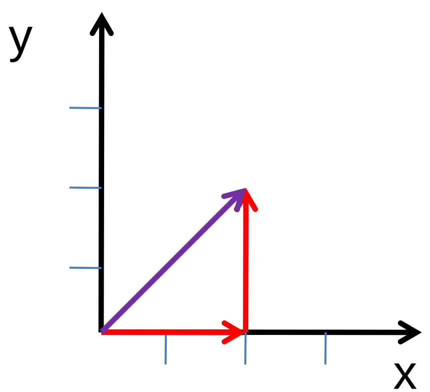
Lecture 3
(Addition and Subtraction of Vectors)

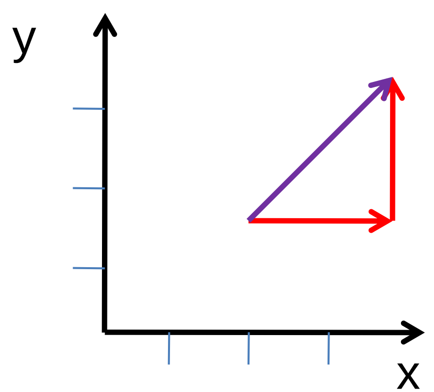
Physics 160-01 Fall 2012

Douglas Fields

Vector Addition

- Let’s give a “real-life” situation to demonstrate how one adds vectors.
- Displacement (change in position) is a vector.
- Let’s say I walk 2m in the +x-direction.
- Then I walk 2m in the +y-direction.
- What is my net displacement?
- It is the vector that points from where I started to where I end up.

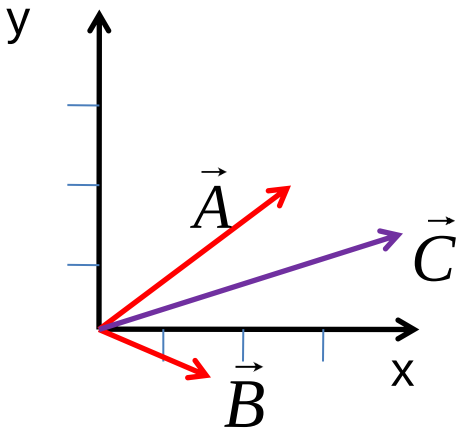


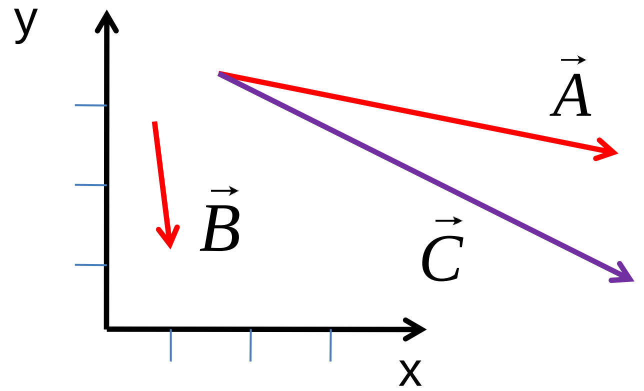


Graphical Vector Addition

- To add vectors graphically, put the tail of one vector to the point of the other.
- The resultant vector starts from the tail of the first and goes to the point of the second:

$$\vec{A}+\vec{B}=\vec{C}$$

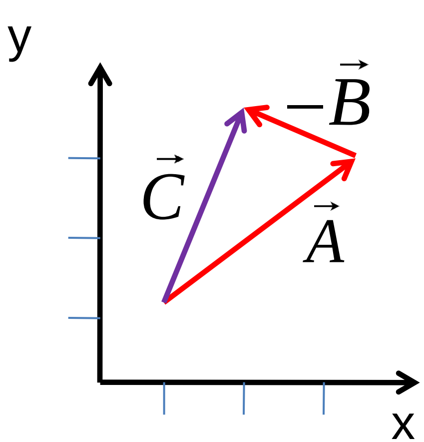




Subtraction

- The negative of a vector is a vector with the same length, but in the opposite direction:

