Fourier Series

– Undamped frequency: $\omega_0 = \frac{2\pi}{T_0} \longrightarrow T_0 =$ fundamental period

Fourier Series of periodic signal $x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}$$

– Fourier coefficient: $X_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)e^{-jk\omega_0 t}dt$

– $X_0 =$ DC value avg. of $x(t)$

Parseval's Power Relation

– Power P_x of periodic signal $x(t)$ of fundamental period T_0:

$$P_x = \underbrace{\frac{1}{T_0}\int_{t_0}^{t_0+T_0}|x(t)|^2dt}_{\text{time domain}} = \underbrace{\sum_{k=-\infty}^{\infty}|X_k|^2}_{\text{frequency domain}}$$

Trigonometric Representations

– The trigonometric Fourier series uses sinusoids rather than complex exponentials as basis functions

$$x(t) = X_0 + 2\sum_{k=1}^{\infty}|X_k|\cos(k\omega_0 t + \theta_k)$$
$$= c_0 + 2\sum_{k=1}^{\infty}[c_k\cos(k\omega_0 t) + d_k\sin(k\omega_0 t)]$$

– dc-component: $X_0 = c_0$, k^{th} harmonic $= \{2|X_k|\cos(k\omega_0 t + \theta_k)\}$

$c_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\cos(k\omega_0 t)dt \longrightarrow Re(k)$, even component of $x(t)$

$d_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\sin(k\omega_0 t)dt \longrightarrow -Im(k)$, odd component of $x(t)$

$X_k = |X_k|e^{j\theta_k} \longrightarrow |X_k| = \sqrt{c_k^2 + d_k^2}$

$\theta_k = -\tan^{-1}\left[\frac{d_k}{c_k}\right] = \angle X_k$

LTI System Frequency Response

– If input $x(t)$ w/ impulse response $h(t)$, steady-state response is:

$$y(t) = X_0|H(j0)| + 2\sum_{k=1}^{\infty}|X_k||H(jk\omega_0)|\cos(k\omega_0 t + \angle X_k + \angle H(jk\omega_0))$$

– frequency response of the system at $k\omega_0$

$$H(jk\omega_0) = |H(jk\omega_0)|e^{\angle H(jk\omega_0)} = \int_0^{\infty} h(\tau)e^{-jk\omega_0\tau}d\tau = H(s)\Big|_{s=jk\omega_0}$$

ex. $x_1(t) = e^{j\omega t} \longrightarrow \boxed{h(t) \text{ System } H(s)} \longrightarrow y_1(t) = H(j\omega)e^{j\omega t} = |H(j\omega)|e^{j(\omega t + \angle H(j\omega))}$

$x_2(t) = e^{-j\omega t}$, $y_2(t) = H(-j\omega)e^{-j\omega t} = H(j\omega)e^{-j\omega t}$

Even/Odd Decomposition

– If Fourier coefficients of aperiodic signal $x(t)$ are $\{X_k\}$, then the Fourier coefficients of $x(-t)$ are $\{X_{-k}\}$

– Even $x(t)$: Fourier coeffs. X_k are real.
Trig. Fourier series:

$$x(t) = X_0 + 2\sum_{k=1}^{\infty}X_k\cos(k\omega_0 t)$$

– Odd $x(t)$: Fourier coeffs. X_k are imaginary
Trig. Fourier Series:

$$x(t) = 2\sum_{k=1}^{\infty}jX_k\sin(k\omega_0 t)$$

– Fourier coeffs: $X_k = X_{ek} + X_{ok}$

$\hookrightarrow X_{ek} = 0.5[X_k + X_{-k}]$
$X_{ok} = 0.5[X_k - X_{-k}]$

Operations of Periodic Signals

– Addition $z(t) = \alpha x(t) + \beta y(t)$

– Same ω_0
$\longrightarrow$ Fourier Coeff. : $Z_k = \alpha x(t) + \beta y(t)$

– diff. ω_0
– If $x(t)$ has period T_1 & $y(t)$ has period T_2 s.t $\frac{T_2}{T_1} = \frac{N}{M}$, then $z(t)$ has period $T_0 = MT_2 = NT_1$

$\longrightarrow$ Fourier Coeff. : $Z_k = \alpha X_{k/N} + \beta Y_{k/M}$

ex. $x(t) = \cos 2\pi t \longrightarrow \omega_1 = 2\pi, T_1 = 1$
$y(t) = \sin 3\pi t \longrightarrow \omega_2 = 3\pi, T_2 = 2/3$
$\frac{T_2}{T_1} = \frac{2}{3} = \frac{N}{M}$ $\therefore T_0 = 2$, $\omega_0 = \pi$

$X_k = \begin{cases} 1/2, & k = \pm 1 \\ 0, & \text{otherwise}\end{cases}$, $Y_k = \begin{cases} \mp 1/2 & k = \pm 1 \\ 0 & \text{, otherwise}\end{cases}$

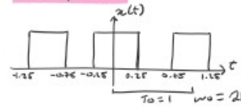
$z(t) = 4\cos 2\pi t + 5\sin 3\pi t$, $z_0 = 0$, $z_{\pm 1} = 0$,
$z_{\pm 2} = 4(1/2) = 2$, $z_{\pm 3} = 5(\mp\frac{1}{2})j = \mp 5/2 j$

– Product $z(t) = x(t)y(t)$

– Fourier coefficients are the convolution sum of fourier coeffs. of $x(t)$ & $y(t)$:

$$Z_k = \sum_m X_m Y_{k-m}$$

Examples



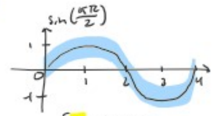
– Complex Exponential Fourier Series

$X_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)e^{jk\omega_0 t}dt$

$X_0 = \frac{1}{1}\int_{-0.25}^{0.25} 2dt = 1 \longrightarrow k = 0$

$X_k = \int_{-0.25}^{0.25} 2e^{-jk\omega_0 t} = \frac{2e^{-jk\omega_0 t}}{-jk\omega_0}\Big|_{-0.25}^{0.25}$

$= \sin\left(\frac{k}{2}\right) \longrightarrow k \neq 0$



$\begin{cases} 0 & \text{if even} \\ (-1)\left(\frac{k-1}{2}\right) & \text{if odd}\end{cases}$

– Trigonometric Fourier Series

– even function

$c_k = \int_{-0.25}^{0.25} 2\cos(k\omega_0 t) = \frac{2\sin(k\omega_0 t)}{k\omega_0}\Big|_{-0.25}^{0.25} = \frac{\sin\left(\frac{k\pi}{2}\right)}{\left(\frac{k\pi}{2}\right)} = X_k$

