Notes on Galileo Galilei Theorems

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Abstract

The purpose of this article is to explore some of *Galileo Galilei* theorems in his famous book on dynamic mechanics *The Two New Sciences*[**?**] and do their demonstrations in a modern way.

Introduction

In his book *The two New Sciences*

1 On the Salviati lemma

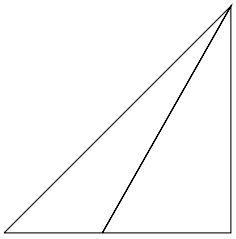
On the *Third day, Change of position, [De Motu Locali]*, Galileo deals with the notions of motions, acceleration, and more precisely, he fonded the *Dynamical mechanics*.

One of the important theorem he stated in this *Third day* is the one that *Salviati* states after the *Scholium* of the *Corollary II.* on page 183-184

Theorem 1 *If a body falls freely along smooth planes inclined at any angle whatsoever, but of the same height, the speeds with which it reaches the bottomare the same.*

Galileo's demonstration of this theorem is very old and might be difficult to apprehend easily, i.e, the expressions and the notations are very old. But hereafter, I'm going to proove that theorem in a modern notation and modern expression which will make the demonstration easy to understand and to apprehend easily.

Proof. Let ABC be a rectangle triangle.



Let us lay down :

$$AC = h;$$
$$AB = x_1;$$
$$AE = x_2;$$

As we know :

$$sin(\alpha) = \frac{h}{x_1};$$
$$sin(\beta) = \frac{h}{x_2};$$

That gives us :

$$h = x_1 sin(\alpha);$$
$$h = x_2 sin(\beta);$$

As we know $x_1 = \frac{1}{2} \cdot a_1 \cdot t_1^2$ where a_1 is the acceleration on the plane AB, and t_1 the time required to travers it; and $x_2 = \frac{1}{2} \cdot a_2 \cdot t_2^2$. where a_2 the acceleration on the plane AE, and t_2 the time required to travers that plane.