Current, Conductors, Boundary Conditions, Dialectric

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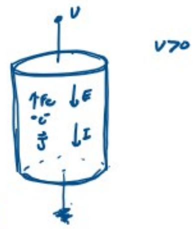
Current

$I = \frac{dQ}{dt}$ A $\longrightarrow$ Motion of pos charge



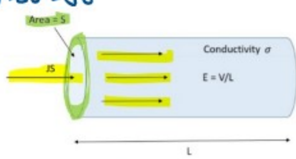
Current Density J

- Total current through surface
  $I = \int_S \vec{J} \cdot d\vec{S} \longrightarrow \vec{J} = \rho_v \vec{v}$ (v = velocity in x)
- Force on electron: $\vec{F} = -e\vec{E}$ (N)
- drift velocity $v_d \longrightarrow$ mobility of electrons in material
  $v_d = -\mu_e \vec{E} \longrightarrow \therefore J = \rho_v \vec{v} = -\rho_e \mu_e \vec{E} = \sigma \vec{E}$
  $\vec{J} = \sigma \vec{E} \longrightarrow \sigma = -\rho_e \mu_e$ conductivity of material $\rho = \frac{1}{\sigma}$

Point Form of Ohm's law (Cylindrical wire segment)

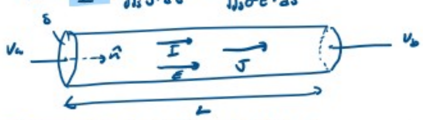
$I = \iint_S \vec{J} \cdot d\vec{S} = \iint_S \vec{J} \cdot \hat{n}\, dS$

- If current $\perp$ cross-section of wire: $I = \int_S \vec{J} \cdot d\vec{S} = JS$
- Potential difference Vab b/w 2 points along wire
  $V_{ab} = -\int_b^a \vec{E} \cdot d\vec{L} = -\int_b^a \vec{E} \cdot \hat{n}\, dL = EL \longrightarrow E = \frac{V_{ab}}{L}$ V/m
  $\because$ moving parallel to $\vec{E}$ field

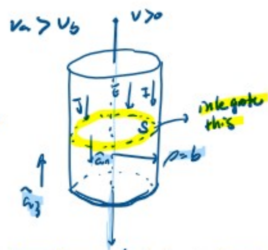


$J = \frac{I}{S} = \sigma E = \sigma \frac{V}{L} \longrightarrow V = \frac{L}{\sigma S} I = RI$

$R = \frac{V_{ab}}{I} = \frac{-\int_b^a \vec{E} \cdot d\vec{L}}{\iint_S \vec{J} \cdot d\vec{S}} = \frac{-\int_b^a \vec{E} \cdot d\vec{L}}{\iint_S \sigma \vec{E} \cdot d\vec{S}}$



$V_a > V_b$



ex. Simple Case: let $\vec{J} = A\rho^2(-\hat{z})$ A/m², solve for I
$I = \iint_S \vec{J} \cdot \hat{n}\, dS = \int_0^{2\pi}\int_0^b A\rho^2(-\hat{z}) \cdot (-\hat{z})\, \rho\, d\rho\, d\varphi$

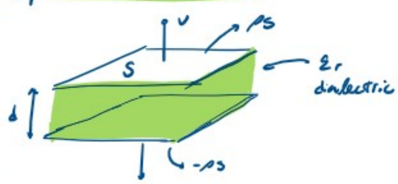
$I = \frac{A\pi b^4}{2}$ A

General Boundary Conditions

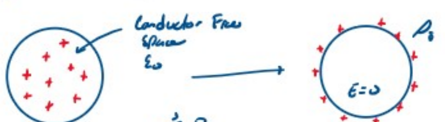
For E: E - tangential is continuous
$E_{T1} = E_{T2}$
$\varepsilon_{r1}\varepsilon_0 \vec{E}_{N1} = \varepsilon_{r2}\varepsilon_0 \vec{E}_{N2} \longrightarrow \varepsilon_1 = \varepsilon_{r1}\varepsilon_0$

