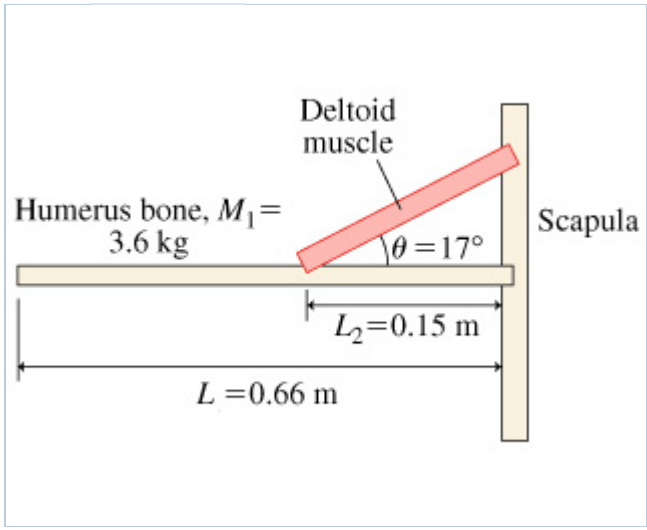
#31 Equillibrium of Rigid Bodies I Post-class

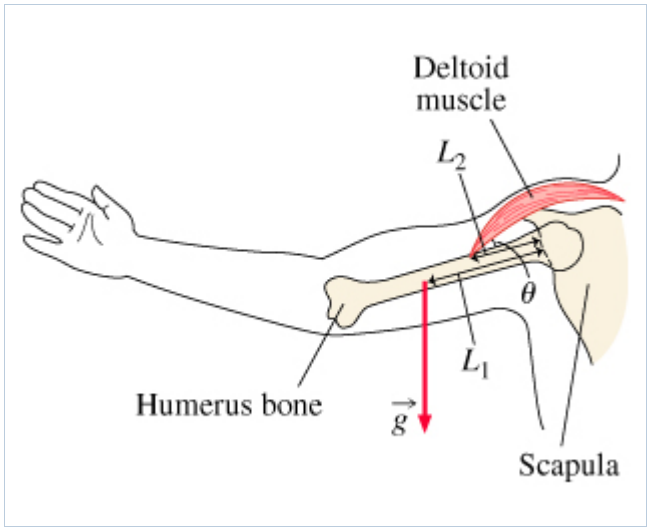
Due: 11:00am on Monday, November 5, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Static Equilibrium of the Arm

You are able to hold out your arm in an outstretched horizontal position because of the action of the deltoid muscle. Assume the humerus bone has a mass $M_1 = 3.6\ \text{kg}$, length $L = 0.66\ \text{m}$ and its center of mass is a distance $L_1 = 0.33\ \text{m}$ from the scapula. (For this problem ignore the rest of the arm.) The deltoid muscle attaches to the humerus a distance $L_2 = 0.15\ \text{m}$ from the scapula. The deltoid muscle makes an angle of $\theta = 17°$ with the horizontal, as shown. Use $g = 9.8\ \text{m/s}^2$ throughout the problem.





Part A

Find the tension T in the deltoid muscle.

Express the tension in newtons, to the nearest integer.

Hint 1. Nature of the problem

Remember that this is a statics problem, so all forces and torques are balanced (their sums equal zero).

Hint 2. Origin of torque

Calculate the torque about the point at which the arm attaches to the rest of the body. This allows one to balance the torques without having to worry about the undefined forces at this point.

Hint 3. Adding up the torques

Add up the torques about the point in which the humerus attaches to the body.

Answer in terms of L_1, L_2, M_1, g, T, and θ. Remember that counterclockwise torque is positive.

ANSWER:

$\tau_{\text{total}} = 0 = L_1 M_1 g - T \sin(\theta) L_2$

ANSWER:

T = 265 N

Correct

Part B

Using the conditions for static equilibrium, find the magnitude of the vertical component of the force F_y exerted by the scapula on the humerus (where the humerus attaches to the rest of the body).

Express your answer in newtons, to the nearest integer.

Hint 1. Total forces involved

Recall that there are three vertical forces in this problem: the force of gravity acting on the bone, the force from the vertical component of the muscle tension, and the force exerted by the scapula on the humerus (where it attaches to the rest of the body).

ANSWER:

$|F_y|$ = 42 N

Correct

Part C

Now find the magnitude of the horizontal component of the force F_x exerted by the scapula on the humerus.

Express your answer in newtons, to the nearest integer.

