Span and Basis

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1.3 Span

- Given vectors $v_1, \dots, v_k$, form subspace of all possible lin. comb. $\longrightarrow$ span $(v_1 \dots v_k)$
  - $\hookrightarrow$ If lin. comb of any two elements of span$(v_1 \dots v_k)$
    ex. $c_1 v_1 + \dots + c_k v_k$ & $d_1 v_1 + \dots + d_k v_k$
    $\rightarrow$ a lin. comb. of these two lin. comb. = lin. comb.

$$S = \left\{ \sum_{j=1}^{k} c_j v_j : c_j \in \mathbb{R} \right\}$$

ex. span of $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \longrightarrow$ whole 3D space

$\because$ every vector is a lin. comb of these.

span $\left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$ = span $\left\{ \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$

ex. let $S = \text{span}\{x^2 - 2x + 3, -2x^2 + 3x + 1\}$
determine if $p(x) = 10x^2 - 17x + 9$ is in S

$\longrightarrow$ Find if exists c_1, c_2 s.t. $c_1(x^2 - 2x + 3) + c_2(-2x^2 + 3x + 1) = 10x^2 - 17x + 9$

ex. span $\left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix} \right\}$ = $x - z$ plane in $\mathbb{R}^3$

1.4 Basis

- set $\{v_1 \dots v_k\}$ in subspace $S \longrightarrow$ basis for that subspace if:
  1. span $\{v_1 \dots v_k\} = S \longrightarrow \forall u \in S \ \exists \ c_1, c_2 \dots c_k$ s.t. $u = c_1 v_1 + \dots + c_k v_k$
     - any vector can be written as a L.C of $v_1 \dots v_k$
  2. $\{v_1 \dots v_k\} \longrightarrow$ lin. indep $\longrightarrow$ choice of c_i's is unique
     - there is one way of doing this:
       $\vec{v} = c_1 v_1 + \dots + c_k v_k$, $\vec{v} = d_1 v_1 + \dots d_k v_k$
       $0 = (c_1 - d_1)\vec{v}_1 + \dots (c_k - d_k)\vec{v}_k$ for L.I, $(c_k - d_k) = 0$
       $\therefore$ $c_k = d_k$

- If $\{v_1, \dots, v_k\}$ = basis of subspace S, $\dim(S) = k$

ex. $\{e_1, e_2, e_3\} \longrightarrow$ basis for $\mathbb{R}^3$

ex. $\left\{ e_1, e_2, \begin{bmatrix}1\\0\\1\end{bmatrix} \right\} \longrightarrow$ NOT a basis for $x - z$ plane

ex. $\left\{ \begin{bmatrix}2\\0\\0\\3\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \right\} \longrightarrow$ L.I., and a basis for $\left\{ \begin{bmatrix}2\\0\\0\\3\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \right\}$ but not for $\mathbb{R}^4$
$\because$ both do not span $\mathbb{R}^4$

ex. $\left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\}$ & $\left\{ \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix} \right\}$ are basis for $\mathbb{R}^2$

ex. check if $\{1, x, x^2\}$ is a basis for vector space P^2 polynomial deg 2

i) span $\{1, x, x^2\} = P^2$:
$= \left\{ \sum_{j=0}^{2} c_j x^j : c_j \in \mathbb{R} \right\} = \{c_0 + c_1 x + c_2 x^2, \ c_j \in \mathbb{R}\} = P^2$

i) span $\{1, x, x^2\} = P^2$:

$= \{ \sum_{j=0}^{2} c_j x^j : c_j \in \mathbb{R} \} = \{ c_0 + c_1 x + c_2 x^2, \ c_j \in \mathbb{R} \} = P^2$

ii) $\{1, x, x^2\}$ is L.I. : for $c_0 + c_1 + c_2 x^2 = 0$, $c_1 = c_2 = c_3 = 0$ if take derivatives