Lab-3: Permanent Magnet Brushed DC Motor

ELEC 343

Lab Experiment: 3

Section: L1B & L1D

Bench #: Home

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1. Data and Parameters

1.1A Task 1 A: Measuring the DC Resistance: Blocked-Rotor Test

Task 1A, Table 1: Armature + Brush Resistance Measurement

V_1, V			I_1, A			**Calculate** R_a, Ω		
2.2	3.82	5.42	3.05	5.11	7.23	0.7213	1.0333	0.7496
						Average = 0.8347		

1.1B Task 1 B: Measuring No-Load Characteristic

Task 1B, Table 2: No-Load Measurement, Tfric(n),

Measurement	**1**	**2**	**3**	**4**	**5**	**6**
$V_1(ave), V$	5.25	12.25	20.56	28.09	38.15	45.25
$I_1(ave), A$	0.6	0.72	0.81	0.87	0.88	0.89
$n_1(ave), rpm$	205	503	871	1208	1660	1967
Calculation						
k_t, $[V \cdot \sec/rad]$	0.2212	0.2211	0.2179	0.2163	0.2152	0.2161
Average k_t	0.2179					
T_{fric}, Nm	0.1308	0.1569	0.1766	0.1896	0.1918	0.1940

1.2A Task 2 A: Load Characteristic – Speed Regulation

Task 2A, Table 3: Load Test Measurement: (up to 6 measurement points)

Measurement #	1	2	3	4	5	6
$V_1(ave), V$	45.37	45.57	45.13	45.62	45.16	45.68
$I_1(ave), A$	1.77	3.12	3.9	4.72	5.4	6.13
$P_1(ave), W$	80.7	142.3	176	215.2	244.2	282
$V_2(ave), V$	45.82	43.52	41.68	40.88	39.4	38.55
$I_2(ave), A$	X	1.32	2.11	2.89	3.58	4.33
$P_2(ave), W$	X	57.3	88.1	118.2	141.1	167.2
n, rpm	1953	1960	1867	1862	1820	1818
T_m, Nm	0.22	0.5	0.68	0.86	1.02	1.19
P_2 / P_1	X	0.4027	0.5006	0.5493	0.5778	0.5929
$SR = \left(n_1 - n_{6,load}\right)/ n_{6,load}$			0.0743			
DC Machine (Generator) Voltage Constant $k_v = V_{2,oc} / \omega_r$			0.2240			

1.2B Task 2 B: Speed Control by Adjusting Voltage

Task 2B, Table 4: Motor Speed Control by Adjusting Voltage

Measurement #	1	2	3	4	5	6
$V_1(ave), V$	7.06	12.51	18.78	28.43	38.75	45.68
$I_1(ave), A$	1.76	2.41	3.23	4.3	5.43	6.11
$P_1(ave), W$	12.5	30.1	60.8	122.5	210	279
$V_2(ave), V$	5.33	9.85	15.09	23.64	32.54	38.48
$I_2(ave), A$	0.59	1.09	1.78	2.69	3.66	4.32
$P_2(ave), W$	3.1	10.7	26.8	63.5	119	166.5
n, rpm	252	470	721	1117	1537	1822
T_m, Nm	0.26	0.38	0.54	0.77	1.01	1.17
P_2 / P_1	0.248	0.3555	0.4408	0.5184	0.5667	0.5968

1.3A Task 3 A: Speed Control by Adjusting the Duty-Cycle (2.4 kHz)

Task 3A, Table 5: Motor Speed Control by Duty-Cycle (2 kHz): Vdc = 45 V, f = 2.4kHz

Measurement #	1	2	3	4	5
Duty-Cycle, d	0.1	0.3	0.6	0.8	0.95
$V_1(ave), V$	4.45	13.14	26.67	35.35	41.78
$I_1(ave), A$	1.4	2.65	4.19	5.23	5.92
Current Ripple $\Delta I_1, A$	1	1.8	2	1.8	1
$P_1(ave), W$	6.7	36.6	113.8	186.7	247.4
$V_2(ave), V$	2.75	10.26	22.24	30.12	35.71
$I_2(ave), A$	0.31	1.16	2.48	3.35	3.97
$P_2(ave), W$	0.9	11.9	55.2	100.9	141.6
n, rpm	139	480	1022	1376	1637
T_m, Nm	0.24	0.08	0.4	0.67	0.75
P_2 / P_1	0.1343	0.3251	0.4851	0.5404	0.5723

1.3B Task 3 B: Speed Control by Adjusting the Duty-Cycle (10 kHz)

Task 3B, Table 6: Motor Speed Control by Duty-Cycle (10 kHz): Vdc = 45.1 V

Measurement #	1	2	3	4	5
Duty-Cycle, d	0.1	0.2	0.6	0.8	0.95
$V_1(ave), V$	4.45	13.48	26.8	35.27	41.7
$I_1(ave), A$	1.4	2.5	4.04	5.02	5.7
Current Ripple $\Delta I_1, A$	0.25	0.45	0.5	0.35	0.15
$P_1(ave), W$	5.9	35	110	176.5	235
$V_2(ave), V$	2.48	10.10	21.55	29.26	34.96
$I_2(ave), A$	0.28	1.18	2.47	3.30	3.92
$P_2(ave), W$	0.7	11.9	53.4	96.6	137.5
n, rpm	126	496	1032	1392	1667
T_m, Nm	0.24	0.42	0.73	0.93	1.085
P_2 / P_1	0.1186	0.34	0.4854	0.5473	0.5851

1.4A Task 4A: Speed Control by Adjusting the Duty-Cycle at 2.5 kHz

Task 4A, Table 7: Motor Speed Control using Universal Inverter Box (at 2.5 kHz)