– odd function

$d_k = \int_{-0.25}^{0.25} 2\sin(k\omega_0 t)dt = \frac{-2\cos(k\omega_0 t)}{k\omega_0}\Big|_{-0.25}^{0.25} = 0$ $\therefore$ function is even

Fourier Series Coefficient from Laplace Transform

– Fourier coefficients of Laplace Transform:

$x_1(t) = x(t)[u(t-t_0) - u(t-t_0-T_0)] \longrightarrow$ signal

$X_k = \frac{1}{T_0}\mathcal{L}[x_1(t)]_{s=jk\Omega} \longrightarrow \omega_0 = \frac{2\pi}{T_0}$

ex. For pulse train: $x_1(t) = 2[u(t) - u(t-0.25) + u(t-0.75) - u(t-1)]$

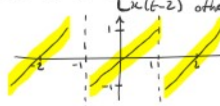
$X_1(s) = 2/s[1 - e^{-s/4} - e^{-3s/4} - e^{-s}]$

$X_k = X_1(jk2\pi) = \frac{2}{jk2\pi}[1 - e^{\frac{-jk\pi}{2}} + e^{-jk\left(\frac{3\pi}{2}\right)} - 1]$

$= \frac{2\sin(k\pi/2)}{k\pi}$

Fourier Series of Triangle Wave (MT Q!)

ex. $x(t) = \begin{cases} t & \text{for } -1 < t \leq 1 \\ x(t-2) & \text{otherwise}\end{cases}$ $\omega_0 = \frac{2\pi}{T_0} = \pi$



Method A

$X_k = \frac{1}{T_0}\int_{-1}^{1} te^{-jk\omega_0 t}dt \longrightarrow X_0 = 0$ (by inspection – avg. of graph)

$= \frac{1}{2}\int_{-1}^{1} te^{-jk\pi t}dt$ IBP: let $u = t$, $du = dt$, $dv = e^{-jk\pi t}$, $v = \frac{e^{-jk\pi t}}{-jk\pi}$

$= \frac{1}{2}\left[\frac{te^{-jk\pi t}}{-jk\pi}\Big|_{-1}^{1} - \underbrace{\int_{-1}^{1}\frac{e^{-jk\pi t}}{-jk\pi}dt}\right]$

$= \frac{j}{k\pi}\left(e^{-jk\pi} - (-1)e^{jk\pi}\right) \quad = 0 \quad k \in \mathbb{Z}$

$\therefore$ For $k \neq 0$, $X_k = \frac{j(-1)^k}{k\pi}$

Method B

$x_1(t) = t[u(t+1) - u(t-1)]$

$\mathcal{L}\{x_1(t)\} = \frac{d}{ds}\left[\frac{e^s}{s}\right] + \frac{d}{ds}\left[\frac{e^{-s}}{s}\right] = \frac{1}{s^2}[e^s(1-s) - e^{-s}(s+1)]$

$X_k = \frac{\mathcal{L}\{x_1(t)\}}{T_0}\Big|_{s=jk\omega_0} = \frac{1}{-2(k\pi)^2}[e^{jk\pi}(1-jk\pi) - e^{-jk\pi}(jk\pi+1)]$

$= \frac{(-1)^k}{-2(k\pi)^2}(-2jk\pi) = \frac{(-1)^k j}{k\pi}$

Example (MT Q!)

– For $x(t) = 4\cos\left(\frac{6\pi}{7}t\right) + \cos\left(\frac{3\pi}{5}t - \frac{\pi}{2}\right)$, Find ω_0 & complex exponential fourier coeffs. $\{X_k\}$

$\omega_0 = \text{GCF}(\omega_1, \omega_2)$: $\omega_1 = 6\pi/7$, $T_1 = 7/3$; $\omega_2 = 3\pi/5$, $T_2 = 10/3$] GFC $\longrightarrow T_0 = \frac{70}{3}$, $\omega_0 = 3\pi/35$

$x(t) = 4\cos\left(\frac{6\pi t}{7}\right) + \sin\left(\frac{3\pi t}{5}\right)$

exp. form: $\underbrace{4/2\left[e^{\frac{j6\pi t}{7}} + e^{\frac{-j6\pi t}{7}}\right]}_{k \;\; \omega_1 = 10\omega_0} + \underbrace{\frac{1}{2j}\left[e^{\frac{j3\pi t}{5}} - e^{\frac{-j3\pi t}{5}}\right]}_{\omega_2 = 7\omega_0}$

$X_k = \begin{cases} \pm\frac{1}{2j} = \mp\frac{j}{2} & \text{for } k = \pm 7 \\ 2 & \text{for } k = \pm 10 \\ 0 & \text{otherwise}\end{cases}$

Webwork 4

	Signal
1	$12 + 10\cos(10\pi t) + 4\cos(30t + \frac{\pi}{5})$
2	$[14 + \cos(2\pi t)]\sin(10\pi t + \frac{\pi}{8})$
3	$2 + \sin(3t + \frac{\pi}{6}) + 8\cos(5t) + 14\cos(3t) + 13\sin(6t)$

① $\omega_1 = 10\pi$ $\omega_2 = 30$, $\longrightarrow$ Aperiodic $\therefore$ No rational common period

② $\omega_1 = 2\pi$, $\omega_2 = 10\pi$, $T_1 = 1$, $T_2 = 1/5$

Fundamental Period: $T_0 = \text{LCM}(1, 1/5) = 1$

Fundamental Frequency: $\omega_0 = 2\pi$

$14\sin(10\pi t + \pi/8) + \cos(2\pi t)\sin(10\pi t + \pi/8) \longrightarrow \sin(\omega t) = \cos(\omega t - \pi/2)$

$= 14\sin(10\pi t + \pi/8) + \cos(2\pi t)\cos(10\pi t - 3\pi/8) \longrightarrow \cos(a)\cos(b) = 1/2(\cos(a+b) + \cos(a-b))$

$= 14\sin(10\pi t + \pi/8) + 1/2[\cos(12\pi t - 3\pi/8) + \cos(-8\pi t + 3\pi/8)] \longrightarrow x(t) = A\cos(\omega t + \theta) = A\frac{e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}}{2}$

OR: $x(t) = A\sin(\omega t + \theta) = A\frac{e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}}{2j}$

