Scalar Product:

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta_{AB} = A_x B_x + A_y B_y + A_z B_z$$

Vector Product:

$$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta_{AB}, \quad \vec{A}\times\vec{B} = \left(A_y B_z - A_z B_y\right)\hat{i} + \left(A_z B_x - A_x B_z\right)\hat{j} + \left(A_x B_y - A_y B_x\right)\hat{k}$$

Equations of motion:

$$v_{fx} = v_{ix} + a_x t$$

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$

$$v_{fx}^{\ 2} = v_{ix}^{\ 2} + 2a_x(x_f - x_i)$$

Radial Acceleration:

$$a_{rad} = \frac{v^2}{r}$$

Newton’s second law

$$\sum \vec{F} = m\vec{a}$$

Magnitude of kinetic friction

$$F_{f_k} = \mu_k F_N$$

Magnitude of static friction

$$F_{f_s} \le \mu_s F_N$$

Definition of work

$$W = \int \vec{F}\cdot d\vec{s}$$

Definition of kinetic energy:

$$KE = \frac{1}{2}mv^2$$

Change in gravitational potential energy:

$$\Delta U_g = mg\Delta y$$

Force of a compressed spring:

$$F_s = -kx$$

Elastic potential energy:

$$U_{el} = \frac{1}{2}kx^2$$

Work-Energy Theorem:

$$W_{net,ext} = \Delta U + \Delta KE$$

Center-of-mass position

$$X_{COM} = \frac{1}{M}\sum_{i=1}^{n} x_i m_i$$

Definition of momentum

$$\vec{p} = m\vec{v}$$

Conservation of momentum

$$\vec{p}_i = \vec{p}_f$$

Definition of torque

$$\vec{\tau} = \vec{r}\times\vec{F}$$

Newton’s second law for rotation

$\sum \vec{\tau} = I\vec{\alpha}$

Rolling: $\begin{aligned} a_{COM} &= \alpha R \\ v_{COM} &= \omega R \end{aligned}$

$$\vec{L} = \vec{r} \times \vec{p}$$

Angular momentum: $\vec{L} = I\vec{\omega}$

$$I = \sum_i m_i r_i^2$$

Newton's Law of Gravitation:

$$F_G = \frac{Gm_1m_2}{r^2}$$

Equation for Simple Harmonic Motion:

$$\frac{d^2x}{dt^2} = \omega^2 x$$

Solution for above equation:

$$x(t) = A\cos(\omega t + \varphi)$$

Where,

$$\omega = 2\pi f = \frac{2\pi}{T}$$

For a spring mass oscillator,

$$\omega = \sqrt{\frac{k}{m}}$$

For a simple pendulum,

$$\omega = \sqrt{\frac{g}{L}}$$

Pressure:

$$p = \frac{F}{A}$$

Bernoulli's Equation:

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Wave Equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v}\frac{\partial^2 y(x,t)}{\partial t^2}$$

Solution to above equation:

$$y(x,t) = A\cos(kx - \omega t)$$

Where,

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \quad v = \lambda f$$

Standing waves on fixed string:

$$y(x,t) = A_{SW}\sin(kx)\sin(\omega t), \quad f_n = n\frac{v}{2L}$$

Doppler Effect:

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

Physics 160-01 Exam #2

Name:____________________Box #_________

1) A car is going around an un-banked curve of radius 500m when it starts skidding off the road. The policeman at the scene had taken a physics class and knew that the coefficient of static friction between the car and the road was 0.75. What was the speed that the car when it started skidding?

A) 3.68 km/hr
B) 60.6 km/hr
C) 218 km/hr
D) 285 km/hr
E) 195 km/hr

The car is in circular motion, so its acceleration is v^2/r directed towards the center of the arc. The force creating that acceleration is the frictional force between the car and the road, F_f, which can have a maximum value of $\mu_s F_N = \mu_s mg$. So,

$$\Sigma \vec{F} = m\vec{a}$$

$$\mu_s mg = m\frac{v^2}{r} \Rightarrow$$

$$v^2 = \mu_s rg$$

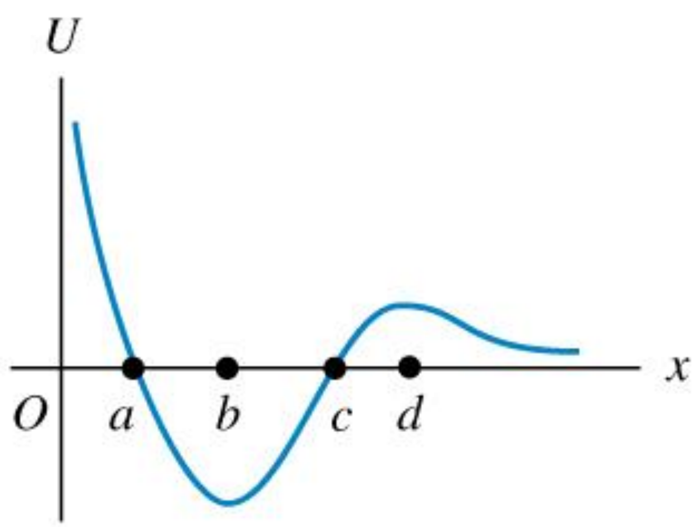
Solving for the velocity gives v = 218 km/hr.

