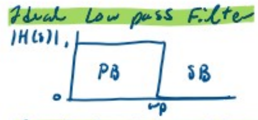
Filters

February 8, 2020 6:45 PM

Definition

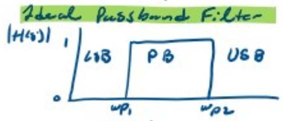
- Filters out unwanted signals
- want "Brick wall" transfer characteristics
  $|Y(\omega)| = |H(\omega)| \cdot |X(\omega)|$

Ideal Low pass Filter

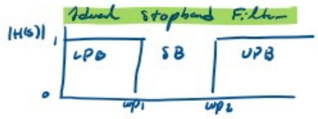


Ideal High Pass Filter

Ideal Passband Filter

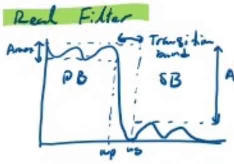


Ideal stopband Filter



- ω_p = cut off f

Real Filter

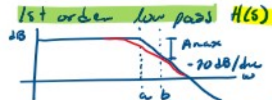


- Amax = max variation in passband
- ω_p, ω_s = selectivity factor
- Amin = min attenuation

Filter classification

- How quickly they stop

1st order low pass $H(s) = \frac{K}{(s+a)}$ → pole @ cutoff f ω_0



cutoff f? → plot? →

HP Prime Plotting:

- ∴ no complex #s & linear axes
  func. ω → func 10^x x → decades

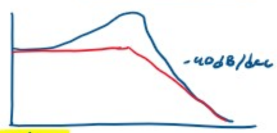
ex. $H(s) = \frac{25000}{(s+6500)}$, $\alpha = 20\log_{10}\frac{2500}{\sqrt{\omega^2+6500^2}}$

$X = \log_{10}\omega \longrightarrow \omega = 10^x$, X = decades

$\alpha = 20\log_{10}\frac{25000}{\sqrt{(10^x)^2+6500^2}}$

- Approximate cutoff:
  - pole on edge of BW = approx. cutoff f
  - Actual cutoff f depends on Amax

ex.

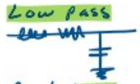


Classification by construction

Passive: only RLC
Active: RLC + opamp

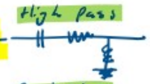
ex.

Low pass



$H(s) = \frac{1}{s^2+s+1}$

High pass

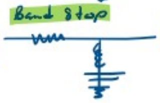


$H(s) = \frac{s^2}{s^2+s+1}$

Band pass

$H(s) = \frac{s}{s^2+s+1}$

Band stop

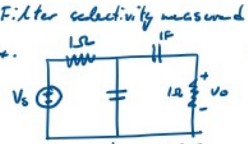


$H(s) = \frac{s^2+1}{s^2+s+1}$

Quality Factor

- For band pass, cutoff f ω_h & ω_L → attenuation goes up by 3dB, "Half power f"
- "Center f" ω_0 → mean of ω_h & ω_L
- Bandwith $BW = \omega_h - \omega_L$
- Filter selectivity measured by quality factor $Q = \frac{\omega_0}{BW}$

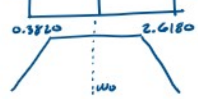
ex.



- Center f? BW? Q=?

$H(s) = \frac{s}{s^2+3s+1}$ → Band Pass

→ proot(1, 3, 1): poles = −0.3820, −2.6180



→ Geometric Mean: $\sqrt[n]{x_1 x_2 \cdots x_n}$

$\omega_0 = \sqrt{0.382 \cdot 2.618}$
$= 1$ rad/s

$BW = 2.618 - 0.382 = 2.236$

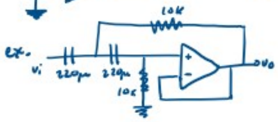
$Q = \frac{\omega_0}{BW} = 0.447227$

Sallen-Key (Active Filters)

- Replace resistors by impedances

$G_v = -\frac{R_2}{R_1} \longrightarrow H(s) = -\frac{Z_2(s)}{Z_1(s)}$

ex.



$H(s) = \frac{1215^2}{1215^2 + 110000s + 25000000}$

Scaling & Recipies

- Filter "cookbooks"
- Scale filters for different components

Scaling & Recipies

- Filter "cookbooks"
- Scale filters for different components

Frequency Scaling

- Want same impedance Z but at K_f times original f

need new values for L' & C'

$$Z_L = j(\omega K_f)L' = j\omega L \rightarrow L' = \frac{L}{K_f}$$

$$Z_C = \frac{1}{j(\omega K_f)C'} = \frac{1}{j\omega C} \rightarrow C' = \frac{C}{K_f}$$

Higher frequency but same impedance

$\therefore$ Divide both L & C by same factor used to multiply f by K_f

Magnitude Scaling

- If components too little/large, scale up/down component values without changing f

