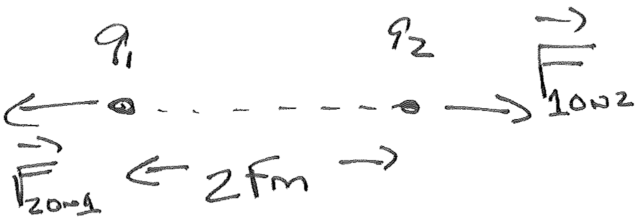
PHYSICS 161, HW #1

#1 Two protons 2fm Apart



Two POSITIVE CHARGES
$\Rightarrow$ Repulsion

protons $\Rightarrow$ $q_1 = q_2 = 1.6\times10^{-19}C$

$r = 2fm = 2\times10^{-15}m$

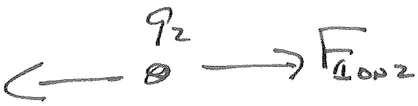
$$F_{1on2} = \frac{K|q_1||q_2|}{r^2} = \frac{(9\times10^9 N\cdot m^2/C^2)(1.6\times10^{-19}C)(1.6\times10^{-19}C)}{(2\times10^{-15}m)^2} = 57.536N$$

$$\Rightarrow \underline{\underline{F_{1on2} = 57.5N}}$$

IN POUNDS? $1N = 0.2248 lb \Rightarrow 57.536N \times \frac{0.2248lb}{1N}$

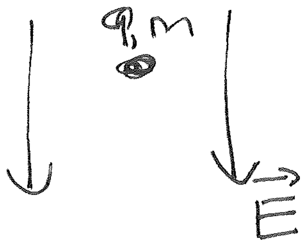
$$\Rightarrow \underline{\underline{F_{1on2} = 12.934lb \cong 13lb}}$$ $\Leftarrow$ DEFINITELY LARGE ENOUGH to feel.

b) PROTONS DON'T FLY OUT? $\Leftarrow$ MUST BE HELD IN PLACE by ANOTHER FORCE.

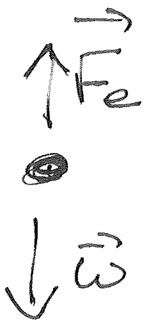


EQUAL but OPPOSITE Force Keeping Proton IN Nucleus

\#2 WHAT charge to keep 1.15g particle stationary in DOWNWARD 750N/C Electric Field?



$m = 1.15g = 1.15 \times 10^{-3}$ Kg



Forces on Particle: DOWNWARD Weight $\vec{\omega}$, $\omega = mg$

So NEED UPWARD Electric Force

$F_e = |q|E$

Stationary $\Rightarrow \sum \vec{F} = 0 \Rightarrow \vec{F_e} + \vec{\omega} = 0$

$\Rightarrow \vec{F_e} = -\vec{\omega} \Rightarrow |q|E = mg \Rightarrow |q| = \frac{mg}{E}$

$\therefore |q| = \frac{(1.15 \times 10^{-3} kg)(9.8 m/s^2)}{750 N/C} \Rightarrow \boxed{|q| = 1.5 \times 10^{-5} C = 15 \mu C}$

UPWARDS $\vec{F_e}$, so $\vec{F_e}$ opposite to $\vec{E}$ $\Rightarrow$ q must be NEGATIVE

$\therefore \boxed{q = -15 \mu C}$

b) WHAT ELECTRIC FIELD to keep proton stationary?

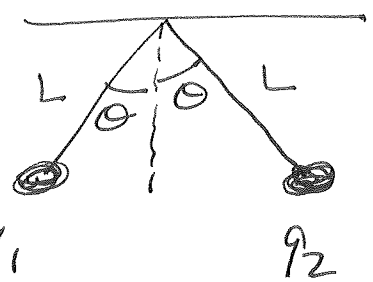
$$F_e = W \Rightarrow |q|E = mg \Rightarrow E = \frac{mg}{|q|}$$

proton $\Rightarrow m = 1.67\times10^{-27}\,kg,\; q = 1.6\times10^{-19}\,C$

$$\therefore\; E = \frac{(1.67\times10^{-27}\,kg)(9.8\,m/s^2)}{1.6\times10^{-19}\,C} = \boxed{1.02\times10^{-7}\,N/C}$$

VERY SMALL.