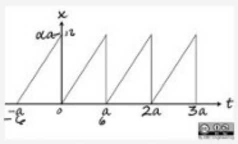
$= \underbrace{7(e^{j10\pi t}e^{j\pi/8} + e^{-j10\pi t}e^{-j\pi/8})}_{X_5 \therefore jk\omega_0 t \to \omega_0 = 2\pi, k=5} + \underbrace{1/4(e^{j12\pi t}e^{-j\frac{3\pi}{8}} + e^{j8\pi t}e^{-j3\pi/8})}_{X_6 \therefore jk\omega_0 t \to \omega_0 = 2\pi, k=6} + \underbrace{1/4(e^{-j8\pi t}e^{j3\pi/8} + e^{j8\pi t}e^{-j3\pi/8})}_{X_4 \therefore jk\omega_0 t \to \omega_0 = 2\pi, k=-4}$

$= 7\left(e^{j10\pi t}e^{j\pi/8} + e^{-j10\pi t}e^{-j\pi/8}\right) + \frac{1}{4}\left(e^{j12\pi t}e^{-j\frac{3\pi}{8}} + e^{-j12\pi t}e^{j3\pi/8}\right) + \frac{1}{4}\left(e^{-j8\pi t}e^{j3\pi/8} + e^{j8\pi t}e^{-j3\pi/8}\right)$

$X_5 \therefore jk\omega_0 t \to \omega_0 = 2\pi, k=5$ $\quad X_6 \therefore jk\omega_0 t \to \omega_0 = 2\pi, k=6$ $\quad X_4 \therefore jk\omega_0 t \to \omega_0 = 2\pi, k=-4$

$\to$ By inspection $t=0$: $X_5 = \frac{14}{2j}e^{j\pi/8}$, $X_6 = \frac{1}{4}e^{-j\frac{3\pi}{8}}$, $X_4 = \frac{1}{4}e^{-j3\pi/8}$, $X_0 = X_1 = X_2 = X_3 = 0$

A periodic signal, $x(t)$ is given in the figure below, where $a = 6$, and $\alpha = 2$.



$T_0 = 6$
$\omega_0 = \frac{\pi}{3}$

a) Find an equation for $x_c(t)$, the signal that describes one cycle of $x(t)$, in terms of the unit step function $u(t)$.
$x_c(t) = 2t[u(t) - u(t-6)]$ (& 2α)

$-e^{-6s}\mathcal{L}\{2(t-6)\}$

b) Find the Laplace transform, $X_c(s)$ of the signal in part **a**.
$X_c(s) = \mathcal{L}\{u(t)g(t)\} \to e^{-cs}\mathcal{L}\{g(t+c)\}$ $\therefore \frac{2}{s^2} - e^{-6s}\frac{2}{s^2} - e^{-6s}\frac{12}{s}$

c) Calculate the Fourier Series coefficients of the signal $x(t)$, X_k for $k \neq 0$ using the Laplace transform from part **b**.
$X_k =$ Let $s = jk\pi/3 \longrightarrow X_k = \frac{1/6 \cdot 36j}{k\pi}$

d) Is it possible to find the Fourier Series coefficient, X_0 using the Laplace transform method? **No** $\therefore$ divide by 0

e) Compute the Fourier Series coefficient, X_0, using the integral definition.
$X_0 = X_0 = \frac{1}{T_0}\int_0^T x_c(t)\,dt \to \frac{1}{6}\int_0^6 2t[u(t)-u(t-6)]\,dt = 6$

The transfer function of an LTI system is given by:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+10}{s^2+5s+8}$$

Given the input $x(t) = 7.5 + \cos(t + \frac{\pi}{5})$, use the eigenfunction property of the LTI system to find the steady-state output.

$y_{ss}(t) =$

$H(s) = \frac{s+10}{s^2+5s+8} \longrightarrow s = j\omega \quad H(\omega) = \frac{j\omega+10}{-\omega^2+5j\omega+8}$

$x(t) = 7.5 + \cos(t + \pi/5) \longrightarrow \omega = 1$

$H(1) = \frac{-75+43j}{-74} = 1.1682743\angle -0.520581$

@ $\omega = 0$, $H_0 = \frac{10}{8}$

$\therefore y_{ss}(t) = 7.5 \cdot \frac{10}{8} + 1.1682743\cos(t + \frac{\pi}{5} - 0.520581)$

① **Fourier Transform** (For Bounded signals w/ finite time support)

- Aperiodic signals have a fourier transform

Fourier Transform: $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ — FT Pair

Inverse Fourier Transform: $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$

ex. $x_1(t) = u(t+1) - u(t-1)$
$X(\omega) = \int_{-1}^{1} x_1(t)e^{-j\omega t} = \int_{-1}^{1} e^{-j\omega t} = \frac{j}{\omega}(e^{-j\omega} - e^{j\omega})$

Find the Fourier transform $X_1(\omega)$, $X_2(\omega)$, and $X_3(\omega)$ of the signals $x_1(t)$, $x_2(t)$, and $x_3(t)$, using the Fourier transform pair
$x(t) = u(t+1) - u(t-1) \longleftrightarrow X(\omega) = 2sinc(\omega)$. Then select the Fourier transform property you used for each signal, from the corresponding drop-down menu.

In your answers, enter "w" for omega.

a) $x_1(t) = -2u(t+2) + 8u(t) - 6u(t-2)$
$X_1(\omega) =$? Time shift

a) Time shift: $x(t-a) \xrightarrow{\mathcal{F}} e^{-ia\omega}$

$x_1 = -2[u(t+2) + u(t)] + 6[u(t) - u(t-2)]$
$X_1 = -2[e^{j\omega}2sinc(\omega)] + 6[e^{-j\omega}2sinc(\omega)]$
$\hookrightarrow x(t) = u(t+1) - u(t-1) \longleftrightarrow X(\omega) = 2sinc(\omega)$

② **FT from LT** (For infinite time signals, contain $j\omega$-axis)

- If $\hat{X}(s) = \mathcal{L}\{x(t)\}$ contains $j\omega$-axis, FT of $x(t)$:

$X(\omega) = \mathcal{F}\{x(t)\} = \hat{X}(s)|_{s=j\omega}$

ex. $x_2(t) = e^{-2t}u(t)$: $\hat{X}_2(s) = \frac{1}{s+2}$

ROC: $Re(s) > 2$ $\therefore$ includes $j\omega$-axis $\longrightarrow X_2(\omega) = \mathcal{F}\{x_2(t)\} = \frac{1}{j\omega+2}$

Parseval's Energy Relation for Energy Signals

$E_x = \int_{-\infty}^{\infty}|x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega$

Frequency Response

- System has **freq. Response**: $H(j\omega) = \mathcal{F}\{h(t)\}$ where $h(t)$ is the **impulse response**
- **Output** of LTI system is $y(t) = (x * h)(t)$ w/ FT $Y(\omega) = X(\omega)H(j\omega)$
- If input $x(t)$ periodic; output has FT: $Y(\omega) = \sum_{k=-\infty}^{\infty} 2\pi X_k H(jk\omega_0)\delta(\omega - k\omega_0)$

