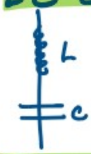
Resonance

February 15, 2020 9:33 AM

LC circuit



- frequency of resonance: $\omega_0 = \frac{1}{\sqrt{LC}}$
- LC circuit looks & behaves like short circuit

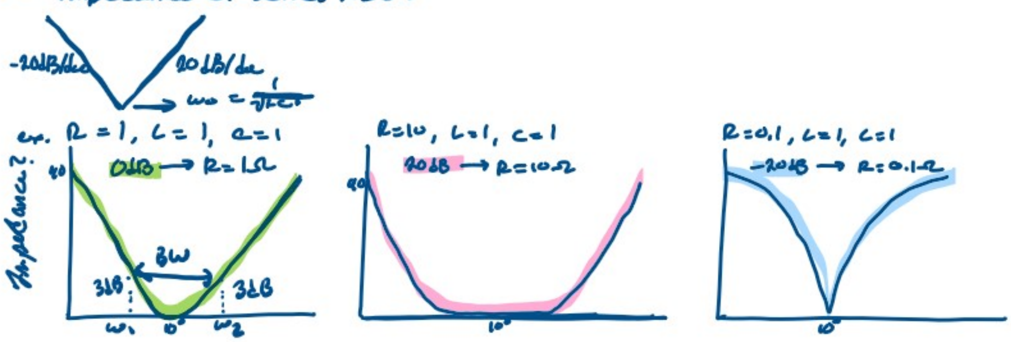
RLC circuits

- @ resonance ω_0, L & C cancel out
- RLC series behaves as resistance

$$Z_T = \frac{LCs^2 + RCs + 1}{Cs} \xrightarrow{\mathcal{L}^{-1}} \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC}$$

Amplitude of Bode Plot

— Impedance of series RLC:



— **Bandwith**: $BW = \omega_2 - \omega_1$ ⟶ **Half power frequencies**

— **Quality Factor**: $Q = \frac{\omega_0}{BW}$

RLC Series

ex. If impedance → 3dB higher: $20\log\left(\frac{Z_{3dB}}{Z_m}\right) = 3$, $Z_{3dB} = Z_m\sqrt{2}$

$Z_m = R$, what freq ω_x does this happen?

$$\sqrt{2}\,R = \sqrt{R^2 + \left(\omega_x L - \frac{1}{\omega_x C}\right)^2}$$

⟶ 2 quadratic eqns in ω_x, each w/ 2 roots, take positive of each

① $\omega_x^2 LC - \omega_x RC - 1 = 0$

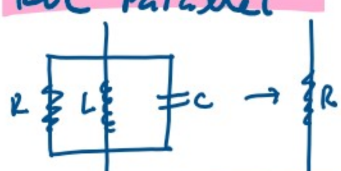
$$\omega_1 = \frac{RC + \sqrt{(RC)^2 + 4LC}}{2LC} \rightarrow \text{greater}: \omega_1$$

② $\omega_x^2 LC + \omega_x RC - 1 = 0$

$$\omega_2 = \frac{-RC + \sqrt{(RC)^2 + 4LC}}{2LC} \rightarrow \text{smaller}: \omega_2$$

$$\left.\begin{aligned} BW &= \omega_2 - \omega_1 = \frac{2RC}{2LC} = \frac{R}{L} \\ Q &= \frac{\omega_0}{BW} = \frac{1}{R}\sqrt{\frac{L}{C}} \end{aligned}\right\} \text{\underline{ONLY} for RLC Series}$$

RLC Parallel



@ resonant frequency, behaves just like R

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\left.\begin{aligned} Q &= \omega_0 RC = \frac{R}{\omega_0 L} = R\sqrt{\frac{C}{L}} \\ BW &= \omega_2 - \omega_1 = \frac{\omega_0}{Q} \end{aligned}\right\} \text{\underline{ONLY} for RLC parallel}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \qquad BW = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

$\therefore$ For Series & Parallel Half power f ω_{12}: $\omega_{12} = \omega_0 \left[\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{1}{2Q}\right]$

Scaling & Recipes

- Filter "cookbooks"
- Scale filters for different components

Frequency scaling

- Want same impedance Z but at K_f times original f
  need new values for L' & C'

$$Z_L = j(\omega K_f)L' = j\omega L \longrightarrow L' = \frac{L}{K_f}$$

$$Z_C = \frac{1}{j(\omega K_f)C'} = \frac{1}{j\omega C} \longrightarrow C' = \frac{C}{K_f}$$

Higher frequency but same impedance

$\therefore$ Divide both L & C by same factor used to multiply f by K_f

Magnitude Scaling

- If components too little/large, scale up/down component values without changing f

$\therefore$ Multiply all impedances by K_m: $(\omega' = \omega)$

$$Z_R' = K_m Z_R = K_m R, \quad Z_L' = K_m Z_L = j\omega K_m L, \quad Z_C' = K_m Z_C = \frac{1}{j\omega C / K_m}$$

$$R' = K_m R \qquad L' = K_m L \qquad C' = \frac{C}{K_m}$$

Magnitude & Frequency scaling

- Increase RLC by K_m, shift f response by K_f:

$$R' = K_m R, \quad L' = \frac{K_m}{K_f} \cdot L, \quad C' = \frac{1}{K_m K_f} \cdot C, \quad \omega' = K_f \omega$$