ANSWERS TO REVIEW PROBLEMS

A. Approximating Integrals

- 1. 41.4 (approximate answer, depends on what you estimated the values at the midpoints to be)
- 2. (a) ≈ 0.555 (b) ≈ 0.513 (c) = 0.50
- 3. (a) = 0.0625 (b) = 0.078125 (c) ≈ 0.083333

B. Double and Triple Integrals

1. (a) Chapter 16 Review, Exercises.

#10:
$$\int_0^4 \int_{y-4}^{4-y} f(x,y) dx dy$$

#12: the region between the two spheres centered at the origin of radius 1 and 2, in the first octant.

#30:
$$\int_0^1 \int_{y+1}^{4-2y} \int_0^{x^2y} dz dx dy = 2.5$$

#42:
$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^6 \sin \phi d\rho d\phi d\theta = \frac{\pi}{14}$$

(b) Section 16.8

24:
$$\int_{0}^{2\pi} \int_{\pi/4}^{\pi/2} \int_{0}^{2} \rho^{2} \sin \phi d\rho d\phi d\theta = \frac{8\sqrt{2}\pi}{3}$$

- 2. $\frac{1}{2} \sin 1$
- 3. $9\pi/2$

4.
$$\int_0^{2\pi} \int_0^{\pi/6} \int_0^2 \rho^2 \sin\phi d\rho d\phi d\theta = \frac{16\pi}{3} (1 - \sqrt{3}/2)$$

5.
$$\int_{0}^{2} \int_{0}^{1-y/2} \int_{0}^{2-2x-y} y \, dz dx dy = \int_{0}^{2} \int_{0}^{2-z} \int_{0}^{1-y/2-z/2} y \, dx dy dz = \int_{0}^{2} \int_{0}^{1-z/2} \int_{0}^{2-2x-z} y \, dy dx dz$$

6. (0,0,3a/8)

7.
$$\int_0^{\pi/2} \int_0^3 \int_0^{\frac{r \cos \theta}{3}} dx r dr d\theta = 9$$

8.
$$\int_0^{\pi/2} \int_0^1 \int_0^{2-r\cos\theta} r\sin\theta dx r dr d\theta = 5/12$$

9. 16/3

$$10. \int_{0}^{2\pi} \int_{0}^{\pi/6} \int_{0}^{2} \rho^{2} \sin \phi d\rho d\phi d\theta = \int_{-1}^{1} \int_{-\sqrt{1-x^{2}}}^{\sqrt{1-x^{2}}} \int_{\sqrt{3x^{2}+3y^{2}}}^{\sqrt{4-x^{2}-y^{2}}} dz dy dx = \int_{0}^{2\pi} \int_{0}^{1} \int_{\sqrt{3}r}^{\sqrt{4-r^{2}}} dz r dr d\theta$$

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