Exercise 23.8

Description: (a) Three equal ##-mu C point charges are placed at the corners of an equilateral triangle whose sides are ## m long. What is the potential energy of the system? (Take as zero the potential energy of the three charges when they are infinitely far...

Part A

Three equal 1.30-μC point charges are placed at the corners of an equilateral triangle whose sides are 0.350m long. What is the potential energy of the system? (Take as zero the potential energy of the three charges when they are infinitely far apart.)

ANSWER:

$U = \frac{3q^2}{4\pi\epsilon_0 L} = 0.130\ \mathrm{J}$

Exercise 23.12

Description: Two protons are aimed directly toward each other by a cyclotron accelerator with speeds of ## km/s, measured relative to the earth. (a) Find the maximum electrical force that these protons will exert on each other.

Two protons are aimed directly toward each other by a cyclotron accelerator with speeds of 1000$\mathrm{km/s}$, measured relative to the earth.

Part A

Find the maximum electrical force that these protons will exert on each other.

ANSWER:

$F = \left(\frac{m_p v^2}{e}\right)^2 \cdot 4\pi\epsilon_0 = 1.21\times10^{-2}\ \mathrm{N}$

Exercise 23.27

Description: A thin spherical shell with radius R_1=## cm is concentric with a larger thin spherical shell with radius R_2. Both shells are made of insulating material. The smaller shell has charge q_1 = + 6.00 (nC) distributed uniformly over its surface, and...

A thin spherical shell with radius R_1 = 2.00cm is concentric with a larger thin spherical shell with radius 4.00cm . Both shells are made of insulating material. The smaller shell has charge $q_1 = +6.00\ \mathrm{nC}$ distributed uniformly over its surface, and the larger shell has charge $q_2 = -9.00\ \mathrm{nC}$ distributed uniformly over its surface. Take the electric potential to be zero at an infinite distance from both shells.

Part A

What is the electric potential due to the two shells at the following distance from their common center: r = 0?

Express your answer with the appropriate units.

ANSWER:

$V = 8.99\cdot10^9\cdot10^{-9}\cdot100\left(\frac{6}{R_1} - \frac{9}{R_2}\right) = 674\mathrm{V}$

Part B

What is the electric potential due to the two shells at the following distance from their common center: r = 3.00cm ?

Express your answer with the appropriate units.

ANSWER:

$V = 8.99\cdot10^9\cdot10^{-9}\cdot100\left(\frac{6}{r_1} - \frac{9}{R_2}\right) = -225\mathrm{V}$

Part C

What is the electric potential due to the two shells at the following distance from their common center: r = 5.00cm ?

Express your answer with the appropriate units.

ANSWER:

$V = 8.99\cdot10^9\cdot10^{-9}\cdot100\left(\frac{6}{r_2} - \frac{9}{r_2}\right) = -539\mathrm{V}$

Part D

What is the magnitude of the potential difference between the surfaces of the two shells?

Express your answer with the appropriate units.

ANSWER:

ΔV = $8.99 \cdot 10^9 \cdot 10^{-9} \cdot 100 \left(\frac{6}{R_1} - \frac{9}{R_2} \right) - 8.99 \cdot 10^9 \cdot 10^{-9} \cdot 100 \left(\frac{6}{R_2} - \frac{9}{R_2} \right)$ = 1350 V

Also accepted: $8.99 \cdot 10^9 \cdot 10^{-9} \cdot 100 \left(\frac{6}{R_2} - \frac{9}{R_2} \right) - 8.99 \cdot 10^9 \cdot 10^{-9} \cdot 100 \left(\frac{6}{R_1} - \frac{9}{R_2} \right)$ = -1350 V

Part E

Which shell is at higher potential: the inner shell or the outer shell?

ANSWER:

- (selected) The inner shell is at higher potential.
- The outer shell is at higher potential.

Exercise 23.38

Description: Two large parallel conducting plates carrying opposite charges of equal magnitude are separated by 2.20 cm. (a) If the surface charge density for each plate has magnitude 47.0 (nC)/m^2, what is the magnitude of E_vec in the region between the plates? ...

Two large parallel conducting plates carrying opposite charges of equal magnitude are separated by 2.20 cm.

Part A

If the surface charge density for each plate has magnitude 47.0 $\mathrm{nC/m^2}$, what is the magnitude of $\vec{E}$ in the region between the plates?

ANSWER:

E = 5310 N/C

Part B

What is the potential difference between the two plates?