③ **FT from FS** (For periodic signals)

Fourier Pair: $x(t) = \sum_k X_k e^{jk\omega_0 t} \longrightarrow X_k(\omega) = \sum 2\pi X_k \delta(\omega - k\omega_0)$

ex. $x_1(t) = A \longrightarrow X_1(\omega) = 2\pi A\delta(\omega)$ (table 5.2)

ex. $x_2(t) = \underbrace{4\cos\left(\frac{6\pi}{7}t\right)}_{T_1 = 7/3} + \underbrace{\cos\left(\frac{3\pi}{5}t - \frac{\pi}{2}\right)}_{T_2 = 10/3} = \underbrace{2\left(e^{j\frac{6\pi}{7}t} + e^{-j\frac{6\pi}{7}t}\right)}_{k=10,\ X_0 = 2} + \underbrace{\frac{1}{2}\left(e^{j\frac{3\pi}{5}t}e^{-j\frac{\pi}{2}} + e^{-j\frac{3\pi}{5}t}e^{j\frac{\pi}{2}}\right)}_{k=7,\ X_7 = \frac{1}{2}e^{-j\pi/2} = j/2}$

$T_0 = LCM(7/3, 10/3) = 70/3$, $\omega_0 = 6\pi/30$

$\therefore X_2(\omega) = 2\pi[X_7(\delta(\omega + 7\omega_0)) + \delta(\omega - 7\omega_0)) + X_{10}(\delta(\omega + 10\omega_0) + \delta(\omega - 10\omega_0)]$
$= 2\pi[\frac{3}{2}(\delta(\omega + \frac{3\pi}{5}) - \delta(\omega - \frac{3\pi}{5})) + 2(\delta(\omega + \frac{6\pi}{7}) + \delta(\omega - \frac{6\pi}{7}))]$

④ **Duality** (For unclassified signals)

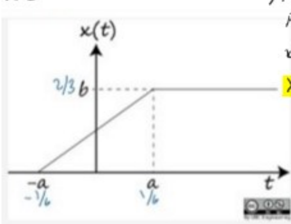
- When $f = \omega/2\pi$ in Hz: $\hat{X}(f) = \int_{-\infty}^{\infty} x(t)e^{-i2\pi ft}\,dt \longleftrightarrow x(t) = \int_{-\infty}^{\infty}\hat{X}(f)e^{i2\pi ft}\,dt$
- $\to$ use table 5.1

ex. Heaviside: $u(t) = \int_{-\infty}^{t}\delta(\tau)\,d\tau \longrightarrow \mathcal{F}\{u(t)\} = \frac{D(\omega)}{j\omega} + \pi D(0)\delta(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$

ex. Sinc: $x_1(t) = u(t+1) - u(t-1) \xrightarrow{\mathcal{F}} X_1(\omega) = 2sinc(\frac{\omega}{\pi})$

$X(\omega) = sinc(t) \xrightarrow{\mathcal{F}} 2\pi x(-\omega) = 2\pi\left[\frac{u(\frac{-\omega}{\pi}+1) - u(\frac{-\omega}{\pi}-1)}{2\pi}\right] = u(\omega+\pi) - u(\omega-\pi)$

Webwork 5



a) Find a closed form expression for the Fourier transform $X(\omega)$ of the signal $x(t)$.

Hint: Use the integration and differentiation properties, as well as the Fourier transf pulse.

$X(\omega) =$

a) FT of a pulse (table 5.2 (12)) $\longrightarrow$ derivative of ramp = pulse

$P(t) = A[u(t+\tau) - u(t-\tau)] \xrightarrow{\mathcal{F}} P(\omega) = 2A\tau\frac{\sin(\omega\tau)}{\omega\tau}$

$x(t) = 2[u(t+1/6) - u(t-1/6)] \xrightarrow{\mathcal{F}} X(\omega) = \frac{4\sin(\omega/6)}{6\cdot\omega/6} \longrightarrow$ pulse

$X(\omega) = \int_{-\infty}^{t}\frac{4\sin(\omega/6)}{\omega}\,d\omega = \frac{4\sin(\omega/6)}{j\omega^2} + \frac{4\pi D(\omega)}{6} \longrightarrow$ ramp

A system is described by the differential equation $\frac{d}{dt}y(t) + 2y(t) = 7\frac{d}{dt}x(t) + x(t)$, where $x(t)$ is the input and $y(t)$ is the output of the system.

In your answers below, enter $D(t)$ instead of $\delta(t)$, and "w" for ω.

a) Find the frequency response of the system, $H(\omega)$.
$H(\omega) =$

b) Find the impulse response of the system, $h(t)$.
$h(t) =$

c) Find the output signal $y(t)$ that corresponds to the input signal $x(t) = e^{-8t}u(t)$. Solve the problem in the time as well as the frequency domain.
$y(t) =$

a) $Ys + 2Y = 7Xs + X$
$H(s) = \frac{7s+1}{s+2}$
$H(\omega) = \frac{7j\omega+1}{j\omega+2}$

b) $h(t) = \mathcal{L}^{-1}\{H(s)\}$
$= 7\delta(t) - (13e^{-2t})u(t)$

c) $Y(s) = X(s)H(s)$
$y(t) = (-\frac{13}{6}e^{-2t} + \frac{55}{6}e^{-8t})u(t)$

Two filters with frequency responses $H_1(\omega) = j\omega$ and $H_2(\omega) = e^{-4j\omega}$ for $-\infty < \omega < \infty$, are cascaded together so that the output of the first filter is fed as the input to the second, as shown in the figure below.

