

## #15 Kinetic Energy Pre-class

Due: 11:00am on Wednesday, September 26, 2012

**Note: You will receive no credit for late submissions.** To learn more, read your instructor's [Grading Policy](#)

### Understanding Work Done by a Constant Force

#### Learning Goal:

To explore the definition of work and learn how to find the work done by a force on an object.

The word "work" has many meanings when used in everyday life. However, in physics work has a very specific definition. This definition is important to learn and understand. Work and energy are two of the most fundamental and important concepts you will learn in your study of physics. Energy cannot be created or destroyed; it can only be transformed from one form to another. How this energy is transferred affects our daily lives from microscopic processes, such as protein synthesis, to macroscopic processes, such as the expansion of the universe!

When energy is transferred either to or away from an object by a force  $\vec{F}$  acting over a displacement  $\vec{d}$ , work  $W$  is done on that object. The amount of work done by a *constant* force can be found using the equation

$$W = Fd \cos \theta,$$

where  $F$  is the magnitude of  $\vec{F}$ ,  $d$  is the magnitude of  $\vec{d}$ , and  $\theta$  is the angle between  $\vec{F}$  and  $\vec{d}$ .

The SI unit for work is the joule, **J**. A single joule of work is not very big. Your heart uses about **0.5 J** each time it beats, and the 60-watt lightbulb in your desk lamp uses **216,000 J** each hour. A joule is defined as follows:

$$1 \text{ J} = 1 \text{ N m} = 1 \frac{\text{kg m}^2}{\text{s}^2}$$

The net work done on an object is the sum of the work done by each individual force acting on that object. In other words,

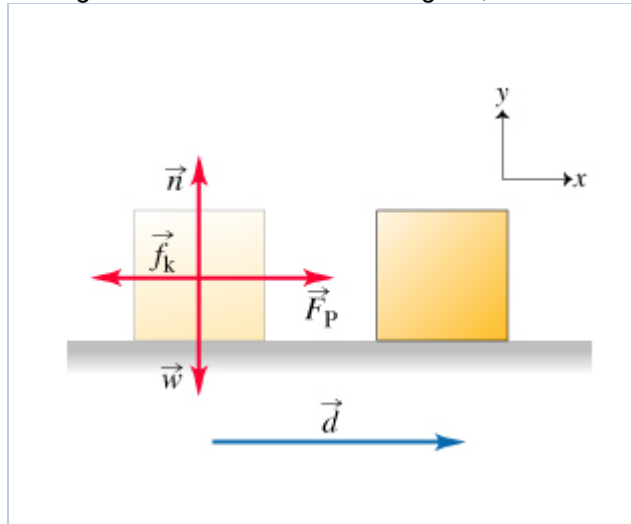
$$W_{\text{net}} = W_1 + W_2 + W_3 + \dots = \sum_i W_i.$$

The net work can also be expressed as the work done by the net force acting on an object, which can be represented by the following equation:

$$W_{\text{net}} = F_{\text{net}} d \cos \theta.$$

Knowing the sign of the work done on an object is a crucial element to understanding work. Positive work indicates that an object has been acted on by a force that transfers energy to the object, thereby increasing the object's energy. Negative work indicates that an object has been acted on by a force that has reduced the energy of the object.

The next few questions will ask you to determine the sign of the work done by the various forces acting on a box that is being pushed across a rough floor. As illustrated in the figure, the box is being acted on by a normal force  $\vec{n}$ , the force due to gravity  $\vec{w}$ , the force of kinetic friction  $\vec{f}_k$ , and the pushing force  $\vec{F}_p$ . The displacement of the box is  $\vec{d}$ .



### Part A

Which of the following statements accurately describes the sign of the work done on the box by the force of the push?

#### Hint 1. Find the angle

The work done on the box by the pushing force depends on the angle  $\theta$  between  $\vec{F}_p$  and the displacement  $\vec{d}$ . What is this angle?

ANSWER:

- ☒ 0 degrees
- ☐ 45 degrees
- ☐ 90 degrees
- ☐ 180 degrees

ANSWER:

- ☒ positive
- ☐ negative
- ☐ zero

**Correct**

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**Part B**

Which of the following statements accurately describes the sign of the work done on the box by the normal force?

