Dielectrics and Capacitors

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Dielectrics (Part 1)

$\vec{P} = q\vec{d}$

↳ electric dipole moment vector

- In $\vec{E}$ Field: Rotation, but no translation
- dipoles rotate & align themselves to field
- Internal polarization creates opposing $\vec{E}$-Field

∴ reduces original external field

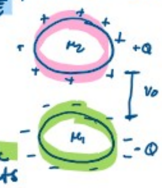
Capacitance

- Capacitance: ratio of charge $|Q|$ over mag. potential diff. b/w 2 conductors

$C = \frac{|Q|}{|V_0|}$ $\quad Q = \int_S \vec{D} \cdot \vec{dS} \longrightarrow \vec{D} = \varepsilon \vec{E}$

$V_0 = -\int_{-q}^{+q} \vec{E} \cdot \vec{dL}$

$\therefore C = \frac{\oint_S \varepsilon \vec{E} \cdot \vec{dS}}{-\int_-^+ \vec{E} \cdot \vec{dL}}$



ex. Parallel plate capacitor w/ dielectric

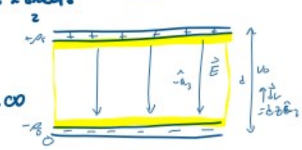
Infinite sheet ⟶ E-field 2 sheets

$C = \frac{|Q|}{|V_0|}$, $\quad \vec{E} = -\frac{\rho_S}{\varepsilon} \hat{a}_z$

$V_0 = -\int_-^+ \vec{E} \cdot \vec{dL} = \int_-^+ \frac{\rho_S}{\varepsilon} \hat{a}_z \cdot dL$

∵ Plates = infinite sheets ∴ $Q = \infty$

$C = \frac{|Q|^{\infty}}{|V_0|} = \infty$

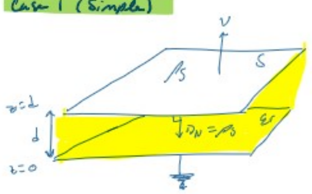


Finite Sheet $S \gg d$

- For finite plates: $Q = \rho_S \cdot S$, $C = \frac{\rho_S S}{\frac{\rho_S}{\varepsilon} d} = \frac{\varepsilon S}{d}$
- Magnitude of C depends on: plate area, dist. d, dielectric material

Capacitors

Case 1 (Simple)



$C = \frac{|Q|}{|V_0|} = \frac{\iint_S \rho_S \, dS}{\int_L \vec{E} \cdot \vec{dL}} = \frac{\rho_S S}{\frac{\rho_S d}{\varepsilon_r \varepsilon_0}} = \frac{\varepsilon_r \varepsilon_0 S}{d}$

$D = \varepsilon E = \varepsilon_r \varepsilon_0 E$

$E = \frac{\rho_S}{\varepsilon_r \varepsilon_0}$

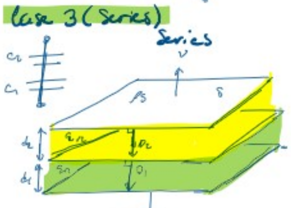
Case 2 (Cylindrical)

↦ E-field infinite line of charge

$\vec{E} = \frac{\rho_L}{2\pi \varepsilon_r \varepsilon_0 \rho} \hat{a}_\rho$ $\quad C = \frac{\rho_L L}{\frac{\rho_L \cdot \ln(b/a)}{2\pi \varepsilon_r \varepsilon_0}} = \frac{2\pi \varepsilon_r \varepsilon_0 L}{\ln(b/a)}$

$V = -\int \vec{E} \cdot d\rho \, \hat{a}_\rho$ $\quad \frac{C}{L} = \frac{2\pi \varepsilon_r \varepsilon_0}{\ln(b/a)}$

Case 3 (Series)



$D_N = \rho_S$

$D_{N1} = D_{N2} = \rho_S$

$D_1 = \varepsilon_{r1} \varepsilon_0 \vec{E}_1$

$D_2 = \varepsilon_{r2} \varepsilon_0 \vec{E}_2$

$D_1 = E_1$

$C_2 = \frac{\varepsilon_{r2} \varepsilon_0 S}{d_2}$

$C_1 = \frac{\varepsilon_{r1} \varepsilon_0 S}{d_1}$

$C_{TOT} = \frac{C_1 C_2}{C_1 + C_2}$

OR:

$C = \frac{Q}{V}$, $Q = \rho_S \cdot S$

$V = \int \vec{E}_1 \cdot dL + \int E_2 \cdot dL$

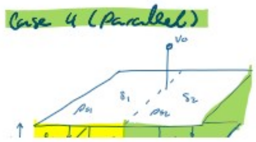
$= E_1 d_1 + E_2 d_2$

$= \frac{\rho_S d_1}{\varepsilon_{r1} \varepsilon_0} + \frac{\rho_S d_2}{\varepsilon_{r2} \varepsilon_0}$

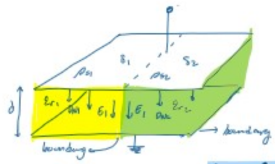
$\therefore C = \frac{\rho_S \cdot S}{\frac{\rho_S d_1}{\varepsilon_{r1} \varepsilon_0} + \frac{\rho_S d_2}{\varepsilon_{r2} \varepsilon_0}}$

⟶ Same Result ⟵

Case 4 (Parallel)



- $E_1 = E_2$ ∵ tangentials = same, D_N continuous
- $D_{N2} = \rho_S$
- If $E_1 = E_2$ but $\varepsilon_{r1} \neq \varepsilon_{r2} \longrightarrow D_1 \neq D_2$, $D = \varepsilon_r \varepsilon_0 \vec{E}$



- $D_{N2} = \rho_S$
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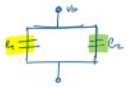
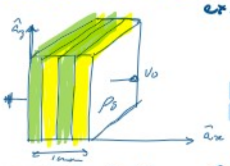


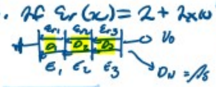
$C_1 = \frac{\varepsilon_{r1}\varepsilon_0 S_1}{d}$, $C_2 = \frac{\varepsilon_{r2}\varepsilon_0 S_2}{d}$

$C_{TOT} = C_1 + C_2$

Case 5 (Graded)



ex. If $\varepsilon_r(x) = 2 + 2\times10^6 x^2$ $C = ?$



$D_1 = D_2 = D_3$

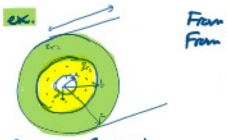
D_N = cts @ dielectric-dielectric boundary

$D = \varepsilon_r \varepsilon_0 E$

$E = \frac{\rho_S}{(2+2\times10^6 x^2)\varepsilon_0}$ V/m

$C = \frac{|Q|}{|V|} = \frac{\rho_S \cdot S}{-\int E \cdot dx} = \frac{\rho_S\, S}{\frac{\rho_S}{\varepsilon_0}\int \frac{1}{(2+2\times10^6 x^2)}dx} = \cdots 451\,pF$

ex.

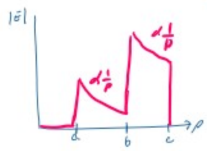


$\varepsilon_{r1} = 10$ $\varepsilon_{r2} = 1$

$a = 1mm$ $b = 3mm$ $c = 4mm$

From BC: $D_1 = D_2$

From $D = \varepsilon_r \varepsilon_0 E$: $E_1 = \frac{\rho_S\, a}{10\varepsilon_0 \rho}\hat{a}_\rho$ V/m $E_2 = \frac{\rho_S\, S}{\varepsilon_0 \rho}\hat{a}_\rho$ V/m



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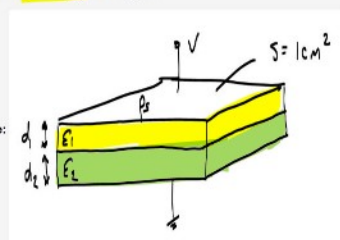
(1 point)
A parallel plate capacitor is shown in the attached figure. Between the two plates sit 2 parallel layers of dielectric material with permittivities $\epsilon_1 = 7\epsilon_0$ and $\epsilon_2 = 4\epsilon_0$. The corresponding layer thicknesses are $d_1 = 1.6$ mm and $d_2 = 1.9$ mm. If $\rho_S = 60$ nC/m^2, what is the potential, V, at the top plate?

Figure:



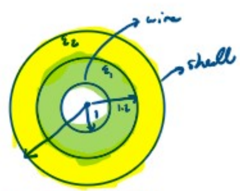
ANSWER: $V =$ Volts

$C = \frac{Q}{V} \longrightarrow Q = \rho_S S$

$C_1 = \frac{\varepsilon_1 S}{d_1}$ $C_2 = \frac{\varepsilon_2 S}{d_2}$

$C_{TOT} = \frac{C_1 C_2}{C_1 + C_2}$
$= 1.2584632\times10^{-8}$

$\therefore V = \frac{\rho_S\, S}{C_{TOT}} = 4.7677197$ V



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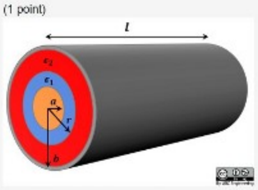
(1 point)



[DL:2/5] A cylindrical capacitor is made using a conducting wire of radius 1 mm and a concentric cylindrical conducting shell of inner radius 5 cm. The capacitor has a length of $l = 0.5$ m.

