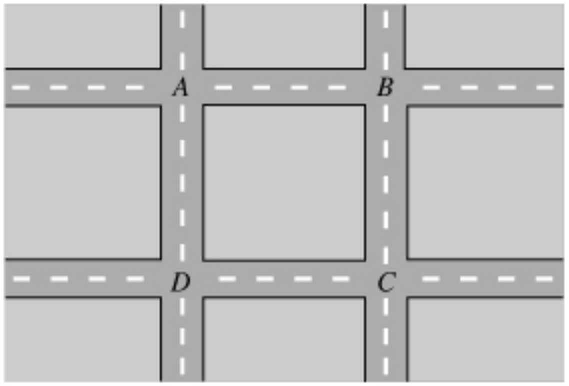
#1 Vectors and Trigonometry Review Pre-class

Due: 11:00am on Wednesday, August 22, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Vector Magnitude and Direction Conceptual Question

A man out walking his dog makes one complete pass around a perfectly square city block. He starts at point A and walks clockwise around the block.



Let $\vec{r}_{AB}$ be the displacement vector from A to B, $\vec{r}_{BC}$ be the displacement vector from B to C, etc.

Part A

Which of the following vectors is equal to $\vec{r}_{AB}$?

Hint 1. Determining a vector

Recall that $\vec{r}_{AB}$ is a vector representing the displacement of the man and his dog as they walk from point A to point B. This vector has a magnitude equal to one block and a direction along the positive *x* axis.

Hint 2. Equal vectors

Two vectors are equal if they have the same *magnitude* and the same *direction*.

ANSWER:

- $\vec{r}_{\rm BC}$ only
- $\vec{r}_{\rm CD}$ only
- $\vec{r}_{\rm DA}$ only
- All of the above
- None of the above

Correct

Recall that, for vectors to be equal, they must have the same *magnitude* and *direction*.

Part B

Which of the following vectors is equal to $-\vec{r}_{AB}$?

ANSWER:

- $\vec{r}_{\rm BC}$ only
- $\vec{r}_{\rm CD}$ only
- $\vec{r}_{\rm DA}$ only
- All of the above
- None of the above

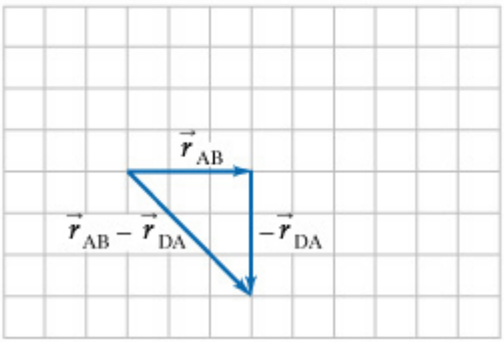
Correct

Part C

Which of the following vectors is equal to $\vec{r}_{AB} - \vec{r}_{DA}$?

Hint 1. Determining the difference of two vectors

$\vec{r}_{AB} - \vec{r}_{DA}$ can be determined by adding the vector $\vec{r}_{\rm AB}$ to the vector pointing opposite to $\vec{r}_{\rm DA}$. Thus $\vec{r}_{AB} - \vec{r}_{DA}$ looks like this:



Carefully perform the vector addition in each of the options and compare the resultant vectors to the one shown above.

ANSWER:

- $-(\vec{r}_{CD}+\vec{r}_{DA})$ only
- $\vec{r}_{AB}+\vec{r}_{BC}$ only
- $\vec{r}_{BC}-\vec{r}_{CD}$ only
- All of the above
- None of the above

Correct

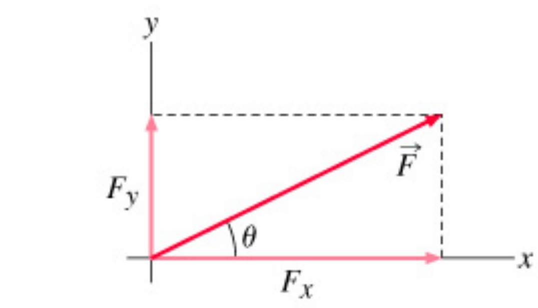
Vector Components--Review

Learning Goal:

To introduce you to vectors and the use of sine and cosine for a triangle when resolving components.

Vectors are an important part of the language of science, mathematics, and engineering. They are used to discuss multivariable calculus, electrical circuits with oscillating currents, stress and strain in structures and materials, and flows of atmospheres and fluids, and they have many other applications. Resolving a vector into components is a precursor to computing things with or about a vector quantity. Because position, velocity, acceleration, force, momentum, and angular momentum are all vector quantities, resolving vectors into components is *the most important skill* required in a mechanics course.