<table>
<tr><td colspan="2">Measurement #</td><td colspan="4">1</td></tr>
<tr><td colspan="2">Duty-Cycle, $\boldsymbol{d}$</td><td colspan="4">0.51</td></tr>
<tr><td colspan="2">$V_1(ave), V$</td><td colspan="4">281</td></tr>
<tr><td colspan="2">$I_1(ave), A$</td><td colspan="4">3.63</td></tr>
<tr><td colspan="2">$P_1(ave), W$</td><td colspan="4">83.4</td></tr>
<tr><td colspan="2">$V_2(ave), V$</td><td colspan="4">18.04</td></tr>
<tr><td colspan="2">$I_2(ave), A$</td><td colspan="4">2.09</td></tr>
<tr><td colspan="2">$P_2(ave), W$</td><td colspan="4">37.8</td></tr>
<tr><td colspan="2">n, rpm</td><td colspan="4">868</td></tr>
<tr><td colspan="2">T_m, Nm</td><td colspan="4">0.65</td></tr>
<tr><td colspan="7">Use the set of measurements complete the following section of the Table</td></tr>
<tr><td colspan="2">Input to Inverter Box</td><td rowspan="4">Input power at the DC Motor terminals
P_{dc-mot}, W
82.4</td><td rowspan="4">Output mechanical power
P_m, W
37</td><td rowspan="4">Efficiency of the Inverter, %
P_{dc-mot}/P_{inv}
94.4%</td><td rowspan="4">Efficiency of the Motor, %
P_m/P_{dc-mot}
45.7%</td><td rowspan="4">Efficiency of Inverter-Motor combined, %
P_m/P_{invt}
43.2%</td></tr>
<tr><td>Voltage, Vdc</td><td>Current, A</td></tr>
<tr><td>45.5</td><td>1.93</td></tr>
<tr><td colspan="2">Total input power to the Inverter Box
P_{inv}, W
87.236</td></tr>
</table>

Consider the chain of the components that you had in this lab including the following: DC Source, **DC-DC Converter**, **DC Machine 1**, **DC Machine 2**, and **Load Resistors Box**.
Using the data you recorded in Table 7, calculate and fill in the energy conversion efficiency at each energy conversion stage in the above chain. Comment on the efficiency of these stages to deliver the energy to the final resistor load.

- The efficiency of the inverter is much higher than that of the motor, and even more efficient than the inverter-motor combined. The Inverter-Motor combined loses more than 50% of its power when delivering energy to the final resistor load.

2. Calculations and Analysis

2.5A Task 5A: Determining Motor Parameters

1. **Calculate the armature resistance Ra and record the value in Table 1.**
    - $Ra = \frac{V1}{I1}$
    - See Task 1A Table 1.

2. **Calculate the motor constant from the no-load data obtained in Table 2, Task 1.**
    - $Kt = Kv = \frac{(Vt - Ia * Ra)}{\omega r}$
    - Average kt = 0.2179
    - See Task 1B Table 2.

    Also, calculate the voltage constant kv for the second DC Machine that was used as a generator and record this number in Table 3.
    - $kv = \frac{V2{,}oc}{\omega r}$
    - From measurement #6, kv = 0.2240
    - See Task 2A Table 3.

    What can you say about these two constants and their values?
    - The motor constant kt and the voltage constant kv are similar, as in the no-load test, the torque and voltage constants are considered to be equal, Kt = Kv.

3. **Calculate the friction torque Tfric in Table 2.**
    - $Tfric = Te = Kt * Ia$
    - See Task 1B Table 2.

    Plot Tfric(n) as a function of rotor speed n.

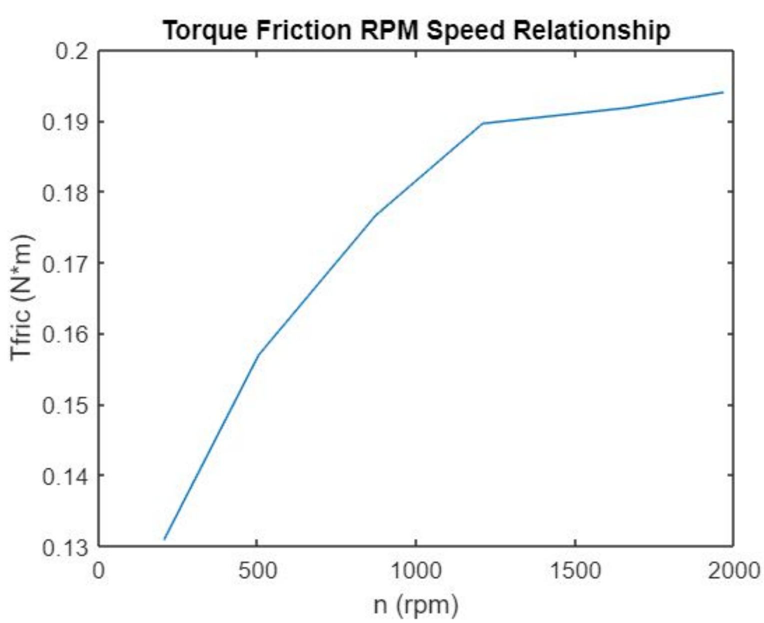

Figure 2.5-1: Torque Friction RPM Speed Relationship

What can you say about the curve?

- As the rotor speed n increases, the torque friction increases considerably when the rotor speed is lower than when it is higher. The curve tends to saturate at the peak when the rotor speed is faster.

Is the loss predominantly from sliding friction, viscous damping, or windage (turbulent airflow)?

- The loss is predominantly from sliding friction, as the torque-speed relationship is linear, and damping would result in a nonlinear relationship.

4. **Calculate the moment of inertia Jrotor using your best estimate of the friction torque at the speed where dEa/dt was measured in Task 1C.**
   - $Jrotor\frac{d\omega r}{dt} = \frac{Jrotor}{kt}\frac{dEa}{dt} = Tfric(\omega r)$
   - From Task 1C: at n = ~1000 rpm = 10π/3, Tfric = ~0.18 N*m
   - Finding rate of change: $\frac{d\omega r}{dt} = \frac{(56.9811-138.007)}{(1.5-1)*1000} = -0.1695\ rad/s$
   - $Jrotor = \frac{Tfric(\omega r)}{0.1695} = \frac{0.18}{0.1695} = 1.0619\ kg * m^2$ at n = 1000 rpm, Tfric = 0.18 N*m
   - See Appendix A for Matlab Code.

