Biot-Savar and Ampere

April 28, 2020 11:24 AM

Biot-Savart

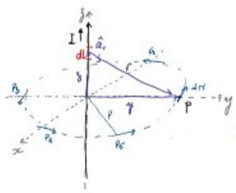
- Current relationship w/ field it generates

$$dH = \frac{I\,dL \times \hat{a}_R}{4\pi R^2}\ A/m \longrightarrow \oint \frac{I\,dL \times \hat{a}_r}{4\pi R^2}\ A/m$$

- $I\,dL$ = current filament, R = dist. filament to point

A) Infinite Filament (non-cross sectional SA)
B) Current loop (assume planar loop)
C) Realistic wire (Ampere's law)

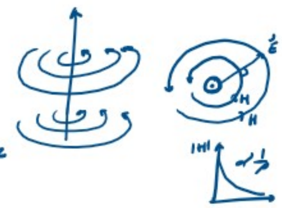
ex. Long straight wire



$$dH = \frac{I\,dL \times \hat{a}_r}{4\pi r^2} \longrightarrow I\,d\vec{L} = I\,dz\,\hat{a}_z$$

$$\hat{a}_R = \frac{-z\hat{a}_z + \rho\hat{a}_\rho}{\sqrt{z^2+\rho^2}} \qquad R = \sqrt{z^2+\rho^2}$$

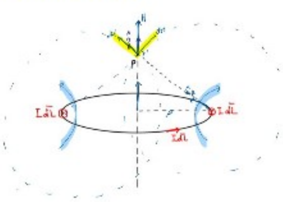
$$dH = \frac{I\,dz\,\hat{a}_z \times (-z\hat{a}_z + \rho\hat{a}_\rho)}{4\pi(z^2+\rho^2)^{3/2}} = \frac{I\rho\,dz\,d\varphi}{4\pi(z^2+\rho^2)^{3/2}}$$



Cross products:

$\hat{a}_\rho \times \hat{a}_\varphi = \hat{a}_z$
$\hat{a}_\varphi \times \hat{a}_z = \hat{a}_\rho$
$\hat{a}_z \times \hat{a}_\rho = \hat{a}_\varphi$

Current loop



$$dH = \frac{I\,dL \times \hat{a}_R}{4\pi R^2} \longrightarrow I\,d\vec{L} = I\rho\,d\varphi\,\hat{a}_\varphi$$

$$\hat{a}_R = \frac{-\rho\hat{a}_\rho + z\hat{a}_z}{\sqrt{z^2+\rho^2}} \longrightarrow R = \sqrt{z^2+\rho^2}$$

$$H = \int \frac{I\rho\,d\varphi\,\hat{a}_\varphi \times (z\hat{a}_z - \rho\hat{a}_\rho)}{4\pi(z^2+\rho^2)^{3/2}} \longrightarrow \begin{array}{l}\hat{a}_\varphi \times \hat{a}_z = \hat{a}_\rho \\ \hat{a}_\varphi \times (-\hat{a}_\rho) = \hat{a}_z\end{array}$$

$$H = \frac{I}{2}\left[\frac{\rho^2}{(z^2+\rho^2)^{3/2}}\right]\hat{a}_z\ A/m$$

Ampere's Circuital Law

- line integral of H about closed path = current enclosed

$$\oint H \cdot dL = I$$

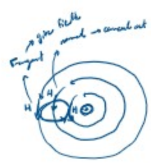
Infinite Current Filament

Ampere Law: $\oint H \cdot dL = I_{enclosed}$

$$\int_0^{2\pi} H\hat{a}_\varphi \cdot (\rho\,d\varphi)\,\hat{a}_\varphi = I$$

$$H_\varphi\,\rho(2\pi) = I$$

$$\longrightarrow \vec{H} = \frac{I}{2\pi\rho}\hat{a}_\varphi\ A/m$$



Current in Coaxial

$\rho < a$: $\oint H \cdot dL = I_{enc.}$ $\quad H_\varphi(2\pi\rho) = \left(\frac{\pi\rho^2}{\pi a^2}\right) I$ — $\pi\rho^2$: SA inner, πa^2: SA total — ratio of 2 circles

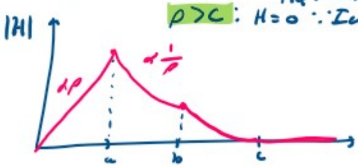
$$H = \frac{\rho I}{2\pi a^2}\hat{a}_\varphi\ A/m$$

$a < \rho < b$: $H = \frac{I}{2\pi\rho}\hat{a}_\varphi\ A/m$

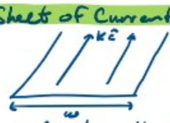
$b < \rho < c$: $\oint \vec{H} \cdot d\vec{L} = I_{enc}$

$$H_\varphi(2\pi\rho) = I - \left(\frac{\pi\rho^2 - \pi b^2}{\pi c^2 - \pi b^2}\right) I$$

