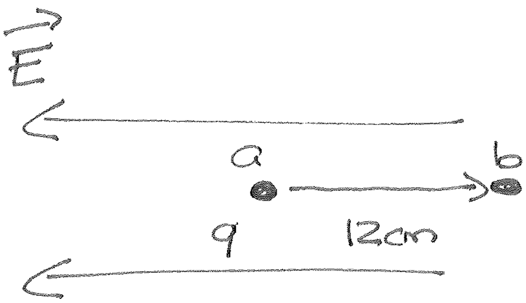
Physics 161, HW #3

#1



$q = 5.5\,nC = 5.5 \times 10^{-9}\,C$

q started from rest

Other force $W_{OTHER} = 6.5 \times 10^{-5}\,J$

At b, $K_b = 2.75 \times 10^{-5}\,J$

a) WHAT IS WORK DONE BY ELECTRIC FIELD?

Electric field and other both do work

$\Rightarrow W_{TOTAL} = \underbrace{W_{a \to b}}_{\text{Electric Field's work.}} + W_{OTHER}$

Work-Energy Thm.: $W_{TOTAL} = \Delta K = K_b - K_a$.

q starts from rest $\Rightarrow K_a = 0 \Rightarrow \Delta K = K_b$

$\therefore W_{a \to b} + W_{OTHER} = K_b$

$\Rightarrow W_{a \to b} = K_b - W_{OTHER} = 2.75 \times 10^{-5}\,J - 6.5 \times 10^{-5}\,J$

$\Rightarrow \boxed{W_{a \to b} = -3.75 \times 10^{-5}\,J}$

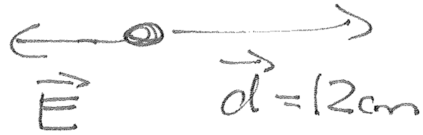
b) WHAT IS POTENTIAL OF STARTING POINT WITH RESPECT TO ENDING POINT?

$\Rightarrow V_{ab} = ?$ $\qquad V_{ab} = \dfrac{W_{a\to b}}{q}$

$$\Rightarrow \boxed{V_{ab} = \frac{-3.75\times10^{-5}\,J}{5.5\times10^{-9}\,C} = -6818.1818\ldots J = -6820\,J}$$

c). WHAT IS MAGNITUDE OF ELECTRIC FIELD?

UNIFORM FIELD, STRAIGHT-LINE MOTION $\Rightarrow W_{a\to b} = qEd\cos\phi$

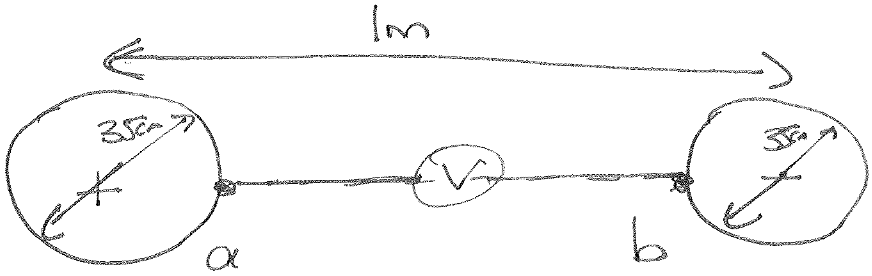


$$W_{a\to b} = qEd\cos 180° = -qEd$$

$$\therefore E = \frac{-W_{a\to b}}{qd} = \frac{-(-3.75\times10^{-5}\,J)}{(5.5\times10^{-9}\,C)(.12\,m)}$$

$$\Rightarrow \boxed{\begin{aligned} E &= 56818.1818\ldots\ V/m \\ &= 5.68\times10^{4}\ V/m \end{aligned}}$$

#2

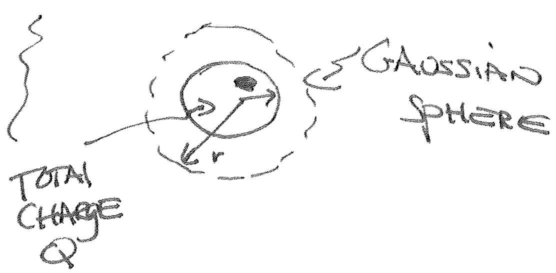


EACH SPHERE HAS 150 μC TOTAL.

a) WHAT IS VOLTMETER'S READING?