ANSWER:

V = 117 V

Part C

If the separation between the plates is doubled while the surface charge density is kept constant at the value in part (a), what happens to the magnitude of the electric field?

ANSWER:

- doubles
- (selected) stays the same
- halves

Part D

If the separation between the plates is doubled while the surface charge density is kept constant at the value in part (a), what happens to the potential difference?

ANSWER:

- (selected) doubles
- stays the same
- halves

Problem 23.52

Description: A small sphere with mass 5.00 * 10^(- 7) (kg) and charge + 3.00 mu C is released from rest a distance of r_0 above a large horizontal insulating sheet of charge that has uniform surface charge density sigma = +8.00 (pC/m)^2. (a) Using energy...

A small sphere with mass 5.00×10^{-7} kg and charge $+3.00\ \mu\mathrm{C}$ is released from rest a distance of 0.550 m above a large horizontal insulating sheet of charge that has uniform surface charge density $\sigma = +8.00\ \mathrm{pC/m^2}$.

Part A

Using energy methods, calculate the speed of the sphere when it is 0.200 m above the sheet of charge?

Express your answer with the appropriate units.

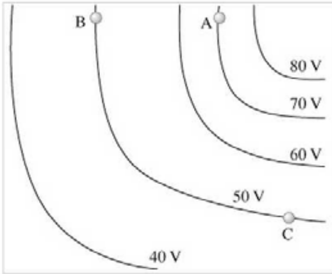
ANSWER:

$v = \sqrt{\frac{2\left(5 \cdot 10^{-7} \cdot 9.8\,(r_0 - r) - 0.4518 \cdot 3 \cdot 10^{-6}\,(r_0 - r)\right)}{5 \cdot 10^{-7}}}$ = 2.23 $\frac{\mathrm{m}}{\mathrm{s}}$

Relationship between Electric Force and Electric Potential Conceptual Question

Description: Simple conceptual question about the electric force vector acting upon a proton or electron released from a certain point within a region with an electric potential. (vector drawing)

Three points (A, B, and C) are located on equipotential lines as shown.



Part A

A proton is released from Point A. Indicate the direction of the electric force vector acting on the proton.

Hints (1)

Hint 1. Determining the direction of the electric force

Electric potential is sometimes visualized as a sort of "elevation." Thus, a region of space where the electric potential is $80\ \mathrm{V}$ is "higher" than a region where the potential is $50\ \mathrm{V}$. Just as a mass will experience a gravitational force pulling it toward lower elevation, a positive charge will experience an electric force pushing it toward lower electric potential. One difference, however, is that there are two types of electric charge (positive and negative) and only one type of gravitational mass (positive). Negative charges behave in an opposite manner to positive charges. For example, negative charges will experience a force moving them toward higher "elevation" (i.e., negative charges feel a force pulling them toward higher electric potential). Therefore, when dealing with negative charges, always remember that they behave opposite to positive charges.

ANSWER:

The electric force vector at Point A

- points upward.
- points downward.
- points to the left. (selected)
- points to the right.
- is zero.

Part B

An electron is released from Point B. Indicate the direction of the electric force vector acting on the electron.

Hints (1)

ANSWER:

The electric force vector at Point B

- points upward.
- points downward.
- points to the left.
- points to the right. (selected)
- is zero.

Part C

An electron is released from Point B and a second electron is released from Point C. What can you say about the electric forces experienced by these electrons the instant they are released?

Hints (1)

Hint 1. Relating electric field strength to change in electric potential

Regions where the electric potential is changing rapidly (i.e., the equipotentials are close together) can be envisioned as analogous to regions where the "elevation" is changing rapidly (i.e., a steeply sloping terrain). On a steep slope, the component of the gravitational force that pulls you down the slope is relatively large. In an analogous way, where the electric potential is "steep," the electric field is relatively large.

ANSWER:

- The electron released at Point B experiences a greater force.
- The electron released at Point C experiences a greater force. (selected)
- Electrons released from Points B and C would experience equal forces.
- The relationship between the two forces cannot be determined.

In conclusion, recall that the electric field is always perpendicular to the equipotential line and points in the direction of decreasing potential. The electric force acting on a charge will then depend on the sign of the charge. On positive charges, the electric force is parallel to the field; on negative charges, the electric force is in the opposite direction as the field.

23.62. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_e = qE$. The potential difference between the plates is $V = Ed$.

SET UP: The free-body diagram for the sphere is given in Figure 23.62.

