Chapter 5: Applying Newtons Laws [Edit]

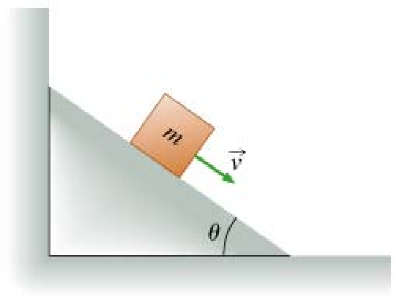
Chapter 5: Applying Newtons Laws

Due: 11:00pm on Tuesday, September 25, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Block on an Incline Adjacent to a Wall

A wedge with an inclination of angle θ rests next to a wall. A block of mass m is sliding down the plane, as shown. There is no friction between the wedge and the block or between the wedge and the horizontal surface.



Part A

Find the magnitude, $F_{\rm net}$, of the sum of all forces acting on the block.

Express $F_{\rm net}$ in terms of θ and m, along with any necessary constants.

Hint 1. Direction of the net force on the block

The net force on the block must be the force in the direction of motion, which is down the incline.

Hint 2. Determine the forces acting on the block

What forces act on the block? Keep in mind that there is no friction between the block and the wedge.

ANSWER:

- ○ The weight of the block and friction
- ◉ The weight of the block and the normal (contact) force
- ○ The weight of the block and the weight of the wedge
- ○ The weight of the block and the force that the wall exerts on the wedge

Hint 3. Find the magnitude of the force acting along the direction of motion

Consider a coordinate system with the *x* direction pointing down the incline and the *y* direction perpendicular to the incline. In these coordinates, what is w_x, the component of the block's weight in the *x* direction?

Express w_x in terms of m, g, and θ.

ANSWER:

$w_x = mg\sin(\theta)$

"Normal," in this context, is a synonym for "perpendicular." The normal force has no component in the direction of the block's motion (down the incline).

ANSWER:

$F_{\rm net}$ = $mg\sin\left(\theta\right)$

Also accepted: $mg\cos\left(\frac{\pi}{2}-\theta\right)$

Part B

Find the magnitude, $F_{\rm ww}$, of the force that the wall exerts on the wedge.

Express $F_{\rm ww}$ in terms of θ and m, along with any necessary constants.

Hint 1. The force between the wall and the wedge

There is no friction between the wedge and the horizontal surface, so for the wedge to remain stationary, the net horizontal force on the wedge must be zero. If the block exerts a force with a horizontal component on the wedge, some other horizontal force must act on the wedge so that the net force is zero.

Hint 2. Find the normal force between the block and the wedge

What is the magnitude, n, of the normal (contact) force between the block and the wedge? (You might have computed this already while answering Part A.)

Express n in terms of m, g, and θ.

ANSWER:

n = $mg\cos\left(\theta\right)$

Hint 3. Find the horizontal component of the normal force

In the previous hint you found the magnitude of the normal force between the block and the wedge. What is the magnitude, $n_{\rm h}$, of the *horizontal* component of this normal force?

Express $n_{\rm h}$ in terms of θ and n.

ANSWER:

$n_{\rm h}$ = $n\sin\left(\theta\right)$

Also accepted: $mg\sin\left(\theta\right)\cos\left(\theta\right)$

ANSWER:

$F_{\rm ww}$ = $\frac{mg\sin\left(2\theta\right)}{2}$

Your answer to Part B could be expressed as either $mg\sin(\theta)\cos(\theta)$ or $mg\sin(2\theta)/2$. In either form, we see that as θ gets very small or as θ approaches 90 degrees ($\pi/2$ radians), the contact force between the wall and the wedge goes to zero. This is what we should expect; in the first limit (θ small), the block is accelerating very slowly, and all horizontal forces are small. In the second limit (θ about 90 degrees), the block simply falls vertically and exerts no horizontal force on the wedge.

A Modified Atwood Machine

Consider the situation depicted in this applet. The red block moves along a rough surface. The system begins at rest. You may assume that the pulley and rope are both massless and frictionless.

Part A

Which of the following forces act upon the red block?

Check all that apply.

ANSWER:

- [x] tension
- [x] gravitational force
- [x] friction
- [x] normal force

Part B

In what direction does the force of tension acting on the red block point?

ANSWER:

- ( ) up
- ( ) down
- ( ) left
- (•) right

Part C

In what direction does the frictional force point?

ANSWER:

- ( ) up
- ( ) down
- (•) left
- ( ) right

Part D

Which of the following forces act upon the light gray block?

Check all that apply.

ANSWER:

- [x] tension
- [x] gravitational force
- [ ] friction
- [ ] normal force

Part E

In what direction does the force of tension acting on the light gray block point?

ANSWER:

- (•) up
- ( ) down
- ( ) left
- ( ) right

Part F

Open the next applet. Now, you can see all of the forces. Note that the gravitational force acting on the red block is represented by $\vec{w}_r$ and that on the light grey block by $\vec{w}_g$. Which of the following shows the relation between the magnitude of the tension $|\vec{T}|$ acting on the light gray block and the magnitude of the tension $|\vec{T}'|$ acting on the red block?

