# Magnetic Boundary Conditions

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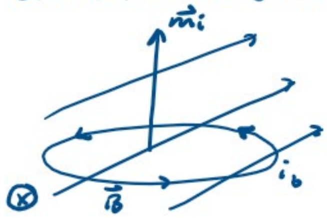
## Magnetic Materials

### Sources of Magnetism at atomic level

Electrons orbit around Nucleus


$\vec{\tau} = \vec{m}_i \times \vec{B}$

- If loop rotates along axis of torque
  → eventually align with $\vec{B}$

  $\therefore \vec{\tau}_i = \vec{m}_i \times \vec{B} = 0$
  → magnetic dipole not rotating

### Electron spin around own axis

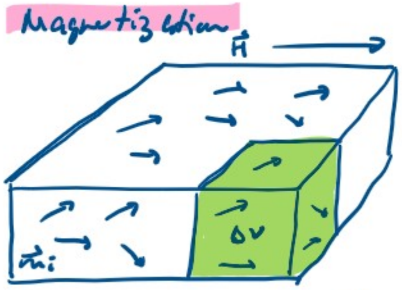
→ electron spin

→ spin magnetic moment: $\pm 9 \times 10^{-24}\ A\,m^2$

## Classifying materials

| Diamagnetics | Paramagnetics | Ferromagnetics |
|---|---|---|
| → $\vec{m}_{orb} + \vec{m}_{spin} = 0$<br>↳ spin & orbit cancel | → $\vec{m}_{orb} + \vec{m}_{spin} = $ small | → $\vec{m}_{spin} \gg \vec{m}_{orb}$ |
| → $\vec{B}_{in} < \vec{B}_{out}$ | → $\vec{B}_{in} > \vec{B}_{out}$<br><br>↳ dipoles align | → $\vec{B}_{in} \gg \vec{B}_{out}$<br><br>↳ some regions align w/ $\vec{B}$ & grow in size<br>↳ when $\vec{B}$ removed, don't go back to initial state<br>∴ remenant magnetic fields (hysterisis) |
| → Superconductors copper, gold, silicon | → potassium, tungsten | → Iron, nickel, cobalt |
| → temp. insensitive | → reduced by temp. | → lost after Curie temp. |

## Magnetization




($\vec{m}$ → magnetic moment)

### n magnetic dipoles/unit volume:

$$\vec{m}_{total} = \sum_{i=1}^{n\Delta v} \vec{m}_i$$

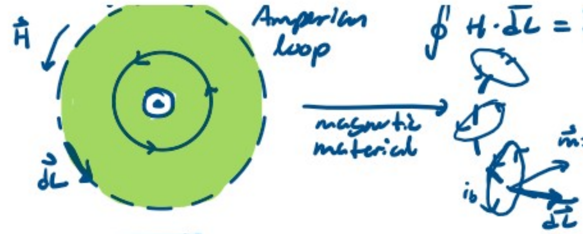
$$m = \frac{\vec{M}}{\# \text{atoms}}$$

### magnetic dipole moment/unit volume:

$$\vec{M} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{i=1}^{n\Delta v} \vec{m}_i \quad A/m$$ → when volume → 0


$\oint H \cdot \vec{dL} = I$

- small atomic dipole moments




$\oint H \cdot \vec{dL} = I$

- small atomic dipole moments contributing.
$\vec{m} = i_b \vec{dS}$ — pointing out of surface

$\therefore$ Total bound current:

Integrate over path & add effect of each $i_b$ entering highlighted surface.

$dI_b = \vec{M} \cdot \vec{dL}$

$I_b = \oint \vec{M} \cdot dL$

$\nabla \times \vec{M} = \vec{J}_b \longrightarrow$ bound current density

$\nabla \times \vec{H} = \vec{J} \longrightarrow$ free current density

Total current passing through highlighted surface:

$I_T = I_b + I$

$\therefore \oint \vec{H} \cdot dL = I_T \longrightarrow$ total current enclosed

$\oint \frac{\vec{B}}{\mu_0} \cdot \vec{dL} = I_T = \oint \vec{M} \cdot \vec{dL} + I$

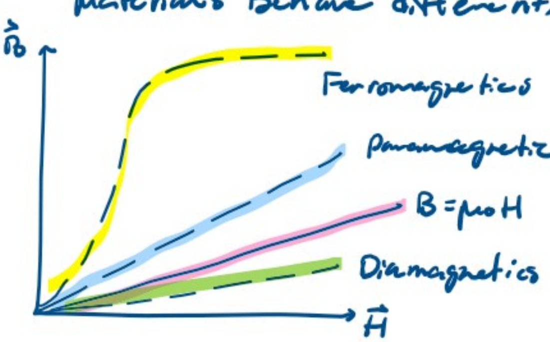
$\oint \left( \frac{\vec{B}}{\mu_0} - \vec{M} \right) \cdot \vec{dL} = I$

$\longrightarrow \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \longrightarrow \vec{B} = \mu_0 (\vec{H} + \vec{M})$

$\vec{M} = \chi_m \vec{H} \longrightarrow \chi_m$ = magnetic susceptibility

$\vec{B} = \mu_0 \mu_r \vec{H} \longrightarrow \mu_r = (\chi_m + 1)$
$\mu = \mu_0 \mu_r$

materials Behave differently at different $\vec{H}$:




## Magnetic Boundary Conditions

$\vec{B} = \mu_r \mu_0 \vec{H}$
- Total of all flux through closed surfaces = 0

$\oiint_S \vec{B} \cdot \vec{dS} = 0$

$\vec{B}_{N1} = \vec{B}_{N2}$ $\quad \vec{H}_{T1} = \vec{H}_{T2}$ $\quad \vec{B} = \mu \vec{H} \longrightarrow \mu = \mu_r \mu_0$