Series And Parallel Connections

Description: Several calculations of increasing complexity that help the students practice finding the equivalent resistance of the circuits combining series and parallel connections.

Learning Goal:

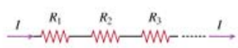
To learn to calculate the equivalent resistance of the circuits combining series and parallel connections.

Resistors are often connected to each other in electric circuits. Finding the **equivalent resistance** of combinations of resistors is a common and important task. Equivalent resistance is defined as the single resistance that can replace the given combination of resistors in such a manner that the currents in the rest of the circuit do not change.

Finding the equivalent resistance is relatively straightforward if the circuit contains only series and parallel connections of resistors.

An example of a **series connection** is shown in the diagram.

For such a connection, the current is the same for all individual resistors and the total voltage is the sum of the voltages across the individual resistors.



Using Ohm's law ($R = \frac{V}{I}$), one can show that, for a series connection, the equivalent resistance is the sum of the individual resistances.

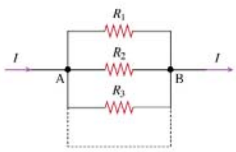
Mathematically, these relationships can be written as:

$$I = I_1 = I_2 = I_3 = ...$$

$$V = V_1 + V_2 + V_3 + ...$$

$$R_{eq-series} = R_1 + R_2 + R_3 + ...$$

An example of a **parallel connection** is shown in the diagram.



For resistors connected **in parallel** the voltage is the same for all individual resistors because they are all connected to the same two points (A and B on the diagram). The total current is the sum of the currents through the individual resistors. This should makes sense as the total current "splits" at points A and B.

Using Ohm's law, one can show that, for a parallel connection, the reciprocal of the equivalent resistance is the sum of the reciprocals of the individual resistances.

Mathematically, these relationships can be written as:

$$V = V_1 = V_2 = V_3 = ...$$

$$I = I_1 + I_2 + I_3 + ...$$

$$\frac{1}{R_{eq-parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

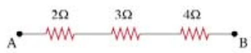
NOTE: If you have already studied capacitors and the rules for finding the equivalent capacitance, you should notice that the rules for the capacitors are similar - but not quite the same as the ones discussed here.

In this problem, you will use the the equivalent resistance formulas to determine R_{eq} for various combinations of resistors.

Part A

For the combination of resistors shown, find the equivalent resistance between points A and B.

Express your answer in Ohms.



ANSWER:

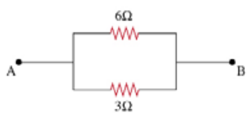
R_{eq} = 9 Ω

These resistors are connected in *series*; the current through each is the same.

Part B

For the set-up shown, find the equivalent resistance between points A and B.

Express your answer in Ohms.



ANSWER:

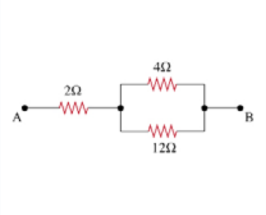
R_{eq} = 2 Ω

This is a *parallel* connection since the voltage across each resistor is the same.

Part C

For the combination of resistors shown, find the equivalent resistance between points A and B.

Express your answer in Ohms.



Hints (2)

ANSWER:

R_{eq} = 5 Ω

In this case, you cannot say that *all three* resistors are connected either in series or in parallel. You have a combination of a series and a parallel connection.

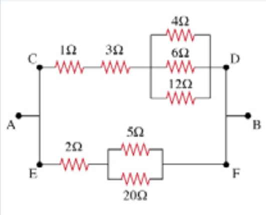
Some circuits may contain a large number of resistors connected in various ways. To determine the equivalent resistance of such circuits, you have to take several steps, carefully selecting the "sub-combinations" of resistors connected in relatively obvious ways. Good record-keeping is essential here.

The next question helps you practice this skill.

Part D

For the combination of resistors shown, find the equivalent resistance between points A and B.

Express your answer in Ohms.



