

To Do Multiply ...

Multiply Example

```
      10111001   (185)
x   11010111   (215)
-----
      10111001
      10111001
      10111001
      00000000
      10111001
      00000000
      10111001
      10111001
      -----
1001101101011111 (39775)
```

```
      10111001
x    11010111
-----
      10111001
0000001000101011 <- running sum
```

```
      10111001
x    11010111
-----
      10111001 <- third PP
0000001000101011
```

```

      10111001
x   11010111
-----
      10111001
0000001100001111 <- running sum

```

```

      10111001
x   11010111
-----
      00000000 <- fourth PP
0000001100001111

```

```
      10111001
    x 11010111
    -----
    00000000
0000001100001111 <- running sum
```

Pollard's Attempt to Explain
Booth's Multiply

First, the Equations

$$\text{Value} = A \times B$$

$$\text{Value} = A \times b_4 b_3 b_2 b_1 b_0$$

$$\text{Value} = A \times (-b_4 \times 2^4 + b_3 \times 2^3 + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0)$$

And the Trick:

$$2^k = 2^{k+1} - 2^k$$

$$2^3 = 2^4 - 2^3$$

$$2^2 = 2^3 - 2^2$$

... and so on ...

$$\begin{aligned} \text{Value} = A \times (& -b_4 \times 2^4 + b_3 \times 2^4 \\ & -b_3 \times 2^3 + b_2 \times 2^3 \\ & -b_2 \times 2^2 + b_1 \times 2^2 \\ & -b_1 \times 2^1 + b_0 \times 2^1 \\ & -b_0 \times 2^0 + 0 \times 2^0 \quad) \end{aligned}$$

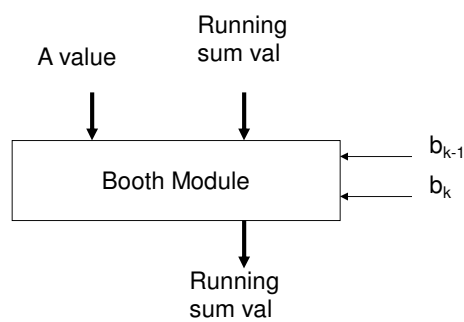
$$\begin{aligned} \text{Value} = A \times (& (-b_4 + b_3) \times 2^4 \\ & (-b_3 + b_2) \times 2^3 \\ & (-b_2 + b_1) \times 2^2 \\ & (-b_1 + b_0) \times 2^1 \\ & (-b_0 + 0) \times 2^0 \quad) \end{aligned}$$

So, Two Bits Per Stage

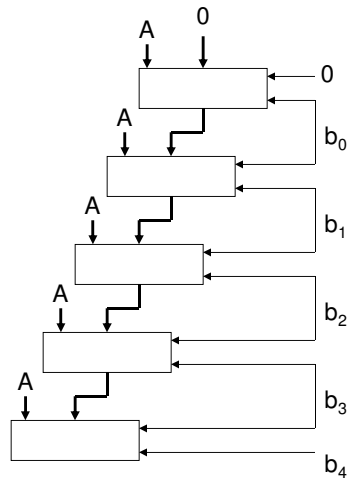
b_{k-1}	b_k	
0	0	$0 - 0 = 0$, pass value
0	1	$0 - 1 = -1$, subtract A
1	0	$1 - 0 = 1$, add A
1	1	$1 - 1 = 0$, pass value

Booth's Algorithm Module

So, system is made of modules that can add (+1), subtract (-1), or pass (0) values of A....




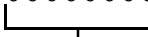
Booth Conceptual Block Diagram



Try Algorithm: 9×-12 in 8-bit 2's Complement

$9 = 00001001 = A$
 $-12 = 11110100 = B$

Step 1: use b_0 and 0, which are 0 and 0: Pass value

$00000000 \leftarrow$ Running sum starts at zero
 \leftarrow Module passes value
 $00000000 \leftarrow$ Result of step 1

 Pass these bits to step 2

00000000 ← Bits passed from step 1
[] ← Step 2 uses $b_1 = b_0 = 0$ so
pass value again
00000000 ← Result of step 2
↓
Pass these bits to step 3

00000000 ← Bits passed from step 2
[] ← Step 3 uses $b_2 = 1, b_1 = 0$ so
subtract A from running sum
11111011 ← Result of step 3
↓
Pass these bits to step 4

11111011 ← Bits passed from step 3
 [] ← Step 4 uses $b_3 = 0$, $b_2 = 1$ so
 add A to running sum
 000000100 ← Result of step 4
 []
 ↓
 Pass these bits to step 5

00000010 ← Bits passed from step 4
 [] ← Step 5 uses $b_4 = 1$, $b_3 = 0$ so
 subtract A from running sum
 111111001 ← Result of step 5
 []
 ↓
 Pass these bits to step 6

11111100 ← Bits passed from step 5
 [] ← Step 6 uses $b_5 = 1$, $b_4 = 1$ so
 pass running sum value
 11111100 ← Result of step 6
 []
 ↓
 Pass these bits to step 7

11111110 ← Bits passed from step 6
 [] ← Step 7 uses $b_5 = 1$, $b_4 = 1$ so
 pass running sum value
 11111110 ← Result of step 7
 []
 ↓
 Pass these bits to step 8

11111111 ← Bits passed from step 7
 [] ← Step 8 uses $b_5 = 1$, $b_4 = 1$ so
 pass running sum value
 11111111 ← Result of step 8
 []
 ↓
 These bits are MSBs of result

Result is found by using one bit from steps 1 to 7,
 then remaining bits from step 8: 1111 1111 1 0 0 1 0 1 0 0