Lecture 39
(AC Circuits)

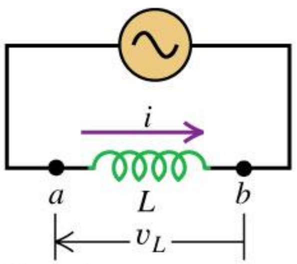
Physics 161-01 Spring 2012

Douglas Fields

CPS 39-1

An inductor is connected across an ac source as shown. For this circuit, what is the relationship between the instantaneous current i through the inductor and the instantaneous voltage v_{ab} across the inductor?

(a) Circuit with ac source and inductor



© 2012 Pearson Education, Inc.

A. i is maximum at the same time as v_{ab}.

B. i is maximum one-quarter cycle before v_{ab}.

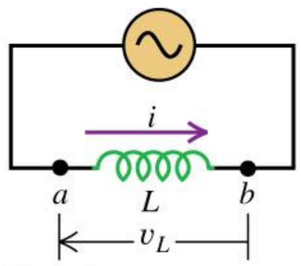
C. i is maximum one-quarter cycle after v_{ab}.

D. not enough information given to decide

CPS 39-1

An inductor is connected across an ac source as shown. For this circuit, what is the relationship between the instantaneous current i through the inductor and the instantaneous voltage v_{ab} across the inductor?

(a) Circuit with ac source and inductor



© 2012 Pearson Education, Inc.

A. i is maximum at the same time as v_{ab}.

B. i is maximum one-quarter cycle before v_{ab}.

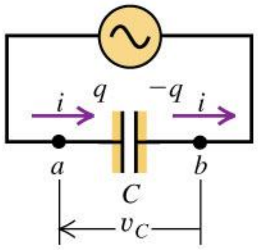
C. i is maximum one-quarter cycle after v_{ab}.

D. not enough information given to decide

CPS 39-2

A capacitor is connected across an ac source as shown. For this circuit, what is the relationship between the instantaneous current *i* through the capacitor and the instantaneous voltage v_{ab} across the capacitor?

(a) Circuit with ac source and capacitor



© 2012 Pearson Education, Inc.

A. *i* is maximum at the same time as v_{ab}.

B. *i* is maximum one-quarter cycle before v_{ab}.

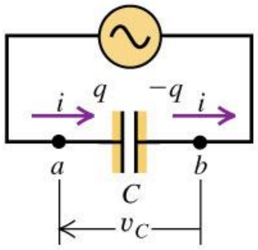
C. *i* is maximum one-quarter cycle after v_{ab}.

D. not enough information given to decide

CPS 39-2

A capacitor is connected across an ac source as shown. For this circuit, what is the relationship between the instantaneous current *i* through the capacitor and the instantaneous voltage v_{ab} across the capacitor?

(a) Circuit with ac source and capacitor



© 2012 Pearson Education, Inc.

A. *i* is maximum at the same time as v_{ab}.

B. *i* is maximum one-quarter cycle before v_{ab}.

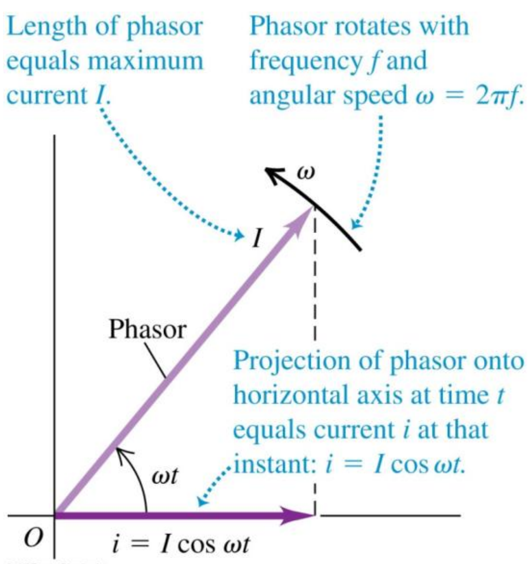
C. *i* is maximum one-quarter cycle after v_{ab}.

