Lecture 32
(Gravitation, Potential Energy and Gauss's Law)

Physics 160-01 Fall 2012

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Gravitational Force

- Up until now, we have said that the gravitational force on a mass m is just mg.
- But remember that we always said that there is a condition on this, that we are at the earth's surface.
- What is the general form of the force due to gravity?

$$F_G = G\frac{m_1 m_2}{r^2}$$

- That is, two particles with mass will attract each other proportionately to their masses and inverse proportionately to the square of the distance between them
- The proportionality constant, G, is known as the universal gravitational constant.

Gravitational Force

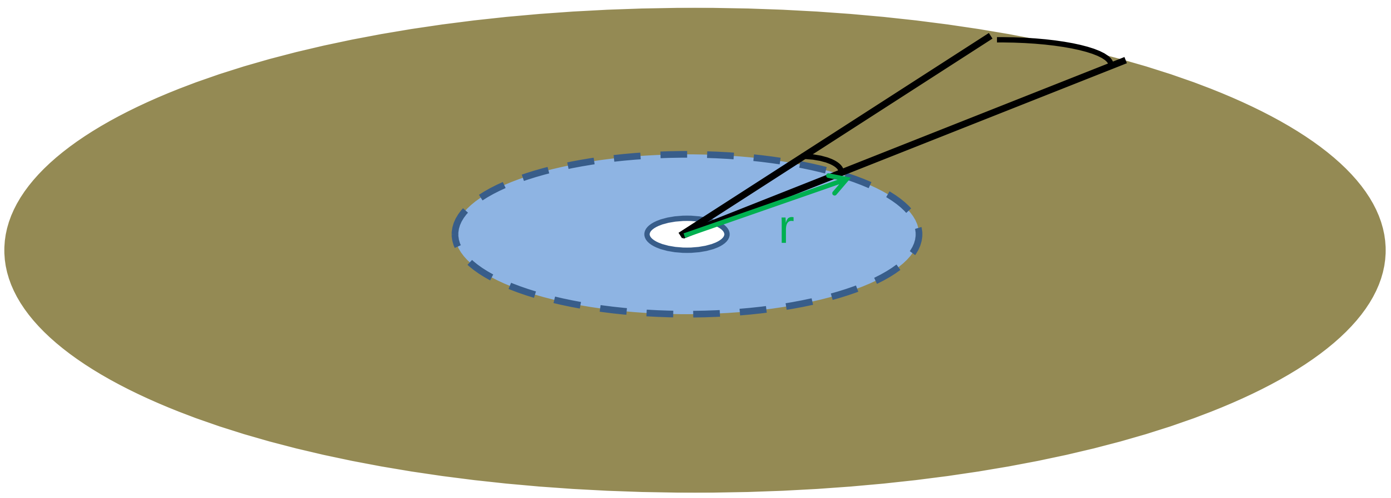
- The gravitational force is very weak (?)…
- For two masses each of 1kg, separated by 1m:

$$F_G = G\frac{m_1 m_2}{r^2} = G\frac{1kg \cdot 1kg}{\left(1m\right)^2} = 6.6742 \times 10^{-11} N$$

- That is, the proportionately constant, G = $6.6742\text{x}10^{-11}\text{Nm}^2/\text{kg}^2$.
- So why is it that we feel such a strong force on us?
- The earth’s mass = $5.98\text{x}10^{24}$kg!!

