Exercise 27.1

Description: A particle with a charge of q is moving with instantaneous velocity v_vec = (v_x)i_unit + (v_y)j_unit. (a) What is the force exerted on this particle by a magnetic field B_vec = (B)i_unit? Find the x-component. (b) Find the y-component. (c)...

A particle with a charge of -1.10×10^{-8}C is moving with instantaneous velocity $\vec{v} = (4.00\mathrm{m/s})\,\hat{i} + (-3.40\mathrm{m/s})\,\hat{j}$

Part A

What is the force exerted on this particle by a magnetic field $\vec{B} = (1.50\mathrm{T})\,\hat{i}$? Find the x-component.

ANSWER:

$F_x =$ 0 N

Part B

Find the y-component.

ANSWER:

$F_y =$ 0 N

Part C

Find the z-component.

ANSWER:

$F_z = -qBv_y = -5.61\times10^{-8}$ N

Part D

What is the force exerted on this particle by a magnetic field $\vec{B} = (1.50\mathrm{T})\,\hat{k}$? Find the x-component.

ANSWER:

$F_x = qBv_y = 5.61\times10^{-8}$ N

Part E

Find the y-component.

ANSWER:

$F_y = -qBv_x = 6.60\times10^{-8}$ N

Part F

Find the z-component.

ANSWER:

$F_z =$ 0 N

27.11. **IDENTIFY** and **SET UP:** $\Phi_B = \int \vec{B}\cdot d\vec{A}$

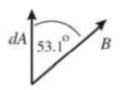
Circular area in the xy-plane, so $A = \pi r^2 = \pi(0.0650\ \mathrm{m})^2 = 0.01327\ \mathrm{m}^2$ and $d\vec{A}$ is in the z-direction. Use Eq. (1.18) to calculate the scalar product.

EXECUTE: **(a)** $\vec{B} = (0.230\ \mathrm{T})\hat{k}$; $\vec{B}$ and $d\vec{A}$ are parallel $(\phi = 0°)$ so $\vec{B}\cdot d\vec{A} = B\,dA$.

B is constant over the circular area so

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\,dA = B\int dA = BA = (0.230\ \mathrm{T})(0.01327\ \mathrm{m}^2) = 3.05\times10^{-3}\ \mathrm{Wb}$$

(b) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11a.



$\vec{B}\cdot d\vec{A} = B\cos\phi\, dA$

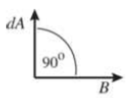
with $\phi = 53.1°$

Figure 27.11a

B and ϕ are constant over the circular area so $\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\cos\phi\, dA = B\cos\phi\int dA = B\cos\phi A$

$\Phi_B = (0.230\ \mathrm{T})\cos 53.1°(0.01327\ \mathrm{m}^2) = 1.83\times10^{-3}$ Wb.

(c) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11b.



$\vec{B}\cdot d\vec{A} = 0$ since $d\vec{A}$ and $\vec{B}$ are perpendicular $(\phi = 90°)$

$\Phi_B = \int \vec{B}\cdot d\vec{A} = 0.$

Figure 27.11b

EVALUATE: Magnetic flux is a measure of how many magnetic field lines pass through the surface. It is maximum when $\vec{B}$ is perpendicular to the plane of the loop (part a) and is zero when $\vec{B}$ is parallel to the plane of the loop (part c).

27.24. **IDENTIFY:** The magnetic force on the beam bends it through a quarter circle.

SET UP: The distance that particles in the beam travel is $s = R\theta$, and the radius of the quarter circle is $R = mv/qB$.

EXECUTE: Solving for R gives $R = s/\theta = s/(\pi/2) = 1.18\text{ cm}/(\pi/2) = 0.751\text{ cm}$. Solving for the magnetic field: $B = mv/qR = (1.67\times10^{-27}\text{ kg})(1200\text{ m/s})/[(1.60\times10^{-19}\text{ C})(0.00751\text{ m})] = 1.67\times10^{-3}\text{ T}$.

EVALUATE: This field is about 10 times stronger than the Earth's magnetic field, but much weaker than many laboratory fields.

27.61. **(a) IDENTIFY and SET UP:** The maximum radius of the orbit determines the maximum speed v of the protons. Use Newton's second law and $a_{\text{rad}} = v^2/R$ for circular motion to relate the variables. The energy of the particle is the kinetic energy $K = \frac{1}{2}mv^2$.