VOLTMETERS READ VOLTAGE, i.e. V_{ab} $\Rightarrow$ $V_{ab} = V_a - V_b = ?$

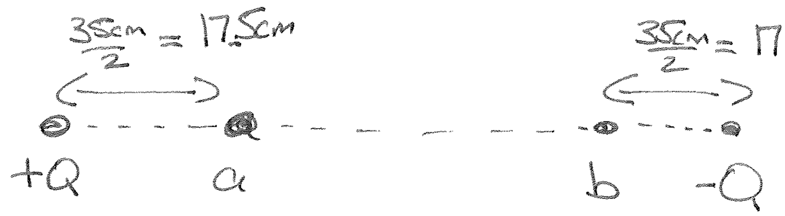
OUTSIDE (OR JUST ON THE SURFACE) OF AN INSULATING SPHERE, GAUSS'S LAW tells US THAT THE ElectRIC FIELD IS EQUIVALENT TO THAT OF A POINT CHARGE LOCATED AT CENTER.



$$EA = \frac{Q_{encl}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{kQ}{r^2}$$ $\leftarrow$ POINT CHARGE AT CENTER

SINCE VOLTMETER READING TWO POINTS ON SURFACE, THIS PROBLEM IS EQUIVALENT TO THE DRAWING ON THE NEXT PAGE.



Diameter of 35cm

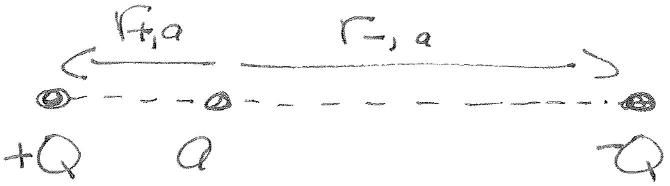
$\Rightarrow$ radius is $\frac{35cm}{2}$

$Q = 150\mu C = 150 \times 10^{-6} C$

Point charges $\Rightarrow$ $V = \frac{Kq}{r}$

For two point charges, $V = \frac{Kq_1}{r_1} + \frac{Kq_2}{r_2}$

$$V_a = \frac{KQ}{r_{+,a}} + \frac{K(-Q)}{r_{-,a}} = KQ\left(\frac{1}{r_{+,a}} - \frac{1}{r_{-,a}}\right)$$



$r_{+,a} = 17.5cm = .175m$

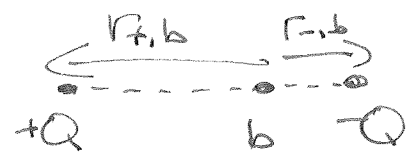
$r_{-,a} = 1m - 17.5cm = 1m - .175m = .825m$

$$\Rightarrow V_a = (9 \times 10^9 N \cdot m^2/C^2)(150 \times 10^{-6} C)\left(\frac{1}{.175m} - \frac{1}{.825m}\right) = 6077922 V$$

$= 6.08 MV$

↳ Mega

$$V_b = \frac{KQ}{r_{+,b}} + \frac{K(-Q)}{r_{-,b}} = KQ\left(\frac{1}{r_{+,b}} - \frac{1}{r_{-,b}}\right)$$



$r_{+,b} = 1m - 17.5cm = .825m$, $r_{-,b} = 17.5cm = .175m$

$\therefore \; V_b = -V_a = -6077922V = -6.08MV$

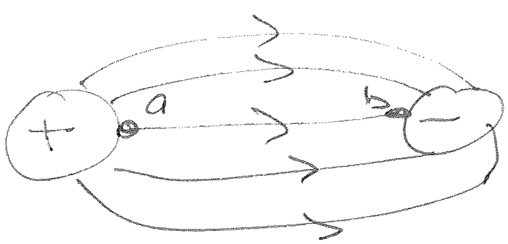
$$\boxed{V_{ab} = V_a - V_b = 6.08MV - (-6.08MV) \cong 12.16MV}$$

b) WHAT POINT IS AT HIGHER POTENTIAL?

