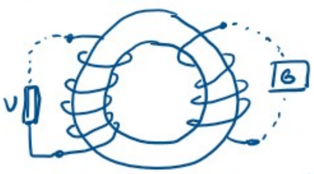
Induced EMF

April 28, 2020 2:49 PM

Faraday's Law

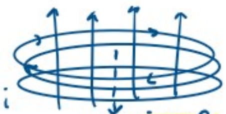


Induced Electromotive Force (EMF)

$$V_{emf} = -\frac{d\varphi(t)}{dt} \longrightarrow \varphi(t) = \iint_S \vec{B}\cdot\vec{dS}$$

↳ effect oppose initial change in external flux

→ Induced emf opposes the change in magnetic flux



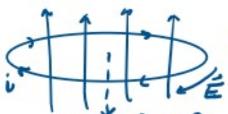
opposing φ

Increasing $\varphi(t)$ → Change B, S, B & S to get time varying flux

→ Induced EMF creates current in CW dir. to reduce passing flux

→ For multiple loops: $V_{emf} = -N\frac{d\varphi(t)}{dt}$

Stationary loop in time varying B-field



opposing φ

Increasing $\vec{B}(t)$

$$\varphi(t) = \iint_S \vec{B}(t)\cdot\vec{dS}$$

$$V_{emf} = -\frac{d\varphi(t)}{dt}$$

$$= -\iint_S \frac{d\vec{B}(t)}{dt}\cdot dS$$

Electric field caused by induced EMF

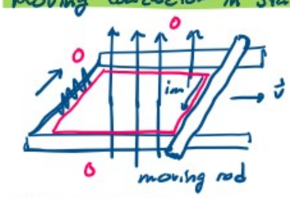
$$emf = \oint_L \vec{E}\cdot\vec{dL} \neq \vec{0}$$

$$-\int \frac{d\vec{B}(t)}{dt}\cdot dS = \oint_L \vec{E}\cdot\vec{dL} = \iint_S (\nabla\times\vec{E})\cdot dS$$

$$\therefore \nabla\times\vec{E} = -\frac{d\vec{B}(t)}{dt}$$ → when have $\vec{B}$ varying by time

↳ $\vec{E}$ = non-conservative

Moving Conductor in Static B-Field



Increasing Area

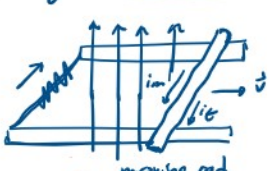
$$\vec{F} = Q\vec{v}\times\vec{B}$$

$$\vec{E}_m = \frac{\vec{F}}{Q} = \vec{v}\times\vec{B}$$

$$emf = \int_L \vec{E}\cdot\vec{dL} = \oint(\vec{v}\times\vec{B})\cdot\vec{dL}$$

↳ integral only non-zero for moving rod $\because \vec{B}$ is static for non-moving rods.

Moving Conductor in time varying B-Field



Increasing $\vec{B}(t)$

Increasing Area

$$emf = \oint \vec{E}\cdot\vec{dL}$$

$$= \iint_S \frac{d\vec{B}(t)}{dt}\cdot dS + \oint_L (\vec{v}\times\vec{B}(t))\cdot\vec{dL}$$

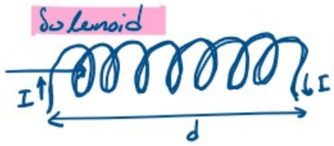
$$\nabla\times\vec{E} = \nabla\times\vec{E}_t + \nabla\times\vec{E}_m$$

$$\nabla\times\vec{E} = -\frac{d\vec{B}(t)}{dt} + \nabla\times(\vec{v}\times\vec{B})$$

↳ Induced $\vec{E}$ = non-conservative

Solenoids & Toroids

Solenoid



→ N-Turns

→ L (inductor)

→ can be filled in the middle with magnetic material

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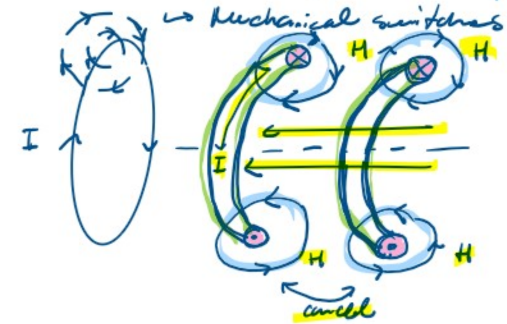
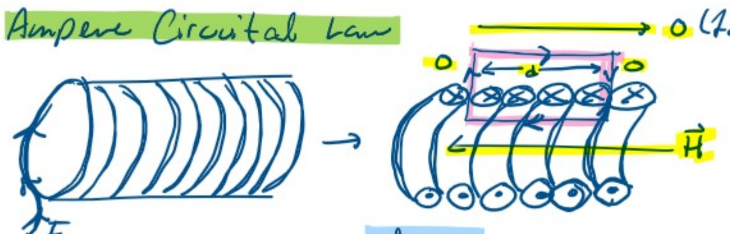
— **Relay**: Field generated in center of structure to pull magnetic material into its core
↳ Solid piece of material pulled into core
↳ Mechanical switch
↳ Core makes contact with conductor at other end of solenoid creates conductive path to flow
↳ Mechanical switches can handle higher loads



∴ H-Fields cancel, uniform fields in middle of solenoid
∴ Field concentrated in center
— Fields low on outside (Ideally 0)

Quantitative Analysis → Solenoids

Ampere Circuital Law

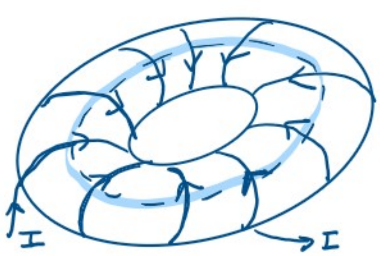


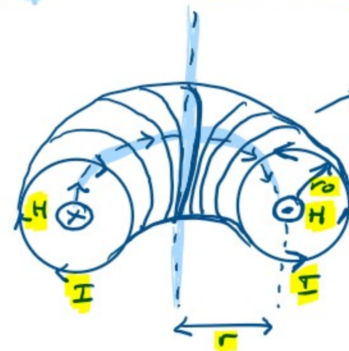
Using ampere's law & treating as sheet current
→ ⊥ to $\vec{H}$ field ∴ = 0
→ Contribution out = 0
→ contribution in = H

$$\oint H \cdot dl = I_{enc.} \rightarrow H(d) = NI$$

$$H = \frac{NI}{d} \quad A/m$$ dir. along axis

Toroids





$$\oint H \cdot dl = I_{enc.} \rightarrow H(2\pi r) = NI$$

$$H = \frac{NI}{2\pi r} \hat{a}_\varphi \quad A/m$$ → in dir. of current flow