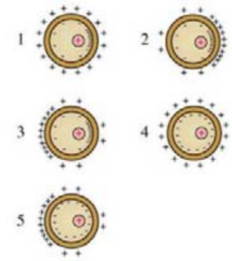
Charge Distribution on a Conducting Shell - 2

Description: Conceptual problem. Positive charge sits off-center inside a conducting shell. What is the charge distribution on the inner and outer surfaces of the shell?

A positive charge is kept (fixed) off-center inside a fixed spherical conducting shell that is electrically neutral, and the charges in the shell are allowed to reach electrostatic equilibrium. The large positive charge inside the shell is roughly 16 times that of the smaller charges shown on the inner and outer surfaces of the spherical shell.

Part A

Which of the following figures best represents the charge distribution on the inner and outer walls of the shell?



Hints (2)

Hint 1. Symmetry inside shell

The field inside the conductor must be zero. The charge inside the shell is off-center, and hence the charge on the inner surface of the shell will arrange itself asymmetrically to cancel the field of the large positive charge.

Hint 2. Symmetry outside shell

The field inside the conductor must be zero. To the charges on the outer surface, it is as if the inside of the conductor were completely neutral. Thus, the charges on the outer surface will feel no force other than their own mutual repulsion, and will therefore have no preferred direction.

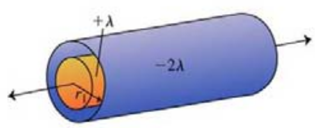
ANSWER:

- (selected) 1
- 2
- 3
- 4
- 5

A Conducting Shell around a Conducting Rod

Description: An infinite charged rod sits at the center of an infinite conducting cylindrical shell. Determine the field between the rod and shell, the field outside the shell, and the surface charge on the inner and outer surfaces of the shell.

An infinitely long conducting cylindrical rod with a positive charge λ per unit length is surrounded by a conducting cylindrical shell (which is also infinitely long) with a charge per unit length of -2λ and radius r_1, as shown in the figure.



Part A

What is $E(r)$, the radial component of the electric field between the rod and cylindrical shell as a function of the distance r from the axis of the cylindrical rod?

Express your answer in terms of λ, r, and ϵ_0, the permittivity of free space.

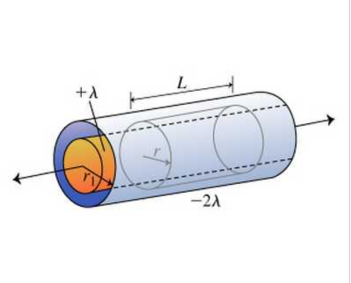
Hints (4)

Hint 1. The implications of symmetry

Because the cylinder and rod are cylindrically symmetric, the magnitude of the electric field *cannot* vary as a function of angle around the rod, nor as a function of longitudinal position along the rod (typically represented by the spatial variables θ and z). By symmetry, the magnitude of the electric field can only depend on the distance from the axis of the rod (the spatial variable r).

Hint 2. Apply Gauss' law

Gauss's law states that $\Phi_E = \frac{q}{\epsilon_0}$, where Φ_E is the electric flux through a Gaussian surface, and q is the total charge enclosed by the surface. Construct a cylindrical Gaussian surface with radius r and length L coaxial with the rod, with $r < r_1$.



Hint 3. Find the charge inside the Gaussian surface

What is the total charge q_{inner} enclosed by the surface?

ANSWER:

$q_{\text{inner}} = L\lambda$

Hint 4. Find the flux

What is Φ_E, the electric flux through the Gaussian surface?

Express your answer in terms of the magnitude of the electric field E and given variables.

ANSWER:

$\Phi_E = 2\pi r L E$

ANSWER:

$E(r) = \dfrac{\lambda}{2\pi r \epsilon_0}$

Part B

What is σ_{inner}, the surface charge density (charge per unit area) on the inner surface of the conducting shell?

Hints (2)

Hint 1. Apply Gauss's law

The magnitude of the net force on charges within a conductor is always zero. This implies that the magnitude of the electric field within the conductor is zero. Think about a cylindrical Gaussian surface of length L whose radius lies at the middle of the outer cylindrical shell. Since the electric field inside a conductor is zero and the Gaussian surface lies within the conductor, the electric flux across the Gaussian surface must be zero. What, then, must q, the total charge inside this Gaussian surface, be?

ANSWER:

$q = 0$

Hint 2. Find the charge contribution from the surface

What is q_{inner}, the total charge on the inner surface of the cylindrical shell that is contained within the Gaussian surface?

Express your answer in terms of L and λ.

