# ELEC 342

Lab Experiment: Lab 2 AC Transformers

Section: L1B & L1D

Bench #: Home

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Date performed: 10/12/2020

Date Submitted: 10/19/2020

# 1. Data and Parameters

## 1.1 Task 1: Measuring the DC Resistance

**Table 1: Measured DC resistance with two and four long wires using the Multimeter.**

| Measurement | Primary Side Resistance, Ohms | Secondary Side Resistance, Ohms |
|---|---|---|
| Windings plus two long wires | 0.2 | 0.6 |
| Windings plus four long wires | 0.235 | 0.645 |

Based on these measurements, calculate resistance of a single wire: $R_{\text{wire}} = \frac{0.235-0.2}{2} = 0.0175\Omega$

## 1.2A Task 2A: Open-Circuit Tests

**Table 2: Open Circuit Measurement: Nominal Condition (100%)**

| | | | | |
|---|---|---|---|---|
| $V_1(rms), V$ | $10.04\angle{-1.4°}$ | | $V_2(rms), V$ | $22.05\angle{-1.2°}$ |
| $I_1(rms), A$ | $0.33\angle{-75.4°}$ | | $I_2(rms), A$ | X |
| $P_1(ave), W$ | 0.6 | | $P_2(ave), W$ | X |
| $\phi_1 = \theta_{V1} - \theta_{I1}$ | 74° | | $\phi_2 = \theta_{V2} - \theta_{I2}$ | X |
| $a = V_1 / V_2$ | | | | 0.455 |

## 1.2B Task 2B: Determining the Transformer Polarity

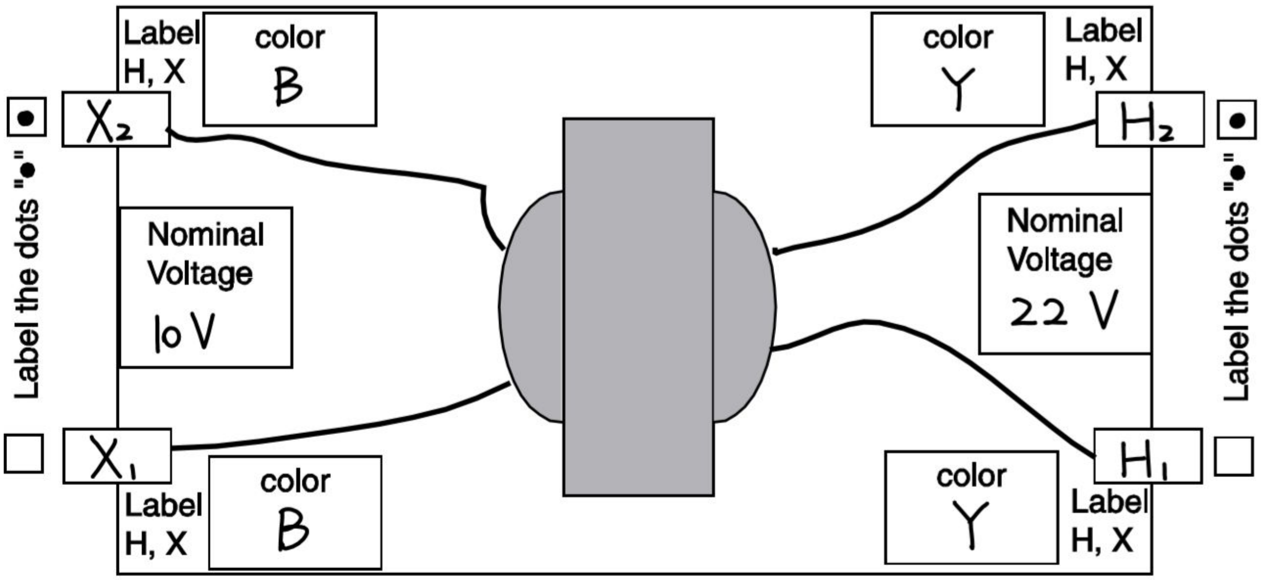



Fig. A: Transformer connection diagram

## 1.2C Task 2C: Open-Circuit Saturated Condition (up to 200%)

**Table 3: Open Circuit Measurement: Increased Voltage (~160%)**

| | | | |
|---|---|---|---|
| $V_1(rms), V$ | 15.07 | $V_2\,(rms), V$ | 32.8 |
| $I_1(rms), A$ | 2.71 | $I_2\,(rms), A$ | X |
| $P_1(ave), W$ | 3.6 | $P_2\,(ave), W$ | X |
| $\phi_1 = \theta_{V1} - \theta_{I1}$ | 83° | $\phi_2 = \theta_{V2} - \theta_{I2}$ | X |
| $a = V_1 / V_2$ | | | 0.459 |

## 1.3 Task 3: Short-Circuit Test

**Table 4: Short Circuit Measurement: Nominal Condition (up to 5A peak current)**

| | | | |
|---|---|---|---|
| $V_1(rms), V$ | $1.05\angle 3.0°$ | $V_2(rms), V$ | X |
| $I_1(rms), A$ | $3.7\angle -29°$ | $I_2(rms), A$ | $1.67\angle -29°$ |
| $P_1(ave), W$ | 3.3 | $P_2(ave), W$ | X |
| $\phi_1 = \theta_{V1} - \theta_{I1}$ | 32° | $\phi_2 = \theta_{V2} - \theta_{I2}$ | X |
| $a = I_2 / I_1$ | | | 0.451 |