ANSWER:

- (selected) $|\vec{T}| = |\vec{T}'|$
- $|\vec{T}| = |\vec{T}' + \vec{f}_k|$
- $|\vec{T}| = |\vec{T}' - \vec{f}_k|$
- $|\vec{T}| = |\vec{T}' + \vec{w}_g|$
- $|\vec{T}| = |\vec{T}' - \vec{w}_g|$

Part G

Find an expression for the acceleration a of the red block after it is released. Use m_r for the mass of the red block, m_g for the mass of the gray block, and μ_k for the coefficient of kinetic friction between the table and the red block.

Express your answer in terms of m_r, m_g, μ_k, and g.

Hint 1. How to approach the problem

Look at each block indivdually, and find the net force on it. These expressions will involve T, so you must find the value of T. Since the blocks must have equal accelerations, you can set the accelerations you find for each block equal. This equation will allow you to find the tension in the string. Substituting this back into the expression for the acceleration of the red block will give you the formula you are looking for.

Hint 2. Find the net force on the red block in terms of T

What is the net force $F_{r,net}$ on the red block? Let the positive direction be to the right.

Express your answer in terms of μ_k, m_r, the tension T, and g.

ANSWER:

$F_{r,net}$ = $T - \mu_k m_r g$

Hint 3. Find the tension in the string

What is the magnitude T of the tension in the string?

Express your answer in terms of μ_k, m_r, m_g, and g.

Hint 1. Find the acceleration of the red block in terms of T

What is the acceleration a of the red block? Let the positive direction be to the right.

Express your answer in terms of μ_k, m_r, the tension T, and g.

Hint 1. Find the net force on the red block

What is the net force $F_{r,net}$ on the red block?

Express your answer in terms of μ_k, m_r, T, and g.

ANSWER:

$F_{r,net}$ = $T - \mu_k m_r g$

ANSWER:

$a = \dfrac{T - \mu_k m_r g}{m_r}$

Hint 2. Find the acceleration of the gray block in terms of T

What is the acceleration a of the gray block? Let the positive direction be downward.

Express your answer in terms of m_g, the tension T, and g.

Hint 1. Find the net force on the gray block

What is the net force $F_{g,net}$ on the gray block?

Express your answer in terms of m_g, T, and g.

ANSWER:

$F_{g,net} = m_g g - T$

ANSWER:

$a = \dfrac{m_g g - T}{m_g}$

Hint 3. Using the accelerations to find T

In the previous two hints, you determined that the accelerations of the red and gray blocks are

$$a = \frac{T - \mu_k m_r g}{m_r}$$

and

$$a = \frac{m_g g - T}{m_g}$$

respectively. Since the two blocks must always have the same acceleration, you can set the two expressions for a equal to one another, obtaining

$$\frac{T - \mu_k m_r g}{m_r} = \frac{m_g g - T}{m_g}.$$

Solve this equation for T to find the magnitude of the tension.

ANSWER:

$T = \dfrac{m_r m_g (1 + \mu_k) g}{m_r + m_g}$

Substitute this into the expression for the net force on the red block and divide by m_r to determine the acceleration of the red block. A good check of your work would be to find the acceleration of the gray block. If it is not the same as the acceleration of the red block, then you have made a mistake somewhere in your calculations.

ANSWER:

$a = \dfrac{g (m_g - \mu_k m_r)}{m_g + m_r}$

Part H

Run the simulation applet introduced in Part F a few times, and use your expression from the previous part to find a relation between the masses of the two blocks in this problem. Type an expression for the mass m_r of the red block in terms of the mass m_g of the gray block.

Notice in the applet that the acceleration is given to you for any value of μ_k that you choose. This can be done in the applet by sliding the control in the top right corner which changes the value of μ_k. Use 10 m/s^2 for the magnitude of the acceleration due to gravity.

Express your answer in terms of m_g.

ANSWER:

m_r = m_g

Part I

Now consider this applet, where the two blocks have different masses. Plug the starting values into your formula from Part G, using 5.0 kg for the mass of the red block and 10 m/s^2 for the magnitude of the acceleration due to gravity. Then, run the applet. Hopefully, the results will look peculiar! Which of the following statements describes an error made by the applet in modeling this situation. You can change the values and run it again to help you determine just what the error is.

Check all that apply.

ANSWER:

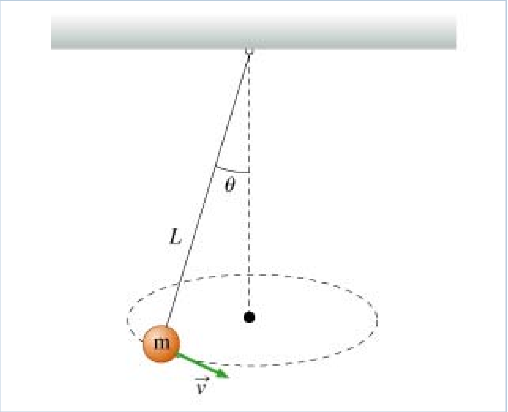
- [ ] It allows the string to transmit compressive (pushing) forces, but a string can only transmit tensile (pulling) forces.
- [x] It does not properly ensure that the friction force points in opposition to the direction of the relative motion of the two objects.
- [ ] It allows gravity to act with an upward force, but gravity always acts with a downward force.
- [ ] It assumes that the normal force is mg, but that is actually the maximum possible normal force.
- [x] It assumes the force of friction always equals $\mu_k n$, but this does not account for static friction.

