Worksheet 12

The Schödinger equation in 3D separates into three equations for functions only of r, θ, φ, when you assume solutions are products of three such functions. We will consider here the function only of φ, the "azimuthal" angle. This part of the wave satisfies

$$\frac{d^2\psi}{d\phi^2} + m^2\psi = 0$$

(I've written m for m_l because I am lazy.)

For $m = \pm 1$ you get the same equation, but that's okay because there are two solutions. We'll take as the solutions

$\psi(\phi) = Ae^{im\phi}$ where A is a normalization constant.

1. Let's ignore the other coordinates. The normalization constant A is determined by the requirement that

$\int_0^{2\pi} P(\phi)d\phi = 1$ where $P(\phi) = \psi^*\psi$.

Find A. $\int_0^{2\pi} Ae^{im\phi} Ae^{-im\phi} d\phi = 1 = \int_0^{2\pi} A^2 d\phi = 2\pi A^2 \quad A = \frac{1}{\sqrt{2\pi}}.$

2. In the absence of a magnetic field, the different m waves all have the same energy. So the sum of two solutions (with the same energy) is also a solution (with that energy.) Remember that solutions of defined energy are **stationary states.**

Define
$\psi_\pm = Ae^{\pm i\phi}$; these are the solutions for $m = \pm 1$.

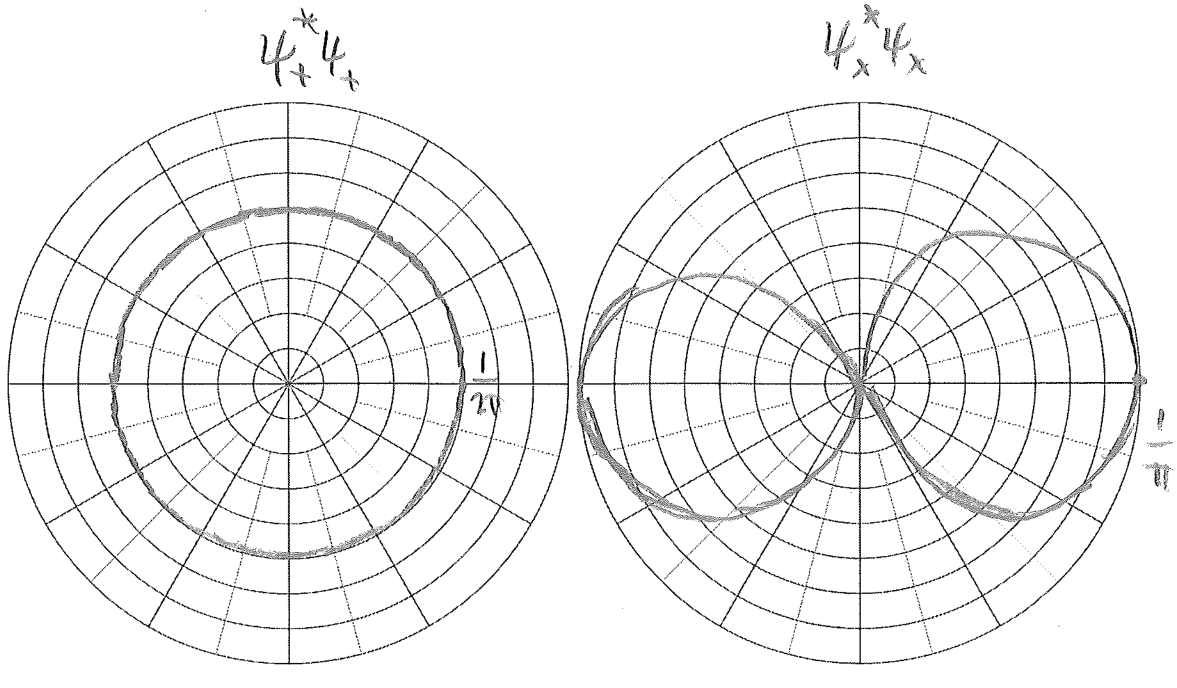
a) Show by substitution into the differential equation that $\psi_x = A'(\psi_+ + \psi_-)$ is a solution.

$$\frac{d^2\psi_x}{d\phi^2} = A'\frac{d^2\psi_+}{d\phi^2} + A'\frac{d^2\psi_-}{d\phi^2} \quad \text{bw} \quad \frac{d^2\psi_\pm}{d\phi^2} = -\psi_\pm$$

$$= A'(-\psi_+ + -\psi_-) = -\psi_x. \checkmark$$

b) Find A'.

c) Make a "polar plot" of $\psi_+^*\psi_+$ and of $\psi_x^*\psi_x$. To make a polar plot, at every angle φ, plot a point that is the appropriate distance away from the origin. To help, polar graph paper is shown below.



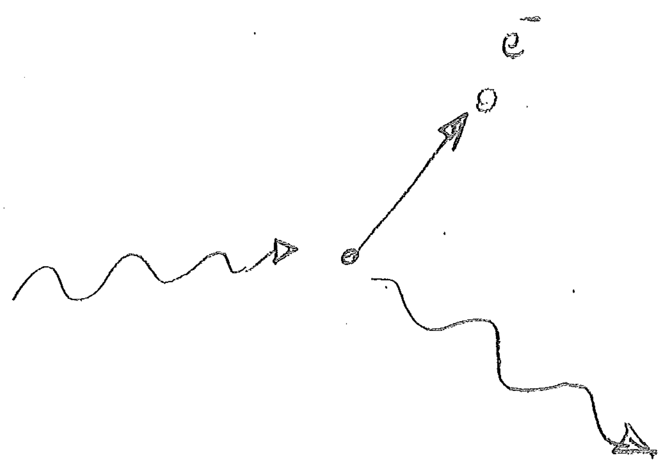
d) For each wave: is there any angle ϕ at which the electron could not be found ?(if you tried to locate it, for example by hitting it with a gamma ray. Note that, you can in principle do an experiment to find out "where the electron is" in an orbit… you will get lots of different answers with probabilities given by $\psi_x^*\psi_x$. But after you make the measurement, the electron won't be in that orbit anymore!!) in ψ_x, e^- is NOT at $\phi = \pm\frac{\pi}{2}$.

e) The ψ_x wave happens to have a very precisely defined x-component of angular momentum, $L_x = 0$. Does this violate the Heisenberg "uncertainty" rule?

No. Since we combined ± 1 L_z states, both L_y & L_z are "uncertain".

3. Compton Scattering Thought Question.

In Compton scattering, we treated both light and the electron as particles with precisely defined momenta. We also drew a picture of a "photon" scattering off an "electron" like this:



What's "wrong" with the picture? (If you understand what's wrong, you will understand why we don't draw a better picture!)

e^- doesn't have precise location and momentum. Neither does photon!