ELEC 301: Electronic Circuits

Mini Project 1: Mathematical and Computer Tools

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1. Introduction

This project report outlines the use of Miller's theorem and the method of open-circuit and short circuit time constants with computer based circuit simulation tools. We verify their accuracy using SPICE (Simulation Program with Integrated Circuit Emphasis) simulations. In Part 1, open circuit (OC) and short circuit (SC) time constants are used to design circuits and plot the Bode plots, and computing its transfer function. In Part 2, the Bode plot for a four pole RC filter is created by running an AC simulation. We determine the low and high 3dB frequencies and calculate the percent error using OC and SC time constants. In Part 3, the mid band gain is calculated, then a linear approximation method is used to find transfer function pole locations.

2. Objective

The objective of this mini project is to strengthen our understanding of computer based circuit simulation tools in the context of designing and solving common electronic circuits such as filters.

3. Project

3.1 Part 1: Three-Pole Low-Pass Filter

<u>A. Circuit Design and AC Simulation</u>

Given the transfer function $T(s) = \frac{V_0(s)}{V_{s(s)}} = 0.125 * \frac{10^5}{s+10^5} * \frac{10^6}{s+10^6} * \frac{10^7}{s+10^7}$, the four 750Ω and two 1.5kΩ resistors must first be correctly configured in the circuit shown in Figure 1.1. Using the known transfer function midband gain of 0.125, the corresponding midband gain circuit is built in Figure 1.2 by trying different combinations of resistor values.

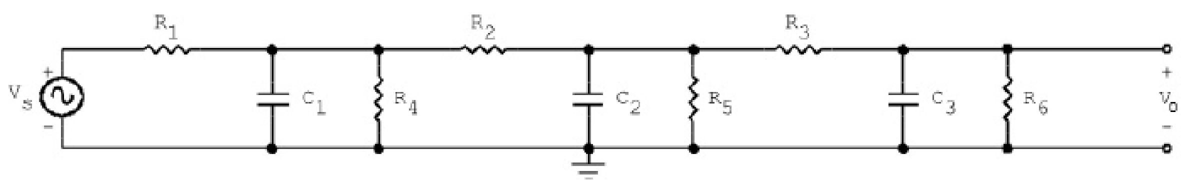

Figure 1.1: A Three-Pole Low-Pass Filter

Setting Vs at an arbitrary voltage of 5V, an output voltage of Vo = 0.625V is measured. The midband gain is calculated as $A_M = \frac{V_o}{V_s} = \frac{0.625}{5} = 0.125$.

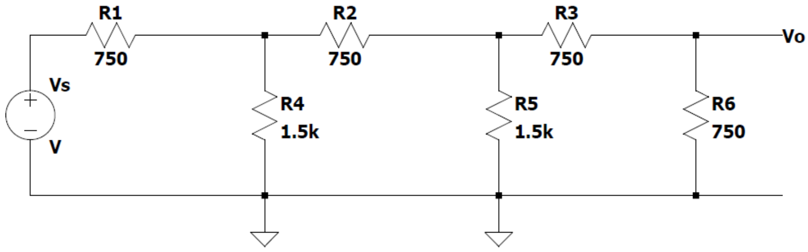

Figure 1.2: Midband Gain Circuit Configuration

Using Open Circuit Time Constant (OCTC) estimation, we must first find the poles frequency in order to solve for each capacitor value. As the denominator of the transfer function terms are $(s + 10^5)$, $(s + 10^6)$, $(s + 10^7)$, and $\tau = \frac{1}{w_p}$, the pole frequency and the time constants are as follow:

$$w_{p1} = 10^5\ rad/sec,\ w_{p2} = 10^6\ rad/sec,\ w_{p3} = 10^7\ rad/sec$$

$$\tau_{p1} = \frac{1}{10^5}\ sec,\ \tau_{p2} = \frac{1}{10^6}\ sec,\ \tau_{p3} = \frac{1}{10^7}\ sec.$$

Since C1 > C2 > C3, C1 will short before C2, and C2 will short before C3 as we move from low to high frequency. We open all capacitors at the exception of a single one for each capacitor and find the equivalent resistance as seen from the capacitor, as shown in Figure 1.3, 1.4, and 1.5. Finally, we solve for the capacitor value using the pole frequencies derived from the transfer function.

