Lecture 37
(L-C & L-R-C Circuits)

Physics 161-01 Spring 2012

Douglas Fields

LC Circuits

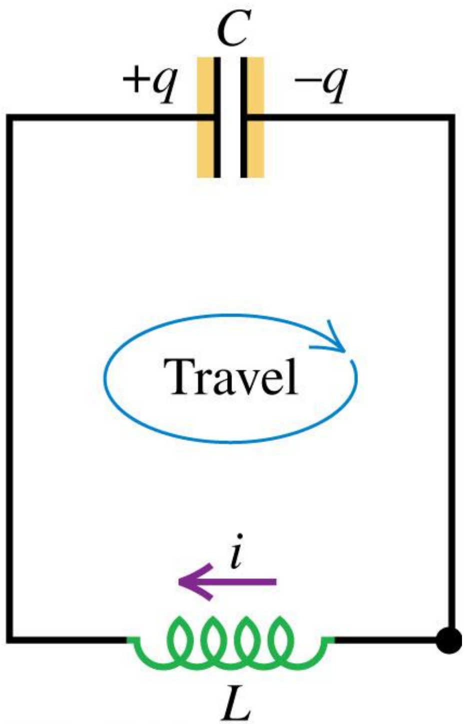
- Let's expand our usage of inductors by introducing a capacitor into the circuit.
- We start with a charged capacitor in series with a switch and an inductor.
- Let's apply Kirchhoff's loop rule:

$$\sum \Delta V = -\frac{q}{C} - L\frac{di}{dt} = 0 \Rightarrow$$

$$\frac{q}{C} + L\frac{di}{dt} = 0 \Rightarrow$$

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0 \Rightarrow$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$



LC Circuits

- Do you recognize this equation?

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

- What if I just changed q to x and renamed the constant?

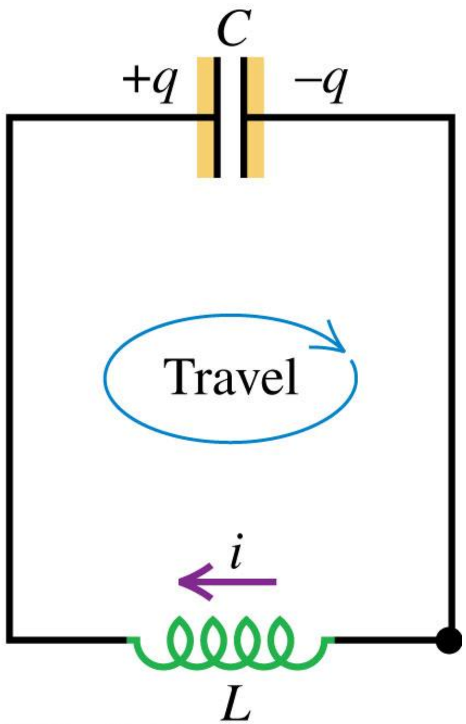
$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

- So the equation just describes oscillations with a frequency:

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \Rightarrow$$

$$q = Q_0 \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$



Analogy to Spring-Mass Oscillations

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \Rightarrow$$

$$x = X_0 \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \Rightarrow$$

$$q = Q_0 \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

Table 30.1 Oscillation of a Mass-Spring System Compared with Electrical Oscillation in an *L-C* Circuit

