Solutions to Homework 2

Problem 1.44 Consider the sinusoidal signal

$$x(t) = A\cos(\omega t + \phi).$$

Determine the average power of $x(t)$.

Solution: As discussed in the solutions to the previous homework, the average power of a periodic signal is given by the ratio of the signal energy of one period E_o, over the length of the period T_o, that is

$$P = \frac{E_o}{T_o},$$

where

$$\begin{aligned} E_o &= \int_{-T_o/2}^{T_o/2} |x(t)|^2 dt, \quad \text{and} \\ T_o &= \frac{2\pi}{\omega}. \end{aligned}$$

So, let's compute the energy E_o:

$$\begin{aligned} E_o &= \int_{-T_o/2}^{T_o/2} |x(t)|^2 dt \\ &= \int_{-\pi/\omega}^{\pi/\omega} |A\cos(\omega t + \phi)|^2 dt \\ &= \int_{-\pi/\omega}^{\pi/\omega} A^2 \cos^2(\omega t + \phi) dt \\ &= \int_{-\pi/\omega}^{\pi/\omega} A^2 \left[\frac{1 + \cos(2\omega t + 2\phi)}{2}\right] dt \\ &= \frac{A^2}{2}\left[\int_{-\pi/\omega}^{\pi/\omega} dt + \int_{-\pi/\omega}^{\pi/\omega} \cos(2\omega t + 2\phi) dt\right] \\ &= \frac{A^2}{2}\left\{\frac{2\pi}{\omega} + \left[\frac{\sin(2\omega t + 2\phi)}{2\omega}\right]_{-\pi/\omega}^{\pi/\omega}\right\} \\ &= \frac{A^2 T_o}{2}. \end{aligned}$$

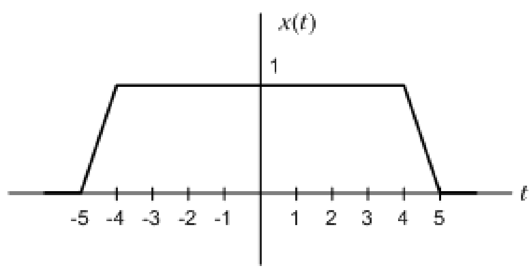
The null value of the second integral is due to the fact that $\sin(2\pi + 2\phi) = \sin(-2\pi + 2\phi)$.

As a conclusion, we get that

$$P = \frac{E_o}{T_o} = \frac{A^2}{2}$$

Problem 1.47 The trapezoidal pulse $x(t)$ shown in the graph below is defined by

$$x(t) = \begin{cases} 5-t, & 4 \le t \le 5, \\ 1, & -4 \le t \le t4, \\ t+5, & -5 \le t \le -4, \\ 0, & \text{otherwise.} \end{cases}$$



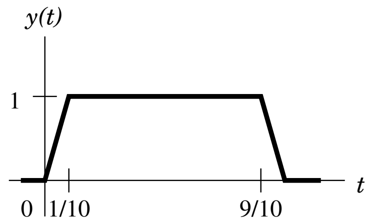
Determine the total energy of $x(t)$.

Solution: By definition

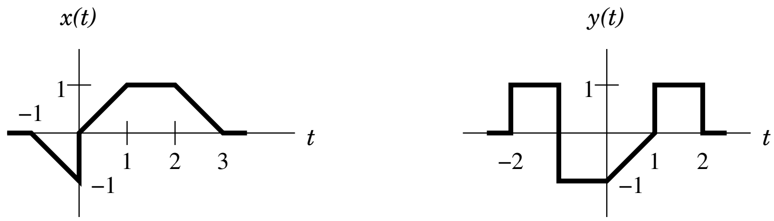
$$\begin{aligned} E &= \int_{-\infty}^{\infty} x^2(t)dt \\ &= \int_{-5}^{5} x^2(t)dt \end{aligned}$$

Since $x^2(t)$ is even, we may write

$$\begin{aligned} E &= 2\int_0^5 x^2(t)dt \\ &= 2\left[\int_0^4 dt + \int_4^5 (5-t)^2 dt\right] \\ &= 2\left\{4 - \left[\frac{(5-t)^3}{3}\right]_{t=4}^{t=5}\right\} = \frac{26}{3} = 8.666 \end{aligned}$$

