11/12
ECE 375
M. [illegible]

Problem Set #5 Solutions

[1] a) Not feasible - not symmetric about real line, & incorrect placement of locus on real line.

b) Not feasible - locus must be to the left of an odd # of poles + zeros.

c) Not feasible - locus must be to the left of an odd # of poles + zeros.

d) Feasible.

[2] $G(s) = \dfrac{1}{(s+4)(s^2+2s+2)}$ $\Rightarrow$ $\Delta(s) = (s+4)(s^2+2s+2) + K\cdot 1$

$= s^3 + 2s^2 + 2s + 4s^2 + 8s + 8 + k$

$= s^3 + 6s^2 + 10s + (8+k)$

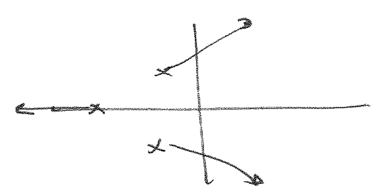
a)

Hurwitz criteria:

$a_1 a_2 - a_3 > 0$

$6\cdot 10 - (8+k) > 0$

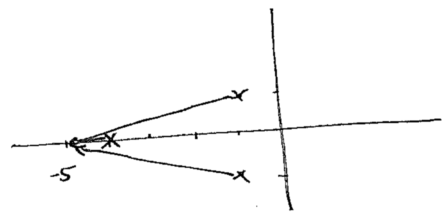
$52 > K$



$\therefore$ $\underline{K = 52}$ for pair of poles to be on imaginary axis & third pole in RHP.

b) For $s=-5$ to be on the locus, it must satisfy the magnitude condition $K = \left|\dfrac{1}{G(s)}\right| = |(s+4)(s^2+2s+2)|$

This is equivalent to the product of the magnitude of vectors drawn from each pole to $s = -5$:



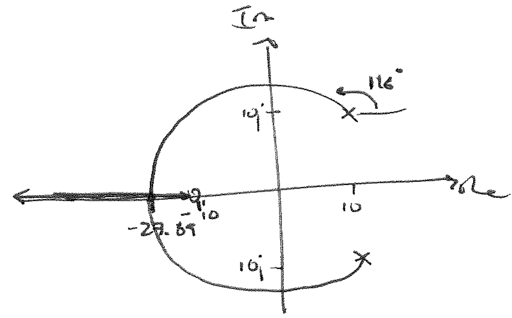
$$\therefore \quad K = |-5+4| \cdot |-5+1+j| \cdot |-5+1-j|$$

$$= |1| \cdot \sqrt{1^2+4^2} \cdot \sqrt{1^2+4^2}$$

$$\underline{K = 17}$$

$\boxed{3}$ $G(s) = \dfrac{s+10}{s^2-20s+200}$ $\Rightarrow$ $m=1$ zero at $s=-10$

$n=2$ poles at $s=+10\pm 10j$



(a) · $n-m=1$ asymptote at $180°$.

· break-in point where

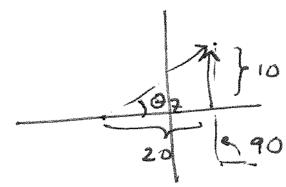
$$0 = \frac{\partial}{\partial s}\left(\frac{1}{G(s)}\right) = \frac{\partial}{\partial s}\left(\frac{s^2-20s+200}{s+10}\right)$$

$$= \frac{(s+10)(2s-20)-(s^2-20s+200)\cdot 1}{(s+10)^2}$$

$$= 2s^2-0\cdot s-200-s^2+20s-200$$

$$= s^2+20s-220 \Rightarrow s = \frac{-20\pm\sqrt{20^2+4\cdot 220}}{2} = -10\pm\sqrt{320} = \underline{\underline{-27.89}},\ +7.89$$

· departure angles



$$\sum \theta_{z_k} - \sum \theta_{p_k} = 180°$$

$$\theta_z - (90° + \theta_{p_1}) = 180°$$

$$\tan^{-1}(1/2) - 90° - 180° = \theta_{p_1}$$

$$-243.4° \simeq \theta_p \quad \Rightarrow \underline{\theta_{p_1} \simeq 116°} \text{ and } \underline{\theta_{p_2} \simeq -116°}.$$

(b) $\Delta(s) = s^2-20s+200+K(s+10)$

$$= s^2 + s\underbrace{(K-20)}_{>0 \text{ for } K>20} + \underbrace{(200+10K)}_{>0 \text{ for any } K>0.}$$

$\Rightarrow$ For stability, $K>20$.

