2nd Order Circuits

Laplace domain:

Inductor:

$v(t) = L\frac{di(t)}{dt} \rightarrow V(s) = sLI(s) - Li_L(0)$

$+$ LS $\quad$ LI_{L0} $-$ → source helps current

$\overrightarrow{I(s)}$

Capacitor:

$\int i(t)dt \rightarrow \frac{I(s)}{s} + \frac{\int_{-\infty}^{0} i_C(t)dt}{s}$

$v(t) = \frac{1}{C}\int i(t)dt \rightarrow V(s) = \frac{I(s)}{sC} + \frac{v_C(0)}{sC}$

$+$ $1/sC$ $\quad$ $\frac{v_0}{s}$ $-$ → source opposes current

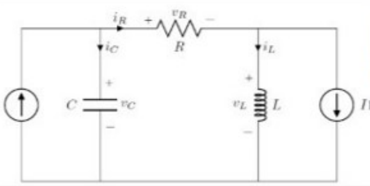
$\overrightarrow{I(s)}$

(8 points)
For the circuit shown below, let $I = 4u(t)A$, $C = 0.7F$, $R = 50\Omega$, $L = 9H$ and $I1 = 11A$. Find:

(a) $i_L(0^+), v_C(0^+), v_R(0^+)$

(b) $\frac{di_L(0^+)}{dt}, \frac{dv_C(0^+)}{dt}, \frac{dv_R(0^+)}{dt}$

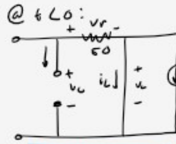
(c) $i_L(\infty), v_C(\infty), v_R(\infty)$



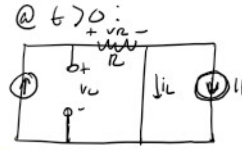
Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$i_L(0^+)$	A	$\frac{di_L(0^+)}{dt}$	A/S	$i_L(\infty)$	A
$v_C(0^+)$	V	$\frac{dv_C(0^+)}{dt}$	V/S	$v_C(\infty)$	V
$v_R(0^+)$	V	$\frac{dv_R(0^+)}{dt}$	V/S	$v_R(\infty)$	V

a) @ $t<0$:



@ $t>0$:

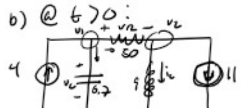


$i_L(0^+) = i_L(0^-) = -11A$

$v_C(0^+) = v_C(0^-) = 0\ V$

$v_R(0^+) = v_R(0^-) = 0\ V$

b) @ $t>0$:



KCL1: $4 = i_C + i_R$

$4 = C\frac{dv_C}{dt} + \frac{v_R}{R}$

$\frac{dv_C(0^+)}{dt} = \frac{4}{0.7}$

$= 5.71429\ V/s$

KCL2: $L\frac{di_L}{dt} = v_L - 0$

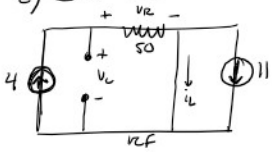
$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{9}$

$= 0\ A/s$

KCL3: $Ri_R = v_R$

$v_R(0^+) = 0\ V/s$

c) @ $t \rightarrow \infty$



$v_R(\infty) = 4 \cdot 50$

$= 200\ V$

$v_C(\infty) = v_R(\infty) = 200V$

$i_L(\infty) = 4 - 11$

$= -7A$

Capacitor

$i = C\frac{dv}{dt}$

$v = \frac{1}{C}\int_{-\infty}^{t} i\,dt$

$v_C(0^+) = v_C(0^-)$

Inductor

$v = L\frac{di}{dt}$

$i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$

$i_L(0^+) = i_L(0^-)$

$5:1 \rightarrow n = \frac{1}{5}$

① $\frac{V_2}{V_1} = \frac{1}{5}$

② $\frac{I_2}{I_1} = 5$

KCL: ③ $I_2 = \frac{V_1 - V_3}{1/3\times10^{-6}S} + I_1$

④ $V_4 = V_1 - V_2$

⑤ $\frac{V_3 - V_4}{50\times10^{-3}S} = I_2 + \frac{V_4 + 2000\angle0^\circ}{20}$

⑥ $\frac{V_1 - V_3}{1/3\times10^{-6}S} = \frac{V_3}{55} + \frac{V_3 - V_4}{50\times10^{-3}S}$

