Problem Set #6 Sol'ns

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11/12
ECE 345

$\boxed{1}$ $T_s \simeq \dfrac{4}{\zeta\omega_n} \le 2$

$2 \le \zeta\omega_n$

(a)

$e^{-\zeta\pi/\sqrt{1-\zeta^2}} \le 0.05$

$\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}} \le \ln\left(\frac{1}{20}\right)$

$\zeta^2\pi^2 \ge \left(\ln\left(\frac{1}{20}\right)\right)^2 \cdot (1-\zeta^2)$

$\zeta^2\left(\pi^2 + \ln\left(\frac{1}{20}\right)^2\right) \ge \ln\left(\frac{1}{20}\right)^2$

$\zeta \ge \dfrac{|\ln(1/20)|}{\sqrt{\pi^2 + \ln\left(\frac{1}{20}\right)^2}} \simeq 0.69$.

$\cos(\zeta) \ge 44.2^\circ$

(b) $G_c(s)G(s)$ has no $\frac{1}{s}$ terms $\Rightarrow$ type # of $\dfrac{G_c(s)G(s)}{1+G_c(s)G(s)} = \dfrac{Y(s)}{R(s)}$ is $\underline{\underline{0}}$.

e_{ss} is infinite in response to a unit ramp.

(c) Poles at $s = -2, -3$. $n = 2$, $m = 0$ $\Rightarrow$ 2 asymptotes at $\pm 90^\circ$

asymptote centroid at $\sigma = \dfrac{-2-3}{2} = -2.5$

Break-away at $\dfrac{\partial}{\partial s}\left(\dfrac{1}{G(s)}\right) = \dfrac{\partial}{\partial s}\left(s^2 + 5s + 6\right) = 2s + 5 = 0 \Rightarrow s = -5/2$.

Departure angles are $+0, +180^\circ$ (on real line).

d) $\Delta(s) = s^2 + 5s + 6 + K$

$5 = 2\zeta\omega_n \Rightarrow \zeta\omega_n = 2.5 > 2$ ∴ Settling time req. met for $K > 0$.

$\omega_n^2 = 6 + K \Rightarrow \omega_n = \sqrt{6+K}$

$$\zeta = \frac{2.5}{\sqrt{6+K}} = 1/\sqrt{2}$$

$$\sqrt{6+K} = 2.5\sqrt{2}$$

$$6 + K = \frac{25}{2}$$

$$K = 7.5$$

overshoot requirement met for $K < 7.5$

$\boxed{12}$ $G_c(s) = K(1 + \frac{1}{s}) = \frac{K(s+1)}{s}$

(a) Type # is now $\underline{\underline{1}}$ ⇒ this system w/ PI control has better ss error than system w/ proportional control. The controller has increased the type # of the closed-loop system by 1.

($e_{ss} = \frac{1}{K_v}$, $K_v = \lim_{s\to 0} s\, G_c(s) G(s) = \lim_{s \to 0} s \cdot \frac{K(s+1)}{s(s^2+5s+6)} = \frac{K}{6}$

(∴ $e_{ss} = 6/K < \infty$.)

(b) $n = 3$ poles at $s = 0, -2, -3$
$m = 1$ poles at $s = -1$

∴ $n - m = 2$ asymptotes at $\pm 90°$

$$\sigma = \frac{(0-2-3)-(-1)}{3-1} = \frac{-4}{2} = -2$$

breakaway point at

$$0 = \frac{\partial}{\partial s}\left(\frac{1}{G_c(s)G(s)}\right) = \frac{\partial}{\partial s}\left(\frac{s(s+2)(s+3)}{s+1}\right)$$

$$= \frac{(s+1)\frac{\partial}{\partial s}\left(s(s+2)(s+3)\right) - s(s+2)(s+3)\cdot 1}{(s+1)^2}$$

$$= (s+1)(3s^2+10s+6) - (s^3+5s^2+6s)$$

$$= 3s^3+13s^2+16s+6 - s^3-5s^2-6s$$

$$= 2s^3+8s^2+10s+6$$

$$= s^3+4s^2+5s+3 \implies s = -2.47,\ -0.77 \pm 0.79;$$

feasible. not feasible since not on real line

(c) Yes, since asymptote of root locus is equivalent to settling time constraint boundary; and at lower gains, overshoot criterion should also be met.

