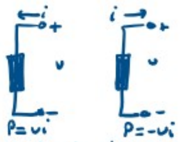
AC Power

April 21, 2020 1:13 PM



Power

- (+) if power absorbed
- (−) if power supplied

Instantaneous Power: $p(t) = v(t)\cdot i(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$

Average Power: $P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

Power Formulas

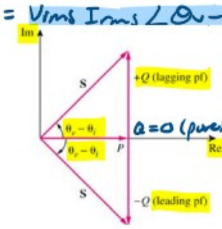
Complex Power: $S = P + jQ = V_{rms} \cdot I_{rms}^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$

Apparent Power: $|S| = V_{rms} I_{rms} = \sqrt{P^2 + Q^2}$

Real Power: $P = Re(S) = S\cos(\theta_v - \theta_i)$

Reactive Power: $Q = Im(S) = S\sin(\theta_v - \theta_i)$

Power Factor: $P/S = \cos(\theta_v - \theta_i)$



Power Triangle

$Q = 0$ (purely resistive)

Power Factor

- power factor: $PF = \cos(\theta_v - \theta_i)$
- lagging: $0 < PF < 1 \rightarrow [0, 90°]$ (+)
- leading: $0 < PF < 1 \rightarrow [-90°, 0]$ (−)
- @ −90°: PF = 0, pure reactive/capacitive, no real power consumption
- @ 90°: PF = 0, pure reactive/inductance, no real power consumption

Complex Power

$S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P} + \underbrace{j V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{jQ}$

- P → avg power delivered to load
- Q → reactive power exchange b/w source & reactive part of load
  - Resistive load (PF = 1) $Q = 0$
  - Capacitive load (leading PF) $Q < 0$
  - Inductive load (lagging PF) $Q > 0$

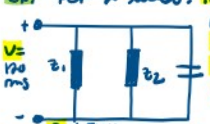
Power Factor Correction

- Increasing the power factor w/o altering the voltage/current to original load

- Conservation of AC Power:
  $S = VI^* = VI_1^* + VI_2^*$

- Add capacitor in parallel to loads
- Capacitor receives (−) reactive power

ex. For 2 loads: Total $P = 24$ kW, $PF = 0.8$ lagging, $\omega = 60$ Hz



Apparent power: $S = P/pf = 30$ kVA

$\theta = \arccos(0.8) = 36.87°$

$S = S\angle\theta = 30\angle 36.87°$ kVA

Load 1: $S_1 = P_1/pf = 2.122$ kVA

$pf \rightarrow \theta_1 = 45°$ (lag)

$S = S_1 \angle \theta_1 = 2.122 \angle 45°$ kVA

$P = 1.5$ kW

$PF_1 = 0.707$ (lag)

Load 2: Power Conservation: $S_2 = S_T - S_1 = 0.948\angle 18.42°$ kVA

$Pf_2 = \cos(18.42) = 0.949$ (lag)

Power Factor Correction: Total Reactive Power: $Q_1 + Q_2 = 1.8$ kVAR

- Add capacitor in parallel: $Q_C = -(Q_1 + Q_2) = -1.8$ kVAR
- Complex Power received by capacitor: $S_C = -j\omega|V|^2 = jQ_C$

$\rightarrow C = \frac{Q_C}{\omega|V|^2} = \frac{1.8\times10^3}{2\pi\cdot 60\cdot 120^2} = 331.6\,\mu F$

Webwork ST2

Homework Sets
ST2
Problem 3
User Settings
Grades

Problems

Problem 1
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6
Problem 7
Problem 8
Problem 9
Problem 10

ST2: Problem 3

Previous Problem | Problem List | Next Problem

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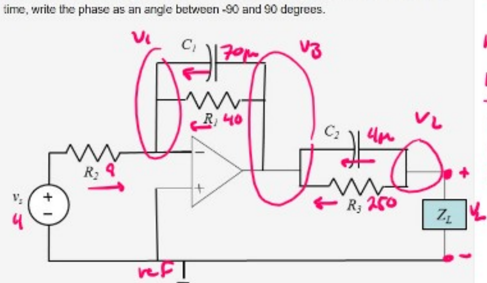
(10 points)

Let $v_s(t) = 4\cos(377t)V$, $R_1 = 40\Omega$, $R_2 = 9\Omega$, $R_3 = 250\Omega$, $C_1 = 70\mu F$ and $C_2 = 4\mu F$. Determine the complex load Z_L to maximize the average power it receives and compute the resulting values for the load: When writing the power as a function of time, write the phase as an angle between -90 and 90 degrees.



