Chapter 11 The Uniform Plane Wave

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11.1 Wave propagation in free space

- transmission line
  - ↳ confine fields while enabling them to travel along as waves

11.1.1 Wave eqn. for Uniform Plane Wave

- for EM waves in free space:

Sourceless Medium:

$\rho_v = \vec{J} = 0$

$\therefore$ **Maxwell's Eqn:**

① $\nabla \times \vec{H} = \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$ → time-varying $\vec{E}$ → $\vec{H}$ curl at pt., $\vec{H}$ varies ⊥ to orientation dir.

② $\nabla \times \vec{E} = -\mu_0 \dfrac{\partial \vec{H}}{\partial t}$ → time-varying $\vec{H}$ → $\vec{E}$

③ $\nabla \cdot \vec{E} = \vec{0}$

④ $\nabla \cdot \vec{H} = \vec{0}$

- Uniform Plane wave (transverse electromagnetic TEM wave)
  - $\vec{E}$ & $\vec{H}$ lie in transverse plane (normal = dir. of propagation)
  - If $\vec{E}$ polarized in x → $\vec{E} = E_x \hat{a}_x$, & wave travel in $\hat{z}$

$\nabla \times \vec{E} = \dfrac{\partial E_x}{\partial z}\hat{a}_y = -\mu_0 \dfrac{\partial H_y}{\partial t}\hat{a}_y$

↳ dir. of curl $\vec{E}$ = dir. of $\vec{H}$

↳ $\therefore$ for TEM, $\vec{E} \perp \vec{H}$

$\nabla \times \vec{H} = -\dfrac{\partial H_y}{\partial z}\hat{a}_x = \varepsilon_0 \dfrac{\partial E_x}{\partial t}\hat{a}_x$

$\therefore \dfrac{\partial E_x}{\partial z} = -\mu_0 \dfrac{\partial H_y}{\partial t}$

$\dfrac{\partial H_y}{\partial z} = -\varepsilon_0 \dfrac{\partial E_x}{\partial t}$

- telegraphist eqn. for lossless transmission line
- **Wave eqn. for x-polarized TEM $\vec{E}$ field in free space:**

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

- **propagation velocity**

$v = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8 \text{ m/s} = c$

- **Wave Eqn. for magnetic field**

$$\frac{\partial^2 H_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 H_y}{\partial t^2}$$

11.1.2 Solutions of wave Eqn

- **Fwd & Bck propagating waves:**

$E_x(z,t) = f_1(t - z/v) + f_2(t + z/v)$

$= \underbrace{|E_{x0}|\cos(\omega t - k_0 z + \varphi_1)}_{\text{fwd } z \text{ travel}} + \underbrace{|E'_{x0}|\cos(\omega t + k_0 z + \varphi_2)}_{\text{bck } z \text{ travel}}$

↳ **Real inst. Form of E-field**

- phase velocity: $v_p = c$
- **wavenumber** in free space: $k_0 = \dfrac{\omega}{c}$ rad/m
- **wavelength** in free space: $k_0 z = k_0 \lambda = 2\pi \rightarrow \lambda = \dfrac{2\pi}{k_0}$
  - ↳ mth crest: $k_0 z = 2m\pi$
  - ↳ cos pt.: $\omega t - k_0 z = \omega(t - z/c) = 2m\pi$
- **Real inst. Form of E-field (phasor form):**

$E_x(z,t) = \frac{1}{2}|E_{x0}|e^{j\varphi_1}e^{-jk_0 z}e^{j\omega t} + c.c = \frac{1}{2}E_{xs}e^{j\omega t} + c.c = \mathrm{Re}[E_{xs}e^{j\omega t}]$

↳ c.c = complex conjugate

↳ Phasor electric field: $E_{xs} = E_{x0}e^{-jk_0 z}$

E_{x0} = complex amplitude

ex. $E_y(z,t) = 100\cos(10^8 t - 0.5z + 30°)$ V/m to phasor form:

$E_y(z,t) = \mathrm{Re}[100 e^{j(10^8 t - 0.5z + 30°)}]$ → exp.