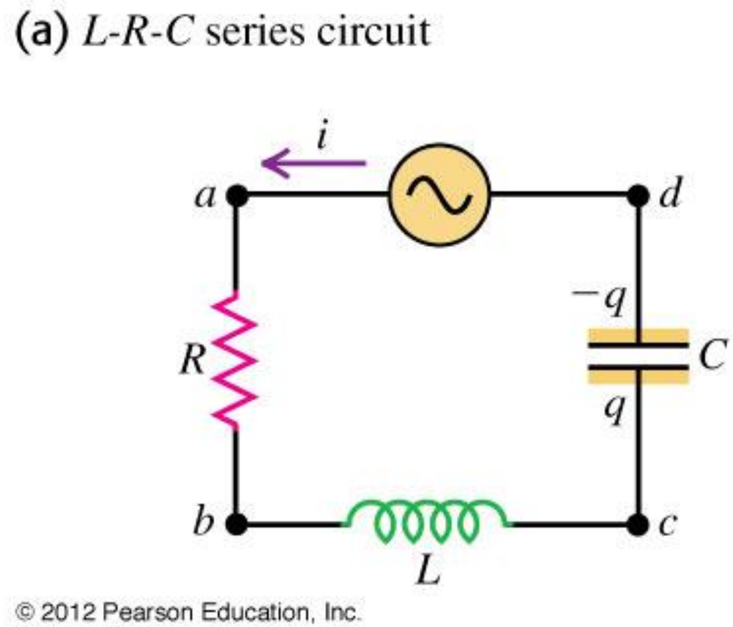
D. not enough information given to decide

LRC Circuit With AC Source

- So, we have examined the potential across individual elements of an LRC circuit with respect to the current through the circuit.
- In particular, the voltage phasors with respect to the current phasor.

$$i = I\cos(\omega t)$$





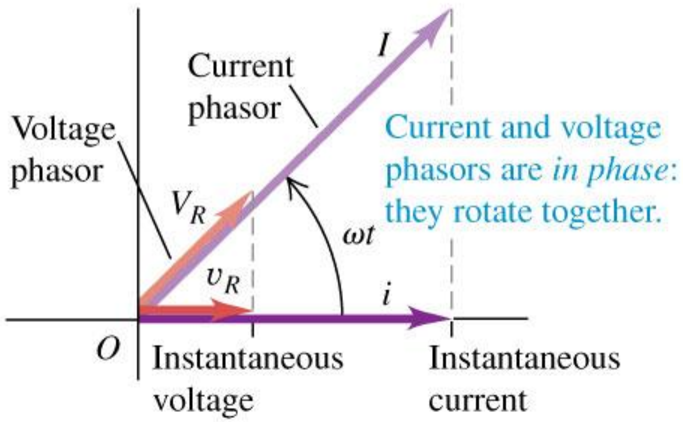
LRC Circuit With AC Source

- The voltage phasor for the resistor is in phase with the current in the circuit, and has a magnitude of IR.

$$i = I\cos(\omega t)$$

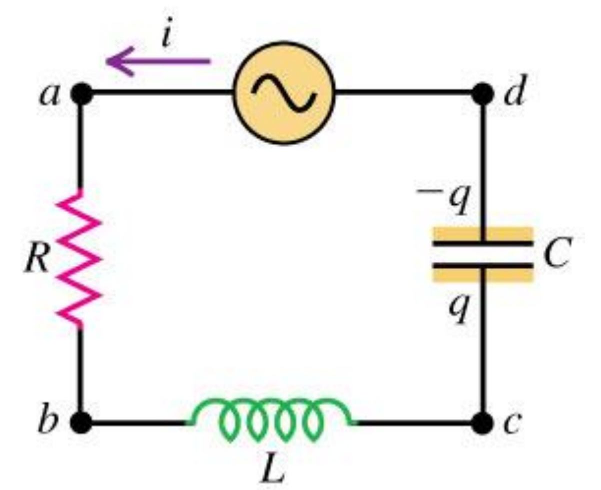
$$v_R = IR\cos(\omega t)$$

(c) Phasor diagram



© 2012 Pearson Education, Inc.

