Physics 160 Solutions - Chapter 1.

1 Vector Components

Given the vector $\vec{A}$ at an angle $\pi/6$ radians and $\vec{B}$ at an angle $-\pi/6$ radians (measured counter-clockwise from the positive x axis), each with the same unknown magnitude:

a. Express the direction of $\vec{A}-\vec{B}$ as an angle in radians measured counter-clockwise from the positive x axis. *Since the vectors $\vec{A}$ and $\vec{B}$ are the same magnitude, they must have the same x component* $(\cos(\pi/6) = \cos(-\pi/6))$, *and opposite signed y components* $(\sin(\pi/6) = -\sin(-\pi/6))$. *Therefore, the y component of $\vec{A}-\vec{B}$ must be positive, and there will be no x component, thus the angle must be $\pi/2$.*

b. Express the direction of $\vec{A}+\vec{B}$ in the same manner. *Similarly to part a, the vector $\vec{A}+\vec{B}$ will have no y component, and a positive x component, thus the angle must be 0.*

c. If $|\vec{A}-\vec{B}| = 10m$, what is $|\vec{A}|$ and $|\vec{B}|$? *We saw in part a that this vector only has a y component, which tells us the y component must be 10 m. Since $\vec{A}$ and $\vec{B}$ have the same magnitude and the same (but reflected) angle, they must have each contributed the same amount to the y component of $\vec{A}-\vec{B}$. Therefore the y component of $\vec{A}$ is 5 m, and the y component of $\vec{B}$ is -5 m. From this we can easily solve $A_y = |\vec{A}|\sin(\pi/6)$ and $B_y = |\vec{B}|\sin(-\pi/6) \to |\vec{A}| = 10m$ and $|\vec{B}| = 10m$.*

d. If $|\vec{A}+\vec{B}| = 10m$, what is $|\vec{A}|$ and $|\vec{B}|$? *Following the same reasoning as part c, $\vec{A}$ and $\vec{B}$ each contributed equally to the x component of $\vec{A}+\vec{B}$. Therefore the x component of both $\vec{A}$ and $\vec{B}$ is 5 m. From this we solve $A_x = |\vec{A}|\cos(\pi/6)$ and $B_x = |\vec{B}|\cos(-\pi/6) \to |\vec{A}| = \frac{20}{\sqrt{3}}m$ and $|\vec{B}| = \frac{20}{\sqrt{3}}m$.*

2 Scalar and Vector Products

Given the vectors $\vec{A} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{B} = -2\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$

a. Compute the scalar product $\vec{A} \cdot \vec{B}$. *From the formula* $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ *we calculate* $\vec{A} \cdot \vec{B} = 4 * (-2) + 3 * 5 + (-1) * 7 = 0$.

b. From your answer to part a, what the angle θ between $\vec{A}$ and $\vec{B}$? *From* $\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \cos\theta$, *the angle* θ *between* $\vec{A}$ *and* $\vec{B}$ *must be* $\pi/2$ *if the dot product is 0.*

c. Compute the vector product $\vec{A} \times \vec{B}$. *Using the formula* $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$ *we can calculate the vector product to be* $26\hat{\imath} - 26\hat{\jmath} + 26\hat{k}$. *Other methods more convenient than the formula, such as determinants, are acceptable and recommended.*

d. Compute the vector product $\vec{B} \times \vec{A}$. *Using the previous formula,* $\vec{B} \times \vec{A} = -26\hat{\imath} + 26\hat{\jmath} - 26\hat{k}$.

e. From the right hand rule, explain the relationship between $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$. *Mathematically,* $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$. *Conceptually, the right hand rule tells us that the resultant vector from the cross product of two vectors in a plane will be normal (orthogonal) to that plane, and the order in which we consider the vectors determines to which side of the plane the vector will point. Since we calculated both cross products, the two vectors will be the same magnitude, but point in opposite (or antiparallel) directions.*