Scalar Product:
$$\vec{A}\cdot\vec{B}=|\vec{A}||\vec{B}|\cos\theta_{AB}=A_xB_x+A_yB_y+A_zB_z$$
Vector Product:
$$\vec{A}\times\vec{B}=|\vec{A}||\vec{B}|\sin\theta_{AB}=\left(A_yB_z-A_zB_y\right)\hat{i}+\left(A_zB_x-A_xB_z\right)\hat{j}+\left(A_xB_y-A_yB_x\right)\hat{k}$$
Equations of motion:
$$v=v_0+at$$
$$x=x_0+v_0t+\frac{1}{2}at^2$$
$$v^2=v_0{}^2+2a(x-x_0)$$
Radial Acceleration:
$$a_{rad}=\frac{v^2}{r}$$
Newton's second law
$$\sum\vec{F}=m\vec{a}$$
Magnitude of kinetic friction
$$F_{f_k}=\mu_kF_N$$
Magnitude of static friction
$$F_{f_s}\le\mu_sF_N$$
Definition of work
$$W=\int\vec{F}\cdot d\vec{x}$$
Definition of kinetic energy:
$$KE=\frac{1}{2}mv^2$$
Change in gravitational potential energy:
$$\Delta U_g=mg\Delta y$$
Elastic potential energy:
$$U_{el}=\frac{1}{2}kx^2$$
Work-Energy Theorem:
$$W=\Delta U+\Delta KE$$
Center-of-mass position
$$X_{COM}=\frac{1}{M}\sum_{i=1}^{n}x_im_i$$
Definition of momentum
$$\vec{p}=m\vec{v}$$
Conservation of momentum
$$\vec{p}_i=\vec{p}_f$$
Definition of torque
$$\vec{\tau}=\vec{r}\times\vec{F}$$
CONTINUED ON BACK!!!

Newton's second law for rotation

$$\sum \vec{\tau} = I\vec{\alpha}$$

Conditions for rolling: $a_{COM} = \alpha R$ and $v_{COM} = \omega R$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$ or $\vec{L} = I\vec{\omega}$, where $I = \sum_i m_i r_i^2$

Newton's Law of Gravitation:

$F_G = \dfrac{Gm_1m_2}{r^2}$ and $U_G = -\dfrac{Gm_1m_2}{r}$ with $U_G = 0$ at infinity

Bernoulli's Equation:

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Equation for Simple Harmonic Motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Solution for above equation:

$$x(t) = A\cos(\omega t + \varphi)$$

Where,

$$\omega = 2\pi f = \frac{2\pi}{T}$$

For a spring mass oscillator,

$$\omega = \sqrt{\frac{k}{m}}$$

For a simple pendulum,

$$\omega = \sqrt{\frac{g}{L}}$$

Wave Equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v}\frac{\partial^2 y(x,t)}{\partial t^2}$$

Solution to above equation:

$$y(x,t) = A\cos(kx - \omega t)$$

Where,

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad v = \lambda f$$

Standing waves on fixed string:

$$y(x,t) = A_{SW}\sin(kx)\sin(\omega t)$$

$$f_n = n\frac{v}{2L}$$

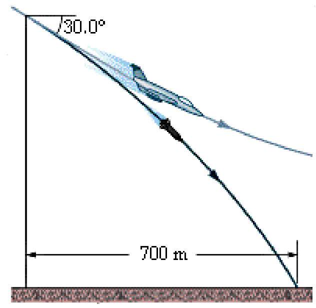
Doppler Effect:

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

Final Exam **Name:**____________________**Box #**_________ **Physics 160-01**

Written Problems

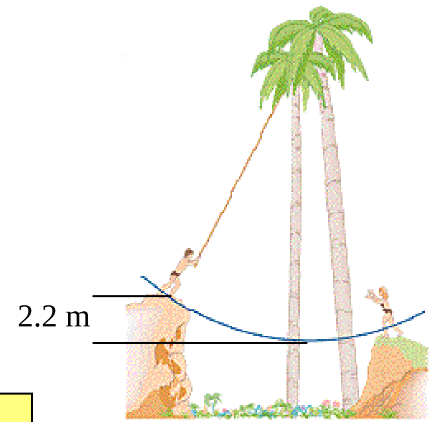
1) A certain airplane has a speed of 325.0 km/h and is diving at an angle of 30.0° below the horizontal when a radar decoy is released. The horizontal distance between the release point and the point where the decoy strikes the ground is 700 m. How high was the plane when the decoy was released? Neglect air resistance.



