Lecture 17
(Gravitational and Elastic Potential Energy)

Physics 160-01 Fall 2012

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Work-Energy Theorem

$$W_{TOTAL} = W_{Gravity} + W_{Elastic} + W_{Other} = \Delta KE$$

- Now, we have just broken the total work on an object up into three terms, the first two being from forces that we understand:

$$W_{Gravity} = \int_{initial}^{final} \vec{F}_{Gravity} \cdot d\vec{s} = -mg\left(y_f - y_i\right)$$

$$W_{Elastic} = \int_{initial}^{final} \vec{F}_{Elastic} \cdot d\vec{s} = \frac{1}{2}kx^2$$

Sign determined by the dot product (+ when pushing in same direction as motion, - when opposite)

Very High Math

- We want to manipulate the work-energy equation:

$$W_{TOTAL} = W_{Gravity} + W_{Elastic} + W_{Other} = \Delta KE$$

Gravitational Potential Energy

- Note that it only depends on the change in the height.

$$W_{Gravity} = -mg\Delta y$$

- Now, let's think of the "system" that we are examining as the object plus the earth.
- If the two separate, then work was done (by some outside force) to do that.
- Where did that energy go?
- Energy is stored in the mass-earth system as potential energy: $\Delta U_{Gravity} = mg\Delta y = -W_{Gravity}$

CPS Question 16-1

- **In which case is the speed of the ball the greatest (assume no air resistance)?**

A) The ball falling straight down.

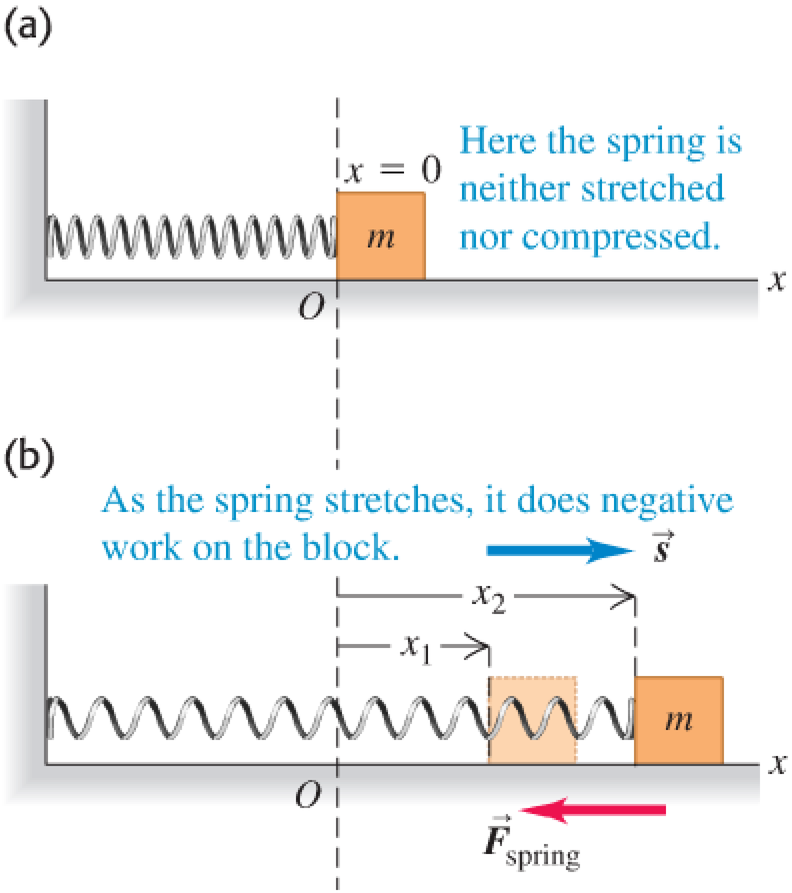
B) The ball on the string.

C) They have the same speed.

D) Not enough information to solve.

