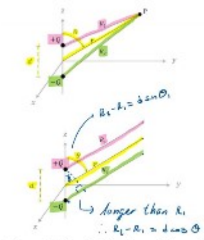
Gauss Law and Electric Dipole

April 25, 2020 5:43 PM

Electric Dipole



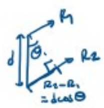


Figure 1: Electric Dipole configuration.

- If $d << r$, R_1 almost $\parallel$ to R_2 $\therefore$ $R_2 - R_1 = d\cos\theta$

$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{Q\,d\cos\theta}{4\pi\varepsilon_0 r^2} \longrightarrow R_1 \approx R_2 \approx r$$

- Electric Field: $E = -\nabla V$

Cylindrical coord.: $E = -\nabla V = -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\hat{a}_\varphi\right)$

$$= \frac{Qd}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right)\ V/m$$

Gauss Law

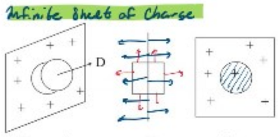
- Electric Flux through closed surface = total charge enclosed by surface

$\Psi = \oint \vec{D}\cdot d\vec{S}$ = charge enclosed = $Q_{enc.}$

- Electric Flux Density D_s

$\oint \vec{D}_s \cdot d\vec{S} = \iiint_v \rho_v\, dv = Q_{enclosed}$

Infinite Sheet of Charge



isometric side front

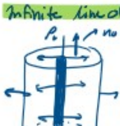
- $\vec{E}$ Field $\perp$ sheet, no flux leaves sides

$2\cdot\iint_{ends} \vec{D}\cdot d\vec{S} + \iint_{side} \vec{D}\cdot d\vec{S} = \iint \rho_s\, dS$

ends $\because$ 2 faces; side $= 0 \because$ no flux

$\therefore 2\vec{D}\iint_{ends} dS = \rho_s \iint dS \longrightarrow 2\vec{D} = \rho_s,\ \vec{D} = \varepsilon_0\vec{E} \longrightarrow \vec{E} = \frac{\rho_s}{2\varepsilon_0}$ ($\vec{E}$-field of sheet)

Infinite line of Charge



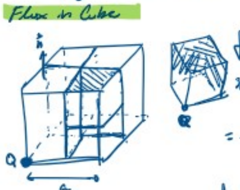
$\iint \vec{D}\cdot d\vec{S} = Q_{enc.}$ $\longrightarrow$ differential volumes

$\int_0^z\int_0^{2\pi} \varepsilon_0 \vec{E}\,\hat{a}_\rho \cdot \rho\, d\varphi\, dz\, \hat{a}_\rho = \rho_L\,\hat{a}_z$

$\varepsilon_0 E_\rho (2\pi) z = \rho_L z$

$\therefore E = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho$ V/m

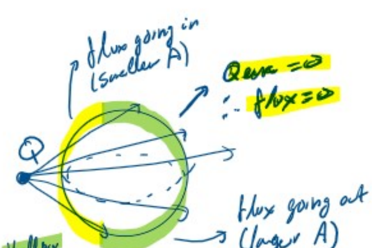
Flux in Cube



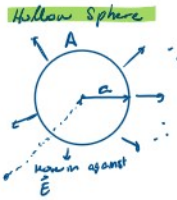
flux Density: $D = \varepsilon_0 \vec{E}$

$$= \frac{Q}{24\varepsilon_0} \longrightarrow \left(E = \frac{Q}{4\pi\varepsilon_0 R^2}\right)$$

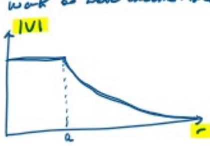
$\int D\cdot ds = \Psi$ = charge enclosed



