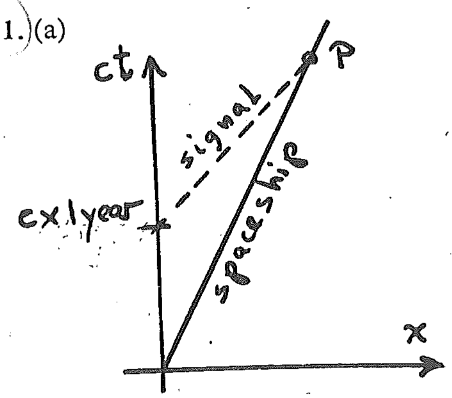
1.(a)



(b) The heavy solid line in the Figure is the worldline of the spaceship; the outward-bound portion has the equation $x = 0.5ct$. The dashed line in the Figure is the worldline of the radio signal; it has the equation $x = c(t - 1 \text{ year})$. The intercept P is given by $0.5ct = c(t - 1 \text{ year})$, from which $0.5t = 1$ year and

$$\boxed{t = 2 \text{ years}}$$

2. (a) The angle between the x,x′ axes is $\theta = \tan^{-1} V/c = \tan^{-1} 0.3 = 17^{\circ}$.
(b) Graphically, $x = 2$, $ct = 4$ corresponds to $x' \simeq 0.84$, $ct' \simeq 3.6$.
(c) The heavy sloping line in the Figure is the worldline of the electron. The slope relative to the x,ct axes is the increment of x divided by the increment of ct, which is 0.3 (this is the slope measured away from the ct axis, that is, away from the vertical). The slope relative to the x′,ct′ axes is obtained by dividing the increment in x′ by the increment of ct′. At the point P, these increments are, respectively $\simeq 0.84$ and $\simeq 3.6$,

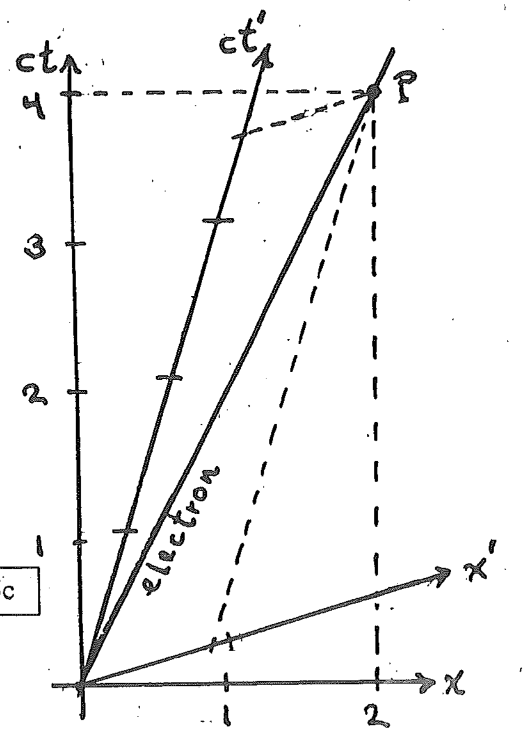
and therefore $v'_x = \dfrac{0.84}{3.6} c = 0.23c$

(d) From Eqs. (12), (13), and (35)

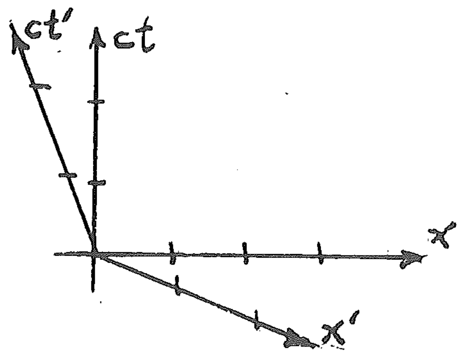
$$x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}} = \frac{2 - 0.3 \times 4}{\sqrt{1 - 0.3^2}} = \boxed{0.839}$$

$$ct' = \frac{ct - Vx/c}{\sqrt{1 - V^2/c^2}} = \frac{4 - 0.3 \times 2}{\sqrt{1 - 0.3^2}} = \boxed{3.56}$$

$$v'_x = \frac{v_x - V}{1 - Vv_x/c^2} = \frac{0.5 - 0.3}{1 - 0.3 \times 0.5} c = \boxed{0.235c}$$



3. The angle between the x,x′ axes is $\theta = \tan^{-1} V/c = \tan^{-1}(-0.4) = -22^{\circ}$. For a negative velocity V, the angle between the ct′,x′ axes is more than 90°.



