#7 Projectile Motion Pre-class

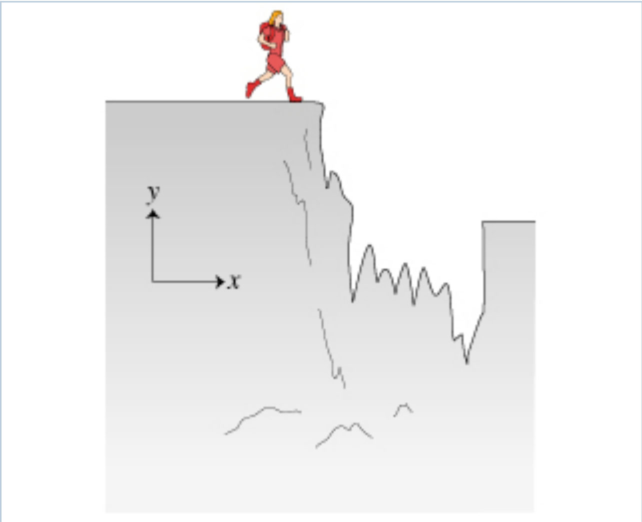
Due: 11:00am on Friday, September 7, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Direction of Velocity at Various Times in Flight for Projectile Motion Conceptual Question

For each of the motions described below, determine the algebraic sign (positive, negative, or zero) of the *x* component and *y* component of velocity of the object at the time specified. For all of the motions, the positive *x* axis points to the right and the positive *y* axis points upward.

Alex, a mountaineer, must leap across a wide crevasse. The other side of the crevasse is below the point from which he leaps, as shown in the figure. Alex leaps horizontally and successfully makes the jump.



Part A

Determine the algebraic sign of Alex's *x* velocity and *y* velocity at the instant he leaves the ground at the beginning of the jump.

Type the algebraic signs of the *x* velocity and the *y* velocity separated by a comma (examples: `+,-` and `0,+`).

Hint 1. Algebraic sign of velocity

The algebraic sign of the velocity is determined solely by comparing the direction in which the object is moving with the direction that is defined to be positive. In this example, to the right is defined to be the positive x direction and upward the positive y direction. Therefore, any object moving to the right, whether speeding up, slowing down, or even simultaneously moving upward or downward, has a positive x velocity. Similarly, if the object is moving downward, regardless of any other aspect of its motion, its y velocity is negative.

Hint 2. Sketch Alex's initial velocity

On the diagram below, sketch the vector representing Alex's velocity the instant after he leaves the ground at the beginning of the jump.

ANSWER:

ANSWER:

+,0

All attempts used; correct answer displayed

Part B

Determine the algebraic signs of Alex's *x* velocity and *y* velocity the instant before he lands at the end of the jump.

Type the algebraic signs of the *x* velocity and the *y* velocity separated by a comma (examples: +,- and 0,+).

Hint 1. Sketch Alex's final velocity

On the diagram below, sketch the vector representing Alex's velocity the instant before he safely lands on the other side of the crevasse.

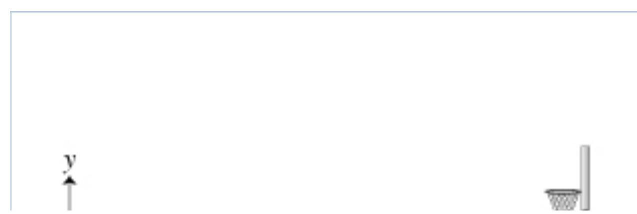
ANSWER:

ANSWER:

+,-

All attempts used; correct answer displayed

At the buzzer, a basketball player shoots a desperation shot. The ball goes in!



Part C

Determine the algebraic signs of the ball's *x* velocity and *y* velocity the instant after it leaves the player's hands.

Type the algebraic signs of the *x* velocity and the *y* velocity separated by a comma (examples: +,- and 0,+).

Hint 1. Sketch the basketball's initial velocity

On the diagram below, sketch the vector representing the velocity of the basketball the instant after it leaves the player's hands.

ANSWER:

ANSWER:

+,+

Correct

Part D

Determine the algebraic signs of the ball's x velocity and y velocity at the ball's maximum height.

Type the algebraic signs of the *x* velocity and the *y* velocity separated by a comma (examples: +,- and 0,+).

Hint 1. Sketch the basketball's velocity at maximum height

On the diagram below, sketch the vector representing the velocity of the basketball the instant it reaches its maximum height.