$\therefore$ Multiply all impedances by K_m: $(\omega' = \omega)$

$$Z_R' = K_m Z_R = K_m R, \quad Z_L' = K_m Z_L = j\omega K_m L, \quad Z_C' = K_m Z_C = \frac{1}{j\omega C / K_m}$$

$$R' = K_m R \qquad L' = K_m L \qquad C' = \frac{C}{K_m}$$

Magnitude & Frequency scaling

- Increase RLC by K_m, shift f response by K_f:

$$R' = K_m R, \quad L' = \frac{K_m}{K_f} \cdot L, \quad C' = \frac{1}{K_m K_f} \cdot C, \quad \omega' = K_f \omega$$

Webwork FLT

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FLT: Problem 6

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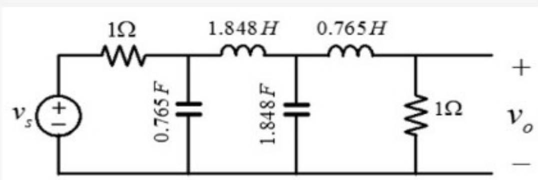
(11 points)
A fourth-order Butterworth low pass filter is shown in the figure below. Using "scaling" replace the two resistors by 7 kilo ohm resistors and determine what must be the values of the inductor and the capacitor for a cutoff frequency of 12 kHz.



Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

L_{left} = ____ H

C_{left} = ____ F

L_{right} = ____ H

C_{right} = ____ F

$\omega_0 = 12\,kHz = \frac{6000}{\pi}$

Scaling → (Filters video)

$R = 1\Omega \rightarrow 7k\Omega$

$\omega = 12kHz \cdot 2\pi = 24k\pi$ rad/s

Magnitude Scaling factor $K_m = 7k$

frequency scaling factor $K_f = 24k\pi$ rad/s

original f: $\omega_0 = 1$ rad/s → needed to plot Bode plot
Find cutoff f & type of filter...

$\therefore \omega_0' = 24k\pi$

$$L_1' = \frac{K_m}{K_f} \cdot L = 0.171569\,H$$

$$C_1' = \frac{1}{K_m K_f} C = 1.4494468 \times 10^{-9}\,F$$

$$L_2' = 7.102289 \times 10^{-2}\,H$$

$$C_2' = 3.50140875 \times 10^{-9}\,F$$

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FLT: Problem 7

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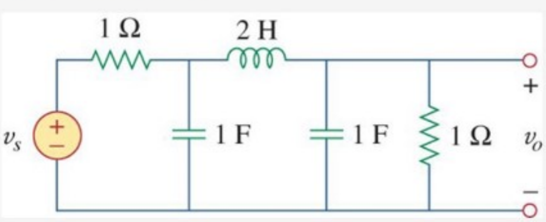
(11 points)
You pick up a third order Butterworth filter from a 'filters cookbook' and want to modify it so the capacitors are 25 nF and it has a cut off frequency of 3 kilo hertz. Determine the values of the inductor and the resistors.



Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

R = ____ Ω

L = ____ H

Scaling:

$C = 1F \rightarrow 25nF$

$\omega_0 = 1 \rightarrow 3kHz \cdot 2\pi = 6k\pi$ rad/s

(Find TF, Bode Plot)

$\therefore K_f = 6k\pi$

$C' = \frac{1}{K_m K_f} C$

$\therefore K_m = 2122.065908$

$R' = K_m R$

$\therefore R = 2122.065907891\Omega$

$L' = \frac{K_m}{K_f} L$

$\therefore L = 0.2251581\,8\,H$

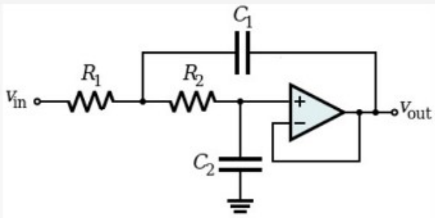
Sets
FLT

FLT: Problem 8

Scaling:

FLT: Problem 8

(11 points)
A Sallen Key low pass filter in its canonical form is shown in the figure, with resistors of one ohm each and where the capacitor on the top is 1.4142 F and the capacitor on the bottom is 0.7071 F. If we want to use 16 kilo ohm resistors instead and we need it to have a cut off frequency of 21 kilo hertz, what should be the values for the two capacitors.



Note: In this problem, you may only submit numerical answers. (i.e. If 4 is the correct answer, 4 will be marked as correct, but 2+2 will be marked as incorrect.)

$C_1 =$ ______ F

$C_2 =$ ______ F

Scaling:

$R = 1\Omega \rightarrow 16k\Omega$

$\omega_0 = 1 \rightarrow 21kHz \cdot 2\pi = 42000\pi$

(Find ω_0 initial: Find TF, draw Bode Plot)

$\therefore k_m = 16k$

$k_f = 42000\pi$

$C' = \frac{1}{k_m k_f} C$

$\therefore C_1 = 6.69871787 \times 10^{-10}$ F

$C_2 = 3.34935893 \times 10^{-10}$ F