Lecture 31
(Ampere’s Law)

Physics 161-01 Spring 2012

Douglas Fields

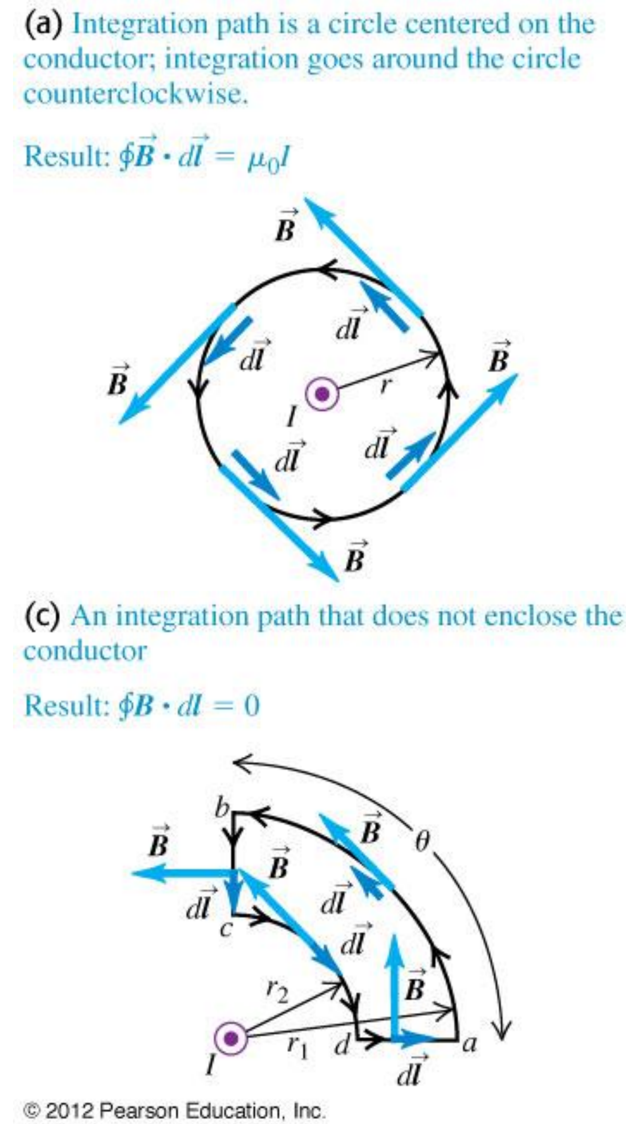
Properties of the Magnetic Field

- So, we found that for two different closed paths, the integral of B dotted into the path element gave two different answers:

$$\int_{\text{Closed Path}} \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\int_{\text{Closed Path}} \vec{B}\cdot d\vec{l} = 0$$



Ampere’s Law

- We can encompass both ideas into one equation:

$$\int_{\text{Closed Path}} \vec{B}\cdot d\vec{l} \equiv \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

- This is known as Ampere’s Law.
- It is fundamental, but incomplete at this point.
- We will have to return to that later.

More General Check

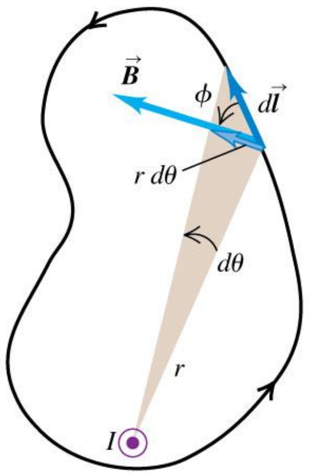
- We can see that Ampere's Law works for any shape closed path since dl is always equal to rdθ.

