Exercices 2

TABLE DES MATIÈRES

1 CONVEXITY

1.1 C1

1.1.1 *Enoncé*

Show that all norms are convex.

1.2 C2

1.2.1 *Enoncé*

$x \mapsto \theta^T x$ is convex on $\mathbb{R}^d$ with $\theta \in \mathbb{R}^d$ (linear form)

1.3 C3

1.3.1 *Enoncé*

if Q is a symmetric definite positive matrix (matrice définie positive) with smallest eigenvalue $\lambda_{min} > 0$, then $x \mapsto x^T Q x$ is $2\lambda_{min}$- strongly convex.

1.4 C4

1.4.1 *Enoncé*

If f is increasing and convex and g is convex, then $f \circ g$ is convex.

1.5 C5

1.5.1 *Enoncé*

Is f in convex and g is linear, then $f \circ g$ is convex.

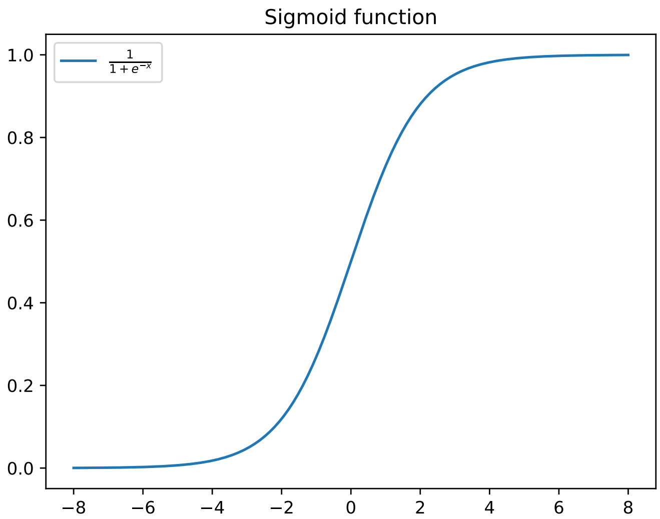
2 LOGISTIC REGRESSION

Definition 1. Cross entropy loss
$l : \mathbb{R}^2 \to \mathbb{R}$

$$l(\hat{y}, y) = y \log(1 + e^{-\hat{y}}) + (1 - y) \log(1 + e^{\hat{y}}) \quad (1)$$

Definition 2. Sigmoid function
$\sigma : \mathbb{R} \to \mathbb{R}$.

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad (2)$$



2.1 L1

2.1.1 *Enoncé*

Show that σ is differentiable and that

$$\forall z, \sigma'(z) = \sigma(z)\sigma(-z) \quad (3)$$

2.2 L2

2.2.1 *Enoncé*

Show that $l(\hat{y}, y)$ is convex in its first argument, which means for fixed y, $\hat{y} \mapsto l(\hat{y}, y)$ is convex.