Current, Conductors, Boundary Conditions, Dialectric

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Current

$I = \frac{dQ}{dt}$ A → Motion of pos charge

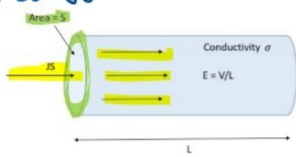
Current Density $\vec{J}$

- Total current through surface
  $I = \int_S \vec{J} \cdot d\vec{S}$ → $\vec{J} = \rho_v \vec{v}$ (v = velocity in x)
- Force on electron: $\vec{F} = -e\vec{E}$ (N)
- drift velocity v_d → mobility of electrons in material
  $v_d = -\mu_e \vec{E}$ → ∴ $J = \rho_v \vec{v} = -\rho_e \mu_e \vec{E} = \sigma \vec{E}$
  $\vec{J} = \sigma \vec{E}$ → $\sigma = -\rho_e \mu_e$ conductivity of material $\rho = \frac{1}{\sigma}$

Point Form of Ohm's law (Cylindrical wire segment)

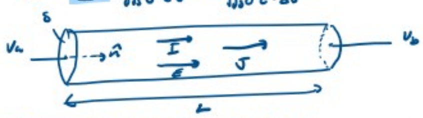
$I = \iint_S \vec{J} \cdot dS = \iint_S \vec{J} \cdot \hat{n}\, dS$

- If current ⊥ cross-section of wire: $I = \int_S \vec{J} \cdot d\vec{S} = JS$
- Potential difference V_{ab} b/w 2 points along wire
  $V_{ab} = -\int_b^a \vec{E} \cdot d\vec{L} = -\int_b^a \vec{E} \cdot \hat{n}\, dL = EL$ → $E = \frac{V_{ab}}{L}$ V/m
  ∵ moving parallel to $\vec{E}$ field

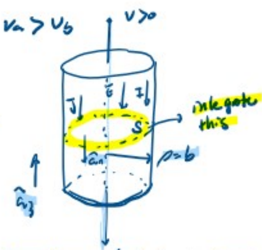


$J = \frac{I}{S} = \sigma E = \sigma \frac{V}{L}$ → $V = \frac{L}{\sigma S} I = RI$

$R = \frac{V_{ab}}{I} = \frac{-\int_b^a \vec{E} \cdot d\vec{L}}{\iint_S \vec{J} \cdot dS} = \frac{-\int_b^a \vec{E} \cdot d\vec{L}}{\iint_S \sigma \vec{E} \cdot dS}$



$V_a > V_b$

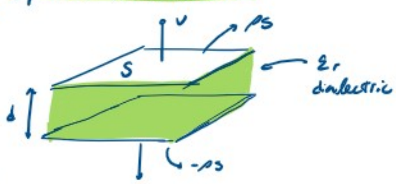


ex. Simple Case: let $\vec{J} = A\rho^2(-\hat{a}_z)$ A/m², solve for I
$I = \iint_S \vec{J} \cdot \hat{n}\, dS = \int_0^{2\pi}\int_0^b A\rho^2(-\hat{a}_z) \cdot (-\hat{a}_z)\, \rho\, d\rho\, d\varphi$
$I = \frac{A\pi b^4}{2}$ A

General Boundary Conditions

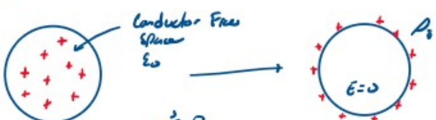
For E: E-tangential is continuous
$E_{T1} = E_{T2}$
$\varepsilon_{r1}\varepsilon_0 \vec{E}_{N1} = \varepsilon_{r2}\varepsilon_0 \vec{E}_{N2}$ → $\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$

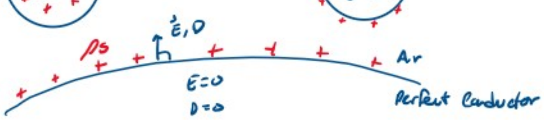
Capacitors & Dialectrics



For D: D-Normal is continuous
$D_{N1} = D_{N2}$
$\varepsilon_{r1}\varepsilon_0 D_{N1} = \varepsilon_{r2}\varepsilon_0 E_{N2}$

