#19 Elastic Potential Energy Post-class

Due: 11:00am on Friday, October 5, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Bungee Jumping

Kate, a bungee jumper, wants to jump off the edge of a bridge that spans a river below. Kate has a mass m, and the surface of the bridge is a height h above the water. The bungee cord, which has length L when unstretched, will first straighten and then stretch as Kate falls.

Assume the following:

- The bungee cord behaves as an ideal spring once it begins to stretch, with spring constant k.
- Kate doesn't actually jump but simply steps off the edge of the bridge and falls straight downward.
- Kate's height is negligible compared to the length of the bungee cord. Hence, she can be treated as a point particle.

Use g for the magnitude of the acceleration due to gravity.

Part A

How far below the bridge will Kate eventually be hanging, once she stops oscillating and comes finally to rest? Assume that she doesn't touch the water.

Express the distance in terms of quantities given in the problem introduction.

Hint 1. Decide how to approach the problem

Here are three possible methods for solving this problem:

1. No nonconservative forces are acting, so mechanical energy is conserved. Set Kate's gravitational potential energy at the top of the bridge equal to the spring potential energy in the bungee cord (which depends on the cord's final length d) and solve for d.
2. Since nonconservative forces are acting, mechanical energy is not conserved. Set the spring potential energy in the bungee cord (which depends on d) equal to Kate's gravitational potential energy plus the work done by dissipative forces. Eliminate the unknown work, and solve for d.
3. When Kate comes to rest she has zero acceleration, so the net force acting on her must be zero. Set the spring force due to the bungee cord (which depends on d) equal to the force of gravity and solve for d.

Which of these options is the simplest, most accurate way to find d given the information available?

ANSWER:

- a
- b
- c

Hint 2. Compute the force due to the bungee cord

When Kate is at rest, what is the magnitude F_b of the upward force the bungee cord exerts on her?

Express your answer in terms of the cord's final stretched length d and quantities given in the problem introduction. Your answer should not depend on Kate's mass m.

Hint 1. Find the extension of the bungee cord

The upward force on Kate is due to the extension of the bungee cord. What is this extension?

Express your answer in terms of the cord's final (stretched) length d and L.

ANSWER:

Extension = $d - L$

Hint 2. Formula for the force due to a stretched cord

The formula for the force due to a stretched cord is

$$F = -k\Delta x,$$

where k is the spring constant of the cord and Δx is the *extension* of the cord.

ANSWER:

$F_b = k(d-L)$

ANSWER:

$d = L + \frac{mg}{k}$

Correct

Part B

If Kate just touches the surface of the river on her first downward trip (i.e., before the first bounce), what is the spring constant k? Ignore all dissipative forces.

Express k in terms of L, h, m, and g.

Hint 1. Decide how to approach the problem

Here are three possible methods for solving this problem:

1. Since nonconservative forces are ignored, mechanical energy is conserved. Set Kate's gravitational potential energy at the top of the bridge equal to the spring potential energy in the bungee cord at the lowest point (which depends on k) and solve for k.
2. Nonconservative forces can be ignored, so mechanical energy is conserved. Set the spring potential energy in the bungee cord (which depends on k) equal to Kate's gravitational potential energy at the top of the bridge plus the work done by gravity as Kate falls. Compute the work done by gravity, then solve for k.
3. When Kate is being held just above the water she has zero acceleration, so the net force acting on her must be zero. Set the spring force due to the bungee cord (which depends on k) equal to the force of gravity and solve for k.

Which of these options is the simplest, most accurate way to find k given the information available?

ANSWER:

- ◉ a
- ○ b
- ○ c

Hint 2. Find the initial gravitational potential energy

What is Kate's gravitational potential energy U_g at the moment she steps off the bridge? (Define the zero of gravitational potential to be at the surface of the water.)

Express your answer in terms of quantities given in the problem introduction.

ANSWER:

U_g = mgh

Hint 3. Find the elastic potential energy in the bungee cord

What is the elastic potential energy U_{el} stored in the bungee cord when Kate is at the lowest point of her first downward trip?

Express your answer in terms of quantities given in the problem introduction.