Note: Use "j" when submitting complex and imaginary numbers (e.g. submit 4+j3 for a complex number with real component 4 and imaginary component 3)

To Maximize Avg power to Z_L, $Z_L = Z_{th}^* = \left(\frac{V_{th}}{I_{oc}}\right)^*$

Thevenin w/ op-amp: 1A/2A test

1A Source $s = 377j$

KCL1: $\frac{4 - V_1}{9} + \frac{V_3 - V_1}{40} + \frac{V_L - V_1}{1/70\mu s} = 0$

KCL2: $1 = \frac{V_L - V_3}{1/4\mu s} + \frac{V_L - V_3}{250}$

NF: $V_1 = 0$

→ solve $V_{2A} = 53.9198 - 139.439068j$

$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} V_{th} \\ R_{th} \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \end{bmatrix} \rightarrow \begin{matrix} V_{th} = \\ R_{th} \end{matrix}$

Z_L maximized when $Z_L = Z_{th}^*$

2A Source $s = 377j$

1A → 2A source

→ solve: 116.24798 − 287.754029j

53.9198−139.439068j

116.24798−287.754029j

O

$S\ 3.7178 - 157.9'87068j$
$116.24798 - 287.754029j$

Note: Use "j" when submitting complex and imaginary numbers (e.g. submit 4+j3 for a complex number with real component 4 and imaginary component 3)

a. Impedance, $Z_L =$ ____ Ω

b. Apparent power received, S = 0.09138 VA

c. Average power received, P = 4.55×10^{-2} W

d. Reactive power received, Q = 3.22×10^{-2} VAR

e. Complex power received, $\mathbf{S}$ = P+Q VA

f. Instantaneous power received $p(t)$ = S + |S| $cos(754t+$? °) W

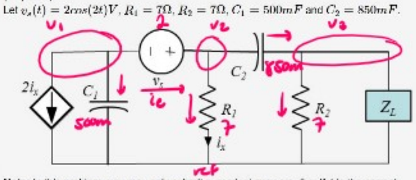
Homework Sets
ST2
Problem 4
User Settings
Grades
Problems
Problem 1
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6
Problem 7
Problem 8
Problem 9
Problem 10

ST2: Problem 4

Previous Problem | Problem List | Next Problem

KCL: $0 = 2i_x$

(10 points)
Let $v_s(t) = 2\cos(2t)V$, $R_1 = 7\Omega$, $R_2 = 7\Omega$, $C_1 = 500mF$ and $C_2 = 850mF$.



Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a. If Z_L is a purely resistive load chosen to maximize the average power it receives, the resistor value is $Z_L =$ ____ Ω

and the resulting average power is $P =$ ____ W.

b. If Z_L is a complex load chosen to maximize the average power it receives, the load value is $Z_L =$ ____ $+j$ ____ Ω

and the resulting average power is $P =$ ____ W.

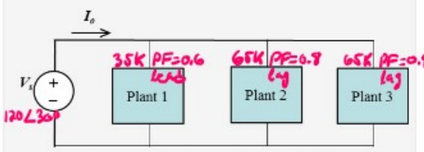
Homework Sets
ST2
Problem 5
User Settings
Grades
Problems
Problem 1
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6
Problem 7
Problem 8
Problem 9
Problem 10

ST2: Problem 5

Previous Problem | Problem List | Next Problem

!!

(10 points)
In the diagram below, the source voltage phasor is $V_s = 120\angle 30° V$ (rms) and measurements show that Plant 1 receives $35kW$ with PF 0.6 leading, plant 2 receives $65kVA$ with PF 0.8 lagging and Plant 3 receives $65kW$ with PF 0.9 lagging. Compute the current phasor I_o and the overall power factor. If the source frequency is $60Hz$, what single passive (time domain) component value should be added in parallel to bring the power factor to unity?



Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$I_o =$ ____ $\angle$ ____ °A (rms)

Overall PF: ____ ?

Component: ____ ?

a) load 1: $|S_1| = \frac{P_1}{pf} = \frac{35}{0.6} = 58.333kVA$
$\theta_1 = a\cos(0.6) = -53.13010$
$S_1 = 58.333\angle -53.13010 = (120\angle 30°)(I_{rms})^*$
$I_1 = 486.1111\angle 83.1301°$

load 2: $S_2 = 81.25\angle 36.86989° \ kVA$
$I_2 = 677.0833\angle -6.86989°$

load 3: $S_3 = 72.2222\angle 25.841933°$
$I_3 = 601.85185\angle 4.15806°$

$I_o = I_1 + I_2 + I_3 =$

b) $pf = \cos(\theta_v - \theta_i) = \cos(30° - 18.50178°) = 0.979931$ (lag)

c) Reactive power $Q = Q_1 + Q_2 + Q_3$ $\quad \omega = 120\pi$

$Q_c = \frac{|V|^2}{Z_c} = \frac{|V|^2}{1/j\omega C}$

$C = \frac{Q_c}{-\omega|V|^2} =$

Homework Sets
ST2
Problem 6
User Settings
Grades
Problems
Problem 1
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6
Problem 7
Problem 8
Problem 9

