ECE 345: Introduction to Control Systems
In-Class Exercise #4

`http://www.ece.unm.edu/course/ece345/`

Dr. Oishi

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1 Introduction

The Hubble telescope was built to facilitate the study of far-away objects without the distortion caused by the atmosphere. Use of the Hubble involves pointing it at a desired object, then gathering data from the light that falls onto the Hubble's scientific instruments (IR sensors, spectrometers, cameras). Hence accurate pointing, that rejects disturbances and unwanted forces acting on the satellite, is paramount to effective operation.

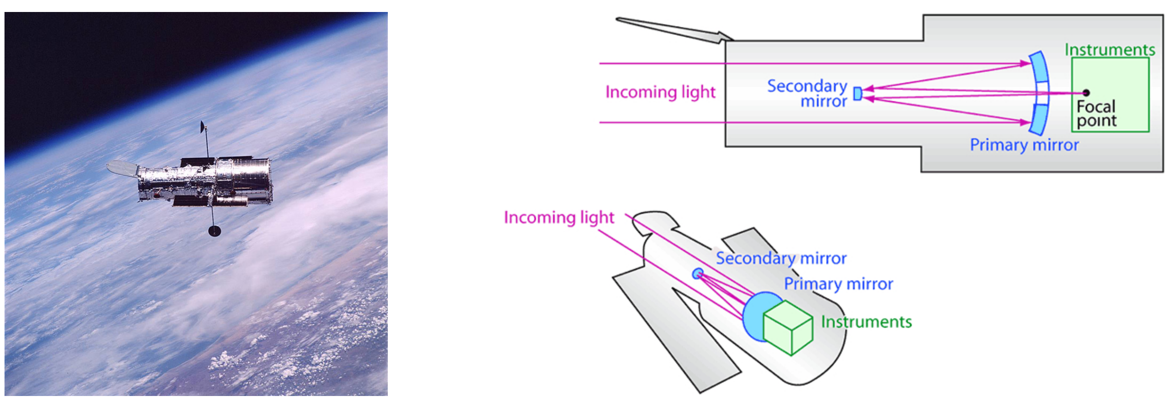

Figure 1: "Hubble is a type of telescope known as a Cassegrain reflector. Light hits the telescope's main mirror, or primary mirror. It bounces off the primary mirror and encounters a secondary mirror. The secondary mirror focuses the light through a hole in the center of the primary mirror that leads to the telescope's science instruments." From `http://www.hubblesite.org`.

The satellite is maneuvered via a simple thruster actuator, which provides torque to the satellite. A controller for the system uses feedback to modify the applied torque based on the positioning of the satellite.

As shown in Figure 2, disturbance forces that act on the spacecraft generate a torque $d(t)$. The objective of this exercise is to design a controller that minimizes the effect of the disturbances on the actual pointing angle, $y(t)$, while also ably tracking a reference pointing command $r(t)$ with certain performance requirements.

2 Pre-Class Work

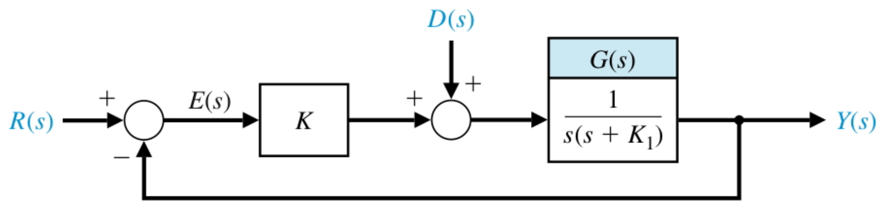

Figure 2: Disturbance forces may act on the spacecraft, generating a net torque $d(t)$. To minimize these effects, an inner loop controller is constructed with gain K_1.

1. Show that the unknown terms $G_R(s)$ and $G_D(s)$ in

$$Y(s) = G_R(s)R(s) + G_D(s)D(s) \tag{1}$$

   have the same denominator $\Delta(s)$. State your results for $G_R(s)$ and $G_D(s)$ in terms of one polynomial divided by another polynomial.

2. Sketch a block diagram for the system when $D(s) = 0$ and the only input to the system is the reference input $R(s)$. What is the type number of the resulting system with input $R(s)$ and output $Y(s)$?