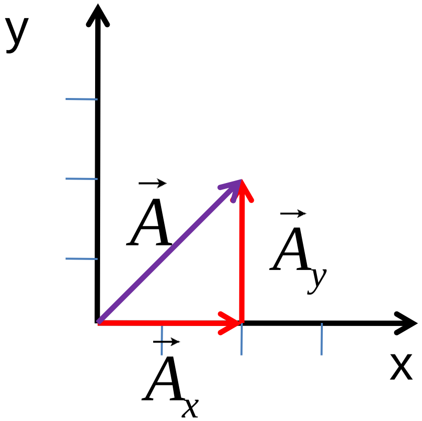
- To subtract vectors, just add the negative.



$$\vec{A}-\vec{B}=\vec{A}+\left(-\vec{B}\right)=\vec{C}$$

Vector Components

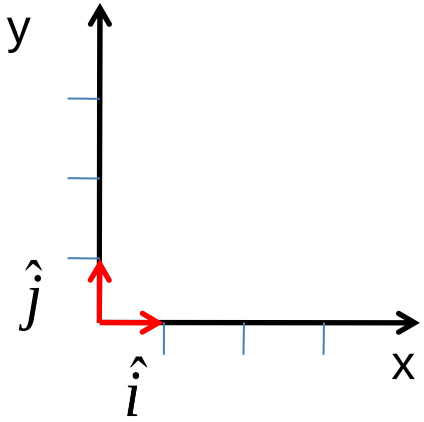
- In many cases, it is better to deal with the components of vectors:



$$\vec{A} = \vec{A}_x + \vec{A}_y$$

Unit Vectors

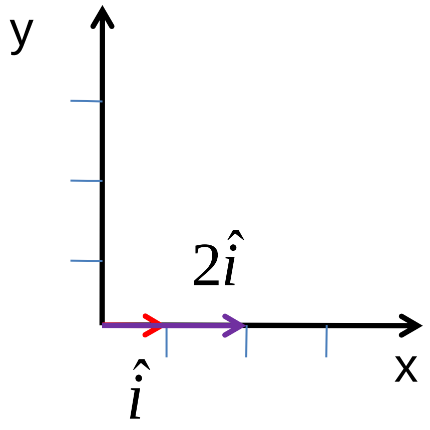
- Unit vectors are vectors in the directions of the axes of the coordinate system whose length is one unit.

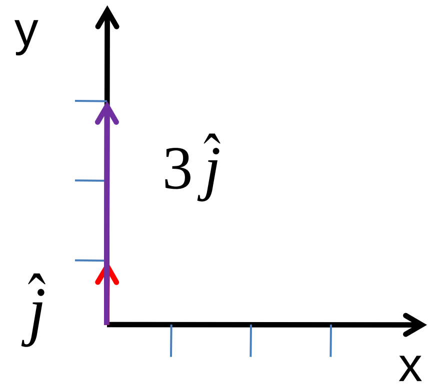


- We will learn how to make a unit vector soon.

Multiplication of a Vector by a Number

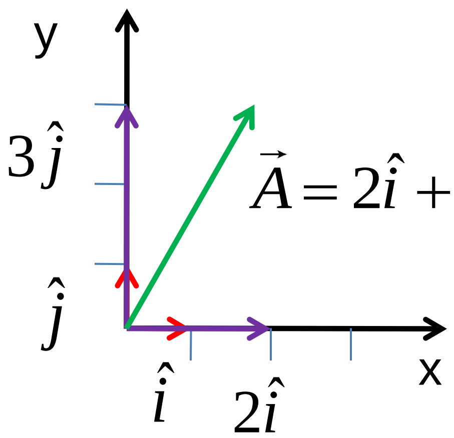
- When multiplying a vector by a number, it’s length is increase by a factor of that number:



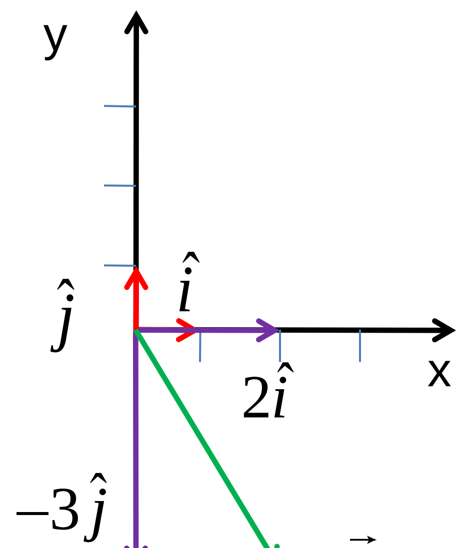


Component Notation

- A vector can then be represented in component notation as:



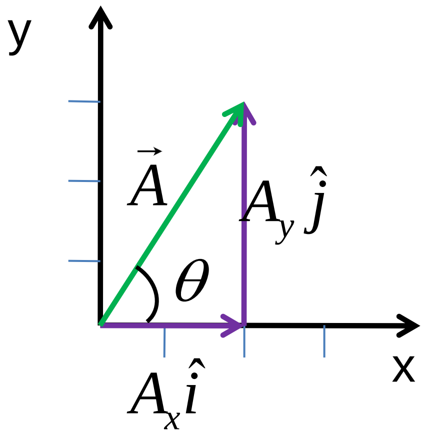
$$\vec{A} = 2\hat{i} + 3\hat{j} = A_x\hat{i} + A_y\hat{j}$$



$$\vec{A} = 2\hat{i} - 3\hat{j} = A_x\hat{i} + A_y\hat{j}$$

Finding Components

- Use trigonometry:



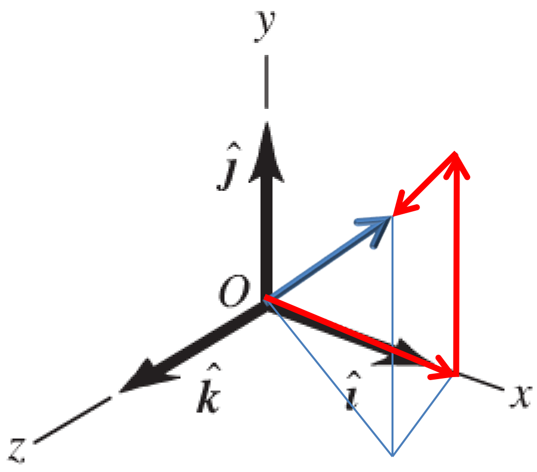
$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$A_x = \left|\vec{A}\right| \cos\theta$$

$$A_y = \left|\vec{A}\right| \sin\theta$$

In 3-dimensions

The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.



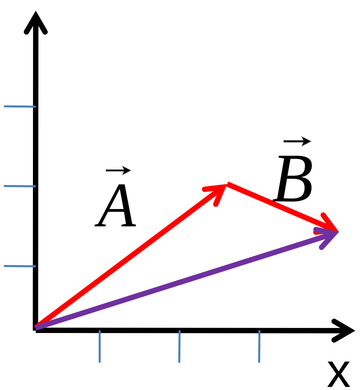
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\left|\vec{A}\right| = \sqrt{\left(A_x\right)^2 + \left(A_y\right)^2 + \left(A_z\right)^2}$$