**Hint 1. Finding theta**

The work done on the box by the normal force depends on the angle  $\theta$  between  $\vec{n}$  and the displacement  $\vec{d}$ . What is this angle?

ANSWER:

- ☐ 0 degrees
- ☐ 45 degrees
- ☒ 90 degrees
- ☐ 180 degrees

ANSWER:

- ☐ positive
- ☐ negative
- ☒ zero

**Correct**

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### Part C

Which of the following statements accurately describes the sign of the work done on the box by the force of kinetic friction?

#### Hint 1. Finding theta

The work done on the box by the force of kinetic friction depends on the angle  $\theta$  between  $\vec{f}_k$  and the displacement  $\vec{d}$ . What is this angle?

ANSWER:

- ☐ 0 degrees
- ☐ 45 degrees
- ☐ 90 degrees
- ☒ 180 degrees

ANSWER:

- ☐ positive
- ☒ negative
- ☐ zero

**Correct**

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### Part D

Which of the following statements accurately describes the sign of the work done on the box by the force of gravity?

#### Hint 1. Finding the angle

The work done on the box by the weight depends on the angle  $\theta$  between  $\vec{w}$  and the displacement  $\vec{d}$ . What is this angle?

ANSWER:

- ☐ 0 degrees
- ☐ 45 degrees
- ☒ 90 degrees
- ☐ 180 degrees

ANSWER:

- ☐ positive
- ☐ negative
- ☒ zero

**Correct**

### Making generalizations

You may have noticed that the force due to gravity and normal forces do no work on the box. Any force that is perpendicular to the displacement of the object on which it acts does no work on the object.

The force of kinetic friction did negative work on the box. In other words, it took energy away from the box. Typically, this energy gets transformed into heat, like the heat that radiates from your skin when you get a rug burn due to the friction between your skin and the carpet. A force that acts on an object in a direction opposite to the direction of the object's displacement does negative work on the object.

The pushing force acts on the box in the same direction as the object's displacement and does positive work on the box.

These generalizations allow physicists to rewrite the equation for work as

$$W = F_{\parallel} d$$

where  $F_{\parallel}$  is the component of  $\vec{F}$  that is either parallel or antiparallel to the displacement. If  $F_{\parallel}$  is parallel to  $\vec{d}$ , as in the case of  $\vec{F}_p$ , then the work done is positive. If  $F_{\parallel}$  is antiparallel to  $\vec{d}$ , as in the case of  $\vec{f}_k$ , then the work done is negative.

### Part E

You have just moved into a new apartment and are trying to arrange your bedroom. You would like to move your dresser of weight 3,500 **N** across the carpet to a spot 5 **m** away on the opposite wall. Hoping to just slide your dresser easily across the floor, you do not empty your clothes out of the drawers before trying to move it. You push with all your might but cannot move the dresser before becoming completely exhausted. How much work do you do on the dresser?

ANSWER:

- ☐  $W > 1.75 \times 10^4 \text{ J}$
- ☐  $W = 1.75 \times 10^4 \text{ J}$
- ☐  $1.75 \times 10^4 \text{ J} > W > 0 \text{ J}$
- ☒  $W = 0 \text{ J}$

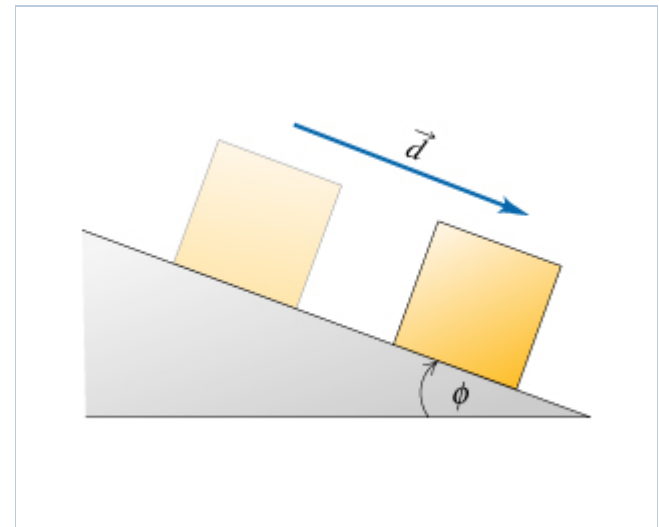
**Correct**

Remember that to a physicist work means something very specific, and since you were unable to move the dresser,  $d = 0$  and therefore  $W = 0$ .

