formed in 10 minutes. Determine the amount of C at any time if the rate of the reaction is proportional to the amounts of A and B remaining and if initially there are 50 grams of A and 32 grams of B. How much of the compound C is present at 15 minutes? Interpret the solution as $t \to \infty$.

Solution Let $X(t)$ denote the number of grams of the compound C present at time t. Clearly $X(0) = 0$ and $X(10) = 30$.

Now for example, if there are 2 grams of compound C, we must have used, say, a grams of A and b grams of B so

$$a + b = 2 \quad \text{and} \quad b = 4a.$$

Thus we must use $a = \frac{2}{5} = 2\frac{1}{5}$ grams of chemical A and $b = \frac{8}{5} = 2\frac{4}{5}$ grams of B. In general, for X grams of C we must use

$$\frac{X}{5} \text{ grams of } A \quad \text{and} \quad \frac{4}{5}X \text{ grams of } B.$$

The amounts of A and B remaining at time t are then

$$50 - \frac{X}{5} \quad \text{and} \quad 32 - \frac{4}{5}X,$$

respectively.

Now we know that the rate at which chemical C is formed satisfies

$$\frac{dX}{dt} \propto \left(50 - \frac{X}{5}\right)\left(32 - \frac{4}{5}X\right).$$

To simplify the subsequent algebra, we factor $\frac{1}{5}$ from the first term and $\frac{4}{5}$ from the second, and then introduce the constant of proportionality:

$$\frac{dX}{dt} = k(250 - X)(40 - X).$$

By separation of variables and partial fractions, we can write

$$\frac{dX}{(250 - X)(40 - X)} = k\,dt$$

$$-\frac{1/210}{250 - X}dX + \frac{1/210}{40 - X}dX = k\,dt$$

$$\ln\left|\frac{250 - X}{40 - X}\right| = 210kt + c_1$$

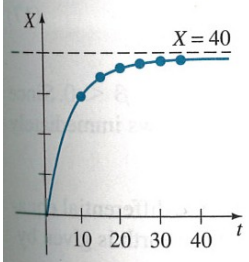
$$\frac{250 - X}{40 - X} = c_2 e^{210kt}. \tag{12}$$

When $t = 0$, $X = 0$, so it follows at this point that $c_2 = \frac{25}{4}$. Using $X = 30$ at $t = 10$, we find that $210k = \frac{1}{10}\ln\frac{88}{25} = 0.1258$. With this information we solve (12) for X:

$$X(t) = 1000\frac{1 - e^{-0.1258t}}{25 - 4e^{-0.1258t}}. \tag{13}$$

The behavior of X as a function of time is displayed in Figure 3.22. It is clear from the accompanying table and equation (13) that $X \to 40$ as $t \to \infty$. This means there are 40 grams of compound C formed, leaving

$50 - \frac{1}{5}(40) = 42$ g of chemical A and $32 - \frac{4}{5}(40) = 0$ g of chemical B.



(a)

t (minutes)	X (grams)
10	30 (measured)
15	34.78
20	37.25
25	38.54
30	39.22
35	39.59

(b)

Figure 3.22