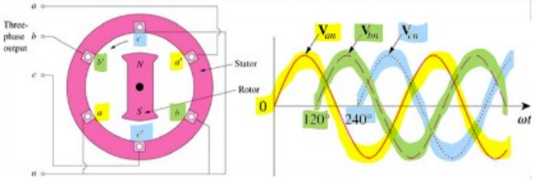
3 Phase Circuits

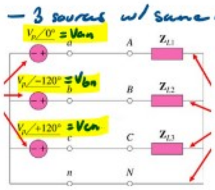
April 22, 2020 2:55 PM

Generation of 3-Phase Voltage

- 3-Phase Generator consists of a rotating magnet surrounded by stationary winding



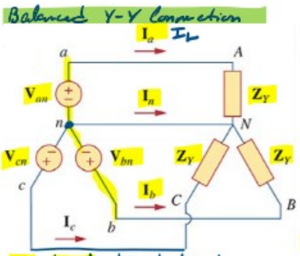
- 3 sources w/ same amplitude & frequency but out of phase by 120°



- V_{an} leads V_{bn} by 120°
- V_{bn} leads V_{cn} by 120°
- V_{cn} leads V_{an} by 120°

OR: V_{an} leads V_{cn} by 240°

Balanced Y-Y Connection



Phase Voltages (Line – neutral)

$V_{an} = V_p \angle 0°$

$V_{bn} = V_p \angle -120°$

$V_{cn} = V_p \angle -240° = V_p \angle 120°$

Line Voltage (Line to line)

$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn} = \sqrt{3}\, V_p \angle 30°$

$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}\, V_p \angle -90°$

$V_{ca} = V_{cn} - V_{bn} = \sqrt{3}\, V_p \angle -210°$

Line Current (Current in each line)

$I_a = \frac{V_{an}}{Z_Y}$, $I_b = \frac{V_{bn}}{Z_Y} = I_a \angle -120°$; $I_c = \frac{V_{cn}}{Z_Y} = I_a \angle -240°$

$V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$

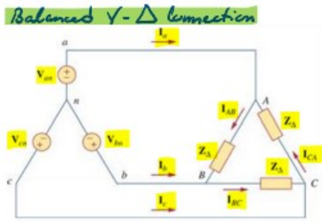
$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}| = \sqrt{3}\, V_p$

$I_a + I_b + I_c = 0$

$I_n = -(I_a + I_b + I_c) = 0$

$V_n = Z_n \cdot I_n = 0$

Balanced Y-Δ Connection



Phase Current

$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{V_{an} - V_{bn}}{Z_\Delta}$

$I_{BC} = \frac{V_{BC}}{Z_\Delta} = I_{AB} \angle -120°$

$I_{CA} = \frac{V_{CA}}{Z_\Delta} = V_{AB} \angle -240°$

Line Current

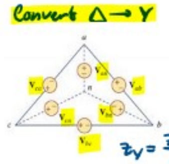
$I_a = I_{AB} - I_{CA} = I_{AB}\sqrt{3} \angle -30°$

$I_L = |I_a| = |I_b| = |I_c|$

$I_P = |I_{AB}| = |I_{BC}| = |I_{CA}|$

$I_L = \sqrt{3}\, I_P$

Convert Δ → Y



$\mathbf{V}_{ab} = V_L \angle 0°$

$\mathbf{V}_{bc} = V_L \angle -120°$

$\mathbf{V}_{ca} = V_L \angle +120°$

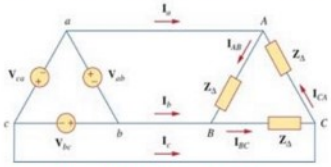
$\mathbf{V}_{an} = \frac{V_L}{\sqrt{3}} \angle -30°$

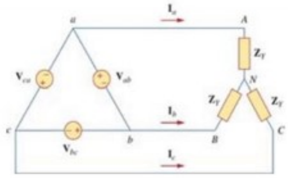
$\mathbf{V}_{bn} = \frac{V_L}{\sqrt{3}} \angle -150°$

$\mathbf{V}_{cn} = \frac{V_L}{\sqrt{3}} \angle +90°$

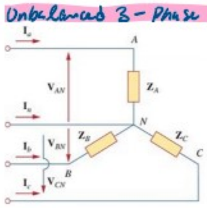
$Z_Y = \frac{Z_\Delta}{3}$, $Z_\Delta = 3 Z_Y$