ST2: Problem 6

Previous Problem | Problem List | Next Problem

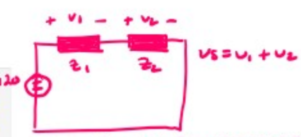
(10 points)
Consider a voltage source, $v_s(t) = 120\sqrt{2}\cos(377t)V$, connected across two loads in series $(v_s = v_1 + v_2)$. If the load voltmeter RMS readings measure $85V$ and $51.75V$, respectively, with the voltage of the first load leading that of the second (by an angle between 0 and 180 degrees), determine the phases for v_1 and v_2. If you also know that load 1 has PF 0.9 lagging, compute the PF of load 2.

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

a. Phase of $v_1(t)$: ____ °

b. Phase of $v_2(t)$: ____ °

c. Load 2 PF: ____ ?



$120\angle 0° = 85\angle\theta_1 + 51.75\angle\theta_2$
Re(): $120 = 85\cos(\theta_1) + 51.75\cos(\theta_2)$
Im(): $0 = 85j\sin(\theta_1) + 51.75j\sin(\theta_2)$
$\longrightarrow$ solve: $\theta_1 = -37.50290° = \theta_{v1}$
$\theta_2 = 21.75587° = \theta_{v2}$
Z_1: $0.9 = \cos(\theta_v - \theta_i) \longrightarrow \theta_v - \theta_i = 25.84193°$
$\theta_i = \theta_{v1} - (\theta_v - \theta_i) = -4.086054° \longrightarrow \theta_{i1} = \theta_{i2}$
Z_2: $\theta_v - \theta_i = \theta_{v2} - \theta_{i2} = -33.41684°$
$Pf_2 = \cos(\theta_v - \theta_i) = 0.83468$ leading

Homework Sets
ST2
Problem 7
User Settings
Grades
Problems
Problem 1
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6
Problem 7
Problem 8
Problem 9
Problem 10

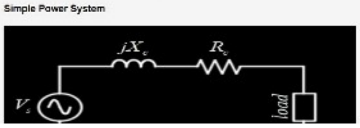
ST2: Problem 7

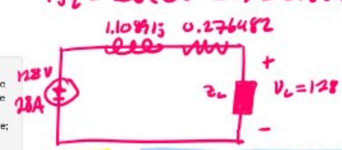
Previous Problem | Problem List | Next Problem

(10 points)
In the quitessential electric power system in the figure, a voltmeter reads the same at the source and at the load, 128 volts. At the source, an ammeter reads 28 amps. The source sees a capacitive circuit. The cable (feeder) has a resistance of 0.278482 ohms and a reactance of 1.10891 ohms. (a) What is the power factor angle, in degrees, at the source; (b) What is the active power delivered by the source; (c) What is the reactive power delivered by the source; (d) What is the power factor angle at the load, in degrees; (e) What is the active power absorbed by the load; (f) What is the reactive power absorbed by the load.

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

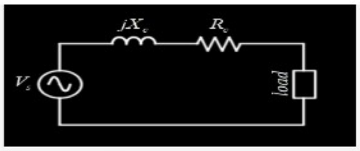
Simple Power System





$V_s\angle\theta_s = V_{th}\angle\theta_{vth} + v_L\angle\theta_{vL} = I_{rms}\cdot Z_{th} + 128\angle\theta_{vL}$
$128\angle\theta_s = (28)(0.276482 + 1.10891j) + 128\angle\theta_{vL}$
$V_s\angle\theta_s = V_s\cos(\theta_s) + jV_s\sin(\theta_s)$
$128\cos(\theta_s) + j128\sin(\theta_s) = 7.741496 + 31.04948j + 128\cos(\theta_{vL}) + j128\sin(\theta_{vL})$
Re(): $128\cos(\theta_s) = 7.741496 + 128\cos(\theta_{vL})$
Im(): $128\sin(\theta_s) = 31.04948 + 128\sin(\theta_{vL})$
a) $\longrightarrow$ solve: $\theta_s = -6.8192254° \longrightarrow$ pf angle @ source
d) $\theta_{vL} = -21.1807438$
b) $|S| = V_{rms}\cdot I_{rms} = (128)(28) = 3584VA$