The strange behavior of this applet comes from its use of a very simplified model of friction. When dealing with friction, you must always remember that it acts in opposition to the relative motion of the two surfaces that are in contact. The relative motion of two objects will never increase because of friction.

When dealing with static friction, recall that its magnitude does not always take the value $\mu_s n$; rather, it takes a value that cancels out the effect of any force with magnitude up to $\mu_s n$. If static friction always took the value $\mu_s n$, then whenever you set a book (or anything else) down on a table and did not apply a horizontal force, the book would begin to accelerate (in some random direction?)! Also, note that the coefficient of static friction is always greater than or equal to the coefficient of kinetic friction.

Conical Pendulum I

A bob of mass m is suspended from a fixed point with a massless string of length L (i.e., it is a pendulum). You are to investigate the motion in which the string moves in a cone with half-angle θ.



Part A

What tangential speed, v, must the bob have so that it moves in a horizontal circle with the string always making an angle θ from the vertical?

Express your answer in terms of some or all of the variables m, L, and θ, as well as the acceleration due to gravity g.

Hint 1. What's happening here?

In this situation, which of the following statements is true?

ANSWER:

- ○ The bob has no acceleration since its velocity is constant.
- ○ The tension in the string is less than mg.
- ◉ A component of the tension causes acceleration of the bob.
- ○ If $\theta = 0$ the tension in the string would be greater than mg.

Hint 2. Find the vertical acceleration of the bob

What is $a_{\rm vertical}$, the vertical component of the acceleration of the bob?

ANSWER:

$a_{\rm vertical}$ = 0

Hint 3. Find the tension in the string

Find the magnitude, T, of the tension force in the string.

Express your answer in terms of some or all of the variables m, L, and θ, as well as the acceleration due to gravity g.

Hint 1. What approach to use

You know the vertical acceleration of the bob, and so you know the net vertical force. The force due to the string has both vertical and horizontal components, and so breaking this force into components should allow you to find the magnitude of the tension force, which is T.

ANSWER:

$T = \frac{mg}{\cos(\theta)}$

Hint 4. Find the horizontal acceleration of the bob

Find a general expression for a, the magnitude of the bob's centripetal acceleration, as a function of the tangential speed v of the bob.

Express your answer in terms of v and some or all of the variables m, L, and θ.

Hint 1. Find the radius of the bob's motion

The bob moves uniformly in a circle of what radius r?

Express your answer in terms of some or all of the variables m, L, and θ.

ANSWER:

$r = L\sin(\theta)$

ANSWER:

$a = \frac{v^2}{L\sin(\theta)}$

Hint 5. Find the horizontal force

Find the magnitude, F_r, of the inward radial force on the bob in the horizontal plane.

Express your answer in terms of some or all of the variables m, L, and θ, as well as the acceleration due to gravity g.

ANSWER:

$F_r = mg\tan(\theta)$

ANSWER:

$v = \sqrt{Lg\sin(\theta)\tan(\theta)}$

Also accepted: $\frac{L\sin(\theta)}{\sqrt{\frac{L\cos(\theta)}{g}}}$

Part B

How long does it take the bob to make one full revolution (one complete trip around the circle)?

Express your answer in terms of some or all of the variables m, L, and θ, as well as the acceleration due to gravity g.

Hint 1. How to approach the problem

Since the speed of the bob is constant, this is a relatively simple kinematics problem. You know the speed, which you found in the previous part, and you can calculate the distance traveled in one revolution (i.e., the circumference of the circle). From these two you can calculate the time required to travel that distance.

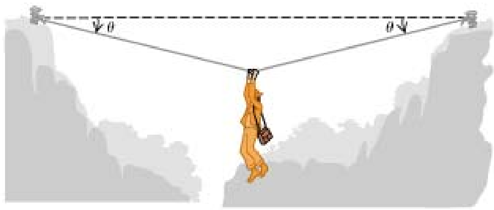
ANSWER:

$2\pi\sqrt{\frac{L\cos(\theta)}{g}}$

Also accepted: $\frac{2\pi(L\sin(\theta))}{\sqrt{gL\tan(\theta)\sin(\theta)}}$

Problem 5.56

An adventurous archaeologist crosses between two rock cliffs by slowly going hand-over-hand along a rope stretched between the cliffs. He stops to rest at the middle of the rope . The rope will break if the tension in it exceeds $2.10\times10^{4}\,\mathrm{N}$, and our hero's mass is $89.4\,\mathrm{kg}$.



Part A

If the angle between the rope and the horizontal is θ = 10.7°, find the tension in the rope.

ANSWER:

$\frac{m \cdot 9.80}{2\sin(\theta)}$ = 2360 N

Part B

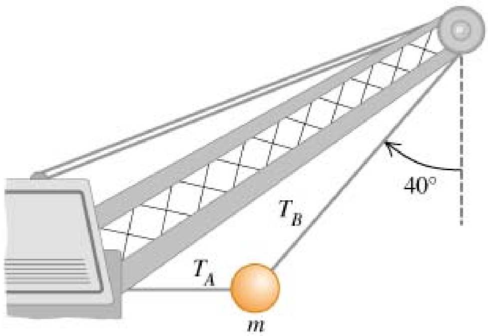
What is the smallest value the angle θ can have if the rope is not to break?

