Sampling

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ADC Sampling

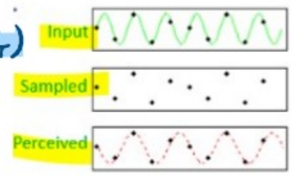
$x(t)$ discrete → [Sampler T_S] → $x_1[n]$ cnt's → [Quantizer] → $x_2[n]$ → [Encoder] → $x_3[n]$

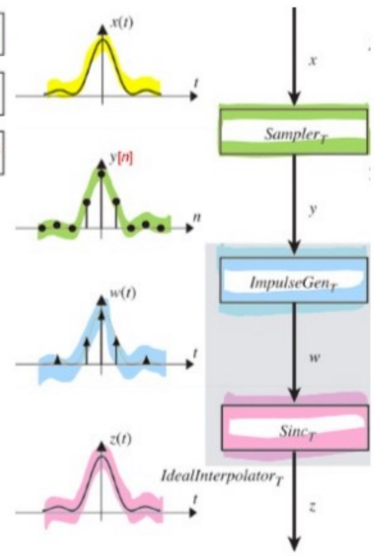
Sampler

- generates DT signal from CT
  - ↳ T_S = sampling interval, $f_S = 1/T_S$ = sampling freq.
- **Aliasing**: causes diff. signals to become indistinguishable when sampled

Reconstruction (from DT signal)

- Ideal Interpolator: $h(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \text{sinc}(t/T)$
  - $\mathcal{F}$ ↳ $H(\omega) = \begin{cases} T, & -\pi/T < \omega < \pi/T \\ 0 & \text{otherwise} \end{cases}$





Nyquist-Shannon Sampling

- Information is preserved by sampled signal $x_s(t)$ w/ samples $x(nT_s)$ if sampling freq. $\omega_s > 2\omega_{max}$ → $\omega_s = 2\pi/T_s$
  OR: $f_s = \frac{1}{T_s} > \frac{\omega_{max}}{\pi}$
- If Nyquist Rate condition satisfied, $x(t)$ reconstructed by passing $x_s(t)$ through ideal low pass filter $H(j\omega) = \begin{cases} T_s & -\omega_s/2 < \omega < \omega_s/2 \\ 0 & \text{otherwise} \end{cases}$
- Nyquist frequency:
  - for sampling rate f_s: $f_{Nyquist} = f_s/2$ ($X(f) = 0$ for $f \geq f_{Nyquist}$)
- Nyquist (sampling) Rate:
  - for max band limited freq. f_{max}: $f_{sampling} > 2 f_{max}$

ex. $x(t) = A\cos(2\pi(96)t + \varphi)$ → $f_{max} = 96\,Hz$, $f_S = 20\,Hz$

$x(t)$ → [Sampler] → $x_s(t)$ → [Ideal Interp.] → $x_r(t)$

$x_r(t) = A\cos(2\pi(96)t + \varphi)$ → $f_s > 2f_{max}$
↳ 76, 56, 36, 16, −4 ✓

$\therefore \quad = A\cos(2\pi(-4)t + \varphi)$

Nyquist Theorem

- To reproduce signal, should be sampled at 2x rate of highest freq
  $\therefore$ Nyquist rate = 2x max signal freq.

ex. $x(t) = \underbrace{1}_{\omega_1 = 0} + \underbrace{\cos(2000\pi t)}_{\omega_2 = 2000\pi} + \underbrace{\sin(4000\pi t)}_{\omega_3 = 4000\pi}$

→ max ω: $\omega_{max} = 4000\pi$

$\because$ $\omega_s > 2\omega_m = 8000\pi$ $\therefore$ Nyquist Rate $= 8000\pi$

☆ Webwork 6

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Problem Set 6: Problem 1

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Suppose that $x(t)$ is a real signal that has a Nyquist rate $\omega_0 = 45\ Hz$. Find the Nyquist rate of each of the signals in the table below.

	Signal, $y(t)$	Nyquist Rate
1	$x(t) - x(t-6)$ is not zero for all t	45
2	$\frac{d^2x(t)}{dt^2}$	45
3	$9x^2(t)$	45+45=90
4	$x(t)cos(5\omega_0 t)$	$45 + \underbrace{(2\times5\times45)}$ → N.R.

- No change to N.R for addition
- No change to N.R for diff.
- Add N.R for multiplication

Problem Set 6: Problem 4

Problem 2
Problem 3

4	$x(t)cos(5\omega_0 t)$	$45+(2\times5\times45)$ N.R.

Problem Set 6: Problem 4

Consider the signal $x(t) = 2cos(8\pi t + \pi/8)$. Determine if the signal is band-limited or not. Then for each of sampling periods $T_s = 0.1, 0.125$ and 1 sec/sample, determine if the Nyquist condition is satisfied, if the sampled signal is aliased, give the expression for the sampled signal, $x_s[n]$, as the simplest discrete-time sinusoid to be used for ideal reconstruction and determine its period.

Sampling Period, T_s	Nyquist condition satisfied?	Signal Aliased?	Sampled Signal $x_s[n]$
0.1	?	?	
0.125	?	?	
1	?	?	

$T_s = 0.1$: $\omega_s = 10$ Hz, $\omega_n = \frac{1}{2}\omega_s = 5$ Hz
$\omega_m = 8\pi$ 2π $= 4$ Hz $\longrightarrow \omega_s > 2\omega_m$ ✓
$T = 8$

$T_s = 0.125$; $\omega_s = 8$ Hz, $\omega_n = 4$ Hz
$\omega_m = 4$ Hz $\longrightarrow \omega_s > 2\omega_m$ ✗
$-8 = -4 \longrightarrow \omega_s > 2\omega_m$ ✓
$T = 2$

Problem Set 6: Problem 5

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A continuous-time signal, $x(t) = 4cos(27\pi t) + 7sin(42\pi t) + 3cos(62\pi t + \pi/7)$ is sampled at 42 Hertz. Determine the signal $\omega(t)$, reconstructed using an ideal interpolator with sampling rate $T = (1/42)$ s.

$\omega(t) =$

$\omega_s = 42$ Hz , $\omega_n = 21$ Hz

$\omega_1 = 27/2 \longrightarrow \omega_s > 2\omega_1$ ✓
$\omega_2 = 21 \longrightarrow \omega_s = 2\omega_2$?
check if $=0$: $7\sin(42\pi \cdot n) = 0$
$\omega_3 = 31 \longrightarrow \omega_s > 2\omega_3$ ✗
$-42 = -11$ ✓

$\therefore x_r(t) = 4\cos(27\pi t) + 3\cos(22\pi t - \pi/7)$

Problem Set 6: Problem 7

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Given the following discrete-time signals, compute their energy and power. If either is infinite, enter "INF".

	Signal, $x[n]$	Energy	Power
1	$14(\frac{1}{4})^n u[n]$		
2	$8e^{j\pi n}u[n]$		

$E_x = \sum_{k=-\infty}^{\infty} |x[n]|^2$, $P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x[n]|^2$

1. $E_x = \sum_{k=0}^{\infty} 14^2(1/4)^{2n} = \sum 14^2\left(\frac{1}{16}\right)^2 \longrightarrow \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} = \frac{14^2}{1-1/16}$
   $P_x = \lim_{N\to\infty} \frac{1}{2N+1}\left(\frac{14^2}{1-1/16}\right) = 0$
2. $E_x = \infty$
   $P_x = a^2/2 = 8^2/2$