Introduction

Linearization is a method used to simplify a nonlinear system by approximating it as a linear system, typically around a specific operating point. This technique is very advantageous in control design, as linear systems are easier to study and control.

Input-State Linearization

Input-state linearization is a control technique that transforms a nonlinear system into a linear one through a change of variables and a suitable control input. By applying this method, the original nonlinear system is represented in a simpler, linear form, enabling the use of well-established linear control strategies.

The process of input-state linearization typically involves:

1. **Change of Variables**: Nonlinear state variables are transformed into new variables that linearize the system dynamics.
2. **Control Input Transformation**: The control input is adjusted to maintain linear behavior after the state transformation.

Feedback Linearization

Feedback linearization is a technique where the original nonlinear system is transformed into an equivalent linear system by applying a feedback control law. This transformation simplifies the system's behavior, making it easier to design controllers such as proportional-integral-derivative (PID) or optimal controllers.

Controllability in Linearized Systems

A system is considered **controllable** if it can be manipulated into a desired state by an appropriate choice of control inputs. After linearizing the system, the controllability condition can often be checked by expressing the system in the following state-space form:

$$\frac{dx}{dt} = Ax + Bu$$

Where:

- x represents the state vector,
- u is the control input,
- A and B are matrices that define the system dynamics after linearization.

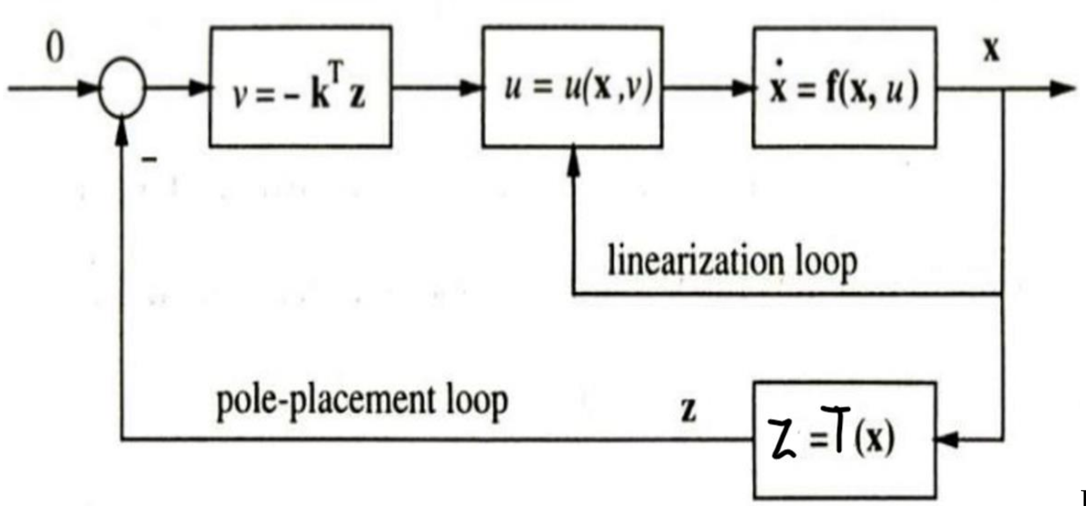
Nonlinear model

$$\dot{x}_1 = x_2 + 4x_1x_2 + x_1^2 + 2x_2^2 - 2(1 + x_2^2)\,u$$

$$\dot{x}_2 = x_2^2 + (1 + x_2^2)\,u$$

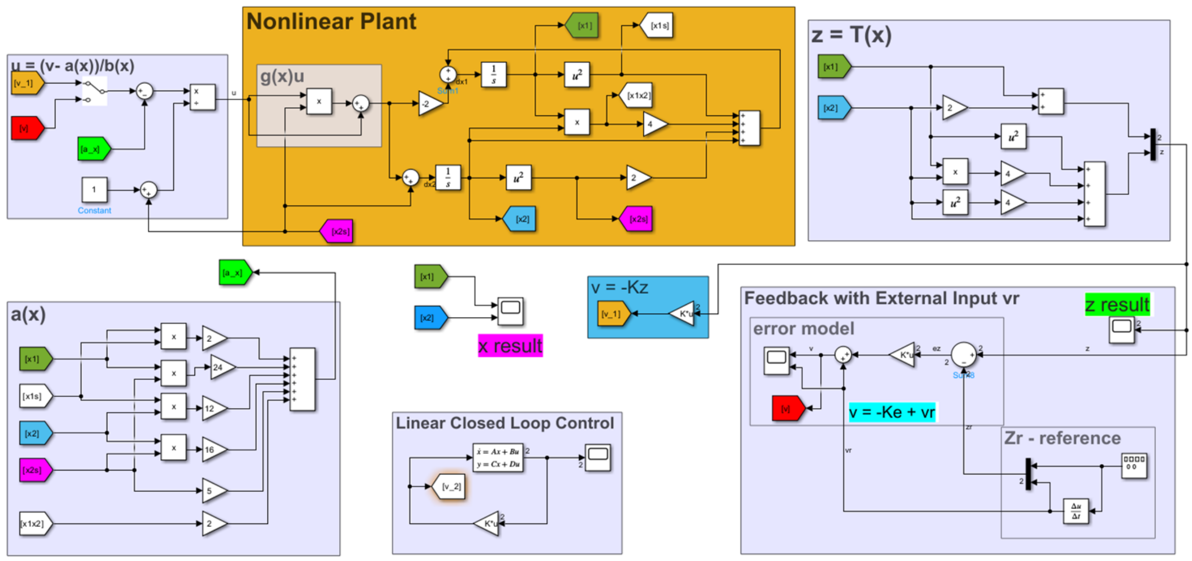
Steps to linearize the above nonlinear system

1. Identify the state transformation:
    - Find **z=h(x)** such that the system dynamics in terms of **z** are linear.
2. Derive the new dynamics:
    - Calculate **dz** and substitute the original nonlinear dynamics.
3. Select the control input transformation:
    - Choose **u=α(x)+β(x)v** to achieve a linear form.
4. Simplify to linear form:
    - Write the dynamics in the form **dz=Az + Bv,** where A and B are matrices, which simplifies the control design.



Block diagram of feedback control

Simulink Model



Conclusion

The **state-input transformation** is a technique used to convert a nonlinear system into an equivalent system that behaves linearly, typically for the purpose of control design. This approach simplifies both the analysis and control of nonlinear systems. In many practical applications, actuators and sensors often exhibit nonlinear behaviors, which can complicate control strategies. State-input transformations effectively linearize these nonlinearities, making control strategies more straightforward and manageable.

This method is widely used in fields such as **robotics** and **aerospace**. In **robotic systems**, where dynamics are often highly nonlinear, state-input transformations help decouple motion, allowing for easier implementation of control laws, such as **trajectory tracking**. Similarly, in **aerospace systems** like drones and missiles, which involve complex nonlinear dynamics, this technique facilitates more precise control of critical parameters, including **pitch, roll, and yaw**.

In general, state-input transformation provides a powerful tool for addressing nonlinear behaviors in a wide range of systems, enhancing both control accuracy and ease of implementation.