ANSWER:

$\mathrm{asin}\left(\frac{m \cdot 9.80}{2T}\right)\frac{180}{\pi}$ = 1.20 °

Exercise 5.6

A large wrecking ball is held in place by two light steel cables .



Part A

If the mass m of the wrecking ball is 4060kg , what is the tension T_B in the cable that makes an angle of 40° with the vertical?

Express your answer using two significant figures.

ANSWER:

$\frac{mg}{\cos(\theta)}$ = 5.2×10^4 N

Part B

If the mass m of the wrecking ball is 4060kg , what is the tension T_A in the horizontal cable?

Express your answer using two significant figures.

ANSWER:

$mg\tan(\theta)$ = 3.3×10^4 N

Exercise 5.16

A 8.90-kg block of ice, released from rest at the top of a 1.28-m-long frictionless ramp, slides downhill, reaching a speed of 2.95$\mathrm{m/s}$ at the bottom.

Part A

What is the angle between the ramp and the horizontal?

ANSWER:

$\phi = \frac{\operatorname{asin}\left(\frac{\frac{\frac{v^2}{2}}{l}}{9.8}\right)}{\pi} \cdot 180$ = 20.3 $^\circ$

Part B

What would be the speed of the ice at the bottom if the motion were opposed by a constant friction force of 10.9N parallel to the surface of the ramp?

ANSWER:

$v = \sqrt{v^2 - \frac{2fl}{m}}$ = 2.36 $\mathrm{m/s}$

Exercise 5.20

A 598-N physics student stands on a bathroom scale in an 871-kg (including the student) elevator that is supported by a cable. As the elevator starts moving, the scale reads 486N .

Part A

Find the magnitude of the acceleration of the elevator.

ANSWER:

$a = \frac{-N + w}{w} \cdot 9.8$ = 1.84 $\mathrm{m/s^2}$

Part B

Find the direction of the acceleration of the elevator.

ANSWER:

- ○ upwards
- ◉ downwards

Part C

What is the acceleration if the scale reads 645N ?

ANSWER:

$a = \frac{N_2 - w}{w} \cdot 9.8$ = 0.770 $\mathrm{m/s^2}$

Part D

If the scale reads zero, should the student worry? Explain.

ANSWER:

3727 Character(s) remaining

n=0 means a_y=-g. The student should worry; the elevator is in free-fall.

Part E

What is the tension in the cable in part A?

ANSWER:

$T = m\left(g+\frac{N-w}{w}\cdot 9.8\right)$ = 6940 N

Part F

What is the tension in the cable in part D?

ANSWER:

T = 0 N

Exercise 5.27

A stockroom worker pushes a box with mass 11.8kg on a horizontal surface with a constant speed of 3.70$\mathrm{m/s}$. The coeflficient of kinetic friction between the box and the surface is 0.250.

Part A

What horizontal force must be applied by the worker to maintain the motion?

ANSWER:

$\mu_k m \cdot 9.80$ = 28.9 N

Part B

If the force calculated in part A is removed, how far does the box slide before coming to rest?

ANSWER:

$\frac{v^2}{2\mu_k \cdot 9.80}$ = 2.79 m

Exercise 5.33

Part A

If the coefficient of kinetic friction between tires and dry pavement is 0.800, what is the shortest distance in which an automobile can be stopped by locking the brakes when traveling at 28.1$\mathrm{m/s}$?

Express your answer using two significant figures.

ANSWER:

$\frac{v^2}{2\mu_k \cdot 9.80}$ = 50 m

Part B

On wet pavement the coefficient of kinetic friction may be only 0.250. How fast should you drive on wet pavement in order to be able to stop in the same distance as in part A? (*Note:* Locking the brakes is *not* the safest way to stop.)

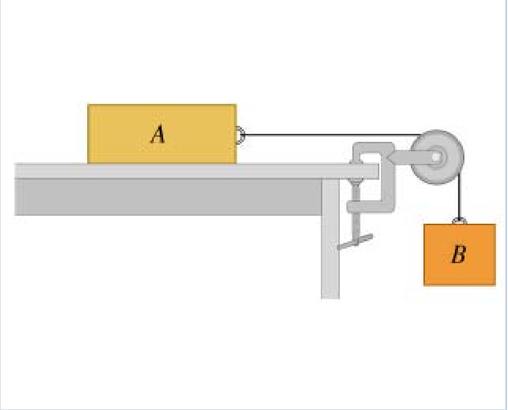
Express your answer using two significant figures.

ANSWER:

$v\sqrt{\frac{\mu_{\text{wet}}}{\mu_k}}$ = 16 m/s

Exercise 5.34

Consider the system shown in the figure . Block A weighs 42.8N and block B weighs 29.8N . Once block B is set into downward motion, it descends at a constant speed.



Part A

Calculate the coefficient of kinetic friction between block A and the tabletop.

ANSWER:

$\mu = \frac{w_B}{w_A}$ = 0.696

Part B

A cat, also of weight 42.8N , falls asleep on top of block A. If block B is now set into downward motion, what is its acceleration magnitude?

