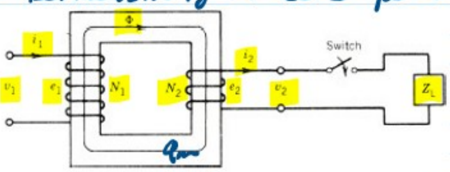
2. Transformers

Tuesday, October 27, 2020 3:47 AM

2.1 Ideal Transformer

- winding $R = 0$
- leakage flux & core losses $= 0$
- Permeability of core $\mu \to \infty$ $\therefore$ net mmf to establish flux in core $= 0$



voltage induced $e_1 = v_1 = N_1 \dfrac{d\varphi}{dt}$

$e_2 = v_2 = N_2 \dfrac{d\varphi}{dt}$

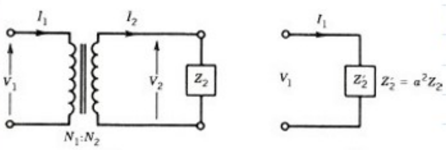
$\dfrac{v_1}{v_2} = \dfrac{N_1}{N_2} = a$ → turn ratio

$\dfrac{i_1}{i_2} = \dfrac{N_2}{N_1} = \dfrac{1}{a}$

- inst. power in = inst power out $N_1 i_1 = N_2 i_2$

$v_1 i_1 = v_2 i_2$ $\because$ no power losses

2.1.1 Impedance Transfer

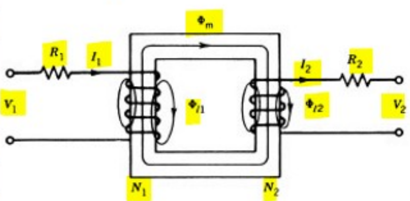


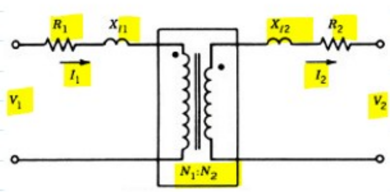
Primary: $Z_1 = Z_2 a^2 = Z_2'$

Secondary: $Z_1' = \dfrac{1}{a^2} Z_1$

2.2 Practical Transformer

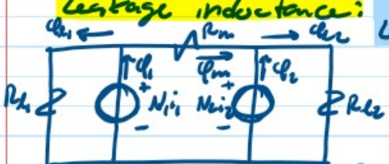
- winding Resistances
- permeability core $\mu \neq \infty$
- Core losses
- windings link $\neq$ same flux





- Mutual flux φ_m in core
- leakage flux φ_l links single winding, varies w/ current

Leakage inductance: $L_{l1} = \dfrac{N_1 \varphi_{l1}}{i_1}$ $L_{l2} = \dfrac{N_2 \varphi_{l2}}{i_2}$



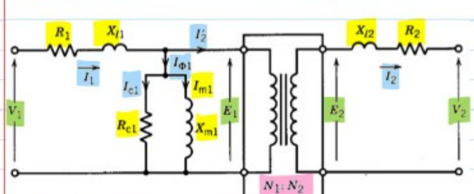
$\varphi_1 = \varphi_{l1} + \varphi_m = \dfrac{N_1 i_1}{R_{l1}} + \dfrac{N_1 i_1 - N_2 i_2}{R_m}$

$\varphi_2 = \varphi_{l2} - \varphi_m = \dfrac{N_2 i_2}{R_{l2}} - \dfrac{N_1 i_1 - N_2 i_2}{R_m}$

$\lambda_1 = N \varphi_1 = L_{l1} + L_{m1} i_1 - L_{m12} i_2$ → $L = \dfrac{N^2}{R}$ $L_{m12} = L_{m21}$