## 1.4A Task 4A: Load Test using Resistive Load

**Table 5: Resistive Load Test Measurement: (up to 5 measurement points)**

| Measurement # | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $V_1(rms), V$ | 10.05∠-1.3° | 10.1∠-1.2° | 10.08∠-1.2° | 10.1∠-1.1° | 10.11∠-1.1° |
| $I_1(rms), A$ | 1.07∠-16.7° | 1.97∠-10° | 2.83∠-8° | 3.67∠-7.5° | 4.47∠-7.5° |
| $P_1(ave), W$ | 10 | 19.4 | 28.1 | 36.5 | 44.8 |
| $\phi_1 = \theta_{V1} - \theta_{I1}$ | 15° | 8.8° | 6.8° | 6.4° | 6.4° |
| $V_2(rms), V$ | 21.6∠-2° | 21.3∠-2.7° | 20.8∠-3.4° | 20.35∠-4° | 20.01∠-4.7° |
| $I_2(rms), A$ | 0.43∠-2° | 0.85∠-2.9° | 1.25∠-3.4° | 1.63∠-4° | 2.0∠-4.9° |
| $P_2(ave), W$ | 9.8 | 18.5 | 26.4 | 33.6 | 40.4 |
| $\phi_2 = \theta_{V2} - \theta_{I2}$ | 0 | 0 | 0 | 0 | 0 |
| $P_2 / P_1$ | 0.98 | 0.95 | 0.94 | 0.92 | 0.90 |
| $R_{load} = V_2 / I_2$ | 50.23 | 25.06 | 16.64 | 12.48 | 10.01 |
| VR=(V2,*oc* -V2,*load*)-V2,*load* (take the values from the last measurement) | 0.102 | | | | |

## 1.4B Task 4B: Load Test using RL (Inductive) Load

**Table 6: RL (Inductive) Load Test Measurement: (up to 5 measurement points)**

| Measurement # | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $V_1(rms), V$ | $10.1\angle -0.8°$ | $10.14\angle -0.8°$ | $10.14\angle -0.8°$ | $10.12\angle -0.9°$ | $10.06\angle -0.9°$ |
| $I_1(rms), A$ | $1.36\angle -82°$ | $1.72\angle -49°$ | $2.39\angle -32.9°$ | $3.12\angle -25°$ | $3.85\angle -20°$ |
| $P_1(ave), W$ | 2 | 11.3 | 20.3 | 28.7 | 36.5 |
| $\phi_1 = \theta_{V1} - \theta_{I1}$ | 81.2° | 48.2° | 32.1° | 24.1° | 19.1° |
| $V_2(rms), V$ | $21.82\angle 0°$ | $21.42\angle 0°$ | $21\angle -1.1°$ | $20.5\angle -2.0°$ | $20\angle -2.7°$ |
| $I_2(rms), A$ | $0.48\angle -84°$ | $0.67\angle -45.2°$ | $1.0\angle -28.5°$ | $1.35\angle -21.2°$ | $1.7\angle -17°$ |
| $P_2(ave), W$ | 1 | 10.4 | 19 | 26.6 | 33.1 |
| $\phi_2 = \theta_{V2} - \theta_I$ | 84° | 45.2° | 27.4° | 19.2° | 14.3° |
| $P_2 / P_1$ | 0.5 | 0.92 | 0.94 | 0.93 | 0.91 |
| Zload=V2/I2 | 45.46 | 31.97 | 21 | 15.19 | 11.76 |
| VR=(V2,*oc* -V2,*load*)-V2,*load* (take the values from the last measurement) | | | 0.1025 | | |

## 1.4C Task 4C: Load Test using RC (Capacitive) Load

**Table 7: RC (Capacitive) Load Test Measurement: (up to 4 measurement points)**

| Measurement # | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $V_1(rms), V$ | $10.1\angle -0.9°$ | $10.14\angle -0.9°$ | $10.11\angle -0.9°$ | $10.09\angle -1.0°$ |
| $I_1(rms), A$ | $2.08\angle 82°$ | $2.31\angle 56°$ | $2.84\angle 38°$ | $3.5\angle 26.6°$ |
| $P_1(ave), W$ | 2.2 | 12.6 | 21.8 | 30.8 |
| $\phi_1 = \theta_{V1} - \theta_{I1}$ | -82.9° | -56.9° | -38.9° | -27.6° |
| $V_2(rms), V$ | $22.92\angle -3.8°$ | $22.48\angle -4.5°$ | $21.91\angle -5.3°$ | $21.38\angle -6.1°$ |
| $I_2(rms), A$ | $1.09\angle 85.7°$ | $1.16\angle 61.2°$ | $1.35\angle 43°$ | $1.63\angle 30.7°$ |
| $P_2(ave), W$ | 0.1 | 10.2 | 20.1 | 28.2 |
| $\phi_2 = \theta_{V2} - \theta_{I2}$ | -89.5° | -65.7° | -48.3° | -36.8° |
| $P_2 / P_1$ | 0.045 | 0.81 | 0.92 | 0.92 |
| *Zload* =V2/I2 | 21.03 | 19.38 | 16.23 | 13.12 |
| VR=(V2,*oc* -V2,*load*)-V2,*load* (take the values from the last measurement) | 0.031 | | | |

### 1.5 Task 5:Recording Inrush Current

- See 2. Calculations and Analysis

# 2. Calculations and Analysis

### 2.6A Task 6A: Determining the Equivalent Circuit Parameters

**How close are the values calculated from the Short-Circuit Test to what you measured in Task 1 using the Multimeter? Explain what might have contributed to possible differences in these results**

- Open Circuit Test:
  - $Rc = \frac{Voc^2}{Poc} = \frac{10.04^2}{0.6} = 168.0027\Omega$
  - $Ic = \frac{Voc}{Rc} = \frac{10.04}{168.0027} = 0.0598A$
  - $Im = \sqrt{Ioc^2 - Ic^2} = \sqrt{0.33^2 - 0.0598^2} = 0.3245A$
  - $Xm = \frac{V1}{Im} = \frac{10.04}{0.3245} = 30.9357\Omega$