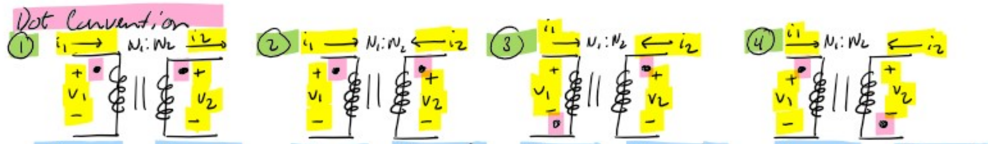
→ $S = 21j$

→ solve: $V_1 = 1833.5\angle -3.13656$ rad

$I_1 = 5.80227\angle -1.4897$ rad

Transformer

Dot Convention



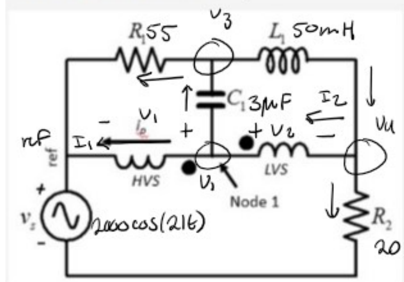
① $\frac{V_2}{V_1} = n$, $\frac{I_2}{I_1} = \frac{1}{n}$ ② $\frac{V_2}{V_1} = n$, $\frac{I_2}{I_1} = -\frac{1}{n}$ ③ $\frac{V_2}{V_1} = -n$, $\frac{I_2}{I_1} = \frac{1}{n}$ ④ $\frac{V_2}{V_1} = -n$, $\frac{I_2}{I_1} = -\frac{1}{n}$

– If current enters dotted terminal, opposite dotted terminal = (+)

– V_1, V_2 act as sources

$n = \frac{N_2}{N_1}$

An ideal transformer circuit.



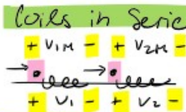
Report the peak value of V_1: V

Report the phase of V_1: rad

Report the peak value of I_1: mA

report the phase of I_1: rad

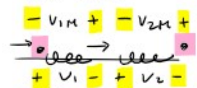
Coils in Series



Voltage Drop: $V = V_1 + V_2 + V_{1M} + V_{2M}$

Total Inductance: $L = L_1 + L_2 + 2M$

→ 2nd coil induces mutual inductance on 1st coil



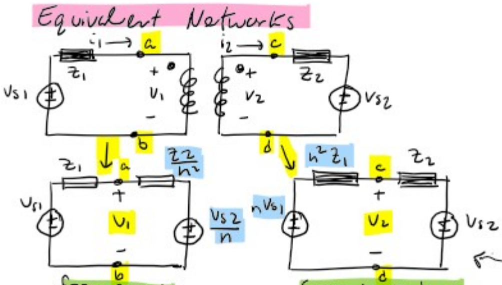
Voltage Drop: $V = V_1 + V_2 - V_{1M} - V_{2M}$

Total Inductance: $L = L_1 + L_2 - 2M$

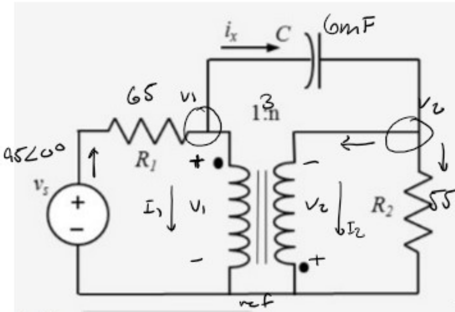
– V_1, V_2 depends on current directions

– V_{1M}, V_{2M} depends on windings of both coils

Equivalent Networks



from source $\quad$ from impedance



$i_x(t) =$ A

$n = \frac{3}{1} = 3$

① $\frac{V_2}{V_1} = 3$ ∵ V_1, V_2 → diff. polarities

② $\frac{I_2}{I_1} = \frac{1}{3}$ ∵ I_1, I_2 → same direction

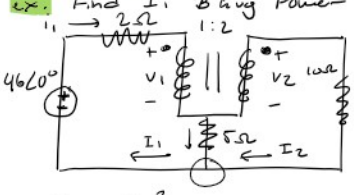
KCL: ③ $\frac{95\angle0^\circ - V_1}{65} = I_1 + \frac{V_1 - V_2}{1/6\times10^{-3}S}$

④ $\frac{V_1 - V_2}{1/6\times10^{-3}S} = I_2 + \frac{V_2}{55}$

→ $S = 7j$

→ solve: $i_x = 0.35307\cos(7t + 0.260354)$

ex. Find I_1 & avg Power absorbed by 10Ω



$n = \frac{2}{1} = 2$ ① $\frac{V_2}{V_1} = 2$ ② $\frac{I_2}{I_1} = \frac{1}{2}$

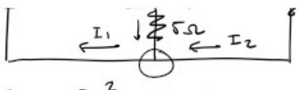
KCL: ③ $\frac{46\angle0^\circ - 2I_1 - V_1}{5} + I_2 = I_1$

