

41.58. IDENTIFY and SET UP: $L_x^2 + L_y^2 + L_z^2 = L^2$. $L^2 = l(l+1)\hbar^2$. $L_z = m_l\hbar$.

EXECUTE: (a) $L_x^2 + L_y^2 = L^2 - L_z^2 = l(l+1)\hbar^2 - m_l^2\hbar^2$ so $\sqrt{L_x^2 + L_y^2} = \sqrt{l(l+1) - m_l^2}\hbar$.

(b) This is the magnitude of the component of angular momentum perpendicular to the z-axis.

(c) The maximum value is $\sqrt{l(l+1)}\hbar = L$, when $m_l = 0$. That is, if the electron is known to have no z-component of angular momentum, the angular momentum must be perpendicular to the z-axis. The minimum is $\sqrt{l}\hbar$ when $m_l = \pm l$.

EVALUATE: For $l \neq 0$ the minimum value of $L_x^2 + L_y^2$ is not zero. The angular momentum vector cannot be totally aligned along the z-axis. For $l \neq 0$, \vec{L} must always have a component perpendicular to the z-axis.