$$T\left({}^{o}C\right)=\frac{5}{9}\left(T\left({}^{o}F\right)-32\right)$$
$$T\left(K\right)=T\left({}^{o}C\right)+273.15$$
$$\Delta L=\alpha L_0\Delta T$$
$$\Delta V=\beta V_0\Delta T$$
$$Q=mc\Delta T=nC\Delta T$$
$$Q_{F/V}=\pm mL_{F/V}$$
$$H=\frac{dQ}{dt}=k\frac{A}{L}\left(T_H-T_C\right)$$
$$pV=nRT$$
$$K_{tr}=\frac{3}{2}nRT$$
$C_V=\frac{3}{2}R$ ideal monatomic gas

$C_V=\frac{5}{2}R$ ideal diatomic gas w/o vibration

$$W=\int_{V_1}^{V_2}pdV$$
$$\Delta U=Q-W$$
$$e=\frac{W}{Q_H}=1-\left|\frac{Q_C}{Q_H}\right|$$
$$e_{Carnot}=1-\left|\frac{T_C}{T_H}\right|$$
$$\Delta S=\int_1^2\frac{dQ}{T}$$
$$S=k\ln w$$
$$R=8.314\,J/mol\cdot K$$
$$N_A=6.02\times10^{23}\,molecules/mole$$
1 atm = 101 325 N / (m^2)

$1/4\pi\varepsilon_0$ = 8.99 x 10^9 Nm^2/C^2

e = -1.602 x 10^{-19} C

$$\vec{F}_E=q\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r}$$
$$\Phi_E=\oint\vec{E}\cdot d\vec{A}=\frac{q_{enc}}{\varepsilon_0}$$
$$V_b-V_a=-\int_a^b\vec{E}\cdot d\vec{l}$$
$$\Delta U=q\Delta V$$
$$\vec{E}=-\left(\hat{i}\frac{\partial V}{\partial x}+\hat{j}\frac{\partial V}{\partial y}+\hat{k}\frac{\partial V}{\partial z}\right)$$
$$Q=CV$$
$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\dots$ series

$C_{eq}=C_1+C_2+C_3+\dots$ parallel

$$U=\frac{1}{2}CV^2$$
$$u_E=\frac{1}{2}\varepsilon_0E^2$$
$$E=\frac{E_0}{K}$$
$$I=\frac{dq}{dt}$$
$$\vec{J}=nq\vec{v}_d$$
$$\rho=\frac{E}{J}$$
$$V=IR$$
$$P=VI$$
$R_{eq}=R_1+R_2+R_3+\dots$ series

$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\dots$ parallel

$q=C\mathcal{E}\left(1-e^{-t/RC}\right)$ charging

$q=Q_0e^{-t/RC}$ discharging

$$\vec{F}=q\vec{v}\times\vec{B}$$
$$\Phi_B=\int\vec{B}\cdot d\vec{A}$$
$$d\vec{F}=Id\vec{l}\times\vec{B}$$
$$\vec{\tau}=\vec{\mu}\times\vec{B},\quad\vec{\mu}=NI\vec{A}$$
$$U=-\vec{\mu}\cdot\vec{B}$$
$$\vec{B}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$
$$d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$
$$\oint\vec{B}\cdot d\vec{l}=\mu_0I_{enc}$$

Calculus

Derivatives:

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \frac{d}{dx}\ln ax = \frac{1}{x} \qquad \frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}\sin ax = a\cos ax \qquad \frac{d}{dx}\cos ax = -a\sin ax$$

Integrals:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1) \qquad \int \frac{dx}{x} = \ln x \qquad \int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax \qquad \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{x^2 + a^2}\right) \qquad \int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} \qquad \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x\,dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Physics 161-001 Spring 2012 Exam 3

Name: ___________________________________ Box#_____________

Multiple Choice (5 points each):

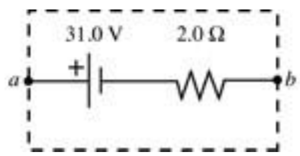
1) A tube of mercury with resistivity 7.84 × 10^{-6} Ωm has an electric field inside the column of mercury of magnitude 15 V/m that is directed along the length of the tube. How much current is flowing through this tube if its diameter is 2.0 mm?