$$\oint_{\text{Closed Path}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

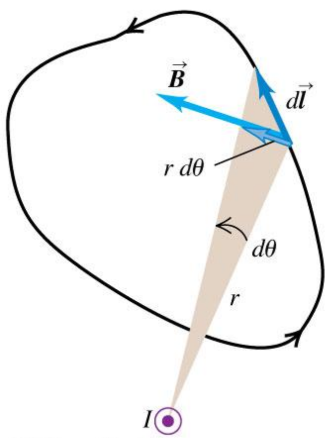
- So, since the magnetic field is proportional to 1/r, those factors cancel.
- And since it is a closed loop, the integral around the path gives no net change in theta.

$$\oint_{\text{Closed Path}} \vec{B}\cdot d\vec{l} = 0$$

(a)

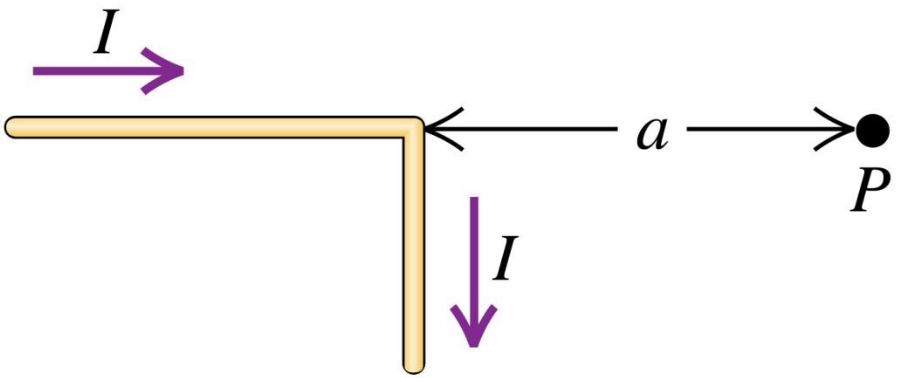


(b)



CPS 31-1

The wire shown here is infinitely long and has a 90° bend. If current flows in the wire as shown, what is the direction of the magnetic field at *P* due to the current?



A. to the right

B. to the left

C. out of the plane of the figure

D. into the plane of the figure

E. none of these

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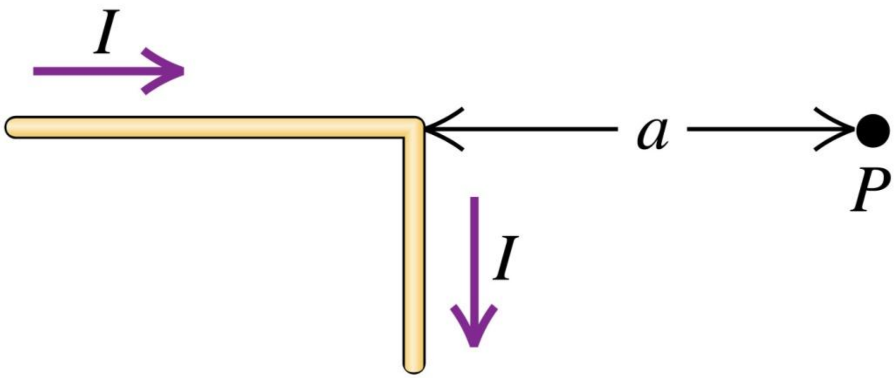

A. to the right

B. to the left

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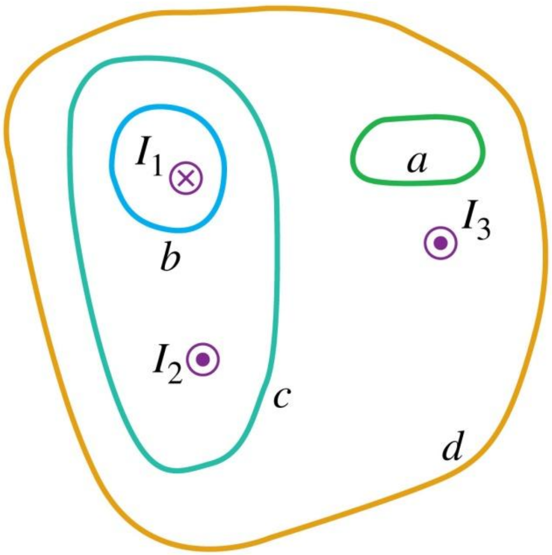
CPS 31-2

The figure shows, in cross section, three conductors that carry currents perpendicular to the plane of the figure.

