Polynomial Fit

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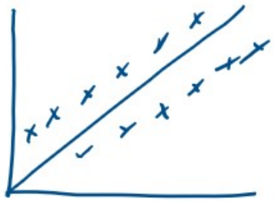

$n=12$ → polynomial approx. of n. datapoints

$m=2$ with a single polynomial of degree $m-1$

(if $m=n$, → Lagrange interpolation)

→ For $m<n$: following data points are given:

$(x_1, y_1), \ldots, (x_n, y_n)$ → look for LS solution

- Fit polynomial $m-1$: $p(x) = a_1 x^{m-1} + a_2 x^{m-2} + \cdots + a_{m-1} x + a_m$
  - → for $p(x_i) = y_i$ and $m=n$, $Va = y$ where:
    - $a = [a_1, \ldots, a_m]^T$ (coeff of $p(x)$)
    - $y = [y_1, \ldots, y_n]^T$ (data values)

    Matrix V has Vandermonde Matrix
  - → for $m<n$, Vandermonde-like Matrix

$$V = \begin{bmatrix} x_1^{m-1} & x_1^{m-2} & \cdots & x_1 \\ x_2^{m-1} & x_2^{m-2} & \cdots & x_2 \\ x_n^{m-1} & x_n^{m-2} & \cdots & x_n \end{bmatrix}$$

∵ $n>m$, this matrix is tall (more rows than colums)

∴ Colums of V are lin. independent as long as $x_i \neq x_j$ when $i \neq j$

Corresponding Least square equation $V^T V a = V^T y$ has unique solution given by $a_{LS} = (V^T V)^{-1} V^T y$

Special Case:

- Best fitting (linear regression) $m=2$, polynomial: $p(x) = a_1 x + a_2$ → polynomial deg. 1

$$V = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ \vdots & \\ x_n & 1 \end{bmatrix}, \quad V^T V = \begin{bmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & n \end{bmatrix}, \quad V_y^T = \begin{bmatrix} \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{bmatrix}$$

- If x_i's distinct, $V^T V$ → invertible and LS solution given by:

$$a_{LS} = \begin{bmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & n \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{bmatrix}$$

→ computable on MATLAB using data points