④ $\frac{46\angle0^\circ - 2I_1 - V_1 + V_2}{10} = I_2$

→ solve: $V_1 = 10V$, $V_2 = 20V$,

$I_1 = 8A$, $I_2 = 4A$

$P = \frac{1}{2} I_2^2 \cdot 10 \ldots$



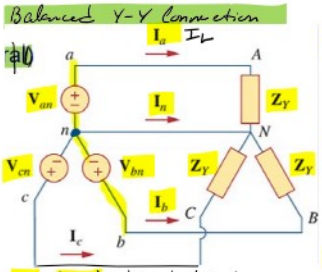
$P = 10\,I_2^2 = 160\,W$

④ $\dfrac{46\angle 0^\circ - 2I_1 - V_1 + V_2}{10} = I_2$

$\rightarrow$ solve: $V_1 = 10V$, $V_2 = 20V$, $I_1 = 8A$, $I_2 = 4A$

3 Phase Circuits

Balanced Y-Y Connection



$V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$
$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}| = \sqrt{3}\,V_p$
$I_a + I_b + I_c = 0$
$I_n = -(I_c + I_b + I_c) = 0$
$V_n = Z_n \cdot I_n = 0$

Phase Voltages (line – neutral)
$V_{an} = V_p\angle 0^\circ$
$V_{bn} = V_p\angle -120^\circ$
$V_{cn} = V_p\angle -240^\circ = V_p\angle 120^\circ$

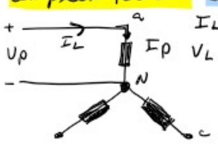
Line Voltage (line to line)
$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn} = \sqrt{3}\,V_p\angle 30^\circ$
$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}\,V_p\angle -90^\circ$
$V_{ca} = V_{cn} - V_{bn} = \sqrt{3}\,V_p\angle -210^\circ$

Line Current (Current in each line)
$I_a = \dfrac{V_{an}}{Z_Y}$, $I_b = \dfrac{V_{bn}}{Z_Y} = I_a\angle -120^\circ$,
$I_c = \dfrac{V_{cn}}{Z_Y} = I_a\angle -240^\circ$

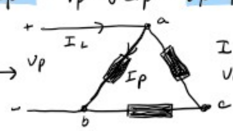
Power

Instantaneous Power
$p(t) = p_a(t) + p_b(t) + p_c(t) = 3V_p I_p\cos(\theta)$

Phase Power
Avg Power: $P_p = V_p I_p \cos\theta$
Reactive power: $Q_p = V_p I_p \sin\theta$
Apparent Power: $S_p = V_p I_p$
Complex Power: $S_p = V_p \cdot I_p^* = P_p + jQ_p = V_p I_p\angle\theta$



$\rightarrow$



Total Power
Avg Power: $P = 3P = 3V_p I_p\cos\theta = \sqrt{3}\,V_L I_L\cos\theta$
Reactive Power: $Q = 3Q_p = 3V_p I_p\sin\theta = \sqrt{3}\,V_L I_L\sin\theta$
Apparent Power: $|S| = 3V_p I_p = \sqrt{3}\,V_L I_L$
Complex Power: $S = 3S_p = 3V_p I_p^* = P + jQ = 3V_p I_p\angle\theta = \sqrt{3}\,V_L I_L\angle\theta$

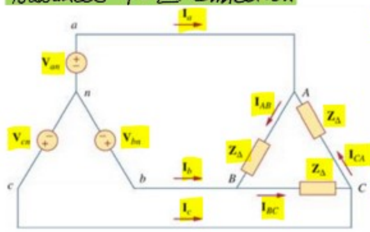
① **Y-Y**
$V_{ab} = V_{an} - V_{bn} = 160\angle 0^\circ - 160\angle -120^\circ = 277.12813\angle 30^\circ V$
$I_a = \dfrac{V_{an}}{Z_f + Z_L} = \dfrac{160\angle 0^\circ}{35+6j+0.4+0.7j} = 4.4409335\angle -10.71734^\circ A$
$I_p = I_L = 4.4409335\angle -10.71734^\circ A$
$S_s = 3V_p \cdot I_p^* = 3(160\angle 0^\circ)\cdot(4.4409335\angle -10.71734^\circ)^*$
$= 2131.65\angle 10.7173^\circ VA$
$S_L = 3|I_L|^2 \cdot Z_L = 47.700889\angle 60.2551^\circ VA$

② **Y-Δ**
$V_{ab} = V_{an} - V_{ab} = 277.12813\angle 30^\circ V$
$I_a = \dfrac{V_{an}}{Z_f + Z_Y} = \dfrac{160\angle 0^\circ}{35+6j+0.1333+0.2333j} = 4.48405\angle -10.0606^\circ$
○ $I_p = \dfrac{V_{ab}/3}{Z_f + Z_Y} = \dfrac{277.12813\angle 30^\circ}{35+6j+0.1333+0.233j} = 2.5888\angle 19.939$
○ $S_s = 2131.64\angle 10.0606^\circ$
$S_L = 3\cdot|I_L|^2\cdot Z_\Delta = 3|4.48405\angle -10.0606^\circ|^2(0.4+0.7j)$
$= 16.2078\angle 60.2577^\circ VA$

