L15 PID Controller Design

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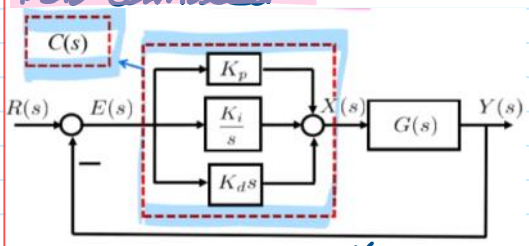
PID Controller



Proportional Gain K_p
Integral Gain K_i
Derivative Gain K_d

$u(t) = \underbrace{K_p e(t)}_{\text{Proportional}} + \underbrace{K_i \int_0^t e(\tau)\,d\tau}_{\text{Integral}} + \underbrace{K_d \frac{de(t)}{dt}}_{\text{Derivative}}$

$X(s) = C(s)E(s) \rightarrow C(s) = \underbrace{K_p + \frac{K_i}{s} + K_d s}_{\text{parallel}} = \underbrace{K_p\left(1 + \frac{1}{T_I s} + T_D s\right)}_{\text{standard}}$

$T_D = \frac{K_d}{K_p}$, $T_I = \frac{K_p}{K_i}$

$K_d \approx \frac{K_d s}{\tau_d s + 1}$ → Avoid high freq & noise amplification

- PI Controller: $C(s) = K_p + \frac{K_i}{s}$ $\quad T_c = \frac{2\pi}{\omega}$
- PD Controller: $C(s) = K_p + K_d s$

P Controller

- Simple
- ↑ Gain = ↓ e_{ss}, ↓ rise Time, ↑ oscillation

$C(s) = K_p$

$e_{ss} = \frac{1}{1 + G(0)C(0)} = \frac{1}{1 + K_p}$

PI Controller

- Zero steady state error
- ↑ Gain K_i = ↓ rise Time, ↑ oscillations

$C(s) = K_p + \frac{K_i}{s}$

$e_{ss} = \frac{1}{1 + G(0)C(0)} = 0$

PID Controller

- Zero steady state error
- ↑ Gain K_d = ↑ damping
  - ↳ too ↑ K_d = ↓ performance

$C(s) = K_p + \frac{K_i}{s} + K_d s$

Ziegler-Nichols PID tuning rules

- Step Response Method
  - ↳ Find L & a

PID parameters:

Type	K_p	T_I	T_D
P	$1/a$		
PI	$0.9/a$	3τ	
PID	$1.2/a$	2τ	0.5τ

$C(s) = K_p\left(1 + \frac{1}{T_I s} + T_D s\right)$

- Ultimate Sensitivity Method
  - ↳ $K_p = K_c$
  - ↳ Find T_c

PID parameters

Type	K_p	T_I	T_D
P	$0.5K_c$		
PI	$0.4K_c$	$0.8T_c$	
PID	$0.6K_c$	$0.5T_c$	$0.125T_c$

$C(s) = K_p\left(1 + \frac{1}{T_I s} + T_D s\right)$

Ch. Eq = $s(s+1)(s+5) + K_c = 0 \rightarrow s^3 + 6s^2 + 5s + K_c = 0$

$-j\omega^3 - 6\omega^2 + 5j\omega + K_c = 0$

Re(): $-6\omega^2 + K_c = 0$
Im(): $-\omega^3 + 5\omega = 0$

solve: $\omega = 2.236$, $K_c = 30$