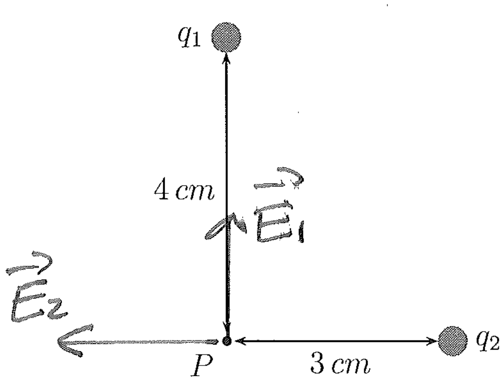
#1 Find the magnitude and direction of the electric field at the point P created by the $q_1 = -50\,\mu C$, $q_2 = 75\,\mu C$ point charges shown below.



$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$E_1 = \frac{K|q_1|}{r_1^2} = \frac{(9\times10^9\,N\cdot m^2/C^2)(50\times10^{-6}C)}{(.04m)^2}$$

$$= 2.8125\times10^8\,N/C$$

q_1 NEGATIVE $\Rightarrow \vec{E_1} = \uparrow$

$$E_2 = \frac{K|q_2|}{r_2^2} = \frac{(9\times10^9\,N\cdot m^2/C^2)(75\times10^{-6}C)}{(.03m)^2} = 7.5\times10^8\,N/C$$

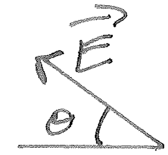
q_2 POSITIVE $\Rightarrow \vec{E_2} = \leftarrow$

$E_x = E_{1,x} + E_{2,x} = 0 + 7.5\times10^8\,N/C = 7.5\times10^8\,N/C$ (LEFT IS POSITIVE)

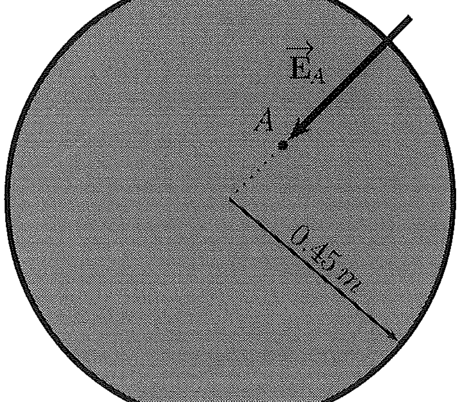
$E_y = E_{1,y} + E_{2,y} = 2.8125\times10^8\,N/C + 0 = 2.8125\times10^8\,N/C$

$E = \sqrt{E_x^2 + E_y^2} \Rightarrow \boxed{E = 8.01\times10^8\,N/C}$ $\quad \theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = 20.556^\circ$

$= 20.6^\circ$



#2 A spherical insulator of radius $0.45\,m$ has a uniform charge density, ρ. If the electric field at the point labeled A, which is $0.15\,m$ away from the center, is $E_A = 6.75 \times 10^6\ N/C$ and points toward the center of the sphere, what is the value of ρ?



GAUSS'S LAW: $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$

BY SYMMETRY, FOR A GAUSSIAN SPHERE $\oint \vec{E}\cdot d\vec{A} = EA = E(4\pi r^2)$

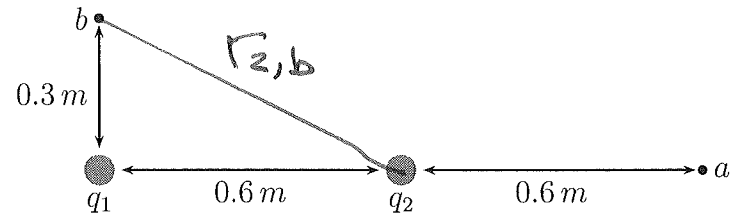
$Q_{encl} = \rho\left(\frac{4}{3}\pi r^3\right)$

$\Rightarrow E(\cancel{4\pi} r^{\cancel{2}}) = \frac{\rho}{\epsilon_0}\frac{\cancel{4\pi} r^{\cancel{3}}}{3}$

$\Rightarrow E = \frac{\rho r}{3\epsilon_0} \Rightarrow \rho = \frac{3\epsilon_0 E}{r} = \frac{3(8.85\times10^{-12}\ C^2/N\cdot m^2)(6.75\times10^{6}\ N/C)}{.15\,m}$