ANSWER:

ANSWER:

+,0

Correct

PhET Tutorial: Projectile Motion

Learning Goal:

To understand how the trajectory of an object depends on its initial velocity, and to understand how air resistance affects the trajectory.

For this problem, use the PhET simulation *Projectile Motion*. This simulation allows you to fire an object from a cannon, see its trajectory, and measure its range and hang time (the amount of time in the air).

Start the simulation. Press **Fire** to launch an object. You can choose the object by clicking on one of the objects in the scroll-down menu at top right (a cannonball is not among the choices). To adjust the cannon barrel’s angle, click and drag on it or type in a numerical value (in degrees). You can also adjust the speed, mass, and diameter of the object by typing in values. Clicking **Air Resistance** displays settings for (1) the drag coefficient and (2) the altitude (which controls the air density). For this tutorial, we will use an altitude of zero (sea level) and let the drag coefficient be automatically set when the object is chosen.

Play around with the simulation. When you are done, click **Erase** and select a baseball prior to beginning Part A. Leave **Air Resistance** unchecked.

Part A

First, you will investigate purely vertical motion. The kinematics equation for vertical motion (ignoring air resistance) is given by

$$y(t) = y_0 + v_0 t - (1/2)gt^2,$$

where $y_0 = 0$ is the initial position (which is 1.2 m above the ground due to the wheels of the cannon), v_0 is the initial speed, and g is the acceleration due to gravity.

Shoot the baseball straight upward (at an angle of 90°) with an initial speed of 20 $\mathrm{m/s}$.

How long does it take for the baseball to hit the ground?

Express your answer with the appropriate units.

ANSWER:

4.10 s

Answer Requested

Notice that this value could be determined from the kinematics equation: Approximating the final height to be $y(t_f) = 0$ (keep in mind that the location where $y = 0$ is inside the cannon, not the ground), the kinematics equation becomes $(v_0 - 0.5gt) = 0$, or $t = 2v_0/g = 4.1\ \mathrm{s}$. This calculation is interesting because it shows that, for vertical motion, the time the ball is in the air is proportional to its initial speed.

Part B

When the baseball is shot straight upward with an initial speed of 20 $\mathrm{m/s}$, what is the maximum height above its initial location? (Note that the ball's initial height is denoted by the horizontal white line. It is initially 1.2 m above the ground. The yellow box that is below the target on the grass is measuring tape that should be used for this part.)

Express your answer to three significant figures and include the appropriate units.

Hint 1. How to approach the problem

Use the measuring tape to determine the height. Align the plus sign at the beginning of the spool with the horizontal white line, and drag the end of the tape to the maximum height of the ball's trajectory. You can zoom in or out using the magnification buttons above the **Fire** and **Erase** buttons.

ANSWER:

20.4 m

All attempts used; correct answer displayed

Notice that this value could be determined from the kinematics equation: Since you found it takes 4.1 s for the ball to reach the ground, it must take roughly 2.1 s to reach the maximum height, which is given by $y(t = 2.1\ \mathrm{s}) = (20\ \mathrm{m/s})(2.1\ \mathrm{s}) - (1/2)(9.8\ \mathrm{m/s^2})(2.1\ \mathrm{s})^2 = 20.4\ \mathrm{m}$. (Since 20.4 m is considerably longer than the initial height above the ground, it is a good approximation to pretend that the initial position of the baseball was the ground.)

Part C

If the initial speed of the ball is doubled, how does the maximum height change?

Hint 1. How to approach the problem

You can use the simulation to help with this question. Set the initial speed to 40 $\mathrm{m/s}$, and again measure the maximum height with the measuring tape. You will likely have to zoom out to see the top of the ball's trajectory.

ANSWER:

- The maximum height increases by a factor of four.
- The maximum height increases by a factor of 1.4 (square root of 2).
- The maximum height increases by a factor of two.

Correct

Since the amount of time it takes to reach the maximum height doubles, and since its average velocity in going upward also doubles (the average velocity is equal to half the initial velocity), the height it reaches before stopping increases by a factor of four (distance is equal to the average velocity multiplied by the time duration).

