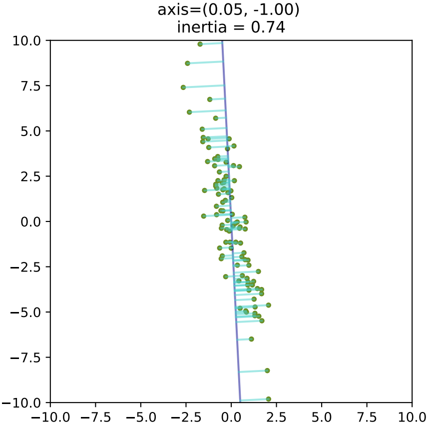
Fondamentaux théoriques du machine learning



First principal component

We look for w, $||w|| = 1$ such that

$$\sum_{i=1}^{n} \left(w^T x_i\right)^2 \tag{1}$$

is maximal.

Proposition

w is the eigenvector of $X^T X$ with largest eigenvalue λ_{max}.

First principal component

We look for w, $||w|| = 1$ such that

$$\sum_{i=1}^{n} \left(w^T x_i\right)^2 \tag{2}$$

is maximal.

Proposition

w is the eigenvector of $X^T X$ with largest eigenvalue λ_{max}.

Exercice 1 : Show the proposition.

First principal component

$$\begin{aligned}\sum_{i=1}^{n}(w^T x_i)^2 &= ||Xw||^2 \\ &= \langle Xw, Xw\rangle \\ &= \langle (X^T X)w, w\rangle\end{aligned}$$

This quantity is always smaller that λ_{max}, and it attained for an eigenvector in the eigenspace with norm 1, since we impose that $||w|| = 1$.