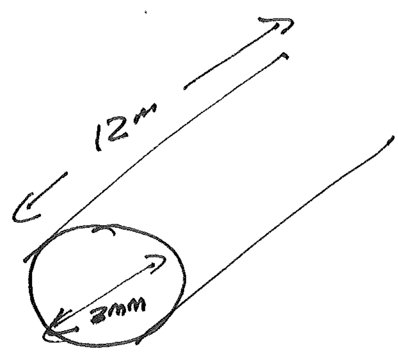
Physics 161, HW #4

#1



$R = .25\Omega$

CIRCULAR AREA OF DIAMETER 3mm

$\Rightarrow r = 1.5\text{mm} = 1.5\times10^{-3}\text{m}$

$A = \pi r^2 = \pi(1.5\times10^{-3}\text{m})^2 = 7.069\times10^{-6}\text{m}^2$

a) WHAT IS RESISTIVITY?

$$R = \rho\frac{L}{A} \Rightarrow \rho = \frac{R\cdot A}{L} = \frac{.25\Omega(7.069\times10^{-6}\text{m}^2)}{12\text{m}} \Rightarrow \boxed{\rho = 1.47\times10^{-7}\Omega\cdot\text{m}}$$

b) IF $E = 1.75\text{V/m}$, WHAT IS TOTAL CURRENT?

$$E = \rho J \Rightarrow J = E/\rho = \frac{1.75\text{V/m}}{1.47\times10^{-7}\Omega\cdot\text{m}} = 1.19\times10^{7}\text{A/m}^2$$

$$J = \frac{I}{A} \Rightarrow I = JA = (1.19\times10^{7}\text{A/m}^2)(7.069\times10^{-6}\text{m}^2) \Rightarrow \boxed{I = 84.2\text{A}}$$

c) IF THE MATERIAL HAS $n = 8.5\times10^{28}$ free electrons/m^3 FIND DRIFT VELOCITY.

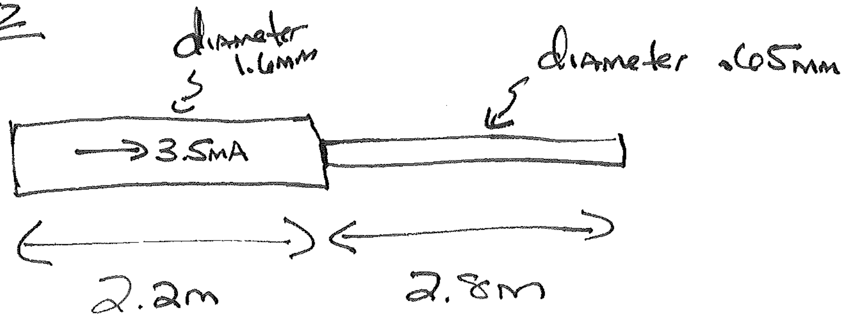
$$J = neV_d \Rightarrow V_d = \frac{J}{ne} = \frac{1.19\times10^{7}\text{A/m}^2}{(8.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\text{C})}$$

UNIT: $\frac{\text{A}}{\text{m}^2}\cdot\frac{\text{m}^3}{\text{C}} = \frac{\text{C/s}\cdot\text{m}}{\text{C}} = \text{m/s}$

$$\Rightarrow \boxed{V_d = 8.75\times10^{-4}\text{m/s} = .875\text{mm/s}}$$

EVEN FOR VERY LARGE Current, THE DRIFT Velocity IS SMALL

#2



Let Larger Section be #1:

$I_1 = 3.5\text{mA}$

$d_1 = 1.6\text{mm},\ L_1 = 2.2\text{m}$

Smaller Section is #2:

$I_2 = ?,\ d_2 = .65\text{mm},\ L_2 = 2.8\text{m}$

a) WHAT IS I_2? EVERY CHARGE THAT LEAVES #1 MUST go INTO #2 (OR VICE-VERSA) $\Rightarrow$ #1 AND #2 ARE CONNECTED IN SERIES $\Rightarrow$ $\boxed{I_2 = I_1 = 3.5\text{mA}}$

b) WHAT IS E_1? $E = \rho J$. Copper at 20°C has $\rho = 1.72\times10^{-8}\,\Omega\cdot\text{m}$

$$\Rightarrow E_1 = \rho J_1 = (1.72\times10^{-8}\,\Omega\cdot\text{m})J_1 = (1.72\times10^{-8}\,\Omega\cdot\text{m})\frac{I_1}{A_1}. \quad A_1 = \pi r_1^2$$

$$r_1 = \frac{1.6\text{mm}}{2} = .8\text{mm}$$

$$\Rightarrow E_1 = \frac{(1.72\times10^{-8}\,\Omega\cdot\text{m})(3.5\times10^{-3}\text{A})}{\pi(.8\times10^{-3}\text{m})^2}$$

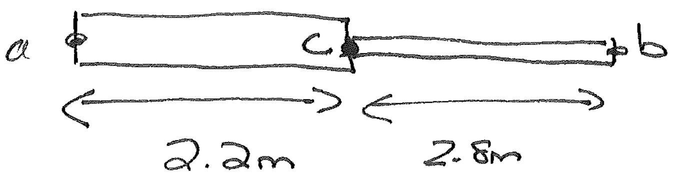
$$\text{Unit: } \frac{\Omega\cdot\text{A}}{\text{m}} = \frac{\text{V/A}\cdot\text{A}}{\text{m}} = \text{V/m}$$

$$\Rightarrow \boxed{E_1 = 2.99\times10^{-5}\,\text{V/m}}$$

c) WHAT IS E_2? $E_2 = \rho J_2 = \rho\frac{I_2}{A_2}$ $\quad A_2 = \pi r_2^2 \quad r_2 = \frac{.65\text{mm}}{2} = .325\text{mm}$

$$\Rightarrow E_2 = \frac{(1.72\times10^{-8}\,\Omega\cdot\text{m})(3.5\times10^{-3}\text{A})}{\pi(.325\times10^{-3}\text{m})^2} \Rightarrow \boxed{E_2 = 1.81\times10^{-4}\,\text{V/m}}$$

d) WHAT IS THE POTENTIAL DIFFERENCE BETWEEN THE ENDS?



$V_{ab} = ?$

Let CONNECTION POINT be C

$V_{ac} = V_a - V_c, \quad V_{cb} = V_c - V_b$

$\Rightarrow V_{ac} + V_{cb} = V_a - V_c + V_c - V_b = V_a - V_b = V_{ab}$

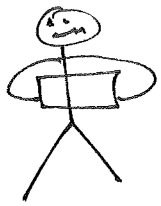
ASSUMING UNIFORM E_1 AND E_2 $\Rightarrow V_{ac} = E_1 L_1, \quad V_{cb} = E_2 L_2$

$\Rightarrow V_{ac} = (2.99 \times 10^{-5}\, V/m)(2.2m) = 6.578 \times 10^{-5} V$

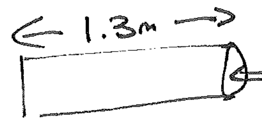
$V_{cb} = (1.81 \times 10^{-4}\, V/m)(2.8m) = 5.068 \times 10^{-4} V$

$\Rightarrow \boxed{V_{ab} = 5.7258 \times 10^{-4} V = .57258\, mV = .573\, mV}$

#3



$\rho = 5\,\Omega\cdot m$



diameter 0.16m ⇒ radios, $r = .08m$

a) WHAT IS RESISTANCE? $R = \rho \frac{L}{A} = \rho \frac{L}{\pi r^2} = \frac{5\,\Omega\cdot m\,(1.3m)}{\pi (.08m)^2}$