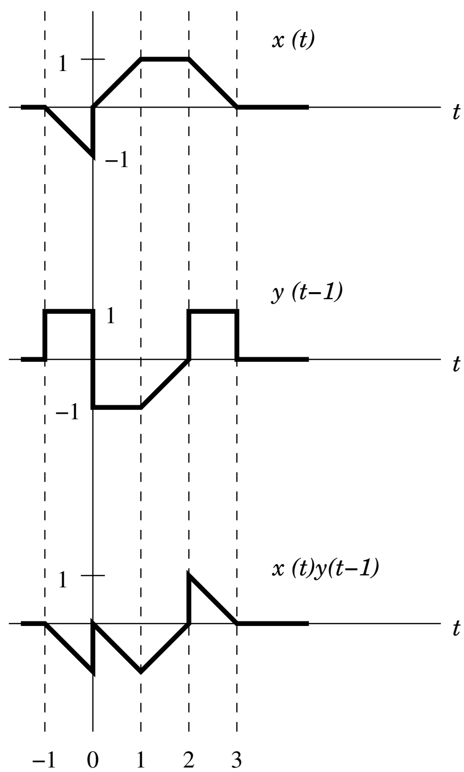


Problem 1.52 Let $x(t)$ and $y(t)$ be given in Fig. below. Carefully sketch the following signals:



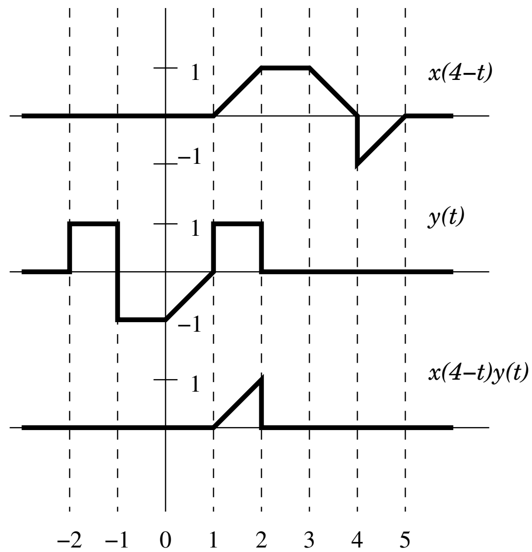
(a) $x(t)y(t-1)$

Solution:



(g) $x(4-t)y(t)$

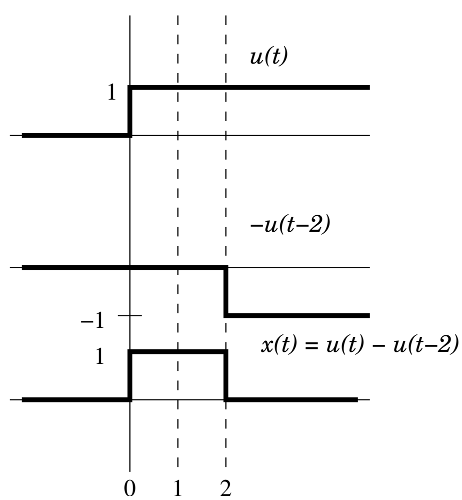
Solution:



Problem 1.54 Sketch the wave forms of the following signals:

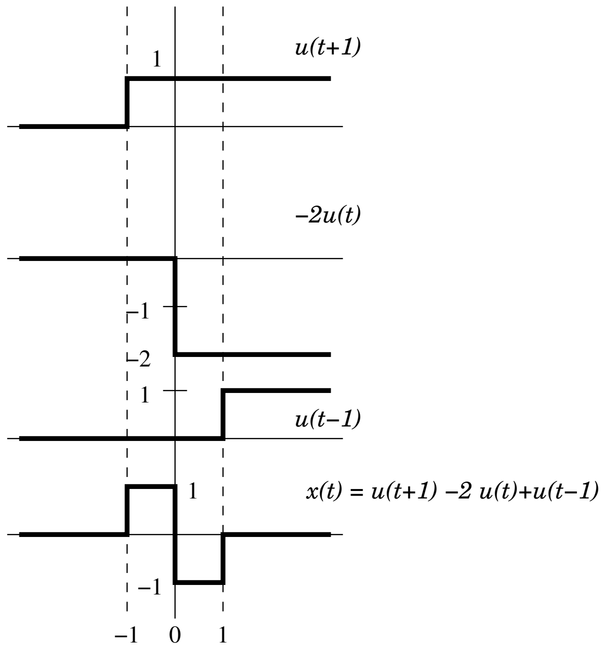
(a) $x(t) = u(t) - u(t-2)$

Solution:

