

# HEI Scheduling constraints problems

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## Notations

- $G$  denotes the set of all group  $g$ .
- $T$  denotes the set of all teacher  $t$ .
- $C$  denotes the set of all course  $c$ .
- $R$  the set of all room  $r$ .
- All element of the set of all awarded courses  $AC$  will be indexed. In other words :

$$ac_i \in AC, i \in \mathbb{N}$$

- Let  $t(ac_i)$  and  $c(ac_i)$  be the corresponding teacher and the corresponding course resp. to  $ac_i$ .
- Let  $STC_{t,c} \subset AC$  be the set of all awarded course which has same teacher and course (Same Course and Teacher):

$$\forall i, j \in \mathbb{N} : (ac_i, ac_j \in STC_{t,c}) \iff [t(ac_i) = t(ac_j) = t] \text{ and } [c(ac_i) = c(ac_j) = c]$$

Example :

$t = DrTokyo$ ,  $c = PROG1$ , and  $G = \{K1, K2, K3\}$ . So, we have  
 $STC_{DrTokyo, PROG1} = \{a_1, a_2\}$

$$\text{where : } \begin{cases} ac_1 = (PROG1, DrTokyo, K1) \\ ac_2 = (PROG1, DrTokyo, K2) \\ ac_3 = (PROG1, DrTokyo, K3) \end{cases}$$

- Let  $STDC_t \subset AC$  be the set of all awarded course which has same teacher but different course (Same Teacher but Different Course) :

$$\forall i, j \in \mathbb{N} : (ac_i, ac_j \in STDC_t) \iff [t(ac_i) = t(ac_j) = t] \text{ and } [c(ac_i) \neq c(ac_j)]$$

Example :

$t = DrToky$ , and  $G = \{K1, K2, K3\}$ . So, we have  
 $STDC_{DrToky} = \{a_1, a_2, a_3, a_4\}$

$$\text{where : } \begin{cases} ac_1 = (PROG1, DrToky, K1) \\ ac_2 = (WEB1, DrToky, K1) \\ ac_3 = (DONNEES1, DrToky, K2) \\ , ac_4 = (PROG2, DrToky, K3) \end{cases}$$

- Let  $AC_g \subset AC$ , the set of all awarded course related to a group  $g$ . Which means that only  $ac_i$  that  $g$  as its group will be in the set  $AC_g$ .

## About the Constraints

**Constraint 1.** *A teacher can't teach two or more different groups at the same time for a same course if it is face-to-face session but can if it is a video conference session.*

Let  $pres(d, STC_{t,c})$  be the binary variable such as :

$$pres(d, STC_{t,c}) = \begin{cases} 1, & \text{if the course related to } STC_{t,c} \text{ is presential at a given } d \text{ day} \\ 0, & \text{otherwise} \end{cases}$$

Let  $M \in \mathbb{N}^*$  be a constant limiting the number of allowed courses.

Therefore :

$$\sum_{r \in R} \left( \sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M(1 - pres(d, STC_{t,c})) \quad (1)$$

### Example:

Let's take an example for course  $c = PROG1$  and teacher  $t = DrTokyo$  and all groups  $K1, K2, K3, K4, K5$ .

We define the set  $ac_i$  as follow :

$$\begin{cases} ac_1 = (PROG1, DrTokyo, K1) \\ ac_2 = (PROG1, DrTokyo, K2) \\ ac_3 = (PROG1, DrTokyo, K3) \\ ac_4 = (PROG1, DrTokyo, K4) \\ ac_5 = (PROG1, DrTokyo, K5) \end{cases}$$

Thus, the set  $STC_{PROG1,DrTokyo}$  is :

$$STC_{PROG1,DrTokyo} = \{ac_1, ac_2, ac_3, ac_4, ac_5\}$$

Which means here that all elements of this  $STC_{PROG1,DrTokyo}$  has the same teacher and same course.

Now, if we say that for a  $d$  day, the  $PROG1$  course will be presential, we

will have :

$$\begin{aligned}
pres(d, STC_{t,c}) = 1 &\implies \sum_{r \in R} \left( \sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M(1 - 1) \\
&\iff \sum_{r \in R} \left( \sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1
\end{aligned}$$

Which tells us that only one session of that course is allowed at that given time slot.

Now, if the course is online :

$$\begin{aligned}
pres(d, STC_{t,c}) = 0 &\implies \sum_{r \in R} \left( \sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M(1 - 0) \\
&\iff \sum_{r \in R} \left( \sum_{ac_i \in STC_{t,c}} o_{ac_i,d,r,s} \right) \leq 1 + M
\end{aligned}$$

which means that multiple session of that course (depending on  $M$ ) is allowed

**Constraint 2.** *A teacher cannot simultaneously teach two or more different courses.*

$$\forall s \in S, \forall d \in D, \forall t \in T :$$

$$\sum_{r \in R} \left( \sum_{ac_i \in STDC_t} o_{ac_i,d,r,s} \right) \leq 1 \quad (2)$$

### Example:

Let's take an example for courses  $c_1 = PROG1$ ,  $c_2 = WEB1$  and teacher

$t = DrToky$  and all groups  $K1, K2$ .

