

L16 Frequency Response

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10:01 PM

Frequency response

- for sin input $r(t)$ & response $y(t)$,
Steady state response $y_{ss}(t)$ = frequency response
- @ Steady state, $r(t)$ & $y(t)$ = same freq

$$y_{ss}(t) = A |G(j\omega)| \sin(\omega t + \angle G(j\omega))$$

↳ freq = ω

↳ Amplitude = A , Gain = $|G(j\omega)|$

↳ Phase = $\angle G(j\omega) = -\frac{T_{shift}}{T_{period}} \times 360^\circ$

First order FRF

$$\text{- FRF } G(j\omega) = \frac{1}{j\omega + 1} \quad \begin{cases} |G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}} \\ \angle G(j\omega) = \angle 1 - \angle(j\omega + 1) = -\tan^{-1}\omega \end{cases}$$

Second order FRF

$$\text{- FRF } G(j\omega) = \frac{2}{(j\omega + 1)(j\omega + 2)} \quad \begin{cases} |G(j\omega)| = \frac{2}{\sqrt{\omega^2 + 1} \sqrt{\omega^2 + 4}} \\ \angle G(j\omega) = \angle 2 - \angle(j\omega + 1) - \angle(j\omega + 2) = 0 - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) \end{cases}$$

Bode Plots

$$M(\omega) = |G(j\omega)| \rightarrow 20 \log |G(j\omega)|$$

$$\phi(\omega) = \angle G(j\omega)$$

- TF needs $\deg(\text{den}) \geq \deg(\text{num})$