A) 8 A
B) 12 A
C) 6 A
D) 18 A
E) 10 A

$$J = \frac{E}{\rho} = \frac{15V/m}{7.84\times10^{-6}\Omega m} = 1.9\times10^{6}\ A/m^{2}$$ and

$$I = J\cdot A = \left(1.9\times10^{6}\ A/m^{2}\right)\left(\pi\left(1\times10^{-3}m\right)^{2}\right) = 6A.$$

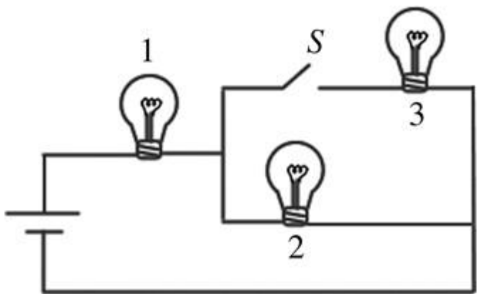
2) The emf and the internal resistance of a battery are as shown in the figure. When the terminal voltage Vab is equal to 24.8 V, what is the current through the battery?



$$V_{ab} = 24.8 = \mathcal{E} - Ir = 31V - I\left(2\Omega\right) \Rightarrow$$
$$I = 3.1A$$

A) 3.1 A
B) 15.5 A
C) 12.4 A
D) 27.9 A
E) 2.4 A

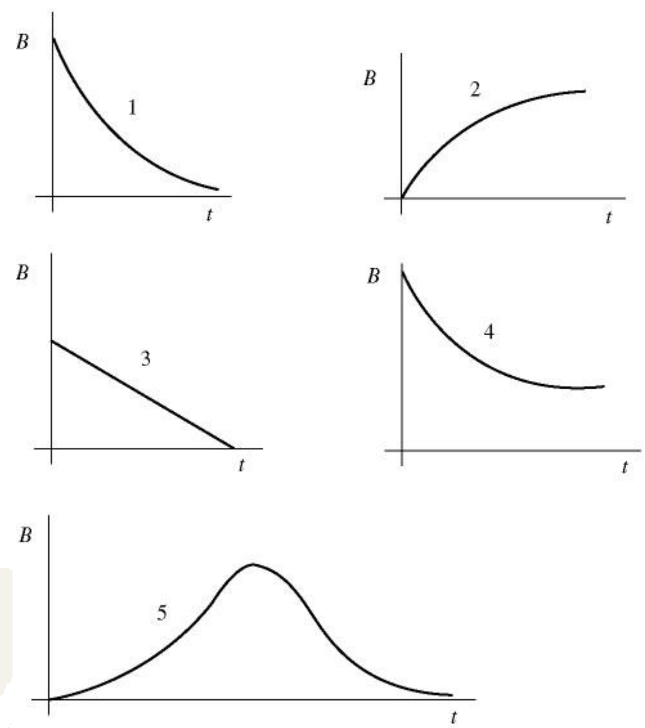
3) The figure shows three identical light bulbs connected to a battery having a constant voltage across its terminals. What happens to the brightness of light bulb 2 when the switch S is closed?

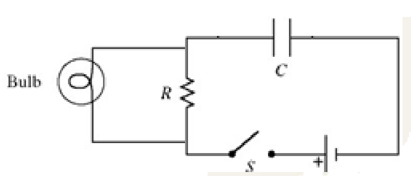


When the switch is open, the current through bulb 2 is the battery voltage divided by 2R. When the switch is closed, the current through the whole circuit is the battery voltage divided by 1.5R, and half of that goes through bulb 2. So, the final current through bulb 2 is the battery voltage divided by 3R - less than before the switch was closed.

A) The brightness remains the same as before the switch is closed.
B) The brightness increases permanently.
C) The brightness will decrease momentarily then return to its previous level.
D) The brightness will increase momentarily then return to its previous level.
E) The brightness decreases permanently.

4) A light bulb is connected in the circuit shown in the figure with the switch S open and the capacitor uncharged. The battery has no appreciable internal resistance. Which one of the following graphs best describes the *brightness* B of the bulb as a function of time t after closing the switch?





A) 1
B) 2
C) 3
D) 4
E) 5

When the switch is current will pass through the bulb and resistor, lessening exponentially with time until the capacitor is fully charged, at which point no more current will flow.

5) An electron moving in the negative z direction enters a magnetic field. If the electron experiences a magnetic deflection in the negative y direction, the direction of the magnetic field in this region points in the direction of the

A) -z axis.
B) +x axis.
C) +z axis.
D) -y axis.
E) -x axis.

$F_B = q\vec{v} \times \vec{B}$. Since the electron is negatively charged, the direction is in the opposite direction given by the right hand rule.

6) A proton is in a region where a uniform electric field of 5x10^4 V/m is perpendicular to a uniform magnetic field of 0.8 T. If its acceleration is zero, then what is its speed?