Now, the statement is that  $DrToky$  cannot teach  $WEB1$  and  $PROG1$  at the same time. So, we have to make sure that **event if we have different courses**,  $DrToky$  don't teach both of those courses at a given slot  $s$ .

$$\text{Let : } \begin{cases} ac_1 = (PROG1, DrToky, K1) \\ ac_2 = (WEB1, DrToky, K2) \end{cases}$$

It is clear that  $ac_1, ac_2 \in STDC_{DrToky}$  and we can only allow session for  $ac_1$  **or** for  $ac_2$ .

It is obvious that if the awarded courses don't have the same teacher but have the same course, we can allow both of them .

**Constraint 3.** *Only one session is allowed in a room at a given time slot.*

$$\forall r \in R, \forall s \in S, \forall d \in D :$$

$$\sum_{ac_i \in AC} o_{ac_i, d, r, s} \leq 1 \quad (3)$$

**Constraint 4.** *A group cannot simultaneously have two or more different course sessions.*

$$\forall g \in G, \forall s \in S, \forall d \in D :$$

$$\sum_{r \in R} \left( \sum_{ac_i \in AC_g} o_{ac_i, d, r, s} \right) \leq 1 \quad (4)$$

### Example:

Let's take for example the group  $J1$ . This group has for example the  $MGT2, PROG3$  courses.

$$\text{Let : } \begin{cases} ac_1 = (MGT2, DrLou, J1) \\ ac_2 = (PROG3, MrRyan, J1) \end{cases}$$

It is obvious that this doesn't concern any other group and we can't allow  $J1$  to have  $ac_1$  and  $ac_2$  at the same time.

**Constraint 5.** *A group should have 2 hours of break per day.*

$\forall g \in G, \forall d \in D :$

$$\sum_{ac_i \in AC_g} \left( \sum_{r \in R} \sum_{s \in S} o_{ac_i, d, r, s} \right) \leq 3 \quad (5)$$

It is clear that this does not concern at all the other groups apart from the given group  $g$ .

**Constraint 6.** *Only one session per course in a day.*

$\forall ac_i \in AC, \forall d \in D :$

$$\left( \sum_{r \in R} \sum_{s \in S} o_{ac_i, d, r, s} \right) \leq 1 \quad (6)$$

**Constraint 7.** *Give one day without the same course after a session of this course.*

$\forall d \in D, \forall ac_i \in AC, i \in I :$

$$\left( \sum_{r \in R} \sum_{s \in S} o_{ac_i, d+1_d, r, s} \right) + \left( \sum_{r \in R} \sum_{s \in S} o_{ac_i, d, r, s} \right) \leq 1 \quad (7)$$

where  $1_d$  is the unit of day. So,  $d + 1_d$  means the next day of  $d$

This inequality states that the number of occurrences of an awarded course in day  $d$  and the following day  $d + 1_d$  should be at most one. Which means that whether the awarded course is scheduled the  $d$  day or it is scheduled in the next day.

**Constraint 8.** *Suitable room for every course session.*

- Let  $g_{ac_i}$  be the corresponding group to a  $ac_i$ .

- Let  $size(g_{ac_i})$  be the group size of a group  $g_{ac_i}$ .
- Let  $capacity(r)$  be the room capacity of a room  $r$ .

$$\forall ac_i \in AC, \forall r \in R, \forall s \in S, \forall d \in D$$

$$o_{ac_i,d,r,s} \cdot capacity(r) \geq size(g_{ac_i}) \quad (8)$$

**Constraint 9.** *Finish the total hour of every course.*

Let  $D(ac_i, d_{start}, d_{end})$  be the needed total duration in hour of a  $ac_i$  within a time interval  $[d_{start}, d_{end}] \subset D$ ,  $d_{start} \leq d_{end}$ .

$$\forall ac_i \in AC :$$

$$2 \cdot \left( \sum_{d=d_{start}}^{d_{end}} \sum_{r \in R} \sum_{s \in S} o_{ac_i,d,r,s} \right) = D(ac_i, d_{end}) \quad (9)$$

**Example:**

$ac_1 = (PROG1, DrTokyo, K1)$ ,  $d_{start} = 10$  march 2025 and  $d_{end} = 17$  march 2025.

If we need to have 10 hours of  $ac_1$ , we have then :  $D(ac_1, d_{end}) = 10$ .

Therefore, we should have :

$$\begin{aligned} 2 \cdot \left( \sum_{d=d_{start}}^{d_{end}} \sum_{r \in R} \sum_{s \in S} o_{ac_1,d,r,s} \right) &= 10) \\ \iff \sum_{d=d_{start}}^{d_{end}} \sum_{r \in R} \sum_{s \in S} o_{ac_1,d,r,s} &= 5) \end{aligned}$$

Which gives us that we should have 5 session of  $ac_1$  during the interval of 10 march and 17 march to get those required 10 hours.