EXECUTE: $T\cos\theta = mg$ and $T\sin\theta = F_e$ gives

$F_e = mg\tan\theta = (1.50\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan(30°) = 0.0085\text{ N}.$

$$F_e = Eq = \frac{Vq}{d} \text{ and } V = \frac{Fd}{q} = \frac{(0.0085\text{ N})(0.0500\text{ m})}{8.90\times10^{-6}\text{ C}} = 47.8\text{ V}.$$

EVALUATE: $E = V/d = 956$ V/m. $E = \sigma/\epsilon_0$ and $\sigma = E\epsilon_0 = 8.46\times10^{-9}\text{ C/m}^2$.

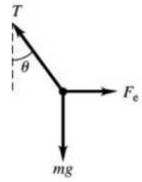


Figure 23.62

23.67. **(a) IDENTIFY and SET UP:** Problem 23.63 derived that $E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}$, where a is the radius of the inner cylinder (wire) and b is the radius of the outer hollow cylinder. The potential difference between the two cylinders is V_{ab}. Use this expression to calculate E at the specified r.

EXECUTE: Midway between the wire and the cylinder wall is at a radius of $r = (a+b)/2 = (90.0\times10^{-6}\text{ m} + 0.140\text{ m})/2 = 0.07004$ m.

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} = \frac{50.0\times10^3\text{ V}}{\ln(0.140\text{ m}/90.0\times10^{-6}\text{ m})(0.07004\text{ m})} = 9.71\times10^4\text{ V/m}$$

(b) IDENTIFY and SET UP: The electric force is given by Eq. (21.3). Set this equal to ten times the weight of the particle and solve for $|q|$, the magnitude of the charge on the particle.

EXECUTE: $F_E = 10mg$

$$|q|E = 10mg \text{ and } |q| = \frac{10mg}{E} = \frac{10(30.0\times10^{-9}\text{ kg})(9.80\text{ m/s}^2)}{9.71\times10^4\text{ V/m}} = 3.03\times10^{-11}\text{ C}$$

EVALUATE: It requires only this modest net charge for the electric force to be much larger than the weight.

23.74. **IDENTIFY:** For $r < c$, $E = 0$ and the potential is constant. For $r > c$, E is the same as for a point charge and $V = \frac{kq}{r}$.

SET UP: $V_\infty = 0$

EXECUTE: **(a)** Points a, b and c are all at the same potential, so $V_a - V_b = V_b - V_c = V_a - V_c = 0$.

$$V_c - V_\infty = \frac{kq}{R} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(150\times10^{-6}\text{ C})}{0.60\text{ m}} = 2.25\times10^6\text{ V}$$

(b) They are all at the same potential.

(c) Only $V_c - V_\infty$ would change; it would be -2.25×10^6 V.

EVALUATE: The voltmeter reads the potential difference between the two points to which it is connected.

23.79. IDENTIFY: Slice the rod into thin slices and use Eq. (23.14) to calculate the potential due to each slice. Integrate over the length of the rod to find the total potential at each point.

(a) SET UP: An infinitesimal slice of the rod and its distance from point P are shown in Figure 23.79a.

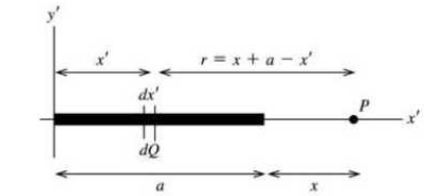


Figure 23.79a

Use coordinates with the origin at the left-hand end of the rod and one axis along the rod. Call the axes x' and y' so as not to confuse them with the distance x given in the problem.

EXECUTE: Slice the charged rod up into thin slices of width dx'. Each slice has charge $dQ = Q(dx'/a)$ and a distance $r = x + a - x'$ from point P. The potential at P due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{dx'}{x+a-x'}\right).$$

Compute the total V at P due to the entire rod by integrating dV over the length of the rod ($x' = 0$ to $x' = a$):

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{(x+a-x')} = \frac{Q}{4\pi\epsilon_0 a}[-\ln(x+a-x')]_0^a = \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x+a}{x}\right).$$

EVALUATE: As $x \to \infty$, $V \to \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x}{x}\right) = 0.$

(b) SET UP: An infinitesimal slice of the rod and its distance from point R are shown in Figure 23.79b.

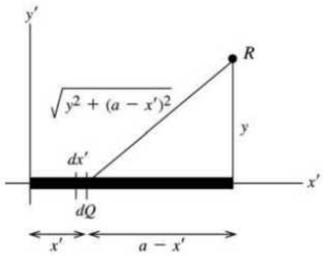


Figure 23.79b

$dQ = (Q/a)dx'$ as in part (a)

Each slice dQ is a distance $r = \sqrt{y^2 + (a-x')^2}$ from point R.

