Complex Numbers

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$z_1 = 2+3i \quad z_2 = 5-4i$

a) $z_1 + z_2 = 7 - i$

b) $z_1 - z_2 = -3 - 7i$

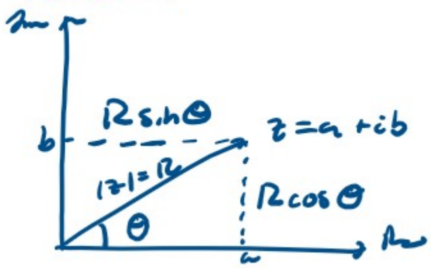
c) $3z_1 = 6 + 9i$

d) $z_1 z_2 = (2+3i)(5-4i) = 10 - 8i + 15i - 12i^2 = 22 + 7i$

- Modulus: $|z| = \sqrt{a^2+b^2} \rightarrow |3+4i| = 5$
- Conjugate: $\bar{z} = a - ib \rightarrow \overline{3+4i} = 3 - 4i$
- $z \cdot \bar{z} = |z|^2$
- $z = a + ib \quad Re(z) = a \quad Im(z) = b$

e) $\frac{z_1}{z_2} = \frac{z_1}{z_2} \cdot \frac{\bar{z}_2}{\bar{z}_2} = \frac{z_1 \bar{z}_2}{z_2 \bar{z}_2} = \frac{z_1 \bar{z}_2}{|z_2|^2}$

Polar Form



- $z = R\cos\theta + iR\sin\theta$
  $= R(\cos\theta + i\sin\theta)$

ex. $z_1 = 3+3i \;;\; |z_1| = 3\sqrt{2},\ \theta = 45° = \pi/4$

$\therefore z_1 = 3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Complex Exponential

Euler's Formula

$e^{i\theta} = \cos\theta + i\sin\theta$

- $\therefore z = Re^{i\theta} \longrightarrow R = |z|$ & $\theta = Arg(z) + 2k\pi,\ k \in \mathbb{Z}$
- $e^{i\theta_1} e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$
- $(e^{i\theta})^n = e^{in\theta}$

Trig Identities

- $\cos(\theta_1 + \theta_2) = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$
- $\sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2$