Electric Flux and Flux Density

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Electric Flux Ψ (C)
- Total Q enclosed

$Q = \Psi$

Electric Flux Density D (C/m²)

$D = \frac{\Psi}{A}$ C/m²

$D = \varepsilon_0 \vec{E}$ (in free space)
- Flow of electric field through surface

ex.

$r<a$: $D = \frac{Q}{4\pi a^2}\hat{a}_r$ C/m²

$a<r<b$: $D = \frac{Q}{4\pi r^2}\hat{a}_r$ C/m²

$r>b$: $D = \frac{Q}{4\pi b^2}\hat{a}_r$ C/m²

☆ Webwork 3

Assignment 3: Problem 2

(1 point)
Let C denote the parametric curve detailed below:

$$x = \sqrt{\frac{\pi}{t}}\cos(t),\ y = \sqrt{\frac{\pi}{t}}\sin(t),\ z = -\frac{1}{\sqrt{x^2+y^2}},\ \pi \le t \le 64\pi.$$

Given $\mathbf{E} = \langle -4z^2e^x,\ 2e^z,\ 2ye^z - 8ze^x \rangle$, evaluate the following line integral.

$$\int_C -\mathbf{E} \bullet d\mathbf{L} = \underline{\qquad}.$$

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You have attempted this problem 0 times.
You have unlimited attempts remaining.

$\nabla f = F$

$\frac{\partial V}{\partial x} = -4z^2e^x \longrightarrow \int -4z^2e^x\,dx = -4z^2e^x + C(y,z)$

$\frac{\partial V}{\partial y} = 2e^z \longrightarrow \int 2e^z\,dy = 2e^zy + C(x,z)$

$\frac{\partial V}{\partial z} = 2ye^z - 8ze^x \longrightarrow \int 2ye^z - 8ze^x\,dz = 2ye^z - 4z^2e^x + C(x,y)$

$-\nabla V = -4z^2e^x + 2e^zy + C$

$V_x = -4z^2e^x + 2e^zy + C = -\vec{E}_x$
$V_y = 2e^z = -\vec{E}_y$
$V_z = -8ze^x + 2e^zy = -\vec{E}_z$

Endpoints: $\pi \le t \le 64\pi$
$t=\pi$: $x=-1,\ y=0,\ z=-1$
$t=6\pi$: $x=1/8,\ y=0,\ z=-8$

$\rightarrow$ plug x,y,z: $f\Big|_{t=\pi}^{t=64\pi} = \left[-4(-1)^2e^{(-1)} + (0)\right] - \left[-4(-8)^2e^{(1/8)} + (0)\right] = 288.61448622$

Assignment 3: Problem 3

(1 point)
Consider the vector-valued function below, in which A, B, and C are constants:

$$\mathbf{E}(x,y,z) = \langle y^3z,\ Axy^2z + Bz^7\cos(yz^7),\ xy^3 + 14z^Cy\cos(yz^7)\rangle. \rightarrow f$$

Given that $\mathbf{E}$ is a true electric field, find the constants in its definition.

ANSWER: $A =$ ___ , $B =$ ___ , $C =$ ___ .

Note: *You can earn partial credit on this problem.*

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$f: \nabla f = F$

$f_x = y^3z\,dx \rightarrow \int y^3z\,dx = y^3zx + C_0(y,z) \xrightarrow{\therefore f \text{ satisfies}} f = y^3zx + C_0(y,z)$

$f_y = Axy^2z + Bz^7\cos(yz^7) = \frac{1}{3}Axy^3z + B\sin(yz^7) + C_0(x,z)$

$f_z = xy^3 + 14z^Cy\cos(yz^7)$

$f_y = 3y^2zx + C_y(y,z) = f_y = Axy^2z + Bz^7\cos(yz^7)$
$\hookrightarrow = 0,\ f(0)dy = C_0(z)$
$\therefore f = y^3zx + C_0(z)$
$f_z = 3y^2x + C_0'(z) = f_z = xy^3 + 14z^Cy\cos(yz^7)$
$\hookrightarrow C_0'(z) = 0,\ \int 0\,dz = C_0$
$\therefore f = y^3zx + C_0$

Assignment 3: Problem 4 ☆!!

bounds

(1 point)
Let C denote the curve of intersection between $x = -3z^3$ and $y = -3z^2$, starting from $A(0,0,0)$ and ending at $B(-24,-12,2)$. Given $\mathbf{G} = \langle 5\sin(x),\ -1\cos(y),\ 9xz\rangle$, evaluate the following line integral.

$$\int_C -\mathbf{G} \bullet d\mathbf{L} = \underline{\qquad}.$$

$\int \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}'(t)\,dt$

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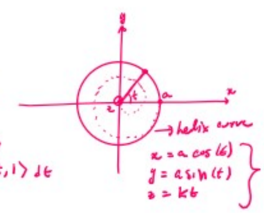
① **Parametrize**
let $z=t$: $x=-3t^3,\ y=-3t^2$
$0\le t\le 2$ $\therefore$ parametric eqn: $\vec{r}(t) = \langle -3t^3, -3t^2, t\rangle$
$\hookrightarrow \because z$ bounds: $(0,0,0) \rightarrow (-24,-12,2)$

② $\vec{r}'(t) = \langle -9t^2, -6t, 1\rangle$

③ $G(r(t)) = \langle 5\sin(-3t^3), -1\cos(-3t^2), 9(-3t^3)(t)\rangle$

④ $\int_0^2 \langle 5\sin(-3t^3), -1\cos(-3t^2), 9(-3t^3)(t)\rangle \cdot \langle -9t^2, -6t, 1\rangle\,dt$

pos? ⑤ $= 170.457467$



$\rightarrow$ helix curve

$$\left.\begin{aligned} x &= a\cos(t) \\ y &= a\sin(t) \\ z &= kt \end{aligned}\right\}\ 0\le t\le\pi \rightarrow \text{parameterization for curve}$$

Assignment 3: Problem 5 ☆!!

