MT 2 Content WW4-8

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Webwork 4 → Q2, Q3, Q5, Q6

→ Gauss law: (line, cylind. spherical shells)

(cylindrical shell) (spherical volume)

$\Psi = Q_{enc}$, $\Psi_L = \int_0^L \rho_L \cdot dl$, $\Psi_S = \iint_S \rho_S\, ds = \int_0^L\int_0^{2\pi} \rho_S \cdot \rho\, d\varphi\, dl$, $\Psi_V = \iiint_V \rho_V \cdot dv = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho_v \cdot \rho^2 \sin\theta\, d\rho\, d\theta\, d\varphi$

$\Psi = \oint \vec{D} \cdot \vec{ds} = \iint_S \vec{F} \cdot \underbrace{\hat{n}\, dS}_{\text{Area}}$

$\vec{D} = \oint \vec{E} \cdot dA = \dfrac{\Psi}{A} = \vec{E} \cdot \varepsilon$ → Electric Flux density

flux 1

→ Flux through Surface: $\int \vec{F} \cdot d\vec{S} = \iint_S \vec{F} \cdot \hat{n}\, dS$ → ① Parametrize: $\vec{r} = \langle x, y, s(x,y) \rangle$ ② partials $\frac{\partial \vec{r}}{\partial x} \times \frac{\partial \vec{r}}{\partial y}$ ③ cross ④ flux $\Psi = \oint \vec{D} \cdot d\vec{S} = \iint_S \vec{F} \cdot \underbrace{\hat{n}\, dS}_{A} = (\vec{F} \cdot \hat{n}) S$ (simple case)

→ Triangle Area/$\hat{n}$: $x + 2y + 3z = 6$ → set 2 coord to zero

$\vec{P_1P_2} \times \vec{P_1P_3}$ → $A_{para.}$

→ Flux through polar surface: ① Parametrize:

$x = r\cos u$, $y = r\sin u$ $\}\ 0 \le u \le 2\pi$

$z = v$ $\}\ 0 \le v \le h$

② $\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}$ ③ $\int_0^{2\pi}\int_0^{h} \vec{F}(\vec{r}(u,v)) \cdot \left(\pm \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right) du\, dv$

Webwork 5 → Q2, Q4, Q5, Q9

flux 2 → Divergence Theorem: (closed surface) $\oiint_S \vec{D} \cdot \hat{n}\, dS = Q_{enc} \xrightarrow{\text{cts. } \rho_v} \iiint_R \rho_v\, dv = \iiint_R (\vec{\nabla} \cdot \vec{D})\, dv$ → $\rho_v = \vec{\nabla} \cdot \vec{D}$

→ Divergence:

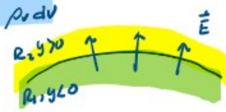
Cartesian: $\vec{\nabla} \cdot \vec{F}(x,y,z) = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}$

Polar/Cyl: $\vec{\nabla} \cdot \vec{F}(\rho, \varphi, z) = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cdot F_\rho) + \frac{1}{\rho}\frac{\partial F_\varphi}{\partial \varphi} + \frac{\partial F_z}{\partial z}$

Spherical: $\vec{\nabla} \cdot \vec{F}(\rho, \theta, \varphi) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(F_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial \varphi}$

→ E, ρ_v, Q_{enc}: $\vec{E} = -\nabla V$, $\rho_v = \nabla \cdot \vec{D}$, $\vec{D} = \varepsilon_0 \vec{E}$, $Q_{enc} = \iiint_V \rho_v\, dv$

→ Boundary Conditions: $\vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$ → $\varepsilon_1 = \varepsilon_r \varepsilon_0$, $\varepsilon_1 \vec{E}_1 = \varepsilon_2 \vec{E}_2$



→ Current Density: $I = \iint_S \vec{J} \cdot d\vec{S} = \iint_S \vec{J} \cdot \hat{n}\, dS$ → dS = diff. surface

Webwork 6 → Q3, Q4, Q5, Q6

→ Boundary conditions: $\varepsilon_1 \vec{D}_1 = \varepsilon_2 \vec{D}_2$

Tangent/Normal components: $\vec{E}_N = \text{Proj}_{\vec{n}} \vec{E} = \frac{\vec{E} \cdot \vec{n}}{\vec{n} \cdot \vec{n}} \cdot \vec{n}$, $\vec{E}_N = \text{proj}_{\hat{n}} \vec{E} = (\vec{E} \cdot \hat{n}) \cdot \hat{n}$

$\vec{E} = \vec{E}_T + \vec{E}_N$, $D = \varepsilon_r \varepsilon_0 \vec{E}$, $D_{N1} = D_{N2}$, $\vec{E}_{T1} = \vec{E}_{T2}$ → cts., $\vec{D} = \vec{D}_T + \vec{D}_N$

→ Capacitors Dialectrics:

Cylindrical Cap: $C_{TOT} = C_1 \| C_2$, $C = \frac{Q}{V}$ → $Q = \rho_L \cdot L$, $\vec{E} = \frac{\rho_L}{2\pi \varepsilon_r \varepsilon_0 \rho} \hat{a}_\rho$, $V = \int \vec{E} \cdot d\rho\, \hat{a}_\rho = \frac{\rho_L \cdot \ln(b/a)}{\pi \varepsilon_r \varepsilon_0}$

Spherical Cap: $V = -\int_a^b \vec{E} \cdot d\rho$, $\vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2}$

→ Conductor Resistance: $R = \frac{V_{ab}}{I} = \frac{-\int_a^b \vec{E} \cdot d\vec{L}}{\iint_S \vec{J} \cdot d\vec{S}}$ → $\vec{J} = \sigma \vec{E}$

Webwork 7 → Q2, Q3, Q6

→ Magnetic Flux: segment: $\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\varphi$ $H_{TOT} = H_1 + H_2$

→ Biot Savart: loop: $\vec{H} = \int \frac{I\, d\vec{L} \times \hat{a}_R}{4\pi R^2}$ → $I d\vec{L} = I \cdot \rho\, d\varphi\, \hat{a}_\varphi$

$\vec{R} = \rho \hat{a}_\rho + z \hat{a}_z$, $\hat{a}_R = \frac{\langle \rho, z \rangle}{\sqrt{z^2 + \rho^2}}$

$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dL \sin\theta}{R^2}$

Webwork 8 → Q2, Q3, Q4, Q5

→ Stokes Theorem: (capping surface) $\oint_C \vec{F} \cdot d\vec{L} = \iint_S (\nabla \cdot \vec{F}) \cdot \hat{n}\, dS$ → $(\nabla \cdot \vec{F}) \cdot \hat{n} \cdot S$

→ current density: $\vec{J} = \vec{\nabla} \times \vec{H}$, $\vec{J} = \sigma \vec{E}$, $\vec{E} = \frac{\Delta V}{d}$

→ Magnetic Flux: $\Phi = \iint_S \vec{B} \cdot d\vec{S} = \iint_S \vec{B} \cdot \hat{n} \cdot dS$ → $B = \mu_0 \frac{I}{2\pi\rho}$