Lecture 35
(Mutual & Self Inductance)

Physics 161-01 Spring 2012

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Applied Physics

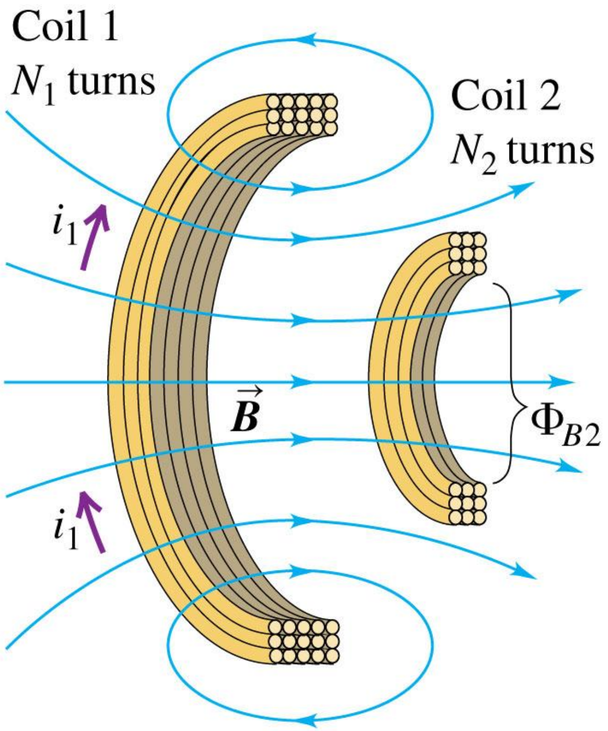
- We are now leaving the realm of "theoretical" physics, and will remain in the realm of applied physics for the remaining lectures.
- This doesn't mean that you won't learn anything new, but all of the things that you will learn now are based on the physics that you already have seen.

Mutual Inductance

- Let's look at two coils of wire, one inside the other.
- We know from Faraday's Law that there will be an EMF induced in the second coil if there is a changing current, and hence a changing magnetic field from the first coil:

$$\mathcal{E}_{2\ \text{Total}} = N_2 \mathcal{E}_{2,\ 1\ \text{Loop}} = N_2 \int_{\text{Loop}} \vec{E} \cdot d\vec{l} = -N_2 \frac{d\Phi_{B2}}{dt}$$

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Mutual Inductance

- Now, if we remember that the magnetic flux through coil 2, Φ_{B2}, is just a function of the geometry and the current:
  - The field from coil 1 is just a function of the geometry of coil 1, and the current through coil 1.
  - The field strength and direction at coil 2 is just a function of the position of coil 2 relative to coil 1.
- So, we can remove all of the geometric factors and write the EMF as:

$$\mathcal{E}_2 = -M_{21}\frac{di_1}{dt}$$

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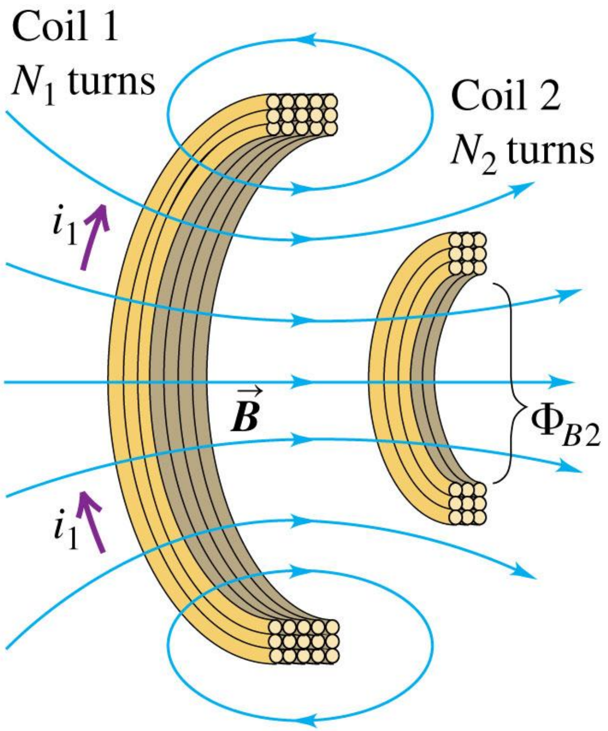


Mutual Inductance

- The mutual inductance, M_{21}, is just all of the stuff (that is not easy to calculate) representing the geometry, relative position and number of turns of each coil.

