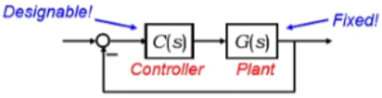
L14 Lead-lag Compensator Design

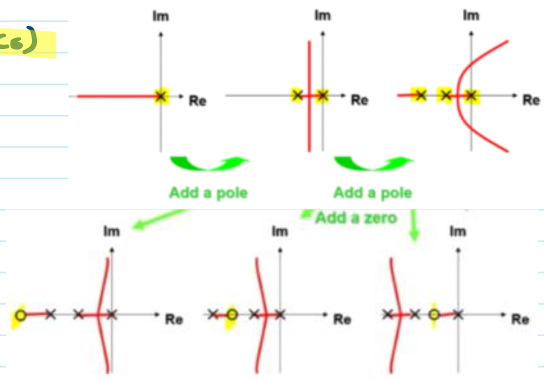
Saturday, June 19, 2021 4:29 PM

Controller Design

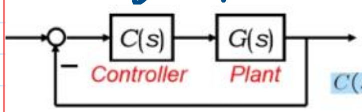
- Place closed-loop poles at desired locations by tuning $C(s) = K$



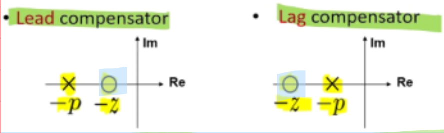
- If RL does not pass through desired location, reshape RL
  - ↳ Add poles/zeros to C(s)
- Adding Poles
  - ↳ pulls RL Right
  - ↳ ↓ stable
  - ↳ slower settle time T_s
- Adding Zeros
  - ↳ pulls RL Left
  - ↳ ↑ stable
  - ↳ faster settle time T_s



Lead lag Compensators

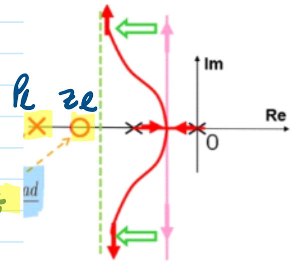


$C(s) = K\dfrac{s+z}{s+p}, \ (z > 0, p > 0)$



Reshaping RL w/ Lead Compensator

- Move centroid/Asymptote Intersection left
  - ↳ Plant: $\dfrac{\sum \text{pole} - \sum \text{zero}}{r}$
  - ↳ Plant & lead: $\dfrac{\sum \text{pole} - \sum \text{zero} + \text{pole}_{lead} - z_{lead}}{r}$

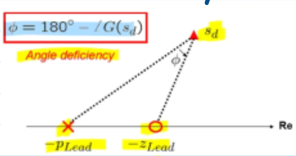


- Improves transient response
- Improves Stability

$C_{lead}(s) = K\dfrac{s + z_{lead}}{s + p_{lead}}$

Design

① Select pole in allowable region (s_d)
Reshape RL to pass through s_d
② Select pole/zero in $C_{lead}(s)$ → $\varphi = 180° - \angle G(s_d)$



For OLTF:
- Angle Condition: $\angle L(s) = 180°(2k+1)$, $k = 0, \pm 1, \pm 2, \ldots$
- Magnitude Condition: $K = \dfrac{1}{|L(s)|}$

- Find φ
  ① $s_d = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$
  ② $G(s_d)$
  ③ $\angle G^*(s_d)$
  ④ $\angle G^*(s_d)$ convert to +Re-axis angle $\angle G(s_d)$ → $\angle G(s_d) = 360° - \angle G^*(s_d)$
  ⑤ $\varphi = 180° - \angle G(s_d)$ ↳ can get $\angle G_d(s_d)$ directly if use angl( ) on HP

ex. $s_d = -2 + 2\sqrt{3}j$, $G(s_d) = \dfrac{4}{(-2+2\sqrt{3}j)2\sqrt{3}j}$

$\angle G^*(s_d) = \angle 4 - \angle(-2+2\sqrt{3}j) - \angle 2\sqrt{3}j = 0 - \left(\tan^{-1}\left(\dfrac{2\sqrt{3}}{-2}\right)\right) - 90° = -210°$

$\angle G(s_d) = 360° - 210° = 150°$

$\varphi = 180° - \angle G(s_d) = 30°$

☆ Use Rad Mode! to solve Angles!!

Evaluate G(s) at the desired pole.

