#24 Angular Velocity and Acceleration Pre-class

Due: 11:00am on Friday, October 19, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Exercise 9.3

The angular velocity of a flywheel obeys the equation $\omega_z(t) = A + Bt^2$, where t is in seconds and A and B are constants having numerical values 2.80 (for A) and 1.60 (for B).

Part A

What are the units of A if ω is in $\mathrm{rad/s}$?

ANSWER:

- ○ $\mathrm{rad^2/s}$
- ◉ $\mathrm{rad/s}$
- ○ $\mathrm{rad/s^2}$
- ○ $\mathrm{rad^2/s^2}$

Correct

Part B

What are the units of B if ω is in $\mathrm{rad/s}$?

ANSWER:

- $\mathrm{rad/s}$
- $\mathrm{rad^2/s^2}$
- $\mathrm{rad/s^3}$
- $\mathrm{rad/s^2}$

Correct

Part C

What is the angular acceleration of the wheel at t = 0.00?

ANSWER:

α_z = 0 $\mathrm{rad/s^2}$

Correct

Part D

What is the angular acceleration of the wheel at t = 7.50s ?

ANSWER:

α_z = 24.0 $\mathrm{rad/s^2}$

Correct

Part E

Through what angle does the flywheel turn during the first 1.00s ? (*Hint:* See Section 2.6 in the textbook.)

ANSWER:

$\theta_2 - \theta_1$ = 3.33 rad

Correct

Angular Motion with Constant Acceleration

Learning Goal:

To understand the meaning of the variables that appear in the equations for rotational kinematics with constant angular acceleration.

Rotational motion with a constant nonzero acceleration is not uncommon in the world around us. For instance, many machines have spinning parts. When the machine is turned on or off, the spinning parts tend to change the rate of their rotation with virtually constant angular acceleration. Many introductory problems in rotational kinematics involve motion of a particle with constant, nonzero angular acceleration. The kinematic equations for such motion can be written as

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

and

$$\omega(t) = \omega_0 + \alpha t.$$

Here, the symbols are defined as follows:

- $\theta(t)$ is the angular position of the particle at time t.
- θ_0 is the initial angular position of the particle.

- $\omega(t)$ is the angular velocity of the particle at time t.
- ω_0 is the initial angular velocity of the particle.
- α is the angular acceleration of the particle.
- t is the time that has elapsed since the particle was located at its initial position.

In answering the following questions, assume that the angular acceleration is constant and nonzero: $\alpha \neq 0$.

Part A

True or false: The quantity represented by θ is a function of time (i.e., is not constant).

ANSWER:

- (selected) true
- false

Correct

Part B

True or false: The quantity represented by θ_0 is a function of time (i.e., is not constant).

ANSWER:

- true
- (selected) false

Correct

Keep in mind that θ_0 represents an initial value, not a variable. It refers to the angular position of an object at some initial moment.

Part C

True or false: The quantity represented by ω_0 is a function of time (i.e., is not constant).

ANSWER:

- ○ true
- ◉ false

Correct

Part D

True or false: The quantity represented by ω is a function of time (i.e., is not constant).

ANSWER:

- ◉ true
- ○ false

Correct

The angular velocity ω always varies with time when the angular acceleration is nonzero.

Part E

Which of the following equations is not an explicit function of time t? Keep in mind that an equation that is an explicit function of time involves t as a variable.

ANSWER:

- ○ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
- ○ $\omega = \omega_0 + \alpha t$
- ◉ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Correct

An equation that is not an explicit function of time is useful when you do not know or do not need the time.

Part F

In the equation $\omega = \omega_0 + \alpha t$, what does the time variable t represent?

Choose the answer that is *always* true. Several of the statements may be true in a particular problem, but only one is always true.

ANSWER:

- the moment in time at which the angular velocity equals ω_0
- the moment in time at which the angular velocity equals ω
- the time elapsed from when the angular velocity equals ω_0 until the angular velocity equals ω

Correct

Consider two particles A and B. The angular position of particle A, with constant angular acceleration, depends on time according to $\theta_A(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$. At time $t = t_1$, particle B, which also undergoes constant angular acceleration, has twice the angular acceleration, half the angular velocity, and the same angular position that particle A had at time $t = 0$.

Part G

Which of the following equations describes the angular position of particle B?