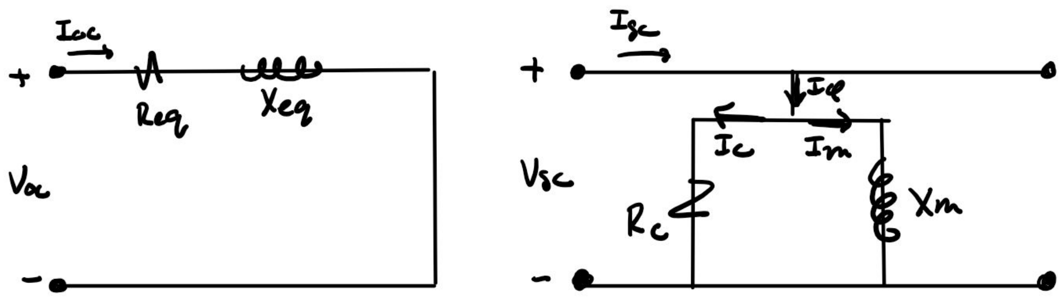



Figure 2.6-1: Open Circuit Test (left) and Short Circuit Test (right)

- Short Circuit Test:
  - $Req = \frac{Psc}{Isc^2} = \frac{3.3}{3.7^2} = 0.2411\Omega$
  - $Zeq = \frac{Vsc}{Isc} = \frac{1.05}{3.7} = 0.2838\Omega$
  - $Xeq = \sqrt{Zeq^2 - Req^2} = \sqrt{0.2838^2 - 0.2411^2} = 0.1497\Omega$
  - $Xl1 = Xl2' = \frac{Xeq}{2} = \frac{0.1497}{2} = 0.0749\Omega$
  - $R1 = R2' = \frac{Req}{2} = \frac{0.2411}{2} = 0.1206\Omega$

- The values calculated from the short circuit test and the values measured in Task 1 using the multimeter are approximately 0.0411**Ω** apart which is quite close. The possible difference to these two values may be due to the long wires added to the DC circuit, adding additional resistance.

**Write down your parameters in Fig. B in appropriate boxes. State whether you have used this transformer as a Step-Up or Step-Down transformer**

- As this transformer goes from LV to HV, this is a step up transformer.

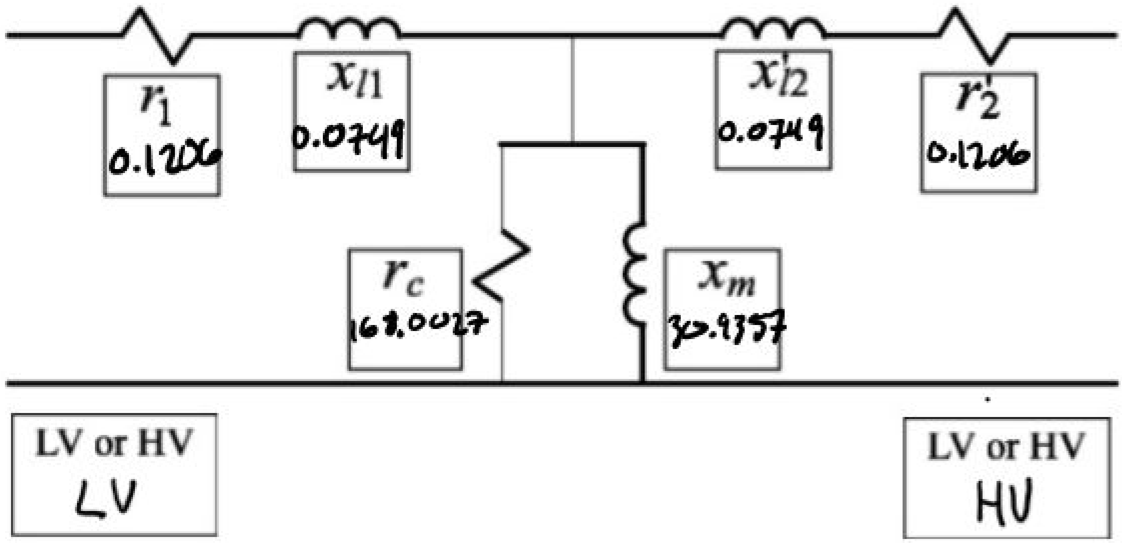
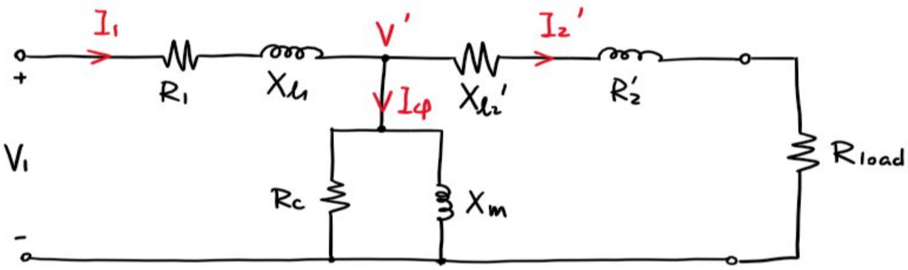



Fig. B: Transformer equivalent circuit diagram (to be completed for report). Here you must write the values referred to the Primary Side, as they come directly from your calculations. Note: the Primary Side (on the left) may be LV or HV Side depending on your initial choice.

## 2.6B Task 6B: Equivalent Circuit vs. Measured Comparison

**Use the equivalent circuit of Fig. B as the model for your Transformer. Assume the same primary voltages V1 and the load resistance Rload as were obtained in Task 4 A, step (c), and recorded in Table 5, as the input values, and then calculate the remaining values in Table 8**




