Dielectrics

Friday, December 4, 2020 8:33 PM

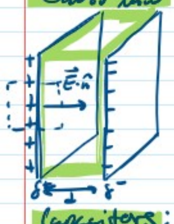
Dielectrics

- Insulator → Large Bandgap, $E_g > 3.2eV$
- Conductivity: Result of e^- & hole movement, depends on density of states $g(E)$, occupation of states in CB $f(E)$ & VB $1-f(E)$ & mobilities of carriers μ_e & μ_h
- Increase Charge storage of capacitors
  - ↳ Piezoelectrics

Fermi Dirac Function

- $(1+\exp[(E-E_F)/kT])^{-1}$
- $f(E)$ at bottom of CB is $\approx 10^{-22}$ → eg. $E_g = 2.5eV \rightarrow f(E)^{(CB)} \approx 10^{-22} = 1-f(E)^{(VB)}$

Gauss law



$\oint \vec{E}\hat{n}\,dA = \frac{Q_{enc}}{\varepsilon}$ ↳ closed S → $\varepsilon = \varepsilon_r\varepsilon_0$ → permittivity (ε_r → dielectric const.)

$\oint \vec{E}\hat{n}\,dA = EA = \frac{\sigma A}{\varepsilon_r\varepsilon_0} \rightarrow E = \frac{\sigma}{\varepsilon_r\varepsilon_0}$

$V = -\int E\,dl = -\frac{\sigma d}{\varepsilon_r\varepsilon_0} = \frac{Q}{A\varepsilon_r\varepsilon_0} \rightarrow Q = \sigma A = V\left(\frac{\varepsilon_r\varepsilon_0 A}{d}\right)$

- @ const. V, ↓d → ↑E

Capacitors: $C = \frac{Q}{V} = \frac{\varepsilon_r\varepsilon_0 A}{d}$

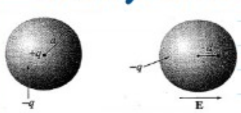
$E = \frac{1}{2}CV^2$

- ↑ε_r, ↑Q for Voltage, ↑E stored

- Dielectric const: $\varepsilon_r = (1+\chi)$ → χ = dielectric susceptibility → deg. of polarization of a dielectric in response to electric field

Atomic level

- (+) charged nucleus (q^+) surrounded by e^- cloud



- When $\vec{E}$ field app. ⟶, nucleus shifts ⟶
  - e^- cloud shifts ⟵
- @ equilibrium, $\vec{E}$ balanced by attraction b/w nucleus & e^- cloud
  - ↳ $\vec{E}$ felt by q^+ towards center: $\vec{E}_q = -\frac{qd}{4\pi\varepsilon_0 a^3}\hat{x}$, ↑d = ↑$\vec{E}$
  - ↳ @ eqm.: $\vec{E}_{ext} = -\vec{E}_q$

- Molecule has induced dipole moment: $\vec{p} = qd$
- dipole moment ∝ to external $\vec{E}$ field: $\vec{p} = 4\pi\varepsilon_0 a^3 \vec{E}_{ext} = \alpha\vec{E}_{ext}$ → α = polarizability
- E field experienced by dipole $\vec{E}_{local} \neq \vec{E}_{ext}$:
  - ↳ When $\vec{E}$ field applied, each atom has induced dipole $p = \alpha\vec{E}$

Polarization

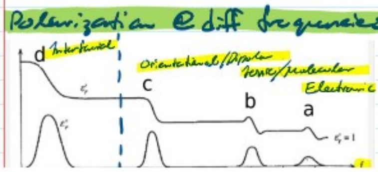
- For n atoms/V, induced dipole moment (Polarization): $P = n\vec{p} = n\alpha\vec{E}$
- Induced dipole moment/V
- @ low fields: $\vec{P} = \chi\varepsilon_0\vec{E}$ → χ in freespace = 0

Polarization: $P = \frac{\Sigma qd}{V} = \frac{12qd}{12(2a)^3} = \frac{qd}{8a^3}$

- Surface Charge Density:
  - Surface charge density: $\sigma = N_m qd = N_m\vec{p}$ → N_m = #molecules/V
  - Plane of polarized dielectric atoms @ surface: $V = Ad_{eq}$
  - dipole moment $\vec{p} = PV = PAd_{eq} = Q_{total}d_{eq}\hat{x}$
  - ∴ Surface charge density from polarization: $\sigma_{bound} = \frac{Q_{total}}{A} = |\vec{P}|$ → eq. to polarization!