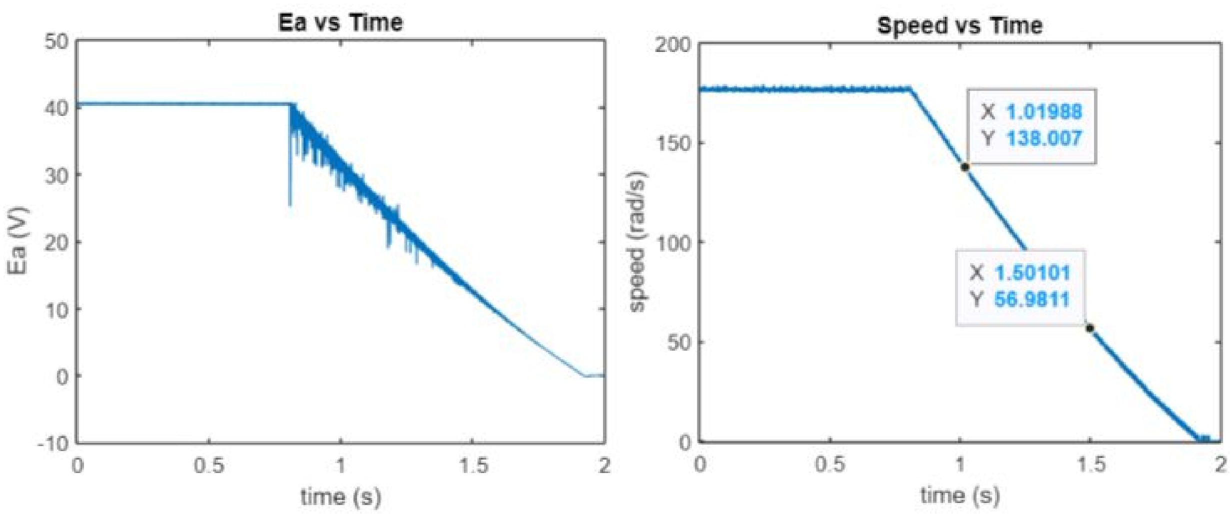

Figure 2.5-1: Ea vs Time (Left) and Speed vs Time (Right) One Machine Relationship

5. **Based on the recorded stopping transient in Task 1D, calculate the combined moment of inertia of both machines Jcombined (when they are coupled), and compare the result with Jrotor obtained in step 4) above.**
    - $Jrotor\frac{d\omega r}{dt} = \frac{Jrotor}{kt}\frac{dEa}{dt} = Tfric(\omega r)$
    - From Task 1C: at n = ~1000 rpm = 10π/3, Tfric = ~0.18 N*m
    - Finding rate of change: $\frac{d\omega r}{dt} = \frac{(16.2769-89.3913)}{(1.5-1)*1000} = -0.1462\ rad/s$
    - $Jrotor = \frac{Tfric(\omega r)}{0.1695} = \frac{0.18}{0.1462} = 1.2309\ kg * m^2$ at n = 1000 rpm, Tfric = 0.18 N*m
    - See Appendix A for Matlab Code.

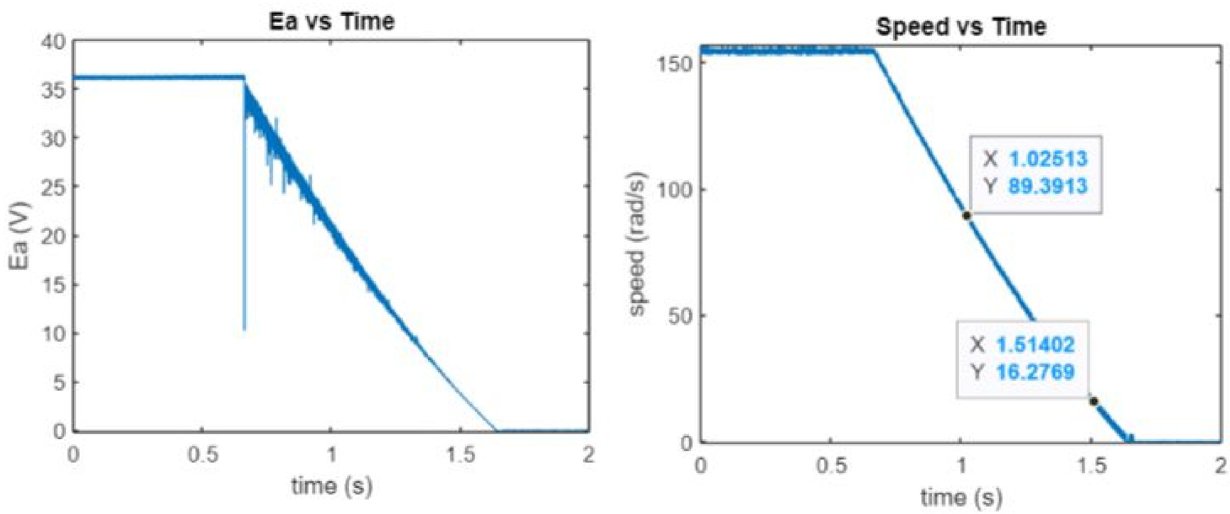

Figure 2.5-2: Ea vs Time (Left) and Speed vs Time (Right) Two Machines Relationship

6. **Based on the recorded data from Task 3, estimate the armature inductance La. First, for a less accurate estimate, you can neglect the resistance Ra and use the equation: va=La*(Δia/Δt)+Ea.**
    - From Task 3A: Va = Vdc = 45 V, Ea = Kt*wr
    - $\frac{dIa}{dt} = 10.6507\ A/s \qquad La = \frac{(Va - Ea) * dt}{dIa} = 2.4756\ mH$
    - See Appendix B for Matlab Code.