A) 0m/s
B) 6.3x10^{4}m/s
C) any value but zero
D) 4.0x10^{5}m/s
E) 1.6x10^{4}m/s

$F_E = qE$ and $F_B = q\vec{v} \times \vec{B}$. For the acceleration to be zero, the net force must be zero, so $F_{Net} = qE - qvB = 0$, so

$$v = \frac{E}{B} = \frac{5\times 10^4 \, V/m}{0.8T} = 6.3\times 10^4 \, m/s$$

7) A wire carries a 2.0-A current along the +x-axis through a magnetic field $\vec{B} = \left(4.0\hat{i} - 2.0\hat{j}\right)T$. If the wire experiences a force of 8.0 N in the positive z-direction as a result, how long is the wire?

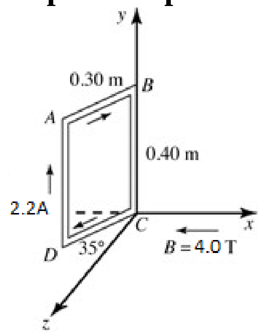
A) 1.1 m
B) 1.8 m
C) 1.0 m
D) 2.0 m
E) 2.5 m

$d\vec{F} = Id\vec{l} \times \vec{B}$. Only the y component of the B-field creates a force on the wire, and since the force is in the negative z direction for the whole wire, the total force is just:

$$\vec{F} = -ILB_y\hat{k} = -(2A)L(2.0T)\hat{k} = -8.0N\hat{k} \Rightarrow$$

$$L = 2.0m$$

8) A rigid rectangular loop, which measures 0.30 m by 0.40 m, carries a current of 2.2 A, as shown in the figure. A uniform external magnetic field of magnitude 4.0 T in the negative x-direction is present. Segment *CD* is in the *xz*-plane and forms a 35° angle with the z-axis, as shown. Find the magnitude of the external torque needed to keep the loop in static equilibrium.



$|\vec{\mu}| = IA = (2.2A)(0.30m \cdot 0.40m) = 0.26Am^2$.

$\vec{\tau} = \vec{\mu} \times \vec{B} = (0.26Am^2)(4.0T)\sin(35°)\,\hat{j} = (0.6N \cdot m)\,\hat{j}$

A) 1.1 N · m
B) 1.4 N · m
C) 0.6 N · m
D) 0.3 N · m
E) 0.9 N · m

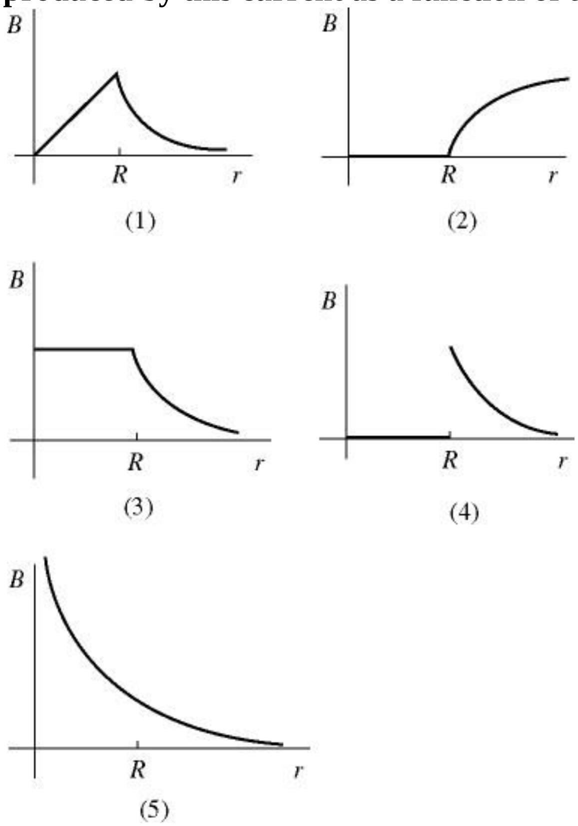
9) Two long straight wires enter a room through a window. One carries a current of 3.0 A into the room while the other carries a current of 5.0 A out. A third wire carries 4A into the room, then loops into a circle 2m from the window and then leaves the room again through the same window. The magnitude (in T·m) of the path integral $\oint \vec{B} \cdot d\vec{l}$ around the window frame is:

A) $3.8\text{x}10^{-6}$ Wb/m
B) $2.5\text{x}10^{-6}$ Wb/m
C) $6.3\text{x}10^{-6}$ Wb/m
D) $1.0\text{x}10^{-5}$ Wb/m
E) none of these

The net current through the window is 3A in – 5A out +4A in – 4A out = 2A out.
So, from Ampere's law,