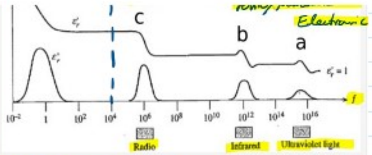
- Field inside ↓ ∵ (+) (−) cancels
  - @ const V: $\sigma_{vac.} = \frac{\varepsilon_0 V}{d} = \varepsilon_0|\vec{E}|$ $\sigma_{total} = \sigma_{free} + \sigma_{bound}$ → due to polarization
    - ↳ due to charge on electrodes
- $\oint \vec{E}\hat{n}\,dA = \frac{Q_{enc}}{\varepsilon_0}$, charge on electrode: $Q = CV$
- $EA = -\frac{\sigma A}{\varepsilon_0}$ $E = \frac{-\sigma}{\varepsilon_0} = -\frac{V}{d}$ ∴ Field inside: $E = -\frac{\sigma_{free} + \sigma_{bound}}{\varepsilon_0}$ → $\sigma_{bound} = P = P/\text{Volume}$
  - → $\sigma_{free} = |\vec{D}| = |\varepsilon\vec{E}| = |\varepsilon_r\varepsilon_0\vec{E}| = |(\chi+1)\varepsilon_0\vec{E}|$
- If field applied, pulls (+) → (−) plate & pull (−) → (+) plate ∴ charging @ material edges
  - ↳ Reduces field inside
- If const. V, ↑σ_{free} to compensate for σ_{bound}
- Electric displacement: free charge density on capacitor plates
  - $|\vec{D}| = |\chi\varepsilon_0\vec{E}| + |\varepsilon_0\vec{E}| = |\vec{P}| + |\varepsilon_0\vec{E}| = \sigma_{bound} + \sigma_{free} = \sigma_{total}$
- Dipole moments can result from polar molecules, ionic polarization, or electronic polarization
  - ↳ ε_r ↓ as f ↑ ∵ Rotations & Nuclei can't keep up w/ freq, e^- inertia too large.

Polarization @ diff frequencies



Dielectric const ε_r: $\varepsilon_r = 1 + \chi_{total} = 1 + \chi_e + \chi_i + \chi_o$

- @ low freq: ↑ε_r, all 3 types can keep up w/ changing $\vec{E}$ field
- @ mid freq: ↓ε_r, $\chi_o = 0$ ∵ can't respond fast enough ∴ $\varepsilon_r = 1 + \chi_e + \chi_o$
- @ high freq: ↓ε_r, $\chi_i = \chi_o = 0$ ∵ can't respond fast enough ∴ $\varepsilon_r = 1 + \chi_e$



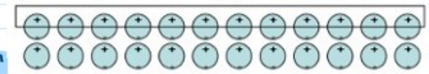
@ low freq: $\uparrow \varepsilon_r$, all 3 types can keep up w/ changing $\vec{E}$ field
@ mid freq: $\downarrow \varepsilon_r$, $\chi_o = 0$ ∵ can't respond fast enough ∴ $\varepsilon_r = 1 + \chi_e + \chi_o$
@ high freq: $\downarrow \varepsilon_r$, $\chi_i = \chi_o = 0$ ∵ can't respond fast enough ∴ $\varepsilon_r = 1 + \chi_e$
@ optical freq: use refractive index n instead of $\varepsilon_r \rightarrow n^2 \approx \varepsilon_r$

- Electronic: All materials consist of ions surrounded by e^- clouds → rapid response to field changes eg. (H_2O)
  - Electronic polarization in solid:
    ↳ Polarization of 1 atom: $\vec{p} = q d_{eq} = 4\pi\varepsilon_0 a^3 E_{ext} = \alpha E_{ext}$
    ↳ Polarization/Volume: $P = 4\pi\varepsilon_0 a^3 E_{ext} \cdot N \rightarrow N = \#\text{atoms}/V$ eg. cubic lattice spacing 2a, $1\,\text{atom} = (2a)^3$