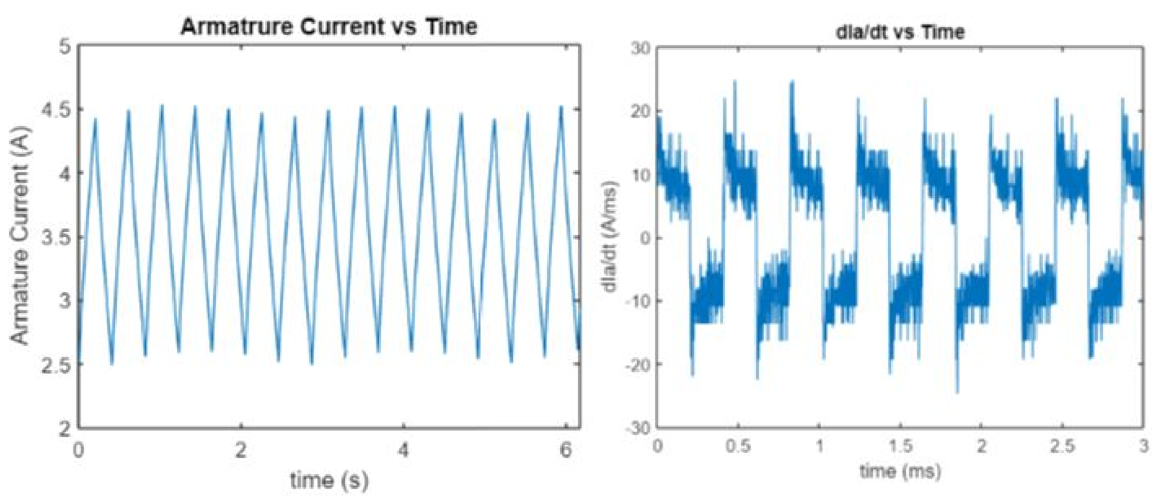

Figure 2.5-3: Ia vs Time (Left) and dIa/dt vs Time (Right) Relationship

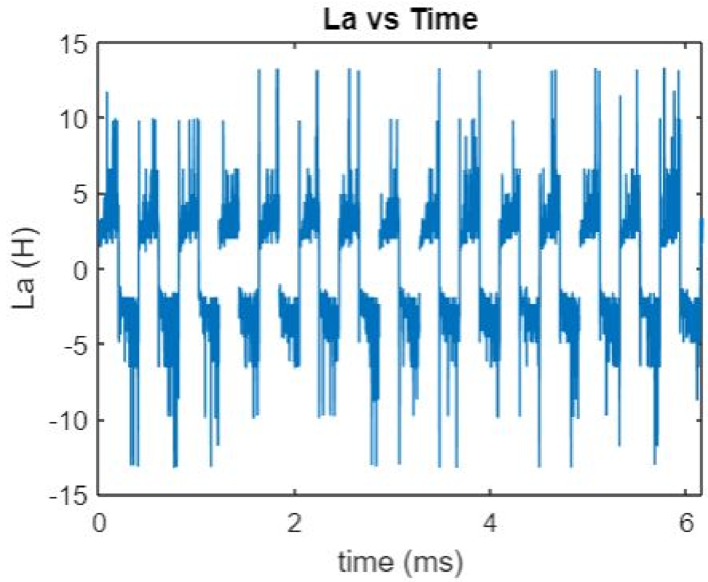

Figure 2.5-4: La vs Time Relationship

Then, you should use more accurate equations that you have derived in your Pre-Lab assuming that you know the value of Ra.
Compare these two results and use the more accurate on Fig. A.

- $Va = Ra * Ia + La * \frac{dIa}{dt} + Kt * \omega r \quad La = \frac{(Va - Ra * Ia - Kt * \omega r) * dt}{dIa}$
- Ra = 0.847**Ω**, Kt = 0.2179, Va = Vdc = 45V, Ea = Kt*wr
- $\frac{dIa}{dt} = 10.6507\ A/s \qquad La = \frac{(Va - Ra * Ia - Kt * \omega r) * dt}{dIa}$ = 2.1958 mH
- The La calculated with the derived formula is slightly lower, and more accurate than when neglecting Ra.

7. **Complete Fig. A by filling-in all remaining machine parameters.**

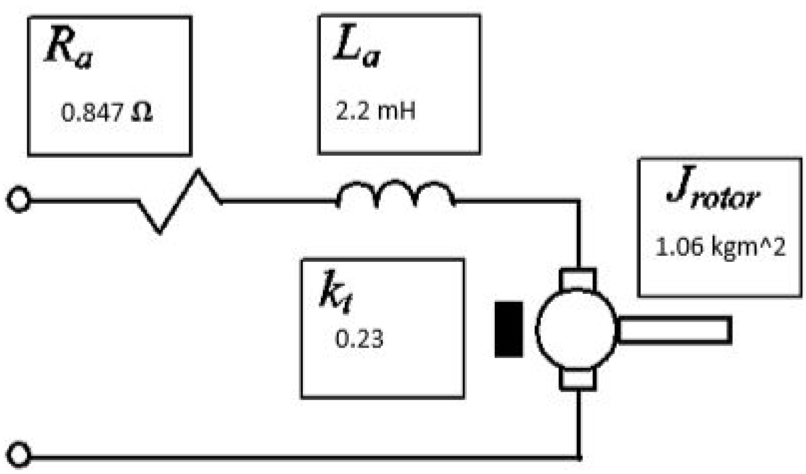

Fig. A. DC Machine Equivalent Circuit. Fill-in the corresponding boxes with machine parameters. Make sure to include the units.

8. **Based on your measurements in Task 2 Table 4, establish the equivalent mechanical load torque vs speed characteristic of the DC machine loaded with Resistor Box (with all resistors connected), Tm-load(n), and plot it.**

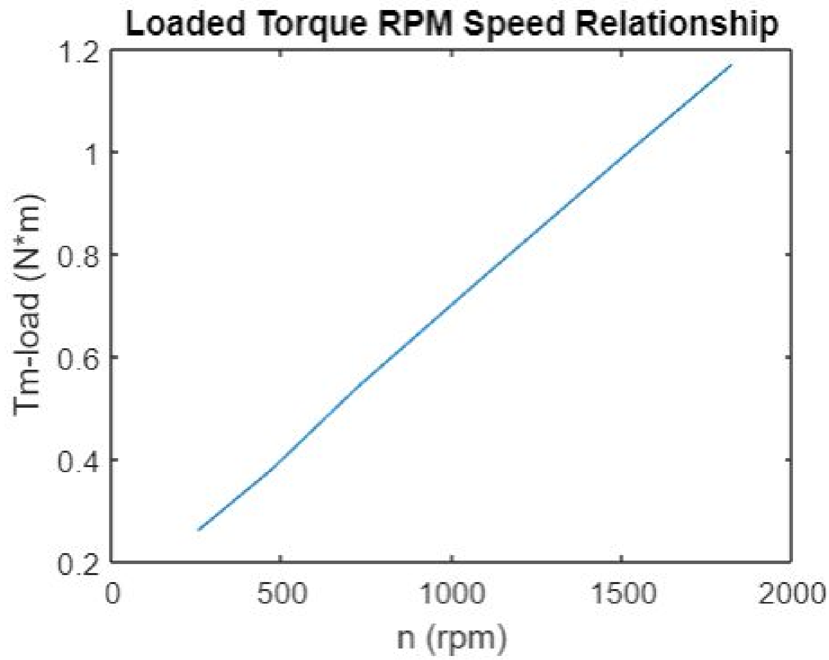

Figure 2.5-5: Loaded Torque Speed Relationship

Comment on or explain the shape of this characteristic. Is it linear or not, and why?