Hints (4)

ANSWER:

R_{eq} = 3 Ω

The next level of analyzing a circuit is to determine the voltages across and the currents through the various branches of the circuit. You will practice that skill in the future.

Of course, there are circuits that *cannot* possibly be represented as combinations of series and parallel connections. However, there are ways to analyze those, too.

Kirchhoff's Rules and Applying Them

Description: Questions elicit the physics behind Kirchoff's rules. Then gives example and suggestions on how to use them.

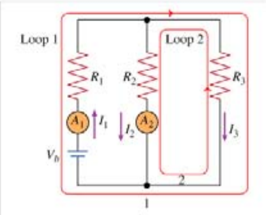
Learning Goal:

To understand the origins of both of Kirchhoff's rules and how to use them to solve a circuit problem.

This problem introduces Kirchhoff's two rules for circuits:

- *Kirchhoff's loop rule*: The sum of the voltage changes across the circuit elements forming any closed loop is zero.
- *Kirchhoff's junction rule*: The algebraic sum of the currents into (or out of) any junction in the circuit is zero.

The figure shows a circuit that illustrates the concept of *loops*, which are colored red and labeled loop 1 and loop 2. Loop 1 is the loop around the entire circuit, whereas loop 2 is the smaller loop on the right. To apply the loop rule you would add the voltage changes of all circuit elements around the chosen loop. The figure contains two junctions (where three or more wires meet)--they are at the ends of the resistor labeled R_3. The battery supplies a constant voltage V_b, and the resistors are labeled with their resistances. The ammeters are ideal meters that read I_1 and I_2 respectively.



The direction of each loop and the direction of each current arrow that you draw on your own circuits are arbitrary. Just assign voltage drops consistently and sum both voltage drops and currents algebraically and you will get correct equations. If the actual current is in the opposite direction from your current arrow, your answer for that current will be negative. The direction of any loop is even less imporant: The equation obtained from a counterclockwise loop is the same as that from a clockwise loop except for a negative sign in front of every term (i.e., an inconsequential change in overall sign of the equation because it equals zero).

Part A

The junction rule describes the conservation of which quantity? Note that this rule applies only to circuits that are in a steady state.

Hints (1)

Hint 1. At the junction

Think of the analogy with water flow. If a certain current of water comes to a split in the pipe, what can you say (mathematically) about the sum of the three water currents at this junction? If this were not true, water would accumulate at the junction.

ANSWER:

- current
- voltage
- resistance

Part B

Apply the junction rule to the junction labeled with the number 1 (at the bottom of the resistor of resistance R_2).

Answer in terms of given quantities, together with the meter readings I_1 and I_2 and the current I_3.

Hints (1)

Hint 1. Elements in series

The current through resistance R_1 is not labeled. You should recognize that the current I_1 passing through the ammeter also passes through resistance R_1 because there is no junction in between the resistor and the ammeter that could allow it to go elsewhere. Similarly, the current passing through the battery must be I_1 also. Circuit elements connected in a string like this are said to be *in series* and the same current must pass through each element. This fact greatly reduces the number of independent current values in any practical circuit.

ANSWER:

$\Sigma I = 0 =$ = $I_3 + I_2 - I_1$

Also accepted: $-I_3 - I_2 + I_1$

If you apply the juncion rule to the junction above R_2, you should find that the expression you get is equivalent to what you just obtained for the junction labeled 1. Obviously the conservation of charge or current flow enforces the same relationship among the currents when they separate as when they recombine.

Part C

Apply the loop rule to loop 2 (the smaller loop on the right). Sum the voltage changes across each circuit element around this loop going in the direction of the arrow. Remember that the current meter is ideal.

Express the voltage drops in terms of V_b, I_2, I_3, the given resistances, and any other given quantities.

Hints (3)

Hint 1. Elements in series have same current

The current through the ammeter is I_1, and this current has to go through the resistor of resistance R_1 because there is no junction in between that could add or subtract current. Similarly, the current passing through the battery must be I_1 also. Circuit elements connected in a string like this are said to be *in series* and the same current must pass through each element. This fact greatly reduces the number of independent current values in any practical circuit.