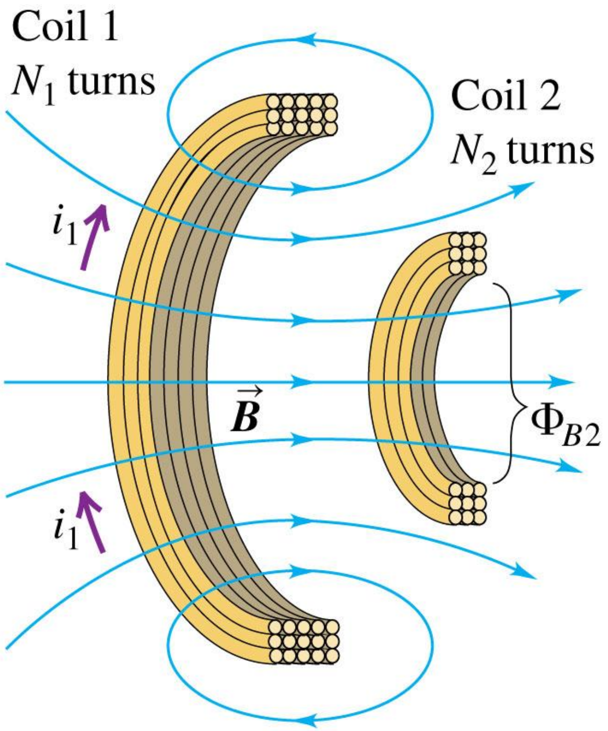
$$\mathcal{E}_2 = -M_{21}\frac{di_1}{dt}$$

- Then, one can write:

$$M_{21} = \frac{N_2 \Phi_{B2}}{i_1}$$

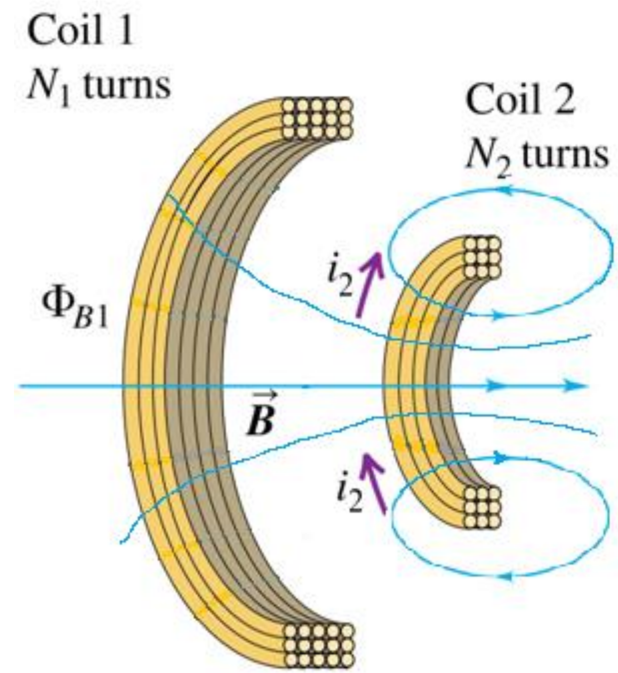
for a given flux through coil 2 from a current in coil 1.

Mutual inductance: If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



Mutual Inductance

- If we repeat the previous discussion for a current through coil 2:
- Now, if we remember that the magnetic flux through coil 1, Φ_{B1}, is just a function of the geometry and the current:
  - The field from coil 2 is just a function of the geometry of coil 2, and the current through coil 2.
  - The field strength and direction at coil 1 is just a function of the position of coil 1 relative to coil 2.
- So, we can remove all of the geometric factors and write the EMF as:



$$\mathcal{E}_1 = -M_{12}\frac{di_2}{dt} \qquad M_{12} = \frac{N_1\Phi_{B1}}{i_2}$$

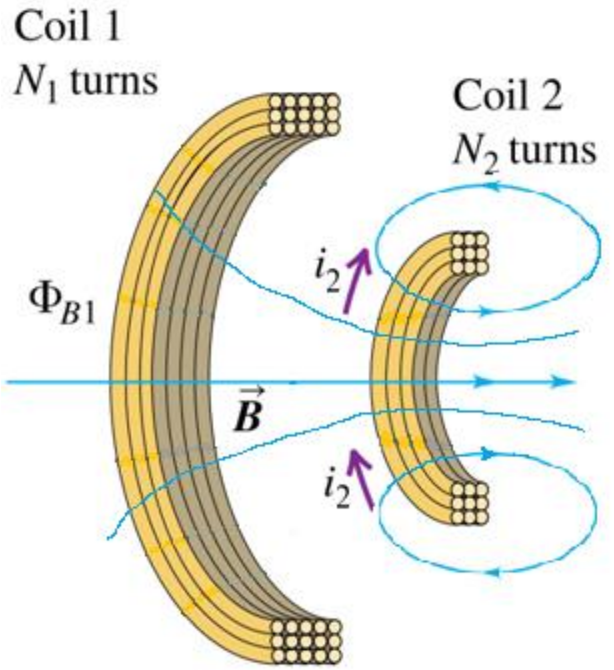
Mutual Inductance

- Now, it turns out that

$$M_{12} = M_{21} = M$$

$$M = \frac{N_1 \Phi_{B1}}{i_2} = \frac{N_2 \Phi_{B2}}{i_1}$$

- Without proving this, one can get a good feel of it since the flux through a coil should be proportional to the current through the other coil and geometries, so, dividing by the current one is just left with geometrical factors.