$s_d = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$

$s_d = -2+2\sqrt{3}j$

$G(\underbrace{-2+2\sqrt{3}j}_{s_d}) = \dfrac{4}{(-2+2\sqrt{3}j)2\sqrt{3}j}$

$\angle G^*(s_d) = \angle 4 - \angle(-2+2\sqrt{3}j) - \angle 2\sqrt{3}j$

Desired pole

Im

- Find K
  ① $z_{lead} = |Re\{s_d\}|$
  ② Find p_{lead} using $\varphi = \angle \dfrac{s_d + z_{lead}}{s_d + p_{lead}}$

- Find K
  1. $z_{Lead} = |Re\{s_d\}|$
  2. Find P_{Lead} using $\phi = \angle \frac{s_d + z_{Lead}}{s_d + P_{Lead}}$
  3. $C_{Lead} = K \frac{s + z_{Lead}}{s + P_{Lead}}$
  4. $K = \frac{1}{|L(s_d)|} \rightarrow L(s) = C(s)G(s)$
- Error Const
  - Step Error: $K_p = \lim_{s \to 0} G(s)C(s)$
  - Ramp Error: $K_v = \lim_{s \to 0} sG(s)C(s)$

$$G(\underbrace{-2+2\sqrt{3}j}_{s_d}) = \frac{}{(-2+2\sqrt{3}j)2\sqrt{3}j}$$

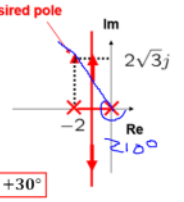
$$\angle G^*(s_d) = \angle 4 - \angle(-2+2\sqrt{3}j) - \angle 2\sqrt{3}j$$
$$= 0° - \left(\tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right)\right) - 90°$$
$$= 0° - (+120°) - 90° = -210° \rightarrow$$
$$\angle G^*(s_d) = -210°$$

Convert "-210°" to an angle from +Re-axis:

$360° - 210° = +150° \rightarrow \angle G(s_d) = = +150°$

Now, use the **angle deficiency formula** to calculate ϕ:

$\phi = 180° - \angle G(s_d) \rightarrow \phi = 180° - 150° \rightarrow \boxed{\phi = +30°}$



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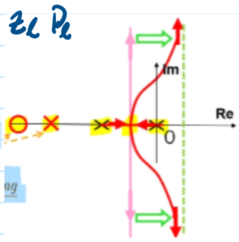
Reshaping RL w/ lag compensator

- Move centroid/Asymptote intersection right
  - ↳ Plant: $\frac{\Sigma pole - \Sigma zero}{r}$
  - ↳ Plant & lag: $\frac{\Sigma pole - \Sigma zero + pole_{lag} - zero_{lag}}{r}$
- Reduce steady state Error
  - $C_{lag}(s) = \frac{s + z_{lag}}{s + P_{lag}}$



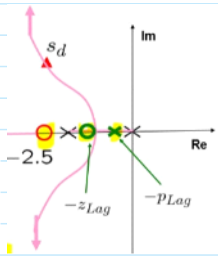
Design

- ↳ Select pole in allowable region (s_d)
- ↳ Reshape RL to pass through s_d
- want small P_{lag} → given

$$K_v = \lim_{s \to 0} sG(s)C_{Lead}(s)C_{Lag}(s) = 2.89 \times \frac{z_{Lag}}{p_{Lag}} \Rightarrow 2.89 \times \frac{z_{Lag}}{p_{Lag}} > 20 \Rightarrow \frac{z_{Lag}}{p_{Lag}} > 6.92$$

$K_v > 20$

$\Rightarrow z_{Lag} > 6.92\, p_{Lag}$



- ↳ $C_{lag}(s_d) \approx 1$
- ↳ $\angle G(s_d)C_{lead}(s_d)C_{lag}(s_d) \approx 180°$

Lead-Lag Compensator

- Takes into account transient & steady state
  - $C_{LL}(s) = K \frac{s + z_{Lead}}{s + P_{Lead}} \cdot \frac{s + z_{lag}}{s + P_{lag}}$

Steady State Error

- **Step $r(t)$:** $K_p = \lim_{s \to 0} L(s) = \infty \Rightarrow e_{ss} = \frac{R}{1 + K_p} = 0$
- **Ramp $r(t)$:** $K_v = \lim_{s \to 0} sL(s) = \frac{3.15K}{0.75} = 4.2K \Rightarrow e_{ss} = \frac{R}{K_v} = \frac{R}{4.2K}$
- **Parabolic $r(t)$:** $K_a = \lim_{s \to 0} s^2 L(s) = 0 \Rightarrow e_{ss} = \frac{R}{K_a} = \infty$