Two Port Networks

April 23, 2020 11:11 AM

Connecting 2-Port Networks

Cascade

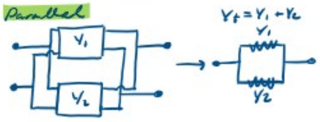
$T_t = T_1 \cdot T_2$

Series

$z_t = z_1 + z_2$

Parallel

$Y_t = Y_1 + Y_2$



Conversion table

to \ from	Z	Y	H	G	T
Z	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta_g}{g_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{11}}{t_{21}} & \frac{\Delta_t}{t_{21}} \\ \frac{1}{t_{21}} & \frac{t_{22}}{t_{21}} \end{bmatrix}$
Y	$\begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ \frac{-g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{22}}{t_{12}} & \frac{-\Delta_t}{t_{12}} \\ \frac{-1}{t_{12}} & \frac{t_{11}}{t_{12}} \end{bmatrix}$
H	$\begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{g_{22}}{\Delta_g} & \frac{-g_{12}}{\Delta_g} \\ \frac{-g_{21}}{\Delta_g} & \frac{g_{11}}{\Delta_g} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{12}}{t_{22}} & \frac{\Delta_t}{t_{22}} \\ \frac{-1}{t_{22}} & \frac{t_{21}}{t_{22}} \end{bmatrix}$
G	$\begin{bmatrix} \frac{1}{z_{11}} & \frac{-z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta_z}{z_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ \frac{-y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{h_{22}}{\Delta_h} & \frac{-h_{12}}{\Delta_h} \\ \frac{-h_{21}}{\Delta_h} & \frac{h_{11}}{\Delta_h} \end{bmatrix}$	$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{t_{21}}{t_{11}} & \frac{-\Delta_t}{t_{11}} \\ \frac{1}{t_{11}} & \frac{t_{12}}{t_{11}} \end{bmatrix}$
T	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta_g}{g_{21}} \end{bmatrix}$	$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$

Port:
- Pair of nodes to connect circuit to
- Current entering a port wire = interaction of 1 voltage & current

$v = Ri, \quad v = L\frac{di}{dt}, \quad i = C\frac{dv}{dt}$

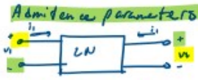
$\rightarrow$ 2 inputs & 2 outputs

Open port 2, Apply 1A port 1: $v_1 = z_{11} i_1 + z_{12} i_2$ $\quad v_1 = z_{11}$

Open port 1, Apply 1A, port ... $v_2 = z_{21} i_1 + z_{22} i_2$ $\quad v_2 = z_{21}$

Impedance parameters: $\left.\begin{aligned} v_1 &= z_{11} i_1 + z_{12} i_2 \\ v_2 &= z_{21} i_1 + z_{22} i_2 \end{aligned}\right]$ in $\xrightarrow{\text{Matrix Form}}$ $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$

Admittance parameters: $\left.\begin{aligned} i_1 &= y_{11} v_1 + y_{12} v_2 \\ i_2 &= y_{21} v_1 + y_{22} v_2 \end{aligned}\right]$ out $\xrightarrow{\text{Matrix Form}}$ $\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$



$\rightarrow$ 2 inputs & 2 outputs

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1}$$

Hybrid Parameters: $\left.\begin{aligned} v_1 &= h_{11} i_1 + h_{12} v_2 \\ i_2 &= h_{21} i_1 + h_{22} v_2 \end{aligned}\right]$ in $\xrightarrow{\text{Matrix Form}}$ $\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$

Inverse Hybrid parameters: $\left.\begin{aligned} v_2 &= g_{11} v_1 + g_{12} i_2 \\ i_1 &= g_{21} v_1 + g_{22} i_2 \end{aligned}\right]$ out $\xrightarrow{\text{Matrix Form}}$ $\begin{bmatrix} v_2 \\ i_1 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}^{-1}$$

Transmission Parameters: $\begin{aligned} v_1 &= Av_2 + B(-i_2) \\ i_1 &= Cv_2 + D(-i_2) \end{aligned}$ $\quad \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Inverse Transmission Parameters: $\begin{aligned} v_2 &= av_1 + b(-i_1) \\ i_2 &= cv_1 + d(-i_1) \end{aligned}$

