#21 Momentum and Impulse Pre-class

Due: 11:00am on Monday, October 8, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Momentum and Internal Forces

Learning Goal:

To understand the concept of total momentum for a system of objects and the effect of the internal forces on the total momentum.

We begin by introducing the following terms:

System: Any collection of objects, either pointlike or extended. In many momentum-related problems, you have a certain freedom in choosing the objects to be considered as your system. Making a wise choice is often a crucial step in solving the problem.

Internal force: Any force interaction between two objects belonging to the chosen system. Let us stress that both interacting objects must belong to the system.

External force: Any force interaction between objects at least one of which does not belong to the chosen system; in other words, at least one of the objects is external to the system.

Closed system: a system that is not subject to any external forces.

Total momentum: The vector sum of the individual momenta of all objects constituting the system.

In this problem, you will analyze a system composed of two blocks, 1 and 2, of respective masses m_1 and m_2. To simplify the analysis, we will make several assumptions:

1. The blocks can move in only one dimension, namely, along the *x* axis.
2. The masses of the blocks remain constant.
3. The system is closed.

At time t, the *x* components of the velocity and the acceleration of block 1 are denoted by $v_1(t)$ and $a_1(t)$. Similarly, the *x* components of the velocity and acceleration of block 2 are denoted by $v_2(t)$ and a_2(t). In this problem, you will show that the total momentum of the system is not changed by the presence of internal forces.

Part A

Find $p(t)$, the *x* component of the total momentum of the system at time t.

Express your answer in terms of m_1, m_2, $v_1(t)$, and $v_2(t)$.

ANSWER:

$p(t) =$ $m_2 v_2(t) + m_1 v_1(t)$

Correct

Part B

Find the time derivative $dp(t)/dt$ of the *x* component of the system's total momentum.

Express your answer in terms of $a_1(t)$, a_2(t), m_1, and m_2.

Hint 1. Finding the derivative of momentum for one block

Consider the momentum of block 1: $p_1(t) = m_1 v_1(t)$. Take the derivative of this expression with respect to time, noting that velocity is a function of time, and mass is a constant:

\frac{dp_1(t)}{dt} = \frac{d\left(m_1 v_1(t)\right)}{dt} = m_1\frac{dv_1(t)}{dt}.

Hint 2. The relationship between velocity and acceleration

Recall the definition of acceleration as a(t)=\frac{dv(t)}{dt}.

ANSWER:

$dp(t)/dt =$ $m_1 a_1(t) + m_2 a_2(t)$

Correct

Why did we bother with all this math? The expression for the derivative of momentum that we just obtained will be useful in reaching our desired conclusion, if only for this very special case.

Part C

The quantity ma (mass times acceleration) is dimensionally equivalent to which of the following?

ANSWER:

- momentum
- energy
- (selected) force
- acceleration
- inertia

Correct

Part D

Acceleration is due to which of the following physical quantities?

ANSWER:

- velocity
- speed
- energy
- momentum
- force

Correct

Part E

Since we have assumed that the system composed of blocks 1 and 2 is closed, what could be the reason for the acceleration of block 1?

Hint 1. Force and acceleration

Since the system is closed, the only object that can affect block 1 is the other block in the system, block 2.

ANSWER:

- the large mass of block 1
- air resistance
- Earth's gravitational attraction
- a force exerted by block 2 on block 1
- a force exerted by block 1 on block 2

Correct

Part F

What could be the reason for the acceleration of block 2?

ANSWER:

- ○ a force exerted by block 2 on block 1
- ◉ a force exerted by block 1 on block 2

Correct

Part G

Let us denote the *x* component of the force exerted by block 1 on block 2 by F_12, and the *x* component of the force exerted by block 2 on block 1 by F_21. Which of the following pairs equalities is a direct consequence of Newton's second law?

ANSWER:

- ◉ F_{12}=m_2 a_2 and F_{21}=m_1 a_1
- ○ F_{12}=m_1 a_1 and F_{21}=m_2 a_2
- ○ F_{12}=m_1 a_2 and F_{21}=m_2 a_1
- ○ F_{12}=m_2 a_1 and F_{21}=m_1 a_2

Correct

Note that both F_12 and F_21 are internal forces.

Part H

Let us recall that we have denoted the force exerted by block 1 on block 2 by F_12, and the force exerted by block 2 on block 1 by F_21. If we suppose that m_1 is greater than m_2, which of the following statements about forces is true?

Hint 1. Which of Newton's laws is useful here?

Newton's third law!

