Work and Potential

April 25, 2020 10:42 AM

Force

Force on Q due to E-Field: $F_e = Q\vec{E}$

Move in dir. $\hat{a}_L$: $\vec{F}_{EL} = \vec{F}_E \cdot \hat{a}_L$, Force applied: $\vec{F}_a = -\vec{F}_{EL}$

$\vec{E}$-Field: $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$

Work

— Force applied · distance moved

$dW = -Q\vec{E}\cdot\hat{a}_L dL = -Q\vec{E}\cdot d\vec{L} \longrightarrow W = -q\int \vec{E}\cdot d\vec{L}$ (J)

— If $Q \perp \vec{E}$, $W = 0$

ex. move q^+ towards Q^+: $W = -q\int_{\infty}^{0}\vec{E}\cdot d\vec{L} = -q\int_{\infty}^{0}\frac{Q\,dL}{4\pi\varepsilon_0 r^2}\,dr$

Potential

- work done by external source moving $q(+)$ from 1 pt. to another in an $\vec{E}$ field

Potential difference: $V_{AB} = -\int \vec{E}\cdot d\vec{L} = V_A - V_B$ (J/C) (V)

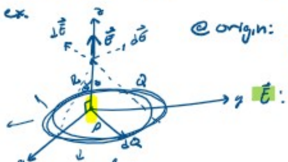
- (+) if work done against field

Electric Field inside Closed Surface: $\oint \vec{E}\cdot d\vec{L} = 0$

Chart

Q	$\lvert\vec{E}\rvert$	$\lvert V\rvert$
Point Q	$\frac{q}{4\pi\varepsilon_0 r^2}$ $\left(\propto \frac{1}{r^2}\right)$	$\frac{Q}{4\pi\varepsilon_0 r}$ $\left(\propto \frac{1}{r}\right)$
Inf. Line ρ_L	$\frac{\rho_L}{2\pi\varepsilon_0 \rho}$ $\left(\propto \frac{1}{\rho}\right)$	$\frac{\rho_L}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$
Inf. Sheet ρ_s	$\frac{\rho_s}{2\varepsilon_0}$ (const.)	$\frac{\rho_s r}{}$ $(\propto r)$

ex.



@ origin: $\vec{E} = 0$ ∴ cancel

$V \neq 0$ ∴ From $\infty \rightarrow$ ring, work done against $\vec{E}$-Field outward from ring

$\vec{E}$: $dE = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta \rightarrow R^2 = \rho^2 + z^2$, $\cos\theta = \frac{z}{R}$

$E(z) = \frac{Q_{total}\cdot z}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}}\,\hat{a}_z$ V/m

V: $V = -\int \vec{E}\cdot d\vec{L}$, $V = 0$ @ ∞

$= -\int_{\infty}^{z}\frac{Q_{total}\cdot z'}{4\pi\varepsilon_0(\rho^2+z'^2)^{3/2}}\,\hat{a}_z\cdot dz\,\hat{a}_z$

$= \frac{Q_{total}}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{\rho^2+z^2}} \longrightarrow$ @ $z=0$ $V = \frac{Q_{total}}{4\pi\varepsilon_0\rho}$

☆ Webwork 1

MAIN MENU
Courses
Homework Sets
Assignment 1
Problem 3
User Settings
Grades

webwork / 2019w2_elec_211__math_264 / assignment_1 / 3

Assignment 1: Problem 3

Previous Problem | Problem List | Next Problem

(1 point)
The electric field at the point $(2, 6, 4)$ is, in SI units of N/C,

$$\mathbf{E} = \frac{10^{-9}}{4\pi\epsilon_0}\langle 8, 2, 8\rangle.$$

Introducing a point charge of -100 nC at some point P will make $\mathbf{E} = \mathbf{0}$ at the point $(2, 6, 4)$. Find P.

ANSWER: $P = ($ ___ , ___ , ___ $)$

Problems

Problem 1 ...
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6
Problem 7
Problem 8
Problem 9

Note: *You can earn partial credit on this problem.*

Preview My Answers | Submit Answers

You have attempted this problem 0 times.
You have unlimited attempts remaining.