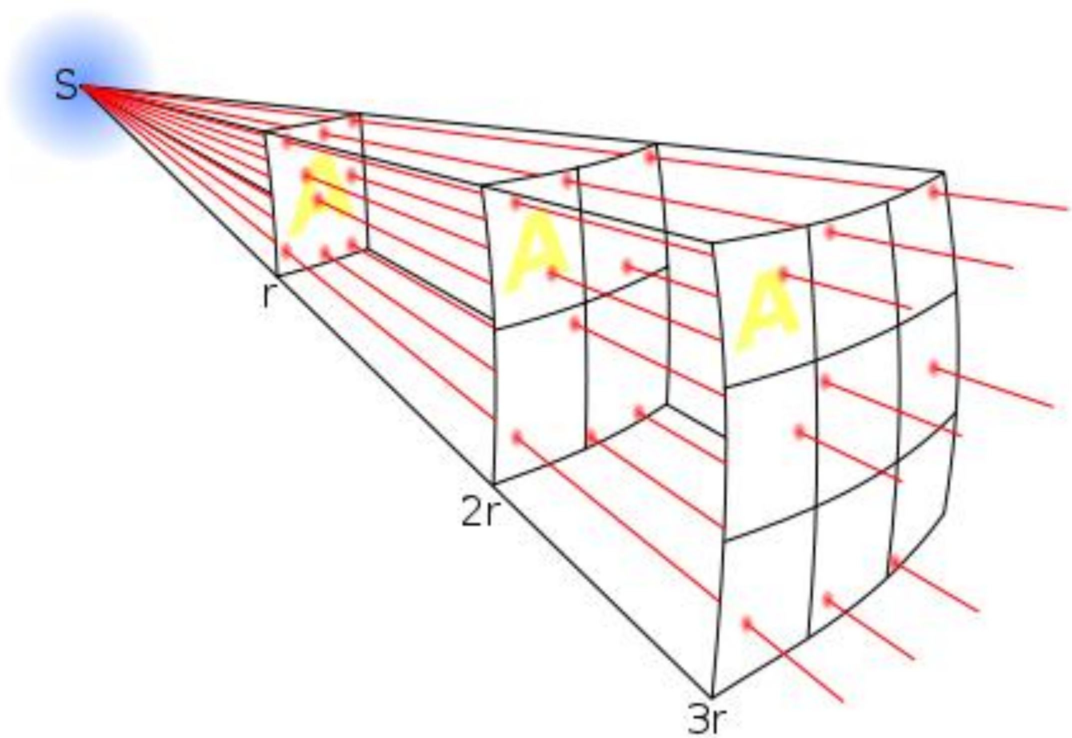
Why $1/r^2$?

- Consider water flowing out of a hole in a level surface, and spreading out evenly along the surface…



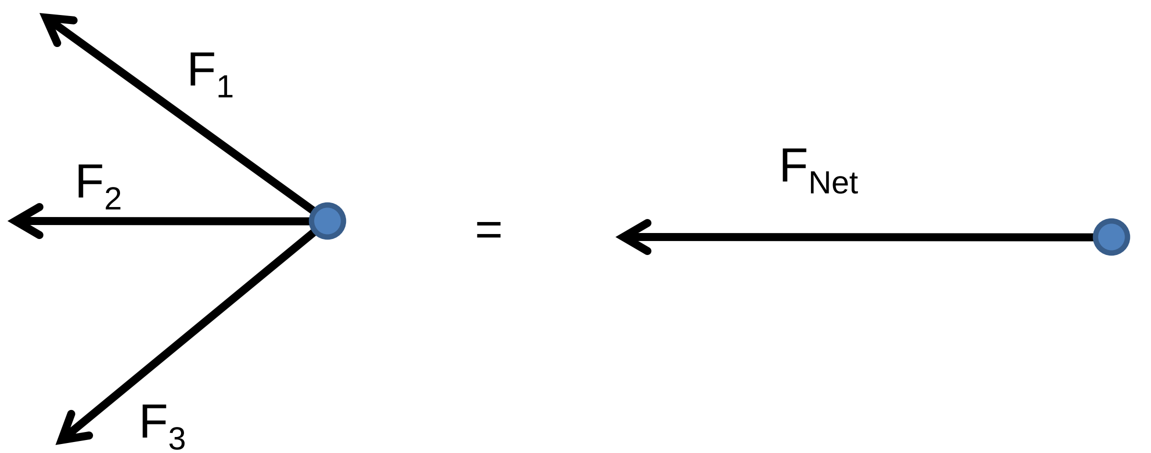
Why $1/r^2$?

- Now, in three dimensions, we examine the flux passing through the surface of a sphere…



Superposition of Force

- Remember that if two (or more) forces are acting on a body, the net force is just the (vector) sum of all the forces:



- The same is true for gravitational forces.