Example

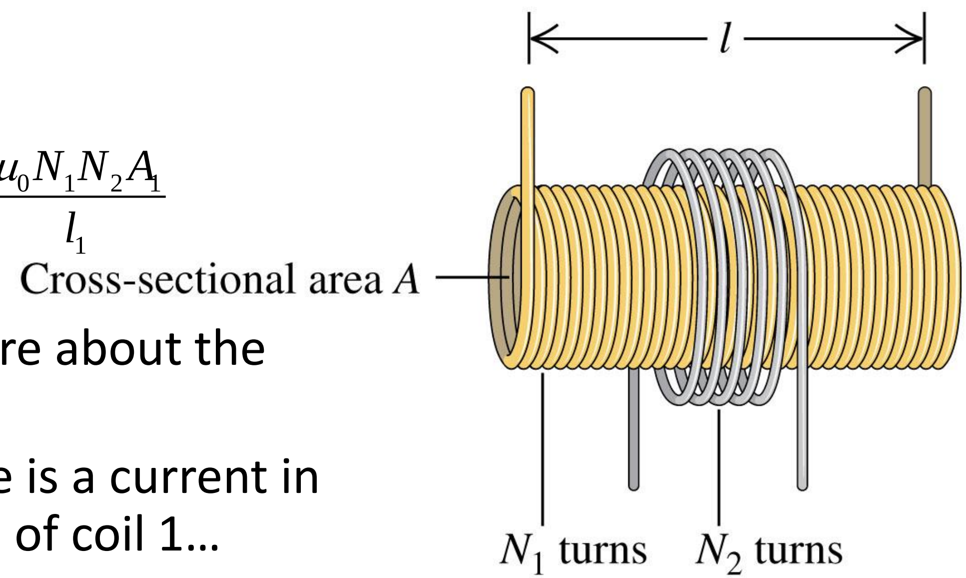
- What is the mutual inductance of the two coils shown below?

$$M = \frac{N_1 \Phi_{B1}}{i_2} = \frac{N_2 \Phi_{B2}}{i_1}$$

$$B_1 = \mu_0 n_1 i_1 = \frac{\mu_0 N_1 i_1}{l_1}$$

$$\Phi_{B2} = B_1 A_1 = \frac{\mu_0 N_1 i_1 A_1}{l_1}$$

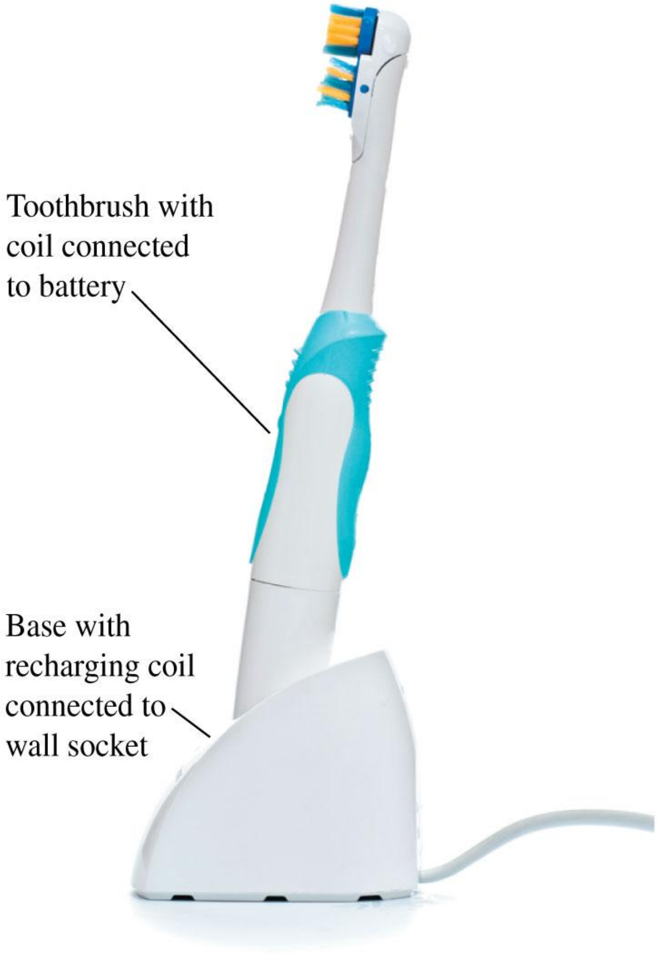
$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2 \dfrac{\mu_0 N_1 i_1 A_1}{l_1}}{i_1} = \frac{\mu_0 N_1 N_2 A_1}{l_1}$$