However, you got tired and sweaty trying to move the dresser, just as you do when you go to "work out" at the gym. Your muscles are not static strips of fibrous tissue. They continually contract and expand a slight amount when you exert them. Chemical energy from food is being transformed into the energy needed to move your muscles. Work is being done inside your muscles, but work is not being done on the dresser.

**Part F**

A box of mass  $m$  is sliding down a frictionless plane that is inclined at an angle  $\phi$  above the horizontal, as shown in the figure. What is the work done on the box by the force due to gravity  $w$ , if the box moves a distance  $d$ ?



**Hint 1. Finding Theta.**

The work done on the box by the force of gravity depends on the angle between the weight and the displacement; this is the angle  $\theta$  that goes into the equation

$$W = Fd \cos \theta.$$

ANSWER:

- ☐  $W = wd \cos \phi$
- ☒  $W = wd \cos(90 - \phi)$
- ☐  $W = 0$
- ☐ None of these

**Correct**

The angle given to you in a problem is not always the same angle that you use in the equation for work!

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**Part G**

The planet Earth travels in a circular orbit at constant speed around the Sun. What is the net work done on the Earth by the gravitational attraction between it and the Sun in one complete orbit? Assume that the mass of the Earth is given by  $M_e$ , the mass of the Sun is given by  $M_s$ , and the Earth-Sun distance is given by  $r_{es}$ .

**Hint 1. Newton's law of universal gravitation**

The magnitude of the force of attraction between two objects of masses  $M_1$  and  $M_2$  that are separated by a distance  $r$  is given by:



$$F = G \frac{M_1 M_2}{r^2}.$$

### Hint 2. Circumference of a circle

The circumference of a circle with radius  $r$  is

$$C = 2\pi r$$

### Hint 3. Finding the angle

The work done on the Earth by the gravitational attraction between it and the Sun depends on the angle between the gravitational force and the displacement of the Earth; this is the angle  $\theta$  that goes into

$$W = Fd \cos \theta.$$

The force of attraction always points from the Earth toward the Sun along the radius of the Earth's orbit. At any instant in time the displacement of the Earth is considered to be tangent to its orbit; perpendicular to the radius.

ANSWER:

- ☐  $W = 2\pi G \frac{M_e M_s}{r_{es}}$
- ☐  $W = G\pi M_e M_s$
- ☒  $W = 0$
- ☐ None of these.

### Correct

An object undergoing uniform circular motion experiences a net force that is directed in toward the center of the circle; this net force is called the centripetal force. This force is always perpendicular to the distance the object moves and therefore never does any work on the object.

**Part H**

A block of mass  $m$  is pushed up against a spring with spring constant  $k$  until the spring has been compressed a distance  $x$  from equilibrium. What is the work done on the block by the spring?

**Hint 1. Hooke's Law**

The force exerted by a spring with spring constant  $k$  is given by

$$\vec{F} = -k \vec{x}$$

where  $x$  is the spring's displacement from equilibrium position  $x_{eq}$ .

ANSWER:

- ☐  $W = kx^2$
- ☐  $W = -kx^2$
- ☐  $W = 0$
- ☒ None of these.

**Correct**

The equation for work presented in this problem requires that the force be constant. Because the force exerted on an object varies with the spring's displacement from equilibrium ( $\vec{F} = k\vec{x}$ ) you cannot use  $W = F d \cos \theta$  to find the work done by a spring. In actuality the work done by a spring is given by the equation

$$W_{\text{spring}} = -\frac{1}{2}kx^2$$

Congratulations! Now that you have the basics down and have been exposed to some tricky situations involving the equation for work, you are ready to apply this knowledge to new situations.

## Exercise 6.12: Some Typical Kinetic Energies

### Part A

How many joules of kinetic energy does a 75-**kg** person have when walking? Assume that a person has speed  $2\text{ m/s}$  when walking.

**Express your answer using two significant figures.**

ANSWER:

$$K = 150 \text{ J}$$

**Correct**

### Part B

How many joules of kinetic energy does a 75-**kg** person have when and when running? Assume that a person has speed  $6\text{ m/s}$  when running.

