

Quicksort

November 22, 2019 7:16 PM

- Recurrence relations

```
double arrMax(double arr[], int size, int start) {  
    if (start == size - 1)  
        return arr[start];  
    else  
        return max( arr[start], arrMax(arr, size, start + 1) );  
}
```

- Merge sort analysis

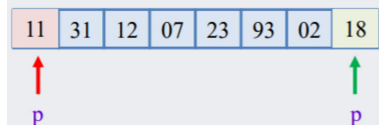
- algorithm: split in half, sort first half, sort second half, merge together
- $T(n)$ $\in O(n \log n)$

- Binary search

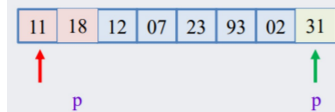
- Inspect midpoint, recursively search left or right half of array
- $T(n)$ $\in O(\log n)$

Quicksort

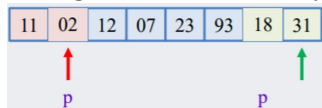
- efficient sorting algorithm than selection or insertion sort
 - sorts array by repeatedly partitioning it
- partitioning is the process of dividing an array into sections (partitions) based on some criteria
 - Big and small values
 - Negative and positive numbers
 - Names that begin with a-m names that begins with n-z
 - Darker and lighter pixels
- partitions roughly equal in size
 - partition array into small and big values using a partitioning algorithm
 - use 3 indices, place 1 at each end of array, low and high. third index starts at low
 - scan high from right to left until $arr[high]$ is less than $arr[p]$, $arr[high]$ (11) is already less than $arr[p]$ (18), swap and set p to high



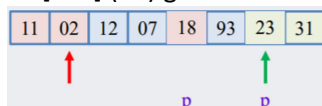
- scan low from left to right until $arr[low]$ greater than $arr[p]$
- $arr[low]$ (31) greater than $arr[p]$ (18) so swap and set p to low



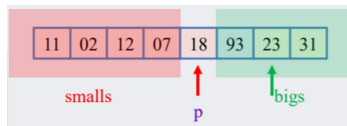
- scan high from right to left until $arr[high]$ less than $arr[p]$
- $arr[high]$ (2) less than $arr[p]$ (18), swap and set p to high



- scan low left-right until $arr[low]$ greater than $arr[p]$
- $arr[low]$ (23) greater than $arr[p]$ (18), swap, set p to low

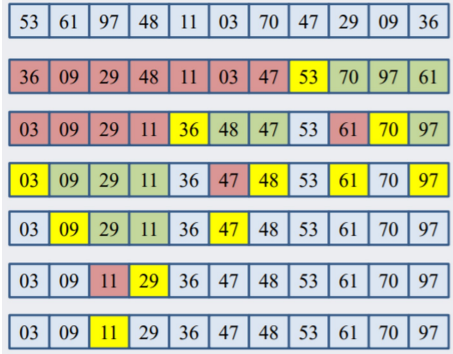


- scan high right-left until $arr[high]$ less than $arr[p]$ or $high = p$
- stop! index p contains pivot value, all elements of left of pivot = smaller values vs right of pivot = larger values, not ordered



Quicksort overview

- quicksort algorithm works by repeatedly partitioning array
- each time subarray partitioned, has sequence of small values, sequence of big values, and pivot value in correct position
- partition small and big values, repeat until each subarray consists of just 1 element



Quicksort Algorithm

- partition is where all comparisons are done, according to the Quicksort process

```
void qsort(int arr[], int low, int high) {
    int p;
    if (low < high) {
        p = partition(arr, low, high);
        qsort(arr, low, p-1);
        qsort(arr, p+1, end);
    }
}
```

```
void quicksort(int arr[], int size) {
    qsort(arr, 0, size-1);
}
```

Quicksort Analysis

- best and worst case
- Best case: each time a sub-array partitioned, the pivot is midpoint of slice, divided in half
 - o each sub array divided in half in each partition, compared
 - o process ends once sub arrays left to be partitioned are size 1
 - o How many times does n have to be divided in half before result is 1?
 - $\log_2 n$ times, Quicksort performs $n \cdot \log_2 n$ operations
- Worst case: array partitioned n times
 - o n comparisons in first partition step, n-1 in second...
 - o $\sum_{i=1}^{n-1} n = n \cdot (n-1)/2$, = around n^2
 - o when nearly sorted in either direction
- Average case: randomize positions of array elements to fix partially sorted case
 - o random scramble complexity: for n permutations: $O(n) + O(n \cdot \log_2 n)$

Merge vs Quicksort

- Quicksort worst case rare, easily avoided, faster than Merge sort
- can be sorted in place using $O(\log n)$ stack space
 - o quicksort

Name	Best	Average	Worst	Stable	Memory
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	challenging	$O(1)$
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$	Yes	$O(1)$
Merge sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$	Yes	$O(n)$
Quicksort	$O(n \log n)$	$O(n \log n)$	$O(n^2)$	challenging	$O(\log n)$