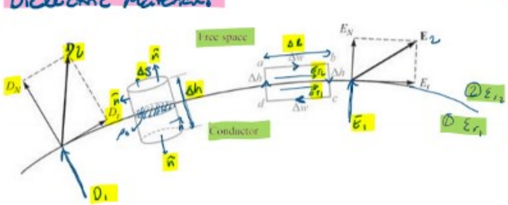
Special Case:





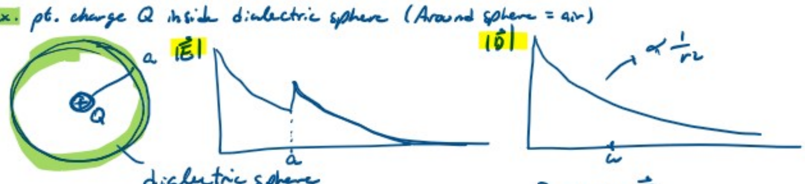
$E_{T1} = E_{T2}$ → $E_{T1} = 0 = E_{T2}$
$D_{N1} = 0$ (∵ E = 0)
$D_{N2} = \rho_S$ + $E_{N2} = \frac{D_{N2}}{\varepsilon_0} = \frac{\rho_S}{\varepsilon_0}$

Dielectric Materials



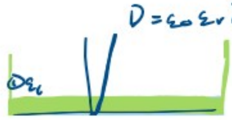
$E_{T1} = E_{T2}$ → $E_{T1} = \frac{D_{T1}}{\varepsilon_1} = \frac{D_{T2}}{\varepsilon_2} = E_{T2}$

ex. pt. charge Q inside dialectric sphere (Around sphere = air)



$D = \varepsilon_0 \varepsilon_r \vec{E}$

ex. $\varepsilon_2 > \varepsilon$ $\varepsilon = \varepsilon_r \varepsilon_0$
D_N = cts, E_T = cts
$D_{N2} = \varepsilon_{r2}\varepsilon_0 E_{N2} = D_{N1} = \varepsilon_{r1}\varepsilon_0 E_{N1}$
$E_{N1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} E_{N2}$ ∴ $E_{N1} > E_{N2}$



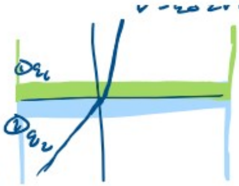
ex. $\varepsilon_2 > \varepsilon \quad \varepsilon = \varepsilon_r \varepsilon_0$

$D_N = cts, \; E_T = cts$

$D_{N2} = \varepsilon_{r2}\varepsilon_0 E_{N2} = D_{N1} = \varepsilon_{r1}\varepsilon_0 E_{N1}$

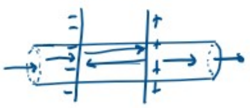
$E_{N1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} E_{N2} \quad \therefore E_{N1} > E_{N2}$

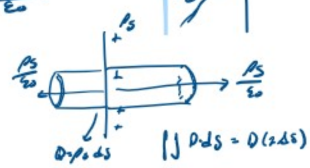
$E_{N2} = E_{T1}$



- For $\vec{E}$ of inf. sheet: $\vec{E} = \frac{\rho_s}{\varepsilon_0}$

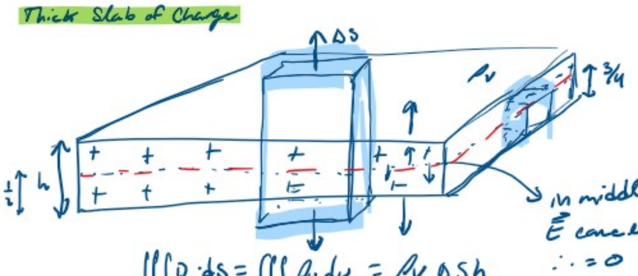
$\therefore E_N = \frac{\rho_s}{\varepsilon_0}$