Hint 1. Formula for elastic potential energy

The elastic potential energy of the bungee cord (which we are treating as an ideal spring) is

$$U_{el} = \frac{1}{2}k(\Delta x)^2,$$

where Δx is the amount by which the cord is stretched beyond its unstretched length.

Hint 2. How much is the bungee cord stretched?

By how much is the bungee cord stretched when Kate is at a depth d_1 below the bridge?

Express your answer in terms of d_1 and L.

ANSWER:

Δx = $d_1 - L$

ANSWER:

$U_{\rm el}$ = $\frac{1}{2}k\,(h-L)^2$

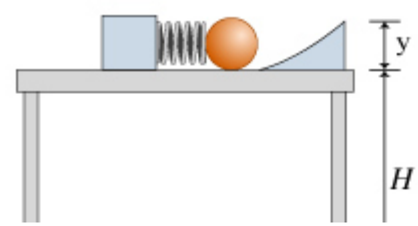
ANSWER:

k = $\frac{2mgh}{(h-L)^2}$

Correct

Spring and Projectile

A child's toy consists of a block that attaches to a table with a suction cup, a spring connected to that block, a ball, and a launching ramp. The spring has a spring constant k, the ball has a mass m, and the ramp rises a height y above the table, the surface of which is a height H above the floor.



Initially, the spring rests at its equilibrium length. The spring then is compressed a distance s, where the ball is held at rest. The ball is then released, launching it up the ramp. When the ball leaves the launching ramp its velocity vector makes an angle θ with respect to the horizontal.

Throughout this problem, ignore friction and air resistance.

Part A

Relative to the initial configuration (with the spring relaxed), when the spring has been compressed, the ball-spring system has

ANSWER:

- gained kinetic energy
- gained potential energy
- lost kinetic energy
- lost potential energy

Correct

Part B

As the spring expands (after the ball is released) the ball-spring system

ANSWER:

- gains kinetic energy and loses potential energy
- gains kinetic energy and gains potential energy
- loses kinetic energy and gains potential energy
- loses kinetic energy and loses potential energy

Correct

Part C

As the ball goes up the ramp, it

ANSWER:

- ○ gains kinetic energy and loses potential energy
- ○ gains kinetic energy and gains potential energy
- ◉ loses kinetic energy and gains potential energy
- ○ loses kinetic energy and loses potential energy

Correct

Part D

As the ball falls to the floor (after having reached its maximum height), it

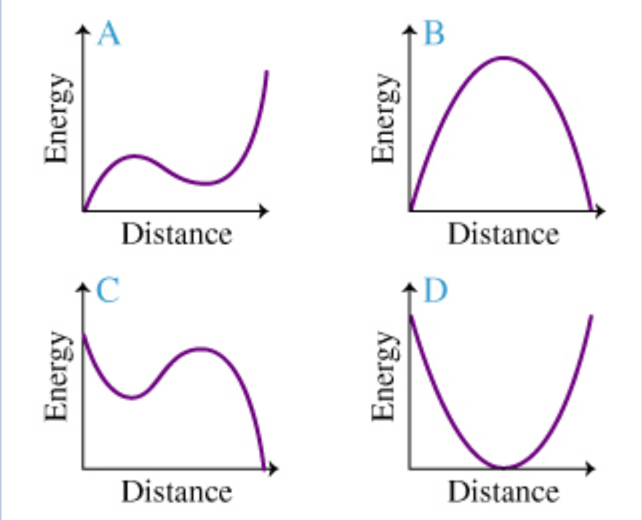
ANSWER:

- ◉ gains kinetic energy and loses potential energy
- ○ gains kinetic energy and gains potential energy
- ○ loses kinetic energy and gains potential energy
- ○ loses kinetic energy and loses potential energy

Correct

Part E

Which of the graphs shown best represents the potential energy of the ball-spring system as a function of the ball's horizontal displacement? Take the "zero" on the distance axis to represent the point at which the spring is fully compressed.
Keep in mind that the ball is *not* attached to the spring, and neglect any recoil of the spring after the ball loses contact with it.



