

1. Magnetic Circuits

Tuesday, October 27, 2020 3:47 AM

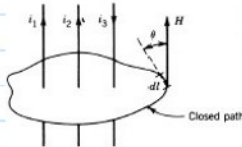
1.1 Magnetic Circuits

i-H Relation

- Conductor \rightarrow current i , Magnetic Field Intensity H
- **Ampere Circuit law**: line integral of H around closed path = $I_{\text{total enclosed}}$

$$\oint H \cdot dl = \sum i = i_1 + i_2 - i_3$$

$$\oint H \cdot dl \cos \theta = \sum i$$



- For single conductor: $\oint H \cdot dl = i \rightarrow dl = 2\pi r$

B-H Relation

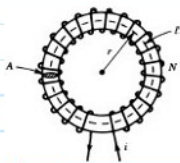
- H produces magnetic flux density B

$$B = \mu H \rightarrow \mu = \mu_0 \mu_r$$

Magnetic Equivalent Circuit

- Magnetic flux mostly in core materials, flux outside \rightarrow leakage flux

$$\oint H \cdot dl = Ni \rightarrow F = Ni \text{ (magnetomotive force mmf)}$$



$$H = \frac{N}{l} i \text{ At/m}$$

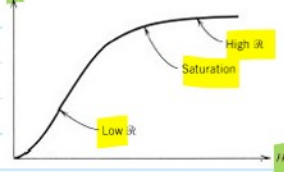
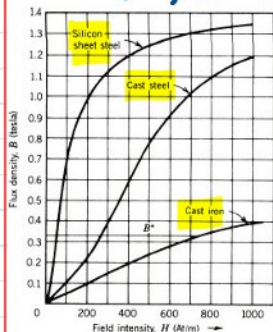
$$B = \frac{\mu Ni}{l} \text{ T}$$

- **Flux** in cross-section of toroid: $\phi = \int B \cdot dA = BA = \frac{F}{R} \text{ wb}$

$$\text{Reluctance: } R = \frac{l}{\mu A}$$

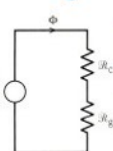
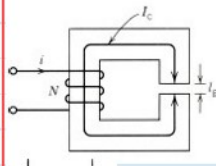
Magnetization Curve

- $\uparrow F \rightarrow \uparrow I, \uparrow B \sim$ linearly when $\downarrow H$, but saturates at $\uparrow H$
- when $\uparrow B, \downarrow R$ and when $\downarrow B, \uparrow R$



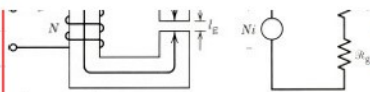
Air gaps

- Air gaps needs \uparrow mmf than core
- $\uparrow F \rightarrow \uparrow \phi$, core may saturate, but air gap unsaturated: $B-H$ air \rightarrow linear

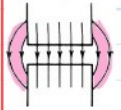


$$R_c = \frac{l_c}{\mu_c A_c} \quad R_g = \frac{l_g}{\mu_0 A_g}$$

$$\phi = \frac{F}{R} \quad F = Ni = H_c l_c + H_g l_g$$



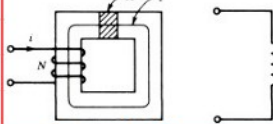
$$\mathcal{L} = \frac{F}{i} \quad F = Ni = H\mu l_c + H\mu l_g$$



- Magnetic flux lines overflow \rightarrow **fringing**
- $\uparrow \mu \downarrow l_g$, neglect $\therefore A_g = A_c \rightarrow B_g = B_c = \frac{\Phi}{A_c}$

Inductance

- Coil wound inductor \rightarrow flux linkage of coil



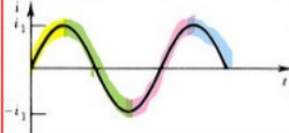
$$\text{Flux linkage } \lambda = N\Phi$$

$$\text{Inductance } L = \frac{\lambda}{i} = \frac{N\Phi}{i} = \frac{N^2}{R_T}$$

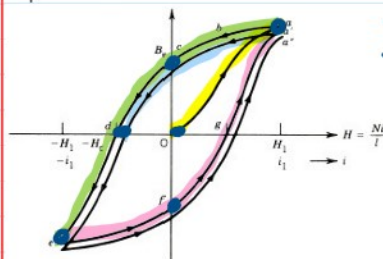
- Can define inductance w/ dimensions (A, L), or magnetic reluctance

1.2 Hysteresis

- Core initially unmagnetized, slowly $\uparrow i$, $\uparrow H$



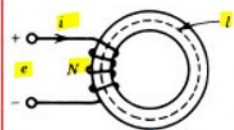
- $\uparrow H$, @ a, $\downarrow H$ until $H=0$
- But core has retained flux density $B_r \rightarrow$ **residual flux density**
- $\uparrow H$ reversed \rightarrow flux becomes, @ H_c , residual flux = 0
- $\uparrow H$ reversed \rightarrow $\uparrow B$ reverse \rightarrow **coercive force**
- $\downarrow H$ to 0 then $\uparrow H$, \rightarrow loop does not close



Hysteresis loss

- $E_{in} > E_{returned}$, $E_{loss} \rightarrow$ core
- \rightarrow Hysteresis loss
- Size Hysteresis loop \propto Hysteresis loss

$$\text{Faraday's law: } e = N \frac{d\Phi}{dt}$$



$$P_{in} = i e = i \frac{d\lambda}{dt}$$

$$dW = \oint P_{in} dt = \oint i \frac{d\lambda}{dt} dt$$

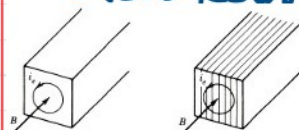
$$dW_h = \oint i d\lambda \rightarrow F = Ni, \lambda = N\Phi = NB_c A_c$$

$$= \oint \left(\frac{H_c l_c}{N} \right) (N A_c dB_c) = \frac{I_c A_c}{V} \oint H_c dB_c = V_{core} \times \text{Area } B-H \text{ loop}$$

$$\text{Power loss: } P_h = K_h \cdot f \cdot (B_{max})^n \rightarrow K_h \text{ } B \text{ } n \text{ determined exp.}$$

Eddy Current Loss

- Flux density changes rapidly in core
- Voltage induced, i.e. eddy current flows around path
- \therefore core Resistance, power loss $i^2 R$

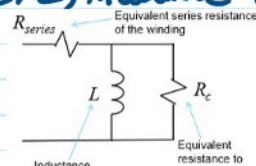


Reducing Eddy Currents:

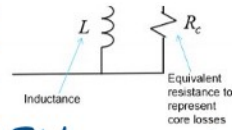
1. \uparrow core resistivity
2. laminated core, insulated from each other

$$\text{Power loss: } P_e = K_e f^2 (B_{max})^2$$

$\rightarrow K_e$ determined exp.



↳ K_c determined exp.



Core Loss

- Core losses = Hysteresis loss + Eddy current loss

$$P_c = P_h + P_e$$

- Core loss can be computed from Area of dynamic B-H loop

$$P_c = V_{\text{core}} \cdot \oint_{\text{dynamic loop}} H \cdot dB \rightarrow \text{volume core} \times \text{frequency} \times \text{Area dynamic loop}$$