Balanced Δ-Δ & Δ-Y Connection





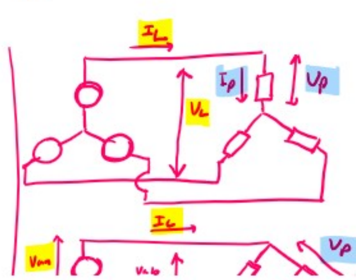
Unbalanced 3-Phase System



- Unbalanced Y-Y Connection
- Unbalanced voltage sources / loads

$I_a = \frac{V_{AN}}{Z_A}$ $I_b = \frac{V_{BN}}{Z_B}$ $I_c = \frac{V_{CN}}{Z_C}$

$I_n = -(I_a + I_b + I_c) \neq 0$



$I_L = I_P$

$V_p = V_L$

Power

Instantaneous Power

$p(t) = p_a(t) + p_b(t) + p_c(t) = 3 V_p I_p \cos(\theta)$

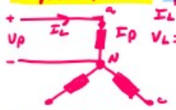
Phase Power

Avg Power: $P_p = V_p I_p \cos\theta$

Reactive Power: $Q_p = V_p I_p \sin\theta$

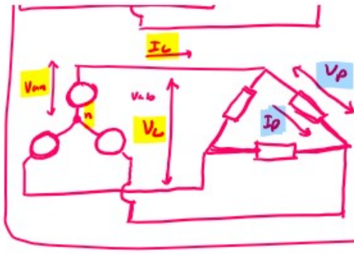
Phase Power

Avg Power: $P_p = V_p I_p \cos\theta$
Reactive Power: $Q_p = V_p I_p \sin\theta$
Apparent Power: $S_p = V_p I_p$
Complex Power: $S_p = V_p \cdot I_p^* = P_p + jQ_p = V_p I_p \angle\theta$



$I_L = I_p$
$V_L = \sqrt{3} V_p$ $\rightarrow$

$I_L = \sqrt{3} I_p$
$V_L = V_p$

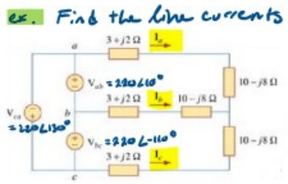


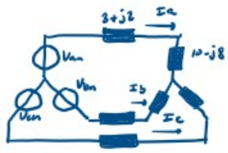
$V_p = V_L$

Total Power

Avg Power: $P = 3P = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta$
Reactive Power: $Q = 3Q_p = 3V_p I_p \sin\theta = \sqrt{3} V_L I_L \sin\theta$
Apparent Power: $|S| = 3V_p I_p = \sqrt{3} V_L I_L$
Complex Power: $S = 3S_p = 3V_p I_p^* = P + jQ = 3V_p I_p \angle\theta = \sqrt{3} V_L I_L \angle\theta$

ex. Find the line currents





$V_{an} = \frac{220}{\sqrt{3}} \angle -20°$
$V_{bn} = \frac{220}{\sqrt{3}} \angle -140°$
$V_{cn} = \frac{220}{\sqrt{3}} \angle 100°$

$I_a = \frac{V_{an}}{Z_L} = \frac{220/\sqrt{3} \angle -20°}{3+2j+10-8j} = 8.8712\angle 4.775°$) $-120°$
$I_b = 8.8712 \angle -115.77514°$
$I_c = 8.8712 \angle 124.77514°$ } $+240° / -120°$

☆ Webwork ST3

ST3: Problem 2

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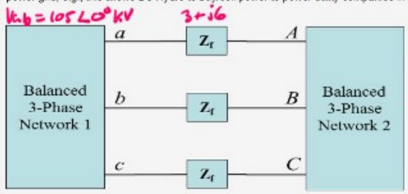
(20 points)
Consider two 3-phase networks connected as shown below (each network consists of its own sources and loads and the connections are solely from the three lines). Given $V_{ab} = 105\angle 0° kV$, $V_{AB} = 105\angle -15° kV$, and feeder line impedance $Z_f = 3 + j6\Omega$, compute the net complex power supplied by each network. [**Hint:** Though it isn't shown, you may wish to consider, w.l.o.g., a neutral reference line from which signal values are measured.] **Aside:** This example demonstrates how controlling the phase between two networks allows power to be transmitted between them (an application would be the power grid, e.g., this allows BC Hydro to buy/sell power to power utility companies in Washington state).