③ **Δ-Δ**
$V_L = V_p = 160\angle 0^\circ$
○ $I_a = \dfrac{V_{an}}{Z_f + Z_\Delta} = \dfrac{92.37604\angle -30^\circ}{35+6j+0.4+0.7j} = 2.5639\angle -40.7123^\circ$
○ $I_p = \dfrac{V_{an}/3}{Z_f + Z_Y} = \dfrac{160\angle 0^\circ/3}{35+6j+0.1333+0.2333j} = 1.4945\angle -10.0606^\circ$
○ $S_s = 717.36\angle 10.0606^\circ$
○ $S_p = 5.4026\angle 60.257^\circ$

④ **Δ-Y**
$V_L = V_p = 160\angle 0^\circ$
$I_a = \dfrac{V_{an}}{Z_f + Z_Y} = \dfrac{92.3604\angle -30^\circ}{35+6j+0.4+0.7j} = 2.5639\angle -40.717^\circ$
$I_p = I_L = 2.5639\angle -40.7173^\circ A$

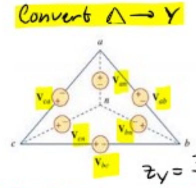
Balanced Y-Δ Connection



Phase Current
$I_{AB} = \dfrac{V_{AB}}{Z_\Delta} = \dfrac{V_{an} - V_{bn}}{Z_\Delta}$
$I_{BC} = \dfrac{V_{BC}}{Z_\Delta} = I_{AB}\angle -120^\circ$
$I_{CA} = \dfrac{V_{CA}}{Z_\Delta} = V_{AB}\angle -240^\circ$

Line Current
$I_a = I_{ab} - I_{cb} = I_{AB}\sqrt{3}\angle -30^\circ$
$I_L = |I_a| = |I_b| = |I_c|$
$I_p = |I_{AB}| = |I_{BC}| = |I_{CA}|$
$I_L = \sqrt{3}\,I_p$

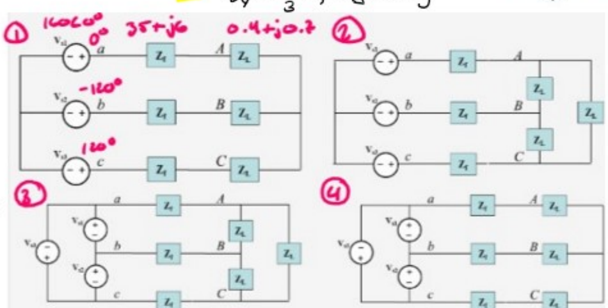
Convert Δ → Y

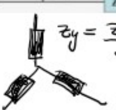


$V_{ab} = V_L\angle 0^\circ$
$V_{bc} = V_L\angle -120^\circ$
$V_{ca} = V_L\angle +120^\circ$

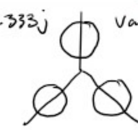
$V_{an} = \dfrac{V_L}{\sqrt{3}}\angle -30^\circ$
$V_{bn} = \dfrac{V_L}{\sqrt{3}}\angle -150^\circ$
$V_{cn} = \dfrac{V_L}{\sqrt{3}}\angle +90^\circ$

$Z_Y = \dfrac{Z_\Delta}{3}$, $Z_\Delta = 3Z_Y$





$Z_Y = \dfrac{Z_\Delta}{3} = 0.1333 + 0.2333j$



$V_{an} = \dfrac{V_L}{\sqrt{3}}\angle -30^\circ$
$= 92.37604\angle -30^\circ$

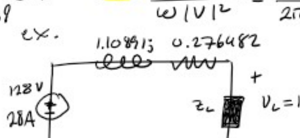
AC Power

Complex Power: $S = P + jQ = V_{rms}\cdot I_{rms}^* = V_{rms}I_{rms}\angle\theta_v - \theta_i$
Apparent Power: $|S| = V_{rms}I_{rms} = \sqrt{P^2 + Q^2}$
Real Power: $P = Re(S) = S\cos(\theta_v - \theta_i)$
Reactive Power: $Q = Im(S) = S\sin(\theta_v - \theta_i)$
Power Factor: $P/S = \cos(\theta_v - \theta_i)$

Power Factor Correction: **Total Reactive Power**: $Q_1 + Q_2 = 1.8\,KVAR$
- Add capacitor in parallel: $Q_C = -(Q_1 + Q_2) = -1.8\,KVAR$
- **Complex Power** received by capacitor: $S_C = -j\omega|V|^2 = jQ_C$

3° A ⟶ $C = \dfrac{Q_C}{\omega|V|^2} = \dfrac{1.8\times 10^3}{2\pi 60\cdot 120^2} = 331.6\,\mu F$

ex.