$\iint D \cdot dS = D(2\Delta S)$

Thick Slab of Charge

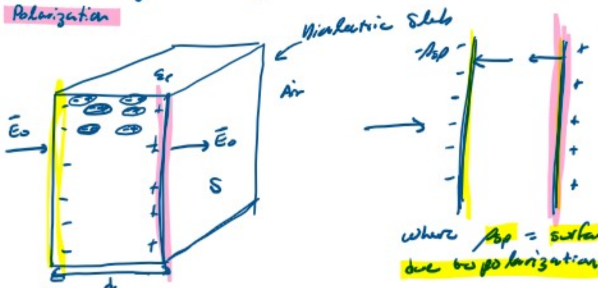


→ hold smaller gaussian surface

in middle, $\vec{E}$ cancel out $\therefore = 0$

$\oiint_S D \cdot dS = \iiint_V \rho_v \, dv = \rho_v \Delta S h$

Polarization



where ρ_{sp} = surface charge due to polarization

- $\vec{E}_d$ = net field
- slab has $p = Qd \rightarrow Q = \rho_s dS$
- polarization $(\vec{P})$ = # of dipole moments / unit volume

$\vec{P} = \frac{p}{v} = \frac{Qd}{Sd} = \frac{Q}{S}\hat{d} = \rho_s p \hat{d}$

$\vec{D}v = \vec{P} \quad \vec{P}_d = \varepsilon_0 \vec{E}_d + \vec{P} \longrightarrow \vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$

☆ Webwork 5

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(1 point)
A spherically symmetric charge distribution in free space defined over the spherical-coordinate region $a < r < \infty$ has a potential function $V(r) = V_0 \frac{a^2}{r^3}$ Volts, where V_0 and a are constants. The charge distribution in the region $0 < r < a$ is unknown.

Find mathematical expressions for the following. Type "Vo" (capital "V", small letter "o") in place of V_0.

(a) Find an expression for the electric field intensity, $\mathbf{E}$, in the region $a < r < \infty$.

$\mathbf{E}$ = ______ $\mathbf{a}_r$ V/m

(b) Find an expression for the volume charge density in the region $a < r < \infty$.

ρ_v = ______ C/m^3

(c) Find the charge enclosed inside a sphere of radius a centred at the origin.

Q_{enc} = ______ C

a) $\vec{E} = -\nabla V = -\left\langle -3V_0 \frac{a^2}{r^4}, 0, 0 \right\rangle = 3V_0 \frac{a^2}{r^4}\hat{a}_r$

b) $\rho_v = \nabla \cdot \vec{D} \longrightarrow \vec{D} = \varepsilon_0 \vec{E}$ (divergence in cylindrical coord.)

$\rho_v = \nabla \cdot (\varepsilon_0 \vec{E}) = \frac{1}{r^2}\frac{d}{dr}\left(r^2\left(3V_0 \frac{a^2}{r^4}\varepsilon_0\right)\right) = -6V_0 a^2 r^{-5}\varepsilon_0$

c) $Q_{enc.} = \oint_v \rho_v \, dv = \int_0^{2\pi}\int_0^{\pi}\int_0^a -6V_0 a^2 r^{-5}\varepsilon_0 r^2 \sin\theta \, dr\, d\varphi\, d\theta$

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0!

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(1 point)
Let S denote the part of the sphere $(x-a)^2 + y^2 + z^2 = 16a^2$ satisfying $z \geq 0$ and $x^2 + y^2 \leq a^2$. Find the upward flux through S for the vector field

$$\mathbf{F}(x, y, z) = \langle y - xy^2,\; x^2y - x,\; zy^2 + 4x^2 - zx^2 \rangle.$$

ANSWER: ______

DISCUSSION: It's easy to calculate the divergence of $\mathbf{F}$. Here that gives such a simple result that one might search for a way to exploit the Divergence Theorem. That theorem only works for closed surfaces, which S is not. Can you invent a 3D solid $\mathcal{R}$ such that (i) S is part of the boundary surface, and (ii) the outward flux of $\mathbf{F}$ through all the *other* parts of the surface is easy? If so, the Divergence Theorem will help you get the desired flux indirectly.

Capping surface = circle → cylindrical coord.