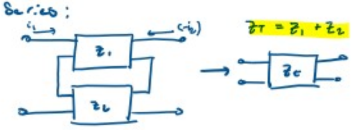
Connecting 2 Port Networks

Cascade:

$T_t = T_1 \cdot T_2$

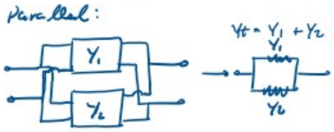
Series:

$z_T = z_1 + z_2$

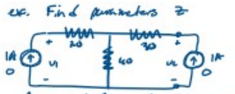


Parallel:

$Y_t = Y_1 + Y_2$



ex. Find parameters z



$v_1 = z_{11} i_1 + z_{12} i_2$
$v_2 = z_{21} i_1 + z_{22} i_2$

Open port 2, apply 1A to port 1: $V_1 = 1 \times (20+40) = 60 = z_{11}$
$V_2 = 1 \times 40 = 40 = z_{21}$

Open port 1, apply 1A to port 2: $V_2 = 1 \times (30+40) = 70 = z_{22}$
$V_1 = 1 \times 40 = z_{12}$

$$z = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix}$$

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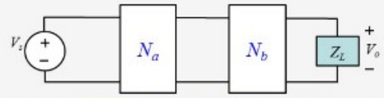
(50 points)

For the individual two-ports shown, the given two-port parameters are known: $[h_a] = \begin{bmatrix} 5\Omega & 6 \\ 6 & -3S \end{bmatrix}$, $[y_b] = \begin{bmatrix} 7 & 6 \\ 4 & -8 \end{bmatrix} S$

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)



a. Determine the transmission and y parameters of the overall two-port. If the y matrix does not exist, contact the teaching staff by private Piazza post to get a different randomized set of parameters.

[$\quad$ $\quad$ Ω]

a) T-Params

Conversion Matrix: $T_h = \begin{bmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{bmatrix}$

$h = \begin{bmatrix} 5 & -6 \\ 6 & -3 \end{bmatrix}$ $\quad \Delta h = -15 - (-36) = 21$

$T_a = \begin{bmatrix} -21/6 & -5/6 \\ 1/2 & -1/6 \end{bmatrix}$

Conversion Matrix: $T_y = \begin{bmatrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\Delta y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{bmatrix}$

$y = \begin{bmatrix} -7 & 6 \\ 4 & -8 \end{bmatrix}$ $\quad \Delta y = 56 - 24 = 32$

$T_b = \begin{bmatrix} 8/4 & -1/4 \\ -\frac{32}{4} & 7/4 \end{bmatrix}$

$T = T_a \cdot T_b \longrightarrow$ Cascade

Y-Params

Conversion Matrix: $Y_t = \begin{bmatrix} \frac{t_{22}}{t_{12}} & -\frac{\Delta t}{t_{12}} \\ \frac{-1}{t_{12}} & \frac{t_{11}}{t_{12}} \end{bmatrix}$

$T = \begin{bmatrix} -0.3333 & -0.5833 \\ 2.3333 & -0.41666 \end{bmatrix}$ $\quad \Delta t = 1.5$

$Y = \begin{bmatrix} 0.714286 & 2.571428 \\ 1.714285 & 0.571428 \end{bmatrix}$

b) Voltage Ratio $G = V_o/V_s \rightarrow Z_L = 6\,\Omega$
- open port 2, Apply 1A to port 1
- open port 1, Apply 1A to port 2

Y-param: $I_1 = Y_{11} V_1 + Y_{12} V_2$
$I_2 = Y_{21} V_1 + Y_{22} V_2$

$Y = \begin{bmatrix} 0.714286 & 2.571428 \\ 1.714285 & 0.571428 \end{bmatrix}$

a. Determine the transmission and y parameters of the overall two-port. If the y matrix does not exist, contact the teaching staff by private Piazza post to get a different randomized set of parameters.