$E_{ys}(z) = 100 e^{-j0.5z + j30°}$ → drop Re & suppress $e^{j10^8 t}$

11.1.3 Vector Helmholtz Eqn. in free space

- taking partial deriv. of any field → multiplying corresponding phasor by $j\omega$

$dH_{ys}(z) \quad j\omega\varepsilon_0 E_{xs}(z)$

11.1.3 Vector Helmholtz Eqn. in free space

- taking partial deriv. of any field $\rightarrow$ multiplying corresponding phasor by $j\omega$

$$\frac{dH_{yo}(z)}{dz} = -j\omega\varepsilon_0 E_{xs}(z)$$

- Maxwell's equ. (Phasor Form):

① $\nabla \times \vec{H}_s = j\omega\varepsilon_0 \vec{E}_s$

② $\nabla \times \vec{E}_s = -j\omega\mu_0 \vec{H}_s$

③ $\nabla \cdot \vec{E}_s = 0$

④ $\nabla \cdot \vec{H}_s = 0$

- Vector Laplacian of $\vec{E}_s$:

$$\nabla^2 \vec{E}_s = \nabla(\nabla \cdot \vec{E}_s) - \nabla \times \nabla \times \vec{E}_s$$

↳ Helmholtz Eqn. in free space:

$$\nabla^2 \vec{E}_s = -k_0^2 \vec{E}_s$$

↳ x-components: $\nabla^2 E_{xs} = -k_0^2 E_{xs}$

$\rightarrow \because$ uniform plane, not vary w/ x or y:

$$\therefore \frac{d^2 E_{xs}}{dz^2} = -k_0^2 E_{xs}$$

↳ solution: $E_{xs}(z) = E_{xo} e^{-jk_0 z} + E_{xo}' e^{jk_0 z}$

11.1.4 Intrinsic Impedance

- $\nabla \times \vec{E}_s = -j\omega\mu_0 \vec{H}_s$

↳ Real Inst. Form:

$$\vec{H}_y(z,t) = E_{xo}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cos(\omega t - k_0 z) - E_{xo}'\sqrt{\frac{\varepsilon_0}{\mu_0}}\cos(\omega t + k_0 z)$$

- $\vec{E}$ & $\vec{H}$ field amplitudes relationship

↳ Fwd. propagating:

$$E_{xo} = \sqrt{\frac{\mu_0}{\varepsilon_0}} H_{yo} = \eta_0 H_{yo}$$

↳ Bck. propagating:

$$E_{xo}' = -\sqrt{\frac{\mu_0}{\varepsilon_0}} H_{yo}' = -\eta_0 H_{yo}'$$

- Intrinsic Impedance of free space:

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \doteq 120\pi\ \Omega$$

↳ Similar to characteristic impedance Z_0 of transmission line

11.2 Wave propagation in Dielectrics

- Propagation in dielectric

↳ homogeneous & isotropic

↳ permittivity ε & permeability μ $\longrightarrow$ invariant

11.2.1 Propagation in Lossy Media

- Helmholtz in homogenous & isotropic medium:

$$\nabla^2 \vec{E}_s = -k^2 \vec{E}_s$$

↳ $\frac{d^2 E_{xs}}{dz^2} = -k^2 E_{xs}$

- wavenumber (func. of material properties):

$k = \omega\sqrt{\mu\varepsilon} = k_0\sqrt{\mu_r \varepsilon_r}$

↳ complex propagation const. k

↳ complex when loss or gain in medium

↳ in terms of Re & Im: $jk = \alpha + j\beta$

- Solution:

$$E_{xs} = E_{xo} e^{-jkz} = E_{xo} e^{-\alpha z} e^{-j\beta z}$$

↳ $E_x = E_{xo} e^{-\alpha z}\cos(\omega t - \beta z)$

↳ Uniform plane wave prop. in z w/ phase const. β

↳ α lose amplitude as $\uparrow z \because e^{-\alpha z}$

↳ (+) $\alpha \rightarrow$ attenuation coeff.

