# Homework 4 Solution

**Sanjay Krishna: University of New Mexico**

**HW 4 Solution, ECE 322L**

Solution 4.25

(a)

$$I_{DQ} = K_p\left(V_{SGQ} + V_{TP}\right)^2$$

$$0.25 = 0.8\left(V_{SGQ} - 0.5\right)^2$$

$$V_{SGQ} = 1.059 \text{ V}$$

$$R_S = \frac{3 - 1.059}{0.25} \Rightarrow \underline{R_S = 7.76 \text{ K}}$$

$$V_D = V_S - V_{SDQ} = 1.059 - 1.5 = -0.441 \text{ V}$$

$$R_D = \frac{-0.441 - (-3)}{0.25} \Rightarrow \underline{R_D = 10.2 \text{ K}}$$

(b)

$$A_v = -g_m\left(R_D \| R_L\right)$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.8)(0.25)} = 0.8944 \text{ mA/V}$$

$$A_v = -(0.8944)(10.2 \| 2)$$

$$A_v = -1.50$$

(c)

$$\Delta V_O = \Delta I\left(R_D \| R_L\right) = 0.25(10.2 \| 2) = 0.418$$

So $\underline{\Delta V_O = 0.836 \text{ peak-to-peak}}$

Solution 4.31

$$A_v = \frac{g_m\left(R_L \| r_o\right)}{1 + g_m\left(R_L \| r_o\right)}$$

$$0.98 = \frac{g_m r_o}{1 + g_m r_o} \Rightarrow g_m r_o = 49$$

$$\text{Also } 0.49 = \frac{g_m\left(R_L \| r_o\right)}{1 + g_m\left(R_L \| r_o\right)} = \frac{g_m\left(\frac{R_L r_o}{R_L + r_o}\right)}{1 + g_m\left(\frac{R_L r_o}{R_L + r_o}\right)}$$

$$0.49 = \frac{g_m\left(R_L r_o\right)}{R_L + r_o + g_m\left(R_L r_o\right)}$$

$$0.49 = \frac{(49)(1)}{1 + r_o + (49)(1)} = \frac{49}{50 + r_o}$$

$$\underline{r_o = 50 \text{ K}}$$

$$\underline{g_m = 0.98 \text{ mA/V}}$$

Solution 4.37

(a) (i) $K_n = \frac{k_n'}{2} \cdot \frac{W}{L} = \left(\frac{0.1}{2}\right)(20) = 1\,\text{mA/V}^2$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(5)} = 4.472\ \text{mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(5)} = 10\,\text{k}\Omega$$

$$r_o \| R_L = 10 \| 4 = 2.857\ \text{k}\Omega$$

$$A_v = \frac{g_m (r_o \| R_L)}{1 + g_m (r_o \| R_L)} = \frac{(4.472)(2.857)}{1 + (4.472)(2.857)} = 0.927$$

(ii) $R_o = \frac{1}{g_m} \| r_o = \frac{1}{4.472} \| 10$

$$R_o = 219\,\Omega$$

(b) (i) $g_m = 2\sqrt{(1)(2)} = 2.828\,\text{mA/V}$

$$r_o = \frac{1}{(0.02)(2)} = 25\,\text{k}\Omega$$

$$r_o \| R_L = 25 \| 4 = 3.448\ \text{k}\Omega$$

$$A_v = \frac{(2.828)(3.448)}{1 + (2.828)(3.448)} = 0.907$$

(ii) $R_o = \frac{1}{g_m} \| r_o = \frac{1}{2.828} \| 25$

$$R_o = 349\,\Omega$$

Solution 4.49

a.

$$I_{DQ} = K_p \left(V_{SG} + V_{TP}\right)^2$$

$$0.75 = \left(0.5\right)\left(V_{SG} - 1\right)^2 \Rightarrow V_{SG} = 2.225 \text{ V}$$

$$5 = I_{DQ} R_S + V_{SG} \Rightarrow R_S = \frac{5 - 2.225}{0.75} \Rightarrow \underline{R_S = 3.70 \text{ k}\Omega}$$

$$V_{SDQ} = 10 - I_{DQ}\left(R_S + R_D\right)$$

$$6 = 10 - \left(0.75\right)\left(3.70 + R_D\right) \Rightarrow \underline{R_D = 1.63 \text{ k}\Omega}$$

b.

$$R_i = \frac{1}{g_m}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{\left(0.5\right)\left(0.75\right)} = 1.225 \ mA/V$$

$$R_i = \frac{1}{1.225} \Rightarrow \underline{R_i = 0.816 \text{ k}\Omega}$$

$$R_o = R_D \Rightarrow \underline{R_o = 1.63 \text{ k}\Omega}$$

c.

$$i_0 = \left(\frac{R_D}{R_D + R_L}\right)\left(\frac{R_S}{R_S + \left[1/g_m\right]}\right) \cdot i_i$$

$$i_0 = \left(\frac{1.63}{1.63 + 2}\right)\left(\frac{3.70}{3.70 + 0.816}\right) i_i$$

$$i_0 = 0.368 i_i = \underline{i_0 = 1.84 \sin \omega t \left(\mu A\right)}$$

$$v_0 = i_0 R_L = \left(1.84\right)\left(2\right)\sin \omega t \Rightarrow \underline{v_0 = 3.68 \sin \omega t \left(\text{mV}\right)}$$

Solution 4.57

(a)

$$I_{DQ} = K_L\left(V_{GSL} - V_{TNL}\right)^2 = K_L\left(V_{DSL} - V_{TNL}\right)^2$$

$$I_D = (0.1)(4-1)^2 = 0.9\ mA$$

$$I_{DQ} = K_D\left(V_{GSD} - V_{TND}\right)^2$$

$$0.9 = (1)\left(V_{GSD} - 1\right)^2 \Rightarrow V_{GSD} = 1.95\ \text{V}$$

$$V_{GG} = V_{GSD} + V_{DSL} = 1.95 + 4 \Rightarrow \underline{V_{GG} = 5.95\ \text{V}}$$

( b.)

$$I_{DD} = I_{DL}$$

$$K_D\left(V_{GSD} - V_{TND}\right)^2 = K_L\left(V_{GSL} - V_{TNL}\right)^2$$

$$\sqrt{\frac{K_D}{K_L}}\left(V_{GG} + V_i - V_o - V_{TND}\right) = V_o - V_{TNL}$$

$$V_o\left(1 + \sqrt{\frac{K_D}{K_L}}\right) = \sqrt{\frac{K_D}{K_L}}\left(V_{GG} + V_i - V_{TND}\right) + V_{TNL}$$

$$A_v = \frac{dV_o}{dV_i} = \frac{\sqrt{K_D/K_L}}{1+\sqrt{K_D/K_L}} \Rightarrow \quad \underline{A_v = \frac{1}{1+\sqrt{K_L/K_D}}}$$

(c) From Problem 4.55.

$$R_{LD} = \frac{1}{2K_L\left(V_{DSL} - V_{TNL}\right)}$$

$$= \frac{1}{2(0.1)(4-1)} = 1.67\ k\Omega$$

…

$$g_m = 2\sqrt{K_D I_{DQ}} = 2\sqrt{(1)(0.9)} = 1.90\ mA/V$$

$$A_v = \frac{g_m\left(R_{LD} \parallel R_L\right)}{1 + g_m\left(R_{LD} \parallel R_L\right)} = \frac{(1.90)(1.67 \parallel 4)}{1 + (1.90)(1.67 \parallel 4)} \Rightarrow \underline{A_v = 0.691}$$