Mass-Spring System

Kinetic energy $= \frac{1}{2}mv_x^2$

Potential energy $= \frac{1}{2}kx^2$

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$

$v_x = dx/dt$

$\omega = \sqrt{\frac{k}{m}}$

$x = A\cos(\omega t + \phi)$

Inductor-Capacitor Circuit

Magnetic energy $= \frac{1}{2}Li^2$

Electric energy $= q^2/2C$

$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$

$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$

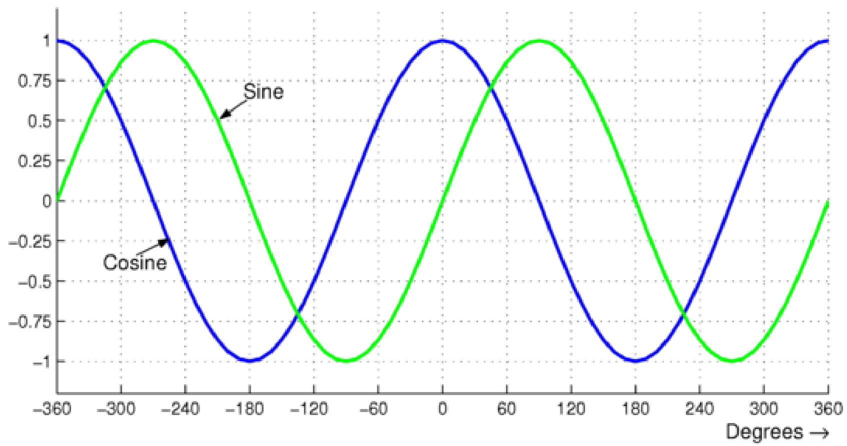
$i = dq/dt$

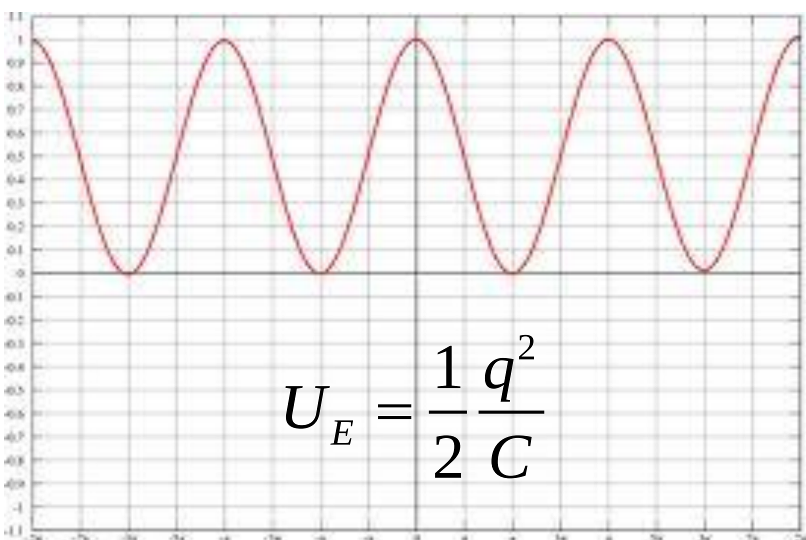
$\omega = \sqrt{\frac{1}{LC}}$