(a) L-R-C series circuit



© 2012 Pearson Education, Inc.

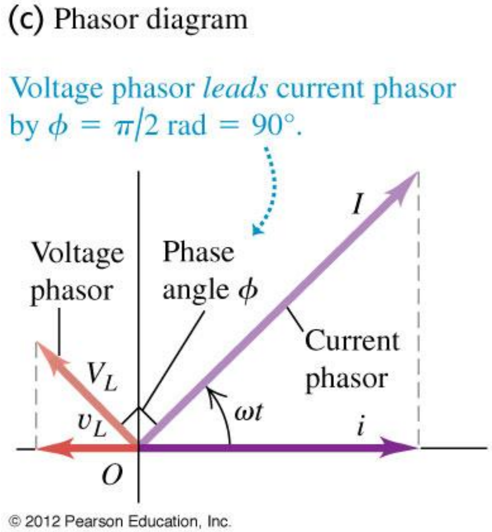
LRC Circuit With AC Source

- The voltage phasor for the inductor is 90 degrees ahead of the phase of the current in the circuit, and has a magnitude of IX_L.

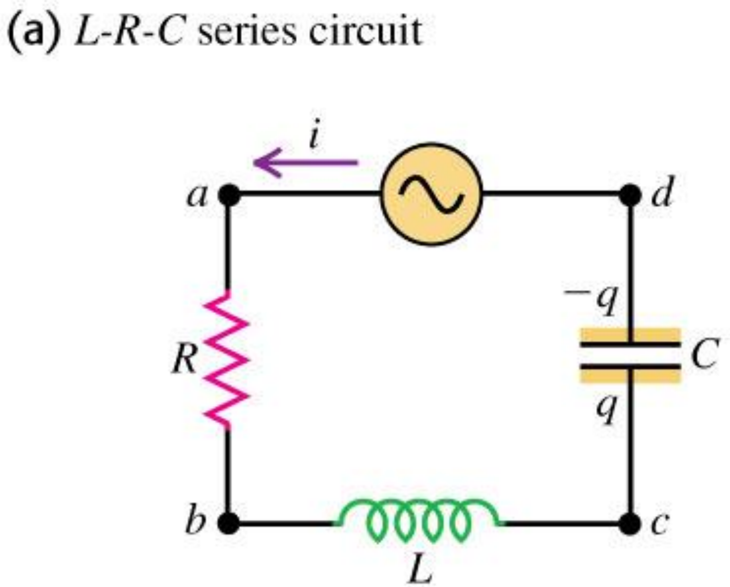
$$i = I\cos(\omega t)$$

$$v_L = IX_L\cos(\omega t + 90^\circ)$$

$$X_L = \omega L$$



© 2012 Pearson Education, Inc.



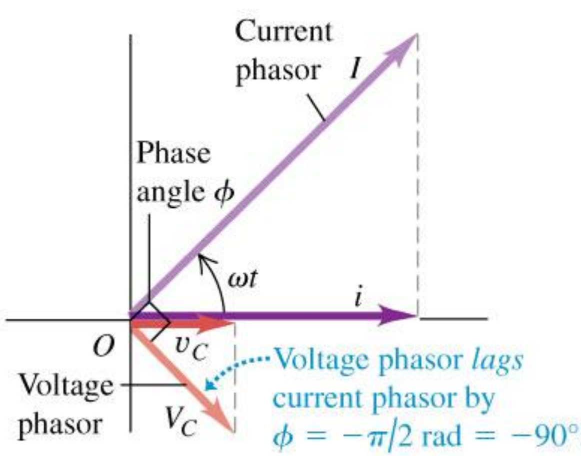
© 2012 Pearson Education, Inc.

LRC Circuit With AC Source

- The voltage phasor for the inductor is 90 degrees behind the phase of the current in the circuit, and has a magnitude of IX_C.

