25.58. Conservation of charge requires that the current is the same in both sections. The voltage drops across each section add, so $R = R_{\text{Cu}} + R_{\text{Ag}}$. The total resistance is the sum of the resistances of each section. $E = \rho J = \dfrac{\rho I}{A}$, so $E = \dfrac{IR}{L}$, where R is the resistance of a section and L is its length.

For copper, $\rho_{\text{Cu}} = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. For silver, $\rho_{\text{Ag}} = 1.47\times10^{-8}\ \Omega\cdot\text{m}$.

(a) $I = \dfrac{V}{R} = \dfrac{V}{R_{\text{Cu}} + R_{\text{Ag}}}$. $R_{\text{Cu}} = \dfrac{\rho_{\text{Cu}} L_{\text{Cu}}}{A_{\text{Cu}}} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(0.8\ \text{m})}{(\pi/4)(6.0\times10^{-4}\text{m})^2} = 0.049\ \Omega$ and

$R_{\text{Ag}} = \dfrac{\rho_{\text{Ag}} L_{\text{Ag}}}{A_{\text{Ag}}} = \dfrac{(1.47\times10^{-8}\ \Omega\cdot\text{m})(1.2\ \text{m})}{(\pi/4)(6.0\times10^{-4}\ \text{m})^2} = 0.062\ \Omega$. This gives $I = \dfrac{5.0\ \text{V}}{0.049\ \Omega + 0.062\ \Omega} = 45\ \text{A}$.

The current in the copper wire is 45 A.

(b) The current in the silver wire is 45 A, the same as that in the copper wire or else charge would build up at their interface.

(c) $E_{\text{Cu}} = J\rho_{\text{Cu}} = \dfrac{IR_{\text{Cu}}}{L_{\text{Cu}}} = \dfrac{(45\ \text{A})(0.049\ \Omega)}{0.8\ \text{m}} = 2.76\ \text{V/m}$.

(d) $E_{\text{Ag}} = J\rho_{\text{Ag}} = \dfrac{IR_{\text{Ag}}}{L_{\text{Ag}}} = \dfrac{(45\ \text{A})(0.062\ \Omega)}{1.2\ \text{m}} = 2.33\ \text{V/m}$.

(e) $V_{\text{Ag}} = IR_{\text{Ag}} = (45\ \text{A})(0.062\ \Omega) = 2.79\ \text{V}$.

For the copper section, $V_{\text{Cu}} = IR_{\text{Cu}} = 2.21\ \text{V}$. Note that $V_{\text{Cu}} + V_{\text{Ag}} = 5.0\ \text{V}$, the voltage applied across the ends of the composite wire.

25.68. Consider the potential changes around the circuit. For a complete loop the sum of the potential changes is zero.

There is a potential drop of IR when you pass through a resistor in the direction of the current.

(a) $I = \dfrac{8.0\ \text{V} - 4.0\ \text{V}}{24.0\ \Omega} = 0.167\ \text{A}$. $V_d + 8.00\ \text{V} - I(0.50\ \Omega + 8.00\ \Omega) = V_a$, so

$V_{ad} = 8.00\ \text{V} - (0.167\ \text{A})(8.50\ \Omega) = 6.58\ \text{V}$.

(b) The terminal voltage is $V_{bc} = V_b - V_c$. $V_c + 4.00\ \text{V} + I(0.50\ \Omega) = V_b$ and

$V_{bc} = +4.00\ \text{V} + (0.167\ \text{A})(0.50\ \Omega) = +4.08\ \text{V}$.

(c) Adding another battery at point d in the opposite sense to the 8.0-V battery produces a counterclockwise current with magnitude $I = \dfrac{10.3\ \text{V} - 8.0\ \text{V} + 4.0\ \text{V}}{24.5\ \Omega} = 0.257\ \text{A}$.

Then $V_c + 4.00\ \text{V} - I(0.50\ \Omega) = V_b$ and $V_{bc} = 4.00\ \text{V} - (0.257\ \text{A})\ (0.50\ \Omega) = 3.87\ \text{V}$.

When current enters the battery at its negative terminal, as in part (c), the terminal voltage is less than its emf. When current enters the battery at the positive terminal, as in part (b), the terminal voltage is greater than its emf.