$V_S\angle\theta_S = V_{th}\angle\theta_{Vth} + V_L\angle\theta_{VL} = I_{rms}\cdot Z_{th} + 128\angle\theta_{VL}$
$128\angle\theta_S = (28)(0.276482 + 1.10891j) + 128\angle\theta_{VL}$
$V_S\angle\theta_S = V_S\cos(\theta_S) + jV_S\sin(\theta_S)$
$128\cos(\theta_S) + j120\sin(\theta_S) = 7.741496 + 31.04948j + 128\cos(\theta_{VL}) + j128\sin(\theta_{VL})$
Re(): $128\cos(\theta_S) = 7.741496 + 128\cos(\theta_{VL})$
Im(): $128\sin(\theta_S) = 31.04948 + 128\sin(\theta_{VL})$

a) ⟶ solve: $\theta_S = -6.8192254^\circ$ ⟶ pf angle @ source
d) $\theta_{VL} = -21.1807438$
b) $|S| = V_{rms}\cdot I_{rms} = (128)(28) = 3584\,VA$
$P = Re(S) = |S|\cos(\theta_v - \theta_i) = 3584\cos(-6.8192254^\circ) = 3558.645788\,W$
c) $Q = Im() = |S|\sin(\theta_v - \theta_i) = 3584\sin(-6.8192254^\circ) = -425.5539376\,VAR$
e) $P = Re(S) = |S|\cos(\theta_v - \theta_i) = 3584\cos(-21.1807438^\circ) = 3341.8839\,W$

$V_L = V_P = 160\angle 0°$

$I_a = \dfrac{V_{an}}{Z_f + Z_Y} = \dfrac{92.3604\angle -30°}{35+6j+0.4+0.7j} = 2.5639\angle -40.717°$

$I_P = I_L = 2.5639\angle -40.7173° A$

$S_S = 717.36\angle 10.7173°$

$S_L = 16.2078\angle 60.2577°$

b) $|S| = V_{rms} \cdot I_{rms} = (128)(28) = 3584 VA$

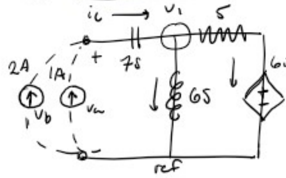
$P = Re(S) = |S| \cos(\Theta_V - \Theta_i) = 3584\cos(-6.8192254°) = 3558.645788 W$

c) $Q = Im(S) = |S| \sin(\Theta_V - \Theta_i) = 3584\sin(-6.8192254°) = -425.5539376 VAR$

e) $P = Re(S) = |S|\cos(\Theta_V - \Theta_i) = 3584\cos(-21.1807438°) = 3341.8839 W$

f) $Q = Im(S) = |S|\sin(\Theta_V - \Theta_i) = 3584\sin(-21.807438°) = -1294.4393783 VAR$

ex. 1A/2A Test source



1A Source

KCL: $1 = \dfrac{V_1}{6S} + \dfrac{V_1 - 6(1)}{5}$

$V_a = V_1 + (1)7S$

$V_a = 10.5424 + 0.44633j$

2A Source

KCL: $2 = \dfrac{V_2}{6S} + \dfrac{V_1 - 6(2)}{5}$

$V_b = V_1 + 2 \cdot 7S$

$V_b = 21.084858 + 0.842678j$

$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} V_{th} \\ R_{th} \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \end{bmatrix} \rightarrow \begin{cases} V_{th} + R_{th} = V_a \\ 2V_{th} + R_{th} = V_b \end{cases}$

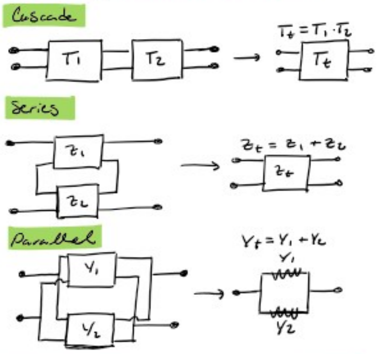
$\rightarrow$ Solve: $R_{th} = 10.542458 + 0.44634j$

AC circuits

Superposition

- Multiple frequencies involved: Solve phasor circuit
- Sum of time-domain responses
- Short V sources Open current source

Connecting 2-Port Networks



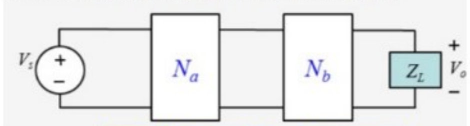
Cascade: $T_t = T_1 \cdot T_2$

Series: $Z_t = Z_1 + Z_2$

Parallel: $Y_t = Y_1 + Y_2$

For the individual two-ports shown, the given two-port parameters are known: $[h_a] = \begin{bmatrix} 5\Omega & -6 \\ 6 & -3S \end{bmatrix}$, $[y_b] = \begin{bmatrix} -7 & 6 \\ 4 & -8 \end{bmatrix} S$

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)



a. Determine the transmission and y parameters of the overall two-port. If the y matrix not exist, contact the teaching staff by private Piazza post to get a different randomize of parameters.