2) In electrodynamics, a magnetic field produces a force on a moving charged particle that is always perpendicular to the direction the particle is moving. How does this force affect the kinetic energy of the particle?

A) Increases it.
B) Decreases it.
C) Doesn't change it.
D) It depends on the amount of time it acts.
E) No way to tell.

Since the force is ALWAYS perpendicular to the direction of motion, it cannot do any work since W = Fdcosθ. Since no work, no transfer of energy, and hence no change in kinetic energy.

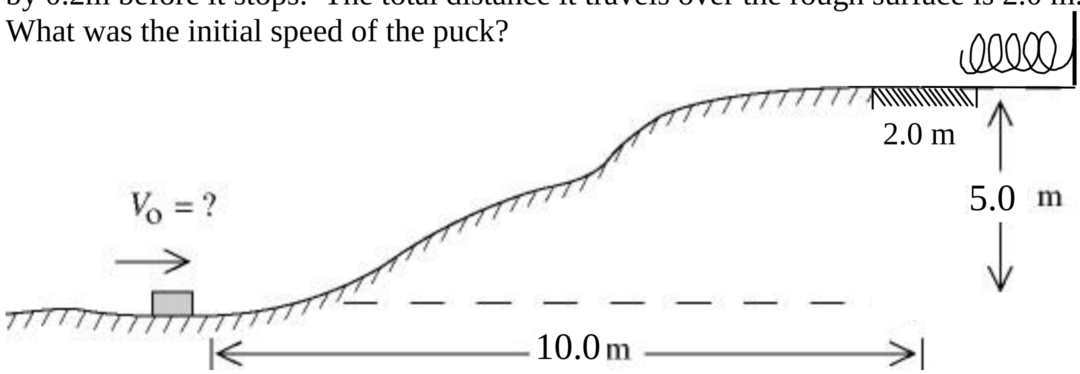
3) A marble moves along the x-axis. The potential-energy function is shown below. At which of the labeled x-coordinates is the force on the marble zero?



A) b
B) b and d
C) a and c
D) d
E) No way to tell.

The force is related to the potential by $F_x = -\frac{\partial U}{\partial x}$. The derivative of the potential function is zero at points b and d, so that's where the force is also zero.

4) A small hockey puck of mass 1 kg slides without friction over the icy hill shown below. At the top of the hill, it encounters a rough horizontal surface (coefficient of kinetic friction = 0.4) and hits a spring with spring constant k = 400N/m, compressing it by 0.2m before it stops. The total distance it travels over the rough surface is 2.0 m. What was the initial speed of the puck?



A) 9.5 m/s
B) 9.8 m/s
C) 10.0 m/s
D) 10.5 m/s
E) 11.4 m/s

From conservation of energy, the net change in mechanical energy is the work done by friction, so

$$\Delta E_{mech} = W_{f_k} = \Delta K + \Delta U_g + \Delta U_{el}$$

$$-F_{f_k} d = \left(1/2\, mv_f^2 - 1/2\, mv_0^2\right) + \left(mgy_f - mgy_0\right) + \left(1/2\, kx_f^2 - 1/2\, kx_0^2\right)$$

$$-\mu_k F_N d = \left(0 - 1/2\, mv_0^2\right) + \left(mgh - 0\right) + \left(1/2\, kx_f^2 - 0\right) \Rightarrow$$

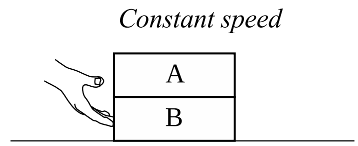
$$-\mu_k mgd = -1/2\, mv_0^2 + mgh + 1/2\, kx_f^2 \Rightarrow$$

$$v_0^2 = 2\mu_k gd + 2gh + \frac{k}{m}x_f^2 = 2(0.4)\left(9.8\frac{m}{s^2}\right)(2m) + 2\left(9.8\frac{m}{s^2}\right)(5m) + \frac{400\frac{N}{m}}{1kg}(0.2m)^2$$

$$v_0^2 = 15.7\frac{m^2}{s^2} + 98\frac{m^2}{s^2} + 16\frac{m^2}{s^2} \Rightarrow$$

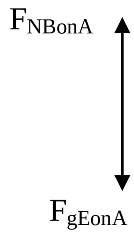
$$v_0 = 11.4\frac{m}{s}$$

5) **WRITTEN** Two identical blocks of mass *m* are stacked as shown at right. A hand exerts a constant force to the right on block B. The blocks move to the right with constant speed, and block A does not move relative to block B.

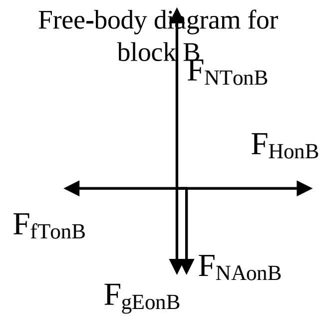


In the spaces provided, draw separate free-body diagrams for blocks A and B. Clearly label each of the forces in your diagrams, identifying the type of force, the object on which the force is exerted, and the object exerting the force.