Work from a Spring

- The same thing can be done with the work from a spring:



$$W_{Elastic} = \int_{initial}^{final} \vec{F}_{Elastic} \cdot d\vec{s} = \int_{initial}^{final} \left(-kx\hat{i}\right) \cdot \left(dx\hat{i}\right)$$

$$= -\left(\int_{initial}^{final} kx\hat{i} \cdot dx\hat{i}\right)$$

$$= -k\int_{initial}^{final} xdx\left(\hat{i} \cdot \hat{i}\right) = -\frac{1}{2}k\left(x_{final}^2 - x_{initial}^2\right)$$

Work from a Spring

- If we take the initial position to be the relaxed position of the spring, then set $x_{initial} = 0$, we have

$$W_{Elastic} = -\frac{1}{2}kx_{final}^2$$

- And we can make the same definition of the elastic potential energy:

$$\Delta U_{Elastic} = -W_{Elastic} = \frac{1}{2}kx^2$$

- Where, again, this is the energy stored in the spring-block system.

Conservation of Energy

- If there are no other forces acting on a body, then we can make a statement of the conservation of mechanical energy:

$$0 = \Delta U_{Gravity} + \Delta U_{Elastic} + \Delta KE \Rightarrow$$

$$U_{Gravity_{final}} - U_{Gravity_{initial}} + U_{Elastic_{final}} - U_{Elastic_{initial}} + KE_{final} - KE_{initital} = 0 \Rightarrow$$

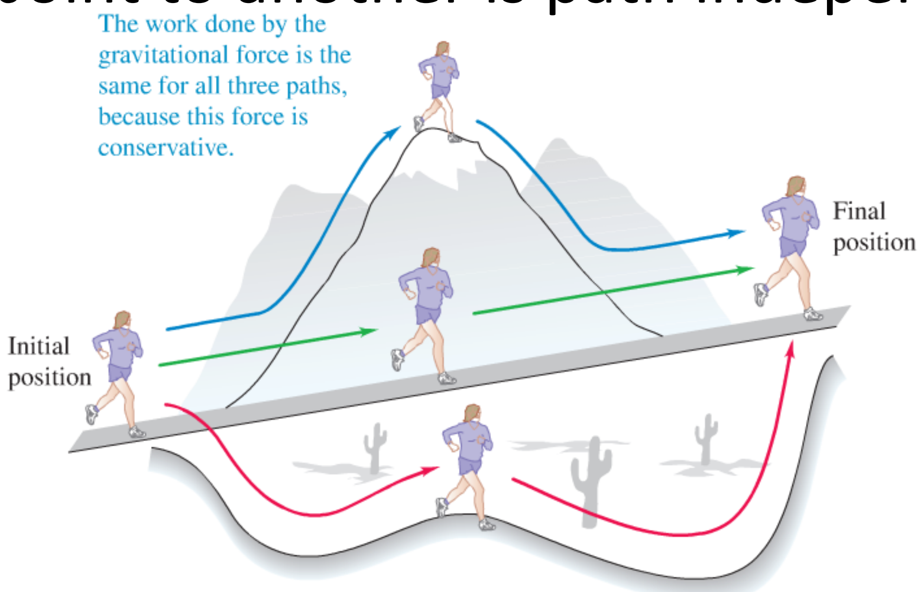
$$U_{Gravity_{final}} + U_{Elastic_{final}} + KE_{final} = U_{Gravity_{initial}} + U_{Elastic_{initial}} + KE_{initital}$$

- Then, we can take the sum of the gravitational, elastic and kinetic energy (mechanical energy) to be unchanging – it is conserved:

$$E = U_{Gravity} + U_{Elastic} + KE$$

Conservative and Non-conservative

- The nice thing about the conservation of mechanical energy is that the change in the potentials only are determined by the initial and final points of the path. That's because potentials always describe conservative forces – forces where the work done by them in going from one point to another is path independent.



Non- conservative forces

- An example of a non-conservative force is friction. The work done by friction is definitely dependent on the path.
- Let's take the example of moving a book on a table with kinetic friction:

- Since path 2 is longer, there will be more work done by friction.