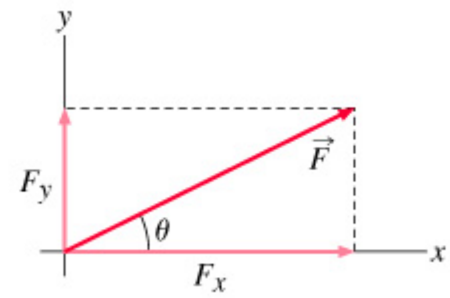
The figure shows the components of $\vec{F}$, F_x and F_y, along the *x* and *y* axes of the coordinate system, respectively. The components of a vector depend on the coordinate system's orientation, the key being the angle between the vector and the coordinate axes, often designated θ.



Part A

The figure shows the standard way of measuring the angle. θ is measured *to* the vector *from* the *x* axis, and counterclockwise is positive.

Express F_x and F_y in terms of the length of the vector F and the angle θ, with the components separated by a comma.



ANSWER:

F_x, F_y = $F\cos\theta, F\sin\theta$

Correct

In principle, you can determine the components of *any* vector with these expressions. If $\vec{F}$ lies in one of the other quadrants of the plane, θ will be an angle larger than 90 degrees (or $\pi/2$ in radians) and $\cos(\theta)$ and $\sin(\theta)$ will have the appropriate signs and values.

Unfortunately this way of representing $\vec{F}$, though mathematically correct, leads to equations that must be simplified using trig identities such as

$$\sin(180^\circ + \phi) = -\sin(\phi)$$

and

$$\cos(90^\circ + \phi) = -\sin(\phi).$$

These must be used to reduce all trig functions present in your equations to either $\sin(\phi)$ or $\cos(\phi)$. Unless you perform this followup step flawlessly, you will fail to recoginze that

$$\sin(180^\circ + \phi) + \cos(270^\circ - \phi) = 0,$$

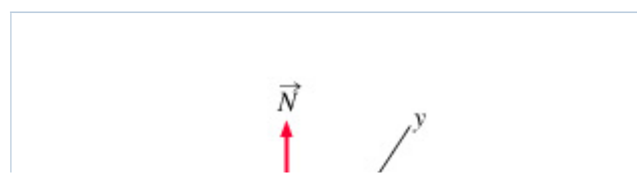
and your equations will not simplify so that you can progress further toward a solution. Therefore, it is best to express all components in terms of either $\sin(\phi)$ or $\cos(\phi)$, with ϕ between 0 and 90 degrees (or 0 and $\pi/2$ in radians), and determine the signs of the trig functions by knowing in which quadrant the vector lies.

Part B

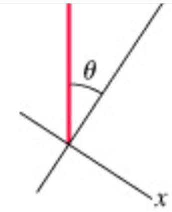
When you resolve a vector $\vec{F}$ into components, the components *must have the form* $|\vec{F}|\cos(\theta)$ or $|\vec{F}|\sin(\theta)$. The signs depend on which quadrant the vector lies in, and there will be one component with $\sin(\theta)$ and the other with $\cos(\theta)$.

In real problems the optimal coordinate system is often rotated so that the *x* axis is not horizontal. Furthermore, most vectors will not lie in the first quadrant. To assign the sine and cosine correctly for vectors at arbitrary angles, you must figure out which angle is θ and then properly reorient the definitional triangle.

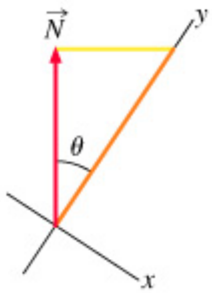
As an example, consider the vector $\vec{N}$ shown in the diagram labeled "tilted axes," where you know the angle θ between $\vec{N}$ and the *y* axis.



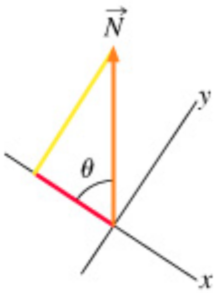
Which of the various ways of orienting the definitional triangle must be used to resolve $\vec{N}$ into components in the tilted coordinate system shown? (In the figures, the hypotenuse is orange, the side adjacent to θ is red, and the side opposite is yellow.)



Tilted Axes



(1)



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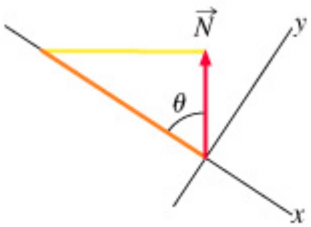

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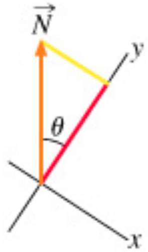


(4)

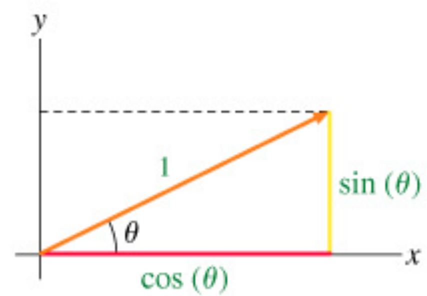
Indicate the number of the figure with the correct orientation.