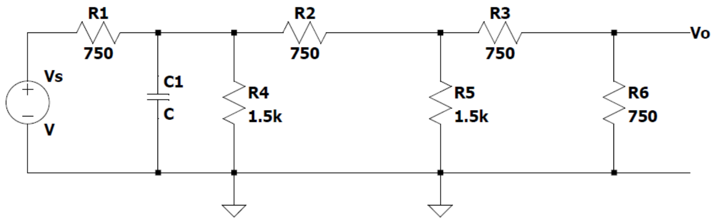

Figure 1.3: Req as seen from C1

$$R_{eq} = R_1||(R_4||(R_2 + (R_5||(R_6 + R_3))) = 375\ \Omega$$

Since C1 > C2 > C3, the lowest pole frequency and time constant τ_{p1} is used to solve for C1:

$$C_1 = \frac{1}{R_{eq}10^5} = 26.667\ nF$$

Since C1 > C2, C1 shorts before C2:

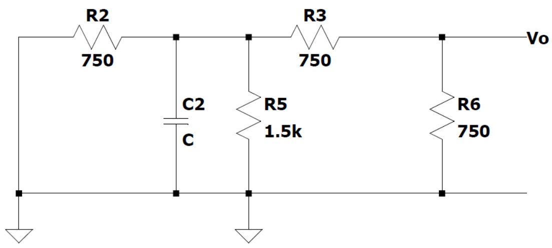

Figure 1.4: Req as seen from C2

$$R_{eq} = R_2 || (R_5 || (R_6 + R_3)) = 375\ \Omega$$

τ_{p2} is used to solve for C2:

$$C_1 = \frac{1}{R_{eq} 10^6} = 2.667\ nF$$

Since C2 > C3, C1 and C2 short before C3:

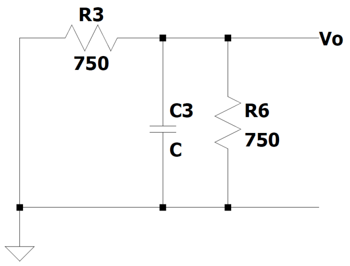

Figure 1.5: Req as seen from C3

$$R_{eq} = R_6 || R_3 = 375\ \Omega$$

τ_{p3} is used to solve for C3:

$$C_1 = \frac{1}{R_{eq} 10^7} = 267\ pF$$

An AC simulation is run using the circuit in Figure 1.6, plotting a Bode plot for the magnitude and phase response as shown in Figure 1.7

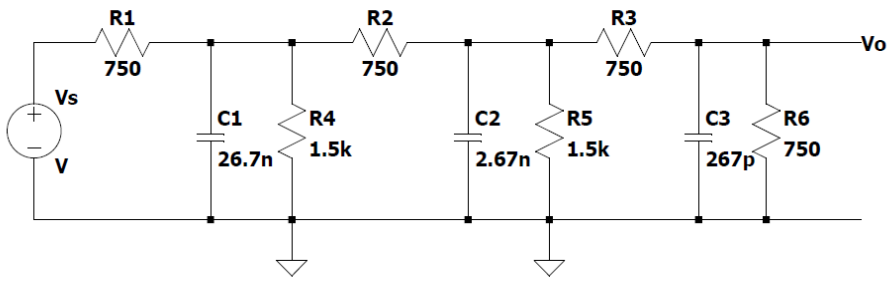

Figure 1.6: Designed Circuit

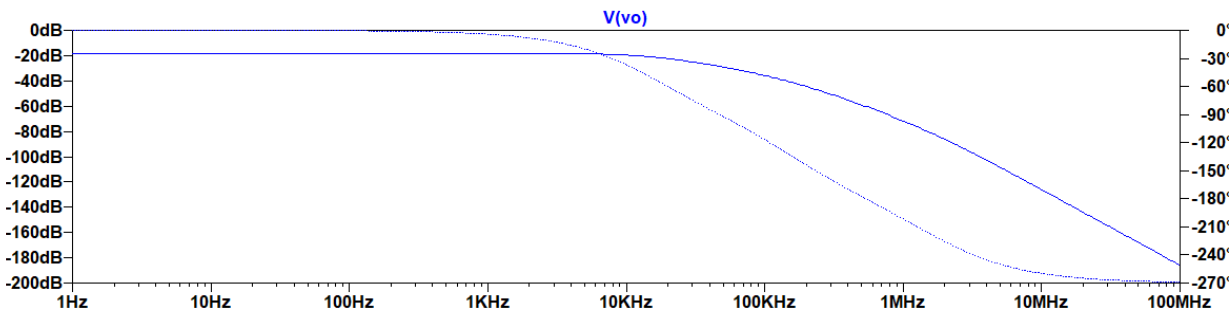

Figure 1.7: Magnitude (solid blue) and Phase (dotted blue) Bode plot of the designed circuit

B. Transfer Function AC Simulation

Using the initial transfer function $T(s) = \frac{V_0(s)}{V_{s(s)}} = 0.125 * \frac{10^5}{s+10^5} * \frac{10^6}{s+10^6} * \frac{10^7}{s+10^7}$, the circuit in Figure 1.8 is modeled using a voltage source element in LTSpice. The transfer function circuit is then simulated as a Bode plot as shown in Figure 1.9.