(NO calculation WAY SINCE obviously $V_{ab} = +12.16MV$

$\Rightarrow$ a HIGHER THAN b)

THE ELECTRIC FIELD POINTS FROM POSITIVE to NEGATIVE

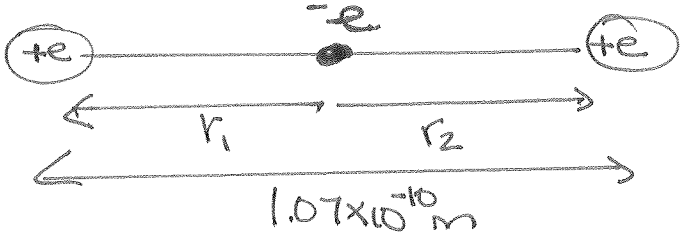


Dipole-like FIELD. PROBABLY A BIT MORE COMPLICATED due to CHARGE DISTRIBUTIONS, but SAME BASIC SHAPE.

$\therefore \; \vec{E}$ from a to b. $\vec{E}$'s IN DIRECTION OF DECREASING Potential $\Rightarrow$ b at lower POTENTIAL THAN a.

#3

[redacted]



electron HALFWAY Between

a) Find electron's potential Energy due to protons

Potential Energy ADDS $\Rightarrow U = U_1 + U_2$

Pot. Energy due to proton on left

proton on right

Point charges $\Rightarrow U_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_e}{r_1} = \frac{1}{4\pi\epsilon_0}\frac{(+e)(-e)}{r_1} = \frac{-1}{4\pi\epsilon_0}\frac{e^2}{r_1}$

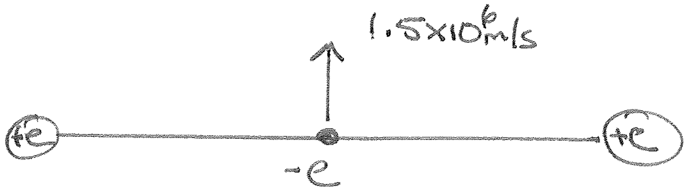
$r_1 = \frac{1}{2}(1.07\times10^{-10}\,m) = .535\times10^{-10}\,m$

$U_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2 q}{r_2} = \frac{-1}{4\pi\epsilon_0}\frac{e^2}{r_2}$. $r_2 = r_1 = .535\times10^{-10}\,m$

$\Rightarrow U = 2\left(\frac{-1}{4\pi\epsilon_0}\right)\frac{e^2}{r_1} = -2(8.99\times10^{9}\,N\cdot m^2/C^2)\frac{(1.6\times10^{-19}\,C)^2}{.535\times10^{-10}\,m}$

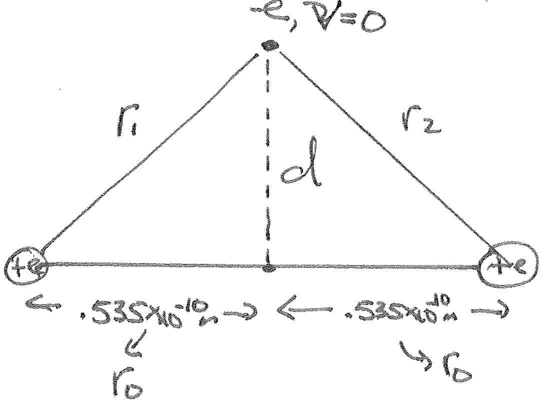
$\Rightarrow \boxed{U = -8.6\times10^{-18}\,J}$

b)



How FAR CAN Electron go?

electron moves until speed is ZERO



AT STOPPING point

$$U = \frac{1}{4\pi\epsilon_0}\frac{-e^2}{\sqrt{r_0^2+d^2}} + \frac{1}{4\pi\epsilon_0}\frac{-e^2}{\sqrt{r_0^2+d^2}}$$

$$= -\frac{2}{4\pi\epsilon_0}\frac{e^2}{\sqrt{r_0^2+d^2}}$$

CONSERVATION OF ENERGY: $K_1 + U_1 = K_2 + U_2$, $K = \frac{1}{2}mv^2$

$$\Rightarrow \frac{1}{2}(9.11\times10^{-31}\,kg)(1.5\times10^{6}\,m/s)^2 + -8.6\times10^{-18}\,J = 0 - \frac{2}{4\pi\epsilon_0}\frac{e^2}{\sqrt{r_0^2+d^2}}$$

