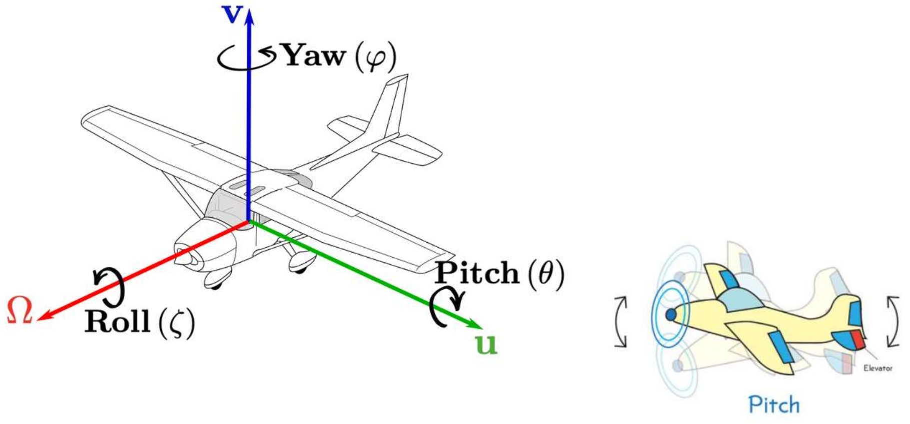
Introduction

The Model Reference Adaptive Control (**MRAC**) block computes control actions to make an uncertain controlled system track the behavior of a given reference plant model. It estimates the feedback and feedforward controller gains based on the real-time tracking error between the states of the reference plant model and the controlled system. It incorporates a reference model to capture the desired closed-loop responses and designs the control law and adaptation algorithm to force the output of the plant to follow the output of the reference model.

Model-Reference Adaptive Control (MRAC)

This exercise is focused on the design of adaptive control design problem in the **MRAC** scheme for **aero-dynamics** the hovering helicopter in the pitch motion to verification of the designed control system in the Matlab-Simulink environment. The adjustment rules derived upon the **Lyapunov stability** analysis which guarantees boundedness and asymptotic convergence of the model-following error in the MRAC control system.

Plant Description



Source: [Yaw Pitch Roll - SadhikaMaja]

Fig 1. Yaw, pitch and roll motion of aero

The dynamics of pitch motion can be approximated in the following equation:

$$\dot{x}(t) = \underbrace{\theta_{a0}}_{a_0} x(t) + \underbrace{(-1)}_{b_0} \cdot \underbrace{\theta_{b0}}_{\lambda_0} \left(u(t) + \underbrace{\frac{-\theta_{c0}}{\theta_{b0}}}_{\theta_0} \underbrace{\left(\frac{360}{\pi} x(t)\right)}_{\phi(x(t))}\right)$$

where

***q*=** pitch rate in rad/s,

u is the control input in rad, while ***θa*0**, ***θb*0**,and ***θc*0** represent the unknown (**true**) parameters of the pitch dynamics.

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Control Method and Description

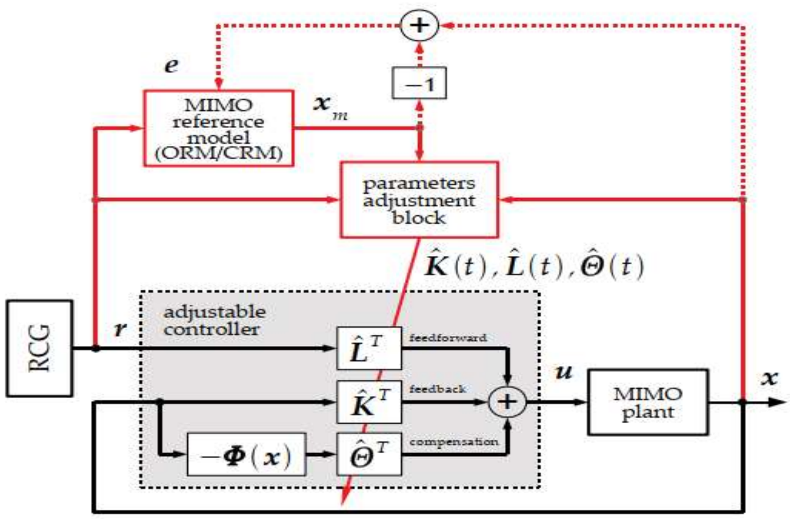

Fig 2a. MRAC scheme source: lecture material

Working Principle

The **working principle** of Model Reference Adaptive Control (MRAC) system:

- The reference model's desired output is compared to the plant's actual output. This difference is called the error signal.
- The adjustable controller uses the error signal to generate a control signal that's sent to the plant.
- The plant's actual output is fed back to the controller and compared to the reference model's output. This creates a new error signal.
- The parameter adjustment block uses the error signal to adjust the parameters of the adjustable controller.