[T] = [____ ____ Ω ; ____ S ____]

[y] = [____ S ____ S ; ____ S ____ S]

b. Find the voltage ratio $G = V_o/V_s$ when $Z_L = 6\Omega$:

G = ____

c) T-Params

Conversion Matrix: $T_h = \begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$

$h = \begin{bmatrix} 5 & -6 \\ 6 & -3 \end{bmatrix}$ $\Delta h = -15 - (-36) = 21$

$T_a = \begin{bmatrix} -21/6 & -5/6 \\ 1/2 & -1/6 \end{bmatrix}$

Conversion Matrix: $T_y = \begin{bmatrix} \frac{-y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$

$y = \begin{bmatrix} -7 & 6 \\ 4 & -8 \end{bmatrix}$ $\Delta y = 56 - 24 = 32$

$T_b = \begin{bmatrix} 8/4 & -1/4 \\ -\frac{32}{4} & 7/4 \end{bmatrix}$

$T = T_a \cdot T_b \longrightarrow$ Cascade

$T = \begin{bmatrix} -\frac{21}{6} & -\frac{5}{6} \\ -\frac{1}{2} & -\frac{1}{6} \end{bmatrix}\begin{bmatrix} -\frac{8}{4} & -\frac{1}{4} \\ -\frac{32}{4} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} -0.3333 & -0.58333 \\ 2.3333 & -0.416666 \end{bmatrix}$

Y-Params

Conversion Matrix: $Y_t = \begin{bmatrix} \frac{t_{22}}{t_{12}} & \frac{-\Delta t}{t_{12}} \\ \frac{-1}{t_{12}} & \frac{t_{11}}{t_{12}} \end{bmatrix}$

$T = \begin{bmatrix} -0.3333 & -0.5833 \\ 2.3333 & -0.41666 \end{bmatrix}$ $\Delta t = 1.5$

$Y = \begin{bmatrix} 0.714286 & 2.571428 \\ 1.714285 & 0.571428 \end{bmatrix}$

b) Voltage Ratio $G = V_o/V_s \rightarrow Z_L = 6\Omega$

- Open port 2, Apply 1A to port 1
- Open port 1, Apply 1A to port 2

Y-param: $I_1 = Y_{11}V_1 + Y_{12}V_2$
$I_2 = Y_{21}V_1 + Y_{22}V_2$

$I_2 = 1.7143V_1 + 0.5714V_2$

$V_2 = -6I_2$ $I_2 = \dfrac{-V_2}{6}$

$-\dfrac{V_2}{6} = 1.7143V_1 + 0.5714V_2$

$-V_2 = 10.2858V_1 + 3.4284V_2$

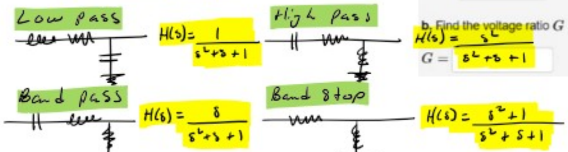
$\dfrac{V_2}{V_1} = -2.32269V$

Classification by construction

Passive: only RLC

Active: RLC + opamp

ex.



Low pass: $H(s) = \dfrac{1}{s^2+s+1}$

High pass: $H(s) = \dfrac{s^2}{s^2+s+1}$

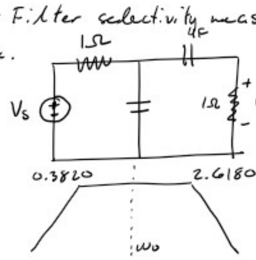
Band pass: $H(s) = \dfrac{s}{s^2+s+1}$

Band stop: $H(s) = \dfrac{s^2+1}{s^2+s+1}$

Quality Factor

- For band pass, cutoff f ω_h & ω_L
- "Center f" $\omega_0 \longrightarrow$ mean of ω_h & ω_L
- Bandwith $BW = \omega_h - \omega_L$
- Filter selectivity measured by quality factor $Q = \dfrac{\omega_0}{BW}$

ex.



$H(s) = \dfrac{s}{s^2+3s+1} \longrightarrow$ Band Pass

cfactor( )

$\longrightarrow$ proot(1, 3, 1) : poles

$\longrightarrow$ Geometric mean: $\sqrt[n]{x_1 x_2 \cdots}$

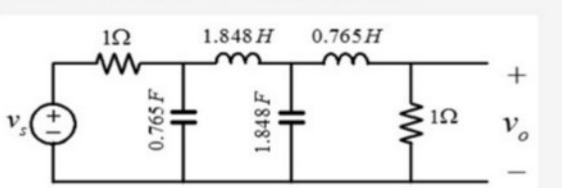
$\omega_0 = \sqrt{0.382 \cdot 2.618}$

$= 1$ rad/s

$BW = 2.618 - 0.382 = 2.236$

$Q = \dfrac{\omega_0}{BW} = 0.447227$

A fourth-order Butterworth low pass filter is shown in the figure below. Using "scaling" replace the two resistors by 7 kilo ohm resistors and determine what must be the valu the inductor and the capacitor for a cutoff frequency of 12 kHz.