Hint 1. Recommended procedure for resolving a vector into components

First figure out the sines and cosines of θ, then figure out the signs from the quadrant the vector is in and write in the signs.

Hint 2. Finding the trigonometric functions

Sine and cosine are defined according to the following convention, with the key lengths shown in green: The hypotenuse has unit length, the side adjacent to θ has length $\cos(\theta)$, and the side opposite has length $\sin(\theta)$. The colors are chosen to remind you that the vector sum of the two orthogonal sides is the vector whose magnitude is the hypotenuse; red + yellow = orange.



ANSWER:

- 1
- 2
- 3
- 4

Correct

Part C

Choose the correct procedure for determining the components of a vector in a given coordinate system from this list:

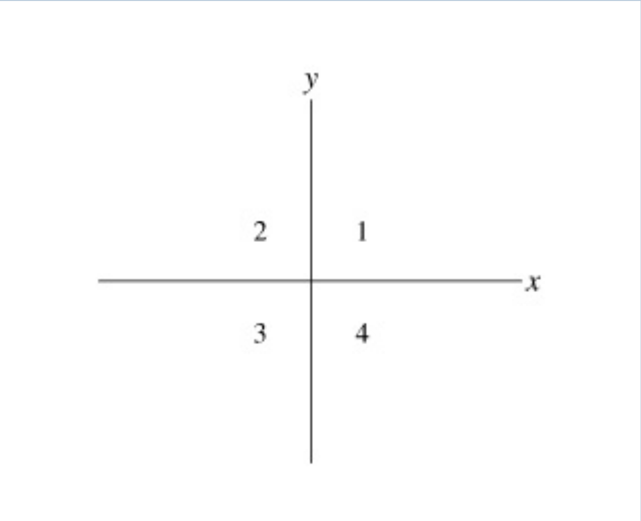
ANSWER:

- ○ Align the adjacent side of a right triangle with the vector and the hypotenuse along a coordinate direction with θ as the included angle.
- ◉ Align the hypotenuse of a right triangle with the vector and an adjacent side along a coordinate direction with θ as the included angle.
- ○ Align the opposite side of a right triangle with the vector and the hypotenuse along a coordinate direction with θ as the included angle.
- ○ Align the hypotenuse of a right triangle with the vector and the opposite side along a coordinate direction with θ as the included angle.

Correct

Part D

The space around a coordinate system is conventionally divided into four numbered *quadrants* depending on the signs of the *x* and *y* coordinates . Consider the following conditions:



1. $x > 0,\ y > 0$
2. $x > 0,\ y < 0$
3. $x < 0,\ y > 0$
4. $x < 0,\ y < 0$

Which of these conditions are true in which quadrants?

Write the answer in the following way: If A were true in the third quadrant, B in the second, C in the first, and D in the fourth, enter "`3, 2, 1, 4`" as your response.

ANSWER:

1,4,2,3

Correct

Part E

Now find the components N_x and N_y of $\vec{N}$ in the tilted coordinate system of **Part B**.

Express your answer in terms of the length of the vector N and the angle θ, with the components separated by a comma.

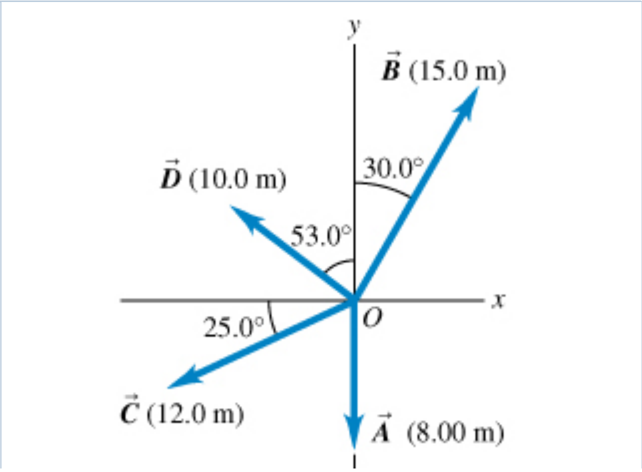
ANSWER:

N_x, N_y = $-N\sin\theta, N\cos\theta$

Correct

Exercise 1.31

Compute the *x*- and *y*-components of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ in the figure .



Part A

Enter your answers numerically separated by a comma.

ANSWER:

A_x, A_y = 0,-8.00 m

Correct

Part B

Enter your answers numerically separated by a comma.

ANSWER:

B_x, B_y = 7.50,13.0 m

Correct

Part C

Enter your answers numerically separated by a comma.

ANSWER:

C_x, C_y = -10.9,-5.07 m

Correct

Part D

Enter your answers numerically separated by a comma.

ANSWER:

D_x, D_y = -7.99,6.02 m

Correct

Score Summary:

Your score on this assignment is 97.5%.
You received 14.62 out of a possible total of 15 points.