Lagrange Interpolation

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2.2 Lagrange Interpolation

- Find polynomial $p(x)$ of lowest possible degree that passes through n points
- n points, n eqns. $p(x_i) = y_i$, $p(x) \rightarrow$ polynomial deg $n-1$

$$p(x) = a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n$$

- Find $a_1, \dots a_n \longrightarrow$ plug in data $p(x_1) = y_1, \dots p(x_n) = y_n$

$$p(x_1) = a_1 x_1^{n-1} + \dots + a_{n-1} x_1 + a_n$$
$$p(x_2) = a_1 x_2^{n-1} + \dots + a_{n-1} x_2 + a_n$$

- Matrix Notation:

$$\begin{bmatrix} x_1^{n-1} & x_1^{n-2} & \cdots & x_1^2 & x_1 & 1 \\ x_2^{n-1} & x_2^{n-2} & \cdots & x_2^2 & x_2 & 1 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ x_n^{n-1} & x_n^{n-2} & \cdots & x_n^2 & x_n & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n-2} \\ a_{n-1} \\ a_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Vandermonde Matrix

Vandermonde Matrix

ex.

$$\begin{bmatrix} 2^2 & 2^1 & 2^0 \\ 3^2 & 3^1 & 3^0 \\ 5^2 & 5^1 & 5^0 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ 9 & 3 & 1 \\ 25 & 5 & 1 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix}\right) = -(x_2 - x_1)(x_3 - x_1)(x_3 - x_1)$$

- $\therefore$ For $i > j$: $\det([\text{Vandermonde}]) = \pm \prod_{i>j} (x_i - x_j)$

MATLAB:

`V = vander(x)`

- coefficients are given by $V\vec{a} = \vec{y} \xrightarrow{\text{solve}} \vec{a} = V \backslash \vec{y}$

`polyval(a, x)`: takes vector $\vec{x}$ of n values $(x_1, \dots x_n)$, returns values $p(x_1), \dots p(x_n)$

↳ More pts = better plot, but too many pts $\longrightarrow$ noise