$\omega_0 = 12kH_3 = \dfrac{6000}{\pi}$

Scaling $\longrightarrow$ (Filters video)

$R = 1\Omega \longrightarrow 7k\Omega$

Magnitube Scaling factor

$K_m = 7k$

Frequency scaling factor

Scaling & Recipies

- Filter "cookbooks"
- Scale filters for different components

Frequency Scaling

Want same impedance Z but at K_f times original f

need new values for L' & C'

$Z_L = j(\omega K_f)L' = j\omega L \longrightarrow L' = \dfrac{L}{K_f}$

$Z_C = \dfrac{1}{j(\omega K_f)C'} = \dfrac{1}{j\omega C} \longrightarrow C' = \dfrac{C}{K_f}$

Higher frequency but same impedance

$\therefore$ Divide both L & C by same factor used to multiply f by K_f

Magnitude Scaling

- If components too little/large, scale up/down component values without changing f

$\therefore$ Multiply all impedances by K_m: $(\omega' = \omega)$

$Z_R' = K_m Z_R = K_m R$, $Z_L' = K_m Z_L = j\omega K_m L$, $Z_C' = K_m Z_C = \dfrac{1}{j\omega C/K_m}$

Scaling → (Filters Video)

$R = 1\Omega \to 7k\Omega$

$\omega = 12kHz \cdot 2\pi = 24k\pi$ rad/s

$\therefore \omega_0 = 24k\pi$

$L_1' = \frac{k_m}{k_f} \cdot L = 0.171569H$

$C_1' = \frac{1}{k_m k_f} C = 1.44944468 \times 10^{-9}$ F

$L_2' = 7.102289 \times 10^{-2}$ H

$C_2' = 3.50140875 \times 10^{-9}$ F

Magnitude scaling factor

$k_m = 7k$

Frequency scaling factor

$k_f = 24k\pi$ rad/s

original f: $\omega_0 = 1$ rad/s

without changing f

$\therefore$ Multiply all impedances by k_m: $(\omega' = \omega)$

$Z_R' = k_m Z_R = k_m R$, $Z_L' = k_m Z_L = j\omega k_m L$, $Z_C' = k_m Z_C = \frac{1}{j\omega C / k_m}$

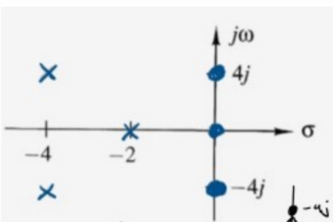
$R' = k_m R$ $\quad L' = k_m L$ $\quad C' = \frac{C}{k_m}$

Magnitude & Frequency scaling

— increase RLC by k_m, shift f response by k_f:

$R' = k_m R$, $L' = \frac{k_m}{k_f} \cdot L$, $C' = \frac{1}{k_m k_f} \cdot C$, $\omega' = k_f \omega$

Bode Plots

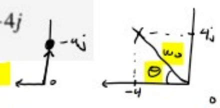


$\therefore H(s) = \frac{9s(s-4j)(s+4j)}{(s+2)(s+4+4j)(s+4-4j)} = \frac{9s(s^2 + 2 \cdot 1 \cdot 4 + 4^2)}{(s+2)(s^2 + 2 \cdot 0.7071 \cdot 5.65685 + 5.65685^2)}$

↳ zeros: 0, 4 (double)

poles: 2, 5.65685 (double)

Starting Pt.: $20 \log \left(\frac{9 \cdot (1) \cdot (4)^2}{(2)(5.65685)^2} \right) = 7.0436$



Complex Roots

$s^2 + 2 \cdot \zeta \omega_0 + \omega_0^2$

$(s-4j)(s+4j)$: $\omega_0 = \sqrt{0^2 + 4^2} = 4$ $\quad \zeta = \cos\theta = \frac{0}{\omega_0}$

$\therefore (s^2 + 2 \cdot 1 \cdot 4 + 4^2)$

$(s+4+4j)(s+4-4j)$ $\quad \omega_0 = \sqrt{4^2+4^2} = \sqrt{32}$ $\quad \zeta = \cos\theta = \frac{4}{\omega_0}$

$\therefore (s^2 + 2 \cdot 0.7071 \cdot \sqrt{32} + \sqrt{32}^2)$

dB Error: $s = 2j$

→ solve $20 \log(H(s)) = 7.487841$

dB Error = |plot value − exact value|

$= 13.0643 - 7.487841$

$= 5.57645$

phase Error: $s = 2j$

→ solve $\arg(H(s))$

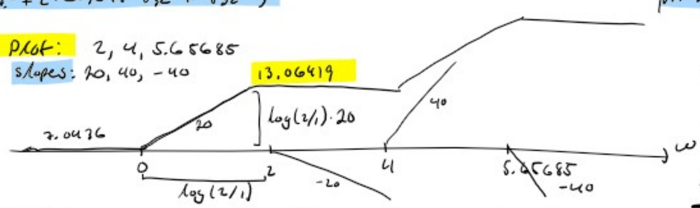
phase Error = |plot value − exact value|