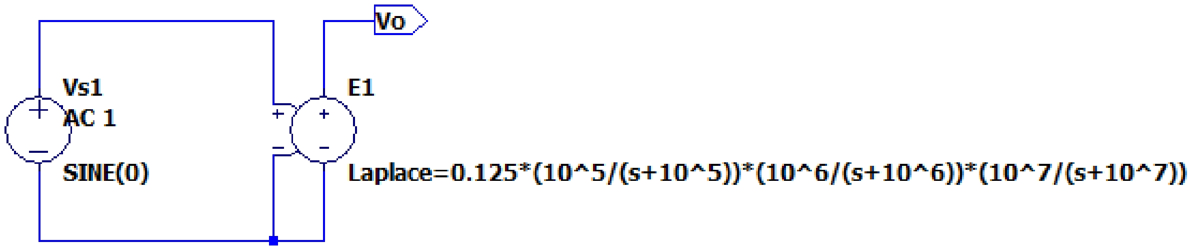

Figure 1.8: Transfer Function s-domain Circuit

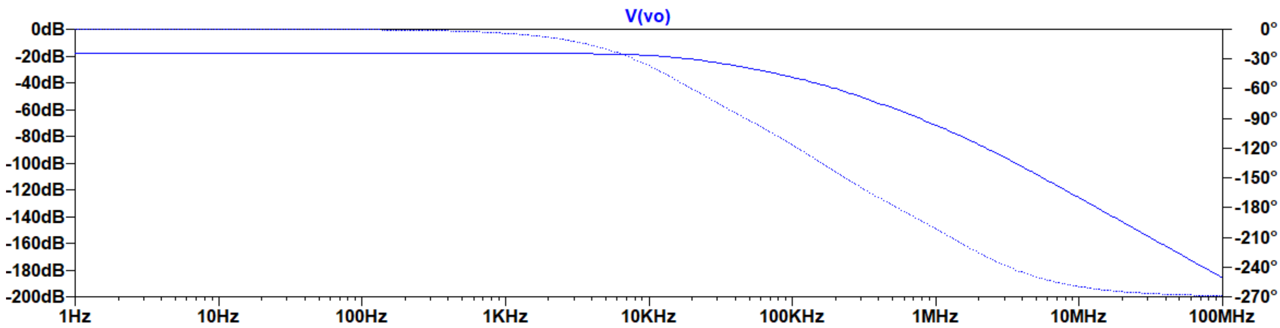

Figure 1.9: Magnitude (solid blue) and Phase (dotted blue) Bode plot of Transfer Function

This Bode plot displays very similar magnitude and phase properties to that of Figure 1.7's designed circuit Bode plot, both of which start at -20dB and 0 degrees respectively, with similar pole placements.

3.2 Part 2: Four-Pole RC Filter

A. AC Simulation of Four-Pole RC Filter

An AC simulation is run on the Four-Pole RC Filter shown in Figure 2.1. The resulting Bode plot in Figure 2.2 is plotted using the circuit's frequency response.

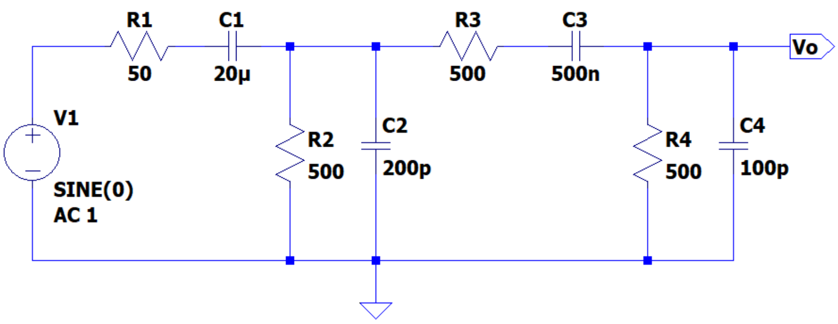

Figure 2.1: Four-Pole RC Filter

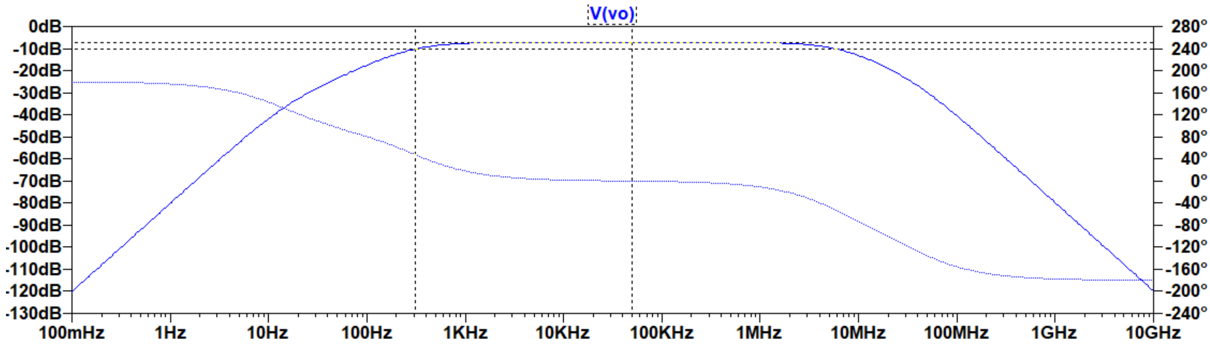

Figure 2.2: Magnitude (solid blue) and Phase (dotted blue) Bode plot of Four-Pole RC Filter

