ECE 345: Introduction to Control Systems
In-Class Exercise #2

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Due Thursday, September 27, 2012 at the end of class

1 Introduction

Robots have become ubiquitous in manufacturing process, to the point of generating concern about replacing human jobs (for example, http://www.nytimes.com/2012/08/19/business/new-wave-of-adept-robots-is-changing-global-industry.html). In contrast, extenders are one class of robot manipulators that extend the strength of the human arm while maintaining human control of the task. The physical contact between the extender and the human allows the direct transfer of mechanical power as the human deems appropriate. Because of this unique interface, control of the extender trajectory can be accomplished directly, rather than through an external device such as a joystick or keyboard. The human acts as a control system for the extender, while the extender actuators provide most of the strength necessary for the task. Such a system needs to be accurate, fast, and versatile in order for the human operator to effectively manipulate objects with the extender.

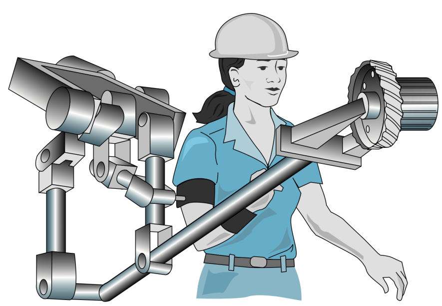


Figure 1: Robot manipulation via an extender. Illustration from *Modern Control Systems*, R. Dorf and R. Bishop, Prenctice Hall, 2011.

We will approximate the actuator dynamics of the extender through the transfer function

$$G(s) = \frac{K}{s^2 + 20s + K} \tag{1}$$

The input $r(t)$ to the system is the position dictated by the human's hand. The output $y(t)$ of the system is the position of the extender in the workspace. The system should have a settling time less than or equal to 0.5 seconds, an overshoot of less than 5%, and a steady-state error of less than 0.01 to a unit step input.

The goal of this exercise is to evaluate the system performance with respect to the above design requirements. *Each group should hand in the answers to the asterisk-marked problems (*).*

2 Pre-lecture work

DC motors will be used in the manipulator joints. Consider the following model of a DC motor that arises from Faraday's law of induction and Ampere's law. The rotation θ and angular rate ω of the load is affected by a back EMF (electromotive force) that develops when the input voltage v is applied. (Because the generated EMF works against the applied armature voltage, it is called the *back* EMF).

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{K^2}{JR} \end{bmatrix}\begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K}{JR} \end{bmatrix} v \tag{2}$$

Motor performance is determined by the coefficients R, the electrical resistance of the motor armature, K, the ratio between the generated back EMF and the speed ω of rotation, and J, the inertia of load that the motor is attempting to drive.

The output of the DC motor is the angular speed ω.

1. What is the transfer function for this system? (You can safely cancel the pole-zero pair in this case.)
2. Sketch the poles and zeros of the transfer function in the complex plane.
3. What is the time constant τ associated with the motor?
4. For a dimensionless load $J = 1e2$, what value should K^2/R have in order to for the motor to have a time constant $\tau = 0.01$ second?

3 In-class assignment

For the first three questions, assume that $K = 500$.

*1. Sketch the location of the poles and zeros of the transfer function. Indicate geometrically the natural frequency, the damped frequency, and the decay rate.

*2. Which of the following most accurately describes relationship between the natural frequency ω_n and the damped frequency ω_d?

   (a) $\omega_d > \omega_n$, and $\omega_n = 10\sqrt{5}$

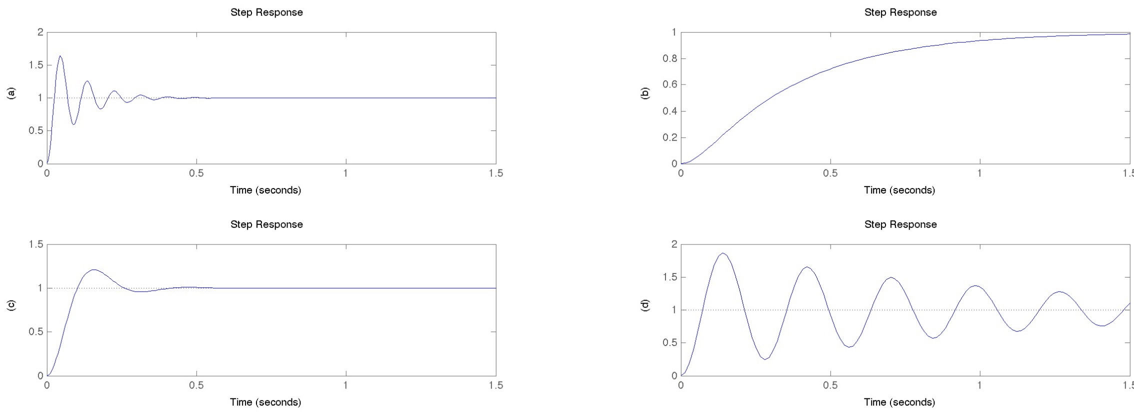
   (b) $\omega_d < \omega_n$, and $\omega_n = 10\sqrt{5}$

   (c) $\omega_d = \omega_n = 10$

   (d) ω_d is not relevant since the system is overdamped, and $\omega_n = 10$

Calculate the settling time and overshoot of the step response. Show that the system meets the settling time requirement but not the overshoot requirement.

*3. Which of the following plots most closely resembles the step response of (1)?



Now consider a generic value for K. Refer back to your complex plane plot of the transfer function poles and zeros for the following two questions.

*4. Which value(s) of K will meet the overshoot requirement? Select all that are correct.

(a) $K = 1000$

(b) $K = 500$

(c) $K = 200$

(d) $K = 100$

*5. Select *all* of the following that are correct. Altering the gain K will:

(a) Make the system overdamped for $K < 100$.

(b) Decrease settling time as K increases.

(c) Reduce peak time as K increases.

(d) Alter overshoot but not settling time for $K > 100$.

(e) Alter overshoot as well as peak time for $K > 100$.

BONUS: 6. The extender model (1) previously presumed that the unmodeled motor dynamics were arbitrarily fast, however this is somewhat optimistic. Consider the effect of the motor dynamics on the overall response of the extender.

(a) Determine whether the motor dynamics (from the pre-lecture work) are faster or slower than the extender dynamics.

(b) Explain (in 1-2 sentences) why the motor dynamics should be significantly faster than the extender dynamics, for good performance.

If your group finishes early...

Other points to consider (not necessary to hand in):

- Describe in your own words the difference between peak time and settling time. Give an example of a system with a fast peak time but slow settling time, and vice versa.
- In questions 4 and 5, we considered the changes in system performance as K varies. Sketch in the complex plane the locations of poles of (1) as K increases from 0 to ∞. (This is a variant of a *root locus* plot.)
- One way of achieving improved performance is by "closing the loop" (which we will discuss in coming chapters). By incorporating proportional feedback, we can change the transfer function from $G(s)$ to $\frac{KG(s)}{1+KG(s)}$. Would closing the loop enable the system to obtain a settling time of 0.1 seconds? Why or why not?
- The steady-state error is defined as the difference between the reference input and the actual output. Will increasing K reduce the steady-state error when the reference input is a unit step? Why or why not?