$q = Q\cos(\omega t + \phi)$

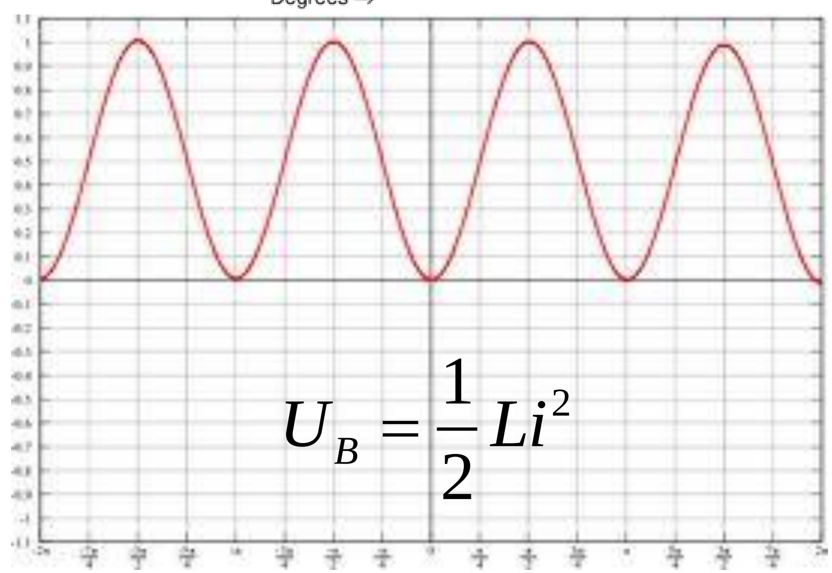
Energy/Current/Charge Oscillations

$q = Q_0 \cos(\omega t + \phi)$ $\qquad$ $i = -Q_0 \omega \sin(\omega t + \phi)$





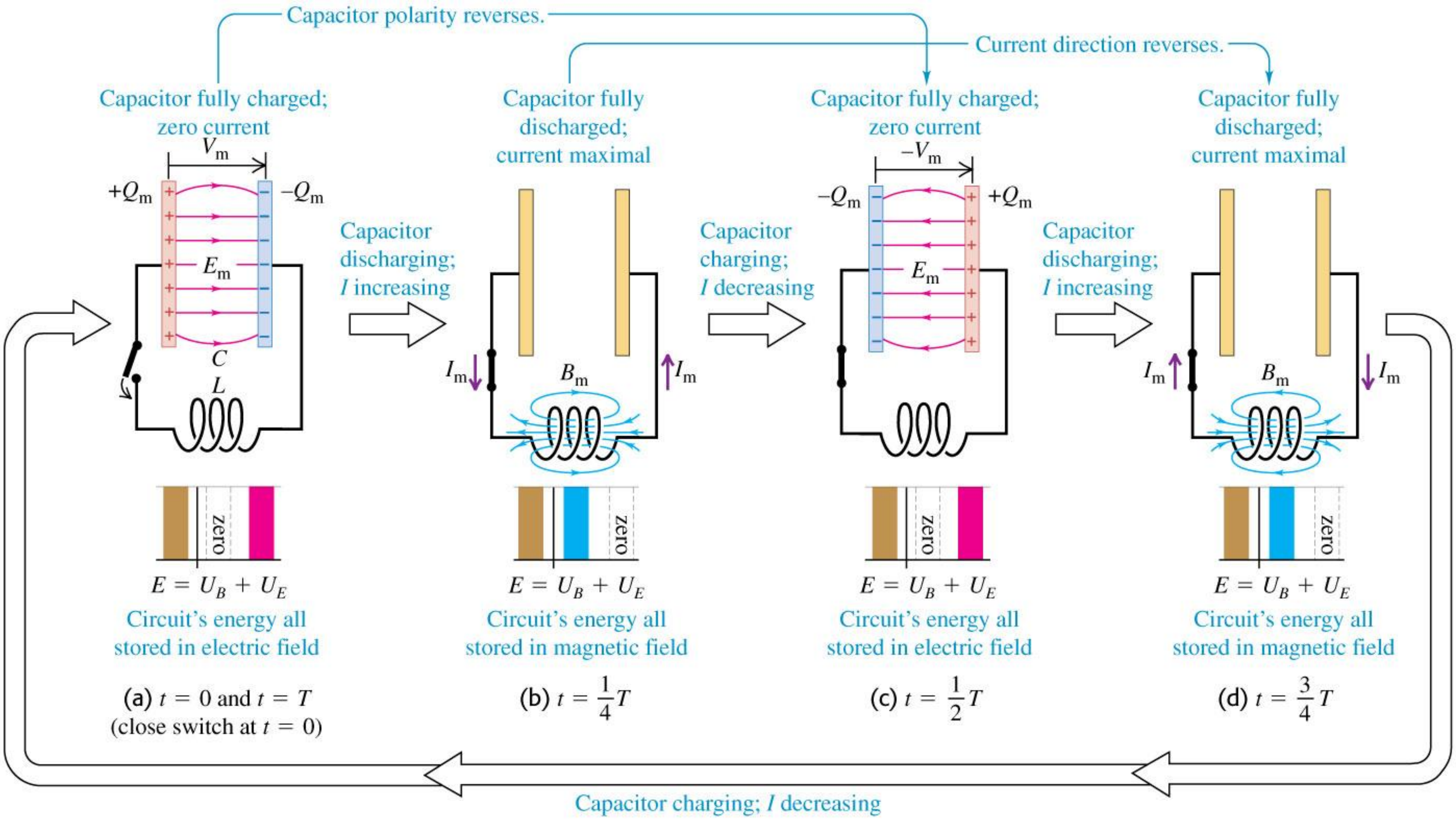
$$U_E = \frac{1}{2}\frac{q^2}{C}$$



$$U_B = \frac{1}{2}Li^2$$

Energy/Current/Charge Oscillations

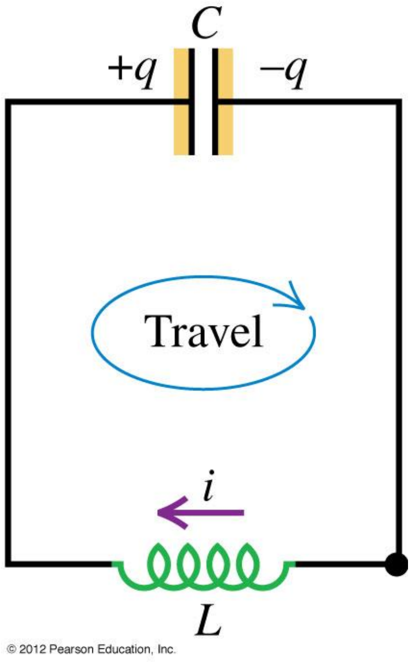
$$E_{\text{Total}} = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2 = \text{Constant} = \frac{1}{2}\frac{Q_{\max}^2}{C} = \frac{1}{2}LI_{\max}^2$$



CPS 37-1