$\Rightarrow \boxed{R = 323\,\Omega}$

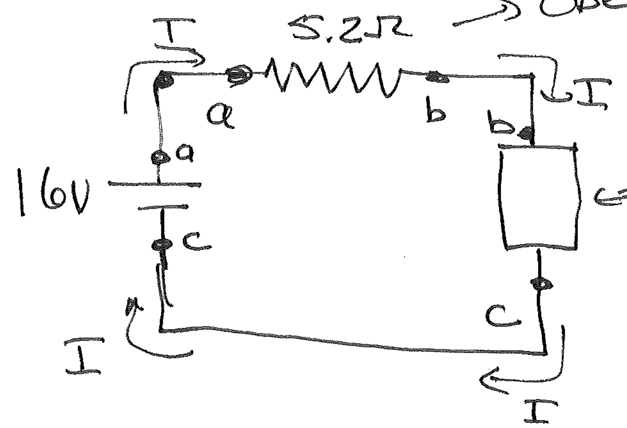
b) $V = ?$ to produce $I = 100mA = .1A$

$V = IR \Rightarrow \boxed{V = (.1A)(323\Omega) = 32.3V}$ ↳ Much less THAN THE 120V MOST APPLIANCES RUN ON

c) $P = IV = (.1A)(32.3V) \Rightarrow \boxed{P = 3.23\,Watt}$

#4

→ Obeys OHM's LAW ⇒ $V_R = I(5.2\Omega)$



← THERMISTOR with $V_T = \alpha I + \beta I^2$

$\alpha = 4\Omega, \quad \beta = 1.8\Omega/A$

LAbel points (a), (b), (c) As Above

$\Rightarrow V_{ac} = V_a - V_c = 16V$

$V_{ab} = V_a - V_b = V_R$

$V_{bc} = V_b - V_c = V_T$

$\Rightarrow V_{ab} + V_{bc} = V_{ac}$

$\Rightarrow V_R + V_T = 16V$

$\therefore \; I(5.2\Omega) + (4\Omega)I + (1.8\Omega/A)I^2 = 16V$

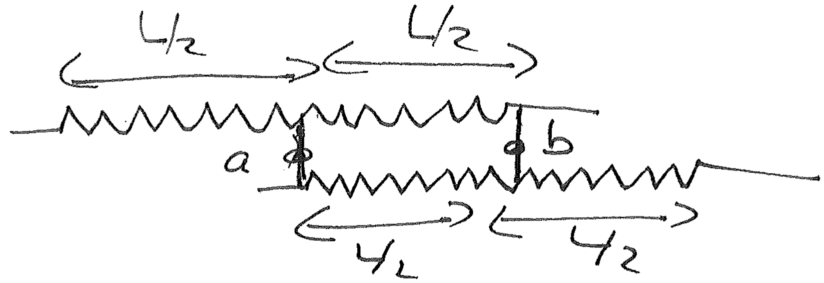
$\Rightarrow (1.8\Omega/A)I^2 + (9.2\Omega)I - 16V = 0$ ← QUADRATIC EQN.

$\Rightarrow I = \dfrac{-9.2\Omega \pm \sqrt{(9.2\Omega)^2 - 4(1.8\Omega/A)(-16V)}}{2(1.8\Omega/A)} = \dfrac{-9.2\Omega \pm \sqrt{199.84\Omega^2}}{3.6\Omega/A}$

