Chapter 9: Rotation of Rigid Bodies

Due: 11:00pm on Tuesday, October 30, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Angular Motion with Constant Acceleration

Learning Goal:

To understand the meaning of the variables that appear in the equations for rotational kinematics with constant angular acceleration.

Rotational motion with a constant nonzero acceleration is not uncommon in the world around us. For instance, many machines have spinning parts. When the machine is turned on or off, the spinning parts tend to change the rate of their rotation with virtually constant angular acceleration. Many introductory problems in rotational kinematics involve motion of a particle with constant, nonzero angular acceleration. The kinematic equations for such motion can be written as

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

and

$$\omega(t) = \omega_0 + \alpha t.$$

Here, the symbols are defined as follows:

- $\theta(t)$ is the angular position of the particle at time t.
- θ_0 is the initial angular position of the particle.
- $\omega(t)$ is the angular velocity of the particle at time t.
- ω_0 is the initial angular velocity of the particle.
- α is the angular acceleration of the particle.
- t is the time that has elapsed since the particle was located at its initial position.

In answering the following questions, assume that the angular acceleration is constant and nonzero: $\alpha \neq 0$.

Part A

True or false: The quantity represented by θ is a function of time (i.e., is not constant).

ANSWER:

- (selected) true
- false

Part B

True or false: The quantity represented by θ_0 is a function of time (i.e., is not constant).

ANSWER:

- true
- (selected) false

Keep in mind that θ_0 represents an initial value, not a variable. It refers to the angular position of an object at some initial moment.

Part C

True or false: The quantity represented by ω_0 is a function of time (i.e., is not constant).

ANSWER:

- ○ true
- ◉ false

Part D

True or false: The quantity represented by ω is a function of time (i.e., is not constant).

ANSWER:

- ◉ true
- ○ false

The angular velocity ω always varies with time when the angular acceleration is nonzero.

Part E

Which of the following equations is not an explicit function of time t? Keep in mind that an equation that is an explicit function of time involves t as a variable.

ANSWER:

- ○ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
- ○ $\omega = \omega_0 + \alpha t$
- ◉ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

An equation that is not an explicit function of time is useful when you do not know or do not need the time.

Part F

In the equation $\omega = \omega_0 + \alpha t$, what does the time variable t represent?

Choose the answer that is *always* true. Several of the statements may be true in a particular problem, but only one is always true.

ANSWER:

- ○ the moment in time at which the angular velocity equals ω_0
- ○ the moment in time at which the angular velocity equals ω
- ◉ the time elapsed from when the angular velocity equals ω_0 until the angular velocity equals ω

Consider two particles A and B. The angular position of particle A, with constant angular acceleration, depends on time according to $\theta_A(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$. At time $t = t_1$, particle B, which also undergoes constant angular acceleration, has twice the angular acceleration, half the angular velocity, and the same angular position that particle A had at time $t = 0$.

Part G

Which of the following equations describes the angular position of particle B?

Hint 1. How to approach the problem

The general equation for the rotation of B can be written as

$$\theta_B(t) = \theta_B(t = t_1) + \omega_B(t = t_1) \cdot (t - t_1) + \tfrac{1}{2}\alpha_B(t - t_1)^2,$$

where $\theta_B(t = t_1)$ and $\omega_B(t = t_1)$ are the position and angular speed, respectively, of particle B at time t_1. Note the time factors of $t_B = (t - t_1)$ and not t. This change is because nothing is known about particle B at time $t = 0$. Instead you have information about particle B at time t_1.

Express the quantities on the right-hand side of this equation for $\theta_B(t)$ in terms of particle A's variables and constants of motion.

ANSWER:

- ○ $\theta_B(t) = \theta_0 + 2\omega_0 t + \frac{1}{4}\alpha t^2$
- ○ $\theta_B(t) = \theta_0 + \frac{1}{2}\omega_0 t + \alpha t^2$
- ○ $\theta_B(t) = \theta_0 + 2\omega_0(t - t_1) + \frac{1}{4}\alpha(t - t_1)^2$
- ◉ $\theta_B(t) = \theta_0 + \frac{1}{2}\omega_0(t - t_1) + \alpha(t - t_1)^2$
- ○ $\theta_B(t) = \theta_0 + 2\omega_0(t + t_1) + \frac{1}{4}\alpha(t + t_1)^2$
- ○ $\theta_B(t) = \theta_0 + \frac{1}{2}\omega_0(t + t_1) + \alpha(t + t_1)^2$

Note that particle B has a smaller initial angular velocity but greater angular acceleration. Also, it has been in motion for less time than particle A.

Part H

How long after the time t_1 does the angular velocity of particle B equal that of particle A?

Hint 1. How to approach the problem

Write expressions for the angular velocity of A and B as functions of time, either by comparison of the above equations with the general kinematic equations or by differentiating the above equations. Then equate the 2 expressions and solve for $t - t_1$.

ANSWER:

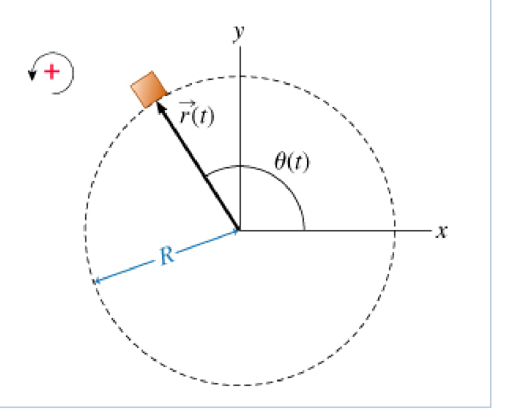
- ○ $\frac{\omega_0}{4\alpha}$
- ○ $\frac{\omega_0 + 4\alpha t_1}{2\alpha}$
- ◉ $\frac{\omega_0 + 2\alpha t_1}{2\alpha}$
- ○ The two particles never have the same angular velocity.

Circular Motion Tutorial

Learning Goal:

Understand how to find the equation of motion of a particle undergoing uniform circular motion.

Consider a particle--the small red block in the figure--that is constrained to move in a circle of radius R. We can specify its position solely by $\theta(t)$, the angle that the vector from the origin to the block makes with our chosen reference axis at time t. Following the standard conventions we measure $\theta(t)$ in the counterclockwise direction from the positive *x* axis.



Part A

What is the position vector $\vec{r}(t)$ as a function of angle $\theta(t)$. For later remember that $\theta(t)$ is itself a function of time.

Give your answer in terms of R, $\theta(t)$, and unit vectors $\hat{i}$ and $\hat{j}$ corresponding to the coordinate system in the figure.

Hint 1. *x* coordinate

What is the *x* coordinate of the particle?

Your answer should be in terms of R and $\theta(t)$.

ANSWER:

x = $R\cos(\theta(t))$

Hint 2. *y* coordinate

What is the *y* coordinate of the particle?

Your answer should be in terms of R and $\theta(t)$.