ANSWER:

$q_{\text{inner}} = -L\lambda$

To obtain the charge density per unit area, divide q_{inner} by the area of the inner surface of the conducting shell that is contained within the Gaussian surface.

ANSWER:

$\sigma_{\text{inner}} = \dfrac{-\lambda}{2\pi r_1}$

Part C

What is $\sigma_{\rm outer}$, the surface charge density on the outside of the conducting shell? (Recall from the problem statement that the conducting shell has a total charge per unit length given by -2λ.)

Hints (1)

Hint 1. What is the charge on the cylindrical shell?

What is $q_{\rm total}$, the total surface charge (the sum of charges on the inner and outer surfaces) of a portion of the shell of length L?

ANSWER:

$q_{\rm total}$ = $-2\lambda L$

Since the charge on the inner surface of the cylinder is $q_{\rm inner}$ and the total charge on the cylinder is $q_{\rm total}$, it is now easy to obtain the charge on the outer surface of the cylinder. Then divide this result by the surface area of the portion of the cylinder that you took to obtain your result.

ANSWER:

$\sigma_{\rm outer}$ = $\frac{-\lambda}{2\pi r_1}$

Part D

What is the radial component of the electric field, $E(r)$, outside the shell?

Hints (3)

Hint 1. How to approach the problem

Apply Gauss's law as you did to find the field between the rod and the shell. Again, choose the Gaussian surface to be a cylinder, with length L and radius r, coaxial with the rod. This time, you need to take $r > r_1$.

Hint 2. Find the charge within the Gaussian surface

What is $q_{\rm outer}$, the total charge contained within the Gaussian surface?

ANSWER:

$q_{\rm outer}$ = $-\lambda L$

Hint 3. Find the flux in terms of the electric field

What is Φ_E, the electric flux through the Gaussian surface?

Express your answer in terms of the magnitude of the electric field E and given variables.

ANSWER:

Φ_E = $2\pi r L E$

ANSWER:

$E(r)$ = $\frac{-\lambda}{2\pi r \epsilon_0}$

Charged Insulating Spheres

Description: Calculate the electric field, both its magnitude and direction, midway between two charged insulating spheres with uniform charge throughout.

Two small insulating spheres with radius $7.50\times10^{-2}\,{\rm m}$ are separated by a large center-to-center distance of $0.520\,{\rm m}$. One sphere is negatively charged, with net charge $-1.40\,\mu{\rm C}$, and the other sphere is positively charged, with net charge $4.30\,\mu{\rm C}$. The charge is uniformly distributed within the volume of each sphere.

Part A

What is the magnitude E of the electric field midway between the spheres?

Take the permittivity of free space to be ϵ_0 = $8.86\times10^{-12}\,{\rm C^2/(N\cdot m^2)}$.

Hints (7)

Hint 1. How to approach the problem

Draw a diagram of the system to keep track of the directions of the fields. Calculate the electric field at the point midway between the charged spheres separately for each sphere, using Gauss's law, and use vector addition to determine the net electric field.

Hint 2. Using Gauss's law

Hint 4. Determine the direction of the electric field from the first sphere

What will be the direction of the electric field due to the negatively charged sphere only?

ANSWER:

- ◉ toward the center of the negatively charged sphere
- ○ away from the center of the negatively charged sphere
- ○ upward perpendicular to the line connecting the centers of the spheres
- ○ downward perpendicular to the line connecting the centers of the spheres

Note that the electric field will point toward a negatively charged sphere, just as it would point toward a negative point charge.

Hint 5. Calculate the field due to the positively charged sphere

Calculate E_2, the magnitude of the electric field at the midway point due to the sphere of charge 4.30 μC only.

Take the permittivity of free space to be ϵ_0 = 8.85×10^{-12} $C^2/(N \cdot m^2)$.

[-] Hints (1)

Hint 1. Using the flux to calculate the field

Since the flux through the surface will be $\Phi = q/\epsilon_0 = EA$ (due to Gauss's law), you can solve for the electric field E_1 using the equation for the surface area of the (Gaussian) sphere, $A = 4\pi R^2$, and the value of the enclosed charge.

[-] Hints (1)

Hint 1. Using the flux to calculate the field

Since the flux through the surface will be $\Phi = q/\epsilon_0 = EA$ (due to Gauss's law), you can solve for the electric field E_1 using the equation for the surface area of the (Gaussian) sphere, $A = 4\pi R^2$, and the value of the enclosed charge.

ANSWER:

$E_2 = \frac{q_2}{\pi \epsilon_0 L^2}$ = 5.72×10^5 N/C

Hint 6. Determine the direction of the electric field from the positively charged sphere

What will be the direction of the electric field due to the positively charged sphere only?

