

EXECUTE: $mg = \dfrac{\pi D_2^2/4}{\pi D_1^2/4}F_1$. $\dfrac{D_2}{D_1} = \sqrt{\dfrac{mg}{F_1}} = \sqrt{\dfrac{(1520\ \text{kg})(9.80\ \text{m/s}^2)}{125\ \text{N}}} = 10.9$

EVALUATE: The diameter is smaller where the force is smaller, so the pressure will be the same at both pistons.

12.30. IDENTIFY: $B = \rho_{\text{water}}V_{\text{obj}}g$. The net force on the sphere is zero.

SET UP: The density of water is 1.00×10^3 kg/m^3.

EXECUTE: **(a)** $B=(1000\ \text{kg/m}^3)(0.650\ \text{m}^3)(9.80\ \text{m/s}^2)=6.37\times 10^3\ \text{N}$

(b) $B=T+mg$ and $m=\dfrac{B-T}{g}=\dfrac{6.37\times 10^3\ \text{N}-900\ \text{N}}{9.80\ \text{m/s}^2}=558\ \text{kg}.$

(c) Now $B=\rho_{\text{water}}V_{\text{sub}}g$, where V_{sub} is the volume of the sphere that is submerged. $B=mg$.

$\rho_{\text{water}}V_{\text{sub}}g=mg$ and $V_{\text{sub}}=\dfrac{m}{\rho_{\text{water}}}=\dfrac{558\ \text{kg}}{1000\ \text{kg/m}^3}=0.558\ \text{m}^3.$

$$\frac{V_{\text{sub}}}{V_{\text{obj}}}=\frac{0.558\ \text{m}^3}{0.650\ \text{m}^3}=0.858=85.8\%.$$

EVALUATE: The average density of the sphere is $\rho_{\text{sph}}=\dfrac{m}{V}=\dfrac{558\ \text{kg}}{0.650\ \text{m}^3}=858\ \text{kg/m}^3.$

$\rho_{\text{sph}}<\rho_{\text{water}}$, and that is why it floats with 85.8% of its volume submerged.

12.36.IDENTIFY: $v_1A_1=v_2A_2$. The volume flow rate is vA.

SET UP: $1.00\ \text{h}=3600\ \text{s}$.

EXECUTE: **(a)** $v_2=v_1\left(\dfrac{A_1}{A_2}\right)=(3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.105\ \text{m}^2}\right)=2.33\ \text{m/s}$

(b) $v_2=v_1\left(\dfrac{A_1}{A_2}\right)=(3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.047\ \text{m}^2}\right)=5.21\ \text{m/s}$

(c) $V=v_1A_1t=(3.50\ \text{m/s})(0.070\ \text{m}^2)(3600\ \text{s})=882\ \text{m}^3.$

EVALUATE: The equation of continuity says the volume flow rate is the same at all points in the pipe.

12.56.IDENTIFY: The buoyant force B equals the weight of the air displaced by the balloon.

SET UP: $B=\rho_{\text{air}}Vg$. Let g_{M} be the value of g for Mars. For a sphere $V=\frac{4}{3}\pi R^3$. The surface area of a sphere is given by $A=4\pi R^2$. The mass of the balloon is $(5.00\times 10^{-3}\ \text{kg/m}^2)(4\pi R^2)$.

EXECUTE: **(a)** $B=mg_{\text{M}}$. $\rho_{\text{air}}Vg_{\text{M}}=mg_{\text{M}}$. $\rho_{\text{air}}\frac{4}{3}\pi R^3=(5.00\times 10^{-3}\ \text{kg/m}^2)(4\pi R^2)$.

$R=\dfrac{3(5.00\times 10^{-3}\ \text{kg/m}^2)}{\rho_{\text{air}}}=0.974\ \text{m}.$ $m=(5.00\times 10^{-3}\ \text{kg/m}^2)(4\pi R^2)=0.0596\ \text{kg}.$

(b) $F_{\text{net}}=B-mg=ma$.

$$B=\rho_{\text{air}}Vg=\rho_{\text{air}}\tfrac{4}{3}\pi R^3g=(1.20\ \text{kg/m}^3)\left(\frac{4\pi}{3}\right)(0.974\ \text{m})^3(9.80\ \text{m/s}^2)=45.5\ \text{N}.$$

$$a=\frac{B-mg}{m}=\frac{45.5\text{ N}-(0.0596\text{ kg})(9.80\text{ m/s}^2)}{0.0596\text{ m}}=754\text{ m/s}^2,\ \text{upward.}$$

