Magnetic Flux

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Magnetic Flux

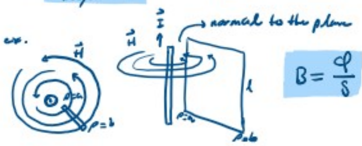
Magnetic Flux Density

$\vec{B} = \mu_0 \vec{H}$ → $\mu_0 = 4\pi \times 10^{-7}$ H/m (Permeability

$\varphi = \iint_S \vec{B} \cdot d\vec{S}$

$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\varphi$

ex.



$B = \frac{\varphi}{S}$

$\varphi = \iint_S \vec{B} \cdot d\vec{S}$ → $d\vec{S} = d\rho\, dz\, \hat{a}_\varphi$

$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\varphi$

$\varphi = \iint_S \frac{\mu_0 I}{2\pi\rho} \hat{a}_\varphi\, d\rho\, dz\, \hat{a}_\varphi = \frac{\mu_0 I}{2\pi} \int_0^l dz \int_a^b \frac{d\rho}{\rho} = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{b}{a}\right)$

Magnetic Flux for Closed Surface

$\varphi = \oiint_S \vec{B} \cdot d\vec{S} = 0$

→ Total Flux = 0

Divergence Theorem

$\oiint_S \vec{A} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{A}$

- Integral of normal comp. of vector over closed surface = Integral of divergence of this vector field throughout volume enclosed by closed surface

$\nabla \cdot \vec{A} = \frac{\partial \vec{A}}{\partial x} + \frac{\partial \vec{A}}{\partial y} + \frac{\partial \vec{A}}{\partial z}$

↳ $Q = \iint_S \vec{B} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{B}$

$\nabla \cdot \vec{B} = 0$ → If divergence of vector field $\neq 0$ then vector field $\neq$ magnetic field

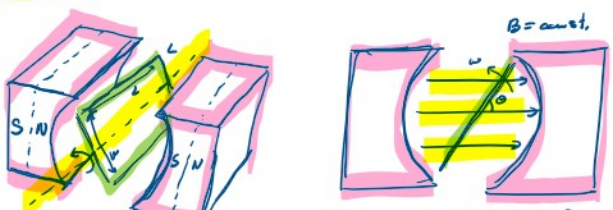
ex. $\vec{A} = a\cos(x)\, \hat{a}_x + by\sin(x)\, \hat{a}_y$

$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$= -a\sin(x) + b\sin(x) + 0 = 0$

$\therefore a = b$ for magnetic field

ex. Motor



→ Flux = greatest @ 90°

☆ webwork 8

$\oint_C \vec{F} \cdot d\vec{l} = \iint_S (\nabla \times \vec{G}) \cdot \hat{n}\, dS$

Stokes Theorem

$\hat{n} = \langle 0, 0, 1 \rangle$

$\nabla \times \vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$

$= \langle -3z^2yx,\ x + 2zy,\ z^3y + 3 - z^2 - 5 \rangle$

$\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS = \iint_S z^3y - z^2 - 2\, dS$

→ set $z = 0$: $-2\rho\pi = -2\pi$

a) $R = \frac{V}{I}$ $H = \frac{I}{2\pi\rho} = 9\times10^5 \rho^2 \hat{a}_\varphi$

→ solve I: $I = 1.2245$ A

$R = 0.1228\,\Omega$

6 b) $\vec{H} = \frac{I}{2\pi\rho}$ $I = 9\times10^5 c^3 2\pi$

c) For current density: $\vec{J} = \nabla \times \vec{H}$:

(6) b) $\vec{H} = \frac{I}{2\pi\rho}$ $I = 9\times10^5 \rho^3 2\pi$

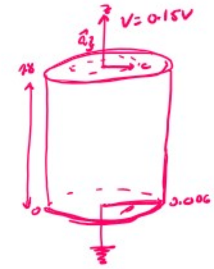
c) For current density: $\vec{J} = \vec{\nabla}\times\vec{H}$:

$$\vec{J} = \frac{1}{\rho}\begin{vmatrix} \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & 9\times10^5\rho^3 & 0 \end{vmatrix} = \langle 0, 0, 3\cdot 9\times10^5\rho\rangle$$

$\rightarrow \hat{a}_z$ direction

d) $\vec{J} = \sigma\vec{E}$ $\vec{E} = \frac{\Delta V}{d}$

$\sigma = J / \frac{\Delta V}{d} = 504000000\rho$



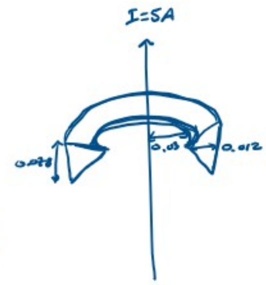
$$\varphi = \iint_S \vec{B}\cdot d\vec{S} = \iint_S \vec{B}\cdot d\vec{S}$$

$\vec{B} = \frac{\mu_0 I}{2\pi\rho}$ $\quad d\vec{S} = d\rho\, dz$ $\quad 0.03 \le \rho \le 0.042$

$$\varphi = 2\int_{0.03}^{0.03+0.012}\int_0^{\frac{0.039}{0.012}} \frac{\mu_0 5}{2\pi\rho^2}\, dz\, d\rho$$

$$= 2\,\frac{\mu_0(5)(0.039)}{2\pi\, 0.012}\left(0.012 - 0.03\ln\left(\frac{0.03+0.012}{0.03}\right)\right)$$

$$= 1.238799\times10^{-8}$$



$$\varphi = \iint_S \vec{B}\cdot dS = \iint_S \vec{B}\cdot\hat{n}\cdot dS \rightarrow \hat{n} = \langle 0, 1, 0\rangle$$

$\therefore$ want $\hat{a}_y$ component

$dS = \rho\, d\rho\, d\varphi$

<u>Parametrize</u>: $x = \rho\cos\varphi$

$y = \rho\sin\varphi = 81$

$z = z$

$$\varphi = \int_0^{2\pi}\int_0^9 \underbrace{\langle 5x-8x,\ 3x^2 - x^2y,\ x^2z-(5y+5z)\rangle}_{\text{Parametrize}}\cdot\langle 0,1,0\rangle\, \rho\, d\rho\, d\varphi$$

$$= \int_0^{2\pi}\int_0^9 \left(3\rho^2\cos^2\varphi - \rho^2\cos^2\varphi\cdot 81\right)\rho\, d\rho\, d\varphi$$

$$= \int_0^{2\pi}\int_0^9 -78\rho^3\cos^2\varphi\, d\rho\, d\varphi$$

$= -401933.793804$ Wb