- The load torque increases with the speed in a linear relationship as speed is directly proportional to supply voltage, and the torque is proportional to the speed of the output shaft

2.5B Task 5B: Equivalent Circuit vs. Measured Comparison

1. **Use the equivalent circuit of the DC machine, and calculate and plot the speed-torque characteristic n(Tm) corresponding to the input voltage in your Task 2A, Table 3. Include the effects of friction in your predicted curve (Te vs. Tm). Superimpose on the same plot the measured and the predicted characteristics. Comment on the results and the expected vs measured speed-voltage regulation.**
   - $T_e = \frac{P_e}{\omega_r} = \frac{P_e}{\frac{n\pi}{30}} = \frac{282}{\frac{1818\pi}{30}} = 1.48$
   - $T_{fric} = V_1 I_1 - T_e \omega_r = V_1 I_1 - P_e = 45.68 * 6.13 - 282 =- 1.98$
   - $T_m = T_e - T_{fric} = 1.48 + 1.98 = 3.46$

   - The above calculations are examples using the last measurement point in Table 3, same for other points.

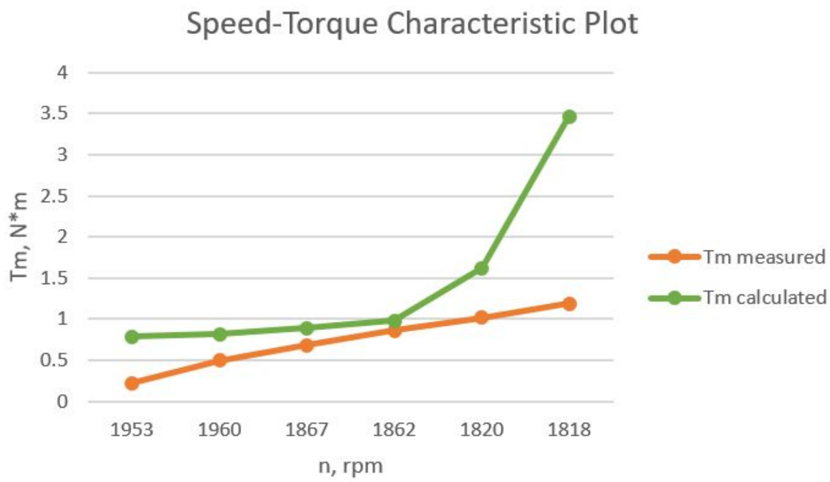

Figure 2.5-6: Speed-Torque Characteristic

- With increasing speed, both measured and calculated mechanical torque show the same trend of increasing. The difference is that the measured torque shows a trend of linearly increasing while the calculated torque increases exponentially.

2. **Calculate the speed-voltage characteristic n(V1) of the loaded machine (use Tm-load(n) that you have established before).**
   Include the effects of friction in your predicted curve.
   Superimpose on the same plot the characteristic measured in Task 2 B, Table 4, and the predicted characteristic.
   Comment on the results.
   - $T_e = \frac{P_e}{\omega_r} = \frac{P_e}{\frac{n\pi}{30}} = \frac{279}{\frac{1822\pi}{30}} = 1.46$
   - $T_{fric} = T_e - T_m = 1.46 - 1.17 = 0.29$
   - $V_1 = \frac{T_{fric}+P_e}{I_1} = \frac{0.29+279}{6.11} = 45.71$
   - The above calculations are examples using the last measurement point in Table 4, same for other points.

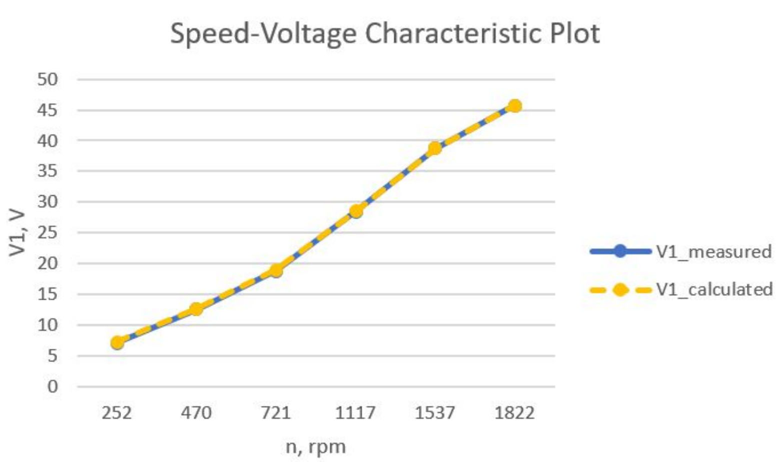

Figure 2.5-7: Speed-Voltage Characteristic

3. **Calculate and plot the motor efficiency vs. current η(I1) characteristics based machine equivalent circuit (plus the measured friction).**
On the same plot, also show the efficiency of the single machine established from the measured date in Task 2 A, Table 3.
Compare the results.
Calculate/note at what current the motor has maximum efficiency.
    - $P_{in} = V_1 I_1 = 45.68 * 6.13 = 280.02$
    - $P_{out} = T_m \omega_r = 1.19 * \frac{1818\pi}{30} = 226.55$
    - $\eta = \frac{P_{out}}{P_{in}} = \frac{226.55}{280.02} = 0.8091$
    - The above calculations are examples using the last measurement point in Table 3, same for other points.

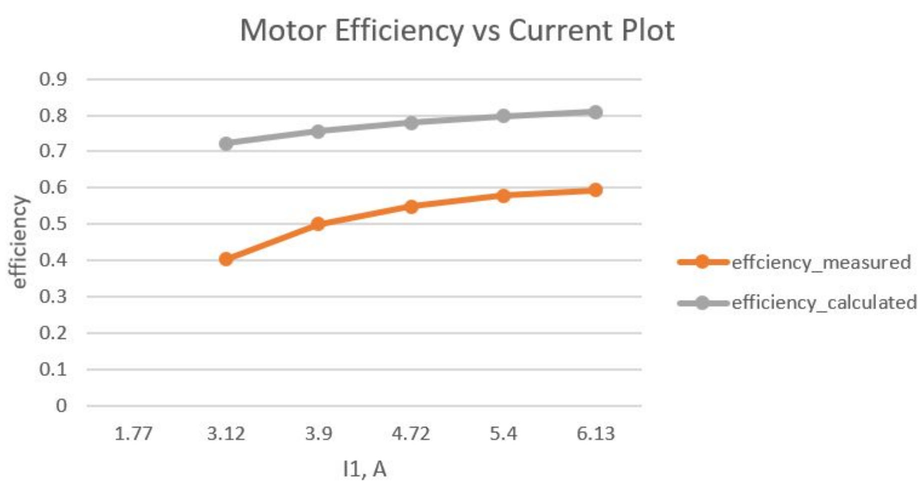

Figure 2.5-8: Motor Efficiency vs Current

    - Both measured and calculated efficiencies follow the same trend where the calculated efficiencies are higher than the measured ones in general. The motor has maximum efficiency at maximum current.