4. In the reference frame of the Earth, $\Delta t = 0$. But in the reference frame of the spaceship

$$\Delta t' = \frac{\Delta t - V\Delta x/c^2}{\sqrt{1 - V^2/c^2}} \simeq 0 - 750 \text{ km/h} \times (c \times 10^3 \text{ years})/c^2$$

$$= \boxed{-2.2 \times 10^4 \text{ s} = -6.1 \text{ h}}$$

Tucana is first, by 6.1 hours.

5. (a) The travel time of the light pulse from tail to nose is

$$t' = (300\text{ m})/(3 \times 10^8\text{ m/s}) = \boxed{1.0 \times 10^{-6}\text{ s}}$$

(b) In the reference frame of the Earth, the difference between the speeds of the light pulse and the spaceship (or the rate at which the light pulse gains on the spaceship) is 0.2c. The length of the spaceship is 300 m × $\sqrt{1 - 0.80^2}$. Hence the travel time from tail to nose is

$$t = \frac{300\text{ m} \times \sqrt{1 - 0.80^2}}{0.2c} = \boxed{3.0 \times 10^{-6}\text{ s}}$$

Alternatively, from the Lorentz transformation,

$$t = \frac{t' + Vx'/c^2}{\sqrt{1 - V^2/c^2}} = \frac{1.0 \times 10^{-6}\text{ s} + 0.8 \times 300\text{ m}/c}{\sqrt{1 - 0.80^2}} = \boxed{3.0 \times 10^{-6}\text{ s}}$$

7. (a) The calculation is best done in the inertial reference frame centered on the Earth (but not rotating). In a time t, the light signal travels a distance ct, and a point on the equator travels a distance ωRt. For a westward signal, the return to the starting point occurs when ct = 2πR - ωRt, so t = 2πR/(c + ωR); for a eastward signal, ct = 2πR + ωRt, so t = 2πr/(c - ωR). The westward signal arrives first, and the delay between the eastward and the westward signals is

$$\frac{2\pi R}{c - \omega R} - \frac{2\pi r}{c + \omega R} \simeq 4\pi \frac{\omega R^2}{c^2} = 4\pi \frac{2\pi}{24\text{ hours}} \frac{(6.4 \times 10^6\text{ m})^2}{(3 \times 10^8\text{ m/s})^2} = \boxed{4.1 \times 10^{-7}\text{ s}}$$

This is the time as reckoned by the clocks of the inertial reference frame. A clock sitting on the equator of the Earth has time dilation relative to the inertial reference frame and registers a time smaller by a factor $\sqrt{1 - \omega^2R^2/c^2}$; but this correction factor is very near 1 and can be ignored for the purposes of this problem.

(b) No. The rotating reference frame of the Earth is not an inertial reference frame, and the Principle of Relativity does not apply.

10. In the x,y,z frame, the corresponding spokes, indicated by the dashed lines, reach the vertical position simultaneously, say, at t = 0. But in the x′,y′,z′ frame, these spokes reach the vertical position at different times, t′ = 0 and

$$t' = \frac{0 - Vx/c^2}{\sqrt{1 - V^2/c^2}} = \frac{-VL/c^2}{\sqrt{1 - V^2/c^2}}$$

The angular velocity of rotation in the x′,y′,z′ frame is related to the angular velocity in the x,y,z frame by the same formula as the y or z components of the velocity,

$$\omega' = \frac{\sqrt{1 - V^2/c^2}\,\omega}{1 - Vv_x/c^2} = \sqrt{1 - V^2/c^2}\,\omega$$

The angular displacement between the spokes equals the angular velocity ω′ multiplied by the time difference,

$$\Delta\phi' = \omega' \frac{-VL/c^2}{\sqrt{1 - V^2/c^2}} \simeq \boxed{-\frac{VL}{c^2}\omega}$$

$$v'_{trans} = \omega' R = \frac{\sqrt{1 - V^2/c^2}}{1 - Vv_x/c^2}\omega R = \boxed{\sqrt{1 - V^2/c^2}\,\omega R}$$

(b) In the above equation, the same R appears on both sides of the equation, since transverse dimensions are unaffected by the Lorentz transformation. Cancellation of R on both sides gives

$$\boxed{\omega' = \sqrt{1 - V^2/c^2}\,\omega}$$

Note that the factor $\sqrt{1 - V^2/c^2}$ is simply the time dilation factor--in the spaceship frame, the rotation of the wheel suffers time dilation.