ANSWER:

$a = \frac{-w_B + \frac{w_B}{w_A}(w_A + w_{\text{cat}})}{w_A + w_{\text{cat}} + w_B} \cdot 9.8$ = 2.53 m/s^2

Part C

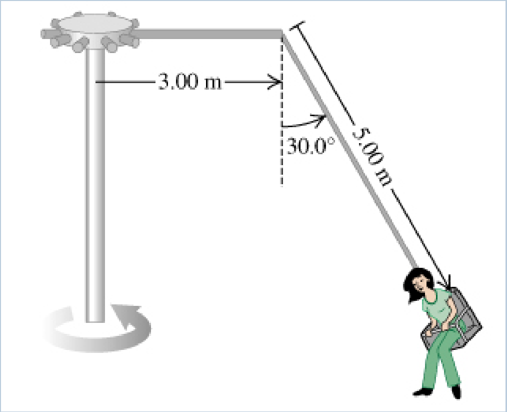
A cat, also of weight 42.8N , falls asleep on top of block A. If block B is now set into downward motion, what is its acceleration direction?

ANSWER:

- upwards
- downwards

Exercise 5.46

The "Giant Swing" at a county fair consists of a vertical central shaft with a number of horizontal arms attached at its upper end. Each arm supports a seat suspended from a cable 5.00 m long, the upper end of the cable being fastened to the arm at a point 3.00 m from the central shaft.



Part A

Find the time of one revolution of the swing if the cable supporting a seat makes an angle of 30.0° with the vertical.

ANSWER:

T = 6.19 s

Part B

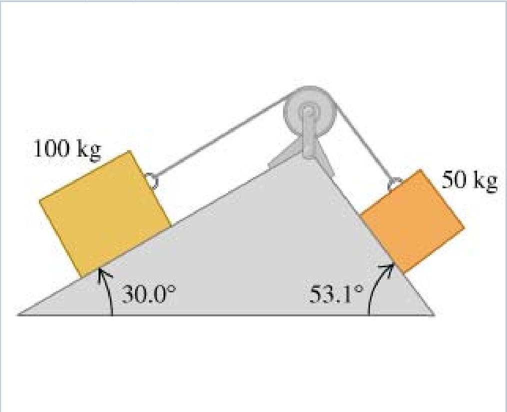
Does the angle depend on the weight of the passenger for a given rate of revolution?

ANSWER:

- ○ Yes.
- ◉ No.

Problem 5.92

Two blocks connected by a cord passing over a small, frictionless pulley rest on frictionless planes (the figure).



Part A

Which way will the system move when the blocks are released from rest?

ANSWER:

- ◉ the blocks will slide to the left
- ○ the blocks will slide to the right

Part B

What is the acceleration of the blocks?

ANSWER:

a = 0.658 $\mathrm{m/s^2}$

Part C

What is the tension in the cord?

ANSWER:

T = 424 N

5.6. IDENTIFY: Apply Newton's first law to the wrecking ball. Each cable exerts a force on the ball, directed along the cable.

SET UP: The force diagram for the wrecking ball is sketched in Figure 5.6.

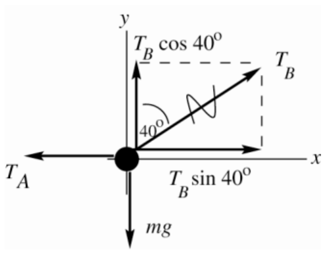


Figure 5.6

EXECUTE: **(a)** $\Sigma F_y = ma_y$

$T_B \cos 40° - mg = 0$

$$T_B = \frac{mg}{\cos 40°} = \frac{(4090\ \text{kg})(9.80\ \text{m/s}^2)}{\cos 40°} = 5.23\times 10^4\ \text{N}$$

(b) $\Sigma F_x = ma_x$

$T_B \sin 40° - T_A = 0$

$T_A = T_B \sin 40° = 3.36\times 10^4\ \text{N}$

EVALUATE: If the angle 40° is replaced by 0° (cable *B* is vertical), then $T_B = mg$ and $T_A = 0$.

5.16. IDENTIFY: In part (a) use the kinematic information and the constant acceleration equations to calculate the acceleration of the ice. Then apply $\Sigma \vec{F} = m\vec{a}$. In part (b) use $\Sigma \vec{F} = m\vec{a}$ to find the acceleration and use this in the constant acceleration equations to find the final speed.

SET UP: Figures 5.16a and b give the free-body diagrams for the ice both with and without friction.

Let $+x$ be directed down the ramp, so $+y$ is perpendicular to the ramp surface. Let ϕ be the angle between the ramp and the horizontal. The gravity force has been replaced by its *x* and *y* components.