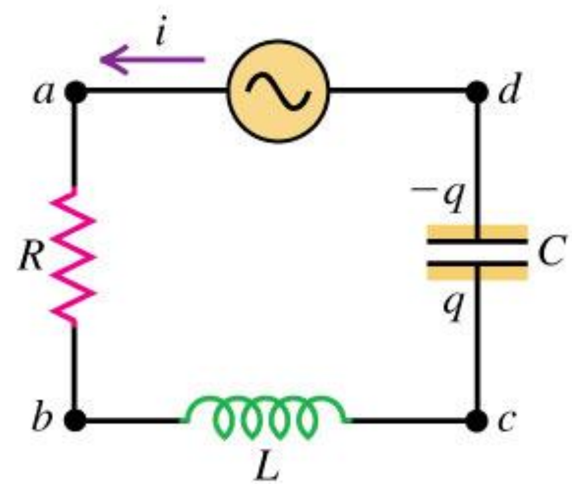
$$i = I\cos(\omega t)$$
$$v_C = IX_C\cos(\omega t - 90°)$$
$$X_C = 1/\omega C$$

(c) Phasor diagram



© 2012 Pearson Education, Inc.

(a) *L-R-C* series circuit



© 2012 Pearson Education, Inc.

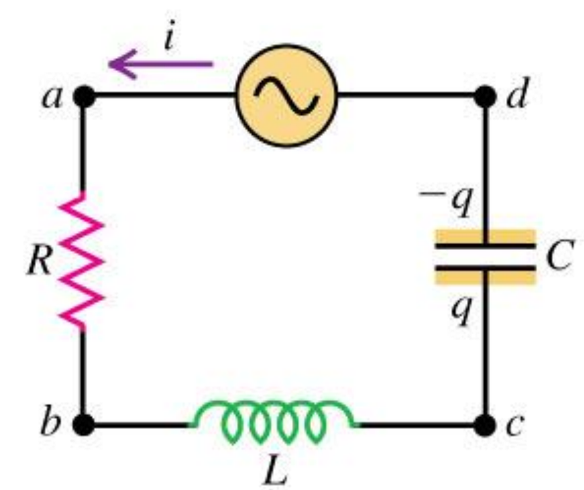
LRC Circuit With AC Source

- How do we now relate these together to find the instantaneous voltage V_{ad} across the source?
- Using Kirchhoff's loop rule, we would just add all of the instantaneous voltages – the projections of the voltage phasors.

$$i = I\cos(\omega t)$$

$$v_R = IR\cos(\omega t)$$
$$v_L = IX_L\cos(\omega t + 90^\circ)$$
$$v_C = IX_C\cos(\omega t - 90^\circ)$$

(a) *L-R-C* series circuit



© 2012 Pearson Education, Inc.

Voltage in a Series LRC Circuit

- But the sum of the projections is the projection of the vector sum.
- So, let's create a phasor, V, which is the vector sum of the voltage phasors.
- The magnitude of this phasor will be:

$$V^2 = V_R^2 + \left(V_L - V_C\right)^2 \Rightarrow$$

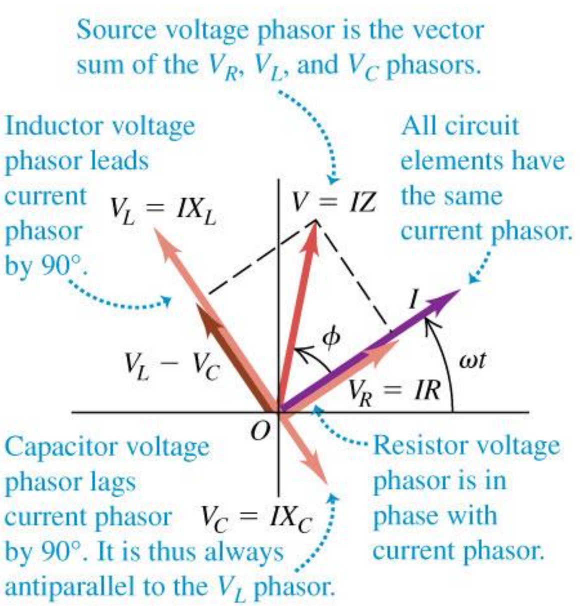
$$V = \sqrt{V_R^2 + \left(V_L - V_C\right)^2} \Rightarrow$$