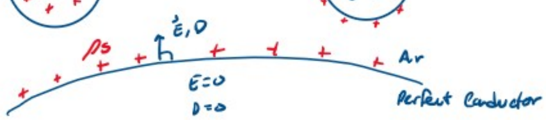
For D: D - Normal is continuous
$D_{N1} = D_{N2}$
$\varepsilon_{r1}\varepsilon_0 D_{N1} = \varepsilon_{r2}\varepsilon_0 E_{N2}$

Capacitors & Dialectrics



Special Case:



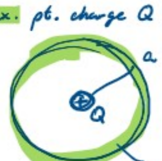


$E_{T1} = E_{T2} \longrightarrow E_{T1} = 0 = E_{T2}$
$D_{N1} = 0 \quad (\because E = 0)$
$D_{N2} = \rho_s \quad + \quad E_{N2} = \frac{D_{N2}}{\varepsilon_0} = \frac{\rho_s}{\varepsilon_0}$

Dielectric Materials

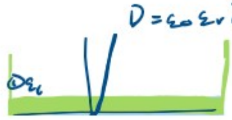
$E_{T1} = E_{T2} \longrightarrow E_{T1} = \frac{D_{T1}}{\varepsilon_1} = \frac{D_{T2}}{\varepsilon_2} = E_{T2}$

ex. pt. charge Q inside dialectric sphere (Around sphere = air)

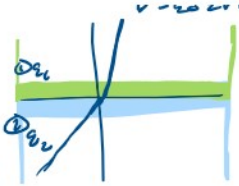


$D = \varepsilon_0 \varepsilon_r \vec{E}$

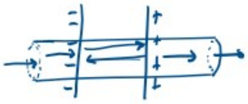
ex. $\varepsilon_2 > \varepsilon \quad \varepsilon = \varepsilon_r \varepsilon_0$
$D_N = cts, \quad E_T = cts$
$D_{N2} = \varepsilon_{r2}\varepsilon_0 E_{N2} = D_{N1} = \varepsilon_{r1}\varepsilon_0 E_{N1}$
$E_{N1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} E_{N2} \quad \therefore E_{N1} > E_{N2}$

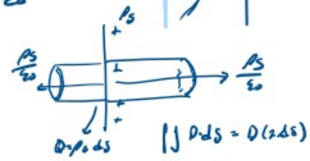


ex. $\varepsilon_2 > \varepsilon$ $\varepsilon = \varepsilon_r \varepsilon_0$
$D_N = cts$, $E_T = cts$
$D_{N2} = \varepsilon_{r2} \varepsilon_0 E_{N2} = D_{N1} = \varepsilon_{r1} \varepsilon_0 E_{N1}$
$E_{N1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} E_{N2}$ $\therefore E_{N1} > E_{N2}$
$E_{N2} = E_{T1}$



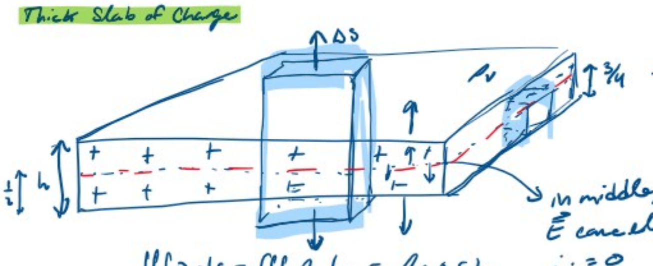
- For $\vec{E}$ of inf. sheet: $\vec{E} = \frac{\rho_s}{\varepsilon_0}$
$\therefore E_N = \frac{\rho_s}{\varepsilon_0}$