EXECUTE: The potential dV at R due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\frac{dx'}{\sqrt{y^2 + (a-x')^2}}.$$

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{\sqrt{y^2 + (a-x')^2}}.$$

In the integral make the change of variable $u = a - x'$; $du = -dx'$

$$V = -\frac{Q}{4\pi\epsilon_0 a}\int_a^0 \frac{du}{\sqrt{y^2+u^2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[\ln\left(u+\sqrt{y^2+u^2}\right)\right]_a^0.$$

$$V = -\frac{Q}{4\pi\epsilon_0 a}\left[\ln y - \ln\left(a+\sqrt{y^2+a^2}\right)\right] = \frac{Q}{4\pi\epsilon_0 a}\left[\ln\left(\frac{a+\sqrt{a^2+y^2}}{y}\right)\right].$$

(The expression for the integral was found in Appendix B.)

EVALUATE: As $y \to \infty$, $V \to \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{y}{y}\right) = 0.$

(c) SET UP: *part (a):* $V = \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x+a}{x}\right) = \frac{Q}{4\pi\epsilon_0 a}\ln\left(1+\frac{a}{x}\right).$

From Appendix B, $\ln(1+u) = u - u^2/2\ldots$, so $\ln(1+a/x) = a/x - a^2/2x^2$ and this becomes a/x when x is large.

EXECUTE: Thus $V \to \frac{Q}{4\pi\epsilon_0 a}\left(\frac{a}{x}\right) = \frac{Q}{4\pi\epsilon_0 x}$. For large x, V becomes the potential of a point charge.

part (b): $V = \frac{Q}{4\pi\epsilon_0 a}\left[\ln\left(\frac{a+\sqrt{a^2+y^2}}{y}\right)\right] = \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{a}{y}+\sqrt{1+\frac{a^2}{y^2}}\right).$

From Appendix B, $\sqrt{1+a^2/y^2} = (1+a^2/y^2)^{1/2} = 1 + a^2/2y^2 + \ldots$

Thus $a/y + \sqrt{1+a^2/y^2} \to 1 + a/y + a^2/2y^2 + \ldots \to 1 + a/y$. And then using $\ln(1+u) \approx u$ gives

$$V \to \frac{Q}{4\pi\epsilon_0 a}\ln(1+a/y) \to \frac{Q}{4\pi\epsilon_0 a}\left(\frac{a}{y}\right) = \frac{Q}{4\pi\epsilon_0 y}.$$

EVALUATE: For large y, V becomes the potential of a point charge.

A Millikan-type Experiment

Description: Find the charge of an oil droplet between two plates, given the potential applied to them and their distance.

Consider an oil droplet of mass m and charge q. We want to determine the charge on the droplet in a Millikan-type experiment. We will do this in several steps. Assume, for simplicity, that the charge is positive and that the electric field between the plates points upward.

Part A

An electric field is established by applying a potential difference to the plates. It is found that a field of strength E_0 will cause the droplet to be suspended motionless. Write an expression for the droplet's charge q. Let g be the acceleration due to gravity.

Express your answer in terms of the suspending field E_0 and the droplet's weight mg.

Hints (2)

Hint 1. A body in static equilibrium

Recall Newton's 1st law, which states that a body is in equilibrium when the net force acting on it is zero. Then, for the droplet to be suspended motionless (i.e., to be in static equilibrium), the net force acting on it has to be zero. Note that the only forces acting on the droplet are the gravitational force (downward) and the electric force (upward).

Hint 2. Find the electric force

Recall the definition of electric field as electric force per unit charge. Write an expression for the electric force F exerted on a point charge q by an electric field E.

Express your answer in terms of E and q.

ANSWER:

$F = Eq$

ANSWER:

$q = \frac{mg}{E_0}$

Part B

The field E_0 is easily determined by knowing the spacing between the plates and measuring the potential difference applied to them. The larger problem is to determine the mass of a microscopic droplet. Consider a mass m falling through a viscous medium in which there is a retarding or drag force. For very small particles, the retarding force is given by $F_{drag} = -bv$, where b is a constant and v is the droplet's speed. The negative sign tells us that the drag force vector points upward when the droplet is falling. A falling droplet quickly reaches a *constant* speed, called the *terminal speed*. Write an expression for the terminal speed v_{term}.