If the currents I_1, I_2, and I_3 all have the same magnitude, for which path(s) is the line integral of the magnetic field equal to zero?



A. path *a* only
B. paths *a* and *c*
C. paths *b* and *d*
D. paths *a*, *b*, *c*, and *d*
E. The answer depends on whether the integral goes clockwise or counterclockwise around the path.

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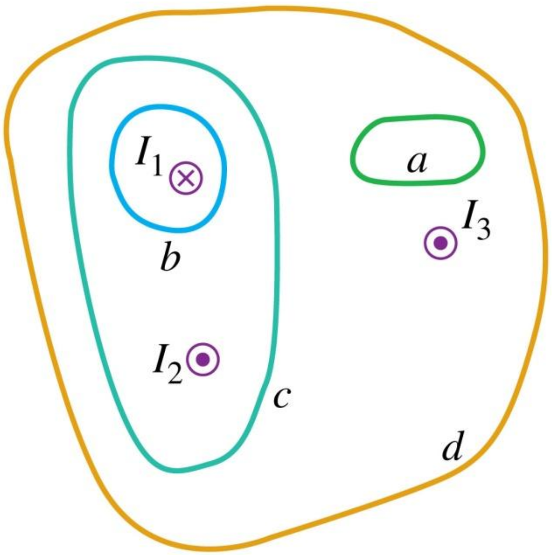

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Applying Ampere's Law

- We can use Ampere's Law now to calculate the magnetic field from certain current configurations.
- Let's start simple – an infinite wire.

$$\int_{\text{Closed Path}} \vec{B} \cdot d\vec{l} = \mu_0 I$$

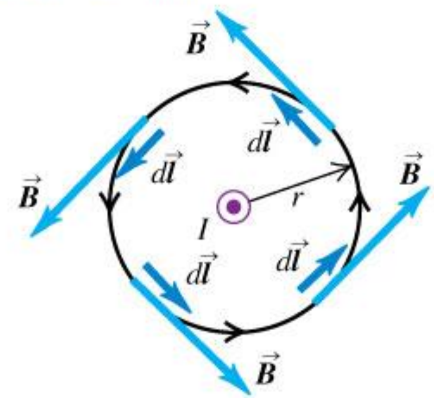
- Because of cylindrical symmetry, we can state that the B-field is the same everywhere on a circular path centered on the current.
- So,

$$\int_{\text{Closed Path}} \vec{B} \cdot d\vec{l} = \int_{\text{Circle}} \vec{B} \cdot d\vec{l} = B \int_{\text{Circle}} dl = 2\pi r B = \mu_0 I \Rightarrow$$

$$B = \frac{\mu_0 I}{2\pi r}$$

(a) Integration path is a circle centered on the conductor; integration goes around the circle counterclockwise.

Result: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

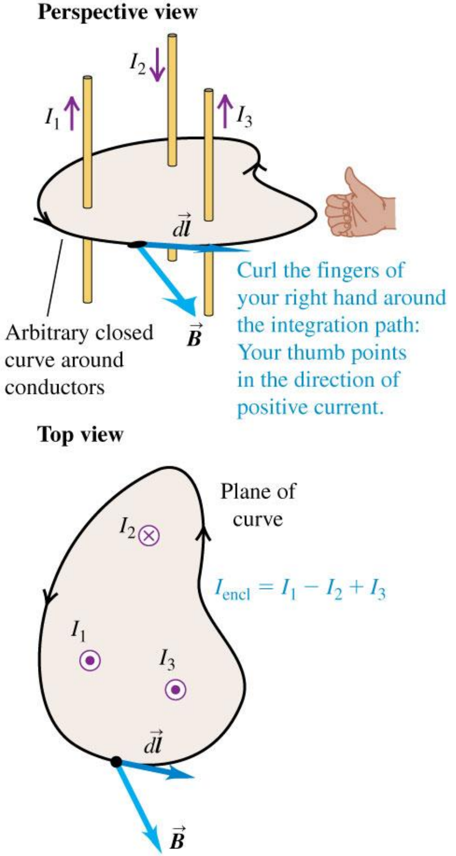


Applying Ampere's Law

- In a similar fashion to Gauss's Law, Ampere's Law is always true, but not always very useful to find the magnetic field from a current.