$\Rightarrow I = 1.37A$ OR $-6.48A$ $\Rightarrow \boxed{I = 1.37A}$

No

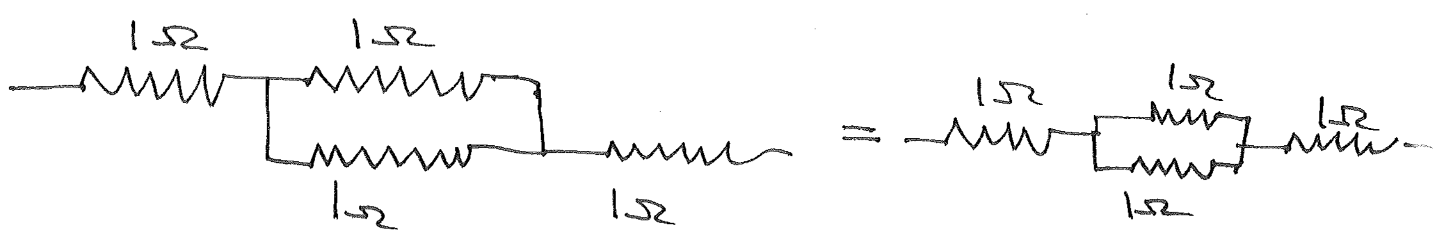
#5



Soldered ⇒ Connected by Metal = Equipotential Surface

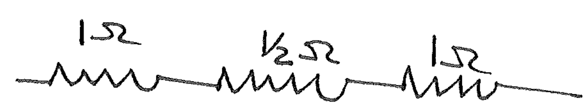
Label a, b ⇒ Half of top resistor and half of bottom resistor at V_{ab} ⇒ in parallel with each other

$R = \rho \frac{L}{A}$ ⇒ $\frac{1}{2}$ of a ~~...~~ 2Ω resistor has $R = 1\Omega$

⇒ 

Find equivalent resistance. For parallel combination

$$R' = \frac{(1\Omega)(1\Omega)}{(1\Omega + 1\Omega)} = \frac{1}{2}\Omega$$

⇒ 

3 resistors in series ⇒ $R_{EQ} = 1\Omega + \frac{1}{2}\Omega + 1\Omega$

$$\Rightarrow \boxed{R_{EQ} = 2\tfrac{1}{2}\Omega = 2.5\Omega}$$

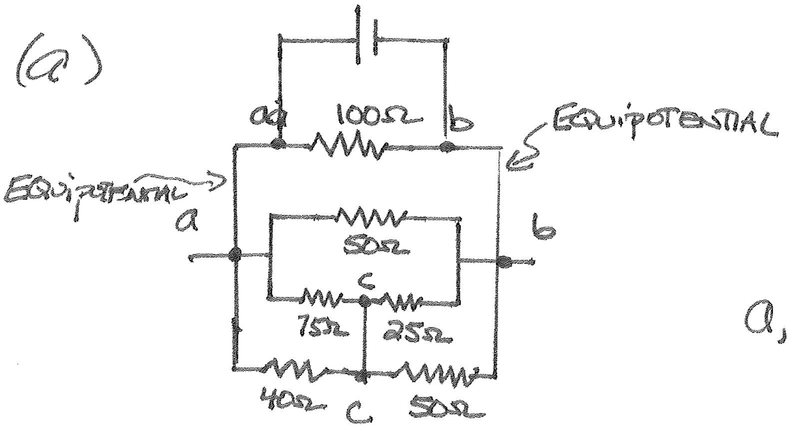
#36

IF OHMMETER CONNECTED to a & b,

WHAT WILL IT READ?

OHMMETER APPLIES VOLTAGE ⇒

(a)

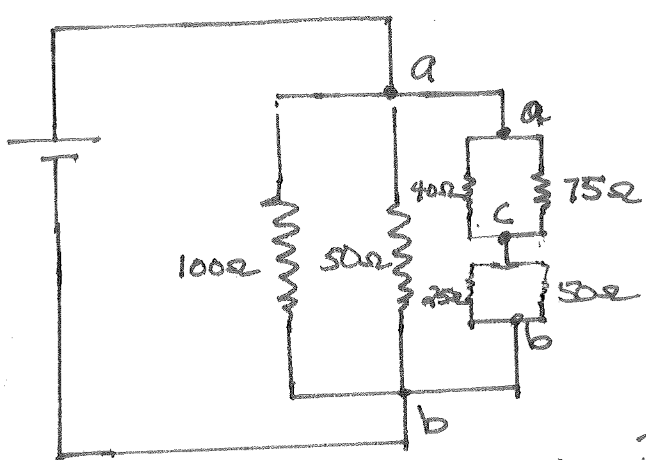


a, b ⇒ 100Ω, UPPER 50Ω, AND REST IN parallel.