eq1: $$I_1 = \frac{V_1 - V'}{R_1 + jX_{l1}}$$

eq2: $$I_2' = \frac{V'}{jX_{l2}' + R_2' + R_{load}}$$

eq3: $$I_\phi = \frac{V'}{R_c \parallel jX_m}$$

eq4: $$I_1 = I_2' + I_\phi$$

Figure 2.6-2: Transformer Equivalent Circuit and Equations

**Table 8: Resistive Load Test: Calculated values based on your equivalent circuit.**

| Calculated points # | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $V_1(rms), V$ | 10.05∠-1.3° | 10.1∠-1.2° | 10.08∠-1.2° | 10.1∠-1.1° | 10.11∠-1.1° |
| *Rload=V2*/I2 | 50.23 | 25.06 | 16.64 | 12.48 | 10.01 |
| $I_1(rms)$, $A$ | 1.05∠-19.6° | 1.94∠-12° | 2.81∠-9.7° | 3.65∠-8.8° | 4.43∠-8.6° |
| $P_1(ave), W$ | 10.02 | 19.25 | 28.01 | 36.53 | 44.4 |
| $\phi_1 = \theta_{V1} - \theta_{I1}$ | 18.3° | 10.8° | 8.5° | 7.7° | 7.5° |
| $V_2(rms), V$ | 21.6∠-1.9° | 21.16∠-2.6° | 20.65∠-3.3° | 20.23∠-3.9° | 19.82∠-4.6° |
| $I_2(rms)$, $A$ | 0.43∠-1.9° | 0.84∠-2.6° | 1.24∠-3.3° | 1.62∠-3.9° | 1.98∠-4.6° |
| $P_2(ave), W$ | 9.29 | 17.77 | 25.61 | 32.77 | 39.24 |
| $P_2 / P_1$ | 0.93 | 0.92 | 0.91 | 0.90 | 0.88 |
| VR=(V2,*oc* -V2,*load*)-V2,*load* (take the values from the last measurement) | 0.113 | | | | |

**Plot the output voltage V2(I2), input power factor angle ϕ1(I2) , and the efficiency η(I2)=P2/P1. On each plot, show and compare the measured and calculated quantities.**

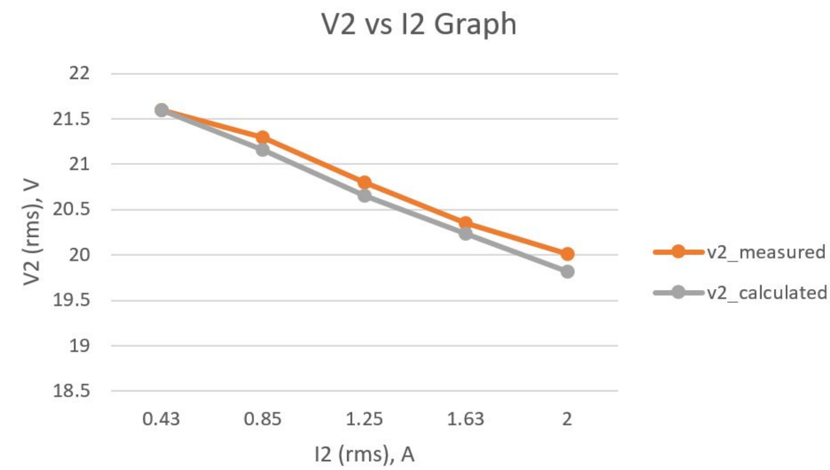



Figure 2.6-3: Measured and Calculated V2 and I2 Comparison Graph

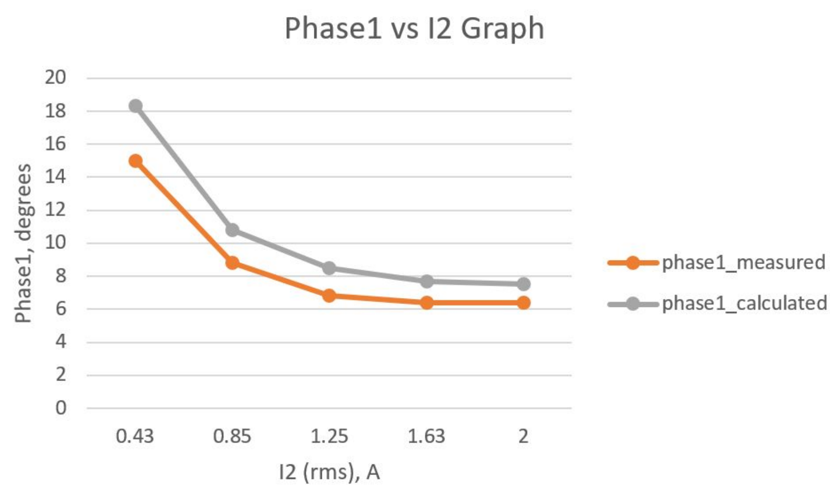



Figure 2.6-4: Measured and Calculated Phase1 and I2 Comparison Graph

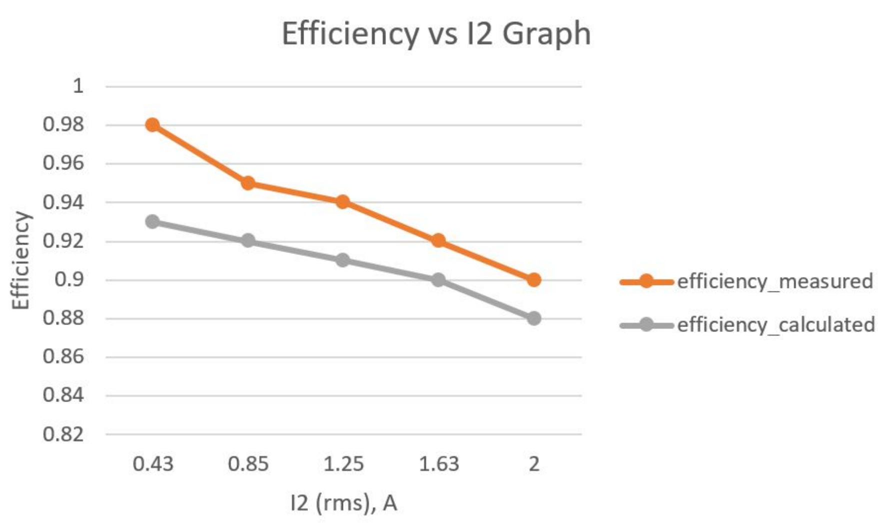



Figure 2.6-5: Measured and Calculated Efficiency and I2 Comparison Graph

**What can you say about accuracy of the equivalent circuit?**

- As all of the calculated values in the equivalent circuit were near the measured values of the transformer, the circuit accuracy is very good.