$= 12.4724 - 15.25512$

$= 2.78269$

Amplitude Plot: 2, 4, 5.65685

slopes: 20, 40, −40



For neg(−) poles $(s-\omega_2) \to (s+\omega_2)$ for dB

$\to \frac{s^2}{(s+\omega_2)}$ for phase

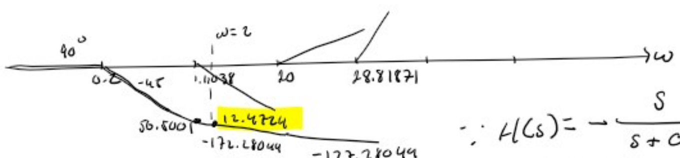
Phase Plot: for complex roots: $\omega_0/10^{\zeta}$, $\omega_0 \cdot 10^{\zeta}$

0.2, 0.4, 1.11038, 20, 28.81871, 40

slopes: −45, 90/ζ, −90/ζ, 45, 90/ζ, −90/ζ

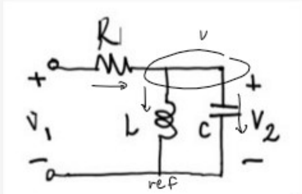
∞ $\quad$ −127.28044 $\quad$ 127.28044 $\quad$ −∞

→ zeros: ︶ poles: ︶



$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{4.81s}{25137.1s^2 + 4.81s + 3900}$

with the circuit in the figure below. Determine and report the value of the capacitor, and the inductor.



$V_{in} = V_{out}$ $\quad \therefore H(s)$, $V_1 = 1$, $H(s) = V_2$

KCL: $\frac{1 - v_2}{R} = \frac{v_2}{Ls} + \frac{v_2}{1/Cs}$

→ solve for v_2:

$v_2 = -\frac{1}{-\frac{r}{\cdot} - 1 - \frac{r}{Ls}} = \frac{-1}{-rCs - 1 - \frac{r}{Ls}} = \frac{-1}{\frac{-rCs^2 L}{Ls} - \frac{Ls}{Ls} - \frac{r}{Ls}}$ → compare w/ $H(s)$:

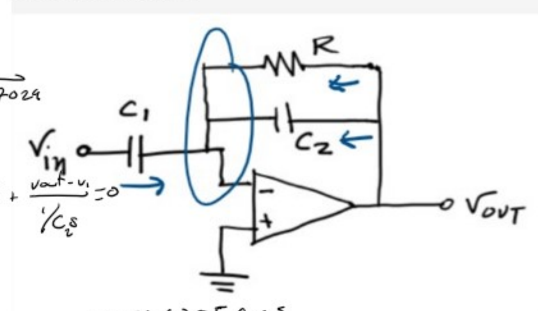
$Ls = \frac{4.81s}{}$

Transfer Functions

Realize the transfer function (where $a = 0.907029$)

$$H(s) = \frac{V_{out}}{V_{in}} = -\frac{s}{s+a}$$

with the circuit in the figure below. If the resistor is $R = 4500\,\Omega$, determine and value of the two capacitors.



$\therefore H(s) = -\frac{s}{s + 0.907029}$

NF: $V_1 = 0$

KCL: $\frac{1 - V_1}{1/C_1 s} + \frac{V_{out} - V_1}{4500} + \frac{V_{out} - V_1}{1/C_2 s} = 0$

→ solve for V_{out}:

$V_{out} = -\frac{4500 \cdot C_1 \cdot s}{4500 \cdot C_2 \cdot s + 1} = -\frac{4081.6305\, C_1 \cdot s}{4081.6305 \cdot C_2 \cdot s + 0.907029}$

$V_{out} = -\frac{s}{s + 0.907029}$

$\therefore C_1 = \frac{1}{4081.6305}$ F $\quad C_2 = \frac{1}{4081.6305}$ F

→ solve for v_2:

$$v_2 = -\frac{1}{-\frac{r}{1/cs} - 1 - \frac{r}{ls}} = \frac{-1}{-rcs - 1 - \frac{r}{ls}} = \frac{-1}{\frac{-rcs^2l}{ls} - \frac{ls}{ls} - \frac{r}{ls}}$$

$$= \frac{-1}{\frac{-rcs^2l - ls - r}{ls}} = \frac{-ls}{-rcs^2l - ls - r} = \frac{ls}{rcs^2l + ls + r}$$

→ compare w/ $H(s)$:

$$\frac{ls}{rcs^2l + ls + r} = \frac{4.81s}{25137.1s^2 + 4.81s + 3900}$$

$$\therefore \; l = 4.81$$

$$r = 3900$$

$$c = \frac{25137.1}{r \cdot l} = 1.34$$