$\Rightarrow \boxed{\rho = .00119475\ C/m^3 = 1.19\times10^{-3}\ C/m^3}$

#3 For the two point charges $q_1 = -50\mu C$ and $q_2 = 25\mu C$ and the points A and B shown below, find the potential of point a relative to b.



$$V_{ab} = V_a - V_b$$

For two point charges $V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$

$$\Rightarrow V_a = k\left(\frac{q_1}{r_{1,a}} + \frac{q_2}{r_{2,a}}\right) \qquad r_{1,a} = 0.6m + 0.6m = 1.2m$$

$$r_{2,a} = 0.6m$$

$$\Rightarrow V_a = (9\times10^9 N\cdot m^2/C^2)\left(\frac{-50\times10^{-6}C}{1.2m} + \frac{25\times10^{-6}C}{0.6m}\right) = 0$$

$$V_b = k\left(\frac{q_1}{r_{1,b}} + \frac{q_2}{r_{2,b}}\right) \qquad r_{1,b} = 0.3m, \quad r_{2,b} = \sqrt{(0.3m)^2 + (0.6m)^2}$$

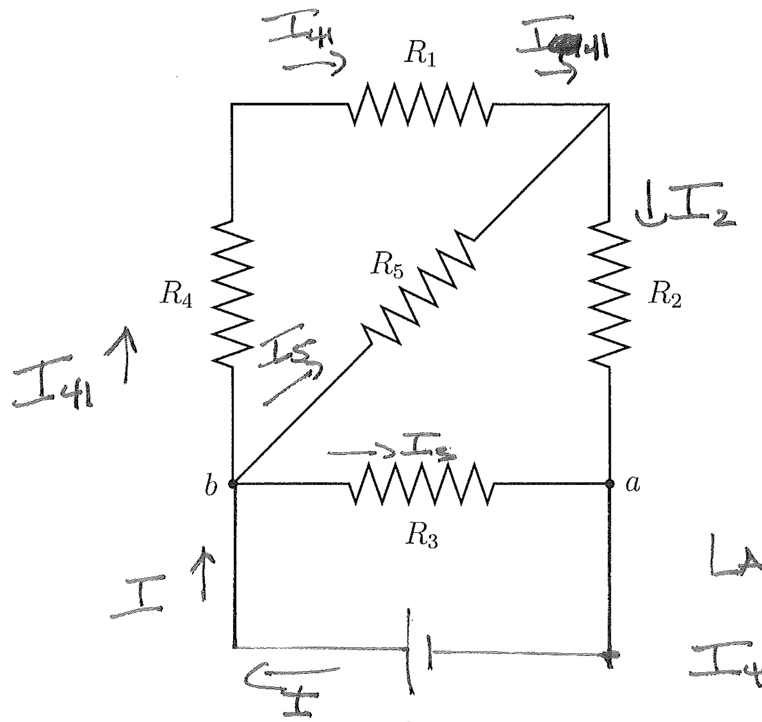
$$= \sqrt{.45m^2} = .67082m$$

$$\Rightarrow V_b = (9\times10^9 N\cdot m^2/C^2)\left(\frac{-50\times10^{-6}C}{0.3m} + \frac{25\times10^{-6}C}{\sqrt{.45m^2}}\right) = (9\times10^9 N\cdot m^2/C^2)(-1.29\times10^{-4} C/m)$$

$$= -1.165\times10^6 V = -1.17MV$$

$$\boxed{V_{ab} = 0 - (-1.17MV) = +1.17MV}$$

#4 If a 10 V battery is connected to the points a and b in the circuit below, how much power will the R_5 resistor dissipate?



$R_1 = 500\,\Omega$	
$R_2 = 200\,\Omega$	
$R_3 = 400\,\Omega$	
$R_4 = 100\,\Omega$	
$R_5 = 350\,\Omega$	