ANSWER:

- ○ |F_{12}|>|F_{21}|
- ○ |F_{21}|>|F_{12}|
- ◉ Both forces have equal magnitudes.

Correct

Newton's third law states that forces F_12_vec and F_21_vec are equal in magnitude and opposite in direction. Therefore, their *x* components are related by F_{12}=-F_{21}

Part I

Now recall the expression for the time derivative of the *x* component of the system's total momentum: dp_x(t)/dt=F_x. Considering the information that you now have, choose the best alternative for an equivalent expression to dp_x(t)/dt.

Hint 1. What is F_x?

F_x = F_{12} + F_{21},

the total (internal) force on the *system* (as a whole). Use the information from the last part to simplify the right-hand side of the above equation.

ANSWER:

- 0
- nonzero constant
- kt
- kt^2

Correct

The derivative of the total momentum is zero; hence the total momentum is a constant function of time. We have just shown that for the special case of a closed two-block system, the internal forces do not change the total momentum of the system. It can be shown that in any system, the internal forces do not change the total momentum: It is conserved. In other words, total momentum is always conserved in a closed system of objects.

Impulse on a Baseball

Learning Goal:

To understand the relationship between force, impulse, and momentum.

The effect of a net force \Sigma \vec F acting on an object is related both to the force and to the total time the force acts on the object. The physical quantity *impulse* J_vec is a measure of both these effects. For a constant net force, the impulse is given by

\vec {J} = \vec {F} \Delta t·

The impulse is a vector pointing in the same direction as the force vector. The units of J_vec are \rm N\cdot s or \rm kg \cdot m/s·

Recall that when a net force acts on an object, the object will accelerate, causing a change in its velocity. Hence the object's momentum (\vec p = m\vec v) will also change. The impulse-momentum theorem describes the effect that an impulse has on an object's motion:

\Delta \vec{p} = \vec{J} = \vec{F} \Delta t·

So the change in momentum of an object equals the net impulse, that is, the net force multiplied by the time over which the force acts. A given change in momentum can result from a large force over a short time or a smaller force over a longer time.

In Parts A, B, C consider the following situation. In a baseball game the batter swings and gets a good solid hit. His swing applies a force of 12,000 N to the ball for a time of 0.70 \times 10^{-3}\; \rm s·

Part A

Assuming that this force is constant, what is the magnitude J of the impulse on the ball?

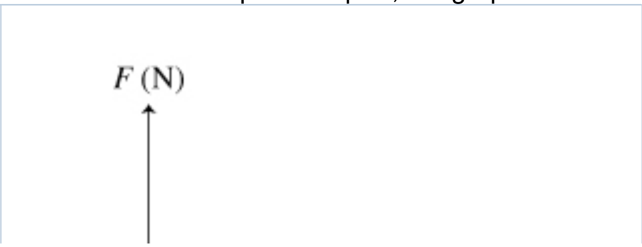
Enter your answer numerically in newton seconds using two significant figures.

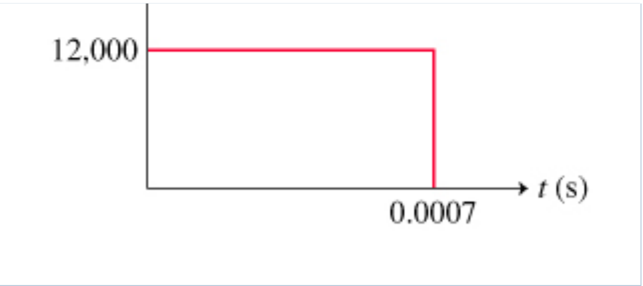
ANSWER:

J = 8.4 {\rm N \cdot s}

Correct

We often visualize the impulse by drawing a graph of force versus time. For a constant net force such as that used in the previous part, the graph will look like the one shown in the figure.





Part B

The net force versus time graph has a rectangular shape. Often in physics geometric properties of graphs have physical meaning.

ANSWER:

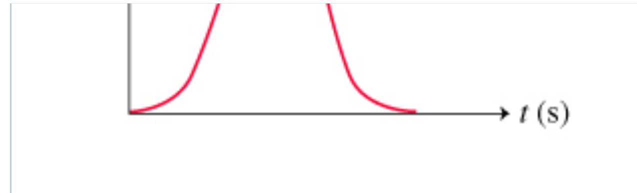
For this graph, the
- ○ length
- ○ height
- ◉ area
- ○ slope

of the rectangle corresponds to the impulse.