Free-body diagram for block A



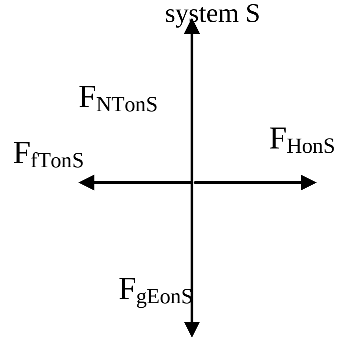
Free-body diagram for block B



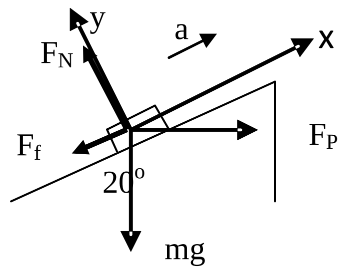
Consider system S, which consists of both blocks together.

In the space provided, draw and label a free-body diagram for system S.

Free-body diagram for system S



6) **WRITTEN** In the figure below, a 100kg box is pushed up a 20^o ramp by a horizontal force F_P. The coefficient of kinetic friction between the ramp and the box is 0.3. If the box is accelerating up the ramp with a = $1 m/s^2$, what is the magnitude of the force, F_P?



$$\sum F_x = F_P \cos 20^0 - mg \sin 20^0 - \mu_k F_N = ma_x$$

$$\sum F_y = F_N - mg \cos 20^0 - F_P \sin 20^0 = 0$$

$$F_N = mg \cos 20^0 + F_P \sin 20^0 \Rightarrow$$

$$F_P \cos 20^0 - mg \sin 20^0 - \mu_k \left(mg \cos 20^0 + F_P \sin 20^0 \right) = ma_x$$

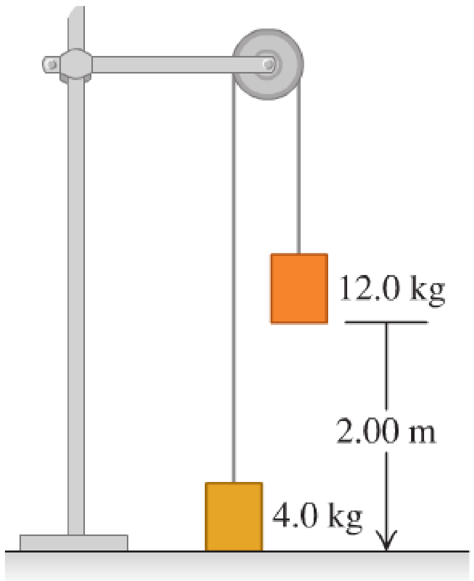
$$F_P \left[\cos 20^0 - \mu_k \sin 20^0 \right] = ma_x + mg \sin 20^0 + \mu_k mg \cos 20^0 \Rightarrow$$

$$F_P = \frac{ma_x + mg \sin 20^0 + \mu_k mg \cos 20^0}{\cos 20^0 - \mu_k \sin 20^0}$$

$$F_P = \frac{100kg \cdot 1 m/s^2 + 100kg \cdot 9.8 m/s^2 \cdot \sin 20^0 + 0.3 \cdot 100kg \cdot 9.8 m/s^2 \cdot \cos 20^0}{\cos 20^0 - 0.3 \cdot \sin 20^0}$$

$$F_P = 850N$$

7) **WRITTEN** A system of two paint buckets connected by a lightweight rope is released from rest with the 12.0 kg bucket 2.00 m above the floor. During the time that the 12.0 kg bucket drops to the floor, friction in the pulley removes 10.0 J of energy from the system. What is the speed of the bucket when it hits the floor?



From conservation of energy, the net change in mechanical energy is the work done by friction, so

$$\Delta E_{mech} = W_{f_k} = \Delta K + \Delta U_g + \Delta U_{el}$$

$$-10J = \left(1/2\,mv_f^2 - 1/2\,mv_0^2\right)^{4kg} + \left(1/2\,mv_f^2 - 1/2\,mv_0^2\right)^{12kg} + \left(mgy_f - mgy_0\right)^{4kg} + \left(mgy_f - mgy_0\right)^{12kg}$$

$$-10J = \left(1/2\,mv^2\right)^{4kg} + \left(1/2\,mv^2\right)^{12kg} + \left(mgy - 0\right)^{4kg} + \left(0 - mgy\right)^{12kg} \Rightarrow$$

$$-10J = 1/2\left(12kg + 4kg\right)v^2 + \left(4kg\right)g\left(2m\right) - \left(12kg\right)g\left(2m\right) \Rightarrow$$

$$v^2 = \frac{-10J}{8kg} - \left(\frac{8kg \cdot m}{8kg}\right)\left(9.8\frac{m}{s^2}\right) + \left(\frac{24kg \cdot m}{8kg}\right)\left(9.8\frac{m}{s^2}\right)$$

$$v^2 = 18.4\frac{m^2}{s^2} \Rightarrow$$

$$v = 4.28\frac{m}{s}$$