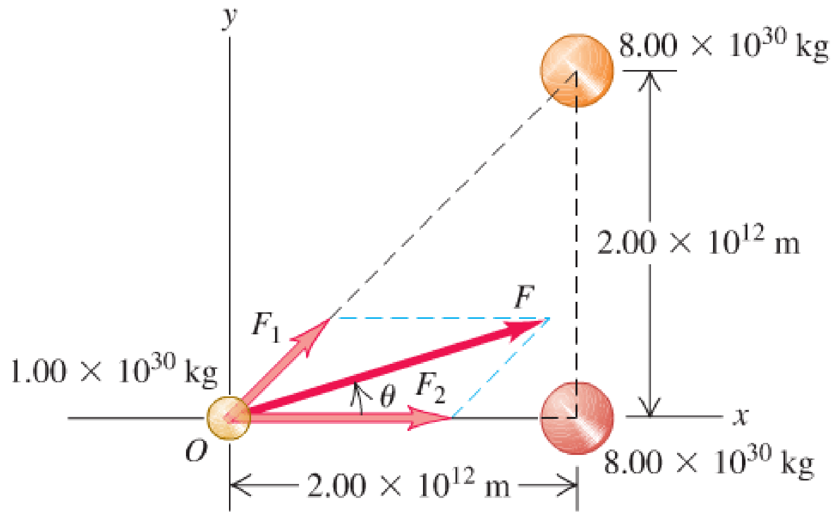
Example

$$F_1 = \frac{\left[\begin{array}{c}(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}) \\ \times (8.00 \times 10^{30}\ \mathrm{kg})(1.00 \times 10^{30}\ \mathrm{kg})\end{array}\right]}{(2.00 \times 10^{12}\ \mathrm{m})^2 + (2.00 \times 10^{12}\ \mathrm{m})^2}$$

$$= 6.67 \times 10^{25}\ \mathrm{N}$$

$$F_2 = \frac{\left[\begin{array}{c}(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}) \\ \times (8.00 \times 10^{30}\ \mathrm{kg})(1.00 \times 10^{30}\ \mathrm{kg})\end{array}\right]}{(2.00 \times 10^{12}\ \mathrm{m})^2}$$

$$= 1.33 \times 10^{26}\ \mathrm{N}$$



$$F_{1x} = (6.67 \times 10^{25}\ \mathrm{N})(\cos 45^\circ) = 4.72 \times 10^{25}\ \mathrm{N}$$

$$F_{1y} = (6.67 \times 10^{25}\ \mathrm{N})(\sin 45^\circ) = 4.72 \times 10^{25}\ \mathrm{N}$$

$$F_{2x} = 1.33 \times 10^{26}\ \mathrm{N}$$

$$F_{2y} = 0$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(1.81 \times 10^{26}\ \mathrm{N})^2 + (4.72 \times 10^{25}\ \mathrm{N})^2}$$

$$= 1.87 \times 10^{26}\ \mathrm{N}$$

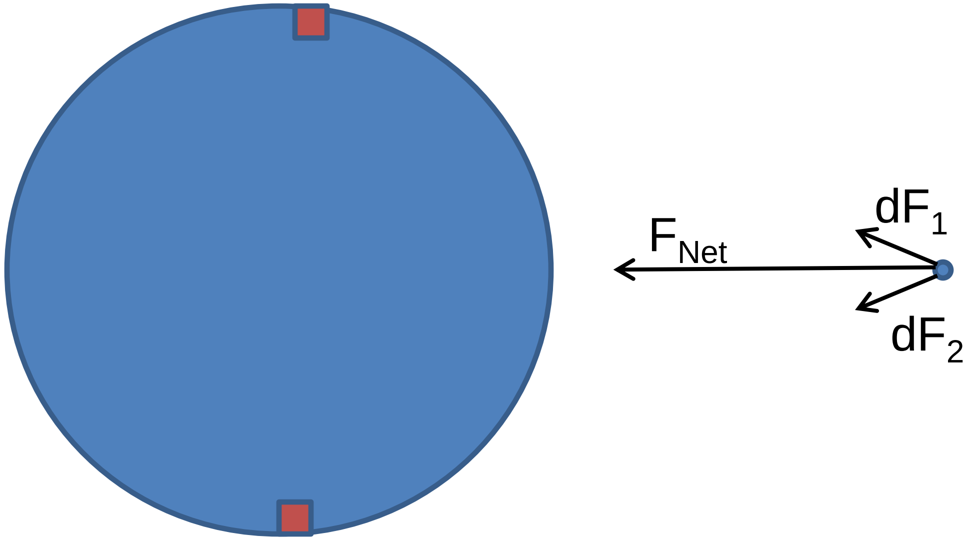
$$F_x = F_{1x} + F_{2x} = 1.81 \times 10^{26}\ \mathrm{N}$$

$$F_y = F_{1y} + F_{2y} = 4.72 \times 10^{25}\ \mathrm{N}$$

$$\theta = \arctan\frac{F_y}{F_x} = \arctan\frac{4.72 \times 10^{25}\ \mathrm{N}}{1.81 \times 10^{26}\ \mathrm{N}} = 14.6^\circ$$