Correct

The assumption of a constant net force is idealized to make the problem easier to solve. A real force, especially in a case like the one presented in Parts A and B, where a large force is applied for a short time, is not likely to be constant.

A more realistic graph of the force that the swinging bat applies to the baseball will show the force building up to a maximum value as the bat comes into full contact with the ball. Then as the ball loses contact with the bat, the graph will show the force decaying to zero. It will look like the graph in the figure.



Part C

If both the graph representing the constant net force and the graph representing the variable net force represent the same impulse acting on the baseball, which geometric properties *must* the two graphs have in common?

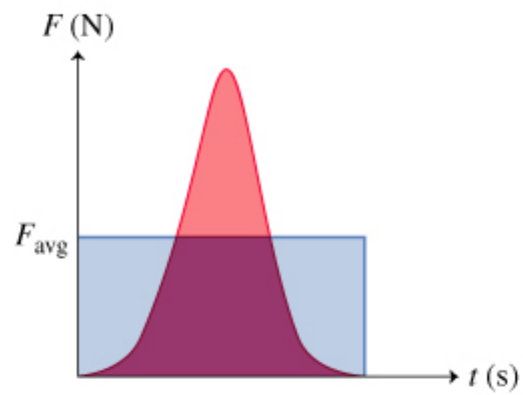
ANSWER:

- maximum force
- area
- slope

Correct

When the net force varies over time, as in the case of the real net force acting on the baseball, you can simplify the problem by finding the average net force F_vec_avg acting on the baseball during time Δt. This average net force is treated as a constant force that acts on the ball for time Δt. The impulse on the ball can then be found as $\vec J = \vec F_{\rm avg} \Delta t$.

Graphically, this method states that the impulse of the baseball can be represented by either the area under the net force versus time curve or the area under the average net force versus time curve. These areas are represented in the figure as the areas shaded in red and blue respectively.



The impulse of an object is also related to its change in momentum. Once the impulse is known, it can be used to find the change in momentum, or if either the initial or final momentum is known, the other momentum can be found. Keep in mind that $\vec J = \Delta \vec p = m(\vec v_{\rm f} - \vec v_{\rm i})$. Because both impulse and momentum are vectors, it is essential to account for the direction of each vector, even in a one-dimensional problem.

Part D

Assume that a pitcher throws a baseball so that it travels in a straight line parallel to the ground. The batter then hits the ball so it goes directly back to the pitcher along the same straight line. Define the direction the pitcher originally throws the ball as the +*x* direction.

ANSWER:

The impulse on the ball caused by the bat will be in the ◯ positive ◉ negative x direction.

Correct

Part E

Now assume that the pitcher in Part D throws a 0.145-kg baseball parallel to the ground with a speed of 32 $\mathrm{m/s}$ in the +x direction. The batter then hits the ball so it goes directly back to the pitcher along the same straight line. What is the ball's *velocity* just after leaving the bat if the bat applies an impulse of -8.4 \;{\rm N \cdot s} to the baseball?

Enter your answer numerically in meters per second using two significant figures.

ANSWER:

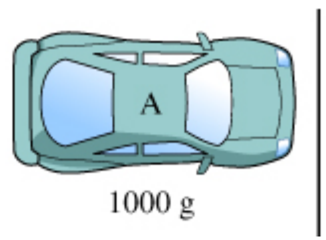
\vec v = -26 $\mathrm{m/s}$

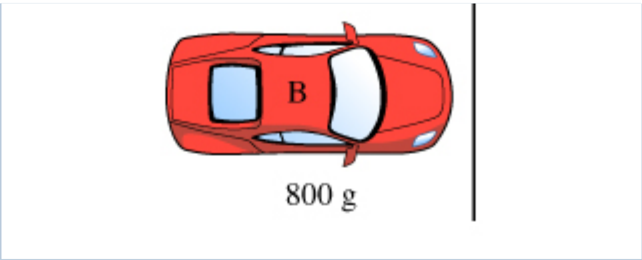
Correct

The negative sign in the answer indicates that after the bat hits the ball, the ball travels in the opposite direction to that defined to be positive.

Kinetic Energy and Momentum Conceptual Question

The two toy cars shown in the figure, with masses as given in the figure, are ready to race. Both cars begin from rest. For each question, state whether the correct answer is car A, car B, or whether the two cars have equal values for the parameter in question.





For the next three parts assume that the cars' motors supply the same force to each car over the course of a 1.0-meter race.

Part A

Which car crosses the finish line 1.0 m away first?