**Express your answer using two significant figures.**

ANSWER:

$$K = 1400 \text{ J}$$

**Correct**

### Part C

In the Bohr model of the atom, the ground-state electron in hydrogen has an orbital speed of  $2190\text{ km/s}$ . What is its kinetic energy?

ANSWER:

$$K = 2.18 \times 10^{-18} \text{ J}$$

Correct

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### Part D

If you drop a 1.0-**kg** weight (about 2  $\text{lb}$ ) from shoulder height, how many joules of kinetic energy will it have when it reaches the ground? Assume that a shoulder height is 1.6 **m**.

**Express your answer using two significant figures.**

ANSWER:

$$K = 16 \text{ J}$$

Correct

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**Part E**

Is it reasonable that a 30-**kg** child could run fast enough to have 100 **J** of kinetic energy?

ANSWER:

- ☒ yes
- ☐ no

**Correct**

## Understanding Work and Kinetic Energy

**Learning Goal:**

To learn about the Work-Energy Theorem and its basic applications.

In this problem, you will learn about the relationship between the work done on an object and the kinetic energy of that object.

The kinetic energy  $K$  of an object of mass  $m$  moving at a speed  $v$  is defined as  $K = (1/2)mv^2$ . It seems reasonable to say that the speed of an object--and, therefore, its kinetic energy--can be changed by performing *work* on the object. In this problem, we will explore the mathematical relationship between the work done on an object and the change in the kinetic energy of that object.

First, let us consider a sled of mass  $m$  being pulled by a constant, horizontal force of magnitude  $F$  along a *rough*, horizontal surface. The sled is speeding up.

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**Part A**

How many forces are acting on the sled?

ANSWER:

- ☐ one
- ☐ two
- ☐ three
- ☒ four

**Correct**

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### Part B

The work done on the sled by the force of gravity is \_\_\_\_\_.

ANSWER:

- ☒ zero
- ☐ negative
- ☐ positive

**Correct**

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### Part C

The work done on the sled by the normal force is \_\_\_\_\_.

ANSWER:

- ☒ zero
- ☐ negative
- ☐ positive

**Correct**

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### Part D

The work done on the sled by the pulling force is \_\_\_\_\_.

ANSWER:

- ☐ zero
- ☐ negative
- ☒ positive

**Correct**

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### Part E

The work done on the sled by the force of friction is \_\_\_\_\_.

ANSWER:

- ☐ zero
- ☒ negative
- ☐ positive

Correct

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**Part F**

The *net* work done on the sled is \_\_\_\_\_.

**Hint 1.** Which force is bigger?

In the situation described, which statement is true?

ANSWER:

- ☒ The magnitude of the pulling force is greater than that of the force of friction.
- ☐ The magnitude of the pulling force is less than that of the force of friction.
- ☐ The magnitude of the pulling force is the same as that of the force of friction.

ANSWER:

- ☐ zero
- ☐ negative
- ☒ positive

Correct



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**Part G**

In the situation described, the kinetic energy of the sled \_\_\_\_\_.

ANSWER:

- ☐ remains constant
- ☐ decreases
- ☒ increases

**Correct**

Let us now consider the situation quantitatively. Let the mass of the sled be  $m$  and the magnitude of the net force acting on the sled be  $F_{\text{net}}$ . The sled starts from rest.

Consider an interval of time during which the sled covers a distance  $s$  and the speed of the sled increases from  $v_1$  to  $v_2$ . We will use this information to find the relationship between the work done by the net force (otherwise known as the *net work*) and the change in the kinetic energy of the sled.

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**Part H**

Find the net force  $F_{\text{net}}$  acting on the sled.

**Express your answer in terms of some or all of the variables  $m$ ,  $s$ ,  $v_1$ , and  $v_2$ .**

**Hint 1.** How to approach the problem

According to Newton's 2nd law,

$$F_{\text{net}} = ma.$$

Therefore, you need to simply find the acceleration of the sled. Once you've found that, multiply it by  $m$  to get the force. You can use kinematics to find the acceleration.

**Hint 2. Find the acceleration**

Find the acceleration  $a$  of the sled.

Express your answer in terms of some or all of the variables  $s$ ,  $v_1$ , and  $v_2$ .