Control performance requirements

- **R1.** pitch rate ***q*(*t*)** follows a time-varying reference in the form of a bounded time-varying signal with bounded time-derivative.
- **R2.** The following **error** asymptotically converges to **zero** as time goes infinitive.

Control System Design of MRAC

The adjustable (adaptive) version of the controller

$$u(t) \triangleq \hat{k}(t)x(t) + \hat{l}(t)r(t) - \hat{\theta}(t)\phi(x(t))$$

where $\hat{k}(t)$, $\hat{l}(t)$, $\hat{\theta}(t)$ adjusted online in a way that guarantees that the error asymptotically converges to **zero** as time goes infinity.

The *adjustment rules* shall be formulated as follows

$$\hat{k}(t) = \hat{k}(0) - \frac{pb_0}{\gamma_x}\int_0^t x(\tau)e(\tau)d\tau$$

$$\hat{l}(t) = \hat{l}(0) - \frac{p\bar{b}_0}{\gamma_r}\int_0^t r(\tau)e(\tau)d\tau,$$

$$\hat{\theta}(t) = \hat{\theta}(0) + \frac{p\bar{b}_0}{\gamma_\phi}\int_0^t \phi(x(\tau))e(\tau)d\tau$$

Simulation Result and Analysis

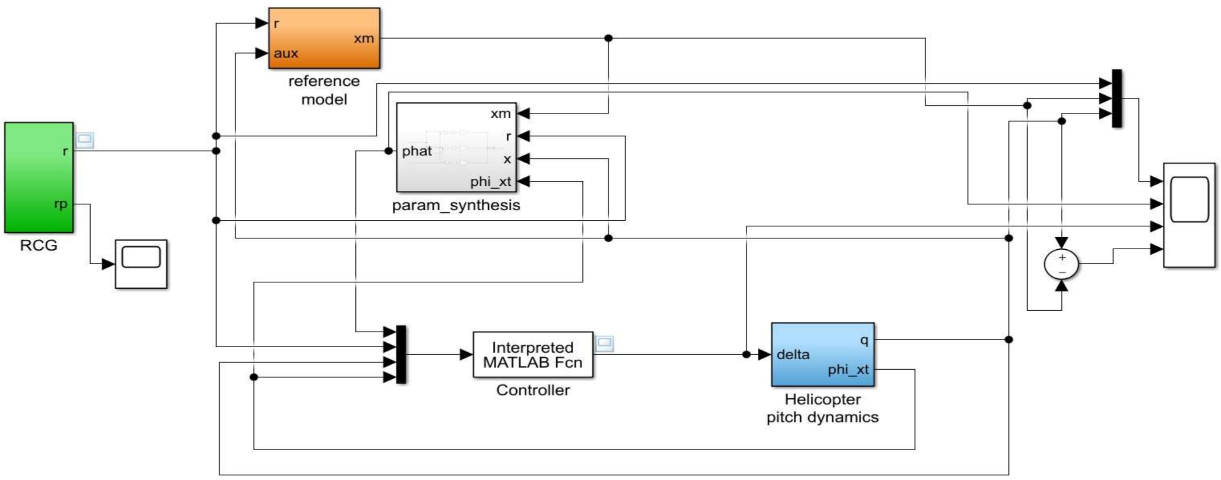

Fig 2b. Simulink model

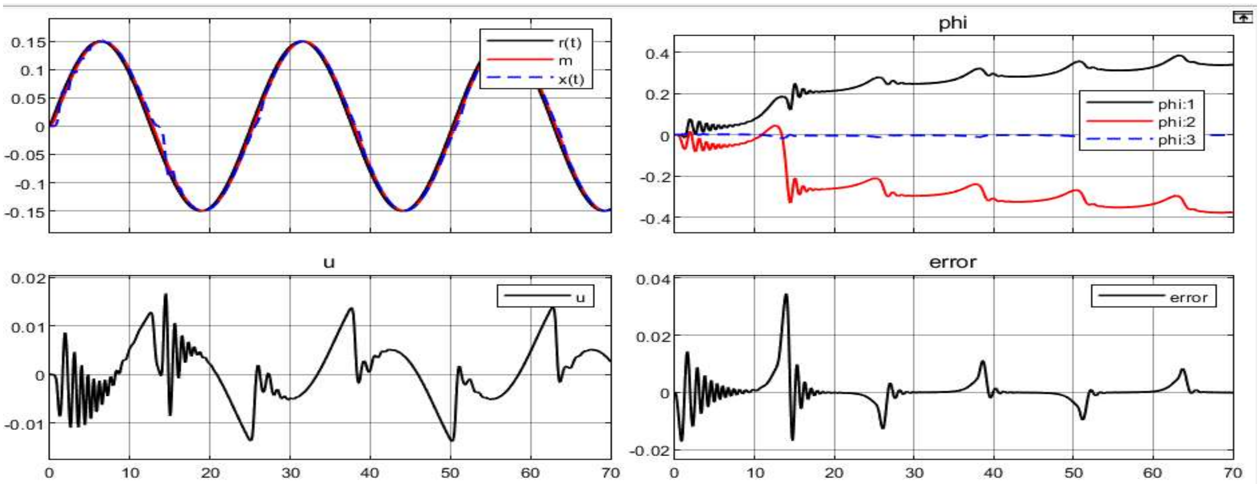

Fig 3. Closed loop adaptation at sigma2e=0 for sinusoidal wave reference input

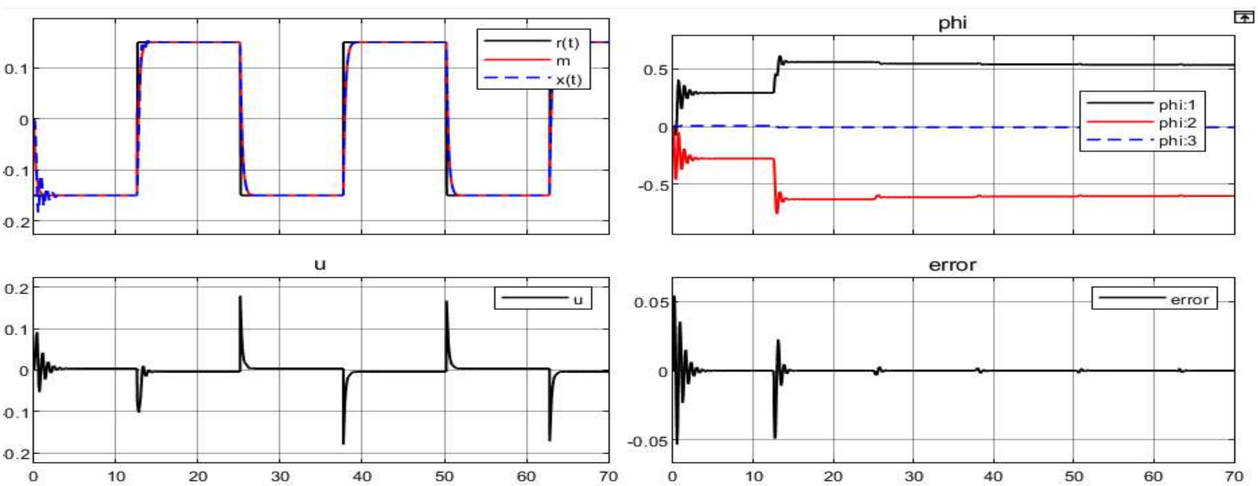

Fig 4. Closed loop adaptation at sigma2e=0 with rectangular reference

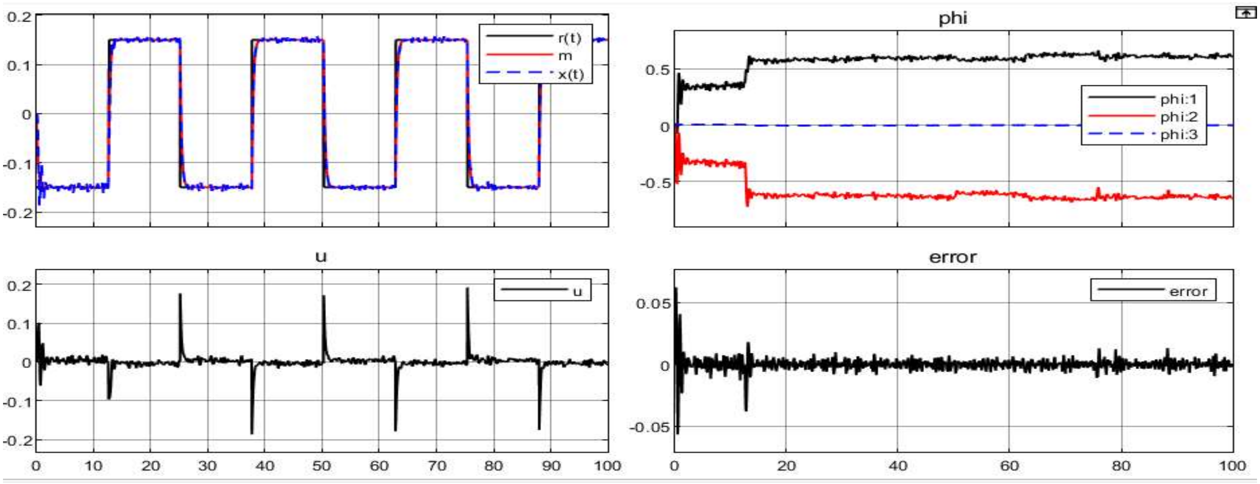

Fig 5. Closed loop adaptation at sigma2e=0.001 with rectangular reference input

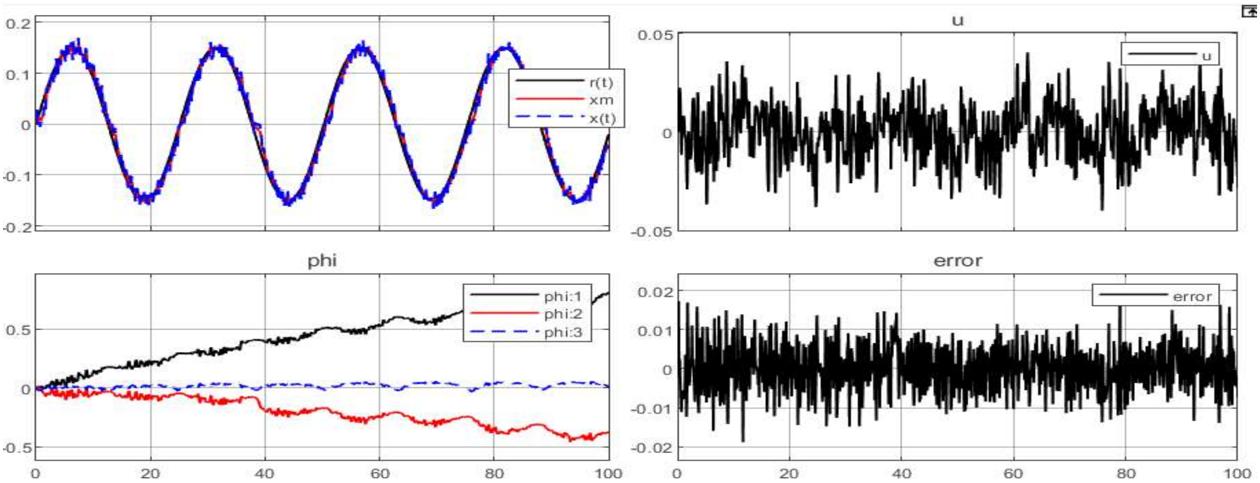