**How well does it predict the loading characteristics of your transformer?**

- The three graphs above (Figure 2.6-3 to Figure 2.6-5) show roughly the same trend for V2 vs I2, phase1 vs I2 and efficiency vs I2. This indicates the transformer with calculated values from Fig B can represent the equipment used in the lab.

**Where is the maximum efficiency of this transformer?**

- The maximum efficiency of this transformer occurs at the first point taken which is at the lowest current I2 of 0.43A. The maximum efficiency has a value of 0.98.

## 2.6C Task 6C: Effect of Load Power Factor on the Loading Characteristics

**Using the measurements from Tasks 4 A, B, and C, plot and superimpose the output voltage V2(P2), input power factor angle $\phi 1$(P2) , and the efficiency $\eta$(P2) = P2/P1 for all three cases.**

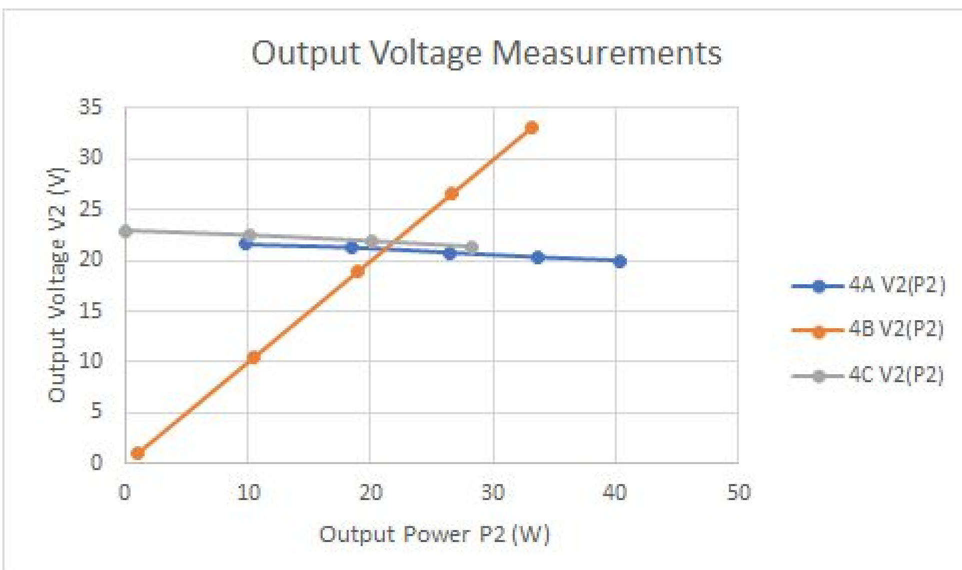



Figure 2.6-6: Output Voltage Measurements V2(P2) for Tasks 4A, 4B, 4C

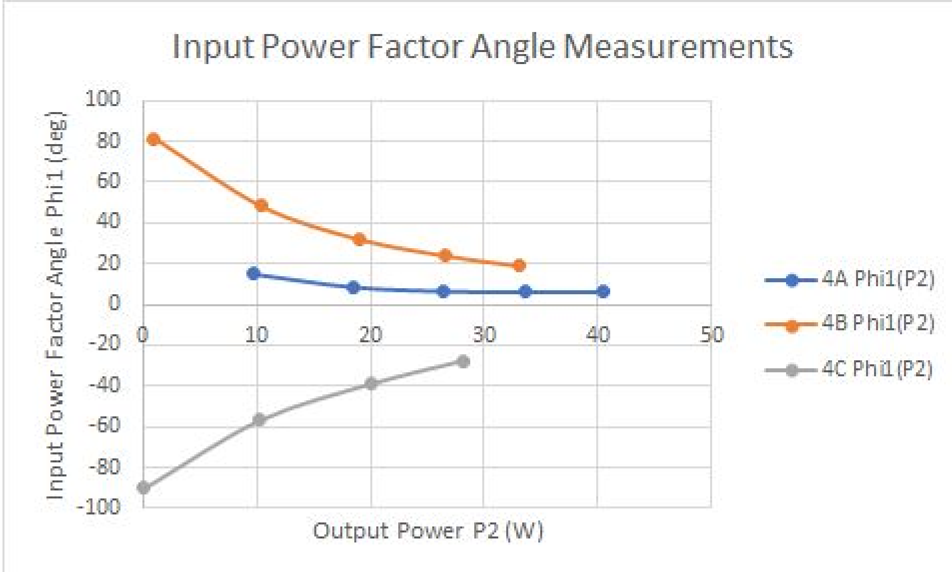



Figure 2.6-7: Input Power Factor Angle Measurements Phi1(P2) for Tasks 4A, 4B, 4C

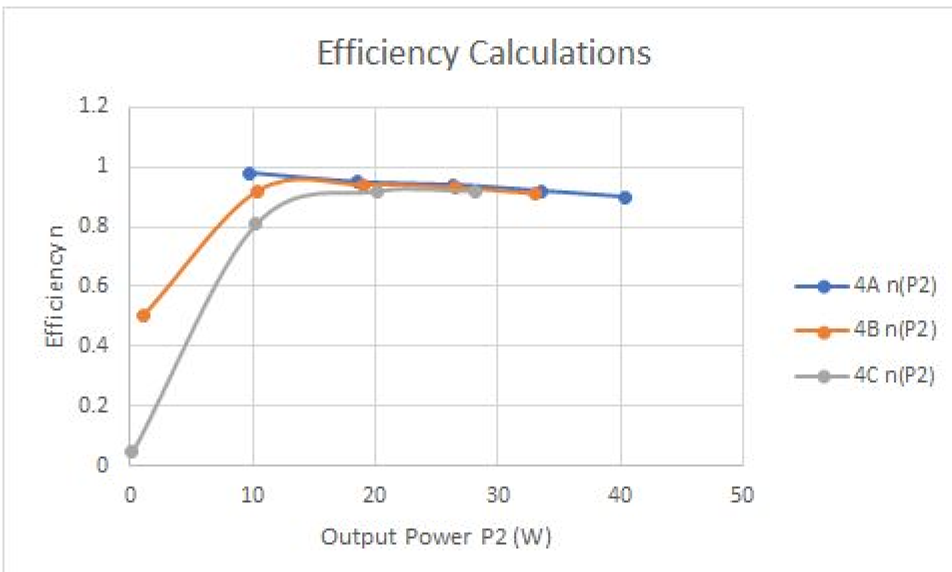



Figure 2.6-8: Efficiency n(P2) for Tasks 4A, 4B, 4C

**How can you explain these results? How does the load power factor affect the output voltage?**

- The load power factor influences voltage regulation, and is a determining factor in the secondary voltage. For a lagging power factor, the output voltage will decrease as large secondary currents will flow, resulting in poor voltage regulation due to greater voltage drops in the winding. On the other hand, a leading power factor will cause the output voltage to rise.