↳ (−) $\alpha \rightarrow$ gain coeff.

- complex permittivity:

↳ Materials physical processes can affect wave $\vec{E}$-field

$$\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon_0(\varepsilon_r' - j\varepsilon_r'')$$

↳ complex permittivity $\rightarrow$ wave loss

- Complex permeability:

↳ Losses from medium response to $\vec{B}$-field

$$\mu = \mu' - j\mu'' = \mu_0(\mu_r' - j\mu_r'')$$

↳ ex. ferrimagnetic

↳ weak $\vec{B}$-response, $\mu \approx \mu_0$

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$$k = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\varepsilon''}{\varepsilon'}}$$

$$\alpha = \mathrm{Re}\{jk\} = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right)^{1/2}$$

$$\beta = \mathrm{Im}\{jk\} = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right)^{1/2}$$

↳ $\frac{\varepsilon''}{\varepsilon'}$ → loss tangent

- Wave phase velocity: $v_p = \frac{\omega}{\beta}$
- Wavelength: $\lambda = \frac{2\pi}{\beta}$

- Magnetic Field (Uniform plane wave):
  $H_{ys} = \frac{E_{x0}}{\eta} e^{-\alpha z} e^{-j\beta z}$

- Intrinsic Impedance (complex)
  $\eta = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}} = \sqrt{\frac{\mu}{\varepsilon'}}\frac{1}{\sqrt{1 - j(\varepsilon''/\varepsilon')}}$
- $\vec{E}$ & $\vec{H}$ fields no longer in phase

- For lossless medium / perfect dielectric:
  ↳ $\varepsilon'' = 0$, $\varepsilon = \varepsilon'$, $\sigma = 0$] free space
  ↳ $\beta = \omega\sqrt{\mu\varepsilon'}$
- $\vec{E}$-field intensity: $E_x = E_{x0}\cos(\omega t - \beta z)$
  ↳ phase velocity: $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon'}} = \frac{c}{\sqrt{\mu_r\varepsilon_r'}}$
  ↳ wavelength: $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon'}} = \frac{1}{f\sqrt{\mu\varepsilon'}} = \frac{c}{f\sqrt{\mu_r\varepsilon_r'}} = \frac{\lambda_0}{\sqrt{\mu_r\varepsilon_r'}}$
    ↳ λ_0 → free space wavelength
- ↓λ & ↓v in real media than freespace
- Magnetic field intensity:
  $H_y = \frac{E_{x0}}{\eta}\cos(\omega t - \beta z)$
  ↳ Intrinsic Impedance: $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

https://gradeup.co/EM-Wave-Propagation-notes-topics-for-gate-ec-2019-i-ade2cda0-738b-11e7-9a49-9f4a78e2f6a9

11.2.2 Propagation in Conducting Media
- Conductive Materials
  ↳ Currents formed → Free e^- or holes from $\vec{E}$ field
- Maxwell curl eqn.:
  $\nabla \times \vec{H}_s = j\omega(\varepsilon' - j\varepsilon'')\vec{E}_s = \omega\varepsilon''\vec{E}_s + j\omega\varepsilon'\vec{E}_s$
  ↳ $\vec{J}_s = \sigma\vec{E}_s$
  ↳ $\nabla \times \vec{H}_s = (\sigma + j\omega\varepsilon')\vec{E}_s = \vec{J}_{\sigma s} + \vec{J}_{ds}$
    ↳ Conduction Current density: $\vec{J}_{\sigma s} = \sigma\vec{E}_s$
    ↳ Displacement Current density: $\vec{J}_{ds} = j\omega\varepsilon'\vec{E}_s$
- For conductive Medium: $\varepsilon'' = \frac{\sigma}{\omega}$

- For small loss in dielectric material:
  ↳ ↓ loss tangent $\varepsilon''/\varepsilon' = \frac{\sigma}{\omega\varepsilon'} = \tan\theta$
  ↳ $\frac{\vec{J}_{\sigma s}}{\vec{J}_{ds}} = \frac{\varepsilon''}{j\varepsilon'} = \frac{\sigma}{j\omega\varepsilon'}$
    ↳ $\vec{J}_{ds}$ leads $\vec{J}_{\sigma s}$ by 90°

