HEI Scheduling constraints problems

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Notations

- G denotes the set of all group g; T denotes the set of all teacher, C denotes the set of all course.
- All element of the set of all awarded courses AC will be indexed. In other words :
$$ac_i \in AC,\ i \in \mathbb{N}$$
- Let t_{ac_i} and c_{ac_i} be the corresponding teacher and the corresponding course resp. to ac_i.
- Let $STC_{t,c} \subset AC$ be the set of all awarded course which has same teacher and course (Same Course and Teacher):
$$\forall\, i, j \in \mathbb{N} \,:\, (ac_i, ac_j \in STC_{t,c}) \Longleftrightarrow \big(t_{ac_i} = t_{ac_j} = t\big) \text{ and } \big(c_{ac_i} = c_{ac_j} = c\big)$$
- Let $STDC_t \subset AC$ be the set of all awarded course which has same teacher but different course (Same Teacher but Different Course) :
$$\forall\, i, j \in \mathbb{N} \,:\, (ac_i, ac_j \in STDC_t) \Longleftrightarrow \big(t_{ac_i} = t_{ac_j} = t\big) \text{ and } \big(c_{ac_i} \neq c_{ac_j}\big)$$
- Let $AC_g \subset AC$, the set of all awarded course related to a group g.

About the Constraints

Constraint 1. *A teacher can't teach two or more different groups at the same time for a same course if it is face-to-face session but can if it is a video conference session.*

Let $pres(d, STC_{t,c}) \in \{-1; 1\}$,be a an indicator constant.

$$pres(d, STC_{t,c}) = \begin{cases} 1 \text{ , if the course related to } STC_{t,c} \text{ is presential at a given } d \text{ day} \\ -1 \text{ , otherwise} \end{cases}$$

$\forall\ s \in S,\ \forall\ d \in D,\ \forall\ t \in T,\ \forall\ c \in C$:

$$2 \cdot pres(d, STC_{t,c}) \cdot \sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq pres(d, STC_{t,c}) + 1 \tag{1}$$

Example:

Let's take an example for course $c = PROG1$ and teacher $t = DrToky$ and all groups $K1$, $K2$, $K3$, $K4$, $K5$.

We define the set ac_i as follow :

$$\begin{cases} ac_1 = (PROG1, DrToky, K1) \\ ac_2 = (PROG1, DrToky, K2) \\ ac_3 = (PROG1, DrToky, K3) \\ ac_4 = (PROG1, DrToky, K4) \\ ac_5 = (PROG1, DrToky, K5) \end{cases}$$

Thus, the set $STC_{PROG1,DrToky}$ is :

$$STC_{PROG1,DrToky} = \{ac_1, ac_2, ac_3, ac_4, ac_5\}$$

Which means here that all elements of this $STC_{PROG1,DrToky}$ has the same teacher and same course.
Now, if we say that for a d day, the $PROG1$ course will be online, we will have :

$$pres(d, STC_{DrToky,PROG1}) = -1 \quad \Longrightarrow \quad -2\sum_{r\in R}\left(\sum_{ac_i\in STC_{t,c}} o_{ac_i,d,r,s}\right) \leq -1+1$$

$$\Longleftrightarrow \quad -2\cdot\sum_{r\in R}\left(\sum_{ac_i\in STC_{t,c}} o_{ac_i,d,r,s}\right) \leq 0$$

$$\Longleftrightarrow \quad \sum_{r\in R}\left(\sum_{ac_i\in STC_{t,c}} o_{ac_i,d,r,s}\right) \geq 0$$

Which tells us that we can allow simultaneous course of $PROG1$ with $DrToky$ for those groups.

Now, if the course is presential :

$$pres(d, STC_{DrToky,PROG1}) = 1 \quad \Longrightarrow \quad 2\cdot\sum_{r\in R}\left(\sum_{ac_i\in STC_{t,c}} o_{ac_i,d,r,s}\right) \leq 1+1$$

$$\Longleftrightarrow \quad 2\cdot\sum_{r\in R}\left(\sum_{ac_i\in STC_{t,c}} o_{ac_i,d,r,s}\right) \leq 2$$

$$\Longleftrightarrow \quad \sum_{r\in R}\left(\sum_{ac_i\in STC_{t,c}} o_{ac_i,d,r,s}\right) \leq 1$$

which tells us that only one session of this course is allowed.