#3



$m = 25g = 0.025 kg$

$L = 1.5m$

when $q_1 = q_2 = q$ (BOTH Negative)

$\theta = 25°$

a) DRAW FREE-BODY DIAGRAM FOR EACH MASS.

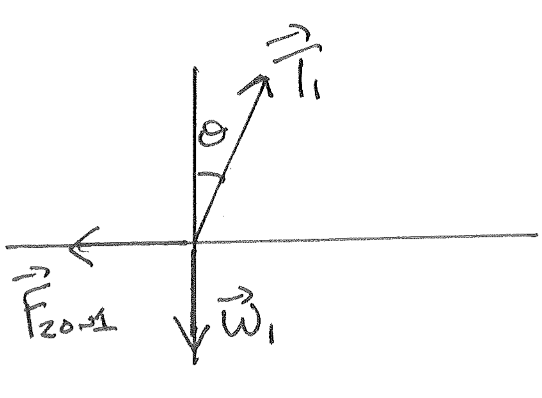
Forces on q_1: GRAVITY ⇒ $\vec{W}_1$ DOWN, $W_1 = mg = (0.025kg)(9.8 m/s^2)$

$= 0.245N$

Tension: $\vec{T}_1$ Along string (Related to θ)

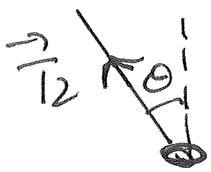
Electric Force: $\vec{F}_{2 on 1}$. q_1, q_2 BOTH Negative ⇒ Repulsion.

EQUAL L AND θ ⇒ q_1 & q_2 AT SAME HEIGHT ⇒ LINE Connecting THEIR CENTERS IS HORIZONTAL ⇒ $\vec{F}_{2 on 1}$ to left

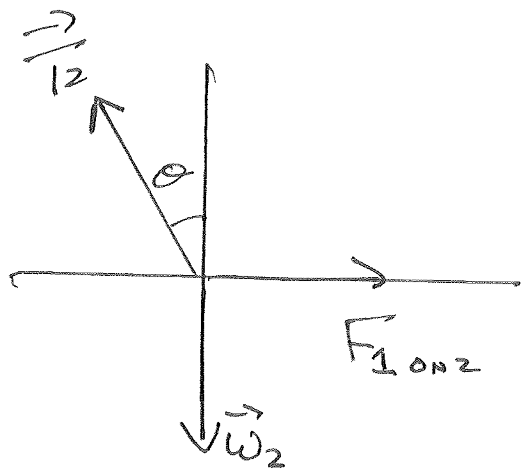


Forces on q_2 mostly THE SAME:

$\vec{W}_2$ DOWN (SAME MASS $\Rightarrow W_2 = 0.245N$ also)

$\vec{T}_2$ along its string $\Rightarrow$ 

$\vec{F}_{1 on 2}$ to Right



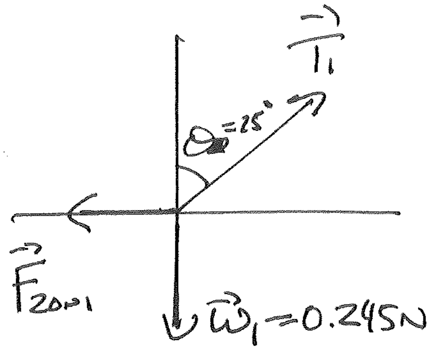
b) Find Value of q

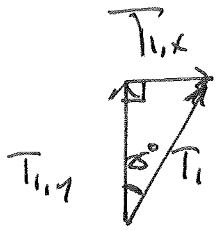
We Know Two Things:

① masses at Rest $\Rightarrow$ Zero Net force $\Rightarrow$ $\sum F_x = 0$, $\sum F_y = 0$ FOR BOTH

② $F_{2 on 1} = F_{1 on 2} = \dfrac{K|q_1||q_2|}{r^2} = \dfrac{K(q)(q)}{r^2} = \dfrac{Kq^2}{r^2}$ { Note: BOTH Negative So multiplying makes them positive, so don't worry about ABS. VALUE