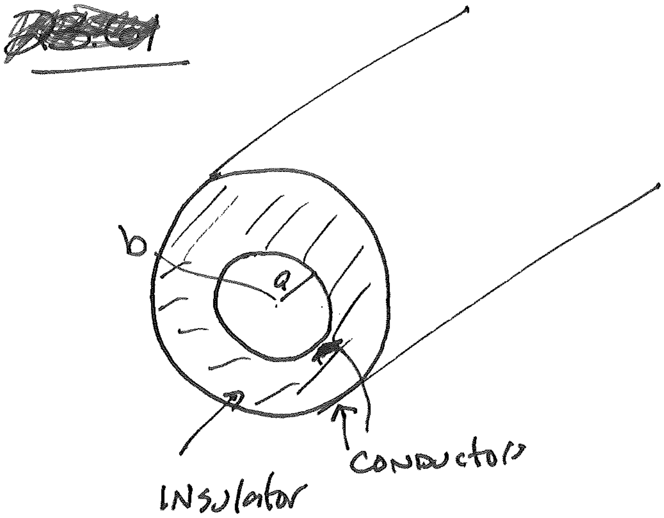
$$\Rightarrow -7.575\times10^{-18}\,J = \frac{-2(8.99\times10^{9}\,Nm^2/C^2)(1.6\times10^{-19}\,C)^2}{\sqrt{r_0^2+d^2}}$$

$$\Rightarrow \sqrt{r_0^2+d^2} = 6.0768\times10^{-11}\,m$$

$$\Rightarrow d^2 = (6.0764\times10^{-11}\,m)^2 - r_0^2 = (6.0764\times10^{-11}\,m)^2 - (.535\times10^{-10}\,m)^2$$

$$\Rightarrow \boxed{d = \sqrt{8.3\times10^{-22}\,m^2} = 2.881\times10^{-11}\,m}$$

#34



INNER CONDUCTOR WITH $+\lambda$
OUTER CONDUCTOR WITH $-\lambda$

a) Find $V(r)$. TAKE $V=0$ at $r=b$. (NOT WHAT I would do! But OK)

FIRST Find the electric field. CYLINDER $\Rightarrow$ RADIAL SYMMETRY AND GAUSSIAN CYLINDER.

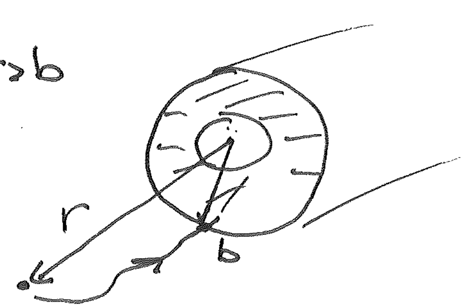
$0<r<a$, INSIDE CONDUCTOR $\Rightarrow E=0$

$a<r<b$, $\Phi_E = Q_{encl}/\epsilon_0 \Rightarrow E(2\pi r l) = \dfrac{\lambda l}{\epsilon_0} \Rightarrow E = \dfrac{\lambda}{2\pi\epsilon_0 r}$

$r>b$, $Q_{encl} = \lambda - \lambda = 0 \Rightarrow E=0$

$W_{a\to b} = q_0 \int_a^b \vec{E}\cdot d\vec{l} = (V_a - V_b)q_0 \Rightarrow V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$

for $r>b$



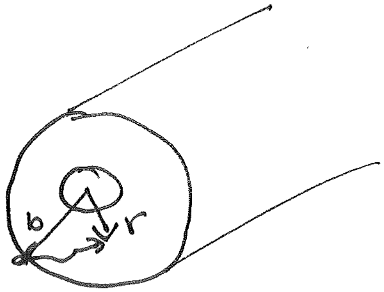
$V_r - V_b = \int_r^b \vec{E}\cdot d\vec{l}$ but $E=0$ for $r>b$

Vat r

$\Rightarrow V_r - V_b = 0 \Rightarrow V_r = V_b$

$V_b = 0 \Rightarrow V_r = 0$

for $a<r<b$



$$V_r - V_b = \int_r^b \vec{E}\cdot d\vec{\ell}$$

$\vec{E}$ is radial $\Rightarrow$ $\vec{E}\cdot d\vec{\ell} = E\,dr$

$$\Rightarrow V_r - V_b = \int_r^b \frac{\lambda}{2\pi\epsilon_0 r}\,dr = \frac{\lambda}{2\pi\epsilon_0}\int_r^b \frac{dr}{r}$$