$$\oint_{\text{Closed Path}} \vec{B}\cdot d\vec{l} = \mu_0 I$$

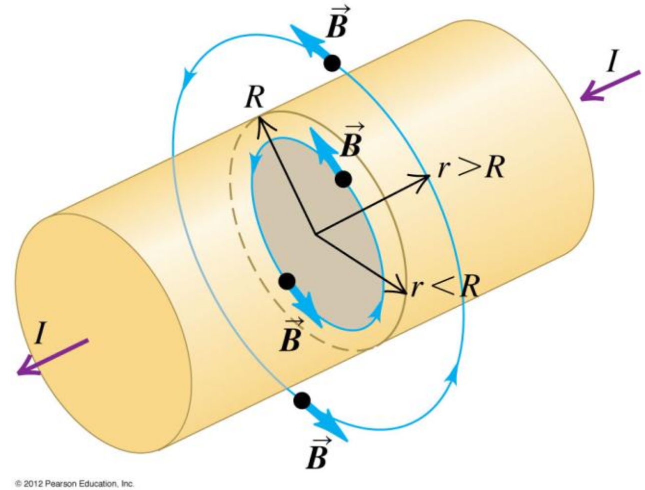
- Notice that in the figure to the right, the magnetic field is not the same at every point on the path (both in magnitude and direction relative to the path).
- But, there are a class of geometries that we can apply Ampere's Law to, where symmetry will allow us to use it to calculate the magnetic field.



Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

Applying Ampere's Law

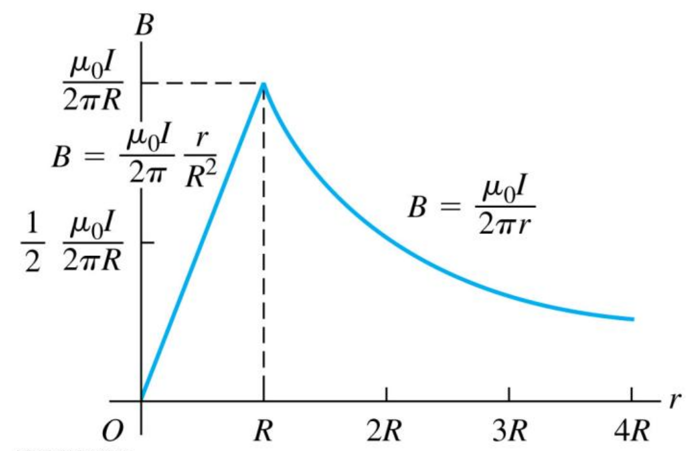
- We can look at the magnetic field inside a conducting wire with some (cylindrically symmetric) current density:



$$\oint_{\text{Closed Path}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}} \Rightarrow$$

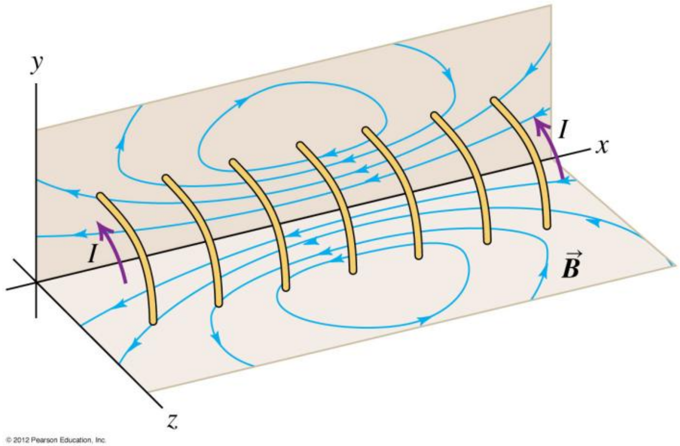
$$\oint_{\text{Circle}} \vec{B}\cdot d\vec{l} = 2\pi r B = \mu_0 I_{\text{enc}} = \begin{cases} \mu_0 J\left(\pi r^2\right) & \text{inside wire} \\ \mu_0 I & \text{outside wire} \end{cases}$$