$x(t) \longrightarrow \boxed{H_1(\omega)} \longrightarrow \boxed{H_2(\omega)} \longrightarrow y(t)$
$\quad j\omega \qquad e^{-4j\omega}$

Suppose that the input to this cascaded system is the signal $x(t) = \cos(\frac{\pi t}{6})[u(t+4) - u(t-4)]$.

a) Find the output, $y(t)$, of this cascaded system.
$y(t) =$

(Table 5.1): **Time diff:** $\frac{d^n x(t)}{dt^n} \to (j\omega)^n X(\omega)$
Time delay: $(t-\tau) \to e^{-\tau j\omega}$

$x(t) = \cos(\frac{\pi t}{6})[u(t+4) - u(t-4)]$
$\therefore y(t) = -\frac{\pi}{6}\sin(\frac{\pi(t-4)}{6})[u(t-4+4) - u(t-4-4)]$
$= -\frac{\pi}{6}\sin(\frac{\pi(t-4)}{6})[u(t) - u(t-8)]$

ADC Sampling

$x(t) \longrightarrow \boxed{\text{Sampler } T_S} \xrightarrow{x_s[n]} \boxed{\text{Quantizer}} \xrightarrow{x_q[n]} \boxed{\text{Encoder}} \xrightarrow{x_b[n]}$
discrete cont's

Sampler
- generates DT signal from CT
  $\hookrightarrow T_s$ = sampling interval, $f_s = 1/T_s$ = Sampling freq.
- **Aliasing**: causes diff. signals to become indistinguishable when sampled

Reconstruction (from DT signal)
- **Ideal Interpolator**: $h(t) = \frac{\sin(\pi t/T)}{\pi t/T} = sinc(t/T)$
  $\mathcal{F} \hookrightarrow H(\omega) = \begin{cases} T, & -\pi/T < \omega < \pi/T \\ 0 & \text{otherwise} \end{cases}$

Nyquist Theorem
- To reproduce signal, should be sample at 2x rate of highest freq
$\therefore$ Nyquist rate = 2x max signal freq.

ex. $x(t) = \underbrace{1}_{\omega_1 = 0} + \underbrace{\cos(2000\pi t)}_{\omega_2 = 2000\pi} + \underbrace{\sin(4000\pi t)}_{\omega_3 = 4000\pi}$
$\longrightarrow$ max ω: $\omega_{max} = 4000\pi$
$\therefore \omega_s > 2\omega_m = 8000\pi$
$\therefore$ Nyquist Rate $= 8000\pi$
— No change to N.R for addition

Nyquist-Shannon Sampling

- Information is preserved by sampled signal $x_s(t)$ w/ samples $x(nT_s)$ if **sampling freq.** $\omega_s > 2\omega_{max} \longrightarrow \omega_s = 2\pi/T_s$
  OR: $f_s = \frac{1}{T_s} > \frac{\omega_{max}}{\pi}$
- If **Nyquist Rate condition** satisfied, **$x(t)$ reconstructed** by passing $x_s(t)$ through ideal low pass filter $H(j\omega) = \begin{cases} T_s & -\omega_s/2 < \omega < \omega_s/2 \\ 0 & \text{otherwise} \end{cases}$
- **Nyquist frequency**:
  — for sampling rate f_s: $f_{Nyquist} = f_s/2$ ($X(f) = 0$ for $f \geq f_{Nyquist}$)
- **Nyquist (sampling) Rate**:
  — for max band limited freq. f_{max}: $f_{sampling} > 2f_{max}$

ex. $x(t) = A\cos(2\pi(96)t + \varphi) \longrightarrow f_{max} = 96$ Hz, $f_s = 20$ Hz

$x(t) \longrightarrow \boxed{\text{Sampler}} \xrightarrow{x_s(t)} \boxed{\text{Ideal Interp.}} \longrightarrow x_r(t)$

$x_r(t) = A\cos(2\pi(96)t + \varphi) \longrightarrow f_s > 2f_{max}$
$\hookrightarrow 76, 56, 36, 16, -4$ ✓

	Signal, $x[n]$	Energy	Power
1	$14(\frac{1}{4})^n u[n]$		

$\longrightarrow$ max ω: $\omega_{max} = 4000\pi$

$\because$ $\omega_s > 2\omega_m = 8000\pi$

$\therefore$ Nyquist Rate = 8000π

- No change to N.R for addition
- No change to N.R for dif.
- Add N.R for multiplication

Webwork 6

Suppose that $x(t)$ is a real signal that has a Nyquist rate $\omega_0 = 45\ Hz$. Find the Nyquist rate of each of the signals in the table below.

	Signal, $y(t)$	Nyquist Rate
1	$x(t) - x(t-6)$ is not zero for all t	45
2	$\frac{d^2x(t)}{dt^2}$	45
3	$9x^2(t)$	$45+45=90$
4	$x(t)cos(5\omega_0 t)$	$45+(2\times5\times45)$ N.R.

$x(t) \longrightarrow$ Sampler $\xrightarrow{x_s(t)}$ Ideal Interp. $\longrightarrow x_r(t)$

$x_r(t) = A\cos(2\pi(96)t+\varphi) \longrightarrow f_s > 2f_{max}$

$\hookrightarrow 76, 56, 36, 16, -4$ ✓

$\therefore\ = A\cos(2\pi(-4)t+\varphi)$

Consider the signal $x(t) = 2cos(8\pi t + \pi/8)$. Determine if the signal is band-limited or not. Then for each of sampling periods $T_s = 0.1, 0.125$ and 1 sec/sample, determine if the Nyquist condition is satisfied, if the sampled signal is aliased, give the expression for the sampled signal, $x_s[n]$, as the simplest discrete-time sinusoid to be used for ideal reconstruction and determine its period.

Sampling Period, T_s	Nyquist condition satisfied?	Signal Aliased?	Sampled Signal $x_s[n]$
0.1	?	?	
0.125	?	?	
1	?	?	

$T_s = 0.1$: $\omega_s = 10\ H_3$, $\omega_n = 1/2\omega_s = 5H_3$

$\omega_m = 8\pi\ 2\pi = 4\ H_3 \longrightarrow \omega_s > 2\omega_m$ ✓

$T=5$

$T_s = 0.125$: $\omega_s = 8H_3$, $\omega_n = 4H_3$

$\omega_m = 4H_3 \longrightarrow \omega_s > 2\omega_m$ ✗

$-8 = -4 \longrightarrow \omega_s > 2\omega_m$ ✓ $T=2$

	Signal, $x[n]$	Energy	Power
1	$14(\frac{1}{4})^n u[n]$		
2	$8e^{j6n}u[n]$		

$E_x = \sum_{k=-\infty}^{\infty} |x[n]|^2$, $P_x = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x[n]|^2$

① $E_x = \sum_{k=0}^{\infty} 14^2(1/4)^{2n} = \sum 14^2(\frac{1}{16})^2 \longrightarrow \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} = \frac{14^2}{1-1/16}$

$P_x = \lim_{N\to\infty}\frac{1}{2N+1}\left(\frac{14^2}{1-1/16}\right) = 0$

② $E_x = \infty$

$P_x = a^2/2 = 8^2/2$

A continuous-time signal, $x(t) = 4cos(27\pi t) + 7sin(42\pi t) + 3cos(62\pi t + \pi/7)$ is sampled at 42 Hertz. Determine the signal $\omega(t)$, reconstructed using an ideal interpolator with sampling rate $T = (1/42)$ s.