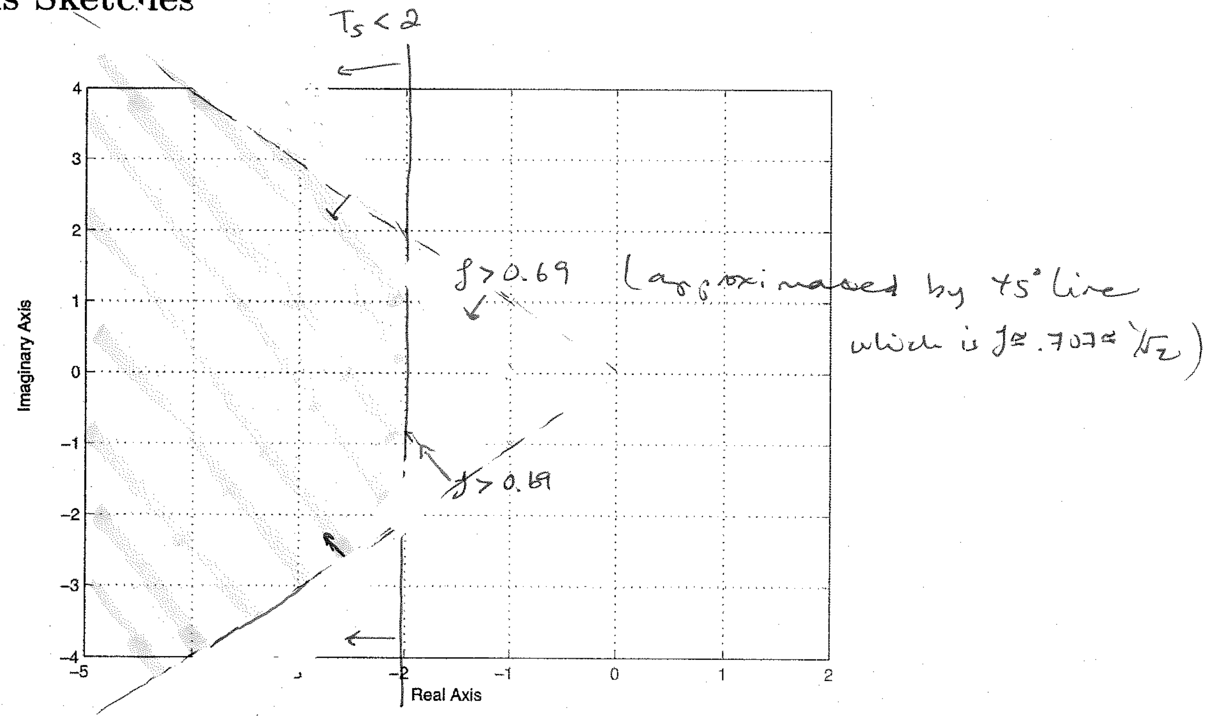
(d) No -- the system has 2 asymptotes, neither of which enter the RHP. The open-loop pole at the origin will yield a LHP root for $K>0$.

$\boxed{3}$ (a) A type 1 system will have a better steady-state response to step & ramp inputs than a type 0 system, hence the system under PI control has better steady-state performance than the system under P control.

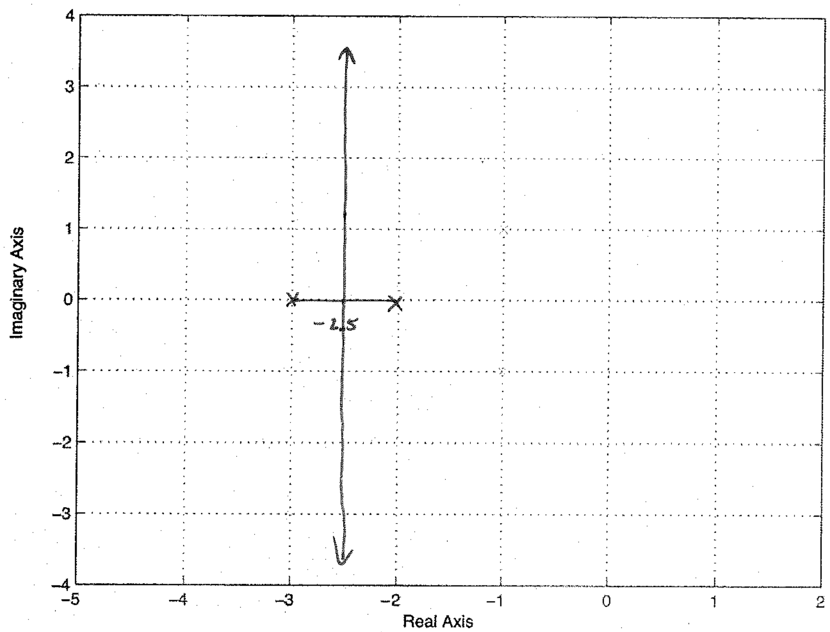
(b) The PI controller will result in a slower transient response than the P controller due to the pole-zero pair near the origin, which results in a time constant of at most $1/\tau = 1$ seconds. The P ctrl will provide at least $1/\tau = 1/2$ second. Hence there is a tradeoff between steady-state & transient performance w/ these 2 ctrllers.

