# Work and Potential

April 25, 2020 10:42 AM

**Force**

Force on Q due to E-Field: $F_e = Q\vec{E}$

Move in dir. $\hat{a}_L$: $\vec{F}_{EL} = \vec{F}_E \cdot \hat{a}_L$, Force applied: $\vec{F}_a = -\vec{F}_{EL}$

$\vec{E}$-Field: $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$

**Work**

— Force applied · distance moved

$dW = -Q\,\vec{E}\cdot\hat{a}_L\,dL = -Q\,\vec{E}\cdot d\vec{L} \longrightarrow W = -q\int \vec{E}\cdot d\vec{L}$ (J)

— If $Q \perp \vec{E}$, $W = 0$

ex. move $q^+$ towards $Q^+$: $W = -q\int_{\infty}^{0} \vec{E}\cdot d\vec{L} = -q\int_{\infty}^{0} \frac{Q\,dL}{4\pi\varepsilon_0 r^2}\,dr$

**Potential**

- work done by external source moving $q(+)$ from 1 pt. to another in an $\vec{E}$ field

Potential difference: $V_{AB} = -\int \vec{E}\cdot d\vec{L} = V_A - V_B$ (J/C) (V)

- (+) if work done against field

Electric Field inside Closed Surface: $\oint \vec{E}\cdot d\vec{L} = 0$

**Chart**

| Q | $\lvert\vec{E}\rvert$ | $\lvert V\rvert$ |
|---|---|---|
| Point Q | $\frac{q}{4\pi\varepsilon_0 r^2}$ $(\propto \frac{1}{r^2})$ | $\frac{Q}{4\pi\varepsilon_0 r}$ $(\propto \frac{1}{r})$ |
| Inf. Line $\rho_L$ | $\frac{\rho_L}{2\pi\varepsilon_0 \rho}$ $(\propto \frac{1}{\rho})$ | $\frac{\rho_L}{2\pi\varepsilon_0}\ln(\frac{b}{a})$ |
| Inf. Sheet $\rho_s$ | $\frac{\rho_s}{2\varepsilon_0}$ (const.) | $\frac{\rho_s r}{}$ $(\propto r)$ |

ex.


@ origin: $\vec{E} = 0$ ∴ Cancel

$V \neq 0$ ∴ From $\infty \to$ ring, work done against $\vec{E}$-Field outward from ring

$\vec{E}$: $dE = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta \longrightarrow R^2 = \rho^2 + z^2$, $\cos\theta = \frac{z}{R}$

$E(z) = \frac{Q_{total}\cdot z}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}}\,\hat{a}_z$ V/m

V: $V = -\int \vec{E}\cdot d\vec{L}$, $V = 0$ @ $\infty$

$= -\int_{\infty}^{z} \frac{Q_{total}\cdot z'}{4\pi\varepsilon_0(\rho^2+z'^2)^{3/2}}\,\hat{a}_z \cdot dz\,\hat{a}_z$

$= \frac{Q_{total}}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{\rho^2+z^2}} \longrightarrow$ @ $z=0$ $V = \frac{Q_{total}}{4\pi\varepsilon_0 \rho}$

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(1 point)
The electric field at the point $(2, 6, 4)$ is, in SI units of N/C,

$$\mathbf{E} = \frac{10^{-9}}{4\pi\epsilon_0}\langle 8, 2, 8\rangle.$$

Introducing a point charge of $-100$ nC at some point $P$ will make $\mathbf{E} = \mathbf{0}$ at the point $(2, 6, 4)$. Find $P$.

ANSWER: $P = ($ ___ , ___ , ___ $)$

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**Note:** *You can earn partial credit on this problem.*

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Superposition:


$\vec{E}_1 + \vec{E}_2 = 0$

$\vec{E}$ field from pt. q (−100) nC:

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r \longrightarrow r = \langle 2-a, 6-b, 4-c\rangle$

$= \frac{-100\times10^{-9}}{4\pi\varepsilon_0 |\langle 2-a, 6-b, 4-c\rangle|^2}\,\frac{\langle 2-a, 6-b, 4-c\rangle}{|\langle 2-a, 6-b, 4-c\rangle|}$

$\frac{10^{-9}}{4\pi\varepsilon_0}\langle 8,2,8\rangle + \frac{(-100\times10^{-9})}{4\pi\varepsilon_0 |\langle 2-a, 6-b, 4-c\rangle|^3}\langle 2-a, 6-b, 4-c\rangle = 0$

$\rightarrow$ solve & compare: $\frac{1}{((2-a)^2+(6-b)^2+(4-c)^2)^{3/2}}\langle -900(2-a), -900(6-b), -900(4-c)\rangle = -\langle 72, 18, 72\rangle$

$\therefore P = \langle -0.0542795, 5.48643, 1.94572\rangle$

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(1 point)
An electrostatic air filter uses an electric field to force charged dust particles to be deflected towards collecting plates as they pass through the unit. For the purposes of this question, model the air filter as two parallel infinite sheets of charge having uniform surface charge densities of $\rho_S$ and $-\rho_S$ respectively. Air containing dust particles flows between these two charged surfaces. If the separation of the plates is $d$ = 2 $cm$, the average dust particle radius is $r$ = 25 $\mu m$, and the density of the dust is 75 $\frac{kg}{m^3}$, estimate the necessary surface charge density $\rho_S$ on the collecting plates if the force on a dust particle having a charge of $q$ = 3 x $10^{-15}$ $C$ must counteract that of gravity in order for the filter to be effective.