EXECUTE: **(a)** $x - x_0 = 1.50$ m, $v_{0x} = 0$. $v_x = 2.50$ m/s. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(2.50\ \text{m/s})^2 - 0}{2(1.50\ \text{m})} = 2.08\ \text{m/s}^2.$$ $\Sigma F_x = ma_x$ gives $mg\sin\phi = ma$ and

$$\sin\phi = \frac{a}{g} = \frac{2.08\ \text{m/s}^2}{9.80\ \text{m/s}^2}.\quad \phi = 12.3°.$$

(b) $\Sigma F_x = ma_x$ gives $mg\sin\phi - f = ma$ and

$$a = \frac{mg\sin\phi - f}{m} = \frac{(8.00\ \text{kg})(9.80\ \text{m/s}^2)\sin 12.3° - 10.0\ \text{N}}{8.00\ \text{kg}} = 0.838\ \text{m/s}^2.$$

Then $x - x_0 = 1.50$ m, $v_{0x} = 0$. $a_x = 0.838\ \text{m/s}^2$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$v_x = \sqrt{2a_x\,(x - x_0)} = \sqrt{2(0.838\text{ m/s}^2)(1.50\text{ m})} = 1.59\text{ m/s}$$

EVALUATE: With friction present the speed at the bottom of the ramp is less.

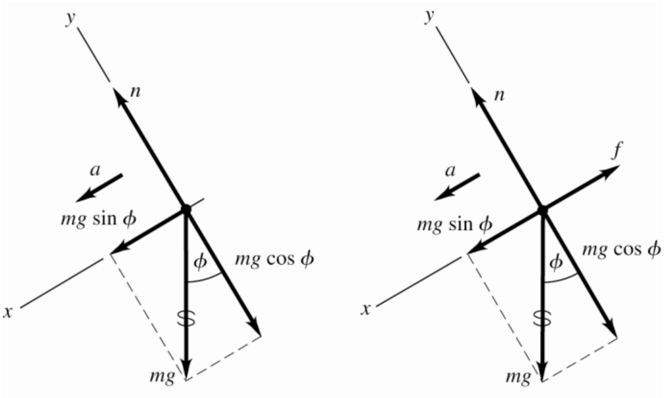


Figure 5.16a, b

5.20.IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of elevator plus student $(m_{\text{tot}} = 850\text{ kg})$ and also to the student $(w = 550\text{ N})$. The elevator and the student have the same acceleration.

SET UP: Let $+y$ be upward. The free-body diagrams for the composite object and for the student are given in Figures 5.20a and b. T is the tension in the cable and n is the scale reading, the normal force the scale exerts on the student. The mass of the student is $m = w/g = 56.1\text{ kg}$.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ applied to the student gives $n - mg = ma_y$.

$a_y = \dfrac{n - mg}{m} = \dfrac{450\text{ N} - 550\text{ N}}{56.1\text{ kg}} = -1.78\text{ m/s}^2$. The elevator has a downward acceleration of 1.78 m/s^2.

(b) $a_y = \dfrac{670\text{ N} - 550\text{ N}}{56.1\text{ kg}} = 2.14\text{ m/s}^2$.

(c) $n = 0$ means $a_y = -g$. The student should worry; the elevator is in free fall.

(d) $\Sigma F_y = ma_y$ applied to the composite object gives $T - m_{\text{tot}}g = m_{\text{tot}}a$. $T = m_{\text{tot}}(a_y + g)$. In part (a), $T = (850\text{ kg})(-1.78\text{ m/s}^2 + 9.80\text{ m/s}^2) = 6820\text{ N}$. In part (c), $a_y = -g$ and $T = 0$.

EVALUATE: In part (b), $T = (850\text{ kg})(2.14\text{ m/s}^2 + 9.80\text{ m/s}^2) = 10{,}150\text{ N}$. The weight of the composite object is 8330 N. When the acceleration is upward the tension is greater than the weight and when the acceleration is downward the tension is less than the weight.

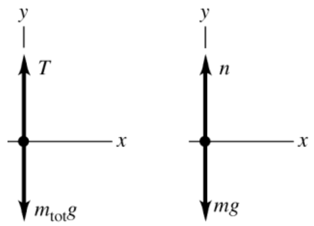


Figure 5.20a, b

5.27. **(a) IDENTIFY:** Constant speed implies $a = 0$. Apply Newton's first law to the box. The friction force is directed opposite to the motion of the box.

SET UP: Consider the free-body diagram for the box, given in Figure 5.27a. Let $\vec{\boldsymbol{F}}$ be the horizontal force applied by the worker. The friction is kinetic friction since the box is sliding along the surface.

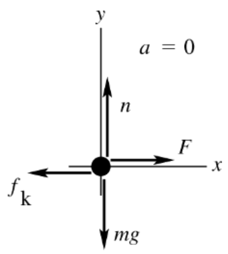


Figure 5.27a

EXECUTE:

$\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

so $f_k = \mu_k n = \mu_k mg$

$\Sigma F_x = ma_x$

$F - f_k = 0$

$F = f_k = \mu_k mg = (0.20)(11.2 \text{ kg})(9.80 \text{ m/s}^2) = 22 \text{ N}$

(b) IDENTIFY: Now the only horizontal force on the box is the kinetic friction force. Apply Newton's second law to the box to calculate its acceleration. Once we have the acceleration, we can find the
distance using a constant acceleration equation. The friction force is $f_k = \mu_k mg,$ just as in part (a).

SET UP: The free-body diagram is sketched in Figure 5.27b.