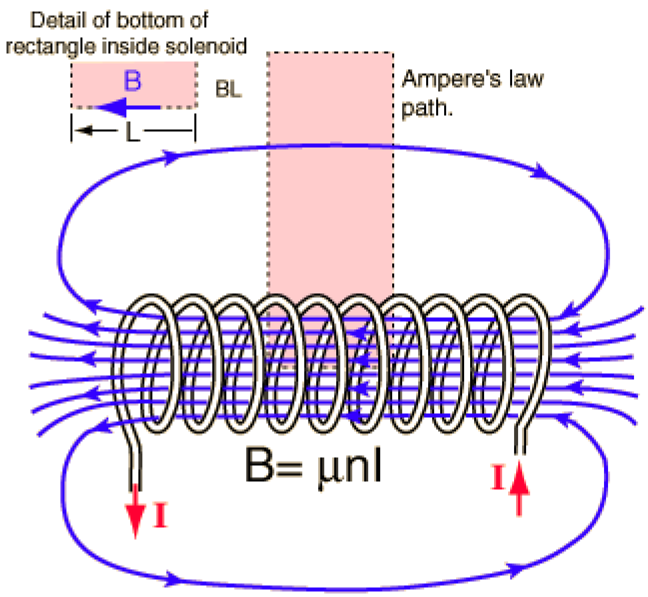
$$B = \begin{cases} \dfrac{\mu_0 J r}{2} & \text{inside wire} \\ \dfrac{\mu_0 I}{2\pi r} & \text{outside wire} \end{cases}$$



Applying Ampere's Law

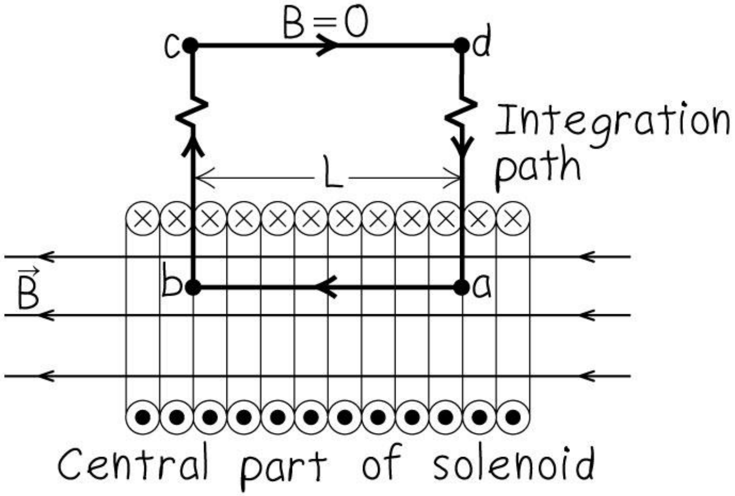
- A solenoid can be considered a stack of current loops.
- As you stack them closer together and increase the number of them, the magnetic field outside the solenoid gets weaker.
- It has cylindrical symmetry, but we want to use the symmetry along the axis for Ampere's Law.