(c) $(s+a)^2 = s^2 + 2as + a^2 = s^2 + (k-20)s + (200+10k)$

for $2a = k-20 \Rightarrow a = \frac{k}{2} - 10$

$a^2 = 200 + 10k$

$\left(\frac{k}{2} - 10\right)^2 = 200 + 10k$

$\frac{k^2}{4} - 10k + 100 = 200 + 10k$

$\frac{k^2}{4} - 20k - 100 = 0 \Rightarrow k = \frac{20 \pm \sqrt{400 + 100}}{2/4} = 40 \pm 20\sqrt{5} \simeq 84.72, 4.72$

$\therefore$ critical damping for $\underline{\underline{k = 84.72}}$

(d) $s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + (k-20)s + (200+10k)$

for $2\zeta\omega_n = k - 20$, $\quad \zeta = 1/\sqrt{2} \Rightarrow k = 20 + \omega_n\sqrt{2}$

$\frac{k-20}{\sqrt{2}} = \omega_n$

$\omega_n^2 = 200 + 10 \cdot k$

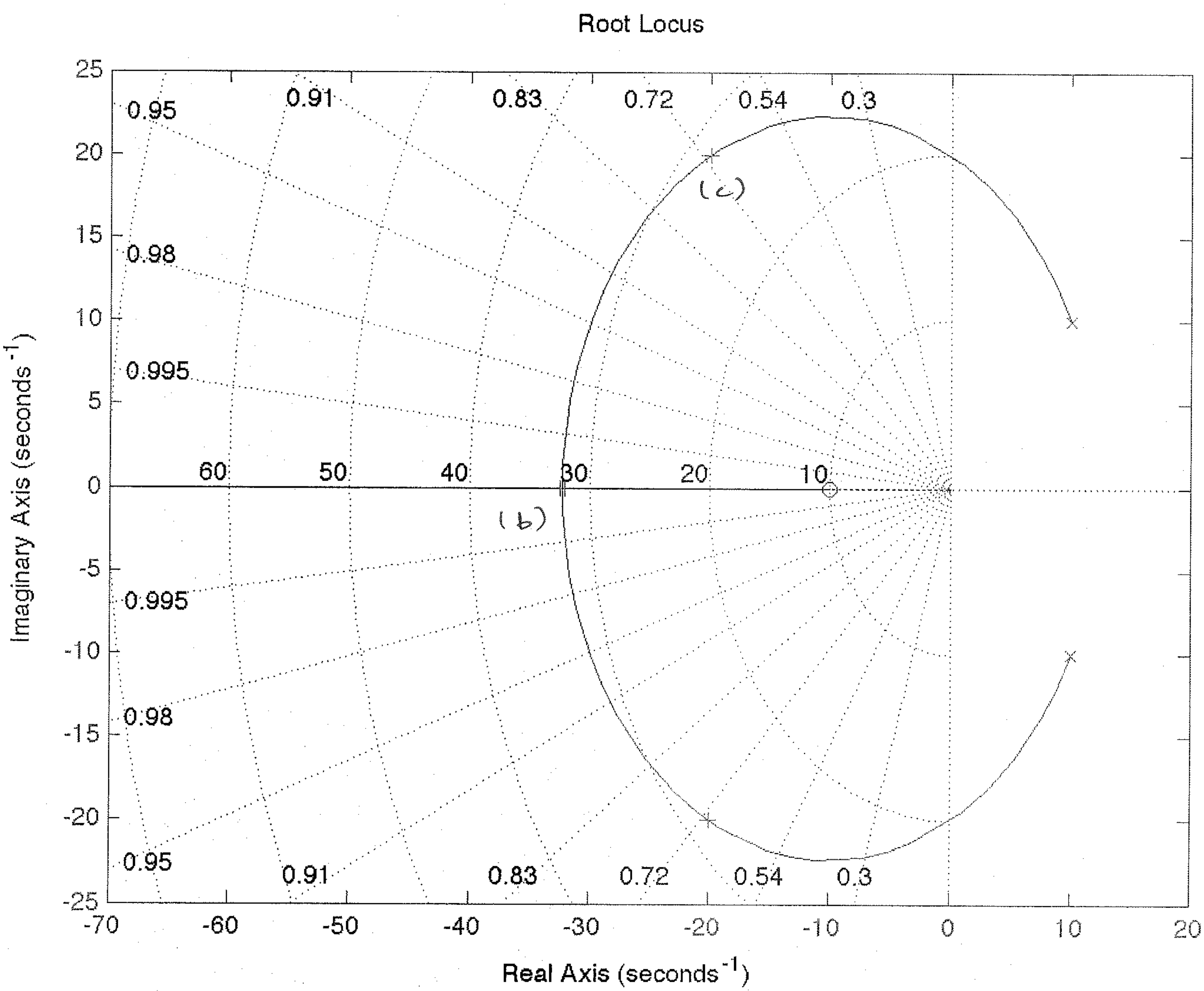
$\left(\frac{k-20}{\sqrt{2}}\right)^2 = 200 + 10k$