SECOND REFORMULATION :

If $pres(d, STC_{t,c})$ is the binary variable such as :

$$pres(d, STC_{t,c}) = \begin{cases} 1 \text{ , if the course related to } STC_{t,c} \text{ is presential ata given } d \text{ day} \\ 0 \text{ , otherwise} \end{cases}$$

which marks whether all awarded courses related to $STC_{t,c}$ are presentials or not.

Let $M \in \mathbb{N}^*$ be a majoration constant.

Therefore :

$$\sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M(1 - pres(d, STC_{t,c})) \tag{2}$$

<u>**Example:**</u>

Now, let's take the previous example again.
With this second inequality, if the course is presential, we have :

$$\begin{aligned} pres(d, STC_{t,c}) = 1 \quad &\Longrightarrow \quad \sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M(1-1) \\ &\Longleftrightarrow \quad \sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 \end{aligned}$$

Which tells us again that only one session of that course is allowed at that given time slot.

Now, if the course is online :

$$\begin{aligned} pres(d, STC_{t,c}) = 0 \quad &\Longrightarrow \quad \sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M(1-0) \\ &\Longleftrightarrow \quad \sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M \end{aligned}$$

which means that multiple session of that course (depending on M) is allowed

Constraint 2. *A teacher cannot simultaneously teach two or more different courses.*

$\forall\ s \in S,\ \forall\ d \in D, \forall\ t \in T$:

$$\sum_{r \in R} \left(\sum_{ac_i \in STDC_t} o_{ac_i,d,r,s} \right) \leq 1 \tag{3}$$

Constraint 3. *A group cannot simultaneously have two or more different course sessions.*

$\forall\ g \in G,\ \forall\ s \in S,\ \forall\ d \in D:$

$$\sum_{r \in R} \left(\sum_{ac_i \in AC_g} o_{ac_i,d,r,s} \right) \leq 1 \tag{4}$$

Constraint 4. *A group should have 2 hours of break per day.*

$\forall\ g \in G,\ \forall\ d \in D:$

$$\sum_{ac_i \in AC_g} \left(\sum_{r \in R} \sum_{s \in S} o_{ac_i,d,r,s} \right) \leq 3 \tag{5}$$

Constraint 5. *Only one session per course in a day.*

$\forall\ ac_i \in AC,\ i \in I,\ \forall\ d \in D:$

$$\left(\sum_{r \in R} \sum_{s \in S} o_{ac_i,d,r,s} \right) \leq 1 \tag{6}$$

Constraint 6. *Give one day without the same course after a session of this course.*

$\forall\ d \in D,\ \forall ac_i \in AC,\ i \in I:$

$$\left(\sum_{r \in R} \sum_{s \in S} o_{ac_i,d+1_d,r,s} \right) + \left(\sum_{r \in R} \sum_{s \in S} o_{ac_i,d,r,s} \right) \leq 1 \tag{7}$$

where 1_d is the unit of day.

Constraint 7. *Suitable room for every course session.*

- Let g_{ac_i} be the corresponding group to a ac_i.
- Let $s_{g_{ac_i}}$ be the group size of a group g_{ac_i}.
- Let rc_r be the room capacity of a room r.

$\forall ac_i \in AC, \forall\ r \in R,\ \forall\ s \in S, \forall\ d \in D$

$$o_{ac_i,d,r,s} rc_r \geq s_{g_{ac_i}} \tag{8}$$

Constraint 8. *Finish the total hour of every course.*

Let $D(ac_i, d_q)$ be the total duration of a ac_i within a time interval $[d_0, d_q] \subset D,\ d_0 \leq d_q$.

$\forall\ ac_i \in AC,\ i \in I :$

$$\sum_{d=d_0}^{d_q} \sum_{r \in R} \sum_{s \in S} o_{ac_i,d,r,s} = D(ac_i, d_q) \tag{9}$$