The space between the wire and the shell is filled with two concentric layers of homogeneous dielectric material as detailed below:

$$\epsilon(r) = \begin{cases} \epsilon_1 = 3\epsilon_0, & \text{for } 1\text{ mm} < r < 1.2\text{cm}, \\ \epsilon_2 = 2\epsilon_0, & \text{for } 1.2\text{ cm} < r < 5\text{cm}, \\ \epsilon_0, & \text{elsewhere.} \end{cases}$$

Find the capacitance of this capacitor. (Note the units supplied.)

$C =$ pF

$C_{TOT} = \frac{C_1 C_2}{C_1 + C_2}$

$C = \frac{Q}{V}$ $Q = \rho_L\, dl$

For Cylindrical Capacitor:

$V = -\int E \cdot d\rho\, \hat{a}_\rho$ $\vec{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\hat{a}_\rho$

$= -\int_a^b \frac{\rho_L}{2\pi\varepsilon\rho}\, d\rho\, \hat{a}_\rho = -\frac{\rho_L \cdot \ln(b/a)}{2\pi\, \varepsilon}\hat{a}_\rho$

$C = \frac{\rho_L \cdot L}{\frac{\rho_L \ln(b/a)}{2\pi\,\varepsilon}}$

$C_1 = \frac{2\pi \cdot 3\varepsilon_0 (0.5)}{\ln\left(\frac{0.012}{0.001}\right)}$ $C_2 = \frac{2\pi \cdot 2\varepsilon_0 (0.5)}{\ln\left(\frac{0.05}{0.012}\right)}$

$C_{TOT} = C_1 || C_2 = 18.040706\,pF$

$6x - 2y + 3z > 6$ $\varepsilon_{r1} = 2.5$

$\varepsilon_{r2} = 4$

$\vec{E}$ = const

$\therefore E_{T1} = E_{T2}$

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(1 point)
Two different ideal dielectrics fill all of space. The relative permittivity in the region where $6x - 2y + 3z > 6$ is $\epsilon_{r1} = 2.5$, while the relative permittivity everywhere else is $\epsilon_{r2} = 4$. In each dielectric region, the electric field intensity is constant. (Of course the

@ (2,1,1): $6x - 2y + 3z > 6 = 13 > 6 \therefore R_1$

@ (-2,6,4): $6x - 2y + 3z > 6 = -12 > 6 \therefore R_2$

① Find $\vec{E}_{N1}$ From $\vec{E}_1$: $\vec{E}_{N1} = \text{proj}_{\vec{n}}\vec{E} = \frac{\vec{E}_1 \cdot \vec{n}}{\vec{n}\cdot\vec{n}} \cdot \vec{n}$
$\vec{n} = \langle 6, -2, 3 \rangle$ $\vec{E}_{N1} = \langle -12, 4, -6 \rangle$

② $\vec{E}_{T1} = \vec{E}_1 - E_{N1}$: $E_{T1} = \langle 88, -6, -180 \rangle$

③ $E_{T1} = E_{T2}$ $\because$ continuous: $D_{T2} = 3\varepsilon E_{T2} = \cdots$

$D_{N2} = D_{N1} = \varepsilon_r \vec{E}_{N1} = \cdots$

$\text{Proj}_{\vec{n}}\vec{E} = \frac{\vec{E}\,\vec{n}}{\vec{n}\cdot\vec{n}}\vec{n} \longrightarrow$ vector

$\text{Proj}_{\hat{n}}\vec{E} = (\vec{E}\cdot\hat{n})\hat{n} \longrightarrow$ unit vector

(1 point)
Two different ideal dielectrics fill all of space. The relative permittivity in the region where $6x - 2y + 3z > 6$ is $\epsilon_{r1} = 2.5$, while the relative permittivity everywhere else is $\epsilon_{r2} = 4$. In each dielectric region, the electric field intensity is constant. (Of course the constant for Region 1 need not be identical to the constant for Region 2.)

Given the electric field intensity $\mathbf{E}(2,1,1) = \langle 76, -2, -186 \rangle$ V/m, find the electric flux density at the point $(-2, 6, 4)$.