2.5C Task 5C: Comparison with Simulink Model

Use the parameters as you have determined for this PM DC motor for the Simulink Model described in Lectures Module 4. Implement this model using MATLAB/Simulink. Implement the Voltage Source with variable duty cycle operation (you can use a pulse generator block or improvise your own way to implement the PWM).

- For Table 5, parameters corresponding to duty cycle of 0.5 are calculated below using trendlines.

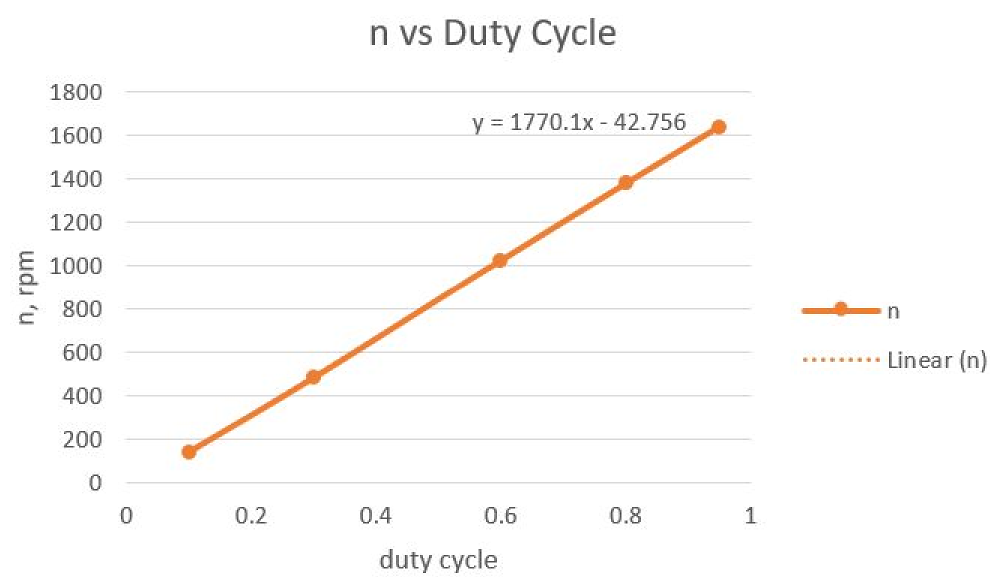

Figure 2.5-9: n vs Duty Cycle Plot

- $n = 1770.1 * 0.5 - 42.756 = 842.3\ rpm$
- $T_{fric}(n) = 0.172\ Nm$
- $T_m(n) = 0.\ 592\ Nm$
- $D_m = \frac{T_{fric}}{\omega_r} = 0.00195\ Nms$
- $R_a = 0.8347\ \Omega$
- $K_t = 0.2179$
- $J = 1.124\ kgm^2$

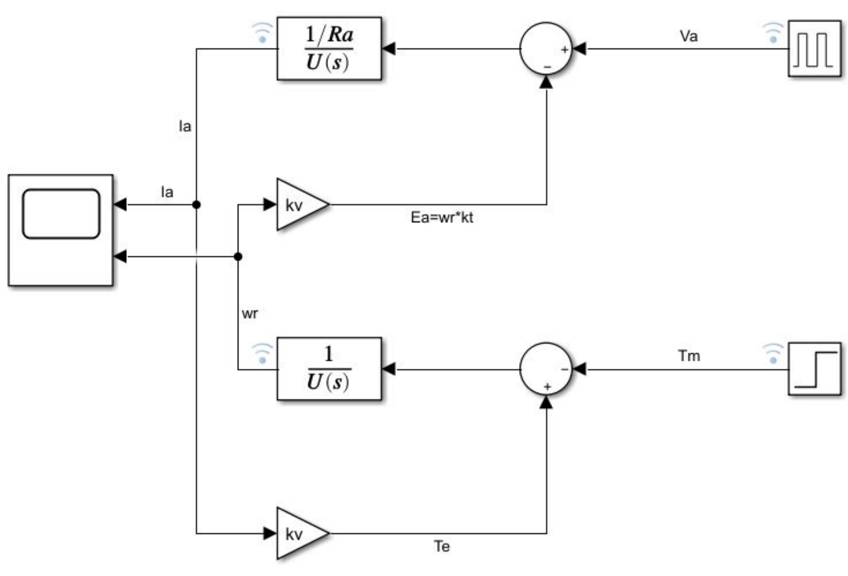

Figure 2.5-10: PM DC motor Simulink

Simulate and plot the current waveforms for the 2kHz and 10kHz operation corresponding to the duty cycle of 0.5 at the same mechanical load torque Tm-load(n).