An inductor (inductance *L*) and a capacitor (capacitance *C*) are connected as shown.

If the values of both *L* and *C* are doubled, what happens to the *time* required for the capacitor charge to oscillate through a complete cycle?



A. It becomes 4 times longer.

B. It becomes twice as long.

C. It is unchanged.

D. It becomes 1/2 as long.

E. It becomes 1/4 as long.

CPS 37-1

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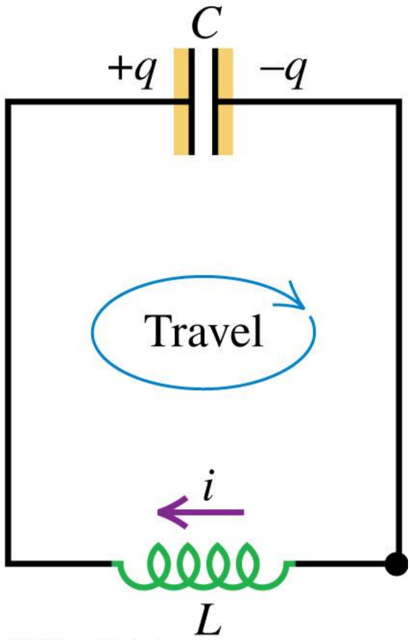


A. It becomes 4 times longer.

B. It becomes twice as long.

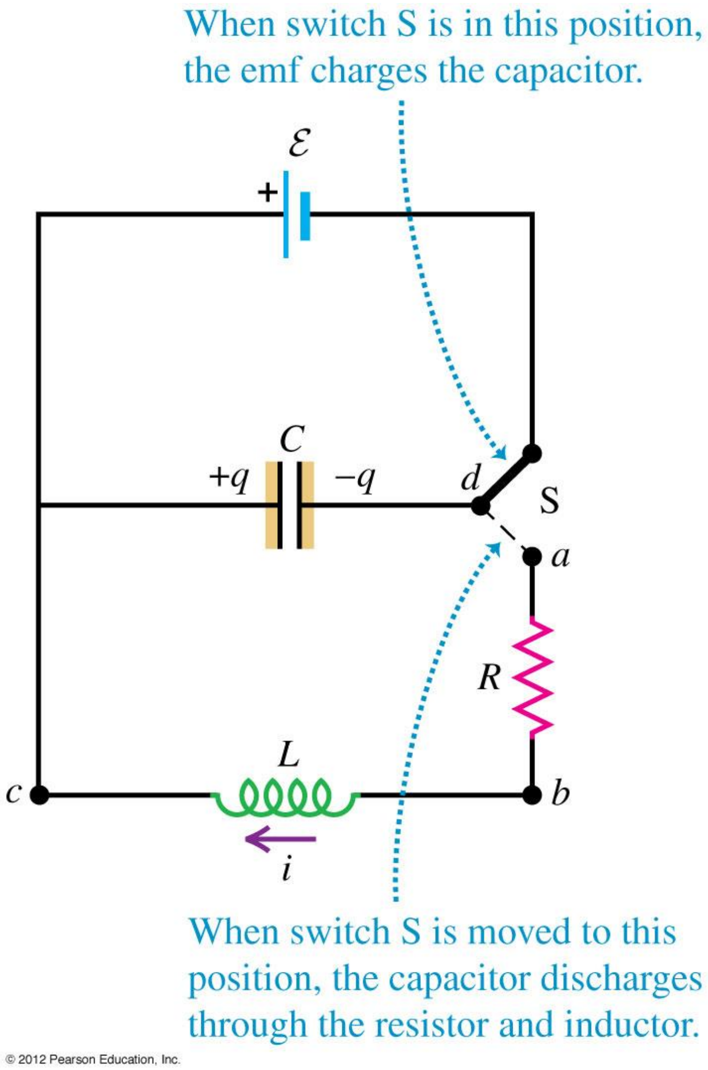
C. It is unchanged.

