#17 Gravitational Potential Energy Pre-class

Due: 11:00am on Monday, October 1, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Exercise 7.12: Tarzan and Jane

Tarzan, in one tree, sights Jane in another tree. He grabs the end of a vine with length 20 m that makes an angle of 45 $^\circ$ with the vertical, steps off his tree limb, and swings down and then up to Jane's open arms. When he arrives, his vine makes an angle of 30 $^\circ$ with the vertical.

Part A

Calculate Tarzan's speed just before he reaches Jane. You can ignore air resistance and the mass of the vine.

Express your answer using two significant figures.

ANSWER:

7.9 m/s

Correct

Introduction to Potential Energy

Learning Goal:

Understand that conservative forces can be removed from the work integral by incorporating them into a new form of energy called potential energy that must be added to the kinetic energy to get the total mechanical energy.

The first part of this problem contains short-answer questions that review the work-energy theorem. In the second part we introduce the concept of potential energy. But for now, please answer in terms of the work-energy theorem.

Work-Energy Theorem
The work-energy theorem states

$$K_f = K_i + W_{all},$$

where W_{all} is the work done by *all* forces that act on the object, and K_i and K_f are the initial and final kinetic energies, respectively.

Part A

The work-energy theorem states that a force acting on a particle as it moves over a ______ changes the ______ energy of the particle if the force has a component parallel to the motion.

Choose the best answer to fill in the blanks above:

ANSWER:

- distance / potential
- distance / kinetic
- vertical displacement / potential
- none of the above

Correct

It is important that the force have a component acting in the direction of motion. For example, if a ball is attached to a string and whirled in uniform circular motion, the string does apply a force to the ball, but since the string's force is always perpendicular to the motion it does no work and cannot change the kinetic energy of the ball.

Part B

To calculate the change in energy, you must know the force as a function of _______. The work done by the force causes the energy change.

Choose the best answer to fill in the blank above:

ANSWER:

- acceleration
- work
- distance (selected)
- potential energy

Correct

Part C

To illustrate the work-energy concept, consider the case of a stone falling from x_i to x_f under the influence of gravity.
Using the work-energy concept, we say that work is done by the gravitational _____, resulting in an increase of the ______ energy of the stone.

Choose the best answer to fill in the blanks above:

ANSWER:

- force / kinetic (selected)
- potential energy / potential
- force / potential
- potential energy / kinetic

Correct

Potential Energy You should read about potential energy in your text before answering the following questions.

Potential energy is a concept that builds on the work-energy theorem, enlarging the concept of energy in the most physically useful way. The key aspect that allows for potential energy is the existence of conservative forces, forces for which the work done on an object does not depend on the path of the object, only the initial and final positions of the object. The gravitational force is conservative; the frictional force is not.

The change in potential energy is the *negative* of the work done by conservative forces. Hence considering the initial and final potential energies is equivalent to calculating the work done by the conservative forces. When potential energy is used, it *replaces the work* done by the associated conservative force. Then only the work due to *nonconservative* forces needs to be calculated.

In summary, when using the concept of potential energy, only *nonconservative* forces contribute to the work, which now changes the total energy:

$$K_{\rm f} + U_{\rm f} = E_{\rm f} = W_{\rm nc} + E_{\rm i} = W_{\rm nc} + K_{\rm i} + U_{\rm i},$$

where $U_{\rm f}$ and $U_{\rm i}$ are the final and initial potential energies, and $W_{\rm nc}$ is the work due *only* to nonconservative forces.

Now, we will revisit the falling stone example using the concept of potential energy.

Part D

Rather than ascribing the increased kinetic energy of the stone to the work of gravity, we now (when using potential energy rather than work-energy) say that the increased kinetic energy comes from the ______ of the _______ energy.

Choose the best answer to fill in the blanks above:

ANSWER:

- ○ work / potential
- ○ force / kinetic
- ◉ change / potential

Correct

Part E

This process happens in such a way that *total mechanical energy*, equal to the ______ of the kinetic and potential energies, is _______.