From q_1's fbd





$T_{1,x} = +T_1 \sin 25°$

$T_{1,y} = +T_1 \cos 25°$

$$\Sigma F_y = 0 \Rightarrow T_{1,y} + w_{1,y} + \cancel{F_{2on1,y}}^{\,0} = 0 \Rightarrow T_1 \cos 25° - w_1 = 0$$

$$\Rightarrow T_1 = \frac{w_1}{\cos 25°} = \frac{0.245N}{\cos 25°} = 0.27N$$

$$\Sigma F_x = 0 \Rightarrow T_{1,x} + \cancel{w_{1,x}}^{\,0} + F_{2on1,x} = 0$$

$$\Rightarrow T_1 \sin 25° - F_{2on1} = 0 \Rightarrow F_{2on1} = T_1 \sin 25°$$

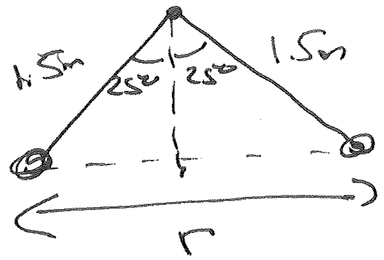
$$\Rightarrow F_{2on1} = \frac{w_1}{\cos 25°} \sin 25° = w_1 \tan 25° = 0.245N \tan 25°$$

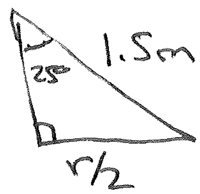
$$= 0.1142N$$

(Looking at q_2 gives $T_2 = 0.27N$ AND $F_{1on2} = 0.1142N$)

Of Course

Finally use $F_{2 on 1} = \frac{kq^2}{r^2}$ after finding r





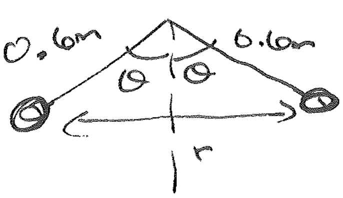
$$\sin 25° = \frac{r/2}{1.5m}$$

$$\Rightarrow r = 2(1.5m \sin 25°) = 1.2679m$$

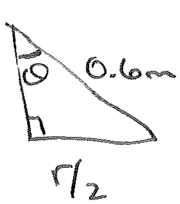
$$\therefore F_{2 on 1} = \frac{kq^2}{r^2} \Rightarrow q^2 = \frac{F_{2 on 1}\, r^2}{k} = \frac{(0.1142N)(1.2679m)^2}{(9\times10^9 N\cdot m^2/C^2)} = 2.04\times10^{-11} C^2$$

$$\therefore \boxed{q = \sqrt{2.04\times10^{-11} C^2} = 4.5\times10^{-6} C = 4.5\,\mu C}$$

c) Strings shortened to 0.6m, what is now angle θ?



Since same string length $\Rightarrow$ same angle for both $\therefore$ $\vec{F}_{2 on 1}$ and $\vec{F}_{1 on 2}$ still horizontal.

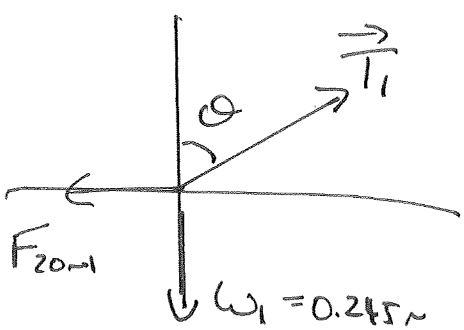


$$\sin\theta = \frac{r/2}{0.6m} \Rightarrow r = 2(0.6 \sin\theta) = 1.2m \sin\theta$$

$$F_{2\,on\,1} = \frac{Kq^2}{r^2} = \frac{(9\times10^9\,N\cdot m^2/C^2)(4.5\times10^{-6}\,C)^2}{(1.2m\,\sin\theta)^2} = \frac{(9\times10^9\,N\cdot m^2/C^2)(4.5\times10^{-6}\,C)^2}{(1.2m)^2\sin^2\theta}$$