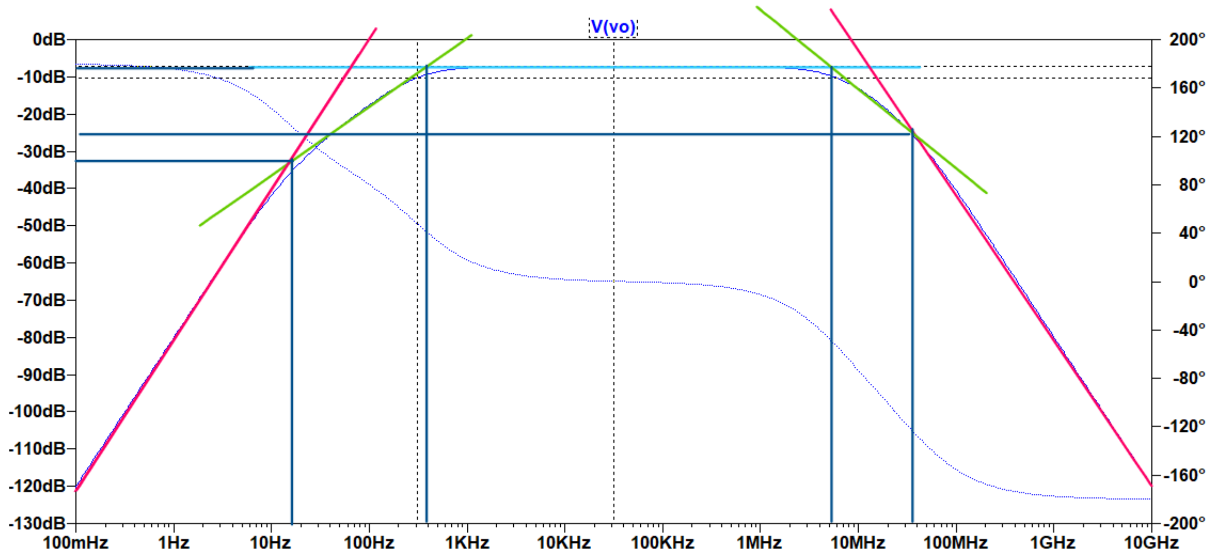

Figure 2.3 Slopes overlapped on the Bode Plot (0dB/dec in light blue, +/- 20dB/dec in green, +/- 40dB/dec in pink)

A linear approximation method is used to approximate the poles of the circuit's transfer function for values of C3. Slopes of +/-40dB/dec, +/-20dB/dec, and 0dB/dec, are drawn, then fitted onto the bode plot as shown in Figure 2.3. It is important to note that identifying pole locations graphically may result in some inaccuracies in our calculations due to cursor inaccuracies and line thicknesses. The approximated poles and low-frequency 3-dB points are described in Table 2.1.

C3 Values	f_{LP1}	f_{LP2}	f_{HP1}	f_{HP2}	f_{L3dB}	f_{H3dB}
500 nF	17.26 Hz	399.20 Hz	5.24 MHz	33.07 MHz	310.50 Hz	33.48 MHz
1 uF	14.12 Hz	157.55 Hz	5.24 MHz	37.79 MHz	157.55 Hz	38.15 MHz
2 uF	14.12 Hz	70.50 Hz	5.83 MHz	38.81 MHz	84.06 Hz	39.24 MHz
5 uF	9.44 Hz	41.59 Hz	6.49 MHz	39.86 MHz	42.65 Hz	40.38 MHz
10 uF	9.21 Hz	28.53 Hz	6.49 MHz	43.19 MHz	30 Hz	43.67 MHz

Table 2.1: Graphically Approximated Poles and 3dB point

B. Percent Error of Low-Frequency 3-dB Point

The Short Circuit Time Constant (SCTC) estimation method is now used to calculate the theoretical low-frequency-3-dB point, before calculating the percent error against the graphically obtained values.

At low frequency response, we open C2 and C4 and short C1 and C3 one by one in order to calculate the resistance as seen from each capacitor, and subsequently the short circuit time constant τ_{sc}.