Choose the best answer to fill in the blanks above:

ANSWER:

- sum / conserved
- sum / zero
- sum / not conserved
- difference / conserved

Correct

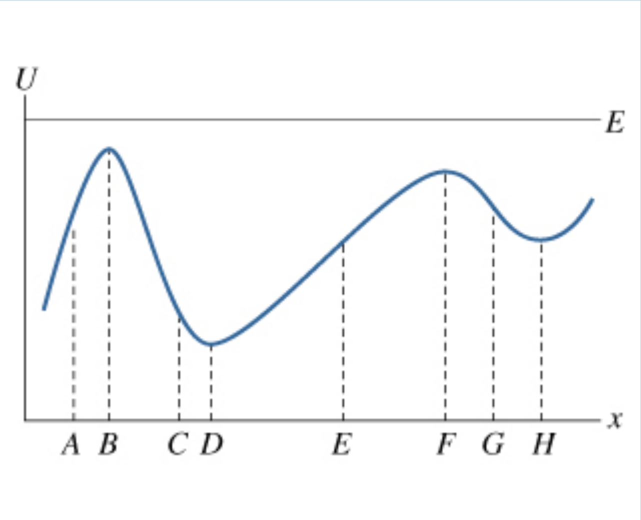
Potential Energy Graphs and Motion

Learning Goal:

To be able to interpret potential energy diagrams and predict the corresponding motion of a particle.

Potential energy diagrams for a particle are useful in predicting the motion of that particle. These diagrams allow one to determine the direction of the force acting on the particle at any point, the points of stable and unstable equilibrium, the particle's kinetic energy, etc.

Consider the potential energy diagram shown. The curve represents the value of potential energy U as a function of the particle's coordinate x. The horizontal line above the curve represents the constant value of the total energy of the particle E. The total energy E is the sum of kinetic (K) and potential (U) energies of the particle.



The key idea in interpreting the graph can be expressed in the equation

$$F_x(x) = -\frac{dU(x)}{dx},$$

where $F_x(x)$ is the *x* component of the net force as function of the particle's coordinate x. Note the negative sign: It means that the *x* component of the net force is negative when the derivative is positive and vice versa. For instance, if the particle is moving to the right, and its potential energy is increasing, the net force would be pulling the particle to the left.

If you are still having trouble visualizing this, consider the following: If a massive particle is increasing its gravitational potential energy (that is, moving upward), the force of gravity is pulling in the *opposite* direction (that is, downward).

If the *x* component of the net force is zero, the particle is said to be in *equilibrium*. There are two kinds of equilibrium:

- *Stable equilibrium* means that small deviations from the equilibrium point create a net force that accelerates the particle back toward the equilibrium point (think of a ball rolling between two hills).
- *Unstable equilibrium* means that small deviations from the equilibrium point create a net force that accelerates the particle further away from the equilibrium point (think of a ball on top of a hill).

In answering the following questions, we will assume that there is a single varying force F acting on the particle along the *x* axis. Therefore, we will use the term *force* instead of the cumbersome *x component of the net force*.

Part A

The force acting on the particle at point A is ___________.

Hint 1. Sign of the derivative

If a function increases (as x increases) in a certain region, then the derivative of the function in that region is positive.

Hint 2. Sign of the component

If x increases to the right, as in the graph shown, then a (one-dimensional) vector with a positive *x* component points to the right, and vice versa.

ANSWER:

- directed to the right
- directed to the left
- equal to zero

Correct

Consider the graph in the region of point A. If the particle is moving to the right, it would be "climbing the hill," and the force would "pull it down," that is, pull the particle back to the left. Another, more abstract way of thinking about this is to say that the slope of the graph at point A is *positive*; therefore, the direction of $\vec{F}$ is *negative*.

Part B

The force acting on the particle at point C is __________.

Hint 1. Sign of the derivative

If a function increases (as x increases) in a certain region, then the derivative of the function in that region is positive, and vice versa.

Hint 2. Sign of the component

If x increases to the right, as in the graph shown, then a (one-dimensional) vector with a positive *x* component points to the right, and vice versa.

ANSWER:

- directed to the right
- directed to the left
- equal to zero

Correct

Part C

The force acting on the particle at point B is __________.

Hint 1. Derivative of a function at a local maximum

At a local maximum, the derivative of a function is equal to zero.

ANSWER:

- directed to the right
- directed to the left
- equal to zero (selected)

Correct

The slope of the graph is zero; therefore, the derivative $dU/dx = 0$, and $|\vec{F}| = 0$.

Part D

The acceleration of the particle at point B is __________.

Hint 1. Relation between acceleration and force

The relation between acceleration and force is given by Newton's 2nd law,

$$F = ma.$$

ANSWER:

- directed to the right
- directed to the left
- equal to zero

Correct

If the net force is zero, so is the acceleration. The particle is said to be in a state of *equilibrium*.

Part E

If the particle is located slightly to the left of point B, its acceleration is __________.

Hint 1. The force on such a particle

To the left of B, $U(x)$ is an increasing function and so its derivative is positive. This implies that the *x* component of the force on a particle at this location is negative, or that the force is directed to the left, just like at A. What can you say now about the acceleration?

