Magnetic Flux

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Magnetic Flux

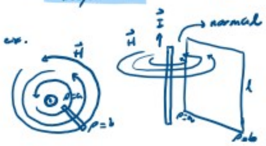
Magnetic Flux Density

$\vec{B} = \mu_0 \vec{H} \longrightarrow \mu_0 = 4\pi \times 10^{-7}$ H/m (Permeability

$\varphi = \iint_S \vec{B} \cdot d\vec{S}$

$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\varphi$

ex.



$B = \frac{\varphi}{S}$

$\varphi = \iint_S \vec{B} \cdot d\vec{S} \longrightarrow d\vec{S} = d\rho\, dz\, \hat{a}_\varphi$

$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\varphi$

$$\varphi = \iint_S \frac{\mu_0 I}{2\pi\rho} \hat{a}_\varphi\, d\rho\, dz\, \hat{a}_\varphi = \frac{\mu_0 I}{2\pi} \int_0^l dz \int_a^b \frac{d\rho}{\rho} = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{b}{a}\right)$$

Magnetic Flux for Closed Surface

$\varphi = \oint_S \vec{B} \cdot d\vec{S} = 0$

→ Total Flux = 0

Divergence Theorem

$\oint_S \vec{A} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{A}$

- Integral of normal comp. of vector over closed surface = Integral of divergence of this vector field throughout volume enclosed by closed surface

$\nabla \cdot \vec{A} = \frac{\partial \vec{A}}{\partial x} + \frac{\partial \vec{A}}{\partial y} + \frac{\partial \vec{A}}{\partial z}$

↳ $Q = \iint_S \vec{B} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{B}$

$\nabla \cdot \vec{B} = 0$ → If divergence of vector field ≠ 0 then vector field ≠ magnetic field

ex. $\vec{A} = a\cos(x)\,\hat{a}_x + by\sin(x)\,\hat{a}_y$

$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$= -a\sin(x) + b\sin(x) + 0 = 0$

$\therefore a = b$ for magnetic field

ex. Motor

→ Flux = greatest @ 90°

☆ webwork 8

Assignment 8: Problem 1

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(1 point)
Let S be the surface formed by combining the cylinder given by $x^2+y^2=1, 0 \le z \le 1$, with the hemispherical cap defined by $x^2+y^2+(z-1)^2=1, z \ge 1$. Notice that S is not a closed surface: it has no "bottom".

Make a reasonable sketch of S.

For the vector field $\mathbf{F} = \langle zx + z^2y + 5y,\ z^3yx + 3x,\ z^4x^2 \rangle$, find the flux integral shown below. Use the upward/outward orientation.

ANSWER: $\iint_S (\nabla \times \mathbf{F}) \bullet d\mathbf{S} =$

$\oint_C \vec{F} \cdot d\vec{l} = \iint_S (\vec{\nabla} \times \vec{G}) \cdot \hat{n}\, dS$

Stokes Theorem

$\hat{n} = \langle 0, 0, 1 \rangle$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \langle -3z^2yx,\ x + 2zy,\ z^3y + 3 - z^2 - 5 \rangle$$

$\iint_S (\vec{\nabla} \times \vec{F}) \cdot \hat{n}\, dS = \iint_S z^3y - z^2 - 2\, dS$

→ set $z = 0$: $-2\rho\pi = -2\pi$

Assignment 8: Problem 2

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(1 point)
A solid conductor occupies the cylindrical region $\rho < b, 0 < z < h$, where $b = 6$ mm and $h = 28$ m. A potential difference of 0.15 Volts between the ends at $z = 0$ m and $z = 28$ m drives a steady current in the direction $\mathbf{a}_z$, and this produces a magnetic field intensity inside the conductor given (in

a) $R = \frac{V}{I}$ $H = \frac{I}{2\pi\rho} = 9 \times 10^5 \rho^2 \hat{a}_\varphi$
→ solve I: $I = 1.2215$ A
$R = 0.1228\,\Omega$

b) $\vec{H} = \frac{I}{2\pi\rho}$ $I = 9 \times 10^5 c^3 2\pi$

c) For current density: $\vec{J} = \vec{\nabla} \times \vec{H}$:

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(1 point)
A solid conductor occupies the cylindrical region $\rho \le b$, $0 \le z \le h$, where $b = 6$ mm and $h = 28$ m. A potential difference of 0.15 Volts between the ends at $z = 0$ m and $z = 28$ m drives a steady current in the direction $\mathbf{a}_z$, and this produces a magnetic field intensity inside the conductor given (in units of A/m^2) by

$$\mathbf{H} = H_0\rho^2\mathbf{a}_\phi; \qquad H_0 = 9 \times 10^5.$$

(a) Find the total resistance between $z = 0$ and $z = h$.

ANSWER: $R =$ ______ Ω

(b) Find a formula involving c for $I(c)$, the current flowing in the region where $\rho < c$, assuming $0 < c < b$.