$\rho > c$: $H = 0 \because I_{enc.} = 0$ ($I \odot$ $I \otimes \rightarrow$ cancel)

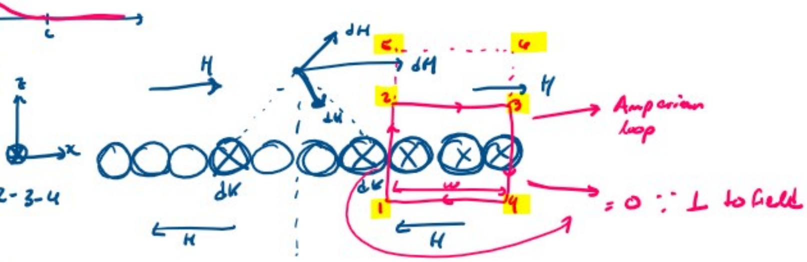


Sheet of Current



- evaluate path 1-2-3-4

$$\oint \vec{H} \cdot d\vec{L} = I_{enc}$$



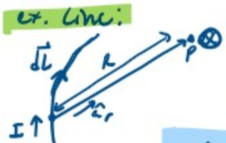
ex. Line:

- evaluate path 1-2-3-4

$\oint \vec{H}\cdot d\vec{L} = I_{enc}$

$= 0 \because \perp$ to field

ex. Line:



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$dH = \dfrac{I\,d\vec{L} \times \hat{a}_R}{4\pi R^2}$ ($I\,d\vec{L} \rightarrow$ differential current; $R \rightarrow$ dist. b/w diff. element to P)

★ $d\vec{L} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z \rightarrow$ Cartesian

$d\vec{L} = d\rho\,\hat{a}_\rho + \rho\,d\varphi\,\hat{a}_\varphi + dz\,\hat{a}_z \rightarrow$ cylindrical

$d\vec{L} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\varphi\,\hat{a}_\varphi \rightarrow$ spherical

$\therefore$ Complete current in circuit:

$$\vec{H} = \oint \frac{I\,d\vec{L} \times \hat{a}_R}{4\pi R^2}$$

ex. Area:

$\vec{K}$ A/m width

$I = |\vec{K}|\,dw$

$I\,d\vec{L} = |\vec{K}|\,dw\,d\vec{L}$

$= \vec{K}\,dw\,dL$

$= \vec{K}\,dS$

$\therefore I\,d\vec{L} = \vec{K}\,dS$

$\rightarrow$ Biot-Savart:

$$dH = \frac{\vec{K}\,dS \times \hat{a}_R}{4\pi R^2} \rightarrow \iint_S \frac{\vec{K} \times \hat{a}_R\,dS}{4\pi R^2}$$ (integral over whole surface)

ex. Volume: $\vec{J}$ A/m² width

$dH = \dfrac{I\,d\vec{L} \times \hat{a}_R}{4\pi R^2}$

$I = |\vec{J}|\,dN$ $dN \rightarrow$ diff. surface element cuts through volume

$I\,d\vec{L} = |\vec{J}|\,dN\,d\vec{L}$ $d\vec{L} \rightarrow$ give direction attribute of $d\vec{L}$ to $\vec{J}$

$= \vec{J}\,dV$

$\therefore I\,d\vec{L} = \vec{J}\,dV$

Biot-Savart:

$$dH = \frac{\vec{J}\,dV \times \hat{a}_R}{4\pi R^2}$$

$$H = \iiint_V \frac{\vec{J} \times \hat{a}_R\,dV}{4\pi R^2}$$

★ Webwork 7

Assignment 7: Problem 2

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(1 point)
A current filament carrying a current 1.9 Amps is formed into a circular loop of radius 0.15 meters centred around the z-axis. Assume that the loop sits in the z = 0 plane, and that the current circulates in the positive-ϕ direction as viewed from a point on the positive z-axis. Find the magnitude of the Magnetic Field Intensity at the point P = (0, 0, 0.1). Assume units of meters for z. Note: this problem is best solved using the Biot-Savart Law.