$\mathcal{E} = 10V$

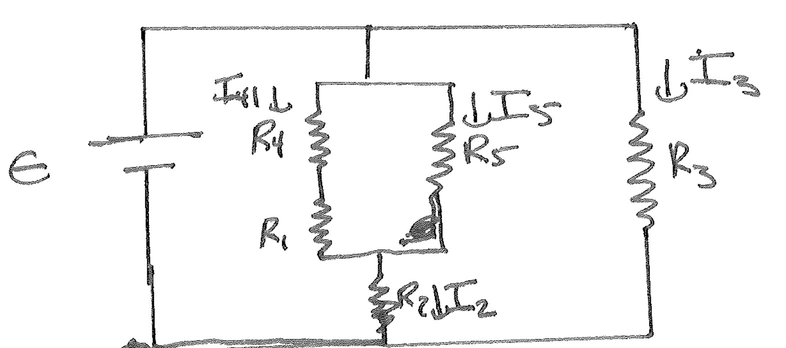
LABEL CURRENTS AS SHOWN

$I_{41} \Rightarrow R_4$ AND R_1 IN SERIES

JUNCTION RULE: $I_{41} + I_5 = I_2 \Rightarrow$

R_2 IN SERIES WITH $R_5/R_4, R_1$ COMBO

Fingertest $\Rightarrow$ R_5 IN Parallel WITH R_4, R_1. R_3 IN PARALLEL WITH EVERYTHING ELSE



R_1, R_4 EQUIVALENT TO

$R' = R_1 + R_4 = 500\Omega + 100\Omega$

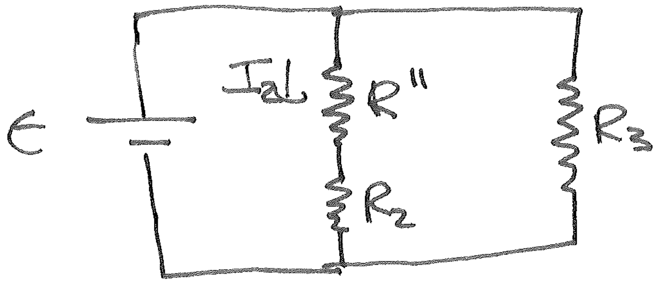
$= 600\Omega$

R', R_5 EQUIVALENT TO $R'' = \dfrac{R' R_5}{R' + R_5}$

(CONT.)

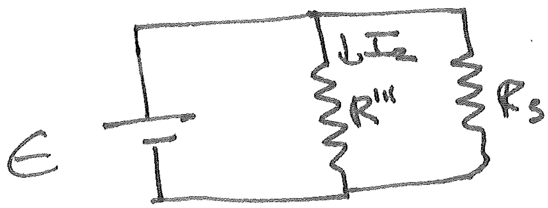
$$R'' = \frac{(600\Omega)(350\Omega)}{(600\Omega + 350\Omega)} = \frac{(600\Omega)(350\Omega)}{(950\Omega)} = 221.05\Omega$$

$\frac{4200}{19}\Omega$ to be exact



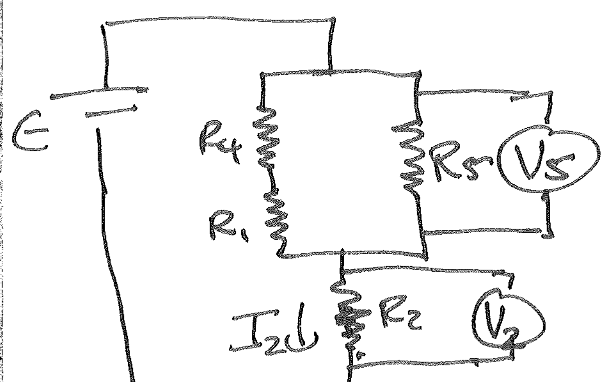
$$R''' = R'' + R_2 = 221.05\Omega + 200\Omega$$
$$= 421.05\Omega$$

$\frac{8000}{19}\Omega$



Across R''', $V = \epsilon = 10V$

$$\therefore I_2 = \frac{\epsilon}{R'''} = \frac{10V}{421.05\Omega} = .02375A$$



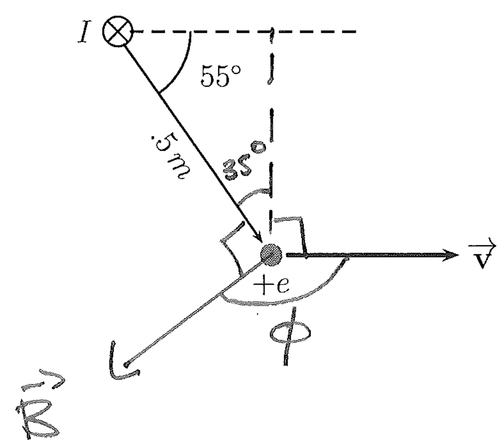
$$V_2 + V_5 = \epsilon$$

$$V_2 = I_2 R_2 = (.02375A)(200\Omega) = 4.75V$$

$$\therefore V_5 = \epsilon - V_2 = 10V - 4.75V = 5.25V$$

$$\boxed{P_5 = \frac{V_5^2}{R_5} = \frac{(5.25V)^2}{350\Omega} = .07875 \text{ Watt}}$$