Hollow Sphere



$\because \vec{E}$ field = 0 in sphere, work to move around inside = 0



Volume Sphere

① $r > a$: $\Psi = \oint \vec{D}\cdot d\vec{S} = \oiint \varepsilon_0 \vec{E}\,\hat{a}_r \cdot \hat{a}_r\, dS = \iiint_v \rho_v\, dv \longrightarrow dS = r^2\sin\theta\, d\theta\, d\varphi$ (diff. surfaces)

$= \varepsilon_0 \int_\varphi\int_\theta E r^2 \sin\theta\, d\theta\, d\varphi = C(4/3\,\pi a^3)$

$\therefore E = \frac{C(4/3\,\pi a^3)}{4\pi\varepsilon_0 r^2}\hat{a}_r \longrightarrow$ Const. C $\longrightarrow$ Q density

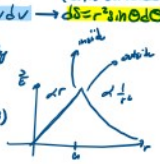
② $r < a$: left side unchanged, $Q_{enc.}$ as func. of r:

If ρ_v = const.:

$\iiint_v \rho_v\, dv = C\iiint_v r^2 \sin\theta\, dr\, d\theta\, d\varphi = C(4/3\,\pi r^3)$

$\varepsilon_0 E(4\pi r^2) = C(4/3\,\pi r^3)$

$E = \frac{C(4/3\,\pi r^3)}{\varepsilon_0(4\pi r^2)} = \frac{C r}{3\varepsilon_0}\hat{a}_r$ V/m



If $\rho_v \neq$ const.:

$Q_{enc.} = \iiint_v \rho_v(r,\theta,\varphi)\,\overbrace{r^2\sin\theta\, dr\, d\theta\, d\varphi}^{dv}$

Field $\neq \parallel$ or $\perp$ to surface

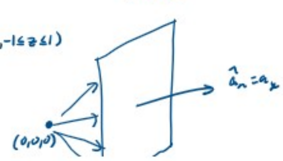
ex. 1 side of cube w/ Q charge in middle:

For pt. Q @ (0,0,0): $\iint_S \vec{D}\cdot d\vec{S} = \Psi$ @ surface on plane $x=1$ w/ $(-1\le y\le 1, -1\le z\le 1)$

$E = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \longrightarrow D = \frac{Q}{4\pi r^2}\hat{a}_r$

$\hat{a}_r = \frac{\langle 1, y, z\rangle}{\sqrt{1^2+y^2+z^2}}$ $\quad r = |\vec{r}| = \sqrt{1^2+y^2+z^2}$ $\quad dS = dy\,dz\,\hat{a}_x$

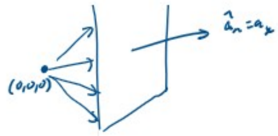
$\Psi = \int_{-1}^{1}\int_{-1}^{1} \underbrace{\frac{Q}{4\pi}\frac{(\hat{a}_x + y\hat{a}_y + y\hat{a}_z)}{(1^2+y^2+z^2)^{3/2}}}_{\vec{D}} \cdot \underbrace{dy\,dz\,\hat{a}_x}_{d\vec{S}}$



$E = \frac{Q}{4\pi\varepsilon_0 r^2} a_r \longrightarrow D = \frac{Q}{4\pi r^2} a_r$

$\hat{a}_r = \frac{\langle 1, y, z\rangle}{\sqrt{1^2+y^2+z^2}}$ $r = |\vec{r}| = \sqrt{1^2+y^2+z^2}$ $dS = dy\,dz\,\hat{a}_x$

$\Psi = \int_{-1}^{1}\int_{-1}^{1} \underbrace{\frac{Q}{4\pi} \frac{(\hat{a}_r + y\hat{a}_y + y\hat{a}_z)}{(1^2+y^2+z^2)^{3/2}}}_{D} \cdot \underbrace{dy\,dz\,\hat{a}_x}_{dS}$



☆ Webwork 4 Charge Densities

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(1 point)

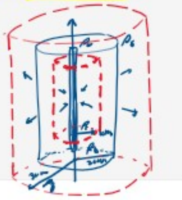
An infinite line charge having uniform line charge density of $\rho_L = -2\,\frac{nC}{m}$ is situated on the z-axis in a cylindrical coordinate system. An infinite, thin, cylindrical shell of charge centered on the z-axis having radius $2cm$ and surface charge density of $\rho_S = 120\,\frac{nC}{m^2}$ also exists.

