Formulas

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Chapter 13:

$\cos\theta_m = \frac{\omega_{cm}}{\omega} = \frac{f_{cm}}{f} = \frac{\lambda}{\lambda_{cm}}$ → f, λ = operating

$v_{gm} = \frac{\partial\omega}{\partial\beta} = \frac{c}{n}\sin\theta_m$, $v_{pm} = \frac{\omega}{\beta} = \frac{c}{n}\sin\theta_m$

$f_{cm} = \frac{m \cdot c}{2nd}$

$f = \frac{c}{\lambda}$

$d = \frac{n\lambda}{2}$

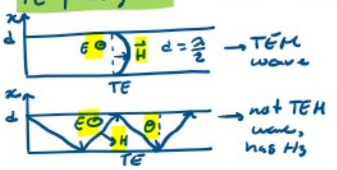
$f_c \rightarrow d = \frac{\lambda_c}{2}$
↳ lowest freq

@ $f = f_c$, $v_g = 0$

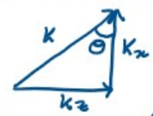
$v_{g\,max} = \frac{c}{n}$

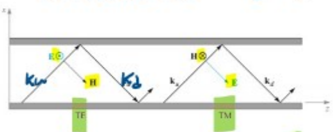
TE-polarized:

$f_{c2} = 2f_{c1}$



↳ TE_1 mode (1 half cycle)
↳ TM_2 mode (2 half cycles)



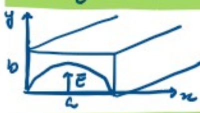


$K = K_u = K_d = \frac{2\pi}{\lambda}$

$K_x = \frac{2\pi}{\lambda_c}$ $K_z = \beta = \frac{2\pi}{\lambda_g}$

→ TM polarized = TM_0 mode
→ no TE_0 mode

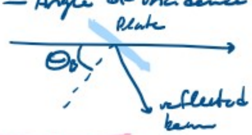
Rectangular wave guides:



$b < a$
TE_{10} mode: $a = \frac{\lambda_c}{2}$

Brewster's Angle

– Angle of incidence → no reflection of p-polarized light (TM), only s-polarized (TE) reflected



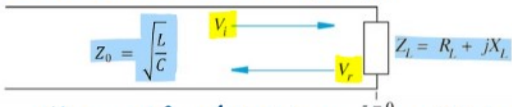
$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\sqrt{\frac{\epsilon'_{r2}}{\epsilon'_{r1}}}$

↳ $n = \sqrt{\epsilon'_r}$

Chapter 10

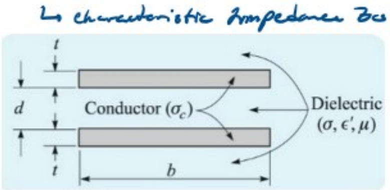
2 wire Transmission line (High freq)

$\lambda = \frac{v}{f} \xrightarrow{\text{air}} = \frac{c}{f}$



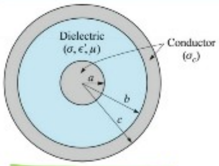
If account for losses in conductor & dielectric
↳ $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$
↳ $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

TEM = Uniform plane wave
↳ characteristic impedance $Z_0 = \frac{E}{H}$ → depends only on x-sectionals, NOT resistance



$Z_0 = \sqrt{\frac{\mu}{\epsilon}}\,\frac{d}{b} = \sqrt{\frac{L}{C}}$

If $d \gg a$



$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\left(\frac{b}{a}\right) = \sqrt{\frac{L}{C}}$

$V_i(z) = V_{i0}e^{-j\beta z}$
$V_r(z) = V_{r0}e^{j\beta z}$

$\beta = \frac{2\pi}{\lambda}$

$s = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$

Smith Chart

– Mapping of Z (impedance) plane onto Γ (reflection coeff) plane

$\Gamma_0 = \frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_L + Y_0}$ → bilinear transformation