Hint 1. Three ways to analyze motion

Newton's second law, the work-energy relation, and the impulse-momentum relation can all be used to draw conclusions about the forces that act on an object and the object's motion. Which approach to use depends on the information given in the problem statement and the information you are trying to determine. In this case, you are asked a kinematic question, so you should use the technique that will give you a useful kinematic quantity such as acceleration or initial velocity.

ANSWER:

- Car A wins
- Car B wins
- Both cars cross the finish line at the same time.

Correct

Part B

Which car has the larger kinetic energy when it crosses the finish line 1.0 m away?

Hint 1. The work-energy relation

Since both cars start from rest and move in the same direction, the work done on each car is equal to its final kinetic energy. Recall that work is given as the product of applied force (parallel to the direction of motion) and the distance over which the force is applied. Notice you are told about both force and distance in the description of the race.

ANSWER:

- Car A
- Car B
- Both cars have the same kinetic energy.

Correct

Part C

Which car has a larger momentum when it crosses the finish line 1.0 m away?

Hint 1. The impulse-momentum relation

Since both cars start from rest, the impulse applied to each car is equal to its final momentum. Recall that impulse is given as the product of applied force and the time interval over which the force is applied. You are given the equality of the forces in the description of the race. The time is the subject of Part A.

ANSWER:

- ◉ Car A
- ○ Car B
- ○ Both cars have the same momentum.

Correct

In the the next three parts we analyze a different type of race, in which the cars' motors supply the same force to each car over the course of a 10-second race.

Part D

Which car has traveled farther after 10 s?

ANSWER:

- ○ Car A
- ◉ Car B
- ○ Both cars travel the same distance.

Correct

Part E

After 10 s which car has a larger kinetic energy?

Hint 1. The work-energy relation

Since both cars start from rest and move in the same direction, the work done on each car is equal to its final kinetic energy. Recall that work is given as the product of applied force (parallel to the direction of motion) and the distance over which the force is applied. You are given the equality of the forces in the description of the second race. The distance is the subject of Part D.

ANSWER:

- ○ Car A
- ◉ Car B
- ○ Both cars have the same kinetic energy.

Correct

Part F

After 10 s which car has a larger momentum?

Hint 1. The impulse-momentum relation

Since both cars start from rest, the impulse applied to each car is equal to its final momentum. Recall that impulse is given as the product of applied force and the time interval over which the force is applied. Notice you are told about both force and time in the description of the second race.

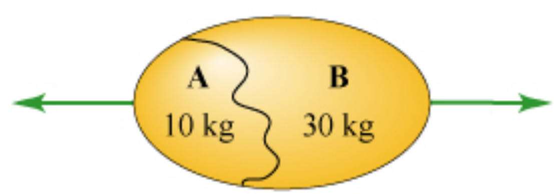
ANSWER:

- Car A
- Car B
- Both cars have the same momentum.

Correct

Momentum in an Explosion

A giant "egg" explodes as part of a fireworks display. The egg is at rest before the explosion, and after the explosion, it breaks into two pieces, with the masses indicated in the diagram, traveling in opposite directions.



Part A

What is the momentum $p_{A,i}$ of piece A before the explosion?

Express your answer numerically in kilogram meters per second.

Hint 1. Initial momentum

The momentum of any object is determined by the product of the object's mass and velocity. The egg is initially at rest. Use this to find the initial velocity.

ANSWER:

$p_{A,i} = 0 \; {\rm kg\cdot m/s}$

Correct

Similarly, piece B has zero momentum before the collision. The total momentum of the "egg," the sum of the two individual momenta, is also zero.

Part B

During the explosion, is the force of piece A on piece B greater than, less than, or equal to the force of piece B on piece A?

Hint 1. Forces in an explosion

The forces specified in this problem must obey Newton's third law, which states that every action has an equal-magnitude and oppositely-directed reaction.

ANSWER:

- greater than
- less than
- equal to
- cannot be determined

Correct

Part C

The momentum of piece B is measured to be 500 \rm kg \cdot m/s after the explosion. Find the momentum p_A,f of piece A after the explosion.

Enter your answer numerically in kilogram meters per second.

Hint 1. Conservation of momentum

The law of conservation of momentum states that the total momentum in an isolated system of objects must remain constant, regardless of the interactions (or collisions) between the objects. Thus, the total momentum of the two pieces of the egg after the explosion must be equal to the total momentum of the two pieces of the egg before the explosion.

ANSWER:

p_A,f = -500 {\rm kg\cdot m/s}

Correct

Score Summary:

Your score on this assignment is 92.3%.
You received 18.45 out of a possible total of 20 points.