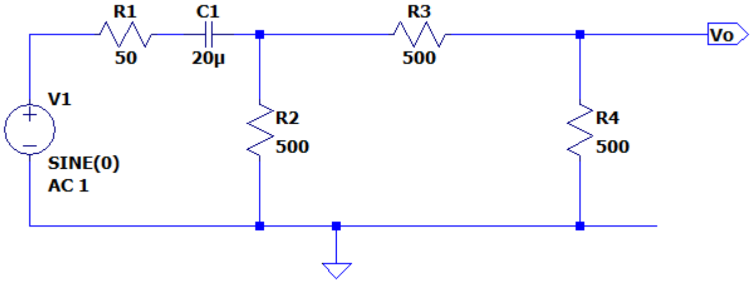

Figure 2.4: Resistance Seen by C1 at Low Frequency

$$\tau_{SC}^{\ C1} = (R_1 + (R_2||(R_3 + R_4)))20\mu = 0.00767\ s$$

$$\omega_{LP1} = \frac{1}{\tau_{SC}} = 130.43\ rad/s \qquad f_{LP1} = \frac{\omega_{LP}}{2\pi} = 20.76\ Hz$$

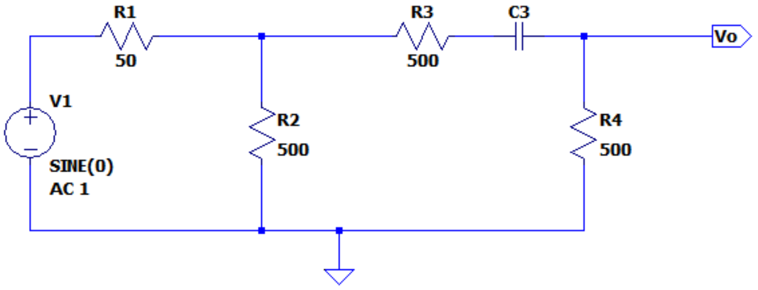

Figure 2.5: Resistance Seen by C3 at Low Frequency

Figures 2.4 and 2.5 show the circuit as seen by C1 and C3 at low frequency respectively. We calculate the equivalent resistance and multiply it to the capacitance to find the time constant. Then, the low-frequency 3dB point is calculated as follows:

$$f_{L3dB} = \sqrt{f_{LP1}^{\ 2} + f_{LP2}^{\ 2}}$$

This process is repeated for every value of C3, with the calculated frequencies displayed in Table 2.2

C3 Values	f_{LP1}	f_{LP2}	f_{L3dB}
500 nF	20.76 Hz	304.47 Hz	305.18 Hz
1 uF	20.76 Hz	152.24 Hz	153.65 Hz
2 uF	20.76 Hz	76.12 Hz	78.90 Hz
5 uF	20.76 Hz	30.45 Hz	36.85 Hz
10 uF	20.76 Hz	15.22 Hz	25.74 Hz

Table 2.2: Calculated Low Frequency Poles and Low 3dB point

	500 nF	**1 uF**	**2 uF**	**5 uF**	**10 uF**
% Error f_{L3dB}	1.74%	2.54%	6.54%	15.74%	16.55%

Table 2.3: Low Frequency 3dB Point Percent Error

The low frequency 3dB point percent error is calculated using the formula:

$$\%\ Error\ = \left|\frac{Theoretical - Experimental}{Theoretical}\right| * 100$$

Discussion

The percent error results between the calculated low frequency 3dB point and measured low frequency 3dB point are shown in Table 2.3. It is observed that while our simulated frequencies are near that of the calculated frequencies, the percentage error increases as the capacitances of C3 increases. This may be an indication of the increasing difficulty of discerning the slopes of the magnitude changes with increasing capacitance, resulting in a larger error margin.

3.3 Part 3: Basic Transconductance Amplifier

A. Midband Gain and Miller's Theorem

Before calculating the midband gain, Miller's theorem must first be used in order to simplify the circuit shown in Figure 3.1.

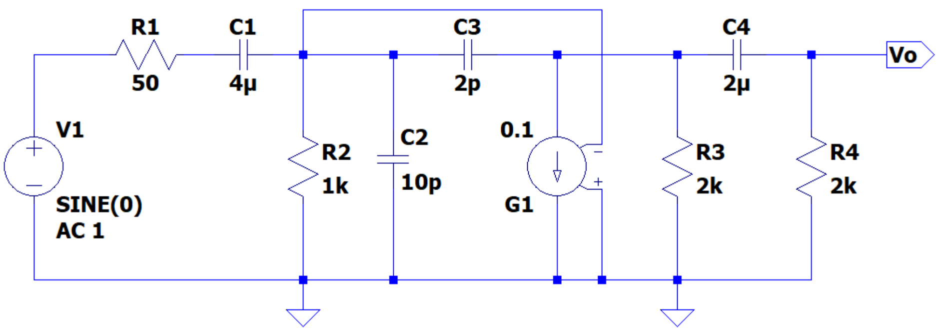

Figure 3.1: Basic Transconductance Amplifier

Using Miller's theorem:

$$k = -g_m r_o || R_c = \frac{V_o}{V_1} = \frac{-0.1V_1(R_3||R_4)}{V_1} = -100 \qquad C_\mu = 2\,pF$$

$$C_{\mu 1} = C_\mu(1 - k) = 202\,pF$$

$C_{\mu 2}$ can be approximated as: $C_{\mu 2} = C_\mu(1 - \frac{1}{k}) \approx C_\mu$

Then, adding C2 and $C_{\mu 1}$ in parallel yields the circuit shown in Figure 3.2.