$$V = \sqrt{I^2R^2 + \left(IX_L - IX_C\right)^2} \Rightarrow$$

$$V = I\sqrt{R^2 + \left(X_L - X_C\right)^2} \equiv IZ$$

$$v = V\cos\left(\omega t + \phi\right)$$

(b) Phasor diagram for the case $X_L > X_C$



© 2012 Pearson Education, Inc.

Impedance

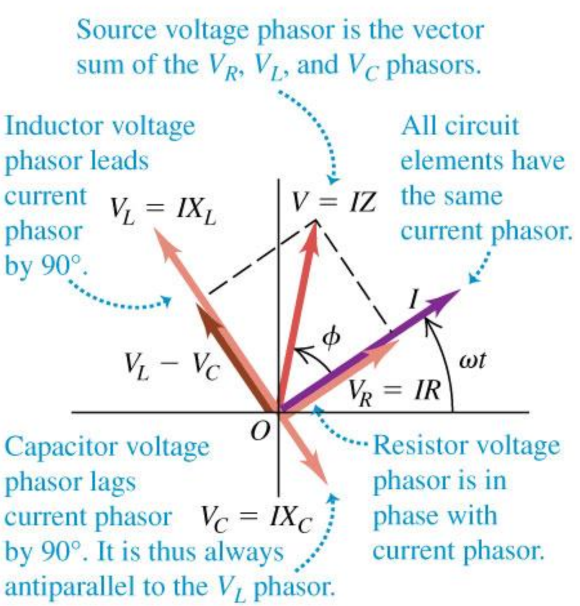
- So, we can define the impedance as the factor that determines the amplitude of the current given an alternating voltage with amplitude V.

$$V = IZ \quad \text{or} \quad I = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

- Notice that the impedance is minimum when $X_L = X_C$.

(b) Phasor diagram for the case $X_L > X_C$

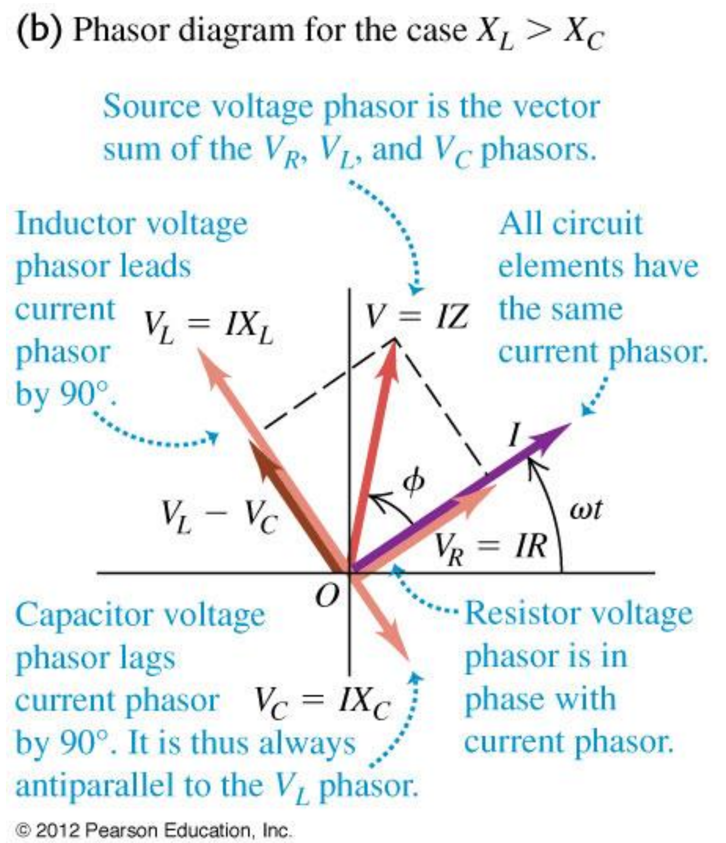


© 2012 Pearson Education, Inc.

