#30 Equilibrium of Rigid Bodies I Pre-class

Due: 11:00am on Friday, November 2, 2012

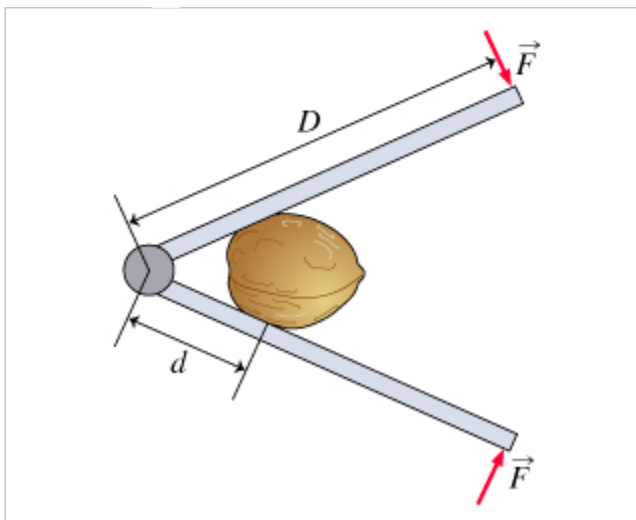
Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

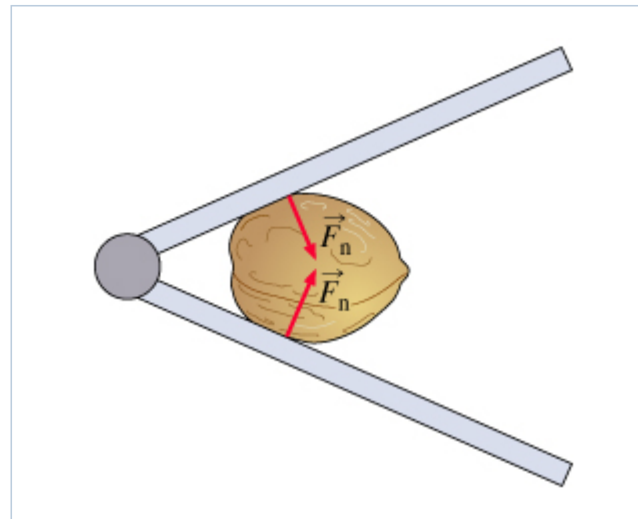
A Tale of Two Nutcrackers

This problem explores the ways that torque can be used in everyday life.

Case 1

To crack a nut a force of magnitude $F_{\rm n}$ (or greater) must be applied on both sides, as shown in the figure. One can see that a nutcracker only applies this force at the point in which it contacts the nut (at a distance d from the nutcracker pivot). In this problem the nut is placed in a nutcracker and equal forces of magnitude F are applied to each end, directed perpendicular to the handle, at a distance D from the pivot. The frictional forces between the nut and the nutcracker are equal and large enough that the nut doesn't shoot out of the nutcracker.





Part A

Find F, the magnitude of the force applied to each side of the nutcracker required to crack the nut.

Express the force in terms of $F_{\rm n}$, d, and D.

Hint 1. Sum of torques

There is no angular acceleration. What will the sum of the torques be about any point?

Hint 2. Sum of torques about a specific point

Take the hinge as the pivot point. Note that the forces at the pivot point give no torque. Since the forces are symmetric about the *x* axis we can just consider the top piece of the nutcracker. What is the sum of the torques about the hinge $\Sigma\tau_{\rm h}$ generated by forces on the top piece (both F and $F_{\rm n}$)? Consider counterclockwise (right-handed) torques to be positive.

Answer in terms of given quantities.

