L13 Controller Design

Saturday, June 19, 2021 4:11 PM

Gain Design to meet Specs

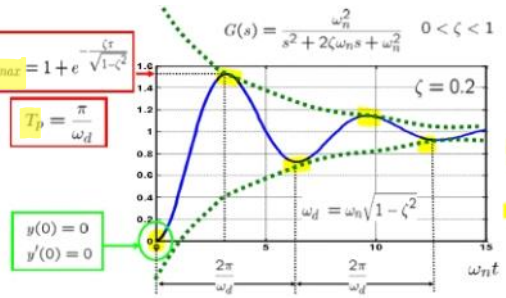
- Design K s.t. OS $\leq 4.32\%$
  2% settle time $\leq 2s$
  Error const $K_p > 1$
- Find min SS error $\rightarrow e_{ss}$ for $u(t)$
  - For 2nd order

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad 0 < \zeta < 1$$

	(5%)	(2%)
Settling time T_s	$\approx \frac{3}{\zeta\omega_n}$ or	$\frac{4}{\zeta\omega_n}$
Peak time T_p	$\frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$	
Peak value y_{max}	$1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}$	
Percent overshoot PO	$100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$	

$PO = 100e^{-\pi/\tan\theta}$ Delay Time $= T_d = \frac{1+0.7\zeta}{\omega_n}$

$y_{max} = 1 + e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$ $\quad T_p = \frac{\pi}{\omega_d}$ $\quad y(0) = 0,\ y'(0) = 0$



- Poles $(0 < \zeta < 1)$

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \qquad \zeta = \cos\theta \qquad \omega_d$$

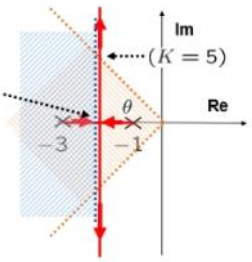
T = Time Constant $= 1/\zeta\omega_n$

- For $PO < 4.32\% \rightarrow \theta < 45°$
- For 2% $T_s \leq 2s \rightarrow \zeta\omega_n \geq 2$
- For $K_p > 1 \rightarrow K_p = K(L(0)) = \frac{K}{8} > 1 \rightarrow K > 8$
- Gain: $K = -\frac{1}{L(s)}$

$s = -2 \rightarrow K = -\frac{1}{L(-2)} = 1$

$s = -2 + 2j \rightarrow K = -\frac{1}{L(-2+2j)} = 5$

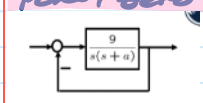
$L(s) = 0$ $\quad (K = 1)$ $\quad = 5$



∴ Acceptable gain: $8 < K < 5$

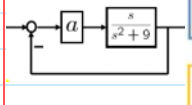
min SS error: $e_{ss} = \frac{1}{1+5/8} = 0.375$

Pole/Zero Location Design



- Draw RL for $q > 0$ $\quad q$
  - Ch. Eqn $= 1 + \frac{q}{s(s+a)} \rightarrow s^2 + sa + q = 0$
  - $1 + a\frac{s}{s^2+q} = 0$ $\quad \frac{s}{s^2+q} \rightarrow L(s)$

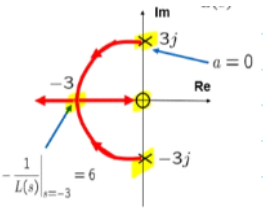
Two CL systems have the same characteristic eq.



$1 + \frac{1}{s(s+p)} = 0 \rightarrow s(s+p) + 1 = 0 \rightarrow s^2 + 1 + ps = 0 \rightarrow 1 + p\frac{s}{s^2+1} = 0$

$1 + \frac{s+z}{s^2} = 0 \rightarrow s^2 + s + z = 0 \rightarrow 1 + z\frac{1}{s(s+1)} = 0$

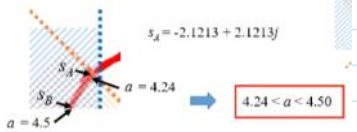
$a = -\frac{1}{L(s)}\Big|_{s=-3} = 6$



- Design pole a: OS $\leq 4.32\%$
  2% settle time $\geq 2s$
  $K_v > 2$

$K_v = \lim_{s\to 0} s\frac{q}{s(s+a)} = q/a > 2 \rightarrow a < 4.5$

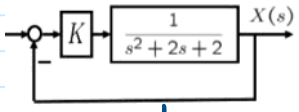
$a < 4.5$



$4.24 < a < 4.50$

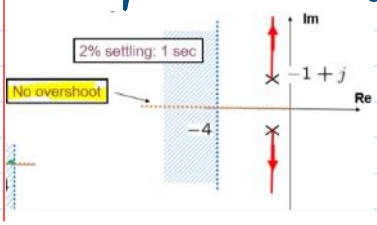
Multiple parameter Design

- Design Controller s.t.: 2% settle $\leq 1s$
  No OS
  e_{ss} for $u(t) < 0.05$



$e_{ss} = \frac{1}{1+K_p} < 0.05 \rightarrow K_p > 19,\ K > 38$

RL poles $s = -1 \pm j$



- Can't achieve design specs w/ pos. feedback