3. Electromechanical Energy Conversion

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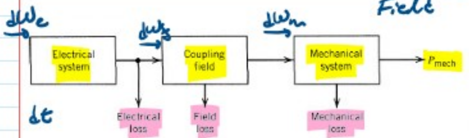
3.1 Energy Conversion Process

- Electromechanical Converter System:
  1. Electric System
  2. Mechanical System
  3. Coupling Field

Elec Energy input source = Mech Energy output + Increase in stored Energy in Coupling Field + Energy loss

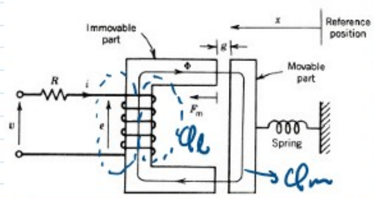


dW_e = Elec E flowing to system
dW_f = E supplied to field → If core losses ↓, $dW_f = \Delta E$ stored field
dW_m = E converted to mech form → If friction & windage ↓, dW_m = E mech output

$dW_e = dW_m + dW_f$

3.2 Field Energy

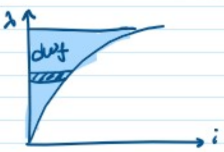
- Faraday + KVL: $v = ri + \overbrace{\frac{d\lambda}{dt}}^{emf} = ri + e$
- Flux Linkage: $\lambda = N\varphi = N(\varphi_m + \varphi_\ell)$ → Magnetizing Flux: $\varphi_m = \frac{Ni}{R_m}$
- Inductances: $\lambda = \left(\frac{N^2}{R_\ell} + \frac{N^2}{R_m}\right)i = (L_\ell + L_m)i$ Flux leakage: $\varphi_\ell = \frac{Ni}{R_\ell}$
  - → Magnetizing $L = \frac{N^2}{R_m}$
  - → Leakage L
- Reluctance: $R_m = R_c + 2R_m$



When Static: $dW_m = 0 \therefore dW_e = dW_f$

$e = \frac{d\lambda}{dt}$ $dW_e = ei\,dt = dW_f$ $dW_f = i\,d\lambda$

flux linkage increased $0 \to \lambda$: $W_f = \int_0^t ei\,dt = \int_0^\lambda i\,d\lambda$



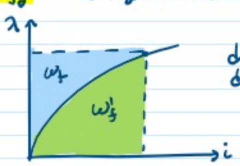
$Ni = H_c l_c + H_g l_g$ → H_c = magnetic intensity of core
H_g = magnetic intensity in gap → $H_g = \frac{B}{\mu_0}$ & ($\because W_{fg} \gg W_{fc}$ $\therefore$ neglect W_{fc})
l_c = length of magnetic core → $H_c = \frac{B_c}{\mu_c}$
l_g = length of gap

$\lambda = N\varphi = NAB$

$W_f = \int\left(\frac{H_c l_c + H_g l_g}{N}\right)A\,dB$
$= W_{fc} + W_{fg}$ → E magnetic material + E in air gap

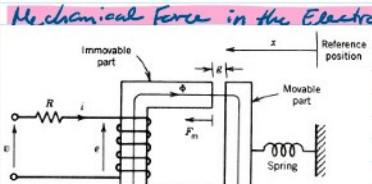
3.2.1 Energy, Coenergy

- ↑ air gap = λ-i ~ linear
- Coenergy: $W_f' = \int_0^i \lambda\,di$
  - ↳ $W_f' > W_f$ if λ-i non-linear
  - ↳ $W_f' = W_f$ if λ-i linear

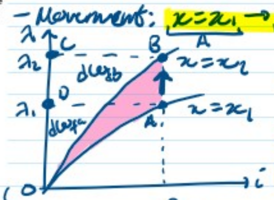


$dW_m = 0$
$dW_e = dW_f$

3.3 Mechanical Force in the Electromagnetic System



- Movement: $\underbrace{x = x_1}_{A} \to \underbrace{x = x_2}_{B}$, $i = \frac{V}{R}$ @ const. λ



$dW_e = \int ei\,dt = \int_{\lambda_1}^{\lambda_2} i\,d\lambda = ABCD$

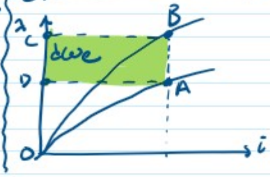
$dW_f = OBC - OAD$

$dW_m = dW_e - dW_f = ABCD - (OBC - OAB)$
$= ABCD - OBC + OAD$

$\therefore dW_m = OAB$ $W_m = \int \lambda\,di$

- If quick movement, flux linkage λ = const.

$dW_e = 0$



Change in Mech Work: $dW_m = dW_e = f_m\,dx$ under const. current; f_m = electromagnetic force

$f_m = \left.\frac{\partial W_f'(i, x)}{\partial x}\right|_{i = const.}$
↳ mech force causing displacement dx

$dW_m = -dW_f = f_m\,dx = OAB$

$f_m = \left.\frac{\partial W_f(\lambda, x)}{\partial x}\right|_{\lambda = const.}$

- For rapid motion: Elec input $= i\,d\lambda = 0 \because \lambda$ = const.
- Mech output E supplied by Field Energy

$f_m = \frac{}{\partial x}\Big|_{\lambda = const.}$

- For rapid motion: Elec input $= i\,d\lambda = 0 \therefore \lambda = const.$
- Mech output C supplied by Field Energy

