

# Scheduling -Phase1

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# Variables and Domains:

- *N: Jobs with processing times  $P_1, P_2, \dots, P_N$ .*
- *M: Machines.*

*Variables: Let  $X_{ij}$  represent the start time of the job  $i$  on the machine.*

*Each  $X_{ij}$  is a non-negative integer.*

*Domains: The domain of each variable  $X_{ij}$  is all non-negative integers, representing the start time of the job  $i$  on the machine  $j$ .*

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# Constraints:

**1- Each Job is Assigned to Exactly One Machine:**

$$\sum_{j=1}^M X_{i,j} = 1 \quad \forall i \in \{1, 2, \dots, N\}$$

This ensures that each job is assigned to exactly one machine.

**2- Processing Time Limit Per Machine :**

Sometimes, you might have a constraint on the maximum processing time allowed for each machine. This is not always a part of the problem but can be represented as:

$$\sum_{i=1}^N p_i \cdot X_{i,j} \leq T_{\max} \quad \forall j \in \{1, 2, \dots, M\}$$

where  $p_i$  is the processing time of job  $i$ , and  $T_{\max}$  is the maximum allowed processing time for any machine.

**3-Consistency Constraint:**

Ensure that the start times are consistent with the processing times.

The "Consistency Constraint" in the context of the scheduling problem refers to ensuring that the start times assigned to jobs on machines are compatible with the processing times of those jobs.

The specific form of the consistency constraint may vary based on the details of the problem, but it generally involves making sure that the start time of a job plus its processing time does not conflict with the start time of the next job on the same machine.

Here's an explanation using a mathematical expression:

$$X_{i,j} + \text{Processing\_Time}(i,j) \leq X_{i,j+1}$$

This expression ensures that the start time ( $X_{i,j}$ ) of job  $i$  on machine  $j$  plus the time it takes to process the job ( $\text{processing\_time}(i,j)$ ) is less than or equal to the start time of the next job ( $X_{i,j+1}$ ) on the same machine. This constraint prevents overlapping or conflicting schedules for jobs on the same machine.



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## Objective Function Explanation:

In a scheduling problem, the objective function is an expression that defines the optimization goal. Here the aim is to reduce the completion time of all tasks across all machines. The function-specific structure focuses on completing the task swiftly to finish all tasks as soon, as possible. When optimising to minimize the completion time the algorithm ensures that no single task prolongs excessively potentially causing delays in the schedule. Essentially the goal is to distribute work, among machines and complete all tasks promptly. By minimizing the completion time) the scheduling algorithm strives for a well-distributed and efficient schedule that finishes all tasks early while meeting constraints.

Let's break down the components of the objective function:

Objective Function: minimize  $\max(X_{i,j} + \text{Processing\_Time}(i,j))$

**max:** The maximum function is used to find the maximum value among a set of values. In this context, it is applied to the completion times of different jobs on different machines.

**$X_{i,j}$  :** This represents the start time of job  $i$  on machine  $j$  . Each term  $X_{i,j}$  is associated with a specific job and machine.

**Processing\_Time( $i,j$ ):** This term represents the processing time required for job  $i$  on machine  $j$ . it is added to the corresponding start time to calculate the completion time for that particular job on that specific machine.