ANSWER:

y = $R\sin(\theta(t))$

ANSWER:

$\vec{r}(t)$ = $R\cos(\theta(t))\hat{i} + R\sin(\theta(t))\hat{j}$

Uniform Circular Motion

A frequently encountered kind of circular motion is *uniform* circular motion, where $\theta(t)$ changes at a constant rate ω. In other words,

$$\omega = \frac{d\theta(t)}{dt}.$$

Usually, $\theta(t=0)=0$.

Part B

For uniform circular motion, find $\theta(t)$ at an arbitrary time t.

Give your answer in terms of ω and t.

ANSWER:

$\theta(t)$ = ωt

Part C

What does $\vec{r}(t)$ become now?

Express your answer in terms of R, ω, t, and unit vectors $\hat{i}$ and $\hat{j}$.

ANSWER:

$\vec{r}(t)$ = $R\cos(\omega t)\hat{i} + R\sin(\omega t)\hat{j}$

Part D

Find $\vec{r}$, a position vector at time $t = 0$.

Give your answer in terms of R and unit vectors $\hat{i}$ and/or $\hat{j}$.

Hint 1. Finding $\vec{r}$

Simply plug $t = 0$ into your expression for the components of $\vec{r}(t)$.

ANSWER:

$\vec{r}$ = $R\hat{i}$

Part E

Determine an expression for the position vector of a particle that starts on the positive *y* axis at $t = 0$ (i.e., at $t = 0$, $(x_0, y_0) = (0, R)$) and subsequently moves with constant ω.

Express your answer in terms of R, ω, t, and unit vectors $\hat{i}$ and $\hat{j}$.

Hint 1. Adding a phase

You can think of changing the initial position as adding a phase angle ϕ to the equation for $\theta(t)$. That is, $\theta(t) = \omega t + \phi$.

Hint 2. Finding a phase

From previous parts you found that $x = R\ \cos(\theta(t))$ and $y = R\ \sin(\theta(t))$. What should the angle θ be for x and y to be equal to 0 and R respectively?

Express your answer as a fraction of the number π, for example $(3/4)\pi$ or $(1/4)\pi$.

ANSWER:

θ = 1.57

This angle θ is now the phase angle ϕ. You can plug it into the equation for $\vec{r}(t)$ and then use trig identities to simplify your result.

ANSWER:

$r_{\mathrm{yaxis}}(t)$ = $-R\sin(\omega t)\hat{i} + R\cos(\omega t)\hat{j}$

From this excersice you have learned that even though the motion takes place in the plane there is only one degree of freedom, angle θ, and that changing the initial coordinates introduces a phase angle in the equation.

Constrained Rotation and Translation

Learning Goal:

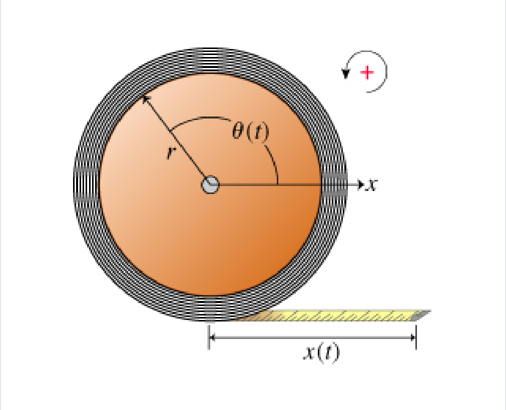
To understand that contact between rolling objects and what they roll against imposes constraints on the change in position(velocity) and angle (angular velocity).

The way in which a body makes contact with the world often imposes a constraint relationship between its possible rotation and translational motion. A ball rolling on a road, a yo-yo unwinding as it falls, and a baseball leaving the pitcher's hand are all examples of constrained rotation and translation. In a similar manner, the rotation of one body and the translation of another may be constrained, as happens when a fireman unrolls a hose from its storage drum.

Situations like these can be modeled by *constraint equations*, relating the coupled angular and linear motions. Although these equations fundamentally involve position (the angle of the wheel at a particular distance down the road), it is usually the relationship of velocities and accelerations that are relevant in solving a problem involving such constraints. The velocities are needed in the conservation equations for momentum and angular momentum, and the accelerations are needed for the dynamical equations.

It is important to use the standard sign conventions: positive for counterclockwise rotation and positive for motion toward the right. Otherwise, your dynamical equations will have to be modified. Unfortunately, a frequent result will be the appearance of negative signs in the constraint equations.

Consider a measuring tape unwinding from a drum of radius r. The center of the drum is not moving; the tape unwinds as its free end is pulled away from the drum. Neglect the thickness of the tape, so that the radius of the drum can be assumed not to change as the tape unwinds. In this case, the standard conventions for the angular velocity ω and for the (translational) velocity v of the end of the tape result in a constraint equation with a positive sign (e.g., if $v > 0$, that is, the tape is unwinding, then $\omega > 0$ also).



Part A

Assume that the function $x(t)$ represents the length of tape that has unwound as a function of time. Find $\theta(t)$, the angle through which the drum will have rotated, as a function of time.

Express your answer (in radians) in terms of $x(t)$ and any other given quantities.

Hint 1. Find the amount of tape that unrolls in one complete revolution of the drum

If the measuring tape unwinds one complete revolution ($\theta = 2\pi$), how much tape, $x_{2\pi}$, will have unwound?

ANSWER:

$x_{2\pi}$ = $2\pi r$

ANSWER:

$\theta(t) = \frac{x(t)}{r}$ radians

Part B

The tape is now wound back into the drum at angular rate $\omega(t)$. With what velocity will the end of the tape move? (Note that our drawing specifies that a positive derivative of $x(t)$ implies motion away from the drum. Be careful with your signs! The fact that the tape is being wound back into the drum implies that $\omega(t) < 0$, and for the end of the tape to move closer to the drum, it must be the case that $v(t) < 0$.

Answer in terms of $\omega(t)$ and other given quantities from the problem introduction.

Hint 1. How to approach the probelm

The function $\omega(t)$ is given by the derivative of $\theta(t)$ with respect to time. Compute this derivative using the expression for $\theta(t)$ found in

Part A and the fact that $\frac{dx(t)}{dt} = v(t)$.

Express your answer in terms of $v(t)$ and r.

ANSWER:

$\omega(t) = \frac{v(t)}{r}$

ANSWER:

$v(t) = \omega(t) r$

Part C

Since r is a positive quanitity, the answer you just obtained implies that $v(t)$ will always have the same sign as $\omega(t)$. If the tape is unwinding, both quanitites will be positive. If the tape is being wound back up, both quantities will be negative. Now find $a(t)$, the linear acceleration of the end of the tape.

Express your answer in terms of $\alpha(t)$, the angular acceleration of the drum: $\alpha(t) = \frac{d\omega(t)}{dt}$.