D. It becomes 1/2 as long.

E. It becomes 1/4 as long.

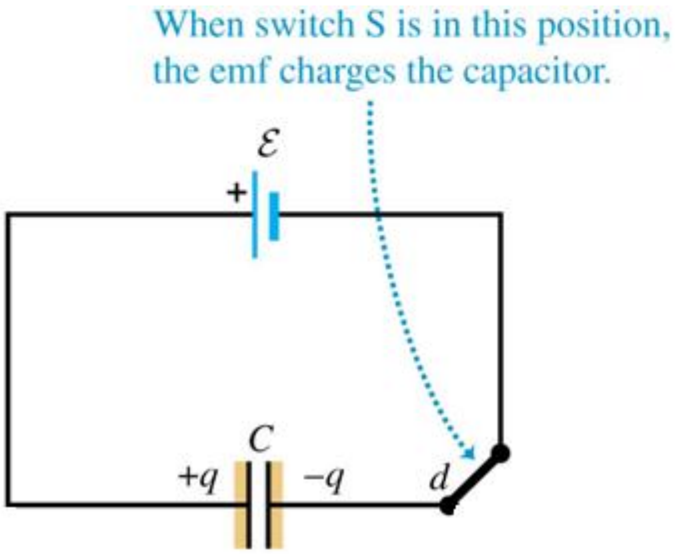
Energy/Current/Charge Oscillations

- Now that you have the spring-mass system in your head as an analogy, what do you think happens if we put a resistor in series with the capacitor and inductor?
- Remember that with the resistor, the total energy will decrease due to the power dissipated in the resistor.



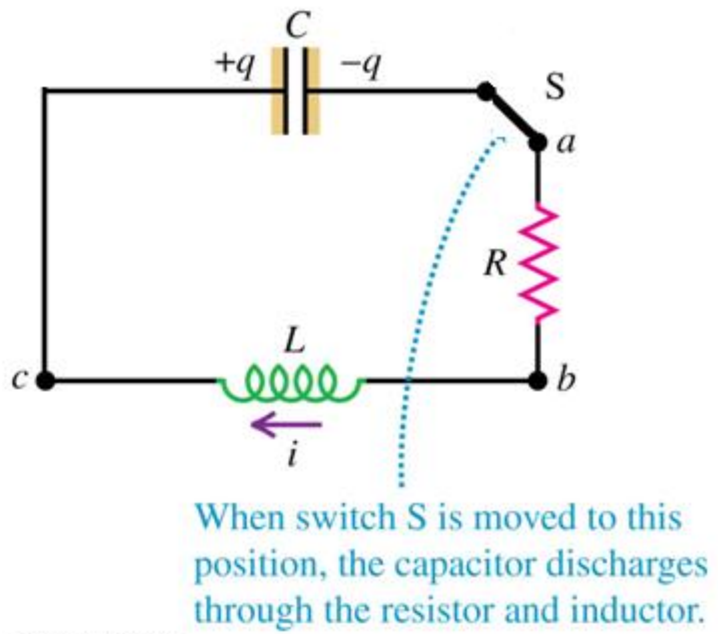
Energy/Current/Charge Oscillations

- Let's first charge the capacitor with an EMF.



Energy/Current/Charge Oscillations

- Then, remove the EMF and hook the capacitor in series to the resistor and inductor.