- But wait! There is nothing in there about the length or area of the coil 2…
- Look at the field lines when there is a current in coil 2, not all go through all turns of coil 1…

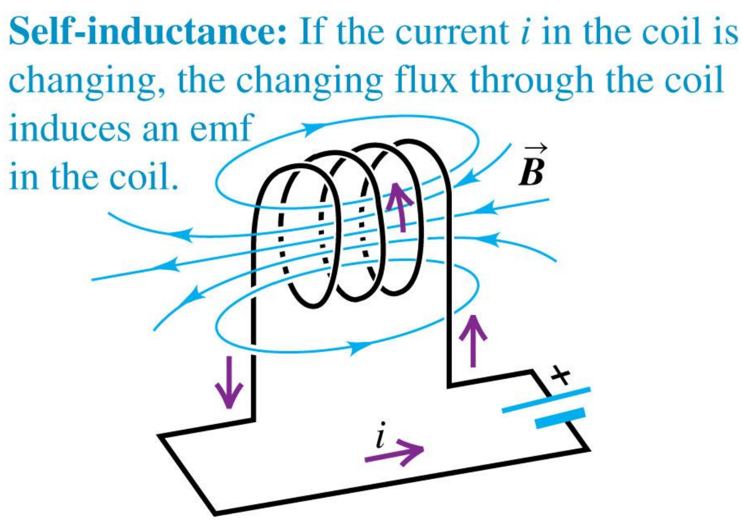
Many Applications

- But perhaps the most important application is keeping my toothbrush charged…



Self Inductance

- Now, if a changing magnetic flux through a coil causes an EMF, does it matter where the changing flux comes from?
- What if the changing flux comes from the coil itself?
- It doesn't matter where it comes from, so a circuit that has a changing current through it will experience an EMF induced from the changing magnetic flux.
- This is known as self-inductance.



Self Inductance

- In the same way as with mutual inductance, the EMF that is self-induced opposes the change in flux as:

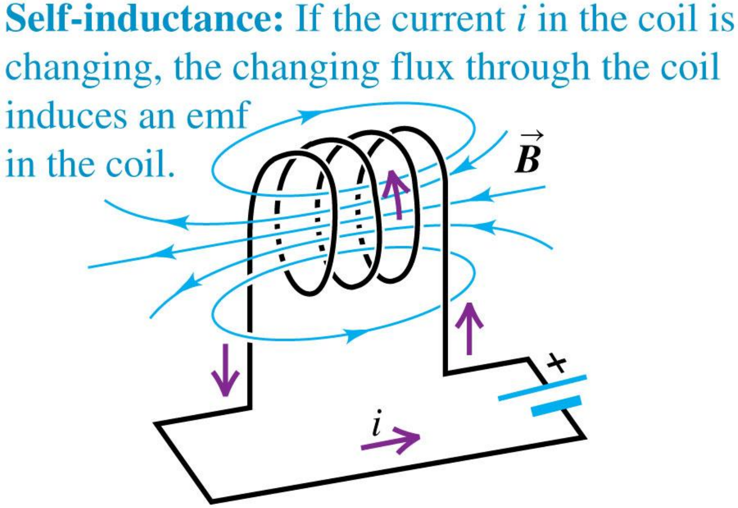
$$\mathcal{E}_{\text{Total}} = N\mathcal{E}_{1\text{ Loop}} = N\int_{\text{Loop}} \vec{E}\cdot d\vec{l} = -N\frac{d\Phi_B}{dt}$$

- And we can remove the geometric factors and put them into one item that describes the geometry:

$$L = N\frac{\Phi_B}{i}$$

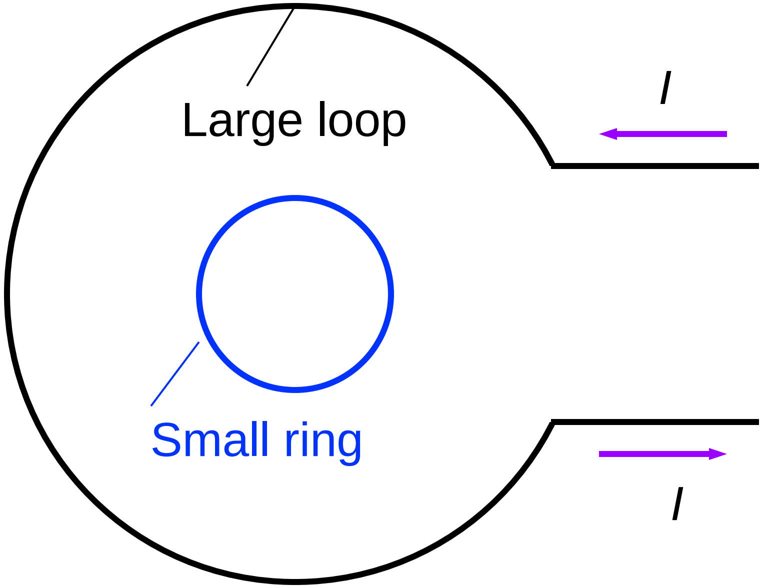
- Leaving us with a simple relationship between the EMF and the changing current:

$$\mathcal{E} = -L\frac{di}{dt}$$



CPS 35-1

A small, circular ring of wire (shown in blue) is inside a larger loop of wire that carries a current *I* as shown. The small ring and the larger loop both lie in the same plane. If *I* increases, the current that flows in the small ring



A. is clockwise and caused by self-inductance.

B. is counterclockwise and caused by self-inductance.

C. is clockwise and caused by mutual inductance.

D. is counterclockwise and caused by mutual inductance.

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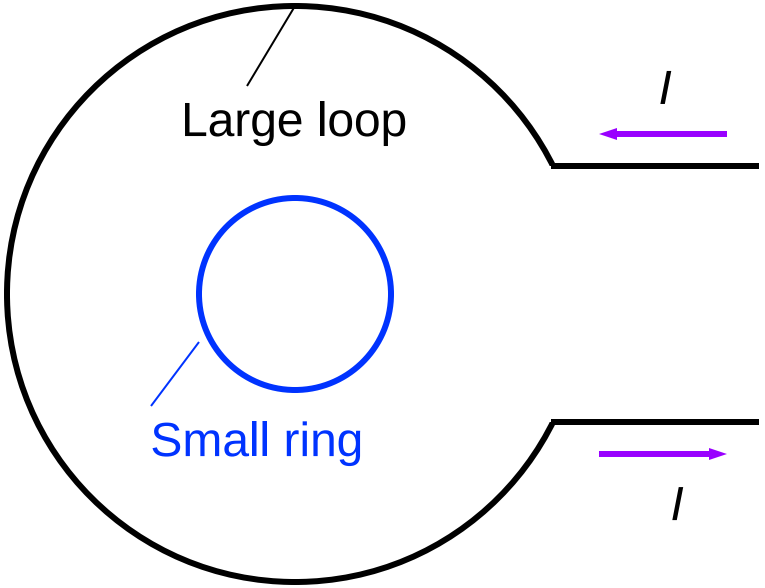


A. is clockwise and caused by self-inductance.

B. is counterclockwise and caused by self-inductance.

C. is clockwise and caused by mutual inductance.

D. is counterclockwise and caused by mutual inductance.

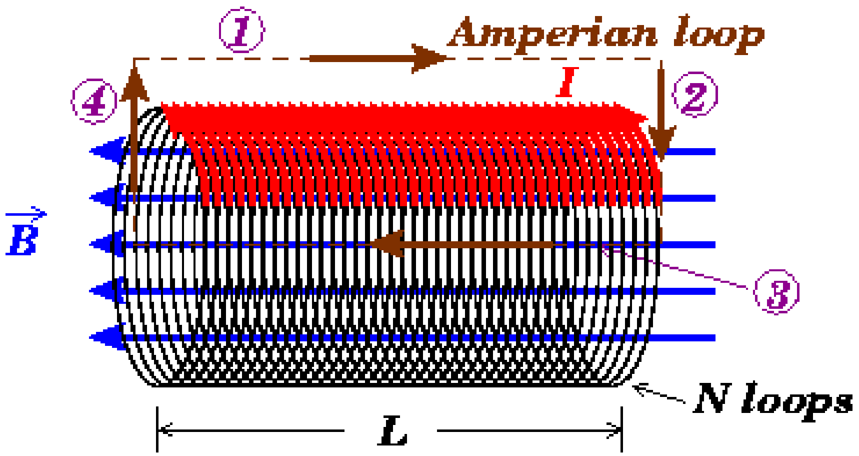
Inductance of a Solenoid

- Let's calculate the self inductance of a solenoid:

$$L = N\frac{\Phi_B}{i} = N\frac{BA}{i},$$

$$B = \mu_0 \frac{N}{L} i \Rightarrow$$

$$L = N\frac{\mu_0 \frac{N}{L} iA}{i} = \frac{\mu_0 N^2 A}{L}$$



- Notice that there are only geometric terms.