Root Locus Sketches

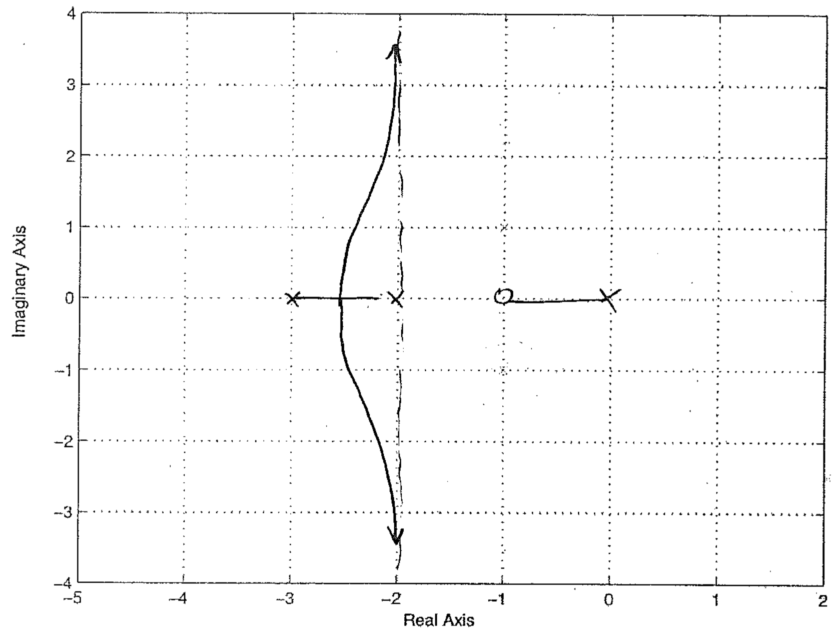
1. (a)



(c)

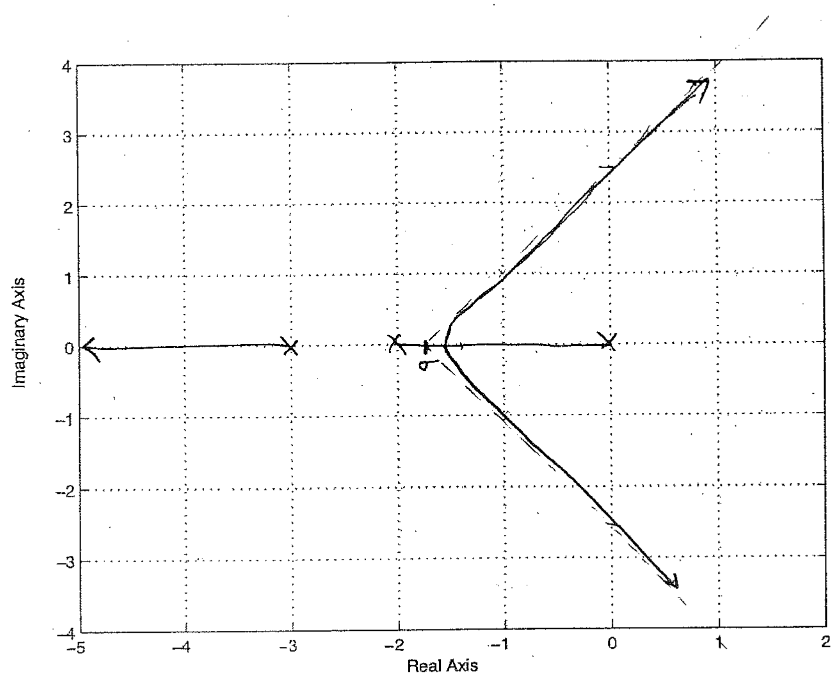


3. (b)



BONUS.

4. (a)



$n=3$ poles at $s=0,-2,-3$

$m=0$ zeros

$\Rightarrow$ 3 asymptotes at $0^\circ, \pm 60^\circ$

$$\sigma = \frac{0-2-3}{3} = -5/3$$

$\Rightarrow$ breakaway point at $\frac{\partial}{\partial s}\left(s^3+5s^2+6s\right) = 3s^2+10s+6=0$

$\therefore s = -2.55, -0.78$

$\times$ not relevant — locus is on Real line

$\Rightarrow j\omega$ crossing at $(s^2+\omega^2)(s+\alpha) = s^3+5s^2+6s+K$

$s^3+\alpha s^2+\omega^2 s+\omega^2\alpha =$

$\omega = \sqrt{6} \simeq 2.45$

(b) Hurwitz criterion for $\Delta(s) = s^3+5s^2+6s+K$

$$a_1 a_2 - a_3 = 5\cdot 6 - K > 0$$

$$30 > K$$

(c) $(s^2+2\zeta\omega_n s+\omega_n^2)(s+\alpha) = s^3+5s^2+6s+K$

$= s^3+s^2(\alpha+2\zeta\omega_n)+(\omega_n^2+2\zeta\omega_n\alpha)s+\omega_n^2\alpha$

with $\zeta\omega_n = 2$

$= s^3+s^2(\alpha+4)+(\omega_n^2+4\alpha)s+\omega_n^2\alpha$

$\Rightarrow \alpha+4=5 \quad \Rightarrow \alpha=1$

$\Rightarrow \omega_n^2+4\alpha = 6 \Rightarrow \omega_n=\sqrt{2}$

$\Rightarrow \omega_n^2\alpha = K \Rightarrow \underline{K=2}$

And with $\zeta\omega_n=2$, $\omega_n=\sqrt{2} \Rightarrow \zeta=\sqrt{2}>1$ $\therefore$ overshoot criteria is also met.

(d) The PI control will have slower response due to pole near the origin. The P and I ctrllers both can meet transient perf. specs, but the I controller can meet them w/ lower gain ($K=2$ instead of $K=7.5$). Hence I ctrl is "best."

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