(1 point) Suppose $\vec{F}(x,y) = x^2\vec{i} + y^2\vec{j}$ and C is the line segment segment from point $P = (1,4)$ to $Q = (-3,-1)$.

(a) Find a vector parametric equation $\vec{r}(t)$ for the line segment C so that points P and Q correspond to $t=0$ and $t=1$, respectively.

$\vec{r}(t) =$ ___

(b) Using the parametrization in part (a), the line integral of $\vec{F}$ along C is

$$\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt = \int_a^b \underline{\qquad}\,dt$$

with limits of integration $a =$ ___ and $b =$ ___

(c) Evaluate the line integral in part (b).

① **Parametrization of line segments**
$\vec{r}(t) = (1-t)\langle x_0,y_0,z_0\rangle + t\langle x_1,y_1,z_1\rangle \longrightarrow 0\le t\le 1$
$F(t) = (1-t)\langle 1,4\rangle + t\langle -3,-1\rangle$

② **Vector Field along curve**
$\vec{r}(t) = \langle 1-t, 4-4t\rangle + \langle -3t, -t\rangle = \langle 1-4t, 4-5t\rangle$
$\vec{F}(x,y) = \langle x^2, y^2\rangle$
$\vec{F}(\vec{r}(t)) = \langle (1-4t)^2, (4-5t)^2\rangle$

③ **Derivative of parametrization**
$\vec{r}'(t) = \langle -4, -5\rangle$

④ **Dot product**
$\vec{F}(\vec{r}(t))\cdot\vec{r}'(t) = \langle (1-4t)^2, (4-5t)^2\rangle\cdot\langle -4,-5\rangle = -4(1-4t)^2 - 5(4-5t)^2$

⑤ **Integrate**
$\int \vec{F}(\vec{r}(t))\cdot\vec{r}'(t) = \int_0^1 -4(1-4t)^2 - 5(4-5t)^2 = -31$

Path independent if:
$\vec{F} = A\hat{i} + B\hat{j} + C\hat{k}$
- for $u(x,y,z)$,

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dt

with limits of integration $a =$ [ ] and $b =$ [ ]

(c) Evaluate the line integral in part (b).

[ ]

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Assignment 3: Problem 6

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(1 point) Determine whether each of the following vector fields $\vec{F}$ is path independent (conservative) or not. If it is path independent, enter a potential function for it. If it is path dependent, enter *NONE*.

(a) If $\vec{F}(x, y) = \langle 3x^2 - y^2, -2xy \rangle$, then
$f(x, y) =$ [ ]

(b) If $\vec{F}(x, y) = \dfrac{4y}{x+6}\vec{i} + 4\ln(x+6)\vec{j}$, then
$f(x, y) =$ [ ]

(c) If $\vec{F}(x, y) = 5y\sin(xy)\vec{i} + 5x\sin(xy)\vec{j}$, then
$f(x, y) =$ [ ]

Note: *You can earn partial credit on this problem.*

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You have attempted this problem 0 times.
You have unlimited attempts remaining.

Path independent if:

$\vec{F} = A\hat{i} + B\hat{j} + C\hat{k}$

– for $u(x,y,z)$,
$\vec{F} = \nabla u$, $\frac{\partial u}{\partial x} = A$, $\frac{\partial u}{\partial y} = B$, $\frac{\partial u}{\partial z} = C$

– Line integral along C from $P_1 \to P_2$: $\int_C \vec{F}(\vec{r}) = \int_C A\,dx + B\,dy + C\,dz = u(P_1) - u(P_2)$

– For closed contour C: $\oint_C \vec{F}(\vec{r}) \cdot d\vec{r} = 0$

– Test for conservative field:
rot $\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A & B & C \end{bmatrix}$

– For conservative field, $\dfrac{\partial F_1}{\partial y} = \dfrac{\partial F_2}{\partial x}$

1

a) $\nabla F = f(x,y) = \langle \frac{\partial F}{\partial y}, \frac{\partial F}{\partial x} \rangle = \langle -2y, -2y \rangle$
$\because \frac{\partial F}{\partial x} = \frac{\partial F}{\partial y}$ $\therefore$ conservative ($f_x = f_y$)

Potential func. for vector field:
$\frac{\partial f}{\partial x} = 3x^2 - y^2$ $\frac{\partial f}{\partial y} = -2xy$

$f = \int 3x^2 - y^2\,dx = x^3 - y^2x + C(y)$
Find $C(y)$:
$\frac{\partial f}{\partial y} = -2yx + C'(y) = -2xy = \frac{\partial f}{\partial y}$
$\therefore C'(y) = 0$ $C(y) = \int 0\,dy = C$
$\therefore f = x^3 - y^2x + C$