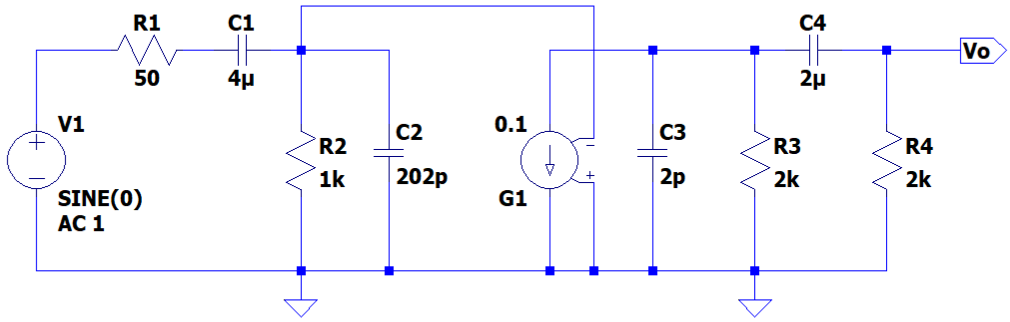

Figure 3.2: Miller's Theorem Applied to Transconductance Amplifier

To find the Midband response, all high frequency capacitors are opened while all low frequency capacitors are shorted, resulting in a fully resistive circuit as shown in Figure 3.3

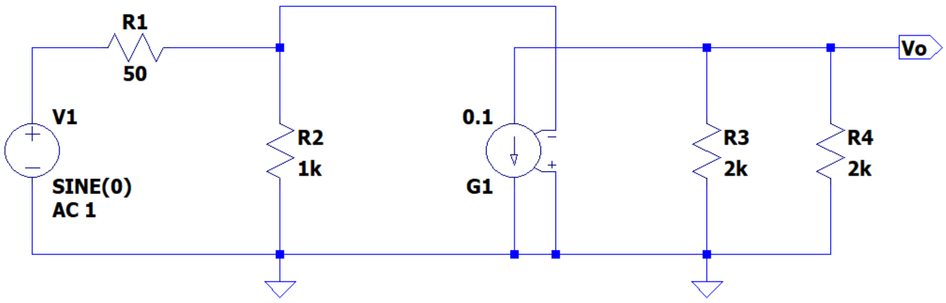

Figure 3.3 Midband Circuit

The midband gain is defined as $A_M = \frac{V_0}{V_s}$ of the resistive circuit found above.

On the input side:

$$V_1 = \frac{1k}{1k+50} V_s \qquad \frac{V_1}{V_s} = 0.9524$$

On the output side:

$$\frac{V_0}{V_1} = -100$$

Therefore, the midband gain is found to be: $A_M = \frac{V_0}{V_s} = \frac{V_1}{V_s}\frac{V_0}{V_1} = -95.24$

To find the location of the poles and zeros of the transconductance amplifier circuit, we analyze the circuit's response at low frequency and high frequency once again using SCTC estimation.

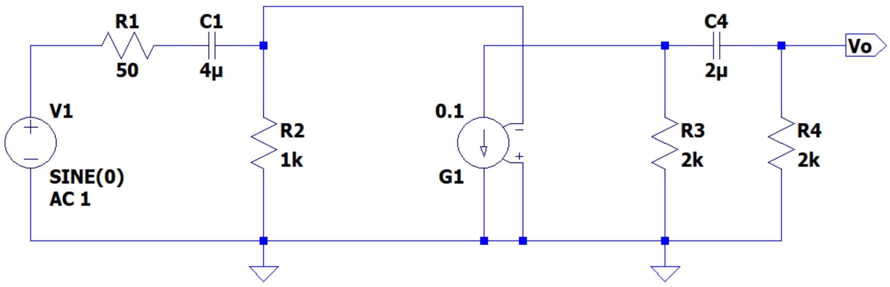

Figure 3.4: Circuit at Low Frequency

At low frequency, we first open all internal capacitors as shown in Figure 3.4, then short C1 and C4 one by one.