$\omega_s = 42H_3$, $\omega_n = 21H_3$

$\omega(t) =$

$\omega_1 = 27/2 \longrightarrow \omega_s > 2\omega_1$ ✓

$\omega_2 = 21 \longrightarrow \omega_s = 2\omega_2$?

check if $=0$: $7\sin(42\pi\cdot n) = 0$

$\omega_3 = 31 \longrightarrow \omega_s > 2\omega_3$ ✗

$-42 = -11$

$\therefore x_r(t) = 4\cos(27\pi t) + 3\cos(22\pi t - \pi/7)$

Region of convergence (ROC)

$ROC(x) = \{z = re^{j\Omega} \in \mathbb{C} \mid x[n]r^{-n}$ is absolutely summable$\}$

- poles cannot belong to the ROC

Finite Support ROC:

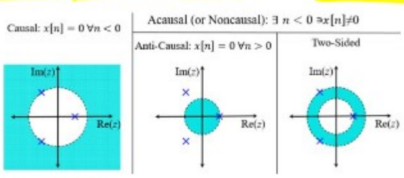
- For $[N_0, N_1]$, ROC is the z-plane excluding $z=0$ & $z=\pm\infty$ depending on N_0, N_1.

Infinite Support ROC:

- Causal signal: $|z| > r_{min} = \max\{|p_i|\} \longrightarrow p_i =$ poles of $X(z)$

ROC = outside circle r_{min}

- Anti-Causal: ROC = inside circle $|z| < r_{max} = \min\{|p_i|\}$
- Two-Sided: ROC $= R_1 < |z| < R_2 \longrightarrow$ torus inside min & max poles



Z-Domain

$x[n]$, $X(z)$ → System $h[n]$, $H(z)$ → $Y[n]$, $Y(z) = H(z)X(z)$

- Laplace Transform: $X_\delta(s) = \sum_n x(nT_s)e^{-nst}$

$\hookrightarrow$ let $z = e^{sT_s}$

- Z-Transform: $Z\{x_s[n]\} = \mathcal{L}\{x_\delta(t)\}|_{z=e^{sT_s}} = \sum_n x_s[n]\cdot z^{-n}$

Z-Plane

- $z = e^{sT} = e^{(\sigma+j\omega)T} = re^{j\Omega} \longrightarrow r = e^{\sigma}$, $e^{j\Omega} = e^{j\omega T}$

One-Sided Z-Trasform

- For signals made causal by multiplying them with $u[n]$

$X_1(z) = Z(x[n]u[n]) = \sum_{n=0}^{\infty} x[n]u[n]z^{-n}$

- Two-sided Z-Transform can be expressed in terms of one-sided transform

$X(z) = Z(x[n]u[n]) + Z(x[-n]u[n])|_{z^{-1}} - x[0]$

$\hookrightarrow z^{-1} \to z \quad \to \cdot^{-1}$

Two-Sided Z-Transform

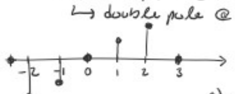
$Z\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$

ex. Z-transform of $x[n] = \sum_{k=-2}^{2} k\delta[n-k]$

$x[n] = -2\delta[n+2] - \delta[n+1] + \delta[n-1] + 2\delta[n-2]$

$X(z) = -2z^2 - z + z^{-1} + 2z^{-2} = \frac{-2z^4 - z^3 + z + 2}{z^2}$

$\hookrightarrow$ double pole @ $z=0$



$ROC(x[n]) = \{\mathbb{C} \setminus z=0\}$

Convolution Sum

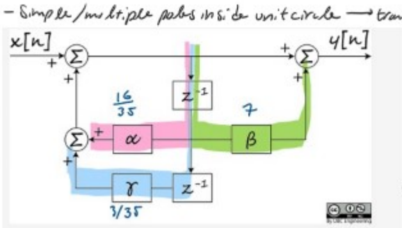
$y[n] = [x * h][n] = \sum_{k=0}^{n} x[k]h[n-k] = \sum_{k=0}^{n} h[k]x[n-k]$

$Y(z) = X(z)H(z)$

Steady State & Transient Responses

$y[n] + \sum_{k=1}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m] \longrightarrow$ difference eqn.

- Simple/multiple poles inside unit circle $\longrightarrow$ transient



a) Find the difference equation that describes this system

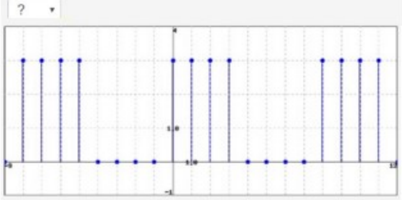
In your answers, enter $z(n)$ for a discrete-time function $z[n]$. WebWork is parse a function that uses square brackets.

b) Find the transfer function, $H(z)$, of this system

$H[z] =$

c) M/C Question: Is the system stable?

?



$X_k = \quad e^{j} \quad \frac{sin(\quad)}{sin(\quad)}$

$x[n] = 4(u[n] - u[n-4])$

$T_0 = 8$, $\omega_0 = \pi/4$

$X_k = \frac{1}{8}\sum_{n=0}^{3} 4e^{-jk\pi/4 n}$

$= \frac{1}{2}\left(\frac{1-e^{-j\pi k}}{\ldots}\right) = \frac{1}{2}(1-e^{-j\pi k}$

Webwork 8

A causal discrete-time LTI system is described by the difference equation, $y[n] - \frac{14}{48}y[n-1] + \frac{1}{48}y[n-2] = 9x[n]$, where $x[n]$ and $y[n]$ are the input and output of the system respectively.

a) Find the system transfer function $H(z)$, and indicate the region of convergence in interval notation (e.g. (-INF,a), (a,b) or (b, INF)).

$H(z) =$ $\quad RoC:$

b) Find the impulse response, $h[n]$, of the system.

$h[n] =$

c) Find the step response, $s[n]$, of the system.