EXECUTE: $\sum\vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$

$v = \dfrac{|q|BR}{m} = \dfrac{(1.60\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{1.67\times10^{-27}\text{ kg}} = 3.257\times10^{7}\text{ m/s}$. The kinetic energy of a proton moving with this speed is $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(3.257\times10^{7}\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.5\text{ MeV}$

(b) The time for one revolution is the period $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(0.40\text{ m})}{3.257\times10^{7}\text{ m/s}} = 7.7\times10^{-8}\text{ s}$.

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2 = \frac{1}{2}\dfrac{|q|^2B^2R^2}{m}$. Or, $B = \dfrac{\sqrt{2Km}}{|q|R}$. B is proportional to $\sqrt{K}$, so if K is increased by a factor of 2 then B must be increased by a factor of $\sqrt{2}$. $B = \sqrt{2}(0.85\text{ T}) = 1.2\text{ T}$.

(d) $v = \dfrac{|q|BR}{m} = \dfrac{(3.20\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{6.65\times10^{-27}\text{ kg}} = 1.636\times10^{7}\text{ m/s}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}(6.65\times10^{-27}\text{ kg})(1.636\times10^{7}\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.5\text{ MeV}$, the same as the maximum energy for protons.

EVALUATE: We can see that the maximum energy must be approximately the same as follows: From part (c), $K = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2$. For alpha particles $|q|$ is larger by a factor of 2 and m is larger by a factor of 4 (approximately). Thus $|q|^2/m$ is unchanged and K is the same.

27.68. **IDENTIFY:** Apply conservation of energy to the acceleration of the ions and Newton's second law to their motion in the magnetic field.

SET UP: The singly ionized ions have $q = +e$. A ^{12}C ion has mass 12 u and a ^{14}C ion has mass 14 u, where $1\text{ u} = 1.66\times10^{-27}\text{ kg}$.

EXECUTE: **(a)** During acceleration of the ions, $qV = \frac{1}{2}mv^2$ and $v = \sqrt{\dfrac{2qV}{m}}$. In the magnetic field, $R = \dfrac{mv}{qB} = \dfrac{m\sqrt{2qV/m}}{qB}$ and $m = \dfrac{qB^2R^2}{2V}$.

(b) $V = \dfrac{qB^2R^2}{2m} = \dfrac{(1.60\times10^{-19}\text{ C})(0.150\text{ T})^2(0.500\text{ m})^2}{2(12)(1.66\times10^{-27}\text{ kg})} = 2.26\times10^{4}\text{ V}$

(c) The ions are separated by the differences in the diameters of their paths. $D = 2R = 2\sqrt{\dfrac{2Vm}{qB^2}}$, so

$$\Delta D = D_{14} - D_{12} = 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{14} - 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{12} = 2\sqrt{\frac{2V(1\text{ u})}{qB^2}}(\sqrt{14}-\sqrt{12}).$$

$\Delta D = 2\sqrt{\dfrac{2(2.26\times10^{4}\text{ V})(1.66\times10^{-27}\text{ kg})}{(1.6\times10^{-19}\text{ C})(0.150\text{ T})^2}}(\sqrt{14}-\sqrt{12}) = 8.01\times10^{-2}\text{ m}$. This is about 8 cm and is easily distinguishable.

EVALUATE: The speed of the ^{12}C ion is $v = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(2.26\times10^{4}\text{V})}{12(1.66\times10^{-27}\text{ kg})}} = 6.0\times10^{5}\text{ m/s}$. This is very fast, but well below the speed of light, so relativistic mechanics is not needed.

27.74. **IDENTIFY:** The force exerted by the magnetic field is $F = ILB\sin\phi$. $a = F/m$ and is constant. Apply a constant acceleration equation to relate v and d.

SET UP: $\phi = 90°$. The direction of $\vec{F}$ is given by the right-hand rule.

EXECUTE: **(a)** $F = ILB$, to the right.

(b) $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v^2 = 2ad$ and $d = \dfrac{v^2}{2a} = \dfrac{v^2 m}{2ILB}$.