$x_0 = 0m$
$x_f = 700m$
$v_{0x} = 90.3m/s*\cos30$
$v_{xf} = $ same
$a_x = 0$
$t = ?$
$x_f = x_0 + v_{0x}t + 1/2a_xt^2$
$700m = 78.2m/s*t$
$t = 8.95s$

$y_0 = h$
$y_f = 0m$
$v_{0y} = -90.3m/s*\sin30$
$v_{xf} = ?$
$a_y = -9.8m/s^2$
$t = 8.95s$
$y_f = y_0 + v_{0y}t + 1/2a_yt^2$
$0m = h - 45.1m/s*(8.95s) - ½(9.8m/s^2)(8.95s)^2$
$h = 796m$

2) Tarzan, who weighs 647 N, swings from a cliff at the end of a convenient vine that is 15 m long. From the top of the cliff to the bottom of the swing, he descends by 2.2 m. What is the greatest force on the vine during the swing?



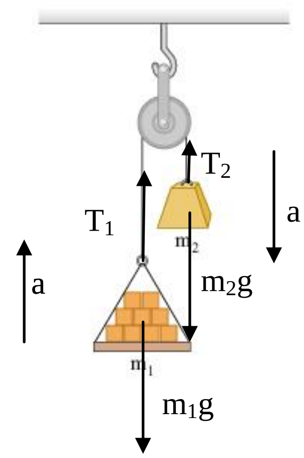
At bottom of swing, $1/2mv^2 = mgh \Rightarrow v = 6.57 m/s$

$\sum F_y = F_V - mg = ma = mv^2/r$

$F_V = mv^2/r + mg = 647N/(9.8m/s^2)(6.57m/s)^2/15m + 647N$

$F_V = 837$ N

3) A load of bricks with mass $m_1 = 15.6$ kg hangs from one end of a rope that passes over a small, frictionless pulley. A counterweight of mass $m_2 = 23.4$ kg is suspended from the other end of the rope, as shown in the figure. The system is released from rest. What is the magnitude of the upward acceleration of the load of bricks?



For mass 1:

$\sum F_y = T_1 - m_1g = m_1a$

For mass 2:

$\sum F_y = T_2 - m_2g = -m_2a$

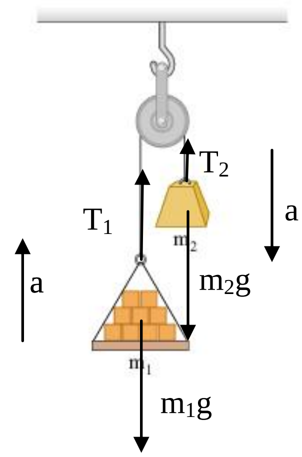
but $T_1 = T_2$, so we can subtract these two equations:

$m_2g - m_1g = m_1a + m_2a$

or:

$a = (m_2 - m_1)g/(m_2 + m_1) = 0.2g = 1.96\text{m/s}^2$.

4) A load of bricks with mass $m_1 = 15.6$ kg hangs from one end of a rope that passes over a pulley of radius 0.1 m and moment of inertia 0.6 kgm^2. A counterweight of mass $m_2 = 23.4$ kg is suspended from the other end of the rope, as shown in the figure. The system is released from rest. What is the magnitude of the upward acceleration of the load of bricks?