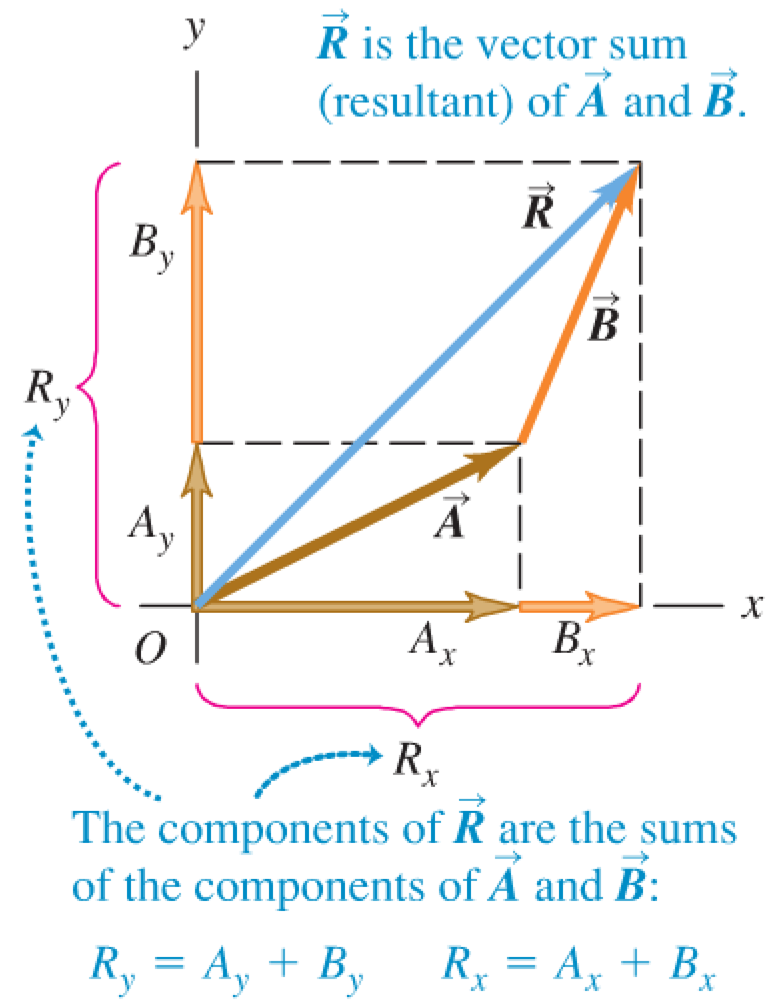
Adding Vectors Using Components

- Then, to add vectors using components, you just have to add the components:

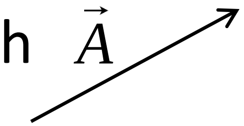
$$\vec{A}+\vec{B}=\left(A_x+B_x\right)\hat{i}+\left(A_y+B_y\right)\hat{j}$$

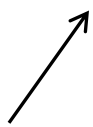


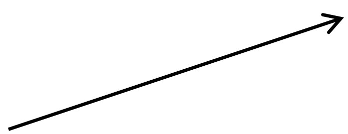
Adding Graphically

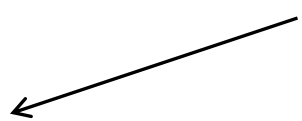


CPS Question 3-1

- Which vector most closely represents $\vec{A}-\vec{B}$ with $\vec{A}$  and  $\vec{B}$?

A. 

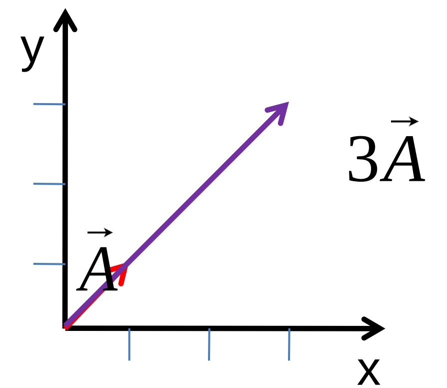
B. 

C. 

D. 

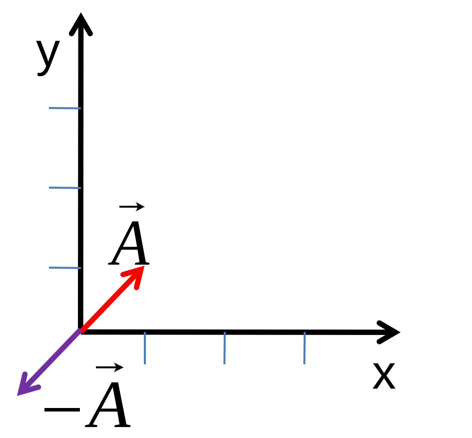
Multiplying scalars and vectors

- How do we determine $3\vec{A}$?



$$3\vec{A} = 3 \cdot \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \right) = 3A_x\hat{i} + 3A_y\hat{j} + 3A_z\hat{k}$$

- How do we determine $-\vec{A}$?



