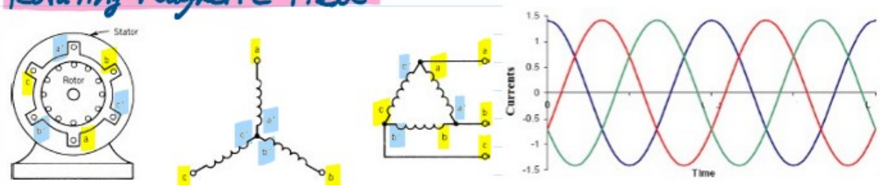
5. AC Induction Machines

Monday, November 16, 2020 3:49 AM

5.1 Rotating Magnetic Field



2 poles

- Applying current @ $f_e = 60Hz$
  - ↳ creates field lines ⊥ axis

$\omega_e = 2\pi f_e$

$F_m = NI_m$

Currents:

$i_a = I_m \cos(\omega_e t)$

$i_b = I_m \cos(\omega_e t - 120°)$

$i_c = I_m \cos(\omega_e t + 120°)$

MMFs in axis:

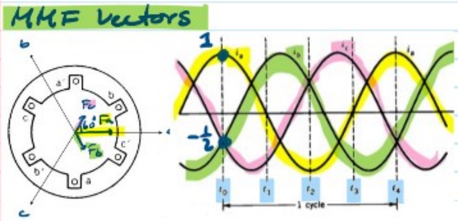
$F_a = F_m \cos(\omega_e t)$

$F_b = F_m \cos(\omega_e t - 120°)$

$F_c = F_m \cos(\omega_e t + 120°)$

$F_{tot} = F_a + F_b + F_c$ → resulting mmf

MMF vectors



@ $t = t_0$: $F_a = 1$, $F_b = F_c = -\frac{1}{2}$

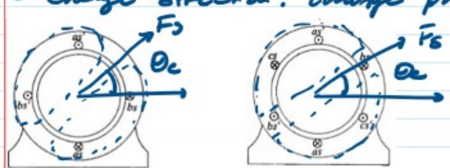
- F_a leads F_b by $60°$
- F_c leads F_a by $60°$

$F_a = F_m$, $F_b = -\frac{1}{2} F_m$, $F_c = -\frac{1}{2} F_m$ → ($F_m = F_{max}$)

$|F_{net}| = F_m + 2 \cdot \frac{1}{2} F_m \cos 60°$

Producing Rotating Magnetic Field

- AC currents shifted in time
- Apply currents to stator windings
  - ↳ Produces rotating vector MMF F_s w/ const. mag.
- Change direction: change phase seq: a,b,c → CCW, b,a,c → CW



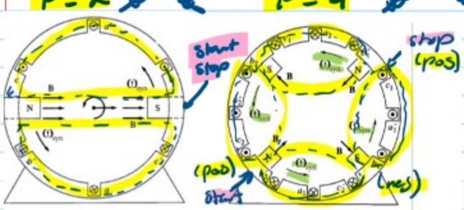
- 2 poles
  - ↳ 2 phases
  - ↳ 3 phases
- $\theta_e = \omega_e t$

2 phase: $F_s = F_m \angle \theta_e$

3 phase: $F_s = (3/2) F_m \angle \theta_e$

Poles

$P = 2$  $P = 4$ 



Syncronous speed ω_{syn}:

- stator magnetic poles rotation speed
- electrical freq./speed ω_e
  - $\theta_e = \omega_e t$ → electrical displacement
  - $\theta_e = 0 \rightarrow 2\pi$

Speed of P-Pole Motor

1 cycle = 1 rev.

$\omega_{syn} = \omega_e$

$n_{syn} = 60 f_e$ (2 poles)

1 cycle = ½ rev.

$\omega_{syn} = \frac{1}{2} \omega_e$

$n_{syn} = 30 f_e$

→ ∴ $\omega_{syn} = \frac{2}{P} \omega_e$

$n_{syn} = \frac{120}{P} f_e$

- Nominal speed (motoring): $n < n_{syn}$ ~50 rpm
- Nominal speed (Generating): $n > n_{syn}$ ~50 rpm
- Can convert P-pole motor to 2-pole using P/2 factor
  - ↳ mech. speed: $\omega_r = \frac{P}{2} \omega_{rm}$ → not const. & depends on mech load
    - ↳ Rotor mech speed

Slip & Async. Speed

- Induction Machine = Async. Machine
  - ↳ Speed rotor → diff. from speed of stator magnetic field/poles
  - ↳ Slip = diff. b/w syn. & r. speed
- Fractional/Inst. Slip: $s = \frac{n_{syn} - n}{n_{syn}} = \frac{\omega_{syn} - \omega_{rm}}{\omega_{syn}} = \frac{\omega_e - \omega_r}{\omega_e}$