ANSWER:

- A
- B
- C (selected)
- D

Correct

Part F

Calculate v_r, the speed of the ball when it leaves the launching ramp.

Express the speed of the ball in terms of k, s, m, g, y, and/or H.

Hint 1. General approach

Find an expression for the mechanical energy (kinetic plus potential) of the spring and ball when the spring is compressed. Then find an expression for the mechanical energy of the ball when it leaves the launching ramp. (v_r will be an unknown in this expression.) Since energy is conserved, you can set these two expressions equal to each other, and solve for v_r.

Hint 2. Find the initial mechanical energy

Find the total mechanical energy of the ball-spring system when the spring is fully compressed. *Take the gravitational potential energy to be zero at the floor.*

Hint 1. What contributes to the mechanical energy?

The total initial mechanical energy is the sum of the potential energy of the spring, the gravitational potential energy, plus any initial kinetic energy of the ball.

ANSWER:

E_i = $\frac{1}{2}ks^2 + mgH$

Hint 3. Find the mechanical energy at the end of the ramp

Find the total mechanical energy of the ball when it leaves the launching ramp. (At this point, assume that the spring is relaxed and has no stored potential energy.) Again, take the gravitational potential energy to be zero at the floor.

Express your answer in terms of v_r and other given quantities.

ANSWER:

$E_{\rm r}$ = $\frac{1}{2}mv_{\rm r}^2 + mg\left(H+y\right)$

Hint 4. Is energy conserved?

Because no nonconservative forces act on the system, energy is conserved: $E_{\rm i} = E_{\rm r}$.

ANSWER:

$v_{\rm r}$ = $\sqrt{\frac{ks^2 - 2mgy}{m}}$

Correct

Part G

With what speed will the ball hit the floor?

Express the speed in terms of k, s, m, g, y, and/or H.

Hint 1. General approach

Find an expression for the initial mechanical energy (kinetic plus potential) of the spring and ball. Then find an expression for the mechanical energy of the ball when it hits the floor. ($v_{\rm f}$ will be an unknown in this expression.) Since energy is conserved, you can set these two expressions equal to each other, and solve for $v_{\rm f}$.

Hint 2. Initial mechanical energy

For the initial mechanical energy, you can use either the expression you found for the mechanical energy of the ball at the top of the ramp or that for the total mechanical energy of the ball plus spring just before the ball was launched. These two expressions are equal.

Hint 3. Find the final mechanical energy

Find the total mechanincal energy E_f of the ball when it hits the floor.

Express your answer in terms of v_f and other given quantities.

ANSWER:

E_f = $\frac{1}{2}mv_f{}^2$

Hint 4. Is energy conserved?

Only conservative forces (gravity, spring) are acting on the ball, so energy is conserved: $E_i = E_r = E_f$.

ANSWER:

v_f = $\sqrt{\frac{ks^2}{m}+2gH}$

Correct

Exercise 7.19

A spring of negligible mass has force constant k = 1500$\mathrm{N/m}$.

Part A

How far must the spring be compressed for an amount 3.40J of potential energy to be stored in it?

Express your answer using two significant figures.

ANSWER:

6.7×10^{-2} m

Correct

Part B

You place the spring vertically with one end on the floor. You then drop a book of mass 1.20kg onto it from a height of 0.900m above the top of the spring. Find the maximum distance the spring will be compressed.

Take the free fall acceleration to be 9.80$\mathrm{m/s^2}$. Express your answer using two significant figures.

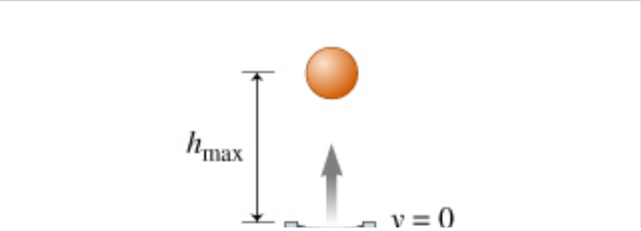
ANSWER:

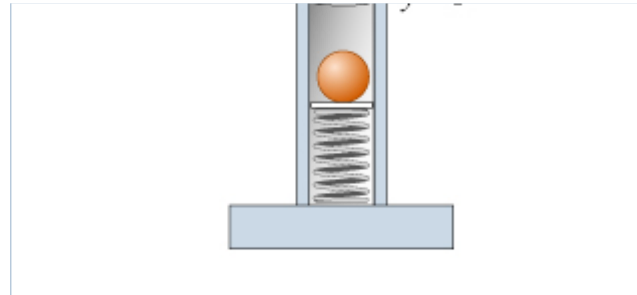
0.13 m

Correct

Fun with a Spring Gun

A spring-loaded toy gun is used to shoot a ball of mass $m = 1.50\ \mathrm{kg}$ straight up in the air, as shown in the figure. The spring has spring constant $k = 667\ \mathrm{N/m}$. If the spring is compressed a distance of 25.0 centimeters from its equilibrium position $y = 0$ and then released, the ball reaches a maximum height $h_{\rm max}$ (measured from the equilibrium position of the spring). There is no air resistance, and the ball never touches the inside of the gun. Assume that all movement occurs in a straight line up and down along the *y* axis.





Part A

Which of the following statements are true?

Check all that apply.

Hint 1. Nonconservative forces

Dissipative, or *nonconservative*, forces are those that always oppose the motion of the object on which they act. Forces such as friction and drag are dissipative forces.

Hint 2. Forces acting on the ball

The ball is acted on by the spring force only when the two are in contact. The force of tension in the spring is a conservative force. Also, the ball is *always* acted on by gravity, which is also a conservative, or nondissipative, force.

ANSWER:

- [x] Mechanical energy is conserved because no dissipative forces perform work on the ball.
- [x] The forces of gravity and the spring have potential energies associated with them.
- [ ] No conservative forces act in this problem after the ball is released from the spring gun.

Correct

Part B

Find $v_{\rm m}$ the muzzle velocity of the ball (i.e., the velocity of the ball at the spring's equilibrium position $y = 0$).

Hint 1. Determine how to approach the problem

What physical relationship can you use to solve this problem? Choose the *best* answer.

ANSWER:

- kinematics equations
- Newton's second law
- law of conservation of energy
- conservation of momentum

Hint 2. Energy equations

Recall that kinetic energy is given by the equation

$$\frac{1}{2}mv^2,$$

where v is the speed of the object and m is the object's mass.

Gravitational potential energy is given by

$$mgy,$$

where y is the object's height measured from $y = 0$.

The elastic potential energy of a spring is given by

$$\frac{1}{2}kx^2,$$

where k is the spring constant and x is the spring's displacement from equilibrium.

Hint 3. Determine which two locations you should examine

Pick the two points along the ball's path that would be most useful to compare in order to find the solution to this problem. Choose from among the following three points:

Check all that apply.

ANSWER:

- [x] y=-25\;\rm{cm}, the location of the ball when the spring is compressed.
- [x] $y = 0$, the equilibrium position of the spring.
- [ ] y=h_{\max}, the maximum height that the ball reaches above the point $y = 0$.

Hint 4. Find the initial energy of the system

A useful statement of mechanical energy conservation relating the initial and final kinetic (K) and potential (U) energies is

$$K_{\text{initial}} + U_{\text{initial}} = K_{\text{final}} + U_{\text{final}}.$$

In this situation, the initial position is $y = -25.0 \text{ cm}$ and the final position is $y = 0$, which is the equilibrium position of the spring. What kind(s) of energy does the system "spring-ball" have at the initial position?

ANSWER:

- kinetic only
- elastic potential only
- gravitational potential only
- kinetic and gravitational potential
- kinetic and elastic potential
- elastic and gravitational potentials

Hint 5. Determine the final energy

A useful statement of mechanical energy conservation relating the initial and final kinetic (K) and potential (U) energies is

$$K_{\text{initial}} + U_{\text{initial}} = K_{\text{final}} + U_{\text{final}}.$$

In this situation, the initial position is $y = -25.0 \text{ cm}$ and the final position is $y = 0$, which is the equilibrium position of the spring. What kind(s) of energy does the system "spring-ball" have at the final position?