Inductance of a Square Toroid

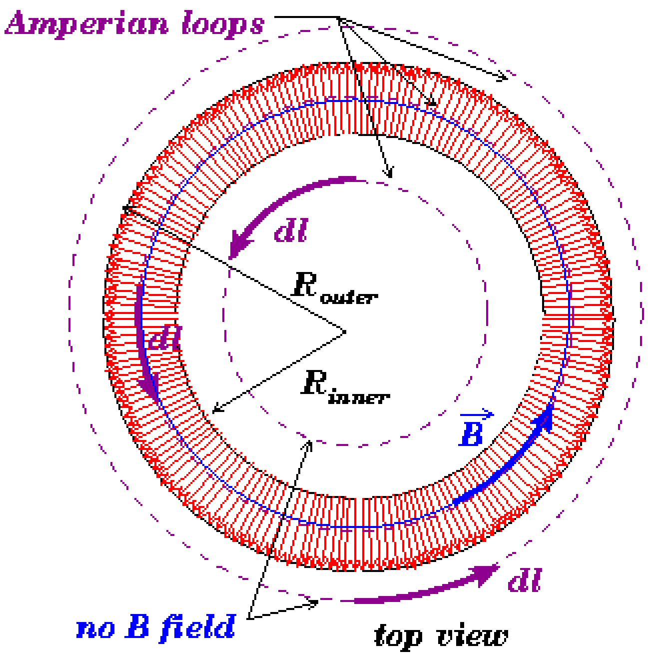
- Can you find the self inductance of a square toroid?

$$L = N\frac{\Phi_B}{i} \neq N\frac{BA}{i}!!!$$

$$L = N\frac{\Phi_B}{i} = \frac{N}{i}\int_{R_{inner}}^{R_{outer}} B dr \int_{bottom}^{top} dz,$$

$$B = \frac{\mu_0 N i}{2\pi r} \Rightarrow$$

$$L = \frac{\mu_0 N^2 h}{2\pi}\int_{R_{inner}}^{R_{outer}} \frac{1}{r} dr = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{R_{outer}}{R_{inner}}\right)$$



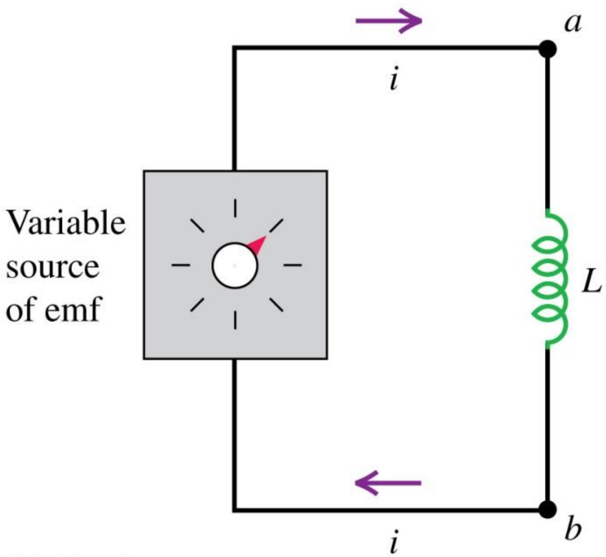
- Can you find the self inductance of a circular toroid?

Inductors

- So, if we have a coil of wire in a circuit, and we change the current through the circuit, then there will be an induced EMF across the inductor equal to

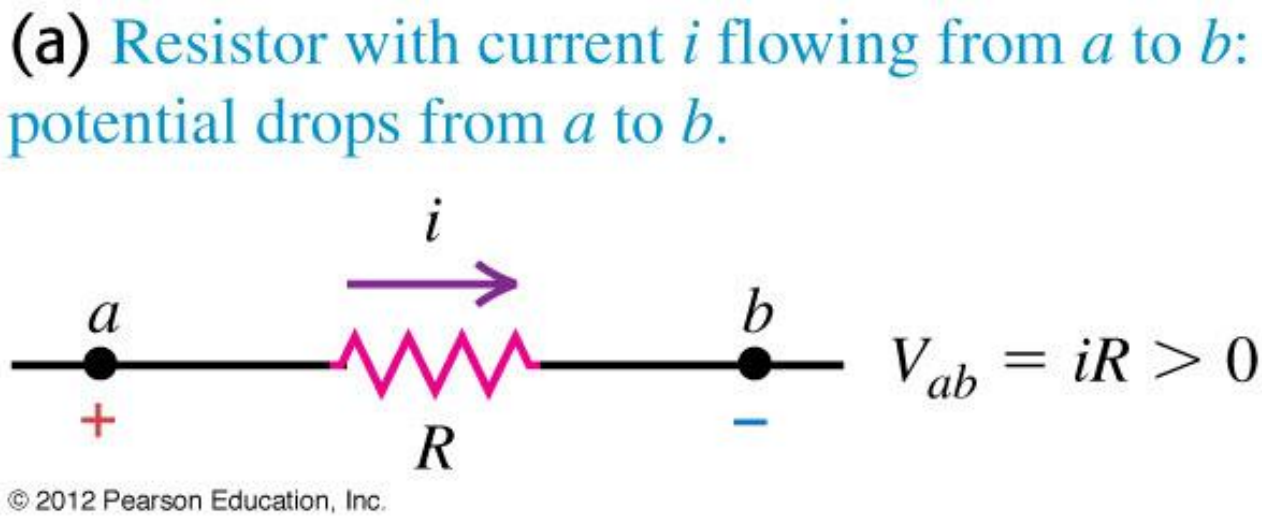
$$\mathcal{E} = -L\frac{di}{dt}$$

- And in a direction that OPPOSES the change in current.
- Let’s look at what this means.