For mass 1:

$\sum F_y = T_1 - m_1g = m_1a$

For mass 2:

$\sum F_y = T_2 - m_2g = -m_2a$

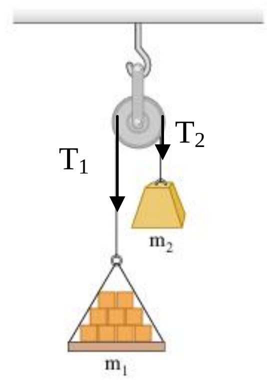
here, T_1 is not equal to T_2,

for the pulley:

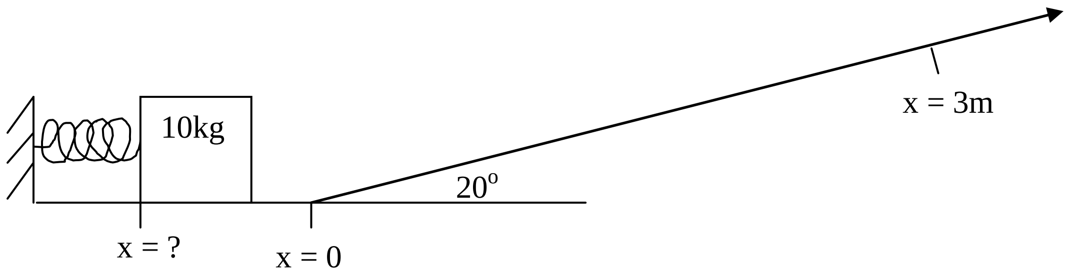
$\sum \tau_z = rT_2 - r\,T_1 = I\alpha$:

$r(T_2 - T_1) = Ia/r \Rightarrow r(m_1a + m_1g - m_2g + m_2a) = Ia/r$

$a = gr(m_2 - m_1)/(I/r + (m_2 + m_1)r) = 0.772 m/s^2$



5) As the figure below shows, a 10kg block is accelerated by a compressed spring whose spring constant is 600N/m. After leaving the spring at the spring's relaxed length, the block travels over a tilted frictionless surface for a distance of 3m before stopping. Through what distance is the spring compressed before the block begins to move?



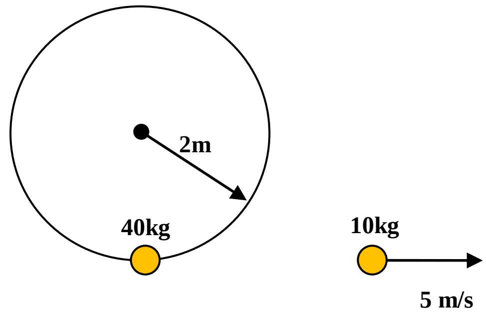
Here, we will use conservation of energy, where the spring potential becomes completely gravitational potential:

$1/2kx^2 = mgh = mgL\sin(20^o)$

and solve for x:

$x = \mathrm{sqrt}(2mgL\sin(20^o)/k) = 0.579$m

6) A merry-go-round with a radius of 2 m and a moment of inertia 100 kgm^2 is initially not rotating. A child of mass 40 kg sits at the edge of the merry-go-round and tosses a 10 kg ball in a line tangent to its edge with a speed of 5 m/s. What is the final angular speed of the merry-go-round?



This involves conservation of angular momentum, $L_i = L_f$.

$$L_i = 0$$

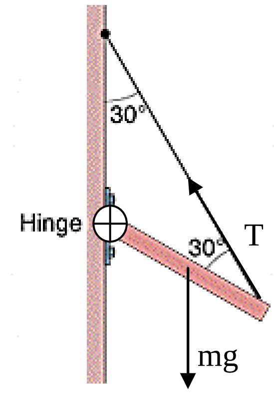
$$L_f = \left[I_{merry-go-round} + I_{child}\right]\omega_f + \left(\vec{r}\times\vec{p}\right)_{ball}$$

$$= \left[\left(100kgm^2\right) + \left(40kg\right)\left(2m\right)^2\right]\left(\omega_f\right) + \left(2m\right)m_{ball}v_{ball} \Rightarrow$$

$$\omega_f = \frac{\left(2m\right)\left(10kg\right)\left(5m/s\right)}{\left[\left(100kgm^2\right) + \left(40kg\right)\left(2m\right)^2\right]} = 0.385\frac{rad}{s}$$

or, about 16s per revolution.

7) One end of a uniform beam that weighs 217 N is attached to a wall with a hinge. The other end is supported by a wire. Find the tension in the wire.



This is an equilibrium problem.