Voltage in a Series LRC Circuit

- Also notice that in general, the voltage phasor (which represents the source voltage) is NOT in phase with the current phasor.
- In fact, it can lead, lag, or be in phase with the current phasor, depending on the value of X_L and X_C.

$$v = V\cos(\omega t + \phi)$$



Voltage in a Series LRC Circuit

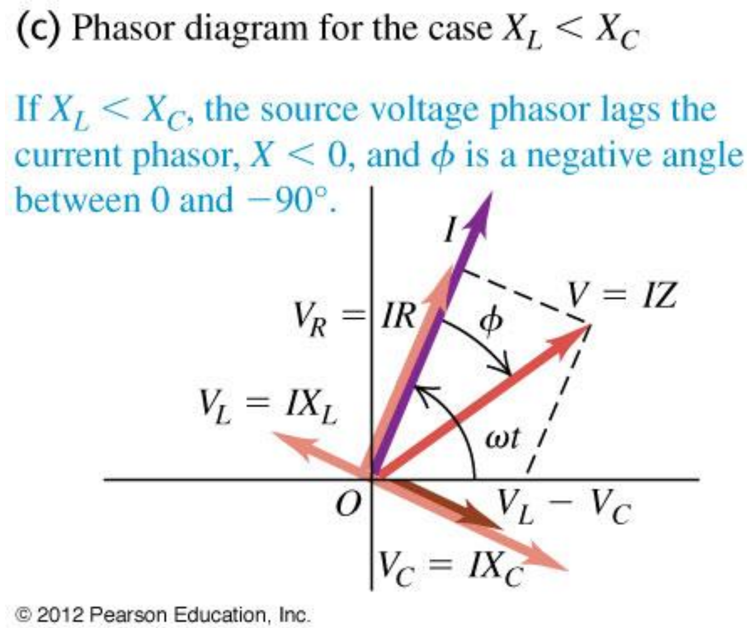
- The angle between the current phasor and the voltage phasor is called the LRC phase angle, and can be determined by its components:

$$V_{\text{I-comp}} = V_R$$

$$V_{\perp \text{ to I}} = V_L - V_C$$

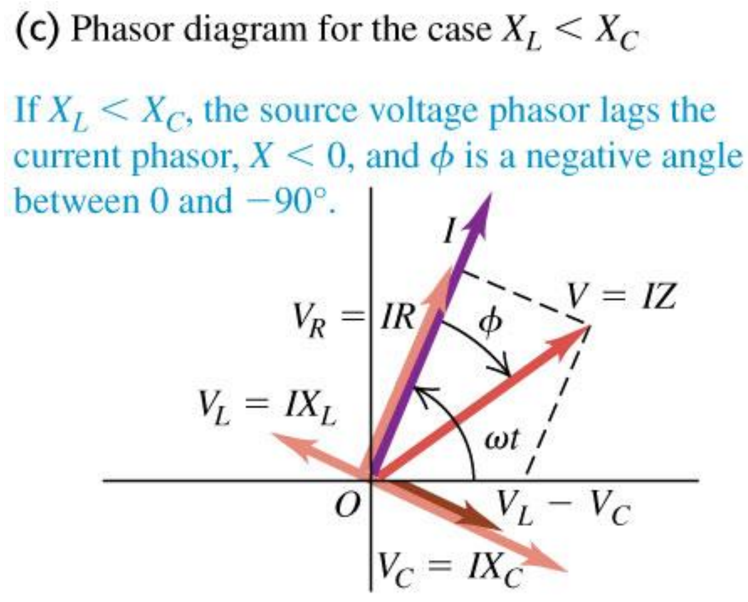
$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - X_C}{IR} = \frac{\omega L - 1/\omega C}{R}$$

$$v = V\cos(\omega t + \phi)$$



Power in an LRC Circuit

- The phase angle of the circuit plays an important role.
- The power in a circuit is just a product of the voltage and the current:



© 2012 Pearson Education, Inc.

$$v = V\cos(\omega t + \phi), \quad i = I\cos(\omega t) \Rightarrow$$

$$p = IV\cos(\omega t + \phi)\cos(\omega t) = IV\left[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)\right]\cos(\omega t)$$

$$= IV\cos^2(\omega t)\cos(\phi) - IV\sin(\omega t)\cos(\omega t)\sin(\phi)$$

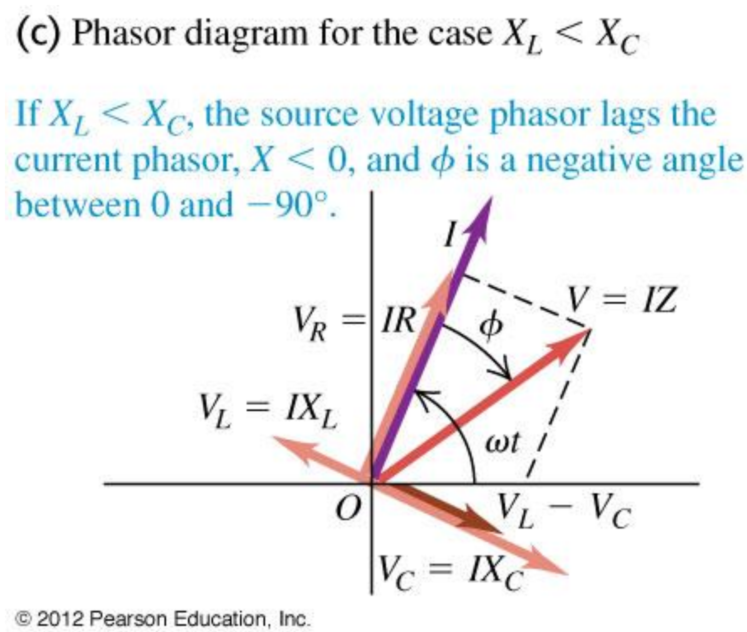
Average Power in an LRC Circuit

- The average power is just:

$$p = IV\cos^2(\omega t)\cos(\phi) - IV\sin(\omega t)\cos(\omega t)\sin(\phi)$$

$$P_{\text{avg}} = \frac{1}{2}IV\cos(\phi) = I_{RMS}V_{RMS}\cos(\phi)$$

- Notice that the average power is largest when the phase angle is zero.