**How did the real power change (in each step) when you connected either inductor or capacitor in parallel to the resistive load?**

- When connecting additional inductors or capacitors in parallel to the resistive load, the real power increased both times.

**What can be used to boost the secondary voltage on the transformer output, inductors or capacitors? Why?**

- The secondary voltage on the transformer output will increase if a capacitor is used as a load, as it will have a leading power factor with an appropriate voltage regulation, causing a voltage rise in the windings.

## 2.6D Task 6D: Hysteresis Loop

**Use the saved data from the Task 2 C (Open-Circuit Test: Saturated Condition) to calculate the flux linkage λ(t). Based on the voltage equation v1=i1*r1+(dλ/dt), the flux linkage can be found as λ(t)=∫ (v1-i1*r1)dt+Cλ. The constant Cλ should be chosen to make the λ(t) centered about zero. To do that, you will need to calculate the average and then subtract it.**

- See Appendix A for Task 6D Matlab script to calculate dλ/dt, λ(t), and Cλ.

**On a separate graph, plot the $\lambda - i1$ loop.**

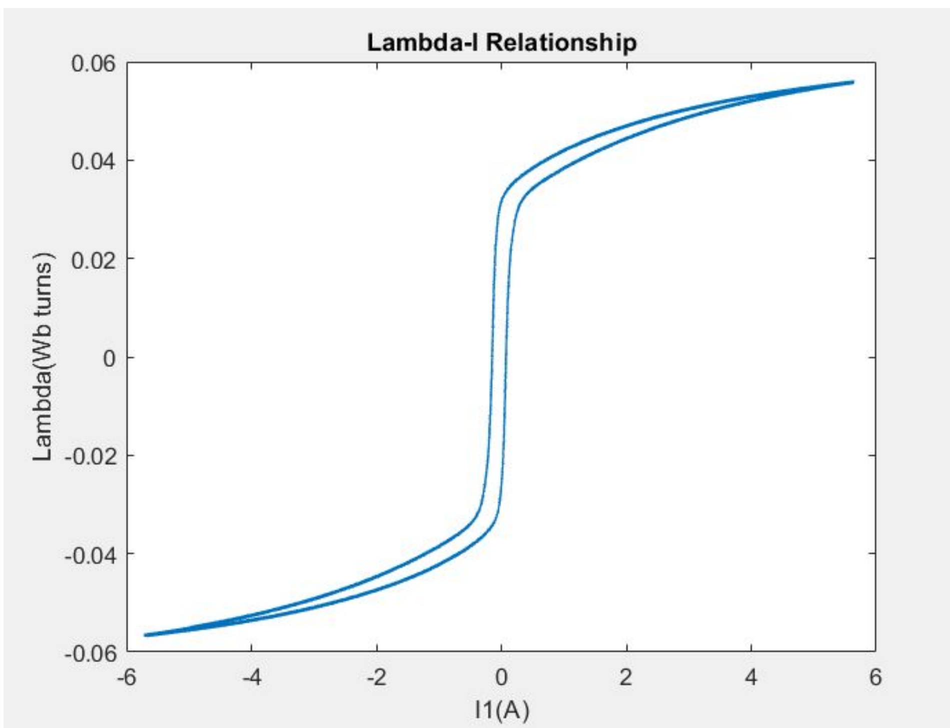



Figure 2.6-9: $\lambda - I1$ Loop plot

**How does this result look similar or different from the Screen Captured view of the $\lambda - i$ curve that you have saved?**

- This result looks very similar to the screen captured view of the $\lambda - i$ curve, the loop is thin, and its endpoints are similarly placed relative to the I1 and Lambda axes.

**Also, calculate the harmonic spectrum for the input current i1 for the saturated condition of Task 2 C using FFT**

- See Appendix B for Matlab script to calculate the harmonic spectrum

**Plot this spectrum on a separate plot and conclude what kinds of harmonics are produced by the transformers if the transformers are heavily saturated?**

- When saturated, a transformer acts as a source of current generating harmonics directly flowing toward the primary and secondary windings. The harmonic content increases significantly when exposed to high magnetic flux densities.

