SOLUTIONS
Midterm #2
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11/12 ECE 345

$\boxed{1}$ Let $G(s) = \frac{100}{s^2}$.

$$Y(s) = G(s)\left(R(s) - Ks \cdot G(s) - G(s) \right)$$

$$Y(s)\left(1 + sK\,G(s) + G(s)\right) = R(s) \cdot G(s)$$

$$Y(s)\left(1 + sK \cdot \frac{100}{s^2} + \frac{100}{s^2}\right) = R(s) \cdot \frac{100}{s^2}$$

$$Y(s)\left(s^2 + 100 \cdot K \cdot s + 100\right) = 100 \cdot R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{100}{s^2 + 100 \cdot K \cdot s + 100}$$

$\boxed{12}$ $G(s) = \dfrac{1}{s(s+1)^2}$

1. Marginally stable, since no poles in RHP, no repeated poles on the imaginary axis, & a non-repeated pole on the imaginary axis at 0.

2. $\Delta_P(s) = s(s+1)^2 + K$

$= s^3 + 2s^2 + s + K$

$a_1 a_2 - a_0 = 1 \cdot 2 - K > 0$

$\underline{2 > K}$ to ensure asymptotic stability

3. $\Delta_{PD}(s) = s(s+1)^2 + K(s+1)$

$= s^3 + 2s^2 + s + Ks + K$

$= s^3 + 2s^2 + (1+K) + K$

$$
\begin{array}{lll}
s^3 & 1 & 1+K \\
s^2 & 2 & K \\
s^1 & -\dfrac{\begin{vmatrix} 1 & 1+K \\ 2 & K \end{vmatrix}}{2} & \cancel{-\dfrac{\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix}}{2}} \\
s^0 & -\dfrac{\begin{vmatrix} 2 & K \\ 1+\frac{K}{2} & 0 \end{vmatrix}}{1+K/2} &
\end{array}
$$

$$-\frac{\begin{vmatrix} 1 & 1+K \\ 2 & K \end{vmatrix}}{2} = -\frac{1}{2}\left(K - (2+2K)\right) = 1 + \frac{K}{2} \quad (>0 \text{ for } K>0)$$

$$= -\frac{1}{(1+K/2)}\left(0 - K(1+K/2)\right) = K \quad (>0 \text{ for } K>0)$$

No sign change in 1st column $\Rightarrow$ no poles of $\Delta_{PD}(s)$ in RHP.

4. $\Delta_P(s)$ has one pole at 0 with $K=2$ $\Rightarrow$ marginal stability & BIBO unstable.
$\Delta_{PD}(s)$ has all poles in open LHP w/ $K=2$ $\Rightarrow$ asymptotic stability & BIBO stable.

(a) With P control, the natural response will have some component that will neither grow nor decay as $t \to \infty$. In contrast, with PD control, the natural response will completely decay to 0 (converging asymptotically) as $t \to \infty$. Hence PD control is a better choice for $K=2$, than P control. (c)

(b) Under P control, a step input could generate an unbounded output. In contrast, under PD control, all bounded inputs generate unbounded outputs.

$\boxed{13}$

1(a) Type number is $\underline{\underline{1}}$ due to $\frac{1}{s}$ in open-loop transfer function $K(s) \cdot G(s)$.

1(b) $e_{ss} = \frac{1}{K_v}$, $K_v = \lim_{s \to 0} s\, K(s)\, G(s)$

$$= \lim_{s \to 0} s \cdot \frac{K}{s} \cdot \frac{2}{s+2} = K$$

$e_{ss} = 1/K$.

2. For a type 1 system, $e_{ss} = 0$ in response to a unit step in $R(s)$.
In addition, when $R(s) = 0$ and $D(s) = \frac{1}{s}$,

$$y_{ss} = \lim_{s \to 0} s Y(s)$$

$$= \lim_{s \to 0} s \cdot \frac{2s}{s^2 + 2s + 2K} \cdot \frac{1}{s} = 0$$

$\Rightarrow$ disturbance rejection.

3. $s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 2s + 2K$

$\Rightarrow 2\zeta\omega_n = 2 \Rightarrow \zeta = \frac{1}{\sqrt{2K}}$

$\Rightarrow \omega_n^2 = 2K \Rightarrow \omega_n = \sqrt{2K}$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\sqrt{2K}\cdot\sqrt{1-\frac{1}{2K}}} = \frac{\pi}{\sqrt{2K}\cdot\sqrt{\frac{2K-1}{2K}}} = \frac{\pi}{\sqrt{2K-1}} \le \pi$$

$1 \le \sqrt{2K-1}$

$1 \le K \Rightarrow$ (d) is correct.