On the input side shorting C1:

$$\tau_{SC}^{C1} = (R_2 + R_1)C_1 \qquad \omega_{LP2} = \frac{1}{\tau} = 238.09\, rad/s$$

On the output side shorting C4:

$$\tau_{SC}^{C4} = (R_3 + R_4)C_4 \qquad \omega_{LP1} = \frac{1}{\tau} = 125\, rad/s$$

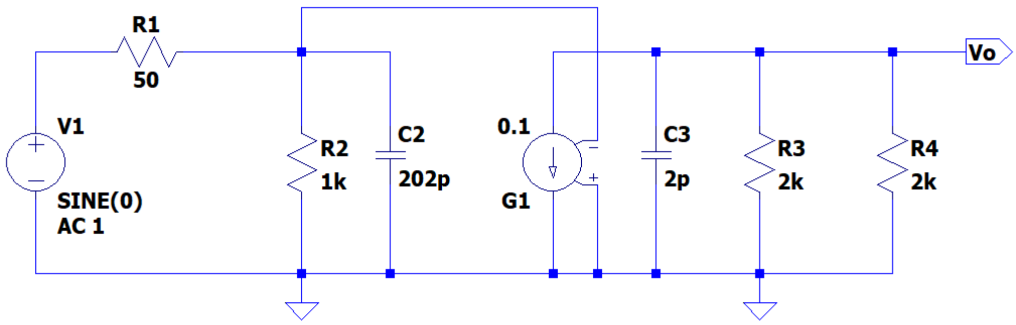

Figure 3.5: Circuit at High Frequency

At high frequency, we short all external capacitors as shown in Figure 3.5, then open C2 and C3 one by one.

On the input side shorting C1:

$$\tau_{SC}^{C2} = (R_2||R_1)C_2 \qquad \omega_{HP1} = \frac{1}{\tau} = 1.0396E8\, rad/s$$

On the output side shorting C4:

$$\tau_{SC}^{C4} = (R_3||R_4)C_3 \qquad \omega_{HP1} = \frac{1}{\tau} = 5E8\, rad/s$$

Combining the midband, low frequency, and high frequency responses, the resulting transfer function is:

$$T(s) = 95.24 * \frac{s}{s+238.09} * \frac{s}{s+124} * \frac{1.0396E8}{s+1.0396E8} * \frac{5E8}{s+5E8}$$

Then, the low frequency and high frequency 3dB point are calculated as follows:

$$f_{L3dB} = \sqrt{f_{LP1}^{\ 2} + f_{LP2}^{\ 2}} = 42.79\, Hz \qquad f_{H3dB} = \sqrt{f_{HP1}^{\ 2} + f_{HP2}^{\ 2}} = 81.28\, MHz$$

B. AC Simulation and Percent Error of Low-Frequency 3-dB Point

An AC simulation is run using the circuit in Figure 3.2, and the Bode plot shown in figure 3.6 is plotted.

