L11-12 Root Locus

Saturday, June 19, 2021 12:10 PM

Root Locus

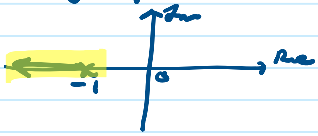
- Poles characterize stability & Transient Properties
- Shows how poles of CL sys. vary as $K > 0 \to \infty$

$L(s) = \frac{1}{s+1}$

$TF = \frac{L(s)}{1+KL(s)} \to$ Characteristic Eqn $= 1 + K\frac{1}{s+1} = 0$

↳ $s+1+K \to s = -1-K$

K	s
0	−1
1	−2
2	−3

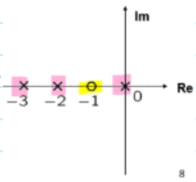


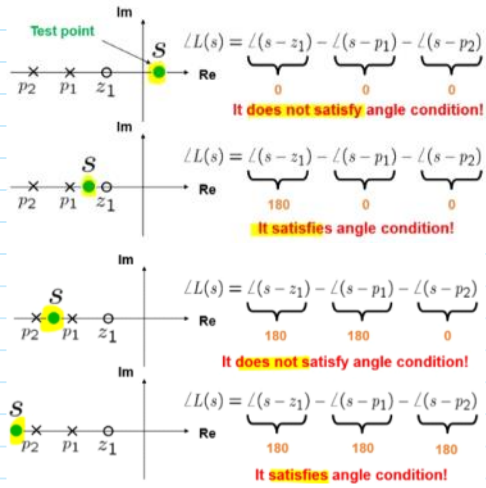
Root Locus Sketching

Step 0:

- OL poles & zeros
  - ↳ Branches → ∞ = # poles − # zeros
  - ↳ # Branches = order L(s)

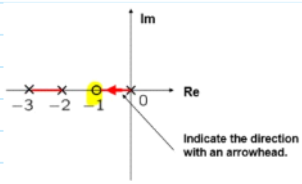
$L(s) = \frac{s+1}{s(s+2)(s+3)}$ → $z = -1$ → $p = 0, -2, -3$





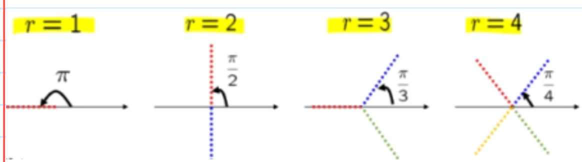
Step 1:

- On the real axis
  - ↳ RL includes pts left of odd # of roots
  - ↳ poles → zeros & Infinity Zeros
    - ↳ Infinity Zeros
    - ∞ zeros = # P − # Z

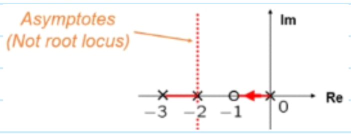


Step 2:

- Asymptotes
  - # Asymptotes = r = deg(den) − deg(num)
  - ↳ Asymptote Angles = $\frac{\pi}{r}(2k+1)$, $k = 1, 2, \ldots (r-1)$

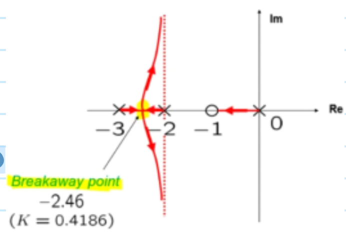


- Intersection of Asymptotes
  - $\alpha = \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{r} = -2$
  - ↳ centroid



Step 3:

- Breakaway points
  - ↳ roots of $\frac{dL(s)}{ds} = 0 \to s = -2.5, -0.8 \pm 0.8$
  - ↳ Check K positivity: $K = -\frac{1}{L(s)} = 0.4$
  - ↳ # RL branches = $N_b = \begin{cases} \#P \geq \#Z \to N_b = \#P \\ \#P < \#Z \to N_b = \#Z \end{cases}$



Stability Condition

- Routh array:

s^3	1	5
s^2	6	K
s^1	$\frac{30-K}{6}$	
s^0	K	

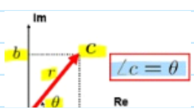
Stability condition
$0 < K < 30$

- When $K = 30$

$6s^2 + 30 = 0 \Rightarrow s = \pm\sqrt{5}j$

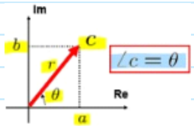
Complex Numbers

- $c = a + bj \to c = re^{j\theta}$
  - $s - z_1$



Complex Numbers

- $c = a + bj \longrightarrow c = re^{j\theta}$



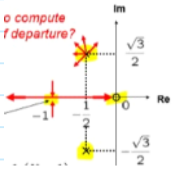
$\angle L(s) = \angle \frac{s - z_1}{(s-p_1)(s-p_2)}$

$= \angle(s - z_1) - \angle(s - p_1) - \angle(s - p_2) = \varphi$

Angle Condition

- For OLTF: $s = s_0$ on RL $\rightarrow \angle L(s_0) = 180^\circ$
  - ↳ exists $K > 0$ s.t. $1 + KL(s_0) = 0$
  - ↳ exists $K > 0$ s.t. $L(s) = -\frac{1}{K}$

RL w/ Complex poles



Step 4:

- Angle of Departure

$\angle L(s) = \frac{s}{s^2 + 2 + 1} \longrightarrow z = 0$, $p = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

$\frac{dL(s)}{ds} \longrightarrow s = \pm 1$

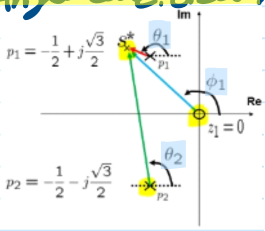
BAP: $s = -1 \rightarrow K = 1$

↳ pt. s^* near p_1

Angle Condition: $\angle L(s^*) = \angle \frac{s^* - z_1}{(s^* - p_1)(s^* - p_2)}$

$= \angle(s^* - z_1) - \angle(s^* - p_1) - \angle(s^* - p)$

$= \varphi_1 - \theta_1 - \theta_2 \approx 180^\circ$



$\therefore \varphi_1 \approx 120^\circ$, $\theta_2 \approx 90^\circ$, $\theta_1 = -150^\circ$

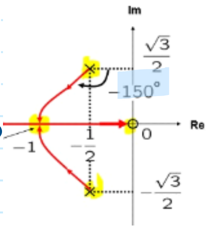
$\angle L(s^*) = \angle \frac{s^* - z_1}{(s^* - p_1)(s^* - p_2)}$

$= \angle(s^* - z_1) - \angle(s^* - p_1) - \angle(s^* - p_2)$

$= \varphi_1 - \theta_1 - \theta_2 \approx 180^\circ$

↳ $\angle s = \frac{\pi}{r}(2k+1)$ BAP

$K = 0, 1, 2, \ldots (r-1)$



If s^* close to p_1:

$\varphi_1 \approx \angle(p_1 - z_1) = 141.54^\circ$

$\varphi_2 \approx \angle(p_1 - z_2) = 150.25^\circ$

$\theta_2 \approx \angle(p_1 - p_2) = 90^\circ$

$\theta_1 \approx \varphi_1 + \varphi_2 - \theta_2 - 180^\circ = 21.59^\circ$