$Q_{bound\,surface} = PA = \frac{\pi}{2}\varepsilon_0 E_{ext} A = \chi\varepsilon_0 E_{ext} A$ ∴ $N = 1/(2a)^3$ ∴ $P = \frac{\pi}{2}\varepsilon_0 E_{ext}$

$EA = \frac{Q_{bound} + Q_{free}}{\varepsilon_0} = \frac{-PA + \sigma_{free} A}{\varepsilon_0} = \frac{-\chi\varepsilon_0 AE + \sigma_{free} A}{\varepsilon_0}$

$E(\varepsilon_0 + \chi\varepsilon_0) = \sigma_{free}$ $\varepsilon_r = 1 + \chi = 1 + \frac{\pi}{2} = 2.57$

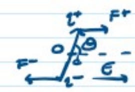


  - Index of Refraction: $v = c/n$, For materials w/ $\downarrow\chi$: $n = \sqrt{\varepsilon_r} = \sqrt{\varepsilon'}$

- Ionic/Molecular: - Lattice ions charged, material has permanent dipole (polar)
  - w/ $\vec{E}$ field, lattice ions are stretched → increase of dipole moment of lattice
  - ∵ ↑mass, ↓response speed, slower than electronic eg. (NaCl & KCl)
  - @ ↓f, force due to E_{app} balanced by electrostatic restoring force
  - @ ↑f, resonant f attached (like mass/spring)
  - Beyond resonant f, Force consumed w/ acc. mass, bd as ↑f ∴ ↓P

- Orientational/Dipolar: - $\vec{E}$ field exerts torque on molecule, aligning them to $\vec{E}$ field dir.
  - Involves stretching & rotating ∴ responds slower than ionic
  Torque: $T = Eq(d/2 \sin\theta + d/2 \sin\theta) = E\vec{p}\sin\theta$ → Apply $\vec{E}$ field, Torque & align w/ Field
  avg deg. of alignment & avg $\vec{p}$: $\bar{p} = \frac{\vec{p}^2 E}{kT}$

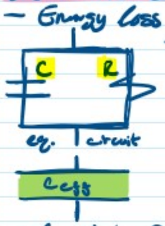
  - If dipole aligned counter to field, additional PE: $U = -\int T d\theta = -\int E\vec{p}\sin\theta\, d\theta = E\vec{p}$
  - Debye Equation: describes freq response, justify drop in ε_r due to orientation polarization
  $\varepsilon' = \frac{(\varepsilon_s - \varepsilon_\infty)\tau}{1 + \omega^2\tau^2} + \varepsilon_\infty$ $\varepsilon'' = \frac{(\varepsilon_s - \varepsilon_\infty)\omega\tau}{1 + \omega^2\tau^2}$ → τ = time const. @ which molecule responds to $\vec{E}$ field
  ε_s = dielectric const. below transition freq. ($\omega \ll 1/\tau$)
  ε_∞ = rel. dielectric const. @ freq above the but < next transition
  ↳ Relative dielectric const. $\varepsilon_r = \varepsilon' - j\varepsilon''$



  - friction due to collisions → can't keep up with $\vec{E}$ field oscillations
  - ↑f, dipole rotates faster, drag force from collisions prevent dipoles from aligning
  - freq dependence of polarization: $ma = F - kx - bv$ → m = ion or e^- cloud mass, a = accel., k = restoring F
  Transfer func. b/w P & $\vec{E}$ (susceptibility): $\frac{x}{F} = \frac{1}{ms^2 + bs + k}$ x = displacement, b = damping coeff, v = velocity
  ↳ @ ↓f: reaches resonance F = force on nuclei & e^- due to $\vec{E}$ field: $\vec{F} = q\vec{E}$
  ↳ @ ↑f: drops @ rate of $1/\omega^2$

Dielectric Loss

- Energy loss from collisions (heating material), Joule heating (small conduction currents)