Hint 2. Sign of voltage across resistors

In determining the signs, note that if your chosen loop traverses a particular resistor in the *same* direction as the current through that resistor, then the end it enters through will have a more positive potential than the end from which it exits by the amount IR. Thus the voltage change across that resistor will be negative. Conversely, if your chosen loop traverses the resistor in the opposite direction from its current arrow, the voltage changes across the resistor will be positive. Let these conventions govern your equations (i.e., don't try to figure out the direction of current flow when using the Kirchhoff loop--decide when you put the current arrows on the resistors and stick with that choice).

Hint 3. Voltage drop across ammeter

An ideal ammeter has zero resistance. Hence there is no voltage drop across it.

ANSWER:

$\Sigma(\Delta V) = 0 =$ = $I_3R_3 - I_2R_2$

Also accepted: $-I_3R_3 + I_2R_2$

Part D

Now apply the loop rule to loop 1 (the larger loop spanning the entire circuit). Sum the voltage changes across each circuit element around this loop going in the direction of the arrow.

Express the voltage drops in terms of V_b, I_1, I_3, the given resistances, and any other given quantities.

ANSWER:

$\Sigma(\Delta V) = 0 =$ = $V_b - I_1R_1 - I_3R_3$

Also accepted: $-(V_b - I_1R_1 - I_3R_3)$

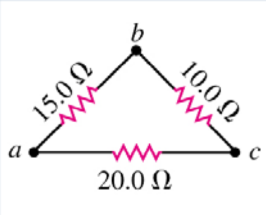
There is one more loop in this circuit, the inner loop through the battery, both ammeters, and resistors R_1 and R_2. If you apply Kirchhoff's loop rule to this additional loop, you will generate an extra equation that is redundant with the other two. In general, you can get enough equations to solve a circuit by either

1. selecting all of the internal loops (loops with no circuit elements inside the loop) or
2. using a number of loops (not necessarily internal) equal to the number of internal loops, with the extra proviso that at least one loop pass through each circuit element.

Exercise 26.5

Description: A triangular array of resistors is shown in the figure . What current will this array draw from a E battery having negligible internal resistance if we connect it across (a) ab, (b) bc, (c) ac? (a) ... (b) ... (c) ... (d) If the battery has an...

A triangular array of resistors is shown in the figure . What current will this array draw from a 45.0 V battery having negligible internal resistance if we connect it across (a) ab, (b) bc, (c) ac?



Part A

ANSWER:

$I =$ $E\left(\frac{1}{15}+\frac{1}{30}\right)$ = 4.50 A

Part B

ANSWER:

$I = E\left(\frac{1}{10}+\frac{1}{35}\right) = 5.79 \text{ A}$

Part C

ANSWER:

$I = E\left(\frac{1}{20}+\frac{1}{25}\right) = 4.05 \text{ A}$

Part D

If the battery has an internal resistance of 5.00Ω , what current will the array draw if the battery is connected across bc?

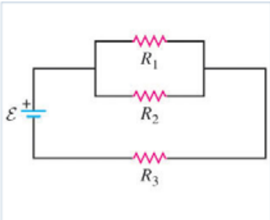
ANSWER:

$I = \frac{E}{r+\frac{1}{\frac{1}{\text{[illegible]}}+\frac{1}{\text{[illegible]}}}} = 3.52 \text{ A}$

Exercise 26.11

Description: In the following figure, R_1=## Omega , R_2=## Omega , and R_3=## Omega . The battery has negligible internal resistance. The current I_2 through R_2 is I_2. (a) What is the current I_1 ? (b) What is the current I_3 ? (c) What...

In the following figure, $R_1 = 6.00 \Omega$, $R_2 = 6.00 \Omega$, and $R_3 = 3.00 \Omega$. The battery has negligible internal resistance. The current I_2 through R_2 is 3.00A .



