

# Linear systems and GE

March 5, 2020 12:58 PM

## System of linear equations

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ x_1 - 5x_2 + x_3 &= 1 \end{aligned} \rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 1 & -5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\rightarrow m$  lin. eqn.,  $n$  unknowns ( $m \times n$  matrix)

$$A\vec{x} = \vec{b} \rightarrow [n \times n][n \times 1] = [m \times 1]$$

### 1.1 Solve general system $A\vec{x} = \vec{b}$

- Form augmented matrix  $[A|\vec{b}]$
- Use gaussian elimination  $\rightarrow$  row echelon form

$$\begin{aligned} \text{ex. } \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & -5 & 1 & 1 \end{array} \right] &\xrightarrow{\text{GE}} \left[ \begin{array}{ccc|c} 1 & 0 & 1 & 2/7 \\ 0 & 1 & 0 & -1/7 \end{array} \right] \rightarrow \begin{aligned} x_3 &= s, x_1 = 2/7 - s \\ x_2 &= -1/7 \end{aligned} \\ \vec{x} &= \begin{bmatrix} 2/7 \\ -1/7 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \end{aligned}$$

### 1.2 Non-singular system (# eqn. = # unknowns)

- Non-singular = invertible
- $\rightarrow$  unique solution

$$\vec{x} = A^{-1}\vec{b}$$

### 1.3 Reduced Row echelon form (singular matrices)

- Solve  $A\vec{x} = \vec{b}$  when  $A$  = singular & not square

Augmented matrix  $[A|\vec{b}] \rightarrow [I|\vec{x}]$   $I$  = Identity Matrix,  $\vec{x}$  = solution

## MATLAB: Solving linear systems

①  $\text{ref}(A, \vec{b})$

②  $\vec{x} = \text{inv}(A) * \vec{b}$  OR:  $\vec{x} = A \setminus \vec{b}$

## Linear system solutions

- $m = n$  (if eqn. = # unknowns)

$$A\vec{x} = \vec{b}$$

① unique soln. if  $\det(A) \neq 0$

② No soln. or infinite soln. if  $\det(A) = 0$

$\rightarrow$  contradiction  $\rightarrow$  dependent var.

} Gaussian elimination

## Identity Matrix & Invertibility

$$I_n = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix},$$

$$\vec{x} \cdot I_n = \vec{x}$$

LO 1.1

- $\vec{x} \cdot I_n = \vec{x}$
- $A \cdot I_n = A$
- If  $A$  non-singular &  $\det(A) \neq 0$ 
  - $\rightarrow A^{-1}$  exists  $[n \times n]$ , invertible
  - $\rightarrow A A^{-1} = A^{-1} A = I_n$
  - $\rightarrow$  soln.  $A \vec{x} = \vec{b}$  is  $\vec{x} = A^{-1}(A \vec{x}) = A^{-1} \vec{b}$

Inverse Matrix

$$[A | I_n] \xrightarrow{GE} [I_n | A^{-1}]$$

Extra: Image Compression

- Send image & store on hard drive
- Reduce image size w/ little loss in image quality