Part D

Erase all the trajectories, and fire the ball vertically again with an initial speed of 20 $\mathrm{m/s}$. As you found earlier, the maximum height is roughly 20 m. If the ball isn't fired vertically, but at an angle less than 90°, it can reach the same maximum height if its initial speed is faster. Set the initial speed to 25 $\mathrm{m/s}$, and find the angle such that the maximum height is roughly 20 m. Experiment by firing the ball with many different angles. You can use the measuring tape to determine the maximum height of the trajectory and compare it to 20 m.

What is this angle?

ANSWER:

- 63°
- 53°
- 30°
- 45°
- 75°

Correct

Notice that the initial speed in the vertical direction is given by $(25\ \mathrm{m/s})\sin(53^\circ) = 20\ \mathrm{m/s}$. The ball launched at this angle reaches the same height as the vertically launched ball because they have the same initial speeds in the vertical direction.

Part E

In the previous part, you found that a ball fired with an initial speed of 25 $\mathrm{m/s}$ and an angle of 53° reaches the same height as a ball fired vertically with an initial speed of 20 $\mathrm{m/s}$. Which ball takes longer to land?

ANSWER:

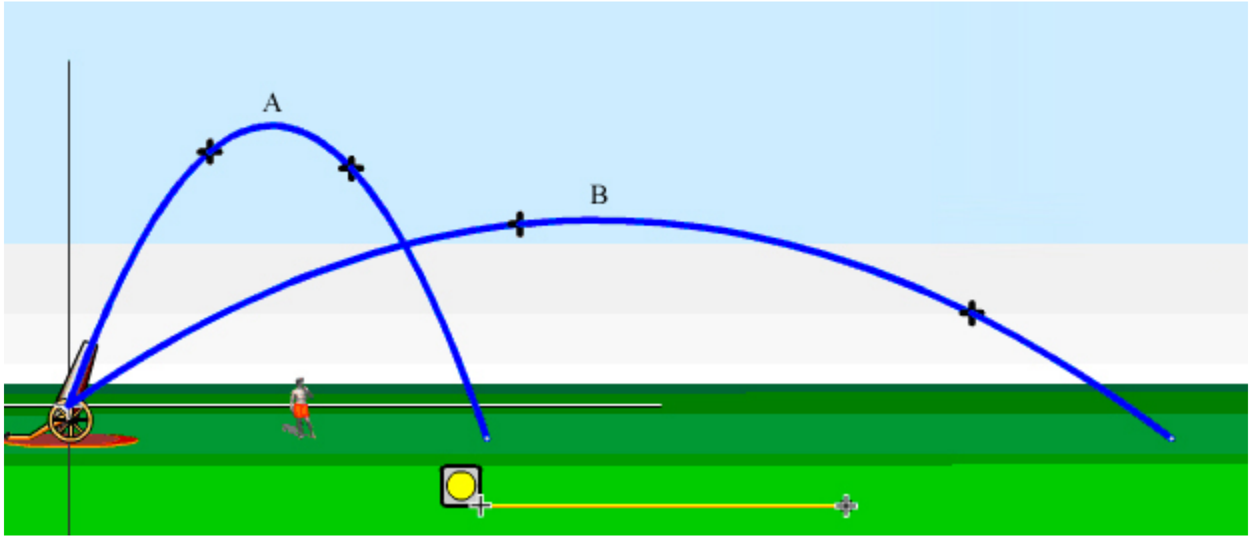
- ○ The ball fired vertically stays in the air longer.
- ◉ Both balls are in the air the same amount of time.
- ○ The ball fired at an angle of 53° stays in the air longer.

Correct

The vertical component of the velocity determines how long the ball will be in the air (and its maximum height). The horizontal component of the ball's velocity does not affect this hang time.

Part F

The figure shows two trajectories, made by two balls launched with different angles and possibly different initial speeds.



Based on the figure, for which trajectory was the ball in the air for the greatest amount of time?

Hint 1. How to approach the problem

Think about the result of Part E (think about the relationship between the maximum height of something thrown upward and the amount of time it is in the air). Does the time in the air depend on the range of the trajectory?

ANSWER:

- Trajectory B
- It's impossible to tell solely based on the figure.
- Trajectory A
- The balls are in the air for the same amount of time.

Correct

All that matters is the vertical height of the trajectory, which is based on the component of the initial velocity in the vertical direction ($v_0 \sin\theta$). The higher the trajectory, the more time the ball will be in the air, regardless of the ball's range or horizontal velocity.

