HEI Scheduling constraints problems

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Notations

- G denotes the set of all group g.
- T denotes the set of all teacher t.
- C denotes the set of all course c.
- R the set of all room r.
- All element of the set of all awarded courses AC will be indexed. In other words :
$$ac_i \in AC,\ i \in \mathbb{N}$$
- Let $t(ac_i)$ and $c(ac_i)$ be the corresponding teacher and the corresponding course resp. to ac_i.
- Let $STC_{t,c} \subset AC$ be the set of all awarded course which has same teacher and course (Same Teacher and Course):

$$\forall\, i, j \in \mathbb{N}\ :\ (ac_i, ac_j \in STC_{t,c}) \Longleftrightarrow [t(ac_i) = t(ac_j) = t] \text{ and } [c(ac_i) = c(ac_j) = c]$$

<u>Example</u> :

$t = DrToky$, $c = PROG1$, and $G = \{K1, K2, K3\}$. So, we can have $STC_{DrToky,PROG1} = \{a_1, a_2, a_3\}$

$$\text{where : } \begin{cases} ac_1 = (PROG1, DrToky, K1) \\ ac_2 = (PROG1, DrToky, K2) \\ ac_3 = (PROG1, DrToky, K3) \end{cases}$$

- Let $STDC_t \subset AC$ be the set of all awarded course which has same teacher but different course (Same Teacher but Different Course) :

  $$\forall\, i, j \in \mathbb{N} \;:\; (ac_i, ac_j \in STDC_t) \Longleftrightarrow [t(ac_i) = t(ac_j) = t] \;\text{ and }\; [c(ac_i) \neq c(ac_j)]$$

  Example :
  $t = DrToky$, and $G = \{K1, K2, K3\}$. So, we can have
  $STDC_{DrToky} = \{a_1, a_2, a_3, a_4\}$

  $$\text{where : } \begin{cases} ac_1 = (PROG1, DrToky, K1) \\ ac_2 = (WEB1, DrToky, K1) \\ ac_3 = (DONNEES1, DrToky, K2) \\ ac_4 = (PROG2, DrToky, K3) \end{cases}$$

- Let $AC_g \subset AC$, the set of all awarded course related to a group g. Which means that only ac_i that g as its group will be in the set AC_g.

About the Constraints

Constraint 1. *A teacher can't teach two or more different groups at the same time for a same course if it is face-to-face session but can if it is a video conference session.*

Let $pres(d, STC_{t,c})$ be the binary variable such as :

$$pres(d, STC_{t,c}) = \begin{cases} 1 \text{ , if the course related to } STC_{t,c} \text{ is presential at a given } d \text{ day} \\ 0 \text{ , otherwise} \end{cases}$$

Let $M \in \mathbb{N}^*$ be a constant limiting the number of allowed courses.

Therefore :

$\forall\ s \in S,\ \forall\ d \in D, \forall\ t \in T,\ \forall\ c \in C$:

$$\sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M(1 - pres(d, STC_{t,c})) \tag{1}$$

<u>**Example:**</u>

Let's take an example for course $c = PROG1$ and teacher $t = DrToky$ and all groups $K1$, $K2$, $K3$, $K4$, $K5$.

We define the set ac_i as follow :

$$\begin{cases} ac_1 = (PROG1, DrToky, K1) \\ ac_2 = (PROG1, DrToky, K2) \\ ac_3 = (PROG1, DrToky, K3) \\ ac_4 = (PROG1, DrToky, K4) \\ ac_5 = (PROG1, DrToky, K5) \end{cases}$$

Thus, the set $STC_{PROG1,DrToky}$ is :

$$STC_{PROG1,DrToky} = \{ac_1, ac_2, ac_3, ac_4, ac_5\}$$

Which means here that all elements of this $STC_{PROG1,DrToky}$ has the same teacher and same course.

Now, if we say that for a d day, the $PROG1$ course will be presential, we will have :

$$pres(d, STC_{t,c}) = 1 \implies \sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M(1-1)$$

$$\iff \sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1$$

Which tells us that only one session of that course is allowed at that given time slot.

Now, if the course is online :

$$pres(d, STC_{t,c}) = 0 \implies \sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M(1-0)$$

$$\iff \sum_{r \in R} \left(\sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M$$

which means that multiple session of that course (depending on M) is allowed

Constraint 2. *A teacher cannot simultaneously teach two or more different courses.*

$\forall\ s \in S,\ \forall\ d \in D, \forall\ t \in T$:

$$\sum_{r \in R} \left(\sum_{ac_i \in STDC_t} o_{ac_i,d,r,s} \right) \leq 1 \tag{2}$$

Example:

Let's take an example for courses $c_1 = PROG1$, $c_2 = WEB1$ and teacher

$t = DrToky$ and all groups $K1$, $K2$.