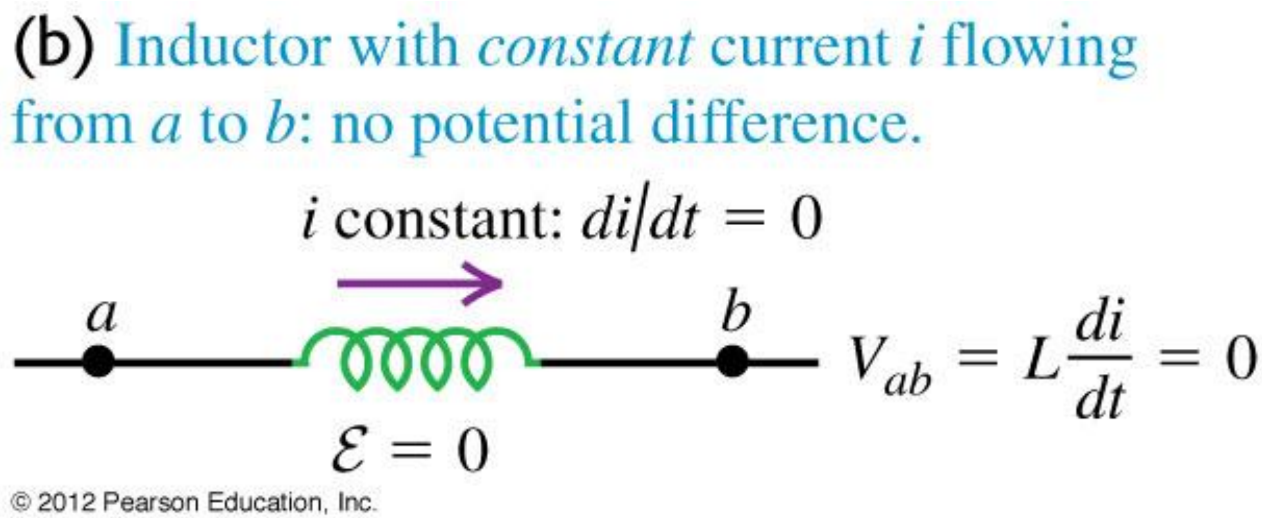
Self Inductance

- For a resistor, the potential across it is always “in the same direction” as the current, in other words, the current points from high to low potential.



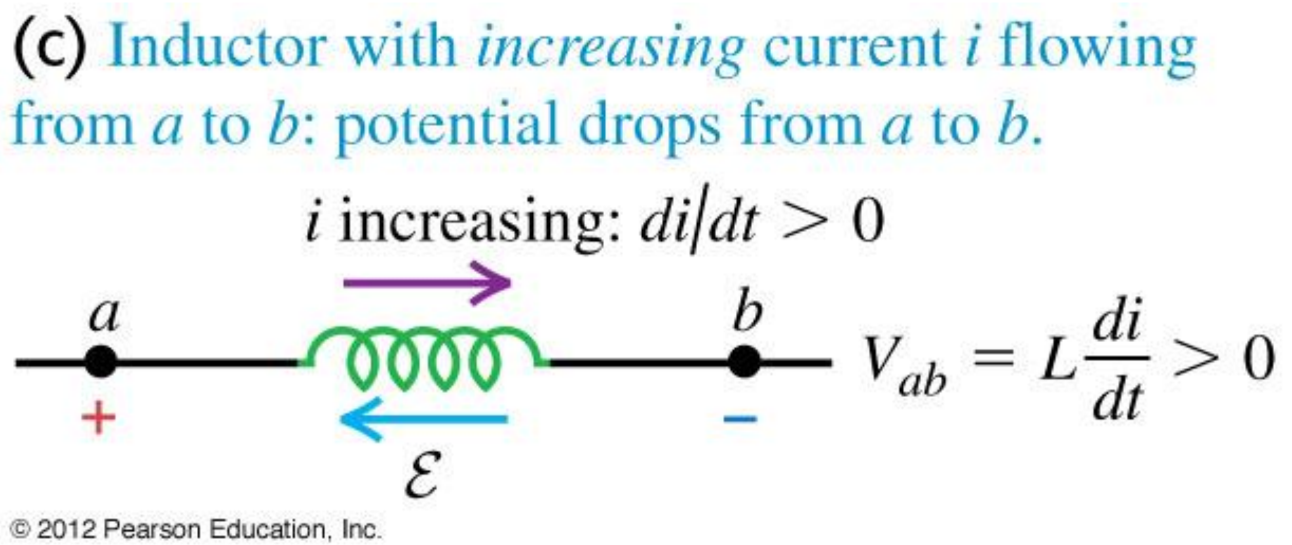
Self Inductance

- Inductors are different!
- With a constant current, there is NO potential difference across the inductor.