ANSWER:

$\sum\tau_{\rm h}$ = $-FD+F_{\rm n}d$

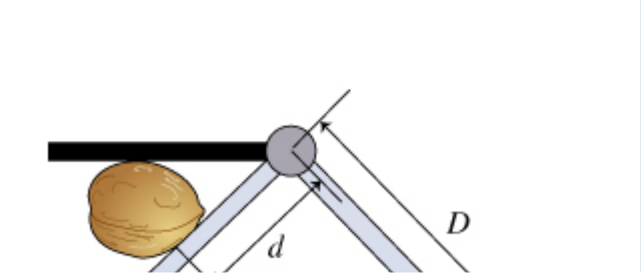
ANSWER:

F = $\frac{F_n d}{D}$

Correct

Case 2

The nut is now placed in a nutcracker with only one lever, as shown, and again friction keeps the nut from slipping. The top "jaw" (in black) is fixed to a stationary frame so that a person just has to apply a force to the bottom lever. Assume that F_2 is directed perpendicular to the handle.



Part B

Find the magnitude of the force F_2 required to crack the nut.

Express your answer in terms of F_n, d, and D.

Hint 1. Sum of torques

Again, the sum of the torques about the hinge will be zero. Also, taking the hinge as a pivot point will again eliminate torques due to the forces between the two pieces of the nutcracker.

ANSWER:

$F_2 = \frac{F_n d}{D}$

Correct

Part C

For the first nutcracker, two applied forces of magnitude F were required to crack the nut, whereas for the second, only one applied force of magnitude F was required. How would you explain this difference?

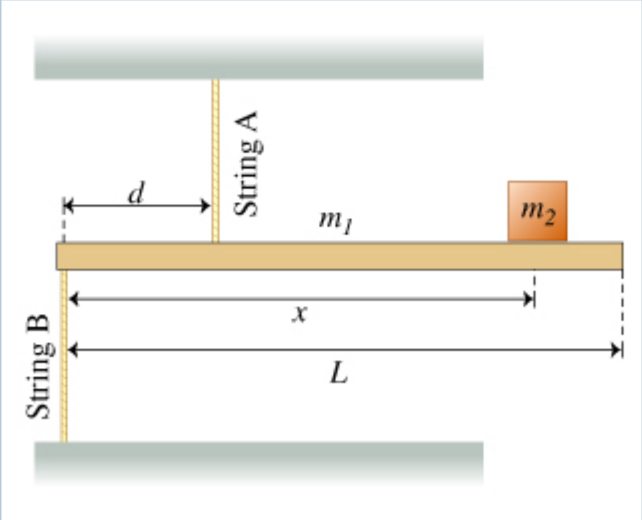
ANSWER:

- In the second case the nutcracker handle is effectively longer and generates twice the torque of that in the first case.
- There is an additional force of magnitude F_n applied to the nut by the fixed jaw in the second case. This jaw is held fixed by external forces (such as the forces due to the frame).
- The net torque on the nutcracker in the second case is non-zero.

Correct

A Bar Suspended by Two Vertical Strings

A rigid, uniform, horizontal bar of mass m_1 and length L is supported by two identical massless strings. Both strings are vertical. String A is attached at a distance $d < L/2$ from the left end of the bar and is connected to the ceiling; string B is attached to the left end of the bar and is connected to the floor. A small block of mass m_2 is supported against gravity by the bar at a distance x from the left end of the bar, as shown in the figure.



Throughout this problem positive torque is that which spins an object counterclockwise. Use g for the magnitude of the acceleration due to gravity.

Part A

Find T_A, the tension in string A.

Express the tension in string A in terms of g, m_1, L, d, m_2, and x.

Hint 1. Choosing an axis

Choose a rotation axis *p*, about which to apply the requirement $\sum \tau_p = 0$. Since the system is in static equilibrium, the choice of rotation axis is arbitrary; however, there is a convenient choice of *p* to find T_A by eliminating the torque from an unknown force.

Hint 2. Find the torque around the best axis

It is convenient to choose the rotation axis to be through the point where string B is attached to the bar. This eliminates any torque from the tension in string B. Find the total torque about this point.

Answer in terms of T_A, m_1, m_2, L, x, d, and g.

ANSWER:

$\sum \tau_B$ = $T_A d - \frac{m_1 g L}{2} - m_2 g x$

Hint 3. Summing the torques

$\sum \tau_p = 0$ for a static system. Solve for T_A.