11.2.3 Good Dielectric Approximation
- ↓ loss tangent ($\varepsilon''/\varepsilon' \ll 1$) → good dielectric
  ↳ Conductive material:
    $jk = j\omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon'}}$ $\left(\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon'} \rightarrow \sigma = \omega\varepsilon''\right)$
  ↳ good dielectric:
    $$\alpha = \mathrm{Re}(jk) \doteq j\omega\sqrt{\mu\varepsilon'}\left(-j\frac{\sigma}{2\omega\varepsilon'}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon'}}$$
    $$\beta = \mathrm{Im}(jk) \doteq \omega\sqrt{\mu\varepsilon'}\left[1 + \frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon'}\right)^2\right] \doteq \omega\sqrt{\mu\varepsilon'}$$

- Intrinsic Impedance:
  $\eta \doteq \sqrt{\frac{\mu}{\varepsilon'}}\left(1 + j\frac{\sigma}{2\omega\varepsilon'}\right)$

11.3 Poynting's Theorem & Wave Power
- Finding EM Power Flow

11.3 Poynting's Theorem & Wave Power

- Finding EM power flow

$$\underbrace{-\oint(\vec{E}\times\vec{H})\cdot d\vec{S}}_{\text{total power flowing out of vol.}} = \underbrace{\underbrace{\int_{vol}\vec{J}\cdot\vec{E}\,dv}_{\text{total inst. power dissipated in vol}} + \underbrace{\underbrace{\frac{d}{dt}\int_{vol}\frac{1}{2}\vec{D}\cdot\vec{E}\,dv}_{\text{Energy stored in }\vec{E}\text{-field}} + \underbrace{\frac{d}{dt}\int_{vol}\frac{1}{2}\vec{B}\cdot\vec{H}\,dv}_{\text{Energy stored in }\vec{B}\text{-field}}}_{\text{rate of inc. in energy stored in vol.}}}_{\text{total power flowing into of vol.}}$$

- Poynting vector: $\vec{S} = \vec{E}\times\vec{H}$ W/m^2
  - ↳ dir. of power flow $\perp$ $\vec{E}$ & $\vec{H}$
  - ↳ Power density amplitude: $S_z = \frac{E_{x0}^2}{\eta}\cos^2(\omega t - \beta z)$

$$\langle S_z\rangle = \frac{1}{2}\frac{E_{x0}^2}{|\eta|}e^{-2\alpha z}\cos\theta_\eta$$

  - ↳ vector form: $\langle\vec{S}\rangle = \frac{1}{2}\,\mathrm{Re}\{\vec{E}_s\times\vec{H}_s^*\}$ W/m^2
    - ↳ $\vec{E}_s = E_{x0}e^{-j\beta z}\hat{a}_x$
    - ↳ $\vec{H}_s^* = \frac{E_{x0}}{\eta^*}e^{j\beta z}\hat{a}_y = \frac{E_{x0}}{|\eta|}e^{j\theta}e^{j\beta z}\hat{a}_y$

11.4 Propagation in good Conductors

- Uniform plane wave established in good conductor
- high loss → $\varepsilon''/\varepsilon' \gg 1$
  - ↳ good conductor → $\sigma/\omega\varepsilon' \gg 1$
- losses occur
- EM waves existing in external dielectric adjoint to conductor surface
  - ↳ waves propagate along surface
  - ↳ field within conductor → dissipative loss from conduction currents
  - ↳ ↓field → ↑distance along surface
    - ↳ resistive transmission line loss

11.4.1 Good Conductor Approximations

- ↑ conductivity, ↑ conduction currents
- ↓ Energy from wave traveling through

$\therefore \alpha = \beta = \sqrt{\pi f\mu\sigma}$

- For E_x component traveling in $\hat{z}$:

$E_x = E_{x0}e^{-z\sqrt{\pi f\mu\sigma}}\cos(\omega t - z\sqrt{\pi f\mu\sigma})$

  - ↳ external field @ conductor surface:
    - good conductor → $z>0$
    - perfect dielectric → $z<0$
    - @ Boundary surface $z=0$:
      - ↳ $E_x = E_{x0}\cos(\omega t)$

- Conduction Current Density in conductor:

$J_x = \sigma E_x = \sigma E_{x0}e^{-z\sqrt{\pi f\mu\sigma}}\cos(\omega t - z\sqrt{\pi f\mu\sigma})$

11.4.2 Skin Effect

- exp. decrease in conduction current density & $\vec{E}$ with penetration in conductor
  - ↳ $e^{-z\sqrt{\pi f\mu\sigma}} = 1 \rightarrow z = 0$ → Power loss over dist. z: $L = e^{-2\alpha z}$ → $\alpha = \frac{1}{\delta}$
  - ↳ $e^{-1} \rightarrow z = \frac{1}{\sqrt{\pi f\mu\sigma}}$
- Skin Depth / depth of penetration

$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}} = \frac{1}{\alpha} = \frac{1}{\beta}$

- EM energy is not transmitted in interior of conductor, travels region surrounding conductor → guides waves
  - ↳ Short skin depth @ Microwave freq → only surface coating of guiding conductor important (ex. glass w/ 3μm silver coating)
- Wavelength: $\beta = \frac{2\pi}{\lambda} \rightarrow \lambda = 2\pi\delta$
- phase velocity: $v_p = \frac{\omega}{\beta} = \omega\delta$

11.4.3 Intrinsic Impedance & Power density in good conductors

- Intrinsic Impedance: $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon'}} = \sqrt{\frac{j\omega\mu}{\sigma + \cancel{j\omega\varepsilon'}}}$ → $\because \sigma \gg \omega\varepsilon'$

$$= \frac{\sqrt{2}\angle 45^\circ}{\sigma\delta} = \frac{(1+j)}{\sigma\delta}$$

- Field Intensities:

$E_x = E_{x0}e^{-z/\delta}\cos(\omega t - \frac{z}{\delta})$

$H_y = \frac{\sigma\delta E_{x0}}{\ldots}e^{-z/\delta}\cos(\omega t - \frac{z}{\delta} - \frac{\pi}{\ldots})$

- **Field Intensities:**
  $E_x = E_{x0} e^{-z/\delta} \cos(\omega t - \frac{z}{\delta})$
  $H_y = \frac{\sigma \delta E_{x0}}{\sqrt{2}} e^{-z/\delta} \cos(\omega t - \frac{z}{\delta} - \frac{\pi}{4})$
- **Poynting vector** (time avg)
  $\langle S_z \rangle = \frac{1}{2} \frac{\sigma \delta E_{x0}^2}{\sqrt{2}} e^{-2z/\delta} \cos(\frac{\pi}{4})$
  $\langle S_z \rangle = \frac{1}{4} \sigma \delta E_{x0}^2 e^{-2z/\delta}$

11.4.4 Skin effect Resistance in conductors

- freq. dependent Resistance in Conductors

- $\downarrow \vec{J} \rightarrow \uparrow z$ as wave attenuates
- **avg Power loss** in width **$0<y<b$** & length **$0<x<L$** in current direction:
  $P_L = \frac{1}{4\sigma} \delta b L J_{x0}^2$
  ↳ current distributed uniformly in 1 skin depth

- Resistance @ ↑ freq w/ skin effect:
  $R = \frac{L}{\sigma \delta} = \frac{L}{2\pi a \sigma \delta}$ → slab width $2\pi a$, thickness δ

11.5 Wave Polarization

- Inst. orientation of field vectors
- time dependent $\vec{E}$ field vector orientation @ fixed pt. in space