$$-\vec{A} = -1 \cdot \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \right) = -A_x\hat{i} - A_y\hat{j} - A_z\hat{k}$$

Example

Example 1.7 **Adding vectors using their components**

Three players on a reality TV show are brought to the center of a large, flat field. Each is given a meter stick, a compass, a calculator, a shovel, and (in a different order for each contestant) the following three displacements:

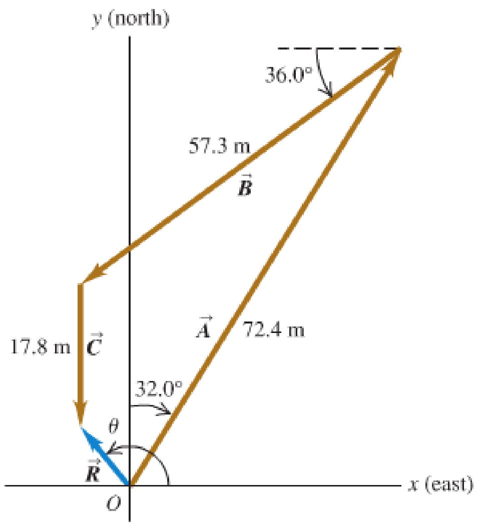
$\vec{A}$: 72.4 m, 32.0° east of north

$\vec{B}$: 57.3 m, 36.0° south of west

$\vec{C}$: 17.8 m due south

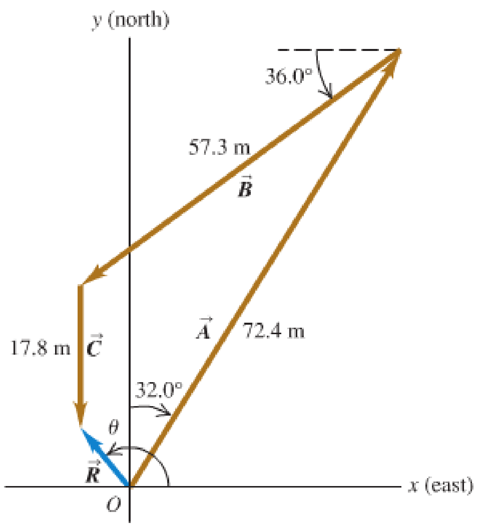
The three displacements lead to the point in the field where the keys to a new Porsche are buried. Two players start measuring immediately, but the winner first *calculates* where to go. What does she calculate?

1.22 Three successive displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$ and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.



Example

1.22 Three successive displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$ and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.



EXECUTE: The angles of the vectors, measured from the $+x$-axis toward the $+y$-axis, are $(90.0° - 32.0°) = 58.0°$, $(180.0° + 36.0°) = 216.0°$, and $270.0°$, respectively. We may now use Eqs. (1.6) to find the components of $\vec{A}$:

$$A_x = A\cos\theta_A = (72.4\text{ m})(\cos 58.0°) = 38.37\text{ m}$$

$$A_y = A\sin\theta_A = (72.4\text{ m})(\sin 58.0°) = 61.40\text{ m}$$

We've kept an extra significant figure in the components; we'll round to the correct number of significant figures at the end of our calculation. The table below shows the components of all the displacements, the addition of the components, and the other calculations.

Distance	Angle	x-component	y-component
$A = 72.4$ m	58.0°	38.37 m	61.40 m
$B = 57.3$ m	216.0°	−46.36 m	−33.68 m
$C = 17.8$ m	270.0°	0.00 m	−17.80 m
		$R_x = -7.99$ m	$R_y = 9.92$ m

$$R = \sqrt{(-7.99\text{ m})^2 + (9.92\text{ m})^2} = 12.7\text{ m}$$

$$\theta = \arctan\frac{9.92\text{ m}}{-7.99\text{ m}} = -51°$$

Comparing to Fig. 1.22 shows that the calculated angle is clearly off by 180°. The correct value is $\theta = 180° - 51° = 129°$, or 39° west of north.

EVALUATE: Our calculated answers for R and θ agree with our estimates. Notice how drawing the diagram in Fig. 1.22 made it easy to avoid a 180° error in the direction of the vector sum.