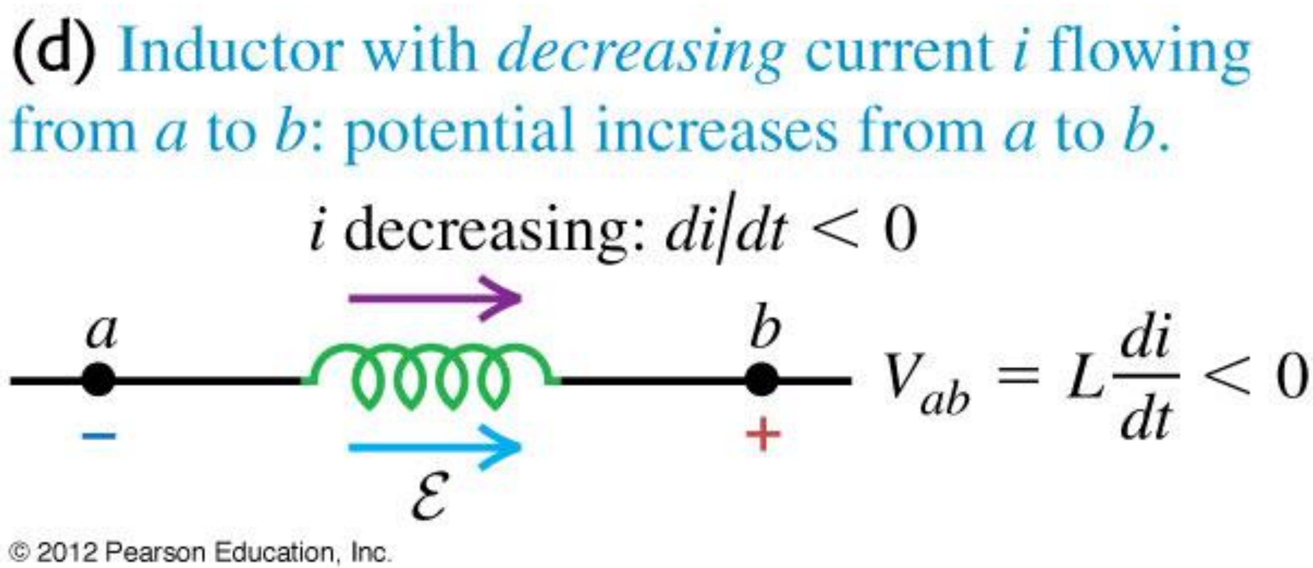
Self Inductance

- If the current is INCREASING, the induced EMF across the inductor will point in the direction opposite to the current.



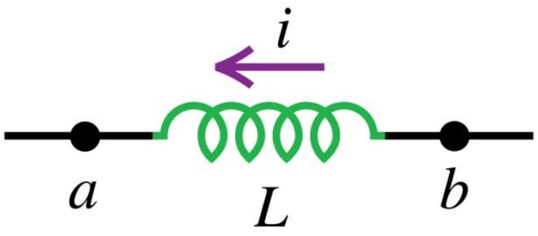
Self Inductance

- If the current is DECREASING, the induced EMF will point in the same direction as the current.
- In all cases, the EMF across the inductor will attempt to create a current that will oppose the CHANGE in current (and thus, the flux).



CPS 35-2

A current *i* flows through an inductor *L* in the direction from point *b* toward point *a*. There is zero resistance in the wires of the inductor. If the current is *decreasing*,



A. the potential is greater at point *a* than at point *b.*

B. the potential is less at point *a* than at point *b.*

C. The answer depends on the magnitude of *di/dt* compared to the magnitude of *i.*

D. The answer depends on the value of the inductance *L.*

E. both C. and D. are correct.

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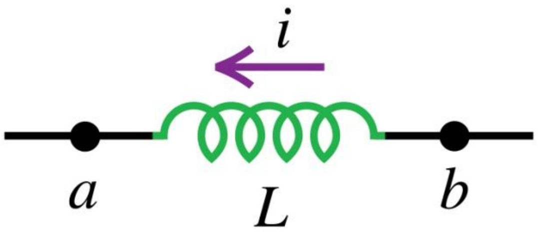

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