ANSWER:

T_A = $\frac{m_1 g \frac{L}{2} + m_2 g x}{d}$

Correct

Part B

Find T_B, the magnitude of the tension in string B.

Express the magnitude of the tension in string B in terms of T_A, m_1, m_2, and g.

Hint 1. Two different methods to find T_B

There are two equivalent ways to find T_B. One way is to balance the torques as was done in the calculation of T_A, except using a different rotation axis. In this case, a convenient axis is through the point where string A is attached to the bar. The second, and easier, method is to use the second equation for static equilibrium, $\sum \vec{F} = 0$.

Hint 2. Direction of forces

Since both strings are vertical, all forces on the bar--the tension forces and the weights of the bar and block--act vertically. Thus, only vertical components of forces need be considered.

ANSWER:

T_B = $T_A - g\,(m_1 + m_2)$

Correct

Part C

If the bar and block are too heavy the strings may break. Which of the two identical strings will break first?

ANSWER:

- string A
- string B

Correct

Part D

If the mass of the block is too large *and* the block is too close to the left end of the bar (near string B) then the horizontal bar may become unstable (i.e., the bar may no longer remain horizontal).

What is the smallest possible value of x such that the bar remains stable (call it $x_{\rm critical}$)?

Express your answer for $x_{\rm critical}$ in terms of m_1, m_2, d, and L.

Hint 1. Nature of the unstable motion

When the bar becomes unstable there are only two points about which the bar can rotate: the points where the strings attach to the bar. About which point will the bar rotate when $x < x_{\rm critical}$?

ANSWER:

- ◉ The point where string A is attached to the bar
- ○ The point where string B is attached to the bar

Hint 2. Tension in string B at the critical point

The tension in string B counteracts the clockwise rotation of the bar about the point where string A is attached to the bar. As x is decreased, T_B is likewise decreased because the clockwise torque about this point decreases. The critical value $x_{\rm critical}$ corresponds to when $T_B = 0$. If x is decreased further, T_B will continue to be zero and the counterclockwise torque due to the weight of the block will be greater than the clockwise torque due to the weight of the bar, causing the system to rotate.

Hint 3. Calculate the torques

Add up the total torque about the point in which string A attaches to the bar when the mass m_2 is at $x_{\rm critical}$. Remember that T_B has a special value at this point and that, owing to the choice of origin, the torque due to string A is 0. Remember to pay attention to the direction of the torques.

Answer in terms of m_2, m_1, d, L, g, and $x_{\rm critical}$.

Hint 1. Find the distance of the center of mass of the bar from string A

What is the distance d_1 of the center of mass of the bar from string A?

Answer in terms of the given variables.

ANSWER:

d_1 = $\frac{L}{2}-d$

Hint 2. Find the distance of m_2 from the string A

What is the distance d_2 of m_2 from the string A?

Answer in terms of the given variables.

ANSWER:

d_2 = $d-x_{\rm critical}$

ANSWER:

$\sum\tau_A$ = 0 = $m_2g\left(d-x_{\rm critical}\right)-m_1g\left(\frac{L}{2}-d\right)$

ANSWER:

x_{critical} = $\frac{dm_1}{m_2} + d - m_1\frac{L}{2m_2}$

Correct

Part E

Note that since x_{critical}, as computed in the previous part, is not necessarily positive. If $x_{\text{critical}} < 0$, the bar will be stable no matter where the block of mass m_2 is placed on it.

Assuming that m_1, d, and L are held fixed, what is the maximum block mass m_{max} for which the bar will *always* be stable? In other words, what is the maximum block mass such that $x_{\text{critical}} \leq 0$?

Answer in terms of m_1, d, and L.

Hint 1. Requirement of stability

If x is calculated to be less than zero, the solution is unphysical. (The bar does not extend there to support it!) The minimum value that x can have is obviously zero. If m is less than the mass that would give $x_{\text{critical}} = 0$ then the bar will be stable for any physical value of x.