Superposition:

$\vec{E}_1 + \vec{E}_2 = 0$

$\vec{E}$ field from pt. q (−100) nC:

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \longrightarrow r = \langle 2-a, 6-b, 4-c\rangle$

$= \frac{-100\times10^{-9}}{4\pi\varepsilon_0|\langle 2-a, 6-b, 4-c\rangle|^2}\,\frac{\langle 2-a, 6-b, 4-c\rangle}{|\langle 2-a, 6-b, 4-c\rangle|}$

$\frac{10^{-9}}{4\pi\varepsilon_0}\langle 8,2,8\rangle + \frac{(-100\times10^{-9})}{4\pi\varepsilon_0|\langle 2-a, 6-b, 4-c\rangle|^3}\langle 2-a, 6-b, 4-c\rangle = 0$

$\rightarrow$ solve & compare: $\frac{1}{((2-a)^2+(6-b)^2+(4-c)^2)^{3/2}}\langle -900(2-a), -900(6-b), -900(4-c)\rangle = -\langle 72, 18, 72\rangle$

$\therefore P = \langle -0.0542795, 5.48643, 1.94572\rangle$

MAIN MENU
Courses
Homework Sets
Assignment 1
Problem 4
User Settings
Grades

webwork / 2019w2_elec_211__math_264 / assignment_1 / 4

Assignment 1: Problem 4

Previous Problem | Problem List | Next Problem

(1 point)
An electrostatic air filter uses an electric field to force charged dust particles to be deflected towards collecting plates as they pass through the unit. For the purposes of this question, model the air filter as two parallel infinite sheets of charge having uniform surface charge densities of ρ_S and $-\rho_S$ respectively. Air containing dust particles flows between these two charged surfaces. If the separation of the plates is d = 2 cm, the average dust particle radius is r = 25 μm, and the density of the dust is 75 $\frac{kg}{m^3}$, estimate the necessary surface charge density ρ_S on the collecting plates if the force on a dust particle having a charge of q = 3 x 10^{-15} C must counteract that of gravity in order for the filter to be effective.

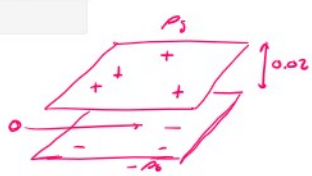
ρ_S = ___ $\frac{C}{m^2}$

Problems

Problem 1 ...
Problem 2
Problem 3
Problem 4
Problem 5
Problem 6
Problem 7
Problem 8
Problem 9

Preview My Answers | Submit Answers

You have attempted this problem 0 times.
You have unlimited attempts remaining.



$F_e = F_g$

$F_g = mg = (25\times10^{-6})^3\frac{4}{3}\cdot 75(9.8) = 4.810564\times10^{-11}$ N

$\vec{E}_s = \frac{\rho_s}{2\varepsilon_0}\hat{a}_z \times 2$ plates

$\vec{F}_e = \vec{E}_s\cdot q = \frac{\rho_s q}{\varepsilon_0}$

$\frac{\rho_s\cdot(3\times10^{-15})}{\varepsilon_0} = 4.810564\times10^{-11}$

$\rightarrow$ solve: $\rho_s = 1.4197848\times10^{-7}$ C/m²

MAIN MENU

webwork / 2019w2_elec_211__math_264 / assignment_1 / 5

MAIN MENU

Courses

Homework Sets

Assignment 1

Problem 5

User Settings

Grades

Problems

Problem 1 ...

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

Problem 7

Problem 8

Problem 9

webwork / 2019w2_elec_211__math_264 / assignment_1 / 5

Assignment 1: Problem 5

Previous Problem | Problem List | Next Problem

(1 point)
The line segment joining $P_0(3, 0, 0)$ to $P_1(9, 0, 0)$ has a uniform linear charge with density $\rho_L = -7$ nC/m. No other charges are present.

(a) Find the electric field at the origin:

$\mathbf{E}(0, 0, 0) =$ ____ N/C.

(b) Find the total charge, Q, on the given segment:

$Q =$ ____ C.

(c) Imagine replacing the given segment with a point charge Q, with Q given in part (b). Where should this point charge be placed so that the electric field at the origin is identical with the one found in part (a)?

$P =$ ____ m.

WeBWorK note: For non-scalar objects, bracket shape matters. WeBWorK interprets $(1, 2, 3)$ as a *point*, but $\langle 1, 2, 3\rangle$ as a *vector*.

a) $\vec{E}$-Field @ origin

$E = \frac{q}{4\pi\varepsilon_0 r^2}\hat{a}_r$ $\quad dE = \frac{dQ}{4\pi\varepsilon_0 r^2}\langle 1, 0, 0\rangle \longrightarrow dQ = \rho_L\, dr$

$\vec{E}_L = \int_3^9 \frac{\rho_L}{4\pi\varepsilon_0 r^2}\, dr\, \hat{a}_x$

$= \langle 14, 0, 0\rangle$ V/m

b) Total charge

$Q_{Total} = \int_3^9 \rho_L\, dr = -4.2\times 10^{-8}$ C

c) $\vec{E} = \langle 14, 0, 0\rangle$ $\quad \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{a}_r$

$14 = \frac{-4.2\times 10^{-8}}{4\pi\varepsilon_0 r^2}$

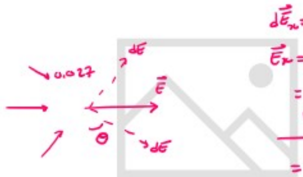
$\longrightarrow$ solve: $r = 5.196152$ m

Note: *You can earn partial credit on this problem.*

Preview My Answers | Submit Answers

You have attempted this problem 0 times.
You have unlimited attempts remaining.

