#43 Sound Waves Post-Class

Due: 11:00am on Wednesday, December 5, 2012

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

Interference of Sound Waves

Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 172 Hz. You are 8.00 m from speaker A. Take the speed of sound in air to be 344 $\mathrm{m/s}$.

Part A

What is the closest you can be to speaker B and be at a point of destructive interference?

Express your answer in meters.

Hint 1. How to approach the problem

Destructive interference occurs when the difference in path lengths traveled by sound waves is a half-integer number of wavelengths. Therefore, to apply the condition for destructive interference you need to know the wavelength of the sound, which can be easily determined given its frequency.

Hint 2. Find the wavelength of the sound wave

Find the wavelength λ of the sound wave emitted by the loudspeakers. Take the speed of sound in air to be 344 $\mathrm{m/s}$.

Express you answer in meters.

Hint 1. Relationship between the wavelength and frequency of a periodic wave

For a periodic wave the speed of propagation v equals the product of the wavelength λ and frequency f of the wave. In symbols,

$$v = \lambda f.$$

ANSWER:

λ = 2.00 m

Hint 3. Find the condition for destructive interference

In general, if d_a and d_b are the paths traveled by two waves of equal frequency that are originally emitted in phase, the condition for destructive interference is

$$d_a - d_b = n\frac{\lambda}{2},$$

where λ is the wavelength of the sound waves and n is any nonzero odd integer.

Given this condition for destructive interference and the situation described in the introduction of this problem, then, what is the value of n that corresponds to the shortest distance d_b?

ANSWER:

n = 7

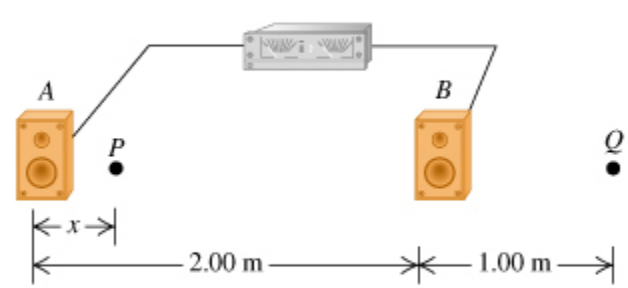
ANSWER:

1.00 m

Correct

Exercise 16.33

Two loudspeakers, A and B , are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. Consider point Q along the extension of the line connecting the speakers, 1.00 m to the right of speaker B. Both speakers emit sound waves that travel directly from the speaker to point Q.



Part A

What is the lowest frequency for which *constructive* interference occurs at point Q?

ANSWER:

f = 172 Hz

Correct

Part B

What is the lowest frequency for which *destructive* interference occurs at point Q?

ANSWER:

f = 86.0 Hz

Correct

Exercise 16.36

Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. The frequency of the waves emitted by each speaker is 172 Hz. You are 8.00 m from A.

Part A

What is the closest you can be to B and be at a point of destructive interference?

ANSWER:

1.00 m

Correct

Two Traveling Waves Beating Together

Learning Goal:

To see how two traveling waves of nearly the same frequency can create beats and to interpret the superposition as a "walking" wave.

Consider two similar traveling transverse waves, which might be traveling along a string for example:

$y_1(x,t) = A\sin(k_1 x - \omega_1 t)$ and $y_2(x,t) = A\sin(k_2 x - \omega_2 t)$.

They are similar because we assume that k_1 and k_2 are nearly equal and also that ω_1 and ω_2 are nearly equal.

Part A

Which one of the following statements about these waves is correct?

ANSWER:

- (selected) Both waves are traveling in the $+\hat{x}$ direction.
- Both waves are traveling in the $-\hat{x}$ direction.
- Only wave y_1 is traveling in the $+\hat{x}$ direction.
- Only wave y_2 is traveling in the $+\hat{x}$ direction.

Correct

The principle of *superposition* states that if two waves each separately satisfy the wave equation then the sum (or difference) also satisfies the wave equation. This follows from the fact that every term in the wave equation is linear in the amplitude of the wave.

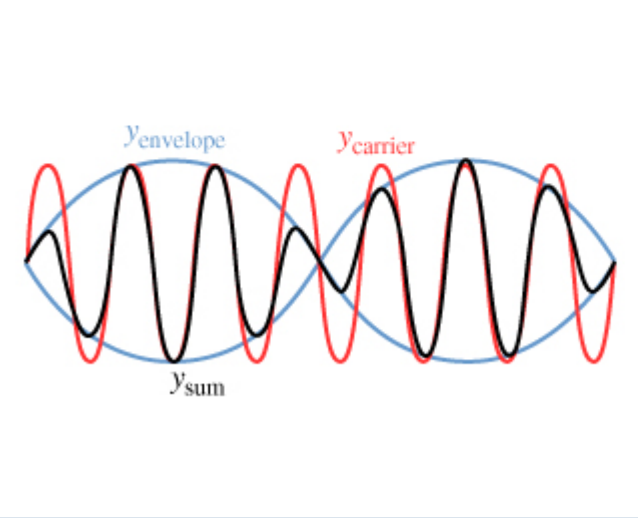
Consider the sum of the two waves given in the introduction, that is,