$\lambda_2 = N_2 \varphi_2 = L_{l2} i_2 + L_{m2} i_2 - L_{m21} i_1$

2.2.2 Determination of equivalent circuit parameters

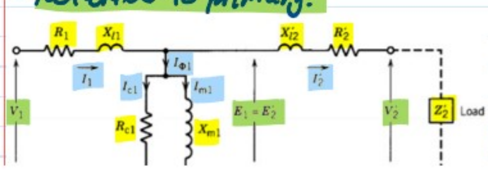


R_{c1} → core losses

$I_2' = I_2 \dfrac{N_2}{N_1} = \dfrac{I_2}{a}$

$X_{m1} = L_{m1} \omega$ → magnetizing reactance

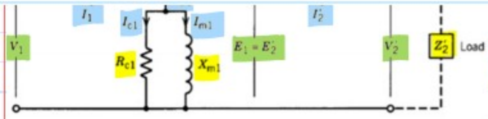
- Referred to primary:



$R_2' = R_2 \left(\dfrac{N_1}{N_2}\right)^2 = R_2 a^2$

$X_{l2}' = X_{l2} \left(\dfrac{N_1}{N_2}\right)^2 = X_{l2} a^2$

$I_{\Phi 1}$ → Exciting current

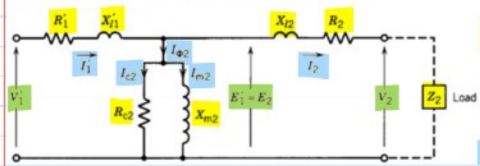


$X_{l2}' = X_{l2}\left(\overline{W_2}\right) = X_{l2}a^2$

$I_{\varphi_1} \rightarrow$ Exciting current
$I_{m_1} \rightarrow$ Magnetizing current

Scaled Secondary values: $V_2' = V_2 a$ $\quad X_{l2}' = X_{l2}a^2$
$I_2' = \frac{I_2}{a}$ $\quad R_2' = R_2 a^2$
$Z_2' = Z_2 a^2$

- Referred to secondary:



Scaled Primary values:

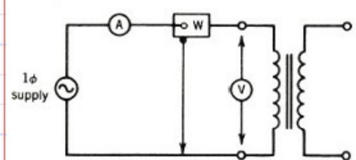
$V_1' = \frac{V_1}{a}$ $\quad X_{l1}' = \frac{X_{l1}}{a^2}$ $\quad X_{m2} = \frac{X_{m1}}{a^2}$

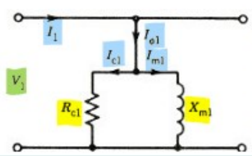
$I_1' = I_1 a$ $\quad R_1' = \frac{R_1}{a^2}$ $\quad R_{c2} = \frac{R_{c1}}{a^2}$

Transformer Rating
- ex. 10kVA, 1100/110 V → voltage windings & turn ratio
  - ↳ kVA rating: each winding designed for 10kVA
    - ∴ current rating = 10000/1100 = 9.09A flows in HV windings
    - = 10000/110 = 90.9A flows in LV windings
- Primary winding carries excitation current (I_φ ≪ rated current)

Tests for Determination of Parameters

Open Circuit Test (No load Test)
- Apply voltage to HV or LV
- Secondary winding is open
- Apply Rated Voltage, Measure V_{oc}, I_{oc}, P_{oc}

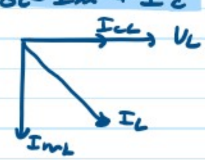




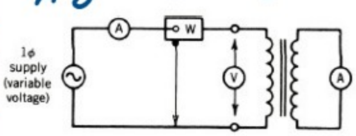
$P_{oc} = \frac{V_{oc}^2}{R_c}$

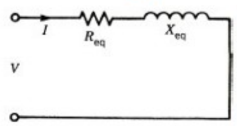
$I_c = \frac{V_{oc}}{R_c}$ $\quad I_{oc}^2 = I_m^2 + I_c^2$ $\quad I_m = \sqrt{I_{oc}^2 - I_c^2}$

$X_m = \frac{V_{oc}}{I_m}$



Short Circuit Test
- Short circuit one winding
- Apply rated current to other winding, Measure V_{sc}, I_{sc}, P_{sc}