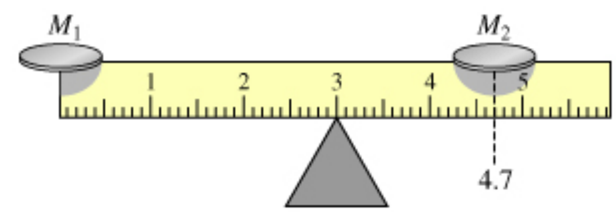
ANSWER:

m_{max} = $m_1\frac{L}{2d} - m_1$

Correct

± Torques on a Ruler

A centimeter ruler, balanced at its center point, has two coins placed on it, as shown in the figure. One coin, of mass $M_1 = 10 \text{ g}$, is placed at the zero mark; the other, of unknown mass M_2, is placed at the 4.7 cm mark. The center of the ruler is at the 3.0 cm mark. The ruler is in equilibrium; it is perfectly balanced.



Part A

Does the pivot point (i.e., the triangle in the diagram upon which the ruler balances) exert a force on the ruler? Does it exert a nonzero torque about the pivot?

Choose the correct series of answers to these two questions.

ANSWER:

- yes/yes
- no/no
- yes/no
- no/yes

Correct

Note that the weight of the ruler itself also does not exert a torque with respect to the pivot point since the rule is uniform and is pivoted at its midpoint.

Part B

Although the pivot exerts a force on the ruler it does not exert a torque with respect to the pivot point. Why not?

ANSWER:

- ○ Because the exerted force is vertical.
- ◉ Because the distance from the exerted force to the pivot is zero.
- ○ Because the exerted force is along the ruler.

Correct

Part C

Find the mass M_2.

Express your answer for the unknown mass numerically, in grams, to two significant digits.

Hint 1. Sum of torques

Because the ruler is in equilibrium, the sum of the torques about any point along the ruler must be zero. Specifically, the sum of the torques about the pivot point must be zero.

Hint 2. Determine the moment arm of M_1

What is the moment arm of M_1 (call it r_1) about the ruler's pivot point? The moment arm is the distance, perpendicular to the direction of the force, from the pivot to the point of application of the force. It is what you multiply the force by to get the magnitude of the torque due to that force.

Hint 1. Finding a moment arm

To find the moment arm for the force exerted by each coin on the ruler, you should measure from the pivot, *not* from the end of the ruler.

ANSWER:

r_1 = 3 cm

ANSWER:

M_2 = 18 g

Correct

The Center of It All

Learning Goal:

To learn the definition of the center of mass for systems of particles and for extended objects and be able to locate it.

Imagine throwing a rock upward and away from you. With negligible air resistance, the rock will follow a parabolic path before hitting the ground. Now imagine throwing a stick (or any other extended object). More likely than not, the stick will rotate throughout its flight; the motion of each point of the stick will be fairly complex. However, there will be one point that will follow the parabolic path, namely, the point about which the stick is rotating.

Such a special point can be located experimentally, for instance, by videotaping the stick in flight and then analyzing its motion. The exciting part is that no matter how you throw the stick, such a special point will always exist, and will always be located at the same spot within the stick! The motion of the entire stick can then be described as a combination of the motion of that point, as if the entire mass of the stick were concentrated there, and the rotation of the stick about that point. Such a point, it turns out, exists for any rigid object or system of massive particles. It is called the *center of mass*.

Locating the center of mass of an object seems a worthy task, because it helps in analyzing the motion of the object. However, it would be annoying and sometimes infeasible to videotape each object in motion in order to locate its center of mass. Fortunately, the location of the center of mass of an object or system can be *calculated* using the following set of formulas (note that we will usually consider cases in which the *z* coordinate is irrelevant):

For a system of massive point particles that have coordinates (x_i, y_i) and masses m_i:

$$x_{\rm cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots},$$

$$y_{\rm cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}.$$

If we replace the quantity $(m_1 + m_2 + m_3 + \cdots)$ by the total mass of the system M, these formulas can be rewritten as follows:

$$x_{\rm cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{M}$$
$$= \frac{1}{M}(m_1x_1 + m_2x_2 + m_3x_3 + \cdots),$$

$$y_{\rm cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{M}$$
$$= \frac{1}{M}(m_1y_1 + m_2y_2 + m_3y_3 + \cdots)$$

In this problem, you will practice locating the center of mass for various systems of particles and for some simple extended objects.