$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} = \left(4\pi \times 10^{-7}\, Wb/_{A \cdot m}\right) \times (2A) = 2.5 \times 10^{-6}\, \frac{Wb}{m}$

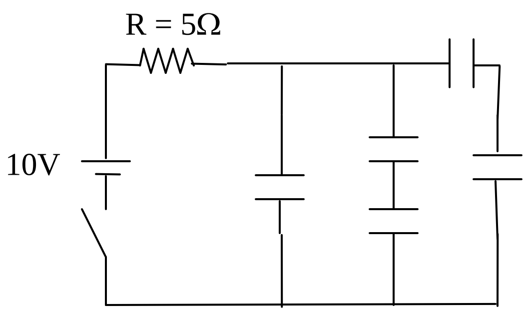
10) A very long, solid, conducting cylinder of radius R carries a current along its length uniformly distributed throughout the cylinder. Which one of the graphs shown in the figure most accurately describes the magnitude B of the magnetic field produced by this current as a function of the distance r from the central axis?



A) 1
B) 2
C) 3
D) 4
E) 5

Written Problems (25 points each) SHOW ALL WORK! No credit for answers without work!

1) In the circuit below, how much power ($P=i^2R$) is being dissipated in the resistor 10 μs after the switch is closed? All capacitors have capacitance of 2μF.



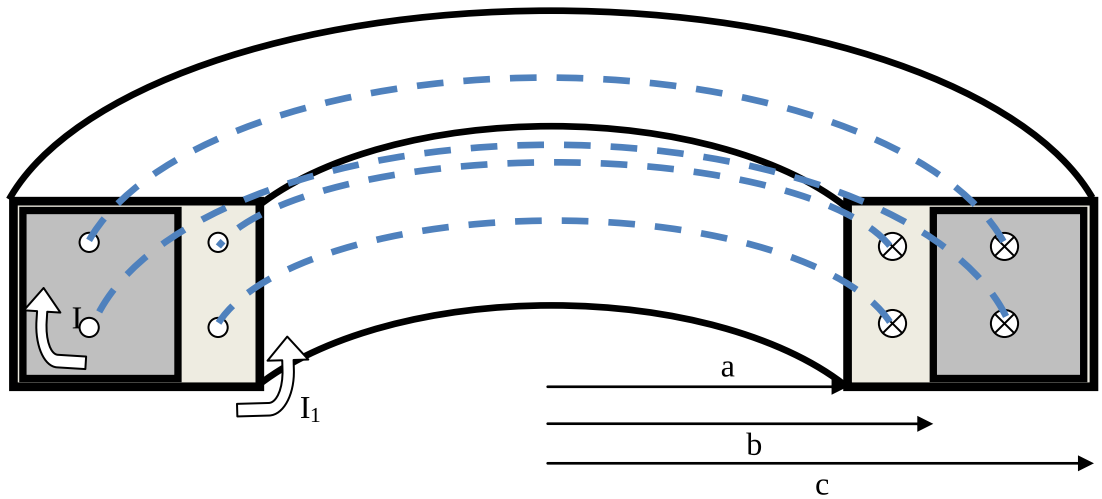
We first have to find the equivalent capacitance. Each series set has an equivalent capacitance of 1μF, and then we have in parallel a 2μF, 1μF and 1μF giving a total capacitance of 4μF. The time constant is then $\tau = RC = (5\Omega)(4\mu F) = 20\mu s$. Now, the current through the resistor as a function of time is given by:

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}\left(e^{-t/RC}\right)$$

$$= \frac{10V}{5\Omega}\left(e^{-10\mu s/20\mu s}\right) = 2A\left(e^{-1/2}\right) = 1.2A$$

So, the power is just: $P = i^2R = (1.2A)^2(5\Omega) = 7.4W$.

2) Two concentric toroids each with 1000 turns carry currents i_1=6A and i_2=3A in opposite directions as indicated on the figure. Using Ampere's law, calculate the magnetic field in the regions a) from r=a to r=b, and b) from r=b to r=c. Indicate the directions of the fields on the drawing.



The B-field inside a torroid can be found using Ampere's law with a circular path of radius r around the torroid centered on the center of the torroid. Inside torroid 1, but before we get to torroid 2, the B-field is $\oint \vec{B}\cdot d\vec{s} = B2\pi r = \mu_0 i_{enc} = \mu_0 i(1000) \Rightarrow B = \dfrac{\mu_0(6A)(1000)}{2\pi r}$.

Once inside torroid 2, the path encloses its current also and the B-field becomes:

$$B = \frac{\mu_0(6A-3A)(1000)}{2\pi r} = \frac{\mu_0(3A)(1000)}{2\pi r}$$