24.60.

With the switch open, each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed, each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in parallel with each other and the two pairs are in series.

(a) With the switch open $C_{\text{eq}} = \left(\left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1} + \left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1}\right) = 4.00\ \mu\text{F}.$

$Q_{\text{total}} = C_{\text{eq}}V = (4.00\ \mu\text{F})(210\ \text{V}) = 8.40\times10^{-4}\ \text{C}.$ By symmetry, each capacitor carries $4.20\times10^{-4}\ \text{C}.$ The voltages are then calculated via $V = Q/C.$ This gives $V_{ad} = Q/C_3 = 140\ \text{V}$ and $V_{ac} = Q/C_6 = 70\ \text{V}.$ $V_{cd} = V_{ad} - V_{ac} = 70\ \text{V}.$

(b) When the switch is closed, the points *c* and *d* must be at the same potential, so the equivalent capacitance is $C_{\text{eq}} = \left(\dfrac{1}{(3.00+6.00)\ \mu\text{F}} + \dfrac{1}{(3.00+6.00)\ \mu\text{F}}\right)^{-1} = 4.5\ \mu\text{F}.$

$Q_{\text{total}} = C_{\text{eq}}V = (4.50\ \mu\text{F})(210\ \text{V}) = 9.5\times10^{-4}\ \text{C},$ and each capacitor has the same potential difference of 105 V (again, by symmetry).

(c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the change in $Q_2 - Q_1.$ With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0.$ After the switch is closed, $Q_2 - Q_1 = 315\ \mu\text{C},$ so $315\ \mu\text{C}$ of charge flowed through the switch.

24.66. **(a)** $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_1}\right)^{-1} = \tfrac{1}{2}C_1 = \tfrac{1}{2}\dfrac{e_0 A}{(d-a)/2} = \dfrac{e_0 A}{d-a}$

(b) $C = \dfrac{e_0 A}{d-a} = \dfrac{e_0 A}{d}\dfrac{d}{d-a} = C_0\dfrac{d}{d-a}$

(c) As $a \to 0,\ C \to C_0.$ The metal slab has no effect if it is very thin. And as $a \to d,\ C \to \infty.$ $V = Q/C.$ $V = Ey$ is the potential difference between two points separated by a distance *y* parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large Q on the plates for a given potential difference. Since $Q = CV$ this corresponds to a very large *C*.

70.

a) The electric field must be in the outward ($+r$) direction. To find it, integrate over a Gaussian cylinder of radius r and length L:

$$\iint \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{A}} = 2\,\pi\,r\,L\,E_r = \frac{\lambda\,L}{\epsilon_0};$$

$$E_r = \frac{\lambda}{2\,\pi\,\epsilon_0\,r} = |\vec{\boldsymbol{E}}|.$$

The energy density is

$$u = \frac{\epsilon_0}{2}\,|\vec{\boldsymbol{E}}|^2 = \frac{\epsilon_0}{2}\left(\frac{\lambda}{2\,\pi\,\epsilon_0\,r}\right)^2 = \frac{\lambda^2}{8\,\pi^2\,\epsilon_0\,r^2}.$$

b)

$$\begin{aligned} U &= \iiint u\,\mathrm{d}V \\ &= \int_0^L \int_0^{2\pi} \int_{r_a}^{r_b} \frac{\lambda^2}{8\,\pi^2\,\epsilon_0\,r^2}\,r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z = \int_0^L \int_0^{2\pi} \frac{\lambda^2}{8\,\pi^2\,\epsilon_0} \ln\frac{r_b}{r_a}\,\mathrm{d}\theta\,\mathrm{d}z \\ &= 2\,\pi\,L\,\frac{\lambda^2}{8\,\pi^2\,\epsilon_0} \ln\frac{r_b}{r_a} = \frac{\lambda^2\,L}{4\,\pi\,\epsilon_0} \ln\frac{r_b}{r_a}; \end{aligned}$$

$$\frac{U}{L} = \frac{\lambda^2}{4\,\pi\,\epsilon_0} \ln\frac{r_b}{r_a}.$$

c) From Example 24.4,

$$\frac{C}{L} = \frac{2\,\pi\,\epsilon_0}{\ln\,(r_b/r_a)} \qquad \text{(for a cylindrical capacitor).}$$

From Eq. 24.9,

$$U = \frac{Q^2}{2\,C} = \frac{(\lambda\,L)^2}{2\,C},$$

so

$$\frac{U}{L} = \frac{\lambda^2\,L}{2\,C} = \frac{\lambda^2}{2\,C/L} = \frac{\lambda^2}{2\,[2\,\pi\,\epsilon_0/\ln\,(r_b/r_a)]} = \frac{\lambda^2}{4\,\pi\,\epsilon_0} \ln\frac{r_b}{r_a},$$

which agrees with part (b).

74.

a) We can view this as two capacitors connected in parallel, the first with dielectric K and plate height h, and the second with dielectric 1 and plate height $L-h$. Both have the same plate width w and separation between plates, which I'm calling d. Then $C_1 = K\,\epsilon_0\,\frac{w\,h}{d}$, $C_2 = \epsilon_0\,\frac{w\,(L-h)}{d}$, and the total capacitance is

$$C = C_1 + C_2 = (K\,h + L - h)\,\epsilon_0\,\frac{w}{d} = \left[(K-1)\,\frac{h}{L} + 1\right]\epsilon_0\,\frac{w\,L}{d}.$$

If there were a single uniform dielectric constant K' in this capacitor, the capacitance would be $C' = K' \epsilon_0 \frac{w\,L}{d}$. Therefore it makes sense to say the effective dielectric constant is $K_{\text{eff}} = (K-1)\frac{h}{L} + 1$ so that $C = K_{\text{eff}} \epsilon_0 \frac{w\,L}{d}$. (This is the same result you get by assuming K_{eff} is a weighted average of K and 1, weighted by the fraction of the plates touching fuel and air, respectively.)

b) $1/4$ full: $K_{\text{eff}} = (1.95-1)\frac{1}{4} + 1 = 1.24$

$1/2$ full: $K_{\text{eff}} = (1.95-1)\frac{1}{2} + 1 = 1.48$

$3/4$ full: $K_{\text{eff}} = (1.95-1)\frac{3}{4} + 1 = 1.71$

c) $1/4$ full: $K_{\text{eff}} = (33.0-1)\frac{1}{4} + 1 = 9.00$

$1/2$ full: $K_{\text{eff}} = (33.0-1)\frac{1}{2} + 1 = 17.0$

$3/4$ full: $K_{\text{eff}} = (33.0-1)\frac{3}{4} + 1 = 25.0$

d) It is more practical using methanol because there will be a greater (and easier to detect) change in capacitance as the tank is emptied.