Vector Spaces

February 6, 2020 11:00 AM

ex. $V = [0,1] \neq$ vector space

$\because$ no additive inverse of any $x \in [0,1]$ for $x \neq 0$; $\frac{3}{4} + \frac{3}{4} \notin [0,1]$

but $3/4 \in [0,1]$

$\therefore$ not closed under addition

1.1 Vector Subspaces

- $V \to$ vector space w/ scalar F, subset $S \subset V$ is a subspace of V if:
  - $\forall$ $u, v \in S$ & $a, b \in F$

$$\begin{cases} ① \ u + v \in S \\ ② \ au \in S \end{cases} \quad \text{OR: } ③ \ au + bv \in S$$

ex1. $S = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \right\}$ is a subspace of $V = \mathbb{R}^3$

let $u = \begin{bmatrix} x_1 \\ y_1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} x_2 \\ y_2 \\ 0 \end{bmatrix}$ be arbitrary vectors in S,

let $a, b \in \mathbb{R}$, then:

$au + bv = \begin{bmatrix} ax_1 + bx_2 \\ ay_1 + by_2 \\ 0 \end{bmatrix} \in S$ $\quad \therefore S$ is a subspace $V = \mathbb{R}^3$

ex2. $V \to$ set of all real value func. on $\mathbb{R}$, $F = \mathbb{R}$

P_n: all polynomials of degree n or less

Is $P_n \to$ subspace of V?

let $p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} \ldots + a_1 x + a_0$, $g(x) = b_n x^n + b_{n-1} x^{n-1} + b_{n-2} x^{n-2} + \ldots + b_1 x + b_0$

be in P_n, let $s, t \in \mathbb{R}$ be two scalars

then $sp(x) + tg(x) = (sa_n + tb_n) x^n + \ldots + sa_0 + tb_0$ is in P_n,

$\therefore P_n =$ subspace of V