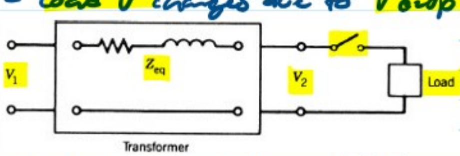
$P_{sc} = I_{sc}^2 R_{eq}$

$Z_{eq} = \frac{V_{sc}}{I_{sc}} = \sqrt{R_{eq}^2 + X_{eq}^2}$

$X_{eq} = X_{l1} + a^2 X_{l2}$ $\quad X_{l1} = X_{l2}' = \frac{X_{eq}}{2}$

$R_{eq} = R_1 + a^2 R_2$ $\quad R_1 = R_2' = \frac{R_{eq}}{2}$

2.3 Voltage Regulation
- Secondary load designed to operate @ const. V
- Load V changes due to V drop in internal impedance (core losses)



w/o secondary load: $V_2|_{NL} = \frac{V_1}{a}$

w/ secondary load: $V_2|_L = V_2|_{NL} \pm \Delta V_2$

- Load terminal V may ↑↓ depending on load nature
- ↑ΔV_2 undesireable ∴ want ↓Z_{eq}, internal impedance
- Voltage Regulation: identify characteristic of ΔV_L w/ loading
  - ↳ $\Delta|V_2|$ as load current changes from no-load → loaded

$$\text{Voltage Regulation} = \frac{|V_2|_{NL} - |V_2|_L}{|V_2|_L}$$

↳ Calculate w/ equivalent primary/secondary circuits

Primary side: $VR = \frac{|V_2'|_{NL} - |V_2'|_L}{|V_2'|_L}$

∵ load voltage → rated voltage $|V_2'|_L = |V_2'|_{rated}$

$V_1 = V_2' + I_2' R_{eq_1} + j I_2' X_{eq_1}$

∵ load voltage → rated voltage $|V_2'|_L = |V_2'|_{rated}$

$V_1 = V_2' + I_2' R_{eq1} + jI_2' X_{eq1}$

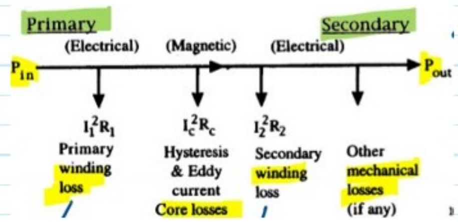
If no load ($I_1 = I_2' = 0$): $|V_2'|_{NL} = V_1$

$$\therefore VR\% = \frac{|V_1| - |V_2'|}{|V_2'|_{rated}} \times 100\%$$

2.4 Efficiency

- Since transformers = static, ↓ losses, ~99% efficiency

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{in} + P_{losses}} = \frac{P_{out}}{P_{in} + P_c + P_{cu}}$$ → P_c core loss, P_{cu} copper loss



- Find P_{cu} w/ winding currents & their resistances

$P_{cu} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{eq1} = I_2^2 R_{eq2}$

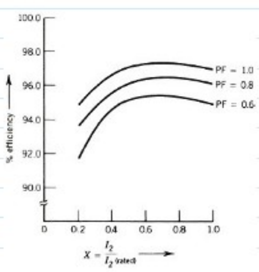
- P_c depends on peak flux density of core → depends on V_{in}
  $P_c \sim$ const., Find using No load test

- $P_{out} = V_2 I_2 \cos\theta_2$

2.4.1 Maximum Efficiency

- @ const. V_2 & load PF θ_2, Max efficiency @ $\frac{d\eta}{dI_2} = 0$
- Conditions: $P_c = I_2^2 R_{eq2}$
  Core loss = copper loss

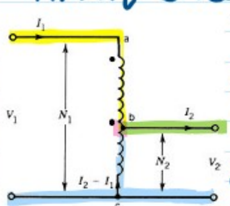
Full load condition → rated: $P_{cu,FL} = I_{2,FL}^2 R_{eq2}$

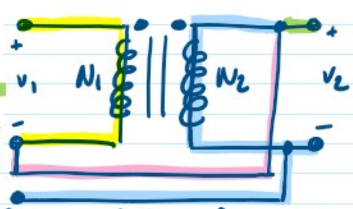


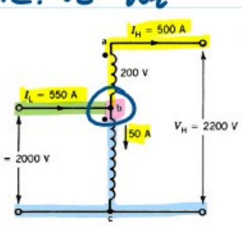
- @ const. V_2 & load current I_2, max η:
  $\frac{d\eta}{d\theta_2} = 0 \rightarrow \theta_2 = 0$, $\cos\theta_2 = 1$
  ∴ PF = 1
  ∴ Max efficiency @ PF = 1 (resistive) & load current → $P_{cu} = P_c$

