L18-19 Nyquist Stability Criterion

Saturday, June 19, 2021 10:42 PM

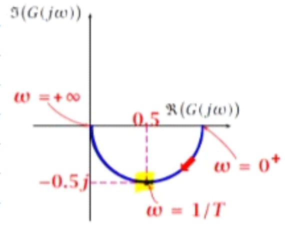
CL Stability Criterion

- CL stability → Ch. Eqn: $1+L(s)=0$, $L(s)=G(s)C(s)$
  - ↳ stable if Ch. Eqn. → all poles in LHP
  - ↳ Nyquist stability criterion

Nyquist Plot

- Amplitude & Phase on single plot
- $Im\{G(j\omega)\}$ vs $Re\{G(j\omega)\}$
1. Start plot → $\omega=0$
2. end plot → $\omega=\infty$
3. plot crosses Re-axis → $Im(G(j\omega))=0$
4. plot crosses Im-axis → $Re(G(j\omega))=0$

	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
$\omega=0^+$	1	0
$\omega=\frac{1}{T}$	$\frac{1}{\sqrt{2}}$	$-45°$
$\omega=+\infty$	0	$-90°$

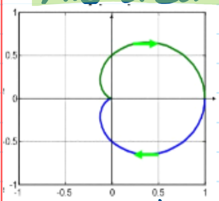


- 1st order $G(j\omega)=\frac{1}{1+j\omega T}$

$|G(j\omega)|=\frac{1}{\sqrt{1+\omega^2T^2}}$, $\angle G(j\omega)=-\tan^{-1}(\omega T)=-\tan^{-1}(\omega T)$

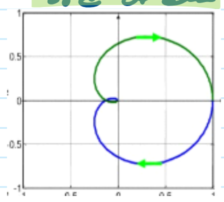
- ↳ Start → $\omega=0^+$: $|G(j\omega)|=\frac{1}{\sqrt{1+0}}=1$, $\angle G(j0^+)=-\tan^{-1}(0^+T)=0°$
- ↳ End → $\omega=\infty^+$: $|G(j\omega)|=\frac{1}{\sqrt{1+\infty}}=0$, $\angle G(j(+\infty))=-\tan^{-1}(+\infty T)=-90°$
- ↳ Mid → $\omega=1/T$: $|G(j\omega)|=\frac{1}{\sqrt{1+1}}=\frac{1}{\sqrt{2}}$, $\angle G(j\,1/T)=-\tan^{-1}(1/T\cdot T)=-45°$

- 2nd order



$L(s)=\frac{1}{(s+1)^2}$

- 3rd order

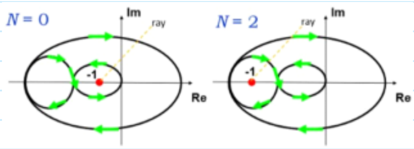


$L(s)=\frac{1}{(s+1)^3}$

- Pros: Can study CL stability w/ OLTF even if FRF unstable or has time delay
- Cons: Controller Design → difficult. Easier on Bode Plot

Nyquist Stability Criterion

- CL stable → $Z=P+N=0$
  - ↳ Z = # CL poles in RHP
  - ↳ P = # OL poles in RHP
  - ↳ N = # CW/CCW encirclements → CW = (−), CCW = (+)
- If Nyquist plot passes pt. −1, CL → pole on Im-axis
  ∴ Marginally stable
- For $P=0$
  - ↳ If $N=0$ → stable

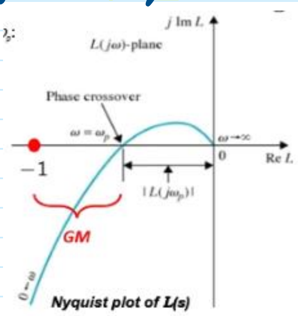


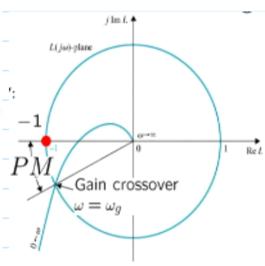
Relative Stability

- How much is CL stable?
  - ↳ Distance from critical pt. −1

Gain Margin

- Additional Gain to make system on verge of instability
- Phase Cross-over freq ω_p
  $\angle L(j\omega_p)=-180°$
- Gain Margin
  $GM=20\log\frac{1}{|L(j\omega_p)|}$
- Sometimes not enough to analyze



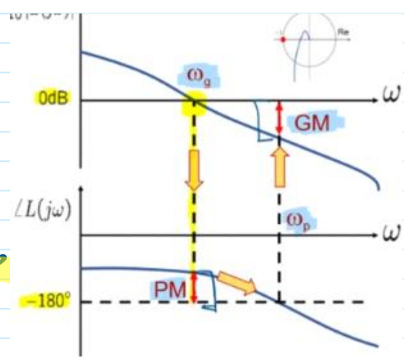


Phase Margin

- Additional phase lag to make system on verge of instability
- Gain Cross-over freq ω_g
  $|L(j\omega_g)|=1$
- Phase Margin
  $PM=\angle L(j\omega_g)+180°$

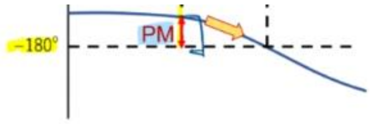
Relative Stability on Bode Plot

1. Find intersection of Gain w/ 0dB → ω_g
2. From ω_g, draw line, intersect Phase, Vertical dist. b/w intersection & −180° → PM
3. In Phase plot, find intersection of phase w/ horizontal −180° line → ω_p
4. From ω_p, find intersection of line, intersect Gain, vertical dist. b/w ... & 0dB → GM



(3) In phase plot, find intersection of phase w/ horizontal $-180°$ line $\rightarrow \omega_p$

(4) From ω_p, find intersection of line, intersect Gain, vertical dist b/w pt. & 0dB $\rightarrow$ GM



Stability w/ GM & PM

- ↑GM = ↑stability
- ↑PM = ↑stability
- Stable : GM & PM = (+)
  OR PM > GM
- Marginally : GM & PM = 0
  OR PM = GM
- Unstable : GM or PM = (−)
  OR PM < GM