Part A

What is the current I_1 ?

Express your answer with the appropriate units.

ANSWER:

$I_1 = \frac{I_2 R_2}{R_1} = 3.00 \text{A}$

Part B

What is the current I_3 ?

Express your answer with the appropriate units.

ANSWER:

$I_3 = I_2\left(1+\frac{R_2}{R_1}\right) = 6.00 \text{A}$

Part C

What is the emf of the battery?

Express your answer with the appropriate units.

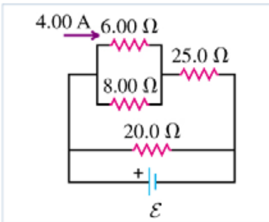
ANSWER:

$\mathcal{E} = I_2\left(1+\frac{R_2}{R_1}\right)\left(R_3+\frac{R_1 R_2}{R_1+R_2}\right) = 36.0 \text{V}$

Exercise 26.16

Description: Consider the circuit shown in the figure . The current through the 6.00-Omega resistor is 4.00 A, in the direction shown. (a) What is the currents through the 25.0-Omega resistor? (b) What is the current through the 20.0-Omega resistor?

Consider the circuit shown in the figure . The current through the $6.00\text{-}\Omega$ resistor is 4.00 A, in the direction shown.



Part A

What is the currents through the $25.0\text{-}\Omega$ resistor?

ANSWER:

$I = 7.00 \text{ A}$

Part B

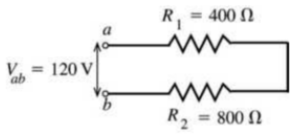
What is the current through the $20.0\text{-}\Omega$ resistor?

ANSWER:

$I = 9.95 \text{ A}$

26.21. **IDENTIFY:** For resistors in series, the voltages add and the current is the same. For resistors in parallel, the voltages are the same and the currents add. $P = I^2R$.

(a) SET UP: The circuit is sketched in Figure 26.21a.



For resistors in series the current is the same through each.

Figure 26.21a

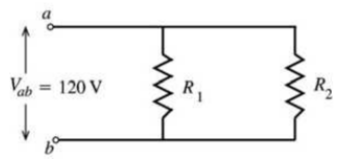
EXECUTE: $R_{eq} = R_1 + R_2 = 1200\ \Omega$. $I = \dfrac{V}{R_{eq}} = \dfrac{120\text{ V}}{1200\ \Omega} = 0.100$ A. This is the current drawn from the line.

(b) $P_1 = I_1^2 R_1 = (0.100\text{ A})^2(400\ \Omega) = 4.0$ W

$P_2 = I_2^2 R_2 = (0.100\text{ A})^2(800\ \Omega) = 8.0$ W

(c) $P_{out} = P_1 + P_2 = 12.0$ W, the total power dissipated in both bulbs. Note that $P_{in} = V_{ab}I = (120\text{ V})(0.100\text{ A}) = 12.0$ W, the power delivered by the potential source, equals P_{out}.

(d) SET UP: The circuit is sketched in Figure 26.21b.



For resistors in parallel the voltage across each resistor is the same.

Figure 26.21b

EXECUTE: $I_1 = \dfrac{V_1}{R_1} = \dfrac{120\text{ V}}{400\ \Omega} = 0.300\text{ A}, I_2 = \dfrac{V_2}{R_2} = \dfrac{120\text{ V}}{800\ \Omega} = 0.150$ A

EVALUATE: Note that each current is larger than the current when the resistors are connected in series.

(e) EXECUTE: $P_1 = I_1^2 R_1 = (0.300\text{ A})^2(400\ \Omega) = 36.0$ W

$P_2 = I_2^2 R_2 = (0.150\text{ A})^2(800\ \Omega) = 18.0$ W

(f) $P_{out} = P_1 + P_2 = 54.0$ W

EVALUATE: Note that the total current drawn from the line is $I = I_1 + I_2 = 0.450$ A. The power input from the line is $P_{in} = V_{ab}I = (120\text{ V})(0.450\text{ A}) = 54.0$ W, which equals the total power dissipated by the bulbs.