3.3.1 Linear System

- If $R_c \ll R_g$, ($\uparrow l_g$), $\lambda - i$ = linear

$\lambda = L(x)\,i \rightarrow L(x)$ = inductance of coil, depends on x

$W_f = W_f' \quad \int i\,d\lambda = \int_0^{\lambda} \frac{\lambda}{L(x)}\,d\lambda = \frac{\lambda^2}{2L(x)}$

$\therefore f_m = -\frac{\partial}{\partial x}\left(\frac{\lambda^2}{2L(x)}\right)\Big|_{i = const.} = \frac{\lambda^2}{2L^2(x)}\frac{dL(x)}{dx} = \frac{1}{2}i^2\frac{dL(x)}{dx}$

$W_i = H_g l_g = \frac{B_g}{\mu_0} l_g$ $\longrightarrow$ volume of air gap

$W_f = \frac{B_g^2}{2\mu_0} \times V_g = \frac{B_g^2}{2\mu_0} A_g 2g$

$f_m = \frac{\partial}{\partial g}\left(\frac{B_g^2}{2\mu_0} A_g 2g\right) = \frac{B_g^2}{2\mu_0}(2A_g)$

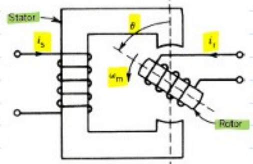
- $\uparrow$: W_e supplies W_f, W_m
- $\downarrow$: W_f, W_m supplies W_e
- $\rightarrow$: W_m supplies W_f
- $\leftarrow$: W_f supplies W_m

- Total cross-sectional A of gap $= 2A_g$
- $\therefore$ Force/unit A of gap $\rightarrow$ Magnetic pressure F_m

$F_m = \frac{B_g}{2\mu_0}$ N/m

3.4 Rotating Machines

- Rotating electromagnetic system: Stator & Rotor
- Current fed into rotor circuit through brushes & slip rings



Find W_f using currents i_s & i_r

$dW_f = e_s i_s\,dt + e_r i_r\,dt$
$= i_s\,d\lambda_s + i_r\,d\lambda_r$

For linear magnetic system, λ_s & λ_r expressed w/ inductances L depending on rotor pos. Θ

$\lambda_s = L_{ss} i_s + L_{sr} i_r$
$\lambda_r = L_{rs} i_s + L_{rr} i_r$

$\rightarrow L_{ss}$ = self inductance stator winding
L_{rr} = self inductance rotor winding
L_{sr}, L_{rs} = Mutual inductances b/w stator & rotor

- For linear sys.: $L_{sr} = L_{ss}$: $\begin{vmatrix}\lambda_s \\ \lambda_r\end{vmatrix} = \begin{vmatrix}L_{ss} & L_{sr} \\ L_{sr} & L_{rr}\end{vmatrix}\begin{vmatrix}i_s \\ i_r\end{vmatrix}$

$dW_f = L_{ss} i_s\,di_s + L_{rr} i_r\,di_r + L_{sr}\,d(i_s i_r)$

$W_f = L_{ss}\int_0^{i_s} i_s\,di_s + L_{rr}\int_0^{i_r} i_r\,di_r + L_{sr}\int_0^{i_s, i_r} d(i_s i_r)$
$= \frac{1}{2}L_{ss} i_s^2 + \frac{1}{2}L_{rr} i_r^2 + L_{sr} i_s i_r$

- Force (Torque) developed in rotational electromagnetic system:

$T = \frac{\partial W_f'(i, \Theta)}{\partial \Theta}\Big|_{i = const.}$

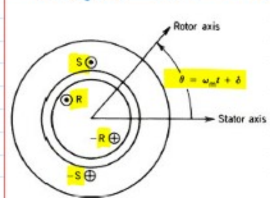
- For linear sys.: $W_f = W_f'$ $\therefore T = \underbrace{\frac{1}{2} i_s^2 \frac{dL_{ss}}{d\Theta} + \frac{1}{2} i_r^2 \frac{dL_{rr}}{d\Theta}}_{} + \underbrace{i_s i_r \frac{dL_{sr}}{d\Theta}}_{}$

Reluctance Torque
Torques produced in machine (self induct)

Variation of mutual inductance b/w stator & rotor windings

3.5 Cylindrical Machines

- 2 pole rotating Machine, uniform airgap
- Reluctance magnetic path independent of rotor position
- L_{ss}, L_{rr} are const, no Reluctance torque produced, L_{sr} varies w/ pos.



Torque $T = i_s i_r \frac{dL_{sr}}{d\Theta}$, $L_{sr} = M\cos\Theta \rightarrow M$ = peak value of mutual inductance L_{sr}

$i_s = I_{sm}\cos\omega_s t \quad i_r = I_{rm}\cos(\omega_r t + \alpha) \rightarrow \omega_s, \omega_r$ = freq of stator, rotor current

Pos. rotor: $\Theta = \omega_m t + \delta \rightarrow \omega_m$ = angular velocity rotor
δ = rotor pos. @ $t = 0$

$T = -I_{sm} I_{rm} M \cos\omega_s t \cos(\omega_r t + \alpha)\sin(\omega_m t + \delta)$

$T_{avg} \rightarrow$ nonzero if: $\omega_m = \pm(\omega_s \pm \omega_r)$

$T_{avg} \rightarrow$ if $|\omega_m| = |\omega_s \pm \omega_r|$

$\omega_m = \int T_m\,d\Theta$