[T] = [___ ___ Ω ; ___ S ___]

[Y] = [___ S ___ S ; ___ S ___ S]

b. Find the voltage ratio $G = V_o/V_s$ when $Z_L = 6\Omega$:

$G =$ ___

$y = [4 \ -8] \quad \Delta y = 56 - 24 = 32$

$$T_b = \begin{bmatrix} 8/4 & -1/4 \\ -\frac{32}{4} & 7/4 \end{bmatrix}$$

$T = T_a \cdot T_b \longrightarrow$ Cascade

$$T = \begin{bmatrix} -\frac{21}{6} & -\frac{5}{6} \\ -\frac{1}{2} & -\frac{1}{6} \end{bmatrix}\begin{bmatrix} -\frac{8}{4} & -\frac{1}{4} \\ -\frac{32}{4} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} -0.3333 & -0.58333 \\ 2.3333 & -0.416666 \end{bmatrix}$$

- open port 2, Apply 1A to port 1
- open port 1, Apply 1A to port 2

Y-param: $I_1 = Y_{11}V_1 + Y_{12}V_2$
$I_2 = Y_{21}V_1 + Y_{22}V_2$

$$Y = \begin{bmatrix} 0.714286 & 2.571428 \\ 1.714285 & 0.571428 \end{bmatrix}$$

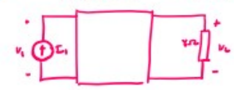
$I_2 = 1.7143V_1 + 0.5714V_2$

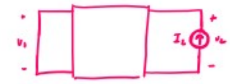
$V_2 = -6I_2 \quad I_2 = \frac{-V_2}{6}$

$-\frac{V_2}{6} = 1.7143V_1 + 0.5714V_2$

$-V_2 = 10.2858V_1 + 3.4284V_2$

$\frac{V_2}{V_1} = -2.32269V$





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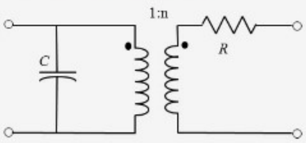
(50 points)
Consider the two-port network shown with $n = 6$, $R = 8k\Omega$ and $C = 3\mu F$.

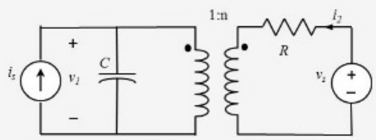
a. Determine the hybrid [h] and impedance [z] parameters (as functions of s; hint: each hybrid parameter is a first order transfer function).



[h] = [___ Ω ___ ; ___ ___ S]

[z] = [___ Ω ___ Ω ; ___ Ω ___ Ω]

b. For sources $i_s(t) = 3cos(700t)mA$ and $v_s(t) = 9u(t)V$, compute $v_1(t)$ and $i_2(t)$ (**Hint:** use superposition on the result from part (a), using phasor domain analysis for one part and Laplace domain analysis for the other).



[**Note:** For the sinusoidal portion of the answer, enter angle in radians instead of degrees. Also, for the Heaviside function, use "u(t)".]

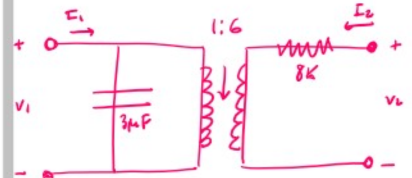
$v_1(t) =$ ___ V

$i_2(t) =$ ___ mA

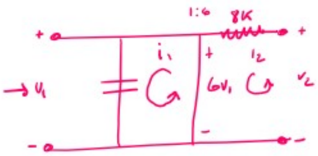
a) $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \longrightarrow \begin{matrix} V_1 = h_{11}I_1 + h_{12}V_2 \\ I_2 = h_{21}I_1 + h_{22}V_2 \end{matrix}$

$V_2 = 0$ (short port) $\longrightarrow h_{11} = \frac{V_1}{I_1}\Omega, \ h_{21} = \frac{I_2}{I_1}$

$I_1 = 0$ (open port) $\longrightarrow h_{12} = \frac{V_1}{V_2}, \ h_{22} = \frac{I_2}{V_2}$