O



a) $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{a}_x$

$d\vec{E}_x = \frac{dQ}{4\pi\varepsilon_0 r^2}\sin\theta \longrightarrow dQ = \rho_L\, ds$

$\vec{E}_x = \int_0^\pi \frac{\rho_L}{4\pi\varepsilon_0 r^2}\sin\theta\, ds \longrightarrow s = \theta r$, $ds = r\,d\theta$

$= \int_0^\pi \frac{\rho_L}{4\pi\varepsilon_0 r}\sin\theta\, d\theta$

$\longrightarrow \rho_L = \frac{Q_{Total}}{\pi r}$ (half circle)

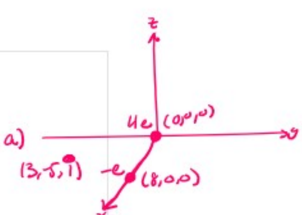
$= \frac{\frac{Q_{Total}}{\pi r}}{4\pi\varepsilon_0 r}\int_0^\pi \sin\theta\, d\theta = 7859.50336$ V/m $\hat{a}_x$

×

b) $\langle 1, 0, 1\rangle$

☆ Webwork 2

×



a)

$V = \frac{Q}{4\pi\varepsilon_0 r}$

$V_{Total} = V_1 + V_2$

$V_1 = \frac{4e}{4\pi\varepsilon_0 r} \longrightarrow r = |\langle 3, -5, 1\rangle| = \sqrt{35}$

$V_2 = \frac{-e}{4\pi\varepsilon_0 r} \longrightarrow r = |\langle 3-8, -5, 1\rangle| = \sqrt{51}$

$V_T = \frac{4e}{4\pi\varepsilon_0\sqrt{35}} - \frac{e}{4\pi\varepsilon_0\sqrt{51}} = 7.71958\times 10^{-10}$ V

O b)

$V_T = 0$

V surface due to 2pt. $Q = 0$

$V_T = V_1 + V_2 = 0 \longrightarrow V = \frac{Q}{4\pi\varepsilon_0 r}$

$r_1 = |\langle x, y, z\rangle|$

$r_2 = |\langle x, y, z\rangle - \langle 8, 0, 0\rangle$

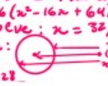
$0 = \frac{4e}{4\pi\varepsilon_0\sqrt{x^2+y^2+z^2}} - \frac{e}{4\pi\varepsilon_0\sqrt{(x-8)^2+y^2+z^2}}$

$\sqrt{x^2+y^2+z^2} = 4\sqrt{(x-8)^2+y^2+z^2} \longrightarrow y=0, z=0$

$x^2 = 16(x^2 - 16x + 64)$

$\longrightarrow$ solve: $x = 32/5$, $32/3$

∴ Circle:



$\frac{x_1 + x_2}{2} =$ origin

∴ O = 128

c) $R = x_2 - O$

$= \frac{32}{3} - \frac{128}{15}$

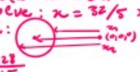
$= \frac{32}{15}$

$\sqrt{x^2+y^2+z^2} = 4\sqrt{(x-8)^2+y^2+z^2} \longrightarrow y=0,\ z=0$

$x^2 = 16(x^2-16x+64)$

$\longrightarrow$ solve : $x = 32/5,\ 32/3$

$\because$ Circle :



$\frac{x_1+x_2}{2} = \text{origin}$

$\therefore a = \frac{128}{15}$

$\vec{E} = \langle y, (x+9z), 9y \rangle$

$V(2,1,1) = 25$

$V(2,6,4) = ?$

① From $(2,1,1) \longrightarrow (2,6,1)$

$\int_1^6 x+9z\, dy \longrightarrow z=1$

$= 55$

② From $(2,6,1) \longrightarrow (2,6,4)$

$\int_1^4 9y\, dz \longrightarrow y=6$

$= 162$

$V(2,6,1) = V(2,1,1) - 55 - 162$

$= 25 - 55 - 162$

$= -192\ V$

O

$|V| = \frac{\rho_s}{2\varepsilon_0}$

$= \frac{(10\times10^{-9})(1.9-0.095)}{2\cdot\varepsilon_0}$

$= 536.469\ V$