#5 A proton is located 0.5 m away from an infinitely long wire whose current, $I = 4.5\,A$, is flowing into the page. What is the magnitude and direction of the force acting on the proton if its speed is $1.2 \times 10^8\,m/s$, and it is moving in the horizontal direction shown below? **Note:** The position and velocity vectors shown below are in the same plane.



By RHR, B = ⟲

$\Rightarrow$ at e, $\vec{B}$ is 90° to $\vec{r}$ as shown & 0.5m

INFINITE WIRE, $B = \frac{\mu_0 I}{2\pi r}$

$$B = \frac{(2\times10^{-7}\,T\cdot m/A)(4.5A)}{0.5m} = 1.8\times10^{-6}\,T$$

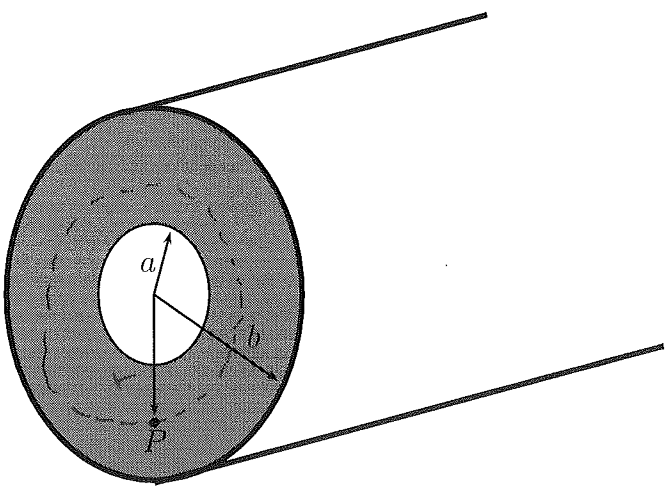
$\vec{F} = q\vec{v}\times\vec{B} = qVB\sin\phi$, ⊗ ← RHR AGAIN

$\phi = 360° - 90° - 35° - 90° = 180° - 35° = 145°$

$F = evB\sin\phi = (1.6\times10^{-19}c)(1.2\times10^{8}m/s)(1.8\times10^{-6}T)\sin 145° = 1.98228\times10^{-17}N$

$$\therefore \boxed{\vec{F} = 1.98\times10^{-17}N, \otimes}$$

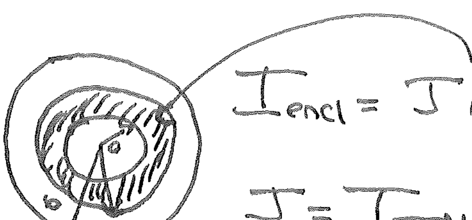
#6 A wire, whose cross-section is shown below, has an empty cylindrical region along its axis and at its center. The conducting region of the wire has inner radius $a = 0.25\,m$ and outer radius $b = 0.80\,m$ and has a total current of $22.5\,A$ flowing through it. Assuming the current density $\vec{\mathbf{J}}$ in the conducting region is constant, use Ampere's Law to find the magnitude of the magnetic field at the point P which is $0.43\,m$ from the center.



Ampere's LAW: $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl}$

FOR A CIRCLE OF RADIUS $r = 0.43\,m$, by symmetry

$\oint \vec{B}\cdot d\vec{\ell} = B(2\pi r)$



$I_{encl} = J\,\text{Area} = J(\pi r^2 - \pi a^2) = J\pi(r^2 - a^2)$

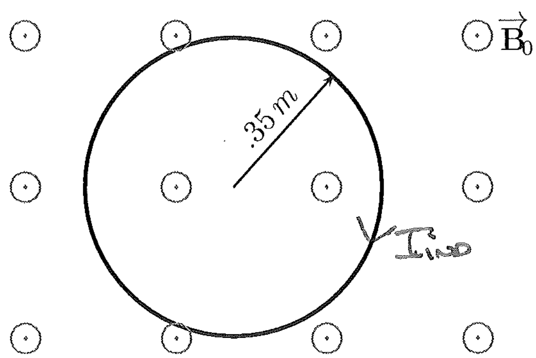
$J = \frac{I_{TOTAL}}{A_{TOTAL}} = I_{TOTAL}/(\pi b^2 - \pi a^2) = \frac{I_{TOTAL}}{\pi(b^2 - a^2)}$