Now, the statement is that $DrToky$ cannot teach $WEB1$ and $PROG1$ at the same time. So, we have to make sure that **event if we have different courses**, $DrToky$ don't teach both of those courses at a given slot s.

$$\text{Let :} \begin{cases} ac_1 = (PROG1, DrToky, K1) \\ ac_2 = (WEB1, DrToky, K2) \end{cases}$$

It is clear that ac_1, $ac_2 \in STDC_{DrToky}$ and we can only allow session for ac_1 **or** for ac_2.
It is abvious that if the awarded courses don't have the same teacher but have the same course, we can allow both of them .

Constraint 3. *Only one session is allowed in a room at a given time slot.*

$\forall\ r \in R,\ \forall\ s \in S,\ \forall\ d \in D :$

$$\sum_{ac_i \in AC} o_{ac_i,d,r,s} \leq 1 \tag{3}$$

Constraint 4. *A group cannot simultaneously have two or more different course sessions.*

$\forall\ g \in G,\ \forall\ s \in S,\ \forall\ d \in D :$

$$\sum_{r \in R} \left(\sum_{ac_i \in AC_g} o_{ac_i,d,r,s} \right) \leq 1 \tag{4}$$

Example:

Let's take for example the group $J1$. This group has for example the $MGT2$, $PROG3$ courses.

$$\text{Let :} \begin{cases} ac_1 = (MGT2, DrLou, J1) \\ ac_2 = (PROG3, MrRyan, J1) \end{cases}$$

It is abvious that this doesn't concern any other group and we can't allow $J1$ to have ac_1 and ac_2 at the same time.

Constraint 5. *A group should have 2 hours of break per day.*

$\forall\ g \in G,\ \forall\ d \in D:$

$$\sum_{ac_i \in AC_g} \left(\sum_{r \in R} \sum_{s \in S} o_{ac_i,d,r,s} \right) \leq 3 \tag{5}$$

It is clear that this does not concern at all the other groups apart from the given group g.

Constraint 6. *Only one session per course in a day.*

$\forall\ ac_i \in AC,\ \forall\ d \in D:$

$$\left(\sum_{r \in R} \sum_{s \in S} o_{ac_i,d,r,s} \right) \leq 1 \tag{6}$$

Constraint 7. *Give one day without the same course after a session of this course.*

$\forall\ d \in D,\ \forall ac_i \in AC:$

$$\left(\sum_{r \in R} \sum_{s \in S} o_{ac_i,d+1_d,r,s} \right) + \left(\sum_{r \in R} \sum_{s \in S} o_{ac_i,d,r,s} \right) \leq 1 \tag{7}$$

where 1_d is the unit of day. So, $d + 1_d$ means the next day of d

This inequality states that the number of occurences of an awarded course in day d and the following day $d + 1_d$ should be at most one. Which means that whether the awarded course is scheduled the d day or it is scheduled in the next day.

Constraint 8. *Suitable room for every course session.*

- Let g_{ac_i} be the corresponding group to a ac_i.

- Let $size(g_{ac_i})$ be the group size of a group g_{ac_i}.
- Let $capacity(r)$ be the room capacity of a room r.

$\forall ac_i \in AC, \forall\ r \in R,\ \forall\ s \in S, \forall\ d \in D$

$$o_{ac_i,d,r,s} \cdot capacity(r) \geq size(g_{ac_i}) \tag{8}$$

Constraint 9. *Finish the total hour of every course.*

Let $D(ac_i, d_{start}, d_{end})$ be the needed total duration in hour of a ac_i within a time interval $[d_{start}, d_{end}] \subset D,\ d_{start} \leq d_{end}$.

$\forall\ ac_i \in AC$:

$$2 \cdot \left(\sum_{d=d_{start}}^{d_{end}} \sum_{r \in R} \sum_{s \in S} o_{ac_i,d,r,s} \right) = D(ac_i, d_{start}, d_{end}) \tag{9}$$

Example:

$ac_1 = (PROG1, DrToky, K1)$, $d_{start} = 10$ march 2025 and $d_{end} = 17$ march 2025.

If we need to have 10 hours of ac_1, we have then : $D(ac_1, d_{start}, d_{end}) = 10$.

Therefore, we should have :

$$2 \cdot \left(\sum_{d=d_{start}}^{d_{end}} \sum_{r \in R} \sum_{s \in S} o_{ac_1,d,r,s} \right) = 10)$$

$$\iff \quad \sum_{d=d_{start}}^{d_{end}} \sum_{r \in R} \sum_{s \in S} o_{ac_1,d,r,s} = 5)$$

Which gives us that we should have 5 session of ac_1 during the interval of 10 march and 17 march to get those required 10 hours since $o_{ac_1,d,r,s}$ gives us 2 hours of sessions.