ANSWER:

- ○ toward the positively charged sphere
- ◉ away from the positively charged sphere
- ○ upward perpendicular to the line connecting the centers of the spheres
- ○ downward perpendicular to the line connecting the centers of the spheres

Note that the electric field will point away from a positively charged sphere, just as it would point away from a positive point charge.

Hint 7. Vector addition

Keep in mind that you need to use *vector* addition in adding the electric fields from the two spheres. Also, keep in mind that the two fields point along the same line.

ANSWER:

$E = \frac{q_2 - q_1}{\pi \epsilon_0 L^2}$ = 7.58×10^5 N/C

Part B

What is the direction of the electric field midway between the spheres?

ANSWER:

- ○ toward the positively charged sphere
- ◉ toward the negatively charged sphere
- ○ upward perpendicular to the line connecting the centers of the spheres
- ○ downward perpendicular to the line connecting the centers of the spheres

Since the electric field will point toward a negative charge and away from a positive charge, the electric field from each sphere separately will point toward the negatively charged sphere, and so the total field will also point in that direction.

Exercise 22.1

Description: A flat sheet of paper of area A is oriented so that the normal to the sheet is at an angle of alpha to a uniform electric field of magnitude E. (a) Find the magnitude of the electric flux through the sheet. (b) Does the answer to part A depend on...

A flat sheet of paper of area 0.350m^2 is oriented so that the normal to the sheet is at an angle of 70° to a uniform electric field of magnitude 14$\mathrm{N/C}$.

Part A

Find the magnitude of the electric flux through the sheet.

Express your answer using two significant figures.

ANSWER:

$\Phi = EA\cos\left(\frac{\alpha\cdot 3.14159}{180}\right) = 1.7\ \mathrm{N\cdot m^2/C}$

Part B

Does the answer to part A depend on the shape of the sheet?

ANSWER:

Part C

For what angle ϕ between the normal to the sheet and the electric field is the magnitude of the flux through the sheet largest?

Express your answer using two significant figures.

ANSWER:

$\phi = 0\ ^\circ$

Part D

For what angle ϕ between the normal to the sheet and the electric field is the magnitude of the flux through the sheet smallest?

Express your answer using two significant figures.

ANSWER:

$\phi = 90\ ^\circ$

Part E

Explain your answers in parts C and D.

ANSWER:

Answer Key:
In part C the paper is oriented to "capture" the most field lines whereas in D the area is oriented so that it "captures" no field lines.

Exercise 22.12

Description: The nuclei of large atoms, such as uranium, with 92 protons, can be modeled as spherically symmetric spheres of charge. The radius of the uranium nucleus is approximately 7.4*10^-15 m . (a) What is the electric field this nucleus produces just...

The nuclei of large atoms, such as uranium, with 92 protons, can be modeled as spherically symmetric spheres of charge. The radius of the uranium nucleus is approximately $7.4\times10^{-15}\ \mathrm{m}$.

Part A

What is the electric field this nucleus produces just outside its surface?

Express your answer using two significant figures.

ANSWER:

$E = 2.4\times10^{21}\ \mathrm{N/C}$

Part B

What magnitude of electric field does it produce at the distance of the electrons, which is about $1.4\times10^{-10}\mathrm{m}$?

Express your answer using two significant figures.

ANSWER:

$E = \frac{8.99\cdot10^{9}\cdot92\cdot1.60\cdot10^{-19}}{r2^2} = 6.8\times10^{12}\ \mathrm{N/C}$

Part C

The electrons can be modeled as forming a uniform shell of negative charge. What net electric field do they produce at the location of the nucleus?

Express your answer using two significant figures.

ANSWER:

$E_{\mathrm{net}} = 0\ \mathrm{N/C}$

Exercise 22.15

Description: Two very long uniform lines of charge are parallel and are separated by d. Each line of charge has charge per unit length lambda. (a) What magnitude of force does one line of charge exert on a L section of the other line of charge?

Two very long uniform lines of charge are parallel and are separated by $0.260\mathrm{m}$. Each line of charge has charge per unit length $5.50\mu\mathrm{C/m}$.

Part A

What magnitude of force does one line of charge exert on a $5.90\times10^{-2}\mathrm{m}$ section of the other line of charge?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

$F = \frac{\lambda^2 L}{2\pi\cdot 8.854 d}\cdot 10^{12} = 0.123\mathrm{N}$

Exercise 22.21

Description: A hollow, conducting sphere with an outer radius of R_1 and an inner radius of R_2 has a uniform surface charge density of +sigma. A charge of q is now introduced into the cavity inside the sphere. (a) What is the new charge density on the outside...