Part G

The range is the distance from the cannon when the ball hits the ground. This distance is given by the horizontal velocity (which is constant) times the amount of time the ball is in the air (which is determined by the vertical component of the initial velocity, as you just discovered).
Set the initial speed to 20 $\mathrm{m/s}$, and fire the ball several times while varying the angle between the cannon and the horizontal. Notice that the digital display near the top gives the range of the ball.

For which angle is the range a maximum (with the initial speed held constant)?

ANSWER:

- ○ $60°$
- ○ $90°$
- ○ $30°$
- ○ $0°$
- ◉ $45°$

Correct

When the ball is launched near a level ground, $45°$ is the optimum angle. If launched with a greater angle, it stays in the air longer, but its horizontal speed is slower, and it won't go as far. If launched with a smaller angle, its horizontal speed is faster, but it won't stay in the air as long and it won't go as far. The product between the horizontal speed and the amount of time in the air is largest when the angle is $45°$.

Part H

How does the range of the object change if its initial velocity is doubled (keeping the angle fixed and less than $90°$)?

ANSWER:

- ○ The ball's range is twice as far.
- ◉ The ball's range is four times as far.
- ○ The ball's range is eight times as far.

Correct

Since the vertical component of the velocity is twice as large, it takes twice as long to hit the ground. The horizontal component of the velocity is also twice as large, and since the range is equal to the horizontal velocity times the amount of time the ball is in the air, the range increases by a factor of four. The results of this question and the previous question can be summarized by the range equation, which is

$$R = 2{v_0}^2 \sin(\theta)\cos(\theta)/g.$$

Part I

Now, let's see what happens when the cannon is high above the ground. Click on the wheel of the cannon, and drag it upward as far as it goes (about 21 $\rm m$ above the ground). Set the initial velocity to 20 $\rm m/s$, and fire several balls while varying the angle.

For what angle is the range the greatest?

ANSWER:

- (selected) $35°$
- $40°$
- $30°$
- $45°$
- $50°$

Correct

Since the cannon is very high off the ground, the ball will be in the air for an appreciable amount of time even if the ball is launched nearly horizontally. Thus, the amount of time the ball is in the air isn't proportional to the vertical component of the initial velocity (as it was when the cannon was on the ground). This means that the initial horizontal velocity is more important, resulting in an optimal angle less than $45°$. You should realize that the range equation given in Part H,

$$R = 2{v_0}^2 \sin(\theta)\cos(\theta)/g,$$

is not valid when the initial height is not zero. You can also verify that, if you change the initial velocity, the optimal angle also changes!

Part J

So far in this tutorial, you have been launching a baseball. Let's see what happens to the trajectory if you launch something bigger and heavier, like a Buick car.
Compare the trajectory and range of the baseball to that of the Buick car, using the same initial speed and angle (e.g., $45°$). (Be sure that air resistance is still turned off.) Which statement is true?

ANSWER:

- The trajectories differ; the range of the Buick is longer than that of the baseball.
- (selected) The trajectories and thus the range of the Buick and the baseball are identical.
- The trajectories differ; the range of the Buick is shorter than that of the baseball.

Correct

Since we are ignoring air resistance, the trajectory of the object does not depend on its mass or size. In the next part, you will turn on air resistance and discover what changes.

Part K

In the previous part, you discovered that the trajectory of an object does not depend on the object's size or mass. But if you have ever seen a parachutist or a feather falling, you know this isn't really true. That is because we have been neglecting air resistance, and we will now study its effects here.

Select **Air Resistance** for the simulation. Fire a baseball with an initial speed of roughly 20 $\mathrm{m/s}$ and an angle of 45°. Compare the trajectory to the case without air resistance. How do the trajectories differ?

ANSWER:

- ○ The trajectories are identical.
- ◉ The trajectory with air resistance has a shorter range.
- ○ The trajectory with air resistance has a longer range.

Correct

Air resistance is a force due to the object ramming through the air molecules, and is always in the opposite direction to the object's velocity. This means the air resistance force will slow the object down, resulting in a shorter range (the simulation assumes the air is still; there is no strong tailwind).

Part L

Notice that you can adjust the diameter (and mass) of any object (e.g., you can make a really big baseball). What happens to the trajectory (with air resistance on) when you increase the diameter while keeping the mass constant?

ANSWER:

- Increasing the size makes the range of the trajectory increase.
- Increasing the size makes the range of the trajectory decrease.
- The size of the object doesn't affect the trajectory.