ANSWER:

- directed to the right
- directed to the left
- equal to zero

Correct

Part F

If the particle is located slightly to the right of point B, its acceleration is __________.

Hint 1. The force on such a particle

To the right of B, $U(x)$ is a decreasing function and so its derivative is negative. This implies that the *x* component of the force on a particle at this location is positive, or that the force is directed to the right, just like at C. What can you now say about the acceleration?

ANSWER:

- directed to the right
- directed to the left
- equal to zero

Correct

As you can see, small deviations from equilibrium at point B cause a force that accelerates the particle further away; hence the particle is in *unstable equilibrium*.

Part G

Name all labeled points on the graph corresponding to *unstable* equilibrium.

List your choices alphabetically, with no commas or spaces; for instance, if you choose points B, D, and E, type your answer as `BDE`.

Hint 1. Definition of unstable equilibrium

Unstable equilibrium means that small deviations from the equilibrium point create a net force that accelerates the particle further away from the equilibrium point (think of a ball on top of a hill).

ANSWER:

BF

Correct

Part H

Name all labeled points on the graph corresponding to *stable* equilibrium.

List your choices alphabetically, with no commas or spaces; for instance, if you choose points B, D, and E, type your answer as `BDE`.

Hint 1. Definition of stable equilibrium

Stable equilibrium means that small deviations from the equilibrium point create a net force that accelerates the particle back toward the equilibrium point. (Think of a ball rolling between two hills.)

ANSWER:

DH

Correct

Part I

Name all labeled points on the graph where the acceleration of the particle is zero.

List your choices alphabetically, with no commas or spaces; for instance, if you choose points B, D, and E, type your answer as `BDE`.

Hint 1. Relation between acceleration and force

The relation between acceleration and force is given by Newton's 2nd law,

$$F = ma.$$

ANSWER:

BDFH

Correct

Your answer, of course, includes the locations of both stable and unstable equilibrium.

Part J

Name all labeled points such that when a particle is released from rest there, it would accelerate to the left.

List your choices alphabetically, with no commas or spaces; for instance, if you choose points B, D, and E, type your answer as `BDE`.

Hint 1. Determine the sign of the *x* component of force

If the acceleration is to the left, so is the force. This means that the *x* component of the force is ___________.

ANSWER:

- positive
- negative

Hint 2. What is the behavior of $U(x)$?

If the *x* component of the force at a point is negative, then the derivative of $U(x)$ at that point is positive. This means that in the region around the point $U(x)$ is ___________.

ANSWER:

- increasing
- decreasing

ANSWER:

AE

Correct

Part K

Consider points A, E, and G. Of these three points, which one corresponds to the greatest magnitude of acceleration of the particle?

Hint 1. Acceleration and force

The greatest acceleration corresponds to the greatest magnitude of the net force, represented on the graph by the magnitude of the slope.

ANSWER:

- A
- E
- G

Correct

Kinetic energy

If the total energy E of the particle is known, one can also use the graph of $U(t)$ to draw conclusions about the kinetic energy of the particle since

$$K = E - U.$$

As a reminder, on this graph, the total energy E is shown by the horizontal line.

Part L

What point on the graph corresponds to the maximum kinetic energy of the moving particle?

Hint 1. K, U, and E

Since the total energy does not change, the maximum kinetic energy corresponds to the minimum potential energy.

ANSWER:

D

Correct

It makes sense that the kinetic energy of the particle is maximum at one of the (force) equilibrium points. For example, think of a pendulum (which has only one force equilibrium point--at the very bottom).

Part M

At what point on the graph does the particle have the lowest speed?

ANSWER:

B

Correct

As you can see, many different conclusions can be made about the particle's motion merely by looking at the graph. It is helpful to understand the character of motion qualitatively before you attempt quantitative problems. This problem should prove useful in improving such an understanding.

Exercise 7.4

A 72.0-kg swimmer jumps into the old swimming hole from a diving board 3.25 m above the water.

Part A

Use energy conservation to find his speed just he hits the water if he just holds his nose and drops in.

ANSWER:

v = 7.98 $\mathrm{m/s}$

Correct

Part B

Use energy conservation to find his speed just he hits the water if he bravely jumps straight up (but just beyond the board!) at 2.50 $\mathrm{m/s}$.

ANSWER:

v = 8.36 $\mathrm{m/s}$

Correct

Part C

Use energy conservation to find his speed just he hits the water if he manages to jump downward at 2.50 $\mathrm{m/s}$.

ANSWER:

v = 8.36 $\mathrm{m/s}$

Correct

Score Summary:

Your score on this assignment is 97.1%.
You received 19.41 out of a possible total of 20 points.