$\rho_S$ = ___ $\frac{C}{m^2}$

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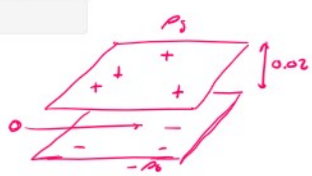


$F_e = F_g$

$F_g = mg = (25\times10^{-6})^3\,\frac{4}{3}\cdot 75\,(9.8) = 4.810564\times10^{-11}$ N

$\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}\,\hat{a}_z \times 2$ plates

$\vec{F}_e = \vec{E}_s \cdot q = \frac{\rho_s q}{\varepsilon_0}$

$\frac{\rho_s\cdot(3\times10^{-15})}{\varepsilon_0} = 4.810564\times10^{-11}$

$\rightarrow$ solve: $\rho_s = 1.4197898\times10^{-7}$ C/m²

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# Assignment 1: Problem 5

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(1 point)
The line segment joining $P_0(3,0,0)$ to $P_1(9,0,0)$ has a uniform linear charge with density $\rho_L = -7$ nC/m. No other charges are present.

**(a)** Find the electric field at the origin:
$\mathbf{E}(0,0,0) =$ ______ N/C.

**(b)** Find the total charge, $Q$, on the given segment:
$Q =$ ______ C.

**(c)** Imagine replacing the given segment with a point charge $Q$, with $Q$ given in part (b). Where should this point charge be placed so that the electric field at the origin is identical with the one found in part (a)?
$P =$ ______ m.

**WeBWorK note:** For non-scalar objects, bracket shape matters. WeBWorK interprets $(1,2,3)$ as a *point*, but $\langle 1,2,3\rangle$ as a *vector*.


a) $\vec{E}$-Field @ origin
$E = \frac{q}{4\pi\varepsilon_0 r^2}\hat{a}_r$  $dE = \frac{dQ}{4\pi\varepsilon_0 r^2}\langle 1,0,0\rangle \longrightarrow dQ = \rho_L\, dr$
$\vec{E}_L = \int_3^9 \frac{\rho_L}{4\pi\varepsilon_0 r^2}\, dr\, \hat{a}_x$
$= \langle 14,0,0\rangle$ V/m

b) Total charge
$Q_{Total} = \int_3^9 \rho_L\, dr = -4.2\times10^{-8}$ C

c) $\vec{E} = \langle 14,0,0\rangle$  $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{a}_r$
$14 = \frac{-4.2\times10^{-8}}{4\pi\varepsilon_0 r^2}$
$\longrightarrow$ solve: $r = 5.196152$ m

**Note:** *You can earn partial credit on this problem.*

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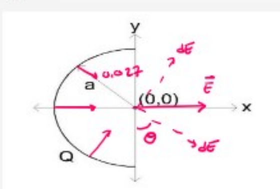
(1 point)
A total charge $Q = 1$ nC is uniformly distributed along a semicircular ring of radius $a = 2.7$ cm and negligible thickness (e.g., a line charge). The semicircle is positioned relative to the origin as shown in the figure provided.

**Figure:**




a) $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{a}_x$
$d\vec{E}_x = \frac{dQ}{4\pi\varepsilon_0 r^2}\sin\theta \longrightarrow dQ = \rho_L\, ds$
$\vec{E}_x = \int_0^\pi \frac{\rho_L}{4\pi\varepsilon_0 r^2}\sin\theta\, ds \longrightarrow s = \theta r$, $ds = r\,d\theta$
$= \int_0^\pi \frac{\rho_L}{4\pi\varepsilon_0 r}\sin\theta\, d\theta$
$\longrightarrow \rho_L = \frac{Q_{Total}}{\pi r}$ (half circle)
$= \frac{\frac{Q_{Total}}{\pi r}}{4\pi\varepsilon_0 r}\int_0^\pi \sin\theta\, d\theta = 7859.50336$ V/m $\hat{a}_x$

**(a)** Find the magnitude of the electric field at the origin $(0,0)$.

$E =$ ______ V/m

b) $\langle 1,0,1\rangle$

**(b)** Give the unit vector that defines the direction of this electric field at $(0,0)$.

______ $\hat{x}$ + ______ $\hat{y}$ + ______ $\hat{z}$


**Note:** *You can earn partial credit on this problem.*

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(1 point)
In free space, a point charge $Q_1 = 4e\ C$ is located at $(0,0,0)$ and a point charge $Q_2 = -e\ C$ is located at $(8,0,0)$.