Hint 1. How to approach the problem

The general equation for the rotation of B can be written as

$$\theta_{\rm B}(t) = \theta_{\rm B}(t=t_1) + \omega_{\rm B}(t=t_1)\cdot(t-t_1) + \frac{1}{2}\alpha_{\rm B}(t-t_1)^2,$$

where $\theta_{\rm B}(t=t_1)$ and $\omega_{\rm B}(t=t_1)$ are the position and angular speed, respectively, of particle B at time t_1. Note the time factors of $t_B = (t-t_1)$ and not t. This change is because nothing is known about particle B at time $t=0$. Instead you have information about particle B at time t_1.

Express the quantities on the right-hand side of this equation for $\theta_{\rm B}(t)$ in terms of particle A's variables and constants of motion.

ANSWER:

- ○ $\theta_{\rm B}(t) = \theta_0 + 2\omega_0 t + \frac{1}{4}\alpha t^2$
- ○ \theta_{\rm B}(t)=\theta_0+\frac{1}{2}\omega_0t+\alpha t^2
- ○ $\theta_{\rm B}(t) = \theta_0 + 2\omega_0(t-t_1) + \frac{1}{4}\alpha(t-t_1)^2$
- ◉ $\theta_{\rm B}(t) = \theta_0 + \frac{1}{2}\omega_0(t-t_1) + \alpha(t-t_1)^2$
- ○ \theta_{\rm B}(t)=\theta_0+2\omega_0(t+t_1) +\frac{1}{4}\alpha (t+t_1)^2
- ○ $\theta_{\rm B}(t) = \theta_0 + \frac{1}{2}\omega_0(t+t_1) + \alpha(t+t_1)^2$

Correct

Note that particle B has a smaller initial angular velocity but greater angular acceleration. Also, it has been in motion for less time than particle A.

Part H

How long after the time t_1 does the angular velocity of particle B equal that of particle A?

Hint 1. How to approach the problem

Write expressions for the angular velocity of A and B as functions of time, either by comparison of the above equations with the general kinematic equations or by differentiating the above equations. Then equate the 2 expressions and solve for t-t_1.

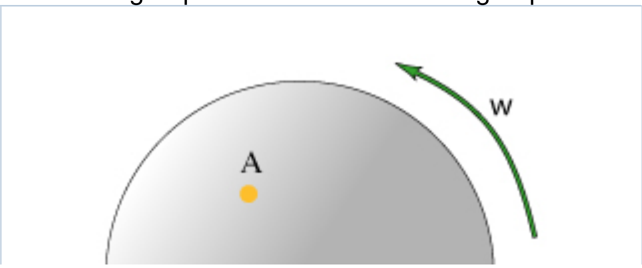
ANSWER:

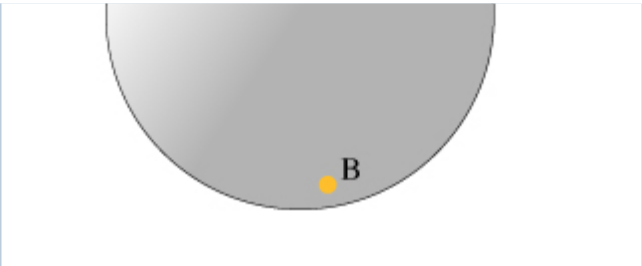
- \frac{\omega_0}{4\alpha}
- \frac{\omega_0+4\alpha t_1}{2\alpha}
- \frac{\omega_0+2\alpha t_1}{2\alpha}
- The two particles never have the same angular velocity.

Correct

Linear and Rotational Quantities Conceptual Question

A merry-go-round is rotating at constant angular speed. Two children are riding the merry-go-round: Alan is riding at point A and Isamu is riding at point B.





Part A

Which child moves with greater magnitude of velocity?

Hint 1. Distinguishing between velocity and angular velocity

Alan’s (or Isamu’s) velocity is determined by the actual distance traveled (typically in meters) in a given time interval. The angular velocity is determined by the angle through which he rotates (typically in radians) in a given time interval.

ANSWER:

- Alan has the greater magnitude of velocity.
- Isamu has the greater magnitude of velocity.
- Both Alan and Isamu have the same magnitude of velocity.

Correct

Part B

Who moves with greater magnitude of angular velocity?

Hint 1. Distinguishing between velocity and angular velocity

Alan's (or Isamu's) velocity is determined by the actual distance he travels (typically in meters) in a given time interval. His angular velocity is determined by the angle through which he rotates (typically in radians) in a given time interval.