Correct

Since the surface area increases if the diameter increases, the object is sweeping through more air, causing more collisions, and a greater force of air drag (in fact, if the diameter is doubled, for a given speed, the force of air drag is increased by a factor of four). This greater force of air drag causes the object to slow down more quickly, resulting in a slower average speed and a shorter range.

Part M

You might think that it is never a good approximation to ignore air resistance. However, often it is. Fire the baseball without air resistance, and then fire it with air resistance (same angle and initial speed). Then, adjust the mass of the baseball (increase it and decrease it) and see what happens to the trajectory. Don't change the diameter.
When does the range with air resistance approach the range without air resistance?

ANSWER:

- The range with air resistance approaches the range without air resistance as the mass of the baseball is decreased.
- The range with air resistance approaches the range without air resistance as the mass of the baseball is increased.
- It never does. Regardless of the mass, the range with air resistance is always shorter than the range without.

Correct

As the mass is increased, the force of gravity on the baseball becomes larger. The force due to air drag just depends on the speed and the size of the object, so it doesn't change if the mass changes. As the mass gets large enough, the force of gravity becomes much larger than the air drag force, and so the air drag force becomes negligible. This results in a trajectory nearly the same as when air resistance is turned off. Thus, for small, dense objects (like rocks and bowling balls), air resistance is typically unimportant, but for objects with a low density (like feathers) or a very large surface area (like parachutists), air resistance is very important.

PhET Interactive Simulations
University of Colorado
http://phet.colorado.edu

Battleship Shells

A battleship simultaneously fires two shells toward two identical enemy ships. One shell hits ship A, which is close by, and the other hits ship B, which is farther away. The two shells are fired at the same *speed*. Assume that air resistance is negligible and that the magnitude of the acceleration due to gravity is g.
Note that after Part B the question setup changes slightly.

Part A

What shape is the trajectory (graph of *y* vs. *x*) of the shells?

ANSWER:

- straight line
- parabola
- hyperbola
- The shape cannot be determined.

Correct

Part B

For two shells fired at the same speed which statement about the horizontal distance traveled is correct?

Hint 1. Two things to consider

The distance traveled is the product of the *x* component of the velocity and the time in the air. How does the *y* component of the velocity affect the "air time"? What angle would give the longest range?

ANSWER:

- The shell fired at a larger angle with respect to the horizontal lands farther away.
- The shell fired at an angle closest to 45 degrees lands farther away.
- The shell fired at a smaller angle with respect to the horizontal lands farther away.
- The lighter shell lands farther away.

Correct

Now, consider for the remaining parts of the question below that both shells are fired at an angle greater than 45 degrees with respect to the horizontal. Remember that enemy ship A is closer than enemy ship B.

Part C

Which shell is fired at the larger angle?

Hint 1. Consider the limiting case

Consider the case in which a shell is fired at 90 degrees above the horizontal (i.e., straight up). What distance x will the shell travel? Now lower the angle at which the shell is fired. What happens to the distance x?

ANSWER:

- (●) A
- ( ) B
- ( ) Both shells are fired at the same angle.

Correct

Part D

Which shell is launched with a greater vertical velocity, v_y?

ANSWER:

- (●) A
- ( ) B
- ( ) Both shells are launched with the same vertical velocity.

Correct

Part E

Which shell is launched with a greater horizontal velocity, v_x?

ANSWER:

- A
- B
- Both shells are launched with the same horizontal velocity.

Correct

Part F

Which shell reaches the greater maximum height?

Hint 1. What determines maximum height?

What determines the maximum height reached by the shell?

ANSWER:

- horizontal velocity
- vertical velocity
- mass of the shell

ANSWER:

- A
- B
- Both shells reach the same maximum height.

Correct

Part G

Which shell has the longest travel time (time elapsed between being fired and hitting the enemy ship)?

Hint 1. Consider the limiting case

If a shell is fired exactly horizontally (0 degrees) the shell hits the ground right away. As the angle above the horizontal increases, what happens to the time of travel? Does this change as the angle becomes greater than 45 degrees?

ANSWER:

- (•) A
- ( ) B
- ( ) Both shells have the same travel time.

Correct

Introduction to Projectile Motion

Learning Goal:

To understand the basic concepts of projectile motion.

Projectile motion may seem rather complex at first. However, by breaking it down into *components*, you will find that it is really no different than the one-dimensional motions that you have already studied.

