State Space

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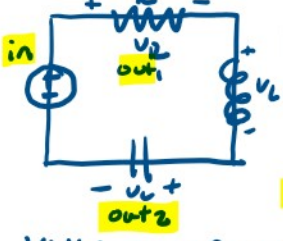
State

– Status of system w/ state known at t_0 for CT

RLC circuit



system M — input M

$\dot{x}(t) = Ax(t) + Bu(t)$ → State diff eqn.

$y(t) = Cx(t) + Du(t)$ → Output eqn.

output M — Feedforward M

$V_L = L\frac{di}{dt}$

$i = C\frac{dV}{dt}$

Inputs: $x_1 = v_c$, $x_2 = i$

Outputs: $y_1 = v_c = x_1$, $y_2 = Ri = Rx_2$

V input: $u_1 =$

KVL: $V = iR + V_L + V_c$

$v = iR + L\frac{di}{dt} + v_c$

$\frac{di}{dt} = \frac{1}{L}V - \frac{R}{L}i - \frac{1}{L}V_c$

$\frac{dv_c}{dt} = \frac{1}{C}i$

$\therefore \dot{x}_2 = \frac{1}{L}u - \frac{R}{L}x_2 - \frac{1}{L}x_1$

$\dot{x}_1 = \frac{1}{C}x_2$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & \frac{1}{C}\\ -\frac{1}{L} & -\frac{R}{L}\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ \frac{1}{L}\end{bmatrix}}_{B}u$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & R\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}u$$