a, c ⇒ 75Ω, 40Ω in parallel

b, c ⇒ 25Ω, 50Ω in parallel

⇒



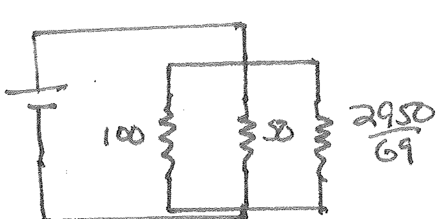
EQUIVALENT for 40Ω, 75Ω IN ~~SERIES~~ parallel

$$R' = \frac{75\Omega(40\Omega)}{(75\Omega+40\Omega)} = \frac{3000}{115}\,\Omega = \frac{600}{23}\,\Omega = 26.09\Omega$$

$$R'' = \frac{25\Omega(50\Omega)}{(25\Omega+50\Omega)} = \frac{1250}{75}\,\Omega = \frac{50}{3}\,\Omega = 16.666\Omega$$

EQUIVALENT for 25Ω, 50Ω

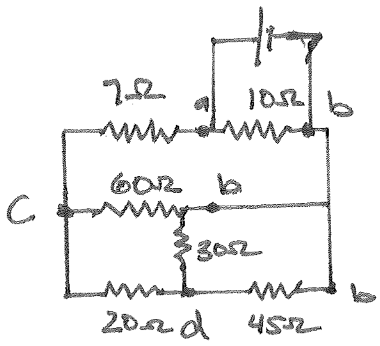
$$R' + R'' = \frac{2950}{69} = 22.753$$ ← R', R'' IN SERIES



$$\frac{1}{R_{EQ}} = \frac{1}{100} + \frac{1}{50} + \frac{1}{2950/69} \Rightarrow \boxed{R_{EQ} = 18.73\Omega}$$

#7

(c)



ab ⇒ 10Ω in parallel to everything else

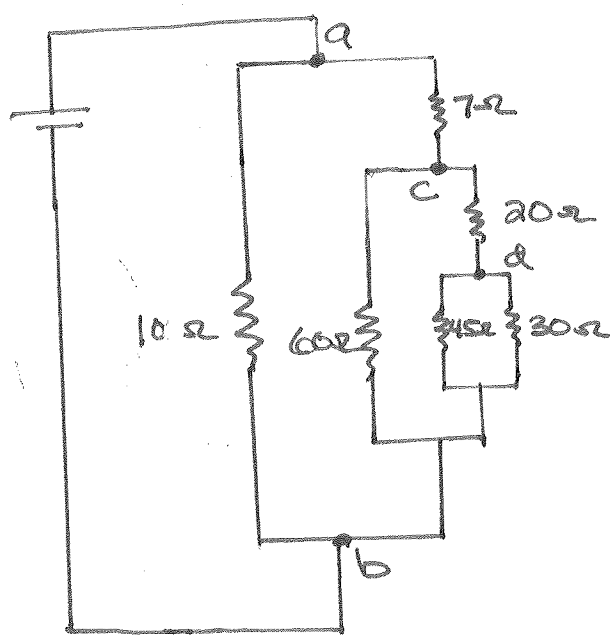
ac ⇒ 7Ω in series with 60, 20, 30, 45 combo