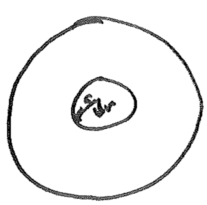
$$= \frac{\lambda}{2\pi\epsilon_0}\ln r\Big|_r^b = \frac{\lambda}{2\pi\epsilon_0}(\ln b - \ln r) = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{r}\right)$$

$V_r - V_b = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{r}\right)$. $V_b = 0 \Rightarrow V_r = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{r}\right)$

WHILE I'M at it HERE, $V_a - V_b = V_{ab} = \int_a^b E\,dr = \frac{\lambda}{2\pi\epsilon_0}\int_a^b \frac{dr}{r}$

$\Rightarrow V_{ab} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$ $\leftarrow$ part b.

for $r<a$



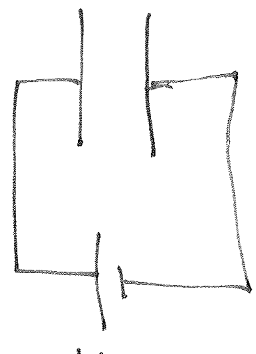
$V_r - V_a = 0$ SINCE $E = 0$ AGAIN

$\Rightarrow V_r = V_a$. SINCE $V_b = 0$ $V_a = V_{ab}$

$$\Rightarrow V_r = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$$

c) $E = \frac{\lambda}{2\pi\epsilon_0 r}$ $V_{ab} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right) \Rightarrow \frac{\lambda}{2\pi\epsilon_0} = \frac{V_{ab}}{\ln(b/a)}$ $\Rightarrow E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}$.

#5



$C = 22.5\mu F = 22.5\times10^{-6} F$

Dielectric MATERIAL WITH $K = 5.2$ PUT BETWEEN PLATES

a) HOW MUCH STORED ENERGY BEFORE AND AFTER?

POWER SUPPLY ENSURES THAT $V_{ab} = 16V$ BOTH BEFORE AND AFTER.

$\Rightarrow$ USE $U = \frac{1}{2} C V_{ab}^2$

BEFORE: $U_1 = \frac{1}{2} C_1 V_{ab}^2 = \frac{1}{2}(22.5\times10^{-6}F)(16V)^2 \Rightarrow \boxed{U_1 = .00288J = 2.88mJ}$

mili

AFTER: INSERTING DIELECTRIC INCREASES CAPACITANCE

to $C_2 = K C_1 = 5.2(22.5\times10^{-6}F) = 117\times10^{-6}F$

$\therefore U_2 = \frac{1}{2} C_2 V_{ab}^2 = \frac{1}{2}(117\times10^{-6}F)(16V)^2 \Rightarrow \boxed{U_2 = .014976J = 14.976mJ}$

b) $\Delta U = ?$ $\boxed{\Delta U = 14.976mJ - 2.88mJ = 12.096mJ}$ $\leftarrow$ INCREASED

NOTE: ENERGY INCREASES BECAUSE IT WOULD REQUIRE WORK to be done to ~~Capacitor~~ INSERT DIELECTRIC.

#6 FLASH LASTS FOR $t = \frac{1}{675}$ s

WITH Power $= 3.1\times10^5$ Watt, AND 89% EFFICIENCY.

a) HOW MUCH ENERGY STORED FOR ONE FLASH?

- SINCE ONLY 89% EFFICIENT, CAPACITOR Needs to deliver a Power

$$\cancel{P} \; \frac{3.1\times10^5\,\text{Watt}}{.89} = 3.483\times10^5\,\text{Watt}$$

Capacitor Loses Energy $\Rightarrow$ ~~P = 3.483×10^5 Watt~~, $P = \frac{W}{\Delta t} = \frac{-\Delta U}{\Delta t}$