ANSWER:

- ○ Alan has the greater magnitude of angular velocity.
- ○ Isamu has the greater magnitude of angular velocity.
- ◉ Both Alan and Isamu have the same magnitude of angular velocity.

Correct

Part C

Who moves with greater magnitude of tangential acceleration?

Hint 1. Distinguishing tangential, centripetal, and angular acceleration

Alan's tangential and centripetal acceleration are components of his acceleration vector. During circular motion, if Alan's speed is changing (meaning the merry-go-round is speeding up or slowing down) he will have a nonzero tangential acceleration. However, even if the merry-go-round is turning at constant angular speed, he will experience a centripetal acceleration, because the direction of his velocity vector is changing (you can't move along a circular path unless your direction of travel is changing!).
Both tangential and centripetal accelerations have units of \rm {m/s^2}, since they are the two-dimensional components of linear acceleration. Angular acceleration, on the other hand, is a measure of the change in Alan's angular velocity. If his rate of rotation is changing, he will have a nonzero angular acceleration. Thus, angular acceleration has units of \rm {rad/s^2}.

ANSWER:

- Alan has the greater magnitude of tangential acceleration.
- Isamu has the greater magnitude of tangential acceleration.
- Both Alan and Isamu have the same magnitude of tangential acceleration.

Correct

Part D

Who has the greater magnitude of centripetal acceleration?

Hint 1. Distinguishing tangential, centripetal, and angular acceleration

Alan's tangential and centripetal acceleration are components of his acceleration vector. For circular motion, if Alan's speed is changing (meaning the merry-go-round is speeding up or slowing down) he will have a nonzero tangential acceleration. However, even if the merry-go-round is turning at constant angular speed, he will experience a centripetal acceleration, because the direction of his velocity vector is changing (you can't move along a circular path unless your direction of travel is changing!).
Both tangential and centripetal accelerations have units of \rm {m/s^2}, since they are the two-dimensional components of linear acceleration. Angular acceleration, on the other hand, is a measure of the change in Alan's angular velocity. If his rate of rotation is changing, he will have a nonzero angular acceleration. Thus, angular acceleration has units of \rm {rad/s^2}.

ANSWER:

- Alan has the greater magnitude of centripetal acceleration.
- Isamu has the greater magnitude of centripetal acceleration.
- Both Alan and Isamu have the same magnitude of centripetal acceleration.

Correct

Part E

Who moves with greater magnitude of angular acceleration?

Hint 1. Distinguishing tangential, centripetal, and angular acceleration

Alan’s tangential and centripetal acceleration are components of his acceleration vector. For circular motion, if Alan’s speed is changing (meaning the merry-go-round is speeding up or slowing down) he will have a nonzero tangential acceleration. However, even if the merry-go-round is turning at constant angular speed, he will experience a centripetal acceleration, because the direction of his velocity vector is changing (you can’t move along a circular path unless your direction of travel is changing!).
Both tangential and centripetal accelerations have units of $\rm {m/s^2}$, since they are the two-dimensional components of linear acceleration. Angular acceleration, on the other hand, is a measure of the change in Alan’s angular velocity. If his rate of rotation is changing, he will have a nonzero angular acceleration. Thus, angular acceleration has units of $\rm {rad/s^2}$.

ANSWER:

- ○ Alan has the greater magnitude of angular acceleration.
- ○ Isamu has the greater magnitude of angular acceleration.
- ● Both Alan and Isamu have the same magnitude of angular acceleration.

Correct

Exercise 9.1

Part A

What angle in radians is subtended by an arc of 1.41 m in length on the circumference of a circle of radius 2.53 m ?

ANSWER:

θ = 0.557 {\rm rad}

Correct

Part B

What is this angle in degrees?

ANSWER:

θ = 31.9 $^\circ$

Correct

Part C

An arc of length 14.8cm on the circumference of a circle subtends an angle of 130°. What is the radius of the circle?

ANSWER:

r = 6.52 cm

Correct

Part D

The angle between two radii of a circle with radius 1.48m is 0.770$_{\mathrm{rad}}$. What length of arc is intercepted on the circumference of the circle by the two radii?

ANSWER:

L = 1.14 m

Correct

Score Summary:

Your score on this assignment is 90.5%.
You received 18.09 out of a possible total of 20 points.