cb ⇒ 60Ω in parallel to 20, 30, 45Ω combination

db ⇒ 30, 45 in parallel

cd ⇒ 20 in series with 30, 45

⇒

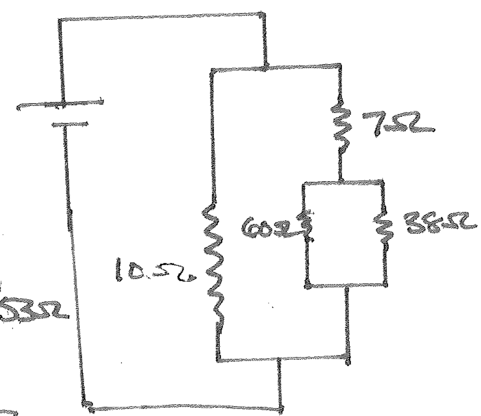


R' = 30Ω, 45Ω in parallel

$\Rightarrow R' = \frac{30\Omega(45\Omega)}{(30\Omega+45\Omega)} = 18\Omega$

$20\Omega + 18\Omega = 38\Omega$

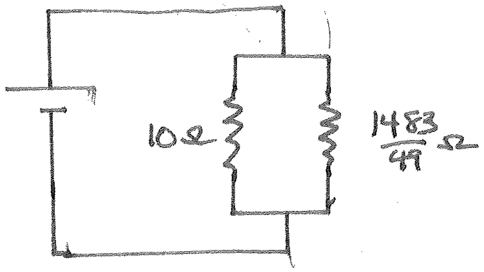
⇒



60, 38 in parallel $\Rightarrow R'' = \frac{60\Omega(38\Omega)}{98\Omega} = \frac{1140}{49}\Omega$

$= 23.2653\Omega$

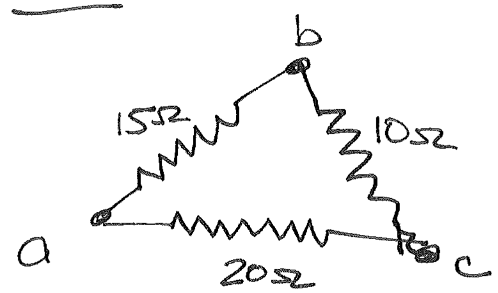
7 in series $\Rightarrow 7 + \frac{1140}{49} = \frac{1483}{49} = 30.2653\Omega$



$$R_{EQ} = \frac{10\Omega\left(\frac{1483}{49}\Omega\right)}{\left[10\Omega + \frac{1483}{49}\Omega\right]}$$

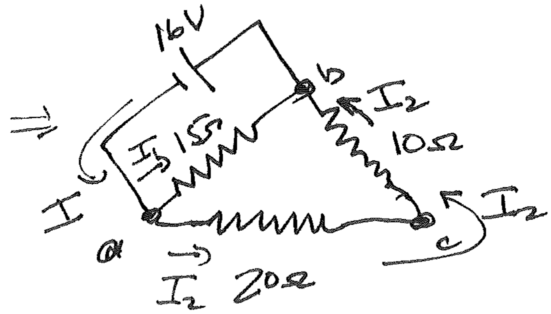
$$\Rightarrow R_{EQ} = 7.5165 = 7.52\Omega$$

#8



WHAT CURRENT FROM 16V battery IF Connected to:

(a) ab ⇒



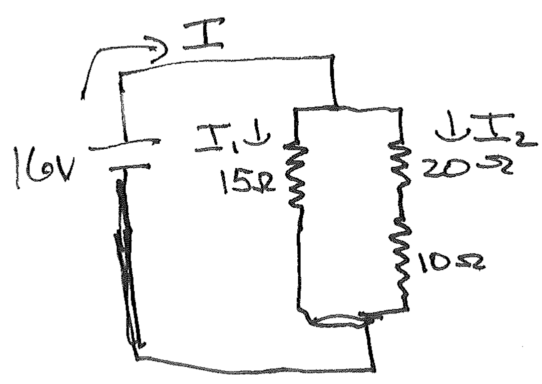
Current splits at a

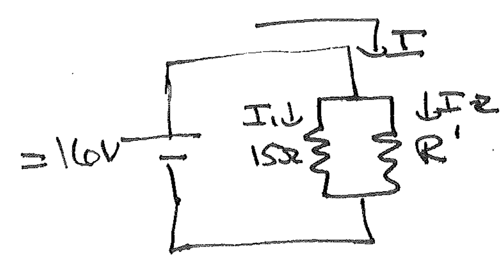
I_1 goes through 15Ω then back to battery