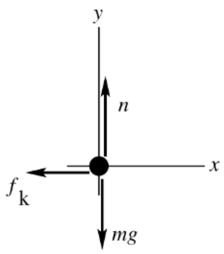


Figure 5.27b

EXECUTE:

$\Sigma F_x = ma_x$

$-f_k = ma_x$

$-\mu_k mg = ma_x$

$a_x = -\mu_k g = -(0.20)(9.80 \text{ m/s}^2) = -1.96 \text{ m/s}^2$

Use the constant acceleration equations to find the distance the box travels:

$v_x = 0, \quad v_{0x} = 3.50 \text{ m/s}, \quad a_x = -1.96 \text{ m/s}^2, \quad x - x_0 = ?$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (3.50 \text{ m/s})^2}{2(-1.96 \text{ m/s}^2)} = 3.1 \text{ m}$$

EVALUATE: The normal force is the component of force exerted by a surface perpendicular to the surface. Its magnitude is determined by $\Sigma\vec{F} = m\vec{a}$. In this case n and mg are the only vertical forces and $a_y = 0$, so $n = mg$. Also note that f_k and n are proportional in magnitude but perpendicular in direction.

5.33.IDENTIFY: Use $\Sigma\vec{F} = m\vec{a}$ to find the acceleration that can be given to the car by the kinetic friction force. Then use a constant acceleration equation.

SET UP: Take $+x$ in the direction the car is moving.

EXECUTE: **(a)** The free-body diagram for the car is shown in Figure 5.33. $\Sigma F_y = ma_y$ gives $n = mg$.

$\Sigma F_x = ma_x$ gives $-\mu_k n = ma_x$. $-\mu_k mg = ma_x$ and $a_x = -\mu_k g$. Then $v_x = 0$ and

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \text{ gives } (x - x_0) = -\frac{v_{0x}^2}{2a_x} = +\frac{v_{0x}^2}{2\mu_k g} = \frac{(28.7 \text{ m/s})^2}{2(0.80)(9.80 \text{ m/s}^2)} = 52.5 \text{ m.}$$

(b) $v_{0x} = \sqrt{2\mu_k g(x - x_0)} = \sqrt{2(0.25)(9.80 \text{ m/s}^2)52.5 \text{ m}} = 16.0 \text{ m/s}$

EVALUATE: For constant stopping distance $\frac{v_{0x}^2}{\mu_k}$ is constant and v_{0x} is proportional to $\sqrt{\mu_k}$.

The answer to part (b) can be calculated as $(28.7 \text{ m/s})\sqrt{0.25/0.80} = 16.0 \text{ m/s}$.

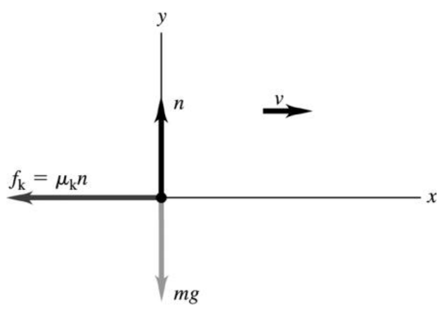


Figure 5.33

5.34. IDENTIFY: Constant speed means zero acceleration for each block. If the block is moving, the friction force the tabletop exerts on it is kinetic friction. Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

SET UP: The free-body diagrams and choice of coordinates for each block are given by Figure 5.34. $m_A = 4.59$ kg and $m_B = 2.55$ kg.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ with $a_y = 0$ applied to block *B* gives $m_B g - T = 0$ and $T = 25.0$ N. $\Sigma F_x = ma_x$ with $a_x = 0$ applied to block *A* gives $T - f_k = 0$ and $f_k = 25.0$ N. $n_A = m_A g = 45.0$ N and $\mu_k = \dfrac{f_k}{n_A} = \dfrac{25.0\text{ N}}{45.0\text{ N}} = 0.556.$

(b) Now let *A* be block *A* plus the cat, so $m_A = 9.18$ kg. $n_A = 90.0$ N and $f_k = \mu_k n = (0.556)(90.0\text{ N}) = 50.0$ N. $\sum F_x = ma_x$ for *A* gives $T - f_k = m_A a_x$. $\sum Fy = ma_y$ for block *B* gives $m_B g - T = m_B a_y$. a_x for *A* equals a_y for *B*, so adding the two equations gives $m_B g - f_k = (m_A + m_B)a_y$ and $a_y = \dfrac{m_B g - f_k}{m_A + m_B} = \dfrac{25.0\text{ N} - 50.0\text{ N}}{9.18\text{ kg} + 2.55\text{ kg}} = -2.13\text{ m/s}^2$. The acceleration is upward and block *B* slows down.

EVALUATE: The equation $m_B g - f_k = (m_A + m_B)a_y$ has a simple interpretation. If both blocks are considered together then there are two external forces: $m_B g$ that acts to move the system one way and f_k that acts oppositely. The net force of $m_B g - f_k$ must accelerate a total mass of $m_A + m_B$.