Parametrization: $x = \cos\theta, \; y = a\sin\theta, \; z = 0$

$\vec{F} \cdot \hat{n} = \vec{F}(x,y,z) \cdot \langle 0,0,1 \rangle = zy^2 + 4x^2 - zx^2 \; \hat{a}_z$

→ use parametrization: $\vec{F}(\vec{r}(\theta)) \cdot \hat{n} = 4(r\cos\theta)^2$

- Integrate: $\int_0^{2\pi}\int_0^a 4r^2\cos^2\theta \, r\, dr\, d\theta = a^4\pi$

Parametrization:

$x = r\cos\theta$
$y = r\sin\theta$
$z = 0$

$0 \leq \theta \leq 2\pi$

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(1 point)
Call the half space defined by $y < 0$ 'region 1', and the half space defined by $y > 0$ 'region 2'.

In region 1, the permittivity is $\epsilon_1 = 9\epsilon_0$ and the electric flux density is $\mathbf{D_1} = 3\mathbf{a}_x + 8\mathbf{a}_y + 1\mathbf{a}_z$ nC/m^2.

Find $\mathbf{D_2}$, the flux density in region 2, given the permittivity $\epsilon_2 = 5\epsilon_0$. (Note the required units.)

ANSWER: $\mathbf{D}_2 =$ [ ] $\mathbf{a}_x$ + [8] $\mathbf{a}_y$ + [ ] $\mathbf{a}_z$ nC/m^2.

$\vec{D}_1 = \langle 3, 8, 1 \rangle \quad \varepsilon_1 = 9\varepsilon_0, \ \varepsilon_2 = 5\varepsilon_0$

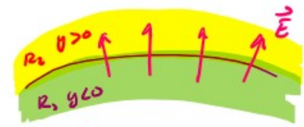
$\vec{D}_1 = \varepsilon_1 \vec{E}_1 \longrightarrow \vec{E}_1 = \frac{1}{9\varepsilon_0}\langle 3, 8, 1 \rangle$

$E_1 \varepsilon_1 = \varepsilon_2 E_2 \longrightarrow E_2 = \frac{E_1 \varepsilon_1}{\varepsilon_2}$

$D_2 = \varepsilon_2 E_2 = \langle 1.6666, 8, 0.5555 \rangle$

$\hookrightarrow \therefore D_{N1} = D_{N2}$

$\hat{n} = \hat{a}_y$



$\vec{D} = \varepsilon \vec{E}$

$\varepsilon = \varepsilon_r \varepsilon_0$

$\varepsilon_1 = \varepsilon_{r1} \varepsilon_0$

$\varepsilon_1 \vec{E}_1 = \varepsilon_2 \vec{E}_2$

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(1 point)
If the flux density is given by

$$D = \frac{18}{r}cos(2\theta)a_\theta \ C/m^2$$

find the total charge within the region $1.6 < r < 3.3\ m$, $1.4 < \theta < 1.7\ rad$, and $1.3 < \phi < 2.7\ rad$.

$Q =$ [ ] C

Hint: $cos^3(\theta) = \frac{1}{4}(3cos\theta + cos3\theta)$

$\psi = \iiint_v \rho_v dv \longrightarrow \rho_v = \nabla \cdot \vec{D}$

Divergence (spherical coord.)

$\nabla \cdot \vec{D} = \frac{18}{r^2}\left[\frac{\cos 2\theta \cos\theta}{\sin\theta} - 2\sin\theta\right]$

$\psi = \int_{1.6}^{3.3}\int_{1.4}^{1.7}\int_{1.3}^{2.7} \frac{1}{r\sin\theta}\cdot\left(-\frac{36}{r}\sin(2\theta)\right) d\varphi\, d\theta\, dr$

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(1 point)
A cylindrical conductor of radius $r = 0.65\ m$ is placed along the z-axis. The current density in the conductor is:

$$J = 8e^{-(1-\rho/r)}a_z \quad A/m^2$$

Find the total current passing through the plane at $z = 0$.

$I =$ [ ] A

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$I = \iint_S \vec{J}\cdot d\vec{S} = \iint_S \vec{J}\cdot\hat{n}\, dS \longrightarrow dS = \rho\, d\rho\, d\varphi$

$I = \int_0^{2\pi}\int_0^{0.65} 8e^{-(1-\rho/0.65)} \rho\, d\rho\, d\varphi \ \hat{a}_z$

$= 7.812716888A$