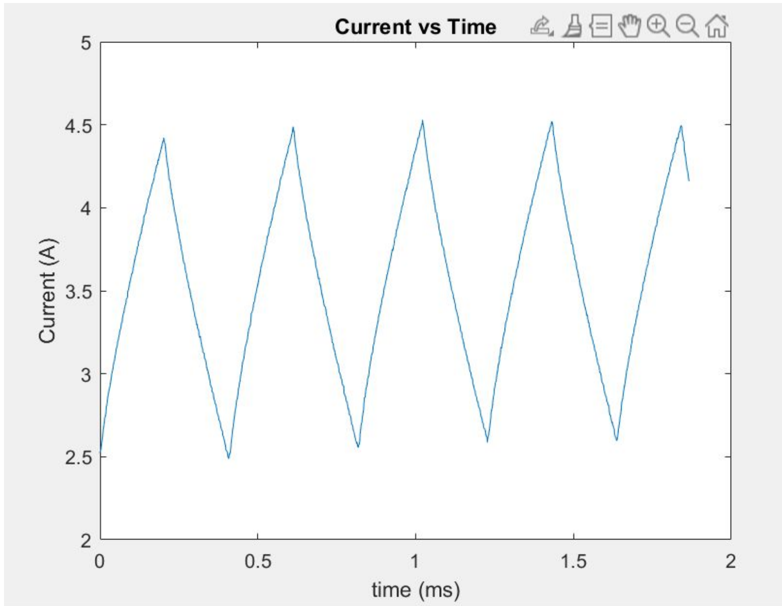

Figure 2.5-11: Current vs Time Plot at 2kHz

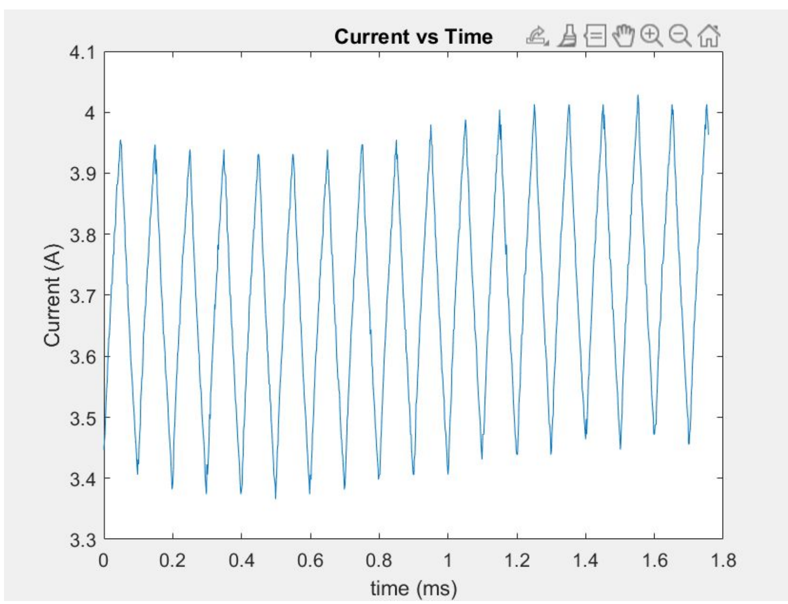

Figure 2.5-12: Current vs Time Plot at 10kHz

Compare these simulated waveforms with the recorded waveforms of the current in Task 3 and Task 4.

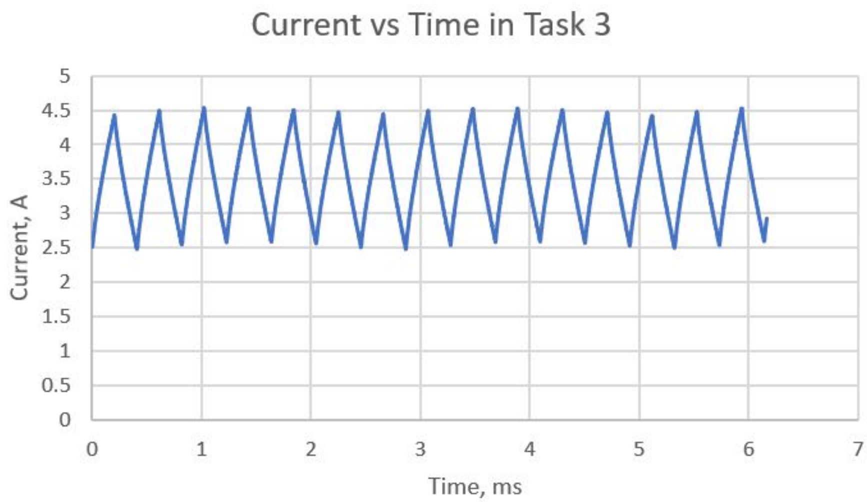

Figure 2.5-13: Task 3 Current vs Time Plot

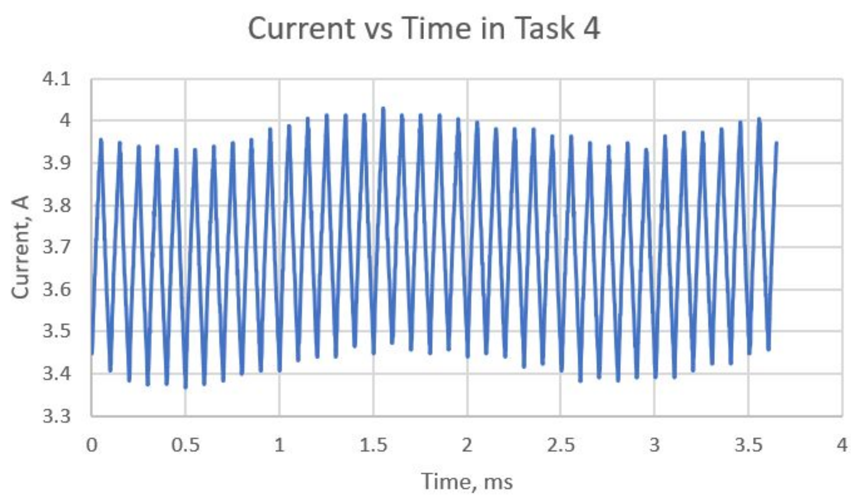

Figure 2.5-14: Task 4 Current vs Time Plot

Comment how good your match is in terms of predicting the current ripple. This should verify how accurately you have determined the machine's armature inductance!

- The simulated waveforms are similar in both shape and amplitude to the recorded waveforms of the current in Task 3 and 4. Therefore, the machine's armature inductance has been determined with a minimal margin of error and is close to the true value.

2.5D Task 5D: General Questions

1. **What can you say about the accuracy of equivalent circuit compared to your measurements?**
   - The accuracy of the equivalent circuit compared to the measurements is quite good, as the motor/constant value is near that of the one obtained from measured values, and the component values are consistent with the measured data.

2. **Which method of the speed control (Task 2 or Task 3/4) in your opinion is more practical for large motors and/or automotive/robotics applications? Briefly explain.**
   - The most practical method of speed control for large motors and automotive/robotics applications is pulse width modulation as it reduces wasted energy used to mechanically control the process. By varying the frequency, the motor speed is also varied, dissipating heat at reduced speeds.The wider the pulse width, the more average voltage applied to the motor terminals and the faster the motor will rotate.

