23.62. Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_e = qE$. The potential difference between the plates is $V = Ed$.

The free-body diagram for the sphere is given in Figure 23.62.

$T\cos\theta = mg$ and $T\sin\theta = F_e$ gives $F_e = mg\tan\theta = (1.50\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan(30°) = 0.0085\text{ N}$.

$$F_e = Eq = \frac{Vq}{d} \text{ and } V = \frac{Fd}{q} = \frac{(0.0085\text{ N})(0.0500\text{ m})}{8.90\times10^{-6}\text{ C}} = 47.8\text{ V}.$$

$E = V/d = 956$ V/m. $E = \sigma/\epsilon_0$ and $\sigma = E\epsilon_0 = 8.46\times10^{-9}\text{ C/m}^2$.

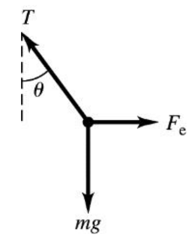


Figure 23.62

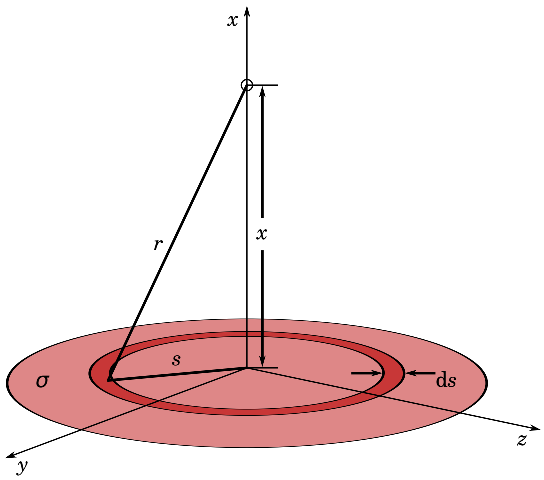
23.64. The wire and hollow cylinder form coaxial cylinders. Problem 23.63 gives $E(r) = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$.

$a = 145\times10^{-6}$ m, $b = 0.0180$ m.

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} \text{ and } V_{ab} = E\ln(b/a)r = (2.00\times10^4\text{ N/C})(\ln(0.018\text{ m}/145\times10^{-6}\text{ m}))0.012\text{ m} = 1157\text{ V}.$$

The electric field at any r is directly proportional to the potential difference between the wire and the cylinder.

68.



a) $\mathrm{d}V = \frac{1}{4\,\pi\,\epsilon_0}\,\frac{\mathrm{d}q}{r}$, $r = \sqrt{s^2 + x^2}$, $\mathrm{d}q = 2\,\pi\,\sigma\,s\,\mathrm{d}s$, so $\mathrm{d}V = \frac{\sigma}{2\,\epsilon_0}\,\frac{s\,\mathrm{d}s}{\sqrt{s^2 + x^2}}$.

$$V = \int_0^R \frac{\sigma}{2\,\epsilon_0}\,\frac{s\,\mathrm{d}s}{\sqrt{s^2 + x^2}} = \frac{\sigma}{2\,\epsilon_0}\,\sqrt{s^2 + x^2}\,\bigg|_{s=0}^{R} = \frac{\sigma}{2\,\epsilon_0}\left(\sqrt{R^2 + x^2} - |x|\right)$$

b)

$$-\frac{\partial V}{\partial x} = -\frac{\sigma}{2\,\epsilon_0}\left(\frac{x}{\sqrt{R^2 + x^2}} - \frac{x}{|x|}\right) = \frac{\sigma}{2\,\epsilon_0}\,\frac{x}{|x|}\left(1 - \frac{1}{\sqrt{R^2/x^2 + 1}}\right).$$

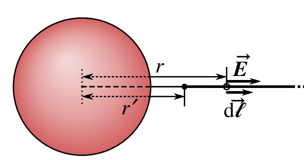
Ignoring $\frac{x}{|x|} = \pm 1$, which just provides the correct sign when $x < 0$, this is the same as the result found for E_x in Example 21.11.

72.

a) From Example 22.9, the field of a uniformly charged sphere with total charge Q and radius R is

$$\vec{\boldsymbol{E}} = \begin{cases} \frac{1}{4\,\pi\,\epsilon_0}\,\frac{Q\,r}{R^3}\,\hat{\boldsymbol{r}}\,, & r < R \\ \frac{1}{4\,\pi\,\epsilon_0}\,\frac{Q}{r^2}\,\hat{\boldsymbol{r}}\,, & r \geqslant R. \end{cases}$$

To find the potential, we can integrate $\vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}}$ along a path from r' to infinity, where r' is the distance from the center of the sphere to the point in question.



For $r' \geqslant R$,

$$\begin{aligned} V(r') &= V(r') - V(\infty) = \int_{r'}^{\infty} \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}} = \int_{r'}^{\infty} \frac{1}{4\,\pi\,\epsilon_0}\,\frac{Q}{r^2}\,\hat{\boldsymbol{r}} \cdot (\hat{\boldsymbol{r}}\,\mathrm{d}r) \\ &= \int_{r'}^{\infty} \frac{Q}{4\,\pi\,\epsilon_0}\,\frac{\mathrm{d}r}{r^2} = -\frac{1}{4\,\pi\,\epsilon_0}\,\frac{Q}{r}\bigg|_{r=r'}^{\infty} = \frac{1}{4\,\pi\,\epsilon_0}\,\frac{Q}{r'}, \end{aligned}$$

and for $r' < R$,

$$\begin{aligned} V(r') &= \int_{r'}^{\infty} \vec{\boldsymbol{E}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}} = \int_{r'}^{R} \frac{1}{4\,\pi\,\epsilon_0} \frac{Q\,r}{R^3} \hat{\boldsymbol{r}} \cdot (\hat{\boldsymbol{r}}\,\mathrm{d}r) + \int_{R}^{\infty} \frac{1}{4\,\pi\,\epsilon_0} \frac{Q}{r^2} \hat{\boldsymbol{r}} \cdot (\hat{\boldsymbol{r}}\,\mathrm{d}r) \\ &= \int_{r'}^{R} \frac{1}{4\,\pi\,\epsilon_0} \frac{Q}{R^3} r\,\mathrm{d}r + \frac{1}{4\,\pi\,\epsilon_0} \frac{Q}{R} = \frac{1}{8\,\pi\,\epsilon_0} \frac{Q}{R^3} r^2 \bigg|_{r=r'}^{R} + \frac{1}{4\,\pi\,\epsilon_0} \frac{Q}{R} \\ &= \left(\frac{1}{8\,\pi\,\epsilon_0} \frac{Q}{R} - \frac{1}{8\,\pi\,\epsilon_0} \frac{Q\,r'^2}{R^3} \right) + \frac{1}{4\,\pi\,\epsilon_0} \frac{Q}{R} = \frac{1}{8\,\pi\,\epsilon_0} \frac{Q}{R} \left(3 - \frac{r'^2}{R^2} \right). \end{aligned}$$

To summarize,

$$V(r) = \begin{cases} \frac{1}{8\,\pi\,\epsilon_0} \frac{Q}{R} \left(3 - \frac{r^2}{R^2} \right), & r < R \\ \frac{1}{4\,\pi\,\epsilon_0} \frac{Q}{r}, & r \geqslant R. \end{cases}$$

b)