$k^2 - 40k + 400 = 400 + 20k$

$k^2 - 60k = 0 \Rightarrow k = 0, 60.$

$\therefore \zeta = 1/\sqrt{2}$ for $\underline{\underline{k = 60}}$.

$\boxed{14}$ See Matlab code & r locus plots, next pages.



Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
25
20
15
10
5
0
-5
-10
-15
-20
-25
-70
-60
-50
-40
-30
-20
-10
0
10
20
0.95
0.91
0.83
0.72
0.54
0.3
0.98
0.995
60
50
40
30
20
10
(c)
(b)

(d)

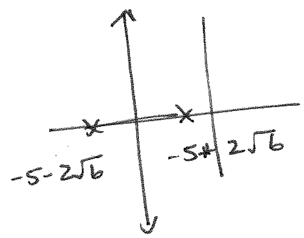
```
sys = tf([1 10],[1 -20 200]);
rlocus(sys);                          ← (a)
rlocfind(sys)
Select a point in the graphics window

selected_point =

 -32.1445 + 0.0776i


ans =                                 ← (b)

   84.7232

grid on
zoom on
rlocfind(sys)
Select a point in the graphics window

selected_point =

 -19.9962 +19.9840i


ans =
                                      ← (c)
   60.0067

zoom out
diary off
```

15

BONUS

$$G_D(s) = \frac{-\frac{1}{2}s+1}{\frac{1}{2}s+1} \quad . \quad G(s) = \frac{-s+2}{(s+2)(s^2+10s+2)} = \frac{-(s-2)}{(s+2)(s^2+10s+2)}$$

(a) $\Delta(s) = s^2+10s+2 = 0$ for $s = \frac{-10 \pm \sqrt{100-4}}{2} = -5 \pm 2\sqrt{6}$



breakaway point at $s = -5$

asymptotes at $\pm 90°$

(b) Delay-free system is stable for all $K > 0$.

(c) Via rlocfind, $K \lesssim 17.86$ results in a stable closed-loop system.