$V_{ab} = 105\angle 0° kV$ 3+j6



Note: In this problem, you may only submit numerical answers. (I.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$S_1 =$ ______ $+j$ ______ VA

$S_2 =$ ______ $+j$ ______ VA

Network 1
$S = 3V_p I_p^*$
$V_{ab} = 105\angle 0° kV$
$V_{an} = 105/\sqrt{3} \angle -30° kV$
$I_{aA} = \frac{V_{an} - V_{AN}}{Z_f} = 2359.12025 \angle -10.93494°$
$S_1 = 3V_{an} \cdot (I_{aA})^*$
$= 405508513.978 - 140142862.795j$

Network 2
$S = 3V_p I_p^*$
$V_{AB} = 105\angle -15° kVA$
$V_{AN} = 105/\sqrt{3} \angle -45° kV$
$I_{Aa} = -I_{aA} = 2359.12025 \angle 169.06505°$
$S_2 = 3V_{AN} \cdot (I_{Aa})^*$
$= -355419478.623 + 240321033.505j$

ST3: Problem 3

ST3
Problem 3
User Settings
Grades

Problems

Problem 1
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(20 points)
In order to compare and contrast the signal relationships for the 4 different configurations of balanced three-phase networks, complete the table below. Take $V_{s1} = 160\angle 0° V$, the load impedance $Z_L = 0.4 + j0.7\Omega$ and feeder line impedance $Z_f = 35 + j6\Omega$.

Note: In this problem, you may only submit numerical answers. (I.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

Config	Voltage (V)		Current (A)		Total Complex Power (VA)	
	Line, V_{ab}	Phase, V_{s1}	Line, I_a	Phase	Supplied by Source, S_S	Received by Load, S_L
Y-Y	∠ °	160∠0 °	∠ °	∠ °	∠ °	∠ °
Y-Δ	∠ °	160∠0 °	∠ °	∠ °	∠ °	∠ °
Δ-Δ	∠ °	160∠0 °	∠ °	∠ °	∠ °	∠ °
Δ-Y	∠ °	160∠0 °	∠ °	∠ °	∠ °	∠ °

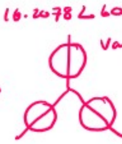
① $160\angle 0°$ $0°$ 35+j6 0.4+j0.7 ②

① **Y-Y**
$V_{ab} = V_{an} - V_{bn} = 160\angle 0° - 160\angle -120° = 277.12813\angle 30° V$
$I_a = \frac{V_{an}}{Z_f + Z_L} = \frac{160\angle 0°}{35+6j+0.4+0.7j} = 4.4409335\angle -10.71734° A$
$I_p = I_L = 4.4409335\angle -10.71734° A$
$S_S = 3V_p \cdot I_p^* = 3(160\angle 0°) \cdot (4.4409335\angle -10.71734°)^* = 2131.65\angle 10.7173° VA$
$S_L = 3|I_L|^2 \cdot Z_L = 47.700889\angle 60.2551° VA$

② **Y-Δ**
$V_{ab} = V_{an} - V_{bn} = 277.12813\angle 30° V$
$I_a = \frac{V_{an}}{Z_f + Z_Y} = \frac{160\angle 0°}{35+6j+0.1333+0.2333j} = 4.48405\angle -10.0606°$
$I_p = \frac{V_{ab}/3}{Z_f + Z_Y} = \frac{277.12813\angle 30°}{35+6j+0.1333+0.233j} = 2.5888\angle 19.9393° A$
$S_S = 2131.64\angle 10.0606°$
$S_L = 3 \cdot |I_L|^2 \cdot Z_\Delta = 3|4.48405\angle -10.0606°|^2 (0.4+0.7j) = 16.2078\angle 60.2572° VA$