(a) Find the value of $\mathbf{D}$ at a radius of $\rho = 1cm$.

$\mathbf{D}$ = ______ $\hat{a}_\rho \frac{C}{m^2}$

(b) Find the value of $\mathbf{D}$ at a radius of $\rho = 3cm$.

$\mathbf{D}$ = ______ $\hat{a}_\rho \frac{C}{m^2}$



a) $\vec{D} = \oint \vec{E}\cdot dA = \frac{\Psi}{A}$

$\because \rho_L = \text{const.} \therefore \vec{D} = \frac{\Psi}{A} \longrightarrow \Psi = \oint_0^l \rho_L\, dL = \rho_L \cdot l$

$\vec{D} = \frac{\rho_L l}{2\pi\rho l} = \frac{-2\times10^{-9}}{2\pi(0.01)} = -3.1830988\times10^{-8}\,\hat{a}_\rho\ C/m^2$

b) $\vec{D} = \oint \vec{E}\cdot dA = \frac{\Psi}{A}$

$\because \rho_L = \text{const.} \therefore \vec{D} = \frac{\Psi}{A} \longrightarrow$ Superposition: $Q_{enc} = \Psi_L + \Psi_S$

$\Psi_L = \int_0^l \rho_L\, dL = \rho_L \cdot l$, $\Psi_S = \iint \rho_S\, dA \rightarrow dA = \rho\, d\varphi\, dz$ $\Psi_S = \int_0^l \int_0^{2\pi} \rho_S \rho_2\, d\varphi\, dl = \rho_S \rho_2 2\pi l$

$\vec{D} = \frac{\Psi_L + \Psi_S}{A} = \frac{\rho_L l + \rho_S \rho_2 2\pi l}{2\pi\rho_3 l} = \frac{(-2\times10^{-9}) + (120\times10^{-9})(0.02)2\pi}{2\pi(0.03)} = 6.938967\times10^{-8}\,\hat{a}_\rho\ C/m^2$

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(1 point)

A spherically symmetric charge distribution in free space is defined by $\rho_V = 6r\,\frac{nC}{m^3}$ for $0 \le r \le 1\,m$ and $\rho_V = 0$ otherwise.

$\because \rho_V$ not const.

(a) Determine the total charge enclosed within a sphere of radius $a = 0.6\,m$.

Q = ______ nC

(b) Determine $\mathbf{D}$ at the radius in part (a).

$\mathbf{D}$ = ______ $\mathbf{a}_r \frac{nC}{m^2}$

(c) Determine the total charge enclosed within a sphere of radius $b = 1.5\,m$.

Q = ______ nC

(d) Determine $\mathbf{D}$ at the radius in part (c).

$\mathbf{D}$ = ______ $\mathbf{a}_r \frac{nC}{m^2}$

(diff. surface)

a) $\Psi = \oint_v \rho_v\, dv \longrightarrow dv = r^2 \sin\theta\, dr\, d\theta\, d\varphi$

$= \int_0^{0.6}\int_0^{\pi}\int_0^{2\pi} 6r^3 \sin\theta\, d\varphi\, d\theta\, dr = 2.442902nC$

b) $D = \frac{\Psi}{A} \longrightarrow A = 4\pi a^2$ $D = \frac{2.442902}{4\pi(0.6)^2} = 0.54\ nC/m^2$

c) $\Psi = \oint_v \rho_v\, dv \longrightarrow dv = r^2 \sin\theta\, dr\, d\theta\, d\varphi$

$= \int_0^{1}\int_0^{\pi}\int_0^{2\pi} 6r^3 \sin\theta\, d\varphi\, d\theta\, dr = 6\pi nC$

d) $D = \frac{\Psi}{A} \longrightarrow A = 4\pi b$ $D = \frac{6\pi}{4\pi(1.5)^2} = 0.66667\ nC/m^2$

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Vector Fields

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(1 point) Calculate the flux of the vector field $\vec{F}(x,y,z) = 5\vec{i} - 9\vec{j} + 2\vec{k}$ through a square of side length 4 lying in the plane $3x + 4y + 4z = 1$, oriented away from the origin.