A hollow, conducting sphere with an outer radius of $0.247\mathrm{m}$ and an inner radius of $0.198\mathrm{m}$ has a uniform surface charge density of $+6.54\times10^{-6}\mathrm{C/m^2}$. A charge of $-0.470\mu\mathrm{C}$ is now introduced into the cavity inside the sphere.

Part A

What is the new charge density on the outside of the sphere?

ANSWER:

$\sigma = \sigma + \frac{q}{4\pi (R_1)^2} = 5.93\times10^{-6}\ \mathrm{C/m^2}$

Part B

Calculate the strength of the electric field just outside the sphere.

ANSWER:

$E = \frac{(\sigma\cdot 4\pi (R_1)^2 + q)\cdot 9\cdot 10^9}{(R_1)^2} = 6.70\times10^5\ \mathrm{N/C}$

Part C

What is the electric flux through a spherical surface just inside the inner surface of the sphere?

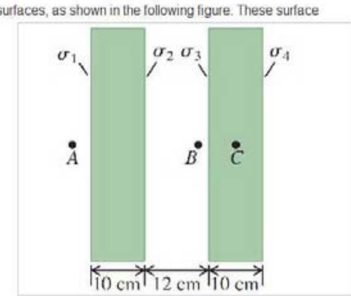
ANSWER:

$\Phi = \frac{q}{8.85\cdot 10^{-12}} = -5.31\times10^4\ \mathrm{N\cdot m^2/C}$

Exercise 22.32

Description: Two very large, nonconducting plastic sheets, each 10.0 cm thick, carry uniform charge densities sigma_1 , sigma_2 , sigma_3 and sigma_4 on their surfaces, as shown in the following figure. These surface charge densities have the values s1=## mu...

Two very large, nonconducting plastic sheets, each 10.0 cm thick, carry uniform charge densities σ_1, σ_2, σ_3 and σ_4 on their surfaces, as shown in the following figure. These surface charge densities have the values $\sigma_1 = -6.00\mu\mathrm{C/m^2}$, $\sigma_2 = 5.00\ \mu\mathrm{C/m^2}$, $\sigma_3 = 2.40\mu\mathrm{C/m^2}$, and $\sigma_4 = 4.00\ \mu\mathrm{C/m^2}$. Use Gauss's law to find the magnitude and direction of the electric field at the following points, far from the edges of these sheets.



Part A

What is the magnitude of the electric field at point A, 5.00 cm from the left face of the left-hand sheet?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

$$E = \frac{\frac{s1+\frac{5}{10^6}+s3+\frac{4}{10^6}}{2}}{8.854}\cdot 10^{12} = 3.05\times 10^5 \frac{\mathrm{N}}{\mathrm{C}}$$

Part B

What is the direction of the electric field at point A, 5.00 cm from the left face of the left-hand sheet?

ANSWER:

- ◉ to the left.
- ○ to the right.
- ○ upwards.
- ○ downwards.

Part C

What is the magnitude of the electric field at point B, 1.25 cm from the inner surface of the right-hand sheet?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

$$E = \frac{\frac{-s1-\frac{5}{10^6}+s3+\frac{4}{10^6}}{2}}{8.854}\cdot 10^{12} = 4.18\times 10^5 \frac{\mathrm{N}}{\mathrm{C}}$$

Part D

What is the direction of the electric field atpoint B, 1.25 cm from the inner surface of the right-hand sheet?

ANSWER:

- ◉ to the left.
- ○ to the right.
- ○ upwards.
- ○ downwards.

Part E

What is the magnitude of the electric field at point C, in the middle of the right-hand sheet?

Express your answer to three significant figures and include the appropriate units.

ANSWER:

$$E = \frac{\frac{-s1-\frac{5}{10^6}-s3+\frac{4}{10^6}}{2}}{8.854}\cdot 10^{12} = 1.47\times 10^5 \frac{\mathrm{N}}{\mathrm{C}}$$

Part F

What is the direction of the electric field at point C, in the middle of the right-hand sheet?

ANSWER:

- ◉ to the left.
- ○ to the right.
- ○ upwards.
- ○ downwards.

22.43. **IDENTIFY:** First make a free-body diagram of the sphere. The electric force acts to the left on it since the electric field due to the sheet is horizontal. Since it hangs at rest, the sphere is in equilibrium so the forces on it add to zero, by Newton's first law. Balance horizontal and vertical force components separately.