I_2 goes through 20Ω then 10Ω then back to battery

⇒ 20Ω + 10Ω IN SERIES. By "Finger Test" 15Ω in parallel to 20Ω/10Ω COMBINATION

⇒

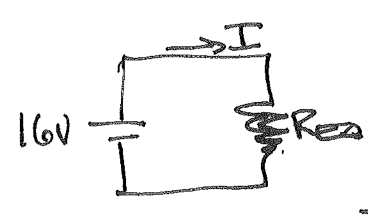




$R' = 20\Omega + 10\Omega$

$= 30\Omega$

=

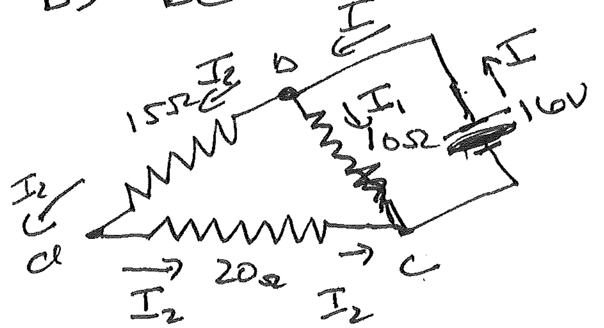


$R_{EQ} = \dfrac{15\Omega(30\Omega)}{(15\Omega + 30\Omega)}$

$= 10\Omega$

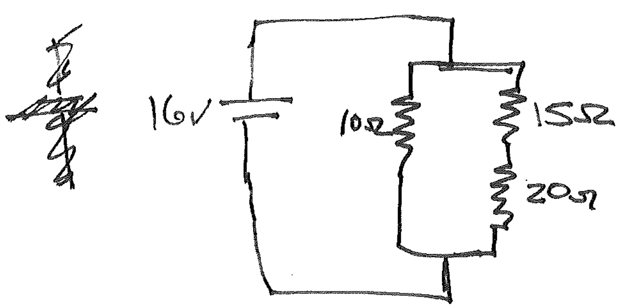
$$\Rightarrow \boxed{I_0 = \frac{16V}{10\Omega} = 1.6A}$$

b) bc



Now when Current splits, I_1 goes Through 10Ω then back to battery. I_2 goes through 15Ω then 20Ω

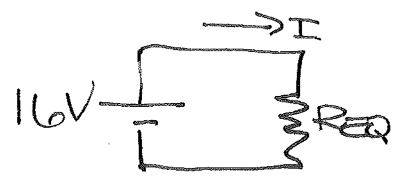
⇒ 15Ω AND 20Ω IN SERIES, 10Ω IN parallel to COMBINATION.



SAME CAlculation for REQ but

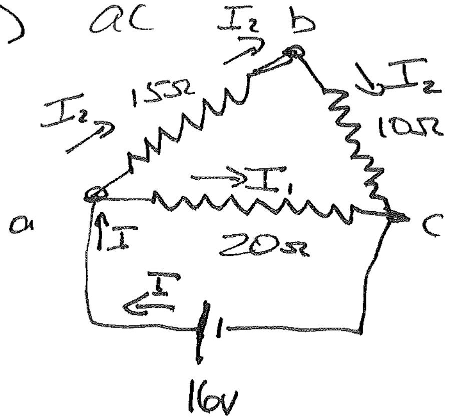
$$R' = 15\Omega + 20\Omega = 35\Omega$$

$$\Rightarrow R_{EQ} = \frac{(10\Omega)(35\Omega)}{(10\Omega + 35\Omega)} = 7.77\ldots\Omega$$

