L17 Bode Plot

Saturday, June 19, 2021 10:38 PM

Complex roots

$s^2 + 2\cdot\zeta\omega_0 + \omega_0^2$

$(s-4j)(s+4j)$: $\omega_0 = \sqrt{0^2+4^2} = 4$ $\zeta = \cos\theta = \frac{0}{\omega_0} = 0$

$\therefore (s^2 + 2\cdot 1\cdot 4 + 4^2)$

$(s+4+4j)(s+4-4j)$ $\omega_0 = \sqrt{4^2+4^2} = \sqrt{32}$ $\zeta = \cos\theta = \frac{4}{\omega_0} = 0.7071$

$\therefore (s^2 + 2\cdot 0.7071\cdot\sqrt{32} + \sqrt{32}^2)$

$\therefore H(s) = \frac{9s(s-4j)(s+4j)}{(s+2)(s+4+4j)(s+4-4j)} = \frac{9s(s^2+2\cdot 1\cdot 4+4^2)}{(s+2)(s^2+2\cdot 0.7071\cdot 5.65685+5.65685^2)}$

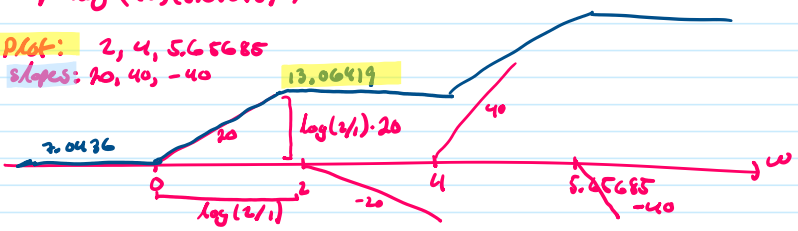
↳ zeros: 0, 4 (double)

poles: 2, 5.65685 (double)

Starting Pt.: $20\log\left(\frac{4\cdot(1)\cdot(4)^2}{(2)(5.65685)^2}\right) = 7.0436$

Amplitude Plot: 2, 4, 5.65685

slopes: 20, 40, −40

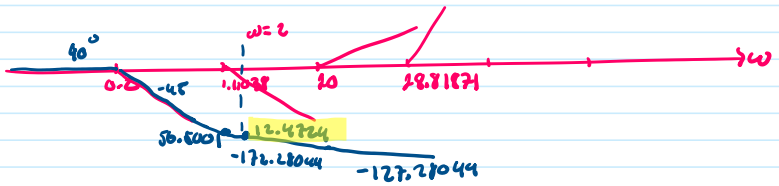


Phase Plot: For complex roots: $\omega/10^{\zeta}$, $\omega_0\cdot 10^{\zeta}$ → zeros: ⌒ poles: ⌣

0.2, 0.4, 1.11038, 10, 28.81871, 40

slopes: −45, $\frac{90}{\zeta}$, $\frac{-90}{\zeta}$, 45, $\frac{90}{\zeta}$, $\frac{-90}{\zeta}$

∞ −127.28044 127.28044 −∞



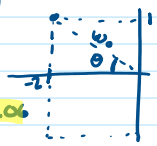
ex. $H(s) = \frac{s^3+6s^2+13s+10}{s^4+11s^3+91s^2+601s+520}$ → zeros → poles

→ Find zeros & poles: HP → cfactor( ) OR: for zeros → proot( ) ↳ (For complex conj)

$H(s) = \frac{(s+2)(s+2+j)(s+2-j)}{(s+1)(s+8)(s+1+8j)(s+1-8j)}$ → Find $s^2+2\zeta\omega_0+\omega_0^2$ → $(s+2)(s^2+2\cdot 0.89\cdot 2.24s+2.24^2)$

↳ $\omega_0 = \sqrt{65}$, $\zeta = \frac{1}{\sqrt{65}}$

$= (s+1)(s+8)(s^2+2\cdot 0.12\cdot 8.06s+8.06^2)$



$\omega_0 = \sqrt{1^2+2^2} = \sqrt{5}$

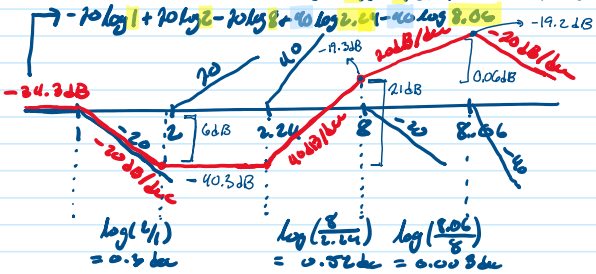
$\zeta = \cos\theta = \frac{-2}{\sqrt{5}}$

→ single poles: 1, 8, double pole: 8.06

→ single zero: 2, double zero: 2.24

Bode Plot: $H(s) = \frac{(s+2)(s^2+2\cdot 0.89\cdot 2.24s+2.24^2)}{(s+1)(s+8)(s^2+2\cdot 0.12\cdot 8.06s+8.06^2)}$

$\to -20\log 1 + 20\log 2 - 20\log 8 + 40\log 2.24 - 40\log 8.06$ → −19.21dB



① Write zeros & poles in order

② Slope contributions
↳ simple: 20dB/dec, double: 40dB/dec
↳ zero: (+), pole: (−)

④ Entry from the left?
↳ zero or pole @ 0 rad/sec?
↳ For s @ zero: ⌒
↳ For s @ pole: ⌣
↳ No pole: —

⑤ Calculate Starting point
↳ Add log of all poles & zeros
↳

⑥ Trace plot
↳ From start point, add/dec by slope contributions

⑦ Calculate Horizontal Amplitude in dB
↳ $\log\left(\frac{f_2}{f_1}\right)\cdot$ #dB/dec = drop in amplitude / increase
(Horizontal dec. · slope)

⑧ Calculate Amplitude at each pole/zero
↳ From original value, subtract (for (−) slope) or add (for (+) slope) the drop/increase

⑧ Calculate Amplitude at each pulse/zero
↳ From original value, subtract (for (-) slope)
or add (for (+) slope) the drop/increase