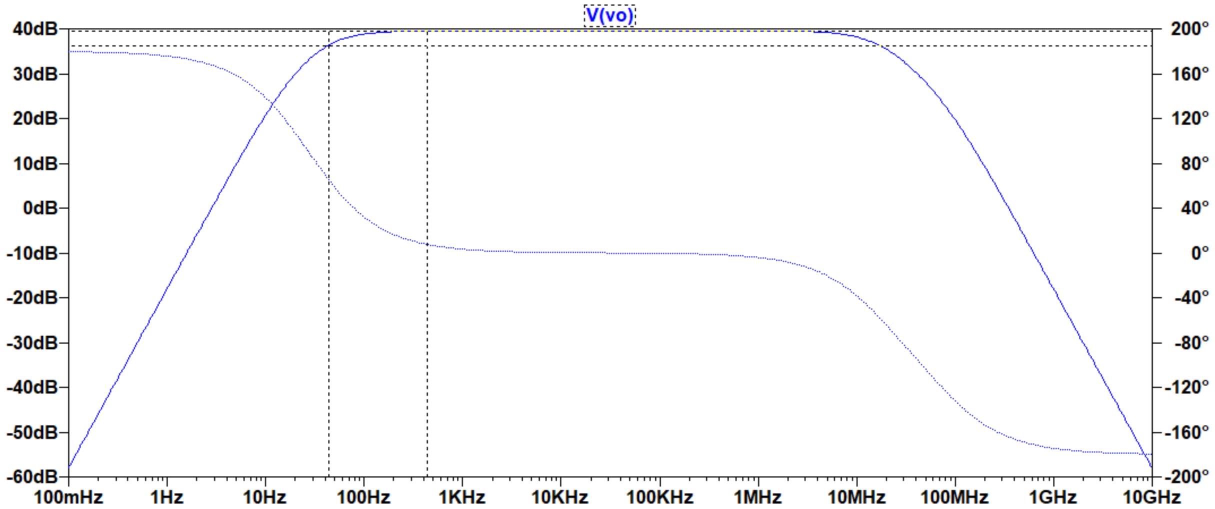

Figure 3.6: Magnitude (solid blue) and Phase (dotted blue) Bode plot of Circuit

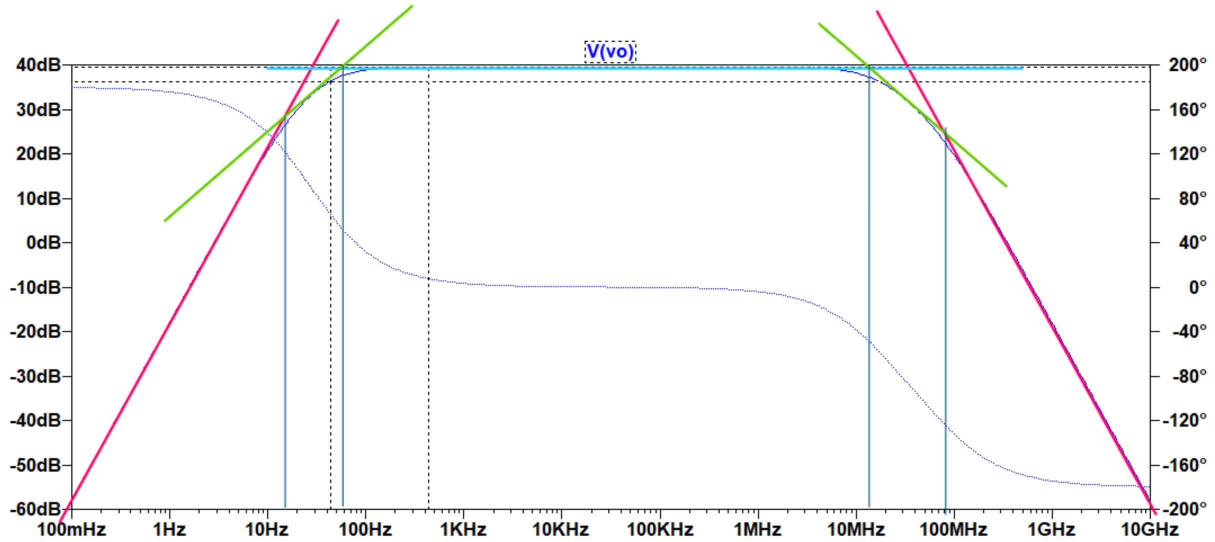

Figure 3.7: Slopes overlapped on the Bode Plot (0dB/dec in light blue, +/- 20dB/dec in green, +/- 40dB/dec in pink)

As in Part 2, A linear approximation method is used to approximate the poles of the circuit's transfer function in Figure 3.7. The approximated and calculated poles as well as 3-dB points are described in Table 3.1.

	f_{LP1}	f_{LP2}	f_{HP1}	f_{HP2}	f_{L3dB}	f_{H3dB}
Calculated	19.89 Hz	37.89 Hz	16.54 MHz	79.58 MHz	42.79 Hz	81.29 MHz
Measured	16.88 Hz	41.49 Hz	14.47 MHz	79.20 MHz	46.12 Hz	75.12 MHz

Table 3.1: Graphically Approximated and Calculated Poles and 3dB Point

The 3dB point percent error is then calculated once again and shown in Table 3.2

	f_{LP1}	f_{LP2}	f_{HP1}	f_{HP2}	f_{L3dB}	f_{H3dB}
% Error	15.13%	9.50%	12.51%	0.48%	7.78%	7.59%

Table 3.2: 3dB Point Percent Error

Discussion

The percent error results between the calculated and measured poles and 3dB points are shown in Table 3.2. Identifying pole locations graphically may result in some inaccuracies in our calculations due to cursor inaccuracies and line thicknesses. It is noted that steeper changes in slopes are more difficult to discern when approximating graphically, as shown with f_{LP1}'s large percentage error.

5. Conclusion

This project consisted in introducing computer based circuit simulation tools and designing common electronic circuits such as filters. Each Part of this project includes an AC simulation component in which a magnitude and phase Bode plot is plotted from a modeled circuit. In Part 1, a Three-Pole Low-Pass filter is designed, calculating and choosing adequate capacitor values. In Part 2, we explored a Four-Pole RC Filter's frequency response, and compared pole values and 3dB frequencies obtained graphically by calculation. Finally, in Part 3, a basic Transconductance Amplifier is analyzed, applying Miller's theorem to simplify the circuit, before calculating percent error for pole values and 3dB frequencies.
This first project provided the base knowledge and tools required to solve a variety of complex circuits, providing different analysis methods theoretically and graphically.

6. References

1. ELEC 301 Course Notes.
2. 2. A. Sedra and K. Smith, "Microelectronic Circuits," 5 th Ed., Oxford University Press, New York.
3. LTSpice™ User's Manual.