(g) The bulb that is dissipating the most power glows most brightly. For the series connection the currents are the same and by $P = I^2R$ the bulb with the larger R has the larger P; the 800-Ω bulb glows more brightly. For the parallel combination the voltages are the same and by $P = V^2/R$ the bulb with the smaller R has the larger P; the 400-Ω bulb glows more brightly.

(h) The total power output P_{out} equals $P_{in} = V_{ab}I$, so P_{out} is larger for the parallel connection where the current drawn from the line is larger (because the equivalent resistance is smaller.)

26.24. **IDENTIFY:** This circuit cannot be reduced using series/parallel combinations, so we apply Kirchhoff's rules. The target variables are the currents in each segment.

SET UP: Assume the unknown currents have the directions shown in Figure 26.24. We have used the junction rule to write the current through the 10.0 V battery as $I_1 + I_2$. There are two unknowns, I_1 and I_2, so we will need two equations. Three possible circuit loops are shown in the figure.

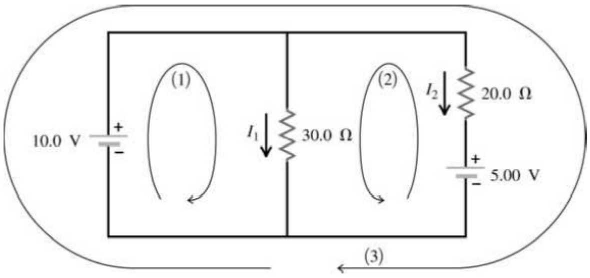


Figure 26.24

EXECUTE: **(a)** Apply the loop rule to loop (1), going around the loop in the direction shown: $+10.0\text{ V} - (30.0\ \Omega)I_1 = 0$ and $I_1 = 0.333\text{ A}$.

(b) Apply the loop rule to loop (3): $+10.0\text{ V} - (20.0\ \Omega)I_2 - 5.00\text{ V} = 0$ and $I_2 = 0.250\text{ A}$.

(c) $I_1 + I_2 = 0.333\text{ A} + 0.250\text{ A} = 0.583\text{ A}$

EVALUATE: For loop (2) we get

$+5.00\text{ V} + I_2(20.0\ \Omega) - I_1(30.0\ \Omega) = 5.00\text{ V} + (0.250\text{ A})(20.0\ \Omega) - (0.333\text{ A})(30.0\ \Omega) =$

$5.00\text{ V} + 5.00\text{ V} - 10.0\text{ V} = 0$, so that with the currents we have calculated the loop rule is satisfied for this third loop.

26.26. **IDENTIFY:** Apply the loop rule and junction rule.

SET UP: The circuit diagram is given in Figure 26.26. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.

EXECUTE: The loop rule applied to loop (1) gives:

$+20.0\text{V} - (1.00\text{ A})(1.00\ \Omega) + (1.00\text{ A})(4.00\ \Omega) + (1.00\text{ A})(1.00\ \Omega) - \mathcal{E}_1 - (1.00\text{ A})(6.00\ \Omega) = 0$

$\mathcal{E}_1 = 20.0\text{ V} - 1.00\text{ V} + 4.00\text{ V} + 1.00\text{ V} - 6.00\text{ V} = 18.0\text{ V}$. The loop rule applied to loop (2) gives:

$+20.0\text{ V} - (1.00\text{ A})(1.00\ \Omega) - (2.00\text{ A})(1.00\ \Omega) - \mathcal{E}_2 - (2.00\text{ A})(2.00\ \Omega) - (1.00\text{ A})(6.00\ \Omega) = 0$

$\mathcal{E}_2 = 20.0\text{ V} - 1.00\text{ V} - 2.00\text{ V} - 4.00\text{ V} - 6.00\text{ V} = 7.0\text{ V}$. Going from b to a along the lower branch, $V_b + (2.00\text{ A})(2.00\ \Omega) + 7.0\text{ V} + (2.00\text{ A})(1.00\ \Omega) = V_a \cdot V_b - V_a = -13.0\text{ V}$; point b is at 13.0 V lower potential than point a.