Problem 7
Problem 8
Problem 9
Problem 10



(a) θ_s = deg

(b) P_s = W

(c) Q_s = VAr

(d) θ_f = deg

(e) P_f = W

(f) Q_f = VAr

Im(): $128\sin(\theta_s) = 31.04448 + 128\sin(\theta_{VL})$

a) $\longrightarrow$ solve: $\theta_s = -6.8192254°$ $\longrightarrow$ pf angle @ source

d) $\theta_{VL} = -21.1807438$

b) $|S| = V_{rms} \cdot I_{rms} = (128)(28) = 3584\,VA$

$P = Re(S) = |S|\cos(\theta_v - \theta_i) = 3584\cos(-6.8192254°) = 3558.645788\,W$

c) $Q = Im() = |S|\sin(\theta_v - \theta_i) = 3584\sin(-6.8192254°) = -425.5539376\,VAR$

e) $P = Re(S) = |S|\cos(\theta_v - \theta_i) = 3584\cos(-21.1807438°) = 3341.8839\,W$

f) $Q = Im(S) = |S|\sin(\theta_v - \theta_i) = 3584\sin(-21.8074380°) = -1294.43937831\,VAR$

Sets
ST2
Problem 9
User Settings
Grades

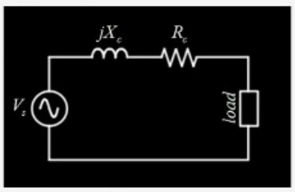
ST2: Problem 9

Previous Problem | Problem List | Next Problem

(10 points)
In the figure, the load absorbs 14.336 kVA at a power factor 0.891007 inductive. For the feeder (cable) R = 0.27 ohms, and X = 3R. The voltage at the source is 265.713 volts. What is the voltage at the load, in volts?

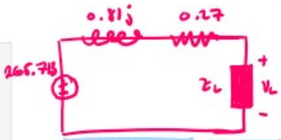
Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

Simple Power System



V_{load} = V

Problems
Problem 1
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6
Problem 7
Problem 8
Problem 9
Problem 10



Inductive $\longrightarrow$ lagging pf

$pf = \cos(\theta_v - \theta_i) \longrightarrow \theta_v - \theta_i = 26.99994°$

$S = 14336\angle 26.99994°$

$265.713\angle\theta_v = I_{rms} \cdot Z_{th} + V_L\angle\theta_L$

$265.713\cos(\theta_v) + 265.713j\sin(\theta_v) = I_{rms}(0.27 + j0.81) + V_L\cos(26.99994°) + V_L j\sin(26.99994°)$

① Re(): $265.713\cos(\theta_v) = I_{rms}(0.27) + V_L\cos(26.99994°)$

② Im(): $265.713\sin(\theta_v) = I_{rms}(0.81) + V_L\sin(26.99994°)$

$|S| = V_{rms} \cdot I_{rms}$

③ $14336 = V_L \cdot I_{rms}$

$\longrightarrow$ solve: $V_L = 224\,V$

Homework Sets
ST2
Problem 10
User Settings
Grades

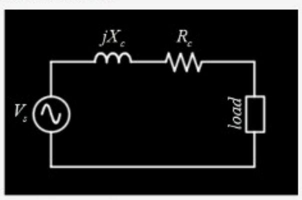
ST2: Problem 10

Previous Problem | Problem List | Next Problem

(10 points)
In the figure, a 337.38 volts source delivers 16.5316 kVA at a power factor 0.715683 inductive through a cable with R = 0.12 ohms and X = 4R to a load. Compute: (a) the voltage at the load; (b) the active power of the load; (c) the reactive power of the load and (d) the power factor angle of the load.

Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

Simple Power System



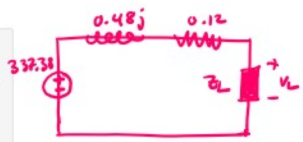
V_{load} = V

P_{load} = W

Q_{load} = VAr

Θ_{load} = deg

Problems
Problem 1
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6
Problem 7
Problem 8
Problem 9
Problem 10



pf: $\theta_v - \theta_i = a\cos(0.715683) = 44.300798°$

a) Apperent Power: $16531.6 = 337.38 \cdot I_{rms} \longrightarrow I_{rms} = 48.99994\,A$

$337.38\angle 44.300798° = (48.99994)(0.12 + 0.48j) + V_L\angle\theta_L$

$V_L\angle\theta_L = V_L\cos(\theta_L) + jV_L\sin(\theta_L)$

$241.457132 + 235.6347j = 5.87999 + 23.519971j + V_L\cos(\theta_v) + jV_L\sin(\theta_L)$

Re(): $241.457132 = 5.87999 + V_L\cos(\theta_L)$

Im(): $235.6347 = 23.519971 + V_L\sin(\theta_L)$

$\longrightarrow$ solve: $V_L = 317.000359\,V$

$\theta_L = 42°$