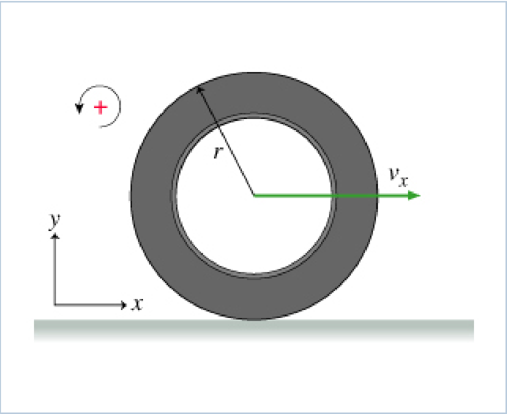
ANSWER:

$a(t) = \alpha(t) r$

Part D

Perhaps the trickiest aspect of working with constraint equations for rotational motion is determining the correct sign for the kinematic quantities. Consider a tire of radius r rolling to the right, without slipping, with constant *x* velocity v_x. Find ω, the (constant) angular velocity of the tire. Be careful of the signs in your answer; recall that positive angular velocity corresponds to rotation in the counterclockwise direction.

Express your answer in terms of v_x and r.



ANSWER:

$\omega = \frac{-v_x}{r}$

This is an example of the appearance of negative signs in constraint equations—a tire rolling in the positive direction translationally exhibits negative angular velocity, since rotation is clockwise.

Part E

Assume now that the angular velocity of the tire, which continues to roll without slipping, is not constant, but rather that the tire accelerates with constant angular acceleration α. Find a_x, the linear acceleration of the tire.

Express your answer in terms of α and r.

ANSWER:

a_x = $-\alpha r$

Introduction to Moments of Inertia

Learning Goal:

To understand the definition and the meaning of moment of inertia; to be able to calculate the moments of inertia for a group of particles and for a continuous mass distribution with a high degree of symmetry.

By now, you may be familiar with a set of equations describing rotational kinematics. One thing that you may have noticed was the similarity between *translational* and *rotational* formulas. Such similarity also exists in dynamics and in the work-energy domain.

For a *particle* of mass m moving at a constant speed v, the kinetic energy is given by the formula $K = \frac{1}{2}mv^2$. If we consider instead a *rigid object* of mass m rotating at a constant angular speed ω, the kinetic energy of such an object cannot be found by using the formula $K = \frac{1}{2}mv^2$ directly: different parts of the object have different linear speeds. However, they all have the same *angular* speed. It would be desirable to obtain a formula for kinetic energy of rotational motion that is similar to the one for translational motion; such a formula would include the term ω^2 instead of v^2.

Such a formula can, indeed, be written: for rotational motion of a system of small particles or for a rigid object with continuous mass distribution, the kinetic energy can be written as

$$K = \frac{1}{2}I\omega^2.$$

Here, I is called the moment of inertia of the object (or of the system of particles). It is the quantity representing the inertia with respect to rotational motion.

It can be shown that for a discrete system, say of n particles, the moment of inertia (also known as *rotational inertia*) is given by

$$I = \sum_{i=1}^{n} m_i r_i^2.$$

In this formula, m_i is the mass of the *i*th particle and r_i is the distance of that particle from the axis of rotation.
For a rigid object, consisting of infinitely many particles, the analogue of such summation is *integration* over the entire object:

$$I = \int r^2\, dm.$$

In this problem, you will answer several questions that will help you better understand the moment of inertia, its properties, and its applicability. It is recommended that you read the corresponding sections in your textbook before attempting these questions.

Part A

On which of the following does the moment of inertia of an object depend?

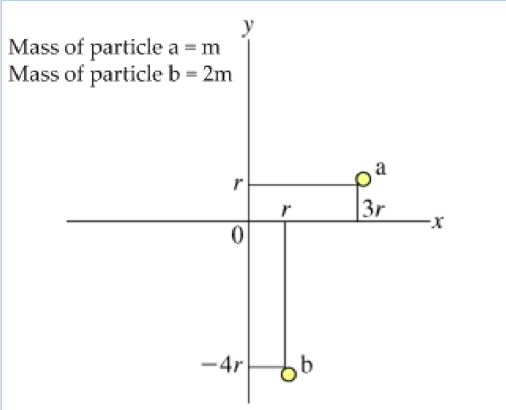
Check all that apply.

ANSWER:

- [ ] linear speed
- [ ] linear acceleration
- [ ] angular speed
- [ ] angular acceleration
- [x] total mass
- [x] shape and density of the object
- [x] location of the axis of rotation

Unlike mass, the moment of inertia depends not only on the amount of matter in an object but also on the *distribution of mass in space*. The moment of inertia is also dependent on the *axis of rotation*. The *same* object, rotating with the *same* angular speed, may have *different* kinetic energy depending on the axis of rotation.

Consider the system of two particles, a and b, shown in the figure . Particle a has mass m, and particle b has mass $2m$.



Part B

What is the moment of inertia I of particle a?

ANSWER:

- ○ mr^2
- ○ $9mr^2$
- ○ $10mr^2$
- ◉ undefined: an axis of rotation has not been specified.

Part C

Find the moment of inertia I_x of particle a with respect to the *x* axis (that is, if the *x* axis is the axis of rotation), the moment of inertia I_y of particle a with respect to the *y* axis, and the moment of inertia I_z of particle a with respect to the *z* axis (the axis that passes through the origin perpendicular to both the *x* and *y* axes).

Express your answers in terms of m and r separated by commas.

ANSWER:

$I_x, I_y, I_z = mr^2, 9mr^2, 10mr^2$

Part D

Find the total moment of inertia I of the system of two particles shown in the diagram with respect to the *y* axis.

Express your answer in terms of m and r.

ANSWER:

$I = 11mr^2$

Part E

Using the total moment of inertia I of the system found in Part D, find the total kinetic energy K of the system. Remember that both particles rotate about the *y* axis.

Express your answer in terms of m, ω, and r.

ANSWER:

$K = \frac{11mr^2\omega^2}{2}$

Part F

Using the formula for kinetic energy of a moving particle $K = \frac{1}{2}mv^2$, find the kinetic energy $K_{\rm a}$ of particle a and the kinetic energy $K_{\rm b}$ of particle b. Remember that both particles rotate about the *y* axis.

Express your answers in terms of m, ω, and r separated by a comma.

Hint 1. Find the linear speed

Using the formula $v = \omega r$, find the linear speed $v_{\rm a}$ of particle a.

Express your answer in terms of ω and r.

ANSWER:

$v_{\rm a}$ = $3\omega r$

ANSWER:

$K_{\rm a}$, $K_{\rm b}$ = $\frac{9m(\omega r)^2}{2}$, $\frac{2m(\omega r)^2}{2}$

Part G

Using the results for the kinetic energy of each particle, find the total kinetic energy K of the system of particles. Remember that both particles rotate about the *y* axis.

Express your answer in terms of m, ω, and r.

ANSWER:

K = $\frac{11m(\omega r)^2}{2}$

Not surprisingly, the formulas $K = \frac{1}{2}I\omega^2$ and $K = \frac{1}{2}mv^2$ give the same result. They should, of course, since the rotational kinetic energy of a system of particles is simply the sum of the kinetic energies of the individual particles making up the system.

Scaling of Moments of Inertia

Learning Goal:

To understand the concept of moment of inertia and how it depends on mass, radius, and mass distribution.