Part A

Two particles of masses m_1 and m_2 ($m_1 < m_2$) are located 10 meters apart. Where is the center of mass of the system located?

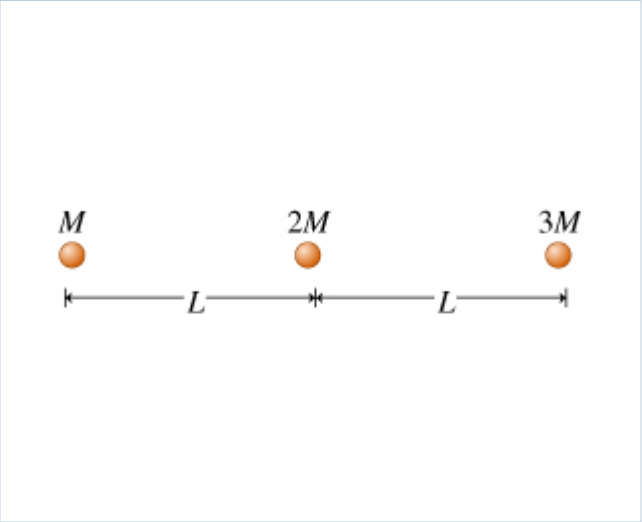
ANSWER:

- less than 5 meters from the particle of mass m_1
- exactly 5 meters from the particle of mass m_1
- more than 5 meters but less than 10 meters from the particle of mass m_1
- more than 10 meters from the particle of mass m_1

Correct

Part B

For the system of three particles shown, which have masses M, $2M$, and $3M$ as indicated, where is the center of mass located?



ANSWER:

- to the left of the particle of mass M
- between the particle of mass M and the particle of mass $2M$
- between the particle of mass $2M$ and the particle of mass $3M$
- to the right of the particle of mass $3M$

Correct

Perhaps you could "feel" that the center of mass should be located to the right of the particle of mass $2M$, since the particle to the right of it has greater mass than the particle to the left. A calculation, however, allows one to pinpoint the exact location of the center of mass.

Part C

For the system of particles described in the Part B, find the *x* coordinate $x_{\rm cm}$ of the center of mass. Assume that the particle of mass M is at the origin and the positive *x* axis is directed to the right.

Express your answer in terms of L.

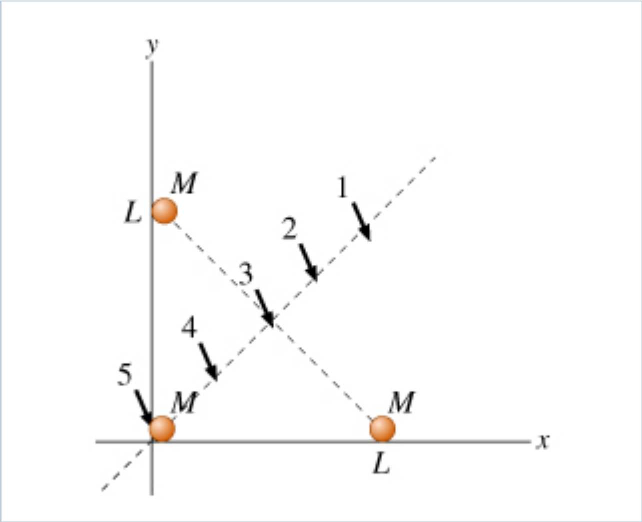
ANSWER:

$x_{\rm cm}$ = $\frac{4L}{3}$

Correct

Part D

Let us now consider a two-dimensional case. The system includes three particles of equal mass M located at the vertices of an isosceles triangle as shown in the figure. Which arrow best shows the location of the center of mass of the system? Do not calculate.