76.

a)

$$\rho(0) = a = 2.25 \times 10^{-8}\,\Omega\cdot\mathrm{m}$$

$$\rho(L) = a + b\,L^2 = 8.50 \times 10^{-8}\,\Omega\cdot\mathrm{m}$$

$$b = \frac{8.50 \times 10^{-8}\,\Omega\cdot\mathrm{m} - a}{L^2} = \frac{8.50 \times 10^{-8}\,\Omega\cdot\mathrm{m} - 2.25 \times 10^{-8}\,\Omega\cdot\mathrm{m}}{(1.50\,\mathrm{m})^2} = 2.778 \times 10^{-8}\,\frac{\Omega}{\mathrm{m}}$$

$$\mathrm{d}R = \rho(x)\,\frac{\mathrm{d}x}{A} = \frac{a + b\,x^2}{A}\,\mathrm{d}x$$

$$A = \pi\,(0.0110\,\mathrm{m})^2 = 3.801 \times 10^{-4}\,\mathrm{m}^2$$

$$\begin{aligned} R &= \int_0^L \frac{a + b\,x^2}{A}\,\mathrm{d}x = \frac{1}{A}\left(a\,x + \frac{b}{3}\,x^3\right)\Big|_0^L = \frac{a\,L + b\,L^3/3}{A} \\ &= \frac{(2.25 \times 10^{-8}\,\Omega\cdot\mathrm{m})\,(1.50\,\mathrm{m}) + (2.778 \times 10^{-8}\,\Omega/\mathrm{m})\,(1.50\,\mathrm{m})^3/3}{3.801 \times 10^{-4}\,\mathrm{m}^2} \\ &= \boxed{1.71 \times 10^{-4}\,\Omega} = 171\;\mu\Omega \end{aligned}$$

b) $\vec{\boldsymbol{E}} = \rho\,\vec{\boldsymbol{J}}$

$$\begin{aligned} \left|\vec{\boldsymbol{E}}\right| &= \rho(x)\,\frac{I}{A} = (a + b\,x^2)\,\frac{I}{A} \\ &= \left[2.25 \times 10^{-8}\,\Omega\cdot\mathrm{m} + \left(2.778 \times 10^{-8}\,\frac{\Omega}{\mathrm{m}}\right)(0.750\,\mathrm{m})^2\right]\frac{1.75\,\mathrm{A}}{3.801 \times 10^{-4}\,\mathrm{m}^2} \\ &= \boxed{1.76 \times 10^{-4}\,\frac{\mathrm{V}}{\mathrm{m}}} \end{aligned}$$

c) Left half:

$$\begin{aligned} R_L &= \int_0^{L/2} \frac{a + b\,x^2}{A}\,\mathrm{d}x = \frac{1}{A}\left[a\,\frac{L}{2} + \frac{b}{3}\left(\frac{L}{2}\right)^3\right] \\ &= \frac{1}{3.801 \times 10^{-4}\,\mathrm{m}^2}\left[(2.25 \times 10^{-8}\,\Omega\cdot\mathrm{m})\,(0.750\,\mathrm{m}) + \frac{2.778 \times 10^{-8}\,\Omega/\mathrm{m}}{3}\,(0.750\,\mathrm{m})^3\right] \\ &= \boxed{5.47 \times 10^{-5}\,\Omega} = 54.7\;\mu\Omega \end{aligned}$$

Right half:

$$\begin{aligned} R_R &= \int_{L/2}^{L} \frac{a + b\,x^2}{A}\,\mathrm{d}x = \frac{1}{A}\left[a\left(L - \frac{L}{2}\right) + \frac{b}{3}\left(L^3 - \left(\frac{L}{2}\right)^3\right)\right] = \frac{1}{A}\left[a\,\frac{L}{2} + \frac{7\,b}{24}\,L^3\right] \\ &= \frac{1}{3.801 \times 10^{-4}\,\mathrm{m}^2}\left[(2.25 \times 10^{-8}\,\Omega\cdot\mathrm{m})\,(0.750\,\mathrm{m}) + \frac{7\,(2.778 \times 10^{-8}\,\Omega/\mathrm{m})}{24}\,(1.50\,\mathrm{m})^3\right] \\ &= \boxed{1.16 \times 10^{-4}\,\Omega} = 116\;\mu\Omega \end{aligned}$$

84.

a) $Q_1 = C_1\,V_{C_1}$, $V_{C_1} = V_{C_2} = V_{R_2} = Q_1/C_1$,

$$Q_2 = C_2\,V_{C_2} = Q_1\,\frac{C_2}{C_1} = (18.0\;\mu\mathrm{C})\,\frac{6.00\;\mu\mathrm{F}}{3.00\;\mu\mathrm{F}} = \boxed{36.0\;\mu\mathrm{C}}.$$

b) When the capacitors have attained their final charges, there is no current through them, so the current through R_1 and R_2 is the same (call it I).

$$\mathcal{E} = I\left(R_1 + R_2\right) = \frac{V_{R_2}}{R_2}\left(R_1 + R_2\right) = \frac{Q_1}{C_1}\left(\frac{R_1}{R_2} + 1\right),$$

$$R_1 = R_2\left(\frac{C_1}{Q_1}\mathcal{E} - 1\right) = (2.00\,\Omega)\left[\frac{3.00\,\mu\mathrm{F}}{18.0\,\mu\mathrm{C}}\,(60.0\,\mathrm{V}) - 1\right] = \boxed{18.0\,\Omega}.$$