12. The first Lorentz transformation is

$$x' = \frac{x - V_1t}{\sqrt{1 - V_1^2/c^2}} \qquad t' = \frac{t - V_1x/c^2}{\sqrt{1 - V_1^2/c^2}}$$

and the second Lorentz transformation is

$$x'' = \frac{x' - V_2t'}{\sqrt{1 - V_2^2/c^2}} \qquad t'' = \frac{t' - V_2x'/c^2}{\sqrt{1 - V_2^2/c^2}}$$

Substituting the first transformation into the second, we find

$$x'' = \frac{1}{\sqrt{1 - V_2^2/c^2}} \frac{1}{\sqrt{1 - V_1^2/c^2}} (x - V_1t - V_2t + V_2V_1x/c^2)$$

$$= \frac{1 + V_1V_2/c^2}{\sqrt{1 - V_2^2/c^2}\sqrt{1 - V_1^2/c^2}} [x - (V_1 + V_2)t/(1 + V_1V_2/c^2)]$$

$$= \frac{1}{\sqrt{1 - (V_1 + V_2)^2/c^2(1 + V_1V_2/c^2)^2}}[x - (V_1 + V_2)t/(1 + V_1V_2/c^2)]$$

This is a Lorentz transformation with velocity

$$\boxed{V = \frac{V_1 + V_2}{1 + V_1V_2/c^2}}$$

Substitution into the equation for t″ gives the same result.

13. If we regard the equations
$x' = \gamma(x - Vt) \qquad ct' = \gamma(ct - Vx/c)$
as two equations for the unknowns x,t, we find that the solution for x is

$$x = \frac{1}{\gamma}\frac{1}{1 - V^2/c^2}(x' + Vt')$$

If, instead, we exchange prime and unprimed coordinates and replace V by - V, we find
$x = \gamma(x' + Vt')$
Comparing these two expressions for x, we see that

$$\frac{1}{\gamma}\frac{1}{1 - V^2/c^2} = \gamma$$

from which $\boxed{\gamma = \frac{1}{\sqrt{1 - V^2/c^2}}}$

Application of the same method to the solution for t gives the same result.

17. The displacement from one end of the meter stick to the other in x,y coordinates is

$\Delta x = 1 \text{ m} \times \cos\phi$
$\Delta y = 1 \text{ m} \times \sin\phi$

The displacement $\Delta x'$at one instant t' is given by

$$\Delta x = \frac{\Delta x' + V\Delta t'}{\sqrt{1 - V^2/c^2}} = \frac{\Delta x' + 0}{\sqrt{1 - V^2/c^2}}$$

or

$$\Delta x' = \sqrt{1 - V^2/c^2}(1 \text{ m} \times \cos\phi)$$

and the displacement $\Delta y'$ is

$$\Delta y' = \Delta y = 1 \text{ m} \times \sin\phi$$

Hence $\tan\phi' = \frac{\Delta y'}{\Delta x'} = \boxed{\frac{\tan\phi}{\sqrt{1 - V^2/c^2}}}$

For $\phi = 30^0$ and $V/c = 0.8$,

$$\tan\phi' = \frac{\tan 30^0}{\sqrt{1 - 0.8^2}} = 0.962$$

and $\boxed{\phi' = 44^0}$

18. The squares of the Lorentz transformation equations are

$$(ct')^2 = \frac{(ct - Vx/c)^2}{1 - V^2/c^2} = \frac{c^2t^2 - 2tVx + V^2x^2/c^2}{1 - V^2/c^2}$$

$$x'^2 = \frac{(x - Vt)^2}{1 - V^2/c^2} = \frac{x^2 - 2tVx + V^2t^2}{1 - V^2/c^2}$$

Hence

$$(ct')^2 - x'^2 = \frac{c^2t^2 + V^2x^2/c^2 - x^2 - V^2t^2}{1 - V^2/c^2}$$

20. Since $c^2(\Delta t)^2 - (\Delta x)^2 = (3 \times 10^8)^2(8.0)^2 \text{ m}^2 - (2.0 \times 10^9)^2 \text{ m}^2 = 1.8 \times 10^{18} \text{ m}^2 > 0$, the two events are separated by a timelike interval. There exists no spaceship frame in which they are simultaneous, but there exists a spaceship frame in which they occur at the same point of space. This spaceship frame has a speed of 2.0×10^9 m/ 8.0 s = 2.5×10^8 m/s relative to the laboratory. The time interval between the events in the spaceship frame is $\sqrt{1.8 \times 10^{18} \text{ m}^2}/c = 1.3 \times 10^9 \text{ m}/c = \boxed{4.4 \text{ s}}$