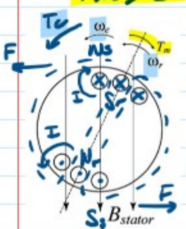
→ speed rotor → diff. from speed of stator magnetic field/poles
↳ Slip = diff. b/w syn. & r. speed

- Fractional/Inst. Slip: $s = \frac{n_{syn} - n}{n_{syn}} = \frac{\omega_{syn} - \omega_{rm}}{\omega_{syn}} = \frac{\omega_e - \omega_r}{\omega_e}$
- Rotor speed vs stator B field (poles) → slip freq: $\omega_s = \omega_e - \omega_r = s\,\omega_e$
- Rotor mechanical Speed: $\omega_{rm} = (1-s)\omega_{syn}$, $n = (1-s)n_{syn}$

Modes of Operation

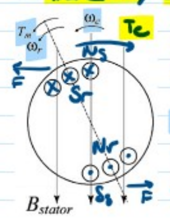
Idle
- Synchronous speed $\omega_r = \omega_e$ → rotor speed = syn. speed = field speed
- $s = \frac{\omega_e - \omega_r}{\omega_e} = 0$
- No flux, No induced current, No torque → $T_e = 0$
- Magnetic field looks stationary

Motor
- $T_m > 0$, $\omega_r < \omega_e$



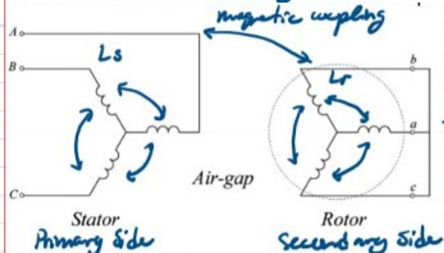
ω_r & ω_e → same dir.
T_e w/ rotation
$T_m > 0$, opp. rotation
$s = \frac{\omega_e - \omega_r}{\omega_e} > 0$
$\omega_s = \omega_e - \omega_r$ → opp dir
- RHR to determine currents
↳ use B & F

Generator
- $T_m < 0$, $\omega_r > \omega_e$



ω_r & ω_e → same dir.
T_e opp. rotation
$T_m < 0$, w/ rotation
$s = \frac{\omega_e - \omega_r}{\omega_e} < 0$
$\omega_s = \omega_e - \omega_r$ → same dir
- RHR

Equivalent Circuit
- Power Factor PF: $PF = \cos\varphi = \frac{P_{in}}{\sqrt{3}\,V_L I_L}$
- Torque: $T_m = \frac{P_{out}}{\omega_{rm}}$
- Rotor current freq: $f_r = f_2 = \frac{P}{120}(n_{syn} - n) = \frac{\omega_s}{2\pi} = \frac{s \cdot \omega_e}{2\pi} = s \cdot f_e$ → $f_e = f_1$



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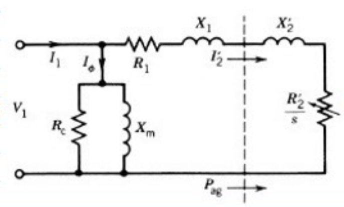

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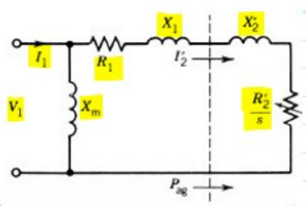
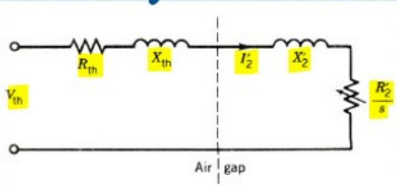


Approximate eq. circuit

Thevenin Equivalent Circuit



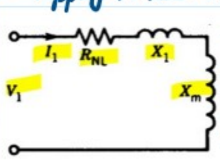
$V_{th} = \frac{X_m}{[R_1^2 + (X_1 + X_m)^2]^{1/2}} \cdot V_1$ → If $R_1^2 \ll (X_1 + X_m)^2$: $V_{th} = \frac{X_m}{X_1 + X_m}$

$Z_{th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} = R_{th} + jX_{th}$

If $R_1^2 \ll (X_1 + X_m)^2$: $R_{th} = \left(\frac{X_m}{X_1 + X_m}\right)^2 R_1 = K_{th}^2 R_1$
$\because X_1 \ll X_m$, $X_{th} \approx X_1$

Determining Circuit Parameters
1. Measure winding Resistance: $R_1 = \frac{V_{dc}}{2I_{dc}}$
2. No Load Test ($P_{rot}, V_1, X_{NL}, Q_{NL}, R_{NL}$)
3. Blocked Rotor Test ($V_1, R_{BR}, X_{BR}, X_1, X_2', X_m, R_2'$)