In rigid-body rotational dynamics, the role analogous to the mass of a body (when one is considering translational motion) is played by the body's moment of inertia. For this reason, conceptual understanding of the motion of a rigid body requires some understanding of moments of inertia. This problem should help you develop such an understanding.

The moment of inertia of a body about some specified axis is $I = cmr^2$, where c is a dimensionless constant, m is the mass of the body, and r is the perpendicular distance from the axis of rotation. Therefore, if you have two similarly shaped objects of the same size but with one twice as massive as the other, the more massive object should have a moment of inertia twice that of the less massive one. Furthermore, if you have two similarly shaped objects of the same mass, but one has twice the size of the other, the larger object should have a moment of inertia that is four times that of the smaller one.

Part A

Two spherical shells have their mass uniformly distrubuted over the spherical surface. One of the shells has a diameter of 2 meters and a mass of 1 kilogram. The other shell has a diameter of 1 meter. What must the mass m of the 1-meter shell be for both shells to have the same moment of inertia about their centers of mass?

ANSWER:

m = 4 kg

It is important to understand how the dimensionless constant c in the moment of inertia formula given in the problem introduction is determined. Consider a disk and a thin ring, both having the same outer radius r and mass m. The moment of inertia of the disk is $\frac{1}{2}mr^2$, while the moment of inertia of the ring is mr^2. (For each object, the axis is perpendicular to the plane of the object and passes through the object's center of mass.)

The factor of $\frac{1}{2}$ for the disk gives an indication of how the mass is distributed in that object. If both the disk and ring are spinning with the same angular velocity ω, the ring should have a greater kinetic energy, because all of the mass of the ring has linear speed ωr, whereas the linear speeds of the different parts of the disk vary, depending on how far the part is from the center, and these speeds vary from zero to ωr.

In general, the value of c reflects the distribution of mass within the object. A number close to 1 indicates that most of the mass is located at a distance from the center of mass close to r, while a number much less than 1 indicates that most of the mass is located near the center of mass.

Part B

Consider the moment of inertia of a solid uniform disk, versus that of a solid sphere, about their respective centers of mass. Assume that they both have the same mass and outer radius, that they have uniform mass distributions, and that the disk is rotated about an axis perpendicular to its face. What is the relation between the moment of inertia of the disk $I_{\rm disk}$ and that of the sphere $I_{\rm sphere}$?

Hint 1. How to approach the problem

Draw a figure of each object and the axis. Consider two "slices" parallel to the axis at different radii and the ratio of masses inside these slices. Which object has a greater percentage of the mass closer to the axis?

ANSWER:

- (selected) $I_{\rm disk} > I_{\rm sphere}$
- $I_{\rm sphere} > I_{\rm disk}$

Visualizing Rotation

Learning Goal:

To be able to identify situations with constant angular velocity or constant angular acceleration by watching movies of the rotations.

Recall that angular velocity measures the angle through which an object turns over time. If a disk has constant angular velocity and it makes a quarter revolution in one second, then it will make another quarter revolution the next second. If the disk turns in the clockwise direction, it has, by definition, negative angular velocity. The magnitude of the angular velocity is the angular speed. This applet, which shows a few rotating disks and lists their angular velocities, should help you to get a feel for how different angular velocities look.

Angular acceleration measures how the angular velocity changes over time. If a disk has constant angular velocity, then it has zero angular acceleration. If a disk turns a quarter revolution one second and a half revolution the next second, then its angular velocity is changing, and so it has an angular acceleration. This applet shows two disks and lists their initial angular velocities and angular accelerations. This should help you to get a feel for how different angular accelerations look. Just as with linear accelerations, if a positive angular velocity decreases, that indicates a negative angular acceleration. If a negative angular velocity becomes more negative (i.e., its magnitude increases), that also indicates a negative angular acceleration.

This applet shows six disks rotating with constant angular acceleration. No two have the same initial angular velocity and angular acceleration. To answer the following questions, number the disks starting from the top. That is, call the yellow disk "1" and go sequentially down to the red disk, which will be "6". In the following questions, you will be asked to determine whether the disks' angular velocities and accelerations are positive, negative, or zero. Keep in mind that angular velocity is considered positive if rotation is in the counterclockwise direction. Angular acceleration is positive if the rotation is in the counterclockwise (positive) direction and the angular speed is increasing, or if rotation is the clockwise (negative) direction and the angular speed is decreasing (thus the angular velocity is becoming less negative). Negative angular acceleration is defined analogously.

Part A

Which of the disks have positive initial angular velocity?

Write down the numbers, in order, that correspond to the disk(s) that you believe are correct, without commas or spaces between them. For example, if you think that the yellow disk and the gray disk are the correct ones, then you should enter 15.

ANSWER:

46

Part B

Which of the disks have negative initial angular velocity?

Write down the numbers, in order, that correspond to the disk(s) that you believe are correct, without commas or spaces between them. For example, if you think that the yellow disk and the gray disk are the correct ones, then you should enter 15**.**

ANSWER:

25

Part C

Which of the disks have zero angular acceleration?

Write down the numbers, in order, that correspond to the disk(s) that you believe are correct, without commas or spaces between them. For example, if you think that the yellow disk and the gray disk are the correct ones, then you should enter 15**.**

ANSWER:

56

Part D

Which of the disks have positive angular acceleration?

Write down the numbers, in order, that correspond to the disk(s) that you believe are correct, without commas or spaces between them. For example, if you think that the yellow disk and the gray disk are the correct ones, then you should enter 15**.**

ANSWER:

12

Part E

Which of the following characterizes the initial angular velocity ω_0 and the angular acceleration α of disk 4?

ANSWER:

- ○ $\omega_0 > 0$ and $\alpha > 0$
- ○ $\omega_0 = 0$ and $\alpha > 0$
- ○ $\omega_0 < 0$ and $\alpha > 0$
- ○ $\omega_0 > 0$ and $\alpha = 0$
- ◉ $\omega_0 > 0$ and $\alpha < 0$
- ○ $\omega_0 = 0$ and $\alpha = 0$

Exercise 9.1

Part A

What angle in radians is subtended by an arc of 1.44m in length on the circumference of a circle of radius 2.56m ?

ANSWER:

$\theta = \frac{L_1}{r_1} = 0.563 \quad \mathrm{rad}$

Part B

What is this angle in degrees?

ANSWER:

$\theta = \frac{L_1}{r_1}\frac{180}{\pi} = 32.2 \quad ^\circ$

Part C

An arc of length 13.8cm on the circumference of a circle subtends an angle of 120°. What is the radius of the circle?

ANSWER:

r = $\frac{L_2}{\theta_1}$ = 6.59 cm

Part D

The angle between two radii of a circle with radius 1.47m is 0.700rad . What length of arc is intercepted on the circumference of the circle by the two radii?

ANSWER:

L = $r_2\theta_2$ = 1.03 m

Exercise 9.8

A wheel is rotating about an axis that is in the z-direction. The angular velocity ω_z is $-6.00\,\mathrm{rad/s}$ at $t=0$, increases linearly with time and is $+8.00\,\mathrm{rad/s}$ at $t=7.00\,\mathrm{s}$. We have taken counterclockwise rotation to be positive.