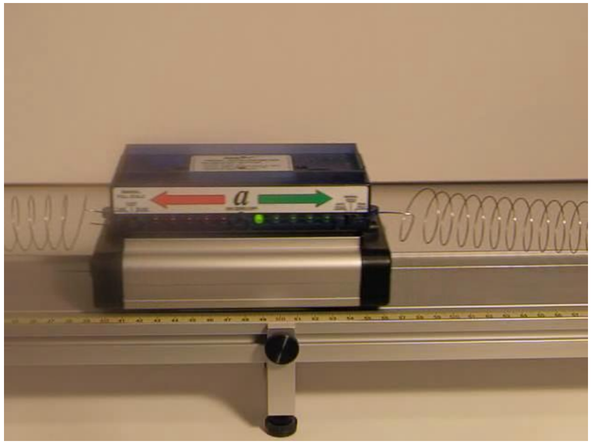
$$\sum \Delta V = -iR - \frac{q}{C} - L\frac{di}{dt} = 0 \Rightarrow$$

$$\frac{q}{C} + L\frac{di}{dt} + iR = 0 \Rightarrow$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

Energy/Current/Charge Oscillations

- This should remind you of the equation for damped simple harmonic oscillations.



Energy/Current/Charge Oscillations

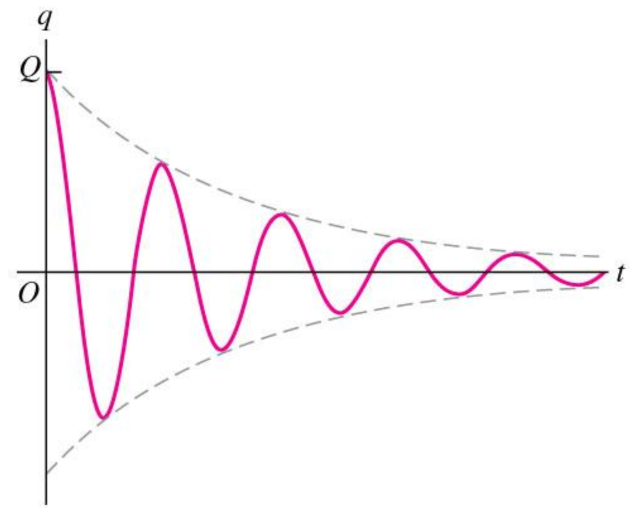
- With a relatively small resistor, $$R^2 < \frac{4L}{C}$$

  then there are oscillations whose amplitude decreases exponentially in time.

$$q = Ae^{-(R/2L)t}\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}t + \phi\right)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

(a) Underdamped circuit (small resistance R)



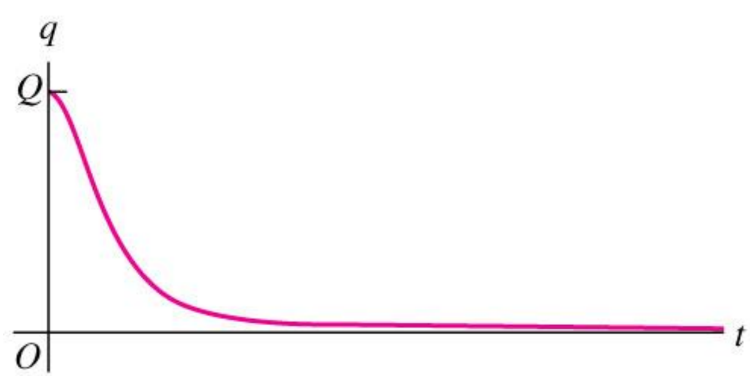
Energy/Current/Charge Oscillations

- When you increase the resistance to
$$R^2 = \frac{4L}{C}$$
then the system no longer oscillates, but instead damps down as quickly as is possible.

$$q = Ae^{-(R/2L)t}\cos\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}t+\phi\right)$$

$$\omega' = \sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} = 0$$

(b) Critically damped circuit (larger resistance R)



Energy/Current/Charge Oscillations

- As you continue to increase the resistance,

$$R^2 > \frac{4L}{C}$$

again, you get no oscillations, but it takes a longer time for the charge on the capacitor to fully dissipate.