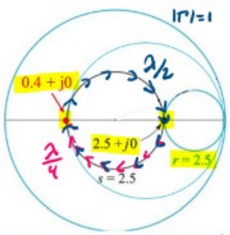
$\Gamma(z) = \Gamma_0 e^{j2\beta z}$
$\Gamma(l) = \Gamma_0 e^{-j2\beta l}$

reverse mapping: $z_L = \frac{Z_L}{Z_0} = \frac{1+\Gamma(z)}{1-\Gamma(z)}$

$|\Gamma| = 1$

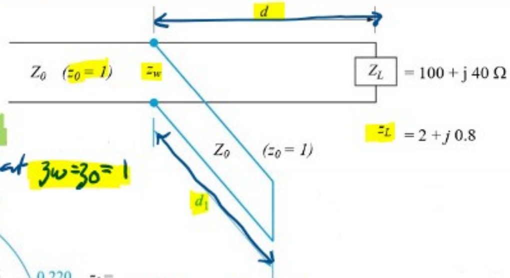
– Half revolution → λ/4 travel (transformer) → $Z_{in} = Z_0^2/Z_L$
– Full revolution → λ/2 travel (transformer) → $Z_{in} = Z_L$

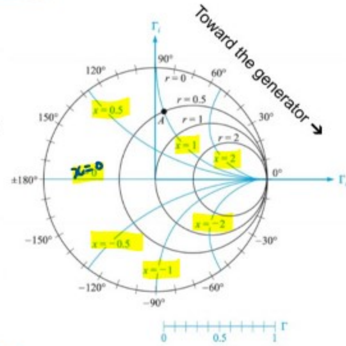
Z_0 $1-\Gamma(z)$



– Half revolution → $\lambda/4$ travel (transformer) → $Z_{in} = Z_0^2 / Z_L$
– Full revolution → $\lambda/2$ travel (transformer) → $Z_{in} = Z_L$

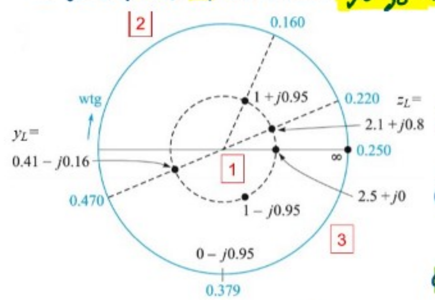
$Z_{in} Z_L = Z_0^2$





Single Shunt Stub Match

– Select d & d_1 such that $z_w = z_0 = 1$



① Convert values → Admittance
  ↳ convert $z = r + xj \rightarrow y = g + jb$

② Find d: (shortest stub dist. from load)
  ↳ towards generator until $g = 1 \pm jb$
  (2 solutions!)

③ Find d_1: (stub length)
  ↳ find stub length for $y = 0 = \mp jb$
  ↳ cancel b

④ If stub ∥ line d, $y_{in} = 1$

ladder-line
$Z_0 = \frac{120\pi}{\pi} \operatorname{acosh} \frac{s}{d}$ → separation btw wires

→ Finding $|\Gamma_L|$: – use λ in bottom scale (RFL coeff, Γ)
  – Read (RFL coeff, E or I)

→ Finding $\angle\Gamma_r$: – plot line from center to normalized $Z_L \rightarrow z_L = \frac{Z_L}{Z_0}$
  – Read (Angle of Refl. Coeff in deg)

Chapter 12 Plane Wave Reflection & Dispersion

Boundary Conditions for E-Fields

charge-free: $D_{n1} = D_{n2}$, $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$

surface charge: $(D_{n1} - D_{n2}) \cdot \hat{n}_{12} = -\rho_s$

$E_{T1} = E_{T2}$

Boundary Conditions for B-Fields

current-free: $H_{t1} = H_{t2}$, $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r2}}{\mu_{r1}}$

current sheet: $(H_1 - H_2) \cdot \hat{n}_{12} = \vec{K}$

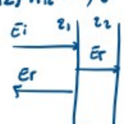
$B_{n1} = B_{n2}$

@ Boundaries

$E_0^i + E_0^r = E_0^T$

$H_0^i - H_0^r = H_0^T$

$\eta_1 = \frac{E_0^i}{H_0^i}$, $\eta_1 = \frac{E_0^r}{H_0^r}$, $\eta_2 = \frac{E_0^T}{H_0^T}$