Using the hinge as the axis of rotation:

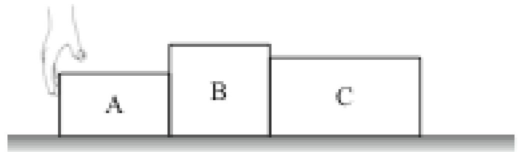
$\sum \tau_z = L/2*mg*\sin 60 - L*T*\cos 60 = 0$

The length cancels (notice I didn't give it to you), and you just solve for T:

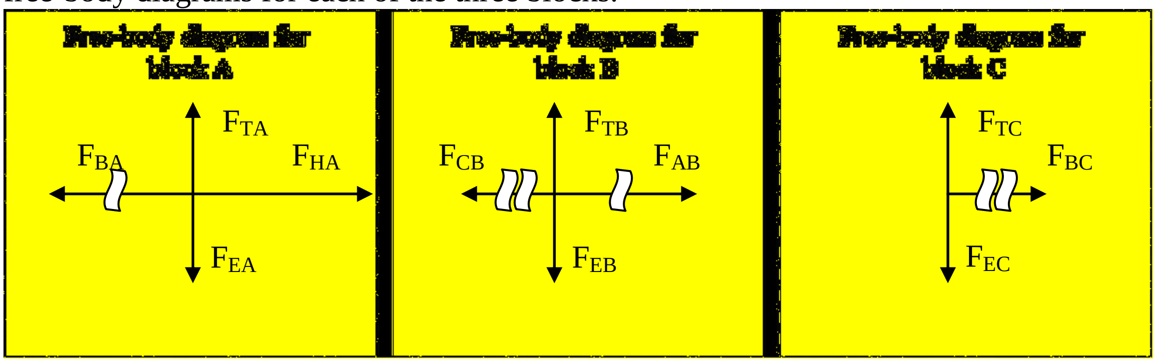
$T = ½*mg*\tan 60 = 188N$

Conceptual Problems

1) Blocks A, B, and C are pushed across a frictionless table by a hand that exerts a constant horizontal force. Block A has mass *M*, block B has mass 2*M*, and block C has mass 3*M*.



a) In the spaces provided, draw and label separate free-body diagrams for each of the three blocks.

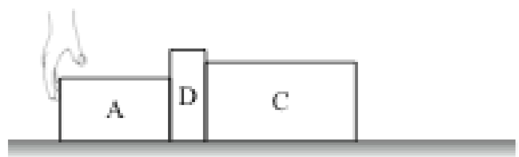


Identify all Newton's third-law force pairs in your diagrams by placing one or more small "~" symbols through each member of the pair.

b) Rank the blocks according to the magnitude of the *net force* on the block, from largest to smallest.

$F_{NetC} > F_{NetB} > F_{NetA}$

Suppose that block B is now replaced by block D, which has a mass that is much smaller than *M*. The hand is still pushing with the same force.



c) Describe the changes (if any) to the motion of the blocks if the hand pushes with the same force. Explain.

From $a = F_{Net}/m$, and since the total mass of the three blocks is now less and the net force is the same as before, they will have a larger acceleration.

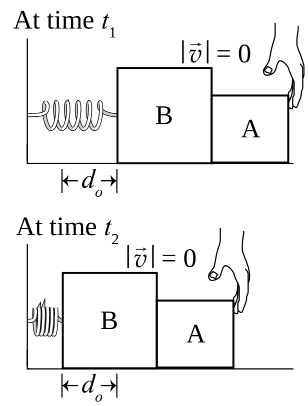
d) Describe any changes to the net force on block A and the net force on block C. Explain.

The net force on both blocks will be larger since $F_{Net} = ma$. The force of block D on A will be smaller and the force of block D on C will be larger.

e) How does the force by block C on block B compare to the force by block C on block D? Explain your reasoning.

The force of block C on block D is larger than block C on block B because it takes a smaller *net* force on block D to accelerate it, and using $F_{NetC} = F_{BC} = F_{CB}$ with block B is less than $F_{NetC} = F_{DC} = F_{CD}$ with block D.

