MATH 264 — R E V I E W E X A M 3 (Chapter 16 - 17.1, 17.5)

A. Approximating Integrals

Approximate double integrals (using definition) by

$$\iint_D f(x,y)\,dA \approx \sum_{k=1}^{n} f(x_k, y_k)\Delta A_k$$

where D has been partitioned into n subrectangles with area ΔA_k and (x_k, y_k) is a point in the kth subrectangle. Approximate triple integrals (using definition) by

$$\iiint_E f(x,y,z)\,dV \approx \sum_{k=1}^{n} f(x_k, y_k, z_k)\Delta V_k$$

where E has been partitioned into n sub-boxes with volume ΔV_k and (x_k, y_k, z_k) is a point in the kth sub-box.

B. Double and Triple Integrals

Set up and evaluate the integrals in cartesian, polar/cylindrical or spherical coordinates
Always sketch the region of integration
Change the oder of integration, if necessary to simplify integration
Change from cartesion to cylindrical or spherical, if necessary to simplify integration
Applications: computing areas, volumes, masses, centroids.

C. Vector Fields

Plot simple vector fields.
Find divergence and curl of vector fields.
Determine whether field is a gradient field. How? (Method 1: show that $\nabla \times \mathbf{F} = 0$ everywhere. Method 2: by finding the potential function)
If conservative, find the potential function.
Know relation between vector field and level curves of potential function. Plot both in one graph.

STUDY PROBLEMS

A. Approximating Integrals

1. Chapter 16 Review, Exercises: 2
2. Approximate $\iint_D x \sin y \, dA$, where $D = [0,1] \times [0, \pi/2]$ using
   (a) 1 subrectangle, midpoint in each subrectangle
   (b) 4 subrectangles, midpoint in each subrectangle
   (c) compare your answers with the exact value.

3. Approximate $\int_0^1 \int_0^1 \int_0^1 xzy^2\, dxdydz$ using
   (a) 1 sub-box, midpoint in each box
   (b) 8 sub-box, midpoint in each box
   (c) compare your answer with the exact value

B. Double and Triple Integrals

1. (a) Chapter 16 Review, Exercises: 3,7,9,10,12,13,17,21,25,30,31,41,42, p.1057: 24
   (b) Section 16.8: 24
2. Evaluate the integral $\int_0^1 \int_y^1 \cos(x^2)\, dxdy$
3. Find the volume bounded by $y = x^2 + z^2$ and $y = 3$.
4. Find the volume above $z = \sqrt{3x^2 + 3y^2}$ and below $x^2 + y^2 + z^2 = 4$.
5. Set up the integral $\int\int\int_T y\, dV$ where T is the tetrahedon bounded by $x = 0$, $y = 0$, $z = 0$ and $2x + y + z = 2$ in the 3 different orders $dV = dzdydx$, $dV = dxdydz$, $dV = dydxdz$.
6. Find the centroid of the hemisphere of radius a given by $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.
7. Find the volume of the region bounded by $y^2 + z^2 = 9$, $x = 0$, $y = 3x$, $z = 0$.
8. Evaluate the integral $\int\int\int_E z\, dV$ where E is bounded by the $y = 0$, $z = 0$, $x = 0$, $x + y = 2$, $y^2 + z^2 = 1$
9. Evaluate the integral $\int\int\int yz\, dV$ where E lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.
10. (a) Set up the integral $\int\int\int_E x^2 + y^2 + z^2\, dV$ where E is the region bounded below by the cone $\phi = \pi/6$ and above by the sphere $\rho = 2$ (i) in cartesian coordinates, (ii) in cylindrical coordinates, (iii) in spherical coordinates.
    (b) Evaluate the integral.

C. Vector Fields

1. Sketch the vector field $\mathbf{F}$ by drawing a representativ set of vectors (make sure to include enough vectors to describe the complete picture).
   (a) $\mathbf{F}(x, y) = x\mathbf{i} + y\mathbf{j}$
   (b) $\mathbf{F}(x, y) = \langle y, 1\rangle$
   (c) $\mathbf{F}(x, y) = y\mathbf{i} - x\mathbf{j}$
   (d) $\mathbf{F}(x, y) = \frac{y\mathbf{i} - x\mathbf{j}}{\sqrt{x^2+y^2}}$
   (e) $\mathbf{F}(x, y) = \dfrac{y\mathbf{i} - x\mathbf{j}}{x^2 + y^2}$
   (f) $\mathbf{F}(x, y) = z\mathbf{j}$
2. Section 17.1: 11-14,15-18
3. Section 17.5: 5,6,12,15,17,19,31
4. Chapter 17 Review, Exercises: 11,12,18,19,
5. Prove that conservative fields are irrotational.

6. (a) Show that the vector field $\mathbf{F}(x, y) = \langle x, -y \rangle$ is conservative.
   (b) Find potential f.
   (c) Plot the level curves of the potential function and the vector field (or if that is easier, the integral curves) on the same graph.
7. Let $\mathbf{F}(x, y) = \frac{\langle x, y \rangle}{x^2+y^2}$.
   (a) Compute $\nabla \times \mathbf{F}$. Explain why you cannot conclude that $\mathbf{F}$ is conservative.
   (b) Find a potential function for $\mathbf{F}$. This shows that $\mathbf{F}$ is conservative.
   (c) Plot the level curves of the potential function and the vector field (or if that is easier, the integral curves) on the same graph.