$\rightarrow V_1$



a) **Transformer:**

$n_s V_1 = n_p V_2$
$n_s I_1 = n_p I_2$

① $6V_1 = V_2$
② $-6I_1 = I_2$
③ $V_1 \cdot Cs = I_1$
④ $\frac{V_2}{6K} = I_2$

$I_1 = -6I_2 + V_1(3\times10^{-6})s$
$V_2 = 6V_1 + 8KI_2$

$$\begin{bmatrix} I_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

g-parameters

$$g = \begin{bmatrix} (3\times10^{-6})s & -6 \\ 6 & 8k \end{bmatrix}$$

h-parameters

$$h = g^{-1} = \frac{1}{36.024}\begin{bmatrix} 8K & 6 \\ -6 & (3\times10^{-6})s \end{bmatrix} = \begin{bmatrix} 222.074 & 0.166555 \\ -0.166555 & 8.32778\times10^{-8}s \end{bmatrix}$$

① $I_1 = -6I_2 + V_1(3\times10^{-6})s$
② $V_2 = 6V_1 + 8KI_2$

z-parameters

$V_1 = z_{11}I_1 + z_{12}I_2$
$V_2 = z_{21}I_1 + z_{22}I_2$

$$z = \begin{bmatrix} \frac{1}{3\times10^{-6}s} & \frac{6}{3\times10^{-6}s} \\ \frac{6}{3\times10^{-6}s} & \frac{36}{3\times10^{-6}s} + 8k \end{bmatrix}$$

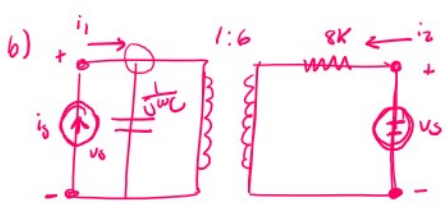
Rearrange:

$V_1 = \frac{1}{3\times10^{-6}s}I_1 + \frac{6}{3\times10^{-6}s}I_2$

$V_2 = 6\left[\frac{1}{3\times10^{-6}s}I_1 + \frac{6}{3\times10^{-6}s}I_2\right] + 8kI_2$

$= \frac{6}{3\times10^{-6}s}I_1 + \left(\frac{36}{3\times10^{-6}s} + 8k\right)I_2$

b)



Superposition

Phasor:

$i_s = 3\cos(700t)$ mA
$V_s = 9u(t)$ V

① $I_1 = -6I_2 + V_1(3\times10^{-6})s$
② $V_2 = 6V_1 + 8kI_2$

$\rightarrow s = j\omega = j700, \ I_1 = i_s(t) = 3\cos(700t), \ V_2 = 0 \rightarrow$ open port 2
$= 3\angle0^\circ$

① $3\times10^{-3}\cos(700t) = -6I_2 + V_1(3\times10^{-6})\cdot(j700)$
② $0 = 6V_1 + 8kI_2$

$\rightarrow$ solve V_1 & I_2:

$V_1 = 0.604122\angle-0.486627$ V, $I_2 = 0.4530915\angle2.70496$ mA

$V_1 = 0.604122\cos(700t - 0.436627)$ V

$I_2 = 0.4530915\cos(700t + 2.70496)$ mA

Laplace:

$I_1 = 0 \longrightarrow$ open port

$V_2 = 9(u(t))$

[illegible]:

$I_1 = 0 \longrightarrow$ open port

$V_2 = 9\,(u(t))$

① $0 = -6 I_2 + V_1 (3\times10^{-6}) s$

laplace domain $\longrightarrow$ ② $\frac{9}{s} = 6 V_1 + 8k I_2$

$\longrightarrow$ solve V_1 & I_2:

$V_1 = \frac{6}{2.6666\times10^{-3} s^2 + 4s} \longrightarrow \mathcal{L}^{-1}\{V_1\} = \left[6\left(-0.25 e^{-1800t} + 0.25\right)\right] u(t)$ V

$I_1 = \frac{3\times10^{-6}}{2.6666\times10^{-3} s + 4} \longrightarrow \mathcal{L}^{-1}\{I_1\} = \left[0.001125 e^{-1500t}\right] u(t)$ mA

Superposition

$V_1 = 0.604122 \cos(700t - 0.436627) + \left[6\left(-0.25 e^{-1800t} + 0.25\right)\right] u(t)$

$I_1 = 0.4530915 \cos(700t + 2.70496) + \left[0.001125 e^{-1500t}\right] u(t)$