EVALUATE: We can also calculate $V_b - V_a$ by going from b to a along the upper branch of the circuit. $V_b - (1.00\text{ A})(6.00\ \Omega) + 20.0\text{ V} - (1.00\text{ A})(1.00\ \Omega) = V_a$ and $V_b - V_a = -13.0\text{ V}$. This agrees with $V_b - V_a$ calculated along a different path between b and a.

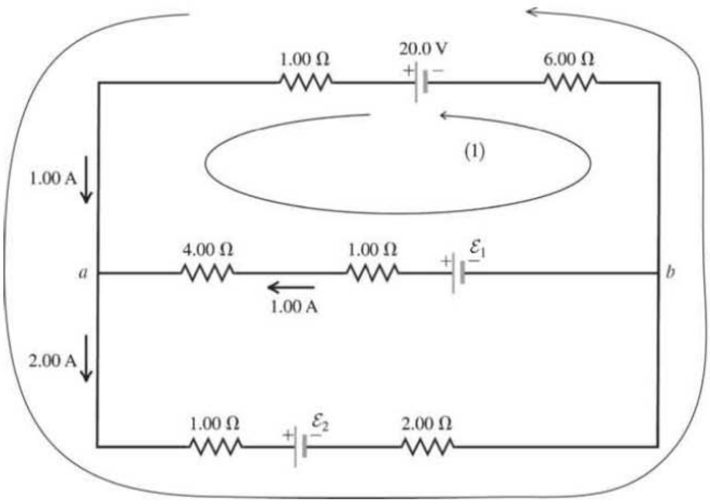


Figure 26.26

26.28. **IDENTIFY:** Use Kirchhoff's rules to find the currents.

SET UP: Since the 10.0-V battery has the larger voltage, assume I_1 is to the left through the 10-V battery, I_2 is to the right through the 5 V battery, and I_3 is to the right through the 10-Ω resistor. Go around each loop in the counterclockwise direction.

EXECUTE: Upper loop: $10.0\text{ V} - (2.00\ \Omega + 3.00\ \Omega)I_1 - (1.00\ \Omega + 4.00\ \Omega)I_2 - 5.00\text{ V} = 0$. This gives $5.0\text{ V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0$, and $\Rightarrow I_1 + I_2 = 1.00\text{ A}$.

Lower loop: $5.00\text{ V} + (1.00\ \Omega + 4.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0$. This gives $5.00\text{ V} + (5.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0$, and $I_2 - 2I_3 = -1.00\text{ A}$.

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$$I_1 = 0.800\text{ A},\ I_2 = 0.200\text{ A},\ I_3 = 0.600\text{ A}.$$

(b) $V_{ab} = -(0.200\text{ A})(4.00\ \Omega) - (0.800\text{ A})(3.00\ \Omega) = -3.20\text{ V}$.

EVALUATE: Traveling from *b* to *a* through the 4.00-Ω and 3.00-Ω resistors you pass through the resistors in the direction of the current and the potential decreases; point *b* is at higher potential than point *a*.

26.48. **IDENTIFY:** Both the charge and energy decay exponentially, but not with the same time constant since the energy is proportional to the *square* of the charge.

SET UP: The charge obeys the equation $Q = Q_0 e^{-t/RC}$ but the energy obeys the equation $U = Q^2/2C = (Q_0 e^{-t/RC})/2C = U_0 e^{-2t/RC}$.