Const. vector field
$\therefore \Psi = (\vec{E}\cdot\hat{n})S$

Flux = ______

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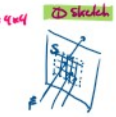
Simple Case: Const. Vector Field

$\therefore \Psi = \oint \vec{D}\cdot dS = \iint_s \vec{F}\cdot\hat{n}\, dS = (\vec{F}\cdot\hat{n})S \longrightarrow$ Area of patch

$\vec{F} = \langle 5, -9, 2\rangle$, $P = 3x + 4y + 4z = 1$, $\hat{n} = \langle 3, 4, 4\rangle$, $S = 4\times4$

① Sketch



☆ ② Parametrize Plane

$x = x$ $y = y$, $z = s(x,y) = \frac{1-3x-4y}{4}$

$\vec{r} = \langle x, y, \frac{1-3x-4y}{4}\rangle$

③ Partials

$\frac{\partial r}{\partial x} = \langle 1, 0, -3/4\rangle$ $\frac{\partial r}{\partial y} = \langle 0, 1, -1\rangle$

④ Cross Product

$\frac{\partial r}{\partial x}\times\frac{\partial r}{\partial y} = \begin{vmatrix} 1 & 0 & -3/4 \\ 0 & 1 & -1 \end{vmatrix} = \langle 3/4, 1, 1\rangle$

$\hat{n} = \frac{\langle 3,4,4\rangle}{\sqrt{3^2+4^2+4^2}} = \frac{\langle 3,4,4\rangle}{\sqrt{41}}$

⑤ Flux

$\Psi = (\vec{F}\cdot\hat{n})S = \langle 5,-9,2\rangle \cdot \frac{\langle 3,4,4\rangle}{\sqrt{41}} \cdot (4\times4) = -32.484142\ C$

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$x = a\cos(t)$
$y = a\sin(t)$
$z = kt$
$0 \le t \le \pi$
$\rightarrow$ parameterization for cu

(1 point) Compute the flux of $\vec{F} = 2x\vec{i} + 2y\vec{j} + 3z\vec{k}$ over the quarter cylinder S given by $x^2 + y^2 = 16$, $0 < x < 4$, $0 < y < 4$, $0 < z < 4$, oriented outward.

Vector field $\ne$ const.
$\Psi = \iint \vec{F}\cdot(\hat{n}\, dS) = \iint \vec{F}(\vec{r}(u,v))\cdot\vec{r}'(u,v)$

flux = ______

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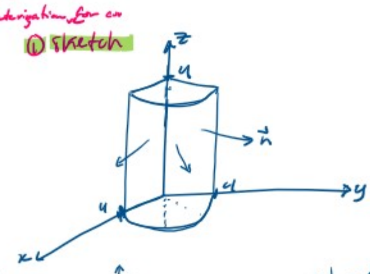
You have attempted this problem 0 times.
You have unlimited attempts remaining.