ANSWER:

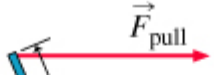
$|F_x|$ = 254 N

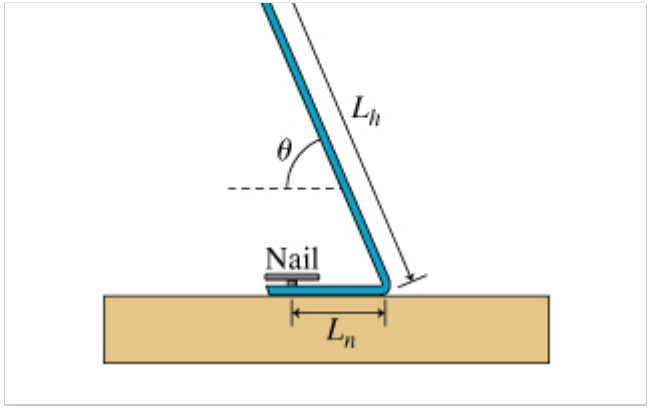
Correct

Pulling Out a Nail

A nail is hammered into a board so that it would take a force F_{nail}, applied straight upward on the head of the nail, to pull it out. (Take an upward force to be positive.)

A carpenter uses a crowbar to try to pry it out. The length of the handle of the crowbar is L_{h}, and the length of the forked portion of the crowbar (which fits around the nail) is L_{n}. Assume that the forked portion of the crowbar is perfectly horizontal. The handle of the crowbar makes an angle θ with the horizontal, and the carpenter pulls directly along the horizontal.





Part A

With what force F_{pull} must the carpenter pull on the crowbar to remove the nail?

Express the force in terms of F_{nail}, L_{h}, L_{n}, and θ.

Hint 1. Torque from applied force

What is the torque the carpenter provides around the pivot point? Recall that counterclockwise torques are positive.

Express the torque in terms of F_{pull}, θ, and either L_{h} *or* L_{n}.

Hint 1. Origin of torque

Consider the torques around the point of contact of the bend in the crowbar and the surface. This eliminates the torque due to the unknown contact force between he crowbar and the surface. Remember to pay attention to signs!

Hint 2. Calculating torque

The distance used for calculating a torque from a force (called the lever arm) is the *perpendicular* distance from the pivot to the "line of action" of the force. The line of action is the line along which the force acts, extending forward and backward from the point of exertion of the force. In this problem, a force is being exerted in the horizontal direction, so the lever arm used to calculate the torque is the vertical distance from the pivot point to the line of action of the force.

ANSWER:

$\tau_{\text{pull}} = -F_{\text{pull}} \sin(\theta) L_h$

Hint 2. Torque from nail

What is the torque around this pivot point due to the force of the nail on the crowbar when the nail is pulled hard enough to remove it? Again, take counterclockwise torques as positive. Remember that the nail is acting on the crowbar in this problem.

Give your answer in terms of F_{nail} and either L_{h} or L_{n}.

ANSWER:

$\tau_{\text{nail}} = F_{\text{nail}} L_n$

Hint 3. Total torque

The torques must sum to zero since the crowbar is static.

ANSWER:

$F_{\text{pull}} = \dfrac{F_{\text{nail}} L_n}{\sin\theta L_h}$

Correct

Now, imagine that F_{pull} is not large enough to dislodge the nail. In other words, the nail stays in place, and, if the surface below the crowbar weren't present, the crowbar would rotate around the point of contact with the nail. This makes it natural to take the pivot point to be the point where the crowbar is in contact with the nail. (But you are always free to choose the pivot point to be *any fixed point*, even one some distance from the object.)

Part B

What is the magnitude of the normal force that the surface exerts on the crowbar, F_{bar}?

Express your answer for the normal force in terms of F_{pull}, θ, L_{n}, and L_{h}. Take the upward direction to be positive.

Hint 1. Torque applied

What is the torque that F_{pull} exerts around the pivot point? Counterclockwise torques are positive as usual.

Answer in terms of F_{pull}, θ, and either L_{n} or L_{h}?

Hint 1. Calculating torque

The distance used for calculating a torque from a force (called the lever arm) is the *perpendicular* distance from the pivot to the "line of action" of the force. The line of action is the line along which the force acts, extending forward and backward from the point of exertion of the force. In this problem, a force is being exerted in the horizontal direction, so the lever arm used to calculate the torque is the vertical distance from the pivot point to the line of action of the force.