EXECUTE: **(a)** The charge is reduced by half: $Q_0/2 = Q_0 e^{-t/RC}$. This gives

$$t = RC\ \ln 2 = (175\ \Omega)(12.0\ \mu\text{F})(\ln 2) = 1.456\text{ ms} = 1.46\text{ ms}.$$

(b) The energy is reduced by half: $U_0/2 = U_0 e^{-2t/RC}$. This gives

$$t = (RC\ \ln 2)/2 = (1.456\text{ ms})/2 = 0.728\text{ ms}.$$

EVALUATE: The energy decreases faster than the charge because it is proportional to the square of the charge.

26.50. **IDENTIFY:** When the capacitor is fully charged the voltage *V* across the capacitor equals the battery emf and $Q = CV$. For a charging capacitor, $q = Q(1 - e^{-t/RC})$.

SET UP: $\ln e^x = x$

EXECUTE: **(a)** $Q = CV = (5.90\times10^{-6}\text{ F})(28.0\text{ V}) = 1.65\times10^{-4}\text{ C}$.

(b) $q = Q(1 - e^{-t/RC})$, so $e^{-t/RC} = 1 - \dfrac{q}{Q}$ and $R = \dfrac{-t}{C\ln(1 - q/Q)}$. After $t = 3\times10^{-3}$ s: $R = \dfrac{-3\times10^{-3}\text{ s}}{(5.90\times10^{-6}\text{ F})(\ln(1 - 110/165))} = 463\ \Omega$.

(c) If the charge is to be 99% of final value: $\dfrac{q}{Q} = (1 - e^{-t/RC})$ gives

$t = -RC\ \ln(1 - q/Q) = -(463\ \Omega)\ (5.90\times10^{-6}\text{ F})\ \ln(0.01) = 0.0126\text{ s}$.

EVALUATE: The time constant is $\tau = RC = 2.73$ ms. The time in part (b) is a bit more than one time constant and the time in part (c) is about 4.6 time constants.

26.54. **IDENTIFY** and **SET UP:** Ohm's law and Eq. (25.18) can be used to calculate *I* and *P* given *V* and *R*. Use Eq. (25.12) to calculate the resistance at the higher temperature.

(a) EXECUTE: When the heater element is first turned on it is at room temperature and has resistance $R = 20\ \Omega$.

$$I = \frac{V}{R} = \frac{120\text{ V}}{20\ \Omega} = 6.0\text{ A}$$

$$P = \frac{V^2}{R} = \frac{(120\text{ V})^2}{20\ \Omega} = 720\text{ W}$$

(b) Find the resistance $R(T)$ of the element at the operating temperature of 280°C.

Take $T_0 = 23.0°\text{C}$ and $R_0 = 20\ \Omega$. Eq. (25.12) gives

$R(T) = R_0(1 + \alpha(T - T_0)) = 20\ \Omega(1 + (2.8\times10^{-3}(\text{C}°)^{-1})(280°\text{C} - 23.0°\text{C})) = 34.4\ \Omega$.

$$I = \frac{V}{R} = \frac{120\text{ V}}{34.4\ \Omega} = 3.5\text{ A}$$

$$P = \frac{V^2}{R} = \frac{(120\text{ V})^2}{34.4\ \Omega} = 420\text{ W}$$

EVALUATE: When the temperature increases, *R* increases and *I* and *P* decrease. The changes are substantial.

26.72. **IDENTIFY:** $P_{\text{tot}} = \dfrac{V^2}{R_{\text{eq}}}$.

SET UP: Let R be the resistance of each resistor.

EXECUTE: When the resistors are in series, $R_{\text{eq}} = 3R$ and $P_{\text{s}} = \dfrac{V^2}{3R}$. When the resistors are in parallel, $R_{\text{eq}} = R/3$. $P_{\text{p}} = \dfrac{V^2}{R/3} = 3\dfrac{V^2}{R} = 9P_{\text{s}} = 9(36\text{ W}) = 324\text{ W}$.

EVALUATE: In parallel, the voltage across each resistor is the full applied voltage V. In series, the voltage across each resistor is $V/3$ and each resistor dissipates less power.