Fig 6 a. Closed loop adaptation at sigma2e=0.01 with sinusoidal wave reference input

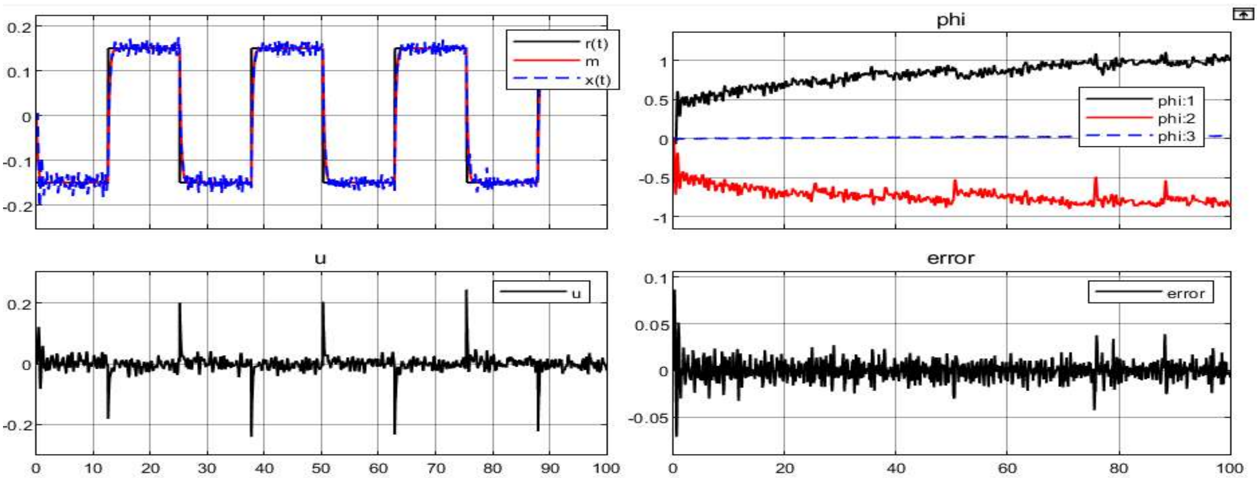

Fig 6 b. Closed loop adaptation at sigma2e=0.01 with rectangular reference input

When the variance of stochastic noise was zero, the transient error was high at the beginning but the error decreased when times went to infinitive as shown in **Fig 3**.

- We used both **rectangular** and **sinuosoidal** reference input , and we observed that **rectangular** wave has rapid transitions from high to low states.
- These fast changes resulted in an increased error during transitions as shown in **Fig 4** .
- However, the sinusoidal wave is continuous and smooth. This resulted in a lower error compared to a rectangular wave as shown in **Fig 3**.