$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}, \quad \frac{H_0^r}{H_0^i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}, \quad \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_1 + \eta_2}, \quad \frac{H_0^t}{H_0^i} = \frac{2\eta_1}{\eta_1 + \eta_2}.$$

	Any Medium $\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon}$	free space	Perfect dielectric (lossless)	Good Dielectric (low loss)	Good Conductor Skin Effect
Intrinsic Imp. η	$\eta = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$	$\eta = \sqrt{\frac{\mu}{\varepsilon}}$	$\eta = \sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\beta} = \sqrt{\frac{j\omega\mu}{\sigma}} = \frac{(1+j)}{\sigma\delta}$
Attenuation coeff. α	$\omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]\right]^{1/2}$		0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$
Phase const. β	$\omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right]\right]^{1/2}$		$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$
σ, ε, μ	$\varepsilon' = \varepsilon$, $\varepsilon'' = \frac{\sigma}{\omega}$	$\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$	$\sigma = 0$	$\sigma \ll \omega\varepsilon$, $\frac{\varepsilon''}{\varepsilon'} \ll 1$ $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$	$\sigma \gg \omega\varepsilon$, $\frac{\varepsilon''}{\varepsilon'} \gg 1$ $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$
v_p	$\frac{\omega}{\beta} = f\lambda$		$\frac{1}{\sqrt{\mu\varepsilon}}$	$\frac{1}{\sqrt{\mu\varepsilon}}$	$\sqrt{\frac{4\pi f}{\mu\sigma}}$
λ	$\frac{2\pi}{\beta} = \frac{v_p}{f}$		v_p/f	v_p/f	v_p/f

Partially conducting: $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$, $\vec{E} = E_0 e^{-\gamma z}\hat{x}$, $\nabla \times \vec{E} = -j\omega\mu\vec{H}$

$\vec{E}(z,t) = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)}\hat{x}$

$\vec{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} e^{j(\omega t - \beta z - \theta)}\hat{y}$

conducting: $\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45°$

$\vec{E}(z,t) = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)}\hat{x}$

$\vec{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} e^{j(\omega t - \beta z - \frac{\pi}{4})}\hat{y}$

Skin Effect

– Conducting mediums → good conductor if $\sigma \gg \omega\varepsilon$

Propagation const: $\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$

$\eta = \sqrt{\frac{\omega\mu}{\sigma}}$ …

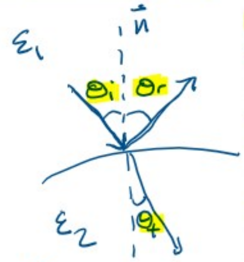
Propagation const: $\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu \sigma}$

$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45°$

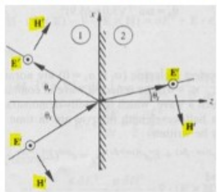
$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\alpha} = \frac{1}{\beta}$

Snell's law



θ_r Law of Reflection: $\theta_i = \theta_r$

θ_t Law of Refraction: $\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}} = \frac{v_1}{v_2}$



@ Boundary b/w Regions

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}.$$

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}.$$

for normal incidence: $\theta_i = \theta_t = 0°$

– If $\mu_1 = \mu_2 \rightarrow$ **No Reflected Wave @ Brewster Angle**

Brewster's Angle $\theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

Standing Wave: Combination of incident & reflected waves

$E(z,t) = [E_0^i e^{j(\omega t - \beta z)} + E_0^r e^{j(\omega t + \beta z)}]\,\hat{x}$

Loss Tangent

$\tan\theta_\delta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon'}$

Chapter 11 Plane Waves

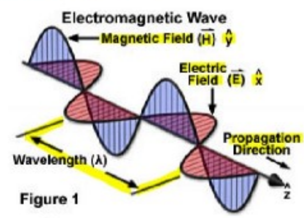


Figure 1

Maxwell's Equations

– w/ time factor $e^{j\omega t}$ & source-free

$\nabla \cdot \vec{E} = 0$

$\nabla \cdot \vec{H} = 0$

$\nabla \times \vec{E} = -j\omega\vec{B} = -j\omega\mu\vec{H} \xrightarrow{\text{free space}} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$