Part A

Is the angular acceleration during this time interval positive or negative?

ANSWER:

- (selected) positive
- negative

Part B

During what time interval is the speed of the wheel increasing?

ANSWER:

t = 4.00 s

Part C

During what time interval is the speed of the wheel decreasing?

ANSWER:

t = 3.00 s

Part D

What is the angular displacement of the wheel at $t=7.00\,\mathrm{s}$?

ANSWER:

θ = 7.00 rad

Exercise 9.10

An electric fan is turned off, and its angular velocity decreases uniformly from 480$\mathrm{rev/min}$ to 230$\mathrm{rev/min}$ in a time interval of length 4.35s .

Part A

Find the angular acceleration in $\mathrm{rev/s^2}$.

ANSWER:

$\alpha = \frac{\frac{-(\omega_1-\omega_2)}{\Delta t}}{60}$ = -0.958 $\mathrm{rev/s^2}$

Part B

Find the number of revolutions made by the motor in the time interval of length 4.35s .

ANSWER:

$N = \frac{\frac{(\omega_1+\omega_2)\Delta t}{60}}{2}$ = 25.7 rev

Part C

How many more seconds are required for the fan to come to rest if the angular acceleration remains constant at the value calculated in part A?

ANSWER:

$t = \frac{\omega_2}{\omega_1 - \omega_2}\Delta t$ = 4.00 s

Exercise 9.17

A safety device brings the blade of a power mower from an initial angular speed of ω_1 to rest in 4.00 revolution.

Part A

At the same constant acceleration, how many revolutions would it take the blade to come to rest from an initial angular speed ω_3 that was three times as great, $\omega_3 = 3\omega_1$?

ANSWER:

$n = 9.00n$ = 36.0

Exercise 9.20

A compact disc (CD) stores music in a coded pattern of tiny pits 10^{-7} m deep. The pits are arranged in a track that spirals outward toward the rim of the disc; the inner and outer radii of this spiral are 25.0 mm and 58.0 mm, respectively. As the disc spins inside a CD player, the track is scanned at a constant linear speed of 1.25 $\mathrm{m/s}$.

Part A

What is the angular speed of the CD when scanning the innermost part of the track?

ANSWER:

ω = 50.0 $\mathrm{rad/s}$

Part B

What is the angular speed of the CD when scanning the outermost part of the track?

ANSWER:

ω = 21.6 $\mathrm{rad/s}$

Part C

The maximum playing time of a CD is 74.0 min. What would be the length of the track on such a maximum-duration CD if it were stretched out in a straight line?

ANSWER:

L = 5.55 km

Part D

What is the average angular acceleration of a maximum-duration CD during its 74.0-min playing time? Take the direction of rotation of the disc to be positive.

ANSWER:

α_{av} = -6.41×10^{-3} $\mathrm{rad/s^2}$

Exercise 9.26

Part A

Derive an equation for the radial acceleration that includes v and ω, but not r.

ANSWER:

a_{rad} = ωv

Part B

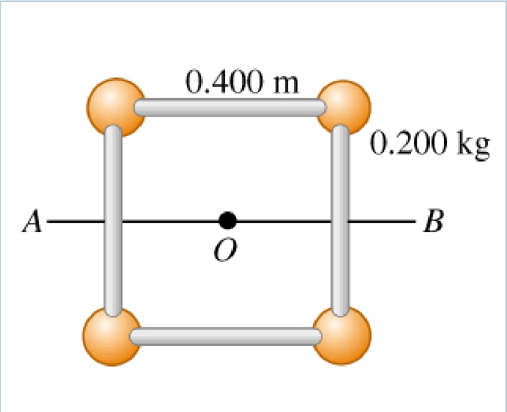
You are designing a merry-go-round for which a point on the rim will have a radial acceleration of 0.500 $\mathrm{m/s^2}$ when the tangential velocity of that point has magnitude 2.00 $\mathrm{m/s}$. What angular velocity is required to achieve these values?

ANSWER:

ω = 0.250 $\mathrm{rad/s}$

Exercise 9.30

Four small spheres, each of which you can regard as a point of mass 0.200 kg, are arranged in a square 0.400 m on a side and connected by light rods .



Part A

Find the moment of inertia of the system about an axis through the center of the square, perpendicular to its plane (an axis through point O in the figure).

ANSWER:

I = 6.40×10^{-2} $\mathrm{kg\cdot m^2}$

Part B

Find the moment of inertia of the system about an axis bisecting two opposite sides of the square (an axis along the line AB in the figure).

ANSWER:

I = 3.20×10^{-2} $\mathrm{kg\cdot m^2}$

Part C

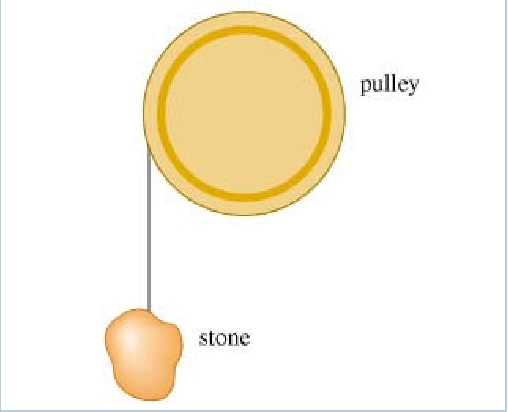
Find the moment of inertia of the system about an axis that passes through the centers of the upper left and lower right spheres and through point O.

ANSWER:

I = 3.20×10^{-2} $\mathrm{kg\cdot m^2}$

Exercise 9.47

A frictionless pulley has the shape of a uniform solid disk of mass 2.50kg and radius 10 cm. A 1.90kg stone is attached to a very light wire that is wrapped around the rim of the pulley (the figure), and the system is released from rest.



Part A

How far must the stone fall so that the pulley has 5.60J of kinetic energy?

ANSWER:

$h = \frac{K\left(2\frac{m}{M_p}+1\right)}{mg}$ = 0.758 m

Part B

What percent of the total kinetic energy does the pulley have?

.

ANSWER:

$\frac{K_p}{K_{tot}} = \frac{1}{2\frac{m}{M_p}+1}\cdot 100$ = 39.7 $\%$

Exercise 9.57

A thin uniform rod of mass M and length L is bent at its center so that the two segments are now perpendicular to each other.

Part A

Find its moment of inertia about an axis perpendicular to its plane and passing through the point where the two segments meet.

Give your answer in terms of given quantities.

ANSWER:

$I = \frac{1}{12}ML^2$

Part B

Find its moment of inertia about an axis perpendicular to its plane and passing through the midpoint of the line connecting its two ends.

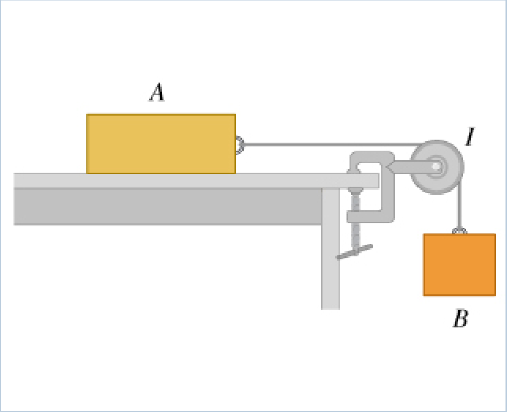
Give your answer in terms of given quantities.