$s[n] =$

a) $y[n] - 14/48\,y[n-1] + 1/48\,y[n-2] = 9x[n]$

Z-transform: $x[n-N] \longrightarrow z^{-N}X(z)$

$Y(z) - 14/48\,z^{-1}Y(z) + 1/48\,z^{-2}Y(z) = 9X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{9}{1 - 14/48\,z^{-1} + 1/48\,z^{-2}}$

ROC: $\{z = re^{j\Omega} : r > \max\{|p_i|\}\}$

Poles: $1/8$, $1/6$ $\therefore$ ROC $= [0.1666, \infty]$

b) Impulse Response $h[n]$: invztrans$(H(z), z, n)$

$h(n) = [36(1/6)^n - 27(1/8)^n]u(n)$

c) Step Response $s[n]$

$x[n] = u[n] \longrightarrow$ convolute w/ $h[n]$

In z-domain: $S(z) = H(z)\cdot\frac{1}{1-z^{-1}} \longrightarrow$ invztrans( )

$s[n] = 1/35(-252(1/6)^n + 135(1/8)^n + 432)u[n]$

a) $\omega[n] = x[n] + (\alpha\omega[n-1] + \gamma\omega[n-2])$

$y[n] = x[n] + \alpha y[n-1] + \gamma\omega[n-2] + \beta x[n-1]$

b) $Y(z) = X(z) + \beta z^{-1}X(z) + \alpha z^{-1}Y(z) + \gamma z^{-2}Y(z)$

$H(z) = \frac{1 + 7z^{-1}}{1 - 16/35\,z^{-1} - 3/35\,z^{-2}}$

c) $(1 - 16/35 - 3/35\,z^{-2} = 0) \times z^2 \longrightarrow z^2 - 16/35\,z - 3/35 = 0$

$z = 7/20 \pm \frac{2\sqrt{41}i}{20}$

$\therefore$ Inside unit circle $\therefore$ stable

Webwork 9

Consider a discrete-time system that has the impulse response $h[n] = (\frac{1}{8})^n u(n)$. Find the response of the system to each of the input signals given in the table below.

	Input signal, $x[n]$	Output signal, $y[n]$
1	$e^{j\pi n/8}$	
2	$cos(\frac{\pi n}{6} + \frac{\pi}{9})$	

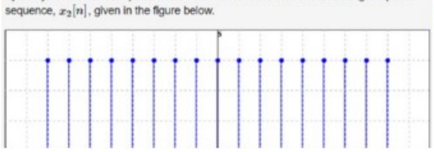
a) $h[n] \longrightarrow H(z) = \frac{1}{1 - 1/8\,e^{-1}}$, $H[\Omega] = \frac{1}{1 - 1/8\,e^{-j\Omega}}$

① $x[n] = e^{j\pi n/8} \longrightarrow \Omega = \pi/8$

$|H(\pi/6)| = \left|\frac{1}{1 - 1/8\,e^{j\pi/8}}\right| = \cdots\ \angle H(\pi/8) = \cdots$

$y[n] = \ldots\cos(\pi n/8 - \ldots) + \cdots j\sin(\pi n/8 - \ldots)$

b) Use your answer from part **a** to find the Fourier transform of the rectangular pulse sequence, $x_2[n]$, given in the figure below.



For the two discrete-time LTI systems described below, find the transfer function $H(z)$, if:

a) In system A, where an input-output signal pair is given by:

$$x[n] = \begin{cases} 4 & n = 0, 1 \\ 0 & otherwise \end{cases}$$

$$y[n] = \begin{cases} 5 & n = 0, 4 \\ 7 & n = 1, 3 \\ 0 & otherwise \end{cases}$$

$H(z) =$

$x[n] = 4(\delta[n] + \delta[n-1])$

$y[n] = 5(\delta[n] + \delta[n-1]) + 7(\delta[n-1] + \delta[n-3])$

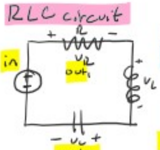
Z-transform: $X(z) = 4 + 4z^{-1}$, $Y(z) = 5 + 5z^{-4} + 7z^{-1} + 7z^{-3}$

$H(z) = \frac{Y(z)}{X(z)}$

State

- Status of system w/ state known at t_0 for CT

RLC circuit



$\dot{x}(t) = Ax(t) + Bu(t) \longrightarrow$ State diff eqn. (system M, input M)

$y(t) = Cx(t) + Du(t) \longrightarrow$ Output eqn. (Feedforward M)

output V_L

$V_L = L\frac{di}{dt}$

$i = C\frac{dv}{dt}$

Inputs: $x_1 = v_c$, $x_2 = i$

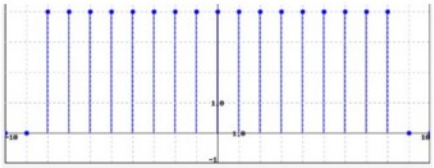
Outputs: $y_1 = V_c = x_1$, $y_2 = Ri = Rx_2$

V input: $u_1 =$

KVL: $V = iR + V_L + V_c$

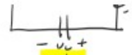
$x[n] = 4(u[n] - u[n-4])$
$T_0 = 8,\ \omega_0 = \pi/4$

$X_k = \frac{1}{8}\sum_{n=0}^{3} 4e^{-jk\frac{\pi}{4}n}$
$= \frac{1}{2}\left(\frac{1-e^{-jkn}}{1-e^{-jk\pi/4}}\right) = \frac{1}{2}\left(\frac{1-e^{-jkn}}{e^{-jk\pi/8}(e^{jk\pi/8}-e^{-jk\pi/8})}\right)$
$= \frac{1}{2}\left(\frac{1-e^{-jkn}}{e^{-jk\pi/8}[\cos(\frac{\pi}{8}k)+j\sin(\frac{\pi}{8}k)-[\cos(\frac{\pi}{8}k)-j\sin(\frac{\pi}{8}k)]]}\right)$
$= \frac{1}{2}e^{-jk\frac{3\pi}{8}}\frac{\sin(k\pi/2)}{\sin(k\pi/8)}$



$X_2(\Omega) =$

$\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \longrightarrow x[n] = 4(u[n+8] - u[n-8])$
$\sum_{n=-8}^{8} 4e^{-j\Omega n} = 4e^{j8\Omega}\left(\frac{1-(e^{-j\Omega})^{r}}{1-e^{-j\Omega}}\right) \longrightarrow$ time shift



$- v_c +$
out 2

$v_L = L\frac{di}{dt}$
$i = C\frac{dv}{dt}$

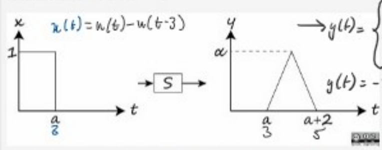
KVL: $V = iR + v_L + v_c$
$v = iR + L\frac{di}{dt} + v_c$
$\frac{di}{dt} = \frac{1}{L}V - \frac{R}{L}i - \frac{1}{L}V_c$ $\quad \frac{dv_c}{dt} = \frac{1}{C}i$
$\therefore \dot{x}_2 = \frac{1}{L}u - \frac{R}{L}x_2 - \frac{1}{L}x_1 \longrightarrow \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1/C\\ -1/L & -R/L\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ \frac{1}{L}\end{bmatrix}}_{B}u$
$\dot{x}_1 = 1/C\, x_2$

$x_2 = i$
Outputs: $y_1 = v_c = x_1$
$y_2 = Ri = Rx_2$
V input: $u_1 =$

$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & R\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + [0]u$

Webwork 3

For an LTI system, S, an input and its corresponding output signal are shown in the figure below. Assume $a = 3$ and $\alpha = -2$.