Spherically Symmetric Bodies

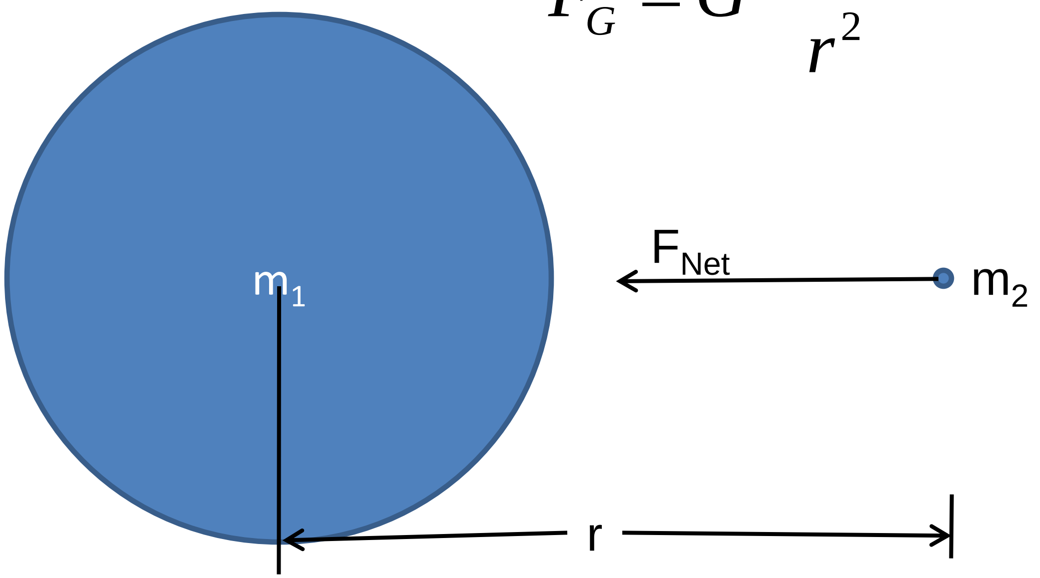
- We can do the same thing for continuous distributions of mass.



Spherically Symmetric Bodies

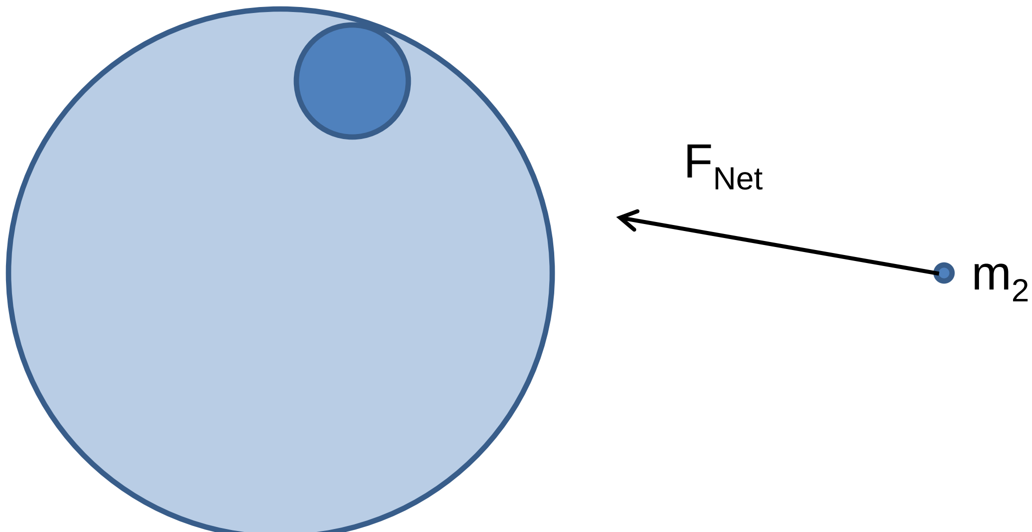
- For a spherically symmetric distribution, the net force is pointed to the center, and has the magnitude as if all the mass was located at the center:

$$F_G = G\frac{m_1 m_2}{r^2}$$



Search For Oil

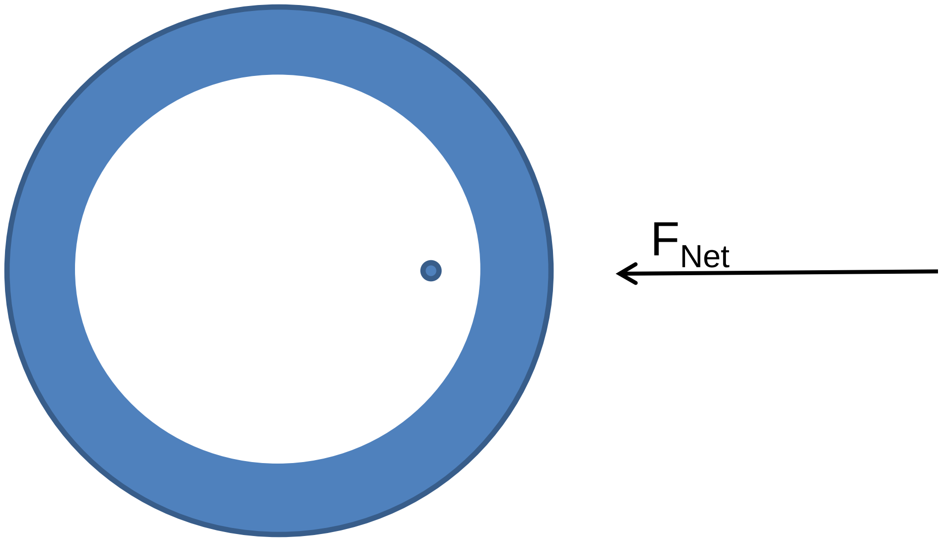
- If the mass is not spherically symmetric, this is no longer the case:



- This can be used to look for non-uniform densities in the earth's crust (oil, uranium, etc.).

Shell Theorem

$$F_G = G\frac{m_1 m_2}{r^2}$$



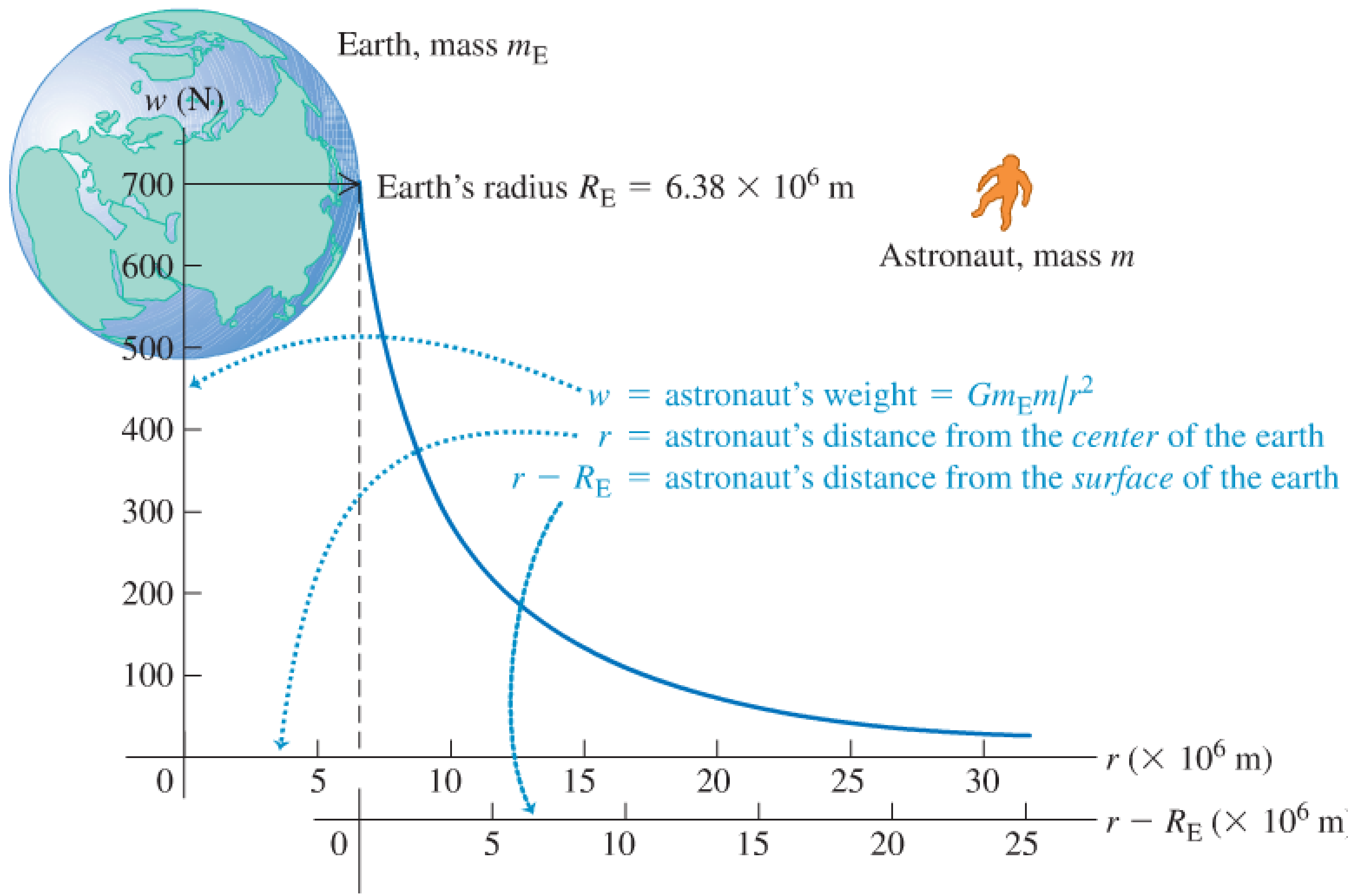
Weight and "Little g"

