Lecture 23
(Angular Velocity and Acceleration)

Physics 160-01 Fall 2012

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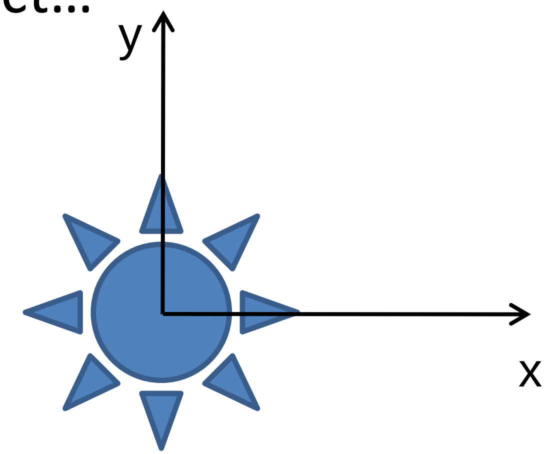
Rotation of Rigid Bodies

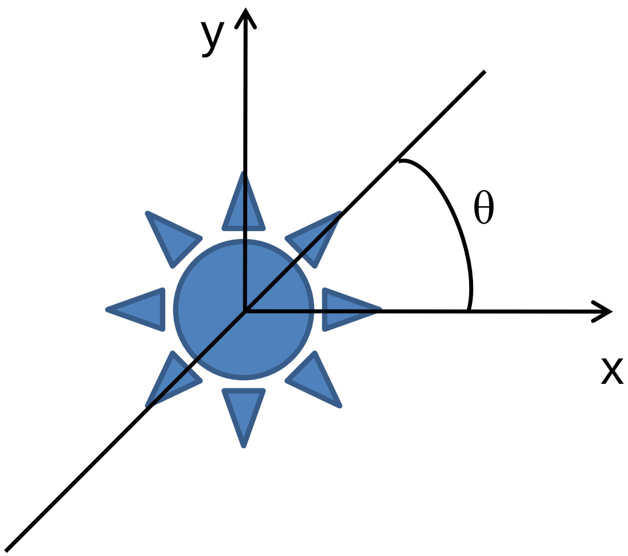
- We know how to describe the motion of the COM of an object…



- Now, we want to understand how to describe the motion of an object ***about*** its COM.
- We could look at each piece of the object as a separate particle and calculate forces and then calculate accelerations…
- Yuk!!!

Rotation of Rigid Bodies

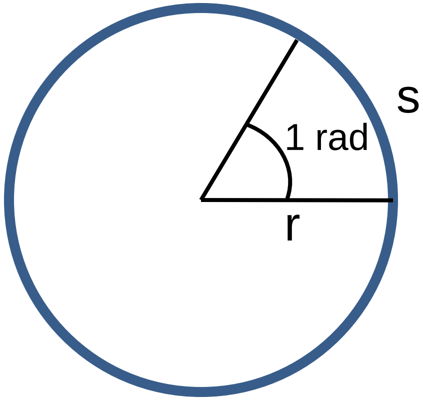
- We need a new information set first:



- So, we can describe the motion about the COM via an angle defined to some point on the body.
- Now, let's define the units for angles…

Radians

- One radian is the angle when the arc length is the same as the radius.



$$\theta = \frac{s}{r}$$

- All the way around, the arc length is $2\pi r$, so there are 2π radians in a complete revolution.

Rotational Variables

- Angular displacement.

$$\Delta\theta = \theta_f - \theta_i$$

- Angular velocity.

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}, \quad \omega = \frac{d\theta}{dt}$$

- Angular acceleration.

$$\alpha = \frac{d\omega}{dt}$$

Constant angular acceleration

- Same as constant linear acceleration:

$$y_f = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

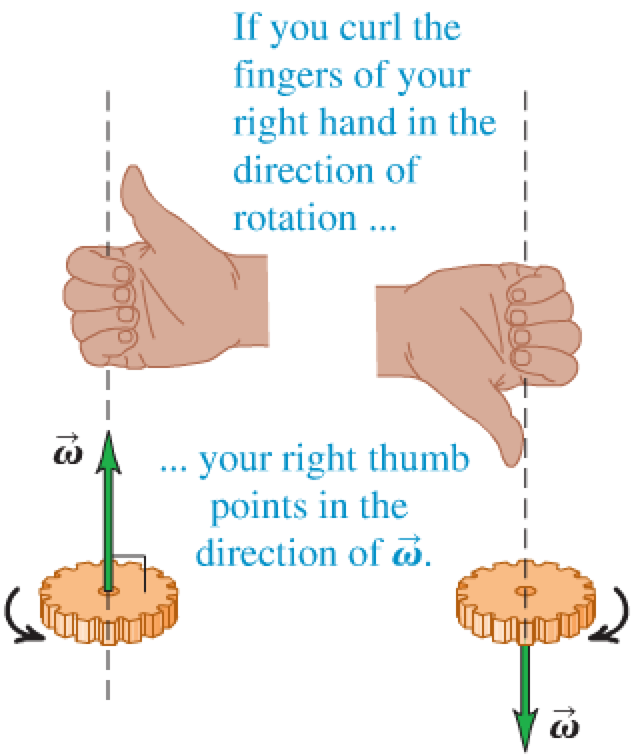
$$v_{fy} = v_{0y} + a_y t$$

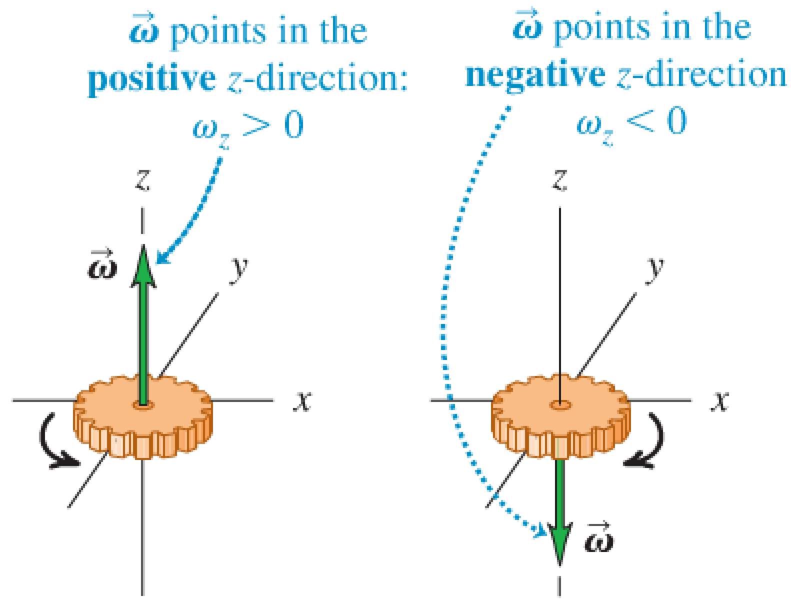
$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega_f = \omega_0 + \alpha t$$

Vector Definitions

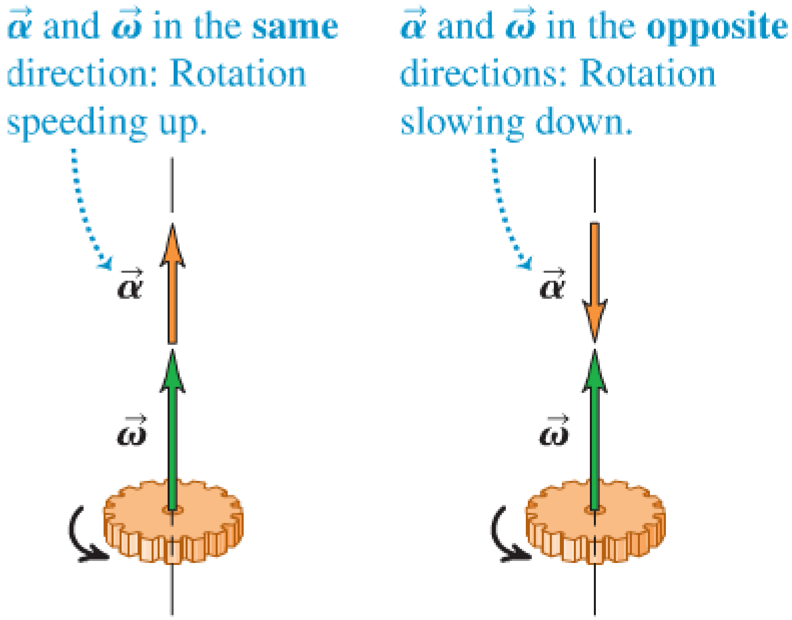
- We will use the right hand rule to define a direction for the angular velocity and the angular acceleration:





Vector Definitions

- and the angular acceleration:



Example

9.8. A wheel is rotating about an axis that is in the z-direction. The angular velocity ω_z is $-6.00\ \mathrm{rad/s}$ at $t = 0$, increases linearly with time, and is $+8.00\ \mathrm{m/s}$ at $t = 7.00\ \mathrm{s}$. We have taken counterclockwise rotation to be positive. (a) Is the angular acceleration during this time interval positive or negative? (b) During what time interval is the speed of the wheel increasing? Decreasing? (c) What is the angular displacement of the wheel at $t = 7.00\ \mathrm{s}$?

Relation to Translational Variables

- Sometimes we may want to relate rotational motion to linear motion…

$$\theta = \frac{s}{r} \Rightarrow s = \theta r$$

$$v = \frac{ds}{dt} = \frac{d\theta}{dt} r = \omega r \Rightarrow v = \omega r$$

$$a_{\text{radial}} = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r \Rightarrow a_{\text{radial}} = \omega^2 r$$

$$a_{\text{tangential}} = \frac{dv}{dt} = \frac{d(\omega r)}{dt} = \alpha r \Rightarrow a_{\text{tangential}} = \alpha r$$

Exercise 9.3

9.3 • **CP CALC** The angular velocity of a flywheel obeys the equation $\omega_z(t) = A + Bt^2$, where t is in seconds and A and B are constants having numerical values 2.75 (for A) and 1.50 (for B). (a) What are the units of A and B if ω_z is in rad/s? (b) What is the angular acceleration of the wheel at (i) $t = 0.00$ and (ii) $t = 5.00$ s? (c) Through what angle does the flywheel turn during the first 2.00 s?

Exercise 9.7

9.7 • **CALC** The angle θ through which a disk drive turns is given by $\theta(t) = a + bt - ct^3$, where a, b, and c are constants, t is in seconds, and θ is in radians. When $t = 0$, $\theta = \pi/4$ rad and the angular velocity is 2.00 rad/s, and when $t = 1.50$ s, the angular acceleration is 1.25 rad/s^2. (a) Find a, b, and c, including their units. (b) What is the angular acceleration when $\theta = \pi/4$ rad? (c) What are θ and the angular velocity when the angular acceleration is 3.50 rad/s^2?

Problem 9.61

9.61 • **CP CALC** A flywheel has angular acceleration $\alpha_z(t) = 8.60\ \text{rad/s}^2 - (2.30\ \text{rad/s}^3)t$, where counterclockwise rotation is positive. (a) If the flywheel is at rest at $t = 0$, what is its angular velocity at 5.00 s? (b) Through what angle (in radians) does the flywheel turn in the time interval from $t = 0$ to $t = 5.00$ s?