② Parametrization (curve)

$x = 4\cos(u)$, $y = 4\sin(u)$, $z = v$

$\vec{F}\langle x,y,z\rangle \longrightarrow \vec{r}\langle u, v\rangle$

① Sketch



③ Partials

$\frac{\partial r}{\partial u} = \langle -4\sin(u), 4\cos(u), 0\rangle$

$\frac{\partial r}{\partial v} = \langle 0, 0, 1\rangle$

④ Cross product

$\frac{\partial r}{\partial u}\times\frac{\partial r}{\partial v} = \begin{vmatrix} -4\sin(u) & 4\cos(u) & 0 \\ 0 & 0 & 1 \end{vmatrix} = \pm\langle 4\cos u, 4\sin u, 0\rangle$

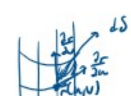
$\because$ radically outward $\therefore \hat{n}\cdot dS = +\langle 4\cos u, 4\sin u, 0\rangle\, du\, dv$

⑤ Flux

$\Psi = \iint G\cdot\hat{n}\, dS \longrightarrow 0 \le u \le \pi/2$, $0 \le v \le 4 \longrightarrow z$

$= \iint G(\vec{r}(u,v))\cdot\hat{n}\, dS = \int_0^4\int_0^{\pi/2} \langle 8\cos u, 8\sin u, v\rangle\cdot\langle 4\cos u, 4\sin u, 0\rangle\, du\, dv$

$= 64\pi$



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Find the upward flux of $\mathbf{F} = \langle -8e^{-z}, 4e^{-z}, 1\rangle$ through the first-octant part of the triangle

$x + 2y + 3z = 6.$

Answer: ______

Points: $P_1(6,0,0)$, $P_2(0,3,0)$, $P_3(0,0,2)$, $\vec{P_1P_2} = \langle -6,3,0\rangle$, $\vec{P_1P_3} = \langle 6,3,-2\rangle$

$\vec{P_1P_2}\times\vec{P_1P_3} = \begin{vmatrix} -6 & 3 & 0 \\ 6 & 3 & -2 \end{vmatrix} = \langle 6, 12, 18\rangle \rightarrow$ vector $\perp \therefore \hat{n}$

$\hat{n} = \frac{\langle 6,12,18\rangle}{\sqrt{14}}$ $S_{triangle} = \frac{1}{2}|\vec{P_1P_2}\times\vec{P_1P_3}| = \frac{1}{2}|\langle 6,12,18\rangle| = \frac{1}{2}\sqrt{14}$

$\Psi = \vec{F}\cdot(\hat{n}\, dS) = \langle -8e^{-18}, 4e^{-18}, 1\rangle \cdot \frac{\langle 6,12,18\rangle}{\sqrt{14}} \cdot \frac{1}{2}\sqrt{14} = 9C$

$\int_0^{2x+2} \int_0^{-\frac{2}{3}y+3}$

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(1 point)
All distances in this question are measured in metres.

Campers find a level spot of ground, with equation $z = 0$, and set up their tent. One end of the tent is a mesh triangle in the xz-plane, with vertices at $A(-12, 0, 0)$, $B(0, 0, 1)$, $C(12, 0, 0)$. The inside of the tent is on the side where $y > 0$.

The velocity of air as a function of position (x, y, z) is $\mathbf{v}(x, y, z) = \left\langle \tan^{-1}\left(\sqrt{x^2 + y^2}\right), x^2 z, e^{-x} \right\rangle$ m/s. Find the net rate at which air flows into the tent through the mesh $\triangle ABC$, in m^3/s.

Note: A full accounting of air flow in the tent would require knowing all about every side. Our goals here are simpler: just quantify the air flow through one given end.

Answer: ______ m^3/s.

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$\therefore$ flux along y axis, $\hat{n} = \hat{a}_y$

slopes: $z = \frac{1}{12}x + 1$, $z = -\frac{1}{12}x + 1$

bounds: $0 \le x \le 12$, $0 \le z \le -\frac{1}{12}x + 1$

$$\Psi = \iint_S \vec{v} \cdot \hat{n}\, dS = \langle \tan^{-1}\sqrt{x^2+y^2}, x^2 z, e^{-x} \rangle \cdot \langle 0, 1, 0 \rangle = x^2 z$$

$$= \int_0^{12} \int_0^{-\frac{1}{12}x+1} x^2 z \, dz \, dx = 57.6$$