ANSWER: $\mathbf{D}(-2,6,4) =$ ______ C/m^2

① Find E_{N1} from E_1: $E_{N1} = \text{proj}_{\hat{n}} E = \frac{\cdots}{\vec{n} \cdot \vec{n}} \cdot n$ $\quad$ $\text{proj}_{\vec{n}} E = \frac{\cdots}{\vec{n}\cdot\vec{n}}$

$\vec{n} = \langle 6, -2, 3 \rangle$ $\quad \vec{E}_{N1} = \langle -12, 4, -6 \rangle$

② $\vec{E}_{T1} = \vec{E}_1 - E_{N1}$: $E_{T1} = \langle 88, -6, -180 \rangle$ $\quad$ $\text{Proj}_{\hat{n}} \vec{E} = (\vec{E} \cdot \hat{n})\hat{n}$ → unit vector

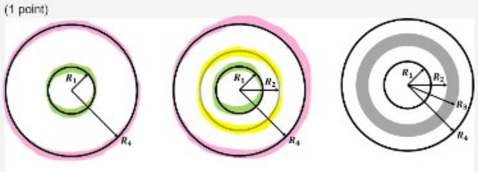
③ $E_{T1} = E_{T2}$ ∵ continuous: $D_{T2} = \epsilon_2 E_{T2} = \cdots$

④ $D_{N1} = \epsilon_1 E_{N1} = \cdots$ $\quad D_{N2} = D_{N1} = \epsilon_1 \vec{E}_{N1} = \cdots$

⑤ $D_2 = D_{T2} + D_{N2} = \langle 2.851048\times10^{-9}, -1.239586\times10^{-10}, -6.507828\times10^{-9} \rangle$

Assignment 6: Problem 6

(1 point)



(a) (b) (c)

[DL:3/5] As shown in Fig. (a), a spherical capacitor is made of two thin concentric shells. The inner shell has radius $R_1 = 0.01$ m and the outer shell has radius $R_4 = 0.33$ m. The inner shell ($r = R_1$) has total charge $Q = 11\ \mu C$, uniformly distributed. Assume that the space between the inner and outer shells has permittivity $\epsilon = \epsilon_0$. Answer the following questions:

a) Find the capacitance between R_1 and R_4, as shown in Fig. (a).

$C =$ ______ pF

b) As shown in Fig. (b), another thin spherical conducting shell is placed at $r = R_2 = 0.13$ m, with $R_1 < R_2 < R_4$. Taking the result from part **a)** for reference, will this new shell make the capacitance between $r = R_1$ and $r = R_4$ increase, decrease, or stay the same? ?

c) What is the total capacitance between $r = R_1$ and $r = R_4$ in part **b)**?

$C =$ ______ pF

d) As shown in Fig. (c), instead of a thin shell, a thick conducting shell defined by $R_2 < r < R_3$ is introduced, where $R_1 < R_2 < R_3 < R_4$; in detail, $R_2 = 0.13$ m and $R_3 = 0.22$ m. Taking the result from part **a)** for reference, will introducing this new shell make the capacitance between $r = R_1$ and $r = R_4$ increase, decrease, or stay the same? ?

e) What is the total capacitance between $r = R_1$ and $r = R_4$ in part **d)**?

$C =$ ______ pF

a) $C = \frac{Q}{V}$ → $V = -\int_a^b E \cdot dl$ $\quad \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}$

$V = -\frac{11\times10^{-6}}{4\pi\epsilon_0}\int_{0.01}^{0.33} \frac{1}{r^2}\, dr = 4586721.9072$ V

$C = \frac{Q}{V} = 1.14742$ pF

b) No change

c) $C = \frac{Q}{V}$ $\quad V = -\int_a^b E \cdot dl$ $\quad \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}$

$V_{12} = -\frac{11\times10^{-6}}{4\pi\epsilon_0}\int_{0.01}^{0.13} \frac{1}{r^2}\, dr$ $\quad C_1 = 1.205371$ pF

$V_{24} = -\frac{11\times10^{-6}}{4\pi\epsilon_0}\int_{0.13}^{0.33} \frac{1}{r^2}\, dr$ $\quad C_2 = 1.147203$ pF

$C_{TOT} = C_1 || C_2 = 1.14742$ pF

d) Increases

e) $C_{12} = \cdots$

$V_{34} = \frac{-11\times10^{-6}}{4\pi\epsilon_0}\int_{0.22}^{0.33} \frac{1}{r^2}\, dr$ $\quad C_{34} = 73.43490$ pF

$C_{TOT} = C_1 || C_2 = 1.1859052$ pF