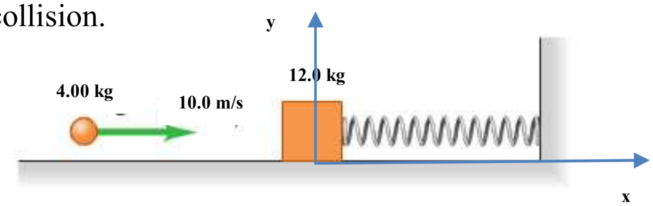
Physics 160 Exam #3 Name:_________________Box#____

Note: All answers must include needed diagrams, coordinate sytems, etc.

1) (20pts) A 12.0 kg block is attached to a very light horizontal spring of force constant 600.0 N/m and is resting on a frictionless horizontal table. Suddenly it is struck by a 4.00 kg stone travelling horizontally at 10.0 m/s to the right. Immediately after the collision, the stone is found to be travelling to the left at 3.00 m/s. Find the maximum distance that the block will compress the spring after the collision.



We first have to apply momentum conservation to the collision process:

$$\vec{p}_i = m\vec{v}_i^{\,stone} = (4.00kg)(10.0\,m/s)\hat{i}$$

$$\vec{p}_f = m\vec{v}_f^{\,stone} + m\vec{v}_f^{\,block} = -(4.00kg)(3.00\,m/s)\hat{i} + (12.0kg)\left|\vec{v}_f^{\,block}\right|\hat{i}$$

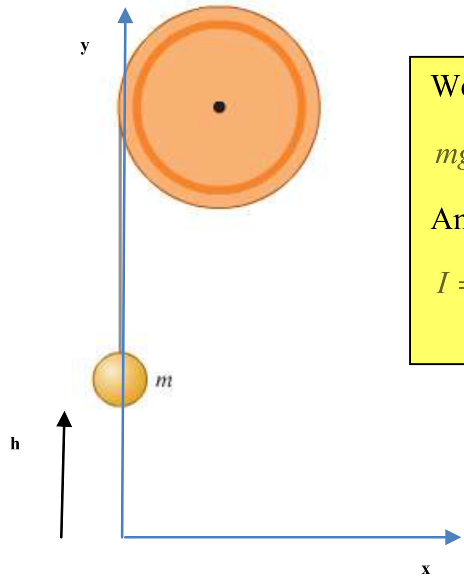
And, solving for the final velocity of the block we get:

$$\vec{v}_f^{\,block} = (4.33\,m/s)\hat{i}$$

Now, the kinetic energy of the block will all be converted to spring potential energy:

$$\frac{1}{2}m\left(\vec{v}_f^{\,block}\right)^2 = \frac{1}{2}k(x)^2 \Rightarrow x = 0.613m$$

2) (20 pts) A mass m is attached to a string which is wrapped around a ***non-uniform*** cylindrical object of radius R, which rotates on a frictionless axle. When the mass is released from rest and falls a distance h, it is seen to have a velocity v. What is the moment of inertia of the rotating object in terms of m, R, h, and v?



We have to apply energy conservation in this process:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$

And, solving for the moment of inertia:

$$I = \frac{2R^2}{v^2}\left(mgh - \frac{1}{2}mv^2\right) = \frac{2R^2 mgh}{v^2} - mR^2$$

3) (20pts) A lawn roller in the form of a thin-walled, hollow cylinder of mass M is pulled horizontally with a constant horizontal force F applied by a handle attached to the axle. If it rolls without slipping, find the acceleration and the friction force.

This is a Newton's Second Law problem:

$$\sum F_x = F_P - F_{fs} = Ma_x$$

and

$$\sum \tau_z = F_{fs}R = I\alpha_z = MR^2\left(\frac{a_x}{R}\right) = MRa_x$$

since it rolls without slipping, and since all the mass is at radius R.

Solving for the friction force gives:

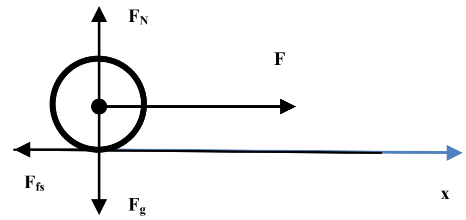
$$F_{fs} = Ma_x$$

and substituting into the first equation gives:

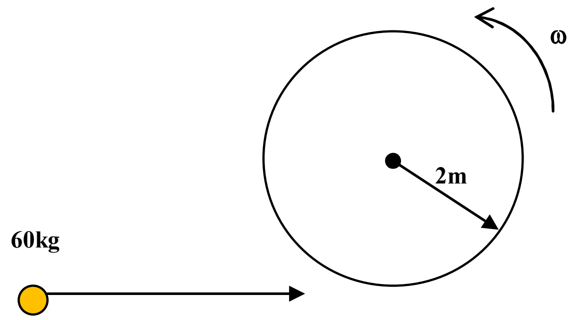
$$\sum F_x = F_P - F_{fs} = F_P - Ma_x = Ma_x \Rightarrow$$

$$F_P = 2Ma_x \Rightarrow$$

$$a_x = \frac{F_P}{2M}$$



4) (20pts) A merry-go-round with a radius of 2m and a moment of inertia 200kgm^2 is rotating at 0.2 revolutions per second. A child of mass 50kg runs at the merry-go-round in a line tangent to its edge, and grabs onto it. After the child is on board, it rotates at 0.4 revolutions per second. How fast was the child running?



This involves conservation of angular momentum, $L_i = L_f$.

$$L_i = (I\omega_i)_{merry-go-round} + (\vec{r} \times \vec{p})_{child}$$

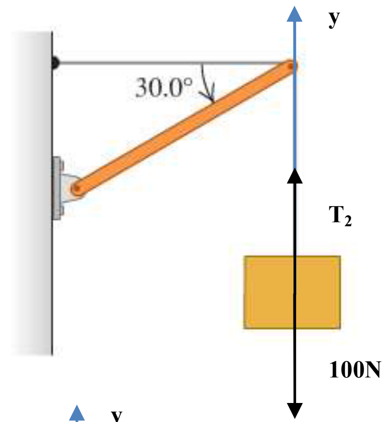
$$= (200kgm^2)\left(0.2\frac{rev}{s}\right)\left(\frac{2\pi rad}{rev}\right) + (2m)(50kg)v$$

$$L_f = \left[I_{merry-go-round} + I_{child}\right]\omega_f$$

$$= \left[(200kgm^2) + (50kg)(2m)^2\right]\left(0.4\frac{rev}{s}\right)\left(\frac{2\pi rad}{rev}\right)$$

Solving for v, we get v = 7.5 m/s

5) (20pts) Find the tension in each cable and the horizontal and vertical components of the force exerted on the strut by the hinge in the figure below. Let the weight of the suspended crate of priceless art objects be 100N. The strut is uniform, 1m long, and also has a weight 100N.



This is an equilibrium problem. First, examine the block:

$$\sum F_y = T_2 - 100N = 0 \Rightarrow$$

$$T_2 = 100N$$

Then examine the beam:

$$\sum F_y = H_y - T_2 - 100N = 0 \Rightarrow$$

$$H_y = 100N + 100N = 200N$$

and

$$\sum F_x = H_x - T_1 = 0$$

and the torques around the hinge are

$$\sum \tau_z = T_2(1m)\cos 30 + F_g(0.5m)\cos 30 - T_1(1m)\sin 30 = 0$$

$$100N(1m)\cos 30 + 100N(0.5m)\cos 30 = T_1(1m)\sin 30 \Rightarrow$$

$$T_1 = 260N$$

so,

$$H_x = 260N$$