- When sigma values increased from 0 to 0.01, the noise added uncertainty to the system's behavior, which made it difficult for the controller to distinguish between the actual system output and the effect of noise.
- This led to a larger error between the system output and the desired reference model output as shown in **Fig 6b** as compared to the error free**(Fig 4)**.

NB. The system satisfy performance requirements of R1 and R2 at sigma= 0 and 0.001, whereas due to higher error at **sigma=0.01**, the system **didn't satisfy** the performance requiremet.

With addition of auxilary feedback signal

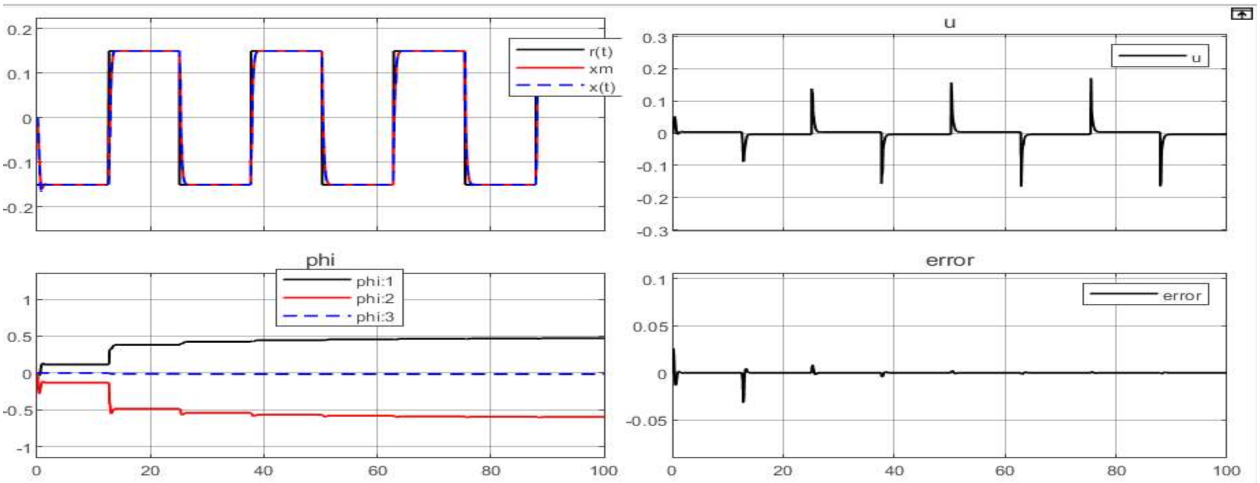

Fig 7. Closed loop adaptation at sigma2e=0.0 with rectangular reference input

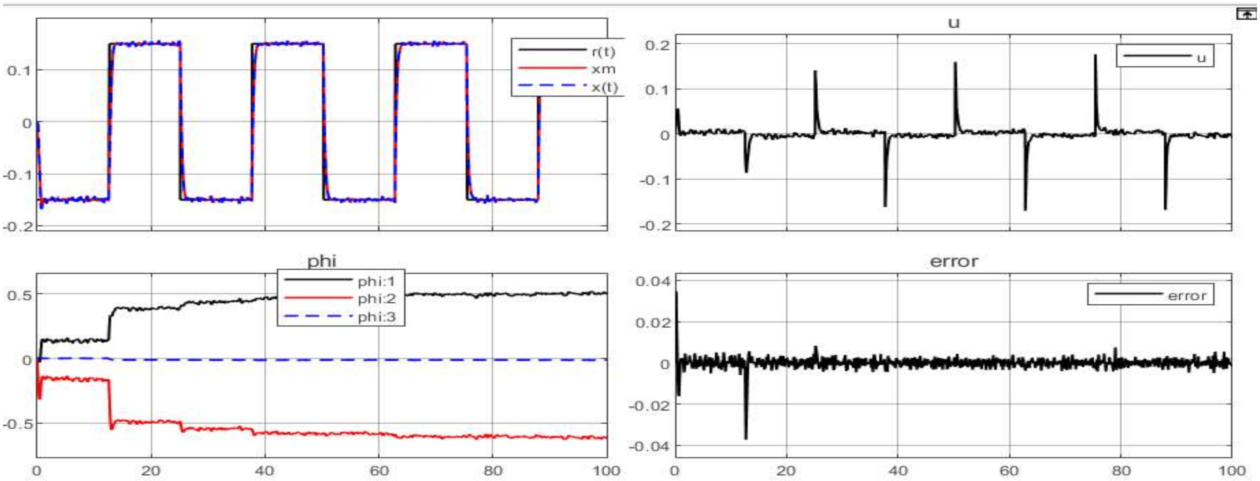

Fig 8. Closed loop adaptation at sigma2e=0.001 with rectangular reference input

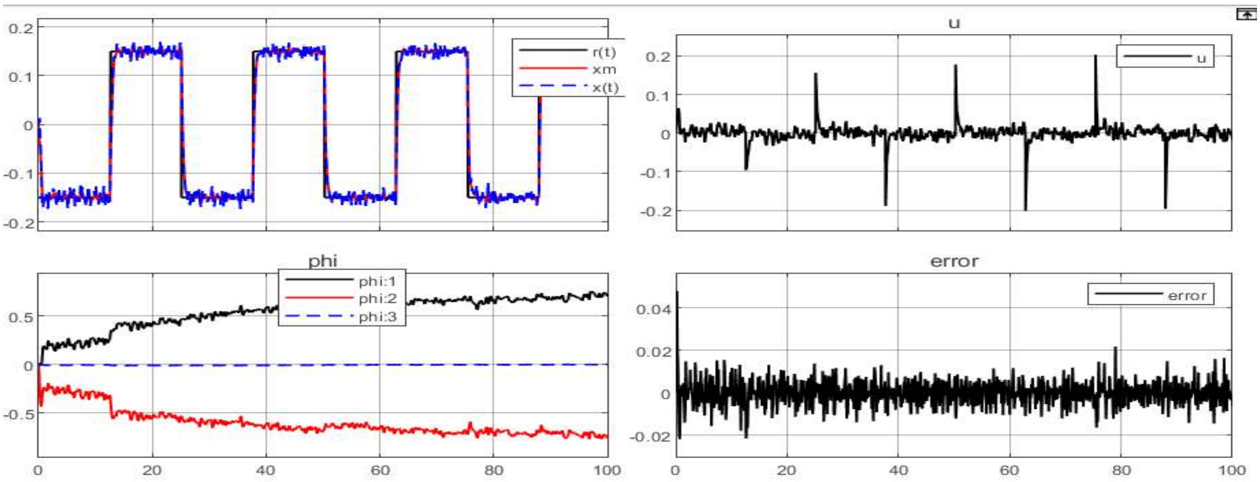

Fig 9. Closed loop adaptation at sigma2e=0.01 with rectangular reference input

- When we introduced the auxilary feedback signal (**Fig 7)**, the adaptation process was faster and the transient response was less disturbed as we compared to the result we got with out the addition of auxilary feedback signal (**Fig 4**).
- As shown from **Fig 7-9,** when we increased sigma value, the uncertainity of the plant dynamics increased even if we introduced the auxiliary feedback signal.
- The system satisfy performance requirements of R1 and R2 in the first two cases (**Fig 7-8**) However, in the last case(**Fig 9**), the performance was better than the result with no auxiliary feedback(**Fig 6 b**). Even if it is designer's choice, we could say that on this condition, the controller satisfied the performance requirements.

Checking the effect of some parameter changes on the plant

Condition 1: Changing the adaptation gains gamma_x, gamma_r, gamma_phi

In **MRAC**, the adaptation gain determines how quickly and effectively the controller learns and adjusts according to the desired behavior of the reference model.

In this activity, we checked both conditions

For lower adaptation gain:

- When we used gamma_x, gamma_y and gamma_phi **half of the given value**, we observed that the adaptive controller quickly changed the parameters in response to the error (**x-xm**). It showed faster convergence, which means that **x** value tracked **xm** more quickly(**Fig 10**).
- However, when we decreased the value of adatation gains too much, it led to instability in the output of the system and made the control signal noisy, this decreased the control performance as shown in **Fig 11**. we zoomed in **Fig 11** to demonstrate the effect.

