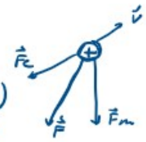
Magnetic Forces and Torque

April 28, 2020 1:05 PM

Magnetic Force

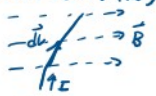
Forces on Moving charges



$\vec{F}_e = Q\vec{E}$, $\vec{F}_m = Q\vec{v}\times\vec{B}$ $\therefore \vec{F} = Q(\vec{E} + \vec{v}\times\vec{B})$

→ $\vec{F}_e$ in direction of $\vec{E}$

→ $\vec{F}_m$ normal to $\vec{v}$ & $\vec{B}$



Force on current element

$\vec{F} = Q\vec{v}\times\vec{B}$
$d\vec{F} = I\,d\vec{L}\times\vec{B}$ → $\boxed{\vec{F} = \oint I\,d\vec{L}\times\vec{B}}$
$\boxed{= -I\oint \vec{B}\times d\vec{L}}$

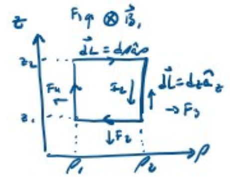
– In uniform field and straight conductor

$\boxed{\vec{F} = I\vec{L}\times\vec{B}}$

ex.

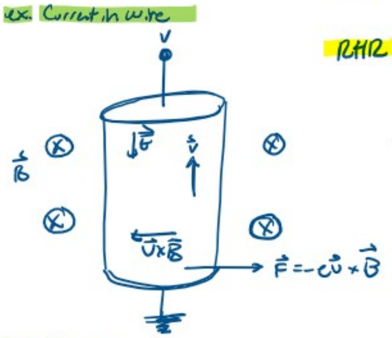
– Find force on wire loop by inf. line of current

$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi\rho}\hat{a}_\varphi$ $\vec{F} = -I_2\oint \vec{B}_1 \times d\vec{L}$



$$\begin{aligned}
\vec{F} &= -\frac{\mu_0 I_1 I_2}{2\pi}\oint \frac{1}{\rho}\hat{a}_\varphi \times d\vec{L} \\
&= -\frac{\mu_0 I_1 I_2}{2\pi}\left(\oint_{\rho_1}^{\rho_2}\frac{1}{\rho}\hat{a}_\varphi \times d\rho\,\hat{a}_\rho + \oint_{z_2}^{z_1}\frac{1}{\rho_2}\hat{a}_\varphi\times dz\,\hat{a}_z \right. \\
&\qquad \left. + \oint_{\rho_2}^{\rho_1}\frac{1}{\rho}\hat{a}_\varphi\times d\rho\,\hat{a}_\rho + \oint_{z_1}^{z_2}\frac{1}{\rho_1}\hat{a}_\varphi\times dz\,\hat{a}_z\right) \\
&= -\frac{\mu_0 I_1 I_2}{2\pi}\left(\int_{\rho_1}^{\rho_2}\frac{d\rho}{\rho}\hat{a}_z + \int_{z_2}^{z_1}\frac{dz}{\rho_2}\hat{a}_\rho + \int_{\rho_2}^{\rho_1}\frac{d\rho}{\rho}\hat{a}_z + \int_{z_1}^{z_2}\frac{dz}{\rho_1}\hat{a}_\rho\right) \\
&= \frac{-\mu_0 I_1 I_2}{2\pi}\left(\frac{(z_1 - z_2)}{\rho_2}\hat{a}_\rho + \frac{(z_2 - z_1)}{\rho_1}\hat{a}_\rho\right) \quad \to z_2 - z_1 = l \\
&= \frac{\mu_0 I_1 I_2 l}{2\pi}\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)\hat{a}_\rho \quad \to \rho_2 - \rho_1 = w \\
&= \frac{\mu_0 I_1 I_2 l}{2\pi}\left(\frac{w}{\rho_2\rho_1}\right)\hat{a}_\rho \\
&= -\frac{\mu_0 I_1 I_2 l w}{2\pi\rho_2\rho_1}\hat{a}_\rho
\end{aligned}$$

ex. Current in wire



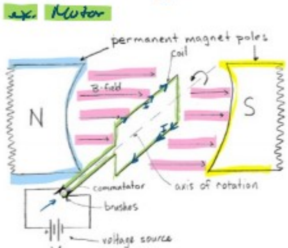
RHR: – Fingers flat pointing to $\vec{v}$
– palm facing $\vec{B}$ field
– thumb facing $\vec{v}\times\vec{B}$

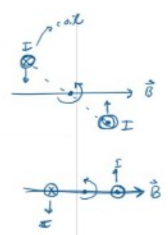
OR:

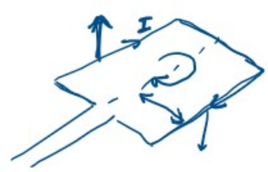
Fingers →	$\vec{B}$
Palm →	F
Thumb →	I →

→ $\vec{I}\times\vec{B}$

ex. Motor

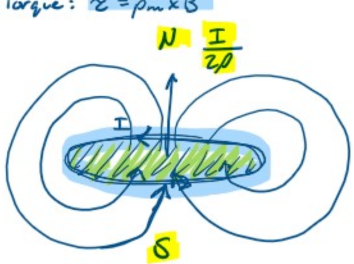


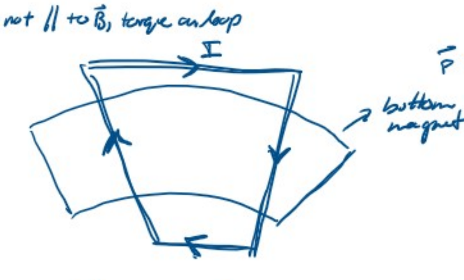




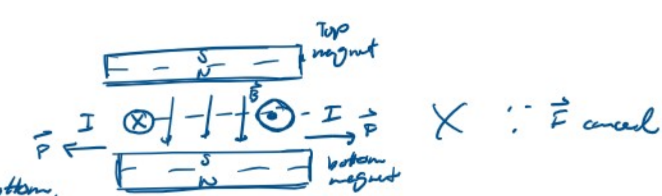
Torque on Current Loop

– If vector ⊥ area enclosed by current loop not ∥ to $\vec{B}$, torque on loop
– Magnetic Dipole Moment: $\boxed{\vec{p}_m = NIS\,\hat{a}_n}$
– Torque: $\boxed{\vec{\tau} = \vec{p}_m\times\vec{B}}$

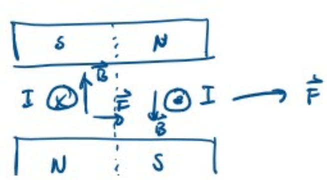




(Top view)



(side view)



if magnet put in $\vec{B}$ Field, BField will adjust