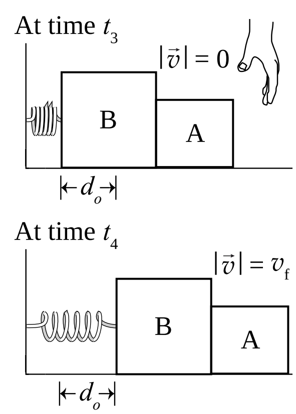
2) Blocks A and B are at rest on a level, frictionless surface. Block B is attached to an ideal massless spring of constant k, which is initially at its equilibrium length. Block B has greater mass than block A ($m_B > m_A$).
During the interval from t_1 to t_2, a hand pushes block A to the left with a constant force of magnitude F_o, compressing the spring a distance d_o, as shown. At time t_2, both blocks again have a speed of zero.
Consider system A consisting of block A and system BS consisting of block B and the spring.



a) During the interval from t_1 to t_2, is the absolute value of the net work done on system BS by external forces, *greater than, less than,* or *equal to* that done on system A by external forces? Explain.

The net work on system BS is greater than that on system A. System A has no net work done on it since system BS is doing negative work on it as the hand is doing positive work. System BS has positive work done on it by system A, but that work is all going into spring potential energy, which is always positive. Hence it has positive external net work done on it.

At some later time, the hand is holding the blocks at rest with the spring compressed a distance d_o. At time t_3, the hand releases the blocks. At time t_4, the spring returns to its equilibrium length and both blocks have speed v_f, as shown.
Consider the interval from t_3 to t_4:



b) Is the change in kinetic energy of system BS *greater than, less than*, or *equal to* that of system A? Explain.

The change in the kinetic energy is greater for system BS than that of system A since both blocks will have the same velocity and block B has more mass (there is a larger net force on block B than on block A over the same distance).

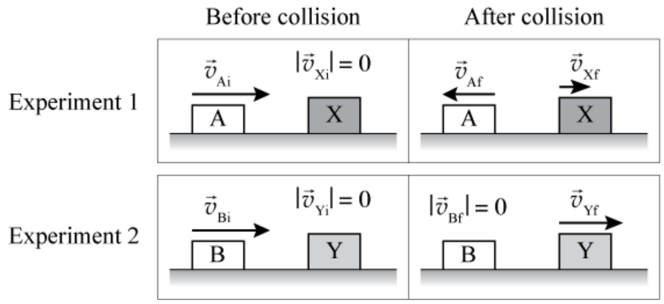
c) Is the absolute value of the net work done on system BS by external forces *greater than, less than*, or *equal to* that done on system A by external forces? Explain.

The net work done by external forces on system BS is negative (the work done by block A on block B), whereas the net work done on system A is positive (the work done by block B on A) and in fact, from Newton's third law the forces have the same magnitude and are applied over the same distance, so the work has equal magnitude.

d) Is the change in total energy of system BS *positive, negative*, or *zero?* Explain.

The *change* in the total energy of system BS is negative, since it had positive spring potential energy, but had negative work done on it during this period from block A. The spring potential (all stored in system BS) is then shared as kinetic between system BS and system A.

3) Two experiments are conducted with gliders on a level, frictionless track. Gliders A and B are both launched toward stationary gliders X and Y as shown. Gliders A and B have the same mass and initial velocities. The mass of glider X is greater than that of glider Y (*i.e.*, $m_X > m_Y$). After the collisions, glider A has reversed direction, and glider B is at rest as shown. The final speed of glider X is less than that of glider Y (*i.e.*, $v_{Xf} < v_{Yf}$).



a) In experiment 1, is the magnitude of the *change in momentum* vector of glider A *greater than, less than,* or *equal to* that of glider X? Explain.

Since momentum is conserved in the collision, the change in momentum of glider A must be equal in magnitude (but opposite in sign) as the change in magnitude of glider X.

b) Is the magnitude of the final momentum of glider X *greater than, less than,* or *equal to* that of glider Y? Explain.

From conservation of momentum, the magnitude of the momentum of glider X after the collision must be equal to the magnitude of glider A before the collision *plus* the magnitude of glider A after the collision. The magnitude of glider Y after the collision is just equal to the magnitude of glider B before the collision. Since the momentum of glider A and B are the same, then the momentum of glider X is greater than momentum of glider Y. Another (briefer) way to say this is that since the momentum before the collision in the two experiments is the same, the momentum in the two experiments after the collision is the same, and since in experiment 1, glider A has momentum to the left, then glider X has more momentum to the right.