ANSWER:

$\tau_{\text{pull}} = -F_{\text{pull}} L_h \sin(\theta)$

Hint 2. Torque from contact

What is the torque about the contact point between the crowbar and the nail due to the contact force with the surface?

Express your answer in terms of F_{bar} and either L_{n} or L_{h}.

Hint 1. Newton's 3rd law

This part relies on Newton's 3rd law. If the bar exerts a force of magnitude F_{bar} on the surface directly downward, what is the magnitude and direction of the force that the surface exerts on the bar? Direction is important!

ANSWER:

$\tau_{\text{surface}} = F_{\text{bar}} L_n$

Hint 3. Total torque

What must be true about the total torque around the pivot point, if the bar does not rotate?

ANSWER:

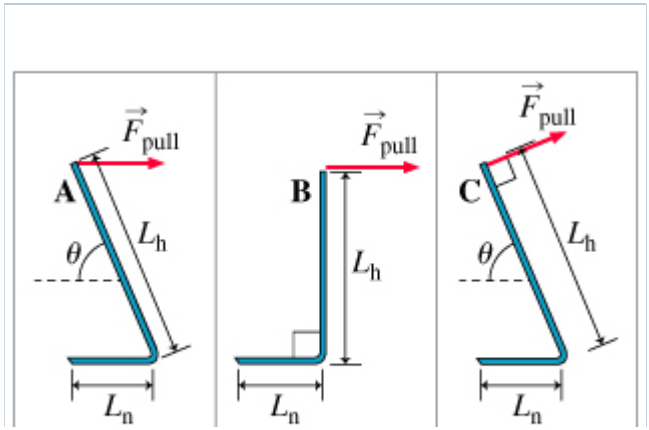
$F_{\text{bar}} = \dfrac{F_{\text{pull}} \sin(\theta)\, L_h}{L_n}$

Correct

As you can see, even with a relatively small force, the force exerted on the surface by a long crowbar can be quite great (especially if L_n is small). This is why a crowbar can damage a surface quite easily if the nail is lodged firmly in the material.

Note that you can also obtain the answer to this part by considering that the sum of the forces in the *y* direction on the crowbar must be zero, and using equations from the first part to find the force exerted on the nail.

Three bars are shown in the figure. Both bars A and B have F_{pull} acting on them in the horizontal direction. Bar C has F_{pull} strictly perpendicular to the bar. L_h, L_n, and θ are the same quantities in each case.



Part C

Let the magnitude of the torque *about the bend* in the crowbars be denoted τ_A, τ_B and τ_C for each of the three cases shown. Which of the following is the correct relationship between the magnitude of of the torques?

Hint 1. Calculating torque

The distance used for calculating a torque from a force (called the lever arm) is the *perpendicular* distance from the pivot to the "line of action" of the force. The line of action is the line along which the force acts, extending forward and backward from the point of exertion of the force.

Hint 2. Find the three torques

What are the *magnitudes* τ_A, τ_B, τ_C of the torques shown in figures A, B, and C respectively?

Express your answer in terms of some or all of the variables F_{pull}, θ, L_{h}, and L_{n}. Answer in the order indicated. Separate your answers using commas as shown.

ANSWER:

τ_A, τ_B, τ_C = $F_{\text{pull}} L_h \sin(\theta)$, F_pull*L_h, F_pull*L_h

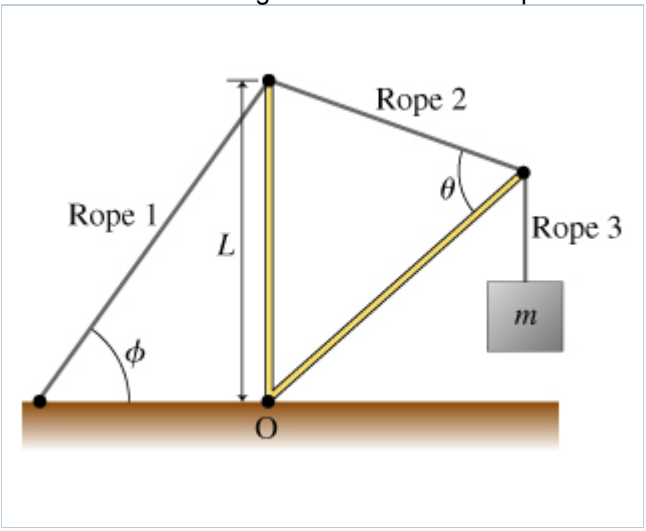
ANSWER:

- $\tau_A > \tau_B > \tau_C$
- $\tau_A > \tau_B = \tau_C$
- $\tau_A = \tau_B = \tau_C$
- $\tau_A < \tau_B = \tau_C$
- $\tau_A < \tau_B < \tau_C$
- $\tau_A = \tau_B > \tau_C$
- \tau_A = \tau_B < \tau_C

Correct

Quarry Crane

A quarry crane is used to lift massive rocks from a quarry pit. Consider the simplified model of such a crane shown in the figure. The ends of two poles are anchored to the ground at the same point (point O). From this point, one pole rises vertically and the second pole rises at an angle. The vertical pole has its free end connected to the ground via an unstretchable, massless rope labeled rope 1. A second rope, labeled rope 2, connects the free ends of the two poles. The angle between the tilted pole and rope 2 is θ. Both poles have length L and can be considered massless for the purposes of this problem. Hanging from the end of the second pole, via rope 3, is a granite block of mass m.



Throughout this problem use g for the magnitude of the acceleration due to gravity.

Part A

Find T_3, the tension in rope 3.

Express your answer in terms of some (or all) of the following quantities: π, m, g, L, θ, and ϕ.

Hint 1. Tension and force

The tension in a rope indicates how much force is applied to an object to which the rope is attached. The direction of the applied force is along the line defined by the rope. In this case, rope 3 pulls upward on the granite block with a force of magnitude T_3.

ANSWER:

$T_3 = mg$

Correct

Part B

Find T_2, the tension in rope 2.

Express the tension in terms of some (or all) of the following quantities: π, m, g, L, θ, and ϕ.

Hint 1. How to approach the problem

Because the tilted pole is not rotating, you know that the total torque on the pole must be zero. Find an expression for the sum of all torques acting on the pole. (If you choose the correct pivot point, these will just be the torques due to the tensions in rope 2 and rope 3.) Set this expression equal to zero, and solve for the magnitude of the tension in rope 2.

Hint 2. Find the torque due to the tension in rope 2

What is the torque τ_2 on the angled pole, about point O, due to the tension in rope 2?

Recall the sign convention for torque: A positive torque would cause counterclockwise rotation of the pole; a negative torque would cause clockwise rotation.

Express your answer in terms of T_2, L, θ, and π.

Hint 1. Find the moment arm of the tension force

The torque on the pole is given by \tau = l_{\rm 2} T_2, where l_2 is the moment arm of the tension force. Find l_2.

Express your answer in terms of L and θ.

ANSWER:

$l_2 = L\sin(\theta)$

ANSWER:

$\tau_2 = T_2 L\sin(\theta)$

Hint 3. Find the torque due to the tension in rope 3

What is the torque τ_3 on the angled pole, about point O, due to the granite block (via the tension in rope 3)?

Recall the sign convention for torque: A positive torque would cause counterclockwise rotation of the pole; a negative torque would cause clockwise rotation.

Express your answer in terms of m, L, g, and θ.

Hint 1. Find the angle between rope 3 and the tilted pole

Use geometry to find the angle between rope 3 and the tilted pole.

Express your answer in terms of θ and π.

ANSWER:

$\angle$ = $\pi - 2\theta$

ANSWER:

τ_3 = -m*g*L*sin(pi-2*theta)

ANSWER:

T_2 = \frac{mg{\sin}\left({\pi}-2{\theta}\right)}{{\sin}\left({\pi}-{\theta}\right)}

Correct

Part C

Find T_1, the tension in rope 1.

Express the tension in terms of some (or all) of the following quantities: m, g, L, θ, and ϕ.

Hint 1. How to approach the problem

Because the vertical pole is not rotating, you know that the total torque on the pole must be zero. Find an expression for the sum of all torques acting on the pole. (If you choose the correct pivot point, these will just be the torques due to the tensions in rope 1 and rope 2.) Set this expression equal to zero, and solve for the magnitude of the tension in rope 1.

Hint 2. Find the torque due to the tension in rope 1

What is the torque tau_1 on the *vertical* pole, about point O, due to the tension in rope 1?

Recall the sign convention for torque: A positive torque would cause counterclockwise rotation of the pole; a negative torque would cause clockwise rotation.

Express your answer in terms of T_1, L, and ϕ.

ANSWER:

tau_1 = T_1*L*cos(phi)

ANSWER:

T_1 = \frac{mg{\sin}\left({\pi}-2{\theta}\right)}{{\cos}\left({\phi}\right)}

Correct

Score Summary:

Your score on this assignment is 104.3%.
You received 31.3 out of a possible total of 30 points.