(c) $B=m_{\text{tot}}g.\quad \rho_{\text{air}}Vg=(m_{\text{balloon}}+m_{\text{load}})g.\quad m_{\text{load}}=\rho_{\text{air}}\frac{4}{3}\pi R^3-(5.00\times10^{-3}\text{ kg/m}^2)4\pi R^2.$

$$m_{\text{load}}=(0.0154\text{ kg/m}^3)\left(\frac{4\pi}{3}\right)(5[0.974\text{ m}])^3-(5.00\times10^{-3}\text{ kg/m}^2)(4\pi)(5[0.974\text{ m}])^2$$

$$m_{\text{load}}=7.45\text{ kg }-1.49\text{ kg}=5.96\text{ kg}$$

EVALUATE: The buoyant force is proportional to R^3 and the mass of the balloon is proportional to R^2, so the load that can be carried increases when the radius of the balloon increases. We calculated the mass of the load. To find the weight of the load we would need to know the value of *g* for Mars.

12.94.IDENTIFY: Apply Bernoulli's equation to points 1 and 2. Apply $p=p_0+\rho gh$ to both arms of the U-shaped tube in order to calculate *h*.

SET UP: The discharge rate is $v_1A_1=v_2A_2$. The density of mercury is $\rho_{\text{m}}=13.6\times10^3\text{ kg/m}^3$ and the density of water is $\rho_{\text{w}}=1.00\times10^3\text{ kg/m}^3$. Let point 1 be where $A_1=40.0\times10^{-4}\text{ m}^2$ and point 2 is where $A_2=10.0\times10^{-4}\text{ m}^2$. $y_1=y_2$.

EXECUTE: **(a)** $v_1=\dfrac{6.00\times10^{-3}\text{ m}^3\text{/s}}{40.0\times10^{-4}\text{ m}^2}=1.50\text{ m/s}.\quad v_2=\dfrac{6.00\times10^{-3}\text{ m}^3\text{/s}}{10.0\times10^{-4}\text{ m}^2}=6.00\text{ m/s}$

(b) $p_1+\rho gy_1+\frac{1}{2}\rho v_1^2=p_2+\rho gy_2+\frac{1}{2}\rho v_2^2.$

$$p_1-p_2=\tfrac{1}{2}\rho(v_2^2-v_1^2)=\tfrac{1}{2}(1000\text{ kg/m}^3)([6.00\text{ m/s}]^2-[1.50\text{ m/s}]^2)=1.69\times10^4\text{ Pa}$$

(c) $p_1+\rho_{\text{w}}gh=p_2+\rho_{\text{m}}gh$ and

$$h=\frac{p_1-p_2}{(\rho_{\text{m}}-\rho_{\text{w}})g}=\frac{1.69\times10^4\text{ Pa}}{(13.6\times10^3\text{ kg/m}^3-1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)}=0.137\text{ m}=13.7\text{ cm.}$$

EVALUATE: The pressure in the fluid decreases when the speed of the fluid increases.

12.98.IDENTIFY: Apply Bernoulli's equation to the fluid in the siphon.

SET UP: Example 12.8 shows that the efflux speed from a small hole a distance *h* below the surface of fluid in a large open tank is $\sqrt{2gh}$.

EXECUTE: **(a)** The fact that the water first moves upward before leaving the siphon does not change the efflux speed, $\sqrt{2gh}$.

(b) Water will not flow if the absolute (not gauge) pressure would be negative. The hose is open to the atmosphere at the bottom, so the pressure at the top of the siphon is $p_{\text{a}}-\rho g(H+h)$, where the assumption that the cross-sectional area is constant has been used to equate the speed of the liquid at the top and bottom. Setting $p = 0$ and solving for H gives $H = (p_a/\rho g) - h$.

EVALUATE: The analysis shows that $H + h < \frac{p_a}{\rho g}$, so there is also a limitation on $H + h$. For water and normal atmospheric pressure, $\frac{p_a}{\rho g} = 10.3\ \text{m}$.