ANSWER:

$I = \frac{1}{12}ML^2$

Problem 9.83

The pulley in the figure has radius R and a moment of inertia I. The rope does not slip over the pulley, and the pulley spins on a frictionless axle. The coefficient of kinetic friction between block *A* and the tabletop is μ_k. The system is released from rest, and block *B* descends. Block *A* has mass m_A and block *B* has mass m_B.



Part A

Use energy methods to calculate the speed of block *B* as a function of the distance d that it has descended.

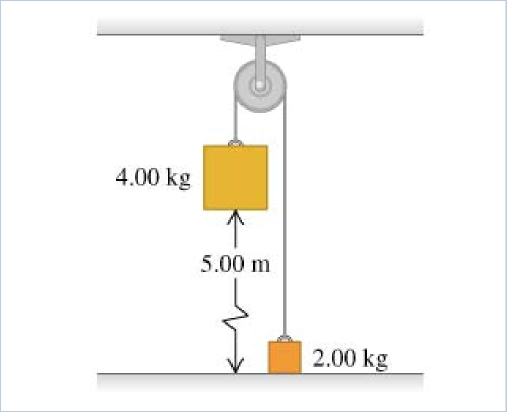
Express your answer in terms of the variables m_A, m_B, R, I, μ_k, d and appropriate constants.

ANSWER:

$v = \sqrt{\frac{2gd\left(m_B - \mu_k m_A\right)}{m_A + m_B + \frac{I}{R^2}}}$

Problem 9.84

The pulley in the figure has radius 0.160 m and a moment of inertia 0.560 $\mathrm{kg \cdot m^2}$. The rope does not slip on the pulley rim.



Part A

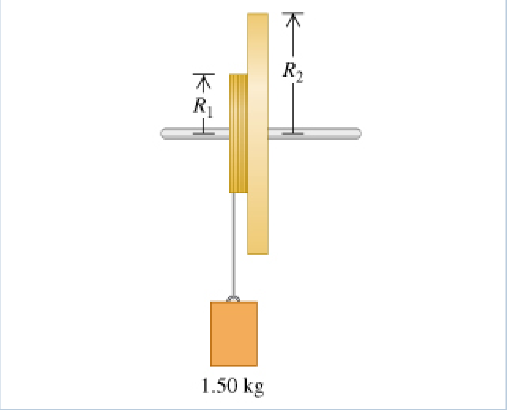
Use energy methods to calculate the speed of the 4.00-kg block just before it strikes the floor.

ANSWER:

v = 2.65 $\mathrm{m/s}$

Problem 9.87

Two metal disks, one with radius R_1 = 2.48cm and mass M_1 = 0.790kg and the other with radius R_2 = 4.94cm and mass M_2 = 1.55kg , are welded together and mounted on a frictionless axis through their common center. .



Part A

What is the total moment of inertia of the two disks?

ANSWER:

I = $\frac{1}{2}\left(M_1{R_1}^2+M_2{R_2}^2\right)$ = 2.13×10^{-3} $\mathrm{kg \cdot m^2}$

Part B

A light string is wrapped around the edge of the smaller disk, and a 1.50-kg block, suspended from the free end of the string. If the block is released from rest at a distance of 1.96m above the floor, what is its speed just before it strikes the floor?

ANSWER:

v = $\sqrt{\frac{2 \cdot 9.8h}{1+\frac{\frac{1}{2}\left(M_1{R_1}^2+M_2{R_2}^2\right)}{1.5{R_1}^2}}}$ = 3.41 $\mathrm{m/s}$

Part C

Repeat the calculation of part B, this time with the string wrapped around the edge of the larger disk.

ANSWER:

v = $\sqrt{\frac{2 \cdot 9.8h}{1+\frac{\frac{1}{2}\left(M_1{R_1}^2+M_2{R_2}^2\right)}{1.5{R_2}^2}}}$ = 4.93 $\mathrm{m/s}$

9.1. IDENTIFY: $s = r\theta$, with θ in radians.

SET UP: $\pi \text{ rad} = 180°$.

EXECUTE: **(a)** $\theta = \dfrac{s}{r} = \dfrac{1.50 \text{ m}}{2.50 \text{ m}} = 0.600 \text{ rad} = 34.4°$

(b) $r = \dfrac{s}{\theta} = \dfrac{14.0 \text{ cm}}{(128°)(\pi \text{ rad}/180°)} = 6.27 \text{ cm}$

(c) $s = r\theta = (1.50 \text{ m})(0.700 \text{ rad}) = 1.05 \text{ m}$

EVALUATE: An angle is the ratio of two lengths and is dimensionless. But, when $s = r\theta$ is used, θ must be in radians. Or, if $\theta = s/r$ is used to calculate θ, the calculation gives θ in radians.

9.8. IDENTIFY: $\alpha_z = \dfrac{d\omega_z}{dt}$. $\theta - \theta_0 = \omega_{\text{av-}z} t$. When ω_z is linear in t, $\omega_{\text{av-}z}$ for the time interval t_1 to t_2 is $\omega_{\text{av-}z} = \dfrac{\omega_{z1} + \omega_{z2}}{t_2 - t_1}$.

SET UP: From the information given, $\omega_z(t) = -6.00 \text{ rad/s} + (2.00 \text{ rad/s}^2)t$.

EXECUTE: **(a)** The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

(b) It takes 3.00 seconds for the wheel to stop $(\omega_z = 0)$. During this time its speed is decreasing. For the next 4.00 s its speed is increasing from 0 rad/s to $+8.00 \text{ rad/s}$.

(c) The average angular velocity is $\dfrac{-6.00 \text{ rad/s} + 8.00 \text{ rad/s}}{2} = 1.00 \text{ rad/s}$. $\theta - \theta_0 = \omega_{\text{av-}z} t$ then leads to displacement of 7.00 rad after 7.00 s.

EVALUATE: When α_z and ω_z have the same sign, the angular speed is increasing; this is the case for $t = 3.00 \text{ s}$ to $t = 7.00 \text{ s}$. When α_z and ω_z have opposite signs, the angular speed is decreasing; this is the case between $t = 0$ and $t = 3.00 \text{ s}$.