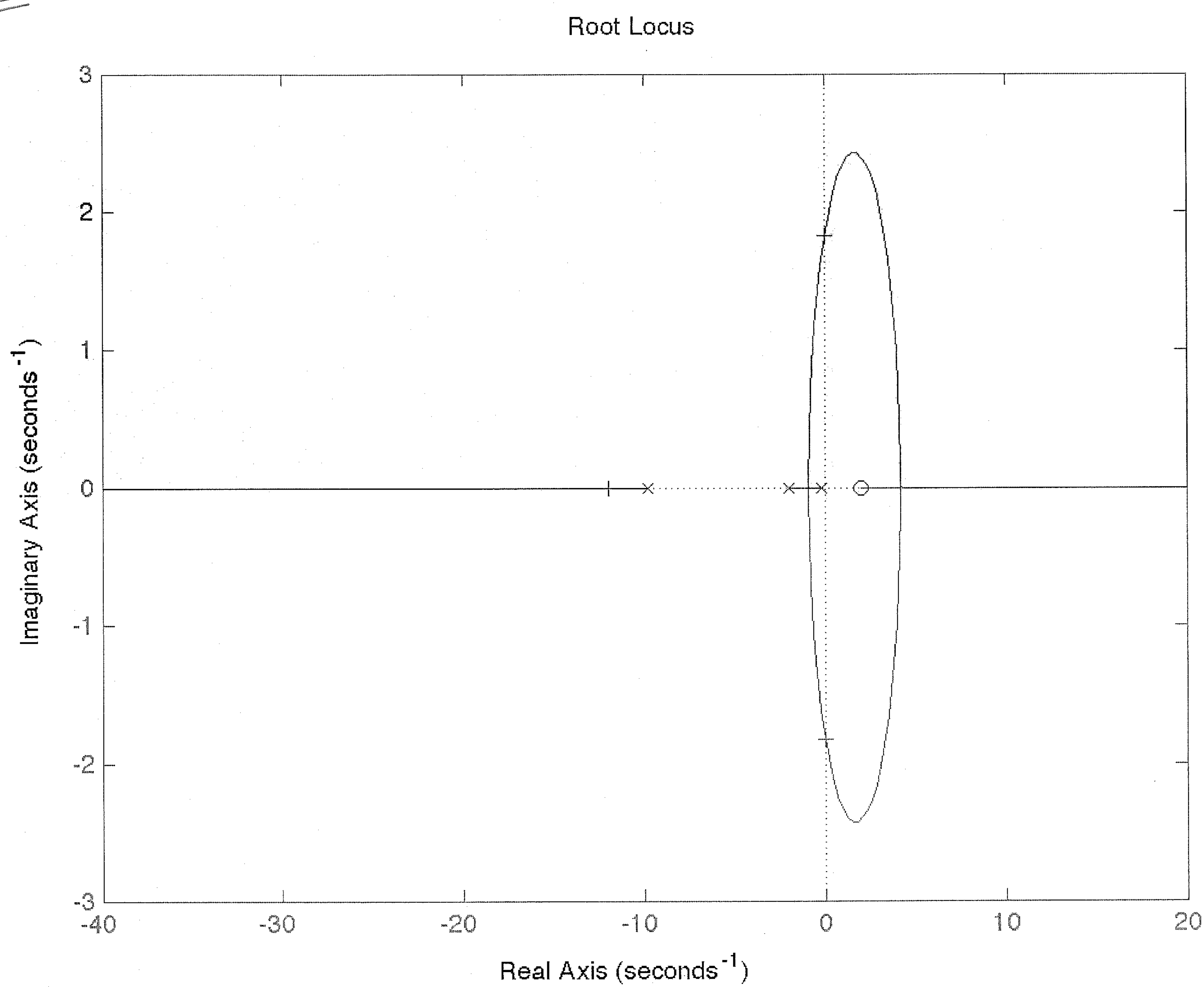
(d) The locus does not obey standard plotting rules because they are based on $K > 0$. With $K < 0$ ($\equiv$ negative sign in front of $G(s)$), the phase condition is

$$\measuredangle G(s) = \measuredangle -1/K > 0$$

$$\Rightarrow \measuredangle G(s) = 0°, \text{ not } 180°.$$

Hence the locus is to the right of an odd # of poles & zeros. Departure/arrival angle rules are also different.

BONUS


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
3
2
1
0
-1
-2
-3
-40
-30
-20
-10
0
10
20