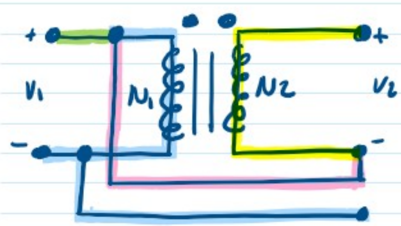
2.5 Autotransformer

- Variable AC voltage obtained @ secondary
- Primary & secondary connected
  - Turns link same flux in core: $\frac{V_1}{V_2} = \frac{N_1}{N_2} = a$









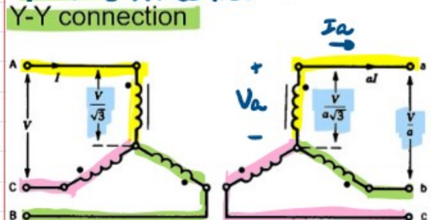
- To compute KVA as autotransformer:
  ↳ Find current rating of windings → $I = \frac{KVA}{V}$
  ↳ Terminal currents for full load → I_H, I_L
  ↳ KVA: $KVA_L = \frac{V_L \times I_L}{a}$ $KVA_H = \frac{V_H \times I_H}{a}$

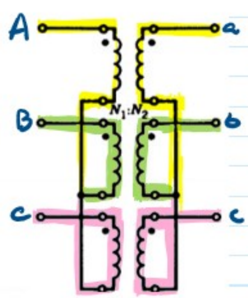
2.6 Three Phase Transformers

- Required to step up/step down voltages in power transmission

Y-Y Connection

Y-Y connection

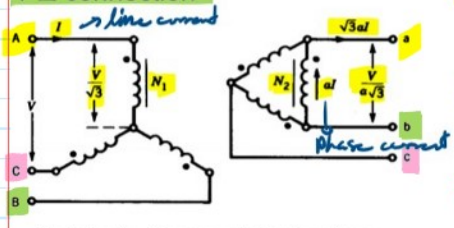


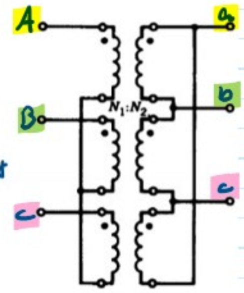


- Rarely used
- Problems exciting currents & induced voltages

Y – Δ Connection

Y-Δ connection



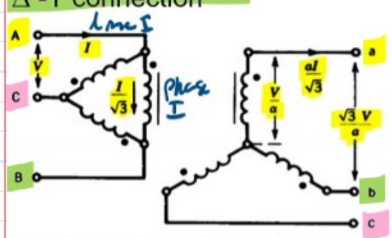


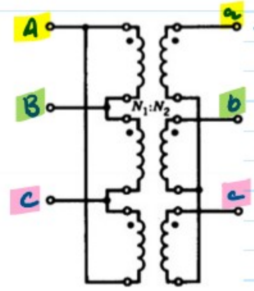
- Neutrals should be grounded on the primary

– Used to step down HV → LV
– Neutral pt. on HV can be grounded

Δ – Y Connection

Δ -Y connection





- Neutrals may be grounded on the secondary

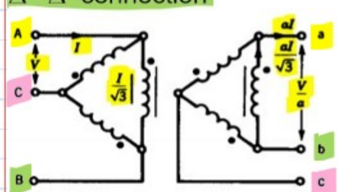
– Step up Voltage

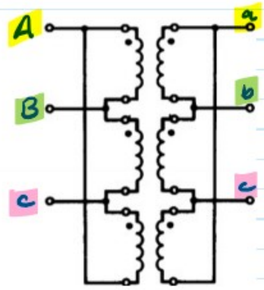
$P_{source} = (I_A \cdot V_{AB})3$

$P_{load} =$

Δ – Δ Connection

Δ - Δ connection



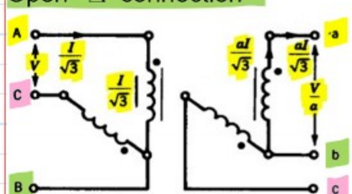


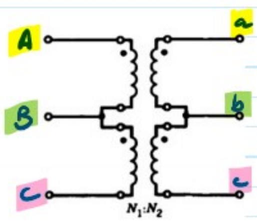
- No phase shift of line voltages
- One transformer can be removed