$\nabla \times \vec{H} = \vec{J}_c + j\omega\vec{D} = (\sigma + j\omega\varepsilon)\vec{E} \xrightarrow{\text{free space}} -\mu_0 \frac{\partial \vec{E}}{\partial t}$

Helmholtz Equations

– In lossless medium (perfect dielectric)

$E_x = Ae^{-jkz} + Be^{jkz}$ V/m

$H_y = \frac{1}{\eta_0}(Ae^{-jkz} + Be^{jkz})$ A/m $\rightarrow k = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c} = \frac{2\pi}{\lambda} = \beta$

$\left|\frac{E_x}{H_y}\right| = \eta_0$

Poynting Vector

– Power flux Density: $S = \frac{1}{2}\mathrm{Re}\{\vec{E} \times \vec{H}^*\}$ W/m²

↳ for e^{-jkz}: $\vec{S} = \frac{A^2}{2\eta_0}\hat{z}$

↳ for e^{jkz}: $\vec{S} = \frac{B^2}{2\eta_0}\hat{z}$

$P(t) = \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = \oint_S \vec{S} \cdot d\vec{S}$

$\vec{S}_{avg} = \frac{1}{2}\mathrm{Re}(E \times \vec{H}^*)$ conj. $(1+j2)^* = 1-j2$

good Conductor:

$\langle S_z \rangle = \frac{1}{2}\frac{E_{x0}^2}{|\eta|}e^{-2\alpha z}\cos\theta_\eta$

$= \frac{1}{4}\sigma\delta E_{x0}^2 e^{-2z/\delta}$

$P_0 = P_i e^{-2\alpha z}$

Chapter 9 Time-Varying Fields & Maxwell's Equations

$\vec{D} = \varepsilon_r\varepsilon_0\vec{E}$

$\vec{B} = \mu_r\mu_0\vec{H} = \nabla \times \vec{A}$ (magnetic vector potential) $\rightarrow \vec{A} = \oint \frac{\mu I\, dl}{4\pi R}$

$\vec{J} = \sigma\vec{E}$

$\vec{E} = -\nabla V$ (grad)

Maxwell's Equations

– point-form

$\nabla \cdot \vec{D} = \rho$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$

– Integral Form

$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho\, dv$

$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$

$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{S}$

$\oint_C \mathbf{E} \cdot d\mathbf{l} = \int_S -\frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$

– Free Space

$\nabla \cdot \mathbf{D} = 0 \longrightarrow \nabla \cdot \vec{E}$

$\nabla \cdot \mathbf{B} = 0 \longrightarrow \nabla \cdot \vec{H}$

$\nabla \times \mathbf{H} = \partial\mathbf{D}/\partial t$

$\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$

& Integral

$\oint_S \mathbf{D} \cdot d\mathbf{S} = 0$

$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$

$\oint_C \mathbf{E} \cdot d\mathbf{l} = \int_S -\frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$

$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$

Divergence Theorem

$\oint_S \vec{D} \cdot d\vec{S} = \int_v (\nabla \cdot \vec{D})\, dv = Q_{enc.}$

Stokes' Theorem

$\oint_C \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{S} = \int_S \vec{B} \cdot d\vec{S} = \varphi$

Displacement Current

Static Fields: $\nabla \times \vec{H} = \vec{J}_c$

time varying Fields: $\nabla \cdot \vec{J}_c = -\frac{\partial \rho}{\partial t}$

$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_d \rightarrow \vec{J}_d = \frac{\partial \vec{D}}{\partial t}$

$\vec{J} = \vec{J}_c + \vec{J}_d = \sigma\vec{E} + j\omega\varepsilon\vec{E}$

$\left|\frac{\vec{J}_c}{\vec{J}_d}\right| = \frac{\sigma}{\omega\varepsilon}$