$$y_{\rm sum}(x,t) = y_1(x,t) + y_2(x,t).$$

These waves have been chosen so that their sum can be written as follows:

$$y_{\rm sum}(x,t) = C y_{\rm envelope}(x,t)\, y_{\rm carrier}(x,t),$$

where C is a constant, and the functions $y_{\rm envelope}$ and $y_{\rm carrier}$ are trigonometric functions of x and t. This form is especially significant because the first function, called the envelope, is a slowly varying function of both position (x) and time (t), whereas the second varies rapidly with both position (x) and time (t). Traditionally, the overall amplitude is represented by the constant C, while the functions $y_{\rm envelope}$ and $y_{\rm carrier}$ are trigonometric functions with unit amplitude.



Part B

Find C, $y_{\rm envelope}(x,t)$, and $y_{\rm carrier}(x,t)$.

Express your answer in terms of A, k_1, k_2, x, t, ω_1, and ω_2. Separate the three terms with commas. Recall that $y_{\rm envelope}$ (the second term) varies slowly whereas $y_{\rm carrier}$ (the third term) varies quickly. Both $y_{\rm envelope}$ and $y_{\rm carrier}$ should be trigonometric functions of unit amplitude.

Hint 1. A useful trigonometric identity

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

Hint 2. Which is the envelope and which is the carrier wave?

Recall that the carrier signal varies much more quickly over time and space than does the envelope signal. Given a choice between $\sin(ax)$ and $\sin(bx)$, where $a > b$, which wave oscillates faster in space?

ANSWER:

- (selected) $\sin(ax)$
- $\sin(bx)$

ANSWER:

C, $y_{\mathrm{envelope}}(x,t)$, $y_{\mathrm{carrier}}(x,t)$ = $2A$, $\cos\left(\frac{(k_1-k_2)x-(\omega_1-\omega_2)t}{2}\right)$, $\sin\left(\frac{(k_1+k_2)x-(\omega_1+\omega_2)t}{2}\right)$

Correct

Part C

Which of the following statements about $y_{\mathrm{carrier}}(x,t)$ is correct?

ANSWER:

- ◉ It is a rapidly oscillating wave traveling in the $+\hat{x}$ direction.
- ○ It is a rapidly oscillating wave traveling in the $-\hat{x}$ direction.
- ○ It is slowly oscillating in time but is standing still.
- ○ It is traveling rapidly but oscillating slowly.

Correct

Part D

Which of the following statements about $y_{\rm envelope}(x,t)$ is correct if you assume that $\delta_k = k_1 - k_2$ and $\delta_\omega = \omega_1 - \omega_2$ are both positive?

ANSWER:

- ◉ It is a slowly oscillating wave traveling in the $+\hat{x}$ direction.
- ○ It is a slowly oscillating wave traveling in the $-\hat{x}$ direction.
- ○ It is slowly oscillating in time but is standing still.
- ○ It is traveling rapidly but oscillating slowly.

Correct

Part E

The envelope function can be written simply in terms of $\delta_k = k_1 - k_2$ and $\delta_\omega = \omega_1 - \omega_2$. If you do so, what is $v_{\rm group}$, the velocity of propagation of the envelope?

Express your result in terms of δ_k and δ_ω.

Hint 1. Substituting δ_k and δ_ω

Substitute $\delta_k = k_1 - k_2$ and $\delta_\omega = \omega_1 - \omega_2$ into the expression for $y_{\mathrm{envelope}}(x,t)$.

Express $y_{\mathrm{envelope}}(x,t)$ in terms of δ_k and δ_ω.

ANSWER:

$y_{\mathrm{envelope}}(x,t)$ = $\cos\left(\frac{\delta_k x - \delta_\omega t}{2}\right)$

Hint 2. Traveling wave velocity

What is the velocity v of a traveling wave $y(x,t) = \sin(kx - \omega t)$?

Express your answer in terms of k and ω.

ANSWER:

v = $\frac{\omega}{k}$

ANSWER:

v_{group} = $\frac{\delta_\omega}{\delta_k}$

Correct

$v_{\rm group}$ is called the group velocity because this ratio is the velocity at which the group of waves under a maximum of the envelope function appears to travel. It is technically defined as a derivative (although you found it for a finite difference):

$$v_{\rm group} = \frac{d\omega}{dk}.$$

This contrasts with the velocity of propagation of the waves themselves:

$$v_{\rm phase} = \frac{\omega}{k}.$$

Information (e.g., applied as amplitude modulation of the wave, that is, as a variation in the envelope) travels at the group velocity. No information can be sent at the phase velocity of a wave, which therefore can exceed the speed of light. (Einstein's Special Theory of Relativity implies that neither a physical body nor information can travel faster than light.)

Score Summary:

Your score on this assignment is 102%.
You received 40.8 out of a possible total of 40 points.