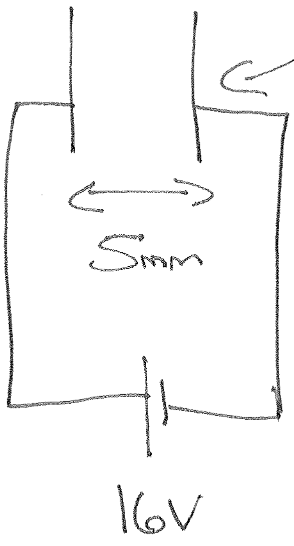
$\Delta U = U_2 - U_1$. $\quad U_2 = 0,\ U_1 = U = ? \Rightarrow P = \frac{-(0-U)}{\Delta t} = \frac{U}{\Delta t}$

$$\Rightarrow U = P\Delta t = (3.483\times10^5\,\text{Watt})\left(\frac{1}{675}\,\text{s}\right) = 516\,\text{J}$$

b) $V_{ab} = 125\,\text{V},\ C = ?$ $\quad U = \frac{1}{2}CV_{ab}^2 \Rightarrow C = \frac{2U}{V_{ab}^2}$

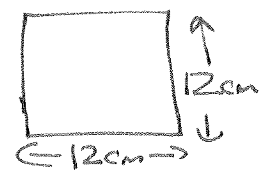
$$\Rightarrow \boxed{C = \frac{2(516\,\text{J})}{(125\,\text{V})^2} = .066\,\text{F}}$$

#7



Plates 12cm SQUARE

$\Rightarrow$



$A = (.12m)(.12m)$
$= .0144m^2$

a) WHAT IS CAPACITANCE?

$$C = \epsilon_0 A/d = \frac{(8.85\times10^{-12} C^2/N\cdot m^2)(.0144m^2)}{(5\times10^{-3}m)} = 2.5488\times10^{-11}F$$

Air-Filled

$$\Rightarrow \boxed{C = 2.55\times10^{-11}F}$$

Unit: $\frac{C^2\cdot m^2}{N\cdot m^2\cdot m} = \frac{C^2}{N\cdot m} = \frac{C^2}{J} = \frac{C}{J/C} = \frac{C}{V} = F$

b) WHAT IS CHARGE? $C = \frac{Q}{V_{ab}}$ battery ensures $V_{ab} = 16V$

$$\therefore Q = CV_{ab} = (2.5488\times10^{-11}F)(16V) \Rightarrow \boxed{Q = 4.078\times10^{-10}C = 4.08\times10^{-10}C}$$

c) WHAT IS E between plates?

ASSUME UNIFORM FIELD $\Rightarrow V = Ed \Rightarrow E = V/d$

$$\boxed{E = \frac{16V}{5\times10^{-3}m} = 3200 V/m}$$

d) WHAT ENERGY IS STORED? $U = \frac{1}{2}CV^2 = \frac{1}{2}(2.5488\times10^{-11}F)(16V)^2$

$$\Rightarrow \boxed{U = 3.26\times10^{-9}J}$$

e) THE Battery IS DISCONNECTED AND d is INCREASED to 7.4mm. Repeat parts (a)-(d).

MOST IMPORTANTLY, DISCONNECTING battery MEANS CHARGE CAN'T LEAVE THE PLATES $\Rightarrow \boxed{Q = 4.08\times10^{-10}C \text{ still}}$ THE POTENTIAL BETWEEN THE PLATES will CHANGE!

$$C = \epsilon_0\frac{A}{d} = (8.85\times10^{-12}C^2/Nm^2)\left(\frac{.0144m^2}{7.5\times10^{-3}m}\right) \Rightarrow \boxed{C = 1.699\times10^{-11}F = 1.7\times10^{-11}F}$$

$$C = \frac{Q}{V_{ab}} \Rightarrow V_{ab} = \frac{Q}{C} = \frac{4.08\times10^{-10}C}{1.699\times10^{-11}F} \Rightarrow \boxed{V_{ab} = 24V}$$

$$E = \frac{V}{d} = \frac{24V}{7.5\times10^{-3}m} \Rightarrow \boxed{E = 3200 V/m} \leftarrow \text{UNCHANGED!}$$

ANOTHER WAY to SEE THIS IS to Remember $E = \sigma/\epsilon_0$. No change in CHARGE OR AREA OF plates $\Rightarrow$ SAME $\sigma \Rightarrow$ UNCHANGED E.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(1.699\times10^{-11}F)(24V)^2 \Rightarrow \boxed{U = 4.89\times10^{-9}J}$$

$\leftarrow$ REQUIRES WORK to pull plates APART