**Hint 1. Some useful kinematics**

The definition of acceleration is

$$a = \frac{v_2 - v_1}{t}.$$

If the acceleration is a constant, the average velocity can be found as

$$v_{\text{avg}} = \frac{v_1 + v_2}{2}.$$

Finally, the distance can be expressed as

$$s = v_{\text{avg}} t.$$

Combining these equations and eliminating  $t$  and  $v_{\text{avg}}$  gives the desired answer.

ANSWER:

$$a = \frac{(v_2)^2 - (v_1)^2}{2s}$$

ANSWER:

$$F_{\text{net}} = \frac{m \left( v_2 - v_1 \right)}{\frac{2s}{v_1 + v_2}}$$

**Correct**

Part I

Find the net work  $W_{\text{net}}$  done on the sled.

Express your answer in terms of some or all of the variables  $F_{\text{net}}$  and  $s$ .

**Hint 1. Work, force, and displacement**

In general, the work done by a constant force  $\vec{F}$  can be found as

$$W = Fs \cos(\theta)$$

where  $\theta$  is the angle between vectors  $\vec{F}$  and  $s_{\text{vec}}$ . However, when the net force and displacement have the same direction (as is the case here),  $\cos(\theta) = 1$ .

ANSWER:

$$W_{\text{net}} = F_{\text{net}} \cdot s$$

**Correct**

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**Part J**

Use  $W = Fs \cos(\theta)$  to find the net work  $W_{\text{net}}$  done on the sled.

Express your answer in terms of some or all of the variables  $m$ ,  $v_1$ , and  $v_2$ .

ANSWER:

$$W_{\text{net}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

**Correct**

Your answer can also be rewritten as

$$W_{\text{net}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

or

$$W_{\text{net}} = K_2 - K_1,$$

where  $K_1$  and  $K_2$  are, respectively, the initial and the final kinetic energies of the sled. Finally, one can write

$$W_{\text{net}} = \Delta K.$$

This formula is known as the Work-Energy Theorem. The calculations done in this problem illustrate the applicability of this theorem in a particular case; however, they should not be interpreted as a proof of this theorem.

Nevertheless, it can be shown that the Work-Energy Theorem is applicable in all situations, including those involving nonconstant forces or forces acting at an angle to the displacement of the object. This theorem is quite useful in solving problems, as illustrated by the following example.

Here is a simple application of the Work-Energy Theorem.

**Part K**

A car of mass  $m$  accelerates from speed  $v_1$  to speed  $v_2$  while going up a slope that makes an angle  $\theta$  with the horizontal. The coefficient of static friction is  $\mu_s$ , and the acceleration due to gravity is  $g$ . Find the total work  $W$  done on the car by the external forces.

**Express your answer in terms of the given quantities. You may or may not use all of them.**

**Hint 1.** How to approach the problem

You are asked to find, in effect, the *net* work done on an object. Use the Work-Energy Theorem.

ANSWER:

$$W = \frac{1}{2}m(v_2^2 - v_1^2)$$

Correct

## ± All Work and No Play

### Learning Goal:

To be able to calculate work done by a constant force directed at different angles relative to displacement

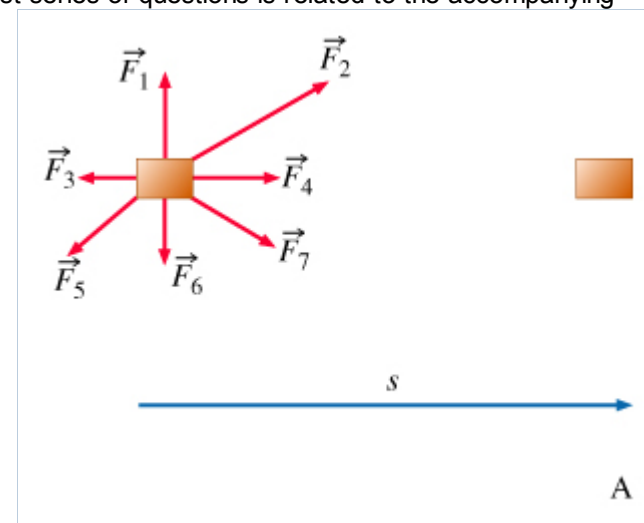
If an object undergoes displacement while being acted upon by a force (or several forces), it is said that *work is being done* on the object. If the object is moving in a straight line and the displacement and the force are known, the work done by the force can be calculated as

$$W = \vec{F} \cdot \vec{s} = |\vec{F}| |\vec{s}| \cos \theta$$

where  $W$  is the work done by force  $\vec{F}$  on the object that undergoes displacement  $\vec{s}$  directed at angle  $\theta$  relative to  $\vec{F}$ .