(c) $d = \dfrac{(1.12\times10^4 \text{ m/s})^2(25 \text{ kg})}{2(2000 \text{ A})(0.50 \text{ m})(0.80 \text{ T})} = 1.96\times10^6 \text{ m} = 1960 \text{ km}$.

EVALUATE: $a = \dfrac{ILB}{m} = \dfrac{(2.0\times10^3 \text{ A})(0.50 \text{ m})(0.80 \text{ T})}{25 \text{ kg}} = 32 \text{ m/s}^2$. The acceleration due to gravity is not negligible. Since the bar would have to travel nearly 2000 km, this would not be a very effective launch mechanism using the numbers given.

27.78. **IDENTIFY:** The torque exerted by the magnetic field is $\vec{\tau} = \vec{\mu}\times\vec{B}$. The torque required to hold the loop in place is $-\vec{\tau}$.

SET UP: $\mu = IA$. $\vec{\mu}$ is normal to the plane of the loop, with a direction given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. $\tau = IAB\sin\phi$, where ϕ is the angle between the normal to the loop and the direction of $\vec{B}$.

EXECUTE: **(a)** $\tau = IAB \sin 60° = (15.0 \text{ A})(0.060 \text{ m})(0.080 \text{ m})(0.48 \text{ T})\sin 60° = 0.030 \text{ N}\cdot\text{m}$, in the $-\hat{j}$ direction. To keep the loop in place, you must provide a torque in the $+\hat{j}$ direction.

(b) $\tau = IAB \sin 30° = (15.0 \text{ A})(0.60 \text{ m})(0.080 \text{ m})(0.48 \text{ T})\sin 30° = 0.017 \text{ N}\cdot\text{m}$, in the $+\hat{j}$ direction. You must provide a torque in the $-\hat{j}$ direction to keep the loop in place.

EVALUATE: **(c)** If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

27.84. **IDENTIFY:** $I = \dfrac{\Delta q}{\Delta t}$ and $\mu = IA$.

SET UP: The direction of $\vec{\mu}$ is given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. I is in the direction of flow of positive charge and opposite to the direction of flow of negative charge.

EXECUTE: **(a)** $I_u = \dfrac{dq}{dt} = \dfrac{\Delta q}{\Delta t} = \dfrac{q_u v}{2\pi r} = \dfrac{ev}{3\pi r}$.

(b) $\mu_u = I_u A = \dfrac{ev}{3\pi r}\pi r^2 = \dfrac{evr}{3}$.

(c) Since there are two down quarks, each of half the charge of the up quark, $\mu_d = \mu_u = \dfrac{evr}{3}$. Therefore, $\mu_{\text{total}} = \dfrac{2evr}{3}$.

(d) $v = \dfrac{3\mu}{2er} = \dfrac{3(9.66\times10^{-27} \text{ A}\cdot\text{m}^2)}{2(1.60\times10^{-19} \text{ C})(1.20\times10^{-15} \text{m})} = 7.55\times10^7 \text{ m/s}$.

EVALUATE: The speed calculated in part (d) is 25% of the speed of light.

27.57. **IDENTIFY:** In part (a), apply conservation of energy to the motion of the two nuclei. In part (b) apply $|q|vB = mv^2/R$.

SET UP: In part (a), let point 1 be when the two nuclei are far apart and let point 2 be when they are at their closest separation.

EXECUTE: **(a)** $K_1 + U_1 = K_2 + U_2$. $U_1 = K_2 = 0$, so $K_1 = U_2$. There are two nuclei having equal kinetic energy, so $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = ke^2/r$. Solving for v gives

$$v = e\sqrt{\frac{k}{mr}} = (1.602\times10^{-19} \text{ C})\sqrt{\frac{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}{(3.34\times10^{-27} \text{ kg})(1.0\times10^{-15} \text{ m})}} = 8.3\times10^6 \text{ m/s}.$$

(b) $\sum\vec{F} = m\vec{a}$ gives $qvB = mv^2/r$. $B = \dfrac{mv}{qr} = \dfrac{(3.34\times10^{-27} \text{ kg})(8.3\times10^6 \text{ m/s})}{(1.602\times10^{-19} \text{ C})(1.25 \text{ m})} = 0.14 \text{ T}$.

EVALUATE: The speed calculated in part (a) is large, nearly 3% of the speed of light.