$Z_Y = \frac{Z_\Delta}{3} = 0.1333 + 0.2333j$

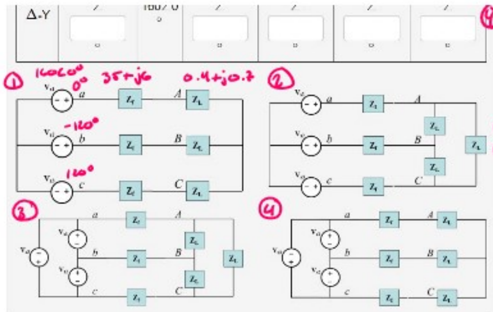
③ **Δ-Δ**
$V_L = V_p = 160\angle 0°$
$I_a = \frac{V_{an}}{Z_f + Z_\Delta} = \frac{92.37604\angle -30°}{35+6j+0.4+0.7j} = 2.5639\angle -40.7123°$
$I_p = \frac{V_{an}/3}{Z_f + Z_Y} = \frac{160\angle 0°/3}{35+6j+0.1333+0.2333j} = 1.4445\angle -10.0606°$
$S_S = 717.36\angle 10.0606°$
$S_p = 5.4026\angle 60.257°$



$V_{an} = \frac{V_L}{\sqrt{3}} \angle -30°$
$= 92.37604\angle -30°$

$Z_Y = \frac{Z_\Delta}{3} = 0.1333 + 0.2333j$

④ **Δ-Y**
$V_L = V_p = 160\angle 0°$
$I_a = \frac{V_{an}}{Z_f + Z_Y} = \frac{92.3604\angle -30°}{35+6j+0.4+0.7j} = 2.5639\angle -40.717°$
$I_p = I_L = 2.5639\angle -40.71734$



④ Δ-Y

$V_L = V_p = 160\angle 0^\circ$

$I_a = \dfrac{V_{an}}{Z_f + Z_Y} = \dfrac{92.3604\angle -30^\circ}{35+6j+0.4+0.7j} = 2.5639\angle -40.717^\circ$

$I_p = I_L = 2.5639\angle -40.7173^\circ A$

○ $S_s = 717.36\angle 10.7173^\circ$

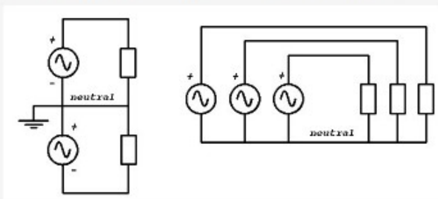
○ $S_L = 16.2078\angle 60.2577^\circ$

ST3: Problem 5

(20 points)

You have to feed a total load of 60 kilowatts with Q/S = 0.2. Option one is to split it in two and connect it to a single-phase three-wire system of voltages 370/740 volts. Option two is to split the load in three, wire it as a wye, and feed it from a three-phsse four-wire system with a line-to-line voltage of 640.859 volts. For each option, compute and report the line current, in amps. The neutral wire in both systems must be of the same caliber of the line wires (for maximum unbalance conditions). If the total weight of copper of a wire is proportional to the current that the wire can carry, what is the ratio of total copper usage between the 1ph/3w and the 3ph/4w sitems?

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)



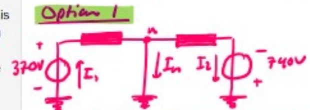
$I_{1-ph-3-w} =$ ____ A

$I_{3-ph} =$ ____ A

Ratio of copper usage 1ph/3ph = ____

$P = 60\,kW,\ Q/S = 0.2 \rightarrow pf$

Option 1



$I_{L1} = \dfrac{P}{V_{L1}\cos\theta} = \dfrac{60\,000}{370\cos(11.5368^\circ)} = 165.5060\,A$

$I_{L2} = \dfrac{P}{V_{L2}\cos\theta} = \dfrac{60\,000}{740\cos(11.5362^\circ)} = 82.7503\,A$

$I_n = I_1 - I_2 = 82.7503A \longrightarrow I_n = I_L$

Option 2

$P = \sqrt{3}\,V_L \cdot I_L \cos\theta \longrightarrow \theta = a\sin(0.2) = 11.5369^\circ$

$I_L = \dfrac{P}{\sqrt{3}\,V_L\cos\theta} = \dfrac{60\,000}{\sqrt{3}(640.859)\cos(11.5369)} = 82.750\,A$