$$\Rightarrow F_{2\,on\,1} = \frac{0.127\,N}{\sin^2\theta}$$

SAME f.b.d. for q_1 but now $\vec{T}_1$ at θ



at rest $\Rightarrow \sum F_x = 0$

$\sum F_y = 0$

$\sum F_y = 0 \Rightarrow T_1\cos\theta = 0.245N$

$\sum F_x = 0 \Rightarrow T_1 \sin\theta = F_{2on1}$

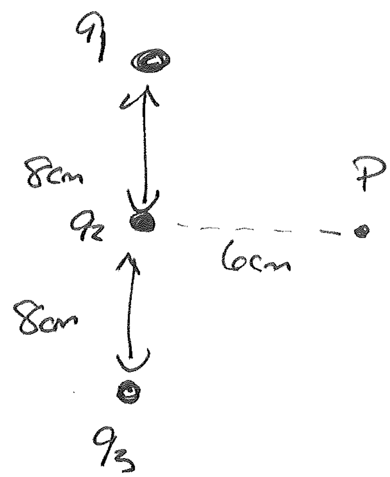
$\therefore T_1 = \frac{0.245N}{\cos\theta}$ AND $T_1 = \frac{F_{2on1}}{\sin\theta} \Rightarrow \frac{0.245N}{\cos\theta} = \frac{F_{2on1}}{\sin\theta}$

$\Rightarrow \frac{0.245N}{\cos\theta} = \frac{\frac{0.127N}{\sin^2\theta}}{\sin\theta} \Rightarrow \frac{0.245N}{\cos\theta} = \frac{0.127N}{\sin^3\theta} \Rightarrow \frac{\sin^3\theta}{\cos\theta} = \frac{0.127N}{0.245N}$

$\Rightarrow \frac{\sin^3\theta}{\cos\theta} = 0.517$

By trial-and-error (or more truthfully using MATLAB), we solve numerically and find $\underline{\theta = 45.5^\circ}$

#4



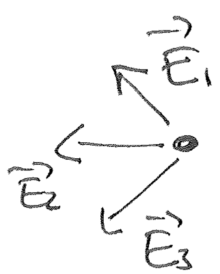
$q_1 = q_3 = -5\mu C$

$q_2 = -2\mu C$

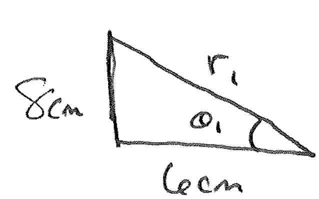
Find Magnitude & Direction of $\vec{E}$ at P

~~at P~~ $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

Since we ~~[illegible]~~ imagine positive charge at P and q_1, q_2, q_3 All Negative



$\vec{E}_1$:

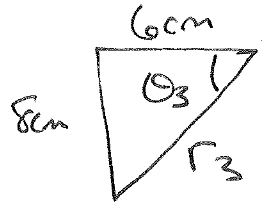


$E_1 = \frac{K|q_1|}{r_1^2}$ $\quad$ $r_1^2 = (0.08m)^2 + (0.06m)^2 = 0.01m^2$

$\therefore E_1 = \frac{(9\times10^9 N\cdot m^2/C^2)(5\times10^{-6}C)}{(0.01m^2)} = 4.5\times10^6 N/C$

$\theta_1 = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°$

$\vec{E}_3$ HAS SAME NUMBERS as $\vec{E}_1$



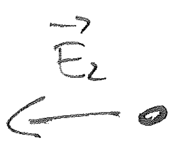
$$E_3 = \frac{K|q_3|}{r_3^2} \qquad r_3^2 = 0.01m^2 \Rightarrow E_3 = 4.5\times10^6 N/C$$

$$\theta_3 = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°$$

So, Given that $E_1 = E_3$ AND they are at SAME Angle Above & Below Axis, y-Components will cancel.

$\therefore$ $\vec{E}$ only HAS X-Component.

$\vec{E}_2$:



$$E_2 = \frac{K|q_2|}{r_2^2} \qquad r_2 = 6cm = 0.06m$$

$$\therefore E_2 = \frac{(9\times10^9 N\cdot m^2/C^2)(2\times10^{-6}C)}{(0.06m)^2} = 5\times10^6 N/C$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \Rightarrow E_x = E_{1,x} + E_{2,x} + E_{3,x}$$

MAke to left positive $\Rightarrow E_{1,x} = E_{3,x} = (4.5\times10^6 N/C)\cos 53.13° =$