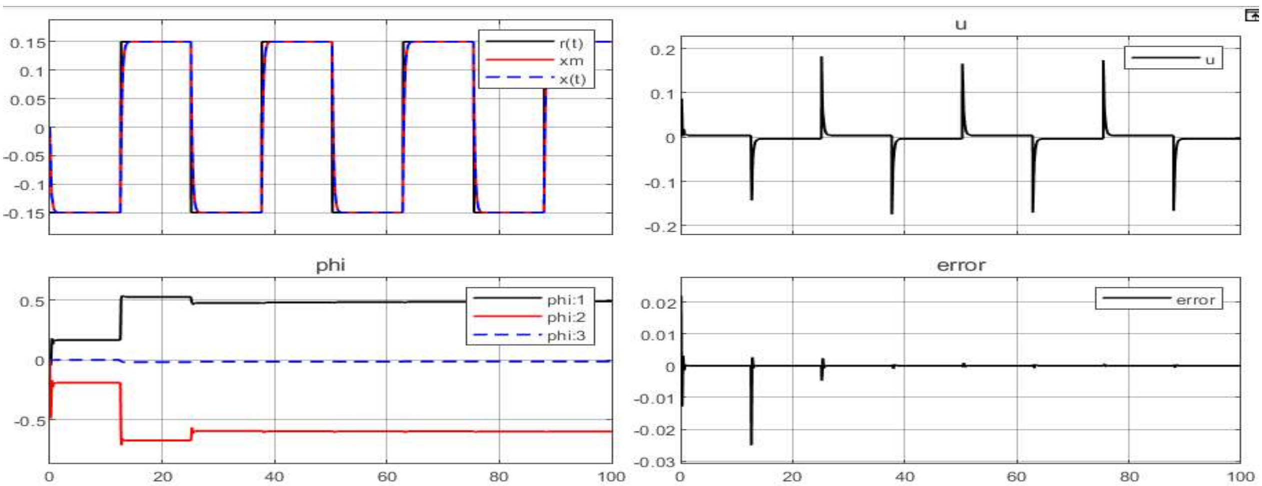

Fig 10. Adaptation gains were **half** times the given adaptation gains

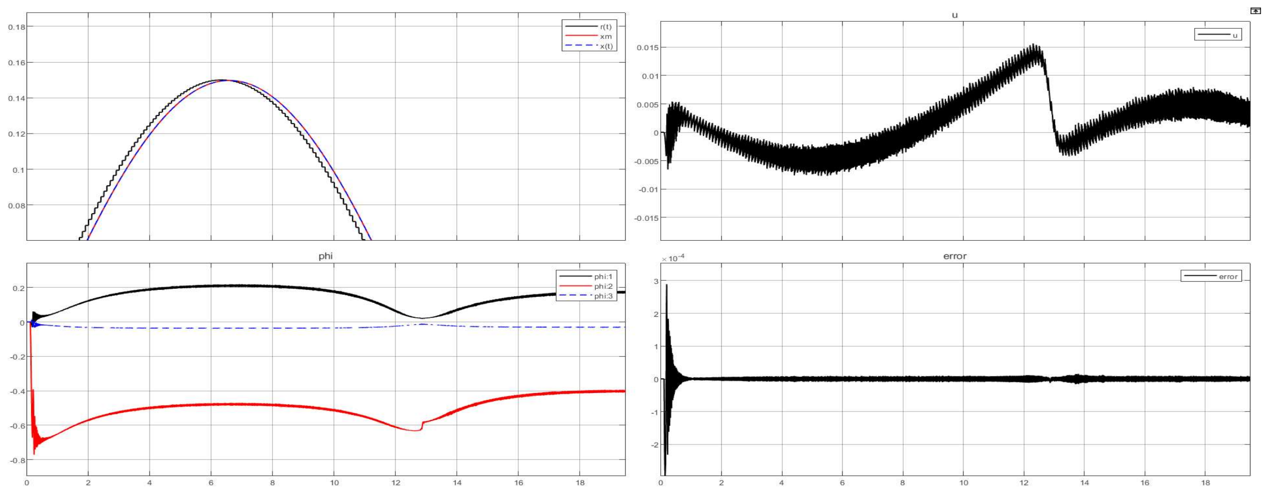

Fig 11. Adaptation gain **one-fifth** times the given adaptation gain

For higher adaptation gain:

- We used a 5 times of the given adaptation gain, it made the adaptation process slower since the controller added smaller adjustments with each iteration. This took longer time for the system's output to converge and accurately track the reference model.

Condition 2: Changing value of reference model parameters am and bm

The values of **am** and **bm** influenced the convergence and tracking capability of our system.

- Appropriate values (in this experiment, i.e., am = -4 and bm = 4) helped the controller adjust its internal parameters, this led to better tracking performance. This result was justified by comparing **Fig 7** and **Fig 12**

Low values of the parameters am and bm

- We used four times less value than the given value. As shown in **Fig 12**, the adaptation process was slow, resulting in low tracking performance.

High values of the parameters am and bm

- when we used a higher value than the given value, the controller was instable.

Based on the above results, we could say that improper valuesselection for **am** and **bm** reduced the adaptation process.