ANSWER:

- 1
- 2
- 3
- 4 (selected)
- 5

Correct

Part E

What is the *x* coordinate $x_{\rm cm}$ of the center of mass of the system described in Part D?

Express your answer in terms of L.

ANSWER:

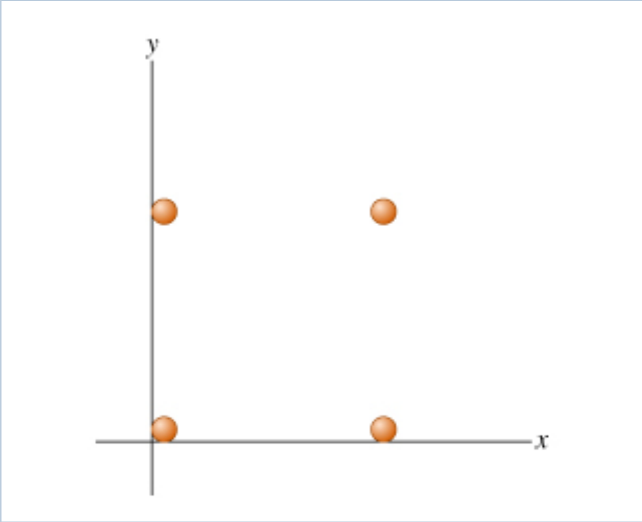
$x_{\rm cm}$ = $\frac{L}{3}$

Correct

From the symmetry of the situation, you can see that the *x* and *y* coordinates of the center of mass are the same.

Part F

A system of four buckets forms a square as shown in the figure. Initially, the buckets have different masses (it is not known how these masses are related). A student begins to add water gradually to the bucket located at the origin. As a result, what happens to the coordinates of the center of mass of the system of buckets?



Hint 1. The movement of the center of mass

As the bucket at the origin gets heavier, would the center of mass be moving closer to the origin or away from it?

ANSWER:

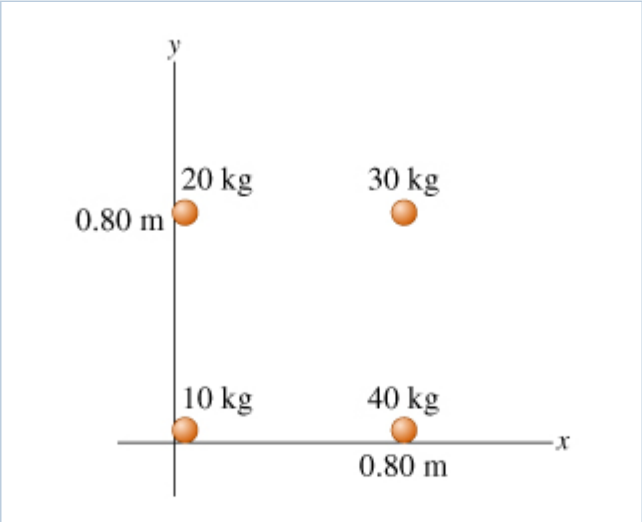
- The *x* coordinate stays the same; the *y* coordinate increases.
- The *x* coordinate stays the same; the *y* coordinate decreases.
- The *x* coordinate increases; the *y* coordinate stays the same.
- The *x* coordinate decreases; the *y* coordinate stays the same.
- The *x* coordinate increases; the *y* coordinate increases.
- (selected) The *x* coordinate decreases; the *y* coordinate decreases.
- The *x* coordinate stays the same; the *y* coordinate stays the same.
- There is not enough information to answer the question.

Correct

Part G

Find the *x* coordinate x_{cm} of the center of mass of the system of particles shown in the figure.

Express your answer in meters to two significant figures.



ANSWER:

x_{cm} = 0.56 m

Correct

Part H

Find the *y* coordinate of the center of mass of the system of particles described in the previous part.

Express your answer in meters to two significant figures.