3. **Also, plot on different plots the armature current and voltage (3 to 6 cycles) for the Task 3 A 3) and Task 3 B 1).**

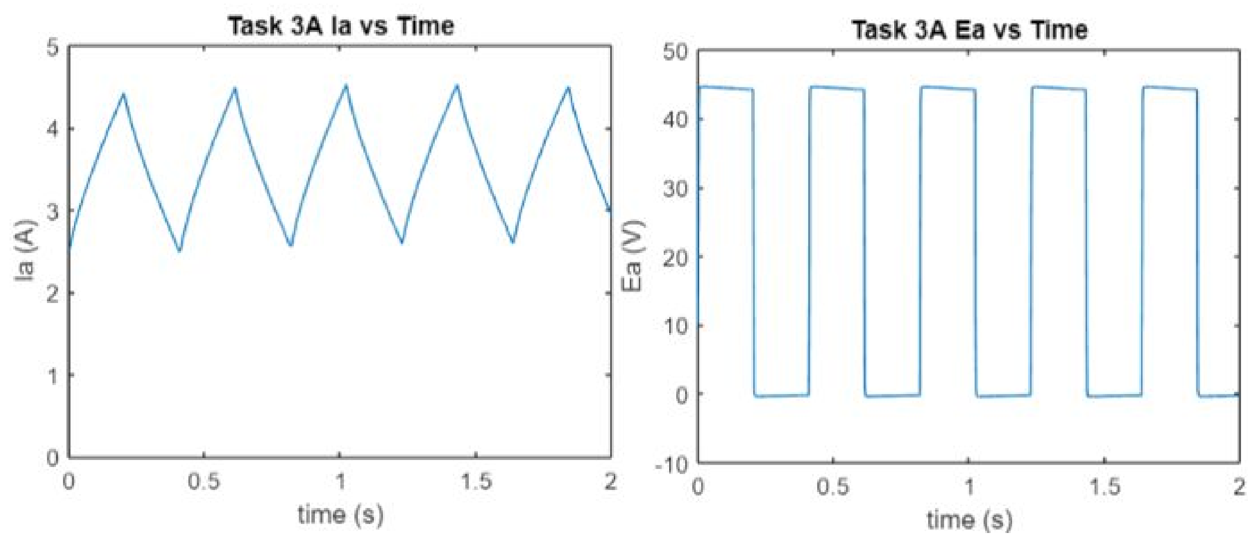

Figure 2.5-15: Task 3A Ia vs Time (left) and Ea vs Time (right) Relationship

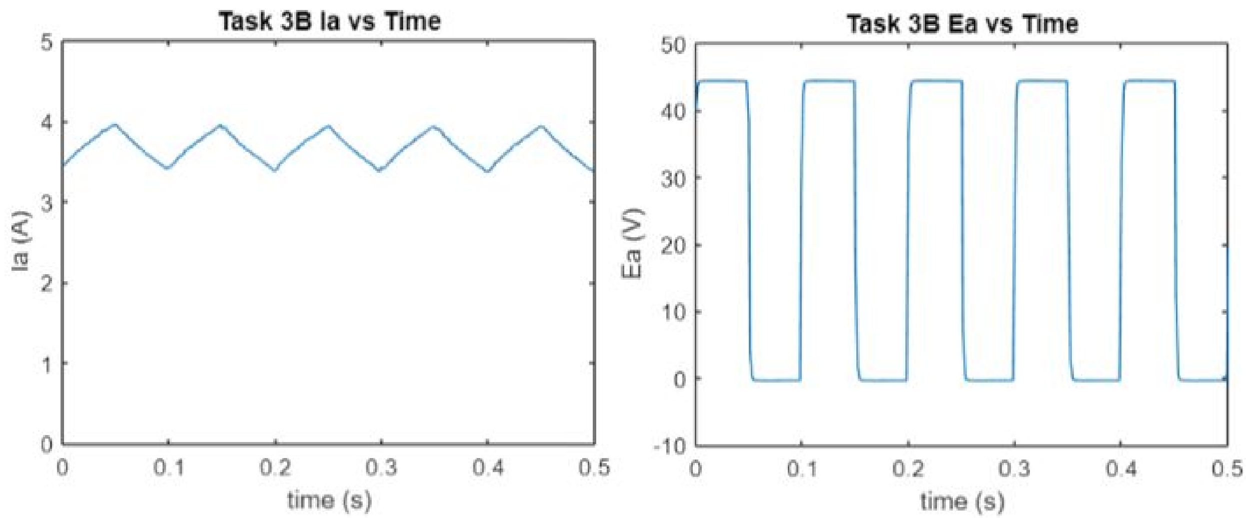

Figure 2.5-16: Task 3B Ia vs Time (left) and Ea vs Time (right) Relationship

What can you say about the effect of switching frequency?

- In Task 2B, the switching frequency is higher, increasing the frequency and duty cycle of the on-off pulses. The voltage pulses become shorter and quicker in Task 2B, however there is no relative change to the on or off timing. The current pulses have a greater magnitude in Task 3A than Task 3B.

4. **In the energy conversion chain presented in Table 7, which element appears to be the least efficient and why?**
    - The efficiency of the Inverter-Motor combined appears to be the least efficient at 43.2% efficiency, as when motors are fed by inverter power, additional losses degrade energy efficiency due to harmonic content.

3. Conclusion

This lab explored the load characteristics when the motor is supplied from a variable DC source and the load and speed characteristics when the motor is supplied from a DC-DC converter with variable duty-cycle and variable frequency. A small industrial 48V Permanent Magnet DC Motor was fully characterized and its circuit parameters were identified using a variety of tests and measurements. By doing a set of measurements, the motor torque/voltage constant Kt/Kv, the armature winding and brush resistance Ra, the armature inductance La, the Torque friction Tfric, and the moment of inertia Jrotor were determined and plotted. A DC-DC converter with PWM was used to control a motor by varying the duty-cycle. Finally, a steady state equivalent circuit of the PM otro was developed then used to predict torque-speed characteristics and compare the measurements.

Appendix A: Task 5A Q4-5 Matlab Code

```
task5A4 = importdata("Task1C-OneMachine.txt");

time = task5A4.data(:,1)/1000;   %time in ms
n = task5A4.data(:,6);

wr = 2*pi*n/60;
Tfric = 0.18;

dwrdt = (55.8327-140.559)/((1.5-1)*1000)
Jrotor = Tfric/dwrdt

figure(1);                  %plot Ea vs time
plot(time,V1);
xlabel('time (s)');
ylabel('Ea (V)');
title('Ea vs Time');

figure(2);                  %plot wr vs time
plot(time,wr);
xlabel('time (s)');
ylabel('speed (rad/s)');
title('Speed vs Time');
```

Appendix B: Task 5A Q6 Matlab Code

```
task5A6 = importdata("Task3A.txt");

time = task5A6.data(:,1);   %time in s
Ia = task5A6.data(:,4);
n = task5A6.data(:,6);
wr = n*2*pi/60;
kt = 0.2179;
Ea = kt .* wr;
Va = 45;

diadt = diff(Ia) ./ diff(time);
Vla = Va - Ea;
%Vla = Va - Ea - (Ia*Ra);
diadt_mean = mean(abs(diadt))
La = abs(Vla(1:end-1)) ./ diadt_mean;
format long
La_final = mean(La)
```