Applying Ampere's Law

- If we have a current, I, and N turns per unit length, L, then:



$$\int_{\text{Closed Path}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}} \Rightarrow$$

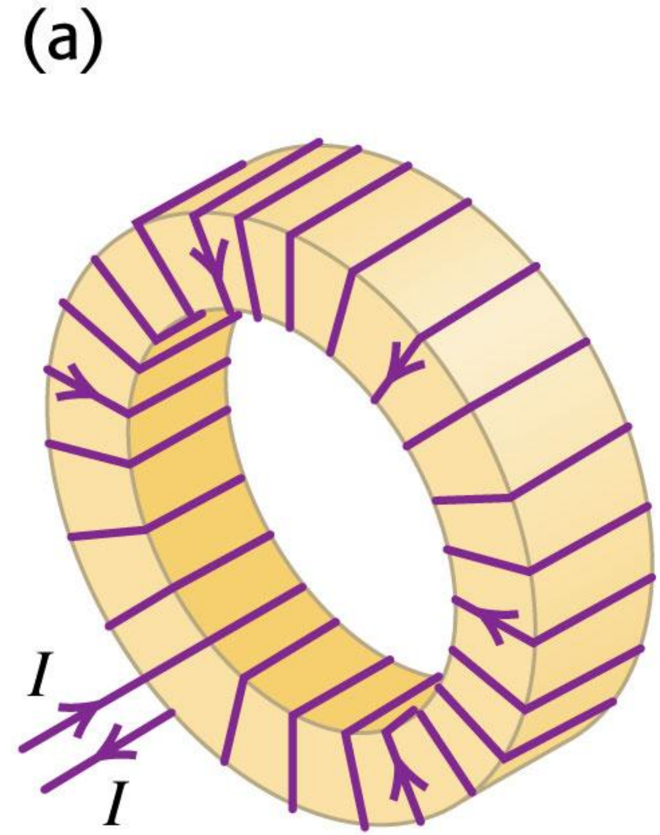
$$\int_{a\to b} \vec{B}\cdot d\vec{l} + \int_{b\to c} \vec{B}\cancel{\cdot} d\vec{l} + \int_{c\to d} \cancel{\vec{B}}\cdot d\vec{l} + \int_{d\to a} \vec{B}\cancel{\cdot} d\vec{l} = \mu_0 N I \Rightarrow$$

$$BL = \mu_0 N I \Rightarrow$$

$$B = \mu_0 \frac{N}{L} I$$

Applying Ampere's Law

- Another Current configuration that has a symmetry that we can take advantage of is a toroid.
- It has a phi-angle symmetry.
- To apply Ampere's Law to calculate the magnetic field, we find a path where we expect the B-field to be constant.



Applying Ampere's Law

- We will use a circle in the interior of the toroid.

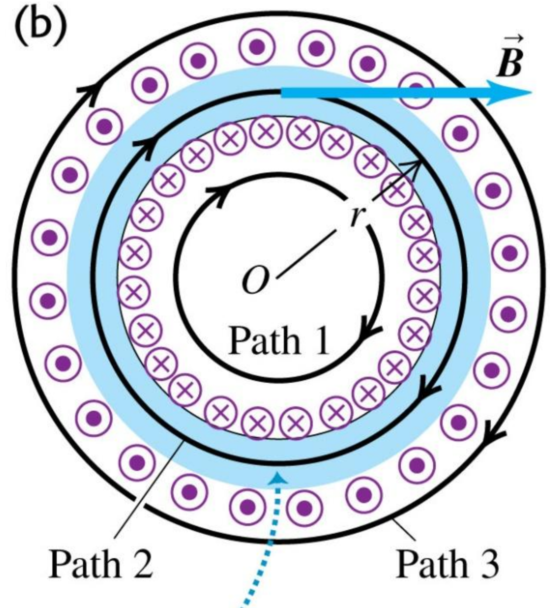
$$\int_{\text{Closed Path}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} \Rightarrow$$

$$\int_{\text{Circle}} \vec{B} \cdot d\vec{l} = \mu_0 NI \Rightarrow$$

$$B 2\pi r = \mu_0 NI \Rightarrow$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

- Note that for a toroid, the field is not constant over the interior.



The magnetic field is confined almost entirely to the space enclosed by the windings (in blue).