- So, what is the force due to gravity on a mass m, at the surface of the earth?

$$F_G = G\frac{m_E m}{r_E^2} = \frac{Gm_E}{r_E^2}m = \frac{\left(6.67\times10^{-11}\,N\cdot m^2/kg^2\right)\left(5.98\times10^{24}\,kg\right)}{\left(6.38\times10^{6}\,m\right)^2}m = \left(9.8\frac{N}{kg}\right)m$$

- Recognize the factor in front of the object's mass?
- Also notice that as the object goes farther from the earth's center, the force of gravity from the earth gets less…

Astronaut’s Weight

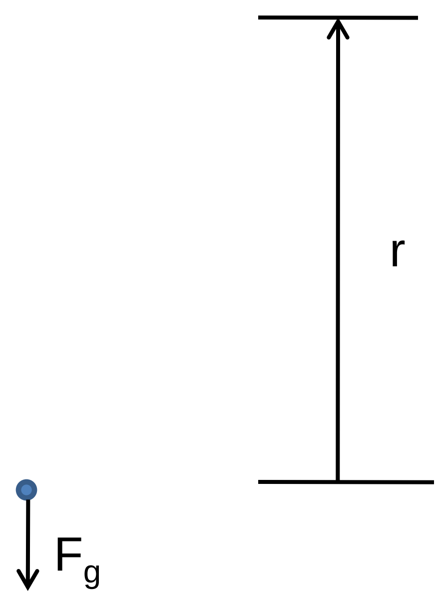


Gravitational Potential Energy

- Remember that we defined the gravitational potential energy as being the work done by gravity when an object is moved from one point to another:

$$\Delta U_g = -W_g = -\int_1^2 \vec{F}_g \cdot d\vec{r}$$

Near the earth's surface, you can take the force to be constant = mg, so the change in potential is just mg (r_2-r_1) = mgh