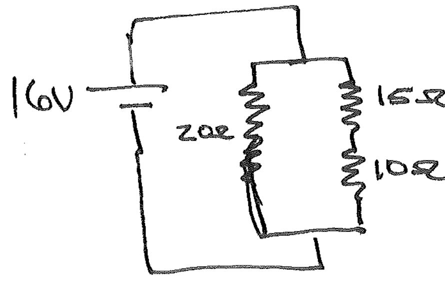


$$\boxed{I = \frac{16V}{7.77\Omega} = 2.057A}$$

c) ac



15Ω AND 10Ω IN SERIES, 20Ω IN Parallel WITH COMBINATION



$$R' = 15\Omega + 10\Omega = 25\Omega$$

$$R_{EQ} = \frac{(20\Omega)(25\Omega)}{(20\Omega + 25\Omega)}$$

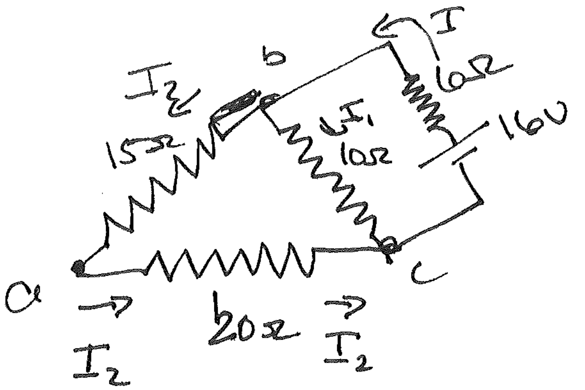
$$= 11.111\ldots\Omega$$

$$\Rightarrow \boxed{I = \frac{16V}{11.111\Omega} = 1.44A}$$

d) If battery has 6Ω of internal Resistance

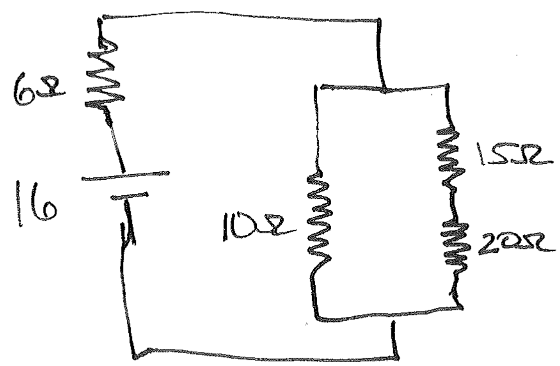
$\Rightarrow$ what current, if connected across bc?

Simply add 6Ω in series with battery.



Still have 15Ω, 20Ω in series.

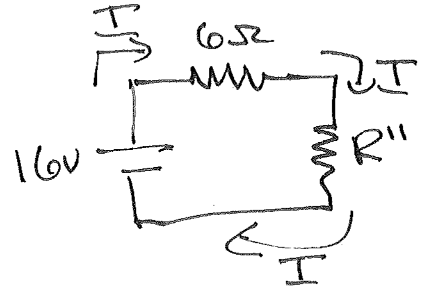
10Ω in parallel with combination



Like before, the 10Ω, 15Ω

and 20Ω behave like

$$R'' = \frac{(10\Omega)(35\Omega)}{45\Omega} = 7.77\ldots\Omega$$



6Ω and R'' in series $\Rightarrow R_{EQ} = 6\Omega + 7.77\Omega$

$\Rightarrow R_{EQ} = 13.777\ldots\Omega$

$$\therefore \boxed{I = \frac{16V}{13.777\Omega} = 1.16129A = 1.16A}$$

In Summary:

	ab	bc	ac	bc with internal R
I	1.6A	2.057A	1.44A	1.16A