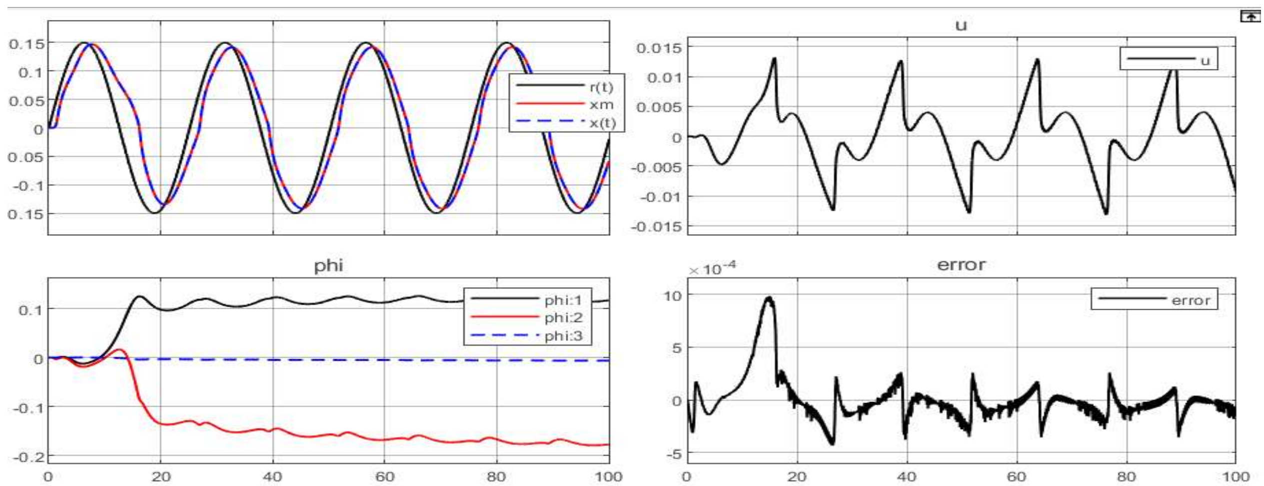

Fig 12. The effect low values of parameter am and bm

Condition 3: Adding auxiliary feedback gain ke

Selecting a high-gain value for the auxilary feedback signal, led to high disturbances at the initial stage(the transient response was highly disturbed), but the steady state response was good, as shown in **Fig 13.**

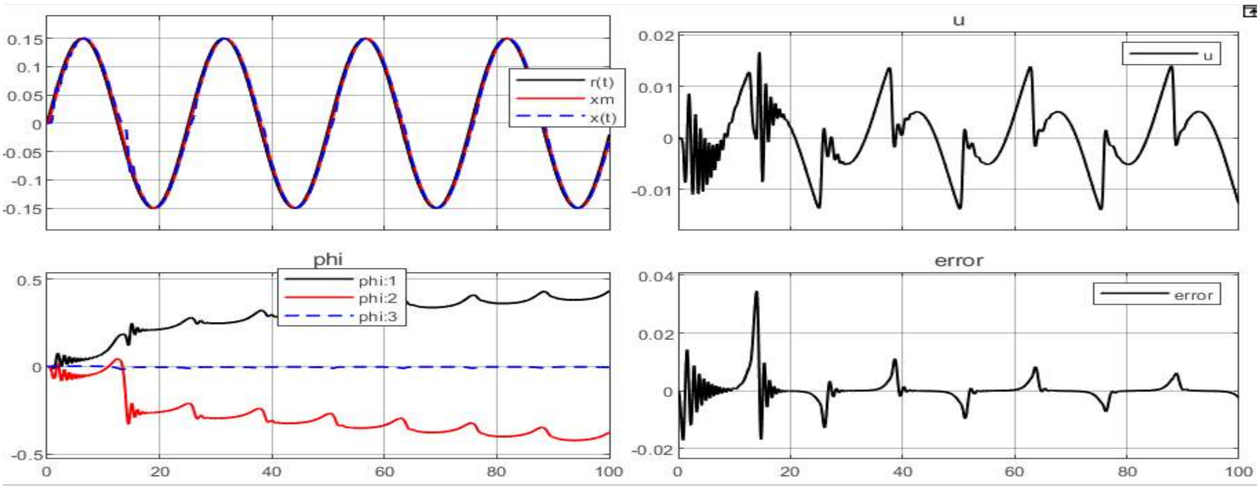

Fig 13. The effect of high auxilary feed back signal

Conclusion

MRAC is a type of adaptive controller that adjusts controller gains and disturbance models to achieve robust tracking performance despite system uncertainties. There is no need of parameter identification in this adaptive scheme, it needs only an approximated reference model based on prior knowledge. MRAC is one of the commonly used control scheme in robot manipulators, aero-dynamic plants and UAVs. We have investigated the performance of MRAC controller using **aero-plant pitch dynamics** and the simulation results demonstrated the effectiveness of MRAC in various applications

Based on our finding from the simulation, we conclude that:

- The appropriate selection of **adaptation gain** is important to ensure good control performance, good tracking, and stability.
- Proper values of adaptive Parameters (am, bm) ensure good tracking and stability.
- Higher noise variance increases tracking error, slows convergence, and requires more control effort.
- Lower noise leads to good control performance and tracking.
- The system satisfy performance requirements of R1 and R2 in all the cases when auxiliary feedback was added.

In general, MRAC is an excellent contol method, especially for highly uncertain and nonlinear dynamics that are not easy to measure their states.