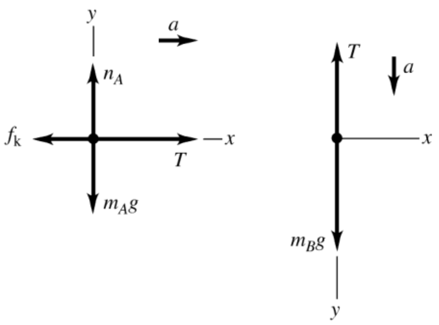


Figure 5.34

5.46. **IDENTIFY:** The acceleration of the person is $a_{rad} = v^2/R$, directed horizontally to the left in the figure in the problem. The time for one revolution is the period $T = \dfrac{2\pi R}{v}$. Apply $\Sigma\vec{F} = m\vec{a}$ to the person.

SET UP: The person moves in a circle of radius $R = 3.00\text{ m} + (5.00\text{ m})\sin 30.0° = 5.50\text{ m}$. The free-body diagram is given in Figure 5.46. $\vec{F}$ is the force applied to the seat by the rod.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $F\cos 30.0° = mg$ and $F = \dfrac{mg}{\cos 30.0°}$. $\Sigma F_x = ma_x$ gives $F\sin 30.0° = m\dfrac{v^2}{R}$. Combining these two equations gives

$v = \sqrt{Rg\tan\theta} = \sqrt{(5.50\text{ m})(9.80\text{ m/s}^2)\tan 30.0°} = 5.58\text{ m/s}$. Then the period is

$T = \dfrac{2\pi R}{v} = \dfrac{2\pi(5.50\text{ m})}{5.58\text{ m/s}} = 6.19\text{ s}.$

(b) The net force is proportional to m so in $\Sigma\vec{F} = m\vec{a}$ the mass divides out and the angle for a given rate of rotation is independent of the mass of the passengers.

EVALUATE: The person moves in a horizontal circle so the acceleration is horizontal. The net inward force required for circular motion is produced by a component of the force exerted on the seat by the rod.

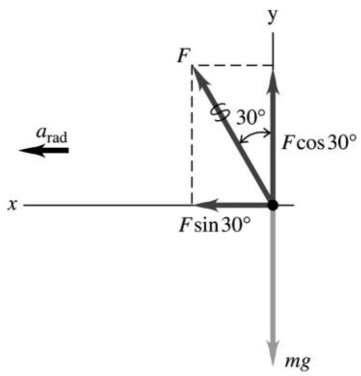
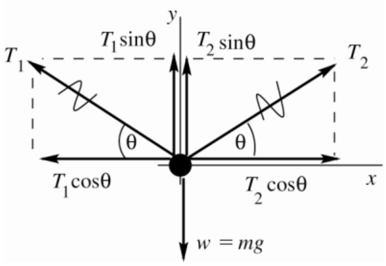


Figure 5.46

5.56. **IDENTIFY:** Apply Newton's first law to the person. Each half of the rope exerts a force on him, directed along the rope and equal to the tension T in the rope.

SET UP: **(a)** The force diagram for the person is given in Figure 5.56.



T_1 and T_2 are the tensions in each half of the rope.

Figure 5.56

EXECUTE: $\Sigma F_x = 0$

$T_2 \cos\theta - T_1 \cos\theta = 0$

This says that $T_1 = T_2 = T$ (The tension is the same on both sides of the person.)

$\Sigma F_y = 0$

$T_1 \sin\theta + T_2 \sin\theta - mg = 0$

But $T_1 = T_2 = T$, so $2T \sin\theta = mg$

$$T = \frac{mg}{2\sin\theta} = \frac{(90.0 \text{ kg})(9.80 \text{ m/s}^2)}{2\sin 10.0°} = 2540 \text{ N}$$

(b) The relation $2T \sin\theta = mg$ still applies but now we are given that $T = 2.50 \times 10^4$ N (the breaking strength) and are asked to find θ.

$$\sin\theta = \frac{mg}{2T} = \frac{(90.0 \text{ kg})(9.80 \text{ m/s}^2)}{2(2.50 \times 10^4 \text{ N})} = 0.01764, \quad \theta = 1.01°.$$

EVALUATE: $T = mg/(2\sin\theta)$ says that $T = mg/2$ when $\theta = 90°$ (rope is vertical). $T \to \infty$ when $\theta \to 0$ since the upward component of the tension becomes a smaller fraction of the tension.

5.92. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. They have the same magnitude of acceleration, a.

SET UP: Consider positive accelerations to be to the right (up and to the right for the left-hand block, down and to the right for the right-hand block).

EXECUTE: **(a)** The forces along the inclines and the accelerations are related by $T - (100 \text{ kg})g\sin 30.0° = (100 \text{ kg})a$ and $(50 \text{ kg})g\sin 53.1° - T = (50 \text{ kg})a$, where T is the tension in the cord and a the mutual magnitude of acceleration. Adding these relations, $(50 \text{ kg} \sin 53.1° - 100 \text{ kg} \sin 30.0°)g = (50 \text{ kg} + 100 \text{ kg})a$, or $a = -0.067g$. Since a comes out negative, the blocks will slide to the left; the 100-kg block will slide down. Of course, if coordinates had been chosen so that positive accelerations were to the left, a would be $+0.067g$.

(b) $a = 0.067(9.80 \text{ m/s}^2) = 0.658 \text{ m/s}^2$.

(c) Substituting the value of a (including the proper sign, depending on choice of coordinates) into either of the above relations involving T yields 424 N.

EVALUATE: For part (a) we could have compared $mg\sin\theta$ for each block to determine which direction the system would move.