ANSWER:

H = ______ $\mathbf{a}_z$ A/m

Biot-Savart

$\vec{H} = \int \dfrac{I\,d\vec{L} \times \hat{a}_R}{4\pi R^2} \longrightarrow I\,d\vec{L} = I\rho\,d\varphi\,\hat{a}_\varphi$

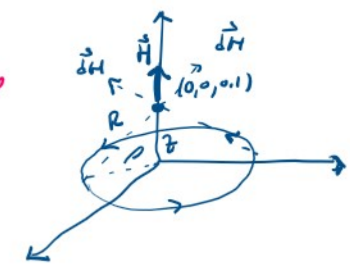
$\vec{R} = \langle \rho, z\rangle \quad |\vec{R}| = \sqrt{z^2+\rho^2}$

$\hat{a}_r = \dfrac{\langle \rho, z\rangle}{\sqrt{\rho^2+z^2}}$

$\vec{H} = \displaystyle\int_0^{2\pi} \frac{I\rho\,d\varphi\,\hat{a}_\varphi \times (z\hat{a}_z - \rho\hat{a}_\rho)}{4\pi\,(z^2+\rho^2)^{3/2}}$

$= \dfrac{I}{2}\left[\dfrac{\rho^2}{(z^2+\rho^2)^{3/2}}\right] = \dfrac{1.9}{2}\left[\dfrac{0.15^2}{(0.1^2+0.15^2)^{3/2}}\right]$

$= 3.64822$ A/m



Assignment 7: Problem 3

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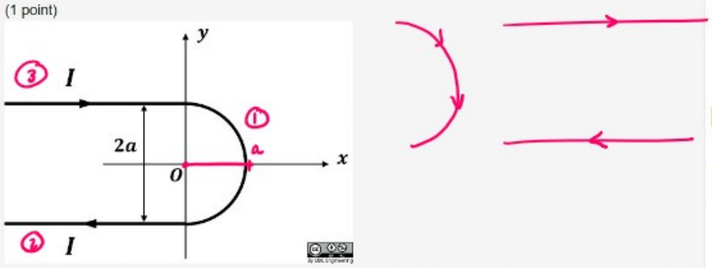
(1 point)



As shown in the figure, an infinite filament carrying current $I = 1.4\ A$ lies in the $z = 0$ plane and forms a U-shape that has a semi-circular bend centered at the origin. The semi-circular bend has a radius $a = 1.4\ m$. Find H at the origin.

$H =$ ______ $a_x +$ ______ $a_y +$ ______ $a_z\ A/m$

① Semicircle

$B = \mu_0 H \quad H = \dfrac{I}{2\pi\rho}\,\hat{a}_\varphi$

$dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,dL\sin\theta}{R^2} \longrightarrow \theta = 90°$

$\vec{B} = \dfrac{\mu_0 I}{4\pi R^2}\displaystyle\int_0^{\pi R} dL \longrightarrow 0 \le L \le \dfrac{2\pi R}{2} \rightarrow$ half circumference

$= \dfrac{\mu_0 I}{4\pi R^2}\,\pi R = \dfrac{\mu_0 (1.4)}{4\,(1.4)} = 3.14159\times10^{-7}\,\hat{a}_z$ wb/m²

$\vec{H} = 0.25\,\hat{a}_z$ A/m $\longrightarrow -0.25\,\hat{a}_z$ A/m

② & ③ Straight line segment

$dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,dL\sin\theta}{R^2}$

$\sin(90°) = 0$

$I\,dL = (1.4)\rho\,d\varphi \quad R = 1.4$

$\vec{B} = \dfrac{\mu_0 (1.4)}{4\pi (1.4)^2}\displaystyle\int_0^{1.5} dL = 9.4999\times10^{-8}$ wb/m²

$\vec{H} = 7.5577\times10^{-2} \times 2 \because 2$ segments

$\vec{H}_{TOT} = \vec{H}_{\text{semi circle}} + 2\,\vec{H}_{\text{segments}}$

As shown in the figure, an infinite filament carrying current $I = 1.4\ A$ lies in the $z = 0$ plane and forms a U-shape that has a semi-circular bend centered at the origin. The semi-circular bend has a radius $a = 1.4\ m$. Find H at the origin.

$H =$ ______ $a_x +$ ______ $a_y +$ ______ $a_z\ A/m$

$H_{TOT} = H_{\underset{circle}{semi}} + 2\,H\ \text{segments}$

Assignment 7: Problem 6

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(1 point)
A long cylindrical conductor of radius 0.0013 meters is centred on the z-axis. The current density in the conductor is defined as $\mathbf{J}(\rho) = \frac{6500}{\rho}\mathbf{a}_z$. Find $\mathbf{H}_1$ at the point 0.0005 and $\mathbf{H}_2$ at the point 0.0024. All coordinate units are in meters.

ANSWER:

$\mathbf{H}_1 =$ ______ $\mathbf{a}_\phi$ A/m

$\mathbf{H}_2 =$ ______ $\mathbf{a}_\phi$ A/m

a) $I = \iint_S \vec{J}\cdot d\vec{s}$ $\quad H = \frac{I}{2\pi\rho}$

$I = \int_0^{2\pi}\int_0^{0.0005} \frac{6500}{\rho}\,\rho\, d\rho\, d\varphi$

$= 20.42035 A$

$H = \frac{20.42035}{2\pi(0.0005)} = 6499.9999\ A/m$