ANSWER:

- kinetic only
- elastic potential only
- gravitational potential only
- kinetic and gravitational potential
- kinetic and elastic potential
- elastic and gravitational potentials

Hint 6. Creating an equation

From the hints you now know what kinds of energy are present at the initial and final positions chosen for the ball in this part of the problem. You also know that

$$K_{\text{initial}} + U_{\text{initial}} = K_{\text{final}} + U_{\text{final}}.$$

It has been determined that K_{initial} is zero and U_{initial} consists of two terms: gravitational potential energy and elastic potential energy. In addition, U_final is zero.

ANSWER:

v_{m} = 4.78 m/s

Correct

Part C

Find the maximum height $h_{\rm max}$ of the ball.

Express your answer numerically, in meters.

Hint 1. Choose two locations to examine

Pick the two points along the ball's movement that would be most useful to compare in order to find a solution to this problem. Choose from among the following three points:

Check all that apply.

ANSWER:

- [x] $y=-25\;\rm cm$, the location of the ball when the spring is compressed.
- [ ] $y=0$, the equilibrium position of the spring.
- [x] $y=h_{\max}$, the maximum height that the ball reaches measured from $y=0$.

Hint 2. Find the initial energy

A useful statement of mechanical energy conservation is

$$K_{\rm inital}+ U_{\rm initial} = K_{\rm final} + U_{\rm final}.$$

Recall that in the problem statement, $y=0$ is set to correspond to the equilibrium position of the spring. Therefore, in this situation, the initial location is at $y=-25\;\rm cm$ and the final position should be taken as $y=h_{\max}$.

What kind(s) of energy does the ball have at the initial location?

ANSWER:

- kinetic only
- elastic potential only
- gravitational potential only
- kinetic and gravitational potential
- kinetic and elastic potential
- elastic and gravitational potentials

Hint 3. Determine the final energy

A useful statement of mechanical energy conservation is

$$K_{\rm initial} + U_{\rm initial} = K_{\rm final} + U_{\rm final}.$$

In this situation, the initial location is at y=-25\;\rm cm, and the final position should be taken as y=h_{\max}. What kind(s) of energy does the ball have at y=h_{\max}?

Hint 1. Find the speed of the ball at the top of its trajectory

What is the speed $v_{\rm top}$ of the ball at the top of its trajectory?

Express your answer numerically, in meters per second.

Hint 1. Motion in the vertical direction

Recall from kinematics that a ball travels upward until its speed decreases to zero, at which point it starts falling back to Earth.

ANSWER:

$v_{\rm top}$ = 0 $\rm m/s$

ANSWER:

- kinetic only
- elastic potential only
- gravitational potential only
- kinetic and gravitational potential
- kinetic and elastic potential
- elastic and gravitational potentials

Hint 4. Creating an equation

From the above hints, you now know what kind of energy is present at the inital and final positions chosen for the ball in this part of the problem. You know that

$$K_{\rm initial} + U_{\rm initial} = K_{\rm final} + U_{\rm final}.$$

It was determined that $K_{\rm initial}$ is zero and that $U_{\rm initial}$ consists of two terms: gravitational potential energy and elastic potential energy. In addition, K_final is zero.

ANSWER:

$h_{\rm max}$ = 1.17 $\rm m$

Correct

In this problem you practiced applying the law of conservation of mechanical energy to a physical situation to find the muzzle velocity and the maximum height reached by the ball.

Part D

Which of the following actions, if done independently, would increase the maximum height reached by the ball?

Check all that apply.

ANSWER:

- [ ] reducing the spring constant k
- [x] increasing the spring constant k
- [ ] decreasing the distance the spring is compressed
- [x] increasing the distance the spring is compressed
- [x] decreasing the mass of the ball
- [ ] increasing the mass of the ball
- [ ] tilting the spring gun so that it is at an angle \theta < 90 degrees from the horizontal

Correct

Score Summary:

Your score on this assignment is 101.7%.
You received 40.68 out of a possible total of 40 points.