ANSWER: $I(c) =$ ______ A

(c) Find a formula involving c for $\mathbf{J}(c)$, the current density at any point where $\rho = c$, assuming $0 < c < b$. Enter your answer (a vector, of course) in Cartesian notation, $\langle J_x, J_y, J_z \rangle$.

ANSWER: $\mathbf{J}(c) =$ ______ A/m^2

(d) Find a formula involving ρ for the position-dependent conductivity $\sigma(\rho)$ inside the conductor. For WeBWorK, type "p" instead of ρ.

ANSWER: $\sigma(\rho) =$ ______ S/m

(Credit: This problem is based on Hayt-Buck, 8/e, problem 7.20.)

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b) $\vec{H} = \frac{I}{2\pi\rho}$ $\quad I = 9\times10^5 c^3 2\pi$

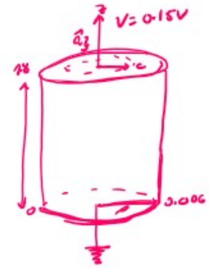
c) For current density: $\vec{J} = \vec{\nabla} \times \vec{H}$:

$$\vec{J} = \frac{1}{\rho}\begin{vmatrix} \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & 9\times10^5\rho^3 & 0 \end{vmatrix} = \langle 0, 0, 3\cdot 9\times10^5\rho \rangle$$

$\rightarrow \hat{a}_z$ direction

d) $\vec{J} = \sigma\vec{E}$ $\quad \vec{E} = \frac{\Delta V}{d}$

$\sigma = J / \frac{\Delta V}{d} = 504000000\rho$



Assignment 8: Problem 4

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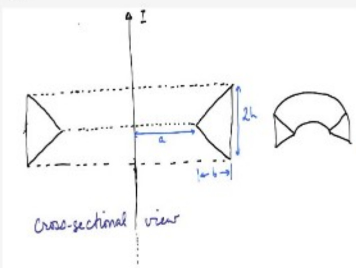
(1 point)
A metallic ring with a triangular cross-section as shown in the figure is centered around a long straight current filament carrying a current 5 Amps in the positive-z direction. If the dimensions on the figure are given as a = 0.03, b = 0.012, and h = 0.039, find the total flux circulating in the metallic ring.

Figure:



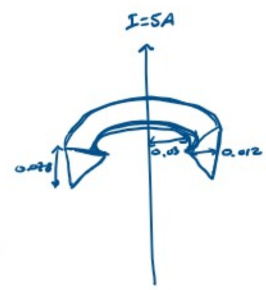
$$\varphi = \iint_S \vec{B}\cdot d\vec{S} = \iint_S \vec{B}\cdot d\vec{S}$$

$\vec{B} = \frac{\mu_0 I}{2\pi\rho}$ $\quad d\vec{S} = d\rho\, dz$ $\quad 0.03 \le \rho \le 0.042$

$$\varphi = 2\int_{0.03}^{0.03+0.012}\int_0^{\frac{0.039}{0.012}} \frac{\mu_0 5}{2\pi\rho^2}\, dz\, d\rho$$

$$= 2\,\frac{\mu_0(5)(0.039)}{2\pi\, 0.012}\left(0.012 - 0.03\ln\left(\frac{0.03+0.012}{0.03}\right)\right)$$

$$= 1.23879\times10^{-8}$$



ANSWER:
$\Phi =$ ______ Webers

Assignment 8: Problem 5

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(1 point)
The magnetic flux density in all of space is given by

$$\mathbf{B} = (5x - 8z)\mathbf{a}_x + (3x^2 - x^2y)\mathbf{a}_y + (x^2z - (5y + 5z))\mathbf{a}_z \text{ Wb/m}^2.$$

Let S denote the paraboloidal surface defined by

$$y = x^2 + z^2, \qquad 0 \le y \le 81.$$

Find the net magnetic flux Φ passing through S in the direction away from the xz-plane.

ANSWER: $\Phi =$ ______ Wb

$\varphi = \iint_S \vec{B}\cdot dS = \iint_S \vec{B}\cdot\hat{n}\cdot dS \longrightarrow \hat{n} = \langle 0, 1, 0\rangle$ $\quad dS = \rho\, d\rho\, d\varphi$

$\therefore$ want $\hat{a}_y$ component

Parametrize: $x = \rho\cos\varphi$

$y = \rho\sin\varphi = 81$

$z = z$

$$\varphi = \int_0^{2\pi}\int_0^9 \underbrace{\langle 5x - 8x, 3x^2 - x^2y, x^2z - (5y + 5z)\rangle}_{\text{Parametrize}} \cdot \langle 0, 1, 0\rangle\, \rho\, d\rho\, d\varphi$$

$$= \int_0^{2\pi}\int_0^9 \left(3\rho^2\cos^2\varphi - \rho^2\cos^2\varphi\cdot 81\right)\rho\, d\rho\, d\varphi$$

$$= \int_0^{2\pi}\int_0^9 -78\rho^3\cos^2\varphi\, d\rho\, d\varphi$$

$$= -401933.793304 \text{ Wb}$$