Resonance in an LRC Circuit

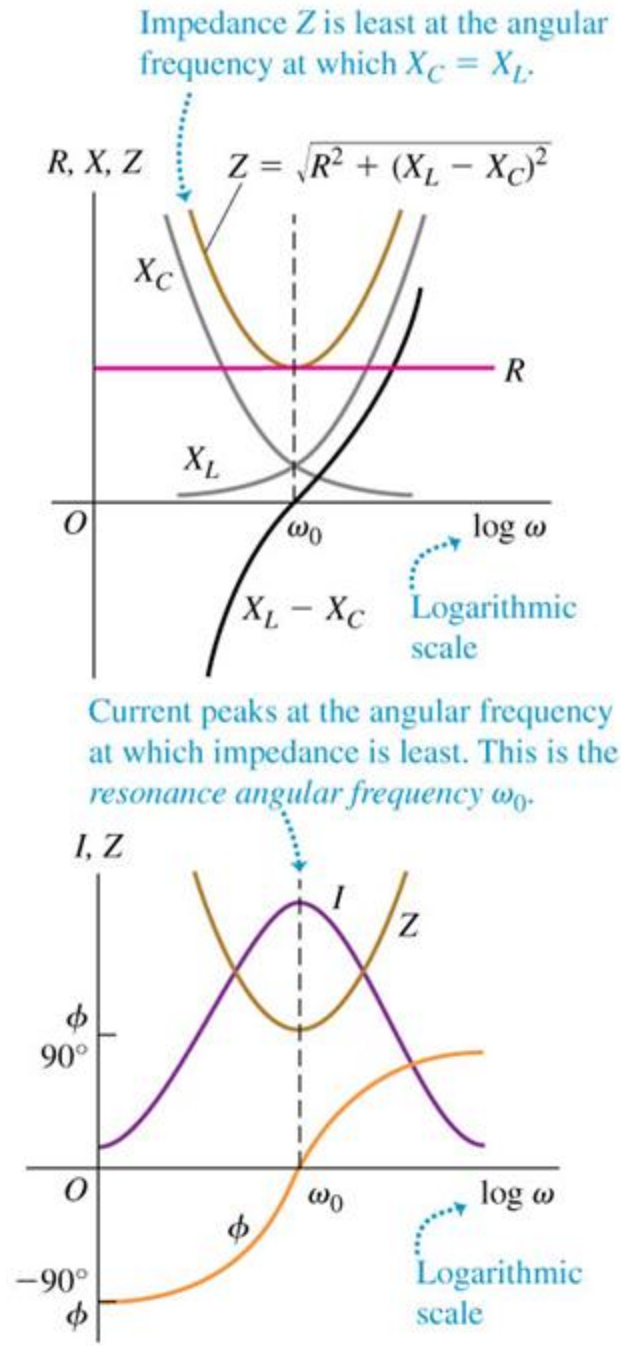
- Let's examine the phase angle, and in particular, what happens when it approaches zero.
- The condition for the phase angle to be zero is the same as for the minimum of the impedance:

$$X_L = X_C \Rightarrow$$

$$\omega L = \frac{1}{\omega C} \Rightarrow$$

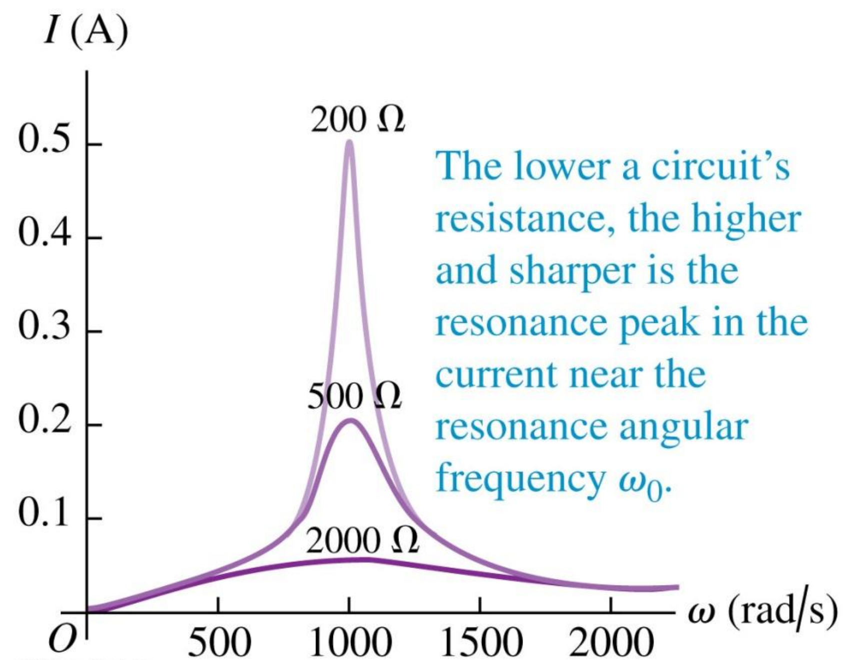
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- Notice that this is also the natural frequency of the LC circuit!



LRC Circuit With AC Source

- In that case, the impedance is just the resistance, and the current in the circuit is just proportional to 1/R.
- This is known as resonance, and can be used, for example, to tune a radio circuit to pick up a particular frequency.



© 2012 Pearson Education, Inc.