$\iint D \cdot ds = D(2ds)$

Thick Slab of Charge

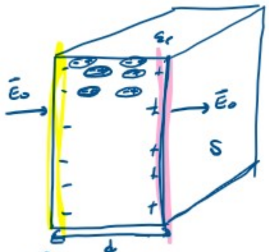


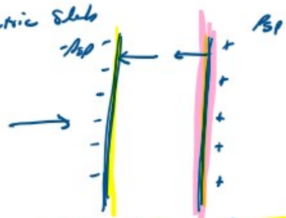
→ held smaller gaussian surface

→ in middle, $\vec{E}$ cancel out $\therefore = 0$

$\oiint_S D \cdot ds = \iiint_V \rho_v\, dv = \rho_v\, \Delta S h$

Polarization





where ρ_{sp} = surface charge due to polarization

- $\vec{E}_d$ = net field
- slab has $p = Qd \rightarrow Q = \rho_s dS$
- polarization $(\vec{P})$ = # of dipole moments / unit volume
$\vec{P} = \frac{p}{v} = \frac{Qd}{Sd} = \frac{Q}{S}\hat{d} = \rho_s p \vec{d}$
$\vec{D}_v = \vec{P}$ $\vec{P}_d = \varepsilon_0 \vec{E}_d + \vec{P} \longrightarrow \vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$

☆ Webwork 5

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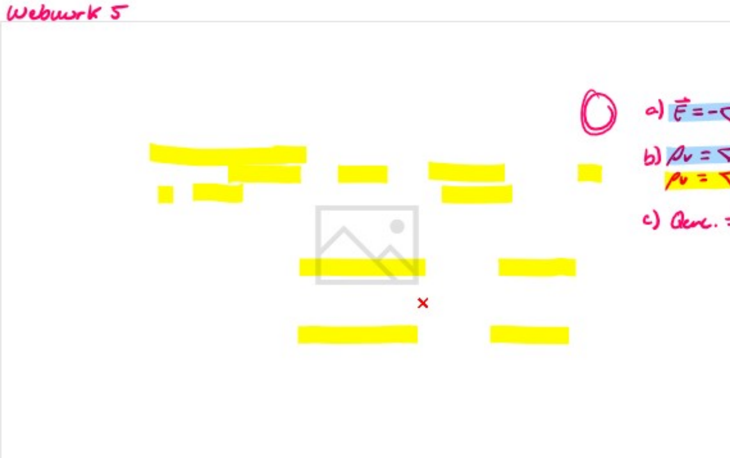


0

a) $\vec{E} = -\nabla V = -\langle -3V_0 \frac{a^2}{r^4}, 0, 0 \rangle = 3V_0 \frac{a^2}{r^4}\, \hat{a}_r$

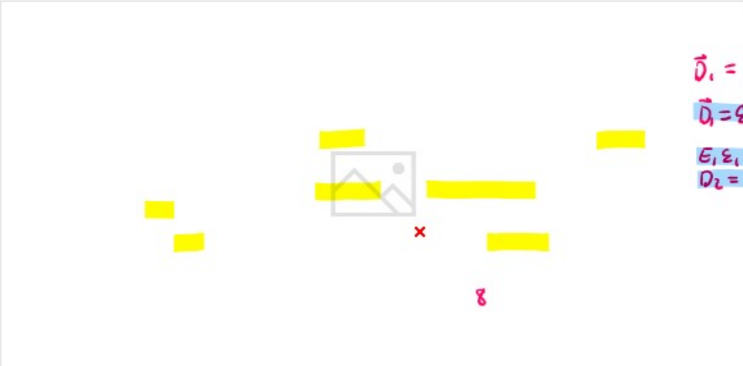
b) $\rho_v = \nabla \cdot \vec{D} \longrightarrow \vec{D} = \varepsilon_0 \vec{E}$ (divergence in cylindrical coord.)
$\rho_v = \nabla \cdot (\varepsilon_0 \vec{E}) = \frac{1}{r^2}\frac{d}{dr}\left(r^2 (3V_0 \frac{a^2}{r^4} \varepsilon_0)\right) = -6V_0 a^2 r^{-5} \varepsilon_0$

c) $Q_{enc.} = \oint_v \rho_v\, dv = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} -6V_0 a^2 r^{-5} \varepsilon_0 r^2 \sin\theta\, dr\, d\varphi\, d\theta$

0!