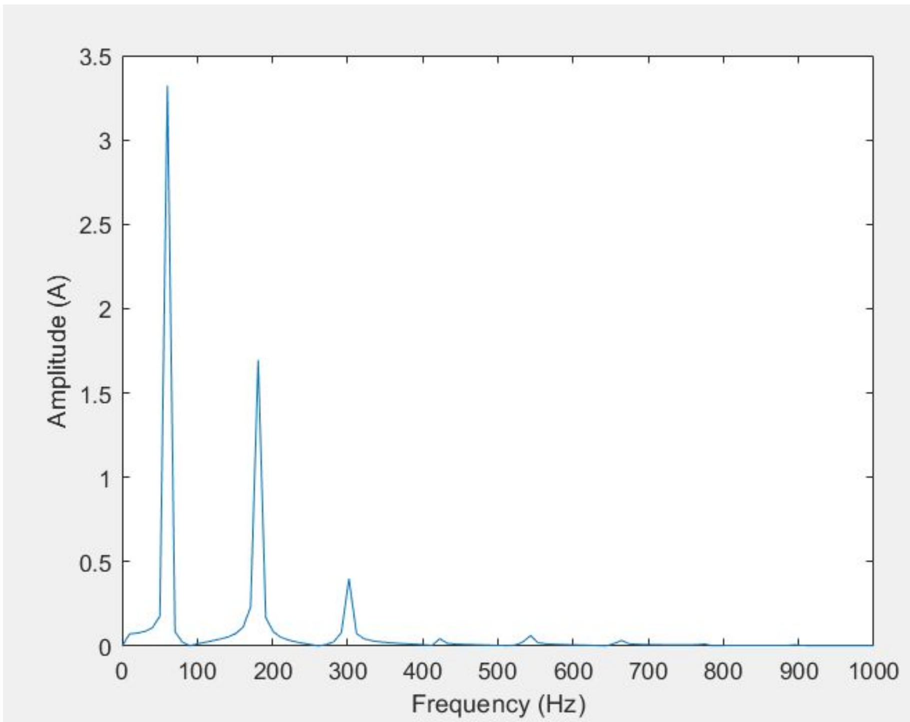



Figure 2.6-10: Harmonic Spectrum Produced by Saturated Transformer

## 2.6E Task 6E: Core Losses

**Calculate the area of one cycle of the $\lambda - i1$ loop.**

- See Appendix A for Matlab Script to calculate the area of one cycle
- Using the data points for one loop, the area of a single cycle loop is approximately 0.0427.

**Relate this area to the core losses and compare it to the measured dissipated input power in Task 2C**

- As windings have some internal resistance, some input power is dissipated as heat. The energy consumed per cycle is equal to the volume of the core times the area of the hysteresis loop. The dissipated input power is proportional to the area of the hysteresis loop. As the transformer in Task 2C was operated with saturated current, the measured dissipated input power was greater.

## 2.6F Task 6F: Analysis of Inrush Current

**Based on your recorded data of the inrush current transient in Task 5, plot the current and voltage for the case with very small inrush current transient**

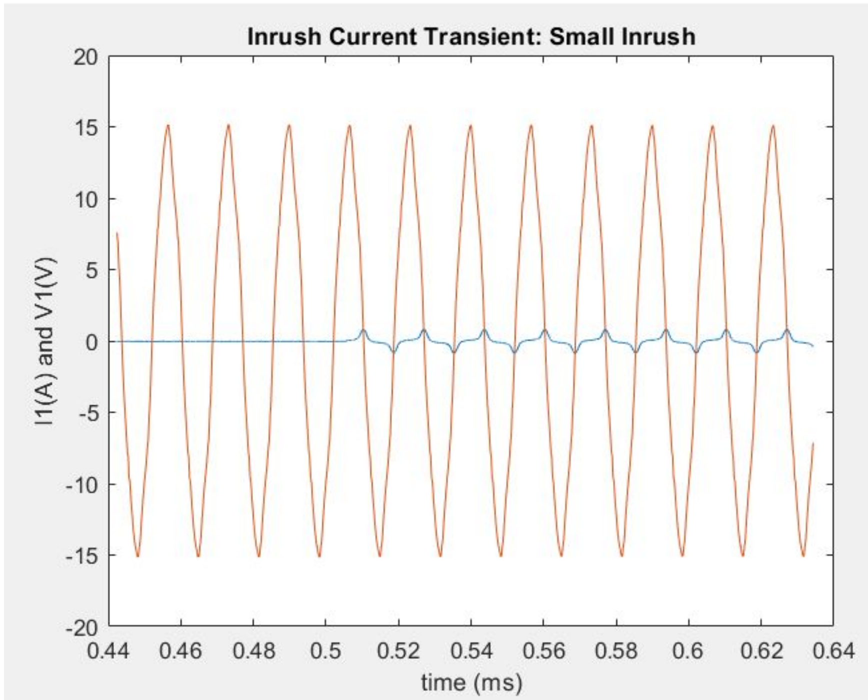



Figure 2.6-11: Small Inrush Current (V1 in Orange, and I1 in Blue)

**Plot current and voltage for the case corresponding to the very high peak of the inrush current.**

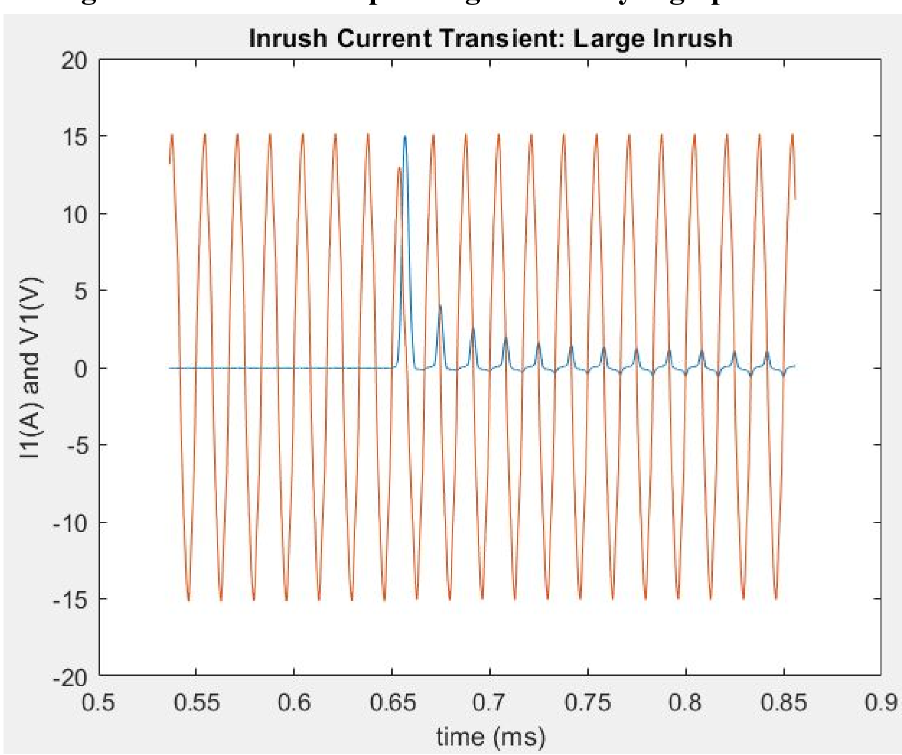



Figure 2.6-12: Large Inrush Current (V1 in Orange, and I1 in Blue)

**Compare the value of voltage at the energizing instance and conclude which case would be more favourable for connecting the transformer to the power source**

- The transformer core is generally saturated above maximum steady state value of flux. In Figure 2.67, the transformer is turned on at v(t) = 0 and the flux jumps much higher than its steady state value, and the current demand increases, resulting in inrush current. To avoid such high inrush currents from occurring, it would be best to connect the transformer to the power source when the voltage is at its peak v(t) = Vpeak, as in Figure 2.66.