9.10. IDENTIFY: Apply the constant angular acceleration equations to the motion of the fan.

(a) SET UP: $\omega_{0z} = (500 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 8.333 \text{ rev/s}$,

$\omega_z = (200 \text{ rev/min})(1 \text{ min}/60 \text{ s}) = 3.333 \text{ rev/s}$, $t = 4.00 \text{ s}$, $\alpha_z = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{3.333 \text{ rev/s} - 8.333 \text{ rev/s}}{4.00 \text{ s}} = -1.25 \text{ rev/s}^2$

$\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z} t + \tfrac{1}{2}\alpha_z t^2 = (8.333 \text{ rev/s})(4.00 \text{ s}) + \tfrac{1}{2}(-1.25 \text{ rev/s}^2)(4.00 \text{ s})^2 = 23.3 \text{ rev}$

(b) SET UP: $\omega_z = 0$ (comes to rest); $\omega_{0z} = 3.333 \text{ rev/s}$; $\alpha_z = -1.25 \text{ rev/s}^2$;

$t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{0 - 3.333 \text{ rev/s}}{-1.25 \text{ rev/s}^2} = 2.67 \text{ s}$

EVALUATE: The angular acceleration is negative because the angular velocity is decreasing. The average angular velocity during the 4.00 s time interval is 350 rev/min and $\theta - \theta_0 = \omega_{\text{av-z}} t$ gives $\theta - \theta_0 = 23.3 \text{ rev},$ which checks.

9.17. **IDENTIFY:** Apply Eq. (9.12) to relate ω_z to $\theta - \theta_0$.

SET UP: Establish a proportionality.

EXECUTE: From Eq. (9.12), with $\omega_{0z} = 0,$ the number of revolutions is proportional to the square of the initial angular velocity, so tripling the initial angular velocity increases the number of revolutions by 9, to 9.00 rev.

EVALUATE: We don't have enough information to calculate $\alpha_z;$ all we need to know is that it is constant.

9.20. **IDENTIFY:** Linear and angular velocities are related by $v = r\omega.$ Use $\omega_z = \omega_{0z} + \alpha_z t$ to calculate α_z.

SET UP: $\omega = v/r$ gives ω in rad/s.

EXECUTE: **(a)** $\dfrac{1.25 \text{ m/s}}{25.0 \times 10^{-3} \text{ m}} = 50.0 \text{ rad/s}, \quad \dfrac{1.25 \text{ m/s}}{58.0 \times 10^{-3} \text{ m}} = 21.6 \text{ rad/s}.$

(b) $(1.25 \text{ m/s})(74.0 \text{ min})(60 \text{ s/min}) = 5.55 \text{ km}.$

(c) $\alpha_z = \dfrac{21.55 \text{ rad/s} - 50.0 \text{ rad/s}}{(74.0 \text{ min})(60 \text{ s/min})} = -6.41 \times 10^{-3} \text{ rad/s}^2.$

EVALUATE: The width of the tracks is very small, so the total track length on the disc is huge.

9.26.IDENTIFY: In part (b) apply the result derived in part (a).

SET UP: $a_{\text{rad}} = r\omega^2$ and $v = r\omega;$ combine to eliminate *r*.

EXECUTE: **(a)** $a_{\text{rad}} = \omega^2 r = \omega^2 \left(\dfrac{v}{\omega}\right) = \omega v.$

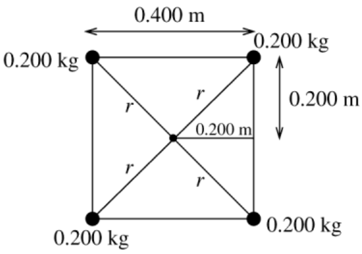
(b) From the result of part (a), $\omega = \dfrac{a_{\text{rad}}}{v} = \dfrac{0.500 \text{ m/s}^2}{2.00 \text{ m/s}} = 0.250 \text{ rad/s}.$

EVALUATE: $a_{\text{rad}} = r\omega^2$ and $v = r\omega$ both require that ω be in rad/s, so in $a_{\text{rad}} = \omega v,$ ω is in rad/s.

9.30. **IDENTIFY** and **SET UP:** Use Eq. (9.16). Treat the spheres as point masses and ignore *I* of the light rods.

EXECUTE: The object is shown in Figure 9.30a.

(a)



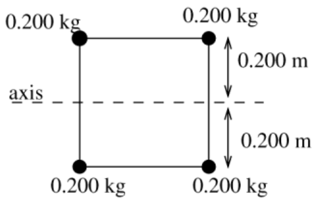
$$r = \sqrt{(0.200 \text{ m})^2 + (0.200 \text{ m})^2} = 0.2828 \text{ m}$$

$$I = \sum m_i r_i^2 = 4(0.200 \text{ kg})(0.2828 \text{ m})^2$$

$$I = 0.0640 \text{ kg} \cdot \text{m}^2$$

Figure 9.30a

(b) The object is shown in Figure 9.30b.



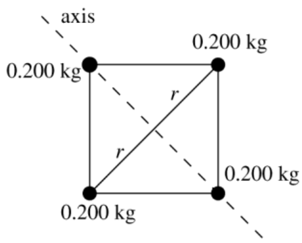
$$r = 0.200 \text{ m}$$

$$I = \sum m_i r_i^2 = 4(0.200 \text{ kg})(0.200 \text{ m})^2$$

$$I = 0.0320 \text{ kg} \cdot \text{m}^2$$

Figure 9.30b

(c) The object is shown in Figure 9.30c.



$$r = 0.2828 \text{ m}$$

$$I = \sum m_i r_i^2 = 2(0.200 \text{ kg})(0.2828 \text{ m})^2$$

$$I = 0.0320 \text{ kg} \cdot \text{m}^2$$

Figure 9.30c

EVALUATE: In general *I* depends on the axis and our answer for part (a) is larger than for parts (b) and (c). It just happens that *I* is the same in parts (b) and (c).

9.47. **IDENTIFY:** Apply conservation of energy to the system of stone plus pulley. $v = r\omega$ relates the motion of the stone to the rotation of the pulley.

SET UP: For a uniform solid disk, $I = \frac{1}{2}MR^2$. Let point 1 be when the stone is at its initial position and point 2 be when it has descended the desired distance. Let $+y$ be upward and take $y = 0$ at the initial position of the stone, so $y_1 = 0$ and $y_2 = -h$, where *h* is the distance the stone descends.

EXECUTE: **(a)** $K_p = \frac{1}{2}I_p\omega^2$. $I_p = \frac{1}{2}M_pR^2 = \frac{1}{2}(2.50\text{ kg})(0.200\text{ m})^2 = 0.0500\text{ kg}\cdot\text{m}^2$.

$\omega = \sqrt{\dfrac{2K_p}{I_p}} = \sqrt{\dfrac{2(4.50\text{ J})}{0.0500\text{ kg}\cdot\text{m}^2}} = 13.4\text{ rad/s}$. The stone has speed

$v = R\omega = (0.200\text{ m})(13.4\text{ rad/s}) = 2.68\text{ m/s}$. The stone has kinetic energy

$K_s = \frac{1}{2}mv^2 = \frac{1}{2}(1.50\text{ kg})(2.68\text{ m/s})^2 = 5.39\text{ J}$. $K_1 + U_1 = K_2 + U_2$ gives $0 = K_2 + U_2$.

$0 = 4.50\text{ J} + 5.39\text{ J} + mg(-h)$. $h = \dfrac{9.89\text{ J}}{(1.50\text{ kg})(9.80\text{ m/s}^2)} = 0.673\text{ m}$.