SET UP: Call T the tension in the thread and E the electric field. Balancing horizontal forces gives $T \sin\theta = qE$. Balancing vertical forces we get $T \cos\theta = mg$. Combining these equations gives $\tan\theta = qE/mg$, which means that $\theta = \arctan(qE/mg)$. The electric field for a sheet of charge is $E = \sigma/2\varepsilon_0$.

EXECUTE: Substituting the numbers gives us

$$E = \frac{\sigma}{2\epsilon_0} = \frac{2.50\times10^{-9}\ \mathrm{C/m^2}}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})} = 1.41\times10^2\ \mathrm{N/C}.$$ Then

$$\theta = \arctan\left[\frac{(5.00\times10^{-8}\ \mathrm{C})(1.41\times10^2\ \mathrm{N/C})}{(4.00\times10^{-6}\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}\right] = 10.2°.$$

EVALUATE: Increasing the field, or decreasing the mass of the sphere, would cause the sphere to hang at a larger angle.

22.46. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that is a sphere of radius r and that has the point charge at its center.

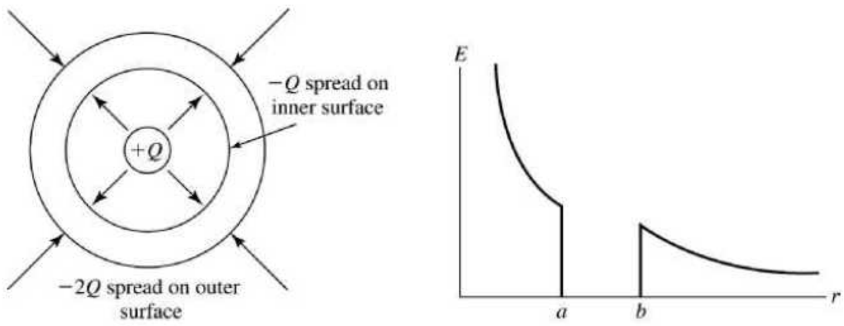
EXECUTE: **(a)** For $r < a$, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, radially outward, since the charge enclosed is Q, the charge of the point charge. For $a < r < b$, $E = 0$ since these points are within the conducting material. For $r > b$, $E = \frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$, radially inward, since the total enclosed charge is $-2Q$.

(b) Since a Gaussian surface with radius r, for $a < r < b$, must enclose zero net charge, the total charge on the inner surface is $-Q$ and the surface charge density on the inner surface is $\sigma = -\frac{Q}{4\pi a^2}$.

(c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\frac{2Q}{4\pi b^2}$.

(d) The field lines and the locations of the charges are sketched in Figure 22.46a.

(e) The graph of E versus r is sketched in Figure 22.46b.



(a) (b)

22.54. **IDENTIFY:** Example 22.9 gives the expression for the electric field both inside and outside a uniformly charged sphere. Use $\vec{F} = -e\vec{E}$ to calculate the force on the electron.

SET UP: The sphere has charge $Q = +e$.

EXECUTE: **(a)** Only at $r = 0$ is $E = 0$ for the uniformly charged sphere.

(b) At points inside the sphere, $E_r = \frac{er}{4\pi\epsilon_0 R^3}$. The field is radially outward. $F_r = -eE = -\frac{1}{4\pi\epsilon_0}\frac{e^2 r}{R^3}$. The minus sign denotes that F_r is radially inward. For simple harmonic motion, $F_r = -kr = -m\omega^2 r$, where $\omega = \sqrt{k/m} = 2\pi f$. $F_r = -m\omega^2 r = -\frac{1}{4\pi\epsilon_0}\frac{e^2 r}{R^3}$ so $\omega = \sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{mR^3}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{mR^3}}$.

(c) If $f = 4.57\times10^{14}\ \text{Hz} = \frac{1}{2\pi}\sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{mR^3}}$ then

$R = \sqrt[3]{\frac{1}{4\pi\epsilon_0}\frac{(1.60\times10^{-19}\ \text{C})^2}{4\pi^2(9.11\times10^{-31}\ \text{kg})(4.57\times10^{14}\ \text{Hz})^2}} = 3.13\times10^{-10}\ \text{m}$. The atom radius in this model is the correct order of magnitude.

(d) If $r > R$, $E_r = \frac{e}{4\pi\epsilon_0 r^2}$ and $F_r = -\frac{e^2}{4\pi\epsilon_0 r^2}$. The electron would still oscillate because the force is directed toward the equilibrium position at $r = 0$. But the motion would not be simple harmonic, since F_r is proportional to $1/r^2$ and simple harmonic motion requires that the restoring force be proportional to the displacement from equilibrium.