Express your answer in terms of m, g, and b.

Express your answer in terms of m, g, and b.

Hints (1)

Hint 1. A body in dynamic equilibrium

From Newton's 1st law, if the net force acting on a body initially in motion is zero, the body continues to move with constant velocity, and the body is said to be in dynamic equilibrium. It follows that the net force acting on a falling droplet that moves at constant velocity is zero. Note that the only forces acting on the falling oil droplet are the gravitational force and the drag force.

ANSWER:

$v_{term} = \frac{mg}{b}$

Part C

A spherical object of radius r moving slowly through the air is known to experience a retarding force $F_{drag} = -6\pi\eta r v$, where η is the *viscosity* of the air. Use this and your answer to Part B to find the radius r of a spherical droplet of density ρ falling with a terminal speed v_{term}.

Express your answer in terms of η, v_{term}, ρ, and g.

Hints (1)

Hint 1. How to approach the problem

In this part of the problem, the situation is simply a specific case of the more general situation analyzed in Part B, where a body of mass m that moves at constant speed through a viscous medium experiences a retarding force F_{drag} proportional to the speed of the body. Here, a droplet of given density falls at constant speed v_{term} through air and experiences a retarding force of magnitude F_{drag}, which is proportional to the droplet speed. The constant of proportionality depends on the viscosity of air and the radius of the droplet. To solve this part of the problem, simply apply Newton's 1st law to the droplet and solve for the droplet's radius.

ANSWER:

$r = \sqrt{\frac{9\eta v_{term}}{2\rho g}}$

Part D

Oil has a density ρ of $860\ \mathrm{kg/m^3}$. An oil droplet is suspended between two plates $1.0\ \mathrm{cm}$ apart by adjusting the potential difference between them to $1177\ \mathrm{V}$. When the voltage is removed, the droplet falls and quickly reaches constant speed. It is timed with a stopwatch and falls $3.00\ \mathrm{mm}$ in $7.33\ \mathrm{s}$. The viscosity of air is $1.83 \times 10^{-5}\ \mathrm{kg/m \cdot s}$. What is the droplet's charge q?

Express the charge in coulombs to two significant figures. Take the free-fall acceleration to be $9.81\ \mathrm{m/s^2}$.

Hints (4)

Hint 1. How to approach the problem

In Part A, you found an expression for the charge of a droplet that is in equilibrium between two plates separated by a potential difference. In particular, you found that the charge of the droplet is given by the weight of the droplet divided by the electric field established between the plates. Therefore, to find the charge of the droplet, you need to know its mass and the value of the electric field between the plates.

Hint 2. Find the mass of the droplet

Recall the definition of density and write an expression for the mass m of a spherical droplet of radius r and density ρ_0.

Express your answer in terms of π, r, and ρ_0.

Hints (2)

ANSWER:

$m = \frac{4}{3}\pi r^3 \rho_0$

Hint 3. Find the droplet's radius

In Part C you found an expression for the radius of a droplet of given density that falls through air at constant speed. In particular, you found that the droplet's radius depends on the viscosity of air, the speed of the droplet, the density of the droplet, and the acceleration due to gravity.

Find the droplet's radius r for the situation described in this part of the problem. Note that the only unknown variable is the droplet's speed.

Express your answer in meters to three significant figures. Take the free-fall acceleration to be $9.81\ \mathrm{m/s^2}$.

Hints (1)

Hint 1. Find the speed of the droplet

The oil droplet falls $3.00\ \mathrm{mm}$ in $7.33\ \mathrm{s}$. Recall the definition of speed and find the droplet's speed v.

Express your answer in meters per second to three significant figures.

ANSWER:

$v = 4.093 \times 10^{-4}\ \mathrm{m/s}$

ANSWER:

r = 2.000×10^{-6} m

Hint 4. Find the electric field between two parallel plates

The potential difference between two parallel charged plates is a function of the electric field established between the plates and the distance between them.

Write an expression for the electric field E between two parallel plates a distance d apart and with a potential difference between them of ΔV.

Express your answer in terms of ΔV and d.

ANSWER:

$E = \frac{\Delta V}{d}$

ANSWER:

q = 2.40×10^{-18} C

Part E

How many units of the fundamental electric charge does this droplet possess?

Express your answer as an integer.

Hints (1)

Hint 1. The fundamental electric charge

The fundamental electric charge is the magnitude of the charge of an electron, which is

$$e = 1.602176462 \times 10^{-19}\ \mathrm{C}.$$

ANSWER:

15