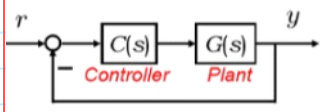
L20 Stability Margin: Frequency Domain Specification

Saturday, June 19, 2021 10:43 PM

Feedback Control System Design

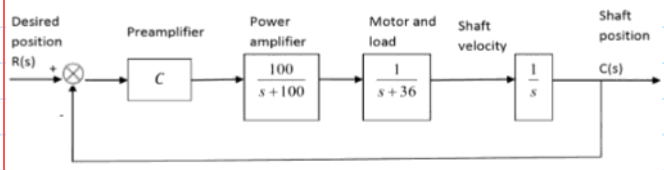


OL: $L(s) = G(s)C(s)$

CL: $T(s) = \frac{L(s)}{1+L(s)}$

- Design C (or K) to yield PO using freq response

1. Find ζ: $\zeta = \frac{|\ln PO/100|}{\sqrt{\pi^2+(\ln PO/100)^2}}$
2. Find PM (PM compensator):
   - ↳ $PM_{comp} = 100\zeta$
   - ↳ $PM_{comp} = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}\right)$
3. Find ω^* → $\angle L(j\omega^*) = -180° + PM_{comp}$
4. Find Gain C: $C = \frac{1}{|L(j\omega^*)|}$



$L(s) = \frac{100}{s(s+36)(s+100)}$ → Find C so that PO = 9.5%:

$\zeta = \frac{\left|\ln\frac{PO}{100}\right|}{\sqrt{\pi^2+\left(\ln\frac{PO}{100}\right)^2}} = \frac{\left|\ln\frac{9.5}{100}\right|}{\sqrt{\pi^2+\left(\ln\frac{9.5}{100}\right)^2}} \rightarrow \zeta \approx 0.6$

$PM_{compensator} = 100\zeta \rightarrow PM_{compensator} = 60°$

$\angle L(j\omega^*) = -180^0 + PM_{compensator} = -180° + 60° = -120° \rightarrow \angle L(j\omega^*) = -120°$

Find ω^* at which $\angle L(j\omega^*) = -120^0$:

$\angle L(j\omega^*) = 0^0 - \{90^0 + \tan^{-1}\left(\frac{\omega^*}{36}\right) + \tan^{-1}\left(\frac{\omega^*}{100}\right)\} = -120^0 \rightarrow \omega^* = 14.45$

At $\omega^* = 14.45 \rightarrow C = ?$

$|L(j \times 14.45)| = \frac{100}{14.45\sqrt{14.45^2+36^2}\sqrt{14.45^2+100^2}} \rightarrow |L(j \times 14.45)| = 1.7656 \times 10^{-3}$

$C = \frac{1}{|L(j\omega^*)|} = \frac{1}{1.7656\times10^{-3}} \rightarrow$ **$C = 566$**

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