Note that depending on the value of  $\cos \theta$ , the work done can be positive, negative, or zero.

In this problem, you will practice calculating work done on an object moving in a straight line. The first series of questions is related to the accompanying figure.



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**Part A**

What can be said about the sign of the work done by the force  $\vec{F}_1$ ?

ANSWER:

- ☐ It is positive.
- ☐ It is negative.
- ☒ It is zero.
- ☐ There is not enough information to answer the question.

**Correct**

When  $\theta = 90^\circ$ , the cosine of  $\theta$  is zero, and therefore the work done is zero.

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**Part B**

What can be said about the work done by force  $\vec{F}_2$ ?

ANSWER:

- ☒ It is positive.
- ☐ It is negative.
- ☐ It is zero.

**Correct**

When  $0^\circ < \theta < 90^\circ$ ,  $\cos \theta$  is positive, and so the work done is positive.

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**Part C**

The work done by force  $\vec{F}_3$  is

ANSWER:

- ☐ positive
- ☒ negative
- ☐ zero

**Correct**

When  $90^\circ < \theta < 180^\circ$ ,  $\cos \theta$  is negative, and so the work done is negative.

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**Part D**

The work done by force  $\vec{F}_4$  is

ANSWER:

- ☒ positive
- ☐ negative
- ☐ zero

**Correct**

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**Part E**

The work done by force  $\vec{F}_5$  is

ANSWER:

- ☐ positive
- ☒ negative
- ☐ zero

**Correct**

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### Part F

The work done by force  $F_{6\_vec}$  is

ANSWER:

- ☐ positive
- ☐ negative
- ☒ zero

**Correct**

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### Part G

The work done by force  $F_{7\_vec}$  is

ANSWER:

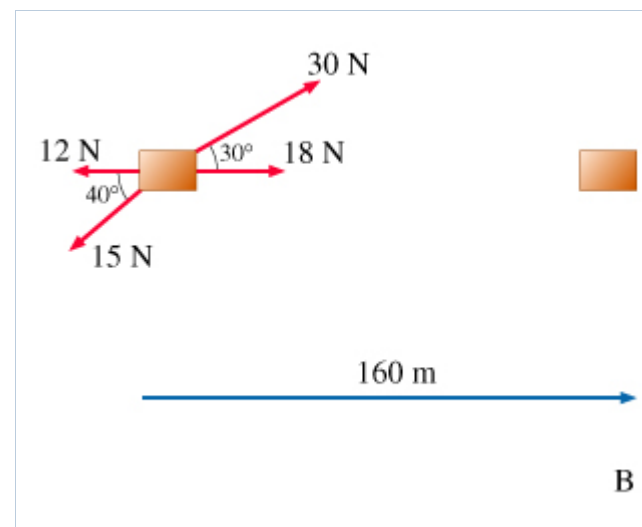


- ☒ positive
- ☐ negative
- ☐ zero

Correct

In the next series of questions, you will use the formula  $W = \vec{F} \cdot \vec{s} = |\vec{F}| |\vec{s}| \cos \theta$

to calculate the work done by various forces on an object that moves 160 meters to the right.



### Part H

Find the work  $W$  done by the 18-newton force.

Use two significant figures in your answer. Express your answer in joules.

ANSWER:

$$W = 2900 \text{ J}$$

Correct

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**Part I**

Find the work  $W$  done by the 30-newton force.

**Use two significant figures in your answer. Express your answer in joules.**

ANSWER:

$$W = 4200 \text{ J}$$

Correct

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**Part J**

Find the work  $W$  done by the 12-newton force.

**Use two significant figures in your answer. Express your answer in joules.**

ANSWER:

$$W = -1900 \text{ J}$$

Correct

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**Part K**

Find the work  $W$  done by the 15-newton force.

**Use two significant figures in your answer. Express your answer in joules.**

ANSWER:

$$W = -1800 \text{ J}$$

**Correct**

### Score Summary:

Your score on this assignment is 96.2%.

You received 19.24 out of a possible total of 20 points.