**a)** Find the potential at point $P\ (3,-5,1)$ due to these two point charges.

$V =$ ______ $V$

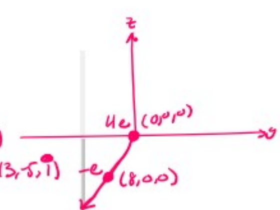
**b)** It is known that the equipotential surface having $V = 0\ V$ is a spherical surface. Find the location of the origin of the equipotential surface having $V = 0\ V$.

$O = ($ ______ , ______ , ______ $)$

**c)** Find the radius $R$ of the equipotential surface having $V = 0\ V$.

$R =$ ______

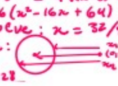
a)




$V = \frac{Q}{4\pi\varepsilon_0 r}$
$V_{Total} = V_1 + V_2$
$V_1 = \frac{4e}{4\pi\varepsilon_0 r} \longrightarrow r = |\langle 3,-5,1\rangle| = \sqrt{35}$
$V_2 = \frac{-e}{4\pi\varepsilon_0 r} \longrightarrow r = |\langle 3-8,-5,1\rangle| = \sqrt{51}$
$V_T = \frac{4e}{4\pi\varepsilon_0\sqrt{35}} - \frac{e}{4\pi\varepsilon_0\sqrt{51}} = 7.71458\times10^{-10}$ V

b)

$V_T = 0$
V surface due to 2pt. Q = 0
$V_T = V_1 + V_2 = 0 \longrightarrow V = \frac{Q}{4\pi\varepsilon_0 r}$
$r_1 = |\langle x,y,z\rangle|$
$r_2 = |\langle x,y,z\rangle - \langle 8,0,0\rangle$
$0 = \frac{4e}{4\pi\varepsilon_0\sqrt{x^2+y^2+z^2}} - \frac{e}{4\pi\varepsilon_0\sqrt{(x-8)^2+y^2+z^2}}$
$\sqrt{x^2+y^2+z^2} = 4\sqrt{(x-8)^2+y^2+z^2} \longrightarrow y=0, z=0$
$x^2 = 16(x^2-16x+64)$
$\longrightarrow$ solve: $x = 32/5,\ 32/3$
$\therefore$ Circle:




$\frac{x_1+x_2}{2}$ = origin
$\therefore O = 128$

c) $R = x_2 - O$
$= \frac{32}{3} - \frac{128}{15}$
$= \frac{32}{15}$

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**c)** Find the radius $R$ of the equipotential surface having $V = 0\ V$.

$R =$

**Note:** *You can earn partial credit on this problem.*

$\sqrt{x^2+y^2+z^2} = 4\sqrt{(x-8)^2+y^2+z^2} \longrightarrow y=0,\ z=0$

$x^2 = 16(x^2-16x+64)$

$\longrightarrow$ solve: $x = 32/5,\ 32/3$

$\because$ Circle: $x_1$ $(0,0,0)$ $x_2$ $\qquad \frac{x_1+x_2}{2} = \text{origin}$

$\therefore O = \frac{128}{15}$

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(1 point)
The electric field in free space is given by

$\mathbf{E} = y\mathbf{a}_x + (x+9z)\mathbf{a}_y + 9y\mathbf{a}_z \quad \mathrm{V/m}.$

Given that $V(2,1,1) = 25$ Volts, find $V(2,6,4)$.

ANSWER: $V(2,6,4) =$ Volts

$\vec{E} = \langle y, (x+9z), 9y \rangle$

$V(2,1,1) = 25$

$V(2,6,4) = ?$

① From $(2,1,1) \rightarrow (2,6,1)$

$\int_1^6 x+9z\, dy \longrightarrow z=1$

$= 55$

② From $(2,6,1) \rightarrow (2,6,4)$

$\int_1^4 9y\, dz \longrightarrow y=6$

$= 162$

$V(2,6,1) = V(2,1,1) - 55 - 162$

$= 25 - 55 - 162$

$= -192\ V$

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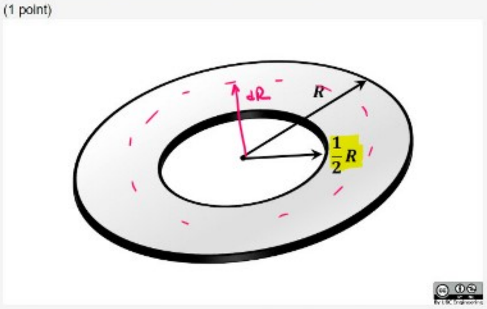
(1 point)




A uniform surface charge density $\rho_S = 10\ nC/m^2$ is present on the region defined by $z = 0,\ 0.95 < \rho < 1.9$ in free space. Find the potential at point $(0,0,0)$.

$V =$ $V$

$|V| = \frac{\rho_s}{2\varepsilon_0}$

$= \frac{(10\times10^{-9})(1.9-0.095)}{2\cdot\varepsilon_0}$

$= 536.469\ V$

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