**Conclude/elaborate on how the inrush current could affect large transformers and the power network?**

- Large transformers have large current demands as they are similar to short circuits until magnetic field and inductive resistance builds. Inrush current sags the system voltage, affecting the power quality of the network in proximity of the transformer, and can lead to failure of circuit components, or damage and shorten the life of the transformer.

## 2.6G Task 6G: Questions

**(a) Why do you think it is not a good idea to operate a transformer above its nominal (rated) voltage?**

- If a transformer is operated too much over its rated voltage, the core will saturate and allow high peak currents to flow in the primary side, burning the transformer.

**(b) How can you describe the waveform of the primary current i1(t) when the transformer is loaded as in Task 4 A (last measurement)? How is this waveform different from i1(t) under the no load case in Task 2 A and C?**

- The waveform of the primary current i1(t) with load has a higher amplitude than the one with no load. The load takes part of secondary current i2, and when secondary current increases, primary current also increases. Since the load is resistive, adding it will not make the waveforms to be out of phase, instead, the two waveforms will just be different in terms of amplitude.

**(c) What can you say about the accuracy of the equivalent circuit? Does it predict all of the phenomena that you have observed during this lab experiment?**

- The accuracy of the equivalent circuit was in general very good. The circuit was able to predict instances such as the effects of the power factor on output voltage and voltage regulation, the effects of adding reactive elements in parallel to a resistive load, as well as transformer efficiency.

**(d) Assuming that you have a two-winding transformer that you have used in this lab, sketch the step-up and step-down auto-transformer connections. In each case, label the input and output voltages based on the nominal voltages found in Task 2.**

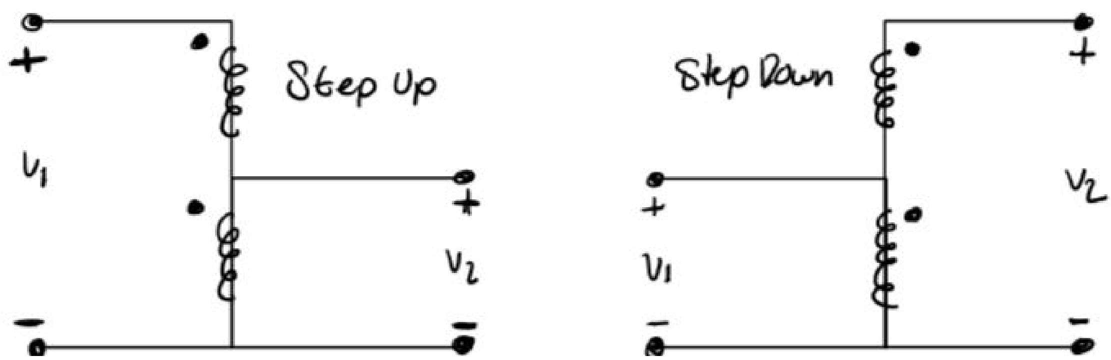



Figure 2.6-13:Step up auto-transformer (left) and Step down auto-transformer (right)

# 3. Conclusion

This lab explored the use of equivalent circuits for a small two winding transformer to analyse and approximate the physical device. Open-Circuit and Short-Circuit Tests were conducted to identify the equivalent circuit parameters and predict the transformer behavior under load. These theoretical predictions were compared with the experimental data and plotted. The basic properties of practical single-phase transformers and their loading characteristics when supplying purely resistive loads and mixed reactive loads were also observed through measurements and waveforms. Furthermore, inrush current was observed and measured when the transformer was energized at different time instances. The accuracy of the calculated transformer equivalent circuit was evaluated, and predicted the steady-state characteristics of the real transformer with an adequately low margin of error.

# Appendix A: MATLAB Code Task 6D $\lambda - I1$ Loop

```
task2Cb = importdata("Task2CStepb.txt");

R1 = 0.12; %resistance of winding
time = task2Cb.data(:,1)/1000;   %time in ms, 1st col
V1 = task2Cb.data(:,2);         %V1, 2nd col
I1 = task2Cb.data(:,4);         %I1, 4th col

fluxLink = V1 - I1*R1;          %dlambda/dt
Clambda = mean(sum(fluxLink),length(fluxLink));     %const Clambda
lambda = cumtrapz(time, fluxLink);
lambda = lambda - mean(lambda);         %get avg and subtract it

area = polyarea(I1, lambda)        %find total area
loopArea = polyarea(I1(1:3665), lambda(1:3665))

figure(1);                  %plot
plot(I1,lambda);
xlabel('I1(A)');
ylabel('Lambda(Wb turns)');
title('Lambda-I Relationship');
```

# Appendix B: MATLAB Code Task 6D Harmonic Spectrum

```
d = importdata("Task2CStepb.txt");

t = d.data(:,1)/1000;    % ms
c = d.data(:,4);     % current
L = length(t);
Ts = mean(diff(t));     % sampling interval (s)
Fs = 1/Ts;          % sampling freq
vc = c - mean(c);   % sub 0hz component

% Fourier Transform
Y = fft(vc);
P2 = abs(Y/L);
P1 = P2(1:L/2+1);
P1(2:end-1) = 2*P1(2:end-1);
f = Fs*(0:(L/2))/L;
plot(f,P1)
xlim([0 1000])
xlabel('Frequency (Hz)')
ylabel('Amplitude (A)')
```