$\longrightarrow y(t) = \begin{cases} 0, & t<3 \\ t, & 3<t<4 \\ 2/3t, & 4<t<5 \\ 0, & t<5 \end{cases}$

$y(t) = -2r(t-3) - (-4)r(t-4) - 2r(t-5)$

For each of the following input signals, determine the corresponding output signal:

a) $x_1(t) = u(t) - u(t-3) - u(t-2) + u(t-5) \longrightarrow \underbrace{[u(t)-u(t-3)]}_{(x(t))} - \underbrace{[u(t-2)-u(t-5)]}_{(x(t-2))}$
b) $x_2(t) = 14.5u(t+3) - 24u(t) + 9.5u(t-3)$
c) $x_3(t) = \delta(t) - \delta(t-3)$

$\therefore y(t) = [-2r(t-3)+4r(t-4)-2r(t-5)] - [-2r(t-5)+4r(t-6)-2r(t-7)]$

$y_1(t) =$
$y_2(t) =$
$y_3(t) =$

$\rightarrow y(t) = -2u(t-3)+4u(t-4)-2u(t-5)$

Use $r(t)$ to represent the ramp function.

$y(t) = Y(s) = H(s)X(s) + H_i(s)I(s)$
$y(t) = y_{zs}(t) + y_{zi}(t) = \mathcal{L}^{-1}\{Y(s)\}$
$y_{zs}(t) = \mathcal{L}^{-1}\{H(s)X(s)\},\ y_{zi}(t) = \mathcal{L}^{-1}\{H_i(s)I(s)\}$
$H(s) = \frac{B(s)}{A(s)},\ H_i(s) = \frac{1}{A(s)}$

In a continuous-time system, the laplace transform of the input $X(s)$ and the output $Y(s)$ are related by $Y(s) = \frac{(s-2)X(s)+4}{(s+2)^2+6}$.

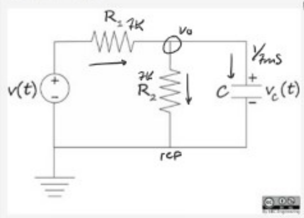
a) If $x(t) = u(t)$, find the zero-state response of the system, $y_{zs}(t)$.
$y_{zs}(t) =$

b) Find the zero-input response of the system, $y_{zi}(t)$.
$y_{zi}(t) =$

c) Find the steady-state solution of the system, $y_{ss}(t)$.
$y_{ss}(t) =$

a) $Y(s) = \frac{(s-2)X(s)}{(s+2)^2+6} + \frac{4}{(s+2)^2+6}$
zero-state: init. cond = 0, $X(s) \rightarrow 1$
$Y(s) = \frac{(s-2)u(t)}{(s+2)^2+6} = \frac{1}{s}\frac{(s-2)}{(s+2)^2+6}$
$y_{zs}(t) = \mathcal{L}^{-1}\{Y(s)\} = [1/5\cos(\sqrt{6}t)e^{-2t} + \frac{7/5e^{-2t}\sin(-\sqrt{6}t)}{\sqrt{6}} - 1/5]u(t)$
b) zero-input: $x(t) = 0 \longrightarrow X(s) \longrightarrow 0$
$Y(s) = \frac{4}{(s+2)^2+6} = \frac{4}{\sqrt{6}}\cdot\frac{\sqrt{6}}{(s+2)^2+\sqrt{6}}$
$y_{zi}(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{4e^{-2t}\sin(\sqrt{6}t)}{\sqrt{6}}u(t)$
c) steady-state: $\lim_{t\to\infty} y(t)$
$y(t) = y_{zs}(t) + y_{zi}(t)$ $\quad \therefore y_{ss}(t) = -1/5$

In the circuit shown in the figure, the input is the voltage source, $v(t)$, and the output is the voltange $v_c(t)$ across the capacitor. Determine the transfer function, $H(s)$, the impulse response $h(t)$, and the step response $d(t)$, of this circuit. Assume $R_1 = 2$ $k\Omega$, $R_2 = 7$ $k\Omega$ and $C = 7$ mF.



Voltage Divider
$V_o = \frac{7K \parallel 1/7mS}{2K + 7K\parallel 1/7mS}\cdot v_i$
$H(s) = \frac{v_o}{v_i} = \frac{7}{98s+9} = \frac{7}{98}\cdot\frac{1}{s+9/98}$
$h(t) = \mathcal{L}\{H(s)\} = \left[\frac{7}{98}e^{-9/98t}\right]u(t)$
Step Response $s(t) = \frac{1}{s}\cdot H(s)$
$Y(s) = \frac{1}{s}\cdot\frac{7}{98s+9} = \frac{7}{98s^2+9s} \longrightarrow$ PFE
$\frac{A}{s} + \frac{B}{98s+9} \longrightarrow A(98s+9) + Bs = 98As + 9A + Bs = (98A+B)s + 9A$
$A = 7/9,\ B = -686/9$
$\therefore s(t) = \mathcal{L}^{-1}\{Y(s)\} = \left[-\frac{7}{9}e^{-9/98t} + \frac{7}{9}\right]u(t)$

$H(s) =$
$h(t) =$ volts
$d(t) =$ volts

Impulse Response	Laplace Transform	Region of Convergence* for $\sigma = Re(s)$	Causality	BIBO stable
$h_0(t) = 10e^{-4t}u(t) + 10e^{8t}u(t)$			?	?
$h_1(t) = -7e^{-9t}u(-t) - 8e^{2t}u(-t)$			?	?
$h_2(t) = 11e^{-3t}u(t) - 19e^{3t}u(-t)$			?	?

① $\mathcal{L}\{h_0(t)\} = \frac{10}{s+4} + \frac{10}{s-8}$ ROC: $(8,\infty) \longrightarrow$ causal, not BIBO
② $\mathcal{L}\{h_1(t)\} = \frac{7}{s+9} + \frac{8}{s-2}$ ROC: $(-\infty,-9) \longrightarrow$ anticausal, not BIBO
③ $\mathcal{L}\{h_2(t)\} = \frac{11}{s+3} + \frac{19}{s-3}$ ROC: $(-3,3) \longrightarrow$ 2-sided, BIBO