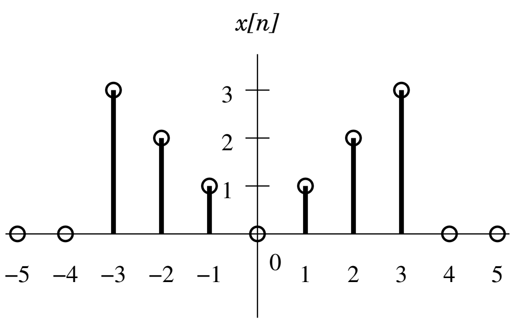


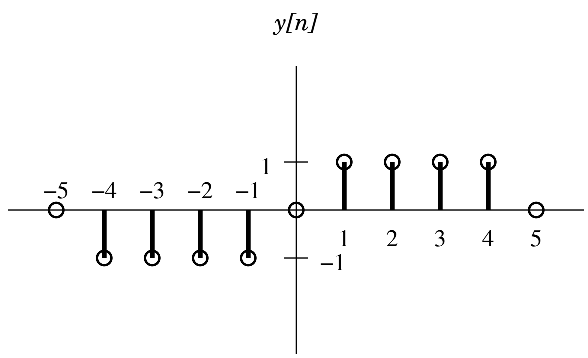
(b) $x(t) = u(t+1) - 2u(t) + u(t-1)$

Solution:



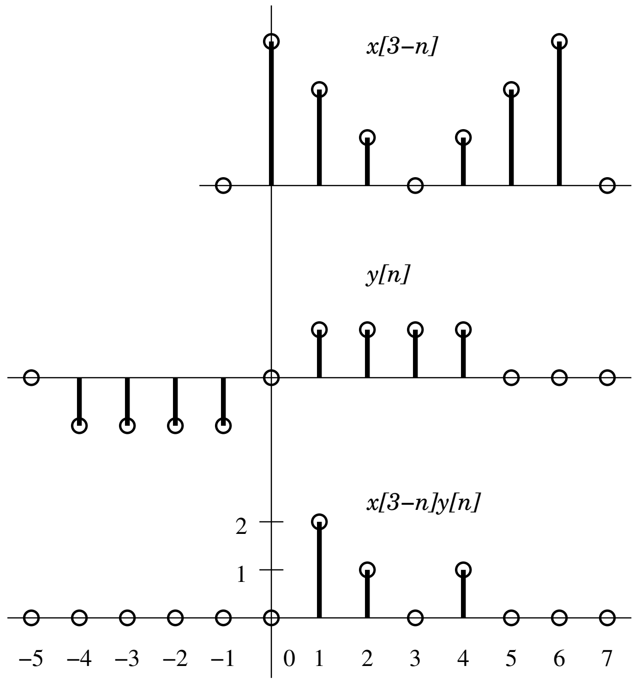
Problem 1.56 Let $x[n]$ and $y[n]$ be given in the figure below. Carefully sketch the following signals:





(h) $x[3-n]y[n]$

Solution:


x[3−n]
y[n]
x[3−n]y[n]
2
1
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
7

Solution: For this system, we can see that

$$\begin{aligned} y[n] &= 2\left(\alpha_1 x_1[n] + \alpha_2 x_2[n]\right) \\ &= \alpha_1(2x_1[n]u[n]) + \alpha_2(2x_2[n]u[n]) \\ &= \alpha_1 y_1[n] + \alpha_2 y_2[n]. \end{aligned}$$

Therefore, the system **IS LINEAR**.

(d) $y(t) = \int_{-\infty}^{t/2} x(\tau)d\tau$

Solution: For this system, we can see that

$$\begin{aligned} y(t) &= \int_{-\infty}^{t/2} \left[\alpha_1 x_1(\tau) + \alpha_2 x_2(\tau)\right] d\tau \\ &= \alpha_1 \int_{-\infty}^{t/2} x_1(\tau)d\tau + \alpha_2 \int_{-\infty}^{t/2} x_2(\tau)d\tau \\ &= \alpha_1 y_1(t) + \alpha_2 y_2(t) \end{aligned}$$

Therefore, the system **IS LINEAR**.

(f) $y(t) = \frac{d}{dt}x(t)$

Solution: For this system, we can see that

$$\begin{aligned} y(t) &= \frac{d}{dt}\left[\alpha_1 x_1(t) + \alpha_2 x_2(t)\right] \\ &= \alpha_1 \frac{d}{dt} x_1(t) + \alpha_2 \frac{d}{dt} x_2(t) \\ &= \alpha_1 y_1(t) + \alpha_2 y_2(t) \end{aligned}$$

Therefore, the system **IS LINEAR**.

(i) $y(t) = x(2-t)$

Solution: For this system, we can see that

$$\begin{aligned} y(t) &= \alpha_1 x_1(2-t) + \alpha_2 x_2(2-t) \\ &= \alpha_1 y_1(t) + \alpha_2 y_2(t) \end{aligned}$$

Therefore, the system **IS LINEAR**.