Capping surface = circle → cylindrical coord.
Parametrization: $x = \cos\theta$, $y = a\sin\theta$, $z = 0$
$\vec{F} \cdot \hat{n} = \vec{F}(x,y,z) \cdot \langle 0,0,1 \rangle = zy^2 + 4x^2 - zx^2\, \hat{z}$
→ use parametrization: $\vec{F}(\vec{r}(\theta)) \cdot \hat{n} = 4(r\cos\theta)^2$
- Integrate: $\int_0^{2\pi}\int_0^{a} 4r^2\cos^2\theta\, r\, dr\, d\theta = a^4\pi$

Parametrization:
$x = r\cos\theta$
$y = r\sin\theta$
$z = 0$
$\} \; 0 \le \theta \le 2\pi$



$\vec{D}_1 = \langle 3, 8, 1 \rangle \quad \varepsilon_1 = 4\varepsilon_0, \ \varepsilon_2 = 5\varepsilon_0$

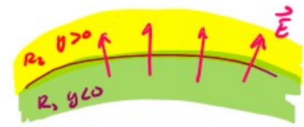
$\vec{D}_1 = \varepsilon_1 \vec{E}_1 \longrightarrow \vec{E}_1 = \frac{1}{4\varepsilon_0} \langle 3, 8, 1 \rangle$

$E_1 \varepsilon_1 = \varepsilon_2 E_2 \longrightarrow E_2 = \frac{E_1 \varepsilon_1}{\varepsilon_2}$

$D_2 = \varepsilon_2 E_2 = \langle 1.6666, 8, 0.5555 \rangle$

$\hookrightarrow \therefore D_{N1} = D_{N2}$

$\hat{n} = \hat{a}_y$



$\vec{D} = \varepsilon \vec{E}$

$\varepsilon = \varepsilon_r \varepsilon_0$

$\varepsilon_1 = \varepsilon_{r1} \varepsilon_0$

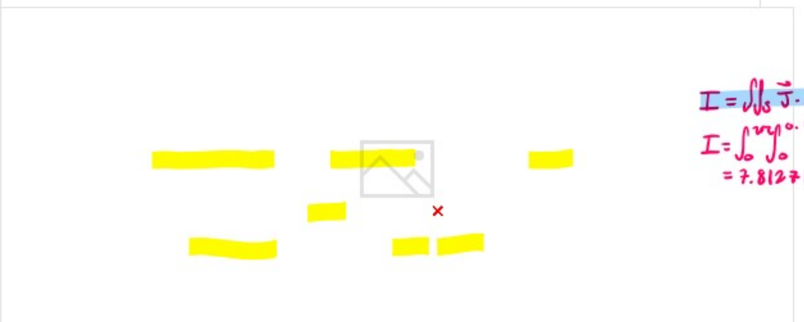
$\varepsilon_1 \vec{E}_1 = \varepsilon_2 \vec{E}_2$

$\psi = \iiint_v \rho_v \, dv \longrightarrow \rho_v = \nabla \cdot \vec{D}$

Divergence (spherical coord.)

$\nabla \cdot \vec{D} = \frac{18}{r^2} \left[\frac{\cos 2\theta \cos\theta}{\sin\theta} - 2\sin\theta \right]$

$\psi = \int_{1.6}^{3.3} \int_{1.4}^{1.7} \int_{1.3}^{2.7} \frac{1}{r \sin\theta} \cdot \left(-\frac{36}{r} \sin(2t) \right) d\varphi \, d\theta \, dr$



$I = \iint_S \vec{J} \cdot dS = \iint_S \vec{J} \cdot \hat{n} \, dS \longrightarrow dS = \rho \, d\rho \, d\varphi$

$I = \int_0^{2\pi} \int_0^{0.65} 8e^{-(1-\rho/0.65)} \rho \, d\rho \, d\varphi \ \hat{a}_z$

$= 7.81271688 A$