**Homework #2 - Due Thursday, June 13**

In the following problems, you will need to use a calculator. If I give you a similar problem on an exam, then you can expect the numbers to be nice enough to where you will not need a calculator. Be sure you understand the methods and can do the calculations by hand!

1. Consider the first-order differential equation

$$\frac{dy}{dt} = 2t(y-1)$$

   with the initial value $y(0) = 0$.

   (a) Use Euler's method to approximate $y(0.6)$ using a step-size of $h = 0.2$.

   (b) Use the improved Euler's method (Heun's Formula) to approximate $y(0.6)$ using a step-size of $h = 0.2$.

   (c) Solve the equation using whichever method you please. Use your solution to calculate $y(0.6)$ exactly. What are the absolute errors of the approximations in parts (a) and (b)?

2. **Radioactive Decay** As mentioned in class, the stability of a radioactive isotope is measured by its *half-life*. Given an initial amount of the isotope, the half-life is the amount of time it takes for half of the atoms to decay into lighter atoms, so the longer the half-life, the more stable the isotope. This is the source of the technique of *carbon dating*. It is known that the half-life of carbon-14 is 5600 years. Given that a fossilized bone contains 1/1000 the original amount of C-14, determine the age of the fossil.

3. **Population Dynamics** A modification of the logistic equation is the *Gompertz* equation:

$$\frac{dP}{dt} = P(a - b\ln P).$$

   It arises as the model for the growth of certain types of populations, but is also encountered in the study of the growth of solid tumors, actuarial tables, and the growth of revenue in the sale of commercial products.

   (a) Find the equilibrium solutions.

   (b) Solve the equation subject to the initial condition $P(0) = P_0$, where $0 < P_0 < e^{a/b}$.

   (c) If $b > 0$, what value does $P$ approach as $t \to \infty$? As $t \to -\infty$? What about if $b < 0$? Sketch a solution curve for each case ($b > 0$ and $b < 0$) separately.

4. **Chemical Concentration** Two chemicals $A$ and $B$ combine to form a chemical $C$ via the reaction

$$2A + B \rightleftharpoons C.$$

   The rate at which the reaction proceeds is proportional to the product of the instantaneous amounts of $A$ and $B$ that have not already been converted to $C$.

   (a) Set up a differential equation that models the amount of chemical $C$ over time. (Hint: First determine equations that express how much of the chemical is left at a given time.)

   (b) Suppose that initially there are 40 grams of $A$ and 50 grams of $B$. If 10 grams of $C$ have formed after 5 minutes, how much is formed after 20 minutes?

   (c) What is the limiting amount of $C$?