$\Rightarrow I_{encl} = \frac{I_{total}}{\pi(b^2-a^2)}\pi(r^2-a^2) = \frac{I_{TOTAL}(r^2-a^2)}{(b^2-a^2)} = \frac{22.5A\,(0.43^2 - 0.25^2)}{(0.8^2 - 0.25^2)} = 4.769A$

$B(2\pi r) = \mu_0 I_{encl} \Rightarrow B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi}\frac{I_{encl}}{r} = \frac{(2\times10^{-7}\,T\cdot m/A)(4.769A)}{0.43m}$

$\Rightarrow \boxed{B = 2.218\times10^{-6}\,T}$

#7 A 0.65-m long, 0.35 m-radius, 120-Ω, 1500-loop solenoid is located in a region of space where there is a uniform magnetic field, $\vec{\mathbf{B}}_0$, pointing out of the page. (One of the solenoid's loops is shown below.) If, for a short period of time, the magnitude of the magnetic field is given by $B_0 = 1.2e^{t/5}$ (where B_0 is in Tesla when t is in seconds), what is the direction and magnitude of the induced magnetic field, $\vec{\mathbf{B}}_{ind}$, in the solenoid at $t = 1.6\,s$? Also, indicate whether the induced current would flow clockwise or counter-clockwise.



FARADAY'S LAW

$$\mathcal{E}_{IND} = -N\frac{d\Phi_B}{dt}$$

UNIFORM $B_0 \Rightarrow \Phi_B = B_0 A$
$= B_0 \pi (.35m)^2$

$$\Rightarrow \Phi_B = 1.2e^{t/5}\pi(.35m)^2 = (0.147)\pi e^{t/5} \Rightarrow \frac{d\Phi_B}{dt} = (0.147)\pi e^{t/5}(1/5)$$

$$\therefore \text{ at } t = 1.6s,\ \mathcal{E}_{IND} = -1500(0.147)\pi e^{(\frac{1.6}{5})}(\tfrac{1}{5}) \doteq 190.79V$$

$$I_{IND} = \frac{\mathcal{E}_{IND}}{R} = \frac{-190.79V}{120\,\Omega} \doteq 1.59A$$ ← Negative sign just for direction

I_{IND} Flowing Through Solenoid $\Rightarrow B_{IND} = \mu_0\left(\frac{N}{\ell}\right)|I_{IND}| = (4\pi \times 10^{-7}\,T\cdot m/A)\left(\frac{1500}{0.65m}\right)(1.59A)$

$\Rightarrow \boxed{B_{IND} = .00461T}$

FLUX INCREASING $\Rightarrow \vec{B}_{IND}$ tries to CANCEL

$\Rightarrow \boxed{\vec{B}_{IND} = .00461T, \otimes}$

By RHR, I_{IND} Flowing Clockwise

BONUS: A resistor is incased in the center of a 0.25 kg block of ice. Luckily, the resistor's leads are free to be connected to a 12-V battery. If the ice melts in 2.3 hours, what is the resistor's value? Assume that the ice absorbs 100% of the heat dissipated by the resistor. Ice's latent heat of fusion is $3.34 \times 10^5\ J/kg$. *Note:* This is the basic design problem for a frost-free refrigerator. (Though all the numbers would be different in real life.)

TO MELT ICE REQUIRES, $Q = ML = (0.25kg)(3.34\times10^5 J/kg)$

$\Rightarrow Q = 83500J$

SO RESISTOR MUST DISSIPATE 83500J OF ENERGY IN

$2.3 hour = 2.3h \times \frac{3600s}{h} = 8280s$

So ITS Power must be $P = \frac{83500J}{8280s} = 10.08 Watt$

FOR RESISTOR, $P = \frac{V^2}{R} \Rightarrow$ $\boxed{R = \frac{V^2}{P} = \frac{(12V)^2}{10.08 Watt} = 14.279\Omega = 14.3\Omega}$