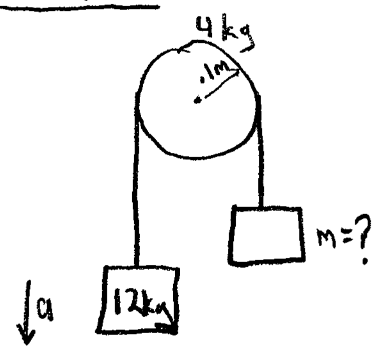
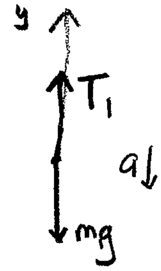
1. Torque



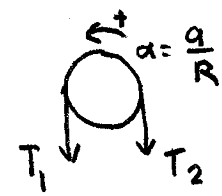
12kg block:



$$\Sigma F_y = T_1 - m_1 g = -m_1 a$$

$$\therefore T_1 = m_1 g - m_1 a$$

4kg pulley:



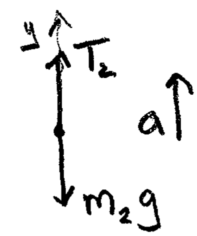
$$\Sigma \tau_z = T_1 R - T_2 R = I\alpha = \frac{1}{2} M_p R^2 \frac{a}{R}$$

$$\Rightarrow T_1 - T_2 = \frac{1}{2} M_p a$$

$$\therefore T_2 = T_1 - \frac{1}{2} M_p a$$

$$= m_1 g - m_1 a - \frac{1}{2} M_p a$$

Unknown mass:



$$\Sigma F_y = T_2 - m_2 g = m_2 a$$

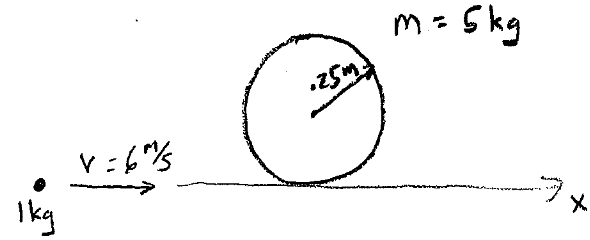
$$T_2 = m_2 (g + a)$$

$$\therefore m_2 = \frac{T_2}{g+a} = \frac{m_1 g - m_1 a - \frac{1}{2} M_p a}{g + a}$$

$$= \frac{(12\,\text{kg})(9.8\,\text{m/s}^2 - 1.0\,\text{m/s}^2) - \frac{1}{2}(4\,\text{kg})(1.0\,\text{m/s}^2)}{(9.8\,\text{m/s}^2 + 1.0\,\text{m/s}^2)}$$

$$= 9.6\,\text{kg}$$

2. Angular Momentum



Linear momentum is conserved:

$$m_c \vec{V}_c = (M_D + M_c)\vec{V}_{D+C}$$

$$(1\,\text{kg})(6.0\,\tfrac{m}{s}\,\hat{i}) = (1\,\text{kg} + 5\,\text{kg})\vec{V}_{D+C}$$

$$\boxed{\vec{V}_{D+C} = 1\,\tfrac{m}{s}\,\hat{i}}$$

Angular momentum is conserved:

The tricky part is deciding which axis of rotation to use!

If you use the center of the disk:

$$L_i = R m_c V_c = L_f = I^{CD}_{C+D}\,\omega_{C+D} + R_{cm}(m_D + m_c)V_{D+C}$$

The second term in L_f is due to the angular momentum taken out by the disk + clay center of mass motion about the axis of the disk.

with $R_{cm} = \dfrac{m_c R}{m_c + m_D}$, we have

$$R m_c V_c = I^{CD}_{C+D}\,\omega_{C+D} + \frac{m_c R}{\cancel{m_c + m_D}}\cancel{(m_c + m_D)}\,V_{D+C}$$

$$\therefore\ \omega_{D+C} = \frac{R m_c V_c - R m_c V_{D+C}}{I^{CD}_{C+D}} = \frac{R m_c V_c\left(\frac{m_D}{m_c + m_D}\right)}{I^{CD}_{C+D}}$$

with $I^{CD}_{C+D} = \frac{1}{2} M_D R^2 + m_c R^2$ then

$$\omega_{D+C} = \frac{m_c V_c\left(\frac{m_D}{m_c + m_D}\right)}{\frac{R}{2}(m_D + 2m_c)}$$

$$= \frac{(1\,\text{kg})(6\,\frac{m}{s})\left(\frac{5\,\text{kg}}{6\,\text{kg}}\right)}{(.125\,\text{m})(7\,\text{kg})} = \boxed{5.7\,\frac{\text{rad}}{s} = \omega_{D+C}}$$

But, that is not the correct way (although that way will be accepted for full credit). The reason is this: the axis of rotation after the collision is the center of mass! Let's do it that way:

$$L_i = (R - R_{cm}) m_c V_c$$

$$L_f = I_{D+c}^{cm} \omega_{D+c}$$

$$\therefore \quad R\left(\frac{m_D}{m_c + m_D}\right) m_c V_c = I_{D+c}^{cm} \omega_{D+c}$$

Now,

$$I_{D+c}^{cm} = \tfrac{1}{2} m_D R^2 + m_D R_{cm}^2 + m_c (R - R_{cm})^2$$

$$= \tfrac{1}{2} m_D R^2 + \frac{m_D m_c^2}{(m_c+m_D)^2} R^2 + \frac{m_c m_D^2}{(m_c+m_D)^2} R^2$$

$$= \frac{m_D R^2}{2(m_c+m_D)^2} \left((m_c+m_D)^2 + 2m_c^2 + 2m_c m_D\right)$$

$$= \frac{m_D R^2}{2(m_c+m_D)^2} \left(3m_c^2 + 4m_c m_D + m_D^2\right)$$

$$= \frac{m_D R^2}{2(m_c+m_D)^2} \left[(m_c + m_D)^2 + 2m_c(m_c + m_D)\right]$$

$$= \frac{R^2}{2}\left[m_D + 2m_c\left(\frac{m_D}{m_c+m_D}\right)\right]$$

Then,

$$\omega_{D+c} = \frac{R\left(\frac{m_D}{m_c+m_D}\right) m_c V_c}{\frac{R^2}{2}\left[m_D + 2m_c\left(\frac{m_D}{m_c+m_D}\right)\right]}$$

$$= \frac{m_c V_c \left(\frac{m_D}{m_c+m_D}\right)}{\frac{R}{2}\left(m_D + 2m_c\left(\frac{m_D}{m_c+m_D}\right)\right)}$$

$$\omega_{D+c} = \frac{(1\,\text{kg})(6\,\tfrac{\text{m}}{\text{s}})\left(\frac{5\,\text{kg}}{6\,\text{kg}}\right)}{(.125\,\text{m})\left(5\,\text{kg} + 2\,\text{kg}\left(\frac{5\,\text{kg}}{6\,\text{kg}}\right)\right)}$$

$$\boxed{\omega_{D+c} = 6\ \frac{\text{rad}}{\text{s}}}$$