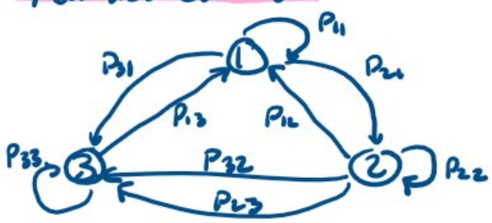
Markov Chains, Google Page Rank

April 12, 2020 12:31 AM

Markov Chains



- P_{ij} = probabilities to transition from state j to state i ⟶ Transition probabilities

a) $0 \le P_{ij} \le 1$

b) $\begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12} & P_{22} & P_{32} \\ P_{13} & P_{23} & P_{33} \end{bmatrix} \begin{matrix} =1 \\ =1 \\ =1 \end{matrix}$

- Let x_{ni} = probability of being at state i at time n
- $0 \le x_{ni} \le 1$ & $x_{n1} + x_{n2} + x_{n3} = 1$
- State vector: $x_n = \begin{bmatrix} x_{n1} \\ x_{n2} \\ x_{n3} \end{bmatrix}$

Find state of system at time n+1 x_{n+1}

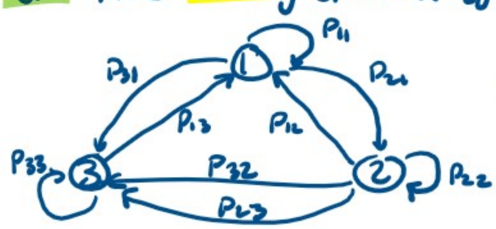
$$x_{n+1} = \begin{bmatrix} x_{n+1,1} \\ x_{n+2,2} \\ x_{n+3,3} \end{bmatrix} = \begin{bmatrix} P_{11} & \cdots & \cdot \\ \vdots & & \vdots \\ P_{13} & \cdot & P_{33} \end{bmatrix} x_n$$

↳ $x_{n+1} = P x_n \longrightarrow x_n = P^n x_0$ ⟶ x_0 = init. state
⟶ n = # time steps
⟶ x_n = curr. state

- For K states, each col. add up to 1

↳ stochastic matrices

ex. Find steady state of system (at ∞ time steps) of system



$$P = \begin{array}{c|ccc} E \backslash S & 1 & 2 & 3 \\ \hline 1 & 3/4 & 0 & 0 \\ 2 & 1/8 & 3/4 & 1/4 \\ 3 & 1/8 & 1/4 & 3/4 \end{array} \qquad x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

- After 10 time steps: $x_{10} = P^{10} x_0 = \begin{bmatrix} 0.06 \\ 0.47 \\ 0.47 \end{bmatrix}$
- After 100 time steps: $x_{100} = P^{100} x_0 = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$

$\therefore P^k x_0 \longrightarrow \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$ as $k \to \infty$

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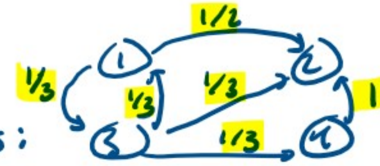
Google Page Rank

- www w/ 4 websites:



\# links out of:
1 : 2
2 : 0
3 : 3
4 : 1

$$P = \begin{array}{c} \\ \\ \end{array}\begin{array}{c|cccc} i\backslash j & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1/3 & 0 \\ 2 & 1/2 & 0 & 1/3 & 1 \\ 3 & 1/2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1/3 & 0 \end{array} \qquad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

↳ if on 2 but no links, randomly choose a website to go next

- For stochastic: $S = \begin{bmatrix} 0 & 1/4 & 1/3 & 0 \\ 1/2 & 1/4 & 1/3 & 1 \\ 1/2 & 1/4 & 0 & 0 \\ 0 & 1/4 & 1/3 & 0 \end{bmatrix}$

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