3. What is the steady-state error when $R(s) = \frac{1}{s}$ and $D(s) = 0$?

4. Equate coefficients of the characteristic equation of a generic second-order system to $\Delta(s)$, to find the damping ratio ζ and natural frequency ω_n for arbitrary values of K, K_1. *(That is, find solutions for ζ and ω_n in terms of K, K_1.)*

3 In-Class Assignment

We first consider the feasibility of various pointing maneuvers for generic positive values of K, K_1. For the following two problems, you may presume that there is no disturbance acting on the system, that is, $d(t) = 0$.

1. Consider your answer to Pre-Class problem #3. As K increases, the steady-state error to a step input in $R(s)$ (assuming that $D(s) = 0$) will

   (a) Decrease

   (b) Remain unchanged

   (c) Increase but remain bounded

   (d) Increase but become unbounded

2. Consider a command to point the spacecraft continuously through a series of desired points of interest, such that $r(t) = t \cdot \mathbf{1}(t)$. Under ideal circumstances, that is, without any disturbance forces acting on the spacecraft, which *one* of the following best describes the spacecraft's ability to track the desired pointing angle trajectory? *Consider your answer to Pre-Class problem #2.*

   (a) Steady-state error will be $1/K$.

   (b) The pointing system will be able to track the desired path without any steady-state error.

   (c) The pointing error will grow over time, becoming unbounded as $t \to \infty$.

   (d) After the transient phenomena has died out, there will be a constant error of magnitude K_1/K in pointing angle.

We now consider the effect of the disturbance input. Using the Final Value Theorem, show that the steady-state response $y_{ss} = \lim_{t\to\infty} y(t)$ due to a step input in $R(s)$ (while $D(s) = 0$) is $y_{ss} = 1$, and that the steady-state response due to a step input in $D(s)$ (while $R(s) = 0$) is $y_{ss} = \frac{1}{K}$.

3. Which of the following statements are correct, presuming positive, constant, finite gains K, K_1?

   (a) The spacecraft pointing system can reject all disturbances of the form of a unit step.

   (b) When commanded via a unit step input, the spacecraft can move to a desired pointing angle without steady-state error.

   (c) The spacecraft will have a error that increases with time, due to a disturbance input in the form of a unit step.

   (d) If there are disturbance forces acting on the spacecraft in the form of a unit step input, the spacecraft will ultimately not point at the desired (reference) angle.

The goal is to choose K_1 and K such that the following performance constraints are met: (a) the settling time of the output to a step command in $r(t)$ is less than or equal to 2 seconds, (b) the steady-state error to a ramp command in $r(t)$ is less than or equal to 0.05, and (c) the steady-state response to a step input in the disturbance $d(t)$ is less than or equal to 0.01.

**4. (Double points) Determine the restrictions on K, K_1 that satisfy

   (a) The decay rate constraint. (Use your answer from Pre-Work problem #4.)

   (b) The steady-state constraint.

   (c) The disturbance response constraint.

   Do there exist positive, finite values of K, K_1 for which all three constraints can be simultaneously satisfied?

5. Notice that K will not affect settling time, but will affect steady-state error and steady-state response to a disturbance. What will be the effect of turning up the gain, that is, in increasing K, on the two steady-state performance goals? Describe the trade-offs (if any) in the overshoot of the transient step response, as K increases (for a fixed K_1). That is, why not simply pick as large a K as possible for the spacecraft?

If your group finishes early...

Other points to consider (not necessary to hand in):

- If K_1 were zero, that is, the inner loop was not used, the type number of $G_R(s)$ would be 2. Why then would the system with input $R(s)$ (assuming that $D(s) = 0$) be *unable* to track a ramp input?
- Both $G_R(s)$ and $G_D(s)$ are Type 1 systems, hence the steady-state error due to a unit step input to either transfer function is 0. Hence the steady-state error to a step input in $R(s)$ (when $D(s) = 0$) is 0. So why will the output $Y(s)$ have a non-zero value in response to a unit step input in $D(s)$ (when $R(s) = 0$)?

Instead of a proportional controller K, now consider a proportional-derivative controller $K(s) = K(s + z)$ for some constant $z > 0$.

- Repeat the Pre-Work questions but with unknown parameters K, K_1, z.
- Repeat In-Class questions #1-#4 for your PD-controlled system.
- Compare your PD controller results to those you obtained earlier via proportional control. Which controller works "better"? Why? Are any of the three criteria met with PD control that were not met with proportional control?