– one transformer can be removed & remaining 2 can deliver 3-phase power @ 58% of original rating

Open – Δ Connection

Open - Δ connection



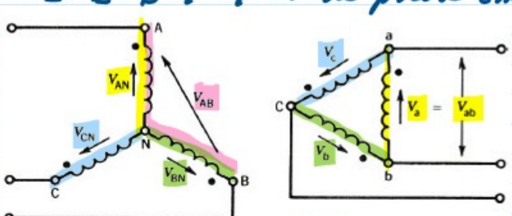


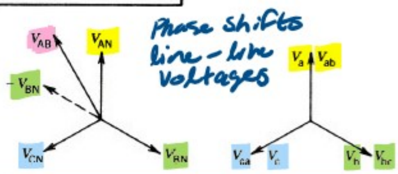
– Open Δ / V connection
– 58% of original 3-phase rating

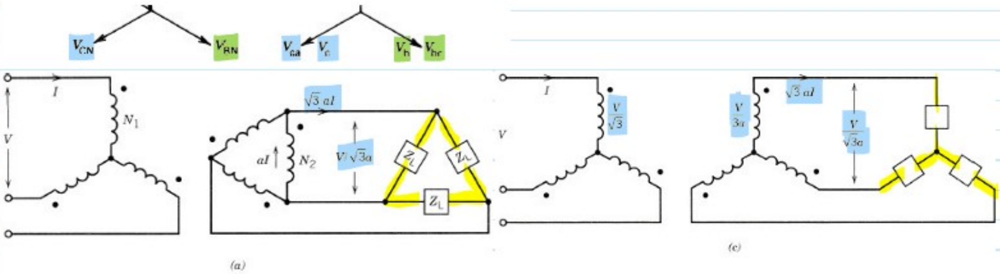
– Voltages same as Δ–Δ
– line current smaller

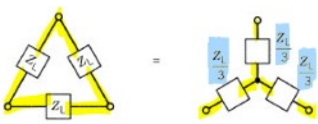
Single Phase Equivalent circuit & Phase Shift

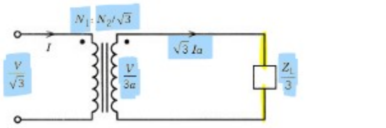
– Y-Δ & Δ-Y provide 30° phase shift b/w line-line Voltage
 ↳ V_{AN} & V_a aligned
 ↳ V_{AB} leads V_{ab} by 30°
– Δ-Δ & Y-Y → no phase shifts





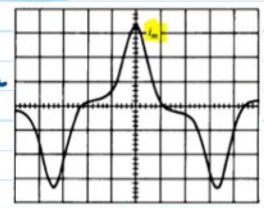






- Turn Ratio a' of Y-Y equivalent transformer: $a' = \frac{V/\sqrt{3}}{V/3a} = \sqrt{3}\,a$

2.7 Harmonics in Transformers



- ↑ flux density → ↓ magnetic material
- ∴ Transformer is designed to operate in saturation region of core
  - ↳ exciting current → non-sinusoidal
  - ↳ will contain fundamental & odd harmonics

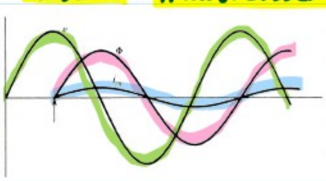
- @ V_{Rated}, 3rd Harmonic in exciting current = 5-10% of fundamental
  @ V_{Rated} 150%, 3rd Harmonic current = 30-40% of fundamental

Inrush Current in Transformer

- Exciting current normally ↓ <5% I_{rated}
- When transformer connected to power system, ↑ Inrush current flows during transient period → 10-20% × I_{rated}
- Important to determine Max Mech. Stress in windings & protective system