$= (4.5\times10^6 N/C)(0.6) = 2.7\times10^6 N/C$

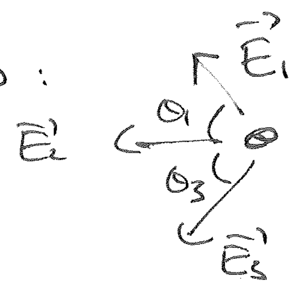
$E_{2,x} = E_2 = 5\times10^{6}\,N/C$

$\therefore \quad E_x = E_{1,x} + E_{2,x} + E_{3,x} = 2.7\times10^{6}\,N/C + 5\times10^{6}\,N/C + 2.7\times10^{6}\,N/C$

$= 1.04\times10^{7}\,N/C$

$E_y = E_{1,y} + E_{2,y} + E_{3,y}$

As Promised:



$E_{1,y} = +E_1 \sin\theta_1$

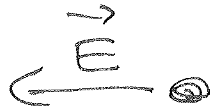
$E_{3,y} = -E_3 \sin\theta_3$

$E_1 = E_3, \quad \theta_1 = \theta_3$

$\Rightarrow E_{1,y} + E_{3,y} = 0$

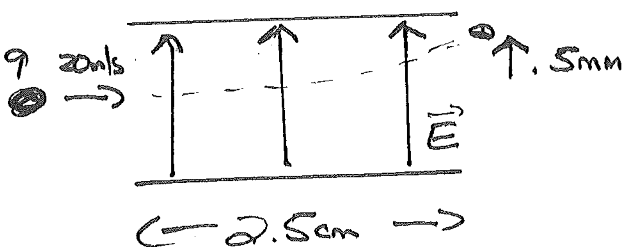
$E_{2,y} = 0 \quad \therefore \quad E_y = 0$

So Finally:



$\boxed{E = 1.04\times10^{7}\,N/C}$

#5



$M = 1\times10^{-8}g = 1\times10^{-11}kg$

$2.5cm = .025m$

$.5mm = 5\times10^{-4}m$

$E = 9\times10^{4} N/C$

WHAT CHARGE q?

FIRST ASSUME GRAVITY IS NEGLIGIBLE. FIND q AND THEN ELECTRIC force to COMPARE WITH WEIGHT TO SEE IF THIS WAS REASONABLE.

ELECTRONS REMOVED $\Rightarrow$ POSITIVE CHARGE. VERTICAL ELECTRIC FIELD $\Rightarrow$ VERTICAL force. IGNORE ALL OTHER forces

$\Rightarrow \sum F_y = qE, \quad \sum F_x = 0$

2ND LAW: $\sum F_y = Ma_y \Rightarrow qE = Ma_y \Rightarrow q = \frac{Ma_y}{E}$

q, E CONSTANT $\Rightarrow$ CONSTANT a_y, THEREFORE WE CAN USE

$y = y_0 + V_{0,y}t + \frac{1}{2}a_y t^2$: $\quad y = 5\times10^{-4}m, \; y_0 = 0, \; V_{0,y} = 0$

but NEED t

Use $\sum F_x = 0 \Rightarrow a_x = 0 \Rightarrow V_x = V_{0,x} = 20\,m/s$

$$x = x_0 + V_{0,x} t$$

$x = 0.025\,m,\ x_0 = 0 \ \therefore\ t = \frac{x}{V_{0,x}} = \frac{0.025\,m}{20\,m/s} = 0.00125\,s$

$y = \cancel{y_0}^{0} + \cancel{V_{0,y}t}^{0} + \frac{1}{2}a_y t^2 \Rightarrow a_y = \frac{2y}{t^2} = \frac{2(5\times10^{-4}\,m)}{(0.00125\,s)^2} = 640\,m/s^2$

So $q = \frac{m a_y}{E} = \frac{(1\times10^{-11}\,kg)(640\,m/s^2)}{9\times10^{4}\,N/C} = 7.11\times10^{-14}\,C$

$\Rightarrow \boxed{q = 0.0711\,pC}$ ← NOT MUCH

Finally: $qE = (7.11\times10^{-14}\,C)(9\times10^{4}\,N/C) = 6.4\times10^{-9}\,N$

$mg = (1\times10^{-11}\,kg)(9.8\,m/s^2) = 9.8\times10^{-11}\,N$

So $mg \ll qE \Rightarrow$ ok to ignore gravity