No Load (NL) Test
↦ 3φ power
- Apply rated $V_{L-L,NL}$, Measure P_{NL}, I_{NL}



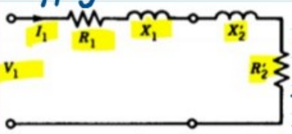
- Rotational losses: $P_{rotational} = P_{NL} - \overbrace{3R_1 I_{NL}^2}^{\text{stator winding resistive loss}}$
- Phase Voltage: $V_1 = \frac{V_{L-L,NL}}{\sqrt{3}}$
- NL Resistance: $R_{NL} = \frac{P_{NL}}{3I_{NL}^2}$, NL Impedance: $Z_{NL} = \frac{V_1}{I_1}$

- Combined Reactance: $X_1 + X_m = X_{NL} = \frac{Q_{NL}}{3I_{NL}^2} = \sqrt{Z_{NL}^2 - R_{NL}^2}$

- Reactive Power: $Q_{NL} = \sqrt{S_{NL}^2 - P_{NL}^2} = \sqrt{(3I_{NL}V_1)^2 - P_{NL}^2}$

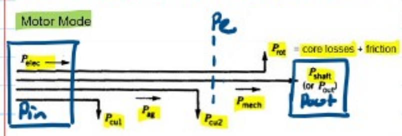
Blocked Rotor Test

- Apply Reduced $V_{L-L,BR}$, Measure P_{BR}, I_{BR} ($P_{BR} \rightarrow 3\phi$ power)

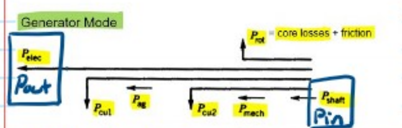


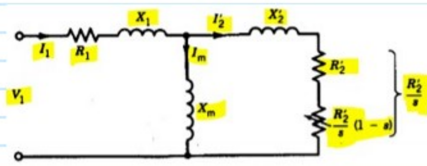
- Phase Voltage: $V_1 = \frac{V_{L-L,BR}}{\sqrt{3}}$
- Combined Resistance: $R_1 + R_2' = R_{BR} = \frac{P_{BR}}{3I_{BR}^2}$, NL Impedance: $Z_{NL} = \frac{V_1}{I_1}$
- Combined Reactance: $X_{BR} = \sqrt{Z_{BR}^2 - R_{BR}^2} = \sqrt{(V_1/I_{BR})^2 - R_{BR}^2}$
- Leakage Reactance: $X_1 = X_2' = \frac{X_{BR}}{2}$
- Magnetizing Reactance: $X_m = X_{NL} - X_1$
- Rotor Resistance: $R_2' = R_{BR} - R_1$

Power Conversion & Flow



- Input Power: $P_{in} = 3\underbrace{V_1 I_1}_{V_{ph} I_{ph}}\cos(\phi)$
- Stator Copper Loss: $P_{cu1} = 3I_1^2 R_1$, $P_{cu2} = 3(I_2')^2 R_2'$] P_{SCL}
- Air-Gap: $P_{ag} = 3I_2'^2 \frac{R_2}{s}$
- $P_{shaft} = T_m \cdot \omega_{rm}$





- Efficiency: $\eta = \frac{P_{out}}{P_{in}}$
- Conversion Power: $P_e = 3I_2'^2 R_2' \left(\frac{1-s}{s}\right) = (1-s)P_{ag}$
- Electromagnetic Torque: $T_e = \frac{P_e}{\omega_{rm}} = \frac{(1-s)P_{ag}}{\omega_{rm}} = \frac{P_{ag}}{\omega_{syn}} = \frac{3I_2'^2 R_2'}{s\omega_{syn}} = \frac{3I_2'^2 R_2'}{\omega_{rm}}$

Torque-Speed Characteristic

$$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')}$$

$$T_e = 3\frac{V_{1th}^2}{\omega_{syn}} \frac{R_2'/s}{(R_{1th} + R_2'/s)^2 + (X_{1th} + X_2')^2}$$

Slip @ Max Torque: $S_{T_{max}} = \pm \frac{R_2'}{\sqrt{R_{1th}^2 + (X_{1th} + X_2')^2}}$

Max Torque (Motoring): $T_{max} = 3\frac{V_{1th}^2}{2\omega_{syn}} \frac{1}{R_{1th} + \sqrt{R_{1th}^2 + (X_{1th} + X_2')^2}}$

Max Torque (Generating): $T_{max} = 3\frac{V_{1th}^2}{2\omega_{syn}} \frac{1}{-R_{1th} + \sqrt{R_{1th}^2 + (X_{1th} + X_2')^2}}$

- For starting Torque, set $s = 1$: $T_{start} = 3\frac{V_{1th}^2}{\omega_{syn}} \frac{R_2'}{(R_{1th} + R_2')^2 + (X_{1th} + X_2')^2}$

Starting Current

- $Z_1 = R_1 + jX_1 + \frac{jX_m(R_2'/s + jX_2')}{R_2'/s + j(X_m + X_2')}$
- $I_1 = \frac{V_1}{Z_1} = I_\phi + I_2'$
- $I_{1\,start} = I_{2\,start}' = \frac{V_{1th}}{(R_{1th} + R_2') + j(X_{1th} + X_2')}$