One of the most often used techniques in physics is to divide two- and three-dimensional quantities into components. For instance, in projectile motion, a particle has some initial velocity $\vec{v}$. In general, this velocity can point in any direction on the *xy* plane and can have any magnitude. To make a problem more managable, it is common to break up such a quantity into its *x* component v_x and its *y* component v_y.

Consider a particle with initial velocity $\vec{v}$ that has magnitude 12.0 $\mathrm{m/s}$ and is directed 60.0 $\mathrm{degrees}$ above the negative *x* axis.

Part A

What is the *x* component v_x of $\vec{v}$?

Express your answer in meters per second.

ANSWER:

v_x = -6.00 $\mathrm{m/s}$

All attempts used; correct answer displayed

Part B

What is the *y* component v_y of $\vec{v}$?

Express your answer in meters per second.

ANSWER:

v_y = 10.4 $\mathrm{m/s}$

All attempts used; correct answer displayed

Breaking up the velocities into components is particularly useful when the components do not affect each other. Eventually, you will learn about situations in which the components of velocity do affect one another, but for now you will only be looking at problems where they do not. So, if there is acceleration in the *x* direction but not in the *y* direction, then the *x* component of the velocity will change, but the *y* component of the velocity will not.

Part C

Look at this applet. The motion diagram for a projectile is displayed, as are the motion diagrams for each component. The *x*-component motion diagram is what you would get if you shined a spotlight down on the particle as it moved and recorded the motion of its shadow. Similarly, if you shined a spotlight to the left and recorded the particle's shadow, you would get the motion diagram for its *y* component. How would you describe the two motion diagrams for the components?

ANSWER:

- ○ Both the vertical and horizontal components exhibit motion with constant nonzero acceleration.
- ◉ The vertical component exhibits motion with constant nonzero acceleration, whereas the horizontal component exhibits constant-velocity motion.
- ○ The vertical component exhibits constant-velocity motion, whereas the horizontal component exhibits motion with constant nonzero acceleration.
- ○ Both the vertical and horizontal components exhibit motion with constant velocity.

Correct

As you can see, the two components of the motion obey their own independent kinematic laws. For the vertical component, there is an acceleration downward with magnitude $g = 10\ \mathrm{m/s^2}$. Thus, you can calculate the vertical position of the particle at any time using the standard kinematic equation $y = y_0 + v_0t + (1/2)at^2$. Similarly, there is no acceleration in the horizontal direction, so the horizontal position of the particle is given by the standard kinematic equation $x = x_0 + v_0t$.

Now, consider this applet. Two balls are simultaneously dropped from a height of 5.0 m.

Part D

How long t_g does it take for the balls to reach the ground? Use 10 $\mathrm{m/s^2}$ for the magnitude of the acceleration due to gravity.

Express your answer in seconds to two significant figures.

Hint 1. How to approach the problem

The balls are released from rest at a height of 5.0 m at time $t = 0\ \mathrm{s}$. Using these numbers and basic kinematics, you can determine the amount of time it takes for the balls to reach the ground.

ANSWER:

t_g = 1.0 s

Correct

This situation, which you have dealt with before (motion under the constant acceleration of gravity), is actually a special case of projectile motion. Think of this as projectile motion where the horizontal component of the initial velocity is zero.

Part E

Imagine the ball on the left is given a nonzero initial speed in the horizontal direction, while the ball on the right continues to fall with zero initial velocity. What horizontal speed v_x must the ball on the left start with so that it hits the ground at the same position as the ball on the right? Remember that the two balls are released, starting a horizontal distance of 3.0 m apart.

Express your answer in meters per second to two significant figures.

Hint 1. How to approach the problem

Recall from Part B that the horizontal component of velocity does not change during projectile motion. Therefore, you need to find the horizontal component of velocity v_x such that, in a time $t_g = 1.0\ \mathrm{s}$, the ball will move horizontally 3.0 m. You can assume that its initial *x* coordinate is $x_0 = 0.0\ \mathrm{m}$.

ANSWER:

v_x = 3.0 $\mathrm{m/s}$

All attempts used; correct answer displayed

You can adjust the horizontal speeds in this applet. Notice that regardless of what horizontal speeds you give to the balls, they continue to move vertically in the same way (i.e., they are at the same *y* coordinate at the same time).

Score Summary:

Your score on this assignment is 21.7%.
You received 4.34 out of a possible total of 20 points.