(b) $K_{tot} = K_p + K_s = 9.89\text{ J}$. $\dfrac{K_p}{K_{tot}} = \dfrac{4.50\text{ J}}{9.89\text{ J}} = 45.5\%$.

EVALUATE: The gravitational potential energy of the pulley doesn't change as it rotates. The tension in the wire does positive work on the pulley and negative work of the same magnitude on the stone, so no net work on the system.

9.57. **IDENTIFY:** Use the equations in Table 9.2. *I* for the rod is the sum of *I* for each segment. The parallel-axis theorem says $I_p = I_{cm} + Md^2$.

SET UP: The bent rod and axes *a* and *b* are shown in Figure 9.57. Each segment has length $L/2$ and

mass $M/2$.

EXECUTE: **(a)** For each segment the moment of inertia is for a rod with mass $M/2$, length $L/2$ and the axis through one end. For one segment, $I_s = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{24}ML^2$. For the rod,

$I_a = 2I_s = \frac{1}{12}ML^2$.

(b) The center of mass of each segment is at the center of the segment, a distance of $L/4$ from each end. For each segment, $I_{cm} = \frac{1}{12}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{96}ML^2$. Axis *b* is a distance $L/4$ from the cm of each segment, so for each segment the parallel axis theorem gives *I* for axis *b* to be

$I_s = \frac{1}{96}ML^2 + \frac{M}{2}\left(\frac{L}{4}\right)^2 = \frac{1}{24}ML^2$ and $I_b = 2I_s = \frac{1}{12}ML^2$.

EVALUATE: *I* for these two axes are the same.

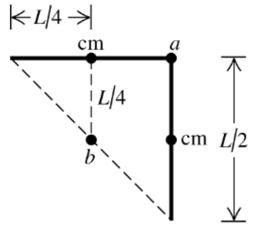


Figure 9.57

9.83. IDENTIFY: Apply conservation of energy to the system consisting of blocks *A* and *B* and the pulley.

SET UP: The system at points 1 and 2 of its motion is sketched in Figure 9.83.

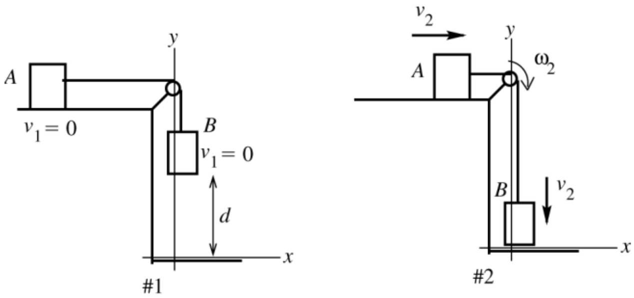


Figure 9.83

Use the work-energy relation $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Use coordinates where $+y$ is upward and where the origin is at the position of block *B* after it has descended. The tension in the rope does positive work on block *A* and negative work of the same magnitude on block *B*, so the net work done by the tension in the rope is zero. Both blocks have the same speed.

EXECUTE: Gravity does work on block *B* and kinetic friction does work on block *A*. Therefore

$$W_{\text{other}} = W_f = -\mu_k m_A g d.$$

$K_1 = 0$ (system is released from rest)

$U_1 = m_B g y_{B1} = m_B g d;\quad U_2 = m_B g y_{B2} = 0$

$$K_2 = \tfrac{1}{2} m_A v_2^2 + \tfrac{1}{2} m_B v_2^2 + \tfrac{1}{2} I \omega_2^2.$$

But $v(\text{blocks}) = R\omega(\text{pulley})$, so $\omega_2 = v_2/R$ and

$$K_2 = \tfrac{1}{2}(m_A + m_B)v_2^2 + \tfrac{1}{2} I (v_2/R)^2 = \tfrac{1}{2}(m_A + m_B + I/R^2)v_2^2$$

Putting all this into the work-energy relation gives

$$m_B g d - \mu_k m_A g d = \tfrac{1}{2}(m_A + m_B + I/R^2)v_2^2$$

$$(m_A + m_B + I/R^2)v_2^2 = 2gd(m_B - \mu_k m_A)$$

$$v_2 = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$$

EVALUATE: If $m_B >> m_A$ and I/R^2, then $v_2 = \sqrt{2gd}$; block *B* falls freely. If *I* is very large, v_2 is very small. Must have $m_B > \mu_k m_A$ for motion, so the weight of *B* will be larger than the friction force on *A*. I/R^2 has units of mass and is in a sense the "effective mass" of the pulley.

9.84. IDENTIFY: Apply conservation of energy to the system of two blocks and the pulley.

SET UP: Let the potential energy of each block be zero at its initial position. The kinetic energy of the system is the sum of the kinetic energies of each object. $v = R\omega,$ where v is the common speed of the blocks and ω is the angular velocity of the pulley.

EXECUTE: The amount of gravitational potential energy which has become kinetic energy is

$K = (4.00\text{ kg} - 2.00\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 98.0\text{ J}.$ In terms of the common speed v of the blocks, the kinetic energy of the system is $K = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2.$

$$K = v^2 \frac{1}{2}\left(4.00\text{ kg} + 2.00\text{ kg} + \frac{(0.560\text{ kg}\cdot\text{m}^2)}{(0.160\text{ m})^2}\right) = v^2(13.94\text{ kg}).$$ Solving for v gives

$$v = \sqrt{\frac{98.0\text{ J}}{13.94\text{ kg}}} = 2.65\text{ m/s}.$$

EVALUATE: If the pulley is massless, $98.0\text{ J} = \frac{1}{2}(4.00\text{ kg} + 2.00\text{ kg})v^2$ and $v = 5.72\text{ m/s}.$ The moment of inertia of the pulley reduces the final speed of the blocks.

9.87. **IDENTIFY:** $I = I_1 + I_2.$ Apply conservation of energy to the system. The calculation is similar to Example 9.8.

SET UP: $\omega = \dfrac{v}{R_1}$ for part (b) and $\omega = \dfrac{v}{R_2}$ for part (c).

EXECUTE: **(a)**

$$I = \frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2 = \frac{1}{2}((0.80\text{ kg})(2.50\times10^{-2}\text{ m})^2 + (1.60\text{ kg})(5.00\times10^{-2}\text{ m})^2)$$

$$I = 2.25\times10^{-3}\text{ kg}\cdot\text{m}^2.$$

(b) The method of Example 9.8 yields $v = \sqrt{\dfrac{2gh}{1 + (I/mR_1^2)}}.$

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(2.00\text{ m})}{(1 + ((2.25\times10^{-3}\text{ kg}\cdot\text{m}^2)/(1.50\text{ kg})(0.025\text{ m})^2))}} = 3.40\text{ m/s}.$$

(c) The same calculation, with R_2 instead of R_1 gives $v = 4.95\text{ m/s}.$

EVALUATE: The final speed of the block is greater when the string is wrapped around the larger disk. $v = R\omega,$ so when $R = R_2$ the factor that relates v to ω is larger. For $R = R_2$ a larger fraction of the total kinetic energy resides with the block. The total kinetic energy is the same in both cases (equal to mgh), so when $R = R_2$ the kinetic energy and speed of the block are greater.