11.5.1 Linear Polarization

- $\vec{E} \rightarrow$ fixed straight orientation @ all time & pos.
- for $\hat{z}$ propagation, wave **$\vec{E}$ field phasors:**
  $\vec{E}_s = (E_{x0} \hat{a}_x + E_{y0} \hat{a}_y) e^{-\alpha z} e^{-j\beta z}$
- **Magnetic Field:**
  $H_s = [H_{x0} \hat{a}_x + H_{y0} \hat{a}_y] e^{-\alpha z} e^{-j\beta z} = \left[-\frac{E_{y0}}{\eta} \hat{a}_x + \frac{E_{x0}}{\eta} \hat{a}_y\right] e^{-\alpha z} e^{-j\beta z}$

- **Power density:**
  $\langle S_z \rangle = \frac{1}{2} Re\left\{\frac{1}{\eta^*}\right\} (|E_{x0}|^2 + |E_{y0}|^2) e^{-2\alpha z} \hat{a}_z$
  $= \frac{1}{2} Re\{\vec{E}_s \times \vec{H}_s^*\} \longrightarrow = \frac{1}{2} Re\{\vec{E}_s \times \vec{H}_s\} e^{-2\alpha z} \cos\varphi \, \hat{a}_z$
  $= \frac{1}{2|\eta|} (E_{x0}^2 + E_{y0}^2) e^{-2\alpha z} \hat{a}_z \rightarrow (\times \cos\varphi \text{ if } \varphi)$
- **Received power:** $P = \iint_{plate} \langle \vec{S} \rangle \cdot d\vec{S} = \frac{1}{2|\eta|} (wh)(E_{x0}^2 + E_{y0}^2) e^{-2\alpha d} \rightarrow (\times \cos\varphi \text{ if } \varphi)$

11.5.2 Phase-Displaced Field components: Elliptical Polarization

- lossless medium prop.
  ↳ **Phasor Form:**
  $\vec{E}_s = (E_{x0} \hat{a}_x + E_{y0} e^{j\varphi} \hat{a}_y) e^{-j\beta z}$
  ↳ **Real Inst. Form:**
  $\vec{E}(z,t) = E_{x0} \cos(\omega t - \beta z) \hat{a}_x + E_{y0} \cos(\omega t - \beta z + \varphi) \hat{a}_y$

- @ $t=0$: $E(z,0) = E_{x0} \cos(\beta z) \hat{a}_x + E_{y0} \cos(\beta z - \varphi) \hat{a}_y$
- E_y leads E_x
- If take length of field vector, @ fixt pos., tip of vector traces **ellipse** over time $t = 2\pi/\omega$

11.5.3 Circular Polarization

- **$E_{x0} = E_{y0} = E_0$**, **$\varphi = \pm \pi/2$** → circular
  $E(z,t) = E_0 [\cos(\omega t - \beta z) \hat{a}_x \mp \sin(\omega t - \beta z) \hat{a}_y]$
- fixed pos. along z ($z=0$)
  ↳ **$\varphi = \pi/2$**: $E(0,t) = E_0 [\cos(\omega t) \hat{a}_x - \sin(\omega t) \hat{a}_y]$
  ↳ **$\varphi = -\pi/2$**: $E(0,t) = E_0 [\cos(\omega t) \hat{a}_x + \sin(\omega t) \hat{a}_y]$
- Field vector Rotates in CCW in xy plane
- **Inst. angle of field:**
  $\theta(z,t) = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \mp(\omega t - \beta z)$

- **Phasor Form:**
  ↳ **Left** Circular polarization
  $E_s = E_0 (\hat{a}_x \pm j \hat{a}_y) e^{-j\beta z}$
  ↳ **Right** Circular polarization
  $E_s = E_0 (\hat{a}_x \pm j \hat{a}_y) e^{+j\beta z}$

- Phasor Form:
  - ↳ Left Circular polarization
    $\vec{E}_s = E_0(\hat{a}_x \pm j\hat{a}_y)e^{-j\beta z}$
  - ↳ Right Circular polarization
    $\vec{E}_s = E_0(\hat{a}_x \pm j\hat{a}_y)e^{+j\beta z}$