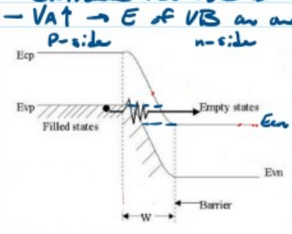
  - Given $\varepsilon_r = \varepsilon' + j\varepsilon''$; Im. of ε_r: $\varepsilon'' = \frac{\sigma}{\omega}$
  $C = Q/V$ $C = \varepsilon\frac{A}{d}$ → Find $C_{effective}$: $\frac{\varepsilon A}{d}$ + ...
  $C_{eff} = \varepsilon_{eff}\frac{A}{d}$ → Find ε_{eff}
  Laplace: $V = I_1 R = \frac{I_2}{sC} \rightarrow I = I_1 + I_2$ ∴ $V = \frac{I - \frac{V}{R}}{sC} \rightarrow V(sC + \frac{1}{R}) = I$, $V(C + \frac{1}{Rs}) = Q$
  ↳ $C + \frac{1}{Rs} = C_{eff} \rightarrow R = \frac{d}{\sigma A}$ ∴ $C_{eff} = \frac{A}{d}\left[\varepsilon + \frac{\sigma}{j\omega}\right]$
  $\varepsilon_{eff} = \varepsilon'\varepsilon_0 - j\varepsilon''\varepsilon_0 \longrightarrow \varepsilon''\varepsilon_0 = \sigma/\omega$
- C ∝ to $Re(\varepsilon_r)$, stored E: $\frac{1}{2}CV^2 = \frac{1}{2}\varepsilon A/d \cdot V^2$
  E density stored in C: $\varepsilon_r\varepsilon_0\vec{E}^2 = \frac{1}{2}\varepsilon(V/d)^2 = \frac{1}{2}\varepsilon E^2$
- To maximize stored E/V, $\uparrow\varepsilon_r$ & $\vec{E}$ (limited by dielectric breakdown) & $\downarrow\varepsilon''$

Dielectric Breakdown

- Sudden increase in Current when V beyond critical level

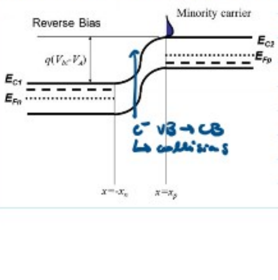
Zener Breakdown

- Extreme reverse bias in diode, tunneling of e^- carriers from VB→CB across depletion region
- VA↑ → E of VB on one side overlap w/ CB on other side



- when bias↑, VB top on p-side is level w/ CB bottom on n-side → tunneling
  ↳ occurs in heavily doped diodes (thin depletion region)

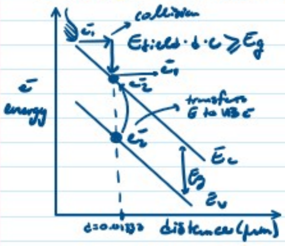
Avalanche Breakdown (same as intrinsic)



- PE gained by minority e^- → to n-side of highly reverse biased diode
  gets more energy than Eg bandgap
  ∴ Transfer Energy to VB e^- through collisions
- ↑carriers through depletion region
- ↑I → breakdown
- Avalanche occurs if: $q(V_{bi} - V_A) \gg E_g$

Intrinsic Breakdown

- Same as avalanche breakdown, but across dielectric
- e^- in CB of dielectric highly accelerated → collide w/ VB ∴ promoted to CB



- e_1^- accelerated by $\vec{E}$ field, lose Energy in collision, then promotes e_2^- into CB
- KE gained b/w collisions ; $KE = \frac{1}{2}mv^2 = Ee\lambda \geq E_g$
- $\# e^-$ promoted $\propto \# e^-$ in CB
- exponential rise : 10^8s
- ↑ depletion width → ↓ probability

Thermal Breakdown:

- Heating → easier promotion of e^- into CB, pos. feedback loop
- ↑e^- in CB → Joule heating
  - ↳ May be due to dielectric loss
- ↑ Heat, ↑ vibration → dielectric failure

Discharge Breakdown

- Gas in porous materials is ionized, damages Material; accelerates Breakdown eg. (ceramics & mica)
  - ↳ From applied Field