Gravitational Potential Energy

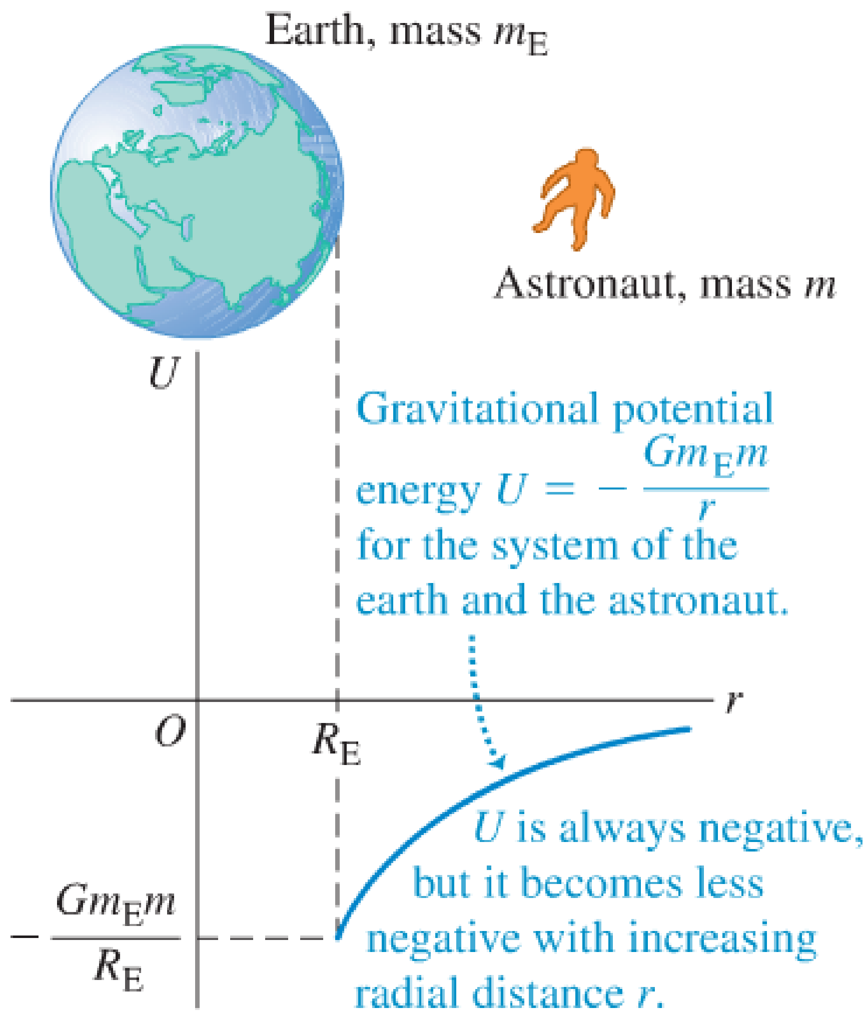
- Now, we have a force that varies with distance:

$$\Delta U_g = -W_g = \int_{r_1}^{r_2} \frac{Gm_1m_2}{r^2}\,dr = \frac{Gm_1m_2}{r_1} - \frac{Gm_1m_2}{r_2}$$

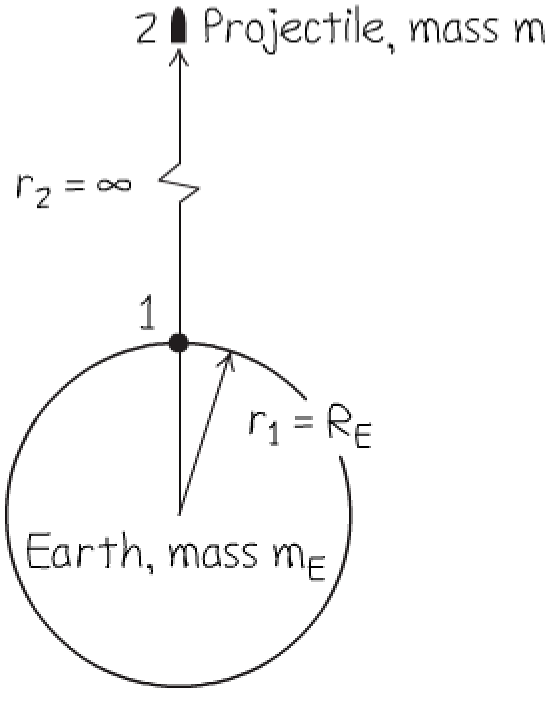
- If we define the zero of the potential now to be at infinity, we can set values for the potential:

$$U_g = -\frac{Gm_1m_2}{r}$$

Gravitational Potential Energy



Escape Velocity



From Conservation of energy:

$$\frac{1}{2}mv_1^{\,2} + \left(-\frac{Gm_{\mathrm{E}}m}{R_{\mathrm{E}}}\right) = 0 + 0$$

$$v_1 = \sqrt{\frac{2Gm_{\mathrm{E}}}{R_{\mathrm{E}}}}$$

$$= \sqrt{\frac{2(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.97 \times 10^{24}\ \mathrm{kg})}{6.38 \times 10^{6}\ \mathrm{m}}}$$

$$= 1.12 \times 10^{4}\ \mathrm{m/s}(= 40{,}200\ \mathrm{km/h} = 25{,}000\ \mathrm{mi/h})$$

Gravitational Potential Energy From More Than One Object

- Since potential energy is just a scalar, it adds just like any other quantity adds:

$$U_{g1} = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_1m_4}{r_{14}} \ldots$$