ANSWER:

y_{cm} = 0.40 m

Correct

For an extended object of mass M, the sum in parentheses is replaced by an integral, and the formulas look like this:

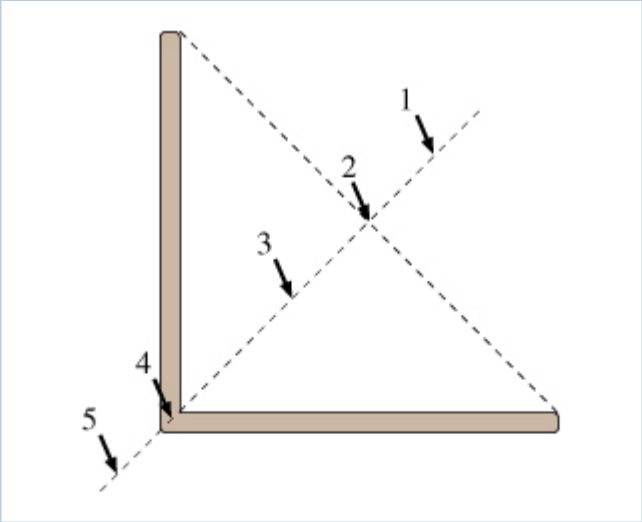
$$x_{\mathrm{cm}} = \frac{1}{M}\int x\,dm,$$

$$y_{\mathrm{cm}} = \frac{1}{M}\int y\,dm,$$

where dm is the infinitesimal mass element (the mass of a small area or volume in the limit as the area or volume goes to zero). For instance, if the density of an object is ρ, then the infinitesimal mass element would be $dm = \rho\,dV$, where dV is an infinitesimal volume element and ρ is evaluated at the point where dV is located. Frequently, the calculations are aided by the very powerful idea of *symmetry*, which plays a major role in all areas of physics. For instance, the center of mass of a uniform circle is at its center; that of a uniform straight rod is at the midpoint; etc.
Note that in dealing with extended objects of irregular shape, it is often useful to use the idea of symmetry to locate the centers of mass of the symmetrical subparts and then treat these subparts as point particles located at their respective centers of mass.

Part I

Let us now consider an extended object. A wire is bent at its midpoint through a right angle as shown in the figure. Which arrow best shows the location of the center of mass of the system? Do *not* calculate.



ANSWER:

- 1
- 2
- 3
- 4
- 5

Correct

For an extended object, the center of mass does *not* have to be within the object itself. This wire is one example; one can think of many others including bagels. The exact location of the center of mass can be found using the symmetry approach: It is located halfway between the midpoints of the two straight segments of the wire (can you see why?).

Part J

A straight rod has one end at the origin and the other end at the point $(L, 0)$ and a linear density given by $\lambda = ax^2$, where a is a known constant and x is the *x* coordinate. Since this wire is not uniform, you will have to use integrtation to solve this part. Use $M = \int_0^L dm$ to find the total mass M. Find $x_{\rm cm}$ for this rod.

Express your answer in terms of one or both of a and L.

Hint 1. How to approach the problem

Evaluate the integral $\int_0^L x\,dm$ and divide it by the mass of the rod. To find that mass, you have to evaluate $\int_0^L dm$.

Hint 2. What is dm?

By definition, the linear density is given by $\lambda = dm/dx$. Therefore, $dm = \lambda\,dx = ax^2\,dx$. Use this expression for dm in both integrals.

ANSWER:

$x_{\rm cm}$ = $\frac{3L}{4}$

Correct

There is another concept closely related to the center of mass, the *center of gravity*. This concept is useful, since gravity is a "distributed force": It is applied to each particle making up an object. Gravity is also the most ubiquitous force of all, which means that the net gravitational force almost always plays a role in determining an object's motion.

The center of gravity is defined as the point at which the net force of gravity acting on the object is applied. The good news is that as long as the gravitational field is uniform, the locations of the center of mass and the center of gravity coincide. In practical terms, thsi means that the center of gravity and the center of mass coincide as long as the object's size is much less than that of Earth (or whatever planet or star provides the force of gravity in a particular situation).

Score Summary:

Your score on this assignment is 101.7%.
You received 20.34 out of a possible total of 20 points.