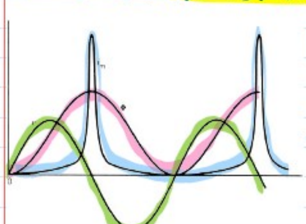
$$\text{Supply } v = \sqrt{2}\,V \sin\omega t = N\frac{d\varphi}{dt} \rightarrow \text{neglect core losses \& winding } R$$
$$\varphi = \frac{1}{N}\int v\,dt$$

- Case 1: Transformer connected when V @ Max



- No Transient in flux
  $\varphi = \varphi_{max}\sin(\omega t - 90°)$ for $\omega t > 90°$
  $\varphi_{max} = \frac{\sqrt{2}\,V}{\omega N}$
- No inrush current, system in steady state

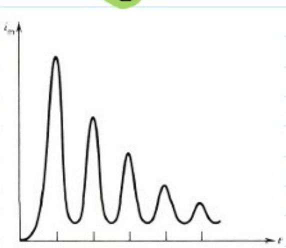
- Case 2: Transformer connected when V @ 0



$$\varphi = \frac{\sqrt{2}\,V}{N}\int_0^t \sin\omega t\,dt = \frac{\sqrt{2}\,V}{\omega N}(1-\cos\omega t) = \varphi_{max} - \varphi_{max}\cos\omega t$$

- Peak flux doubled, ↑peak magnetizing current ∵ Core saturation
- ∵ Winding Resistance, Inrush current decays rapidly

- Effects of winding Resistance on inrush current



2.8 Per-Unit (PU) System

- Select Base Values, convert params to PU
- Calculate & convert back to original units

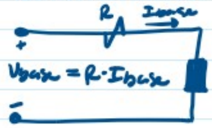
28 # Per-Unit (PU) System

- Select Base Values, convert params to PU
- Calculate & convert back to original units

$$\text{Quantity (PU)} = \frac{\text{Actual Value}}{\text{Base Value}}$$

Per-Unit for Single Phase

- Select Base power S_{base} & Base Voltage V_{base}
- Calculate Base current & Impedance



$$R_{pu} = \frac{R}{Z_{base}} = R\frac{I_{base}}{V_{base}} = R\frac{I_{base}^2}{S_{base}}$$

- Transformation from One base → Another

$$\text{Quantity (base 2)} = \text{Quantity (base 1)}\,\frac{\text{Value Base 1}}{\text{Value Base 2}}$$

$$V_{pu}(base2) = V_{pu}(base1)\,\frac{V_{base1}}{V_{base2}} \qquad I_{pu}(base2) = I_{pu}(base1)\,\frac{I_{base1}}{I_{base2}}$$

$$Z_{pu}(base2) = Z_{pu}(base1)\,\frac{Z_{base1}}{Z_{base2}} = Z_{pu}(base1)\,\frac{S_{base2}}{S_{base1}}\left(\frac{V_{base1}}{V_{base2}}\right)^2$$

- Given S_{base}, $V_{base,L,H}$
Find $I_{base,L,H}$ & $Z_{base,L,H}$

divide by 8kV divide by 80kV

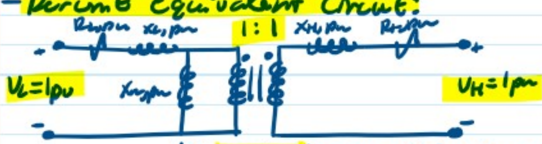
∴ scale both sides 1:1

$$a = \frac{8kV}{80kV} = \frac{1}{10}$$

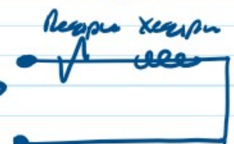
- Calculate Per-Unit: $R_{L,pu} = \frac{R_L}{Z_{base,L}}$ $R_{H,pu} = \frac{R_H}{Z_{base,H}}$

$X_{L,pu} = \frac{X_L}{Z_{base,L}}$ $X_{H,pu} = \frac{X_H}{Z_{base,H}}$ $X_m = \frac{X_m}{Z_{base,L}}$

- Per-unit Equivalent Circuit:



→



↳ neglect magnetizing branch

$$Z_{eq,pu} = \sqrt{R_{eq,pu} + X_{eq,pu}}$$